Fostering Student Agency in a Maths Classroom: A Qualitative Practitioner Study

A thesis submitted in partial fulfilment of the requirements for the Degree of Doctor of Education (EdD)

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Abstract

This thesis explores and problematises student agency in a New Zealand primary maths classroom. Although the term is widely used, the concept of student agency is less understood and definitions vary. There is an urgency for a common definition of student agency in maths to realise its potential for learners. This research focuses on understanding how student agency can be fostered during collaborative maths challenge lessons in a typical classroom from a teacher-researcher perspective.

Student agency is explored through the experiences of twenty-five students using a qualitative case study design. The main data sources were participant observations, focus group interviews (using stimulated recall) and student agency reflections written by the students. Data were collected during 11 maths lessons, over a 9-month period and analysed using a thematic analysis process.

The students exercised influence and control in the maths lessons by making many choices. The justifications for their decisions included personal learning orientations, maths learning intentions and intentions to fulfil the classroom norms. Thinking and talking about mathematical choices supported the students to become more aware of their decisions and actions in future maths lessons.

My findings highlight the conditions necessary to foster agency in maths. These findings pertain to deliberate teacher strategies. The first is paying attention to learner intentions associated with social relationships and maths choices. The second relates to planning and setting up the mathematical task, the lesson structure and routines, and the establishment of a maths learning community to reinforce student agency. The remaining finding is about teacher responsiveness and in-the-moment encounters with students to further their agency. Within this context, examples include the encouragement of talk for maths sense-making, students’ sense of belonging and the co-construction of classroom norms.

This study contributes to the understanding of the concept of student agency in the maths classroom. It demonstrates possible actions by classroom teachers to create the conditions for student agency and students’ responses through the choice of maths methods, representations and equipment.
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Chapter 1: The Concept of Student Agency

Introduction

As a classroom practitioner, I have been drawn to the concept of student agency. My interest stems from realising that there is more to teaching than teaching subject material. In my role, I devote attention to how students make decisions, work in groups and take responsibility for their actions. I consider the concept of student agency presents potential for both teachers and students.

The New Zealand Curriculum places emphasis on key competencies, one of which is managing self. This is the closest alignment with student agency. My study seeks to answer: In what ways can student agency be fostered in the teaching of maths? in order to support students to take increased agency.

An example of my reflections on how to teach for agency stems from the following critical incident:

It’s another day at school. I take a few seconds to stand back and survey my busy classroom … learners going about their daily tasks, some talking and interacting with others, some intent on individual projects and still others on the periphery. I recognise a group of students working together, using their group skills to get along with others and making decisions to solve maths problems. Some students are very focused and intentional about their actions in this communal learning environment. They appear to be making progress towards their goals and they possess an inner confidence to approach a task from different angles. I wonder … what is it that ensures students’ success despite their individual circumstances?
What drives or motivates students to take full advantage of learning opportunities and the work that I do as a teacher?

Student agency has attracted my attention as a classroom practitioner working in a New Zealand primary school because I see the enormous potential to empower learners. The ability of students to act with agency encompasses every learning area and affects classroom interactions such as group work (Ministry of Education, 2016). Student agency presents many exciting possibilities for myself, my peers and most of all my students.

Sections on student agency are included in The New Zealand Curriculum Online (Ministry of Education, 2016) which offers information, advice and guidance for New Zealand teachers. In addition, a monthly publication entitled the Education Gazette has featured articles on the concept of student agency. One recent example is Maker Culture Meets Learner Agency (Education Gazette Editors, 2020). The concept of student agency has also appeared as a topic of interest for professional support networks (e.g., the Professional Leadership Network of Mathematics). Professional networks offer opportunities for school maths leaders to discuss policy (including agency) and implications for teaching practice within schools. Despite these two examples (feature article and professional network), there is still a lack of an agreed definition of student agency (what this means and looks like in classroom teaching and learning) and limited advice for teachers about how student agency can best happen in the maths classroom.

**Student Agency is a Valued Concept**

Student agency is a valued concept in education, but it appears in many guises. In New Zealand, The Ripple Effect: Student Agency, Well-Being and Learning (a
student agency is associated with improved well-being and enhanced learning (Martin et al., n.d.). The researchers suggested that certain teaching strategies that promote student agency (such as keeping students' interests central, listening to students and co-constructing curriculum design) result in increased engagement which improves well-being. Further to this, learner agency is portrayed as an important concept on the Enabling e-learning website, the Ministry of Education’s online ‘hub’ written to support teachers in using digital technologies (Ministry of Education, n.d.). Here student agency is related to students being aware of their actions and the impact on others, increased ownership and motivation, the development of self-regulation to make decisions and emerging student voice to make contributions.

Student agency is also a topic of interest beyond New Zealand. A study by Fluckiger et al. (2018) investigated the voices of 200 Australian students (aged 3-8) to identify their views on learning. Results suggested that student agency and positive dispositions towards learning reduced once students moved from kindergarten to school. The authors challenged educators to create programmes where students are actively engaged, motivated and able to exercise agency in their learning. Another international study was conducted by Vaughn et al. (2020) (USA). They undertook a systematic review of ‘student agency’ in empirical research within the literacy field (1975-2017). Their work (discussed in chapter 2) provided a synthesis of ideas and understandings of agency and what has gone before. This was in response to challenges associated with understanding student agency and aligning teacher practices.

Working on the assumption that agency benefits maths learning, the problem prompting this investigation is around understanding student agency (and students’
perspectives of agency) to inform the work teachers need to do to foster agency through their maths teaching.

1.1 Student Agency in the Classroom

I begin by considering several definitions of student agency and what research says about agency in the classroom. Student agency is firstly considered in general terms (as the ‘power to act’) and then more specifically, in relation to mathematics.

Student Agency as the ‘Power to Act’

Firstly, “Agency refers to the power or capacity to act and make choices” (21st Century Learning Reference Group, 2014, p.7). When applied to students in an educational context, “Agency is about children exercising influence over their environment. Agency implies active participation, which includes choice” (Martin et al., n.d., p.6). Two matters especially important in these definitions are student participation and choice within their environment. Another definition from a Ministry of Education source, highlighting the words ‘influence’ and ‘control,’ explains that “students have a sense of agency when they feel in control of things that happen around them; when they feel that they can influence events” (Ministry of Education, 2016). Wenmoth (2014) explained in more detail in a NZC Online blog post, reinforcing the ideas of students having power to act and being actively involved:

One way of thinking of learner agency is when learners have the ‘power to act’... When learners move from being passive recipients to being much more active in the learning process, and actively involved in the decisions about the learning, then they have greater agency (What is agency? section, para. 2).

This explanation implies different levels of agency, with students taking lesser or greater agency at different times in their learning and choosing when to act.

Explaining further, Wenmoth argued the importance of student initiative in response
to the transfer of power from the teacher to the student. In addition, writers for The Organization for Economic Co-operation and Development (2019), explained that students acting with agency have the will to influence things around them. Student agency is not a personality trait and does not mean: acting in social isolation or solely in self-interest, nor students choosing whatever they want to learn. However, student agency is something that can be exercised in different contexts and is learnable.

In a mathematical context, Gresalfi et al. (2009) defined agency as the way an individual acts (or refrains from acting) and how their actions contribute to the joint actions of the group. The authors described students developing patterns of agency and expectations when they interact with others. A person does not ‘have’ or ‘lack’ agency, but rather the ways that agency can be exercised change in different contexts. In a similar way, Sengupta-Irving (2016) pointed out that “agency is available to everyone” (p. 211). These definitions suggest that all individuals have the potential to act with agency in the mathematical classroom.

**Conditions for Student Agency**

An implication of student agency is that some situations are more likely to encourage students to exercise agency than others. It is the role of the teacher to create these conditions. Wenmoth argued, “agency isn’t simply about handing control over to the learner” (Wenmoth, 2014, paras. 2,4). However, it involves “a far greater tapestry” of deliberate actions by schools and teachers to create environments and contexts where students “are actively involved in the moment-by-moment learning”. The term ‘tapestry’ implies complexity and depth in design. An understanding of these
conditions is necessary for a teacher to design the learning environment to promote student agency.

The concept of student agency in the classroom has been discussed in general terms. How does student agency relate more specifically to the learning area of maths?

### 1.2 Student Agency in Maths Learning

Different types of agency have been identified in maths. Gresalfi et al. (2009) (in USA) drew on Pickering’s (1995) idea of mathematicians using *conceptual agency* and *disciplinary agency*. *Conceptual agency* involves an individual making decisions, exploring and strategising, for example, when they explore and choose representations. In this way, students are involved in taking initiative to construct meaning and understanding. *Disciplinary agency* is used when decisions are made to use established methods (following prescribed steps), for example, memorising facts and executing procedures. Their findings in a middle school classroom suggested that different maths tasks and structures of participation, present different opportunities for student agency.

Boaler (2003) described high school students (in the USA) acting with agency as being more active participants in the classroom community during engagement with maths problems. Also drawing on Pickering’s (1995) work, she described students participating in a ‘dance of agency’ moving between “the standard methods and procedures they know and the new situations to which they would apply them” (p11). Students in her study had opportunities to talk about their own ideas, as well as talk about, adapt and extend established methods. Students were encouraged to *represent* problems using pictures and symbols, and to justify their solutions. This
contrasts with approaches involving students taking on more passive roles (described as demonstration and practice). In this way, there was an interchange between known mathematical procedures and students’ own ideas.

Key researchers are in agreement that student agency has positive effects for learning maths. Boaler and Greeno (2000) suggested when students act with agency, they are not just learning maths, but learning the identity of being mathematicians. By participating and negotiating ideas with others in a maths community, students are more able to use their maths knowledge in new and different situations. Their participation affects their sense of belonging, their beliefs, and their identification as maths learners. And finally, research studies on student agency suggest a lack of agency (as found in more traditional textbook approaches) can have negative consequences for achievement (Boaler, 1997, 1998; Boaler & Greeno, 2000). Students in conditions lacking agency were less able to apply their mathematical knowledge to new and unfamiliar situations.

Student agency is a concept that can be researched in maths. For example, Sengupta-Irving’s (2016) study in the USA focused on organising for agency and provides direction for this investigation. Her study explored the designing and implementation of an algebra intervention (during a summer school) to promote student agency. Here agency was simply interpreted as actively “doing things”, and the pedagogy and curriculum design aimed at reflecting mathematicians-in-action, rather than passive learning. She drew on Pickering’s (1995) ideas of the dance of agency, where students were using their own ideas or extending established ones, as well as the standard procedures of mathematical proof. Although this study involved a 5 week summer intervention for middle school students, Sengupta-Irving’s experimental design is important because it presents an example of a study focused
Deliberate aspects of the study design included student collaboration, the task selection and the establishment of class norms which contributed to the positive impacts for the students. Study findings suggested that teachers can deliberately organise for agency to benefit learning. While some dilemmas of classroom design are addressed in Sengupta-Irving’s (2016) study, the context differs from a typical New Zealand primary school classroom. The design of her study involved a team of teachers, rather than a single teaching situation, and was not limited to the curriculum constraints imposed when operating within a school environment e.g., resourcing and space. Sengupta-Irving’s study has spurred me on to investigate in my own classroom over a longer period of time.

**Other Concepts Related to Agency**

Other concepts are associated with student agency such as autonomy and self-regulation. Writers for the OECD (2019) explained that student agency is more than the concept of ‘student autonomy’ which can be mistakenly used as a synonym. Autonomy is associated with both agency and positive attitudes in maths, such as when students take charge of their own learning, use questions, make connections and seek solutions (Liu et al., 2016). In this way, students take a more active role and act with independence. Yackel and Cobb (1996) used the term ‘autonomy’ to describe students making mathematical choices and judgements during participation in their classroom community, being aware of and drawing on “their own intellectual capabilities” (p. 478). Students’ autonomy can be fostered by teachers and is in contrast to individuals relying on direction to know how to act.

Another concept associated with agency is self-regulation. Self-regulation describes a student taking active control during problem-solving using skills including
“planning, monitoring, control and reflection” (Mevarech & Kramarski, 2014 p. 37). They recommended that teachers should incorporate metacognitive competencies, including self-regulation, into their teaching (discussed further in 2.5).

In summary, opportunities for purposeful student choice are a common theme of studies related to agency. Examples of ‘taking agency’ in the maths classroom include students choosing from multiple mathematical methods, talking about methods used with others, exploring and trying out their own ideas, using a variety of representations (rather than prescribed ones) and justifying solutions to others. The use of appropriate tasks and other aspects of lesson structure and organisation have enabled students to make and justify mathematical choices to other learners when they participate. Realising the potential of increased student agency, how can the opportunities for agency be fostered in maths?

The Definition of Agency for This Study

For the purpose of this study, student agency in the maths classroom means active participation in choosing methods, materials and representations to solve maths challenges. In my study I use the term ‘maths challenge’ to refer to a particular type of maths task suitable for group problem solving, that is, can be solved in different ways using a variety of methods and representations. A teacher needs to make many decisions about how and what they present as maths challenges to foster student agency. By investigating student agency in a typical New Zealand primary school, I aim to better understand the nature of student agency in maths teaching and learning, and how I can foster this. I will also need to understand how the context of maths learning contributes to opportunities for student agency.
1.3 The Context of New Zealand Maths Learning

In this section, I locate this study within a New Zealand school maths learning context, which involves *The New Zealand Curriculum*, along with the school and class context.

**The New Zealand Curriculum**

In New Zealand, curriculum design is guided by *The New Zealand Curriculum (NZC)* (Ministry of Education, 2007), *Tātaiako* (Education Council New Zealand, 2011) and *Tapasā* (Ministry of Education, 2018). The NZC identifies five key competencies considered important for participating in society: thinking, using language symbols and texts, managing self, relating to others, and finally, participating and contributing (Ministry of Education, 2007 p. 12), in which agency is implicitly embedded (Ministry of Education, 2016). These competencies underpin learning and are developed in social contexts, aligning with maths social-constructivist learning theories (Taylor & Bailey, 2011). Maths is about sense-making with students involved in using symbols, graphs, relationships and models of both real-life and hypothetical situations (Ministry of Education, 2007 p. 28). It is a subject requiring students to think creatively and logically as learners are involved in predicting outcomes, seeking patterns, and justifying and communicating ideas to others.

**The School and Class Setting**

My study is located in a state contributing primary school (catering for students from Years 1-6) within a city environment, with a roll of under 300. The class involved in the study is a mixed Year 5 and 6 class of 29 students, ranging from 9-11 years of age. All subjects including reading, writing, maths, social science and science are taught in the same class. The class works at a range of curriculum levels within the
NZ Curriculum. As I am both the researcher and teacher of the class, I now consider the challenge and opportunity that this position presents.

1.4 Researcher Positionality

I begin by considering my position as the researcher, and the beliefs and assumptions which guide my research actions (Creswell, 2007; Mutch, 2013). For this study, I take on the dual roles of teacher and researcher. The research context is my classroom setting, and specifically, the daily maths lessons that make up the maths programme for my class. My perspective is that of a middle-class European New Zealander, with a professional history as a full-time teacher, working within New Zealand Curriculum policy. I have taught in a variety of New Zealand primary schools and at the time of this study, have been at my present school for 8 years. These experiences influence my decisions about how best to organise my maths classroom to maximise learning. An example of an assumption I hold is what successful learning looks like; I explicitly address assumptions within my investigation and am open to having them challenged.

As both the researcher and teacher (classroom practitioner), I am curious about whether agency can be shown by students’ actions, but also as part of a cognitive process (e.g., thinking) or a social process (e.g., talking and negotiating)? I wonder if there is a transactional aspect to agency. Can a teacher give agency (the power to act) or are students taking agency (using initiative and taking the opportunity to act) and we see evidence of their actions? From my perspective as a teacher attempting to foster student agency in maths, further questions come to mind. What practices can I implement to support student agency? What is my role and how do I scaffold students? I am also curious about what the learning experience is like for my
students. What do my students think about agency and what decisions and choices are they making?

I offer the perspective of a New Zealand teacher negotiating the politics and practice of what is currently topical, focusing on the concept of student agency. With the acknowledgement of my research position, I present the questions guiding my investigation.

1.5 Research Questions

My investigation focus is student agency. I aim to capture students’ experiences of agency in the context of solving maths challenges to support teaching practices. My main research question is:

- In what ways can student agency be fostered in the teaching of maths?

This question is designed to understand how a teacher can organise for agency, and support students to take agency to benefit their maths learning. Specifically, my interest is to explore how students respond to opportunities for choice when solving mathematical challenges set by the teacher, the actions associated with that choice and how students justify their actions. As a result, my study is guided by the following sub-questions:

- How do students respond (do and say) when there are opportunities for choice?
- How do students explain and justify their actions in choosing methods and equipment?
- What teaching strategies (or teacher actions) promote students to take agency in maths?
• What are the implications for practice and beyond the immediate context of my particular maths classroom?

Having established a gap in the research about student agency, my investigation is guided by this main question and sub-questions. I now outline the structure of this thesis.

1.6 Thesis Overview

My thesis is presented in eight chapters. The next chapter reviews research and scholarly literature. I focus on research involving student agency within the maths classroom and begin with a discussion of the literature exploring the dimensions of agency. A selected review of research studies reveals opportunities for organising for agency. These include aspects of the classroom such as curriculum design, the role of the teacher, student self-efficacy and theories of learning maths. These theories underpin the maths programme design, as well as the classroom environment.

Chapter 3 explains the theoretical framework that underpins my study and the features of its design. Here, I justify my choice of an interpretivist approach and qualitative case study design. I consider the ethical considerations that are relevant to my investigation with children and explain how I address each of these. The process of gaining ethical approval and informed consent from the participants is described. In a later section, I explain the methods of data collection, their strengths and limitations, and why they are relevant and useful for my study. I address the trustworthiness of my design and choices I have made to increase the reliability and usefulness of my research.
Chapter 4 continues with further explanation of the context of my study, including the maths programme and the maths challenges chosen for the data gathering lessons. Processes involved in the setting up of the learning environment are explained including the deliberate work of the teacher to establish norms that will enable student learning in maths. The initial analysis of the data collected is described, along with the development of identified themes and categories.

Chapter 5 reports the data analysis in more depth, following the analysis of Theme 1, "Social and Personal Ways of Working". This involves the interpretation and discussion of the data collected from the range of sources including student reflections, interviews and observations. Lastly, patterns of agency observed throughout a lesson are discussed.

Chapter 6 follows with the analysis of the data following the mathematical theme. This includes an exploration of student views on choice of methods, representations and use of resources and equipment. Further analysis of data follows groups of students and the changes for these students over time. The chapter ends with a discussion of my study’s methodological reflections.

Chapter 7 presents the twelve findings of my study through three frameworks. These findings are compared with the work of other researchers in the maths field to identify similarities and differences and critically reflect on the student agency observed.

Finally, Chapter 8 presents a summary of my findings in response to my research questions and the conclusions of this investigation. The implications of my study are considered, as well as, suggestions for future work in this area.
Chapter 2: Strategies for Enabling Student Agency in Maths Classrooms - Key Research Themes

Introduction

This literature review examines research and scholarly writing about student agency in the primary classroom and its application within maths teaching. Student agency is explored as a general concept and then related to teaching and learning in the maths classroom. A school environment is a complex place with many variables affecting student outcomes. While aspects of this environment are considered separately, they are then combined to highlight the possibilities and challenges for the deliberate teaching of student agency through the lens of one teaching area, maths. This literature chapter is organised into eight sections. Section one discusses the concept of agency and what this means for maths classrooms. Section two describes research informing pedagogy in maths lessons, including curriculum design, organisational structures and the class community. Section three discusses studies informing the role of the teacher. In section four, literature focused on the expectations of learners is reviewed including studies on classroom norms. Section five includes research related to student math methods and representations, and section six presents literature concerning student efficacy and affective factors. Section seven discusses literature explaining the theoretical lens informing this study, followed by the summary in section eight.

2.1 Exploring the Concept of Student Agency

In this section, I examine literature to identify features of agency in general and then more specifically student agency in the context of learning maths. While student
agency is the term favoured in my study, the literature uses a variety of terms including personal agency and human agency.

Human agency allows individuals to shape their life circumstances, instead of just being products of them (Bandura, 2006). Bandura described human agency as having four core properties that are relevant to my study: intentionality, forethought, self-reactiveness and self-reflectiveness. According to Bandura (2001 p. 6), “an intention is a representation of a future course of action to be performed.”

Intentionality involves people forming intentions and deliberate plans of action to achieve future outcomes. Forethought means anticipating the likely outcomes and involves visualising goals which in turn provides direction and meaning, promoting purposeful behaviour. The third core property is self-reactiveness, which means giving shape to an appropriate course of action and involves regulating the plan being carried out. Lastly, self-reflectiveness is when individuals reflect on their thoughts and actions to examine their performance. Being self-reflective allows individuals to make adjustments as necessary and adapt to different circumstances.

Summarising literature on agency, Paris and Lung (2008) highlighted Bandura’s core properties, adding that people with agency demonstrate certain qualities including recognising possibilities, being willing to act, taking initiative to originate actions and being persistent. Another writer acknowledging Bandura’s ideas defined agency as, the quality of “self-reflective and intentional action and interaction with their environment” (Klemencic, 2015 p. 11). This definition encompasses the notions of agentic possibility and orientation (will) which are temporary, shaped by consideration of the past, present alternatives and future projections. These ideas lead to agency in the classroom with students.
Vaughn et al. (2020) identified dimensions of student agency in the classroom with many similarities to Bandura’s work on human agency. Their study (focused on literacy) was based on 182 articles, reduced to 51 after their search criteria was applied. The concept analysis of the literature identified five dimensions of student agency: **self-perception, intentionality, choice-making, persistence** and **interactiveness**. Self-perception “expresses how students perceive their roles” (p. 727) and involves learner attitudes, individuals seeing themselves as capable of carrying out actions. In a similar way, Bandura included people’s beliefs in their capability (using the term efficacy) as important within the core property of self-reflectiveness. Like Bandura, Vaughn et al. claimed that individuals with agency have **intentionality**, their beliefs and desires driving their actions. Agency requires students to **make choices** and decisions which influence things around them.

Choices and regulating the execution of action plans are a part of Bandura’s property of self-reactiveness. The dimension of **persistence** is necessary to overcome difficulties when faced with challenges, and this relates to Bandura’s notion of self-reflectiveness, that is, individuals determining what challenges to undertake and how long to persevere. Finally, the dimension labelled ‘**interactiveness**’ highlights how individuals exert influence within social contexts. Bandura acknowledged people operating in social systems in more depth. He described three modes of agency: personal, proxy (relying on others to act) and collective agency exercised through interdependent effort. Vaughn et al.’s **interactiveness** dimension is more in line with collective agency. The authors connected agency to learning spaces where teachers build on students’ ideas, needs, interests, and strengths. Their analysis also identified barriers for teachers to foster agency such as following prescribed literacy programmes, limiting students’ self-monitoring, as well as students’ limited content
knowledge. This study, although focussing on literacy, is one of few research syntheses related to student agency. It presents common understandings about student agency that can be applied to the maths classroom.

Structures and supports can foster student agency within classrooms. Drawing on her own and scholarly research, Vaughn (2020) presented a conceptual model of agency in practice suggesting teachers focus on three areas of the classroom including the disposition of students, their motivation and their position within social contexts. The first area, learner dispositions, involves students’ understanding of themselves, having intentions and purpose to act and transform environments. Using vignettes from empirical research (from a variety of classrooms), this area was depicted by students who were willing and eager, focused on acting and pursuing their ideas. Motivation, the second area explores how students make decisions and choices, particularly how they overcome difficulties when presented with problems. The remaining area is interaction and negotiation within the social context of the classroom. Agency is co-created and dynamic. Individuals make choices to act and exert their influence, therefore, learning environments can be structured to support agency by increasing the opportunities for students to exert influence.

The intentional and purposeful nature of agency (in Bandura, 2006; Vaughn, 2020; Vaughn et al., 2020) has been a similar focus for Leadbeater (2017) in England. He claimed it is not enough for students to be active rather than passive learners, terms that are commonly used within maths literature. Activity with no purpose that is ill-structured and chaotic is not good for learning or developing a sense of agency. Agency is not the same as child-centred programmes, where students choose what they want to do, but is about students making choices and putting in effort towards pursuing their goals. He explains that at times this may involve uncertainty and
taking risks, or students being unsure of the outcomes. This distinction concerning active and passive learning is important and aligns with the ‘intention’ and ‘purpose’ associated with agency.

Student voice is sometimes linked with student agency. Cook-Sather (2020) in USA argued that student voice can influence educational practices. From empirical studies and her previous work (Cook-Sather, 2006), she identified approaches to student voice that in turn foster student agency. These include changes in pedagogical practice such as students understanding how they learn so they can determine learning pathways alongside the teacher. Another teacher action includes listening to students’ interpretations of what and how they learn to inform future teaching approaches. In her view, the aim is not to replace teacher authority and responsibilities but to maximise and democratise education for those involved to support engagement and value contributions (Cook-Sather, 2020). Therefore, fostering student agency includes creating opportunities for student voice so that students’ ideas and perspectives inform classroom practices.

Co-construction is a dimension of agency identified by several other studies (Brown, 2020; Vaughn, 2020). The term co-construction involves collaboration, something being built together involving back-and-forth reiterating and redefining between parties (Mueller, 2009). Vaughn (2020) suggested that “agency is not solely developed by an individual, but is co-created with other individuals such as peers and teachers and across a variety of social interactions and contexts” (p. 112). Co-construction involves interactions and negotiations, individuals deciding when or how to act and exerting their influence. As agency is socially constructed, it is constantly changing and being continually reshaped through social interactions. Similarly, Brown (2020) proposed a model of shared agency between the students and teacher
in the maths classroom. He claimed that it is through “teacher-student negotiation of coming to know and do mathematics that the development of student agency can be actualised” (p. 139). In this way, student agency involves back-and-forth negotiations between the teacher and student shaping what actions are taken, and by whom. Brown’s model stemmed from his earlier work in the mathematics discourse field (e.g., Brown, 2009; Brown & Renshaw, 2006). His 2009 study mapped the development of student agency in a Year 7 Australian classroom while students participated in a community of practice. He analysed journal descriptions from two students in response to teacher posed open-ended questions such as “Why did you do the maths that way?” He found that student agency did not happen by chance, but by collective practices scaffolded over time. His findings support the development of student agency when students co-construct their mathematical solutions and are supported to produce explanations and arguments to convince others within a collaborative environment. However, as a result of the nature of co-construction, tensions exist between the people involved, as well as the discipline. Brown (2020) recognised tensions between the agency of teachers, learners, and the discipline, such as the conventions and ways of knowing and doing maths. Building agreement, that brings about deep learning, should be explicitly recognised and students given opportunities to re-present group solutions to the class for verification. Therefore, fostering student agency involves promoting interactions and negotiations between students and the teacher. Having identified important dimensions associated with student agency, I now focus on literature and research about recognising student agency in classroom contexts.
Recognising Student Agency in Action

Student agency in classrooms can be identified by observing students’ participation and actions. Vaughn’s (2020) literacy study described examples of agency such as students’ questions and decisions driving discussions, students interacting with others to share ideas and students making choices in how to present their ideas. The students had a say in lesson topics, but also a sense of purpose and used their intentions to solve problems and inform their world. Students being purposeful and intentional involve more implicit expressions of agency.

In maths, agency in problem-solving involves critical engagement such as “choosing particular tools and interrogating their impact in attaining desired ends” (Gresalfi et al., 2012 p. 249). This is opposed to procedural engagement (using procedures accurately) and conceptual engagement (understanding why an equation works). In an Australian study, Durksen et al. used the term agentic engagement to describe observed student responses during maths lessons (Durksen et al., 2017). They linked student agency in teacher-student interactions with positive student motivational engagement, where motivation means a student’s willingness to act and engagement means a student’s involvement. Durksen et al. used a multi-case process with participants from six upper primary/secondary maths classes with higher-than-average motivation and engagement. Their investigation focused on teacher-student interactions and how teachers maintained high levels of motivation and engagement. Data gathering methods included one video recorded lesson and pre- and post-lesson teacher interviews (semi-structured). They describe observed examples of agentic engagement presented through three classifications: 

*behavioural*, *cognitive* and *emotional*. Behavioural examples (represented more often) included students volunteering answers or responses and choosing to sit
close to the board. *Cognitive* examples included students being comfortable to say they didn’t understand, students asking themselves questions and watching the board. *Emotional* examples of agentic engagement included observed student looks (puzzled expressions) and students saying they were enjoying the activity. Table 1 (next page) highlights what students do and say that might be considered indicators of student agency.

**Table 1**

*Possible Indicators of Student Agency*

<table>
<thead>
<tr>
<th>Behavioural examples</th>
<th>Cognitive examples</th>
<th>Emotional examples</th>
</tr>
</thead>
<tbody>
<tr>
<td>Asking questions</td>
<td>Watching the board/ others explanations</td>
<td>Puzzled looks</td>
</tr>
<tr>
<td>Volunteering e.g., hand up</td>
<td>Asking themselves questions</td>
<td>Saying they enjoyed participating in an activity</td>
</tr>
<tr>
<td>Proximity to the teacher/centre of discussion</td>
<td>Admitting they don’t understand</td>
<td></td>
</tr>
</tbody>
</table>


Agency is more commonly understood as activity choice, however, students can demonstrate and express agency in solving maths challenges in a variety of ways such as through thought processes and feelings e.g. forethought and self-perception (Bandura, 2006; Vaughn et al., 2020).

Davis (2019) provided further classroom examples of observed behaviours associated with student agency. Writing from a professional development perspective within a New Zealand context, she explored aspects of agentic practice using principles of intrinsic motivation (see section 2.6 for more detail). Student agency can be observed through students’ attitudes including their enthusiasm towards learning tasks and positive feelings towards the class climate. Students
show self-efficacy when they believe they are capable of successfully carrying out a
task and reaching a solution. Another behaviour associated with student agency is
when students view themselves (self-concept) as making progress. In Davis’s view,
student agency in classrooms is connected to intrinsic motivation.

The Mathematical Classroom Presents Opportunities for Agency

The context of the mathematical classroom is important for agency. Relating agency
to the field of maths education, Brown (2020) considered agency as a concept that is
central to maths learning because opportunities for agency are embedded in the
activities. The opportunities presented for students to actively participate in their
learning affect the way agency is enacted. Pedagogies described in his study to
improve active participation and agency in maths contexts include incorporating
students’ interests, using group work and being sensitive to developing learner
identities. Identity means how students think about themselves in relation to maths,
such as its value and being committed to maths (Cobb et al., 2009). Brown
highlighted the importance of classroom organisation to guide student actions. This
has clear connections with Vaughn’s (2020) work and the creation of contexts,
including dialogue and materials, for students to exercise agency.

The circumstances in which students learn maths also define the maths knowledge
produced. Boaler and Greeno (2000) referred to different ways of knowing maths,
depending on different learning theories and environments. They considered maths
learning as a result of participation in social contexts, where students try to make
sense of problems, are involved in communication and use representations and
methods as resources, in contrast to an emphasis on mathematical procedures.
Boaler and Greeno analysed interviews with 48 high school students (USA) to
understand students’ perceptions of the maths class environment and their beliefs about maths learning and participation. They found that the students involved in the discussion-based programmes were more able to use maths in new and different situations compared with students in more traditional textbook programmes. Boaler and Greeno favour shifting authority and agency from teachers to students, claiming this enhances student learning in maths (Boaler & Greeno, 2000).

To summarise, student agency involves:

- dimensions including student choice, self-perception, intentions, persistence and interactions;
- opportunities embedded in classroom activities;
- decisions and actions that are socially constructed through interactions and negotiations;
- deliberate teaching practices to support students over time; and above all
- empowerment of students as learners.

The following section explores studies identifying deliberate teaching practices required for students to develop agency in maths.

2.2 Deliberate Decisions and Actions to Foster Agency in Maths

How the learning context is organised and managed affects opportunities for students to take agency. A number of international research projects highlight conditions within the maths classroom necessary for learning, providing opportunities for student choice. These conditions suggest a variety of interwoven factors including; curriculum design and task selection, organisational structures and routines within maths lessons, and the development of a class community. Each factor will now be addressed in turn.
Curriculum Design to Enable Student Agency

A curriculum that is adaptable and allows for flexibility can support agency. In Vaughn’s (2020) view, agentic learning experiences are co-constructed and generative, moving beyond a transactional approach with prescriptive programmes emphasising a single approach. The unpredictability involved in co-constructing lessons with students can however, create difficulties for teachers working within school and legislative constraints e.g., assessment, timetabling, limited resources and school planning expectations. To foster student agency and enable flexibility in curriculum design, teachers can focus on the ‘big’ overarching ideas of maths to support coherence and connections within the curriculum (Tout & Spithill, 2015). Findings from international studies suggest content areas such as shape, change and quantity (as big ideas) can guide the design of learning opportunities (Barclay & Barnes, 2013; Boaler et al., 2017; Tout & Spithill, 2015). Flexibility in curriculum design and the co-construction of learning experiences with students can support agency. My next task is to review research on task selection to foster agency for classroom programmes.

Deliberate Teaching Actions: Selection of the Maths Task

The selection of the maths task is critical for shaping opportunities for students to exercise agency (Gresalfi et al., 2009). Key task selection criteria to support agency in maths are outlined in a range of studies. When focussing on organising for agency, Sengupta-Irving (2016) selected tasks with high cognitive demand, allowing collaboration and opportunities for visual or symbolic representations. Sengupta-Irving attributed the increased student agency to this conscious task selection from the outset. Other literature suggests tasks should be selected by teachers with a
clear idea of the learning goals and key ideas, with the mathematical purpose also being understood by the students (Leinwand et al., 2014; Sullivan et al., 2009). Other studies have focussed on the open-endedness of tasks (having high cognitive demand), providing opportunities for multiple starting and end points, and allowing different strategies (Anthony & Walshaw, 2007; Leinwand et al., 2014; Lowe, 2011; Sengupta-Irving, 2016). Further considerations for task selection include the need for meaningful contexts (Anthony & Walshaw, 2007), as well as, ‘group worthiness’ allowing shared responsibility (Boaler, 2006), ease of access by all students, possibilities for different solution pathways and extension to higher levels (Boaler, 2016; Schoenfeld & Teaching for Robust Understanding Project, 2016).

Task selection alone is not enough to ensure students show agency. Gresalfi et al. (2012) (USA) argued that the task alone does not ensure deep student engagement. Rather it is the teacher practices in supporting student engagement and actions, such as the organisation, presentation and the questions asked. Gresalfi et al. (2012) noted the usefulness of Gibson’s (1979) theory of perceptual affordances to explain what contributes to students’ developing sense of agency. This is defined by the environment, students’ intentions and their ability to realise the opportunity. It is the understanding of the conditions to ensure all learners are engaged in the task that has significance for my study on fostering agency.

Several studies suggest pedagogical teacher actions can maximise learning opportunities and ensure student engagement in tasks. One study is from Sullivan et al. (2006) in Australia. They explored actions to address barriers for learners in heterogeneous classrooms by analysing observations of teacher actions and students’ work (aged 11-12) during 10 lessons. Their findings suggest the potential of enabling prompts, as well as prompts to extend student learning. In New Zealand,
Hunter’s case study (Ministry of Education, 2010), is a further classroom study involving a different age group (two Year 4-6 classes). Learnings from her study include the need for teachers to allow time for task completion, as well as provide scaffolds and learning supports (e.g., peers and visuals). With these examples of deliberate teacher actions to develop student agency, the next section continues with a discussion of studies highlighting lesson organisational structures.

**Organisational Structures and Routines**

The organisation of the classroom and routines affect how the lesson is experienced by students. Durksen et al.’s (2017) study suggests classroom organisation is effective when there is some predictability, structure and simplicity. One focus for studies promoting student agency is distributed authority. Gresalfi et al. (2009) explored the distribution of authority (whether the teacher or students make decisions) in a class and its link to student agency. Examples of distributed authority from their study include decisions involving the completion of a task and the pace of learning. These decisions include choice of problem-solving methods (discussed in a later section) and having joint responsibility to determine the accuracy of others’ contributions. Here, lesson structures and routines are viewed as tools to guide student interactions, decisions and actions as the learners are involved in making choices. Their findings question whether students are given opportunities to be involved in the decision-making and the priorities in class.

Structures and routines for group problem-solving represent a second focus for research on student agency in mathematical activity. Sullivan et al. (2016) described the benefits of a clearly articulated lesson structure for lessons involving maths problem-solving tasks. Their project involved students (8-10 years) in an Australian
maths learning context, with online surveys for teachers and students. They identified three deliberate components for lesson organisation including an introduction to present the task, group problem-solving, followed by the lesson conclusion. Sullivan et al. found this lesson organisational structure enabled student group collaborations, participation in discussions and the sharing of ideas during the lesson conclusion. Whilst endorsing this lesson organisational structure, Jackson et al. (2012) contended several other elements should be considered. These relate to how the task is introduced, naming the key maths ideas and describing the context of the problem, all the while not suggesting procedures. Sengupta-Irving’s (2016) study highlighted a further organisational strategy, the use of activity stations. Here the classroom was organised into activity stations, with students making decisions on the time spent at a particular station.

Each of these aforementioned studies has identified strategies teachers have used to develop student agency. In the next section, I turn to the social aspects of the classroom including literature focused on collaboration, relationships and classroom discourse.

**The Classroom Community Encouraging Collaboration**

Consistent ideas in the literature associated with agency are the important role of classroom interactions (Vaughn et al., 2020), and the development of a learning community (Ministry of Education, 2010). This section discusses studies related to building a maths learning community, the role of collaboration and its relationship to facilitating student agency.

Three terms warrant definitions: *community, ecology* and *collaboration*. Firstly, a classroom maths *community* can be thought of as “nested within an evolving
systems network. This system might be described as an ecology in which the activities of the teacher and students ... are mutually constituted through the course of interactions” (Anthony & Walshaw, 2007 p. 21). Secondly, the term ecology involves the people, their relationships, as well as, their interaction with the physical class environment. Thirdly, collaboration can be defined as “groups of learners working together to solve a problem, complete a task, or create a product” instead of competing (Laal & Ghodsi, 2012 p. 486). This involves students co-operating and working towards a common goal, through consensus building. Individuals are responsible for their own actions, as well as respecting the contributions and abilities of their peers. Sengupta-Irving (2016) linked collaboration to student agency through deliberately designed “collaborative mathematical activity” (p. 210), which is dependent on group relationships, the focus of the next section.

**Valuing Relationships**

Relationships, such as interactions between students, can support the development of student agency (Mevarech & Kramarski, 2014; Ontario Ministry of Education, 2018; Vaughn, 2020). Anthony and Walshaw (2007) suggested teachers can work to *help students establish a web of relationships* such as a maths community to support learning.

Building class communities with student collaboration has many benefits (Gresalfi, 2009; Laal & Ghodsi, 2012; Lampert, 1990a). In England, Boaler (2000a) studied the impact of teaching methods on classroom communities. By analysing 76 student interviews (Year 8 and 9) from six schools, she found that students themselves identified that social interactions contributed to their positive learning experiences. In New Zealand, Hunter focused on developing mathematical inquiry communities.
(Ministry of Education, 2010). Her findings support the benefits of student collaboration, rather than simply working in seat or social groupings, including support for student well-being, a better understanding of maths and increased confidence. As well as these benefits, students developed more productive dispositions and maths identities, seeing the value of learning maths, wanting to learn, and believing they could succeed with effort. However, the Ontario Ministry of Education (2018) in Canada cautioned that these maths communities take time to build.

Another aspect of the class community involves teacher-student relationships. Relationship building is based on the principle of caring, teachers needing to know their students’ current mathematical knowledge and interests to plan effectively and build on what they already can do (Anthony & Walshaw, 2009). In addition, Attard (2013) found positive teacher-student relationships and teachers’ enthusiasm were the foundation for engagement in a case study involving middle school maths in Australia.

The concept of classroom accountability within a maths learning community is what a student is expected to know and who the student is expected to convince (Gresalfi et al., 2009). When a student is able to exercise agency and is accountable to their classmates, they need to engage in more work to convince their peers than having to just demonstrate understanding to the teacher. In this situation, the learners have opportunities to revise their mathematical ideas with members of their group as they participate in communal discussions and respond to questions (Gresalfi et al., 2009; Sullivan et al., 2006). Sengupta-Irving’s (2016) study deliberately included accountability to others, as well as disciplinary norms. She found students were
engaged in the maths, used evidence, developed arguments and generated questions when interacting with others.

Several researchers have focused on the composition of student groups. The teaching practice of using a mixed-ability maths approach (including students working at a range of levels), encourages effective communication and cultural appreciation between class members, attending to students’ social and emotional needs (Sellars & Lowndes, 2013). This is different to ability grouping (students grouped by maths level), found by some researchers to polarize students and communicate a message of different social class (Attard, 2013; Boaler, 2008). Children working in lower ability groups are seriously disadvantaged by being shut out of the curriculum, leading to the development of negative attitudes towards maths (Education Review Office, 2018). Teachers have been advised to group students without assuming what they can and cannot do (Skinner et al., 2019), hereby showing belief in the students and challenging the power and privilege that are present in any classroom. The regular reshuffling of groups is also a teacher strategy to develop high expectations (Rubie-Davies, 2018). These research studies present options for the grouping of students in order to strengthen relationships and collaboration, and by implication, to encourage agency.

Once grouped, a variety of factors contribute to the group’s ability to work collaboratively. Dyer et al.’s (2013) research within an adult context, identified factors contributing to team performance. Findings suggest a team’s success is determined by a variety of factors: the context (culture, structure and support systems), the team composition (skills, experience and team motivation), competencies (team’s ability to communicate, resolve conflict and make decisions), as well as the team’s ability to monitor their performance. Several of these ideas can be related to collaborative
group work within the maths classroom including, systems to access help (e.g., asking questions), aspects of classroom talk (e.g., turn-taking, listening), students’ maths knowledge (e.g., ability to contribute), and the ability to explain/represent ideas and reach a consensus (e.g., agree/disagree respectfully).

To summarise, deliberate teacher actions that foster student agency focus on productive social interactions with others in the class. Building a class community involves valuing relationships, encouraging student collaboration and designing for students to be accountable to other students. This leads to literature focussing on the field of mathematical discourse.

**Mathematical Discourse**

Mathematical discourse is...

> used to highlight the ways in which knowledge is constructed and exchanged in classrooms. Who talks? About what? In what ways? What do people write down and why? What questions are important? Whose ideas and ways of knowing are accepted and whose are not? What makes an answer right or an idea true? What kinds of evidence are encouraged or accepted? (Ball, 1991 p. 44)

The importance of discourse within a maths learning community is well recognised (e.g., Anthony et al., 2014; Attard, 2013; Fuson et al., 2015; Hunter, 2008; Leinwand et al., 2014; Mevarech & Kramarski, 2014; Ontario Ministry of Education, 2018; Stein, 2007). This section explores the role of maths discourse in the classroom and potential teaching actions to enable student agency.

Participating and contributing to mathematical discussions presents an opportunity to take agency. Anthony and Walshaw (2007) referred to students learning ‘from’ and
‘with’ peers, as well as, building confidence when they are involved in explaining and justifying solutions to others. International studies (USA) linked to primary aged students found maths discussion allows learners to become engaged when they share mistakes, make suggestions (Lampert, 1990a), test ideas, develop and ask questions, as well as agree and disagree with others (Allen & Schnell, 2016; Stein, 2007). Encouraging discussion is identified as a way to promote equity and engagement in the classroom (Boaler, 2016), and to value others’ ideas (Faria-Mitchell, 2009). The qualities of discourse in the classroom that lead to improved learning, however, can be difficult to implement and manage, and supportive class norms (the focus of a later section) take time to establish (Stein, 2007).

Deliberate teacher actions can support rich maths discussion. Hunter (2014) found the role of the teacher changed to a facilitator when they acted as a participant using questions, prompts and guiding students. The mathematical discussion is targeted to address the learning goals, such as to identify similarities and differences in methods, why something works or the most efficient method (Kazemi & Hintz, 2014). Teachers may use whole class discussions, small groups or sharing with a close neighbour, depending on the situation (Chapin et al., 2009). By paying attention to the social aspects of the class such as listening to each other, working co-operatively, and being able to agree and disagree respectfully, teachers can support students and intervene appropriately. Establishing the expectations for talk is important, including the students understanding why discussions are important (to share strategies, clarify a classmate’s thinking and make connections) (Faria-Mitchell, 2009).

A range of teaching tools can help teachers to orchestrate productive maths talk (to promote deeper learning). Firstly, Chapin et al. (2009) established five ‘talk moves’ to
support students to think mathematically including *revoicing, restating, agreeing or disagreeing, adding on* and *wait time*. The first, *re-voicing* (e.g., “So you are saying …”), is used to respond to a student whose ideas are unclear and involves the teacher attempting to repeat what they are saying. *Restating* is when the teacher asks another student to repeat or rephrase what a student has said. The teacher can ask a student whether they *agree or disagree* with a peer’s explanation and why. *Adding on* involves prompting students for further ideas or participation in the discussion. Lastly, *wait time* involves waiting for at least 10 seconds to allow students time to think before asking for a response from other class members. Actions such as these shift authority in the classroom, increasing opportunities for students’ influence and control in their maths learning.

A second study to develop tools to promote student agency is from Stein et al. (2008). They presented a model for managing class discussions after an open-ended task. This involves, anticipating the responses and ways to solve the maths challenge, followed by monitoring students as they work, selecting solutions to share with the larger group, sequencing these solutions from simple to more complex reasoning and finally, making connections with different responses and the key ideas being taught.

To summarise, researchers agree about the importance of maths discourse for student learning saying it enables many opportunities for purposeful participation (and student choice). Researchers have suggested actions teachers can take to work towards constructive maths talk such as asking appropriate questions, using prompts and attending to relationships. The role of the teacher in organising and designing the maths lessons is clearly crucial to this process.
2.3 The Role of the Teacher in Supporting Student Agency

Teachers play an important role in creating a learning environment that values agency (OECD, 2019). Wenmoth (2014) reminded readers it is not simply about handing over control, but working to create a supportive context and environment. In this section, the role of the teacher is considered both in longer-term planning and organising of lessons and in-the-moment responding during maths lessons.

Teachers can meaningfully focus on creating a learning environment for student agency (Louie, 2020). Louie’s study analysed teacher discussions about agency (from five USA elementary schools), aimed at supporting students to see themselves as capable and to engage in mathematical sense-making. She claimed opportunities for agency can be limited by ‘traditional’ teacher-centred class programmes. To promote and support agency, Louie suggested that teachers have a clear vision, make concrete instructional decisions (e.g., choose the task and how to structure student participation) and choose what to notice. Teachers need to give attention to all students in the class to ensure equitable experiences, such as not giving students messages of being smarter or more capable. Other studies emphasise aspects of the teacher’s role relevant to planning for agency, including clear learning objectives, helping students know what they need to do to improve and using formative assessment practices (Absolum et al., 2010; Newton & Winches, 2013). In addition, belief in students is important for fostering student agency (Allen & Schnell, 2016; Boaler, n.d.-b).

As well as planning for agency, responding ‘in the moment’ can support student agency during lessons. Vaughn (2020) advocated working alongside students during lessons, allowing some adaptability and flexibility in how the lesson plays out. She
identified specific actions that teachers used including roving around the classroom to listen, prompting (e.g., “tell me more”), and asking open-ended questions (e.g., “What do you think? ... and why?; What might happen if ... ?”). Teachers in Hunter’s study listened to and observed students working to assess their progress towards learning goals, their observations helping them decide in-the-moment both how to respond and when to intervene (Ministry of Education, 2010). The timing of teacher support is important, with recommendations not to offer unsolicited help and target only low achieving students (Ontario Ministry of Education, 2018). Further teacher actions ‘in-the-moment’ include supporting students to design good questions to help themselves (Newton & Winches, 2013), teaching vocabulary, offering encouragement, making suggestions and giving support for revisions (Mevarech & Kramarski, 2014). By working alongside students and responding in this way, teachers make decisions on what to tell students and what to let them figure out (Louie, 2020), as opposed to being seen as “authorities who provide answers or procedures” (Lampert, 1990a p. 256).

To summarise, key points pertaining to the crucial role of the teacher and their work to foster student agency include what they do to organise both the learning context and student participation. This involves deliberate teacher actions both in longer-term planning and organisation, as well as in-the-moment responses during lessons. I now move to consider how a teacher can design setting up classroom norms that guide student actions and enable agency.

**2.4 Classroom Norms: Expectations for Student Choice and Control**

Classroom norms play an important role in environments where students are able to take agency. In Boaler’s (2003) view, the “success of teaching rests upon carefully
and slowly established class norms” (p.12). This section identifies the definition of classroom norms, explores how key researchers have referred to classroom norms, their effects and how these norms have been established.

A norm is defined as a “recurrent pattern in joint activity” that is regulated by expectations (Cobb et al., 2009 p. 44). Classroom norms, therefore, are patterns of behaviour that take place within the classroom, “frequently outside the participant’s awareness” (Voigt, 1985, as cited in Yackel et al., 1991 p. 397). Examples of these patterns of behaviour are the way students share maths solutions with their group and use representations to demonstrate mathematical thinking to others.

Researchers discuss classroom norms using different terminology, for example, Yackel et al. (1991) used the terms: norms, classroom norms and social norms to mean the expectations and implicit obligations that students and teachers have that guide the way they interact with each other, contribute and take action.

Classroom norms relate to different aspects of the classroom, such as expectations from a group or an individual perspective. For example, Yackel et al. (1991) observed maths lessons set in a second-grade classroom (7-year-old children) involving small group collaborative problem-solving followed by whole class discussions. Their analysis identified the negotiation of social norms for group work which were students cooperating to solve problems and explaining their solutions to their group, valuing meaningful activity over correct answers and quantity of maths completed, valuing persistence and working to reach a group consensus. In addition to these group norms, they observed the negotiation of norms from an individual’s perspective when students worked with a peer on a maths activity. These norms included expectations that individuals figure out solutions meaningful to them,
explain their solutions to their partner and try to make sense of their partner’s solutions (Yackel et al., 1991).

Classroom norms influence learning in the school maths classroom in a variety of ways. During lessons, learning involves knowing when to do things and what to do (Yackel et al., 1991). The classroom norms provide established boundaries and encourage productive working relationships. Another role of classroom norms is in shaping students’ expectations of doing maths (Gresalfi et al., 2012). For example, learning maths in a classroom where they are expected to make decisions and actively participate to solve a maths task, rather than expecting to be told what to do by the teacher. The personal experiences of working in an environment governed by classroom norms, in turn, form beliefs of what engagement in maths involves.

Classroom norms are not uniform, however, because they are not a prescribed list of rules but change according to the situation, and are established through teacher and student interactions (Cobb et al., 2009).

Classroom norms need to be established, reinforced and modified within class environments and this involves a process of negotiation. Yackel et al.’s (1991) study described the establishment of classroom norms that involved the co-construction of expectations the teacher and students had for each other, as well as obligations towards each other. In this example, the teacher deliberately “initiated and guided discussion of obligations and expectations to make possible the mutual construction of classroom norms for cooperative learning” (p. 390). These discussions took place within the classroom when the students were interacting and involved in collaboratively solving maths tasks requiring negotiation and renegotiation over time. In their research, the negotiation of classroom norms was mostly at the beginning of the year, but the norms were renegotiated throughout the year in response to
students’ actions in concrete situations (Yackel et al., 1991). So far, these classroom norms relate to general social situations and could be applied to any subject, which leads to a discussion of norms specifically for maths.

Sociomathematical norms are a specific type of norm, setting up expectations about qualities of aspects of mathematical activity (Yackel & Cobb, 1996). They include understandings of what counts as mathematically different or an efficient solution, and what constitutes a suitable explanation or justification. Sociomathematical norms are important because they regulate mathematical activity (such as reasoning and discussions) and “influence the learning opportunities” (p. 461), such as when to contribute a solution because it is mathematically different. Yackel and Cobb noted that the understanding of mathematical difference was interactively established by the teacher and students through deliberate teacher questioning. The students were involved in comparing their solution with others’ ideas and making judgements about its accuracy and worth. Therefore, classroom norms and more specifically sociomathematical norms form an important part of a maths classroom that encourages student agency.

Classroom norms have been studied within New Zealand classrooms. Hunter’s research (Ministry of Education, 2010) (introduced in 2.2) described the teachers’ attention to classroom norms through developing ‘rules for talk’, scaffolding students to construct mathematical explanations, using thinking time, considering student pairings to encourage risk-taking and giving specific feedback. Other teacher strategies included expecting students to be accountable for their opinions and allowing students time and space to think about their mathematical explanations and to seek clarification from others (Hunter, 2008; Ministry of Education, 2010).
In summary, classroom norms form a key message from the research literature. The establishment of classroom norms is supported by many researchers and is seen as an important part of maths classrooms (Cobb et al., 2009; Ministry of Education, 2010; Yackel & Cobb, 1996; Yackel et al., 1991). Focusing on the classroom norms forms an important part of my study into understanding and organising for student agency because the norms influence student actions. So for the purposes of my study, I will use the general term classroom norms for expectations and obligations established for participation within the classroom. More specifically, these classroom norms could be:

- social norms for general social interaction (used across subjects);
- sociomathematical norms – referring to mathematical understanding involved in carrying out explanations, justifications and creating representations; or
- norms of mathematical activity – used to describe organisational structures or routines.

Operating within a collaborative maths classroom shaped by classroom norms involves students making choices in maths methods and representations, the focus of the next section.

**2.5 Choice in Maths Methods and Representations**

Learners use maths methods and representations to solve maths tasks and help make sense of mathematical concepts. In this section, literature is discussed related to opportunities to choose between different maths methods or representations.
Choosing Methods and Solution Pathways

An opportunity for agency involves choosing from different methods and solution pathways to solve maths tasks. A number of research findings support the benefits of students being involved in choosing their mathematical methods. For example, Gresalfi et al.’s (2009) study found students exercised agency when they were involved in making decisions, such as choosing methods and making sense of different maths concepts. Sengupta-Irving (2016) deliberately organised and encouraged student authority to make decisions about solution pathways and methods during problem-solving. Students were taught the norms for choosing from a variety of methods using Boaler’s daily number talks (Boaler, n.d.-a). Students were given a problem to solve and then asked to identify the different methods of reaching mathematical solutions. These were recorded visually by the teacher, compared and emphasised, demonstrating teacher actions to enable students to describe and discuss different solution methods. Other writers agree that supportive teaching practices include presenting opportunities for students to debate, negotiate and justify solutions (Leinwand et al., 2014), helping students to understand problems can be solved in more than one way and that some methods are more efficient than others (Ontario Ministry of Education, 2018).

Choice of Mathematical Representations

Representations offer further opportunities for choice-making in maths. Representations provide visual information or understanding of a task and may take the form of:

- physical representations (e.g., a model using manipulatives);
- pictorial representations (e.g., a diagram); or
• symbols (e.g., an equation) (Brown, 2020).

Representations support problem-solving processes. Selling’s (2016) case study analysed the learning of three boys (7th and 8th grade, USA) when they created and used representations as problem-solving tools. Data analysis included written work from student journals, as well as small group and class videos. She observed changes in each student’s communication as they reasoned with representations, such as generalising and justifying claims. Selling noticed the students used multiple representations for the same problem (pictures, words and a table) showing connections between the representations and used more sophisticated representations over time. The students’ representations enabled them to collaborate and persist in solving the problems. Her findings suggest the negotiation of different representations involved opportunities to act with agency, when students made choices and judgements about their usefulness. Similarly, Gresalfi et al. explained that choosing representations and recognising their usefulness is associated with agency and critical engagement in problem-solving when students interpret the meaning behind symbolic representations such as why an equation works (Gresalfi et al., 2012).

Representations are tools or resources to support social interactions. Students may use representations to share ideas, monitor understanding to assist their contribution in mathematical discourse and to create questions for others (Brown, 2020). In this way, representations provide something that can be summarised, compared or re-represented by other class participants. Teachers can support students to make connections between representations with others by selecting tasks allowing multiple representations, introducing students to useful visual supports and allowing time for making connections between different representations (Leinwand et al., 2014).
Some researchers, however, present conflicting views recognising that not all materials used for representations are helpful for student understanding. Anthony and Walshaw (2007) reminded readers that the type of artefact selected to model maths ideas affects a student’s reasoning and understanding. Attempting to maximize the benefits of using materials (e.g., blocks), Laski et al. (2015) argued that learning is dependent on how they are used. Their literature review highlighted four principles to consider when planning and selecting equipment to make available to learners. To be effective the materials need to be used over a long time period. Learners also need to move to more abstract materials over time. The relationship between the object and the maths concept needs to be made explicit to learners. And finally, objects that have features that are irrelevant and could be distracting, should be avoided, such as colour and shape that are not relevant. Teachers therefore can make decisions about the features of the materials available to learners.

**Self-Regulation**

Self-regulation can support student agency when students have authority to make decisions such as choosing maths methods and representations. Self-regulation is an individual’s processing of information to evaluate and adjust courses of action (Bandura, 2001), and is linked to student agency by both Bandura (2001) and Vaughn (2020). Hunter’s research found that students became increasingly more autonomous when they were prompted to check parts of a problem (moving to the whole problem) or to identify an efficient strategy (Ministry of Education, 2010). In support, Mevarech and Kramarski (2014) suggested that metacognitive strategies (involving self-monitoring thinking and self-regulation of learning) positively affect student learning and students can be taught to ask themselves questions during an
activity. What is the maths task about? What problem-solving strategies are appropriate? Can the task be solved in a different way? Am I stuck? and Why? Using questions such as these can guide students to be aware of the task requirements, appropriate methods for the task and their personal capabilities. In this way, teachers can foster self-regulation to support student agency.

To summarise, research suggests that students take agency when they choose maths methods and representations to solve maths tasks. There are key teacher actions that support student choices of methods and representations. These actions are to help students understand there are different ways to reach a solution (some being more efficient than others), make sense of different ideas through discussion (including the negotiation and justification of different choices) and self-regulate.

2.6 Self-Efficacy and Affective Aspects

*Self-efficacy* is a student’s judgement of their capability to solve a problem successfully (Bandura & Schunk, 1981), such as in maths. Bandura argued “unless people believe they can produce desired effects by their actions they have little incentive to act or to persevere” (Bandura, 2006, p. 170). Self-efficacy is therefore foundational for human agency, impacting affective, cognitive and decisional aspects. *Affective* aspects include motivation, beliefs and emotion which are all recognised in shaping maths learning (Hannula, 2019), such as a sense of belonging (Cargnelutti et al., 2017). Motivation is defined as the intentions or willingness of a student to take action (Gettinger & Walter, 2012). Factors that influence motivation can be understood using Ryan and Deci’s (2017) Self Determination Theory framework. Their framework proposed three core needs that promote intrinsic motivation (motivation driven by internal rewards): competence (to feel like we have
done a good job), autonomy (to have control over what we do) and relatedness (meaningful interactions with others). When applied to education and the maths classroom, these core needs support students’ motivation to make choices and act, resulting in engagement and satisfaction. Davis’s (2019) work within a New Zealand context aligns with Ryan and Deci’s theory. She argued that a focus on intrinsic motivation can lead to increased student agency giving learners power and control in their learning and is an “essential element of effective classroom instruction” (p. 42). Student agency can be strengthened by students being engaged in learning that is relevant and useful to them and by experiencing positive emotions.

Positive emotions are important for both learning maths and enabling student agency. Debellis and Goldin (2006) identified affective pathways, one positive and one negative, that describe how feelings change during problem-solving, either empowering or disempowering the learner. For example, curiosity can motivate students to explore leading to better understanding, whereas, a negative pathway may lead to feelings of frustration, anxiety and despair, and act as a barrier to the problem-solving experience. Positive emotions allow the long-term recall of lessons, while negative emotions can result in disrupted processing in the brain, causing difficulties in recalling the lesson (Dumont et al., 2012). Concerned with students’ emotions and maths anxiety, Boaler argued that learning takes time and “cannot be accelerated by methods that encourage speed at expense of understanding” (Boaler, 2014b p. 473). Boaler suggested students are better to decompose and regroup numbers with confidence (and show positive emotions) rather than being placed under time pressure. Positive attitudes and dispositions play an essential role in learning maths because student beliefs influence what they say publicly, the
questions they ask and the risks that they are willing to take, such as when making maths choices (Whitin, 2007).

Feelings of belonging in maths have been related to enhanced self-efficacy (or confidence), a reduction in anxiety and increased maths achievement (Barbieri & Miller-Cotto, 2021; Dasgupta et al., 2022; Good et al., 2012). A student’s sense of belonging comes from having their presence and contributions valued (Good et al., 2012). Dasgupta et al. (2022) found feelings of belonging enhanced learning outcomes in maths. They carried out a longitudinal field study involving 8th-grade students (2,939) from 10 schools in the USA. Data sources included surveys and final course grades. They found middle school classes that emphasised collaboration and the social relevance of maths, predicted stronger self-concept, more engagement and better performance. In another study, Barbieri and Miller-Cotto (2021) (USA) identified a sense of belonging as being a significant predictor of maths learning.

**Persevering When Faced With Difficulties**

Efficacy beliefs determine how opportunities and obstacles are viewed when taking agency (Bandura, 2006), and whether a student will give up when faced with difficulties. A number of researchers have focused on ‘productive struggle’ which involves working hard and persevering to make progress in solving difficult maths tasks (Hiebert & Grouws, 2007; Smith, 2017; Warshauer, 2011). Literature suggests teachers can help students understand that struggle is expected (Leinwand et al., 2014; Ontario Ministry of Education, 2018; Warshauer, 2015), and mistakes are important for learning (Boaler, 2014a). Yeager and Dweck (2012) suggested that focusing on a growth mindset can help students be more resilient to both social and
academic challenges. Other deliberate teacher actions to support productive struggle in solving maths tasks include using questions to help focus students’ thinking, encouraging effort and giving time to think through problems (Warshauer, 2015). Teachers can avoid giving step-by-step instructions for solving the problem (Roche & Clarke, 2014; Schoenfeld & Teaching for Robust Understanding Project, 2016) and value perseverance (Hattie & Timperley, 2007). Rather than stepping in to do the work for them, teachers can scaffold student thinking by asking questions and providing adequate time for student thinking (Leinwand et al., 2014).

To summarise, researchers’ findings about how motivation and emotions support student agency suggest that teachers can create a positive maths classroom environment where students feel safe, enjoy participating and contributing, and are willing to persevere. Teachers can deliberately focus on student problem-solving processes rather than speed and correct answers and develop a shared understanding that we learn from mistakes. Teachers can encourage meaningful interactions with others, scaffold student thinking using questions and help students to see that they are competent.

2.7 Theoretical Lens – Learning Maths in Primary School

In this section, I discuss social constructivism and its link to important aspects of the maths classroom already outlined in this chapter.

Social constructivism considers maths knowledge as actively constructed and created in connection to people’s lives (Ernest, 1991). Within this view, attention is given to participation in activities (doing maths) in meaningful contexts (Sfard, 1998). The role of social constructivism in learning maths involves both a process of students actively constructing knowledge (constructivism) (Von Glasersfeld, 1984),
as well as “acculturation into the mathematical practices of wider society” involving classroom social interactions (Bauersfeld 1993, cited in Yackel & Cobb, 1996 p. 460).

Social constructivism informs many studies relevant to this investigation of agency (e.g., Gresalfi et al., 2012; Gresalfi et al., 2009; Sengupta-Irving, 2016; Yackel & Cobb, 1996; Yackel et al., 1991). Social constructivism is illustrated in Cobb et al.’s (1992) study, recognising the importance of social interactions and participation in maths learning within a class of 7-year-old students (USA). A ten-minute episode of maths group work was analysed, exploring the interactions of the individual learners and the group solution. Students actively interpreted and adapted the ideas of other group members. Mathematical learning occurred through social interactions and cyclic events involving student participation, rather than a linear chain. Students explained and justified their maths ideas and influenced each other through a process of continual mutual adjustment. Individual and group development were interdependent because the students’ understanding of the maths concepts increased through interaction with others. Gresalfi et al. (2012; 2009) contributed to theories of maths learning in a social setting organised by norms, highlighting the interactional nature of learning and the social context. The organisation of social settings such as in these examples, have particular relevance for this study on agency. They support the argument that student agency enhances maths learning in the social context of the classroom.

Social constructivism can connect with a situated perspective of learning, where knowledge is co-constructed between people, activities and their environment that has been created and deliberately shaped for opportunities and sense-making (Boaler, 2000b; Boaler & Greeno, 2000; Cobb, 2000). Learners are involved in
activities within the maths classroom, constructing their own understanding by participating in discussions with others, adapting procedures and modelling ideas.

Social constructivism underpins learning in a classroom where agency is valued when students recognise opportunities, see themselves as capable and take purposeful action to seek mathematical solutions. Earlier in the chapter (2.1), social constructivism was recognised in the ‘interactiveness’ dimension of student agency where individuals exert influence in social situations (Vaughn et al., 2020). This was strengthened further with the exploration of maths learning contexts (2.2) to allow individuals time and resources (social and physical) to construct maths understanding. The role of the teacher (2.3) to nurture this social constructivist learning environment was discussed, with the co-construction of class norms (2.4) to guide student actions. Constructivist learning opportunities were further recognized in contexts where students have the authority to choose meaningful math methods and representations to make sense of their own interpretations and influence others (2.5). Finally, self-efficacy and the affective aspects of social constructivist learning environments were considered including motivation and emotions (2.6).

2.8 Summary

Student agency is a term commonly used in education but the understanding of the concept is not universal. Its meaning within the primary maths classroom is associated with particular student behaviours involving purposeful choices and actions. Findings from research studies agree that student agency is a positive aspect of the maths classroom and can support maths learning. Many concepts that foster student agency overlap (e.g., discourse, participation and engagement). They are linked and form part of the current assumptions about effective pedagogy in
maths (Anthony & Walshaw, 2009). Findings from research studies provide many mathematical findings and examples that support student agency. However, a teacher faces many dilemmas in organising for and fostering agency, aiming for distributed authority where students have influence and opportunities for choice in solving maths tasks. Many studies agree that the learning context is key, as well as the teacher's role in responding in-the-moment to foster agency. I now move to Chapter 3 where I explain my research design for my journey to investigate students' perspectives of agency in my classroom context.
Chapter 3: Methodology

Introduction

The purpose of this chapter is to describe and justify my research design decisions. I aim to soundly represent my investigation on student agency in the maths classroom by making these methodological and data-gathering decisions explicit. This involves explaining the reasons for my decisions, as well as clearly describing the context of my study. By doing so I add to the robustness of my design and the credibility of my findings. I begin this section with the theoretical framework outlining the key features of my research design, followed by my chosen case study design. Ethical considerations form the next section, along with how these are addressed within my study. Data collection methods are described including my intention, the process and ethical reflections of each method. I end this chapter with the timeline of my study, data management and my initial data analysis.

3.1 Theoretical Framework

In this section, I outline key features of my theoretical framework and my methodological approach to practitioner research. I draw upon methodological literature that has influenced the research process.

Interpretivist Approach

This study draws upon an interpretivist approach to studying human behaviour. An interpretivist approach "tries to show how choices are made by participants … in social situations within the process of interaction" (Burton & Bartlett, 2005, p. 10). The understanding of students’ choices within a social classroom context as they interact with each other to learn is relevant to interpretivism in a school context. An
interpretivist approach is characterised by concern for the individual rather than groups and society (Cohen et al., 2011). A researcher starts with the individual and aims to understand their interpretations of the world in which they interact. Interpretivist approaches focus on people’s actions that can be thought of as “behaviour with meaning” because they are intentional and future-focused (Cohen et al., 2011, p. 17). This aligns with my focus on students’ agency within the maths classroom. An interpretivist approach acknowledges some degree of subjectivity in the researcher and participants (compared with a positivist, more objective approach), and is therefore a typical approach used in small-scale educational research (Atkins & Wallace, 2012; Cohen et al., 2011). Those involved can be treated as participants rather than subjects being studied (Atkins & Wallace, 2012). An interpretivist approach offers an apt theoretical lens for my study because it enables me to interrogate the web of relationships that exist within a typical classroom. An interpretivist approach can involve an insider researcher (Cohen et al., 2011), and be written in the first person (Atkins & Wallace, 2012). From a practitioner researcher’s stance, a study can be aimed at understanding a particular case or situation such as my classroom. An interpretivist researcher is concerned with methods that are qualitative and provide accurate and detailed accounts from participants rather than large-scale research involving statistical comparisons for reliability purposes (Burton & Bartlett, 2005; Cohen et al., 2011). The data methods favoured in an interpretivist study include informal interviews and observations of people in their natural environments (Burton & Bartlett, 2005; Mutch, 2013; Taylor et al., 2015). Therefore, an interpretivist approach aligns with the aim of my research which is to understand student actions in the maths classroom, through their reflections and interpretation of what has happened for them as learners.
In summary, I have adopted an interpretivist stance to my research design because of my interest in students’ interactions in collaborative learning situations, the unpredictable nature of students’ actions and the focus on solving teaching dilemmas in the classroom. I now look to the characteristics of qualitative design.

**Qualitative Research**

The characteristics of qualitative research approaches inform my research design decisions (e.g., Atkins & Wallace, 2012; Burton & Bartlett, 2005; Creswell, 2018; Mutch, 2013; Taylor et al., 2015). Qualitative methods focus on understanding the meaning people attach to things in their lives (Creswell, 2018; Edmonds & Kennedy, 2017; Taylor et al., 2015). Researchers take a naturalistic approach, observing in a natural setting and analysing data to understand the voices of the participants and their views (Taylor et al., 2015). Insights and understandings are developed from patterns in the data collected using inductive analysis rather than assessing preconceived ideas (Creswell, 2018; Mutch, 2013; Taylor et al., 2015). The people and the setting can be viewed holistically, with researchers exploring how people think and act during face-to-face interaction, in order to answer how or why questions of human behaviour (Creswell, 2018; Taylor et al., 2015). The understandings obtained from qualitative research are emphasised rather than its reliability and ability to be replicated (Taylor et al., 2015). “Biases are accepted as part of the process” (Edmonds & Kennedy, 2017, p. 2), so the researcher needs to be aware of their assumptions, values and world views (Creswell, 2018).

My decision to use qualitative research to explore students’ perspectives about their sense of agency in school maths is based on its strengths aligning with my research question, context and researcher positionality. Qualitative research has its roots in
the social sciences and is concerned with interpreting people’s views, interactions or values which are based on feelings and human responses of a more subjective nature (Atkins & Wallace, 2012; Mutch, 2013). As such, it enables me as the researcher to examine both my participants and the classroom setting in which there are many student actions and interactions. “Qualitative research is about making the world visible in ways that implement the goals of social justice…” (Denzin et al., 2010, p. 15), and aims to make a difference in everyday lives through change (Creswell, 2018). This aligns with the aim of my research which is to improve classroom maths learning opportunities. As a people-centred approach, a qualitative research design is preferrable because it affords interpretations of behaviour from participants who are naturally not static and predictable in their actions (Atkins & Wallace, 2012; Sargeant & Harcourt, 2012). A qualitative researcher can also interact with the participants (Mwita, 2022), and openly acknowledge their positionality as I have done as the classroom teacher engaging in a research study in my own classroom and drawing on years of teaching experience (Creswell, 2018; Mutch, 2013). Qualitative research acknowledges my insider knowledge of the context which enables a rich body of familiarity not possible from an outsider researcher. My research involves a researcher relationship based on trust (involving empathy and interactions) to enable understanding of my students’ views (Mutch, 2013; Taylor et al., 2015). Typically, a qualitative design will use methodologies such as case study (investigation of a phenomenon within a specified context) (Cohen et al., 2011; Creswell, 2018; Edmonds & Kennedy, 2017; Mutch, 2013). Due to my limited resources as a sole researcher, a qualitative design enables me as the researcher to focus on a small number of participants who are representative of a larger whole (the class) and to continue the practice of teaching. Other qualitative
data-gathering methods can be used. These include participant observations, semi-structured interviews, and document analysis (Creswell, 2018). A range of methods allows the researcher increased flexibility in how the research is undertaken, and provides opportunities to probe deeper to gather detailed information through one or other methods (Mwita, 2022). The researcher can continue to collect data until they consider they have sufficient, presenting the opportunity to further clarify understandings in the data analysis. These methods enable the analysis of students’ perspectives, their actions and their maths activity.

Qualitative research also has weaknesses of which a researcher needs to be aware. Firstly, qualitative research is interpretive (Mwita, 2022). As not all researchers interpret things the same, it is more subjective to researchers’ interpretations. As a result, this challenge can be addressed by clearly explaining my position and providing detail of the context and data analysis. Secondly, management of the data collection process may be a further limitation as interviews and observations can be time intensive to implement (Mwita, 2022). The careful consideration of data collection methods and how other researchers have managed this challenge within classrooms will ensure I select methods that are manageable, and appropriate for the classroom and for myself as a sole researcher. Thirdly, the findings of a qualitative study are difficult to replicate due to the varied feelings, experiences and backgrounds of those involved. Measures will be taken to address the trustworthiness of my study, based on literature and how other researchers have responded to this limitation. Fourthly, the researcher’s bias may influence the data collection as they are human and may exaggerate, understate, be sympathetic to participants or subconsciously affect the data collected. As a result, I acknowledge my position within the research and my aims for the study and approach this
investigation with honesty and integrity. I deliberately use strategies in my research design to ensure the validation of my findings (Creswell, 2018).

To summarise, my chosen research design adopts an interpretivist approach, using a qualitative research methodology. This methodology enables me to uncover, interpret and construct meaning from the experiences of students during maths challenge lessons, as a part of in-the-moment teaching and learning. My students’ views of the classroom matter and result in decisions to manage themselves, work with others and make choices in maths methods and representations to solve maths challenges. An awareness of the weaknesses of this chosen approach will ensure that I address these through my design decisions.

**Practitioner Research**

My research design is built from an insider (emic) perspective, also referred to as practitioner research (Hamilton et al., 2013). The advantages of being an insider researcher can be associated with my situation, including ease of access to the site and participants, the possibility of fitting the research around workplace commitments and access to resources such as the classroom and maths equipment (Atkins & Wallace, 2012; Burton & Bartlett, 2005; Mutch, 2013). For me, as the practitioner-researcher and the class teacher, my knowledge of the participants and my established rapport are clear advantages. However, that knowledge also serves as a challenge because I already have professional knowledge of the context which could be seen to bias my perspective as a researcher (Harcourt & Sargeant, 2012). Needless to say, this is a challenge that can be addressed when “the researcher acknowledges their own viewpoint, values and preconceptions and explains the
measures they have taken to prevent these from contaminating the data” (Atkins & Wallace, 2012 p.12).

3.2 Case Study Design

A case study methodology provides me with the opportunity to delve deeply into the case of investigating student agency in maths in my classroom setting. Stake describes a case study as the investigation of the “particularity and complexity” of a single case (as cited in Patton, 2002, p. 297). In a single case study, the researcher can explore a bounded system (such as my maths classroom) over time through in-depth data collection (Cohen et al., 2011; Creswell, 2007; Mutch, 2013). Stake’s work (1995) identified two main forms of case studies: an intrinsic case study attempts to understand the case in order to identify an issue, and an instrumental case study focuses on the issue of the case (as cited in Hamilton et al., 2013). In an instrumental case study, the themes and issues, revealed by the researcher from the data collected, are specific to the situation and can be included in the findings section. The conclusions formed by the researcher about the overall meaning can be seen as general lessons learned and as in my situation, inform future teaching to foster agency in maths. The potential of case study is reinforced by Vaughn et al. (2020) who found it was the most common research design encountered during a systematic literature review of agency (focused on literacy instruction).

3.2.1 Informing Studies

Literature on qualitative case studies and examples built within similar contexts informed my research design. The first example of case study methodology is seen in Mills’ (2016) case study. Mills explored fraction understanding of students in a Year 5 and 6 class, within a New Zealand context. Procedures for gathering data
outlined pre- and post-unit assessments and observations within the maths lessons. These included three lessons within a 6-week unit. To help validate lesson observations, field notes were written, photos taken, and the lessons videoed enabling the researcher to closely examine the data at a later date. Also, the researcher could compare and contrast the variety of forms including visual information from the photographs and recordings, as well as the researcher journal entries.

In a second research example, Fielding-Wells et al. (2017) explored motivation and engagement in an inquiry-based maths classroom (using design-based research as the methodology for a larger project). This incorporated a variety of data collection methods. While this study involved a team of researchers, and therefore was on a bigger scale than my study, the first author was a practitioner-researcher with similar challenges to myself in this dual role. Their data collection methods included videos of students working (collaboratively in groups, as well as whole class discussions), students’ maths work and researcher logs. They used three central questions to organise students’ ideas about motivation during the data analysis:

- “Can I do this task?”
- “What do I have to do to succeed?” and
- “Do I want to do this task and why?”

The questions enabled them to refine the data from the transcripts to excerpts that illustrated their categories. The results section of their study includes selected excerpts to illustrate each of the framing questions. The researchers note that they did not include any instances that could be considered negative, demonstrating their control of what was reported. Data collection methods that have potential for my study include the range of views provided by the video recordings and the student
research logs providing easy-to-collect information on their views. Also, the data analysis description provides a possible model for analysis in my study.

In a third example (another case study), Lampert (2001) investigated dilemmas and the choices made in her classroom (the case) to support student engagement and learning as students worked on problems. From her perspective as a practitioner-researcher, she used a variety of data sources to analyse her teaching which added to the reliability of her work. These included some records from normal classroom practice, such as chalkboard graphics (copied), student tests, parent report comments, students' work samples (copied) and classroom seating plans. In these cases, the researcher needed to ensure the data was gathered and copied in a timely manner. Other data sources, in addition to normal classroom practice, planning and organisation, included video and audio recordings of lessons, structured field notes by an observer and researcher journal entries. Her study highlights the range of possibilities for data collection within a classroom setting. The advice I glean from these studies is a need to use multiple forms of data collection as I gather students' views on agency.

**Strengths and Challenges of Case Study Design**

A clear strength of a case study is the opportunity to use an interpretivist approach to observe participants in their natural setting solving maths problems as learners within a class (Cohen et al., 2011; Mutch, 2013). The results provide an in-depth description of the bounded case (Creswell, 2018). Case studies allow flexibility and can be used to explore a variety of contexts (Atkins & Wallace, 2012; Cohen et al., 2011), such as a person, setting or concept (Mutch, 2013). Yin (2009) claimed that case studies suit investigations involving ‘how’ or ‘why’ questions, a real-life context
and where the researcher has little control over the events such as how activities play out in a classroom. The case study methodology also allows a variety of data collection methods (Cohen et al., 2011; Mutch, 2013). Case studies are useful because they are “down-to-earth and attention-holding” (Stake, 2009, p. 2), in line with the reader’s understanding and experiences, and can provide examples to learn from for use in similar contexts. They are considered a popular approach in educational research because they enable practitioners, such as teachers, to examine aspects of their classroom environment without being dependent on large sample sizes (Atkins & Wallace, 2012; Burton & Bartlett, 2005).

My justification for using case study design is based on its strengths aligning with my context and research questions. My study aligns with Stake’s instrumental case study because I use my class as the case, my students as the participants (Year 5 and 6) and fostering student agency in the maths programme (during the 2019 school year) as the issue under investigation. I utilise a popular strategy in education practice which is to examine, generate accounts of and provide knowledge about important aspects of school environments to bring about curriculum change (Burton & Bartlett, 2009; Miles, 2015). A case study approach enables me to explore a bounded system (my maths classroom) (Cohen et al., 2011; Creswell, 2007; Mutch, 2013), and in my position as practitioner-researcher, I also bring knowledge of the recent history of the school, students, families and the community to my research.

Case study design presents challenges that nevertheless need consideration in the planning of a research study. Critics have highlighted that the findings are specific to a particular context (place and time) and are therefore unable to be generalised to wider populations (Burton & Bartlett, 2005; Miles, 2015). Some researchers view them as a less demanding methodology because the findings lack validity, reliability
and are less rigorous e.g., unreliable evidence and biased views influencing findings (Burton & Bartlett, 2005; Miles, 2015; Mutch, 2005; Yin, 2009). Validity refers to the accuracy of data, ensuring the data measures what it set out to measure, whereas, reliability is the extent the study can be repeated. Swain (2017) argued that ethical issues are more likely to arise in case studies due to their flexible design and the ability of the researcher to respond to contingent events in particular contexts. But in Stake’s view, readers of case studies recognise essential elements of interest or similarity to their own experiences, the basis for natural generalisation (Stake, 2009). Natural generalisation is “a process where readers gain insight by reflecting on the details and descriptions presented in case studies” (Melrose, 2009). An awareness of these challenges of case study, enables me to address them through appropriate research design.

I now select and clearly describe my case, which I consider worthy of study and representative of a typical operating classroom within the New Zealand public school system.

3.2.2 The Research Case: My Classroom Within an Urban School

In this section, I introduce the research case by situating it in a school setting to explain aspects such as the physical layout, student organisation and maths learning.

Research Context - The School Setting

The case study class was in an urban school. Warranting particular mention was the case study school’s values, which closely linked to the key competencies of The New Zealand Curriculum (NZC) (Ministry of Education, 2007). These underpinned all planning and lesson delivery and focused on supporting students to be life-long
learners. The school values included being able to adapt to new situations, work hard, think about situations, respect others, and be able to take sensible risks. My study narrowed these transferable values down to one learning area, maths which is explored through a focus on student agency.

**The Classroom Setting – The Physical Layout**

The physical environment of the study was a single-cell classroom, with a small breakout area. At times during the year, the class worked collaboratively with another similar classroom. Sound tiles had recently been installed on the ceiling in an attempt to alleviate the poor sound quality inside the room when students were engaged in collaborative tasks. This was a necessary improvement and relevant to my study with its focus on student talk as a way to explore mathematical understandings and the development of student agency. It illustrates the need for the physical environment to be an enabler, not a hindrance for students to interact with one another.

The layout of the classroom provided students with individual desks grouped around the room (used as shared learning spaces during maths), as well as access to small tables and floor space for collaborative learning. Resources were available for student use including a range of concrete materials within easy reach on a maths shelf, maths games, measuring equipment, small whiteboards and IT equipment (an interactive whiteboard and individual laptops). Students had maths workbooks (squared and plain paper) for recording ideas and computers for individual digital work. Directly outside the class was an asphalt courtyard and grass area available for work outside the classroom.
The Classroom Organisation

The majority of students had moved up through the school, coming from several Year 4 and 5 classrooms (the previous year), so they were familiar with each other, but had not necessarily worked together before. A small number of Year 6 students had been in my class the previous year as Year 5 students. The student numbers were stable during the time of the study. A teacher aide was available to assist students during the maths lessons.

Maths Pedagogy: The Maths Challenge Lessons

The focus lessons for my study were maths challenge lessons that took place as part of the whole maths programme. Data were collected from one of these challenge lessons each week. The lesson tasks were suitable for group problem-solving and were chosen from a variety of sources to address students’ next steps and encourage students’ choice of solution pathways. The lessons were aimed at level 3 of The New Zealand Curriculum and the lesson material is summarised in section 4.1.2. Having described my specific case, I now give attention to processes in my research design to address the trustworthiness of my investigation.

3.2.3 Addressing Trustworthiness

Trustworthiness refers to the quality and degree of confidence placed on a research study including aspects of credibility (is believable), transferability, dependability and confirmability (Guba & Lincoln, 1982). Three objectives of the researcher are deemed necessary by Yin to convey trustworthiness and credibility (2011). The first objective is to make procedures transparent. Transparency involves allowing readers to access and scrutinise the data, as well as the evidence leading to findings and conclusions. To this effect, my diligence as a researcher is demonstrated through a
thorough account of my decisions regarding design steps, options and processes which have been informed by the careful reading of a broader research literature, thereby satisfying the criteria of being trustworthy (Cohen et al., 2011; Creswell, 2018; Mutch, 2013). A second objective is about the need to be methodical and follow a set of research procedures (Yin, 2011). This includes avoiding any distortion of the research and unexplained bias. The third and remaining objective of researchers is to base the study’s conclusions on an explicit set of data. I acknowledge my responsibility to the people involved (participants, parents, Board of Trustees, school community) to ensure my study is viewed as trustworthy (Creswell, 2007; Mutch, 2013; Yin, 2011).

Further aspects for consideration to ensure readers trust the design and decisions have been articulated by researchers (Cohen et al., 2011; Creswell, 2018; Johnson et al., 2020; Mutch, 2013). One aspect is validity - ensuring that the study “measures what it sets out to measure” (Mutch, 2013, p. 109). Burton and Bartlett (2005) suggested that validity is an important consideration in practitioner research. Another aspect of research is reliability, meaning that the study could be replicated with similar results (Cohen et al., 2011; Mutch, 2013). But as my study is a qualitative case study, the aim is for the participants to represent themselves. Therefore, it is not possible to replicate the study with similar results (and to generalise). Instead, transferability is a criteria for judging the quality of the study, allowing readers to apply the information to their own situation (Guba & Lincoln, 1989). These objectives of trustworthiness and credibility can be addressed through deliberate strategies which I now outline.

Credibility involves the researcher using strategies to ensure findings are meaningful to those familiar with the setting or case (Creswell, 2018; Mutch, 2013). Strategies
used in qualitative research to enhance rigour and credibility related to detailed description include, accounts of methods, procedures and decisions, as well as descriptions of the context. Strategies related to aspects of data include, member checking, ensuring that enough data is collected and deliberately seeking variation in sample selection. Strategies related to researcher practices include triangulation, researcher reflexivity and discussing the study process with colleagues. Providing careful description (e.g., including time, place and context) can address transferability, so others can judge whether to apply the study to their own situation (Guba & Lincoln, 1989). By carefully defining the terms, attributes and how data are analysed, a researcher can ensure their study is trustworthy (Creswell, 2018; Mutch, 2013). Member checking involves sharing the understandings from the data with the participants to check interpretations (Guba & Lincoln, 1989; Mutch, 2013). Spending adequate time collecting data (persistent observation), as well as deliberately seeking sample variation such as different views, ensures that enough views have been considered to identify elements relevant to the inquiry and to reflect the study focus. Prolonged engagement at the site in this way, ensures immersion in the context, helping to overcome misinformation and distortion (Guba & Lincoln, 1989; Johnson et al., 2020). Triangulation involves the inclusion of multiple sources of data, data collection methods or investigators (Cohen et al., 2011; Johnson et al., 2020; Mutch, 2013; Patton, 2002). It enables a researcher to gain a better understanding by approaching the research from many angles and perspectives (Burton & Bartlett, 2005; Creswell, 2018). Another strategy involves the researcher taking part in critical self-reflection (reflexivity) to consider their assumptions, biases, and relationship to the study (Cohen et al., 2011; Johnson et al., 2020; Mutch, 2013). Guba and Lincoln (1989) described peer debriefing (with a disinterested party -
someone not involved in the study) as a way for a researcher to discuss the findings and conclusions and to ‘test out’ ideas.

Planned actions to ensure my case study is viewed as robust and the findings trusted include consideration of the aptness of the study’s focus (Creswell, 2007; Creswell et al., 2007; Johnson et al., 2020). Student agency is relevant and topical in the classroom today (see Chapter 1). From professional experience, I acknowledge that the dilemmas I face as a practitioner-researcher fostering student agency are experienced by other New Zealand primary school maths teachers. The findings of my study are aimed at benefiting the learning experiences of students both in my classroom and beyond. The study explores variations in teaching actions e.g., groupings, giving instructions, class organisation, norms etc. within a normal operating classroom (ensuring transferability) (Guba & Lincoln, 1989). I define the terms used within my study to achieve clarity of meaning and the data being analysed, providing examples of the data collected and coding decisions. I deliberately include triangulation of methods and data sources e.g., documents and interviews, as well as different participants. My aim is to approach the topic from many angles and avoid being vulnerable to the weaknesses of individual methods and perspectives (e.g., biased responses and loaded questions). This gives me the opportunity to ensure validity by testing for consistency in data and gaining deeper insight (Burton & Bartlett, 2005; Cohen et al., 2011; Johnson et al., 2020; Mutch, 2005).

Throughout my study, I address further challenges. I openly acknowledge my perspective as a practitioner-researcher and discuss my possible influence on its reported findings (Patton, 2002). It is why a reflexive approach is useful for helping me to be more conscious of my role as a practitioner-researcher, taking into account
issues such as assumptions about children’s learning and behaviour (Graham et al., 2013; Johnson et al., 2020). Maintaining dialogue with my supervisors and using a research journal are two strategies enabling my reflexivity (Creswell, 2018). This assists me to be self-aware, revealing decisions, issues, theories and my influence during the process (Street, 1990). To address the challenge of missing aspects of classroom activity whilst teaching and researching simultaneously (Alton-Lee, 2001; Hamilton et al., 2013; Mutch, 2013), I select data-gathering methods which provide the means to review and assess data after the lessons are taught (e.g., recorded observations and stimulated recall). My aim is to avoid leading questions (Creswell, 2007), and continue to be aware of my position and biases when analysing data and selecting examples to communicate (Miles, 2015). Creswell (2007) added further aspects for consideration such as difficulties with colleagues and pedagogical priorities. To address this challenge, I maintain communication with my colleagues ensuring that I uphold the priorities of my teaching position and obligations to my students. By maintaining an awareness of these potential issues throughout my research, I address them through research practices that are ethical and trustworthy.

In summary, I have named strategies utilised to increase the validity of my study (Burton & Bartlett, 2005; Cohen et al., 2011; Creswell, 2018; Mutch, 2005). Likewise, trustworthiness is demonstrated through the clear documentation of decisions made for the research design, data collection and analysis techniques of my findings. Having explained my consideration of the strengths and weaknesses of using case study as a research methodology, my involvement as a practitioner-researcher and my response to trustworthiness, I now outline the ethical considerations within my study.
3.3 Ethical Considerations

In my quest to develop a robust research design, I have a duty of care as a researcher to be guided by sound ethical principles. “An ethic is a moral principle or code of conduct” that governs how people behave and is a part of the research process (Mutch, 2005 p.76).

Careful consideration of ethical principles is necessary because collecting personal information places researchers in positions of power over their participants (Creswell, 2018; Swain, 2017). When teachers collect information about student learning (e.g., assessment data) as a normal part of their job, they are protected by legislation such as the Education Act (Mutch, 2005). This differs from a research context where the purpose of the information collected is to answer research questions, therefore the information must be given voluntarily. Participants need to be able to make an informed decision about whether it is in their interests to participate in a research study, understanding that they can decline the invitation without consequences or harm to what might be a professional relationship. When involved in research, classroom teachers (as researchers) and their participants (the children they teach), need to be protected through adherence to ethical guidelines. As a consequence, practitioner researchers need to determine when they are collecting information for research purposes as opposed to teaching episodes and ensure they act ethically.

3.3.1 Ethical Principles

In general, acting ethically involves treating research participants “with consideration, fairness and respect” (Mutch, 2005 p.79), but requires deeper thought to a number of guiding ethical principles. Amongst methodological research authors, there are
variations in terminology to address ethical principles (e.g., Atkins & Wallace, 2012; Burton & Bartlett, 2005; Hamilton et al., 2013; Punch, 2000). My discussion of key ethical concepts is structured around three broad categories as outlined by Mutch (2005):

- protecting participants' identity - confidentiality, anonymity and privacy;
- ensuring the researcher and participant relationship - no harm, voluntary participation, no coercion and the right to withdraw; and
- providing clarity of expectations - informed consent and no deceit.

**Protecting the Identity of the Participants**

Ethical practice as a researcher means safeguarding the identity of participants. This requires promising confidentiality and anonymity for participants (Alderson & Morrow, 2011; Alton-Lee, 2001; Burton & Bartlett, 2005). Confidentiality involves an assurance that the data collected for the study will remain confidential to the researcher (Mutch, 2005). Anonymity guarantees that any names and identifying features will be changed to protect the participants being identified (Swain, 2017). Pseudonyms are often used to provide this protection ensuring the participants’ anonymity.

**The Relationship Between the Researcher and Participants**

The relationship between the researcher and participants must be safe for both the researcher and those researched. This means avoiding harm including physical, cultural, psychological or emotional harm (Atkins & Wallace, 2012; Cohen et al., 2011; Mutch, 2005; Swain, 2017). Within a classroom, harm could include low self-esteem, stress, loss of privacy, burden or disruption to learning opportunities. If harm is likely, then the researcher needs to act as a responsible practitioner by either
removing the stress the participant is encountering, or not proceeding with the research (Graham et al., 2013). The researcher also needs to consider their own safety through an awareness of the dual roles encountered as both the classroom teacher and researcher (Creswell, 2007; Cullen, 2005; Mutch, 2005). Examples involve not positioning themselves in situations involving physical or emotional distress such as feeling overburdened.

One concept, expressed as beneficence, involves the researcher considering the harms alongside the benefits in order to justify and maximise the purpose of the research (Alderson & Morrow, 2011; Creswell, 2007; Cullen, 2005). For example, researchers need to be clear there is a need for the research, realising that research can be intrusive and cause distress to participants. As my study involves educational research, it needs to have the potential to improve the well-being and status of the participants who are involved (Alderson & Morrow, 2011; Cullen, 2005; Graham et al., 2013). The rationale for children being included in a research context such as mine is the right to their voice (Fitzgerald, 2020; Graham et al., 2013; Kyritsi, 2019; Loveridge, 2010). Arguing for including children’s perspectives in research, Sargeant and Harcourt (2012) suggested it “positions children as competent social actors, being, in their own right” (p.2). Importantly, children’s views can challenge adult-centred assumptions. Alton-Lee (2001) argued the data gathered illuminates aspects of teaching and learning invisible to teachers in their day-to-day activities. As a result of the research, policies and practices can be strengthened to advance children’s rights and well-being (Graham et al., 2013), and make a positive contribution to teaching practice (Alton-Lee, 2001). In light of this, another obligation of the researcher to minimise harm is to ensure the research is methodologically and ethically sound, relevant and rigorous (Graham et al., 2013), so that it does not
waste participants’ time. This responsibility requires academic integrity and honesty by researchers (Cohen et al., 2011; Hamilton et al., 2013; Punch, 2000).

A further consideration regarding the relationship between the researcher and participants is the need for voluntary participation (Alderson & Morrow, 2011; Alton-Lee, 2001; Burton & Bartlett, 2005). To ensure participation is voluntary, ethical principles require participants to have the right to withdraw at any stage during the research without fear of consequences. The researcher has a responsibility to avoid any coercion (e.g., persuasion to participate) which is especially relevant if the researcher is in a position of power over the participants, as is the situation for a practitioner-researcher (Harcourt & Sargeant, 2012; Mutch, 2005).

**Clarity of Expectations**

Participants should have a clear understanding of what is expected of them should they agree to the research invitation (Creswell, 2018; Mutch, 2005). In this way, there should be no deceit concerning the purpose or methods used within the study. Providing clarity of expectations involves a process of obtaining genuine informed consent from the participants (including parental/caregiver and gate-keeper consent) (Alderson & Morrow, 2011; Alton-Lee, 2001; Burton & Bartlett, 2005).

Advice from John et al.’s (2008) study within a medical context, indicated that many children did not understand the concepts involved in a simple study and the risks involved, with some being more capable than others to make responsible decisions. This impacts the ways researchers introduce their requests to participate with school-aged children. Alderson and Morrow (2011) reminded researchers that there is another layer of participant acceptance because the children’s parents need to provide consent for their children. Here the issue of whose interests are being
prioritised needs consideration, e.g., the child’s, parents’ or the interests of the research. From these informing studies, I have accepted my responsibility to provide clear information so that parents can guide and support their children when decisions are made whether to participate or not knowing what the expectations are for involvement. Further to this, Loveridge (2010) acknowledged, that at the beginning of a project it can be difficult for a researcher to know the full extent of participant involvement. Therefore, ‘active’ informed consent, involving ongoing negotiation with participants using simple language is necessary to promote understanding of what participation involves throughout the research process (Alton-Lee, 2001). Mutch (2005 p.77) reminded readers that “the consent and understanding of those participating,” are also necessary when disseminating the study findings.

3.3.2 The Process of Gaining Informed Consent for my Study

I sought and gained ethical approval from the University of Canterbury Human Ethics committee prior to starting my data collection. The documentation included my plan, describing what participation would involve as realistically as possible, including design decisions to ensure the safety of my participants e.g., methods and data sources. Once I obtained approval (early in Term 1, 2019), I then sought written permission from the ‘gatekeepers’ of my research site, including the school Board of Trustees and Principal (Hamilton et al., 2013).

As children were the participants of my study, both parental consent and student assent were required (Finch, 2005). I approached the parents and students in my class about what participation involved so parents were fully informed in order to give voluntary informed consent (Finch, 2005; Hamilton et al., 2013). Informing studies emphasised the need to give careful detail of information to ensure students have a
genuine understanding of their involvement in the research (e.g., Finch, 2005). Loveridge (2010) advised giving consideration to the different values and traditions of the diverse cultures within a New Zealand classroom leading to different approaches to issues such as consent e.g., individual versus collective decision making. Alton-Lee (2001) suggested explaining to the class that the research helps teachers understand how they learn and give time to answer questions. Following this advice, I made myself available to consult with the parents, being sensitive to the cultures within my classroom during the 2019 school year. I used information letters, and a presentation (see appendices A and B) to explain who I was (a practitioner-researcher), the purpose of my research and the students' involvement (what they were expected to do and for how long). Students had the right to withdraw from my research at any stage by communicating this with me (or my teacher aide). Any complaints that arose should be addressed to the ethics committee. The data would be stored safely to protect participant confidentiality and anonymity and destroyed at the completion of my study. Anonymity and confidentiality would be maintained in the dissemination of the study and a summary of the results would be shared if requested. After clearly communicating the aim of my study and my intentions, I provided a consent form to be signed by parents (see appendix C) and a form for assent from my students (simplified version). As a result of this process, I received assent from 25 out of 29 students in my class.

3.3.3 Issues and Challenges When Researching With Young Children

Research involving young children introduces particular ethical considerations due to the power disparities involved (Alderson & Morrow, 2011; Graham et al., 2013; Swain, 2017). As school is compulsory, Loveridge (2010) warned it can be easy to disregard children’s confidence. Children as research participants may feel a need to
please the teacher researcher. Power also affects the relationships between children, so it is important to ensure different views are represented rather than those of a few more articulate participants. Furthermore, young children may be accustomed to others telling them what to do rather than making their own decisions and explaining their actions, which could affect data collection. Researchers acknowledge that issues involving ethical research with children are complex, with no easy answers and ready solutions (Alderson & Morrow, 2011; Bourke, 2017; Graham et al., 2013). Various authors provided knowledge and guidance to address dilemmas when researching with young children which have informed my choices (e.g., Fitzgerald, 2020; Graham et al., 2013; Harcourt & Sargeant, 2012; Loveridge, 2010).

3.3.4 Addressing Ethical Dilemmas as a Practitioner-Researcher

To address ethical dilemmas as a practitioner-research, I sourced literature to find out how other researchers had responded to similar challenges. Four matters arose from Fitzgerald’s (2020) research with young children. Her qualitative research within an Irish school setting was with similarly aged participants (9-12 years), but from an outsider perspective. Firstly, critical reflection enabled her to address power relationships. A reflective approach helped her to ‘think’ her way through the research process and remain aware of her adult biases. Secondly, she took deliberate actions to ensure the safety of participants including, informed consent, pseudonyms for participants, and spending time building relationships and trust. Thirdly, she tailored data collection methods to fit in with class activities including audio recordings, focus groups and group interviews. Fourthly, she reassured students by using non-verbal cues and communicated she was not carrying out a formal assessment of their achievement. As a result, these four deliberate actions
provide useful insights and recommendations for conducting research with young children in formal settings such as mine.

Ethical practice with children concerns the authentic representation of participants' ideas. To this effect, further learnings were gleaned from Harcourt and Sargeant (2012). They suggested students be involved in the analysis of the data, for example, samples of students' work be offered back to participants with the question “What are all these ideas about?” Another strategy they used was observing whether the students were fully engaged in the data collection when communicating their views and perspectives. As a result of this interpretivist stance to data where the teacher and students interact, the co-construction of meaning took place through both the data collection and its analysis.

Two other research studies offered me insights through the incorporation of visual methods. A study by Alderson and Morrow (2011) incorporated methods such as artwork, images and photography. They claimed these methods have the potential to open up children’s responses to research questions when written language can become difficult or laborious. Similarly, Wall (2017) explored the use of three visual methods (e.g., using templates and storyboards) with children aged 4-14 years in compulsory schooling. She found the inclusion of visual data-gathering methods (drawing-based) had the potential to gather children’s perspectives because the tasks were familiar, more inclusive, age-appropriate and the process encouraged thinking time. The incorporation of visual methods presents a further strategy for me to ensure the authentic representation of students’ ideas within my study.

There are also ethical risks to consider when including photographs as a data-gathering method (Cohen et al., 2011). Loveridge (2010) explained that photographs
may create ethical problems since the research involves electronic images that can be easily shared. The purpose of using photographs in my study is to allow students to communicate their maths choices. As normal practice within my classroom includes the use of photographs for learning tasks, I consider photos to be a useful data collection method allowing me to analyse and understand students’ views of agency, such as whether students are aware of choices they have made during a lesson.

I have addressed further ethical dilemmas through **deliberate actions** to safeguard my participants and myself as practitioner-researcher throughout the research process. Firstly, concerning the identity of individuals, I accept that my participants were aware of each other as members of the class so anonymity could not be guaranteed. Neither could confidentiality be guaranteed as my research was carried out within a shared social situation (busy class). To increase the chance that confidentiality would not be breached, there has been a substantial time period between the data collection and reporting making it unlikely for individuals in the class to remember specific conversations. Likewise, I have used pseudonyms for reporting what people did and said in my reported findings. Secondly, the need to keep the children involved in my research safe and ensure no harm was of paramount importance throughout my research process, both in my decisions and actions. I was aware that when I gathered information for research purposes I should follow the ethical principles discussed above to protect both myself and my students (Cullen, 2005; Graham et al., 2013; Mutch, 2005). The consideration of ethics throughout my study (see Appendix D overview) has led to deliberate actions summarised in Table 2 below, grouped into actions to protect the participants’ identity, their relationships and expectations.
Table 2

Deliberate Actions to Address Ethical Dilemmas

<table>
<thead>
<tr>
<th>Ethical Concepts</th>
<th>Deliberate Actions to Address Ethical Dilemmas I have:</th>
</tr>
</thead>
</table>
| Protecting the identity of the participants          | • represented a composite picture of the classroom (Creswell, 2007);
|                                                      | • avoided identifying information e.g. demographics (Alton-Lee, 2001; Cullen, 2005);
|                                                      | • used pseudonyms (Graham et al., 2013);
|                                                      | • restricted access to the data;
|                                                      | • been clear about what was photographed e.g. equipment (no faces);
|                                                      | • taken supervisors’ advice on how to write myself into the study (Creswell, 2007).                                    |
| Protecting the relationship between the researcher   | • used data collection methods to suit classroom activities;
| and the participants                                 | • included methods of visual communication e.g., photos and representations;
|                                                      | • continued with the maths challenge routines as I typically would;
|                                                      | • prepared for the data collection in my own time;
|                                                      | • been mindful of any stress caused (Cullen, 2005) e.g., low self-esteem and negative changes in behaviour;
|                                                      | • listened to what students had to say during data collection (Scott, 2008);
|                                                      | • established a rapport;
|                                                      | • interviewed on home ground;
|                                                      | • allowed ‘don’t know’ answers;
|                                                      | • used multiple data sources to avoid bias;
|                                                      | • maintained a researcher journal to assist in critical reflection;
|                                                      | • altered groups regularly ensuring students were supported;
|                                                      | • involving students in collaboratively reviewing and interpreting the data (Burton & Bartlett, 2005) e.g., through clarifying during interviews;
|                                                      | • respected individuals’ cultural experiences and capabilities (Graham et al., 2013);
|                                                      | • regularly updated my principal;
|                                                      | • made my colleagues aware of data collection lessons;
|                                                      | • represented a range of student views;
|                                                      | • ensured non-participants were not excluded from learning or put under stress or peer pressure for their decision (Alton-Lee, 2001; Cullen, 2005); and
|                                                      | • avoided non-participants in group recordings.                                                                     |
| Clarity of expectations                              | • communicated honestly what my research involved (Bourke, 2017);
|                                                      | • included the use of photos in information letters to parents and children; and
|                                                      | • made participants aware of when they were participating in research (Graham et al., 2013).                           |

The class numbers were stable during the time of the data collection with no children leaving or arriving into the class. I catered for student absences on the days of the data collection by adjusting the composition of the maths groups (normal practice), but the data collection was able to go ahead as planned.
3.4 Data Collection Methods

In this section, I explain my data collection methods and their relevance related to fostering student agency in the maths classroom. My choice of data collection methods is informed by how other researchers collected data around similar concepts such as agency in the classroom. Consideration of the strengths and weaknesses of each method enables me to identify possibilities and make choices to strengthen my study design. I also comment on any ethical considerations of particular data collection methods.

My aim was to use multiple sources of data to develop an in-depth understanding of my research topic from various angles, as multiple data sources add to the robustness of a case study design (Creswell, 2018; Creswell, 2007; Yin, 2009). Many forms of data already existed as part of my normal classroom programme (e.g., student maths workbooks) with the potential to be included in my consent process for research purposes. The data collection methods selected included:

- video recordings
- participant observation;
- focus group interviews using lesson videos to stimulate recall; and
- student-generated data (including student agency reflections and photographs, students’ work samples and recorded student voice); and
- researcher journaling (field notes).

I now discuss the strengths and limitations of these data collection methods, justify the reasons for choosing them and reflect on the data gathering process. I begin with the first of five observation strategies, video recordings of the lessons.
3.4.1 Video Recordings of Lessons

**Intent**

Video was used to record the whole class during the maths challenge lessons, selected maths groups and student voice. The purpose was to collect data about student choices, thoughts and actions, during the maths challenges. Video recordings provided two different views of the classroom activity, as well as students' reflections and explanations (on iPad), enabling analysis after the lesson had taken place.

The work of other researchers informed my decision, providing examples and possibilities of how videos can be utilised within similar classroom settings. Firstly, Gresalfi et al. (2009) collected data using video recordings and supporting observation notes of 10-12 lessons from three urban middle school classrooms (6th, 7th and 8th grades). Recordings were taken in each classroom from two viewpoints, one focused on a group, and the second recording took an overall view of the working classroom. A similar set-up is seen in the Yackel et al. (1991) study with younger students, where their recordings were reviewed daily and enabled the modification and adjustment of future lessons. It is this iterative process of analysis that aligns closely with my teaching practice of planning for learners' next steps. An example of gathering student voice using iPad apps is demonstrated by Larkin and Jorgensen (2016) with Australian students in Year 3 and 6. They used a video diary technique where students verbally recorded their ideas using an iPad application to gather data on the students’ feelings and attitudes in maths. Using the iPad in this way allowed the children control over the data collection process and was "a mechanism to reduce the power imbalance and minimise the tendency of children to..."
seek to please adults” (p. 8). The majority of the students in their study were comfortable sharing their ideas on the iPad. Recording thoughts on the iPad removed the requirement for students to record ideas in written form, so increased the opportunities to express thoughts and feelings, and the device was easily operated by the students. Noyes (2004) warned that the richness of the data produced can be more difficult to analyse requiring more knowledge of the context and students.

The use of recording devices has strengths and weaknesses. They capture the lesson, so that the data can be analysed post-lesson (Taylor et al., 2015), and enhance the participant observer’s lens which is restricted by their primary role of teaching (Alton-Lee, 2001). As a result, video recordings enable me to quickly review the lessons to inform future teaching decisions and to remember what happened when writing post-lesson reflections. A study of several recordings presents the possibility of observing multiple class interactions, helping raise my awareness of any issues. Whole class videos enable the corroboration of information with other data such as researcher journal reflections and student documents. The small group recordings can be used within post-lesson interviews to assist students’ recall of what happened (stimulated recall, discussed later in 3.4.4), adding to the trustworthiness of my study. The use of video recordings as a data source has strengths, as well as limitations. Challenges of video (or sound recordings) are the resulting change in behaviours of the children and teacher who are being observed (Burton & Bartlett, 2009), the recording captured on camera is dependent on the positioning and focus of the camera, and students may unintentionally block the camera’s view (Cohen et al., 2011). As a result, actions need to be taken to ensure that the recordings capture as ‘natural’ and useful a scene as possible.
A trial lesson was undertaken to ensure the management and organisation of video equipment (e.g., iPad, video, computer). The aim was to minimise technological glitches in the recordings, to practise managing the equipment and to socialise the students to the devices. As a result, a video recorder was used to record whole lessons, strategically placed computers (running Photo Booth) were used to record group interactions and iPads, to record individual student voice (using Explain Everything). As laptop computers were equipment normally present in the classroom, they were available, easy to access and use to review different parts of the recording during the interviews. In this way, the presence of the recording devices were less likely to change students’ behaviour during the data-gathering lessons (Alton-Lee, 2001). The recording devices, therefore, became a normal feature of the classroom instead of something novel.

**The Process of Video Recording**

The main video camera was set up prior to each lesson at the back of the classroom. A student pushed ‘record’ at the start of the lesson. Group videos were recorded using computers focused on specific groups working on the maths challenges (consented students only). The composition of student groups was regularly altered in response to student needs, as is normal class practice, and to enable the students to work with different people. Groups of students were enthusiastic to be videoed and later share their ideas in an interview situation, so I kept an updated list to ensure different groups each had a turn. A quick check-in with the students prior to recording ensured they were happy to talk about the video of their group in a post-lesson interview (ensuring ongoing voluntary participation).
Student voice was collected using the iPad application *Explain Everything* (e.g., setting up the class, L1-3, 5, 8). Student voice recordings involved students talking into an iPad application (*Explain Everything*) in response to teacher prompts, on aspects of the lessons that were a focus e.g., How well did your group support each other? and How did you work out today’s solution? Several children were selected from volunteers, handed the iPad with the application ready to record, given a prompt and allowed outside to record their responses. For example, when the lesson focus was on establishing small-group norms, several students recorded their reflections on how well their group had worked during that day. On other days, students were asked to explain (using *Explain Everything*) how they had worked out their solutions. The verbal recordings provided assessment information and supported future teacher planning, enabling me to transcribe the data for later analysis.

**Ethical Reflections on the use of Video**

Viewing the class video recordings contributed to my reflections and they were not shared beyond this. The group videos were replayed during the focus group interviews (discussed in 3.4.3) and also used for researcher reflections. Below is an example of a short extract from a transcribed group video recording illustrating student group interactions during Lesson 10 (using pseudonyms). In this way, the visual identities of students in my class were protected. I reassured the students and their caregivers that pseudonyms would be used in my reported findings.

16.30 minutes - interaction between a focus group and a visiting student (Anne)

Anne  *I get it, I get it. Do you get it?* (Interaction between the group and a visiting student from another group)

Nora  *No. The score.*

Maya  *So you got 11.7 is that right?*
Anne: I got it.

Maya: So do we plus all these together? We plus all these together to get the total score? I think we wait for Miss.

Anne: I am still trying to work out my total score. Mine looks like that. We need help with the scores ...

Teacher: Right, what is your question? (Teacher arrives).

iPads were usually used in the classroom as a digital learning tool with students making recordings as an activity to encourage explanations, reflections, and the communication of ideas, such as sharing learning with families. Used in this way, iPad applications presented an authentic and easy way to gather student ideas (expressed verbally) on aspects of this study within lessons. The student recordings were transcribed and not shared beyond this.

3.4.2 Participant Observation

**Intent**

Participant observations involve the researcher as a participant (in some way) in the activities being observed (Mutch, 2013; Yin, 2009), such as a teacher making observations in their own class lessons. The observer makes detailed notes of what they see, hear and do (Mutch, 2013), leading to an in-depth description of “the natural setting of the case” (Yin, 2009 p.109). This could involve plans of the classroom layout, a description of the environment, discussions and interactions as influenced by and seen in Lampert’s (2001) research. For my study, my position as the teacher meant I could observe student interactions and actions in response to my teaching decisions in real time and context. For example, participant observation in the maths classroom enabled me to consider students acting naturally in an authentic context during lessons, when they responded to others in their group, the types of discussions that ensued and the representations utilised to think about and
communicate their ideas to others. Observations also allow flexibility in their purpose (Burton & Bartlett, 2005), as they can focus on one-off activities, specific behaviours or immersion in the setting (Mutch, 2013). For example, the observations could be either unstructured or carried out using an observation schedule to order and collect data according to the focus of the study (Burton & Bartlett, 2005; Creswell, 2018). By using participant observation, I could also use other forms of data collection to strengthen the data collected, such as interviews.

When using participant observation, my aim was to be aware of and use strategies to mitigate the limitations of this data collection method. I was aware of possible power implications in the way the observations were carried out e.g. their formality and openness (Burton & Bartlett, 2005). For example, in order to avoid the children under scrutiny feeling burdened, I endeavoured to remain as relaxed and natural as possible. I paid attention to matters of sensitivity and respected the research site by showing interest in all students’ ideas and responses (Mutch, 2013). In terms of my own presence on the research site, I was aware that I too could influence the situation by behaving differently or pre-empting students’ ideas, so reflexivity was important (Cohen et al., 2011; Johnson et al., 2020; Mwita, 2022). In my role as the teacher, I endeavoured to make students aware of opportunities and to encourage their thinking and ownership when practising solving maths challenges. But in doing so, my aim was not to put ideas into their heads prior to data collection or unwittingly direct students’ decisions. This was important, as the purpose was to observe students’ agency and perspectives in a classroom environment that include the teacher’s explicit and implicit guidance. Furthermore, as a teacher and researcher in my own classroom, I acknowledge that the busyness of classroom life presented me
with the continuing challenge of juggling the demands of teaching whilst researching (Alton-Lee, 2001).

**The Process of Observing**

Observations contributed data about how students were acting in response to maths challenges. I made observation notes post-lesson in a research journal by writing what I noticed. In addition, these observations were supported by the use of video recordings which could be reviewed post-lesson. I designed two observation schedules (see Appendix E) to help focus on aspects of the maths lessons associated with agency. The first schedule was unstructured including questions to help focus on student actions and the second was structured to identify student and teacher actions at intervals (10 minutes) throughout a lesson. The observation data, therefore, includes both field notes and data from the two observation schedules.

**Ethical Reflections on the use of Observations**

In providing a rich description of what was taking place, the challenge I faced was how to observe closely and not take things for granted. The observations were supported with video recordings. The position as observer allowed me to describe the environment, discussions, interactions and behaviours of groups of students and the class in general through detailed note-taking. Being present, gave me the advantage of being able to interpret meaning from the tone of voice of students, the speed of speech, gestures and other non-verbal communications, as well as the levels of student engagement in the moment. This is the same kind of challenge facing teachers in their daily practice. Observing can also be considered as snooping (Burton & Bartlett, 2005), so I made the students aware that they were sharing their ideas on agency in maths to inform research on teaching and learning. The observed
maths challenge lessons were purposely spaced out (once a week) to allow students a break between lessons when they were under scrutiny. Participant observations were supported by focus group interviews, which I now discuss.

3.4.3 Focus Group Interviews (Using Video-Stimulated Recall)

Interviews are a suitable way of collecting data in a qualitative study because they are opportunities for participants to talk about their lived experiences (Seidman, 2013). Several studies informed my decision to use interviewing, demonstrating how they could be used in combination with other data-gathering methods and the possibility of using focus groups. In the first example, Sengupta-Irving (2016) interviewed students in pairs using semi-structured interviews of about 20 minutes in length. The interviews gathered data on the students’ reflections of their experiences of the maths lessons and their views of learning. The interview data was transcribed and coded (for students’ views on maths learning opportunities and agency). In another example, Wang and Fan (2021) used focus group interviews (5-7 peers) to gather students’ views on their use of textbooks in maths, and this data supplemented classroom observations and questionnaires.

As a data collection method, interviewing has a number of strengths. The researcher uses open-ended questions which can be either unstructured or semi-structured and non-standardised (compared to opinion polls, questionnaires, and surveys) (Taylor et al., 2015). The respondent’s explanations provide insight into the topic (Creswell, 2007; Yin, 2009), interviewers may pick up non-verbal clues such as pleasure or disappointment (Burton & Bartlett, 2009), and reading ability is not a concern (Mutch, 2013). Another strength is that the interviewer has the flexibility to assess and adapt
to responses and to follow unexpected lines of inquiry (Burton & Bartlett, 2009; Taylor et al., 2015).

Interviews are not without challenges for the researcher. When interviews are unstructured or semi-structured, there can be more variation in data which becomes difficult to analyse later (Burton & Bartlett, 2009). The data collected depends on students’ willingness “to tell the truth” or remember details of the learning experience (Wang & Fan, 2021 p. 681). When students are involved in school-based research projects, they are generally willing participants but may be easily persuaded and at times naïve (Groundwater-Smith, 2011). It is possible for the interviewees to respond with what they think the interviewer wants to hear. Researchers may put students in positions of vulnerability, but by not consulting students about their experiences, researchers risk losing their valuable contributions. When designing research studies, there is also the question of how many interviews to conduct. Seidman (2013) explained that after completing several interviews, the researcher comes to a point when new interviews reveal no new insights.

Focus groups present the possibility of interviewing students in small groups. Mutch (2013) explained that these generally have pre-set questions, but allow some negotiation of responses. She described focus groups as a useful tool for busy practitioners as they take less time compared to individual interviews. Focus groups have the benefits of group interaction, gathering many participants’ views in a limited time and possibly encouraging some students who may be reluctant to talk (Cohen et al., 2011; Creswell, 2007). There are disadvantages of using focus group interviews. Here the researcher needs to be aware that some participants may dominate the interview, making it difficult to know if the group dynamics have
changed the data (Burton & Bartlett, 2009). In addition, the interviews can be difficult to transcribe as there are more people speaking (Mutch, 2013).

**Intent**

I selected focus group interviews (semi-structured) to gather students’ views after the maths lessons had taken place. My rationale was that teachers carry out informal interviews every day as part of their work. I considered interviewing to gather student views as a potentially valuable data source, especially if done immediately after the maths lesson. My purpose was to help me gain an understanding of the students’ experiences of the lesson (Seidman, 2013), in particular, their choices and the reasons for these choices. By utilising some prepared questions, I could guide the conversation, but also respond to students’ ideas and information. Small focus group interviews enabled me to take advantage of the benefits of group interaction (Creswell, 2007), as the students had just spent the last hour working together on the same collaborative task and felt supported by their peers, encouraging quieter students to share their ideas. Also, focus groups made better use of the limited class time I had available to carry out the interviews.

I decided to strengthen the group interviews using video-stimulated recall. Playing back video in a semi-structured interview can support students to remember and focus on specific phases of a lesson (Meier & Vogt, 2015). As a result, video-stimulated recall acts as a stimulus for discussion and can help students reveal the meaning they are making from the experience (DeWitt, 2008). By gathering data in this way, the students’ views about their thinking and actions are respected. Open-ended questions such as: what were you doing? And why? can be used to gather students’ thoughts on their own learning.
The Process of Interviewing

As a result of this decision, a total of 13 in-depth interviews were carried out throughout my study. I interviewed 1-2 groups of 2-3 students (from my pool of volunteers) post-lesson about their choices and what they were thinking at the time. I interviewed different groups after each of 8 separate lessons, to gather a range of student views. A rapport was already established with the students and the interviews took place within the classroom space while others were working independently. The group video recordings were played back to the students to stimulate their recall of their group task and help them reflect on the experience. A set of key questions (see Appendix F) were used that allowed for open-ended answers. To ensure understanding of students' ideas, meanings were clarified throughout the interviews using prompts e.g., tell me more, and revoicing e.g., “You are saying…”, “Is that what you are saying?” (Lesson 1 interview). In this way, the key ideas were confirmed (Hamilton et al., 2013). Before closing each interview, the content was summarised and confirmed with the students. The interviews were audio recorded to allow me to concentrate on my students’ responses and the challenge of multi-tasking (Burton & Bartlett, 2009; Seidman, 2013), which I later transcribed for analysis. Although offered the opportunity (at the end of each interview), no students asked to view transcripts.

Ethical Reflections on Focus Group Interviews

I considered the example of Sengupta-Irving (2016), where the team of teacher-researchers avoided interviewing students from their own class. But despite the possibility of researcher bias, in consultation with my supervisors, I decided it was preferable for me to be the interviewer. This was because as the teacher I knew the
content and sequence of the lesson, had established a rapport with the students and possessed background knowledge of the case study literature. During the interviews, video-stimulated recall helped the students to remember what had taken place during the lessons. It also confirmed that the students in my study acted as they described, increasing the credibility of the interview conversations. The prepared interview schedule helped guide me as the interviewer, providing a starter, prompt, probe and ordered questions to follow up (Creswell, 2007). I was aware of the need to remain focused on the students’ responses to the questions, showing interest and listening carefully. As a practitioner-researcher, I also needed to avoid exerting dominance in my role as the classroom teacher. The prompts helped me avoid introducing bias of poorly constructed questions and influencing my respondents during the interviews (Burton & Bartlett, 2009; Yin, 2009).

3.4.4 Student-generated data

Intent

I decided to collect two forms of student-generated data relevant to my study. The two methods were:

- student responses to reflective questions including photographs, which I call student agency reflections (SARefls); and
- individual student documents.

The range of techniques involved different methods of communication (visual and written), utilising students’ different communication strengths. The student-generated data were collected at different times throughout the data collection period, with the potential to identify changes in students’ thinking over time and student responses to different lesson prompts. In this way, collecting student-generated data from different
individuals, in different forms and times, added depth to the data gathered and contributed to the dependability of my study. Each data source is now described in detail.

**Student Agency Reflections (SARefls)**

Student agency reflections (SARefls) were a form of student-generated data that included both photographs and written reflections. I asked students to gather photographic evidence (using the computer) that illustrated 2 good choices they had made during the maths challenge, and to insert the photos into their digital journal presentations (see student SARefl instructions below):

*Maths Journal Task: Thinking about our learning in maths*

*Insert 2 photos that you believe show evidence of good choices you have made to help solve today’s maths problem.*

Students were then asked to write 3 sentences (using sentence starters I provided) to explain what each choice was, why they made it and how it helped them find mathematical solutions:

*Use the following sentence starters to explain your photos:*

*Today, choices I made during problem-solving were …*

*I made these choices because …*

*This helped me to solve the maths problem/s by …*

In this way, the student agency reflection (SARefl) included photographs taken by the students and a written response to the sentence starters to explain and justify their choices (see the example in Figure 1 below).
Chapter 3: Methodology

Figure 1

Example of a SARefl From Lesson 2

Today, choices I made during problem solving were using times tables and written multiplication.

I made these choices because it could help me work out everything easier.

This helped me to solve the maths problems by adding up faster to the answer.

The design of the SARefl task was informed by several studies. Firstly, Burton and Bartlett (2005) viewed a research log (a type of document) as a useful way of creating written records of participants’ feelings and reactions to chosen aspects of classroom life. Fielding-Wells et al. (2017) used research logs within a maths classroom (see section 3.2.1) involving students writing a short diary to investigate motivation and engagement. Student self-reflection was used in my normal classroom programme, so I considered that the completion of the SARefl documents would fit in with classroom practice. The SARefl documents provided insight into student choices to solve maths challenges, their explanations and justifications for these choices, and the documents could be gathered to review at a later date.

Use of photographs

Similarly, my decision to include photos as a part of the SARefl was informed by literature. Burton and Bartlett (2009) suggested photos can deepen a researcher’s understanding of children’s experiences in classroom-based research. Photos can support a student explanation and illustrate what has been observed by the researcher. By involving the participants in taking the photos, student insights can be
brought into the research (Creswell, 2016), giving them power or voice (Mills et al., 2010). The use of photography can be a positive and empowering experience for children as they are engaged and their contribution is valued (Whiting, 2015). However, incorporating students’ photographs as key documents introduces other strengths and weaknesses. To ensure the quality of the data, Loveridge (2010) advises that the task needs to be understood by the students and the images be of reasonable quality. When analysing the photo, it is also necessary to be aware of the ‘snapshot’ nature of the images and that the picture has been framed with part of the scene left out of the photos (Burton & Bartlett, 2005). As students within my classroom normally used photographs within digital presentations to support their learning, I asked students to take photographs of their choices as a way to support their written reflections (making up the SARefl document). It also gave students that found written communication difficult an alternative method to share their ideas.

**Individual Student Documents**

Student documents, the second student-generated data, includes students’ written work (writing and drawing) in their maths books (hard copies), as well as digital records of students’ work on the computer (Padlet posts). Documents were a suitable and easy way to gather data from students as part of a case study (Mutch, 2013). My decision to use students’ documents was informed by several studies indicating the usefulness of these documents when used in combination with other data collection methods to strengthen studies, as well as how they were analysed. Selling (2016) examined individual student's written work from their maths journals showing the representation of their maths ideas. In a similar way, Lampert (2001) used student work samples copied from their notebooks, in combination with records of assessment as data sources.
Collecting document data has strengths and challenges for researchers. Documents can be collected unobtrusively, provide information over a long time span and allow for later review (Burton & Bartlett, 2009; Yin, 2009). But, a limitation of documents by themselves is that they do not show how they are interpreted by the writer of the document (Burton & Bartlett, 2009). As a consequence, triangulation by other data sources is advisable. Document data can be used to enhance or corroborate other data and as stimuli for interviews. Yin (2009) also warned that bias can be introduced by focussing on selected students’ work samples. As a result, care needs to be taken not to deliberately select or withhold some students’ work.

**The Process of Gathering Student-Generated Data**

I collected student-generated data from 25 students (consented). SARefls were collected from each of my student participants in order to understand whether students were aware of their choices and how they perceived their choices when they solved maths challenges in the classroom. During the trial lesson, the SARefl document was posted to all students as a shared document on the computer, allowing individuals in the class to view others’ responses. After considering the data that was collected, modifications were made to encourage students to communicate their original ideas. As a result, the reflection documents for Lessons 1-6 were placed into personal computer folders for each student, so responses were private to each individual. This ensured that the responses were from individuals allowing them to communicate their own ideas, rather than adopting a collaborative approach. For ease of storing and ongoing access, these individual responses were later collated back into a single document. Some of the SARefls included photographs taken by the students, as not all students took up this opportunity. I collected 163 SARefl
samples over the course of the data collection period. Students' documents (132) were collected at the end of lessons (L1-4, 7 and 8), and copied.

**Ethical Reflections on Student-Generated Data**

In line with the rest of the maths programme, I showed respect for students’ views by showing an interest in their ideas. I deliberately encouraged students to show their thinking through mathematical representations, both as a learning tool and to help communicate their ideas to others. Sharing ideas was a normal part of classroom practice due to the classroom norms. As the study proceeded the students became familiar with the reflection document and instructions (SARefl), and were happy to participate. It was important that the reflections represented individuals’ views, so I encouraged honesty in their responses, taking care not to direct their thinking (beyond the use of the prompts) or put ideas into their heads. Continual review of the data collected led to the modification of the SARefl instructions from Lesson 3 to read, “Today equipment and methods I used to help me...”, encouraging students to focus more on choices of mathematical methods and equipment, rather than choices related to personal and group management. I now explain the last data collection method, researcher journaling.

### 3.4.5 Researcher Journaling

**Intent**

Durdella (2019 p.2) described researcher journaling as a strategy to “mitigate the researcher effects on the study’s design and methods” and a way to build trustworthiness into a study involving a practitioner-researcher. The researcher journal acts as a reflective tool to support reflexivity/ critical engagement with the role of researcher and influence of the practitioner-researcher during the research
process (Durdella, 2019; Mutch, 2013). When involved in qualitative research “your subjectivities – identity categories, personal characteristics, and perspectives – all shape who you are in the field and the roles that you assume in your study” (Durdella, 2019 p.6), influencing the design, data collection and interpretation. So, the act of journaling can provide a record of thoughts for further analysis, reflection or charting of research progress and be used as a data source (Street, 1990). The process of writing can help to make conscious the unconscious, which is an opportunity to recognise taken-for-granted assumptions, identify values and feelings, and reveal issues and theories at work. Mutch (2013) described the practice of keeping a researcher journal as helpful to articulate frustrations, work out problems as well as highlight successes, presenting possibilities for the transparency of my study. An example of how researcher journaling can be used to strengthen a study is seen in Lampert’s (2001) work. She used journaling in her role of practitioner-researcher to document issues and decisions she had made in the maths classroom. Using a narrative style, and first-person, her study described the problems and decisions she faced daily in the classroom. This allowed her to acknowledge both her position as researcher and teacher in the classroom.

I made the decision to use research journaling as a data collection method within my study based on my reading of how other practitioner-researchers had approached their research. As a practitioner-researcher, I needed to be aware of my influence within my case, both in decisions around data collection, as well as ethical considerations. Researcher journaling provided me with an opportunity to establish a list of guiding principles and to keep reviewing these. It allowed me to think through when I was collecting data for my study and the consequences of this for my teaching. Another reason for including researcher journaling was the resulting written
record of thoughts and decisions throughout the research process that could be referred to at a later date.

**The Process and Ethical Reflections on Researcher Journaling**

Records were regularly updated in my research journal including teaching decisions to address student needs, thoughts and decisions on data gathering and ethical considerations to ensure the trustworthiness and rigour of my study. Examples of researcher journal entries below show the nature of this reflection/reflexivity. The first indicates addressing students’ needs through deliberate teacher decisions to include a wall display, the facilitation of a class discussion about choice, and questions/prompts to encourage maths talk about choices students had made.

> 30.4.19 I made a wall display titled “Student Agency – having the power to act.” A question was posed: Making choices but what choices? I co-constructed ideas with the students on what this meant and added the students’ ideas to the display. I deliberately modelled using vocabulary associated with multiplication when working with small groups and encouraged students to talk/think about the choices they made.

A second entry on the same date records my thoughts as I transcribed data from a post-lesson interview. I noted the need to use wait time before asking the next question to improve my interviewing technique and encourage students to share their perspectives. The need to ensure that one student speaks at a time was also noted, so that I could transcribe their responses appropriately.

> 30.4.19 continued…

> I transcribed the group interviews. I need to take care that I don’t butt in with the next question immediately after a student has responded. Also, that one student answers at a time. The tape was easy to hear.
Another reflection was made when reviewing the student agency reflections (SARefls) concerning the timing of the lesson and data collection.

30.4.19 continued…

I looked at the students’ work in response to the question prompts – some students found it difficult to identify the choices they had made. Photos were difficult for students to take because they were completing the reflection questions after the lesson and had already packed up their equipment etc.

As a result of this entry, I was more aware of the timing of the lesson and ensured the students had the opportunity to take photos of their choices before the maths materials were packed up at the end of the lesson.

3.5 The Timeline

The timeline for the main data collection was lessons during Terms 2-3, 2019. These lessons were all mixed group maths challenge lessons involving all of the class. The original plan was for the main data collection, including all of the methods, to be collected from consented students weekly, over Lessons 1-4 (Term 2, 2019). This was due to my limited resources, my concern to focus on my primary role of teaching and to minimise the disruption to the class programme. The original four lessons went very well and the participant consent continued into Term 3, so I was able to continue to gather data to follow themes of interest in response to my research questions. As a result, the data collection continued in some forms only, throughout the remainder of Term 2 to the end of Term 3 (making a total of 11 Lessons) to extend lines of thinking and to add more depth to the data collection. Additional data were collected through student voice recordings, explanations on Explain Everything (iPad), interviews, student agency reflections and the collection of students’ work documents. The timeline of the investigation is outlined in Table 3 below.
Table 3

Timeline of my Study

<table>
<thead>
<tr>
<th>2018</th>
<th>Term 1 2019</th>
<th>Term 2 2019</th>
<th>Term 3 2019</th>
<th>Term 4 2019 - 2022</th>
</tr>
</thead>
<tbody>
<tr>
<td>Proposal</td>
<td>Researcher journal entries</td>
<td>Trial lesson</td>
<td>Lesson 7-11</td>
<td>Researcher journal entries</td>
</tr>
<tr>
<td>Ethical</td>
<td>Observations</td>
<td>Lesson 1-6</td>
<td>Researcher journal entries</td>
<td>Data analysis</td>
</tr>
<tr>
<td>approval</td>
<td>Collection of student voice</td>
<td>Researcher journal entries</td>
<td>Focus group interviews</td>
<td>Findings</td>
</tr>
<tr>
<td></td>
<td>Informed consent</td>
<td>Video - whole class</td>
<td>SARefs</td>
<td>Writing thesis</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Focus group interviews</td>
<td>Collection of student voice</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>SARefs</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>Collection of student voice</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

3.6 Data Management

Following Creswell’s (2007) advice, all data collected throughout my study was organised, stored on a computer and backed up. A list of all the data collected was maintained and updated (see Table 4 below).
Table 4

Summary of Data Collection Methods

<table>
<thead>
<tr>
<th>Kinds of data</th>
<th>Quantity / content</th>
<th>Research question addressed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Participant Observation</td>
<td>Observations of 11 lessons</td>
<td>How do students respond when there are opportunities for choice?</td>
</tr>
<tr>
<td></td>
<td></td>
<td>What teaching strategies (or actions) promote students to take agency in maths?</td>
</tr>
<tr>
<td>Researcher Journaling</td>
<td>Planning decisions</td>
<td>How do students respond when there are opportunities for choice?</td>
</tr>
<tr>
<td></td>
<td>Field notes (post-lesson)</td>
<td>What teaching strategies (or actions) promote students to take agency in maths?</td>
</tr>
<tr>
<td></td>
<td>Reflections</td>
<td>What are the implications for my practice and beyond the immediate context of the particular</td>
</tr>
<tr>
<td></td>
<td>Reflexivity</td>
<td>maths classroom?</td>
</tr>
<tr>
<td>Video recordings</td>
<td>4 whole class lessons</td>
<td>How do students respond when there are opportunities for choice?</td>
</tr>
<tr>
<td></td>
<td>13 group recordings</td>
<td>What teaching strategies (or actions) promote students to take agency in maths?</td>
</tr>
<tr>
<td></td>
<td>21 Student voice recordings</td>
<td></td>
</tr>
<tr>
<td>Focus group interviews</td>
<td>1-2 interviews after each</td>
<td>How do students respond when there are opportunities for choice?</td>
</tr>
<tr>
<td></td>
<td>of 7 lessons (13 in total)</td>
<td>How do students explain and justify their actions in choosing methods and equipment?</td>
</tr>
<tr>
<td></td>
<td>2-3 different students in</td>
<td>What teaching strategies (or actions) promote students to take agency in maths?</td>
</tr>
<tr>
<td></td>
<td>each focus group</td>
<td></td>
</tr>
<tr>
<td>Student generated data:</td>
<td>22-25 students’ SARelfs</td>
<td>What teaching strategies (or actions) promote students to take agency in maths?</td>
</tr>
<tr>
<td></td>
<td>(photos and reflections)</td>
<td>How do students respond when there are opportunities for choice?</td>
</tr>
<tr>
<td></td>
<td>Work samples (L1-4,7,8)</td>
<td>How do students explain and justify their actions in choosing methods and equipment?</td>
</tr>
</tbody>
</table>

3.7 Completion of data collection

During the data collection, a continual reflection and evaluation process took place that informed future lessons. The 163 SARelfs, 132 documents, 13 group recordings and interview transcripts, 5 videos and 21 voice recordings were listed in a table.
Key phrases were highlighted from the interviews and SAREfls to identify the main ideas and indications of students taking agency. The process of analysing the data and updating the tables (see Table 6 and Table 18, Appendix G) gave an overview of the ideas gathered and indicated when the data collection was complete. These indicated few new ideas were being revealed in consequent lessons. After discussion with my supervisors, the decision was made to move to the analysis stage at the end of Term 3. Table 5 (next page) provides a summary of the complete data set for this study.
### Table 5

**Summary of the Complete Data Set**

<table>
<thead>
<tr>
<th>Researcher Journal Lesson</th>
<th>SARefs (including photos and written reflections)</th>
<th>Student Documents</th>
<th>Focus group Interviews and Group Recordings</th>
<th>Whole class Video</th>
<th>Student Voice Recordings</th>
</tr>
</thead>
<tbody>
<tr>
<td>Setting up the class</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Trial lesson</td>
<td>15 students</td>
<td>Interview a</td>
<td>13 min video</td>
<td></td>
<td>11 Explain everything</td>
</tr>
<tr>
<td></td>
<td>1</td>
<td>Interview b</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>21 students</td>
<td>12 documents</td>
<td>Interview a</td>
<td>46 min video</td>
<td>2 Explain everything</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Interview b</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>20 students</td>
<td>23 documents</td>
<td>Interview a</td>
<td>57 min video</td>
<td>2 Explain everything</td>
</tr>
<tr>
<td></td>
<td>2</td>
<td>Interview b</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>18 students</td>
<td>30 documents</td>
<td>Interview a</td>
<td>60 min video</td>
<td>2 Explain everything</td>
</tr>
<tr>
<td></td>
<td>3</td>
<td>Interview b</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>20 students</td>
<td>27 documents</td>
<td>Interview a</td>
<td>46 min video</td>
<td></td>
</tr>
<tr>
<td></td>
<td>4</td>
<td>Interview b</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>16 students</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>20 students</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>23 documents</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>17 documents</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>18 students</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>17 students</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>16 students</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Total: Trial + 11 lessons</td>
<td>163 SARefs</td>
<td>132 documents</td>
<td>13 interviews</td>
<td>5 videos</td>
<td>21 voice recordings</td>
</tr>
</tbody>
</table>

#### 3.8 Initial data analysis

The data collected in my investigation were analysed in a variety of ways using a thematic analysis process described by Mutch (2013). This qualitative data analysis process involved:
• reduction - browsing with an open mind, highlighting things of interest (e.g. notes in the margin), coding into loose categories, comparing, contrasting;
• display of the categories - grouping and labelling, attempting to understand what the data is showing;
• conclusion drawing – identifying patterns, themes and implications; and
• verification – returning to the data, checking if the themes are valid and consistent (Hamilton et al., 2013; Mutch, 2005).

I began by transcribing the recorded interviews. As I listened to the recordings, I was alert to possible codes that could be used to group ideas (e.g., methods, equipment, representations, where in the class and who with) and noted things of interest such as students’ justifications for making a particular choice (e.g., clarity, accuracy, speed, communication). Below is an example of a partial transcript from a focus group interview after Lesson 1, using video-stimulated recall (L1 Int Keith, Anne, Maya):

Teacher     OK. What choices did you make in this lesson?
Keith        Good choices.
Teacher      Can you tell me some more?
Anne         We made the choice of a table to write out the answers.
Teacher      OK.
Keith        We’ve respectfully disagreed.
Teacher      With each other? So, you were watching and explaining and asking questions. Cool.
Maya         We were talking with each other and not fighting.
Teacher      Awesome. What methods and equipment helped you to solve this problem?
Anne         A table… to help us remember what we had been working on and to see the answers.
I started to analyse the data by identifying choices and actions that the students described, such as making a table and choosing to disagree with their peers. The analysis of the interview data used a similar approach to Sengupta-Irving (2016). She created codes to identify students’ perceptions of teaching practices in maths. The frequency of each code was counted and displayed using tables to give a sense of the importance of each idea within and across the interviews. In a similar way, Mutch (2013) described using raw data grids (p.155) as a way of reducing the data into totals or columns. I began the analysis process by using tables to display and summarise the range, and later, frequencies of possible codes. Actions indicated by the students were identified, to give an idea of the range of responses to opportunities for choice, and then displayed in tabular form (see Table 6 below). Student actions were recorded if they were described in a different way, but not if they were repeated. Once in the table form, the student actions were compared and contrasted across lessons and data sources to identify patterns.
### Table 6

**Establishing Initial Codes From Focus Group Interviews (Video Stimulated Recall)**

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Quantity/Content</th>
<th>How do Students Respond to Opportunities for Choice?</th>
<th>Range of Actions Described by Students</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2 group interviews</td>
<td>listened, worked out together, didn’t give up, table, disagreed/ agreed, talked, skipped counted, carried on from what just done, added, wrote down, explained, pattern, showed.</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>2 group interviews</td>
<td>times tables, adding, multiplication, what I already knew, drawing, fingers, listening, writing, argue, talking, thinking, telling, written multiplication, table, subtract, written form, equations, showed, disagreeing.</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>2 group interviews</td>
<td>area and perimeter, working together, use the square sheet, pencils, colour, 17 rows of six, times tables, got the numbers, measured, figured it out, times it, looking at sheet, using our books, reading, put down information, writing, (look for) similarities, let people speak, listen, try it out, agree, tell, equations, pictures, explaining, looking, listening.</td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>2 group interviews</td>
<td>both participated, times tables, drawing pictures, pencil and paper, used fractions, reading the question, discussing our ways, agreeing and disagreeing, debate, realised we did something wrong, halve the numbers and halve them again.</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>No interviews</td>
<td>Worked it out in our heads, counted them at the start, wrote down, looked at the pictures, reading, showing them our working, made a mistake – crossed it out and did it again, asking each other, double the numbers, draw a picture, draw a line in the middle, half, 15 x 4, 20 x 2 + 1.</td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>No interviews</td>
<td>adding, used things that we already knew, columns, add, the difference, wrote, question, spoke clearly, talking, bit of fighting.</td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>1 interview</td>
<td>The students described many choices and actions that they made during the focus group interviews including both choices to interact with their group and mathematical choices. In this way, the data was reduced and possible initial codes were identified. Further grouping and labelling into themes and categories is explained in Chapter 4.3.</td>
<td></td>
</tr>
</tbody>
</table>
Another form of data – the student agency reflections (SARefs), (completed post-lesson) were analysed in a similar way to the group interview transcripts. The students written responses and photographs communicated a range of student choices during the maths challenges. Below is an example of the data collected from one student’s reflection:

**Figure 2**

*Example of a SARefl Response (L7)*

Today, equipment and methods I used to help me during problem solving were blocks, cubes, and thinking about volume.

I made these choices because they were helpful and educational.

These helped me to solve the maths problems by making my math easier and faster.

I began by highlighted student actions and the reasons given for these actions to begin the analysis of this sample. The initial analysis of the SARefs involved reading the students' reflections and photographs and comparing these with my observation data. Most of the student photographs matched what they wrote and my observations, but as these were chosen by the students to demonstrate a particular choice, they signified students’ thinking and understanding about choice. Points of interest were identified and I compiled a table that displayed the identified data in the form of a list. The possible codes included ideas such as; using small times tables to figure out big numbers, counting, thinking hard, allocating tasks and splitting numbers (see Appendix G).
As with the group interview data, students’ ideas were listed if they were written in a different way, but not if they were repeated by different individuals. This gave an indication of the range of actions, maths vocabulary used and student thinking across the class. Once displayed in this form, a process of comparing and contrasting of ideas took place, and I noted points of interest. The data was further reduced as these points were aggregated into themes and categories, which are described in Chapter 4.3.

The analysis of the student document data was informed by Selling (2016). She analysed student documents generated in her study in combination with supporting videos that had recorded the student interactions as they worked. Student representations were coded by representation type (e.g., tabular, numerical, graphical etc.) and whether students had made any connections between the representations. In a similar way, I grouped students’ representations establishing initial codes e.g., drawings, tables and equations. As the students’ work documents involved mathematical representations aligned with Theme 2: Maths choices, further data analysis is reported in Chapter 6.2.

Participant observations were strengthened by the post-lesson analysis of class video recordings using the prepared observation schedules to identify student and teacher actions. Table 7 illustrates a short extract of observed actions from a video recording (Lesson 1) using observation schedule 2.
Table 7

Extract From Lesson 1 Video - Observation Notes Using Schedule 2

<table>
<thead>
<tr>
<th>Time</th>
<th>Teacher</th>
<th>Student 1</th>
<th>Student 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Moving to new group.</td>
<td></td>
<td>29 min. Sitting talking with group.</td>
</tr>
<tr>
<td></td>
<td>Questioning e.g. Can you show me…? Prompting e.g., Where is your working out?</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

In this way, the data collected from my investigation were analysed in a variety of ways.

3.9 Chapter Summary

In summary, my study draws upon an interpretivist approach with the aim of understanding students’ and teachers’ actions which reflect their interpretation of events. The characteristics of qualitative methodology were selected as best suited to my study and have allowed me to use a case study methodology and qualitative data-gathering methods. I carried out a case study in my classroom to explore and deepen my understanding of student agency in the maths classroom. My research design, built from a practitioner-researcher perspective, includes deliberate decisions to ensure its transferability and trustworthiness, and is based on ethical principles. A variety of data collection methods have generated qualitative data and allowed the triangulation of methods and sources. Data collection methods included video recordings, participant observation, focus group interviews, student-generated data and researcher journal notes by myself as a way to be reflexive. The video recordings of the classroom, groups and individuals supported me to observe the
actions of students after the lessons, increased the dependability of my teacher observations and the group video recordings were used to stimulate recall during post-lesson interviews. I used focus group interviews to collect students’ views of their actions within the lessons (different groups for each interview). Student-generated data included SAREfIs with photos selected by the students showing evidence and choices, as well as documents that were collected and analysed. I now report on the data analysis, the focus of Chapter 4.
Chapter 4: Results

Introduction

Chapter 4 is the first of three chapters that report the data analysis to understand students’ perspectives when given choice in maths. Chapter 5 and 6 provide a thematic analysis with a focus on one theme per chapter. For now, I introduce Chapter 4 with contextual information describing the school and class maths programme, including a summary of the lessons that took place during the data collection. Then I describe setting up the class at the beginning of the school year with an emphasis on developing and reinforcing classroom norms to guide student actions during maths challenge lessons. Next, I describe my methods of analysis. Within this section, I identify the range and frequency of student actions in response to opportunities for choice in the maths classroom. I compare and contrast student ideas within the data, explaining the development and classification of my themes and categories. I end Chapter 4 by investigating the reasons the students have provided to explain their choices.

4.1 Context Information

Planning was carried out collaboratively by the school teaching team (2019) to meet the requirements of the New Zealand and school curricula. Maths topics for the year were documented in a long-term plan and included *Number and Algebra* throughout the year, along with other maths topics. Table 8 outlines maths topics taught over the year of the investigation, with data for this study collected in Terms 2 and 3.
Table 8

Outline of Maths Planning Over the Year

<table>
<thead>
<tr>
<th>Term 1</th>
<th>Term 2</th>
<th>Term 3</th>
<th>Term 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Addition/ subtraction</td>
<td>Multiplication/ division</td>
<td>Proportions/ ratios</td>
<td>Revision of Number</td>
</tr>
<tr>
<td>Statistics</td>
<td>Measurement</td>
<td>Algebra</td>
<td>Geometry</td>
</tr>
</tbody>
</table>

Integration with other topics and events:

Getting to know the class  
School camp  
Cycling  
Cross country  
Science: forces  
School production: drama, art, music  
Science: light  
Swimming  
Athletics  
Famous Artists

Teachers were expected to use a range of teaching strategies, student groupings and tasks set in real-life contexts in response to students’ needs and interests. Opportunities to integrate maths with other topics or events happening within the school were also planned.

4.1.1 The Class Programme – Maths Lessons

Maths was the daily learning focus during the morning block, for 50 minutes to 1 hour during the morning. Lesson structures and organisation varied according to the topic and students’ needs including, whole-class teaching, flexible use of small group explicit teaching (mini-lessons), stations and independent work. Independent work involved assigned tasks, as well as some time for students to choose from a range of maths activities. Students used their individual maths books for written recording (blank or squared paper), as well as digital math journals accessed by the computer (1 per student). Time was spent helping students to understand their learning goals (Researcher journal entry, April 2019). These were specific maths learning intentions
e.g. to use a variety of strategies to multiply. In this way, the students developed an understanding of the learning focus and their progress within this learning area.

Maths challenge lessons focused on solving maths problems and occurred several times a week during the year. These maths challenge lessons followed a structure that became a norm related to the maths activity (Sullivan et al., 2016). This comprised of different phases, a lesson introduction, time spent solving the maths challenges in collaborative groups, followed by a teacher-led class discussion to conclude, allowing students to share solutions and make connections.

During the introduction, whole class teaching was aimed at task clarification and engagement with the task (Researcher journal entry, May 2019). Students were then allocated any resources they needed for the task, such as instruction sheets (1-2 copies for each group) and sent off with their assigned group to begin working on the maths challenge. In the collaborative group phase, students were given time to think and attempt their solutions (Anthony et al., 2014), and then share their ideas within their groups. Once each group had reached a consensus, a check-in conversation with the teacher took place. This provided an opportunity for the teacher to listen to group members’ explanations and use questions and prompts to extend learning. Finally, the lesson concluded with a whole class discussion where selected groups shared their solutions, and students were involved in discussing maths concepts and making connections.

**4.1.2 The Focus of Each Maths Challenge Lesson**

The maths challenge lessons are summarised in Table 9, along with the main content of each challenge and the reference source of the activity.
Table 9

Summary of the Lessons

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Title</th>
<th>Main ideas/ concepts</th>
<th>Reference</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>That Old?</td>
<td>Calculating the age of cats and dogs</td>
<td><em>Figure it out: Number 3:2 pp.12,13</em> (Ministry of Education, 2004a)</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(patterns, addition, multiplication)</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>Sweet Thoughts</td>
<td>Calculating the cost and profits from selling chocolates (multiplication, addition)</td>
<td><em>Figure it out: Number 3:2 p.15</em> (Ministry of Education, 2004a)</td>
</tr>
<tr>
<td>3</td>
<td>Area and Perimeter</td>
<td>Calculating area and perimeter</td>
<td><em>NRICH</em> (University of Cambridge, n.d.)</td>
</tr>
<tr>
<td>4</td>
<td>Is There Room?</td>
<td>Calculating area using larger numbers (multiplication, comparing)</td>
<td><em>Figure it out: Gala 2-3, p. 7</em> (Ministry of Education, 2001)</td>
</tr>
<tr>
<td>5</td>
<td>Cut it Out</td>
<td>Calculating volume (addition, multiplication)</td>
<td><em>Figure it out: Measurement 3 p.11</em> (Ministry of Education, 2000b)</td>
</tr>
<tr>
<td>6</td>
<td>Cooking Time</td>
<td>Three short challenges involving time</td>
<td><em>Figure it out: Measurement 3 p. 23</em> (Ministry of Education, 2000b)</td>
</tr>
<tr>
<td>7</td>
<td>How Many</td>
<td>Calculating fractions of a set</td>
<td><em>Figure it out: Number 2-3 p. 20</em> (Ministry of Education, 1999)</td>
</tr>
<tr>
<td>8</td>
<td>Stacking Up</td>
<td>Adding fractions</td>
<td><em>Figure it out: Number 3:3 p. 20</em> (Ministry of Education, 2004b)</td>
</tr>
<tr>
<td>9</td>
<td>Marble Marvels</td>
<td>Fractions of a set, comparing sizes</td>
<td><em>Figure it out: Number 3:3 p. 21</em> (Ministry of Education, 2004b)</td>
</tr>
<tr>
<td>10</td>
<td>Target Decimals</td>
<td>Adding and subtracting decimals to reach a target number</td>
<td><em>Figure it out: Number 3:2 p. 16</em> (Ministry of Education, 2004a)</td>
</tr>
<tr>
<td>11</td>
<td>Stick Patterns</td>
<td>Identifying the rule in stick patterns, calculating nth item</td>
<td><em>Figure it out: Algebra 3 p. 2</em> (Ministry of Education, 2000a)</td>
</tr>
</tbody>
</table>

Lessons 1-4 were the main focus for data collection, where data was collected using video recordings, observations, focus group interviews, student generated data (SARefls and student work samples) and researcher journaling. The lessons were selected mostly from Ministry of Education resources, to deepen understanding of multiplication and division, and later, to explore fractions.
4.2 Establishing Classroom Norms for Maths Lessons

During Term 1, 2019, time was spent getting to know the students and developing working relationships (teacher-student and student-student). Assessment data was collected and students were observed working on maths challenges to identify what individuals could do, as well as their next steps. I wanted to understand students’ confidence levels, willingness to take risks and their social skills, as they interacted with others. Social norms were established including expectations and obligations for working within our class maths community. The social norms included whole-class norms for working with the class, small-group norms for students working in groups (2-3 students) and individual norms. A researcher journal entry reflects my thoughts and attention given to establishing these norms at the beginning of the year e.g., students getting to know each other, where to access resources and the expectations for small-group and whole-class discussions (Researcher journal entry, Term 1, 2019).

4.2.1 Small-Group Norms

I decided to have maths activity as group tasks rather than individual tasks, and I planned the composition of the small mixed-ability groups (2-3 students). Small-group norms were established early in the year (prior to the data collection lessons) using tasks to enable students to form relationships with each other. In small groups, the students collected statistical information about the class to answer student inquiry questions (about topics such as family size, age and favourite subjects) and graphed the information. Social aspects e.g., feeling comfortable sharing ideas with others and working towards a common goal, were the focus of these lessons and as such, the topic of many reflective comments in the Term 1 researcher journal. As a
result, deliberate work within the class took place to encourage the inclusion and acceptance of all group members. The class discussed how it felt to work with others co-operatively and have ideas valued, what this looked like in our classroom space and what to do if they disagreed with others. Common understandings of small-group norms were recorded (Figure 3 below) and displayed as a reference for future maths lessons throughout the year (Researcher journal entry, March 2019).

**Figure 3**

*Establishing Group Norms – Group Member Responsibilities*

When asked, “How do you feel about working in a group?” one student responded, “I like working in a group… because I get to talk, to agree or disagree with the answers” (Student voice recording, March 2019). Desirable social behaviours were encouraged, for example, active listening, respect for others and commitment to the group effort. Students self-assessed their group’s performance against the agreed guidelines and had the opportunity to compliment others. When asked, “How well has your group worked together?” one student responded, “Our group listened to each other, helped each other, supported each other and stayed with each other. We … talked about what we were going to do…” (Student voice recording, March 2019).
The initial emphasis on small-group social norms and working collaboratively was reinforced as necessary throughout the year. Researcher journal entries note aspects of consolidating or strengthening these social norms. For example, working on:

- all group members contributing (Researcher journal entry, March 2019);
- active listening: what it looks like, feels like and how it helps us learn (Researcher journal entry, May 2019);
- respecting others (Researcher journal entry, August 2019); and
- what it means to persevere (Researcher journal entry, August 2019).

In response, the social skills under development were the need to take turns, explain ideas to others in a way they could understand and check on the feelings of other group members. After reading ideas on helping teachers to recognise students’ mathematical strengths (Skinner et al., 2019), a discussion was initiated on “What being smart at solving math challenges looks like?” (Researcher journal entry, May 2019). Figure 4 shows a visual summary of students’ ideas including: working with others, participating, being able to talk, explain and listen, being accountable to the group, as well as using pictures and diagrams. These examples illustrate the ongoing discussion around classroom social norms for maths.
The analysis of researcher journal entries illustrates the responsive nature of teacher actions to encourage students' co-operative group work (Researcher journal entries, April, September 2019). For example, I trialled different group combinations to ensure students were having a successful experience. After observing students working and listening to student voice, I grouped students who worked at similar speeds together. This response was aimed at avoiding some students becoming frustrated at the pace of maths challenge completion or lack of understanding by others.

Having documented the establishment of expectations, obligations and routines for working in small groups, students also needed to be able to participate in whole class discussions involving whole-class norms.

4.2.2 Whole-Class Norms

Whole-class norms supported students to participate in the class discussions at the end of the maths challenge lessons. Students were expected to be able to explain
Chapter 4: Results

their solutions to others. This expectation was not immediately apparent to all students and needed reinforcement. A researcher journal reflection identifies the need for students to be accountable to their maths challenge group and prepared for sharing at the end of the lesson (Researcher journal entry, March 2019). Several students were selected to present their ideas at the end of each lesson. Students were encouraged to participate in this teacher-led discussion using ‘talk moves’ such as ‘adding on’ or ‘agreeing or disagreeing’ (Chapin et al., 2009). After the initial focus on whole class norms, specific aspects were reinforced throughout the year as needed.

4.2.3 Struggling and Making Mistakes

Another classroom norm relates more to individuals and involves the expectations and routines when faced with difficulty. Students were asked how they felt about making mistakes in maths. One student responded, “I feel like I need to work harder on the mistake I just made” (Student voice recording, March 2019). Another stated, “It’s fine making mistakes because it helps me learn the answers because I can rework it” (Student voice recording, March 2019). The analysis of teacher researcher journal entries illustrate class discussions on what it means to learn and what this looks like in maths. In one student’s opinion, “you could use another strategy to find the same answer” (Researcher journal entry, March 2019). Another student suggested, “you can always get smarter.” It was accepted that making mistakes was a normal part of learning. Students were taught by teacher modelling and prompting (with whole-class and group contexts), how to agree or respectfully disagree with others without causing offence (Researcher journal entry, March 2019). When this became a teaching focus, appropriate student actions were noticed and
celebrated. Words such as “agreeing and disagreeing” were recorded many times within the data collection, illustrating the norms in action and reflecting the language that was used when teaching and negotiating these norms.

When struggling, students were expected to take some sort of action to help themselves. Knowing what to do and how to access help, was therefore an important classroom norm that was built up over time. There was a continual emphasis on self-help strategies (Researcher journal entries, May, September 2019). For example, the CUBES strategy (mnemonic for teaching actionable steps to help engage with a story problem such as Circle the key numbers, Underline the question) was modelled to help get started on a maths challenge. This involves identifying the important numbers and words, as well as eliminating unnecessary information. Students were encouraged to direct their questions to other group members. One student commented, “if you need help you can always ask a … classmate.” If the students within the group still needed help, the teacher and teacher aide were roving and available to answer questions. At times, students were observed accessing help from other students (from different groups) within the room (e.g., L10 Group 1 video, see Appendix H). Teacher prompts were used to encourage self-help strategies including drawing a picture, acting out the problem, and accessing available equipment and resources to ‘show your thinking’.

Having reported the math learning context and described the establishment and emphasis on classroom norms, I now move to report on the data analysis.

4.3 What Choices? Students’ Responses to Opportunities for Choice

In this section, I focus first on the range of ideas provided by the students in response to the SAREfl and interview prompts. Secondly, I analyse the frequency of
the ideas within the data collected. I display the data as loose categories, group and label the data, and explain the development of my themes and categories.

4.3.1 The Range of Choices and Development of Themes

The initial data analysis process (described in Chapter 3) included the identification of student choices written in the SARefls, displayed in photographs and verbalised during the group interviews. Student choices were listed in table form for each lesson and for each data source. An overview of the tables (Tables 6 and 18) revealed the choices made were connected to the focus for each lesson, individual group attributes and personal preferences. For example, individuals described skip counting, not giving up and writing things down. Some responses indicated an awareness of the social aspects of their group work, such as demonstrating and taking turns – skills that had been a focus when classroom norms were developed and reinforced during the year. When students described the choices they made, some ideas related directly to solving the maths challenges, whereas others were more related to general study skills within the classroom environment, easily transferable to other curriculum areas. These showed two distinct themes developing from the student responses, namely: choices around maths calculations and those around individual and group self-management. Comparing and contrasting the data produced two main ideas, which I now refer to as themes. The first theme relates to individual and group self-management, ‘Social and Personal Ways of Working’. The second theme ‘Maths Choices’, includes codes directly related to mathematical thinking.
4.3.2 Developing Categories Within the Themes

Students’ ideas describing actions and their decisions, were identified and grouped into loose categories within the two themes, then given a possible category label. This led to the development of working definitions for each of the categories. At this stage, analysis triangulation was undertaken to confirm the clarity of the definitions and to test researcher agreement on sorting the data. The following two categories are set out in Table 10.

Table 10

Theme 1: Social and Personal Ways of Working

<table>
<thead>
<tr>
<th>Theme 1: Social and Personal Ways of Working</th>
</tr>
</thead>
<tbody>
<tr>
<td>Choices within the lesson not directly related to solving the maths challenges.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Category 1: Social/Co-operative</th>
<th>Category 2: Self-management</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working collaboratively in a group within the maths challenge lessons by participating and contributing. Involves interacting with others e.g., supporting others, agreeing and disagreeing, dealing with frustration.</td>
<td>Involved with personal intentions, self-regulation and self-organisation. Personal choices to lead or follow, and where, when and how to act e.g., to persevere, engage with the task, monitor progress.</td>
</tr>
</tbody>
</table>

Theme 2 categories related to maths choices and thinking and were named ‘Student Methods’, ‘Representations’, and ‘Resources and Equipment’. During the data analysis, decisions needed to be made if data could be placed in more than one category, as some ideas overlapped the Student Methods and Representations categories. For example, when a student identified written multiplication as a maths choice, this could be interpreted as a method or representation. My decision was to adjust the definition of Student Methods to mental strategies, compared to Representations, which is defined as a visual diagram or symbolic representation.
My justification for the classification ‘Representations’ is based on a student having described written multiplication as a choice and involving written recording (an algorithm) to calculate the answer. I have classified this choice as a Representation. Students’ ideas have been included in the Student Methods category if they involved mental strategies with no reference of written recording. After feedback, and further adjustment, the definitions for each of the themes and categories, are set out in Table 11 below:

**Table 11**

*Theme 2: Maths Choices*

<table>
<thead>
<tr>
<th>Category 3: Student Methods</th>
<th>Category 4: Representations</th>
<th>Category 5: Resources and Equipment</th>
</tr>
</thead>
<tbody>
<tr>
<td>Includes mental strategies used to work with numbers and maths concepts to help solve maths challenges e.g., multiplying, subtracting, estimating.</td>
<td>Using a diagram or symbolic representation for representing and communicating maths information and ideas e.g., written numbers, a visual diagram, table, number line or inscription (doodling).</td>
<td>Physical items chosen by the students to model maths ideas e.g., blocks, abacus.</td>
</tr>
</tbody>
</table>

I used data from Lesson 2 (from 20 student agency reflections (SARefls)) to test these category definitions. The classification of students’ ideas within the chosen themes and categories is set out in Table 12 below.
### Table 12

**Theme 1 and 2 Categories for Responding to Opportunities for Choice (L2)**

<table>
<thead>
<tr>
<th>Theme 1: Social and Personal Ways of Working</th>
<th>Theme 2: Maths Choices</th>
</tr>
</thead>
<tbody>
<tr>
<td>Social/ Co-operative</td>
<td>Student Methods</td>
</tr>
<tr>
<td>Respectfully agreeing and disagreeing</td>
<td>Repeated addition</td>
</tr>
<tr>
<td>Working with a good partner</td>
<td>(x2)</td>
</tr>
<tr>
<td>Working with my group</td>
<td>Multiplying (x7)</td>
</tr>
<tr>
<td>Helped my teammates</td>
<td>Adding (x4)</td>
</tr>
<tr>
<td>Talking and explaining how (x4)</td>
<td>Splitting it (x2)</td>
</tr>
<tr>
<td>Listening (x2)</td>
<td>Tidy numbers</td>
</tr>
<tr>
<td>Shared my answers</td>
<td>Use my times tables (x2)</td>
</tr>
<tr>
<td>Comparing our answers</td>
<td>Representations</td>
</tr>
<tr>
<td>Understand my buddies working out</td>
<td>Written addition</td>
</tr>
<tr>
<td></td>
<td>(x2)</td>
</tr>
<tr>
<td></td>
<td>Written equations</td>
</tr>
<tr>
<td></td>
<td>(x2)</td>
</tr>
<tr>
<td></td>
<td>Drawing pictures</td>
</tr>
<tr>
<td></td>
<td>(x4)</td>
</tr>
<tr>
<td></td>
<td>Table (x2)</td>
</tr>
<tr>
<td></td>
<td>Written multiplication (x4)</td>
</tr>
<tr>
<td></td>
<td>Resources and equipment</td>
</tr>
<tr>
<td></td>
<td>Fingers</td>
</tr>
</tbody>
</table>

Up to this point, the analysis has focussed on the *range* of choices that students described within the maths classroom. I now return to the complete data set to identify the *frequency* of student ideas about choices.

### 4.3.3 The Frequency of Choices

Students described similar actions using a variety of vocabulary. A word count analysis was carried out to identify the frequency of ideas. First, I identified keywords that represented choices made by the students and their synonyms e.g., multiply, multiplication, times, times tables and lots of. These synonyms all refer to ideas of multiplication. After identifying keywords, frequency counts were made of these words in the interview, SARefl and student work document data. The full word count...
analysis is displayed in Appendix I. A summary of the word count analysis, related to students' choices, can be seen in Table 13 below:

Table 13

*Frequency of Words Describing Choices in Interview and SARrefl Data*

<table>
<thead>
<tr>
<th>Word</th>
<th>Count</th>
<th>Addition</th>
<th>Multiplication</th>
<th>Table</th>
<th>Diagram</th>
<th>Patterns</th>
<th>Explained</th>
<th>Agreed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synonyms</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Counting</td>
<td>24</td>
<td>80</td>
<td>131</td>
<td>21</td>
<td>57</td>
<td>10</td>
<td>14</td>
<td>18</td>
</tr>
<tr>
<td>Adding</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plus</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Multiplying</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Times</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Times tables</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Lots of</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Chart</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Pictures</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Draw</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Drawing</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Explaining</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Agree</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

The analysis of vocabulary shows the largest word group was ‘multiplication’, followed by ‘addition’, then ‘diagram’. These data reflect the focus of the lessons during the school year, with multiplication being the learning goal in the long-term plan. The frequency of ‘diagrams’ was corroborated by my Term 2 researcher journal entry which reads, “Drawing pictures and diagrams to help think about problems was a common strategy identified by students for helping them think and communicate while solving problems” (Researcher journal entry, Term 2 2019). The word count within the data collection reflects common ideas communicated within the classroom at the time. Grouping and labelling the students’ responses to opportunities for choice in this way, has helped to identify and confirm patterns in student thinking and actions (both maths and social), but does not address their reasons for making the choices. Other questions have arisen: Were students aware of their choices? and did they justify their choices?

4.4 How do Students Explain and Justify Choice?

The analysis led me to investigate the explanations and justifications students gave for making maths and social/ personal choices during the maths challenge lessons. Responses from each of the lessons were analysed to identify how students
explained and justified their choice of actions. I was interested in both the range of justifications from students’ perspectives and the frequency, to understand ways student agency can be fostered in maths. In particular, this understanding would help to answer my research question: “How do students’ explain and justify their actions in choosing methods and equipment?” and to understand student responses to opportunities for choice as they worked within this social environment.

4.4.1 The Range of Explanations and Justifications

The students’ explanations and justifications for their choices were grouped into categories. Definitions for each of the categories were established, reviewed by a third party and adjusted to increase the trustworthiness of the analysis process:

**Category 1 - Shared Responsibility:** involves participation, respect and obligation to the group.

**Category 2 - Personal:** related to individual goals and achievement.

**Category 3 - Understanding:** helped with thinking/ understanding the numbers, method or maths challenge.

**Category 4 – Efficiency:** working quickly and effectively in a more organised way to make the task easier.

An example of the range of data using this classification can be seen in Table 14 below (using Lesson 2 SARefl data, 20 student responses).
Table 14

Display of Categories Justifying Actions (L2)

<table>
<thead>
<tr>
<th>Shared responsibility</th>
<th>Personal</th>
<th>Understanding</th>
<th>Efficiency</th>
</tr>
</thead>
<tbody>
<tr>
<td>Working with my buddy/group</td>
<td>Staying on task (x2)</td>
<td>Helped me work my problem</td>
<td>Timesing the profits</td>
</tr>
<tr>
<td>Resperfully agreeing and disagreeing</td>
<td>Try my best (x2)</td>
<td>So we could figure out the problem</td>
<td>Adding up faster</td>
</tr>
<tr>
<td>My partner (JP)</td>
<td>Good for me</td>
<td>It was hard</td>
<td>Easier to find the profit (x3)</td>
</tr>
<tr>
<td>I don’t want to let my teacher down</td>
<td>Get as much done as possible</td>
<td>Helped us work the profit out</td>
<td></td>
</tr>
<tr>
<td>I wanted to help</td>
<td>Actually doing it</td>
<td>Knowing the profit of each carton</td>
<td></td>
</tr>
<tr>
<td>Comparing to see if we had the same answer and worked it out the same way</td>
<td>I needed help (x2)</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Telling what I thought</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>I wanted to do good maths</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Getting me more confident</td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>Helps me work with someone my speed</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Students’ reasons vary, showing depth in thinking and a range of ideas related to personal and social processes in learning maths. Some ideas are related to aspects of the classroom social norms such as their responsibilities when working as part of a group and comparing answers. Students identified personal goals such as trying their best, staying on task and increasing confidence. One student explained, it helped them work with someone at their speed. Other justifications involved working on their maths solutions and completing their work, for example, understanding the maths concepts, and being efficient with their time and effort.

The analysis of how students explain and justify their choices has so far looked at the range of ideas given by the students. I now review the data set to identify how frequently some common explanations and justifications were given by the students.
4.4.2 Frequency of Student Explanations and Justifications

Two common ideas came out in the word count analysis (Appendix I) of the interview and SARefl data, both related to efficiency. I have chosen to show these ideas because of the high frequency they appeared in the data compared to other ideas. The results are summarised in Table 15 below.

Table 15
Word Count Analysis for Student Justifications of Choices

<table>
<thead>
<tr>
<th>Word</th>
<th>Easier</th>
<th>Faster</th>
</tr>
</thead>
<tbody>
<tr>
<td>Synonyms</td>
<td>Easy, Easiest</td>
<td>Quicker, Quickly, Finishing, Finished, Wasted less time</td>
</tr>
<tr>
<td>Frequency</td>
<td>45</td>
<td>22</td>
</tr>
</tbody>
</table>

The frequency of students choosing ‘easier’ ways was 45. The idea was used to explain and justify choices throughout most of the lessons but was highest in Lesson 3 (calculating area), Lesson 5 (calculating volume using models and counting blocks) and Lesson 6 (solving time challenges). The idea of speed (‘faster’, ‘quicker’, ‘quickly’, ‘finishing’, ‘finished’, ‘wasted less time’) was mentioned 22 times in total. The idea came up in most of the lessons, with speed being given as a reason for making particular choices, for example, four times in both Lessons 5 and 6. This data suggests that the students were prioritising more efficient methods.

A question to consider is whether analysing the data in this way has removed the meaning and purpose from students’ choices. By separating the actions of students from their explanations and justifications, the data and students’ experiences of responding to opportunities for choice in maths may be diluted. In summary, having established my themes:
Theme 1: Social and Personal Ways of Working;

Theme 2: Maths Choices;

and categories within each theme, I am aware of the range of student actions and the frequency that these ideas appeared in the data. In the next chapter, I report the analysis of student responses within each theme as a whole.
Chapter 5: Theme 1: Social and Personal Ways of Working

Introduction

This chapter reports on the data that aligns with the theme of ‘Social and Personal Ways of Working’ to identify how students respond when there are opportunities for choice. Two categories were identified within this theme (refer to Table 10). The first relates to student interactions with others within the social learning environment. Section 5.1 reports on the Category 1 data analysis relating to student choices as they worked with others. The chapter is informed by student-generated data, student interviews, the researcher journal and video observations. Vignettes of quotes from SAREfls and interviews have been selected to illustrate my analysis. Each vignette is labelled with the lesson number, data source and student identification (pseudonym) e.g., (L1 SAREfl Nora). This is followed by Section 5.2 reporting the Category 2 data analysis focused on self-management.

5.1 Social/Cooperative

This section is devoted to the first category Social/cooperative relating to student choices when they worked collaboratively in a group, participating and contributing during the maths challenge lessons. This category includes the following ideas: participating and contributing, relationships, giving and receiving support, agreeing and disagreeing, and dealing with frustration.

5.1.1 Participating and Contributing

Participating and contributing in group work involved a variety of student choices and actions. Students identified choices related to talking, listening, explaining, agreeing and disagreeing with others. For example, one student responded that his partner
“disagreed … and explained how he got his answer and it was right” (L1 SARefl Lance). Talking between students was seen as important because it enabled him to identify his misconceptions and to make modifications to his answers. Another student explained that “listening to my group … it helped me because they told me what they were thinking … by giving me pointers” (L1 SARefl Nora). In this example, listening helped by providing some prompts for understanding and solving the maths challenge, by giving her access to others’ thinking. Anne described the benefits of participating in group work as being an easier way to work when solving maths challenges (L4 SARefl Anne).

Contributing in a group involved sharing responsibility for the groups’ solution. The concepts of fairness and taking shared responsibility were evident throughout Lesson 1 from Len’s interview response. He explained, “[I did] not sit back and let my buddy do everything” (L1 Int1 Len). It was important for Len to take on a fair amount of the work, as he had chosen to participate and engage with the task. An interesting statement, taken from Lesson 3, described a choice made by a group to allocate tasks between members in the interests of efficiency. One student intimated, “[My partner] did the area and I did the perimeter and counting the squares … because it wasted less time of doing area and perimeter together at the same time” (L3 SARefl Lance). In this way, each student took responsibility for part of the maths challenge to help manage the group’s time. The group was aware that time was limited and were focused on completing the task.

**5.1.2 Relationships**

Some choices identified by the students indicated their awareness of maintaining good relationships, particularly through displays of courtesy and respect. One
student described how her group chose to take turns and let each other speak (L4 Int 1 Cathy). Showing respect to group members is apparent in many responses, with words such as “respectful” and “kindly” being used. One student explained how they were “listening to each other’s ideas and respectfully disagreeing” (L2 Int 2 Dave). Another student wrote, “We talked to each other and we kindly disagree if we think it is wrong” (L4 SAREfl Amber). In an interview one student explained, “If anyone had an idea we would listen and try it out. If it was wrong, we would just kindly tell them that we didn’t agree” (L4 Int 1 Cathy). These examples demonstrate students were making deliberate choices to be kind to others and working to build strong relationships.

5.1.3 Support – Giving and Receiving

In several statements, students described giving support and receiving support from others. Ideas include being able to talk to and share ideas, ask questions and help group members to understand the correct methods. One student reported, that talking to their buddy helped them to work out and solve the maths challenge (L6 SAREfl Keith). Another valued the support of group members by saying, “working together with … and checking the questions … because the questions were hard. I needed the help” (L6 SAREfl Kaiden). Sharing ideas helped a student to get the answers “by knowing the correct methods” (L6 SAREfl Nadine). The classroom norm that students make choices towards reaching a maths solution to solve maths challenges encouraged individuals to show interest in the methods used by others. Several statements from Lesson 2 suggest that students valued the opportunity to support others during their maths lessons. Reviewing a group video during an interview, one student said, “I showed them how I did it” (L2 Int 2 Mark) indicating a variety of representations. This was in response to others in the group showing
confusion, and in particular, one student who had been absent for a few days, was trying to follow the group’s reasoning. Another student, Rose, explained how group support had enabled the testing of ideas and comparison of answers. “This helped ... by seeing if we had the same answer and worked it out the same way” (L2 SARefl Rose). Being able to ask questions was seen as another form of support. In a SARefl from Lesson 6, a student described being able to “ask the teacher questions and my buddy ... because they really helped me ... it was confusing and I had support” (L6 SARefl Maya). The opportunity to ask questions of other group members and the teacher was a choice she had deliberately made to get help and support. These responses show that group members were giving encouragement and reassurance to others as they worked through the maths challenges. Thus, reciprocal support was evident between students.

Working together as a group benefitting individual students is summarised by one student’s response, “explaining my ideas and working together ... helped me” (L11 SARefl Dave). However, were all students committed to the group? Was there a sense of responsibility and obligation to the collective effort of the group? Two further SARefl responses describe in detail the benefits of group work to both them and other group members, suggesting a sense of duty and obligation to others. One response claimed, “My group was helpful and explained stuff twice to each other if we didn’t hear it the first time ... I got all of the information I needed and we all helped each other” (L4 SARefl Dave). Another stated, “We helped each other figure out the answer and they waited for me when I didn’t know the answer” (L1 SARefl Ken). The same student explained that they were “sitting in a sort of triangle and not blurtling out the answer, because we needed to see the book and I didn’t want to ruin the others’ learning.” These reflections illustrate students having patience with each
other and persevering to help other group members understand. When a student needed to hear the explanation again, others in the group were able to repeat their ideas. Access to this information source helped the students to get all the information they needed. Group members showed respect, an understanding that others were learning too and allowed their peers thinking time to understand what others were saying within the group. Further evidence of this commitment was found from video data that showed students sitting in groups, some at desks and some on the floor. Occasionally a student moved around the room to get equipment but remained committed to their group as they returned and continued working.

### 5.1.4 Agreeing and Disagreeing

Negotiating to reach an agreement with everyone in the group was evident from in the data. One student explained her choice to speak clearly, “so that [the] other person could understand what was going on” (L10 Int 1 Maya), and an expectation for their partner’s opinion. Describing his group’s procedures, Luke stated, “We just did our ideas and then we like disagree or agree” (L1 Int 2 Luke). Several other interview responses mention students disagreeing with others’ ideas (e.g., L1 Int 1 Keith, L2 Int 2 Dave). Some disagreement was signalled by another who said, “We were talking about how to add it up and we had a bit of fighting” (L10 Int 1 Nora). Although the term ‘fighting’ was used by the student, when I viewed the video data, I interpreted this interaction as a disagreement between group members in line with a healthy classroom interaction. It shows the conviction of students when defending their ideas within their group. These responses suggest that students were spending time and effort to convince others of their thinking.
5.1.5 Dealing With Frustration

The data analysis reveals both positive feelings about working in groups and negative student feelings. At the beginning of the year, students were asked how they felt about working in groups. The general consensus was summarised by one student’s response, “Yes, I like working in a group” (Student voice recording, March 2019). Working with others was the preferred method of tackling maths challenges compared with working independently. However, student data also revealed some negative feelings about their experiences. In these cases, working together in groups posed issues for the students. Expressing feelings of frustration, one student explained, “My buddy didn’t let me do anything, they just gave me the answers ... I couldn’t make any choices because they wouldn’t let me ... I just got told the answers” (L1 SARefl Amber). The student’s partner was choosing to contribute by telling or showing but this may not have been requested or received as useful. This example illustrates how this student showed the expectation and desire to participate, and an awareness of their responsibility to understand what was happening in the group but became frustrated when this didn’t happen. Written responses from two students’ SARefl indicated further frustration. “My buddies were going too fast and I didn’t understand anything ... it was hard to understand” (L4 SARefl Rose). Here the student was referring to the speed that others in the group were making decisions and working out calculations. She explained how there was not enough time to follow their thinking. Another student explained, “It was hard to understand my buddies working out ... I did manage to get it in the end” This student continued, pointing out “it helps me when I work with someone my speed” (L2 SARefl Amber). This could also mean that the group members needed to slow down when they were explaining their ideas to others.
In summary, students identified choices they made while solving maths challenges related to participating and contributing to their maths group. Key ideas include students taking shared responsibility, maintaining relationships with others in class, giving and receiving support, commitment to the group, negotiating with others and finally, dealing with frustration. Some students also gave reasons for their actions.

5.2 Self-Management

Self-management (Category 2) includes choices related to personal intentions, self-regulation and self-organisation. Ideas classified in this category include self-organisation, engaging with the task, monitoring progress, persevering and personal intentions.

5.2.1 Self-Organisation

After the introduction of the maths challenge, students organised themselves to work on the task. A classroom norm set up at the beginning of the year focused on choosing a suitable place to work together that allowed access to appropriate equipment and where students could communicate clearly with each other. Working in a good place was described as a choice by one individual (L2 SARefl Evan). Another student explained how he had deliberately chosen to sit in a spot where no one else could annoy or talk to him, besides his buddy (L10 SARefl Dave). This statement is indicative of increasing self-awareness and understanding of his effect on other learners. A review of the class video during Lesson 1 shows that this student was typically talkative and appeared to be off-task for significant periods – but despite this, the challenge had been satisfactorily completed with time spent explaining his thinking to his group. His self-organisation and engagement in further maths challenge lessons were observed and are indicated by his description of
“thinking”, drawing pictures, “listening to each other’s ideas” (L2 Int 2 Dave), making “good decisions” (L2 SARefl Dave), explaining stuff, helping each other and getting “all the information” (L4 SARefl Dave).

5.2.2 Engaging With the Task

Reading the task instructions was an explicit choice described throughout the data by various students. During one interview, a student explained what she was doing as she watched the video of her group working. “I am reading out what the sheet says to do” (L4 Int 1 Cathy). Another example suggests the priority of actions, “first we read through the questions thoroughly” (L1 Explain Everything Rose). Other students identified reading the instructions as a self-help strategy when they responded to difficulty. One student wrote that he “read the questions carefully because the questions were really complicated,” and he “did not get the questions that much” (L6 SARefl Lance). This confirms students were deliberately choosing to reread the instructions when they were having difficulties.

Approaching a maths challenge by selecting the important information was a choice described by Cathy in Lesson 4, “We were putting down the information that it gave us already” (L4 Int 1 Cathy). The same student explained in data from two lessons that she was “getting the right information” (L3,4 SARefl Cathy). This student described the deliberate action of selecting important information and rejecting unnecessary ideas to help her focus on the maths challenge. Another student described her choice to look at the task to see if it was the “same kind” (L4 Int Amber), her approach being dependent on prior experiences with a similar challenge.
5.2.3 Monitoring Progress

Self-monitoring skills are utilised in different curriculum areas and enable the regulation of future behaviours e.g., deciding to slow down and reread to check decoding accuracy when unsure of a word in literacy and rereading to check for meaning in writing. Interview and SARRefl data showed that students were choosing to go through a process to monitor and review their progress on a maths challenge. Lesson 6 provided Ken with an opportunity to apply his self-check skills when he double-checked his work (L6 SARRefl Ken). Other students were involved in self-assessment and evaluating their performance. Examples of statements include, “I did manage to get it in the end” (L2 SARRefl Amber), “because I needed help” (L2 SARRefl Michael), and “I did not get the questions that much” (L6 SARRefl Lance). The first statement suggests that the challenge was understood, despite some struggle. In the second, Michael admits that he needed support, and in the third statement, Lance communicates difficulty understanding the task. All three students were involved in assessing their progress and reflecting on their own performance. Another student demonstrated this skill when he reflected (post-lesson) how his group had chosen to make “good decisions” for the maths challenge (L10 SARRefl Evan). In this way, self-monitoring and self-evaluation (critical reflection and evaluation of progress) processes helped the students to make decisions about what to do next or in future maths challenge situations.

Evidence from the data suggests students were learning to manage their time and efficiency during maths challenges. Some were aware of time limitations when completing tasks and were making decisions accordingly. For example, one student explained making a particular choice “because it wasted less time” (L3 SARRefl Lance). Natalie made another choice of strategy because it was “easier and quicker”
(L3 SARefl Natalie). And finally, one student who held high standards for himself was aiming to “finish all the questions”, get “everything” or “a lot” of work done (L1, L2 SARefl Evan).

5.2.4 Persevering

Persevering to remain focused on a task when things get hard is a further beneficial learning skill. In reviewing their video, Cathy said, “We got that wrong. Then we mistakenly found out that we weren’t supposed to plus or add, we were supposed to times it … then we figured it out” (L4 Int 1 Cathy). In this situation, Cathy and her group returned to the maths challenge when they realised they had made some mistakes. This perseverance led to understanding and self-corrections. In a further illustration of perseverance, Luke explained how they had “kept getting the dots wrong … so just kept on doing it” (L7 SARefl Luke). This student was referring to the maths diagrams they were creating to help think about equal sharing. Such explanations show students learning from making mistakes. It illustrates the struggle some students face when working to complete the maths challenges and reach their solutions.

5.2.5 Personal Intentions

Students aligned their choices with their personal goals. One student wanted to “learn how I’m going to make good decisions”, particularly in maths (L10 SARefl Nora). Another explained wanting to “learn more things” (L10 Amber). A third student’s goal was to try to stay on task and not be distracted (L2 SARefl Dave). The same student also chose to make a task more challenging (L10) as he was trying to become more comfortable dealing with decimal numbers. This student became more
aware of the choices that he had available to him to solve maths challenges and worked towards meeting his personal goals.

5.3 Patterns in Student Choices Throughout a Maths Challenge

I now turn my attention to focus on the patterns in student choices during the collaborative group phase of maths challenge lessons. I analyse the data (group videos, interviews and SAREfIs) from one group of students from Lesson 8 and one group from Lesson 10 to identify how the students exercised choice during their small-group work. What choices did students make during this part of the maths challenge lessons?

The groups began by organising themselves as part of a group, managing themselves into a workspace and gathering their resources. Examples of this can be seen on video as students settled themselves within the classroom where they could see the task, had access to resources (such as pen paper and blocks) and were ready to focus. One group chose to use paper, scissors, rock to help make decisions (L10).

Figure 5

*Students Organised Themselves for Learning*

Next, students focused on and engaged with the task, making decisions on how to proceed. Examples of observed actions include rereading the challenge, looking at
the pictures to help visualize the situation, pointing, verbalizing ideas, repeating others’ ideas, and making suggestions. One group was deciding how to proceed and involved in a disagreement, “You do it yourself.” Her partner patiently replied, “No, you do the same as your partner,” and she repeated this several times to allow her partner thinking time before there was mutual agreement (Nora, Maya L10 Group Video). Once decisions on how to proceed were made, the students began to manipulate the numbers.

Students manipulated the numbers, choosing to apply various methods and representations while using equipment to record and model ideas. Students used methods such as mental calculations, known facts, counting, adding, subtracting, dividing and comparing amounts (L8, L10). A variety of representations were used to record scores, equations and for written addition. Students were involved in discussions with their peers to agree, disagree, explain their reasons, give suggestions and ask questions. For example, one student said, “I think it is $\frac{3}{8}$. Do you agree?” (L8). Another student stated, “You should put it here.” and “It’s 0.8 because …” (L10) (Appendix H). Other social aspects from the data included taking turns, waiting for other group members, being aware of what others in the group were thinking and the progress they were making with their calculations. Another example illustrates student-teacher interactions. After agreeing they did not understand, a group of students (L10) asked the teacher a question, which led to a short discussion to help them move forward. A few minutes later, a student from a different group appeared on the video saying she did not understand and could this group help her? (Anne, Nora, Maya L10 Group video). This student was seeking out an expert peer, other than the teacher for advice.
Finally, the students *summarised* their groups’ solutions, made decisions to check their understanding and practised sharing their solutions within their group. One student (L10) showed encouragement and support to her partner when they added their final scores stating, “If you need help, I’ll be right here next to you” (Nora) (Appendix H). Each student in the group took responsibility for their own answers but showed an awareness of what their partner was thinking and supported them if needed.

During the collaborative group phase of maths challenge lessons, students made choices to *organise themselves, engage with the task, manipulate the numbers* and *summarise* their solutions. Data suggest some students spent little effort deciding how to organise themselves or how to proceed on a challenge, the social norms being well embedded. Whereas for others, these were distinct patterns that were observed through their recorded actions, by listening to their conversations and points of view. Students spent the most time *manipulating numbers* during the collaborative group phase of these maths challenge lessons.

### 5.4 Chapter Summary

The close analysis of student data indicates that students made many choices and actions within the maths challenge lessons when they took up opportunities. Many choices can be classified as social and personal choices. These were guided by the classroom norms and are skills, attitudes and values that are transferable to other curriculum areas. During the study, students became more aware of the choices that they were making and some students justified their reasons for the choices. In the next chapter, I report the data analysis for Theme 2: Maths Choices, involving numbers, quantity and space.
Chapter 6: Theme 2: Maths Choices

Introduction

The purpose of this chapter is to report the data analysis within the theme Maths Choices to understand students’ views and justifications for choices they made. This is followed by the analysis of group data, and finally, my study’s methodological reflections. I begin using the definitions previously established for Theme 2, grouping students’ ideas into three categories (see Table 11). When analysing the data, I follow through each of the lessons identifying significant statements from the students. I identify examples of statements from the students that reveal their views of the maths classroom without breaking up and removing meaning from the statements.

6.1 Student Methods

I now report the data analysis for Category 3, Student Methods chosen to solve the maths challenges, under the following subheadings: manipulation of numbers, making connections and using prior knowledge, and easier and faster methods.

6.1.1 Manipulation of Numbers

Students described choosing a variety of methods involving the manipulation of numbers e.g., reorganising numbers within equations, and recomposing and decomposing numbers. These methods included counting, adding, subtracting, multiplying, and dividing. Sometimes a variety of methods involving the manipulation of numbers were used within one lesson during several parts of a maths challenge. For example, after one lesson a student explained, “I used quite a few different ways to solve the problems. First of all, I used some adding and subtracting, then I went...
onto times tables and then I did multiplication” (L2 Int 2 Dave). At other times, several strategies involving number manipulation were used within the same equation. One student explained, “I used ... my addition and jumping the line ... because it helped me for adding and jumping up to my answer” (L10 SAREf! Anne). This student first reorganised the numbers within the equation, choosing to add instead of subtract. This strategy enabled her to jump a number line in sections and then add the parts together to calculate the difference between the two numbers in the equation.

Within multiplication (the focus during the data collection period), students referred to various strategies using mathematical terms such as skip counting, doubles, repeated addition, place value and tidy numbers (rounding to a multiple of 10 or 100 and then compensating). For example, a student explained that his group “counted in fours” and carried on from what they had just done (L1 Int 1 Keith). His choice to skip count involved identifying and continuing the number pattern. This same strategy was explained by another group as “just doing the pattern” (L1 Int 2 Len). The decomposition of numbers was used by another student when she multiplied larger numbers in an equation (L2 Nadine). She chose to split one number, multiply the parts and then add the parts together.

The analysis of students' work samples illustrates the use of the strategy of repeated addition throughout Lessons 1-4.
6.1.2 Making Connections and Using Prior Knowledge

Some students identified using their prior knowledge and making connections with what they already knew. Prior knowledge of basic facts was a common thread in student responses. For example, “I used my times tables. I figured out the pattern first because it helped me to figure out the answers” (L11 SARefl Anne). Another
student explained, “I used … my times tables and counted the squares. These helped me … by showing me some clues” (L3 SARefl Maya). And finally, a student identified, “One of the choices I made was use what I already knew to work out some of the answers … multiplication helped us and what we already knew about smaller numbers with multiplication” (L2 Int 1 Rebecca). This student was making connections and using what she already knew about multiplication with smaller numbers and applying this to the multiplication of larger numbers.

Other statements indicated that students were making connections to their prior knowledge and previous learning. For example, “We used our times tables and our knowledge from area” (L4 SARefl Amber). In a new situation, this group choose to draw on their area knowledge from previous lessons. Another group also explained using their prior knowledge. “We were trying to figure out together … if the two shapes had any similarities to each other” (L4 Int 1 Cathy). These students were trying to make connections by thinking about similarities between the two shapes and what they had already worked out. In a different lesson, a doubling strategy was used to calculate the perimeter of rectangles (L3), when a student recognised that the opposite sides of the rectangles were the same length and that they had already worked out the length of one side. Several students suggested what they already knew was a good place to start when solving a challenge. A student gave two instances when she had used this strategy. She used, “what I knew about area and perimeter” (L3 SARefl Cathy). Later in Lesson 5, the same student described a choice to “think about volume.”

“Thinking outside the box”, was a choice described by one student (L6 SARefl Evan) in an attempt to solve the Lesson 6 math challenges involving time intervals for cooking different quantities of food. He was referring to a challenge where the
process to get the solution was not obvious and he was unable to follow what he already knew and had experienced. He was deliberately trying to think about the maths challenge in new ways. As a result, he used diagrams, multiplication and addition to help visualise and play around with the clock times given in the maths challenge.

6.1.3 Easier Methods

Some individuals justified a choice of method as being easier for them, inferring a desire to work efficiently. For example, “choices I made … were … tidy numbers … because I thought it would be a lot easier to find the profit” (L2 SARefl Mark). This student recognised when numbers were close to a multiple of ten; he used rounding, multiplied and then subtracted the extra from his written recording. In his view, this was an easier way to perform the calculation than using other strategies to multiply large numbers. Another student associated the words ‘easier’ and ‘quicker’ when she explained, “I counted and timesed it, but it depended on the shape …These helped me … by making it easier and quicker so I didn’t have to draw all the squares” (L3 SARefl Natalie). This student chose to use counting to find the area of an irregular shape, and multiplication, as a more efficient method, when the shape was rectangular. Choosing the easier method that required less effort also helped her complete the maths challenge in less time. She switched between the two methods of calculation depending on the shape.

Several students explained choosing strategies that were more manageable for them. One student explained she was “using small times tables to figure out big numbers … because it made my work easier” (L4 SARefl Maya). Another student explained their group was “using multiplication by splitting it … so we could figure out
the problem” (L2 SARefl Ken). This student was referring to multiplying 2-digit
numbers and was using a method to split one number, multiply the parts and then
add the parts together to make the calculation manageable for him. Another chose to
“times to the closest ten and then take away the number that we were supposed to
... because I did not know the times that we were supposed to do” (L4 SARefl
Lance). Using a rounding and compensating strategy enabled him to round one
factor to the closest 10 and then multiply. He then needed to subtract one group of 6
(e.g., $6 \times 9 = 6 \times 10 - 6$). These examples illustrate that students had different views
about what a manageable strategy was.

6.1.4 Faster Methods

Some students identified their choice of methods as being faster. For example, in the
volume challenge, “I counted 14 rows of 14 on the bottom and then doubled it …
because it was quicker” (L5 SARefl Dave). The student was calculating one layer of
cubes and then multiplying the answer by the number of layers. Another student
explained, “I … times the tens, then the ones … These helped me by working it out
faster” (L5 SARefl Lance). From these students’ perspectives, their choice of
calculation strategy was related to the speed of completion.

Students were also being strategic. An individual described “thinking ahead to know
where the best spot would be and using strategy to put my numbers in the correct
place” (L10 SARefl Evan). This student was throwing a dice and placing his 4 digits
in the squares on the template in Figure 7 below, to add up to a total closest to the
target number.
Thinking ahead and performing mental calculations enabled him to identify the effect, before placing his numbers on the maths challenge template, because once placed, the numbers could not be moved. In a different situation, Evan described using guess and check to help him get the answer to parts of a maths challenge (L3 SARefl Evan). He was referring to making a guess at the possible answer, working through the calculation to check if his guess was correct and then making appropriate adjustments.

To summarise, the data analysis has identified a rich variety of maths methods (strategies) utilised by students within the Student Methods category. Some students described using one method, while others used several methods for the same maths challenge. Some students have justified their chosen methods by responding to the prompts in the SARefl post lesson (I made these choices because … ; They helped me to solve the maths problems because … ) or within the interview.

### 6.2 Representations

Representations (Category 4) presented another choice for students when they went about solving the math challenges and made decisions on how to respond. I have defined this category as using a diagram or symbolic representation for representing and communicating mathematical information and ideas, compared with mental
strategies (as in Category 3: Student Methods). When I reviewed the data, some students’ explanations were brief, with little detail, and overlapped across Categories 3 and 4. In the case where a student has talked about forming columns, I could interpret this to mean thinking about the place value of digits and visualising the tens digits placed underneath each other to help add (which I classify as a method) or the physical act of recording the numbers in columns and carrying out an addition algorithm (classified as using representations). In these cases, I have looked at the statement and interpreted meaning from the whole statement, so the same piece of data could be in both categories. I now report the data analysis for the category Representations.

Students utilised a variety of representations to help solve the maths challenges and examples from one challenge (L1) can be seen below. This challenge consisted of several parts, calculating the age of a dog (and cat) in equivalent human years.

**Figure 8**

*Variety of Representations (L1)*

Dana Lance
Luke

Amber

Dana produced a vertical list, the human years followed by a dot, and then alongside listed the associated age in dog years. Lance started a horizontal number sequence and then used an equation to represent 10 groups of 4. Luke’s recording shows his use of repeated addition and equations to keep track of his thinking. Amber’s representation resembles the development of a table of values. I now analyse the variety of representations used by students classified as drawings and diagrams, tables, identification of important information, equations and algorithms.

6.2.1 Drawing and Diagrams

A variety of drawings and diagrams were used by students to represent and communicate their mathematical ideas. Some of these involve sketching freehand lines to indicate connections. In a SARefl from Lesson 10, a student described choosing “smiley faces and rainbows” to help figure it out (L10 SARefl Dave).
The student was referring to sketching lines (see Figure 9) to help make connections to the place value of digits in a horizontal equation when he used a mental place value addition strategy. In the example above, a line was used to connect 1 tenth to 6 tenths to help keep track of the addition of the digits. The student thought of this line as a smile, on a smiley face. This student had been exposed to the place value strategy in Term 1, but the origin of the smiley face analogy he was choosing to use is unclear. Lines were also used by other students to represent a number line (e.g., Figure 10).

**Figure 10**

*Jumping the Line Strategy (L10 Anne)*

In this example, Anne sketched an empty number line to record the jumps up to a given number (jumping the line), to find the difference between her score and the target number. She then calculated the total of the jumps. This jumping strategy had been modelled and explicitly taught earlier in the year, and the student chose to use the strategy in the context of the Lesson 10 maths challenge.
Some students chose to draw shapes while they were solving the challenges. One example is from a group working and thinking about the concept of profit that was introduced during Lesson 2. One student in the group drew boxes (Figure 11) and used labels.

Figure 11

*Drawing Shapes (L2 Dave)*

The maths challenge involved a box of chocolates sold by the school PTA. The box contained 20 bars, to be sold for $2 each - all details recorded on the student’s diagrams. The picture on the right appears to represent a box with the smaller bars inside. The labels on the left show the money received for selling each box ($40), the cost of the box for the PTA ($23) and the calculated profit for selling one box ($17). The group of students chose to use these diagrams to show the important information and as a discussion point so the group could refer to them when they were working.

Students chose to draw shapes, to help solve the area and perimeter challenges in Lesson 3. Resources within the class included a supply of grid paper available for students to use if they needed. One student explained, “I used the square sheet ...
because the sheet had squares to make it easier to do the area and the perimeter (L3 SArfl Natalie). Work samples (Figure 12 below) show different strategies used by three individual students.

**Figure 12**

*Symbols and Text (L3)*

![Work samples showing different strategies for counting](image)

Kaiden  | Ella  | Dana

The first shows lines and numbers around the outside of the shape to record the counting of each unit around the perimeter. Pencil marks have been left in individual squares, possibly as the student was counting units of area. The second example has number symbols inside each square, suggesting one-to-one counting of the square units to calculate the area of the shape. The third diagram shows lines in each row, suggesting the individual has marked the counted rows when they used a skip counting strategy to calculate the area. Each student has made sense of the maths challenges using their own ideas.

Within this same lesson (L3), some students described choosing highlighters, coloured pencils and square grid paper as drawing tools, an indication of original ideas as their method was unique within the class and had not been modelled by the teacher. A student explained, “I used coloured pencils and counting ... It made it easier to work with my buddy ... coloured pencils are easier to see my work” (L3
SARefl Nora). Her work (Figure 13) shows she used a different colour to shade in the squares outside of the irregular shapes, to form rectangles.

**Figure 13**

*Choosing Colour and Grid Paper (L3 Nora)*

![Image](Image)

The shading of these extra squares suggests she was thinking about the area of rectangles when she worked through the challenges. Another work sample from a different student (Figure 14 below) shows the same idea of using different colours to indicate units.

**Figure 14**

*Using Colour to Indicate Units (L3 Amber)*

![Image](Image)

On close examination, the student appears to have some misunderstanding between units of area and perimeter – the same colour has been used to mark corner units which could lead to miscalculations. In this way, the student has experimented with colour, trying to clarify their ideas and solutions.
Work samples from two different students (Figure 15 below) show diagrams of shapes, including annotated measurement labels and sketched lines dividing the shapes into sections (L4).

**Figure 15**

*Sketched Shape Representations (L4 Dave, Mark)*

In the first diagram, it appears Dave was thinking about area and what that means. He has written the word area and marked in some ‘squares’ on the inside of the shapes, but these have no relevance to the measurements given in the challenge. The triangle is divided into sections of varying shape – not squares. The idea of splitting the irregular shapes into rectangles (to calculate the square units) was discussed during the introduction to the lesson – Dave has noted this keyword on his diagram. The second shape diagram above was used by Mark to calculate the area. Analysis suggests he has recognised the relationship between the opposite sides on the irregular shape (i.e., $100 = 75 + 25$ and $30 + 30 = 60$). He has marked the known measurements onto the shape diagram. The diagram was used by his group to focus on the challenge (this individual taking a leadership role), possibly as a way to help visualise and track progress through the parts of the challenge and to support the decision of what to do next. Mark did not mention the choice to draw a diagram in his
SARefl, focussing only on his use of written multiplication to solve the maths challenge.

Students chose to use diagrams to help solve challenges involving stick patterns. For example, a student took a photo of her working out (L11 SARefl Natalie). She drew the following diagram during Lesson 11 (Figure 16).

**Figure 16**

*Diagram of a Stick Pattern (L11 Natalie)*

![Diagram of a Stick Pattern](image)

By annotating her stick diagram with symbols to represent the position in the sequence (above) and to represent the number of sticks (below), her representation resembles the construction of a table. Her work sample reveals that as she continued further down the page, her working drawings developed into using tables with no diagrams. Possibly, as she gained understanding and confidence, she moved to a more efficient strategy. Another student, Maya, suggests diagrams made the maths challenges easier to solve, by helping her “to figure out the pattern … because she knew how to do this.” She continued to explain that it kept her “on the right track” (L11 SARefl Maya). From her point of view, she felt capable of drawing diagrams and the diagrams helped track her thinking while she continued the stick pattern. In this way she could view all the figures, check she had maintained the pattern and use annotated symbols to represent quantities.
Work samples show a variety of diagrams, symbols and notations were used by students to help think about groups of objects and equal sharing. For example, different representations were chosen by four students (Figure 17) to solve the fraction challenges in “Marble Marvels” (L9). Students have represented their ideas in different ways e.g., using numbers, dots, sections of shapes and sets.

**Figure 17**

*Diagrams, Symbols and Notations Used to Represent Equal Sharing*

The first diagram shows the work of one second language student, Eva, who was learning to understand English. She chose to draw individual dots to represent numbers and drew these in sets. This was a time-consuming process while the rest of her group raced ahead, but she was observed persevering with her chosen method and showed satisfaction reaching her solution. Work from Nora in the second picture shows segments of a circle suggesting she broke the number into parts (85 into 50 and 35) to divide it into five sections i.e., \( \frac{2}{5} \times 85 = 34 \). The third
sample diagram was drawn by Dave as he solved $\frac{2}{3} \times 78$. It appears he distributed the tens evenly across 3 groups (3 lots of 20), and then the remaining 18 into groups of 6. The final student diagram records Keith’s sharing of 232 into 4 equal sets. The recording suggests he mentally broke the number up into 200 and 32. He then divided 200 into four 50s, which he connected with lines. When dividing 32, Keith has recorded the digit 5, four times, then followed this with a 3, possibly realising that $(4 \times 5) + (4 \times 3) = 32$. The diagrams provide a glimpse into how students have chosen to represent the groups of marbles in a variety of ways that made sense to them. Digit symbols have been used as labels to keep track of their calculations.

And finally, the work sample below shows an example of doodling (Figure 18) that was a part of a diagram (Figure 11) already discussed.

**Figure 18**

*Doodling (L2 Dave)*

Although it does not include space or numbers, the illustration suggests the student was thinking about the transportation of chocolates – all part of making sense of the presentation of the maths challenge to the students (L2).
6.2.2 Tables

A variety of tables, involving numbers systematically displayed in columns, were observed in student work samples. (These have been classified in the Category: Representations). The tables illustrate students’ different choices of layout when they were making sense of the information, adapting the format to fit their thinking processes. Some recording looks more like a list, which students have extended while they worked. Other tables are more structured, requiring visualisation of the completed format when the student began recording. The student has adapted the table to fit the information they wanted from it, making the table do some ‘work’. For example, in Figure 19, the student appears to add on numbers as they progressed through the task.

Figure 19

Constructing a Table (L1 Paul)

This challenge involved calculating the age of a dog in human years. At one year of age the dog was equivalent to 21 human years, then as it continued to age, each dog year was equivalent to 4 human years. Paul has carefully kept the ‘pairs’ of numbers together and then circled them, constructing a table in his own way. In the second part of the task the table has been extended using the same pattern to find the solution.
Figure 20 shows another example of a table from the same lesson (L1). Dana selected a vertical layout (similar to the example in Figure 8), ruling boxes and included some column labels.

**Figure 20**

*Table Construction Showing Vertical Layout (L1 Dana)*

In the first column, the first number followed by a dot indicates the human years, and the adjacent number, the cat’s equivalent age in years. The first column was completed, and then she continued to work out the dog ages using a new column. She has aligned the years of both animals across the page. The rule or pattern has been noted in each column. The construction of the table in this way suggests she was aware of the patterns (+5, +4) and also the relationships between the animals at the same age. Her choice to use a dot to separate the numbers, which in maths could be confused with a decimal point, may suggest an unfamiliarity with decimal numbers. Alternatively, Dana could be responding to the opportunity to choose a method by inventing her own ‘code’. The tidiness, organisation of information and overall construction of the table in this example suggests she was aware she needed to show her reasoning and communicate her ideas so others could understand it.
The next example (Figure 21), illustrates Mark’s use of lines to form a table during Lesson 1. This diagram suggests he was aware of the opportunity that table formats presented; he developed his grid horizontally across the page and drew lines to form the columns and rows.

**Figure 21**

*Table Construction Showing Lines and Horizontal Layout (L1 Mark)*

The digits in the bottom row represent the animal age in human years. The row above (marked with a lowercase d) is the equivalent dog years, and the top row, the equivalent cat years. Mark identified his choice to make a table to solve the maths challenge and included a photo in his student agency reflection (SARefl) at the end of the lesson.

Many students justified their choice to construct a table. Making a table was a deliberate choice to keep track of their thinking and progress through a challenge (from SARefls). For example, Maya explained, “I made a year chart ... because I would get muddled up. This helped me to solve the maths problem by adding 4” (L1 SARefl Maya). The table helped her keep track of the last number as she continued to add 4 on each new line. Another student wrote he chose to “make a table … telling me all the ages” (L1 SARefl Paul). In this way, he could refer to the table to see what the equivalent age of the pet was in human years. And in a different
lesson, Lance described using “a table to help me work out the answers. I made this choice because it helped me to see the pattern” (L11 SARefl Lance). Lance chose the table as an organising tool to review all of the numbers so he could identify any pattern. Another student, Ella, chose to use a table representation to make a task easier to solve. She created a list as a whiteboard display and took her own photo to support her SARefl (Figure 22).

**Figure 22**

*Table Construction (L1 Ella)*

[Image of a table showing the relationship between human and dog years]

Ella has aligned the numbers in columns, using a dash to show the relationship between the number of human years and dog years. She also recorded the rule she used to skip count, perhaps for herself or for others to see. Another student, Rose, included a photo of a table in her SARefl and stated, “I explained my ideas and showed my working out and I wrote it all down in my book so it made it easier to work out” (L1 SARefl Rose). Rose’s explanation suggests that the explaining and systematic recording of ideas, both contributed to making the calculation easier to work out. And finally, two students described a table as a quick way to find a maths challenge solution. First, Mark explained he chose to “make a table so it would be quicker” (L2 SARefl Mark). It seemed that he viewed a table as a tool to help manage his time efficiently when recording his thinking and calculations. In the last
example (Figure 23), Natalie explained that she used a table because it was both quicker and easier (L11 SARrefl Natalie).

**Figure 23**

*Using a Table to Make Things Quick and Easy (L11 Natalie)*

Natalie formed a table horizontally across the page, aligning the numbers in columns to show the relationships. The first row represents the pattern sequence, and the row below represents the number of sticks for each representation. She used the information in the table to find the solution to the maths challenge.

**6.2.3 Important Information**

Some students chose to use a method involving the identification and highlighting of essential information in the written version of the maths challenges, using symbols and notations to help focus. Figure 24 shows an example of a student’s work on the chocolate maths challenge in Lesson 2. They have circled both the important numbers and words in the challenges while calculating the money received from chocolate sales and the profit.
Figure 24

Identifying and Highlighting Important Information (L2 Nadine)

For part b, the number of items (3) has been circled. The student made some notations, including an arrow and some numbers, adding in more information and recording their progress through the challenge. Since each carton was sold for $40, it appears she has made the calculation $3 \times 40 = 120$ and noted 120 above the quantity. In part d, the same student circled the word half, and drew a line to 20 - noting that half of a carton would be sold for $20. Nadine used a line to connect the 80 (cost of 2 cartons) and 20. I interpreted this to represent addition, and another line leads to the total price of $100. In this way, Nadine recorded her ideas and progress while her group worked.

When interviewed post-lesson while he watched a replay of his group working on video (stimulated recall), Evan explained that he was “writing down the different characteristics of the thing, problem” (L4 Int 2 Evan). A snapshot of his work can be seen in Figure 25 below (the maths challenge was to compare the area of two fields).
Evan has recorded the field shape, measurements and his area calculations. He has also written the unit km, which could be his measurement unit of choice, because kilometres were unrelated to this particular challenge. Methods used by his group included multiplication, equations and diagrams to help.

Taking notes and recording important information as a strategy in solving maths challenges was a deliberate choice for many students. Students explained their reasons for their choice; for example, “I …write it down to remember … helped me … by making it easier” (L3 SAREf Keith). Another student justified their notetaking in Lesson 3 to make the solving process easier. Making notes on the page was seen by students as a way to help them remember the numbers and record their progress throughout multistep maths challenges.

6.2.4 Equations

This section reports the analysis of student responses involving equations, where equations are maths expressions showing equality. Students’ written recording illustrates some individuals chose to represent their ideas using maths symbols to
form statements, including an equal sign. Figure 26 illustrates Paul’s work sample from Lesson 2, where equations were used to represent his ideas.

**Figure 26**

*Using Equations (L2 Paul)*

The challenge required the students to find the value of selling $2\frac{1}{2}$ cartons of chocolate (a full carton cost $40$ and 10 bars cost $2$ each). Interpreting the student’s notations, it appears Paul has used multiplication (first $40 \times 2$, then $10 \times 2$) to solve the challenge. He has been systematic in his recording, keeping the products from the left-hand and right-hand equations in the same order on the final line, and then calculated the total. Figure 27 illustrates a snapshot of Dana’s work involving equations.

**Figure 27**

*Using Equations (L2 Dana)*

The notations suggest Dana chose to use repeated addition, the symbols helping her to keep track of her calculations when adding 2. The dot underneath each 2 suggests she has counted the number of groups of 2 along the line, or skip counted up to the total, leaving a mark as she counted each one. I cannot know the order that
these equations were written – whether 15 x 2 = 30 was recorded first, and then the
next equation written to check mental calculations. Or alternatively, 15 x 2 = 30 was
recorded later to summarise findings from adding 2, fifteen times.

These examples of students’ work indicate their choices to use equations using
maths symbols to help solve the maths challenges. The analysis of other data
sources shows some students have described using equations in their SARefs (e.g.,
L2 SAR Dave, L2 SAR Mark), but their justifications are not clear. Could this be as a
systematic way of working to record progress through a task? Or could this be a
summary of thinking and calculations to communicate to other group members? And
finally, one student explained, “I used equations and pictures. I made these choices
because they were the easiest way for me to do the area of the problem...making the
explaining quick and efficient” (L4 SARrefl Evan). This student’s statement suggests
equations were his preferred method because they helped him to communicate his
ideas efficiently and effectively to others.

6.2.5 Algorithms

An algorithm is a set of written steps to calculate the answer to a maths equation.
Mark’s work sample below shows his choice to use a written algorithm (Figure 28) to
multiply larger numbers.
He explained, “The method I used was written multiplication ... because it’s the best way to do multiplication with large numbers ... giving me the structure of my answers” (L4 SARefl Mark). For Mark, the algorithm gave him a procedure to deal with the larger numbers that were difficult for him to multiply using other methods. Another student Nora also used algorithms. “I did columns. That helped me to add my scores up ... wrote stuff on the paper that helped me” (L10 SARefl Nora). She suggested that writing the numbers in columns was a useful way to help her keep track of the numbers and place value of the digits as she added up the scores.

To summarise, a wide variety of diagrams and symbolic representations were utilised by the students showing examples of students making choices about how they solved the maths challenges. Strategies for representing maths ideas were drawn from a range of sources including learning from peers, previous maths experiences and original ideas (such as using colour for marking measurement units). Representations often showed evidence of mental strategies (Category 3) e.g., an equation involving multiplication. Some students described using diagrams and symbolic representations as a choice they made during the lessons. Student work samples show different approaches taken by individuals across the same
challenge, and individual students used different representations for different tasks. Some students justified their choice to others, “easier” and “faster” being common codes (from interview and SARefl data). Other reasons given include helping with understanding, memory, visualisation, providing a structure to follow and supporting communication.

6.3 Resources and Equipment

I now report the data analysis for Resources and Equipment to model maths ideas. A student quote from Lesson 2, “fingers sometimes” (L2 Int 1 Ken), suggests fingers were a useful tool. Many student SARefls from Lesson 5, described choices to use blocks and square grid paper to make models. For example, “We used blocks to help us. We didn’t have enough little cubes so we used the hundreds and tens to help ... these helped me by making it easier to count” (L5 SARefl Amber). The task involved calculating volume and introduced opportunities for equipment to make models of boxes and to use cubes to measure the space on the inside. Student photographs (Figure 29) show the variety of equipment used.

Figure 29

Resources and Equipment (L5)

Dana    Ella    Natalie
Many students chose blocks and grid paper to model during this lesson, because; “they were helpful” (L5 SAR efl Nadine), to be “more accurate” (L5 SAR efl Ken) and it made the maths “easier” and “faster” (e.g., L5 SAR efl Cathy, Natalie, Dave, Lance).

One student sourced a piece of classroom equipment to help understand and carry out a Lesson 6 maths challenge. This task involved working with time, and a classroom timer was used to visualise how an egg timer worked. The student later mentioned in his SAR reflection using an “egg timer to help imagine and buddy to talk to ... It helped me work out the times” (L6 SAR efl Paul). The equipment helped to model the situation so the student could understand what he was working out and strategies to apply.

To summarise, analysis of the lesson data indicates some use of resources and equipment to model maths ideas. The equipment used varied according to the lesson. Some resources were prepared by the teacher and made ready for the lesson, and at other times were unplanned, with students choosing to access them (e.g., from the maths shelf), utilising what was available within the classroom environment.

6.4 Further Analysis: Maths Thinking of Different Groups

I now shift focus to look at the collective maths activity of groups using mainly Theme 2: Maths Choices data. Data was collected from 8 groups (2 groups of students from each of lessons 1-4). The group composition was different in each observed lesson, as I regularly mixed the student groups. These were mostly random social combinations, with a few students carefully placed in response to specific needs. As a result, I am unable to compare the same group over time as a complete unit. I now compare and contrast the data collected from the student interviews and SAR efls for
these 8 groups with the purpose of identifying what choices were made by these students within and across the lessons. I looked for any patterns or trends in the group data.

6.4.1 Patterns in Data Collection Method Across Groups (Interview vs SARefl)

Many ideas in the SARefl data from each of the maths challenge lessons were repeated and expanded on within the interview (held after completion of the SARefl and a 20-minute break). For example, one student explained in her SARefl (completed at the end of the lesson), that she was “making a table to help … work out the answers.” She later expanded on her idea in the post-lesson interview; “We made the choice of a table to write out the answers … to help us remember what we had been working on and to see the answers” (L1 Int Amber). This pattern of connection between choice of strategy was observed across the four lessons. In general, the interview data provided more information and detail about the students’ ideas.

6.4.2 Patterns Within Groups for the Same Lesson

The SARefls from individual students who had worked in the same group contained many of the same ideas, reflecting the shared maths lesson experience. This pattern supports the communication, sharing of methods and ways of working between group members. For example, one group member wrote, “Choices I made were using multiplication by splitting it” (L2 SARefl Ken). Another student from the same group independently recorded, “Choices I made during problem solving were adding and multiplying” (L2 SARefl Nora). Both statements, from different group members, refer to multiplying a large number using the strategy of splitting one number into two, multiplying the more manageable parts, then adding the two parts together to
get the product. The statements, written in different words, reflect two perspectives of the same group experience.

6.4.3 Patterns in Group Data Across the Lessons

Examination of the interview and SARefl data reveals there were many choices of students’ methods, and representations across the 4 main lessons. Within the group data, between two and five different Student Methods (Category 3) have been identified by students within each of the individual lessons e.g., times tables, adding, counting, skip counting and doubles (L3 Nadine, Dave). Choices involving representations range from between one and five different ideas within the same lesson e.g., pictures, equations, written addition, a table (L2 Mark, Amber, Dave). Some of the representations used show evidence of the mental strategies (Category 3) being carried out e.g., mental multiplication being recorded in an equation using symbols, to keep track of calculations. Only a few choices classified as Resources and Equipment (Category 5) were recorded in Lesson 2, 3 and 4 e.g., fingers, blocks and a ruler. The choices about resources and equipment depended on the nature of the task in any particular lesson.

6.4.4 Following One Group Through One Lesson

I followed one group through one lesson (L1) to determine the choices made and the reasons given for their decisions. Lesson 1 data from three students (Group 1: Keith, Anne, Maya) demonstrates they were choosing a variety of methods including: skip counting in fours, adding, multiplying (10 lots of 4), and continuing on from what they had just done. Using their opportunity to make choices, the group used a symbolic representation to display their answers in table form “to help them remember what they had been working on” and so they could see all of the numbers together. For
this group, some aspects of Theme 1 data appear important for their learning experience. One group member admitted that they needed help to get started on the challenge (Anne). They first tried to understand the challenge by going back to the task information. The group explained in the interview that they worked the solutions out together, they talked to each other and listened, respectfully disagreeing and not fighting. In Keith’s opinion, he helped by doing some working out. This statement suggests he was aware of the importance of his active participation within the group. He went on to explain that the rest of the group worked with him and helped him to understand. Responding to an interview question, he explained that the group didn’t give up, suggesting some difficulty (for the group) and perseverance throughout the lesson.

6.4.5 Changes for These Three Students Over Time (Anne, Keith, Maya)

I follow the same three students (as they worked with other groups) to identify changes during three consecutive lessons (L2 to L4).

Student 1 (Anne)

During Lesson 2, Anne identified using times tables (Category 3: Student Methods) and written multiplication (Category 4: Representations) in her SARefl. This was a different strategy and representation to Lesson 1. She explained that she made these choices to help her work everything out more easily and that it was faster adding everything up to calculate the answer. Analysis of her work sample reveals little about what she was adding up: perhaps her working was on a communal document and not attributed to her personally.

A week later in Lesson 3, Anne said that using times tables and also using paper helped her work stuff out. Analysis of her work sample (Figure 30) shows neatly
sketched diagrams with annotated symbols representing area and perimeter calculations. These representations suggest that counting was another strategy she used.

**Figure 30**

*Student 1 Work Sample (L3 Anne)*

In Lesson 4, Anne described using her times tables, paper and measurements to solve the area challenges because it helped her to know what to work out. I take from her use of the word 'measurements', that she was using measurement units rather than physically measuring with a ruler, because the diagrams in the maths challenge were of fields and not drawn to scale. Examination of her work sample (Figure 31 below) shows steps in her working out, involving addition and multiplication of multiples of ten. She has calculated the area of a rectangular field (90m x 60m) using her times table knowledge (which she acknowledged) and has compared this with the area of the irregular shaped field in the challenge. The area of the second irregular shaped field was worked out by splitting the shape into rectangles and adding the parts (6000 + 750 + (75 + 75 + 75)), which can be seen in the equations below. This sample shows a variety of strategies to manipulate numbers (decomposing and recomposing) utilised within the same challenge. The equations and written recording have helped her to keep track of her thinking and progress through the task. She has used a horizontal layout for her maths expressions, as well as an addition algorithm.
From Anne’s perspective, she chose to use a range of mental strategies in the first four lessons, as well as times tables to complete her work in three of the four lessons. She described using some form of representation in each of the lessons including; a table, written multiplication, paper (with shapes and symbols), and paper with measurements. Lesson 1 is the only lesson that she described having difficulty getting started.

**Student 2 (Keith)**

Keith (from the initial interview group) described using addition and multiplication to solve the Lesson 2 maths challenges when he worked and contributed to his new group. It is unclear what Keith was adding and multiplying, as his working out was not recorded on his work sample.

In his Lesson 3 SARefl, Keith explained he used squares, times tables and wrote ideas down to remember them. The concept of writing ideas down to remember them had been previously identified by a peer (Anne) in his Lesson 1 group interview, carried out 2 weeks before. His working diagram (Figure 32 below) shows he used square paper to draw a rectangular shape and sketched circles on each
square resembling an array (3 x 6). He has recorded his calculation of the area and
perimeter at the top using symbols.

**Figure 32**

*Student 2 Work Sample (L3 Keith)*

During Lesson 4, Keith described using his maths book because he was thinking his
hardest, but he did not explain what he was using his book for. Analysis of his work
sample (see Figure 33) shows that he was using multiplication to find the smaller
areas (smaller rectangles as parts of the irregular field) and adding them together to
get a total of 6,975 metres squared. The second equation (75 x 30 = 225) is
recorded inaccurately (missing a zero), but the final area is correct. Perhaps, he
decided and agreed on the answer with his group and then recorded the groups’
working inaccurately by placing a zero after the 3. He has chosen to add the
products in parts, first adding 6,000 + 225, and then adding the extra 750. The final
equation (60 x 90) to compare the area of the second field was not completed,
possibly due to time constraints during the lesson.
To summarise, the analysis of Keith’s responses over the four lessons indicate that he used representations in 3 of the four lessons, because he wanted a record to help remember his thinking. He identified using some form of multiplication in each of the lessons (described as times tables in L3).

**Student 3 (Maya)**

Maya described using written multiplication in her Lesson 2 SAREfl. She explained that she needed support during the lesson, so was talking with her group. It helped her to listen to other group members’ ideas. In Lesson 3, Maya identified using times tables and counting the squares, describing how these strategies showed her some clues. Although not identified as a choice, her work sample (Figure 34 below) shows her use of symbolic representations. In the first part of the challenge, the sketched lines suggest skip counting in 5s to find the area. When calculating the perimeter, she indicates some confusion. She has counted and annotated the square units around the outside of the square (16). As well as this, she has marked a 5 on the edge of each side to correctly calculate the unit lengths around the perimeter (5 + 5
+ 5 + 5). I infer from this that her initial misunderstanding was clarified with group support. In the second part of the maths challenge (Figure 34), she has calculated the area of the irregular shape (9 square units) by counting and recording the symbol in each square. Numbers have been recorded around the outside to help calculate the perimeter (suggesting progress from the first maths challenge), but on closer inspection, it looks like she has miscalculated, counting 15 rather than 16 units around the outside of the shape.

Figure 34

Student 3 Work Sample (L3 Maya)

Maya described using times tables to figure out big numbers in Lesson 4, to make her work easier. She explained that times tables helped her to solve the maths challenges by telling her the first part of the number, and then she ‘added’ a zero. While the strategy described in her SAR has been classified within Category 3 (Student Methods), a sample from her work shows she also used representations of equations. These helped her to make connections and to show a record of her strategy and calculations (also ensuring the trustworthiness of the SAR data).
Analysing Maya’s choices over time shows she has solved maths challenges utilising a range of mental strategies. Multiplication is a common method, reflecting the class focus and task selection. She has used some form of symbolic representation throughout the four lessons including a table, written multiplication, shapes, symbols, lines and equations.

To summarise, this group’s responses were just one example from within the whole class Maths Challenge data. They demonstrate the variety of choices that the students were making during their maths challenge lesson experiences, involving student methods, representations, resources and equipment when they were responding to the task and expectations. Students who were working in one group, then carried that experience and expertise with them to share with other groups in later lessons. With the completion of the group data, I reflect on the data analysis process in the next section.

6.5 Methodological Reflections

My study’s data analysis process included careful and detailed reporting with illustrative examples from data. This has contributed to the dependability audit process. I have provided the four aspects of trustworthiness such as transparency, analysis of an explicit data set, methodical procedures and descriptive detail to allow for transferability. My perspective as a practitioner researcher has given me the
advantage of immersion in practical teaching dilemmas leading to findings grounded in real teaching and learning situations. The rich descriptions of normal daily classroom practice reveal how knowledge of the participants and context contributed to my analysis processes and interpretations of what students said and did during the maths challenges. For example, my awareness of how much effort the students put into their responses has added to the data analysis revealing the progress and steps necessary for increasing student agency. The range of data collection methods utilised has enabled data triangulation for the same event (each maths challenge lesson) and provided a depth of detail about the complex nature of the classroom. The SARefl tool was designed as a data-gathering method to scaffold students’ reflections and identification of choices made during a lesson. This reflection process may have contributed to students’ awareness of the choices they had during lessons, hereby, acting as a learning tool to encourage more agency in future lessons. The dependability of this design is strengthened through my clear description of the case, the acknowledgement of my position within the research and my researcher decisions during the time of the study. As the practitioner researcher I had to make decisions about the composition of the groups during my data gathering. In line with the normal class programme, I opted to reshuffle the groups on a regular basis. This did mean however, that it was not possible to track the progress and interactions of a complete group over different lessons in my study. This might be considered as a limitation.

6.7 Chapter Summary

The analysis of student Maths Choice data has identified many maths choices made by students, as well as the range of choices during the maths challenge lessons. Data analysis has illustrated the depth of students’ thinking and persistence to reach
solutions that made sense to them. The student data reinforces the work of the classroom norms to guide student actions and interactions within the maths classroom.
Introduction

In this discussion chapter, there are six sections. Student agency is initially examined and discussed through three frameworks: choice through a Social Relationship Framework, choice through a Maths Framework, and The Role of the Teacher Framework. I next bring together the Framework of Student Agency in Section 4 which illustrates the interrelated nature of classroom activity when students form intentions and make choices. Having presented and discussed my findings, Section 5 links my study to literature. The chapter concludes with a summary of my findings to address my research question: In what ways can student agency be fostered in the teaching of maths?

7.1 Choice Through a Social Relationship Framework

The first analysis framework provides a lens on the role of social relationships in developing student agency. Social relationships refer to students’ participation and interactions within the class community. When examining student choices through this social relationship framework, many choices and actions can be attributed to learners being placed in a social learning situation, where they are required to participate and contribute as they work with others.

Social Relationships Were Crucial for Students to act With Agency

There are a number of findings from my study of student agency that endorse the importance of social relationships. The first finding relates to social relationships
within the class. These are a necessary pre-requisite for students if they are to act with agency. When students focused on the maths activities, they interacted with each other, and it was during these interactions that showed them making many collective choices including the sharing of ideas, allocation of jobs, listening, talking, comparing answers and actions to support each other. These social processes contributed to the development of student agency, recognising the advantages of collaborative work with their peers when working towards maths solutions. Students communicated the benefits of working in peer groups in the data; some individuals chose to share their reasoning. One example is when Nora (L11) figured out the rule to answer a particular problem herself and subsequently explained her answer to the group. This example illustrates how one student’s thinking about the maths solution helped others. In another situation (L1), Nora explained how and why she had made the decision to listen to her group. It was because she wanted to hear what they were thinking so she could gain “pointers” on how she could tackle the challenge. Another example from Maya described how her choice to speak clearly to her group (L10), enabled others’ understanding. Students also felt supported by their peers, as was the case when Amber described interacting and contributing to a group as an easier way to work (L4). Such examples illustrate a dual focus, not simply just on the maths activity, but also working effectively to assist their peers. Reciprocity was evident in these student-to-student relationships with students assisting others, as well as being willing to be assisted themselves. Over the time of the study, students were working together to solve the maths challenges and attributing the solutions to their group rather than themselves as individuals. In this way, it was the norms of the mathematical activity (structures and routines) and social norms, including small-group and whole-class norms, that provided the opportunity to talk, listen, test ideas,
and to give and receive support from others. All of these actions demonstrate examples of student choices and their commitment, obligation and shared responsibility.

**Students Desired a Sense of Belonging**

A second social relationship finding was how students’ actions conveyed their desire to belong within their small maths group. Belonging involved having their presence and contributions valued by others. Students identified actions including taking turns, showing respect for others’ ideas, patiently repeating explanations (so others could follow their thinking) and being kind to other group members. Students showed consideration while they waited and assisted others (e.g., L1). At other times, students indicated their sense of belonging or comfort within the group by acknowledging others in their SARefl using compliments such as “working with a good buddy” (e.g., L1) and “worked well”. Conversely, individuals reported social behaviours that made group acceptance difficult, when other group members worked too fast and took over all of the decisions (e.g., L4). In these situations, individuals did not feel valued by their group. Consequently, the students chose to negotiate with their group, sometimes with teacher intervention. The desire of being accepted in this social learning environment and trusted by other group members encouraged students to act with agency.

**Classroom Norms Enabled and Encouraged Student Agency**

The third social relationship finding is the important role of classroom norms (including social norms for general social interaction, sociomathematical norms for maths understanding and norms of the maths activity) to enable and encourage students to be agentic. Norms were established, by co-construction where possible,
in effect building a ‘landscape’ in which the students could operate. The data analysis provided many examples of the students forming intentions and adopting the roles and expectations of numerous classroom norms, such as the expectation to participate, make choices, take responsibility, and respond to differences in solutions. I now discuss these four aspects of classroom norms in turn.

**The Expectation to Participate**

One important classroom social norm was the expectation that students participate in solving the maths challenges with their group. Students’ intentions and awareness of their participation were illustrated in statements such as, “I wanted to help” (L2 SARefl Kaiden) and “helped me learn to participate” (L1 SARefl Rebecca). Students described their participation using action words such as: tell, share, compare, speak, explain, find out, ask, check and realise, to name a few, describing many social processes and illustrating agency in many ways. Several students viewed participation in group work as an easier way to work compared to working individually (L4).

**The Expectation to Make Choices**

The second important classroom norm was the expectation that students make choices towards reaching a mathematical solution. This was reflected in different statements from many students. For example, “We were trying to figure out …”, “[We] found out a way to do it,” and, “We made the choice of a table” (Int 1 L1). Students made different choices and as a result, agency was observed in different ways as they took up a variety of opportunities. Choices were made at different times throughout lessons and at different frequencies, giving the effect of some students taking greater or lesser agency. I initially defined agency as being indicated by
actions, but it could also involve a cognitive or social process. When students were observed taking action or expressing choices, this was inferred as agency in action.

**The Expectation to Share Responsibility**

The third important classroom norm involved the expectation for group members to share responsibility for their group’s outcomes. One aspect of this responsibility related to sharing the workload during the maths challenge lesson. The second aspect of responsibility was being aware of other group members’ progress or understanding throughout a maths challenge. The third aspect of responsibility was to understand the group’s decisions and solutions and to be ready to communicate these. I now discuss each of these responsibilities in turn.

Student statements showed an understanding of having the responsibility, as well as the intention to share the workload e.g., “not sit back and let my buddy do everything” and “I didn’t want anyone, including me, to catch a ride” (L1). Nora explained that she was trying to participate in the maths challenge and not let her partner do everything. Here the students were acting with agency by contributing to their group’s solution to share the workload. The concept of sharing the work had broken down in an incident already discussed (belonging), when one student communicated dissatisfaction that her group just told her everything.

Another shared responsibility was to be aware of how other group members were progressing through a maths challenge. This awareness was communicated in student responses through questions, comments and visual cues. For example, video data showed Nora reassuring her partner, “So, if you need help, I’ll be right here next to you,” (L10) as she gave her partner physical space and thinking time to complete the calculations. Another video episode showed two students thinking and
writing independently but keeping pace and sharing ideas as they progressed through the maths challenge (L8 Video, Natalie and Rose). Ken described how his group “waited for me when I didn’t know the answer” (L1). These examples illustrate how students were considering others in their group and were aware of what their group peers were doing and feeling, their maths understanding and progress through a problem. Showing a kind of empathy, they took responsibility, making intentional decisions to support each other. In doing so, students’ actions reinforced their sense of belonging and maths understanding within the social learning environment.

Another responsibility of group members was to understand and be able to explain their choice of solution pathway to others. This responsibility is related to participating, but ensured individuals were aware of their groups’ decisions about solving the maths challenge. This expectation was reflected in several student statements. For example, Dave responded, “I got all of the information I needed and we all helped each other” (L4), and Ken explained, “We helped each other figure out the answer” (L1). Other groups were comparing answers, for example, “Seeing if we had the same answer and worked it out the same way” (L2 SARefl Rose). In an interview, Cathy’s group said that they would try out everyone’s ideas and explain if they didn’t agree (L4). This expectation encouraged individuals to take the time to ensure they understood the solution processes involved and avoided some students relying on more confident group members for the group’s solution.

**The Expectation to Respond to Differences**

The fourth finding relates to the final classroom norm of the expectation to respond to differences and difficulties within their group. The classroom norms placed students in a position where they were able to first think about a maths challenge,
then test ideas and seek others’ opinions. One finding was that students chose to listen and compare their thinking and calculations with others, to affirm or disagree with others’ ideas. Evidence of students involved in maths discussions involving explanations, justifications and negotiation can be seen in each type of data; the video observations, interview transcripts and in student SARefl responses. For example, students described, “talking to each other and not fighting,” and respectfully disagreeing with others’ maths ideas (L1 int). During a video replay of her group working, Rebecca described what was happening, “this is where we started to argue a little bit because ...” (L2 int). Finally, Nora explained they had had a little fight (L10) when they worked to reach an agreement. In this way, students were involved in friendly arguments when they responded to differences in maths thinking. They resolved conflicting ideas by explaining and justifying their thinking to others, making further decisions to demonstrate and model drawing on mathematical equipment and representations. The expectation to respond and resolve differences placed students in a situation where they were acting with agency and extending their maths understanding, choosing to lead or following others in the group as they made sense of the maths challenge.

To summarise, the three main findings within the Social Relationship Framework are:

- social relationships were crucial for students to act with agency providing opportunities to interact and make many collective choices;
- students acted with agency when they fulfilled their sense of belonging to their group and ensured others’ sense of belonging; and
- the classroom norms contributed to students’ intentions and willingness to be agentic as they made and carried out decisions. These norms included the expectation to participate in the maths challenge, make choices towards
reaching a maths solution, take shared responsibility for the groups’ solutions, and to respond to differences that the group encountered.

Fostering student agency through a Social Relationship Framework involved a number of aspects illustrated in Figure 36 below.

**Figure 36**
*Social Relationship Framework for Fostering Student Agency*

In this way, social relationships set up a platform where students are able to be agentic. This implies that teachers can foster student agency by creating an environment where students collaborate to solve maths challenges, feel a sense of belonging within the class community, and classroom norms guide expectations and obligations. Deliberate teacher actions include raising students’ awareness of their participation in maths group tasks, promoting a sense of belonging through the
acceptance of others and valuing of their ideas, and negotiating classroom norms as students work.

7.2 Choice Through a Maths Framework

This section presents and discusses findings related to student agency through a maths framework. I discuss my findings about observed student maths choices as well as their justifications. I argue that many choices of maths methods, tools and representations were related to the students’ intentions. These intentions were shaped by individual learning orientations and the social learning environment guided by classroom norms when individuals worked collaboratively with others. I begin by highlighting findings related to students’ choices of maths methods, followed by tools and representations when students worked to solve maths challenges within the classroom.

Maths Methods

The discussion of maths methods comprises of four sections:

- different methods were chosen by individuals and groups of students to solve each of the maths challenges;
- students justified their choices of methods in a variety of ways related to their intentions;
- students chose to make connections to their prior knowledge and experiences; and
- students were exposed to others’ ideas which influenced their future thinking both within one maths challenge and across different maths challenges.
**Different Methods**

The first finding is that, within the same maths challenge lesson, different methods were chosen by individuals within a group and by different groups of students. An example from one group of students (L1 Keith, Anne, Maya) is when they chose a variety of maths methods such as skip counting in fours, adding, multiplying (10 lots of 4), and recognising and continuing a pattern, to solve parts of one maths challenge. In Lesson 2, another student indicated that she used adding, multiplying and finding a pattern to solve parts of the maths challenge (Nora). These students used a variety of methods for calculations and for explaining their ideas to others in their group. These student choices suggest that the selected maths tasks did, in fact, enable students to use a range of methods. The range of methods above describe different approaches to multiplying numbers, some being more efficient than others. Students had chosen a method that made sense to them and were also exposed to methods used by others when they explained and justified their solutions. In this way, connections were being made and students recognised methods that were ‘easier’ and required less effort.

**Variety of Student Justifications for Maths Choices**

The second finding is that students justified their choices in a variety of ways related to their intentions. Some students clearly communicated their justification for the choices they made and others did not give a reason. Justifications for method selection are grouped into three clusters: personal learning orientations (tendency to act in a certain way due to preferences, feelings and attitudes); students’ maths learning intentions (what they wanted to understand and do); and students’ intentions to fulfil the classroom norms.
The first cluster includes justifications of methods related to personal learning orientations, students tending to act in certain ways according to their preferences, attitudes and beliefs. Several students utilised a variety of methods and tools in different lessons but gave the same reason for their choices across the lessons. For example, Evan’s choices of methods were often based on efficiency and time management when he explained, “This helped me to solve the maths problems by finishing all the questions”, “staying on task,” to “get as much done as possible,” and it “wasted less time” (L1,2,4 SARefl). Whereas Nora’s main justification for her choices made across the lessons, was to help her understand more easily (L3, L5 SARefl). She preferred to work slowly and methodically through the tasks, often writing more so that she could see her work. Student justifications were often related to their preferences and understanding as learners, for example, whether it was easier, faster, or helped their understanding. Student choices were related to their personal learning orientations and what helps them learn (e.g., their confidence, memory and speed of decoding).

The second cluster involves justifications of method related to personal maths learning intentions. For example, Nora described making maths choices within a task to help her become better at multiplying (L4). Cathy explained that she “wanted to do this style of maths more easily” (L10). These students understood that participation in the task was supporting and strengthening the achievement of their personal maths learning intentions.

The third justification cluster relates to the students’ intentions to fulfil the classroom norms, such as collaboration, sense of belonging and responsibility. At times, students deliberately chose to use less efficient methods. For example, one student used one-to-one counting to ensure understanding and accuracy, as well as to
model and communicate ideas to others in the group (L3 Rose, area challenges). In this way students were involved in choosing their solution pathways, using a variety of different methods within the same task. Student choices reflected their knowledge of methods and their intentions to fulfil their social responsibility to support their group.

**Choices to Make Connections with Prior Knowledge and Experiences**

A third finding is that student choices were connected to their prior knowledge and experiences. Examples are choosing to apply maths methods and facts already learned (e.g., splitting and reorganising the numbers, and instantly recalling times tables (Ken L2, Nadine L2, Ella L3)). Some students chose to use a familiar strategy across consecutive lessons, for example, repeated addition was observed as a regular student choice (Len L1, Keith L2, Zoe L3 etc.). One student made the connection to real-world experiences when he was calculating the profit in a challenge (Mark L2). Students were drawing on their prior knowledge in order to solve the maths challenges.

**Exposure to Other Ways of Thinking**

A fourth finding is that students were exposed to others’ ideas which influenced their future thinking both within one maths challenge and across different maths challenges. Reflecting and thinking about others’ methods helped students to identify methods that could help them as they worked together in their group. For example, when Keith watched a replay of his group video and scanned his workbook, he explained the difficulties his group had. He then reflected that his group “could have just halved the numbers and then halved them again,” to find a quarter of the quantity (L7 Int). He communicated these ideas as a result of being exposed to the
strategies of others during the class discussion at the end of the lesson. In this way, students became more aware of choices they could make through interacting with peers, participating in class discussions and the reflection process. Interaction with other groups provided another learning opportunity. A video recording from lesson 10 captured a student ‘visiting’ another group actively seeking out others’ ideas while waiting for the teacher, evidence of a learning ‘network’ and students from different groups supporting each other. This illustrates the migration of ideas across groups. Data analysis also found the migration of ideas between lessons. One student (Keith) picked up on the idea to write things down to help remember them (as discussed by his group in the Lesson 1 interview). He then repeated the idea in his Lesson 3 SARefl, when working within a new group.

**Student Choice of Tools and Representations**

The next group of findings relates to student choice of maths tools and representations. Tools refers to resources and equipment (physical items) chosen to model maths ideas, whereas the term representations, refers to using diagrams or symbols to communicate maths information and ideas. Students intentionally made choices in tools and representations to support their own and other group members’ thinking. There are three main findings related to the choice of tools and representations:

- maths representations were a common student choice and important for student learning;
- students’ choices to adopt the classroom norms encouraged variety in tools and representations; and
• choices of tools and representations related to students’ intentions to find solutions in this collaborative environment.

**Maths Representations Were a Common Choice**

The first finding is that representations were a common student choice and appeared important for student learning. This was indicated by a variety of data including student group interviews, individual reflections, student work samples and observations of students working within the class. The word count of terms used by students within the interviews and SARefls (described in the initial analysis, Chapter 4) revealed students used ‘picture’, ‘draw’ and ‘drawing’ 57 times to explain their choices. The use of representations appeared to be important for student learning and there was a wide variety of representations, for example, sketched lines, use of colour, drawn shapes, recorded equations, drawn tables, as well as, doodling within each lesson. These representations helped students to focus on important ideas (L4 Int 2 Evan), record their thinking for themselves and others (e.g., Fig 24), make connections between numbers (e.g., Fig 9), refer to calculations already made (L4 SARefl Evan), organise information (L11 SARefl Lance), visualise problems (L6 SARefl Paul) and remember information (L3 SARefl Keith).

**Variety in Tools and Representations**

The second finding is that students’ choices to participate in the classroom norms encouraged variety in the use of tools and representations. Variety is seen in the range of tables designed by students to support their thinking (e.g., L1, 2 and 11) as they participated in their group, made choices and shared responsibility. Tables varied from a list of numbers to numbers organised into rows and columns divided by lines (a vertical layout). Students were involved in designing diagrams to help them
make sense of situations resulting in intentional choices in the layout of figures and diagrams.

The choice and design of representations can be attributed to different reasons through the data analysis (documents and participant observation), my inferences and knowledge of the context. These included:

- known/ familiar representations;
- group influences on the choice of representation; and
- trialling and adaptation of others’ representations.

The first reason, namely known or familiar representations, relates to representations that had been explicitly taught, discussed and practised during previous maths lessons as part of the class maths programme. Examples are the procedure used in the multiplication algorithm (L2) and the jumping-the-line method to find the difference between two numbers (e.g., Figure 10, Chapter 6). Another was writing equations to represent and record information such as the different mental operations used when solving maths challenges (Nadine, Dave). Some familiar resources and equipment were chosen by students including fingers (Ken) and place value blocks (L5), but the usefulness of objects was dependent on the maths challenge and maths concepts involved.

The second reason for choosing particular representations came from group influences. Examination of student work samples revealed individuals from within a group had all chosen a similar table layout (L1 Anne’s group, L4 Cathy’s group). In Lesson 3, several students from different groups chose to use coloured pencils to illustrate shapes to help calculate the area. This idea originated in class and individuals observed others and then decided to use the idea themselves. These
suggest students were choosing to either lead or follow initiatives from others within their groups and also ideas were migrating between groups.

The third reason for the variety in tools and representations was students trialling and adapting others’ representations. One illustration of trialling original ideas was Nora’s use of coloured pencils to shade squares outside of the irregular shapes (L3). Her explanation revealed her intention to make the units easier to see and to communicate her thinking with her group (Figure 13, Chapter 6). Students trialled and adapted an idea when they used hundreds and tens blocks to fill in the volume of a 3D shape when they had run out of unit blocks (L5). In this instance, they were able to explain that their choice made it easier to count. Some representations suggest that individuals were adapting their diagrams to fit their thinking as they progressed through a maths challenge. For example, Paul added sketched lines to separate numbers in a table (L1). Another student’s work shows labels on a stick pattern that developed into the construction of a table (Natalie, L11).

In summary, these findings illustrate the varied choices students were making in using tools and representations. The students used ideas from their peers, from prior learning experiences, and trialled and adapted their own designs utilising opportunities within the learning context.

**Choice of Tools and Representations Relates to Student Intentions**

The third finding is that decisions about the type of tools and style of representations related to the students’ intentions. Some students communicated the reasons for their choice of tools and representations. The range of reasons included, “so I wouldn’t become muddled”, “telling me all the ages”, “to help me see the pattern”, it was easier to work out, to make it quicker and “so I wouldn’t forget”. Evan (L4
SARefl) chose equations and pictures to represent his ideas to help communicate his ideas quickly and efficiently to others, illustrating the classroom norm of an expectation for him to convince the group of his ideas. Students therefore chose different forms of representations to assist their own and others’ thinking in their social learning environment.

To summarise, within a maths framework there are four findings related to maths methods, and three findings related to tools and representations. The findings are:

- different methods were chosen by individuals and groups of students to solve each of the maths challenges;
- students justified their choices of methods in a variety of ways related to their intentions;
- students chose to make connections with their prior knowledge and experiences;
- students were exposed to others’ ideas which influenced their future thinking both within one maths challenge and across different maths challenges;
- maths representations were a common student choice and important for student learning;
- students’ choice to adopt the classroom norms encouraged variety in tools and representations; and
- choices of tools and representations related to students’ intentions to find solutions in this collaborative environment.

Student agency when viewed through a maths lens, has identified important aspects, summarised in Figure 37 below.
7.3 The Role of the Teacher: Strategies to Promote Student Agency in Maths

The third and final framework for understanding student agency is a teacher framework. A lens on the teacher recognises the important role they play in creating the conditions to support classroom learning through deliberate actions. There are two findings related to the role of the teacher. The first finding is that deliberate teacher actions across lessons are crucial to set conditions to promote agency, involving curriculum design and the establishment of classroom norms. The second finding is that teacher responses ‘in-the-moment’ during lessons are important for fostering student choices in methods and representations. I also identify aspects of monitoring and maintaining the classroom environment to actively promote student agency.
Deliberate Teacher Planning

The first group of findings highlights the deliberate acts of teacher planning needed across lessons to promote student agency. These acts of longer-term planning involved:

- curriculum design;
- the establishment of norms for maths activity (organisational structures and routines); as well as
- the establishment of small-group and whole-class norms.

Deliberate Teacher Planning – Curriculum Design

Deliberate acts of teacher curriculum planning supported students to be agentic and make contributions during maths challenges. The maths challenges were deliberately selected for their potential for group work at a range of levels and student choice of methods and representations (e.g., L2 could be solved using multiple methods). Tasks were deliberately sequenced to enable the consolidation of maths concepts learned. This supported some students to display greater agency and confidence by carrying over learning from one activity through to future lessons (e.g., repeated addition in L1-4). Deliberate teacher planning included anticipating how students could tackle a maths challenge. This enabled planning to allow time, and to make equipment and resources available to promote greater agency e.g., squared paper. Other planned support included visual wall displays, for example, the CUBES strategy poster to help engage with a task, and a poster summarising possible multiplication strategies (L2). The use of mini-lessons between the data-gathering lessons provided opportunities for students to become more familiar with
some concepts as needed e.g., equal sharing, and to become fluent with a particular strategy e.g., place value partitioning.

The features of some tasks appeared to be more suitable for fostering student agency. Some tasks provided more observable examples of students taking agency because students were more active and could be seen ‘doing’ things such as manipulating equipment, making models or heard explaining their ideas. In others, student choices e.g., mathematical methods, were more apparent when students’ work samples were analysed later. For example, Lesson 5 involved the use of equipment that others in the classroom could see and hear. These more overt displays of agency may have influenced others. The more difficult multiplication challenges in Lesson 2, while providing lots of opportunities for agency, resulted in choices that were less obvious to neighbouring groups. The Lesson 3 area challenge involved different parts that ranged in difficulty before moving to open-ended situations, enabling all students to easily engage with the task.

**Deliberate Teacher Planning – Lesson Structure**

The second aspect of deliberate teacher planning involved the norms for the maths activity. The planned lesson structure and routines (described in Chapter 4) provided the social environment and foundation for learners to understand and engage with each math challenge and then collaborate in small groups to reach solutions. Finally, sharing solution processes at the end of the lesson (during the whole class discussion) was important, allowing student communication and exposure to others’ ideas. This also gave me an opportunity to make students aware of specific choices made by groups or students, connect ideas and celebrate significant students’ work.
Deliberate Teacher Planning – Small-Group and Whole-Class Norms

The third aspect of deliberate planning was the negotiation of small-group and whole-class norms to guide social interactions and promote student agency in maths. Deliberate actions included taking time to focus on strategies and expectations for respectful interactions and discourse, as these were seen as important for maths learning e.g., eye contact, taking turns, listening, asking questions, explaining clearly and disagreeing in a respectful manner. Ideas were made explicit (e.g., using talking frames), discussed, modelled, practised in context and reflected upon. A video recording illustrated the testing of ideas when a student asked her partner, “So do we plus all these together to get the total score?” (Maya, L10). These social norms enabled students to talk and test ideas, make connections with others’ working out and for more efficient strategies to be explored.

One classroom norm was that students were expected to take some sort of action when things got difficult. I focussed on conveying the message that the students could learn and ‘do’ maths during student-teacher interactions by the careful use of prompts and questions to build confidence. A researcher journal entry (May 2019) reflects my thinking and effort not to take over and intervene unnecessarily. I incorporated, “Do you have a question for me?” as a deliberate conversation starter when I approached a new group during a maths challenge. In this way, there was a shift in responsibility for solving the maths challenge rather than the assumption that the teacher would provide solution processes or answers.

Further teacher actions involved organising for students in the study class to carry out regular reflections on student agency. The SARefl (Student Agency Reflection) was a new addition to my maths programme. The students were presented with a
scaffold to use and time to reflect both on the choices they had made and what had helped them solve the maths challenge. This teacher action increased the value and visibility of student agency, endorsing it as part of the class culture. The direct consequence of this activity was an increased awareness of agency which resulted in more deliberate student choices. The students also participated in interviews, with the aid of stimulated recall, to reflect on their participation and contributions to lessons. Students were exposed to the term ‘student agency’ during the research consent process, as well as within the title of the SAREfl. But, more often the term ‘choices’ was used, for example, in the interview questions. Over time this contributed to a feedback cycle - students becoming more confident in articulating their choices and justifications, hereby promoting more deliberate choice.

The deliberate organisation of students into groups was important for the social nature of learning because it placed students in a position to interact and make choices with others. Data analysis found that students received support and encouragement from other group members. For example, one student explained, “I got an answer and then we showed it to each other to see if it was wrong.” Another stated, “We worked it out together “(L1 int). Providing encouragement and reassurance to a group member, one student was recorded as saying, “So if you need help, I’ll be right here next to you.” Occasionally, teacher intervention was required to help students persevere, maintain relationships and move on, for example, when one student became discouraged, complaining that she couldn’t make any choices because her group wouldn’t let her (Amber).
Teacher Responses ‘in-the-Moment’

The second group of findings relate to my deliberate teacher responses ‘in-the-moment’ as the lessons unfolded. The deliberate noticing of choices ‘in-the-moment’ enabled me to encourage students to act with agency. Students were supported in their agency when I allocated sufficient time for the maths challenges to enable students to explore solutions without teacher intervention. Some student choices (such as methods they trialled), were unsuccessful and required further attempts to reach a solution. Students made mistakes and reworked solutions, sometimes using less efficient strategies. As a result, this took time. Care was needed by the teacher about deciding when it was necessary to intervene. This was illustrated when a student recorded a choice to ask the teacher questions (L6 SARefl Maya) as I roved the classroom. The ability to be flexible with time and to extend some lessons supported the continuing development of student agency. For example, extra time was needed for the manipulation of blocks (in L5) because students first created the box model with paper, then used blocks to assist their volume calculations. Continuing with the lesson avoided frustrations with packing up and having to redo calculations. Allowing time after asking students a question (wait time) provided time for them to think about and formulate their response. This teacher action transferred authority to students, communicating the expectation for them to contribute and gave time for the students to actually do this.

To summarise this section, deliberate teacher actions fostered student agency. The Role of the Teacher Framework has led to two important findings listed below:
• deliberate teacher planning across lessons including curriculum design, lesson structure and organisation, and negotiation of classroom norms fostered student agency; and
• teacher responses ‘in-the-moment’ as the lessons unfolded, contributed to greater agency.

Specific aspects of these teacher actions are represented visually in Figure 38 below.

**Figure 38**

*Role of the Teacher Framework for Fostering Student Agency*

These findings show that the teacher has an important role to plan for student agency, as well as to notice and respond to encourage agentic acts by students ‘in-the-moment’.
7.4 Bringing Together the Framework of Student Agency

This study has 12 important findings across the three frameworks. The findings are listed below and numbered for ease of reference in the discussion.

1. Social relationships were crucial for students to act with agency providing opportunities to interact and make many collective choices.
2. Students acted with agency when they fulfilled their sense of belonging to their group and ensured others’ sense of belonging.
3. The classroom norms shaped students’ intentions and willingness to be agentic when they made and carried out decisions. These norms included the expectation to participate in the maths challenge, make choices towards reaching a maths solution, take shared responsibility for the group’s solutions, and to respond to differences that they encountered.
4. Different methods were chosen by individuals and groups of students to solve each of the maths challenges.
5. Students justified their choices of methods in a variety of ways related to their intentions.
6. Students chose to make connections with their prior knowledge and experiences.
7. Students were exposed to others’ ideas which influenced their future thinking both within one maths challenge and across maths challenges.
8. Maths representations were a common student choice and important for student learning.
9. Students’ choices to adopt the classroom norms encouraged variety in tools and representations.
10. Choices of tools and representations related to students’ intentions to find solutions in this collaborative environment.

11. Deliberate teacher planning across lessons including curriculum design, lesson structure and organisation, and the negotiation of classroom norms fostered student agency.

12. Teacher responses ‘in-the-moment’ as the lessons unfolded, contributed to greater agency.

The three frameworks, Social Relationships, Maths and The Role of the Teacher are interrelated, helping to form student intentions and choices, as represented in Figure 39 below. Intentions as it is used here, is a broad term, and could refer to social goals or the intent to make sense of a maths challenge. Student intentions precede choice, for example, the intention to help a group member understand maths ideas leads to choices in representations to illustrate their thinking. Intentions are important for students to make purposeful decisions and take action. Hence, student intentions are an important aspect of agency.
Findings from my study suggest that setting appropriate classroom conditions and the teacher’s role in this (Findings 11 and 12) were crucial for student agency in the maths challenge lessons. The teacher’s influence in organising these conditions is represented by the top circle with two-way arrows leading to (and feeding back from) the social conditions (left circle) and maths conditions (right circle). The Social Relationship Framework identifies the organisation of student interaction and collaboration (Finding 1), a sense of belonging (Finding 2) and the co-construction of classroom norms to guide behaviour (Finding 3), as being important for students to be agentic. The Maths Framework identifies fostering student choices of maths methods (Finding 4), tools and representations (Finding 8) as important to enable student agency. Aspects of these conditions are interrelated and help form student agency.
intentions, indicated by a two-way arrow. For example, the development of classroom norms helped the students to work together and influenced their intentions to choose suitable methods and representations (Findings 5 and 10). Expectations to resolve differences, resulted in students explaining and justifying why something made sense mathematically to their group. This helped them to realise opportunities to demonstrate and convince others of their thinking from the maths perspective e.g., to utilise representations or use a different method.

7.5 Linking Findings to Literature

I now relate these findings back to the findings of other researchers addressing student agency within maths classrooms. I set out literature that aligns with the findings of my study. My findings have highlighted many interrelated aspects which contribute to creating conditions for student agency. Dominant themes have arisen in response to common maths classroom teaching dilemmas e.g., encouraging purposeful talk, and the effects of allowing students to choose methods. These main ideas are discussed in the following six sections:

1. Actively participating and learning together.
2. Students forming intentions about their future actions.
3. Classroom norms to guide actions.
4. Students acting by representing their maths thinking.
5. Thinking and talking about agency.
6. The role of the teacher to foster agency.

7.5.1 Actively Participating and Learning Together

Students actively participating and learning together formed the foundation for students to act with agency in maths. Aspects of this learning environment, social
relationships, student collaboration and sense of belonging are discussed, forming the focus of this section.

Social relationships within the maths learning environment were foundational for enabling student agency (Finding 1). Students within my study were choosing to participate and interact with others to share maths experiences and ideas, some suggesting that it was an easier way to work. Relationships such as these, between students and teachers, form the basis of social constructivist maths learning theories described by Cobb et al. (1992) and Yackel et al. (1991), where language and communication between students play an important role. Cobb et al. (1992) observed students making adaptions to their maths ideas when they worked together to reach a consensus towards their solutions for the tasks, and as a result, they were constructing solutions that they probably would not have done if working alone. In this environment, the students “participated in the establishment of the situations in which they learned” (p.99). Further, the participation in social practices involving interactions with others within the maths classroom e.g. sense-making through discussion, determine an individual’s maths learning, as well as their identity as a maths learner (Boaler & Greeno, 2000). They found that maths knowledge and understanding is the result of participation in social activities where people are involved in maths talk and use maths representations and concepts to make sense of maths challenges.

Specific aspects of the importance of social relationships include students choosing to give support and receive support from others. In my study, students described talking, listening, trialling ideas and helping each other. In this way, students were involved in communicating choices to organise themselves, sharing ideas with others, comparing answers, checking their understanding of the questions,
explaining, justifying and reasoning together. The importance of social relationships involving interactions is supported by other researchers (e.g. Boaler, 2000a; Brown, 2020; Vaughn, 2020). Vaughn (2020 p.112) suggested that agency is “co-created with other individuals such as peers and teachers and across a variety of social interactions and contexts.” Students are part of a socially interconnected system, and they make decisions and act, negotiating and drawing on resources around them to carry out their plans. Similarly, Brown (2020) viewed students’ ways of exercising agency are embedded in the context of the mathematical activity where they actively participate with others, applying what they know and can do. He suggested that agency is negotiated between the students, the teacher, as well as the conventions of the mathematical discipline. As such, Brown’s view has similarities with the three frameworks of my study (Figure 39), summarising my findings for creating conditions for fostering student agency.

Students made choices with the desire to be accepted and belong to their group (Finding 2). In my study, students indicated their intention to share responsibility and take on a fair amount of the workload. Some made decisions to assign parts of the task to different group members, sharing responsibility and ownership. These examples indicate students collaborating together, working towards the same goal and using each other for help and support. The benefits of student collaboration were in common with existing research. Collaboration involves shared authority and responsibility when group members work towards a common goal as opposed to individuals competing against each other (Laal & Ghodsi, 2012). Group members are responsible for their actions including respecting the abilities and contributions of their peers. Gresalfi (2009) investigated aspects of classroom systems where the teacher focused on what it meant to work collaboratively in a group. She found
collaboration was important for maths learning, where the students engaged collectively in groups working towards a common goal and took responsibility for the learning of others in their group. In a collaborative environment, students have opportunities to ask for help, to challenge and support each other, take more responsibility for understanding and engage with the content at a deeper level (Cobb et al., 2009; Gresalfi et al., 2009). Collaboration is a key aspect of curriculum design within research literature to inform teachers (e.g., Dumont et al., 2012; Ontario Ministry of Education, 2018). Therefore, collaboration and social relationships can be thought of as resources to support learning – or in this case, resources to support student agency.

Belonging refers to feelings of being connected and respected. A sense of belonging (Finding 2) in my study was when students took care to “kindly” or “respectfully” disagree with others’ ideas showing they valued their contributions and did not want to cause offence. Students’ sense of belonging and comfort within their group influenced their ability and willingness to make further choices to contribute mathematically. This finding resonates with research about how a sense of belonging in the maths learning environment can enhance learning (Barbieri & Miller-Cotto, 2021; Dasgupta et al., 2022; Good et al., 2012). A sense of belonging can be associated with student well-being leading to students’ satisfaction in the learning environment. Students in my study both received and showed respect for others. Students felt valued as others waited for them and assisted them. Student compliments such as “a good buddy” reflected their positive feelings and attitudes within the learning environment supporting their willingness to act with agency. Aligning with this finding, Gresalfi (2009) found affective factors (emotional and motivational) to be inseparable from the learning process.
7.5.2 Students Forming Intentions About Mathematical Actions

Intentions about mathematical actions often led to more specific goals and plans of action, such as the choice to use maths methods and representations. Many students justified their choices in terms of their intentions (Findings 5 and 10) e.g., to organise their numbers, to recognise a pattern, to select a particular maths method so they could manage the calculations. Intentions are identified as a property of agency and are a common theme in the literature (e.g., Bandura, 2006; Klemencic, 2015; Vaughn et al., 2020), although these sources are not specific to maths.

Some student justifications for choices were attributed to personal learning orientations e.g., specific individuals deliberately made decisions (on more than one occasion) to use methods that they viewed as more efficient or easier. This is consistent with Bandura’s (2006) view, that self-interests need to be accommodated if an individual is to achieve “unity of effort” within a collective situation (p.164). Boaler and Greeno (2000) also considered learners negotiate a sense of themselves, as well as being learners of maths. Through this negotiation they become aware of their capabilities, learning what they can do, as well as their preferences. The opportunities for students to form intentions and act on them enabled learners to manage themselves and establish self-belief as learners, fulfilling the intent of the key competencies in the national curriculum (Ministry of Education, 2007).

Some students explained their personal intentions were to work towards achieving the maths learning intention, which they had made their own. In these cases, the student’s understanding of the maths purpose of the activity was important. When a student chose to multiply larger numbers, the student was purposefully focused on
multiplication, taking up the opportunity presented by the task and paying more attention to mathematical methods that were efficient to handle larger numbers. In a similar way, Absolum et al. (2010) found students understanding of the learning purpose enabled them to be more intentional and to take more responsibility in general classroom practice. More specifically in maths, literature recognises students’ understanding of mathematical learning intentions as important (Leinwand et al., 2014; Sullivan et al., 2009). The effects of understanding the mathematical purpose of an activity on intrinsic motivation could also be explained using Ryan and Deci’s (2017) Self Determination Theory framework (see Chapter 2). Understanding the lesson learning intention helped to make the activity more purposeful from a student’s perspective, facilitating their motivation to act, resulting in higher engagement and satisfaction.

7.5.3 Classroom Norms to Guide Actions

The classroom norms in my study, shaped students’ intentions to participate, make choices, share responsibility and resolve differences (Finding 3). Firstly, the norm for student participation and engagement in the group activity was expected as part of the classroom routines and important for their learning. Students were expected to work with their group to select and try out a method for solving the maths challenge. During the course of my study, other participants’ reactions (both the teacher and students) helped to affirm and encourage appropriate actions or discourage unhelpful contributions. When students made contributions to their group, this led to choices in methods (Finding 4) and representations (Finding 8). This is consistent with many researchers who recognise that active participation is essential to a successful and equitable maths programme (e.g., Boaler & Greeno, 2000; Engle & Conant, 2002; Lampert, 1990b). The role of the classroom norms to guide these
actions to support maths learning has been found in other studies (e.g., Boaler, 2003; Yackel & Cobb, 1996; Yackel et al., 1991).

Participating with agency involved the expectation for students to talk about maths solutions with group members, the class and the teacher. When students in my study were selecting from different maths methods and representations to solve the maths challenges, they did not always agree with each other on the solution or best way to approach a task. Consequently, discussion ensued requiring individuals to explain and justify their thinking to others. Often, they explained why a particular choice of method made sense. Groups of students resolved differences in solutions and responded to difficulties (Finding 3) resulting in students being exposed to other ways of thinking (Finding 7) and drawing on different methods and representations (Findings 4 and 8). In this way, the group and class discussions provided important opportunities for taking agency. Similarly, Boaler (2014a) highlighted how resolving differences through maths talk and struggling with solutions, can serve as learning opportunities. Other maths researchers support students talking to learn by explaining and justifying solutions with others and being agentic. For example, students need to be active participants when they negotiate solutions, engaging in explanations and justifications to convince others in their group, rather than just convincing the teacher of their understanding (Boaler, 2003; Gresalfi et al., 2009). Brown’s (2020) model of shared agency included the co-construction of arguments, class discussion, consensus through explanations and construction of agreement (p.141). Maths talk is therefore one way that students can be accountable to other group members for contributing, understanding and taking ownership of group solutions (Gresalfi et al., 2009).
Secondly, the classroom norm that students make mathematical choices was necessary for students to be agentic. In my study, students made choices in maths methods (Finding 4) and representations (Finding 9) to reach solutions. Choosing methods and representations required individuals to determine which method would best suit their intentions. This involved students visualising and trialling their chosen method’s suitability in working towards a solution. Students were exposed to new ideas when they shared their ideas with other group members (Finding 7) and were influenced by others’ choices of methods and representations. As a result, students developed confidence and self-belief as learners to make further mathematical decisions. Choosing maths methods can be associated with Bandura’s (2006) second property of agency, being self-reactive, where people actively create and implement their action plans. Placing students in situations where they need to choose maths methods is supported by other researchers who found maths learners used maths representations, methods and concepts as resources for sense-making (Boaler & Greeno, 2000; Gresalfi, 2009; Gresalfi et al., 2009). The exploration of maths methods could be likened to the ‘dance of agency’ (Boaler, 2003; Pickering, 1995), described as students moving between using standard methods of the discipline and their own ideas. However, Yackel and Cobb (1996) suggested that certain types of maths choices, involving sociomathematical norms to compare solutions and make judgments on their usefulness, are important for maths understanding.

Opportunities to choose a variety of solution pathways sometimes resulted in less efficient methods. One consequence of enabling choice was that in some cases, students chose to use a strategy that required more effort or took longer to complete. At times this was deliberate, for example, when a student used one-to-one counting
of squares to explain the area to others. But, at other times it was not, as in the case when an ESOL student was working slowly and methodically to understand a challenge (L2). Yackel et al. (1991) considered it a norm that students use methods meaningful to them, and that they also try to make sense of their group’s methods. Stein et al. (2008) drew attention to the range of reasoning in solutions, including more efficient methods. Their model suggests sequencing simple to more complex thinking during class discussions. This provides an opportunity for teachers to respond to student choices, to acknowledge more efficient methods and to make connections.

Acting with agency by choosing methods and representations encouraged student self-regulation. Self-regulation involved students’ organising their thinking, reflecting on, and monitoring their progress towards their intentions, supporting them to manage what skills and knowledge to apply. An implication of being given choice in maths methods was the need for individuals to monitor and establish whether their choice helped reach a solution. In my study, students used self-checking, estimation, and knowledge of numbers to make decisions on whether to, and how to proceed. This monitoring aligns with Bandura’s (2006) third property of agency - self-reflectiveness. Other literature supports both self-regulation and reflection as general dimensions of agency (e.g., Bandura, 2001; Vaughn, 2020).

7.5.4 Students Acting by Representing Their Mathematical Thinking

In my study, students acted by representing their mathematical thinking for themselves and others (Finding 8). Student-generated representations of symbols, diagrams, and models, were a common choice and provided many opportunities to for student choice. Sometimes resources and equipment were used as tools to
create representations e.g., blocks, square paper. Some students recorded ideas as a graphic organiser to establish their progression through a maths challenge. Others deliberately used representations to prove their ideas to others. The classroom norms, encouraging choice of maths methods, translated into a variety of representations (Finding 9). The analysis of representations indicates that students drew from a range of ideas, the social nature of the classroom providing exposure to other ways of thinking. Students were also involved in interpreting the usefulness of representations. In this way, student representations provided important opportunities for student choice and the different representations showed student agency in action. The importance of tools and representations to support learning in maths is supported by studies (e.g., Boaler & Chen, 2016; Gresalfi et al., 2012; Sellars & Lowndes, 2013; Selling, 2016). Representations are also recognised as important for learning in a national teacher guiding document (e.g., Ministry of Education, 2007), where they are described as a key competency: using language, symbols and texts. Brown (2020) viewed representations as an opportunity to contribute ideas to maths discussions, summarise ideas or re-represent ideas to others, echoing students making choices related to their intentions. Similarly, Selling (2016) identified students using representations as a way to encourage persistence and justify their thinking to other members of their group.

7.5.5 Thinking and Talking About Agency

Students’ involvement in a conversation about agency and reflecting on their choices and actions within lessons, supported some individuals to be more agentic in later lessons compared to the start of my study. They became more aware of choices in maths as they justified their decisions (Finding 5), were exposed to other ways of thinking (Finding 7), and shared responsibility with others (Finding 3). The post-
lesson discussions and data collection methods (student reflections and focus group interviews) led to an increase in students’ awareness of their intentions and actions. For example, students recognised choices made within the lessons, both by themselves and others, and became more aware of opportunities for future lessons. The students were seeing themselves as capable of making choices and acting. As a result, they exhibited an increase in self-confidence, enabling or activating their capability in this area. In support of these findings, several writers suggest that everyone has the potential to act with agency. For example, agency is “available to everyone” (Sengupta-Irving, 2016 p.211). Bandura (2001) viewed people as not simply onlookers or undergoers of experiences but agents who are exploring, manipulating and influencing what happens. Bandura (2006) identified a self-reflective property of agency involving individuals thinking about their performance and making appropriate adjustments to future actions that resonates with my study. However, Bandura considered self-belief and confidence to perform matter, which has important links with a maths learning context because “unless people believe they can produce desired effects by their actions, they have little incentive to act, or to persevere in the face of difficulties” (Bandura, 2006 p.170). Self-efficacy as a dimension of agency is also supported by Vaughn (2018).

7.5.6 The Role of the Teacher to Foster Agency

The role of the teacher to deliberately create conditions for agency and remove ‘roadblocks’ was critical. Deliberate teacher planning (Finding 11) included curriculum design, maintaining lesson structure and routines, and the establishment of classroom norms. Also, planned actions such as modifying student groups were crucial for supporting a sense of belonging and to maintain working student relationships. Research studies recognise the fostering of appropriate conditions for
agency through planned actions. Vaughn et al. (2020) recognised antecedents of student agency in literacy that have similarities to creating the conditions for fostering student agency in my study (see Figure 39). They suggested the instructional practice of allowing flexibility and choice in literacy materials and resources which aligns with my maths choices. Vaughn et al. also identified actions to encourage student dialogue and thinking together, which has similarities with my social relationships sphere. Sengupta-Irving (2016) demonstrated deliberate planning and organisation (e.g., social organisation, task selection) to encourage agency within a different mathematical learning context. Cobb and associates focused on the important role of the teacher to establish and maintain classroom norms for maths lessons (Cobb et al., 1992; Yackel & Cobb, 1996; Yackel et al., 1991). These norms included the development of both social norms and sociomathematical norms through teacher-initiated discussions and guided co-construction as students were working. In this way, the role of the teacher, as represented in Figure 39, is influencing (and being influenced by) both social relationships and maths choices.

Secondly, the role of the teacher to provide ‘in-the-moment’ support (Finding 12) was critical for students to act with agency. In my study, this involved in-the-moment teacher actions in response to observed student behaviours. For example, the nature of the classroom changed when a group member’s suggestion or action then required a response from their peer. As a result, the teacher was engaged in deliberate acts such as questioning and prompting to support students to recognise mathematical opportunities, to provide feedback, to encourage perseverance and confidence for students to contribute to their group. In this way, the role of the teacher acting in-the-moment influenced both the social relationships and maths choices. In a similar way, Gresalfi et al. (2012) recognised that the relationship...
between the maths learning environment and the individual learner was constantly changing. Vaughn et al. (2020, p. 723) described agency as a “dynamic construct”, suggesting that it can be used as a tool to adapt to changing surroundings and contexts, as well as to cultivate identities. Consequently, teaching for agency in a literacy study involved “adopting an adaptive stance to instruction” (Vaughn, 2018, p.66). This required some teacher decisions in-the-moment to modify their responses according to what the students were doing, in order to maximise student opportunities. Similarly, an adaptive teacher stance in maths involves listening, recognising opportunities for students to make choices to apply maths skills and knowledge, then responding to encourage and acknowledge these actions.

### 7.6 Summary of my Findings: Fostering Student Agency in Maths

In summary, Chapter 7 presented and discussed my study findings using three frameworks important for fostering student agency. First, the Social Relationship Framework highlighted the critical role of social interaction and collaboration, students’ sense of belonging and classroom norms to guide behaviour. Second, the Maths Framework identified aspects of maths choices involving methods, tools and representations. Thirdly, the last framework set out The Role of the Teacher to plan and organise suitable conditions to enable student agency, as well as to support learners “in-the-moment”. These three frameworks are interrelated and contribute to students forming intentions and making choices during maths challenge tasks. The model “Fostering Conditions for Student Agency” in Figure 39 brought together the three frameworks and illustrates these as important aspects of agency, crucial for fostering student agency in the maths classroom. Lastly, I continued discussing the findings from my study in relation to the work of other researchers. Important aspects of an agentic classroom including active participation, classroom norms and student
intentions to drive actions are highlighted. I discussed the effects of students’
involveent in talking and thinking about agency, and how this can lead to increased
awareness and promotion of further student choices. Finally, the crucial role of the
teacher to foster suitable conditions for student agency was discussed and related to
literature. This leads to Chapter 8, the final chapter which concludes my study.
Chapter 8: Conclusion

Introduction

In this chapter, I reflect on my doctoral journey and revisit my research questions to identify how students respond to opportunities for choice and justify their actions. I suggest teacher strategies for fostering student agency in the primary school classroom and the implications for students, myself and other teachers. Finally, I discuss the limitations of my study and ideas for future research.

8.1 Personal Reflections as a Practitioner-Researcher

Understanding the concept of student agency and recognising agency in action have been priorities for me in my professional role as a teacher and prompted me to take this interest forward as a topic for research. I began my study wanting to explore how students responded to opportunities for choice when solving maths challenges, the actions associated with their choices and how students justified their choices. My study has provided another layer of depth to my classroom practice and honed my observation skills, requiring me to let go of what I think I know and leaving me questioning and wondering. I have learnt many things. Students make many choices, for a variety of reasons. The maturity and insight of some students’ responses have both inspired and amazed me, reinforcing the importance of nurturing agency within the upper primary school age group.

I began my study with a personal definition of student agency which was influenced by my professional teacher knowledge. This was presented as active student participation in choosing maths methods, materials and representations to solve maths challenges. I started by using the umbrella term ‘choice’ to focus on students’
decision-making and actions during maths lessons. My subsequent research findings suggest that student agency in the maths classroom requires a number of important aspects, which are not sufficiently captured in that initial definition. One aspect I wrestled with was the language to use. For me this necessitated a search for clarity in the relationship between choice and student agency. Another aspect was the recognition that students made different kinds of choices, some in response to the maths challenge and some in response to social relationships. A final aspect was that some students were more aware of their choices. They were driven by their intentions to reach a solution, used chosen methods and could justify their decisions e.g., choosing methods that were easier, rather than reacting in response to others and being told what to do. As a practitioner-researcher, my work as a classroom teacher and doctoral researcher has been to help students become more aware of their decisions and actions, so that they recognise and fulfil their learning goals and intentions. In this respect, student intentions matter for the development of student agency. My study provides examples of active student participation highlighting the kinds of teaching strategies needed to shape student intentions and foster student choices to complete the maths challenges and at the same time, strengthen students’ connections with their peers as co-learners. In particular, my study has shown that the students needed a sense of belonging with their peers where their contributions were valued, and they felt comfortable and supported in their struggle to reach solutions. I have reworked my definition of agency to recognise the need for students to monitor that they are moving forward through a challenge e.g., self-checking calculations, and self-reflecting to evaluate and inform future maths decisions. My definition now reads: Student agency involves acting on intentions to
choose methods, materials and representations to collaboratively solve maths problems. Student self-regulation and self-reflection inform future actions.

8.2 Revisiting my Research Questions

My research questions were designed to understand how I, as a practitioner-researcher, could organise for agency and support students to take agency while learning maths. To advance my quest, I formulated one major research question supported by some sub-questions. These questions are reiterated here:

Main question: In what ways can student agency be fostered in the teaching of maths?

Supplementary questions:
- How do students respond (do and say) when there are opportunities for choice?
- How do students explain and justify their actions in choosing methods and equipment?
- What teaching strategies (or teacher actions) promote students to take agency in maths?
- What are the implications for practice and beyond the immediate context of my particular maths classroom?

Firstly, I address students’ reactions to opportunities for choice.

How do Students Respond When There are Opportunities for Choice?

When choice was introduced as an important classroom norm, students responded by making many choices involving a range of actions towards solving the maths challenges. I concentrated on three particular categories resulting from my data analysis. The first category was social choices affecting others in the group. The second was personal choices to enable students to apply themselves to the maths challenges and make contributions to their own and others’ learning, and thirdly, maths choices to manipulate numbers and represent maths ideas. Social choices included sharing ideas with group members and checking on others’ progress.
Examples of personal choices included ideas such as “trying my hardest” and “staying on task”. Maths choices included recomposing and decomposing numbers and using representations to record progress or explain maths solutions to other group members. These choices were identified from the data analysis through student actions and student communication e.g., recordings, reflections, representations and models, as well as students’ emotional responses. Individuals bring their personal orientations and intentions into their learning so the student agency identified was not the same for all students. Some students appeared to take greater or lesser agency at different times. During the time of the study, students became more comfortable with the idea of making choices. Talking and thinking about their choices had the effect of shaping intentions, promoting more confidence and further choice-making in future lessons.

**How do Students Explain and Justify Their Actions in Choosing Methods and Equipment?**

Most students in my study explained their actions at different times. Students explained chosen maths methods to others by describing their ideas and actions in detail, responding to the classroom norms. For example, students explained the details of splitting a number, multiplying the parts and then adding the parts together to reach a solution. The clarity and detail of students’ explanations varied and students often utilised visual and symbolic maths representations to make their thinking clear to others. As well as this, students were responding to differences in solutions within their group (enacting another class norm). This resulted in further explanations as students compared their thinking and calculations with others utilising additional methods, tools and representations that would help to
communicate their ideas. In this way, the classroom norms encouraged students to explain their choices.

Some students in my study justified their choices by providing a reason for their actions. The reasons were often to make the task easier to understand or quicker to solve, as in the case when a student chose a table in order to explain their ideas clearly to others. Other students had personalised the maths learning intention and were focused on improving their skills and understanding of specific maths concepts. Yet others made choices to support their group with the desire to belong, their sense of belonging to the learning community increasing their willingness to engage and make further choices. This suggests that students’ social interactions with others and their intentions, along with the role of the classroom norms in forming these intentions, were crucial in encouraging the students to act with agency.

8.3 In What Ways can Student Agency be Fostered in the Teaching of Maths?

Fostering student agency involves deliberate teacher actions to set the conditions and create the opportunities for agency. This includes expectations for students to participate in collaborative maths tasks and to make choices, as well as encouraging students’ sense of belonging within the maths classroom. The assumption that all students are capable of taking agency in some form introduces the question of what helps different individuals to make choices within any class.

Findings from my study suggest teaching strategies, both planned and in-the-moment can support student agency. My study highlights three teaching strategies crucial to planning and creating appropriate conditions and opportunities for agency. The first teaching strategy involves the selection and organisation of the maths
challenge. Aspects of planning and preparing the task include selecting a challenge that has a clear learning purpose so that students are more willing and motivated to act and make purposeful choices. The maths challenge needs to be suitable to be solved collaboratively and in different ways to enable choice in solution pathways, presenting the opportunity to utilise a variety of skills and knowledge. The selection of a challenge at an appropriate level is important to enable all students to contribute in some way to their group’s solution, to persist and wrestle with the solution.

Students need access to appropriate resources to model, demonstrate and represent their ideas. In this respect, it is helpful for the teacher to work through challenges prior to teaching to anticipate methods students may use to approach a problem, as well as the resources needed. Concrete materials e.g., blocks and squared paper, need to be readily accessible for encouraging agency. Access to information resources e.g., times tables wall charts, removes barriers for some students. Another important resource for students includes time. Time is needed to think about, trial and test ideas, because working collaboratively with other students to compare solutions and attend to differences requires both time and effort. The teaching strategy to be flexible with lesson timing encourages students to follow through to explore their ideas and make further maths choices.

A second teacher strategy necessary to support student agency is to establish and maintain the lesson structure and routines. This enables students to be confident of the expectations and obligations of working in this environment so they can make appropriate choices at different times during the lesson, with safety and increasing capability. The lesson introduction enables understanding of the problem and thinking time for individuals to engage with the challenge. During this time, the teacher’s goal is to encourage student choice of maths methods. The middle of the
lesson allows for collaborative work with peers to explore different solutions. This is an important opportunity for teacher-student interactions to co-construct social norms and sociomathematical norms (e.g., expectations for maths discussion). The final part of the lesson involves a teacher-facilitated class discussion enabling selected groups to explain their solutions and justify chosen methods to the class. Students are accountable to each other for making sense of the solution. During this element of the lesson, students have the opportunity to make contributions and ask questions, extending the sociomathematical norms.

The third teacher strategy crucial for student agency is the establishment of a learning community. The organisation of mixed ability groups (2-3 students) positions students so that they interact with and are accountable to others for sense-making, not just relying on the teacher. In this way, individuals form intentions and make choices to act when they contribute knowledge and skills to the group effort. A sense of belonging ensures students feel safe to make choices and that their contributions are valued by others. Student actions are guided by classroom norms deliberately co-constructed by the teacher and students during lessons. The first essential norm to foster student agency in maths includes the expectation and obligation to participate and contribute to their group’s solution. The second essential norm is an expectation for students to make choices about methods, tools and representations to solve maths challenges. This results in students trialling different methods and representations to reach a solution and checking their reasoning makes sense. The third norm is the expectation to share responsibility for the maths challenge within the group. With this expectation, it is not socially acceptable to sit back and let other group members do all of the work. Sharing responsibility for group outcomes ensures that students share ideas and understand the group’s decisions and
solutions. Students listen to others’ ideas about how to solve a task and support each other to make sense of the ideas. And finally, the *fourth essential norm* involves the expectation to act to address differences in solutions between group members. This norm encourages further choices of methods and representations when students negotiate, explaining and justifying why something is reasonable, and aim to reach a consensus. In this learning environment, classroom talk for maths sense-making is pivotal for communicating ideas to others. Therefore, student agency is supported by the establishment of norms to communicate their maths ideas clearly, for example, to take turns, listen and to agree/disagree respectfully with others.

With these three deliberately planned strategies in place that set the conditions for student agency, what actually plays out is influenced by teacher actions and interactions *during the time* of the maths challenge. The recognition of agentic acts in-the-moment enables the teacher to promote further student choices in maths methods and representations. The teacher makes decisions on when and how to intervene in student learning conversations (e.g., to encourage self-monitoring), understanding that student choices are a priority. Reinforcing class routines in-the-moment involves checking in with students regularly to see how they are progressing through a maths challenge so they feel supported. The teacher provides in-time support and reinforces expectations for small and large group interactions. Teacher actions in-the-moment include the deliberate use of questions, prompts, thinking aloud, modelling and specific feedback to enable students to have an influence on their maths learning. All of these teacher actions develop student intentions to interact positively and to persist in maths sense-making.

These deliberate teacher strategies are represented in Figure 40 below forming teacher practice to foster student agency. The left side of the diagram highlights the
three key strategies in setting up the learning context. The right side of the diagram highlights areas for decision making in-the-moment through interaction and co-construction.

**Figure 40**

*Teacher Practice to Foster Student Agency in Maths*

Arrows connect the aspects of teacher planning on the left side of the diagram with the right side of the diagram. These arrows represent the implementation of these aspects during the lesson, dependent on teacher interactions and co-construction of expectations and obligations as the lesson plays out. Vertical arrows connect each of the boxes on the right-hand side, to show they are interrelated. For example, the
teacher selects a maths challenge with the potential for student choice. For students to act with agency, they need to work in collaboration with others and understand the shared expectations of how to act according to the classroom norms. This understanding includes the expectation to make choices in methods and representations, to explain and justify their thinking to others and to have their contributions valued by others.

Extending the model of *Teacher Practice to Foster Student Agency* (Figure 40), I present deliberate teacher actions from my findings that encourage student choice in Table 16 (next page). Establishing classroom norms at the beginning of the year is followed by reinforcement of these norms and the active teaching of learning strategies throughout the year.
Table 16

**Deliberate Teacher Actions to Foster Student Agency in Maths**

<table>
<thead>
<tr>
<th>Social Aspects</th>
<th>Maths Aspects</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Establish classroom norms at the beginning of the year:</strong></td>
<td><strong>Teach routines for the maths activity e.g., lesson structure – what to do and when;</strong></td>
</tr>
<tr>
<td>• Co-construct expectations to participate and for group responsibilities;</td>
<td>• Co-construct expectations to choose methods, tools and representations;</td>
</tr>
<tr>
<td>• Deliberately talk about students’ choices and the reasons e.g., to explain ideas;</td>
<td>• Establish students’ accountability to their peers e.g., to convince others of their maths ideas.</td>
</tr>
<tr>
<td>• Negotiate norms to listen, take turns and explain ideas clearly; and</td>
<td>• Provide equitable and inclusive access to resources e.g., concrete materials, and ensure students are aware of them.</td>
</tr>
<tr>
<td>• Build relationships between students, as well as the teacher.</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Recognise and acknowledge student agency throughout the year:</strong></th>
<th><strong>Link maths learning with other topics/ events e.g., athletics, to make it purposeful;</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Reinforce classroom norms;</td>
<td>• Deliberately select the maths challenge e.g., to build confidence over several lessons;</td>
</tr>
<tr>
<td>• Check-in with groups regularly e.g., “Do you have a question for me?”;</td>
<td>• Anticipate student methods, resources etc</td>
</tr>
<tr>
<td>• Focus on student collaboration;</td>
<td>• Plan to allow time to think and trial methods.</td>
</tr>
<tr>
<td>• Encourage students to ask questions of their peers (before the teacher);</td>
<td>• Share learning intentions;</td>
</tr>
<tr>
<td>• Choose what to notice and give specific feedback e.g., I noticed … chose to…;</td>
<td>• Use teacher questions/ prompts: Show me …, Can you think of a different way?</td>
</tr>
<tr>
<td>• Encourage students’ sense of belonging and feeling valued;</td>
<td>• Value the use of visual representations.</td>
</tr>
<tr>
<td>• Provide opportunities to work with different people e.g., change groups;</td>
<td>• Ask students if they need more time and adjust as necessary;</td>
</tr>
<tr>
<td>• Listen to student voice e.g., interests, pace of learning; and</td>
<td>• Facilitate discussions to enable explanations and justifications of maths ideas; and</td>
</tr>
<tr>
<td>• Provide reflection opportunities e.g. What helped you solve the challenge today? What choices did you make? E.g., SAREf.</td>
<td>• Support students to make connections between concepts e.g., repeated addition and multiplication methods.</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>Actively teach learning strategies to add to students’ repertoires and knowledge:</strong></th>
<th><strong>Introduce strategies to solve maths challenges: use a table, draw a picture etc.;</strong></th>
</tr>
</thead>
<tbody>
<tr>
<td>• Teach self-help strategies to get started: reread the question, identify the numbers, keywords etc.; and</td>
<td>• Teach maths methods through mini-lessons as needed e.g., splitting one number;</td>
</tr>
<tr>
<td>• Model agreeing/ disagreeing respectfully with others.</td>
<td>• Introduce students to different maths representations; and</td>
</tr>
<tr>
<td></td>
<td>• Teach self-monitoring/ self-checking e.g. Is my method helping reach a solution? Does that look right?</td>
</tr>
</tbody>
</table>
8.4 What are the Implications for Practice and Beyond the Immediate Context of my Particular Maths Classroom?

Fostering agency requires more than handing over control to students. My study suggests that maths teaching and learning provide a valuable context and opportunity for promoting student agency in the classroom. I now discuss implications for students, myself as a teacher and researcher, as well as other teachers of maths.

**Students**

An implication of my study suggests that shaping student intentions and encouraging choice in solving maths challenges has positive effects on maths learning. In solving maths challenges, students can demonstrate and express agency in many ways, as opposed to the more commonly understood definitions of agency, such as activity choice. Agency by students is therefore identified by students’ actions, conversations, feelings, observations and exploration. Students were observed explaining and justifying chosen methods and representations to their peers. In the process of convincing others, they were using maths vocabulary and applying maths concepts to consolidate and deepen their own knowledge and capability. As well as this, students were learning to persist when faced with difficulties, value the support of others and gain a sense of belonging within the class. Students expressed satisfaction and were engaged working in this type of social learning environment. Students became more aware of the decisions and choices they were making throughout my study. Some students’ intentions led to particular choices, for example, one student explained he deliberately chose a table because it enabled him to demonstrate a relationship to his peers. Students’ confidence to talk about
and make choices increased. Therefore, certain teaching strategies supported students to act with agency within the maths classroom to support their learning.

**Myself as a Teacher**

Student agency is implicit in the key competencies of the *New Zealand Curriculum* (Ministry of Education, 2007). A teacher can carefully create conditions for student agency by paying attention to curriculum design to encourage maths choices, social relationships and classroom norms to guide interactions. Focusing on and recognising the subtle ways that student agency can be demonstrated and expressed (the doing, saying, feeling, noticing and exploring) can affirm teaching decisions. There are common aspects in creating suitable conditions for agency that apply to different learning contexts. A suitable task to enable agency is one that provides a maths challenge as well as opportunities to apply different maths methods, tools and representations. Social interactions between students are necessary for students to share responsibility for the solution, and be involved in explaining and justifying their solutions to others. An important role of the teacher involves the co-construction and negotiation of classroom norms to guide student intentions and choices through deliberate planning and in-the-moment interactions during lessons. Examples include helping students to understand the expectation to reach a group consensus and to encourage students to ask questions when they do not follow a peer’s solution.

My study suggests that the students became more aware of their choices to solve maths challenges. Certain teaching strategies served to increase students’ awareness of their decisions, shaping intentions and leading to student choices and actions in future lessons. Two particular strategies included talking about the choices students made during the post-lesson interview (as they watched a replay of
themselves working) and using the SAREfl tool to reflect on the lesson. While these two strategies were additional to a normal maths lesson, they gave students the opportunity to reflect on their participation and choices. Making particular choices was also a class focus at particular times in response to students’ needs. For example, foci at the beginning of the school year included choosing a good place to sit to enable communication with others and how to respectfully agree and disagree. The deliberate conversation around agency and making choices helped students within my class to value agency and see themselves as capable of taking agency in maths.

Another implication is that a commitment to setting up conditions for student agency benefits learners. Important beliefs include student agency is available to all, as well as students can learn to act with agency, which influences future agency. This age group appears to be a good time to focus on agency in schooling (9-10 year olds), to learn perseverance and coping strategies when solving maths challenges, with the potential of influencing future learning.

**Myself as a Researcher**

Being a researcher in my own classroom has been a learning process. My research design was carefully planned to overcome the challenges of being an insider researcher and the ethical dilemmas arising when researching with my own students. These issues are taken for granted in my normal teaching role. An example was the selection of data-gathering methods including photos and representations, to allow a variety of communication forms and to fit in with normal classroom activities. The experience of undertaking this research has taught me data analysis techniques and revealed the enormous amount of work and attention to
detail that is required to analyse data and bring a study to its conclusion. Three
different frameworks have assisted me to analyse my data from different angles,
helping to make sense of my findings. These include the lenses of social
relationships, maths choices and the role of the teacher. Within the social
relationship framework, the significance of a sense of belonging within a maths
learning environment where agency is valued has been highlighted. Undertaking the
research process has also taught me to be more critical when analysing other
research studies, highlighting aspects of which I would previously be unaware.

Other Teachers of Maths

Implications beyond the immediate context of my maths classroom include applying
these teacher strategies to nurture student agency in similar mathematical contexts.
The work of teachers to facilitate a conversation about agency with students and
opportunities to reflect has the potential to increase students’ awareness and
confidence to make choices and act. A recommendation is for teachers to allow time
for students to explore methods and representations. Teacher professional
development is necessary to ensure an understanding of the conditions for agency,
recognition of agency in action and teacher strategies for in-the-moment support of
students during maths lessons. A resource bank of suitable maths tasks to
encourage agency would support teachers to provide opportunities for students to
make choices in supportive environments with their peers. Recommendations for
school leaders are to ensure students have access to suitable resources. Actions
include providing suitable learning spaces that allow for communication between
groups of students, as well as physical materials to model and represent ideas. Work
with parents to encourage student agency includes the recognition and celebration of
student choices to solve maths challenges in both informal communications between
home and school, and formal reporting. Action by policymakers to ensure national and local curricula value student agency through explicit documentation would provide direction for teachers. A shared understanding of the aim to foster student agency and choice of solution pathways is important to equip students to solve a variety of maths challenges, and apply their knowledge to future maths learning.

8.5 Limitations

My study has explored student agency within a specific maths classroom where the students’ growth in agency was dependent on my work as the practitioner-researcher. There is no guarantee that what the students and I experienced will transfer to new classroom situations. Given the complexity of classroom environments and interwoven nature of aspects of agency, it is difficult to conclude with a definitive list of strategies to respond to the question: in what ways can student agency be fostered in the teaching of maths? What my study is able to convey is the kind of work that teachers can deliberately employ to increase student agency. Also, my study has focussed on maths challenges, which were just one part of my classroom maths programme. The work to negotiate classroom norms and act with agency continued throughout the school day across other curriculum areas. The challenge remains for the profession to realise the intentions of the curriculum so that student agency permeates throughout all learning areas, not just maths.

8.6 Ideas for Future Exploration

This case study has provided valuable insight into a single class of 9-11 year students solving maths challenges. All of the students with whom I worked demonstrated agency through purposeful actions in some way. The ongoing effects of student agency as these participants move through the New Zealand school
system is of interest. I wonder whether learning to act with agency requires continuous deliberate teaching or whether being taught with this focus in the upper primary is enough. The sense of belonging in the maths classroom appears to be important for agency, for students to be willing to act. This is a concept with limited research studies during my literature search and could be investigated further. The focus of this study was on one classroom and one teacher. Other teachers’ perspectives of implementing programmes to foster agency, both in maths and other curriculum areas, would expand and extend the findings of this study. Further research could explore schools’ awareness and readiness for agency, identifying existing barriers and the removal of constraints within these learning environments. This includes local curriculum expectations as well as the physical learning environment e.g., classroom spaces that are suitable for student discussions.

8.6 Concluding Thoughts.

Learners having agency is an expectation (21st Century Learning Reference Group, 2014). My study of student agency has explored fostering conditions for agency and identified what agency looks like in the maths classroom. Along with this, students’ increased awareness of their intentions and possibilities for choice have supported further agency. To end this thesis, I give voice to my students. I include a vignette from an interview in which a student recalled his group’s choices and actions while watching a video replay of a lesson. This example highlights the thinking, decisions, actions and self-reflection of one group during a maths challenge:

We worked together ... both participate ... drawing pictures and adding stuff inside of them ... used fractions ... reading down the activity... discussing our ways ... I got the question wrong ... I realized that was around the wrong way ... having a little debate ... I made mine a bit messy ... we realised that we did something wrong ... we could halve the
numbers and stuff, like there was 60. We could have just halved them and then halved them again (Int L7 Keith).

And finally, another student’s comments illustrate how students valued social relationships within the classroom as the research site to foster agency.

“We talked to each other, and we kindly disagree if we think it is wrong”
“I think it is easier to work like this, to work it out” (L4 SARrefl, student voice Amber).
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[https://doi.org/10.4324/9781315070131](https://doi.org/10.4324/9781315070131)


[https://doi.org/10.1016/j.jmathb.2015.10.001](https://doi.org/10.1016/j.jmathb.2015.10.001)


Appendix A

Information Letter

Telephone:

Email: xxxxxxx

2.2.19

Student Agency in the Maths Classroom: Information Sheet for Parents and Students

As a classroom teacher, I am involved in educational research through the University of Canterbury. The purpose of the research is to help understand student perspectives of learning maths in the classroom. My research is about understanding how teachers can organise maths learning experiences where students can take agency (have ownership of their learning and the power to make choices) to maximise their engagement and achievement.

I would like to invite your child to participate in the study. This will involve participation in normal classroom maths lessons (videoed), including the collection of students’ work showing their thinking (with photographic evidence) and methods of calculations to be analyse later. A limited number of students will be involved in post lesson interviews (taped) to help understand their views of the maths experiences.

The project will be carried out within a normal operating class programme. Participation in the research project is voluntary and students have the right to withdraw at any stage with no penalty. If a student withdraws, I will do my best to remove any information relating to them, provided this is practically achievable.

Confidentiality and anonymity will be maintained by reporting on ideas as a collective group where possible, using pseudonyms and avoiding any identifying details of individuals or the school during reporting. The data will be securely stored and access limited to myself and my university supervisors. The material will be destroyed after 10 years.

Risks associated with the research for the participants include social pressure whether they are included in the study or not, the presence of a video at times within the classroom, the use of photos being used to show evidence of learning. These risks will be addressed by ensuring that I continue to deliver a normal maths programme (including equitable practices), notifying students when they are being recorded, avoiding classroom documents that identify individuals and keeping the focus on student maths learning.

The results of the study will be reported in a thesis through the University of Canterbury. Participants can receive a report on the study when it is completed by including their contact email on the consent form.

Participants may contact myself (and my supervisor) if they have any questions about the study at any stage. Contact details are at the top of this information sheet.
My project has received ethical approval from the University of Canterbury Educational Research Human Ethics Committee, and that participants should address any complaints to The Chair, Educational Research Human Ethics Committee, University of Canterbury, Private Bag 4800, Christchurch (human-ethics@canterbury.ac.nz).

Potential participants need to complete the consent form if they understand and agree to take part in the study. The form needs to be returned to me at school by 16.2.19.

Tracey Mitchell

Appendix B

Excerpts From Information Presentation

Aim: to understand student views of their choices in maths lessons to help teachers improve students' maths learning.

Parents: The purpose of the research is to help understand student perspectives of learning maths in the classroom. My research is about understanding how teachers can organise maths learning experiences where students can take agency (have ownership of their learning and the power to make choices) to maximise their engagement and achievement.
Participant involvement:

- Typical class maths programme
- Collecting some of your work to analyse
- Video recording some maths lessons to look carefully at what is happening in the classroom
- A few groups of students will be asked some questions about their maths learning and the choices they made in an interview after the lesson (audio recorded, approximately 30 minutes).

Parents: This will involve participation in 4 classroom maths lessons (videoed) over term 2, including the collection of students’ work showing their thinking (with photographic evidence) and methods of calculations to be analyse later. A limited number of students will be involved in a post lesson interview (taped, 30 minutes duration) to help understand their views of the maths experiences. The participants will be offered the opportunity to check the transcript of the interview.
Your protection (confidentiality, anonymity)

- If you choose not to participate in the study you will still be expected to participate in the maths lesson.
- Only Mrs Mitchell and her teacher will listen to or watch the recordings.
- Mrs Mitchell will not tell anyone your name.

Parents: Confidentiality and anonymity will be maintained by reporting on ideas as a collective group where possible, using pseudonyms and avoiding any identifying details of individuals or the school during reporting. The data will be securely stored (at researcher’s home, either in a locked cupboard or stored digitally under password protection) and access limited to myself and my university supervisors (Jane McChesney). The material will be destroyed after 10 years.

Risks associated with the research for the participants include possible social pressure about whether they are included in the study or not, the presence of a video at times within the classroom, and the use of photos being used to show evidence of learning. These risks will be addressed by ensuring that I continue to deliver a typical maths programme (including equitable practices), notifying students when they are being recorded, allowing “don’t know” answers, avoiding classroom documents that identify individuals, and keeping the focus on student maths learning.
Appendix C

Consent Form

Tracey Mitchell
Telephone: 
Email: xxxxxxxxxxxx

Supervisor: Jane McChesney
Telephone: xxxxxxxx
Email: xxxxxxxxxxx

Student Agency in the Maths classroom

Consent Form for Parents/ Caregivers

• The research project has been explained to my child and he/she has been given an opportunity to ask questions.

• I understand what is required of my child if he/she agrees to take part in the research, and understand that their participation is voluntary and they may withdraw at any stage without penalty by communicating this with xxxxxxxxx.

• I/we understand that any information or opinions provided will be kept confidential to the researcher (and supervisor) and that any published or reported results will not identify the participant (or the school).

• I/we understand that all data collected for the study will be kept in locked and secure facilities and/or in password protected electronic form and will be destroyed after ten years.

• I/we understand the risks associated with taking part and how they will be managed.

• I/we understand that we are able to receive a summary report on the findings of the study. If you would like to receive a summary report please give your email address below:

__________________________________________________________________________

• I/we understand that we can contact the researcher (or supervisor) for further information.
• I/we understand we can contact the Chair, University of Canterbury Educational Research Human Ethics Committee, if we have any complaints.

• I/we give consent for my child to be videoed during the maths lessons.

• I/we give consent for our child to be included in this study.

Student’s name ______________________________

Parent/ Caregiver’s name ______________________ Signature ______________________

Date ____________________.

Please return the consent form to Room x or the school office by 1 March, 2019

Tracey Mitchell
## Appendix D

### Summary of Ethical Considerations

#### Table 17

**Summary of Identified Ethical Considerations Throughout My Research**

<table>
<thead>
<tr>
<th>Where in research process</th>
<th>Ethical considerations</th>
<th>How consideration was addressed</th>
</tr>
</thead>
<tbody>
<tr>
<td>Study preparations</td>
<td>Obtain University approval and permission.</td>
<td>Carefully planned research. Followed Human Ethics Policy of Canterbury University (University of Canterbury, n.d.). Sought Ethical approval. Permission was obtained from the school principal and BOT.</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Start of the study</td>
<td>Obtain informed consent.</td>
<td>Information session, consent form.</td>
</tr>
<tr>
<td></td>
<td>Communicate the purpose of the study.</td>
<td>Participants were made aware of their voluntary participation.</td>
</tr>
<tr>
<td></td>
<td>Avoid pressurising participants.</td>
<td>Maintained primary teaching role.</td>
</tr>
<tr>
<td></td>
<td>Respect classroom norms.</td>
<td>Obtained informed consent from children and parents.</td>
</tr>
<tr>
<td></td>
<td>Consider vulnerable classroom members (e.g., non-participants, culture etc.).</td>
<td>Sensitive to those not involved.</td>
</tr>
<tr>
<td>Collecting the data</td>
<td>Avoid disruption of the class routines.</td>
<td>Ensured ongoing voluntary consent.</td>
</tr>
<tr>
<td></td>
<td>Avoid deceiving the students.</td>
<td>Discussed the study purpose.</td>
</tr>
<tr>
<td></td>
<td>Acknowledge power imbalance between myself and the students.</td>
<td>Gave careful instructions for children’s work.</td>
</tr>
<tr>
<td></td>
<td>Avoid collecting information that could be harmful e.g. cause stress, not relevant to study.</td>
<td>Avoided leading questions.</td>
</tr>
<tr>
<td></td>
<td>Avoid bias.</td>
<td>Used planned interview questions, probes.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Involved participants in the interpretation of data.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Checked and clarified understandings.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Avoided the disclosure of sensitive information.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Used random representative selection from voluntary participants.</td>
</tr>
<tr>
<td>Analysing the data</td>
<td>Maintain privacy and anonymity of students and school.</td>
<td>Interpreted findings honestly.</td>
</tr>
<tr>
<td></td>
<td>Avoid acknowledging only positive results.</td>
<td>Included different perspectives.</td>
</tr>
<tr>
<td>Storing and reporting the data</td>
<td>Honestly report data, findings and conclusions.</td>
<td>Used pseudonyms.</td>
</tr>
<tr>
<td></td>
<td>Communicate authors.</td>
<td>Built up composite profiles of participants.</td>
</tr>
<tr>
<td></td>
<td>Communicate findings clearly.</td>
<td>Reported findings honestly.</td>
</tr>
<tr>
<td></td>
<td>Store the data safely.</td>
<td>Used composite stories to avoid identifying individuals.</td>
</tr>
<tr>
<td></td>
<td>Do not report information that would harm participants.</td>
<td>Took care to use unbiased language.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Materials and data will be stored for 10 years and then destroyed.</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Credit given to the people involved i.e. participants and advisers.</td>
</tr>
<tr>
<td>Publishing the study</td>
<td>Protect the confidentiality of participants and school.</td>
<td>Will provide a report summary to participants and stakeholders.</td>
</tr>
<tr>
<td></td>
<td>Share the report.</td>
<td>Cited other published work.</td>
</tr>
</tbody>
</table>

*Note:* Adapted from *Research design: Qualitative, quantitative, and mixed methods approaches* by Creswell (2018).
Appendix E

Observation Schedules

Observation Schedule 1: Actions of Students During Maths Lesson

How do students participate?
What actions are evident?
Are they engaged throughout the lesson?

Observation Schedule 2: Actions of Teacher and 2 Identified Pupils During Maths Lesson

School:
Date/ time:
Subject: Maths
Class: Year 5/6

<table>
<thead>
<tr>
<th>Teacher</th>
<th>Student 1</th>
<th>Student 2</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
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<tr>
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</tr>
<tr>
<td>10</td>
<td>10</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td>20 etc.</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

(e.g., listening, discussing, walking, watching, writing, praising, prompting, drawing)
Appendix F

Interview Questions

Interview Protocol (Creswell, 2007)

Focus Group/ Stimulated Recall Interviews

Time of interview:

Date:

Interviewer:

Interviewees:

Position of interviewees:

Recording/ storing information about interview:

Introduction: purpose of study, consent, structure of interview i.e., stimulated recall, ask if interviewee has any questions)

Interview content questions:

1. What did you do in this lesson? (Ice breaker)

2. What choices have you made in this lesson?
   Probes: tell me more. Please explain.

3. What methods and equipment helped you to solve this problem?
   Probes: tell me more. Please explain.

4. What were you doing here? Why did you choose these?

5. How did you explain your ideas to your group? Was it easy to get them to understand?
   Probes: tell me more. Please explain.

6. Today we were learning … How are you feeling about your maths understanding? What are your next steps?

Closing instructions:

Thank you, assure confidentiality. Read back and confirm content. Check if participants would like to read transcripts.
Appendix G

Establishing Possible Initial Codes From SARefls

Table 18

Establishing Possible Initial Codes From SARefls (Photos and Reflections)

<table>
<thead>
<tr>
<th>Lesson</th>
<th>Quantity/Content</th>
<th>How do students respond to opportunities for choice? Range of actions</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>21 students</td>
<td>added, multiplied, made a table, stay on-task, write in a book, good buddy, pattern, sit in a triangle, waited, helped, disagreed, explained, suggestions, listened, showed</td>
</tr>
<tr>
<td>2</td>
<td>20 students</td>
<td>equations, written addition, draw picture, written equations, on-task, repeated addition, buddy, agree/ disagree, multiply, add, good place, good partner, splitting it, timesing, tidy numbers, written multiplication, talking, table, recording, listening, telling, comparing, shared</td>
</tr>
<tr>
<td>3</td>
<td>18 students</td>
<td>used what I knew about area and perimeter, highlighters and square paper drawing around the edge, paper and pen, draw pictures, guess and check, multiplication, equations, doubles, times tables, counted the squares, write it down, allocating tasks, my book, coloured pencils, counting, getting the right information, ruler, grid paper, counted one by one</td>
</tr>
<tr>
<td>4</td>
<td>20 students</td>
<td>ruler and my book, made sure I had the right information, my group explained stuff, helped each other, got all the information I needed, times tables, pen helped me to draw, paper to help remember numbers, working together, equations and pictures, making explaining quick and efficient, splitting, my book, thinking hard, multiplication, group that helped, timesing to the nearest 10 then take away the number, written multiplication, gave me the structure of the answers, addition, add on the number I had, using small times tables to figure out big numbers, work with my biddy, doubles, adding, work out the area with times, measurements, knowledge from area, talked, kindly agree if wrong</td>
</tr>
<tr>
<td>5</td>
<td>16 students</td>
<td>blocks, cubes, thinking about volume, square paper and blocks, times tables, counted 14 rows of 14, doubled, made models, multiplication, pencil, paper, times the tens then the ones, addition, subtraction, cubes, counting, draw, skip counting, tape, scissors, hundreds and tens cubes</td>
</tr>
<tr>
<td>6</td>
<td>20 students</td>
<td>ruler, add, dividing, rubber, times tables, diagrams, thinking outside the box and multiplication, drawing a picture, read the question, double check for mistakes, pencil, my buddy to talk to, draw the problem, working together, checking the questions, read carefully, drawing a diagram, subtracting, ask the teacher questions and my buddy, book to write, sheet to see what I had to do, pen, paper, sharing ideas, egg timer</td>
</tr>
<tr>
<td>7</td>
<td></td>
<td>(no SARefls were collected during this lesson)</td>
</tr>
<tr>
<td>8</td>
<td></td>
<td>(no SARefls were collected during this lesson)</td>
</tr>
<tr>
<td>9</td>
<td></td>
<td>(no SARefls were collected during this lesson)</td>
</tr>
<tr>
<td>10</td>
<td>17 students</td>
<td>adding facts, thinking ahead (predicting), using strategies to put my numbers in the correct place, addition, jumping the line, book and columns, adding, smiley faces and rainbows (place value), dice, decimals, sit in a spot where no one else could annoy me besides my buddy, decimals out of 100, my knowledge of decimals, times tables,</td>
</tr>
<tr>
<td>Lesson</td>
<td>Quantity/Content</td>
<td>How do students respond to opportunities for choice? Range of actions</td>
</tr>
<tr>
<td>--------</td>
<td>------------------</td>
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<td>subtracting, columns, adding tens and adding the decimals numbers together, book, decimals, taking away</td>
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<td>table, times, times tables, skip counting, explaining, working together, pencil, ruler, addition, subtraction, talk to my group, share our ideas, pictures, finding the pattern, multiplying, working out shapes, picture, figure out the rule, explain, book, draw</td>
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**Appendix H**

**Lesson 10 Video Transcript**

**Excerpts of Transcript L10 M and L**

1.17 Minutes

M  We are trying to see who is winning or not. Oh wow ...a one. I still got a one.
L  It doesn't really matter where exactly
M  That was from one. OK
L  And me getting a two.
M  Six. Where are you going to put it? You should put it here... because it equals 7 and then to get 6.

16.30 minutes. Interaction between groups

Z  I get it, I get it. Do you get it? (Interaction between a student visiting from another group)
M  No. The score.
L  So you got 11.7 is that right?
Z  I got it.
L  So do we plus all these together? We plus all these together to get the total score? I think we wait for Miss.
Z  I am still trying to work out my total score. Mine looks like that. We need help with the scores...
T  Right, what is your question? (Teacher arrives)

30.00 minutes

L  No, it's 0.8 because ... coz ... look ... um. What we have to get to is 8.
M  Yeah, I'm saying that it is 8.8 for here.
L What’s the difference to it.
M Yes, its 0.8. Yeah, I’m just saying is it 0.8 or 18.8. Scribble it out. Yeah 0.8.
L This one is lower. This one is lower coz what’s the difference.
M This one is lower?
L This one has …
M 3.3
L We have a struggle
M Yes 3.3. It’s 3.3. It’s 3.3 I know.
L 7.5 and this one has a one percentage bigger so I mean 0.1 so...
M So, if you need help, I’ll be right here next to you … coz I am next to you. (31.30mins)
L So 5.7 0.8 3.3 and 0.1. That one is a one percentage.
M Are you done?
L So
M Need help?
L Yep
M Done all of the scores?
Appendix I

Word Count Analysis

Table 19

*Full Word Count Analysis*

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<th>Data source</th>
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