Improved Hazard Model for Performance-Based Earthquake Engineering

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ABSTRACT: An improved ground motion seismic hazard model for use in performance based earthquake engineering is presented. The model is an improved approximation of the so-called 'power law' model, which is linear in log-log space. The mathematics of the model and uncertainty incorporation is briefly discussed. Various means of fitting the approximation to 'exact' hazard curves developed by seismologists are discussed, including the limitations of the model. Based on 'exact' hazard data for major centers in New Zealand, the parameters for the proposed model are calibrated. To illustrate the significance of the proposed model, a simplified performance-based assessment is conducted on a typical bridge pier. The new hazard model is compared to the current power law relationship to illustrate its effects on the loss estimation assessment. The propagation of seismic hazard uncertainty to drift hazard is also considered.

1 INTRODUCTION

Certain applications of Performance Based Earthquake Engineering (PBEE) require a relationship describing the occurrence over time of a given ground motion intensity measure. This relation, commonly in the form of a ground motion intensity measure and annual rate of exceedance, is typically obtained by conducting Probabilistic Seismic Hazard Analysis (PSHA). A relationship is then required to represent this so-called 'seismic hazard curve' so that PBEE assessments can be made using closed form solutions or numerical integration techniques.

Luco and Cornell (1998) proposed the following power law expression for the relationship between annual rate of exceedance and ground motion intensity:

$$v(IM) = k_0 IM^{-k} \tag{1}$$

Where IM = ground motion intensity; $v(IM)$ = annual rate of exceedance of a ground motion of intensity IM; and k_0 and k are empirical constants. As seismic hazard curves are typically plotted on a log-log scale, Equation 1 is linear in log-log space. This form of parametric equation - primarily used when combined with similar power laws relating seismic intensity to demand and seismic demand to loss - permits closed form PBEE solutions to be obtained. It was proposed that the curve be fitted through the seismic hazard curve at the Design Basis Earthquake (DBE) and Maximum Considered Earthquake (MCE) intensity levels which have a 10% and 2% probability of exceedance in 50 years, respectively. Constraining the curve to pass through these points yields the following parameter values:

$$k = \frac{\ln\left(v_{DBE} / v_{MCE} \right)}{\ln\left(IM_{MCE} / IM_{DBE} \right)} \quad , \quad k_0 = v_{DBE} \left(IM_{DBE} \right)^k \tag{2}$$

where IM_{DBE}, IM_{MCE}, v_{DBE}, v_{MCE} are the ground motion intensities and annual rates of exceedance at the DBE and MCE intensity levels; and ln() is the natural logarithm. A typical comparison of a seismic hazard curve for a Wellington (NZ) site obtained by performing PSHA and Equation 1 is given in Figure 1. It can be seen that due to the 'concave from below' shape of the hazard curve that the approximation of Equation 1 significantly over estimates and under estimates the hazard for ground motion intensities below the DBE and above the MCE intensity levels, respectively. Equation 1 also slightly underestimates the hazard for intensities between the DBE and MCE levels. Hence, while the power law is an adequate local approximation to the hazard curve in the vicinity of the DBE and MCE , over several orders of magnitude the rate of exceedance, ν , is poorly approximated (i.e. from $\nu = 10^0$-10^{-6}). Previous researchers (Solberg et al., 2007) have tried to alleviate this inaccuracy for the more frequent earthquake events by only considering rates of exceedance up to a certain threshold value when using the power law model. The value of this threshold is arbitrary (and consequently not applicable in general) and in order to accurately conduct a probabilistic financial risk assessment, ground motions of all intensities are required to be considered. Hence, the use of Equation 1 is unsuitable.

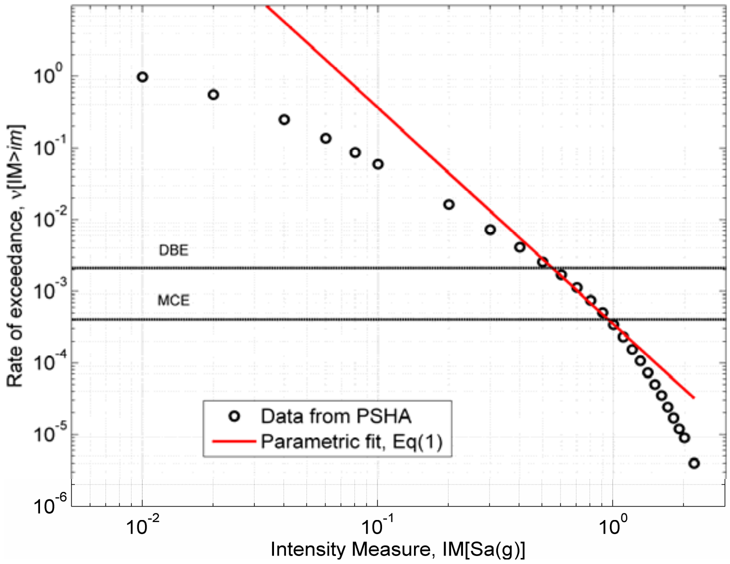


Figure 1: Comparison on hazard data from PSHA fitted by Equation 1

While it is possible to perform PBEE calculations using the raw data from the seismic hazard curve directly, in order to consider the propagation of uncertainty in the ground motion seismic hazard a parametric relationship is required. Therefore it can be seen that a parametric curve which is non-linear in log-log space is required. This paper aims to develop improved seismic hazard curves based on the above objectives.

2 HYPERBOLIC MODEL IN LOG-LOG SPACE

2.1 *Model Development*

As the shape of the hazard curves typically have a 'concave from below' global shape then it would seem reasonable to approximate the curve with a hyperbola of the form y=α/x. Figure 2 illustrates the use of a reference origin that can be used to envisage how the hyperbola can be expressed in the ν–IM plane. The parametric curve has both vertical and horizontal asymptotes and is given by:

$$\ln(\nu) - \ln(\nu_{asy}) = \frac{\alpha}{\ln(IM) - \ln(IM_{asy})} + \varepsilon \tag{3}$$

Where ν_{asy} and IM_{asy} are the horizontal and vertical asymptotes, respectively; α is constant; and ε = a random variable representing uncertainty with mean zero and standard deviation β_H. Hence by rearranging, Equation 3 can be expressed as either a function of ν or IM, the expected values of which are given in Equation 4. The three unknown parameters v_{asy}, IM_{asy}, and α can then be determined using data fitting techniques as described in the following section.

$$\hat{v} = v_{asy} \exp\left[\alpha\left\{\ln\left(\frac{IM}{IM_{asy}}\right)\right\}^{-1}\right] \quad ; \quad \widetilde{IM} = IM_{asy} \exp\left[\alpha\left\{\ln\left(\frac{v}{v_{asy}}\right)\right\}^{-1}\right] \tag{4}$$

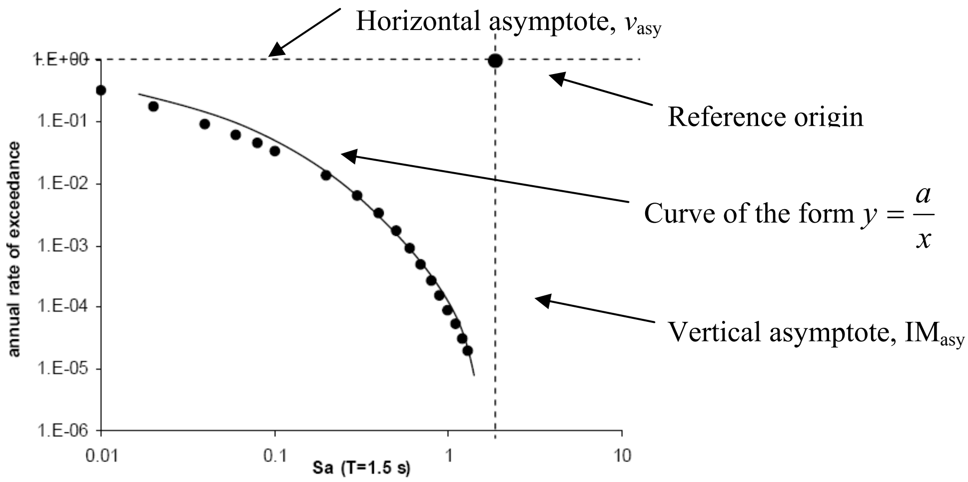


Figure 2: Concept of hyperbolic curve fitted to hazard data

2.2 *Methods of fitting to SHA data*

The parametric equation can be fitted to hazard data using several techniques. The first and most likely used technique is that of (non-linear) least-squares regression. The hyperbolic model is used with v the dependent variable, so that errors are measured as deviations of v between the data and the model. As the overall shape of the hazard curve is of interest then is it is desired to minimise the relative error between the data and the proposed curve and not the absolute error. The later would lead to very accurate prediction of the data with large values of v, but poor prediction of small values. As an alternative, it is typical to minimise the logarithms of the error; that is the least squares problem becomes:

$$\text{Minimise } R = \sum_{i=1}^{n} r_i = \sum_{i=1}^{n}\left[\ln\left(v(IM_i)\right) - \ln\left(v_i\right)\right] \tag{5}$$

where v_i = data points obtained via PSHA; $v(IM_i)$ = value of v obtained from parametric equation; and r_i = the least square residual for each data point. A second fitting method is that of Maximum Likelihood Estimation (MLE) (Kay, 1993), which determines the parameters of the underlying distribution (Eq 4a), which are most likely to have produced the data observed. Both the above two methods produce similar curves to fit the data.

2.3 *Probability of occurrence*

PSHA gives the annual rate of exceedance of an event of specific intensity; however the probability of exceedance is typically more insightful. The probability of occurrence of a given earthquake event over a specific period in time can be obtained based on the Poisson assumption. For a Poisson random variable, the probability of x occurrences is given by:

$$P(X = x) = \frac{e^{-\lambda}\lambda^{x}}{x!} \tag{6}$$

where λ = mean rate of occurrence. Hence the probability of occurrence (i.e. $x \geq 1$) over t years can be obtained as unity minus the probability of no occurrence over t years. That is:

$$P_{occurrence} = 1 - P(X=0) = 1 - e^{-\nu t} = \nu t - \frac{(\nu t)^2}{2} + \frac{(\nu t)^3}{6} - .... \quad (7)$$

where ν = the annual rate of occurrence. The same statements can be made for relating rate of exceedance and probability of exceedance. The final part of the equation is obtained via the Taylor series expansion of the exponential term. For small values of νt, the higher order terms are insignificant and the rate and probability of exceedance are numerically very similar, however for larger values of νt there is a significant difference.

2.4 *Incorporation of uncertainty*

There are several sources of uncertainty in the seismic hazard model. These can be grouped into uncertainty associated with obtaining the data points and the additional uncertainty introduced by fitting the curves parametrically. Firstly, the (aleatoric) uncertainty in approximating the hazard data with a parametric curve can be obtained from the least squares regression. As in general, it is assumed that the uncertainty can be represented by a lognormal random variable, ε with a mean of zero and constant dispersion (lognormal standard deviation) of β_U. The value of β can be obtained by determining the standard deviation of the residuals r_i. The (epistemic) uncertainty in obtaining the seismic hazard data points, β_R, via PSHA arises from several assumptions (such as the use of the truncated Gutenberg-Richter law) is far more difficult to quantify and is typically done via logic tree weightings of different attenuation models. The two uncertainties can be combined to give the total uncertainty associated with the seismic hazard curve (Kennedy et al., 1980):

$$\beta_H = \sqrt{\beta_U^2 + \beta_R^2} \quad (8)$$

APPLICATION TO SEISMICITY DATA

To illustrate the applicability of the proposed hyperbolic model, seismic hazard curve data for the main centres in New Zealand was obtained from Stirling et al. (2002). When the least squares fits are performed for both peak ground acceleration (PGA) and spectral acceleration at a period of 1.5 seconds then Figures 3 and 4 result. It can be seen that the accuracy of the hyperbolic model is maintained over the full range of data for both high seismic regions which have hazard curves with large 'curvature' and for low seismic regions with smaller 'curvature', where curvature refers to the second derivative of the curve in log-log space. Only the PGA seismic hazard curve for Dunedin is poorly fitted by the parametric curve due to its large 'curvature' for large IM values and then smaller curvature at lower IM values. In this case it was selected to fit the data best for the higher values of IM, and hence the first three data points were removed from the least-squares regression. The values of the three parameters for each of the PGA seismic hazard curves in Figure 3 and the associated dispersions are presented in Table 1.

Table 1: Hazard curve parameters for various regions to be used in Equation 4 for PGA

Region	ν_{asy}	IM_{asy}	α	β_U
Auckland	98450	126	121.6	0.12
Wellington	6617	81.7	75.9	0.20
Christchurch	1221	29.8	62.2	0.06
Otira	9.95	10.5	20.5	0.14
Dunedin	1.8	10.3	26.3	0.13

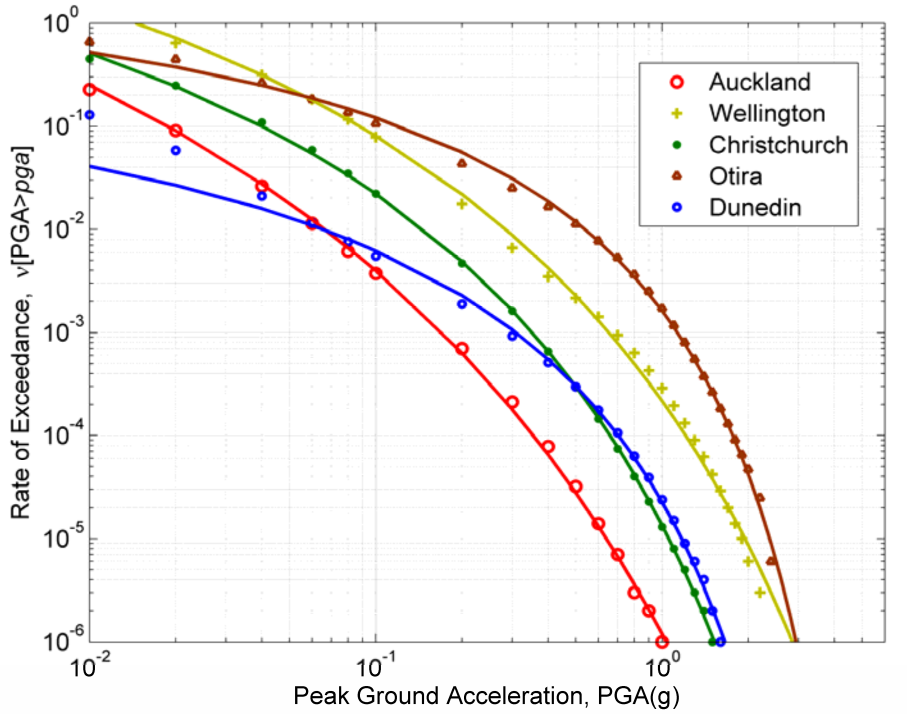


Figure 3: Seismic hazard data for PGA fitted using Equation 4

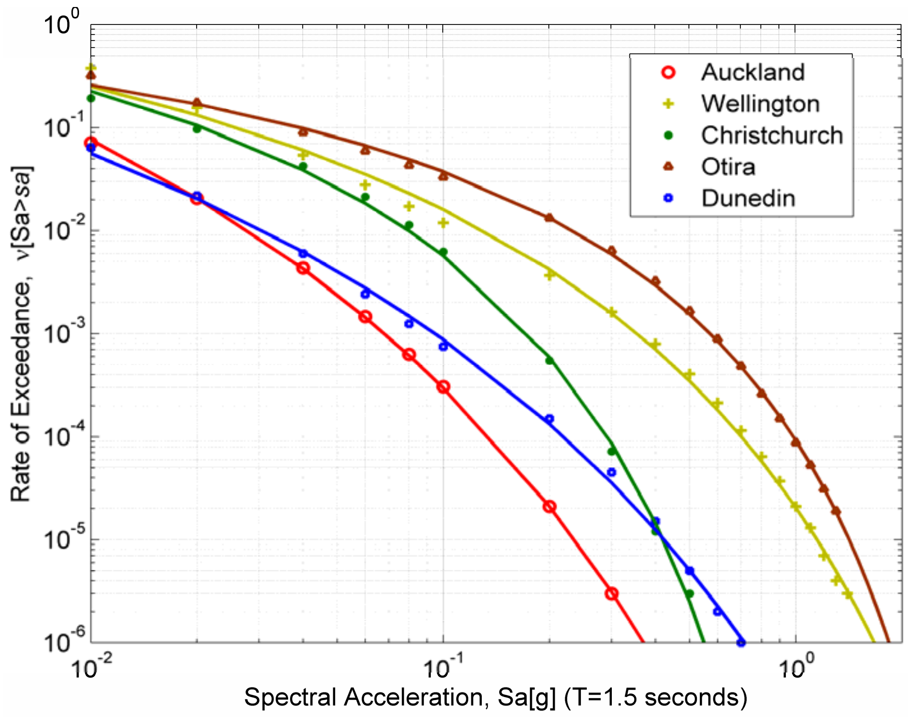


Figure 4: Seismic hazard data for Sa (T=1.5 s) fitted using Equation 4

3 LIMITATIONS OF THE PARAMETRIC EQUATION

As with any curve fitting of data the primary limitation of the parametric curve given by Equation 4 is its use in extrapolation. Asymptotes on the maximum rate of exceedance and ground motion intensity are requirements based on physical principles. The parametric relationship proposed has both horizontal and vertical asymptotes. However, because the parameters of the relationship are determined based on the data points within a specific range, the values of the asymptotes may not be consistent with those of different regions. Overall the range of hazard up to return periods of one million years (ν = 1x10^{-6}) would be considered as a relatively large upper value to use for the assessment integration, and therefore in the opinion of the authors no extrapolation of the parametric curve is required to obtain suitably accurate results when conducting performance based assessments.

4 APPLICATION TO PERFORMANCE BASED RISK ASSESSMENTS

In this section the propagation of the effects of the seismic hazard curve is investigated by computing the displacement hazard curve for a typical bridge pier designed to New Zealand standards.

The prototype bridge pier is 7m high and taken from a typical 'long' multi-span highway bridge on firm soil with 40m longitudinal spans and a 10m transverse width. The seismic weight of the superstructure was calculated to be 7000 kN. Further design details and experimental modelling of the pier can be found elsewhere (Mashiko, 2006).

The bridge was assumed to be located in Wellington, New Zealand, The fundamental period of the pier was 0.6 seconds, and hence the seismic hazard data for a spectral acceleration of 0.6 seconds was used, as it typically gives rise to the least dispersion in the structural demand-response assessment. Damping was assumed to be 5% of critical. From the hazard data, both power law (Equation 1) and hyperbolic (Equation 4) parametric equations were fitted to the data, as shown in Figure 5a.

By conducting Incremental Dynamic Analysis (IDA) (Vamvatsikos and Cornell, 2002) using the spectral acceleration at the fundamental period of vibration as the intensity measure (IM), and the deck drift as the engineering demand parameter (EDP) the following IDA data shown in Figure 5b was obtained using ground motion records from the SAC project (SAC, 1995). The conditional IM-EDP relationship was then parameterised using Equation 9 developed by Jayaler (2002), which is based on separating the mutually exclusive and collectively exhaustive cases of structural collapse and non-collapse:

$$P(EDP > edp|IM) = P(EDP > edp|IM, NC)\left[1 - P(C|IM)\right] + P(C|IM) \tag{9}$$

where $P(C)$ = the probability of collapse; and $P(EDP>edp|IM,NC)$ = $\Phi\left[\left(\ln edp - \ln\{aIM^b\}\right)/\beta_s\right]$, where aIM^b = median response, β_s = logarithmic standard deviation, and $\Phi[]$ = standard normal Cumulative Distribution Function. The 10th, 50th and 90th percentile curves are shown on Figure 5b.

Using both the ground motion seismic hazard and IDA parametric curves the displacement hazard of the pier can be obtained using the convolution integral presented by Deierlein et al. (2003):

$$\nu(EDP) = \sum P(EDP > edp \mid IM)\Delta\nu(IM) \tag{10}$$

where the summation is over a range of IM values which have significant influence on the solution.

Equation 10 is then computed using standard numerical integration, the results of which are presented in Figure 6. It can be seen that in the immediate region surrounding the DBE and MCE levels the drift hazard is relatively similar. This is as to be expected considering the power law curve is fitted through the DBE and MCE data points. However, as expected the power law relationship significantly over-predicts the drift hazard in the region of $\nu > \nu$(DBE). While the power law relationship also over-predicts the EDP for larger earthquake events ($\nu < 5$x10^{-4}), it is not as significant as would be expected based on the shape of the seismic hazard curves. The reason for this can be attributed to the fact that for these larger earthquakes, the onset of structural collapse is occurring, which is illustrated by the fattening of the drift hazard curves around $\nu \sim 2$x10^{-4}. Therefore it can be said that the over-prediction of the power law relationship in the region of large ground motion IM's is a function of the seismic capacity of the structure.

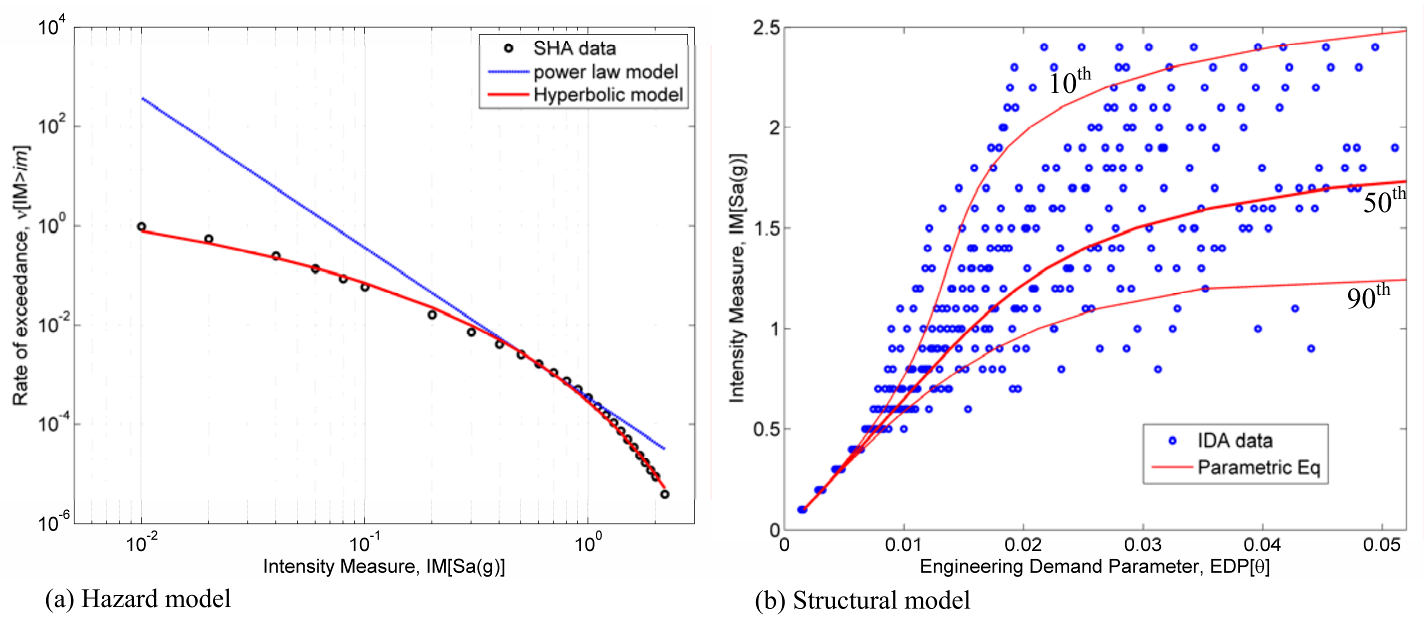


Figure 5: Hazard model and IDA curves

The mean annual drift (MAD) can be determined by first converting the annual rate of drift exceedance (Figure 6a) to annual probability of drift exceedance via the Poisson assumption (Equation 7), then by integrating the area below the resulting curve. The result is not that meaningful in terms of the drift hazard curve as the values are typically very small. This is far more insightful when losses are considered in the performance-based assessment, in which case the Expected Annual Loss (EAL) is obtained. In this study, the MAD is computed for the purpose of quantifying the over-prediction of the power law model. Using the power law model gives a MAD of 0.16% while using the hyperbolic model gives 0.05%, a 69% reduction.

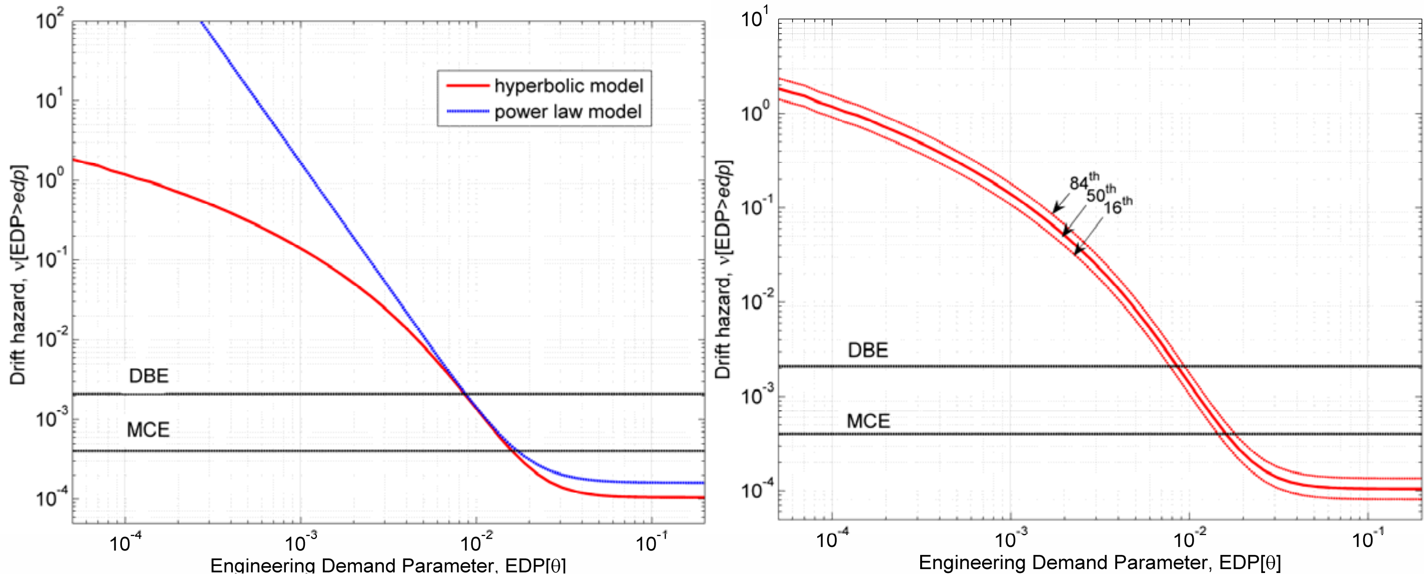
As mentioned in Section 2.4 uncertainty can also be incorporated into the seismic hazard model. It was assumed that the uncertainty in the values of the SHA data points was $\beta_R = 0.2$. The additional (epistemic) uncertainty due to fitting the data parametrically (Figure 5a) was $\beta_U = 0.16$, giving a total dispersion of $\beta_H = 0.26$ (Equation 8). By then using the 16th, 50th, and 84th percentile seismic hazard curves (84th and 16th percentiles are ± one standard deviation), the corresponding drift hazard curves shown in Figure 6b are obtained. Although the uncertainty appears insignificant, note that the curves are plotted in log-log space. For example, at the DBE intensity level, the 16th percentile response is 0.88% while the 84th percentile is 1% drift. Correspondingly at the MCE intensity levels the values are 1.5% and 1.8% at the 16th and 84th percentile levels, respectively. The most noticeable effect of the uncertainty is the occurrence probability at which collapse occurs. This value is approximately $\nu \sim 8\text{x}10^{-5}$ (12500 year return period) for the 16th percentile, but only $\nu \sim 1.5\text{x}10^{-4}$ (6670 year return period) for the 84th percentile. This is a difference of almost two-fold.

5 CONCLUSIONS

Based on the findings of this research the following conclusions can be drawn:

1. A novel parametric hazard model was developed which is non-linear in log-log space. The model was fitted to seismic hazard data via least squares regression, and allows for the incorporation of uncertainty.
2. The applicability of the model to seismic hazard data in New Zealand was illustrated for both PGA and Spectral acceleration and results for PGA were tabulated.
3. Propagation of the effects of the hyperbolic model were investigated via a performance based

assessment of a bridge pier designed to New Zealand standards, indicating the over-approximation of the power law hazard model in computing the drift hazard.



(a) Using different hazard models

(b) Effect of hazard model uncertainty on drift hazard

Figure 6: Drift hazard curves

6 ACKNOWLEDGEMENTS

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