Developing fully compatible taper and volume equations for all stem components of

_Eucalyptus globoidea_ Blakely trees

in New Zealand

A thesis submitted in partial fulfilment of the requirements for the Degree of Doctor of Philosophy in Forestry by

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Like so many things, it is not what’s outside,

but what is inside that counts. – Grandma Maria Nowak

Jak w przypadku wielu różnych rzeczy, nie ważne, co jest na zewnątrz,

ale to, co jest w środku. – Babcia Maria Nowak
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- **Boczniewicz, Daniel**, Euan G. Mason, and Justin A. Morgenroth (2022). “Developing fully compatible taper and volume equations for all stem components of *Eucalyptus globoidea* Blakely trees in New Zealand”. New Zealand Journal of Forestry Science 52. ISSN: 11795395. DOI: [https://doi.org/10.33494/nzjfs522022x180x](https://doi.org/10.33494/nzjfs522022x180x)


PRIZES

- Participant in the International Visualise Your Thesis Competition (Melbourne, Australia, 2021)
- 1st place in the University of Canterbury Visualise Your Thesis Competition (Christchurch, New Zealand, 2021) [Click here to see video]
- 2nd place in the University of Canterbury Visualise Your Thesis Competition (Christchurch, New Zealand, 2020) [Click here to see video]
- 1st place at the International Society for Modeler’s Choice Poster Award at Ecological Modelling Global Conference (Salzburg, Austria, 2019)
- Finalist in Thesis in Three Competition at the University of Canterbury (Christchurch, New Zealand, 2019)
- 2nd place in the College of Engineering at the University of Canterbury Thesis in Three Competition (Christchurch, New Zealand 2019)
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CONFERENCES

- Poster presentation "Developing fully compatible taper and volume equations for all stem components of Eucalyptus globoidea Blakely trees in New Zealand" at Ecological Modelling Global Conference (Salzburg, Austria, 2019)
- Oral presentation "Developing fully compatible taper and volume equations for all stem components of Eucalyptus globoidea Blakely trees in New Zealand" at Southern Mensurationists Conference (Asheville, USA, 2019)
- Poster presentation "Developing fully compatible taper and volume equations for all stem components of Eucalyptus globoidea Blakely trees in New Zealand" at the Australia and New Zealand Institutes of Forestry Conference (Christchurch, New Zealand 2019)

VIDEOGRAPHY

- “Guidelines for Eucalyptus growers” miniseries (6 episodes) for NZ Dryland Forests Initiative (2022)
- “Eucalyptus post peeler trials” miniseries (2 episodes) for the School of Forestry at the University of Canterbury (2021)

[Click here to see videos]
ABSTRACT

Background: Individual tree taper and volume equations are essential for forest management. They provide estimates of volume that are incorporated into plot level volume equations and growth and yield models to estimate volumes per hectare in forest crops. Moreover, taper equations allow forest managers to estimate the dimensions of logs that can be cut from stems in their forests when they have measured diameters at breast height outside bark (dbhobs) and heights of trees in inventories. Compatible taper and volume equations have the property that the same individual tree volume can be estimated either from the tree volume equation or by integrating the taper equation. Durable eucalypt species such as *Eucalyptus globoidea* Blakely have especially valuable heartwood, so managers require estimates of the volumes and shapes of heartwood zones within trees. Simple overall wood taper and volume equations would therefore be inadequate.

Methods: 74 *Eucalyptus globoidea* trees were destructively sampled in eight trials throughout New Zealand. Tree ages were 7 to 29 years old, the diameters at breast height (dbh) were 11 to 67.6 cm, and the heights were 7.2 to 35.4 m. All trees were felled, and lengths and taper diameters outside bark were measured. Discs were cut at irregular intervals along the stems to measure heartwood and sapwood's taper diameters. Heartwood and sapwood components were identified by applying methyl orange dye and quantified using image analysis. In this study, compatibility was extended so that sums of estimated volumes of separate components of stems, bark, sapwood and heartwood would equal overall tree volume estimates. In addition, taper equations were made for outside bark, inside bark and heartwood that were compatible with their respective volume equations. Parameters of five volume equations for the whole stem, whole wood, bark, sapwood, and heartwood were simultaneously estimated. Compatible taper equations for the whole stem, stem wood and heartwood were estimated to be compatible with
the volume equations, creating a fully compatible system. Different model variants were tested using dbhob and tree height as independent variables and heartwood ratio, diameter and height as additional independent variables. Three techniques were used to obtain heartwood-related measurements: the electric resistance tomograph PiCUS TreeTronic scan simulation, increment core simulation and measurements from the bottom stump parts of felled trees. The main goal was to create compatibility between all volume and taper equations using primary predictor inputs. Another purpose was to improve the prediction of the heartwood volume and taper using additional independent variables.

Results: For model with two independent variables (dbh, tree height), root mean squared error (RMSE) of volume models were: heartwood (0.13 m³), sapwood (0.06 m³), wood inside bark (0.15 m³), bark (0.11 m³), wood including bark (0.14 m³). RMSE of taper models predicting diameter were: heartwood (2.57 cm), wood inside bark (2.35 cm), and wood including bark (2.47 cm).

For models with three independent variables (dbh, tree height, heartwood ratio), the best RMSE results were obtained from increment core simulation for heartwood volume 0.11 m³ and heartwood taper 2.59 cm. For models with four independent variables (dbh, tree height, heartwood ratio and diameter), the best RMSE results were for heartwood volume 0.07 m³ and heartwood taper 2.14 cm. For models with five independent variables (dbh, tree height, heartwood ratio, heartwood diameter and height), the best RMSE results were for heartwood volume 0.07 m³ and heartwood taper 1.96 cm.

Conclusions: A compatible system of multiple taper and volume equations was created by simultaneously fitting parameters with minimal bias and precision levels of ±0.06-0.15 m³ for volume equations and ±2.35 to 2.57 cm for taper equations for a model with individual tree dbhob and height as independent variables. By using the additional independent variables
(heartwood ratio, diameter and height), heartwood volume and taper predictions were improved by up to: ± 0.02 m³ (models with three independent variables), ± 0.06 m³/ ± 0.43 cm (models with four independent variables), ± 0.06 m³/ ± 0.61 cm (models with five independent variables). Leave-one-out cross-validation of the fitted models yielded very similar levels of precision and bias to those encountered when fitting models with the entire dataset.

The taper and volume equations were incorporated into an interactive tool to provide volume and taper estimates of durable *Eucalyptus globoidea* trees for forest managers. The interactive tool produces complex taper and volume information for all stem components using the independent variables as model inputs. Connecting all tree stem wood and bark components’ taper and volume and ensuring their compatibility is novel in forest mensuration. Moreover, future development and improvements for similar projects are suggested.

**Keywords:** Volume, taper, mensuration, *Eucalyptus*, heartwood
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If you have ever climbed a mountain, you know the feeling before you start. You gaze at it and wonder if it is possible to reach the summit. The peak is so far away that you cannot see the whole path from the bottom. You feel the hike will be challenging and unpredictable, but you are optimistic and excited about the upcoming adventure. You begin your long and arduous journey with a single step. Four years ago, I took my first step to reach the highest mountain in my life, my doctoral thesis. I would never have accomplished it and probably would have vanished under one of many avalanches, if not for fantastic guides who constantly kept me safe and brought me back onto the right path in critical moments.

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Now let’s all enjoy the beautiful view from the peak, looking for other mountains to climb…
CHAPTER 1

Introduction
CHAPTER 1 – AN INTRODUCTION

1.1. BACKGROUND

1.1.1. New Zealand forestry history

Humanity has relied on natural resources throughout history. Over the centuries, there has been an increasing demand for wood, oil, and coal due to the agricultural revolution, urbanization, and industrialization. Those resources played a significant role as key energy sources to advance human endeavours (Daly & Townsend, 1992). Though wood, oil, and coal can all be used to produce energy, only wood is sustainable. Wood is a renewable resource created and maintained in forest ecosystems over time (Hill, 2012). The forestry sector in New Zealand is modern and innovative (De Gouw et al., 2020; Morgenroth & Visser, 2013) and serves humanity by providing wood products as sustainable natural resources.

New Zealand is the last major landmass settled by humankind (Belich, 2002). It was almost entirely forested except for high mountain regions and areas affected by volcanic eruptions or deficient rainfall. First, Māori settled the land about 1000 years ago (Taylor & Smith, 1997). In 1769, when the first Europeans arrived, the forest cover was still dense and thick. In their reports, James Cook and Joseph Banks wrote about an environment of "immense woods, lofty trees and the finest timber" (Taylor & Smith, 1997). However, by 1840, when the Treaty of Waitangi was signed by representatives of the British Crown and New Zealand's indigenous Māori chiefs, forest cover decreased from 85% to 53% (Roche, 1990; Taylor & Smith, 1997).

Due to the rapid expansion of British colonies, the demand for timber increased. The colonists harvested trees in New Zealand's forests for construction purposes. The slash-and-burn technique was often used to enlarge and prepare the land for farming. It was not carried out reliably, so many lands caught fire where burning had not been prescribed. These were the main reasons behind the rapid deforestation in New Zealand in the 19th century (McDonald, 1952; Wynn, 2002). Today there is a total of 10.1 million hectares of forest left in New Zealand,
covering 38% of the land (8 million ha native forest; 2.1 million ha plantation forest) (Ministry for Primary Industries, 2022). The milling of extensive native forest was one of the first industries developed in New Zealand. The most promoted and selected tree species were kauri (Agathis australis), rimu (Dacrydium cupressinum), tōtara (Podocarpus totara), mataī (Prumnopitys taxifolia) and miro (Prumnopitys ferruginea) (Roche, 1990).

1.1.2. Exotic tree species plantation introduction in New Zealand

The 1850s were a crucial time in the history of New Zealand forestry. An exotic tree species from Central Coastal California was introduced – *Pinus radiata*, known as the Monterey or radiata pine (Roche, 1990). The first study was reported by Thomas William Adams, who experimented with *Pinus radiata* trials in Canterbury in the 1870s with promising results showing fast growth in infertile acidic soil conditions (McKelvey, 1991). Based on this information and due to a timber shortage and exhausted resources of slow-growing native forests, the government introduced exotic species plantations in 1899 at Whakarewarewa, near Rotorua (McKinnon, 2009). In the 1930s, relief workers planted vast areas with *Pinus radiata*, including the 188,000-hectare Kaingaroa forest, the largest plantation forest in New Zealand (McKinnon, 2009). Many different species were planted over thousands of hectares, as radiata pine was often killed by out-of-season frosts and plantations of that species failed. Among them *Pseudotsuga menziesii*, *Pinus ponderosa*, *Pinus contorta*, *Pinus nigra* and *Larix decidua* (Marden et al., 2012).

The next big step in New Zealand's forest history occurred in 2002. The environmental movement had been campaigning against harvesting operations in native forests for many decades. As a result, in 2002, the logging of native forests on public land was ultimately forbidden (Bensemann, 2018). Nowadays, the central core of the New Zealand timber industry is in exotic plantations. In 2020, the total area of 1.665 million hectares of plantations was covered with 90% *Pinus radiata* and 6% *Pseudotsuga menziesii* (New Zealand Forest Owners
Association, 2021). These numbers highlight that New Zealand's forest industry over-relies on one species, *Pinus radiata* (Maclaren, 1993). Monterey pine has multiple properties and benefits but does not perform well with dry conditions and has some wood property limitations (Apiolaza et al., 2011). Its biggest constraint is a lack of natural wood durability. Many timber products from exotic species (especially radiata pine) are used outdoors and therefore exposed to biotic and abiotic factors that cause the wood to decay quickly. The lack of natural durability makes the wood unsuitable for outdoor purposes unless its durability properties can be improved. The use of treatments such as chromated copper arsenate (CCA) can make timber last for at least 40 years, exposed to the weather and in direct contact with the ground or fresh/salty water (Read, 2003). However, despite improving radiata pine's durability, CCA-treated wood can leach copper, chromium, and arsenic into the surrounding soil, and there are concerns about contamination of waterways and soils (Vogeler et al., 2005).

1.1.3. Problems with CCA and disposal of toxic waste

To resolve the lack of natural wood durability, non-durable *Pinus radiata* wood can be treated with toxic preservatives. Chromated copper arsenate is the most common wood preservative (created from a mixture of chromium, copper and arsenic), which is used to impregnate non-durable timber and wood products (Hullinger et al., 1998). Concerns over the release of toxic components resulted in a ban on CCA-impregnated timber for residential and recreational settings in the early 2000s in the United States, Canada and the European Union (Read, 2003). In New Zealand, there is a growing trend towards excluding CCA-treated timber from the market; for example, the Auckland Regional Council has put regulations in place preventing the use of treated timber in particleboard production (Rhodes & Dolan, 2010).

However, in New Zealand's Marlborough region, CCA-treated vineyard posts are still widely used. New Zealand is among the world's greatest CCA-treated timber users (per capita) (Christmas, 2002). The wine industry's standard posts are usually made from CCA-treated
radiata pine timber, and they cause many problems. Due to the softwood properties of the posts, they have to be replaced frequently because of high breakage rates (the result of mechanical harvesting). Their toxicity does not allow vineyard managers to dispose of them safely on-site (Altaner et al., 2017). During burning, the preservative components of CCA-treated timber accumulate in ash and cause unfavourable environmental effects (Dobbs & Grant, 1976; Hedley, 1997). This is why burning is prohibited. Secure landfills or controlled incineration facilities are the only legitimate methods to dispose of toxic timber (Rhodes & Dolan, 2010).

New Zealand's vineyard and farm disposal and storage options for CCA-treated posts are minimal (Marlborough District Council, 2016), resulting in significant technical and financial constraints (Rhodes & Dolan, 2010). Marlborough District Council requested reports from HortResearch about the environmental risk of CCA leaching from treated vineyard posts. An experiment in Rarangi, New Zealand, confirmed that the leachate from CCA-treated posts was present in the soil during the first six months. In some cases, the concentration in soils surrounding these posts exceeded recommended guidelines (100 mg/kg) for arsenic and chromium in agricultural soils in the National Environmental Protection Measures (National Environmental Protection and Heritage Council, 1999; Vogeler et al., 2005). Based on modelling, after about 25 years, the concentration of arsenic in the drainage water vertically entering the Rarangi Shallow Aquifer will reach the New Zealand Drinking Water Standard limit, which is 0.01 mg/L of spatially-based averaged arsenic concentration (Vogeler et al., 2005). Despite these problems, CCA-treated Pinus radiata remains the most commonly used timber species in agricultural and horticultural industries (Rhodes & Dolan, 2010; Richardson, 1993; Smith & Williams, 1973). This current trend is changing, and some managers are moving to an alternative to treated radiata pine in the form of plastic, steel or naturally durable timber posts (Millen & Altaner, 2017). Plastic and steel posts come with their issues; they are non-renewable, easy to bend under the pressure of the taut metal wire, and producing them is carbon
intensive. However, naturally durable timber posts provide a sustainable alternative to CCA-treated posts.

### 1.1.4. Durable *Eucalyptus* as a natural alternative

In timbers without resistance to biological degradation, preservation is achieved with toxic treatments (e.g., CCA), making them decay- and termite-resistant. One potential alternative is to use naturally durable timber that does not require CCA treatment. Some *Eucalyptus* has naturally durable heartwood (Bootle, 1983) and produces much stiffer wood with more excellent growth rates than radiata pine (Walker, 2006). Due to this characteristic, it is possible to create structural wood products without applying toxic chemicals while maintaining high resistance to biodegradation. Moreover, *Eucalyptus* timber products do not pose a significant waste disposal problem compared to chemically treated wood, as they can be burned or buried readily (Townsend & Solo-Gabriele, 2006). Many regions with *Eucalyptus* resources recognise that the species are well suited to supply wood for structural timber products, for example, Australia (McGavin, 2016; Ozarska, 1999), Brazil (de Carvalho et al., 2004), Europe (Konnerth et al., 2016) and China (Arnold et al., 2013). This makes *Eucalyptus* timber an excellent alternative to current timber market solutions.

### 1.1.5. The New Zealand Dryland Forests Initiative

The New Zealand Dryland Forests Initiative (NZDFI) identified this opportunity in the timber industry as the catalyst for its efforts and development strategy. The primary purpose of the NZDFI is for New Zealand to become the international leader in breeding naturally ground-durable *Eucalyptus* species. The two main goals of the initiative are to produce genetically improved planting stock and provide vital information for growers. All the effort optimises the economic potential of ground-durable *Eucalyptus* (Altaner et al., 2017). From over 700 different *Eucalyptus* species, researchers selected five main species with potential for durable wood
(Brooker 2000): *E. argophloia*, *E. bosistoana*, *E. globoidea*, *E. tricarpa* and *E. quadrangulata*. Their wood properties are elite: the dynamic modulus of elasticity ranges from 14 to 21 GPa, air-dry density from 880 to 1130 kg/m$^3$ and in-ground life expectancy (without treatment) from 15 to more than 25 years (Bootle, 1983). Each of those values is almost double those of *Pinus radiata* timber (9 GPa, 480 kg/m$^3$, 0-5 years) (Sharma & Altaner, 2017). In addition to more excellent durability, *Eucalyptus* wood is also highly stiff – stiffness increases with height in the lower stem section (butt log) and distance from the pith (Ozarska, 2009).

The NZDFI conducts many parallel research projects in dryland areas of New Zealand. To achieve that goal, widespread trials in representative sites of the research area have been established. As the project's founding partner, the New Zealand School of Forestry formed the science team's core. There are six main themes of the research which fulfil the future wood industry idea of the whole project: wood quality; site species matching; modelling and silviculture, insect pest and disease monitoring; tree breeding and genetics. Research on wood quality focuses on improving and establishing a new heartwood forest industry, where growing naturally durable *Eucalyptus* is the central objective (Li & Altaner, 2017). The modelling and silviculture theme aims to provide information to ensure that growers understand how to manage stands silviculturally and predict product outturns from dryland eucalypt plantations. To obtain these results, it is essential to develop growth and yield models and survival models of selected durable *Eucalyptus* species (Salekin et al., 2017). The tree breeding and genetics theme's primary purpose is to choose the most suitable and durable *Eucalyptus* genotypes and species, which will provide genetic gain in the next generation (Apiolaza et al., 2011). Despite proactive biosecurity, threats to *Eucalyptus* are present in New Zealand forests, and it is essential to prepare a pest management approach to control these risks. That objective pre-occupies the insect pests and disease monitoring theme (Murray & Lin, 2017).
Many studies support durable eucalypts as natural alternatives and reveal their potential in New Zealand’s timber market. One of the research projects supported by the NZDFI was to provide the foundations of a new second-generation breeding population of *E. bosistoana*, *E. argophloia*, *E. quadrangulata* and *E. tricarpa* with low-growth strain. As a result, superior genotypes with lower growth strains have been identified and will soon be made commercially available through clonal propagation from coppice. Also, the potential for laminated veneer lumber (LVL) production using NZDFI *Eucalyptus* species has been demonstrated (Altaner, 2019; Riley, 2016). Based on another study, New Zealand's existing LVL industry could immediately use *E. bosistoana* veneers, proving that markets for higher stiffness LVL products exist. To produce enough *E. bosistoana* veneers to substitute 10% of New Zealand's LVL production in 10 to 15 years, annual plantings of 150 to 750 ha would be required (Altaner, 2020).

As heartwood is essential for durable timber alternatives, many researchers have focused on assessing its structure and quality. Many trees deposit a wide range of chemical compounds into heartwood tissue (Hillis, 1987), and those extractives contribute to the durability of timber. One study was able to quantify 30 compounds in the heartwood of *E. bosistoana* and *E. globoidea* using a promising method of gas chromatography (GC) of silylated ethanol extracts (Schroettke et al., 2017). It proved that this approach could successfully assess variation in the composition of heartwood extracts. Heartwood quality assessment with current technology in a research context is resource-intensive and not feasible commercially. However, pilot studies by Kuwabara (2017) and Li (2018) showed that near-infrared (NIR) imaging could be a quick and contactless method to assess heartwood in durable eucalypts. The studies focused on *E. globoidea* and *E. bosistoana*.

Another research topic explored with NZDFI trees was to test the feasibility of manufacturing posts from NZDFI *Eucalyptus* species with existing New Zealand post-manufacturing machinery. Two post-peeler machines were used with *E. bosistoana*, *E. quadrangulata* and *E.
**globoidea** logs (obtained from 14- and 17-year-old tree plantations). The results showed that pre-barked *Eucalyptus* logs could be easily turned into durable vineyard posts using a peeler machine commercially used for *Pinus radiata* (Boczniewicz et al., 2021). That means the entry point for small and medium sawmills to adopt *Eucalyptus* durable post production is feasible. They could use old technology and methods to create new high-quality products.

Another study provides an overview of *Eucalyptus* research and reviews the potential of growing and processing naturally durable eucalypts in New Zealand based on Australian experience. Nine of the twelve species examined appear to have opportunities to be grown in New Zealand to produce naturally durable wood. All five NZDFI *Eucalyptus* species are among them, and by Australian standards, they meet the criteria to be classified as ‘durable’ species as they are classified as Class 1 or 2 (Poole et al., 2013).

Another study supported by the NZDFI assessed durable eucalypt vineyard posts’ performance in five different vineyards in Marlborough managed under organic standards. None reported post failure due to decay; however, three said a few posts had broken during harvesting due to large knots in the posts. The feedback was generally positive, and the central aspect was that the posts were naturally durable and acceptable for their organic standards. From 1065 durable eucalypt posts installed by Marlborough vineyard owners at least 11 years before testing, just 14 were found broken. That equates to 1% of the total, giving an annual breakage rate of 0.1%, compared to up to 5% yearly breakage in conventional CCA-treated pine posts (Millen & Altaner, 2017).

### 1.1.6. General aims of the research

This PhD project will contribute to the overall body of NZDFI research. The study is focused on *Eucalyptus globoidea* (white stringybark), specifically modelling its stem components’ volume and taper, with a focus on the relationship between heartwood and sapwood
development for the species. The first objective was to estimate the volume and taper of heartwood, sapwood, wood inside bark, bark and wood including bark from two independent variables, such as dbh and height. The second objective was to improve heartwood taper and volume prediction by using heartwood ratio, heartwood diameter and heartwood height as additional independent variables. This knowledge will help forest owners and managers to plan and manage future uses for their *Eucalyptus globoides* resource.
CHAPTER 2

Literature review
2.1. WOOD ANATOMY

The tree trunk is comprised of (starting from the outer layer) bark, phloem, cambium, xylem (water-conducting sapwood and non-conductive heartwood), and pith (Figure 1) (Panshin & deZeeuw, 1980). The bark is dead tissue and protects the inner layers from external factors (such as mechanical injury, pests, etc.). The phloem is the thin layer of living cells and transports and distributes organic nutrients (photosynthesis products) in the tree. The cambium is an even thinner layer of living tissue and produces new phloem (outer side) and new xylem (inner side). The most significant components of the trunk are sapwood and heartwood, which comprise the xylem. Sapwood transports water and dissolved minerals between the root and the crown and stores energy reserves. Heartwood was previously sapwood but is no longer conductive. It is the supporting pillar of a tree, filled with extractives, tannins and minerals. The pith placed in the very centre of the tree is its oldest part (Aguilera & Zamora, 2009; Meerts, 2002; Taylor et al., 2002).

Early in a tree’s life, all wood consists of sapwood; the cambium produces bark cells outside and sapwood cells inside. Over the years, the tree’s girth increases, the sapwood accumulates inside the trunk, and the cambium and bark move outward. However, the sapwood cross section in older trees is too large to transport the sap (Kampe and Magel 2013) efficiently. Hence, the tree reduces it by converting the more senior and ageing sapwood (nearest to the pith) to heartwood. This change also accommodates a shift in its function. The layer ceases to transport water and store energy reserves (Song et al., 2014). The chemical compounds (resins, terpenes and phenols) transported towards dying sapwood change it to heartwood and make it more resistant to insects and decay (Bertaud & Holmbom, 2004). Heartwood comprises the inner rings of wood, which do not contain living cells where reserve materials such as starch have been recycled back to the sapwood (Bamber, 1961) or converted into excretory substances (International Association of Wood Anatomists, 1964; Taylor et al., 2002). Those substances
are transported in concentrations below the toxicity limit along with the ray cells in the direction of the pith. They will accumulate with other materials until the toxicity concentration reaches a lethal level. That will cause the death of the innermost parenchyma cells and the formation of the first cylinder of the heartwood (Stewart, 1966). Heartwood provides structural support for the tree and develops proportionally with tree height (Panshin & deZeeuw, 1980). More rapidly growing trees tend to have a more significant proportion of sapwood (Dünisch et al., 2010). For example, slow-growing *Robinia pseudoacacia* often has less than 2.5 cm sapwood, while faster-growing *Acer* species have wide sapwood band and relatively narrow heartwood core (Dünisch et al., 2010).

![Figure 1](image)

Figure 1. The cross-section view of a 29-year-old *Eucalyptus globoidea* disc with tree trunk elements labelled.

### 2.2. WOOD DURABILITY

#### 2.2.1. Wood natural durability

Wood – one of the most durable cellulosic materials – owes its durability to high levels of lignin (Zabel & Morrell, 2012). Lignin resists degradation in the humic acid fraction of soil as it is a
highly recalcitrant polymer. However, it does not render wood immune to decay – it is still susceptible to degradation under adverse environmental conditions. Dead sapwood is usually susceptible to degradation and has a propensity to infestation by Lyctine beetles (Lyctus brunneus) (Peters et al., 2002). The main commercial timbers that are susceptible are tawa (Beilschmiedia tawa), rewarewa (Knightia excelsa), beech (Nothofagus), oak (Quercus), ash (Fraxinus), elm (Ulmus), and eucalpyt (Eucalyptus) (Bain, 1978; Bletchly, 1967). The insect is a common timber pest in New Zealand and Australia (Rosel, 1962, 1969). However, some trees produce heartwood – the inner part of the tree - that is exceptionally resistant to biological degradation (Scheffer, 1966; Taylor et al., 2002). More durable heartwood helps trees to last longer and extend the potential reproduction time, giving species that develop heartwood an advantage over their competitors (Morrell & Lipeh, 2017). Heartwood durability can vary between trees and within a given tree (Hillis, 1987; Scheffer, 1966). This is why many countries have created durability classification systems. New Zealand, Australia and the European Union divide materials into classes based on laboratory and field studies (Kutnik et al., 2014; Morrell & Lipeh, 2017).

Due to durability, heartwood has been widely used for centuries for house and marine construction, resulting in local or regional deforestation due to high demand and popularity. For example, the Lebanese cedar (Cedrus libani) was nearly wiped out in the Levant region by overharvesting (Graham, 1973). Naturally durable woods were the only practical option for prolonging the life of structures until the end of the 19th century. After that, the rapid growth of infrastructure created a massive demand for durable materials that could not be met by only naturally durable heartwood. That period led to the development of synthetic wood preservatives, methods for effective wood treatment and the usage of synthetically protected wood substitutes (Graham, 1973; Morrell & Lipeh, 2017).
2.2.2 Lack of durability in sapwood

Sapwood is a portion of the wood that contains living cells and reserve materials. Its main functions are to support the tree's stable development, to conduct water and mineral salts from the roots to the leaves, and to store food reserves mainly in the form of starch (Bamber, 1961). The presence of food materials and the absence of protective substances make it susceptible to fungi and wood-destroying insect attacks. For example, the powder post beetles (classified in the insect subfamily Lyctinae) lay eggs in sapwood, an attractive food source for newly hatched offspring (Batt & Ahmed, 2018). Where untreated poles are used for in-ground purposes, sapwood is removed up to 45 cm above the ground level to prevent insects and fungal growth. As time passes, some species can transform sapwood into heartwood (Bamber, 1961).

2.2.3 The importance of heartwood for durability

The heartwood of certain tree species has considerable natural resistance against insect attack and fungal colonization and degradation (Schultz & Nicholas, 2000). Heartwood's natural durability results from the death of the parenchyma; it leaves only passive mechanisms of resistance to pathogens, unlike sapwood (Shain, 1995). Heartwood has reduced attractiveness to decay organisms because it lacks the necessary nutrients (Scheffer, 1966). Heartwood extractive content also protects it against fungal colonization (Backa et al., 1992; Schultz et al., 1995). As a mechanical barrier, heartwood prevents and slows wood decay in some species by physically blocking insects (De Vries & Kuyper, 1990). Aged heartwood is usually less decay resistant than younger heartwood due to the slow degradation of extractives and its undergoing slow autoxidation, lowering its toxicity (Scheffer, 1966; Schultz & Nicholas, 2000).

The quality and amount of heartwood in different tree species strongly affect their timber properties, and potential end uses. For example, Robinia pseudoacacia usually has a large proportion of durable heartwood, while Pinus species have a small proportion of low to
moderately durable heartwood. Therefore, *Robinia* timber can be used for outdoor applications without any treatment, unlike pine timber (Bijak & Lachowicz, 2021; Brischke & Alfredsen, 2022).

In New Zealand, the Australasian natural durability classification system AS 5604 is used to measure durability (Standards Australia, 2003). It classifies timbers into four classes: class one (very durable), class two (durable), class three (moderately durable), and class four (non-durable). To test the durability of locally grown tree species, the Forest Research Institute (Scion) conducted long-term experiments (1982, 1997). Multiple heartwood stakes (base equals 20 or 50 mm\(^2\)) were obtained from different tree species. They were simultaneously installed in the same plot in two ways, in the ground and above. Thus the exposure conditions for all groups were the same. The number of stakes from each tree species varied. Table 1 presents New Zealand tree species and their in-ground natural durability based on 50 mm\(^2\) stake samples. Table 2 presents the life expectancy for naturally durable timbers in-ground and above-ground (based on 20 and 50 mm\(^2\) stake samples) (Forest Research Institute, 1982, 1997; Page & Singh, 2014). There are more heartwood durable species (among others, *Eucalyptus bosistoana* (class 1) and *Eucalyptus quadrangulata* (class 2)), which will be commercially available shortly due to the efforts of NZDFI (Altaner et al., 2019; McConnochie & Apiolaza, 2021).

Durability can be affected even within the same tree species by growing conditions, genetics, climate and wood age (Page & Singh, 2014). Climate and soil conditions (warmer and moist environments) decrease the durability of wood in contact with the ground. In the case of above-ground uses, unshaded north-facing slopes are exposed to greater temperature ranges than south-facing slopes. In the case of horizontal decking, there is a greater possibility of absorbing water than near vertical surfaces, such as fence palings (Page & Singh, 2014). The durability variation (within and between species), based on Forest Research Institute experiment with 20 mm\(^2\) *Eucalyptus* stakes (in-ground exposure), is presented in Table 3.
Table 1. New Zealand grown heartwood tree species and their in-ground natural durability (50 mm² stake study samples) (Forest Research Institute, 1982, 1997; Page & Singh, 2014).

<table>
<thead>
<tr>
<th>Class 1 (very durable) &gt;25 years</th>
<th>Class 2 (durable) 15-25 years</th>
<th>Class 3 (moderately durable) 5–15 years</th>
<th>Class 3 (moderately durable) 5–15 years</th>
<th>Class 4 (non-durable) &lt;5 years</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Hardwoods</strong></td>
<td><strong>Hardwoods</strong></td>
<td><strong>Hardwoods</strong></td>
<td><strong>Softwoods</strong></td>
<td><strong>Hardwoods</strong></td>
</tr>
<tr>
<td>Eucalyptus cladocalyx</td>
<td>E. amygdalina</td>
<td>Black beech</td>
<td>Japanese cedar</td>
<td>Paulownia elongata</td>
</tr>
<tr>
<td>Robinia</td>
<td>E. botryoides</td>
<td>Blackwood</td>
<td>Kaikawaka</td>
<td>Paulownia tomentosa</td>
</tr>
<tr>
<td>E. cornuta</td>
<td>E. globulus</td>
<td>Kauri</td>
<td>Poplar</td>
<td></td>
</tr>
<tr>
<td>E. globoidea</td>
<td>E. sieberi</td>
<td>Larch</td>
<td>Tawa</td>
<td></td>
</tr>
<tr>
<td>E. muelleriana</td>
<td>Gleditsia</td>
<td>Lawson cypress</td>
<td>Silver wattle</td>
<td></td>
</tr>
<tr>
<td>E. pilularis</td>
<td>Southern rata</td>
<td>Lusitanica</td>
<td></td>
<td></td>
</tr>
<tr>
<td><strong>Softwoods</strong></td>
<td>E. radiata (p)</td>
<td>E. fastigata</td>
<td>Macrocarpa</td>
<td><strong>Softwoods</strong></td>
</tr>
<tr>
<td>Silver pine</td>
<td>E. saligna</td>
<td>E. delegatensis</td>
<td>Matai</td>
<td>P. nigra</td>
</tr>
<tr>
<td>Totara</td>
<td>European oak</td>
<td>E. fraxinoides</td>
<td>Redwood</td>
<td></td>
</tr>
<tr>
<td>Hard beech</td>
<td>E. obliqua</td>
<td>Rimu</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mountain beech</td>
<td>E. pyrocarpa</td>
<td>Tanekaha</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Red beech</td>
<td>E. viminalis</td>
<td>Western red cedar</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Sweet chestnut</td>
<td>E. regnans</td>
<td>Douglas fir</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Hinau</td>
<td>Mangeao</td>
<td>Miro</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pukatea</td>
<td>Pinus contorta</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Silver beech</td>
<td>P. muricata</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>P. radiata</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>P. strobos</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>P. ponderosa</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

1 Species with durability towards the lower end of the range;
2 Species with durability towards the upper end of the range;
(p) Provisional classification, species still under testing
Table 2. In-ground and above-ground life expectancy (years) for naturally durable timbers (based on 20 and 50 mm² stake samples) (Page & Singh, 2014).

<table>
<thead>
<tr>
<th>Durability Class</th>
<th>In-ground stakes</th>
<th>Above–ground stakes</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>50 mm</td>
<td>20 mm</td>
</tr>
<tr>
<td>Class 1</td>
<td>&gt; 25</td>
<td>&gt; 10</td>
</tr>
<tr>
<td>Class 2</td>
<td>15 – 25</td>
<td>6 – 10</td>
</tr>
<tr>
<td>Class 3</td>
<td>5 – 15</td>
<td>2 – 6</td>
</tr>
<tr>
<td>Class 4</td>
<td>0 – 5</td>
<td>0 – 2</td>
</tr>
</tbody>
</table>

Table 3. The average durability variation (in years) within and between three *Eucalyptus* species (class 2 durability) based on a 20 mm² stakes experiment in the ground variant (Page & Singh, 2014).

<table>
<thead>
<tr>
<th>Species</th>
<th>Tree 1</th>
<th>Tree 2</th>
<th>Tree 3</th>
<th>Tree 4</th>
<th>Tree 5</th>
<th>Average</th>
</tr>
</thead>
<tbody>
<tr>
<td><em>E. pilularis</em></td>
<td>13.3</td>
<td>6.6</td>
<td>10.9</td>
<td>5.7</td>
<td>10.4</td>
<td>9.9</td>
</tr>
<tr>
<td><em>E. muelleriana</em></td>
<td>10.7</td>
<td>6.2</td>
<td>6.6</td>
<td>8.2</td>
<td>9.4</td>
<td>9.2</td>
</tr>
<tr>
<td><em>E. globoidea</em></td>
<td>12.4</td>
<td>14.3</td>
<td>12.0</td>
<td>9.4</td>
<td>14.3</td>
<td>9.8</td>
</tr>
</tbody>
</table>

2.2.4. Factors influencing heartwood development

A lot of different factors influence heartwood formation and development. Still, heartwood formation is not entirely understood (Taylor et al., 2002). One of the factors is gas accumulation. Carrodus (1971) applied carbon dioxide to Australian acacia (*Acacia mearnsii*) sapwood and noticed that heartwood-type extractive compounds were formed. That research suggests that heartwood extractive formation is related to high carbon dioxide levels. However, Shain & Hillis (1973) indicated that ethylene gas was responsible for that process. The time of year of heartwood formation is also the subject of debate, depending on tree species. Much evidence suggests that it forms in the dormant season. This has been confirmed for *Pinus radiata* (Shain...
& Hillis, 1973), Juglans nigra, and Prunus serotina (Nelson, 1975). The macro-distribution of heartwood has also been shown to vary significantly between species. The amount of heartwood within and between trees and species has received proper study. However, patterns of variation in heartwood quality are less well understood (Taylor et al., 2002). There appears to be significant genetic control of heartwood proportion within species. Wei & Borralho (1997) proved solid genetic control of heartwood area in mature Pinus and some diffuse-porous species. Site quality differences can influence the heartwood growth rates in trees. Hillis (1972) found that poor site may delay the initiation age of heartwood formation or decrease heartwood proportion. In terms of abiotic factors affecting heartwood formation, the effect of water seems to be significant. Harris (1954) found that adequate water supply promoted heartwood development in Pinus radiata. However, based on Climent’s study (1993), annual water surplus is related more to sapwood width and diameter growth rates than to heartwood width for Canary Island pines. The same author reported for the same species based on five different climate type models; the heartwood was narrower in wet and high-altitude (supra-nubius) climate, than in drier climate types (Climent et al., 2002). Another abiotic factor influencing heartwood formation is the effect of temperature. The absence of dry conditions promoted heartwood development in Pinus radiata (Harris, 1954) in contrast to Climent’s study (2002), where the heartwood of Canary Island pines was wider in drier climate types. Frost may induce irregular heartwood formation in the wood of European oaks (Quercus robur and Quercus petraea) called moon rings or frost hearts (Krapiec, 1999). This wood defect is caused by traumas (such as severe frost) which affect the tree’s capacity to transform phenolic substances in sapwood during heartwood formation (Dujesiefken et al., 1984). As a result, false sapwood appears as clear halos surrounding the central heartwood, affecting the durability and quality of the total heartwood area within the tree (Charrier et al., 1995). Another abiotic factor is wind. Based on Climent’s study (1993), the effect of trade winds is related more to sapwood width and diameter
growth rates than to heartwood width for Canary Island pines. Soil chemistry may also be a factor in heartwood formation, as Guyette (2007) proved that the mineral elements present in heartwood were related to their abundance in the soil. The sapwood basal area of a tree stem is strongly associated with the tree crown and foliage size based on Populus tremuloides (Yang & Hazenberg, 1991). Therefore, artificial or natural environmental factors can affect crown size (pruning, thinning, competition) or directly improve growth rate (irrigation, fertilisation). All these processes influence sapwood basal area and indirectly influence heartwood formation by affecting tree crown size (Margolis et al., 1988; Mörling & Valinger, 1999; Sellin, 1994; Takei, 1996). In conifer species, genetic factors play an essential role in controlling heartwood content. Significant genetic effects were proven by different authors on the clonal, progeny and origin levels, including for Pinus radiata (Downes et al., 2002), Pinus sylvestris (Ericsson & Fries, 1999), Pinus banksiana (Magnussen & Keith, 1990) and Pseudotsuga menziesii (Lausberg et al., 1995). Heartwood formation can also be affected by biotic factors. In the Abies genus infected by the insect Balsam woolly adelgid (Adelges piceae), an increase in heartwood area was observed (Hillis, 1987; Hollingsworth et al., 1991).

2.3. STEM STRUCTURE

Trees come in a variety of shapes and sizes. Tree stems could be described as having different forms, for example, conoid, cylinder, neiloid, and paraboloid. To investigate the exact shape of a stem, its diameter has to be plotted at various height points. If the tree diameter (d) is on the x-axis and tree height (H) is on the y-axis, the resulting curve can be divided into three parts. The topmost part is a conical section (A), the middle part is a paraboloid section (B), and the bottom part is a neiloid section (C) (Figure 2). Each of these sections, in different shapes, has other equations relating to its heights and diameters. In the case of the conical section, y=kx or $y^2=lx^2$, where x is the distance from the apex, y is the diameter, and k is the constant; in the
case of the paraboloid section $y^2=lx$; and in the case of the neiloid section $y^2=lx^3$ (Figure 2) (Demaerschalk & Omule, 1982; Husch, 1963; Rosner, 2004). The shape of the stem could be described as comprising stem form and taper. Stem form refers to the shape of the solid, the diameter or height curve, which is determined by the power index ($n$) of $x$ in the equation $y^2=lx^n$. The taper refers to the rate of narrowing in the diameter in relation to the increase in height for a given shape or form.

Figure 2. The shape of the stem divided into three parts with corresponding equations.

2.3.1. Tree form

Three tree form theories can explain differences in various tree forms. The first theory is the nutritional and water-conducting theory (Daum, 1967). It postulates that stem form is optimised to transport water and nutrients within the tree. The second is the hormonal theory (Chailakhyan, 1937), which states that growth substances (hormones) originate in the crown and are distributed around and down the bole, causing radial growth and affecting tree form. These two theories are physiological and do not receive much credence as they are difficult to prove. The third theory, Metzger’s beam theory (mechanistic) (Larson, 1963), is one of the most accepted. It explains the shape of a tree as the most efficient design to counter wind forces. It considers the
tree stem a beam of uniform resistance to bending, anchored at the base. The stem profile shape of an individual tree is also affected by its position within the stand, the site conditions, genetic attributes, and silvicultural treatments such as stand density and fertiliser treatment (Flewelling & Raynes, 1993; Morris & Forslund, 1992). Knowledge of these variables can help achieve desired tree forms. For instance, genetic attributes can be selected via tree breeding and by selecting extraordinary trees for seed collection. Field situations can also be altered in a stand by regulating the amount of thinning done in the stand, by controlling the amount of nutrients that the tree is getting in this stand, or by fertilisation.

2.3.2. The volume of individual tree stems

The computation of tree stem volume is an essential mensurational tool in forestry. It estimates biomass (when the wood density is known), which is necessary information in forest management for timber production and carbon sequestration (Van Laar & Akça, 2007). Volume measurement of an individual tree is essential for constructing volume tables and estimating the parameters of tree volume equations. It is a necessary first step to obtain a stand volume estimate. Volume can be calculated in different ways, depending on the precision and scale of the task.

2.3.2.1. Whole piece

The most popular and easy way to calculate the volume of an individual tree is presented in Equation 1:

\[ V = \frac{1}{4} \pi D^2 H f \]

where \( V \) is calculated stem volume, \( D \) is the diameter at breast height (1.4 m), \( H \) is the tree's height, and \( f \) is a form factor. It is a volume estimate and can be used only for well-researched species, as the form factor \( f \) is needed.

Another approach to calculating single-tree volume is the quarter–girth formula (Hoppus, 1738). It is used to quickly estimate volume using the girth of the solid \( (G) \) measured at the mid-length
of the log length \(l\). It calculates the volume of the cuboid inside the potential log, which is a usable portion of the log, excluding the outside cut portions potentially lost in future sawing operations. This equation is one of the first attempts to include only the usable volume of the final product, excluding edging waste \((Equation\ 2)\).

\[\text{Equation 2. } v = \left(\frac{6}{4}\right)^2 l\]

Xylometry can be used to calculate the volume of an individual or multiple logs \((Van\ Laar\ &\ Akça,\ 2007)\). A log is submerged in a tank of water, and the change in water level can be converted into volume. It produces accurate estimates. However, some logs may absorb the water, which should also be considered.

\subsection*{2.3.2.2. Dividing stems into smaller sections}

Another method to calculate the volume of an individual tree is direct calculation through sections. The tree is divided into several sections, and each section’s volume is calculated using an optimal formula. All sections are then added together to provide the tree’s total volume. Each section’s volume can be calculated as a cylinder, a cone, or using Smalian’s formula \((Equations\ 3,\ 4,\ and\ 5,\ respectively)\).

Cylinder \((a\ prism\ with\ a\ circle\ as\ its\ base)\) – when both section ends are circular, the radius \(r\) of both section ends is the same, and the section length \(l\) is known, then Equation 3 can be used:

\[\text{Equation 3. } v = \pi r^2 l\]

Cone \((a\ three-dimensional\ geometric\ shape\ that\ tapers\ smoothly\ from\ a\ flat\ base\ to\ a\ point\ called\ the\ apex\ or\ vertex)\) – when the topmost section of a log, usually ending with a terminal bud \((cross-section \approx 0)\), the bottom cross-sectional area \(s\), and log length \(l\) are known, then Equation 4 can be used:
Equation 4. \( v = \frac{s}{3} l \)

Smalian’s formula (Van Laar & Akça, 2007) – Paraboloid (a quadric surface that has precisely one axis of symmetry and no centre of symmetry) – when the radius of the section ends differs and we can distinguish large end \((s_1)\) and small end \((s_2)\) cross-sectional areas, and the section length \((l)\) is known, then the Equation 5 can be used:

Equation 5. \( v = \frac{(s_1+s_2)}{2} l \)

Paraboloid section volume could also be calculated using Huber’s formula (Van Laar & Akça, 2007), where only the middle cross-sectional area \((s_m)\) is used (Equation 6):

Equation 6. \( v = s_m l \)

Newton’s formula (Van Laar & Akça, 2007) – Neiloid (the surface of revolution obtained by rotating a semicubical parabola, or Neile parabola, around its axis of symmetry) – to calculate the volume of this shape Equation 7 is used:

Equation 7. \( v = \frac{(s_1+4s_m+s_2)}{6} l \)

Hossfeld’s formula (Van Laar & Akça, 2007) – requires measurement of the cross-sectional area at 1/3rd \((s_1)\) of the stem, and Equation 8 can be used:

Equation 8. \( v = \frac{3s_1+s_1}{3} l \)

Simony’s formula (Van Laar & Akça, 2007) requires the cross-sectional area at 1/4 \((s_1)\), 3/4 \((s_2)\) of the stem and Equation 9 can be used:

Equation 9. \( v = \frac{2s_1-s_m+2s_3}{4} l \)

Hohenadal’s formula (Van Laar & Akça, 2007) – requires measurements of cross-sectional areas at 10%, 30%, 50%, 70%, and 90%, and of the total length, and Equation 10 can be used:
Equation 10. \[ v = \frac{s_{0.1} + s_{0.3} + s_{0.5} + s_{0.7} + s_{0.9}}{5} l \]

Equations presented by Huber, Smalian, Newton, Hossfeld, Simony and Hohenadel are widely applied to estimate the total and merchantable volume of felled trees using a limited number of diameter measurements (de León & Uranga-Valencia, 2013). Smalian’s, Huber’s and Newton’s are commonly used in forest mensuration. Newton’s formula is the most accurate and is used to find errors in other formulas (Biging, 1988). However, it is less used in the field due to difficulties with measuring mid-diameter in stacked logs. Smalian’s formula requires only end cross-section measurements, so it is easier to apply in field situations for stacked logs or logs on the ground. It is accurate for cylindrical and paraboloid log shapes. However, it tends to overestimate the volume (Biging, 1988). Huber’s formula requires only one measurement. However, it is a middle cross-section, making it difficult in the case of stacked logs. It is accurate for cylinder and paraboloid log shapes; however, it tends to underestimate volume (Anuchin, 1970).

2.3.2.3. Volume equations

To create a volume equation, Bruce (1982) compared volume estimates of butt logs obtained from the cross-sectional areas of the log for the lower and upper ends. The result was presented by following Equation 11:

Equation 11. \[ v = 0.00007854 L \left( 0.25 DL^2 + 0.75 DS^2 \right) \]

where \( L \) = length of butt log in meters, \( DL \) = diameter at the large end in cm and \( DS \) = diameter at the small end in cm.

To estimate volumes of butt logs, Grosenbaugh (1952) developed Equation 12:

Equation 12. \[ v = 0.00007854 DL^2 \left( DS \times DL + \frac{(DL - DS)^2}{k} \right) \]

with \( k = 2 \) for a paraboloid, \( k = 3 \) for a cone and \( k = 4 \) for a subneiloid.
Grosenbaugh’s formula (Equation 12) was combined with Baisiger’s modification by Bruce (1987) for estimating the butt log volumes and is presented in Equation 13:

**Equation 13.** \[ v = 0.00007854 DL^2 * \frac{DS}{DL} + c (1 - (DS - DL^2)) \]

where \( c = -0.96 \).

### 2.3.3. Stand volume

Stand volume is often estimated from forest inventories. It can be expressed as stem or total tree volume, total or merchantable volume, over or under the bark. Many measurements are considered to calculate stand volume, including basal area, mean height, and individual or average tree form. The information provided in 2.3.3.1. – 2.3.3.6. below has all been sourced from (Van Laar & Akça, 2007).

#### 2.3.3.1. Standard tree volume tables and functions

The estimated volume for trees with a specific diameter and height can be obtained using a two-entry tree volume table, where analogous volume functions use dbh and height as independent variables. Based on the table's assumptions, the tree's form factor is not affected by provenance, site, stand treatment or other external factors. The mean tree method (which requires estimating the quadratic mean diameter and its regression height) is applied.

#### 2.3.3.2. Volume estimation with form height and volume series

The form height series was created by Laer (1964). The author derived form factors from German tree tables (Grundner & Schwappach, 1952) and heights from standardised height curves created by Wiedemann (1949). Tables (for different tree species) present the estimated stand form height as a function of the mean height of the stand.
2.3.3.3. Stand volume tables and functions

In stand volume tables, estimated stand volume per hectare is a function of stand basal area and mean height.

2.3.3.4. Estimation with yield tables

Yield tables represent the average growth pattern within a large region. They are constructed as a function of age, site class and stocking density. The following data are required to estimate stand volume using yield table information: stand age, mean or top height, and stocking density (basal area per hectare).

2.3.3.5. From felled sample trees

The volume estimate of the stand can be calculated based on measurements of felled sample trees. The sampling technique is applied to obtain the diameter and mean height stand. The stem form factor is obtained from a table or estimated from regression equations (where height and diameter are independent variables). The selection of sample trees can be random, quasi-random or based on probability proportional to prediction (with basal area or volume used as a size variable). This method has a long history and tradition, especially in European forestry (Dubey, 1967; Gehrhardt, 1909).

2.3.3.6. Critical height sampling

The stand volume per hectare acquired from critical height sampling is calculated by multiplying the basal area factor with the sum of critical tree heights.
2.3.4. Tree volume tables and equations

2.3.4.1. With one predictor variable

a) Simple tariff functions

The early equations estimated volume based on dbh. Tarif rapide, proposed by Algan (1894), was built on a relationship between tree volume, its dbh and the assumed volume of a forty-five-year-old tree. A related idea was presented by Schaeffer (1949) as tarif lent. Another approach to early tariff development was proposed by Huffel (1919), who used graphic methods to construct them. However, the more modern single-entry volume tables are established using regression analysis with log (dbh) as the independent variable and log (volume) as the dependent variable or with volume as the dependent and squared dbh as the predictor variable (Van Laar & Akça, 2007). Volume Equation 14, based on the following model, was proposed by Meyer (1955). Ordinary unweighted least squares assume independently and normally distributed residuals. The model is based on the condition of homoscedasticity.

\[ \ln(v) = b_0 + b_1 \ln(d) + e \]

where \( b_0, b_1 \) are coefficients, \( d \) is diameter, and \( e \) is the standard error.

Another model (Equation 15), which assumes a linear relationship between tree cross-sectional area at breast height and stem volume, was proposed by Hummel (1953), who derived it from Kopezky’s (1899) idea. It also assumes independently and normally distributed residuals and produces unbiased estimates of the stem volume. However, the assumption of homoscedasticity does not hold and requires weighting, for example, \( w = 1/d \) or \( w = 1/d^2 \).

\[ v = b_0 + b_1 d^2 \]
b) Incorporation of height into a tariff function

In single-entry volume tables, the parameters are not related to age, site or stand treatment. This idea could generate biased stand volume estimates. The following three methods were created to reduce it.

- **Hummel’s method**

Hummel’s (1953) set of volume lines is constructed based on the relationship between basal area and stem volume (Equation 16) for trees with dbh greater than 10 cm.

\[ \text{Equation 16. } \nu = b_0 + b_1 g \]

where \( g \) is the basal area.

After rewriting Equation 16, its final version is presented in Equation 17.

\[ \text{Equation 17. } \nu = \frac{TN}{0.913} (g - 0.087) \]

where \( \frac{TN}{0.913} = b_1, 0.087 = \text{the common intercept on the abscissa (0.087 $\text{ft}^2$).} \)

- **Stoffels' method**

A different method was presented by Stoffels (1953), which was based on the allometric relationship between diameter and volume (Equation 18). The main assumption was that \( b_1 \) is not influenced by stand characteristics. However, \( b_0 \) relates to the quadratic mean diameter and the mean height of the stand (Equation 19).

\[ \text{Equation 18. } \ln(\nu) = b_0 + b_1 \ln(d) \]

\[ \text{Equation 19. } \ln(b_0) = c_0 + c_1 \ln(\bar{d}) + c_2 \ln(\bar{h}) \]

- **Brister's method**

Brister (1985) studied *Pinus taeda* in 36 stands, where trees were felled and measured to establish a volume basal area line. Brister’s proposed tariff is based on Stoffels’ earlier study (Stoffels, 1953) and links them to Hummel’s volume basal area line (Hummel, 1953). The
assumption was that tree diameter predicts stem volume with the mean diameter and mean height of the dominants introduced as covariates. The final model is presented in Equation 20.

\[
\ln(\nu) = d_0 + d_1 \ln(d) + d_2 \ln(h) + d_3 \ln(d)
\]

2.3.4.2. With two predictor variables

a) Graphical methods

Before the development of computers, one method to estimate the tree volume from dbh and height was to use alignment charts proposed by Bruce (1919). It was a time-consuming graphic method of construction, and regression methods replaced it.

b) Regression equations

Regression methods have the parameter estimates obtained from a multiple-regression equation and fitted by ordinary or weighted least squares. The equation, linearised by a logarithmic transformation of the dependent and predictor variables (Equation 21), was proposed by Schumacher (1933). The primary assumption is that the residuals are normally distributed with constant variance. Due to the slightly biased volume estimates (because of the logarithmic transformation of the target variable), some authors prefer to use a non-linear algorithm to estimate the parameters, adding some weighting procedure (Boeckmann & Kramer, 1990; Deadman & Goulding, 1978; Lockow, 1977; Scott, 1981).

\[
\ln(\nu) = a_0 + a_1 \ln(d) + a_2 \ln(h)
\]

Many models (linear in their parameters) were proposed with d, d², h, h², dh, d²h, interaction terms as predictor variables and volume as the dependent variable (Bruce & DeMars, 1974; Murphy, 1983; Näslund, 1947; Newcomer & Myers, 1984; Spurr, 1952; Stoate, 1945).

2.3.4.3. With more than two predictor variables

Volume equations with two predictor variables assume that the form of the tree is sufficiently controlled by diameter and height (as predictor variables). However, to obtain more accurate
tree volume estimates, many authors suggested adding a third variable, such as an upper-stem diameter, height above ground of the base of the live crown, diameter at 30% of the tree height, etc. (Hann et al., 1987; Pollanschütz, 1965; Rustagi & Loveless Jr, 1991; Smalley, 1973; Wagner, 1982).

2.3.4.4. Merchantable Volume

Total stand volume estimates are essential. However, merchantable stand volume is even more significant for forest managers. Nowadays, tables and functions are constructed for a variable upper diameter. To estimate the stem volume \( (v) \) up to a specific upper diameter \( (d_m) \), Burkhart and others proposed Equation 22, linearised to estimate its parameters by ordinary least squares.

Equation 22. \[ \frac{v - v_m}{v} = b_0 + b_1 d_m^{b_2} + b_2 \]

where \( v_m \) is the volume above the upper diameter \( (d_m) \).

Another Equation 23, proposed by Newberry (1989), estimated the ratio of merchantable volume to total volume based on geometric solids.

Equation 23. \[ R = 1 - b_0 \left( \frac{d_i}{d} \right)^{b_2} \left( \frac{h - h_i}{h - h_{st}} \right)^{b_2} \]

where \( h_{st} \) = height of stump diameter, \( d_i \) = diameter limit for merchantability, \( h_i \) = corresponding height above ground and \( (h - h_{st}) \) = total height minus stump height.

2.3.3. Tree taper

The diameter of the stem decreases moving from the bottom to the top of the tree. The rate of this decrease is called a taper. The rate of change in stem diameter is calculated as the change in stem diameter between two points/length of the stem between the same two points. If the lower diameter = \( d_1 \) at lower height = \( h_1 \) and the higher diameter = \( d_2 \) at higher height = \( h_2 \), then the taper is given by equation \((d_1 - d_2)/(h_2 - h_1)\) (Figure 3). Its units are centimetres per meter of stem length. A tree with a large taper will resemble a cone, while a tree with a small taper (nearly
equal to zero) will resemble a cylinder. Large taper is characteristic for a solitary tree, a widely spaced plantation tree or a tree in a heavily thinned stand. In contrast, the low taper is distinct for trees in groups, in closely spaced plantations, and lightly thinned stands.

In its simplest form, the taper can be mathematically expressed as follows (Equation 24) (Ormerod, 1973):

\[
\frac{d_2}{d_1} = \left(\frac{H-h_2}{H-h_1}\right)^{b_n}
\]

where \(d_2\) = diameter measured at the height \(h_2\), \(d_1\) = diameter measured at a reference height (usually breast height), \(h_2\) = height of the diameter \(d_2\), \(h_1\) = reference height (usually breast height), \(H\) = total tree height, \(b_n\) = taper coefficient (Figure 4).

Based on Equation 24, the diameter \((d_2)\) measured at the height \(h_2\) is the function of the diameter measured at the breast height \((d_1)\), total tree height \((H)\) and height \((h_1)\) of diameter \(d_1\) (Equation 25).

\[
d = f(d_1, H, h_1)
\]
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Figure 3. Graphical illustration of taper concept.

Figure 4. Graphical illustration of Ormerod’s taper equation.
The taper equations could be used to predict: stem diameter at any point of the stem, individual log volume, total stem volume, merchantable stem height and merchantable stem volume. This function (Equation 25) can be used to predict the volume of any tree, as it is essentially the equation of the form of a tree. Nevertheless, for a tree's shape, the formula predicts the diameter of a particular form at a different height in terms of the diameter at the breast height and total tree height. For instance, in the case of a cylinder, the volume formula is calculated in Equation 26:

\[ v = \pi r^2 h = \pi \left(\frac{d}{2}\right)^2 h = \frac{\pi}{4} d^2 h \]

To calculate a small section of the stem shaped as a cylinder, the following formula (Equation 27) will be used:

\[ dv = \frac{\pi}{4} d^2 hi \]

To calculate the total volume of that tree, the integral of Equation 27 will be used (Equation 28):

\[ v = \int_{h_1}^{h_2} \frac{\pi}{4} d^2 hi \]

where \( h_1 = 0 \) m, \( h_2 = \text{max. tree height} \)

In Equation 28, there are two variables: selected diameter (\( di \)) and height (\( hi \)). The formula for Equation 25 could be used as a substitute for diameter (\( di \)) and constant values \( D_r, H, \pi \) could be simplified (Equation 29):

\[ v = \int_{h_1}^{h_2} \frac{\pi}{4} [f(D_r, H, h)]^2 hi = \int_{h_1}^{h_2} g(h)^2 hi \]

Now the simplified Equation 29 can be integrated and provide the tree volume.

Taper equations still have a few limitations. One of them is the infinite variety of shapes in nature. All these volumes and merchantable heights could be calculated only for a regular shape like a cone or a portion of a paraboloid. If a tree is crooked (or has an unusually irregular shape), considerable error in volume estimation may occur. The second limitation is variation in
physical features, such as branching. The presented equations only calculate the volume of the main stem, and they do not include branches with a large diameter that could be used economically. The third limitation is that equations do not consider unexpected effects, such as different site conditions or pathology within the same stand. For instance, there could be large and small trees in the same stand. Several other conditions could be found in the same stand, and the equations cannot describe them in general terms.

Many different factors influence tree taper. One of them is the effect of spacing. Trees grown at higher stockings will have slighter tapers than those grown at lower stockings (Mason, 2000). Reducing initial stocking (even for the same final crop stocking) should increase log taper; furthermore, higher stocking during a longer period of rotation could reduce taper, even if the final crop stocking number was low (Mason, 2000). Greater taper is observed on the trees on the edges of shelter belts compared to those within shelter belts (Chavasse, 1982).

2.3.4. Taper equations

Taper functions can be classified as parametric or non-parametric based on the approach adopted for their development. The taper function classification provided below has all been sourced from Salekin et al. publication (2021), of which I am a co-author.

2.3.4.1. Parametric Taper Equations

Through different regression approaches, a variety of taper equations have been fitted.

a) Static Taper Equations

Static taper functions do not include time and age factors in the model and are based on the assumption that changes in diameter at breast height reflect changes in upper stem diameters. Equation 30 calculates tree volume sections, which predicts the ratio of diameter or radius at a specific distance from the topmost point of the main stem (Kershaw Jr et al., 2016).
Equation 30. \( y = k \sqrt{x^r} \)

where \( y \) is the radius or diameter at a specific distance \( x \) from the tip, \( k \) is a constant, and \( r \) is a form exponent that changes with the geometric solids that reflect different parts of the stem.

- **Polynomial Form models**

The first taper equation (Equation 31) was reported by Behre (1923) and designed for *Picea abies*.

Equation 31. \( \frac{d}{D} = C \log \left( \frac{c-l}{c} \right) \)

where \( d \) is the diameter inside the bark at distance \( l \) from the tip, \( D \) is the diameter at breast height, and \( C \) and \( c \) are constants.

It was later improved with the hyperbolic form (Equation 32):

Equation 32. \( \frac{d}{D} = \frac{(H-h)}{b_0 + b_1(h)} \)

where \( d \) is the diameter inside the bark at height \( h \), \( D \) is the diameter at breast height, \( H \) is the total height, and \( b_0 \) and \( b_1 \) are parameters.

A simple paraboloid Equation 33, which results in a lower standard error, was developed by Kozak and Smith (1966):

Equation 33. \( d = D \sqrt{a + b \left( \frac{h}{H-4.5} \right)} \)

where \( d \) is the diameter inside the bark at height \( h \), \( D \) is the diameter at breast height, \( H \) is the total height, and \( a \) and \( b \) are parameters. The units of this equation were feet, and breast height is usually at 4.5 feet in imperial units.

Another polynomial taper equation (Equation 34) was introduced by Bruce et al. (1968) for *Alnus rubra* and has been used for many hardwoods and softwoods worldwide. It includes a series of exponents for representing different parts of the stem.
Equation 34. \[
\frac{d^2}{D^2} = b_1x^\frac{3}{2} + b_2\left(x^\frac{3}{2} - x^3\right)D + b_3\left(x^\frac{3}{2} - x^3\right)H + b_4\left(x^\frac{3}{2} - x^{32}\right)H \times \\
D + b_5\left(x^\frac{3}{2} - x^{32}\right)\sqrt{H} + b_6\left(x^\frac{3}{2} - x^{40}\right)H^2
\]

where \(d\) is the diameter inside the bark at height \(h\), \(D\) is the diameter at breast height, \(H\) is the total height \(x = \left(\frac{H-h}{H-4.5}\right)\), and \(b_1\) to \(b_6\) are parameters.

- **Sigmoid Taper Equations**

A simple sigmoid equation (Equation 35) was used by Ormerod (1973) to describe the form of the tree stem. Other transformations of this equation have been presented by Bryne and Reed (1986):

Equation 35. \[
d = b_1D \left(\frac{H-h}{H-BH}\right)^{b_2}
\]

where \(d\) is the diameter inside the bark at height \(h\), \(D\) is the diameter at breast height, \(H\) is the total height, \(BH\) is breast height, and \(b_1\) and \(b_2\) are parameters.

A simple sigmoidal equation (Equation 36) was developed by Forslund (1991) for *Populus tremuloides* Michx.

Equation 36. \[
Y = (1 - X^a)^\left(\frac{1}{b}\right)
\]

where \(Y = d/D\), \(d\) is the diameter at the upper position, \(D\) is the basal diameter, \(X = h/H\), \(h\) is the height to the measurement position from the base of the stem, \(H\) is the total height, and \(a\) and \(b\) are parameters.

- **Segmented Polynomial Taper Equations**

Another equation by Ormerod (1973) using inflexion points for different stem sections was a geometrically segmented taper equation (Equation 37).

Equation 37. \[
d = (d_j - C_i) \left(\frac{H_i-h}{H_i-h_j}\right)^{P_i} + C_i
\]
where \( d \) is the estimated diameter at the upper position, \( h \) is the height to the estimated diameter position from the base of the section, \( H_i \) is the height to the top of the section, \( h_i \) is the height to the measured diameter \( d_i \), \( p_i \) is the fitted exponent for the section, and \( C_i \) is the intercept of sectional diameter.

Another interesting approach was introduced by Max and Burkhart (1976), who developed a statistical segmented taper model (Equation 38) for \textit{Pinus taeda} L., which has been used extensively for conifers and broadleaves. By restricting the continuous and smooth functions at the join points, the sum of squared error for sub-models was minimised.

\[
\text{Equation 38. } d = \sqrt{D^2 \left[ b_1 \left( \frac{h}{H} - 1 \right) + b_2 \left( \frac{h^2}{H^2} - 1 \right) + b_3 \left( a_1 - \frac{h}{H} \right)^2 I_1 + b_4 \left( a_2 - \frac{h}{H} \right)^2 I_2 \right]}
\]

where \( d, D, H, \) and \( h \) have been defined previously; \( a_1 \) and \( a_2 \) are joined points, and \( I_1 \) and \( I_2 \) have the values of 1 or 0 (dummy variables).

- **Variable Exponent Form Models**

Newnham (1988, 1992) introduced a new type of taper equation named a variable exponent taper equation (Equation 39). To compensate for different shapes, the continuous function has an exponent that varies from the ground to the tip.

\[
\text{Equation 39. } Y = x^{1/k}
\]

where \( Y = \left( \frac{d}{D} \right) \); \( x = \left( \frac{H-h}{H-BH} \right) \); \( k = b_0 + b_1 \left( \frac{D}{H} \right) + b_2 x \left( \frac{D}{H} \right)^2 + b_3 \left( \frac{1}{H} \right)^b_4 \). In this equation, \( d \) is the diameter inside the bark at height \( h \), and \( k \) depends on the diameter at breast height (\( D \)), total height (\( H \)), and breast height (\( BH \)).

Based on Newnham’s equation (1988), Kozak (1988) developed a variable exponent equation (Equation 40)

\[
\text{Equation 40. } d = a_0 DD^{a_1} a_2 D \left( \frac{1-\sqrt{2}}{1-\sqrt{D}} \right)^{b_1 z^2 + b_2 ln(z+0.001)+b_3 \sqrt{z}+b_4 e(z)+b_5 (D/H)}
\]

38
where \( d \) is the diameter inside the bark at height \( h \); \( D \) is the diameter at breast height; and \( Z \) is \( h/H \), with \( h \) and \( H \) described previously, \( p = HI/H \), where \( HI \) is the inflexion point that can be fitted depending on the species, and \( a_0 \) and \( a_1 \) as well as \( b_1 \) to \( b_5 \) are parameters.

- **Trigonometric Models**

  Thomas and Parresol (1991) included trigonometric functions to describe stem form by creating a trigonometric taper equation (Equation 41).

  \[
  \text{Equation 41. } \quad d = \sqrt{D^2 \left[ b_1(z - 1) + b_2 \sin (c\pi z) + \frac{b_3}{-\tan (\pi z/2)} \right]}
  \]

  where \( z \) is the relative height \( h/H \) and \( \pi \) is Pi, the mathematical constant, and \( b_1, b_3, \) and \( c \) are parameters. The variables \( d, D, h, \) and \( H \) have been described previously.

b) **Complex Taper Functions**

- **Compatible Taper Models**

  Another critical step in taper function progress was the development of compatible taper equations (Equation 42) developed by Demaerschalk (1972) for sixteen tree species.

  \[
  \text{Equation 42. } \quad \frac{a^2}{D^2} = a + b \frac{h}{H} + c \frac{n^2}{H^2}
  \]

  The innovative part was that after integration of Equation 42, the volume could be calculated following Equation 43, and the ratio of volume to the basal area could be derived when the constant of integration was set to 0 (Equation 44) by solving for \( K_1 \) (Equation 45) and \( K_2 \) (Equation 46):

  \[
  \text{Equation 43. } \quad V = K_1 D^2 H
  \]

  \[
  \text{Equation 44. } \quad \frac{V}{B} = K_2 H
  \]

  \[
  \text{Equation 45. } \quad K_1 = 0.00545415(a + \frac{b}{2} + \frac{c}{3})
  \]

  \[
  \text{Equation 46. } \quad K_2 = \left(a + \frac{b}{2} + \frac{c}{3}\right)
  \]
where \( D, V, h, \) and \( H \) have been described previously; and \( a, b, \) and \( c \) are regression coefficients from Equation 42, \( d \) is the diameter inside the bark at the point \( l \), that is the distance from the tip; \( k = \pi/40,000 \) numeric units or English units in the case of Equation 45; and \( b_2 \) to \( b_n \) are parameters and \( n \) is the number of the parameter.

- **Whole-Bole Systems Models**

This system was created by the cooperation between Demaerschalk and Kozak (1977). It consists of two functions linked at the stem's inflexion point. Equation 47 predicts the diameter inside the bark at the tree's top and Equation 48 for the calibrated diameter at the tree's bottom.

**Equation 47.**
\[
d = \left( \frac{h}{RH} \right)^{b_1} b_2 \left( 1 - \frac{h}{RH} \right)^{b_4} DI
\]

**Equation 48.**
\[
d = \left[ b_3 - (b_3 - 1) \left( \frac{h}{RH} \right)^{b_4} \right] DI
\]

where \( d \) is the diameter inside bark at point \( h \), \( DI \) is the diameter inside bark at the inflexion point, \( RH \) is the distance of the inflexion point from the tip, \( RHI \) is the distance of the inflexion point from the ground level, \( h \) and \( H \) have been described previously, \( b_1 \) and \( b_2 \) are regression parameters, and \( b_3 \) and \( b_4 \) are coefficients in the conditioned tree bottom model.

- **Dynamic Taper Models**

The first author to include time in the diameter estimation was Muhairwe (1994), who developed a dynamic taper equation (Equation 49):

**Equation 49.**
\[
\hat{d}_{ijt} = a_0 \hat{D}_{it} \left[ a_1 \left( 1 - \sqrt{\hat{Z}_{ijt}} \right)^{b_1 Z_{ijt}^2 + b_2 \sqrt{Z_{ijt} + b_3 \hat{D}_{it} + b_4 \hat{H}_{it} + b_5 Q D_{50K}^2 + b_6 \hat{A}_{it} + b_7 \hat{H}_{ijt} + 0.001} \right]
\]

where \( \hat{d}_{ijt} \) is the estimated diameter at breast height for the tree \( i \), section \( j \), and time \( t \); \( \hat{D}_{it} \) is the predicted diameter at breast height for the tree \( i \) at time \( t \); \( \hat{H}_{it} \) is the predicted tree height for the tree \( i \) at time \( t \); \( h_{ijt} \) is the height for the tree \( i \), section \( j \), and time \( t \); \( \hat{A}_{it} \) is the breast height age
of the tree i at time t; \( \hat{Z}_{ijt} \) is the predicted relative height; \( \overline{QD}_{50k} \) is the quadratic mean diameter at age 50 years for plot k, and \( a_0, a_1, \) and \( b_1 \) to \( b_7 \) are parameters.

- Other Complex Taper Models

A taper equation that is a simultaneous model containing all the diameters measured at different relative heights in the stem (Equation 50) was developed by Laasasenaho (1982).

Equation 50. \[
\frac{d_l}{d_{0.2h}} = c_1 x^1 + c_2 x^2 + c_3 x^3 + c_4 x^5 + c_5 x^8 + c_6 x^{13} + c_7 x^{21} + c_8 x^{34}
\]

where \( d_l \) is the diameter inside bark at distance \( l \) from the ground level, \( d_{0.2h} \) is the diameter at 20% of the tree height, \( x = (1 - \left( \frac{l}{h} \right)) \) or the relative distance from the top, and \( c_1 \) to \( c_8 \) are the fitted parameters.

Another interesting taper equation was a linear-mixed model for predicting stem forms that are represented by diameter and height using polar coordinates created by Lappi (1986) (Equation 51):

Equation 51. \[
d_{ki}(u) = a_0(u) + a_1(u)s_{ki} + a_2(u)s_{ki}^2 - a_3(u)(s_{ki} - \overline{s_k}) + v_k(u) + e_{ki}(u)
\]

where \( a_1(u) = a_1(u) + a_3(u); d_{ki}(u) \) is the logarithmic diameter i at angle \( u \) for the stand \( k; u \) is the angle measured from the ground level; \( s_{ki} \) is the logarithmic size of the tree; \( \overline{s_k} \) is the average size for the stand \( k; v_k \) and \( e_{ki} \) are the random stand and tree effect, respectively, and \( a_0 \) to \( a_3 \) are fixed parameters.

- Contemporary Taper Models

Özçelik et al. (2010) introduced the use of artificial intelligence tools – a regression tree algorithm named random forest and an artificial neural network (Nunes & Görgens, 2016; Özçelik et al., 2010). Both test a range of possible values and then verify through fitting statistic Field (Feng, Huang, Lin, & Gay, 2009) through a trial-and-error method.
2.3.4.2. Non-Parametric Taper Equations

The core of this popular solution is ensemble methods such as machine learning or deep learning algorithms, which create multiple models and combine them to produce results. Even though nonparametric approaches generally have superior predictive quality over traditional parametric approaches (Nunes & Görgens, 2016; Yang & Burkhart, 2020), McTague and Weiskittel (2021) observed that they are highly data-sensitive, can overpredict and cannot explain underlying biological processes.

2.3.5. Examples of previous work on volume and taper for Eucalyptus species

While all the equations reported above for volume and taper were developed for a range of species, some work has been conducted specifically on Eucalyptus, the subject of this thesis. A study from northern Tasmania, Australia, used a hybrid growth model to predict height and volume growth in young Eucalyptus globulus plantations (Battaglia et al., 1999). Even though height growth was predicted accurately, the volume growth estimates were biased. This may result from applying a growth model developed for Eucalyptus nitens to Eucalyptus globulus plantation (Battaglia et al., 1999). This example shows that suitable volume equations should be developed for every Eucalyptus species to maximise the accuracy of the prediction.

In another study from Western Kenya, tree and stand volume equations were developed for Eucalyptus saligna plantations (Shiver & Brister, 1992). In that case, a logarithmic model was used. The system of equations indicates that the mean annual increment culminates at approximately age five to six across the range of site indices present in these plantations (Shiver & Brister, 1992).

There is an interesting study conducted by Gomat et al. (2011) about factors influencing the stem taper of Eucalyptus grown in Congo. The study concludes that the tree shape stabilised after distinct changes between years 1 and 2. The authors claimed that the planting density had
no significant influence on the stem profile. Except for some slight differences, the global form of the trees remains the same, whatever the stand density and age. The study's main conclusion was that environmental conditions and genetics impacted stem shape.

A study conducted in Australia by the State Forest of New South Wales focused on collecting and analysing data describing *Eucalyptus pilularis* (blackbutt) and *Eucalyptus grandis* (flooded gum) (Muhairwe, 1999). In this study, new taper models (Equation 52 and Equation 53) were developed and then compared with earlier well-tested taper models created by Max and Burkhart (1976), Kozak (1988), and Gordon (1983). Equation 52, which used total height in addition to dbh and sectional height, had the best results for describing the stem profile and predicting stem volume for both species; it is also a well-fitted taper model. Therefore Equation 52 is the recommended model for blackbutt (Muhairwe, 1999). For the flooded gum, Muhairwe (1999) recommended Equation 53, which is a modification of the Gordon (1983) taper model. Although this model was biased, it was still the most precise model for *Eucalyptus grandis* (Muhairwe, 1999).

**Equation 52.**  
\[ d = a_0 dbh^{a_1} a_2^{dbh} [1 - \sqrt{Z}]^c \]
where \( c = b_1 Z^2 + \frac{b_2}{Z} + b_3 D + b_4 H + b_5 \left( \frac{D}{H} \right) \), \( a_i \) and \( b_i \) are regression coefficients, \( I = 0 \) to \( 6 \), \( D = \) diameter at breast height (1.30 m) outside bark (cm), \( h = \) height along the stem (m) from the ground, \( H = \) total tree height (m), \( n = \) number of observations used in the calculations, and \( Z = \frac{h}{H} \).

**Equation 53.**  
\[ d^2 = \frac{V}{(KH)} \left[ b_1 X + b_2 X^2 + b_3 X^3 + b_4 X^4 + b_5 X^8 \right] \]
where \( V = \) total stem volume, inside bark (\( m^3 \)), \( X = \frac{(H-h)/H}{H} \), \( K = 7.853981634 \times 10^{-5} \), and \( b_1 \) to \( b_5 \) are fitted parameters.

A study conducted in Brazil with four different genetic families of *Eucalyptus* faced challenges of short rotations and dramatic changes in tree size during the rapid growth of *Eucalyptus*
plantations (Scolforo et al., 2016). The study was focused on finding the best equation fit for predicting both stem form and volume. The conclusion was that the PMS (Penalized Mixed Spline) methodology was highly efficient and relatively easy to fit for that purpose and performed the best among all tested approaches (Scolforo et al., 2018), (Pedan, 2003).

Various authors have reported on studies of heartwood taper. They took into consideration different aspects and species in their studies. Ojansuu & Maltamo (1995) modelled heartwood and total wood taper for Scots pine (*Pinus sylvestris*) regarding the sapwood pipe theory, but their taper and volume models were not compatible. Brown (2019) worked on models to predict the heartwood radius for Northern Red Oak (*Quercus rubra* L.). Heartwood radius (HR) was a function of wood radius (WR) and height above the stump (H); HR=f(WR, H). Another model to predict the heartwood radius for Canary Island Pine (*Pinus canariensis*) was developed by Climent et al. (2003). They described heartwood radius (HR) as a function of heartwood radius measured at breast height (HRbh) and height of a tree (H); HR=f(HRbh, H). Another interesting study was conducted by Cardoso and Pereira (2017). In this study, the heartwood of Douglas Fir (*Pseudotsuga menziesii*) was characterised as a percentage; the vertical taper of the heartwood was described but not modelled.

**2.4. GAPS IN THE LITERATURE**

Recent studies show that there is a growing interest in modelling heartwood taper. Some models for future *Eucalyptus* plantation managers in New Zealand have been developed. A site suitability model for durable *Eucalyptus* species and a growth and yield model for *Eucalyptus globoidea* have been developed (Salekin et al., 2018). Serajis Salekin created a set of plot-level growth and yield equations, while the focus of this study was tree-level, volume and taper equations. Both systems were combined by Professor Euan Mason in a separate study looking at creating plot-level volume equations from individual tree volume equations. Another step
 CHAPTER 2 – LITERATURE REVIEW 

will be to develop the *Eucalyptus globoidea* growth and yield model into a framework that allows the prediction of diameter distributions. Having individual tree distributions, will enable applications of the taper and volume equations to individual trees within the growth and yield model. The user of the model could then have estimates of log dimensions and the amount of heartwood within each log. My study goal was to provide the missing models for individual tree taper and volume equations, especially heartwood taper and volume equations. Even though plenty of research has been done on *Eucalyptus globoidea*, we require compatible taper and volume equations for both stem and heartwood. It is a novel and promising research idea with considerable potential in the future *Eucalyptus* durable heartwood for New Zealand Forestry and timber market.

2.5. **RESEARCH OBJECTIVES**

The research objectives of this thesis were to develop equations for *Eucalyptus globoidea*:

1) to estimate the volume of heartwood, sapwood, wood inside bark, bark and wood including bark from two independent variables (dbh and height),

2) to estimate the taper of heartwood, wood inside bark and wood including bark from two independent variables (dbh and height),

3) to create compatibility between taper and volume equations,

4) to improve heartwood taper and volume prediction by using heartwood ratio, heartwood diameter and heartwood height as additional independent variables,

5) to assess impacts of different heartwood ratio estimation measurement techniques on fits of the taper and volume models, thereby making them sensitive to variation in heartwood ratio produced by using different genotypoes or different sites.
CHAPTER 3

Developing fully compatible taper and volume equations for all stem components of *Eucalyptus globoidea* Blakely trees in New Zealand

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3.1. INTRODUCTION

A plan to diversify New Zealand’s planted forests by expanding the establishment of *Eucalyptus* species in dryland areas requires information for decision-making based on data from forest inventory and techniques to evaluate these data. Information obtained from forest inventories is needed so that forest managers can understand the current growing stock and growth potential. These data can provide accurate and precise estimates of volume production per tree, log types, and, in association with stand inventories, stand value (Casnati, 2016). Taper and volume equations can provide diameter estimates at any point on a stem, height estimates at which a given diameter occurs along the stem, total stem volume, merchantable volume to any merchantable height and individual log volumes (Methol, 2001). This information is crucial as it can determine the wood products and timber produced from forest stands. Taper and volume equations can provide estimates of tree stem volumes and shapes with input data of only tree height and diameter at breast height.

3.1.1. Tree stem volume

Measurement of stem volume describes the amount of wood in a tree’s stem or, collectively, in a stand of trees. The unit of measure is cubic meters (m$^3$) per tree or cubic meters per hectare (m$^3$/ha). Volume can be calculated in two variants: over-bark or under-bark (Avery & Burkhart, 1994). The stem volume of the individual standing trees is characterised as shown in *Equation 54*.

*Equation 54.*

\[ v = \frac{1}{4} \pi D^2 H f \]

where \( v \) is calculated stem volume, \( D \) is the diameter at breast height (1.4 m), \( H \) is the tree's height, and \( f \) is a form factor.

Volume is usually estimated from equations with the diameter at breast height (D), dbh and height, or dbh, height and an upper-stem diameter as predictor variables (Van Laar & Akça,
The shape of a tree bole or stem can be estimated approximately with three mathematical solids: a conoid, a paraboloid or a neiloid. However, to provide better volume results, Equation 55, Equation 56, and Equation 57 are specific to different parts of a tree stem shape (Vanclay, 1994):

\[ v = \frac{\pi}{12} l \left( d_1^2 + d_2^2 + d_1 d_2 \right) \]  
\[ v = \frac{\pi}{8} l \left( d_1^2 + d_2^2 \right) \]  
\[ v = \frac{\pi}{16} l \left( d_1^2 + \frac{3}{2} d_1^2 d_1^2 + \frac{1}{2} d_1^2 d_2^2 + d_2^2 \right) \]

where \( v \) is the calculated stem volume, \( l \) is the length of the tree section, \( d_1 \) is the large-end diameter of the section, and \( d_2 \) is the small-end diameter of the section.

### 3.1.2. Tree stem taper

Measurement of tree stem taper describes the shape of the tree bole, which influences volumes of specific products from a forest (Gomat et al., 2011). The most challenging part of developing a taper equation is to describe changing shapes along the stem. The first attempt in this area was conducted at the beginning of the 20th century, and since then, significant progress has been made (Loetsch et al., 1973). Different factors should be considered to find the best taper equation fit, including tree species, available data, tree size classes, climatic conditions and region (Li et al., 2012). The long history of taper and volume equations compatibility began in the early 1970s when Demaerschalk (1971, 1972) published papers on converting individual tree volume equations to compatible taper equations. The advantage of this system is that volume can be calculated by both the tree volume equation and by integrating the taper function, with both methods providing the same answer (Casnati, 2016). Compatible taper and volume models are widely used in the forestry sector (Brooks et al., 2008; Cao et al., 1980; Fang et al., 2000; Jiang et al., 2005; Jordan et al., 2005; Özçelik & Brooks, 2012).
3.1.3. Heartwood and sapwood components of tree stems

Heartwood and sapwood are two components within a tree’s stem. Heartwood is the central core of the tree stem, while sapwood describes the newer growth rings being found between the heartwood and the bark. The sapwood transports water and minerals upward, while the phloem transports photosynthates downward between leaves and other components of the tree. Sapwood is lighter, often less durable, softer, and contains more moisture than heartwood. As trees age and new sapwood rings are formed, older sapwood changes into heartwood (Bamber, 1961). Heartwood is an inner layer of the wood, which does not contain living cells (Taylor et al., 2002). Heartwood is naturally strong, often durable and more resistant to deterioration by insects and microorganisms than sapwood (Taylor et al., 2002). Heartwood has many vital functions in living trees. It provides structural support for a tree, although it does not differ structurally from the original sapwood. Significant radial strength differences between sapwood and hardwood result from radial changes in wood density and cell wall ultrastructure (Panshin & deZeeuw, 1980). As the heartwood is newly formed, it recycles nutrients back to sapwood (Bamber, 1961) while building up anti-decaying substances (Stewart, 1966), which provides its natural durability (Shain, 1995). In some species, heartwood can be distinguished from sapwood by its darker colour, lower permeability, different moisture content and increased decay resistance (Taylor et al., 2002). Due to its relatively high density and durability, the heartwood is used to produce furniture, flooring, roofing and other outdoor applications. Due to its lower density and non-durability, sapwood is often used as pulp or for indoor applications.

Durable heartwood is a valuable commodity and a vital timber component in the wood industry. Due to its natural durability can be an effective supplement and potential replacement for non-durable woods treated with toxic preservatives. Estimating the amount and shape of the heartwood in a tree stem is challenging since it is an internal part of the tree and cannot be measured from the outside without special tools or destructive intervention within the tree (such
as collecting discs or cores). For growers of *Eucalyptus globoidea* Blakely, which grows heartwood rated at class 2 durability (Bootle, 1983; Nguyen et al., 2020), estimation of dimensions of heartwood zones within stems is crucial for estimating product types and financial value.

### 3.1.4. Objective

The objective of this study was to develop compatible taper and volume equations for *Eucalyptus globoidea* to estimate the following:

- stem taper from dbh and height,
- stem volume from dbh, height, and taper,
- heartwood taper and volume from dbh and height.

There are five volume components; heartwood, sapwood, wood inside bark, bark, and wood including bark and three taper components; heartwood, wood inside bark, and wood including bark. In Brown’s (2019) study, the pattern of heartwood development was examined concerning several tree characteristics. Cross-sectional discs of northern red oak (*Quercus rubra* L.) were analysed to detect changes in heartwood radius. The study showed that the tree’s age, dbh size class, height, and inside bark radius were significantly related to heartwood diameter. In the study described here, a similar cross-sectional analysis approach was applied. However, the research context was extended to five tree components, including heartwood. The goal of this study was to create compatibility within taper and volume models for all components of stems of *Eucalyptus globoidea*. Estimates of heartwood and sapwood volume combined produce the same estimate as wood inside bark, and estimates of wood inside bark and bark combined to create the same estimate as wood including bark. Integrating these taper equations produces identical volume estimates to those obtained with tree volume equations. Compatibility at this
level, where all stem volume components depend on each other, is a new approach to taper and volume modelling.

3.2. METHODS

3.2.1. Study sites

New Zealand is situated between 34ºS and 47ºS, between the South Pacific Ocean and the Tasman Sea, and comprises three main islands: the South Island (151,215 km²), the North Island (113,729 km²), and Stewart Island (1,746 km²) (Moot et al., 2009). The total land area is almost 270,000 km² (McKinnon et al., 1997). Mean annual temperatures in New Zealand range from 10ºC in the south to 16ºC in the north. The warmest month is usually January or February and the coldest month is usually July. Most areas of New Zealand have between 600 and 1600 mm of rainfall, with increased rainfall during the winter months, although some extreme areas have rainfall as high as 4000 mm/annum (Mackintosh, 2001). The desired area for future large-scale Eucalyptus plantations is located in the dryland areas of New Zealand, which are spread along the east coasts of the North and the South Islands. Drylands are defined and characterised by a water deficiency; over the long term, natural moisture inputs such as precipitation are outweighed by moisture losses through plants' evaporation from surfaces and transpiration. This potential water deficit affects both natural and managed ecosystems and constrains the production of crops, forage, and other plants and trees (Safriel et al., 2005). To study growth and species acclimatisation in these areas, multiple trial plantations of *Eucalyptus globoidea* were established in different years throughout the dryland areas of New Zealand. For this study, data were collected from eight various study sites, as shown in Figure 5.
Figure 5. Locations of study sites in New Zealand (red dots). Scales are in latitude and longitude.

The climate data presented in Table 4 came from Land Environments of New Zealand (LENZ). The data represented the year 2009 and were mainly obtained from summaries of climate observations published by the New Zealand Meteorological Service.
Table 4. Description of the study sites includes location and climate data, which were sourced from the Land Environments of New Zealand (LENZ 2020).

<table>
<thead>
<tr>
<th>Site No.</th>
<th>Establishment Date</th>
<th>Plantation Age (as at 2019)</th>
<th>Location</th>
<th>Coordinates</th>
<th># of trees Sampled</th>
<th>Mean annual temperature (ºC)</th>
<th>Mean minimum temperature of the coldest month (ºC)</th>
<th>Mean annual solar radiation (MJ/m²/day)</th>
<th>Mean winter solar radiation (MJ/m²/day)</th>
<th>Mean annual water deficit (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>1990</td>
<td>29</td>
<td>South Island, Canterbury, Okuti Valley</td>
<td>43°47'32.0&quot;S 172°50'20.8&quot;E</td>
<td>28</td>
<td>11.7</td>
<td>3.4</td>
<td>13.8</td>
<td>4.5</td>
<td>176</td>
</tr>
<tr>
<td>2</td>
<td>1998</td>
<td>22</td>
<td>North Island, Bay of Plenty, Welcome Bay, Tauranga</td>
<td>37°43'55.8&quot;S 176°13'20.9&quot;E</td>
<td>13</td>
<td>14.1</td>
<td>5</td>
<td>15.1</td>
<td>6.0</td>
<td>18</td>
</tr>
<tr>
<td>3</td>
<td>2003</td>
<td>16</td>
<td>South Island, Marlborough, Havelock</td>
<td>41°17'27.9&quot;S 173°48'56.0&quot;E</td>
<td>1</td>
<td>10.6</td>
<td>0.5</td>
<td>15.1</td>
<td>4.7</td>
<td>5</td>
</tr>
<tr>
<td>4</td>
<td>2006</td>
<td>13</td>
<td>South Island, Marlborough, Fairhall</td>
<td>41°32'29.2&quot;S 173°52'20.5&quot;E</td>
<td>5</td>
<td>12.5</td>
<td>2.2</td>
<td>15</td>
<td>4.9</td>
<td>274</td>
</tr>
<tr>
<td>5</td>
<td>2009</td>
<td>10</td>
<td>South Island, Marlborough, Lower Wairau Valley</td>
<td>41°26'32.7&quot;S 173°56'07.6&quot;E</td>
<td>8</td>
<td>12.5</td>
<td>1.7</td>
<td>15.1</td>
<td>4.8</td>
<td>231</td>
</tr>
<tr>
<td>6</td>
<td>2009</td>
<td>10</td>
<td>South Island, Marlborough, Seddon</td>
<td>41°43'08.4&quot;S 174°02'02.4&quot;E</td>
<td>5</td>
<td>12.2</td>
<td>2.9</td>
<td>14.9</td>
<td>5.5</td>
<td>222</td>
</tr>
<tr>
<td>7</td>
<td>2010</td>
<td>9</td>
<td>South Island, Marlborough, Lower Wairau Valley</td>
<td>41°26'31.7&quot;S 173°56'17.5&quot;E</td>
<td>4</td>
<td>12.5</td>
<td>1.7</td>
<td>15.1</td>
<td>4.8</td>
<td>231</td>
</tr>
<tr>
<td>8</td>
<td>2011</td>
<td>7</td>
<td>North Island, Stronvar, Masterton, Wellington region</td>
<td>41°02'42.7&quot;S 175°52'37.2&quot;E</td>
<td>10</td>
<td>12.2</td>
<td>3.7</td>
<td>14.0</td>
<td>4.6</td>
<td>107</td>
</tr>
</tbody>
</table>
CHAPTER 3 – DEVELOPING FULLY COMPATIBLE TAPER AND VOLUME …

3.3. DATA

3.3.1. Tree selection standards

To select study trees on each site, preliminary assessments were conducted to find a diverse range of diameter combinations at breast height (dbh) and height among the same-aged groups. They were based on two measurements: the dbh at 1.4 m height, measured by a dbh tape, and the tree's height measured by a Vertex IV Hypsometer (Haglöf, Sweden). The most important criteria for tree selection were straight stems, lack of stem damage, absence of forking, and no epicormic branches. The selected study trees had straight stems without deformation (Figure 6). Before felling, each tree was spray-painted in two places: at breast height around the stem at the height of 1.4 m and in a straight line from the ground up to the dbh, which was needed later for the correct orientation of the discs. After this preparation, trees were felled. 74 Eucalyptus globoidea trees were destructively sampled across the eight trial sites throughout New Zealand. Tree age varied from 7 to 29 years old, with a mean value of 19. Dbh ranged from 11 to 67.6 cm with a mean value of 32.3 cm and a standard deviation of 15.1 cm. Height varied from 7.2 to 35.4 m with a mean value of 19 m and a standard deviation of 8.2 m. The total volume ranged from 0.036 to 4.859 m$^3$, with a mean value of 0.88 m$^3$ and a standard deviation of 0.82 m$^3$.

3.3.2. Measurement of felled stems

The first step after felling was to measure the length of the stem using a tape measure. The tape was extended along the stem, from the base of the felled stem to the tip of the apically dominant leader in a straight line, and was pulled tightly before taking the final measurement. Then, the height of the stump was measured by a ruler. In the case of sloping land, the stump measurement was taken from the highest ground level. The tree's total height was obtained from the sum of the felled stem length and the height of the stump.
The next step was to measure diameters along the stem from the bottom to the top, including the measurement from the stump, located as close to ground level as possible. Two diameter measurements at 90-degree angles were obtained using a calliper and repeated at defined increments along the stem up to the tip of the apically dominant leader. All diameter measurements had corresponding height measurements obtained from the tape extended along the stem. Diameter measurement points were spray-painted along the stem, making the painted markings visible for subsequent disc cutting (Figure 7).

Distances between stem diameter measurements, called the taper step distance, were established depending on the diameter (D) of the stem. Where D ≤ 25 cm, stem diameter was measured at 2.5 cm taper increments; where 25 cm < D < 50 cm, stem diameter was measured at 5 cm taper intervals; and where D ≥ 50 cm, stem diameter was measured at 7.5 cm taper intervals.

The taper step distance was treated as a guideline for measurements. During fieldwork, the taper step distance was adjusted, particularly near the base of the felled stem and towards the tip of the apically dominant leader, to find the exact height at which the heartwood ended. In Brown’s (2019) study of northern red oak, the author produced satisfactory results for heartwood taper based on 5-6 discs per tree. In the study reported here, there were 993 diameter measurements collected from 74 trees, resulting in a mean value of 13 diameter measurements per tree.

3.3.3. Bark measurement

The bark of *Eucalyptus globoidea* is flaky and tends to fall off, especially for older trees and lower trunk sections. For this reason, bark measurements were taken before cutting discs from felled stems or using a ruler just after cutting stem sections. For every disc, four bark measurements were collected at intervals of 90 degrees around the stem in the same orientation.
3.3.4. Disc collection and preparation

After collecting diameter measurements along the stem, the next step was to collect tree discs corresponding to the location of each diameter measurement. Selecting discs within a tree was based on taper, the most commonly used procedure for similar studies (Ojansuu & Maltamo, 1995). The first disc was cut from the stump; after felling, the tree chainsaw operator cut a disc from the remaining stump. Subsequent discs were cut from the felled log at the spray paint marks, where the previous taper measurements had been made (Figure 8). The disc thickness was between 3 – 5 cm. Every disc was labelled with a unique number after being cut (Figure 9). During the initial research stage, the discs were transported to the wood technology laboratory at the University of Canterbury and analysed in a green state. However, due to transport logistics, some discs were dyed and measured in the field (Figure 10).

To help with heartwood and sapwood identification, an indicator dye was applied with a brush to the discs to dye the heartwood pink and the sapwood bright yellow. The indicator dye was 1% methyl yellow diluted in 40% ethanol, which after correct preparation, is clear orange. Colour differences in the indicator dye arise from differences in pH between sapwood and heartwood. The methyl yellow indicator turns pink below pH 2.9 and yellow above pH 4.0 (Tawarah & Abu-Shamleh, 1991). When both heartwood and sapwood were visible, the next step was to take photographs. All discs were placed on a level surface in order. Each disc was photographed in good light to minimise shadow and under/overexposed areas. In subsequent steps, every image included a ruler to scale the image size (Figure 11).

3.3.5. Data pre-processing - image analysis

Images of the discs were analysed using ImageJ software version 1.52a (Rasband 2018). The images were loaded into the software and analysed individually. The first step was to set a scale for the image. Using a “set scale” tool, the software calculated the area occupied by a pixel
based on the scale visible on each image. The software calculated any pixel unit with this information and converted it to the unit of interest, such as cm or cm$^2$. The next step was manually selecting the heartwood, sapwood and bark circuit area using a “polygon selection” tool. After completing each circuit, the area inside it was calculated in cm$^2$ using the “analysis measure” tool. From the area measured in cm$^2$, the diameter and radius of the different tree components were calculated and used for the subsequent analysis.
Figure 6. Examples of selected study trees from the trial plot located in the Okuti Valley, Canterbury.

Figure 7. Paint marks showing the diameter measurement points along a sample log.
Figure 8. Cutting a disc from the sample stem with a visible “wavy” thick bark structure typical of *E. globoidea* at this site.

Figure 9. Labelling the discs with a unique number after being cut to maintain the order.
Figure 10. An example of discs dyed and measured in the field.

Figure 11. An example of a disc with a visible scale after applying dye.
3.4. VOLUME MODELLING

This study modelled volumes of five tree components: heartwood, sapwood, wood inside bark, wood including bark, and bark. To calculate sectional volumes of heartwood, wood inside bark and wood including bark components, the Smalian formula (Equation 56) was used. To calculate sapwood and bark volume, simple subtractions of the above-calculated components were applied:

- sapwood volume = wood inside bark volume – heartwood volume;
- bark volume = wood including bark volume – wood inside bark volume.

At the beginning of the modelling process, it was essential to ensure that all variables were normally distributed. Modelling was attempted with various dependent variables, including tree diameter squared times height. Still, after a preliminary evaluation of bias, each volume component was modelled as a function of tree diameter times height (dh) with both volume components and dh was transformed using scaled power transformations (Sakia, 1992) (Equation 58), with λ values chosen to make distributions of values as normal as possible. Normality of frequency distributions of correlated variables makes relationships between them as linear as possible and generally helps to avoid heteroscedasticity of residuals if dependent variables are transformed.

\[
\text{Equation 58.} \quad x^{(\lambda)} = \begin{cases} 
(x^\lambda - 1)/\lambda & \lambda \neq 0 \\
\log(x) & \lambda = 0
\end{cases}
\]

where \( x^{(\lambda)} \) is the scaled power transformation of \( x \), \( \lambda \) is a parameter that defines the curvature of the relationship between \( x \) and \( x^{(\lambda)} \).

After applying transformations, the following assumptions were fulfilled:

- the Y values could be expressed as linear functions of X values;
• variation of observations around the regression line was homoscedastic;
• for given values of X and Y, error values were normally distributed.

The next step was to run a linear regression using the command `lm` in R (R Core Team, 2021) for each tree volume component separately to determine the starting values for a non-linear regression. Non-linear regression was required when all models were fitted simultaneously because all transformation terms had to be on the right-hand sides of equations to ensure the additivity of stem components. In the next step, the obtained starting values were used as the starting coefficients for non-linear models (nls) (Hamann, Henningsen et al. 2007) for each tree volume component separately. Both sides of volume equations were weighted by 1/dh (tree breast height diameter times height), which ensured that the residuals of small values were not biased. The weighting reduced the impacts of heteroscedasticity on the bias for small estimates. Right-hand sides of equations were back-transformed during fitting so that component volume predictions could be made compatible, and so this weighting was necessary to avoid bias.

The next step was to create compatible volume equations. The following dependencies were created between heartwood, sapwood, wood inside bark, bark, and wood including bark:
• the volume of wood inside bark = heartwood volume + sapwood volume
• the volume of wood including bark = volume of wood inside bark + bark volume

The R library `systemfit` (Hamann et al., 2007) and command `nlsystemfit` were used to simultaneously fit equations to the data using the Ordinary Least Squares method (OLS). Different `systemfit` methods were considered in this study (Weighted Least Squares (WLS), Seemingly Unrelated Regression (SUR), Two-Stage Least Squares (2SLS), Weighted Two-Stage Least Squares (W2SLS) or Three-Stage Least Squares (3SLS)). However, the OLS method converged more efficiently and effectively than other methods.
CHAPTER 3 – DEVELOPING FULLY COMPATIBLE TAPER AND VOLUME …

3.5. TAPER MODELLING

Taper modelling of heartwood, wood inside bark and wood including bark was based on Demaerschalk’s (1971, 1972, 1973) method for estimating tree taper and volume. The idea is to create compatibility between taper and volume equations. Taper and volume equations are compatible when integration of the taper equation yields the same total volume as that given by the volume equation (Demaerschalk, 1973). Taper equations were polynomial (Equation 43- Equation 46). The constraint is that the coefficients of the taper equation fit must be restricted so that they sum to one. To solve it in the non-linear fitting procedure, one of the coefficients was set to one minus the others.

3.6. HEARTWOOD HEIGHT PREDICTION

Predicting the height at which the heartwood stops is vital for applying the heartwood taper model. To predict the heartwood height, a linear mixed-effects model was executed by the lme function from R package nlme (Pinheiro et al., 2019). The heartwood height prediction was modelled based on the correlation between heartwood height and tree height. No transformation was applied as there was a linear relationship between those variables.

3.7. SITE EFFECT ON HEARTWOOD DEVELOPMENT

To establish if there was a site effect on heartwood development, a linear model was created using the lme function from the R package nlme (Pinheiro, Bates et al. 2019). The site effect on heartwood development was modelled by examining the correlation between heartwood volume % within a tree and total tree volume + site as a fixed effect. In all other models site was a random effect. Still, to examine whether the site significantly affected heartwood development in principle, it was helpful to apply it as a fixed effect in this case. As the
relationship between these variables was not linear, scaled-power transformations were applied.

3.8. VALIDATION

The leave-one-out cross-validation technique was used to validate taper and volume models. To find estimators of parameters, each observation has to be systematically left out of the dataset to calculate estimates and fit new models. Then residuals for the record left out are computed. This is a cross-validation method (Refaeilzadeh et al., 2009).

Due to many observations and the complexity of the compatible taper and volume fitting process, a loop was created in R to obtain new models with each successive tree left out and to compute residual values for that tree. Each time from the dataset of 74 trees, one tree was excluded, and the analyses were conducted using the 73 remaining trees.

3.9. STATISTICAL INTERPRETATION

All taper and volume models were compared statistically with root mean squared error (RMSE), mean absolute bias (MAB), and model efficiency (EF).

\[
\text{RMSE} = \sqrt{\frac{\sum(Y - Y')^2}{N}}
\]

\[
\text{MAB} = \frac{\sum|Y - Y'|}{N}
\]

\[
\text{EF} = 1 - \frac{\sum(Y - Y')^2}{\sum(Y - \bar{Y})^2}
\]

where \(N\) = number of observations, \(Y\) = observed value, \(Y'\) = expected value, \(\bar{Y}\) = overall mean.
3.10. RESULTS

3.10.1. Volume modelling

The volume equations were all of the form shown in Equation 59:

\[
\frac{\nu_i}{dh} = \left( \frac{\lambda_i a_0 + a_1 \left( \frac{dh}{\lambda_i dh} \right) + 1}{\lambda_i} \right)
\]

where \(a_0\) and \(a_1\) are fitted coefficients, \(\lambda_i\) is a value unique to each variable, \(i\) is the heartwood, wood inside bark or wood including bark component.

Parameters, \(\lambda\) values for each volume component and standard errors are shown in Table 5. The wood inside bark and wood including bark values were obtained using initial non-linear regression; the parameters were used as the starting values in systemfit modelling. The wood inside bark and wood including bark were created as simple sums of tree components in the systemfit model: heartwood + sapwood = wood inside bark, and wood inside bark + bark = wood including bark. Plots of residuals are presented in Figure 12.
<table>
<thead>
<tr>
<th>Component</th>
<th>$\lambda$</th>
<th>$a_0$</th>
<th>$a_1$</th>
<th>$a_2$</th>
<th>RMSE (m$^3$)</th>
<th>MAB (m$^3$)</th>
<th>EF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heartwood</td>
<td>0.22</td>
<td>-5.1118424</td>
<td>0.2089528</td>
<td>-</td>
<td>0.1248298</td>
<td>0.06782255</td>
<td>0.905</td>
</tr>
<tr>
<td>Sapwood</td>
<td>0.38</td>
<td>-2.5616701</td>
<td>0.0673768</td>
<td>-</td>
<td>0.05496807</td>
<td>0.0382446</td>
<td>0.805</td>
</tr>
<tr>
<td>Wood inside bark*</td>
<td>0.25</td>
<td>-1.114294*</td>
<td>0.050221*</td>
<td>0.064701*</td>
<td>0.1539625</td>
<td>0.08887412</td>
<td>0.91</td>
</tr>
<tr>
<td>Bark</td>
<td>0.12</td>
<td>-4.9560452</td>
<td>0.1991522</td>
<td>-</td>
<td>0.1108201</td>
<td>0.06808297</td>
<td>0.905</td>
</tr>
<tr>
<td>Wood incl. bark*</td>
<td>0.19</td>
<td>-0.743484*</td>
<td>0.054786*</td>
<td>-</td>
<td>0.1439786</td>
<td>0.08347402</td>
<td>0.971</td>
</tr>
<tr>
<td>$dh$</td>
<td>0.28</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>Tree height</td>
<td>0.32</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

*Wood inside bark and wood including bark values with an asterisk (*) were calculated individually rather than by systemfit, and within systemfit they were assigned as the sums of their components to ensure compatibility. For the wood inside bark model, transformed tree height was added as an independent variable (coefficient $a_2$).
Figure 12. Residual values versus fitted values for compatible volume equations for heartwood (a), sapwood (b), wood inside bark (c), bark (d), wood including bark (e) and the leave-one-out cross-validation of those equations.
Graphs in Figure 12 present residual vs fitted values. Residuals were also plotted against explanatory variables. Overall the residuals were well distributed in the volume range, and all models converged without difficulty. As can be seen, back-transformed residuals are heteroscedastic. Still, transformations and weighting used during fitting ensured that relationships were relatively unbiased and that residuals for small predictions had similar impacts on fitting to those of large predictions. Back-transformed equations were required so that volume estimates could be summed to ensure compatibility. The residual range around the x-axis varied from -0.15/0.15 m$^3$ (for sapwood) to -0.6/0.6 m$^3$ (for wood including bark). The rest of the components were between -0.4/0.5 m$^3$ (heartwood), -0.4/0.6 m$^3$ (wood inside bark), and -0.3/0.5 m$^3$ (bark).

Model validations showed minor differences between leave-one-out cross-validation techniques and full model fits. Both methods yielded normal residuals.

A summary of fitting statistics for five compatible volume equations is presented in Table 5. Differences between the components were small. The RMSE value varied from 0.05496807 m$^3$ (for sapwood) to 0.1539625 m$^3$ (for wood inside bark). The MAB value ranged from 0.0382446 m$^3$ (for sapwood) to 0.08887412 m$^3$ (for wood inside bark). The EF value varied from 0.805 (for sapwood) to 0.971 (for wood including bark).

### 3.10.2. Heartwood height

The heartwood height was found to be a linear function of tree height. The residuals versus fitted values of heartwood height prediction are visible in Figure 13. The model is presented in Equation 60.

\[ H_h = -6.2994 + 1.1208H \]

where $H_h$ is heartwood height, $H$ is tree height, -6.2994 and 1.1208 are model coefficients. The root mean squared error (RMSE) was 1.6461 m. In the heartwood height prediction model, one
visible outlier is the tree with an unusual characteristic: heartwood height is substantially smaller than the tree height. It was unusual, but I decided to keep the record.

Figure 13. Heartwood height prediction residual values versus fitted values.

### 3.10.3. Taper modelling

Compatible taper equations were created, and the final version is presented in Equation 61. This equation predicts diameter at any height up the stem.

**Equation 61.**
\[
d = \sqrt{\frac{(b_1z_1 + b_2z_2 + b_4z_4 + b_5z_5)V_c}{0.00007854H}}
\]

where \( V_c \) is the volume component predicted by the appropriate volume function using Equation 59.

The essential parts needed to create Equation 61 are \( z \) values (\( z_n \)) and height ratio (HR), which is the relative height at which the diameter was predicted (Equation 62). The value of HR is in the range from 0 (the small-end diameter) to 1 (the large-end diameter).

**Equation 62.**
\[
HR = \frac{H-h}{h}
\]

where \( H \) is the tree's height, and \( h \) is the height of the diameter measurement.
Z values \((z_n)\) are power functions of height ratio (HR), representing the different shapes along the stem. To calculate \(z\) values \((z_n)\) Equation 63 was used. The \(z\) values of larger power are responsible for shape starting from the large-end stem, while the \(z\) values of smaller power are responsible for shape ending to the small-end stem. In the fitting process, to ensure the taper and volume compatibility, the best results were obtained with \(z\) values of power 1, 3, 4 and/or 5 kept and the \(z\) value of power 2 excluded.

\textit{Equation 63.}\hspace{1cm} z_n = (n + 1)HR^n

where \(z\) values are powers of the height ratio used in the multilinear equation, \(n =1,3,4,\) and/or 5.

Equation 64 was created using Equation 61. It predicts any log length volume and demonstrates compatibility between taper and volume models. This volume equation created from the taper equation (integration of the taper equation) yields the same total volume as given by the volume equation (obtained by summation of sections).

\textit{Equation 64.}\hspace{1cm} V_l = V_{chS}^{hL}[b_1HR^2 + b_3HR^4 + b_4HR^5 + b_5HR^6]

where \(h_S\) is the height to the small-end diameter, \(h_L\) is the height to the large-end diameter.

Taper function parameters are shown in Table 6 and are all statistically significant. The best taper functions for heartwood included coefficients for 1st, 3rd, 4th and 5th powers, while for wood inside bark and wood including bark included coefficients for 3rd, 4th and 5th powers. Residuals for diameter predictions by the taper function are shown in Figure 14.

Table 6. Parameter estimates of three compatible taper equations fitted for Eucalyptus globoidea.

<table>
<thead>
<tr>
<th>Component</th>
<th>(b_1)</th>
<th>(b_3)</th>
<th>(b_4)</th>
<th>(b_5)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heartwood</td>
<td>0.2419945</td>
<td>4.9174316</td>
<td>-7.9903269</td>
<td>3.830901</td>
</tr>
<tr>
<td>Wood inside bark</td>
<td>-</td>
<td>6.416621</td>
<td>-9.975695</td>
<td>4.559074</td>
</tr>
<tr>
<td>Wood including bark</td>
<td>-</td>
<td>6.001335</td>
<td>-9.781425</td>
<td>4.780089</td>
</tr>
</tbody>
</table>
Figure 14. Residual values versus fitted values for compatible taper equations for heartwood (a), wood inside bark (b), wood including bark (c) and the leave-one-out cross-validation of those equations.
Figure 14 presents the three compatible taper models' residual versus fitted values graphs. In all the graphs, we can observe heteroscedastic patterns, which means that variance is increasing; larger predicted values are associated with larger errors of residuals. Smaller diameters had lower residuals compared to larger diameters, as the values of residuals and diameter increased proportionately. The residual range around the x-axis was similar for all three components: -11/16 cm for heartwood, -12/15 cm for wood inside bark and -14/16 cm for wood including bark.

The model validation showed differences between the leave-one-out cross-validation technique and full model fits. Still, the ranges and distributions of fitted and validation residuals were similar. Both methods showed normal residuals.

A summary of the fitting statistics for three compatible taper equations is presented in Table 7. The RMSE values varied from 2.348552 cm for wood inside bark to 2.572765 cm for heartwood. The MAB value ranged from 1.585148 cm for wood inside bark to 1.816375 cm for heartwood. The EF value varied from 0.959 for heartwood to 0.977 for wood inside bark.

<table>
<thead>
<tr>
<th>Component</th>
<th>RMSE (cm)</th>
<th>MAB (cm)</th>
<th>EF</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heartwood</td>
<td>2.572765</td>
<td>1.816375</td>
<td>0.959</td>
</tr>
<tr>
<td>Wood inside bark</td>
<td>2.348552</td>
<td>1.585148</td>
<td>0.977</td>
</tr>
<tr>
<td>Wood including bark</td>
<td>2.474088</td>
<td>1.647877</td>
<td>0.972</td>
</tr>
</tbody>
</table>

### 3.10.4. Site effect on heartwood development

The site significantly influenced heartwood development after controlling for tree size. After running the Anova analysis (type 3), the site was statistically significant (P = 0.001). The heartwood volume % within a tree was found to be a linear function of total tree wood volume after transformation (Figure 15). The observations appear clustered in the site groups (each
colour in Figure 15 represents a different site). The residuals versus fitted values of heartwood volume % within a tree are visible in Figure 16.

![Site effect on heartwood % within a tree](image)

Figure 15. The site effect on heartwood development illustrated as a linear function between heartwood volume % within a tree* and total tree volume* (*T stands for transformed values); different colours represent different sites (1-8).
Figure 16. The residuals versus fitted values of heartwood volume % within a tree; different colours represent different sites (1-8).

3.10.5. Taper and volume interactive tool

The above results from taper and volume modelling were combined to create an interactive tool that can graph tree shapes, including three components of heartwood, wood inside bark and wood including bark. The tool is in an Excel spreadsheet (Figure 17). The user of the tool inputs tree dbh and height. If the inputs are within the range model will show “OK” in the adjacent cell. If inputs are beyond the range of the model, then “Extrapolated” will appear. The tool predicts the height of the heartwood and plots the graphs. Examples of three different trees
that vary in size are shown in Figure 18. Information about the trees is provided in Table 8 below.

Table 8. Information about trees projected in Figure 18.

<table>
<thead>
<tr>
<th>Tree no.</th>
<th>1</th>
<th>2</th>
<th>3</th>
</tr>
</thead>
<tbody>
<tr>
<td>dbh (cm)</td>
<td>11</td>
<td>32</td>
<td>62</td>
</tr>
<tr>
<td>Tree height (m)</td>
<td>8</td>
<td>19</td>
<td>34</td>
</tr>
<tr>
<td>Heartwood height (m)</td>
<td>2.7</td>
<td>15</td>
<td>31.8</td>
</tr>
<tr>
<td>Heartwood proportion in wood inside bark (%)</td>
<td>13</td>
<td>57</td>
<td>79</td>
</tr>
<tr>
<td>Heartwood volume (m$^3$)</td>
<td>0.004</td>
<td>0.2</td>
<td>1.62</td>
</tr>
<tr>
<td>Wood inside bark volume (m$^3$)</td>
<td>0.03</td>
<td>0.35</td>
<td>2.05</td>
</tr>
<tr>
<td>Wood including bark volume (m$^3$)</td>
<td>0.05</td>
<td>0.57</td>
<td>3.57</td>
</tr>
</tbody>
</table>
Figure 17. Screenshot of interactive tool interface designed for the model with two independent variables (dbh, tree height).
Figure 18. Projection of three different trees using a tool based on taper and volume models.
3.11. DISCUSSION

This study used a compatible taper and volume equation approach developed by Demaerschalk (1971, 1972, 1973). Its simple approach allowed the integration of many simultaneously harmonised components in one whole system of compatible models. In this study, the total volume of heartwood, wood inside bark and wood including bark, obtained by summation of sections whose volumes are defined using the taper equation, is almost identical to the volume defined by the volume equation. Integration of the taper equation of those components yields the same total volume as given by the volume equation.

Compatible heartwood, sapwood, and bark volume equations were built using non-linear regression. The compatible volume equations of wood inside bark and wood including bark were created using a simple summary of the above components (wood inside bark = heartwood + sapwood; wood including bark = wood inside bark + bark). The compatible taper equations of heartwood, wood inside bark and wood including bark were built using non-linear regression.

From a silvicultural and economic point of view, the most valuable part of the log is the lower end, with larger diameters. Still, residuals are larger for predictions of that part of the tree. This is unavoidable, and I can best say that predictions across the entire range of diameters are relatively unbiased.

There are a few publications about *Eucalyptus* species taper and volume equations. Rachid’s study compared several taper and volume equations, emphasising testing the suitability of compatible taper equations (Casnati et al., 2014). Among two species, one of them is *Eucalyptus grandis* W.Hill. Muhairwe (1999) developed taper equations for *Eucalyptus pilularis* Sm. and *Eucalyptus grandis*. A comparison of statistics of wood including bark volume equations’ fit from three studies (the study reported here, Casnati and Muhairwe),
provides the following results. The *Eucalyptus grandis* volume equation (Casnati et al., 2014) with its RMSE value of 0.0422 m$^3$ was comparably more precise than the *Eucalyptus globoidea* model developed herein (RMSE values 0.1439786 m$^3$). The *Eucalyptus globoidea* MAB value was higher than the lowest (0.0245 m$^3$) reported for *Eucalyptus grandis*. Model efficiency was the best for *Eucalyptus globoidea* based on EF value (0.971). The *Eucalyptus globoidea* taper equation was a less precise model as it had a larger RMSE value (2.474088 cm) compared to Rachid’s model (0.8937 cm) (Casnati et al., 2014). The MAB value of 1.647877 cm was also higher than the lowest (0.6747 cm) reported for *Eucalyptus grandis*. Model efficiency was similar for *Eucalyptus globoidea* based on EF value (0.972) compared with *Eucalyptus grandis* (0.989).

The study described here had only 74 *Eucalyptus globoidea* trees (aged from 7 to 29 years old) compared to 932 *Eucalyptus grandis* trees (age from 2 to 23 years old) (Casnati, 2016) or 526 *Eucalyptus pilularis* trees and 645 *Eucalyptus grandis* trees (varied in age; *Eucalyptus pilularis* dbh range 11.2 – 192.4 cm; *Eucalyptus grandis* dbh range 5.9 to 94.3 cm) (Muhairwe, 1999).

Another interesting finding was obtained by analysing the predictions of the interactive tool. By increasing dbh and keeping tree height constant at harvest, we observed that the proportion of heartwood in wood inside bark increased with dbh. We deduced the silvicultural implication that lower stockings might result in higher proportions of heartwood. This implication was borne out in a study by Gominho (2005). He studied 27 *Eucalyptus globulus* Labill. trees and found that the proportion of heartwood volume increased with spacing from 20% to 40% of tree volume, respectively, for 2 × 1 and 3 × 3 spacings. Another study found that the heartwood percentage of *Eucalyptus grandis* was also positively correlated with dbh; however, it didn’t find this correlation in the case of *Eucalyptus grandis × Eucalyptus urophylla* S.T.Blake (Brito et al., 2019).
The results of this study are specific to *Eucalyptus globoidea* within ranges of dbhs of 11-67.6 cm, tree heights of 7.2-35.4 m, and volumes of 0.036-4.86 m$^3$. Using the models outside the ranges of fitting data is not recommended, as they can predict unrealistic values. The standard error tends to be small for smaller trees and bigger for big trees. The models shouldn’t be generalised to other species or beyond the range of tree sizes included in the training data. Future research could be focused on making the models presented herein more specific by incorporating spatial effects and different genotypes of the species. Sites with varying relations of water, for instance, may influence the retention of sapwood, and sites or genotypes with greater leaf area index may also result in greater retention of sapwood based on the sapwood-pipe theory.

The sample size of eight sites was too small to examine causes of site effects. To include site effects on the formation of heartwood in the future many more sites will be required in the input data set, along with a more balanced data set with a wide range of tree sizes on each site. This study involved collection of the best data on available sites for *Eucalyptus globoidea* in New Zealand.

### 3.12. CONCLUSIONS

The taper and volume equations presented in this paper were created using a simultaneous fitting with the following constraints: the wood inside bark is the sum of the heartwood volume and sapwood volume; both of these components never exceed the volume of the wood inside bark; and the wood including the bark is the sum of the heartwood, the sapwood and the bark volumes. These three components never exceed the volume of the wood including bark. These two constraints ensure that the system is compatible and correct. Moreover, taper equations compatible with the volume equations were created for the entire stem, the wood within the stem and the heartwood. These models performed well during leave-one-out cross-validation, presenting similar results to those obtained during fitting with the entire dataset.
CHAPTER 4

Extending fully compatible taper and volume equations for all stem components of *Eucalyptus globoidea* Blakely trees in New Zealand with a focus on heartwood prediction improvement
CHAPTER 4 – EXTENDING A FULLY COMPATIBLE TAPER AND VOLUME …

4.1. INTRODUCTION

As mentioned previously, the primary goal of the NZDFI is for New Zealand to become the international leader in breeding naturally ground-durable Eucalyptus species. The most valuable tree part, which creates this opportunity, is heartwood, with an in-ground life expectancy of up to 25 years (Bootle, 1983). The previous chapter describes the creation of compatible taper and volume equations for Eucalyptus globoidea (model 1). They enable the user of the interactive tool to predict the taper of heartwood, wood inside bark, wood including bark and volume of heartwood, sapwood, wood inside bark, bark, and wood including bark of any given tree based on measurements of dbh and height. Heartwood height was predicted as a function of tree height. As heartwood is used to create durable products, it is crucial to maximize the efficiency of model predictions of this component and reduce uncertainty related to external factors which may influence heartwood development, such as the site effect. Therefore the objective of the research described in this chapter was to improve heartwood volume and taper predictions, enhancing model 1 from Chapter 3 by using additional independent variables. In this chapter, I will focus only on changes that lead to model improvements, as the basic model-building steps are described in detail in Chapter 3.

4.2. METHODS - HEARTWOOD PREDICTIONS IMPROVEMENT STEPS

Several steps were applied to find the best model (Figure 19). During every step, newly created models were examined and compared to the initial model when a new model performed worse than the initial one; it was not promoted to the next step and was excluded from further examination (Figure 27). This approach of eliminating and promoting newly created models led to determining the most precise heartwood prediction model.
CHAPTER 4 – EXTENDING A FULLY COMPATIBLE TAPER AND VOLUME …

MODEL 1: two predicting variables (dbh, tree height)
Re-fitting and adjusting data to focus on improvement of heartwood equations;
Re-grouping the trees based on the age

MODEL 1: A (all trees), B (≥ 8 years old), C (< 8 years old)
Adding 3rd independent variable: heartwood ratio (HR) calculated using three different methods

MODEL 2: Ax (1.4m core), Bx (1.4m scan), Cx (0m stem)
Adding more independent variables: 4th heartwood diameter (D_h) and 5th heartwood height (H_h) in different configuration;
including diameter at breast height (D) and tree height (H) and different model weighting

MODEL 3x:

<table>
<thead>
<tr>
<th>Model</th>
<th>Component</th>
<th>Heartwood</th>
<th>Remaining (without heartwood)</th>
<th>All</th>
<th>Model weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>3A/3Aa</td>
<td>D_h * H_h</td>
<td>D*H</td>
<td>-</td>
<td>D*H</td>
<td></td>
</tr>
<tr>
<td>3B/3Bb</td>
<td>-</td>
<td>-</td>
<td>D_h * H_h</td>
<td>D*H</td>
<td></td>
</tr>
<tr>
<td>3C/3Cc</td>
<td>D_h * H_h</td>
<td>D*H</td>
<td>-</td>
<td>D*H</td>
<td></td>
</tr>
<tr>
<td>3D/3Dd</td>
<td>D_h ¹* H_h</td>
<td>D²*H</td>
<td>-</td>
<td>D*H</td>
<td></td>
</tr>
<tr>
<td>3E/3Ee</td>
<td>-</td>
<td>-</td>
<td>D_h * H_h</td>
<td>D*H</td>
<td></td>
</tr>
<tr>
<td>3F/3Ff</td>
<td>-</td>
<td>-</td>
<td>D_h * H_h * H_h</td>
<td>D*H</td>
<td></td>
</tr>
<tr>
<td>3G/3Gg</td>
<td>-</td>
<td>-</td>
<td>D_h ²* H_h</td>
<td>D*H</td>
<td></td>
</tr>
<tr>
<td>3H</td>
<td>D_h * H</td>
<td>D*H</td>
<td>-</td>
<td>D*H</td>
<td></td>
</tr>
<tr>
<td>3I</td>
<td>D_h ²* H</td>
<td>D²*H</td>
<td>-</td>
<td>D*H</td>
<td></td>
</tr>
<tr>
<td>3J</td>
<td>-</td>
<td>-</td>
<td>D_h ²* H</td>
<td>D*H</td>
<td></td>
</tr>
<tr>
<td>3K</td>
<td>-</td>
<td>-</td>
<td>D_h * H</td>
<td>D*H</td>
<td></td>
</tr>
</tbody>
</table>

Figure 19. Heartwood prediction improvement steps in the applied order (from top to bottom).
4.2.1. Database enhancement

The first enhancement step was optimising the database with an emphasis on heartwood prediction performance. The initial database used in Chapter 3 consisted of 74 trees and 993 discs from eight study plots in the age range from 7 to 29 years old. The two initial independent variables were dbh and tree height. As a result of adding new independent variables strictly related to heartwood diameter and/or heartwood height, a new database was created where all discs not having heartwood present were excluded. That way, the new models predicted heartwood components based on heartwood information and bark and total wood volumes and tapers were estimated from all available tree information. Due to the use of systemfit for fitting volume equations and the relationship between taper and volume equations based on Demaerschalk’s method (1973), all equations were compatible.

4.2.2. Tree age groups

The dynamics of heartwood development can vary due with age of a tree and so the tree database was divided into three age groups:

a) Model 1A – all trees in full age range (74 trees),
b) Model 1B – trees eight years old and older (64 trees),
c) Model 1C – trees younger than eight years old (10 trees).

4.2.3. Adding a 3rd independent variable – heartwood ratio (HR)

Heartwood ratio (HR) was added as a 3rd independent variable to the heartwood volume equation. To calculate the heartwood ratio, Equation 65 was used:

\[
\text{Equation 65. heartwood ratio} = \frac{\text{heartwood diameter}}{\text{wood inside bark diameter}}
\]

Improved heartwood volume equations were all of the form shown in Equation 66:
CHAPTER 4 – EXTENDING A FULLY COMPATIBLE TAPER AND VOLUME …

Equation 66. \[ \frac{V_h}{dh} = \frac{\lambda_h a_0 + a_1 \left( \frac{dH_R j}{\lambda_h^{a-1}} \right) + a_2 \left( \frac{H_R j}{\lambda_H R_j} \right) + 1}{\lambda_h} \]

where \( V_h \) is the heartwood volume, \( a_0, a_1, a_2 \) are fitted coefficients, \( \lambda_h \) is a heartwood lambda value, and \( HR \) is the heartwood ratio obtained by different measurement techniques \( j \).

Three different measurement techniques were used to obtain heartwood diameter and wood inside bark diameter, which was necessary to calculate the heartwood ratio.

a) Model 2A/2Aa – heartwood ratio was estimated from simulated tree coring (Figure 20) to extract a wood core (Figure 21) from the tree stem using a specially modified drill. To simulate this process, heartwood diameter and wood inside bark diameter were measured from randomly oriented transects from images of discs at 1.4 m tree height (Figure 22).

b) Model 2B/Bb – heartwood ratio was estimated from simulated electric resistance tomography from PiCUS TreeTonic device (Figure 23), which produces electric resistance tomography scan (Figure 24). To simulate this technique, images of the discs at 1.4 m tree height were analyzed in ImageJ software; after manual selection of the area of heartwood and wood inside bark using a “polygon selection” tool, the area was calculated in cm\(^2\) using an “analysis measure” tool (Figure 25). Heartwood and wood inside bark diameters were calculated from the area measured in cm\(^2\) based on Equation 67.

Equation 67. \[ r = \frac{\sqrt{A}}{\pi} \]

where \( r \) is the radius (0.5 diameter) and \( A \) is the area of the component.
Figure 20. An example of a tree coring measurement.

Figure 21. An example of a wood cores.
Figure 22. An example of simulated tree coring measurement; heartwood and wood inside bark diameter measurement from randomly oriented transect from image of disc at 1.4 m tree height.

Figure 23. An example of electric resistance tomography measurement using a PiCUS TreeTonic device.
Figure 24. An example of electric resistance tomography scan.

Figure 25. An example of simulated tree coring measurement; heartwood and wood inside bark diameter measurement from randomly oriented transect from image of disc at 1.4 m tree height.
c) Model 2C/2Cc – heartwood ratio was estimated by measuring the bottom stump of the felled tree at 0 m tree height. After felling the tree, the remaining stump was cut parallel to the ground as close as possible (Figure 26). Two randomly orientated diameter measurements of heartwood and wood inside bark were collected and recorded using a ruler.

![Figure 26](image)

Figure 26. An example of heartwood ratio measurement from the bottom stump at 0m tree height.

### 4.2.4. Adding a 4\textsuperscript{th} independent variable – heartwood diameter

Heartwood diameter ($D_h$) was tried as a 4\textsuperscript{th} independent variable. $D_h$ was tried in different places in the models, and additional variants were considered. The volume equations were all of the form shown in Equation 68 (heartwood), Equation 69 (wood inside bark), and Equation 70 (wood including bark):
CHAPTER 4 – EXTENDING A FULLY COMPATIBLE TAPER AND VOLUME …

\[ \frac{V_h}{B} = \left( h_0 + h_1 \left( A_{A-1} \lambda_A \right) + h_2 \left( \frac{H_{HR_j}^{2HR_{j-1}}}{\lambda_{HR_j}} \right) + 1 \right) \frac{1}{\lambda_h} \]

Equation 68.

\[ \frac{V_w}{B} = \left( w_0 + w_1 \left( A_{A-1} \lambda_A \right) + w_2 \left( H_{HR}^{2H_{HR-1}} \right) + 1 \right) \frac{1}{\lambda_w} \]

Equation 69.

\[ \frac{V_t}{B} = \left( t_0 + t_1 \left( A_{A-1} \lambda_A \right) + 1 \right) \frac{1}{\lambda_t} \]

Equation 70.

where \( h \) is heartwood, \( w \) is wood inside bark, \( t \) is wood including bark, \( H \) is tree height, \( A \) is the independent variable variant, and \( B \) is the weight variant - both presented in Table 9.

4.2.5. Adding a 5\textsuperscript{th} independent variable – heartwood height

Another step was to add measured heartwood height (\( H_h \)) as a 5\textsuperscript{th} independent variable. The process is the same as the one described in section 4.2.4, except for the different variants of \( A \) and \( B \), which are presented in Table 10.

Table 9. Independent variable (A) and model weight (B) and their variants used in different configurations depending on the examined models (case of adding heartwood diameter as the 4\textsuperscript{th} independent variable), where \( D_h \) is heartwood diameter, and \( H \) is tree height.

<table>
<thead>
<tr>
<th>Model</th>
<th>Heartwood ((\text{Equation 68}))</th>
<th>Wood inside bark and wood including bark ((\text{Equation 69 and Equation 70}))</th>
<th>Model weight</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>( A )</td>
<td>( A )</td>
<td>( B )</td>
</tr>
<tr>
<td>3H</td>
<td>( D_h #H )</td>
<td>( D #H )</td>
<td>( D #H )</td>
</tr>
<tr>
<td>3K</td>
<td>( D_h #H )</td>
<td>( D_h #H )</td>
<td>( D #H )</td>
</tr>
<tr>
<td>3J</td>
<td>( D_h #2H )</td>
<td>( D_h #2H )</td>
<td>( D #H )</td>
</tr>
<tr>
<td>3I</td>
<td>( D_h #2H )</td>
<td>( D #2H )</td>
<td>( D #H )</td>
</tr>
</tbody>
</table>

Table 10. Independent variable (A) and model weight (B) and their variants used in different configurations depending on the examined models (case of adding heartwood height as the 5\textsuperscript{th}
independent variable), where $D_h$ is heartwood diameter, $H$ is tree height, and $H_h$ is heartwood height.

<table>
<thead>
<tr>
<th>Model</th>
<th>Heartwood (Equation 68)</th>
<th>Wood inside bark and wood including bark (Equation 69 and Equation 70)</th>
<th>Model weight</th>
</tr>
</thead>
<tbody>
<tr>
<td>3A/3Aa</td>
<td>$D_h \times H_h$</td>
<td>$D \times H$</td>
<td>$D \times H$</td>
</tr>
<tr>
<td>3B/3Bb</td>
<td>$D_h \times H_h$</td>
<td>$D_h \times H_h$</td>
<td>$D \times H$</td>
</tr>
<tr>
<td>3C/3Cc</td>
<td>$D_h \times H_h$</td>
<td>$D \times H$</td>
<td>$D^2 \times H$</td>
</tr>
<tr>
<td>3D/3Dd</td>
<td>$D_h^2 \times H_h$</td>
<td>$D^2 \times H$</td>
<td>$D \times H$</td>
</tr>
<tr>
<td>3E/3Ee</td>
<td>$D_h \times H_h$</td>
<td>$D_h \times H_h$</td>
<td>$D^2 \times H$</td>
</tr>
<tr>
<td>3F/3Ff</td>
<td>$D_h \times H_h$</td>
<td>$D_h \times H_h$</td>
<td>$D_h^2 \times H_h$</td>
</tr>
<tr>
<td>3G/3Gg</td>
<td>$D_h^2 \times H_h$</td>
<td>$D_h^2 \times H_h$</td>
<td>$D \times H$</td>
</tr>
</tbody>
</table>

### 4.2.6. Heartwood height prediction

To predict heartwood height, a linear mixed-effects model was fitted using the lme function from R package nlme (Pinheiro, Bates et al. 2019). No transformation was applied as there was a linear relationship between selected variables. Stand was the random effect in each model. Heartwood height prediction was modelled based on the correlation between heartwood height and:

- tree height for model 1A, 1B,
- tree height and heartwood ratio (obtained from core, scan or tree base) for models 2Aa-Cc, 2A-C,
- tree height, heartwood ratio and heartwood diameter for model 3A-D.
Heartwood height prediction was modelled only for the well-performing models presented in Table 13 and 14. In the case of models 3H, 3K and 3J, the actual heartwood height (from field measurement) was applied.

4.3. RESULTS

Heartwood prediction improvement was divided into five main steps described in section 4.2. Figure 27 presents the examination steps and models with adequate heartwood volume and taper root mean square error (RMSE). Figure 28 shows the best volume, taper, and overall models (with their RMSEs). A statistical examination of all models is presented in Table 11 (for volume) and Table 12 (for taper). Selected well-performing models with their parameters are shown in Table 13 (for volume) and Table 14 (for taper).

As shown in Figure 27, model 1 (coloured blue) used dbh and tree height as independent variables to predict heartwood volume. In the second step, three new models 1x (targeting different tree ages) were created (grey colour). Another step was to add a third independent variable – the heartwood ratio (obtained from three other techniques) and create models 2x (in yellow). After that, the fourth and fifth independent variables were added, creating models 3x (in green). The models with the small RMSE values (both volume and taper) are presented in solid colour, while those with large RMSE values (for volume, taper, or both) are presented in a transparent colour. The best overall models are presented in Figure 28 (with their volume and taper RMSEs):

- 1A (different age groups – all trees; in grey) 0.12 m³/ 2.66 cm,
- 2A (3rd independent variable – heartwood ratio from the core; in yellow) 0.12 m³/ 2.59 cm,
- 3H (4th independent variable – heartwood diameter; in green) 0.07 m³/ 2.14 cm,
- 3B (5th independent variable – heartwood height; in green) 0.07 m³/ 1.96 cm.
Figure 27. All researched models presented in examination order (from top to bottom). The values inside the white rectangles show heartwood volume RMSE (m$^3$) and heartwood taper RMSE (cm). The models promoted and selected for further examination are shown in a solid colour, while those eliminated are shown in a transparent colour.
Figure 28. Summary of the best models with the smallest RMSE for: volume, taper, and overall.
4.3.1. Tree age groups

Dividing the trees into three different age groups: 1A (all trees), 1B (eight years old and older), and 1C (younger than eight years old), showed that the difference between models 1A and 1B was insignificant. RMSE values are almost the same for 1A vs 1B: for volume 0.12 m$^3$ vs 0.13 m$^3$; for taper 2.66 cm vs 2.68 cm. Consequently both models were promoted to the next step. Model 1C’s RMSE values were significantly smaller (volume 0.002 m$^3$, taper 1.55 cm), as the young trees’ heartwood volume and stem diameters were very small compared to the older trees. As a result, comparing RMSE values between the three models was not useful. However, in Figure 29, model 1C presented worse results than models 1A & B. It tended to under-predict higher-end values (notice that the x and y axis’ scale was adjusted to smaller values so that the 1C results could be visible), and leave-one-out cross-validation values coloured blue did not overlap exactly with predicted values. In model 1A, a discrepancy for validation vs predicted values was observed as the values increased. The best fit was visible for 1B, which resulted from excluding younger trees (1C) from all trees (1A).
Figure 29. Residual vs fitted values for models 1A/B/C; heartwood volume (left side), heartwood taper (right side); predicted values (blue colour), leave-one-out cross-validation values (orange colour).
4.3.2. Adding a 3rd independent variable – heartwood ratio (HR)

Models 1A and B were adjusted by adding the heartwood ratio as an independent variable. That led to models 2A/B/C (model 1B + HR), and models 2Aa/Bb/Cc (model 1A + HR).

Testing three different heartwood ratios: 2A/Aa (core 1.4 m), 2B/Bb (scan 1.4 m), and 2C/Cc (stem 0 m), showed that there were small differences between models. RMSE values for 2A/2B/2C were: for volume 0.12/0.13/0.13 m³; and for taper 2.59/2.69/2.69 cm. A similar result was found for models 2Aa/2Bb/2Cc, where RMSE values were: for volume 0.11/0.12/0.12 m³; for taper 2.88/2.97/2.94 cm. The best result (for all age groups) was obtained from the heartwood ratio core simulation technique 2A & Aa.

The taper and volume RMSE range in the 2A-C and 2Aa-Cc did not vary much (2A-C 0.12-0.13 m³/ 2.59-2.69 cm; 2Aa-Cc 0.11-0.12 m³/ 2.88-2.97 cm). However, a greater difference was apparent when comparing the range 2A-C (younger trees excluded) vs 2Aa-2Cc (all tree ages): for volume 0.12-0.13m³ vs 0.11-0.12m³; for taper 2.59-2.69cm vs 2.88-2.94cm.

The fit was relatively unbiased for heartwood volume (Figure 30) and taper models (Figure 31), and predicted values almost entirely overlapped leave-one-out cross-validation values. For heartwood taper models 2Aa/2Bb/2Cc (all tree ages) with an increase of x-axis values, the predicted values tended to be under or over-predicted (Figure 31). As a result, models 2A/2B/2C (younger trees excluded) provided more reliable results for larger diameter trees.
Figure 30. Heartwood volume residual vs fitted values for models 2A/B/C & 2Aa/Bb/Cc; predicted values (blue colour), leave-one-out cross-validation values (orange colour).
Figure 31. Heartwood taper residual vs fitted values for models 2A/B/C & 2Aa/Bb/Cc; predicted values (blue colour), leave-one-out cross-validation values (orange colour).
4.3.3. Adding a 4th independent variable – heartwood diameter

Model 2A was adjusted by adding heartwood diameter. That led to the creation of the following models:

- 3H, where heartwood components used heartwood diameter*tree height (D_h*H) as an independent variable, while the rest of the components used dbh*tree height (D*H),
- 3K, where all components used D_h*H as an independent variable,
- 3J, where heartwood components used heartwood diameter^2*tree height (D_h^2*H) as an independent variable, while the rest of the components used dbh^2*tree height (D^2*H),
- 3I, where all components used D_h^2*H as an independent variable.

Models 3H/3K/3J were relatively unbiased and precise, and their RMSE values were quite similar: for volume 0.07/0.08/0.07 m^3; for taper 2.14/2.12/2.19 cm. Predicted values almost entirely overlapped leave-one-out cross-validation values (Figure 32). For volume models, the predicted values tend to be under-predicted for 3H/3J and over-predicted for 3K with an increase in the x-axis values. Model 3I is not recommended due to a large RMSE value, and biased residual vs predicted values.

4.3.4. Adding a 5th independent variable – heartwood height

Adding a 5th independent variable resulted in 14 different models being tested. A detailed description of each model is presented in Table 10 (independent variable variants) and Figure 27 (volume and taper RMSE). Models 3Aa-Gg (built with an all-ages dataset) had larger RMSEs than models 3A-3G (built with data from eight-year-old and older trees). Although the median value of heartwood volume RMSE was 0.07 m^3 for both groups, the heartwood taper RMSE median value for 3Aa-Gg vs 3A-3G was 5.29 vs 1.99 cm (models were compared using medians due to the vast RMSE of biased models from both groups). Using the model weights
D*H and D^2*H did not change the models’ performances. However, using D_h^2*H_h significantly reduced model performance.

The best results were obtained from models 3A/3B/3D with RMSE: for volume 0.07 m^3; for taper 1.99/1.96/1.97 cm. Predicted values almost entirely overlapped leave-one-out cross-validation values (Figure 33). Heartwood volume models tended to slightly under-predict with increasing x-axis values, while heartwood taper models tended slightly over-predict. Overall all models had homogenous distributions of residuals.
Figure 32. Residual vs fitted values for models 3H/K/J; heartwood volume (left side), heartwood taper (right side); predicted values (blue colour), leave-one-out cross-validation values (orange colour).
Figure 33. Residual vs fitted values for models 3A/B/D; heartwood volume (left side), heartwood taper (right side); predicted values (blue colour), leave-one-out cross-validation values (orange colour).
Table 11. All volume models with their RMSE, MAB and EF values; well-performing models are bolded.

<table>
<thead>
<tr>
<th>Model</th>
<th>Heartwood</th>
<th>Sapwood</th>
<th>Wood inside bark</th>
<th>Bark</th>
<th>Wood including bark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSE</td>
<td>MAB</td>
<td>EF</td>
<td>RMSE</td>
<td>MAB</td>
</tr>
<tr>
<td>1</td>
<td>0.12</td>
<td>0.07</td>
<td>0.91</td>
<td>0.05</td>
<td>0.04</td>
</tr>
<tr>
<td>1A</td>
<td>0.12</td>
<td>0.07</td>
<td>0.90</td>
<td>0.09</td>
<td>0.06</td>
</tr>
<tr>
<td>1B</td>
<td>0.13</td>
<td>0.08</td>
<td>0.89</td>
<td>0.11</td>
<td>0.07</td>
</tr>
<tr>
<td>1C</td>
<td>0.002</td>
<td>0.001</td>
<td>0.10</td>
<td>0.02</td>
<td>0.01</td>
</tr>
<tr>
<td>2A</td>
<td>0.12</td>
<td>0.07</td>
<td>0.91</td>
<td>0.10</td>
<td>0.07</td>
</tr>
<tr>
<td>2Aa</td>
<td>0.11</td>
<td>0.06</td>
<td>0.92</td>
<td>0.09</td>
<td>0.06</td>
</tr>
<tr>
<td>2B</td>
<td>0.13</td>
<td>0.08</td>
<td>0.89</td>
<td>0.10</td>
<td>0.07</td>
</tr>
<tr>
<td>2Bb</td>
<td>0.13</td>
<td>0.08</td>
<td>0.89</td>
<td>0.10</td>
<td>0.07</td>
</tr>
<tr>
<td>2Cc</td>
<td>0.12</td>
<td>0.07</td>
<td>0.91</td>
<td>0.09</td>
<td>0.06</td>
</tr>
<tr>
<td>3A</td>
<td>0.07</td>
<td>0.05</td>
<td>0.97</td>
<td>0.18</td>
<td>0.10</td>
</tr>
<tr>
<td>3Aa</td>
<td>0.07</td>
<td>0.04</td>
<td>0.97</td>
<td>0.16</td>
<td>0.08</td>
</tr>
<tr>
<td>3B</td>
<td>0.07</td>
<td>0.05</td>
<td>0.97</td>
<td>0.08</td>
<td>0.06</td>
</tr>
<tr>
<td>3Bb</td>
<td>0.07</td>
<td>0.04</td>
<td>0.97</td>
<td>0.11</td>
<td>0.07</td>
</tr>
<tr>
<td>3C</td>
<td>0.07</td>
<td>0.05</td>
<td>0.97</td>
<td>0.18</td>
<td>0.10</td>
</tr>
<tr>
<td>3Cc</td>
<td>0.07</td>
<td>0.04</td>
<td>0.97</td>
<td>0.15</td>
<td>0.08</td>
</tr>
<tr>
<td>3D</td>
<td>0.07</td>
<td>0.05</td>
<td>0.97</td>
<td>0.21</td>
<td>0.12</td>
</tr>
<tr>
<td>3Dd</td>
<td>0.06</td>
<td>0.04</td>
<td>0.98</td>
<td>0.18</td>
<td>0.10</td>
</tr>
<tr>
<td>3E</td>
<td>0.07</td>
<td>0.05</td>
<td>0.97</td>
<td>2.06</td>
<td>1.25</td>
</tr>
<tr>
<td>3Ee</td>
<td>0.07</td>
<td>0.05</td>
<td>0.97</td>
<td>0.48</td>
<td>0.38</td>
</tr>
<tr>
<td>3F</td>
<td>0.23</td>
<td>0.17</td>
<td>0.67</td>
<td>0.49</td>
<td>0.35</td>
</tr>
<tr>
<td>3Ff</td>
<td>0.50</td>
<td>0.35</td>
<td>-0.35</td>
<td>1.03</td>
<td>0.81</td>
</tr>
<tr>
<td>3G</td>
<td>1.72</td>
<td>1.13</td>
<td>-17.26</td>
<td>4.59</td>
<td>2.66</td>
</tr>
<tr>
<td>3Gg</td>
<td>0.06</td>
<td>0.04</td>
<td>0.98</td>
<td>0.10</td>
<td>0.07</td>
</tr>
<tr>
<td>3H</td>
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<td>0.05</td>
<td>0.97</td>
<td>0.18</td>
<td>0.10</td>
</tr>
<tr>
<td>3I</td>
<td>0.5</td>
<td>3.00</td>
<td>-158.00</td>
<td>20.00</td>
<td>12.00</td>
</tr>
<tr>
<td>3J</td>
<td>0.08</td>
<td>0.05</td>
<td>0.96</td>
<td>0.21</td>
<td>0.11</td>
</tr>
<tr>
<td>3K</td>
<td>0.07</td>
<td>0.05</td>
<td>0.97</td>
<td>0.10</td>
<td>0.06</td>
</tr>
</tbody>
</table>

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Table 12. All taper models with their RMSE, MAB and EF values; well-performing models are bolded.

<table>
<thead>
<tr>
<th>Model</th>
<th>Heartwood</th>
<th>Wood inside bark</th>
<th>Wood including bark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>RMSE</td>
<td>MAB</td>
<td>EF</td>
</tr>
<tr>
<td>1</td>
<td>2.57</td>
<td>1.82</td>
<td>0.96</td>
</tr>
<tr>
<td>1A</td>
<td>2.66</td>
<td>1.86</td>
<td>0.94</td>
</tr>
<tr>
<td>1B</td>
<td>2.68</td>
<td>1.87</td>
<td>0.94</td>
</tr>
<tr>
<td>1C</td>
<td>1.52</td>
<td>1.26</td>
<td>0.57</td>
</tr>
<tr>
<td>2A</td>
<td>2.59</td>
<td>1.81</td>
<td>0.95</td>
</tr>
<tr>
<td>2Aa</td>
<td>2.88</td>
<td>2.07</td>
<td>0.93</td>
</tr>
<tr>
<td>2B</td>
<td>2.69</td>
<td>1.88</td>
<td>0.94</td>
</tr>
<tr>
<td>2Bb</td>
<td>2.97</td>
<td>2.09</td>
<td>0.92</td>
</tr>
<tr>
<td>2C</td>
<td>2.69</td>
<td>1.88</td>
<td>0.94</td>
</tr>
<tr>
<td>2Cc</td>
<td>2.94</td>
<td>2.07</td>
<td>0.92</td>
</tr>
<tr>
<td>3A</td>
<td>1.99</td>
<td>1.51</td>
<td>0.97</td>
</tr>
<tr>
<td>3Aa</td>
<td>6.40</td>
<td>4.82</td>
<td>0.64</td>
</tr>
<tr>
<td>3B</td>
<td>1.96</td>
<td>1.47</td>
<td>0.97</td>
</tr>
<tr>
<td>3Bb</td>
<td>6.31</td>
<td>4.75</td>
<td>0.65</td>
</tr>
<tr>
<td>3C</td>
<td>1.99</td>
<td>1.51</td>
<td>0.97</td>
</tr>
<tr>
<td>3Cc</td>
<td>5.29</td>
<td>3.99</td>
<td>0.75</td>
</tr>
<tr>
<td>3D</td>
<td>1.97</td>
<td>1.45</td>
<td>0.97</td>
</tr>
<tr>
<td>3Dd</td>
<td>3.57</td>
<td>2.63</td>
<td>0.89</td>
</tr>
<tr>
<td>3E</td>
<td>1.99</td>
<td>1.50</td>
<td>0.97</td>
</tr>
<tr>
<td>3Ee</td>
<td>5.94</td>
<td>5.03</td>
<td>0.75</td>
</tr>
<tr>
<td>3F</td>
<td>4.15</td>
<td>4.26</td>
<td>1.00</td>
</tr>
<tr>
<td>3Ff</td>
<td>7.90</td>
<td>7.05</td>
<td>0.56</td>
</tr>
<tr>
<td>3G</td>
<td>14.86</td>
<td>11.32</td>
<td>-0.80</td>
</tr>
<tr>
<td>3Gg</td>
<td>4.70</td>
<td>3.93</td>
<td>0.85</td>
</tr>
<tr>
<td>3H</td>
<td>2.14</td>
<td>1.56</td>
<td>0.96</td>
</tr>
<tr>
<td>3J</td>
<td>2.19</td>
<td>1.56</td>
<td>0.96</td>
</tr>
<tr>
<td>3K</td>
<td>2.12</td>
<td>1.53</td>
<td>0.96</td>
</tr>
</tbody>
</table>
Table 13. Selected well-performing volume models* with their parameters for *Eucalyptus globoidea.*

<table>
<thead>
<tr>
<th>Model</th>
<th>Heartwood</th>
<th>Wood inside bark</th>
<th>Wood including bark</th>
</tr>
</thead>
<tbody>
<tr>
<td>h0</td>
<td>h1</td>
<td>h2</td>
<td>w0</td>
</tr>
<tr>
<td>1A</td>
<td>-5.24361957</td>
<td>0.21483597</td>
<td>-4.11631374</td>
</tr>
<tr>
<td>1B</td>
<td>-5.24891234</td>
<td>0.21518529</td>
<td>-4.15681590</td>
</tr>
<tr>
<td>2A</td>
<td>-4.08830894</td>
<td>0.17860407</td>
<td>3.35907476</td>
</tr>
<tr>
<td>2Aa</td>
<td>-4.09138316</td>
<td>0.17917087</td>
<td>3.36197004</td>
</tr>
<tr>
<td>2B</td>
<td>-5.07643754</td>
<td>0.20812360</td>
<td>0.14482525</td>
</tr>
<tr>
<td>2Bb</td>
<td>-5.06181000</td>
<td>0.20764000</td>
<td>0.14483000</td>
</tr>
<tr>
<td>2Cc</td>
<td>-5.09271968</td>
<td>0.20804290</td>
<td>0.02078991</td>
</tr>
<tr>
<td>2Cc</td>
<td>-5.09143479</td>
<td>0.20776818</td>
<td>0.02078991</td>
</tr>
<tr>
<td>3A</td>
<td>-2.68935335</td>
<td>0.06333545</td>
<td>2.45765838</td>
</tr>
<tr>
<td>3B</td>
<td>-2.69791652</td>
<td>0.06319889</td>
<td>2.46348500</td>
</tr>
<tr>
<td>3C</td>
<td>-2.68935335</td>
<td>0.06333545</td>
<td>2.45765838</td>
</tr>
<tr>
<td>3D</td>
<td>-2.68938294</td>
<td>0.04049975</td>
<td>2.39568347</td>
</tr>
<tr>
<td>3H</td>
<td>-2.67319440</td>
<td>0.03598767</td>
<td>2.39701496</td>
</tr>
<tr>
<td>3J</td>
<td>-2.72777552</td>
<td>0.03158268</td>
<td>2.40007446</td>
</tr>
<tr>
<td>3K</td>
<td>-2.68954824</td>
<td>0.03622452</td>
<td>2.39881980</td>
</tr>
</tbody>
</table>

*Sapwood and bark model parameters were calculated individually rather than by systemfit, and within systemfit they were assigned as the subtraction of different components to ensure compatibility. For 1A, 2Aa, 2Bb, 2Cc sapwood (s0= -2.555952, s1= 0.067681) and bark (b0= -4.953766, b1= 0.199150). For all remaining models, sapwood (s0= -2.631999, s1= 0.07117) and bark (b0= -4.936205, b1= 0.198392).
Table 14. Selected well-performing taper models with their parameters for *Eucalyptus globoidea*.

<table>
<thead>
<tr>
<th>Model</th>
<th>Heartwood</th>
<th>Wood inside bark</th>
<th>Wood including bark</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>a1</td>
<td>a2</td>
<td>a3</td>
</tr>
<tr>
<td>1A</td>
<td>0.30522</td>
<td>4.68199</td>
<td>-7.74882</td>
</tr>
<tr>
<td>2Aa</td>
<td>-0.43017</td>
<td>7.41260</td>
<td>-10.64490</td>
</tr>
<tr>
<td>2Cc</td>
<td>0.56665</td>
<td>3.52836</td>
<td>-6.32324</td>
</tr>
<tr>
<td>3A</td>
<td>0.17021</td>
<td>0.62614</td>
<td>2.99385</td>
</tr>
<tr>
<td>3B</td>
<td>0.10509</td>
<td>0.62835</td>
<td>3.30978</td>
</tr>
<tr>
<td>3C</td>
<td>0.17021</td>
<td>0.62614</td>
<td>2.99385</td>
</tr>
<tr>
<td>3H</td>
<td>0.18083</td>
<td>0.68871</td>
<td>2.92208</td>
</tr>
<tr>
<td>3K</td>
<td>0.15502</td>
<td>0.66897</td>
<td>3.11509</td>
</tr>
</tbody>
</table>
4.3.5. Heartwood height prediction

Heartwood height prediction equations with corresponding parameters are presented in Table 15. Heartwood height was found to be a linear function of studied variables.

Table 15. Heartwood height prediction equations with corresponding model parameters for *Eucalyptus globoidea*, where a-d are parameters, \( H_h \) is heartwood height prediction, \( H \) is tree height, HR is heartwood ratio, and \( D_h \) is heartwood diameter.

<table>
<thead>
<tr>
<th>Model</th>
<th>Equation</th>
<th>a</th>
<th>b</th>
<th>c</th>
<th>d</th>
</tr>
</thead>
<tbody>
<tr>
<td>1A</td>
<td>( H_h = a \times b \times H )</td>
<td>-6.299399</td>
<td>1.120759</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1B</td>
<td>( H_h = a \times b \times H )</td>
<td>-5.919826</td>
<td>1.108069</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2A</td>
<td>( H_h = a \times b \times H \times c \times HR )</td>
<td>-7.076568</td>
<td>1.073812</td>
<td>2.405761</td>
<td></td>
</tr>
<tr>
<td>2Aa</td>
<td>( H_h = a \times b \times H \times c \times HR )</td>
<td>-7.739758</td>
<td>1.046321</td>
<td>3.995553</td>
<td></td>
</tr>
<tr>
<td>2B</td>
<td>( H_h = a \times b \times H \times c \times HR )</td>
<td>-6.703268</td>
<td>1.110244</td>
<td>0.935943</td>
<td></td>
</tr>
<tr>
<td>2Bb</td>
<td>( H_h = a \times b \times H \times c \times HR )</td>
<td>-6.749137</td>
<td>1.120620</td>
<td>0.629203</td>
<td></td>
</tr>
<tr>
<td>2C</td>
<td>( H_h = a \times b \times H \times c \times HR )</td>
<td>-7.396203</td>
<td>1.110353</td>
<td>1.763300</td>
<td></td>
</tr>
<tr>
<td>2Cc</td>
<td>( H_h = a \times b \times H \times c \times HR )</td>
<td>-7.386339</td>
<td>1.121555</td>
<td>1.396120</td>
<td></td>
</tr>
<tr>
<td>3A-D</td>
<td>( H_h = a \times b \times H \times c \times HR \times d \times D_h )</td>
<td>-7.920699</td>
<td>1.107775</td>
<td>3.631842</td>
<td>-0.038925</td>
</tr>
</tbody>
</table>

4.3.6. Taper and volume interactive tool

The interactive tool from chapter 3 was also adjusted to improve the heartwood volume and taper predictions. As the result there are four different interactive tools (with different variants):

- Models with two independent variables: dbh and tree height, introduced in Chapter 3 (Figure 17),
- Models with three independent variables: dbh, tree height and heartwood ratio (Figure 34),
- Models with four independent variables: dbh, tree height, heartwood ratio and diameter,
- Models with five independent variables: dbh, tree height, heartwood ratio, heartwood diameter and height (Figure 35).
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Figure 34. Screenshot of interactive tool interface designed for the model with three independent variables (dbh, tree height, heartwood ratio).
Figure 35. Screenshot of interactive tool interface designed for the model with five independent variables (dbh, tree height, heartwood ratio, heartwood diameter and height).
4.4. DISCUSSION

The objective of improving heartwood prediction was achieved. At each step of the process, a heartwood model provided a better fit with improved results with smaller RMSEs for volume and taper equations. Volume RMSE improved from 0.12 to 0.06 m$^3$, while taper RMSE improved from 2.66 to 1.96 cm.

Five main independent variables were used in this study: dbh, tree height, heartwood ratio, heartwood diameter, and heartwood height. All those variables are relatively easy to measure except heartwood height which any non-destructive technology cannot measure cheaply. This study involved harvesting 74 trees destructively, and so heartwood height was located by cutting multiple discs from the top part of the tree and examining where heartwood ended. Due to this opportunity, this study examined models using heartwood height as a 5th independent variable are of academic interest only. The best performing model from an academic point of view was 3B with volume/taper RMSE 0.07 m$^3$/1.96 cm. This examination demonstrated what improvement in heartwood prediction could be expected if the heartwood height could be measured more easily in future.

However, for practical use in current forest management, models using up to four independent variables are recommended (dbh, tree height, heartwood ratio, heartwood diameter). If a forest manager decided to obtain a heartwood ratio, heartwood diameter is measured simultaneously. The most precise and unbiased model for practical use was 3H with RMSE of 0.07 m$^3$ and 2.14 cm for volume and taper, respectively.

The quality of heartwood is as important as its quantity. Taking increment cores is a fast technique to obtain heartwood samples without seriously damaging a tree (Estapa et al., 2017; Jones et al., 2008) or decreasing log value. The extractive content in the heartwood is responsible for the natural durability of wood (Bush et al., 2011; Harju & Venäläinen, 2006),
and it can be effectively predicted from cores by near-infrared spectroscopy (Li, 2018; Stackpole et al., 2011). The coring process can improve volume predictions and allow chemical analyses of heartwood quality.

Models developed during this study could be used to estimate product yields during pre-harvest inventory. They can estimate volumes of total wood, wood inside bark and heartwood; they can also predict log types likely to be produced and sizes of trees and logs.

There is a need for a model to show how heartwood develops through time so that the coring of young trees can be used to estimate the heartwood content of older trees. The dataset was separated into young and old tree groups in this study. A taper and volume model would be mainly used for old stands at the time of harvest. Therefore, age groups were tested in this study to provide the most precise model prediction. However, with future research on heartwood development through time and a better understanding of the process, there may be no need to split the dataset into different age groups.

Valverde et al. (2022) focused on individual tree taper and volume equations of *Eucalyptus* varieties (*E. nitens, E. badjensis, E. smithii, E. camaldulensis x globulus, E. globulus*, and *E. nitens x globulus*) under constrained irrigation regimes (Valverde et al., 2022). The authors evaluated four non-linear taper equations: Kozak (2004), Kozak et al. (1969), Ormerod (1973), and Max and Burkhart (1976). They found that genotype, irrigation regime, and their interaction were not statistically significant for all evaluated taper and volume equations. Kozak’s (2004) taper equation presented the best fit and adaptation to irregular boles. All volume equations underestimated volume in trees with a volume greater than 0.22 m³. Reduced bias (from validation of the equations) suggested that the equations could be used to predict taper and volume regardless of *Eucalyptus* genotype x irrigation regime combinations.

Following those results, the *Eucalyptus globoidea* compatible taper and volume equation described in this chapter may be used in New Zealand as a generalized taper and volume
equation and could simplify *Eucalyptus* estimates required for stand management and projection. However, future research in this area is recommended in order to confirm this inference.

Since compatible systems of taper and volume equation were first introduced by Demaerschalk (1971, 1972), several approaches to developing them have been proposed, and the idea of compatibility has evolved with time. The first definition of compatible systems states that compatible taper equations, when integrated, produce an identical estimate of total volume to that given by existing volume equations (Demaerschalk, 1971, 1972, 1973). In Clutter’s study (1980), taper functions were developed from variable-top merchantable volume equations, and compatibility implied that integrating taper equations along the length of any section of the tree bole has to produce the volume of that section obtained from the corresponding variable-top merchantable volume equations. In a recent study (Zhao et al., 2019), the concept of compatibility in a system of taper and volume equations has been enlarged so that taper, total volume, and merchantable volume equations are algebraically compatible with each other. There are two typical ways to develop algebraically compatible taper and volume equations: to develop taper equations first, then integrate the taper equation to get merchantable volume and total volume equations (Kozak, 2004; Li & Weiskittel, 2010); or to derive compatible taper equations from volume equations by solving a differential equation after differentiating that volume equation (Bailey, 1994; Clutter, 1980) or by differentiating the volume equation only (Lynch et al., 2017; Van Deusen et al., 1981).

In the present study of *Eucalyptus globoidea*, compatible taper and volume ensure numeric consistency among component equations in an algebraically compatible system. This was achieved by fitting a volume equation and a taper equation individually. However, the coefficients of the taper equation were produced using volume predictions at an early stage so that integrating the taper equation squared/4 * pi produced a tree volume estimate that was
identical to that obtained from the volume equation. In this study, the novel part was that the
modelling system provided estimates of volumes and shapes of five different stem parts that
were all compatible with each other (heartwood, sapwood, wood inside bark, bark, wood
including bark) and three taper equations (heartwood, wood inside bark, wood including bark)
simultaneously.

Moreover, in this chapter, three new independent variables were introduced (heartwood ratio,
heartwood diameter, and heartwood height), which made the system even more precise and
potentially applicable to specific genotypes or sites where heartwood production may be
enhanced or diminished. The novelty of this *Eucalyptus globoidea* compatible taper and
volume system is built on multiple levels and through its complexity, provides high precision
predictions. This approach could be used for any heartwood durable tree species in future,
providing forest managers with an effective way of predicting heartwood volume without
destructive tree measurement. In future, reliable heartwood measurements could be available
through non-invasive scanning techniques. However, this *Eucalyptus globoidea* compatible
taper and volume system may be the most effective and efficient option for forest mensuration
focusing on the heartwood component.

### 4.5. CONCLUSIONS

Heartwood prediction improvement in compatible taper and volume equations was
accomplished by incorporating additional independent variables into the model. By using the
additional independent variables, heartwood volume and taper RMSEs were improved by up
to: ± 0.02 m³ (with 3rd independent variable – heartwood ratio), ± 0.06 m³/ ± 0.43 cm (with 4th
independent variable – heartwood diameter), ± 0.06 m³/ ± 0.61 cm (with 5th independent
variable – heartwood height). These results demonstrate that for *Eucalyptus globoidea*, adding
heartwood-related independent variables increased the model's prediction accuracy and
broadened its applicability. Future research could focus on different heartwood durable tree species and the possibility of using compatible taper and volume equations with additional independent variables proposed in this study. For now, the author suggests using the heartwood ratio and heartwood diameter. However, heartwood height could be considered if non-destructive technologies allow it to be measured readily in the future.
CHAPTER 5

Conclusion
5.1. Overall synopsis

The findings of this thesis contribute to advancing the understanding of modelling stem properties and creating compatible taper and volume equations for all stem components for *Eucalyptus globoidea* Blakely in New Zealand’s dryland environments.

The research objectives fulfilled in this thesis were to develop equations for *Eucalyptus globoidea*:

1) to estimate the volume of heartwood, sapwood, wood inside bark, bark and wood including bark from two independent variables (dbh and height),

2) to estimate the taper of heartwood, wood inside bark and wood including bark from two independent variables (dbh and height),

3) to create compatibility between taper and volume equations,

4) to improve heartwood taper and volume prediction by using heartwood ratio, heartwood diameter and heartwood height as additional independent variables.

The created taper and volume equations were the basis for *Eucalyptus globoidea* plantation forest mensuration. The initial model predicted:

- individual tree volume from tree diameter (measured at breast height outside bark (dbhob)) and tree height,
- stem shape, which allowed log shapes to be estimated and to get a volume of logs or stems by the integration of $\pi r^2$.

The created taper and volume equations were compatible, which means that the volume estimated from the volume equation was equal to the volume estimated from the taper equation.

This system presented multiple compatibilities of taper and volume within eight equations:

- five compatible volume equations: heartwood, sapwood, bark, wood inside bark (heartwood + sapwood), wood including bark (wood inside bark + bark),
CONCLUSION

- three compatible taper equations: heartwood, wood inside bark, and wood including bark.

To improve heartwood volume and taper prediction, the initial model was expanded. The information about heartwood was obtained from three additional independent variables: heartwood ratio (from core simulation, scan simulation, or tree stump), heartwood diameter and heartwood height. Those predictors allowed expanded models to improve heartwood taper and volume prediction and made them less susceptible to bias from effects of factors such as genotype, site, or spacing in stands.

This study used compatible taper models developed by Demaerschalk (1972) among many possible taper modelling techniques. Different approaches were considered in the initial assessment, such as variable exponent form models developed by Kozak (1988) or dynamic taper models by Muhairwe (1994). However, due to the complex simultaneous fit of five volume equations and corresponding taper equations, including multiple compatibilities, Demaerschalk’s approach was the easiest to implement due to its less complicated structure (compared to other approaches). However, alternative modelling options can be implemented for similar studies and can be considered for future research.

5.2. Overall limitations

NZDFI created the concept of establishing large-scale durable *Eucalyptus* plantations in New Zealand at the beginning of the 2000s. Since then, many trial plots of *Eucalyptus* species have been established. To provide an understanding of modelling stem properties of *Eucalyptus globoidea*, it was necessary to analyse tree size within different age groups and established on various sites. Therefore, data were acquired from plantations that were accessible within New Zealand. These data include data from before and after the NZDFI was established. As a result, the collected data was imbalanced regarding age, site, genetics and spacing within the stand. However, Demaerschalk’s compatible taper and volume approach does not include time and
age factors in the model and is based on the assumption that changes in diameter at breast height reflect changes in upper stem diameters. Therefore, mentioned above factors may influence the prediction results, especially for heartwood volume and taper initial model (with dbhob and tree height as the independent variables), less for models with three and more independent variables. Despite all, the collected data was the best available at the time. Even if not perfect, it can be used as a foundation for future research for durable plantation forest mensuration.

A limitation associated with imbalanced data is the potential effect of site on heartwood percentage within a tree. However, simply because there is a significant correlation between those site and heartwood percentage it does not necessarily mean that one is the cause of the other. There is also a correlation between tree size and site, so the effect of the site in that study should be accepted only tentatively. Heartwood volume percentage within a tree may be influenced by site, plantation age, tree size, genetics, spacing within the stand, or a combination of those variables. To define which factor and to what extent each influences the heartwood volume percentage, a dedicated series of silvicultural experiments would have to be established using clones, and trees would have to be sampled at a range of times after planting to create a carefully balanced dataset.

In the absence of a dedicated silvicultural experiment, this research mitigated the uncertainty mentioned above by adding additional independent variables to inform the models on heartwood development within the tree. This approach helped increase the precision of the predictions by examining heartwood development based on individual tree information rather than external influencing factors.

A final limitation is a technique used to measure heartwood height. Currently, the only way to precisely estimate the heartwood height is to cut the tree and destructively examine the log. The taper and volume equations, which used the heartwood height measured in the procedure
as the independent variable, were created to investigate the potential benefits of adding the measured heartwood height to the model. The result provides academic and scientific value. However, it is not practical to use this approach in large-scale durable plantation forest mensuration unless heartwood height measurement techniques become less destructive in the future.

5.3. Future research

Taper and volume equations are the basis of almost all forest mensuration. In terms of compatible taper and volume equations for *Eucalyptus globoidea*, future directions can improve the development of durable plantation forest mensuration.

Firstly, it is vital to understand *Eucalyptus globoidea* heartwood formation and how its development is influenced by site, plantation age, tree size, genetics, spacing within the stand and environmental effects. A series of long-term silvicultural experiments is recommended to provide more understanding of the effects of these factors on heartwood taper and volume for *Eucalyptus globoidea*.

Secondly, this study provides results at the individual tree level. The next step would be to scale it to the plot level to help in large-scale plantation forest mensuration. This step has already been achieved and is presented as part of the tool “Growth and yield model for *Eucalyptus globoidea* Blakely in New Zealand” by Professor Euan Mason (available here).

This *Eucalyptus globoidea* study could be a foundation for future research of other durable *Eucalyptus* species selected by NZDFI, such as *E. bosistoana, E. argophloia, E. tricarpa* and *E. quadrangulata*. Little is known about heartwood development in these species, so the methods developed in this research could be applied to those species.
Finally, the development of non-invasive heartwood height measurement within the tree will allow the use of heartwood height as an independent variable and improve the heartwood taper and volume predictions.

Despite its limitations and opportunities for future research, the work presented in this thesis meets the research objectives. It fills important gaps in the literature concerning heartwood taper and volume modelling. This system of fully compatible taper and volume equations describing different wood and bark components of tree stems is the first of its kind.
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