Forecasting Electricity Consumption: A Comparison of Models for New Zealand and the Maldives

Zaid Mohamed, Pat S. Bodger, and David J. Hume

Abstract-- **This paper compares the forecasting ability of six electricity forecasting models for New Zealand and the Maldives. They are three growth curve models (Logistic, Harvey Logistic and Harvey models), a multiple linear regression model proposed using economic and demographic factors referred to as the Combined model, a growth curve model that uses economic and demographic factors referred to as the Variable Asymptote Logistic (VAL) model and a Box-Jenkins ARIMA model. The models are compared using goodness of fit to the historical consumption data and their forecasting accuracy in the short, medium and long term. The analyses of the six presented forecasting models showed that the best overall model is the Harvey model that generally gave more accurate or comparable forecasts than the more complex ARIMA and Combined models. This shows that the simple Harvey model could play a significant role in forecasting electricity.**

Index Terms-- **ARIMA models, forecasting, power demand, error analysis, time series analysis.**

I. Introduction

FORECASTING electricity consumption has been undertaken using many theoretical methods including growth curves [1]-[5], multiple linear regression methods that use economic, social, geographic and demographic factors [6]-[8], and Box-Jenkins autoregressive integrated moving average (ARIMA) techniques [9]-[11].

This paper compares the effectiveness of six forecasting models developed for electricity consumption in New Zealand and Maldives. Firstly, a Logistic model [1], [2] based on the growth curve is developed. The fitting of logistic curves to the historical data employs a Fibonacci search technique to establish optimum asymptotes [1], [2]. In a second model, the influence of selected economic and demographic variables on the annual electricity consumption in New Zealand is investigated. The study uses population, price of electricity and gross domestic product (GDP). The resulting Combined models are developed using multiple linear regression analysis. The third model uses an ARIMA technique in developing electricity forecasting models. Fourthly, two other models, Harvey models and Harvey Logistic models, based on growth curves are developed [3].

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Finally, the paper discusses a Variable Asymptote Logistic (VAL) model for electricity consumption in New Zealand. The saturation levels of the logistic curve are estimated using the Fibonacci search technique. Correlation of the estimated saturation level with population, price of electricity and gross domestic product (GDP) is determined. Multiple linear regression is used in studying the correlation between the explaining variables and electricity consumption.

The developed models are compared for these two countries using the criteria of fit to the historical data, and forecasting accuracy in the short, medium and long term.

II. Proposed Models

In all the model equations to follow, Y represents the annual electricity consumption in GWh.

A. Logistic model

The proposed Logistic model is [1], [2],

$$Y = \frac{F}{1+\exp(C_0 + C_1 t)} \quad (1)$$

where F is the asymptotic value in GWh, t is time in years and C_0 and C_1 are constants.

B. Harvey Logistic and Harvey models

The Harvey Logistic model is based on the Logistic model. The proposed Harvey Logistic Model is [3]:

$$\ln y_t = 2\ln Y_{t-1} + \delta + \gamma t + \varepsilon_t, \quad t = 2 \ldots T \quad (2)$$

where $y_t = Y_t - Y_{t-1}$, ε_t is a disturbance term and δ and γ are constants to be found by regression.

The Harvey model is based on general modified exponentials. The proposed Harvey model is [3]:

$$\ln y_t = \rho \ln Y_{t-1} + \delta + \gamma t + \varepsilon_t \quad (3)$$

where $\rho = \frac{k-1}{k}$, $\delta = \ln\left(k\beta\alpha^{1/k}\gamma\right)$, and ρ, β and γ are parameters to be estimated. The value of k determines the form of the exponential function. When k = -1, it is Logistic and when k = 1 it is a simple modified exponential [3].

C. Combined model

The proposed Combined model using multiple linear regression is of the form:

$$Y = a + b_1 X_1 + b_2 X_2 + b_3 X_3 + u \quad (4)$$

where X_1 is GDP ($NZ millions), X_2 is electricity price (cents/kWh), X_3 is population and u is the error term.

D. ARIMA model

ARIMA models are generally written as ARIMA(p,d,q), where p represents the order of the autoregressive (AR) part, d denotes the degree of first differencing (I) involved and q denotes the order of the moving average (MA) part. The autoregressive (AR) part of the model with order p is of the form:

$$Y_t = c + \phi_1 Y_{t-1} + \phi_2 Y_{t-2} + \dots\dots\dots + \phi_p Y_{t-p} + e_t \quad (5)$$

The moving average (MA) part of the model consists of the past errors as the explanatory variable. A moving average model of order q is of the form:

$$Y_t = c + \theta_1 e_{t-1} + \theta_2 e_{t-2} + \dots\dots\dots + \theta_p e_{t-q} + e_t \quad (6)$$

where ϕ and θ are the maximum likelihood estimates of the respective models and e_t is the error series. The Box-Jenkins methodology for modeling time series consists of identification, estimation, testing and forecasting [12].

E. VAL model

In the VAL model, the saturation level F of the Logistic model (Eq.1) is estimated using economic and demographic variables ($X_1 \dots X_n$) and used as a variable asymptote $F(X)$ in Eq.1. The proposed VAL model takes the form,

$$Y = \frac{F(X_i)}{1 + \exp(a_0 + a_1 t)} \quad (7)$$

$$F(X_i) = c_0 + \sum_{i=1}^{n} (c_i . X_i) \quad (8)$$

where $F(X_i)$ is the saturation level expressed as a function of n variables and c_0 and c_i are the parameters obtained from the explaining variables

F. Statistical Measures

Mean absolute percentage error (MAPE) is used to measure the model fit and forecasting accuracy of each of the models to the other. It is defined as:

$$MAPE = \frac{1}{n} \sum_{i=1}^{n} \left| \left(\frac{Y_i - \hat{Y}_i}{Y_i} \times 100 \right) \right| \quad (9)$$

where $\hat{Y}_i$ is the forecasted consumption data at time i and n is the number of data points considered. In addition, the developed models are tested against various statistical measures including autocorrelation analysis, Durbin-Watson (DW) statistic, F-test, t-test, residual plots and autocorrelation (ACF) and partial autocorrelation (PACF) plots of the data and residuals [12].

III. Application to Electricity Consumption

The developed models are applied to the Domestic, the Non-Domestic and the Total electricity consumption data in New Zealand [13], [14] and Maldives [15]-[17]. Fig.1. shows the electricity consumption data for the two countries. The data for New Zealand are shown from 1943 to 1999 (March year 2000). At the time of this study any later data for New Zealand are not confirmed by the Ministry of Economic Development [14] and therefore are not used in this research. The data for Maldives are from 1980 to 2002. The data for the Maldives represent the electricity consumption in the capital island Male' where about 70% of the total electricity in the country is consumed.

There is an increase in trend in the consumption data for both the countries. The rate of growth of electricity consumption in New Zealand is slower than that for Maldives, especially in the recent years. The growth in the Maldives shows more of an early phase of development. Although the pattern of growth is similar in some cases, the amounts of electricity consumed in the two countries are significantly different from each other due to the differences in size of the population and strength of the economy. Therefore, application of the developed models to these two different levels of electricity consumption will give an overall idea of the performance of these models and the applicability of the models to widespread levels of consumption.

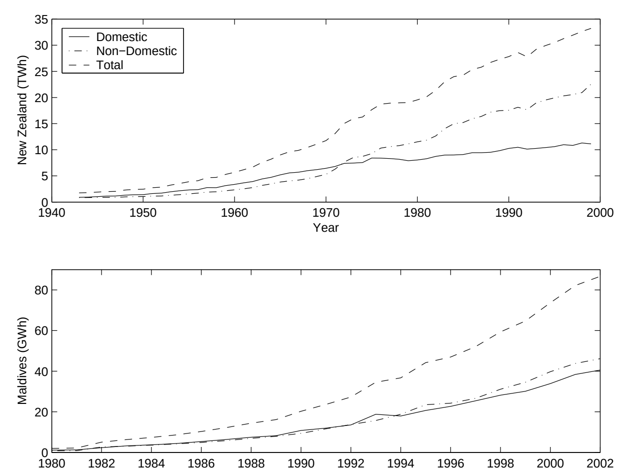

Fig.1. Electricity Consumption for New Zealand and Maldives

Population and GDP data for New Zealand are obtained from Statistics New Zealand [18], [19]. Electricity price data are obtained from the Ministry of Economic Development [14], [20]. The population and GDP data for Maldives are obtained from the Ministry of Planning and National Development [15] and [17]. Details of the developed models for electricity consumption in New Zealand are discussed in [1] and [2] for the Logistic models, in [3] for the Harvey Logistic and the Harvey models, and in [21] for the Combined models. A summary of the application of the six models for New Zealand including those for ARIMA and VAL models are described in [22]. Although the VAL model is found to be very effective in the Domestic and the Total electricity consumption of New Zealand, it was not suitable in describing the Non-Domestic electricity consumption in New Zealand due to the inconsistencies in the saturation levels obtained. This is caused by possible immaturity in the electricity consumption of the Non-Domestic sector [22].

All the three growth curve models are found to be suitable in forecasting electricity consumption in the Maldives. The Combined models for Maldives are proposed using the explaining variables population and GDP. The Combined models effectively satisfied all the required statistical tests. In the ARIMA modeling for Maldives, each data set is treated independently for stationarity, identification, estimation and diagnostic checking of residuals [12], in a similar fashion to those for New Zealand. The Domestic sector required first order differencing while the Non-Domestic sector and the Total electricity consumption required second order differencing to achieve stationarity. The proposed ARIMA models for Maldives are ARIMA(0,1,2) for the Domestic sector, ARIMA(0,2,1) for the Non-Domestic sector and ARIMA(2,2,0) for the Total consumption. The limited amount of data available for Maldives made the application of VAL model unsuitable as the saturation levels are inconsistent when some data points are discarded in the modeling procedure. Therefore, the VAL model is not used in the comparison of models for electricity consumption in Maldives.

IV. MODEL FIT AND FORECASTING ACCURACY

The developed models for each of the countries are compared using their ability to fit to the historical data and the forecasting accuracy. Forecasting accuracy is measured from one year ahead through to nine years ahead using MAPE for New Zealand, and from one through to five years ahead for the Maldives. The number of actual data points available in the Maldives is far less than New Zealand. Therefore, if 9 years of data are held out for accuracy comparisons, the developed models using the remaining few data points are unstable. Thus, for the Maldives the forecasting accuracy is calculated for five years ahead only. To compare the forecasting accuracy, models are developed by holding out some actual data at the end of the series. Forecasts obtained by the models over the held out data period are then used to calculate the MAPE value. The forecasting accuracy of the developed models for New Zealand and Maldives are shown in Fig. 2 and Fig. 3 respectively.

The models are ranked according to the criteria of model fit, and short term (1 to 3 years), medium term (4 to 6 years) and long term (7 to 9 years) forecasting accuracy for New Zealand and short term (1 to 3 years) and medium term (4 to 5 years) forecasting accuracy for Maldives. The average of the MAPE values over the short, medium and long term is calculated. The overall rankings for the Domestic, the Non-Domestic and the Total electricity consumption are calculated by taking the average of the MAPE values over the whole period compared. Models are ranked from 1 (best model) with lowest MAPE value to 5 or 6 (worst model) with the highest MAPE value. Table I summarizes the ranking of the models for all countries.

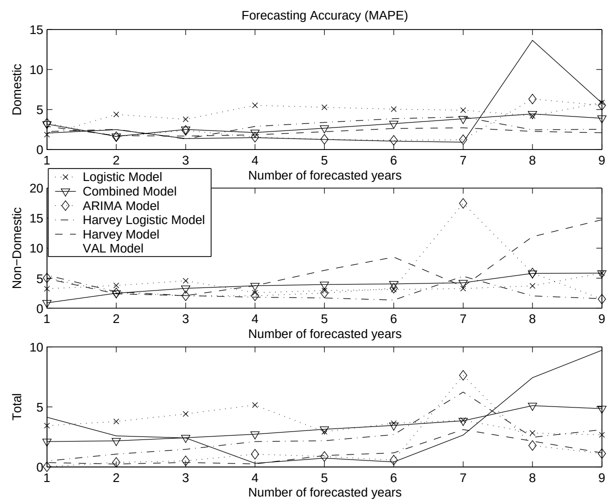

Fig. 2. Forecasting accuracy of the developed models for New Zealand.

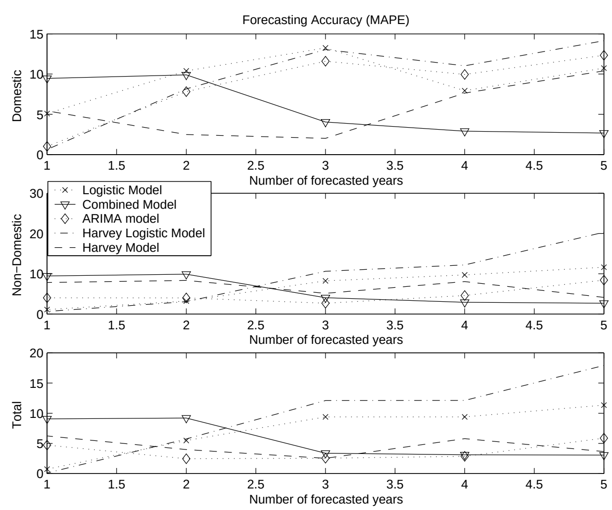

Fig. 3. Forecasting accuracy of the developed models for Maldives.

A. *New Zealand*

In the Domestic sector, the best model fit is given by the Combined model. The VAL model gave the best short and medium term forecasts, while it is ranked the worst in forecasting the long term. The best model in forecasting the long term is the Harvey model. The worst forecasts for the short and medium term are given by the Logistic model. Overall, the Harvey model gave the best accuracy in the Domestic sector followed by the ARIMA model.

In the Non-Domestic sector, the best model fit is given by the Harvey model. The Combined model gave the best short term forecast while the Harvey Logistic gave the best medium and long term forecasts. The worst forecasts are given by the

Logistic model for the short term and by the Harvey model for the medium and long term forecasts. Overall, the Harvey Logistic model gave the best accuracy in the Non-Domestic sector followed by the Logistic model.

In the Total electricity consumption, the best fit is again given by the Harvey model. The Harvey and ARIMA model gave very low MAPE values from year 1 to year 6 (Fig. 2). Thus, the ARIMA model was ranked the best for short term forecasting. The VAL model was ranked the best to forecast the medium term. The Harvey model gave the best long term forecast. The Logistic and VAL models gave the worst Total electricity consumption forecasts. Overall, the Harvey model gave the best forecasts in the Total electricity consumption followed closely by the second best ARIMA model.

TABLE I
RANKINGS OF MODEL FITS AND FORECASTING ACCURACY FOR NEW ZEALAND AND MALDIVES

Country	Model	Domestic					Non-Domestic					Total				
			Accuracy					Accuracy					Accuracy			
		fit	Short	medium	long	**Overall**	**fit**	Short	medium	long	**Overall**	**fit**	Short	medium	long	**Overall**
New Zealand	Logistic	6	6	6	5	**6**	4	5	3	2	**2**	4	6	6	2	**6**
	Combined	1	5	4	3	**4**	5	1	4	3	**3**	5	4	5	5	**4**
	ARIMA	4	4	2	4	**2**	3	3	2	4	**4**	3	1	3	3	**2**
	Har_Log	2	2	5	2	**3**	2	4	1	1	**1**	2	3	4	4	**3**
	Harvey	3	3	3	1	**1**	1	2	5	5	**5**	1	2	2	1	**1**
	VAL	5	1	1	6	**5**	-	-	-	-	-	-	5	1	6	**5**
Maldives	Logistic	4	5	3	-	**5**	4	2	4	-	**4**	4	3	4	-	**4**
	Combined	5	3	1	-	**2**	5	4	1	-	**2**	5	5	1	-	**3**
	ARIMA	3	2	4	-	**3**	3	1	3	-	**1**	1	1	2	-	**1**
	Har_Log	2	4	5	-	**4**	1	3	5	-	**5**	2	4	5	-	**5**
	Harvey	1	1	2	-	**1**	1	5	2	-	**3**	2	2	3	-	**2**

B. Maldives

The best model fit of the historical data is given by the Harvey model for the Domestic and Non-Domestic sectors and the ARIMA model for the Total consumption. In general the worst model fit is given by the Combined model in all cases. The Harvey Logistic model gave the best fit for the Non-Domestic sector.

In the Domestic sector, the best short term forecast is given by the Harvey model. The best medium term forecast is given by the Combined model. The Harvey model is also ranked the second best in forecasting medium term Domestic consumption while the ARIMA model is ranked as the second best in the short term forecasting. The worst forecast in the Domestic sector is given by the Logistic and Harvey Logistic models. In general, the best model to forecast the Domestic electricity consumption in Male' is the Harvey model, while the overall second best model is the Combined model.

In the Non-Domestic sector, the best short term forecast is given by the ARIMA model and the best medium term forecast is given by the Combined model. The second best Non-Domestic forecast is given by the Logistic model for the short term and the Harvey model for the medium term. Overall, the worst forecast is given by the Harvey Logistic model. The best Non-Domestic overall forecast is given by the ARIMA model and the second best forecast is given by the Combined Model.

In the Total consumption, the best short term forecast is given by the ARIMA model and the medium term by the Combined Model. The second best forecasts are given by the Harvey model for the short term and ARIMA model for the medium term. Once again the worst overall Total consumption forecast is given by the Harvey Logistic model. The best overall forecast is given by the ARIMA model and the second best by the Harvey model.

V. COMPARISON OF FORECASTS

In New Zealand, national electricity consumption forecasts are published by the Centre for Advanced Engineering (CAE) [23] and the Ministry of Economic Development (MED) [24]. The CAE forecasts are modeled using an annual load growth of 1.8% as the baseline estimates. The MED forecasts are made by the Ministry of Economic Development [24] using its Supply and Demand Energy Model (SADEM). SADEM is a descriptive market equilibrium model focusing on the entire energy sector. The model determines equilibrium in the energy market by projecting demands for a given set of prices and comparing this with the modeled cost of supplying this level of demand [24]. In the Maldives, no such official forecasts are available. Therefore, the forecasts for Maldives are compared using the five models only while the forecasts for New Zealand are compared for the applied models as well as the against the national forecasts. The forecasts obtained by all the models from the year 2000 to 2015 for the Domestic, the Non-Domestic and the Total electricity consumption of New Zealand are shown in Fig. 4 to Fig. 6 respectively.

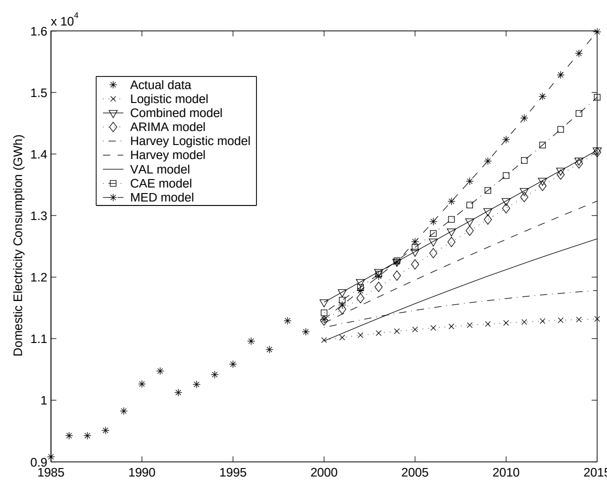

Fig. 4. Comparison of Domestic forecasts for New Zealand.

For the Domestic sector, the highest forecasts are given by the MED model followed by the CAE model. The forecasts by the Combined model and ARIMA models are very similar especially at the long term forecasts. The Harvey model forecasts approximately an average of all the other models.

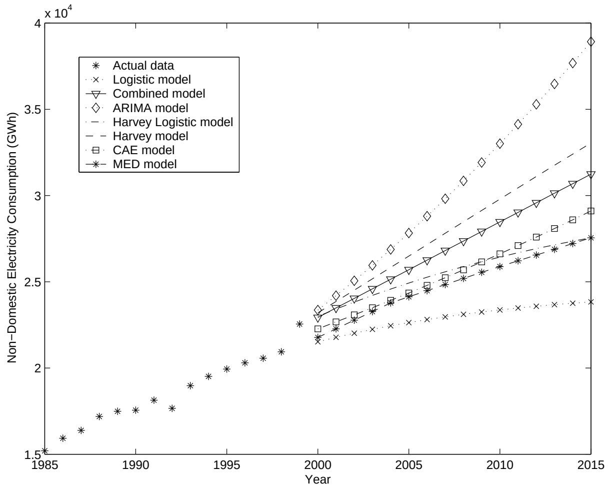

Fig. 5. Comparison of Non-Domestic forecasts for New Zealand.

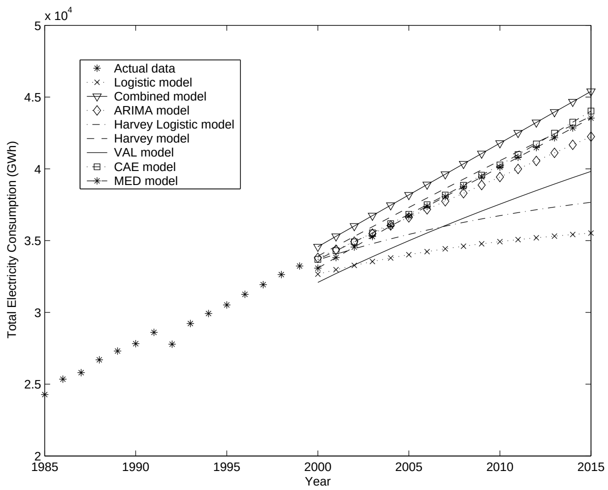

Fig. 6. Comparison of Total consumption forecasts for New Zealand.

For the Non-Domestic sector, the ARIMA model predicted the highest consumption values followed by the Harvey model. CAE model and MED model forecasts are very similar at the early years while CAE and Harvey Logistic model forecasts are very similar at the latter years. The forecasts of the Combined model are more similar to but less than the Harvey model forecasts.

For the Total consumption, forecasts given by the Harvey model are very comparable with the CAE and the MED model forecasts. Forecasts by the ARIMA models are also very similar to these three models especially at the early years. Although the Combined model forecasts are also close to these forecasts, its forecasts are a little higher on average than these models. The VAL model initially started with lower predictions than the Logistic model, but ultimately predicted higher consumption values than the Logistic and Harvey Logistic models. In all cases, the Logistic models have predicted the lowest consumption values followed by the Harvey Logistic models.

The forecasts obtained by all the applied models from 2003 to 2012 for the Domestic, Non-Domestic and Total electricity consumption for the Maldives are shown in Fig. 7 to Fig. 9 respectively.

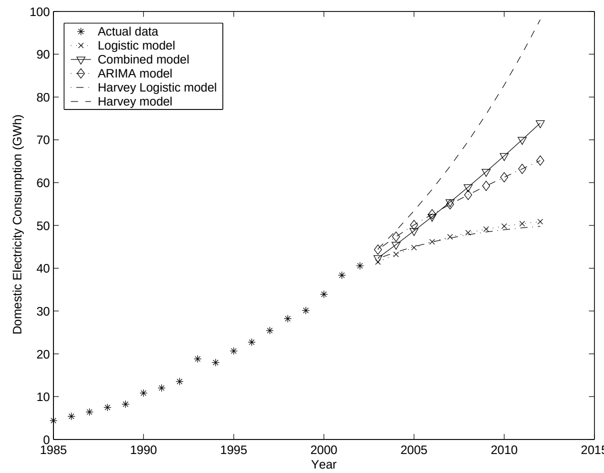

Fig. 7. Comparison of Domestic forecasts for Maldives.

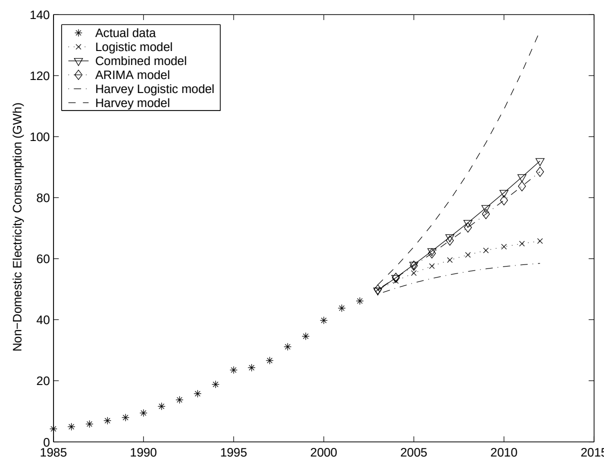

Fig. 8. Comparison of Non-Domestic forecasts for Maldives.

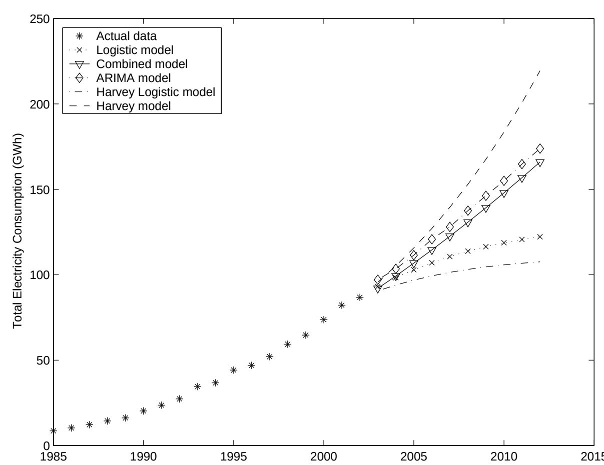

Fig. 9. Comparison of Total consumption forecasts for New Zealand.

For the Maldives, a fifteen year forecast as for New Zealand is not projected due to the few historical data points available to develop the models. For the Domestic sector, the

Harvey model has given rise to the highest forecasts while the Logistic and Harvey Logistic has given rise to the lowest forecasts. The forecasts by the Combined and ARIMA models are in the middle. The forecasts for the Non-Domestic sector and Total consumption follow a similar pattern as for the Domestic sector.

VI. Discussion of results

The analysis of model fit and forecasting accuracy for the two countries indicates that although the Combined and ARIMA models have given comparable accuracies to the Harvey model, no single model has performed better than the Harvey model. The Harvey model has given relatively more accurate historical forecasts for both the countries. In addition, the Harvey models also gave very low MAPE errors over the period compared. For the Total consumption of New Zealand and the Maldives the average errors are 1.08% and 4.43% respectively by the Harvey models, 3.3% and 3.3% respectively by the Combined models and 1.54% and 3.69% respectively by the ARIMA models. The relatively high errors by all models for the Maldives are due to the small number of data points available as compared to New Zealand. For New Zealand, where some national forecasts are available for comparison, it was found that the forecasts by the Harvey models are comparable with the national forecasts. This comparison revealed that the models applied to a developed country like New Zealand can be well applied to a small developing country like Maldives that has many differences in terms of geography, climate, population and economy. This study also showed that although the ARIMA and Combined models have been used frequently in electricity forecasting, the Harvey models have generally given rise to better or very competitive forecasts. This implies that the simple Harvey models could play a significant role in electricity forecasting over the traditional ARIMA and regression techniques.

VII. Conclusions

This paper has compared six forecasting models for electricity consumption in New Zealand and five of the six models for Maldives. The models were compared for both the countries using goodness of fit to the historical data and forecasting accuracy in the short, medium and long term for each of the Domestic and the Non-Domestic sectors and for Total electricity consumption. Overall, the best forecasts for New Zealand are given by the Harvey and Harvey Logistic models. For the Maldives, the best overall forecasts are given by the ARIMA and Harvey models. This study has revealed that while ARIMA and regression techniques using economic and demographic factors are well known in electricity forecasting, the more simple growth curve models may give rise to similar or more accurate electricity consumption forecasts.

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