Theoretical investigation of the performance of an Alpha Stirling engine for low temperature applications

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Abstract

The purpose of this paper is to explore the applicability and peculiarities of Alpha engines at low heat source temperature levels of 100 to 200°C. A parameter study of an Alpha engine has been carried out using the commercial Stirling software Sage. The obtained results revealed some interesting insights into the peculiarities of low-temperature Alpha-SEs. A method for optimising the system design-parameters for a SE is described in this paper.

The compact design of a double-acting Alpha engine helps to reduce not only engine size and complexity but also costs. At low temperature differences single-cylinder displacer-type (gamma) engines are well documented, while little is known about the performance of Alpha multi-cylinder engines. In order to achieve the highest possible power output not only the thermodynamic side but also the mechanical side of the engine has to be optimised. This is especially important at low temperature differentials, where the conversion efficiencies are inherently small and the driving force for heat absorption and rejection by the working gas is low. It is mandatory not only to convert as much heat as possible to indicated work but also to transfer as much of this hard gained work to usable power output by minimising internal friction, when trying to keep the engine size as small as possible.

The indicated power output of an engine of a specific swept volume is dependent on the temperature and pressure levels, the frequency, the phase angle between the two pistons, the working fluid, and the design of the heat exchangers and the regenerator. A simple model of an Alpha engine was created using the commercial SE simulation tool Sage. In order to explore the relations between those parameters a model was developed, where the design parameters (temperature, mean pressure, frequency, and phase angle) were varied, and the heat exchangers and the regenerator were then optimised for each of the parameter combinations to reach the maximum power output. Temperature and mean pressure are shown to have the expected positive influence on the power output. For the frequency and the phase angle, optimum values can be found that differ significantly from those found for high temperature engines. Helium is used as the benchmark working gas. It can be shown that the use of Nitrogen instead cuts the power output in half, whereas Hydrogen doubles the achievable power output.

The mechanical efficiency of a kinematic SE is largely dependent on the load that is transferred from the pistons to the crankshaft during expansion and vice versa during compression, as it increases frictional losses. In double-acting engines an identical pressure oscillation acts on the opposing faces of a piston, although it is out of phase. The resulting forces can balance each other to some extent depending on the phase, and thus the net force applied to the crankshaft can be reduced. For a four cylinder engine it is shown that the Siemens arrangement balances internal forces on the pistons to a larger extent than the Franchot arrangement, so that the mechanical losses are inherently smaller and thus the brake power is larger even though the indicated work is identical for both engine types.

The contribution of this work is the provision of a modelling methodology, and the identification of a number of insights for system-design considerations for low-temperature applications.

Nomenclature

α = phase angle

P = power

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θ = crank angle
f = frequency
m = gas mass
p = pressure
R = specific gas constant
T = temperature
V = volume
W = work

1. Introduction

Although at a thermodynamic unfavourable temperature level low exergy heat sources such as industrial waste heat and geothermal brine possess a large potential for power generation due to their large availability. As a consequence of increasing energy costs and demand the development of technology for the exploitation of low temperature heat sources becomes more and more viable.

The idea to use double-acting pistons in Stirling engines of the Alpha type (engines that use only pistons to achieve volume and pressure variations) dates back to as early as 1853 when French engineer C. Franchot patented the concept [1]. A more detailed design of a double acting multi-cylinder was given by Englishman W. Siemens in 1863 [2] but was never realised. It took another eighty years for the first engine to actually be built at Philips in the Netherlands. Since then the four cylinder double-acting design was the design of choice for all following Alpha engines with power outputs larger than a few kilowatts.

The great advantage of double-acting Alpha engines is their simplicity compared to displacer type engines and the economical use of the cylinders, as the whole space on both sides of the pistons acts as active swept volume. Figure 1 shows two possible arrangements for a four cylinder engine. According to Walker [2] engines with pistons having solely expansion or compression spaces on both sides of the pistons are referred to as Franchot arrangements (a). Where an expansion space is separated from a compression space by a piston it is referred to as Siemens arrangement (b).

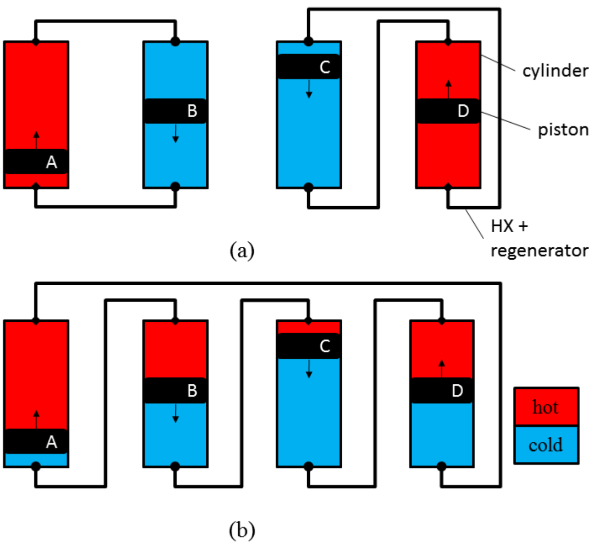


Fig. 1 Franchot (a) and Siemens (b) piston arrangement

Until recently double-acting Alpha multi-cylinder engines were only used for high temperature heat sources. A prototype engine of the Franchot type for industrial waste-heat recovery was reported by Hoeg et al. [3]. The practical advantage of the Franchot arrangement over the Siemens arrangement is the fact that a minimum of two pistons is enough to create a working engine with a freely adjustable phase angle. In the Siemens arrangement with its interconnected cylinders the phase angle is always a result of the number of cylinders so that at least three cylinders are needed in a working engine. Because of the low heat source temperature it is possible to have pistons running in cylinders with expansion (hot) gas spaces on both sides as it is the case for the Franchot arrangement. This has the advantage of symmetric thermal expansion on both sides of the pistons and cylinders as well as omitted heat conduction. However, as the temperature difference is low these effects are much less pronounced than at high temperature differences.

Although little is reported, double-acting Alpha engines seem to be an interesting option also at smaller temperature differences due to their inherent simplicity. The purpose of this paper is to explore the limitations of such engines in terms of the power and efficiency and to investigate the implications of the two different arrangements described above on performance and design.

2. Material and methods

In order to optimise the power output of a Stirling engine two subsystems have to be understood and optimised: first, the thermodynamic and the fluid dynamic parts which involve the heat transfer to and from the gas as well as the transfer of the gas itself within the engine in order to maximise the indicated work; second, the minimisation of the mechanical losses when the generated indicated work is transferred into a usable power output.

A meaningful optimisation of a Stirling engine relies on a detailed and validated model of the heat transfer mechanisms and the fluid dynamics. Thus a third order model should be used for any optimisation task of the heat exchangers and the regenerator. For this study the commercial SE simulation tool Sage is used [4]. Figure 2 shows the set-up of the main model on top and the corresponding sub-models for each component at the bottom. All gas flow path are indicated by the mass flow symbols (m_{Gt}) and matching numbers, the established heat transfer paths are indicated by the capital Qs. For all optimisation purposes, the set-up of this model was not modified; only the values of the different parameters within the components were. The key specifications of the model can be seen in Table 1. Note that the thickness of the regenerator walls changes according to the mean pressure and the regenerator inner diameter to compensate for the stress in the regenerator shell. The simulations performed with the described model are independent of the actual physical set-up; it is valid for single-acting Alpha engines as well as for double-acting engines of any arrangement as only the indicated power and efficiency is calculated.

Little is known about the exact influence of frequency, phase angle, pressure level and working gas on engine performance at low temperature heat sources of 100 - 200°C. Therefore an experimental design was set-up to investigate the influence of these factors. In order to reduce the number of runs necessary for the simulation a central-composite design was chosen, after Montgomery [5]. For the engine of the given dimensions restrictions were made in the allowed regenerator diameter and in the maximum number of tubes to be used for the heat exchangers, according to Table 1. For each combination of parameters the geometry of the heat exchangers and the regenerator is optimised in order to receive the highest possible power output. The performance of each component is mapped in a separate model by varying its geometry with the geometry of the other two components unchanged. For the three components, this can be done simultaneously in three parallel running simulations. Then, the geometry that results in best performance is selected and implemented into the other two simulations. When all three models converge to the same power output, the simulation can be stopped as each component is now mapped with the other two optimized. Usually two or three iterations are enough to find the best overall performance possible with a set of parameters.

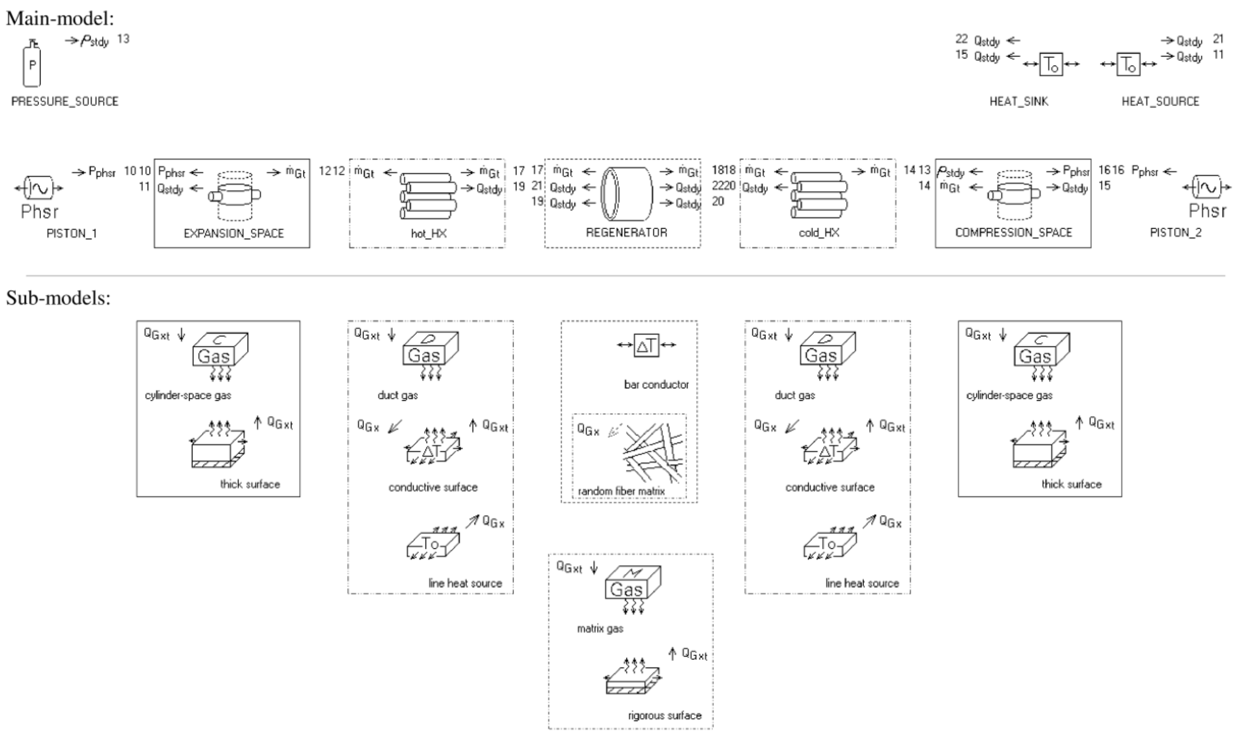


Fig. 2 The used Sage model and corresponding sub-models

Table 1 Simulation parameters

	Value	unit
Bore	0.2	m
Stroke	0.2	m
Material	steel	-
Heat source temperature	150	°C
Heat sink temperature	40	°C
Mean pressure	5 E6	Pa
Max. regenerator diameter	0.5	m
Max. number of tubes	1000	-
Wall thickness regenerator	7.5 E-10 p_{mean} d_{reg}	m
Gas model	Redlich Kwong	-

For the comparison of the Siemens and the Franchot arrangement as well as for all the following efficiency and pressure considerations the much simplified isothermal model was used. In depth information on this analysis can be found for example in Walker's book [2], as well as in many others. The idea behind it is very basic and uses drastic simplifications but it allows a good insight in the Stirling cycle as long as heat transfer and fluid dynamics can be left aside, as it is possible when two thermodynamic identical setups are compared. The basic equations needed for the isothermal analysis are equations for the volume and pressure change in the engine. The volume is defined by the constant volume of the heat exchangers and the regenerator (V_d) and the variable volume in the expansion and compression space depending on the crank-angle (θ) and the phase angle between the two volumes (α). If sinusoidal piston motion is assumed the gas volume becomes:

$$V(\theta) = V_e(\theta) + V_d + V_c(\theta) = V_{0,e}\,(1 - \cos\theta) + V_d + V_{0,c}\,(1 - \cos(\theta - \alpha)).$$

With the constant dead volume chosen to a realistic size:

$$V_d = V_{0,e} + V_{0,c}$$

There is no pressure drop within the engine, so that the pressure is identical in all parts of the engine but varies with the crank angle. The ideal gas law is applied to describe the reaction of the working fluid to volume and temperature changes:

$$p\,V = m\,R\,T$$

Considering the instantaneous gas mass distribution and the constancy of the gas mass in the engine the gas pressure can be found to be

$$p_{gas}(\theta) = m_{gas}\,R_{gas}\,(V_c(\theta) / T_c + V_d / T_d + V_e(\theta) / T_e)$$

In order to gain a general formulation of the results pressure and volume are brought into the non-dimensional form. Thus:

$$V^* = V(\theta) / V_{total} \text{ and } p^* = p(\theta) / p_{max}$$

with V_{total} being the swept volume in the cylinders and the volume in the heat exchangers and the regenerator and p_{max} being the maximum pressure in the cycle. The non-dimensional power output is then found to be

$$P^* = p^*\,dV^*.$$

In order to find the mechanical efficiency of the engine a simple analysis similar to the one presented by Senft [6] is conducted. The mechanical efficiency of the engine ($\eta_{mechanical}$) is given by the ratio of the brake work to the indicated work:

$$\eta_{mechanical} = W_{brake} / W_{indicated} = W^*_{brake} / W^*_{indicated}$$

Not all the work generated by the thermodynamic cycle can be used externally. In fact a large fraction of the generated work has to be recycled internally for the compression of the gas. Thus work is transferred from the pistons to the crankshaft and vice versa depending on the gas forces on the piston. In double-acting arrangements only the net work of the two gas cycles acting on one piston is transferred to the crankshaft. A constant efficiency of the mechanism of the crankshaft is assumed regardless of the direction of the power transfer or the crank-angle. The brake work is then calculated by reducing the work transferred from the piston to the shaft (W_-*) by the efficiency of the mechanism ($\eta_{mechanism}$) and increasing the work transferred back to the pistons (W_+*) by the reciprocal value of that efficiency, as more work has to be supplied from the shaft to overcome the friction.

$$W^*_{brake} = \eta_{mechanism} W_-^* + 1 / \eta_{mechanism} W_+^*$$

With p_{eff} being the difference of the two pressures acting on one piston the brake power can be calculated:

$$W^*_{brake} = \eta_{mechanism} \int_- p^*_{eff} dV^* + 1 / \eta_{mechanism} \int_+ p^*_{eff} dV^*$$

3. Results and Discussion

First the results of the simulations with Sage are presented to show the thermodynamically achievable performance in terms of indicated power and efficiency followed by a comparison of the Siemens and Franchot arrangement using isothermal analysis to show the impact of the compounding on the brake power.

Indicated power optimisation using Sage simulations

It can be shown that the influence of the pressure level on the power output is linear. Thus the pressure level is set to a level of choice for all further optimisations. If a different pressure level has to be considered later the power output can be adjusted accordingly without further need for optimisation of the thermodynamic set-up. The temperature shows the expected strong influence on performance and efficiency. A detailed optimisation of the parameters frequency and phase angle is given in Figure 3 for Hydrogen (a, b), Helium (c, d), Nitrogen (e, f) as working gases at a heat source temperature of 150°C and 5 MPa mean pressure. It can be seen that the maximum achievable power output is doubled if Helium (~ 12 kW) is used instead of Nitrogen (~ 6 kW), and again if Hydrogen (~ 23 kW) is used instead of Helium. This is partly due to the decreasing molecule size (N2 > He > H2) which allows for higher frequency and thus more power, and partly due to the thermal properties of the different gases.

At these low temperature differences, the heat transfer to and from the working fluid has a much smaller driving force than at higher temperatures as e.g. in fossil fired SEs. Therefore the achievable frequency is much lower than at high temperature differences. This has also an impact on the preferable phase angle. Iwabuchi et al. [7] showed experimentally that the heat transfer between the heat exchanger and the working gas in reciprocating flow improves when the phase angle between two opposed pistons is changed from 90 degrees to 180 degrees, so that the complete gas volume from one piston is swept to the other. In the case discussed here the 180 degree phase difference corresponds to a phase angle of zero degrees, as the backside of the piston is used as compression piston (see Figure 2). Depending on the used working fluid, the preferred phase angle lies in between 30 and 45 degrees which correspond to a 12, 10 or 8 cylinder engine in the case of a Siemens arrangement (the phase angle of an Alpha multi-cylinder engine can be calculated by: $\alpha_{piston} = 360°/n$, with n being the number of cylinders). These small phase angles lead to small volumetric changes as the pistons move very close to parallel motion, but the enhanced heat exchange with heat source and sink makes them the preferable choice.

The West number (W_n) as a means to evaluate the power output of a SE relates the pressure level, the frequency, the volumetric change, and the temperature levels [8]:

$$W_n = P / (p\ f\ V\ (T_{hot}-T_{cold}) / (T_{hot}+T_{cold}))$$

Typical values range from 0.2 to 0.25. Not considering the mechanical efficiency, at maximum power, the simulation gives values between 0.2 and 0.24 depending on the working fluid, which prove to be within the predicted range even though the temperature difference is very low.

In the considered range the efficiency drops as the frequency rises. The higher the frequency the higher the temperature difference between the working gas and the heat exchangers and thus the smaller the temperature difference in the working gas and so the efficiency. As the optimisation of the heat exchanger components was done in order to achieve the highest possible power output, the efficiencies could be improved at the expense of power density. At the maximum power point the efficiency is independent of the working gas around 8%, which is 30% of the Carnot efficiency.

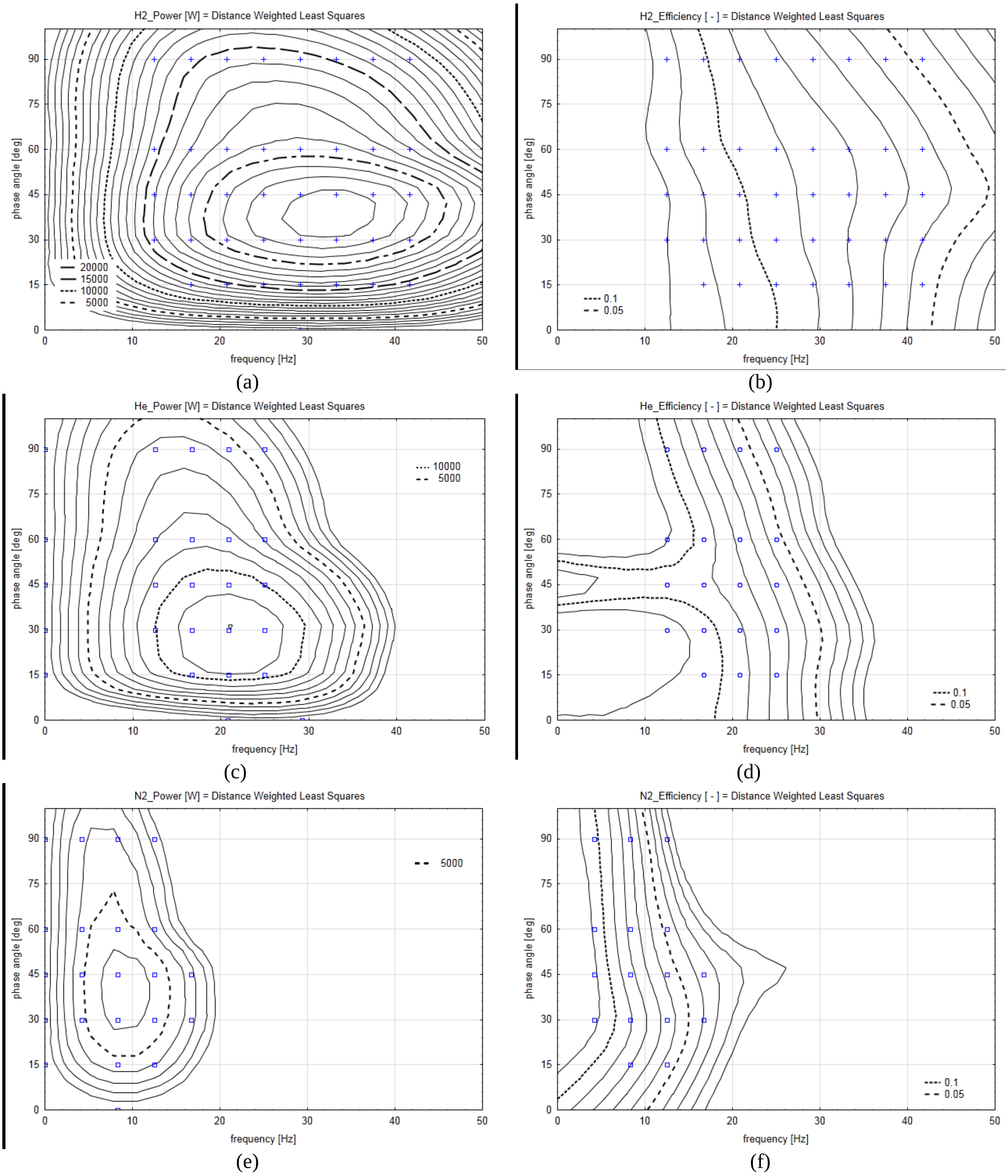


Fig. 3 Power and efficiency for Hydrogen (a,b), Helium (c,d), and Nitrogen (e,f) at T_{hot}=150°C and p_{mean} = 5 MPa

Brake power reduction as a result of the piston arrangement

Low temperature differences between heat source and sink in Stirling engines result in an unfavourable ratio of compression work to expansion work. Figure 4 shows this relation clearly using pV-plots for the expansion, compression, and total gas volume for 450 K (~ 180°C) heat source and 300 K (~ 30°C) sink temperature. It can be seen that a large portion of the expansion work is consumed for compression. The net work (total) is largely diminished and the resulting pV-plot very narrow.

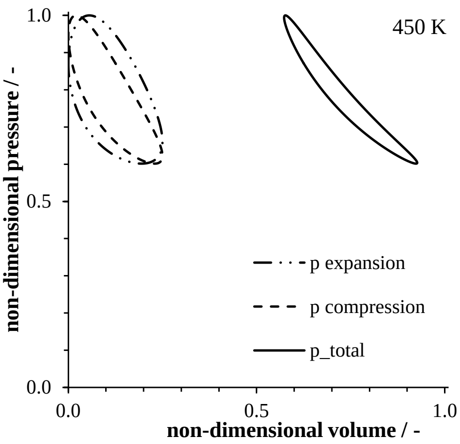
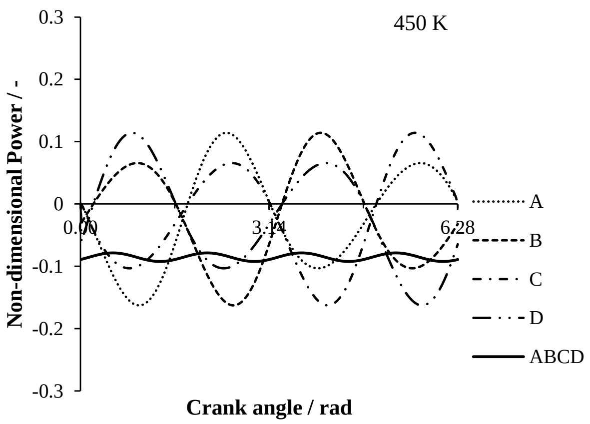
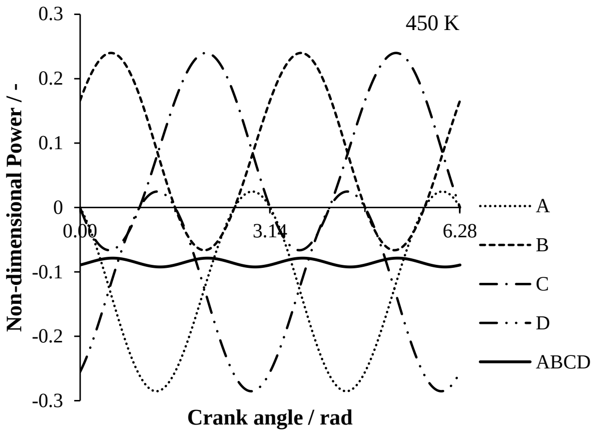


Fig. 4 Pressure - volume variations for the expansion and compression space and the total gas volume

In a double-acting Alpha arrangement two variable gas spaces are combined on the two sides of a piston. These can be a compression and an expansion space in the case of the Siemens arrangement or two gas spaces of the same kind in the case of the Franchot arrangement (expansion – expansion; compression – compression). Figure 5 shows the non-dimensional power for each piston in a four cylinder engine. For each of the two different arrangements the power is given over one crankshaft revolution. In both cases the net indicated power of all four pistons (P_{ABCD}) is identical, which is not the case for the individual pistons. In the case of the Siemens arrangement (a) each piston shows the same characteristic. The combination of compression and expansion on one single piston leads to smaller power amplitudes and thus loads on the piston rods as parts of the power is directly transferred between the two gas cycles. In the case of the Franchot arrangement (b) two pistons supply the crankshaft with power (A, D) and two pistons consume power (B, C) most of the time. This results in larger power amplitudes and loads on the crankshaft as the generated power has to be transferred from the expansion pistons to the crankshaft first and then back to the compression pistons.



(a)



(b)

Fig. 5 Non dimensional power (P*) Siemens (a) and Franchot (b) arrangement

The magnitude of the load transferred from the pistons to the crankshaft and back has a direct influence on the mechanical efficiency of the engine. Applying the above explained analysis of the mechanical efficiency on the two arrangements, the benefits of internal balancing of expansion and compression forces can be seen clearly. Figure 6 shows the overall mechanical efficiency as a function of the efficiency of the crankshaft mechanism. As expected the overall mechanical efficiency drops as the efficiency of the crank mechanism drops. A less efficient crank mechanism proves to be less detrimental in the case of the Siemens arrangement. But even at high mechanism efficiencies of for example 0.95 the Siemens arrangement provides 18% more power or mechanical efficiency than the Franchot arrangement, a notable difference.

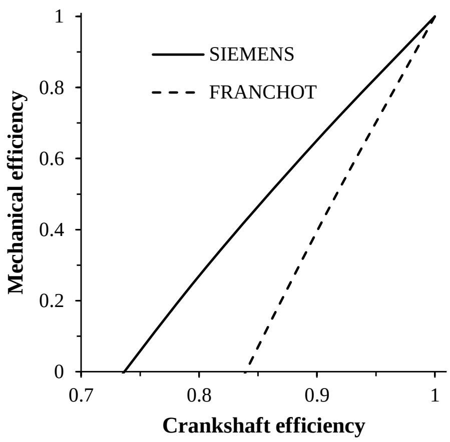


Fig. 6 Overall mechanical efficiency versus crankshaft efficiency

The type of compounding also has an influence on the pressure difference between the gas spaces separated by the piston seal. A larger amplitude in pressure difference leads to increased leakage into the adjacent gas cycle. Even though there is no net loss of the working gas as the direction of the pressure difference changes there is a loss in power. Figure 7 shows the pressure difference acting on one piston for each of the two arrangements. For the Franchot arrangement the amplitude of this difference is almost 30% higher and leakage increases accordingly. For an identical piston diameter this pressure difference is also directly proportional to the force on the piston rod and the crankshaft.

Although the indicated power is identical and both arrangements effectively reduce the engine size by using the double acting principle the Siemens arrangement shows a number of advantages. In addition to the above stated, the reduced load on the crankshaft has additional benefits. Having to sustain less stress, the piston rods can be made from less or lighter material. This reduction of mass reduces the inertial forces of the crank and thus side forces and piston ring friction (for slider crank assemblies). Reduced load on the bearings and reciprocating seals also increase longevity of the engine. A smaller rod diameter also decreases leakage to and from the crankcase as the sealed gap becomes smaller and the power output is increased.

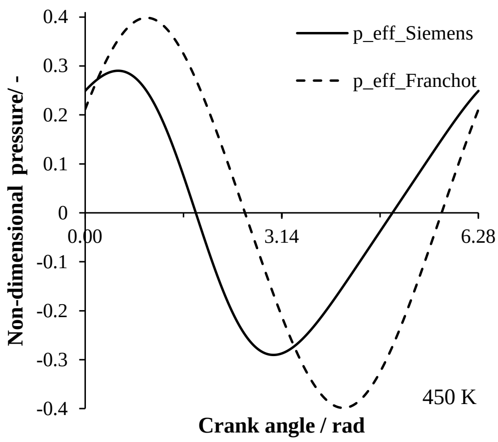
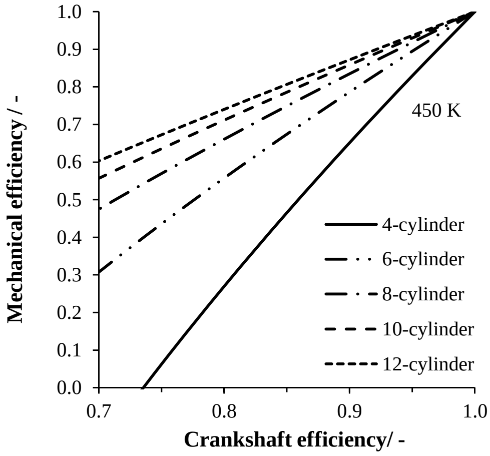
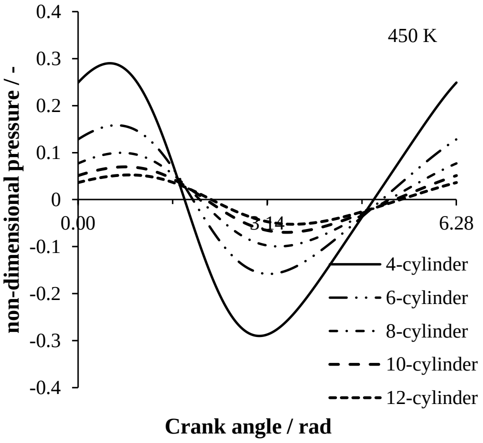


Fig. 7 Effective pressure oscillation for the Siemens and the Franchot arrangement over one cycle

So far the mechanical implications were discussed only for four cylinder engines. It could be shown that the combination of a compression space and an expansion space on one piston is favourable in terms of mechanical efficiency, seal leakage, and internal loads. The thermodynamic optimisation with Sage implies that phase angles between pistons of 30 – 45 degrees are favourable which correspond to piston numbers between 8 and 12 in the case of interconnected cylinders. In Figure 8 the above made analysis is expanded to higher piston numbers for the better balanced Siemens arrangement. The higher the number of interconnected pistons the higher the overall mechanical efficiency as the gas forces are balanced even better (Figure 8 (a)). The amplitude of the pressure difference on the piston seal decreases with the number of pistons as the pressure amplitude in each gas cycle decreases (Figure 8 (b)).



(a)



(b)

Fig. 8 Mechanical efficiency and effective pressure for different cylinder numbers in a Siemens arrangement

4. Conclusion

The results of the two different analyses to optimise the indicated power as well as the mechanical efficiency both indicate favourable working conditions in a similar region, thus no compromise has to be found to reach maximum performance, a well appreciated fact when dealing with low enthalpy heat sources. The simulation shows the possibility to reach high power outputs from Alpha Stirling engines operating at low temperature differentials if the operation conditions (phase angle, frequency, working fluid) are chosen appropriately and if the mechanical set-up is designed aptly. As discussed above the Siemens configuration is the superior double-acting set-up for a number of reasons.

Acknowledgements

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