Harmonic State Estimation: Error Analysis and Optimal Location for Power Quality Measurements

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ABSTRACT

The harmonic levels in a distribution system are becoming more important as the use of non-linear loads and new technology is increasing. Monitoring of the harmonic levels is important in order to manage them. Direct measurement of harmonics at all locations is too costly, yet obtaining a system-wide view is important.

Harmonic State Estimation (HSE) is a cost-effective alternative to direct monitoring of the harmonics. To date, it has been applied mostly to transmission systems but in this work, it is applied to an actual distribution system using actual measurements. The aim of this research is to find out how the measured power quality data can be utilised and used for the efficient and accurate estimation of harmonics by using HSE technique, and installing equipment at optimal locations of power distribution networks.

HSE is performed based on a robust mathematical technique, Singular Value Decomposition (SVD). The SVD based HSE algorithm is first validated with the simulation results from the harmonic penetration study on the model of an actual distribution feeder using Open Distribution System Simulator (OpenDSS) and MATLAB. The estimated results and the measured values are compared and found to be extremely close to each other.

The validated algorithm is then applied to the actual distribution system. Harmonics are measured at eight locations in one of the feeders of an actual distribution system. Using the measurements and the system model, HSE is performed. The estimates and the actual measurements are compared. Comparison is performed for several scenarios assuming different number of measuring equipment available for taking measurements.

The observability of the system is also checked. Locations of the measuring equipment, which can give maximum observability with the least errors in the estimated values, are found. Such locations are taken as the optimal locations for the placement of the measuring equipment.

A power quality assessment tool has been developed to evaluate and visualise the power quality indices based on the measurements taken by the power quality monitors in Unison Networks Limited.

It has been established that harmonic state estimation using SVD method can be applied to the distribution systems to estimate the harmonics at unmonitored locations using few measurements and the system information. Further, it has been shown how the optimal location of the measuring equipment can be found using the observability of the system and the minimum error

in the estimates. A new and simple method of adding new equation to the measurement equation in order to improve observability has also been applied for the non-triplen odd harmonics.
This research has thus been successful in providing the experimental evidence for the application of HSE (based on SVD) to the distribution systems for the first time, finding optimal location for the placement of measuring equipment to monitor harmonics, and developing Power Quality Assessment and Visualization Tool for the Power Quality Management System in a distribution company.

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Chapter 1

INTRODUCTION

1.1 BACKGROUND

Power Quality is a very important issue in the performance of a power system as many loads are sensitive to power quality. Their performance, security and efficiency are affected by poor power quality. It can also raise regulatory issues. In the context of competitive deregulated market and increasing demands for more efficient and smarter operation of power system, it is of utmost importance to keep compliance with power quality requirements. Hence, it becomes mandatory for any utility to have an intelligent Power Quality Management System for monitoring, analysing and keeping compliance with power quality standards.

1.1.1 What is Power Quality?

Consumers of electricity are concerned not only with the continuity of electrical power but also with the certainty of its characteristics like voltage waveform and frequency. Such expected nature of electrical power can be called as power quality which has been defined in various ways in literature. Power Quality has been defined in [Heydt 1991] as the measure, analysis and improvement of bus voltage, usually a load bus voltage, to maintain that voltage to be a sinusoid at rated voltage and frequency. As defined by IEEE, Power Quality is the concept of powering and grounding electronic equipment in a manner that is suitable to the operation of that equipment and compatible with the premise wiring system and other connected equipment. Power Quality (more accurately voltage quality) is a measure of the deviation of the voltage waveform from the ideal sinusoidal waveform (at rated voltage and frequency) and encompasses such phenomena as: Steady-state-voltage magnitude (under-voltage or over-voltage), Variations in the peak or RMS voltage (sag and swells), Harmonics (and inter-harmonics), Voltage fluctuations (light flicker), and Transients (spikes, impulses or surges) [Watson 2010].

1.1.2 Necessity for Maintaining Good Power Quality

Modern distribution system has many sensitive and smart equipment. The topology and nature of operation of these modern distribution systems have also deviated significantly from the conventional one. They are no more passive in nature. There are distributed generations (DG) with unique features and mode of operations. Technological advancement and emphasis on renewable energy-based electricity market growth in New Zealand and everywhere in the world today is encouraging installation of numerous DG's of various types. Government authorities have also been emphasizing and encouraging consumer participation for power production through DG's. The increasing number of such renewable energy-based DG's in distribution systems is causing alterations in the magnitude and direction of power-flow. Voltage profile at buses and losses in the lines are certainly going to be affected. Operation and control of power distribution systems will also be affected with these emerging technologies.

Advanced innovative technologies have significantly involved the use of power electronics based components. Their increasing use in the system has contributed further to the power quality issues. The harmonic levels in a distribution system are also becoming more important.

Utilities have legal obligations also to comply with the power quality standards. All of these factors have significantly raised the concern for power quality.

1.1.3 Power Quality Monitoring and Power Quality State Estimation (PQSE)

"Measurement is the first step that leads to control and eventually improvement of any system. If you can't measure something, you can't understand it. If you can't understand it, you can't control it. If you can't control it, you can't improve it" [H.James Harrington]. Assessment of power quality indices, as can be interpreted from this quote, is facilitated by monitoring through modern measuring equipment which operate often in synchronization with a common time frame of reference like global positioning system (GPS). However, these power quality monitors are sophisticated and costly. Direct measurement of harmonics deploying large number of such equipment at all locations is too costly, yet obtaining a system-wide view is important. Hence, there must be a smarter way of getting information of the variables of concern through some intelligent methods of estimation. If such a method can provide the required information within a reasonable accuracy, it can be applied to supplement the information obtained through direct measurement for the system-wide evaluation of power quality. Power Quality State Estimation (PQSE) is one such intelligent technique which can be used to estimate power quality indices. It is the method of determining the unmeasured system state variables through mathematical estimation techniques based on the measured data at various locations of the power system network and the system information. Although the computation techniques for determining system states and the power quality indicators have been discussed in literature, very little has been contributed for their application in practice, particularly in distribution systems. Various algorithms and strategies to get such solutions actually need to be tested and applied to practical systems. Once the required system state variables are estimated, they can be used to evaluate power quality indices. Types of PQSE are:

- Harmonic State Estimation (HSE)

- Voltage Sag State Estimation (VSSE)
- Transient State Estimation (TSE)

Among the types of PQSE, harmonic state estimation (HSE) is also one. Harmonics are the signals with frequency other than the fundamental frequency. These are present in modern power systems due to increasing use of power electronic components, non-linear loads, several disturbances, unbalanced operation, distributed generations, etc. Harmonics have often been the cause of failures severely affecting the performance and efficiency of power systems. Measurement of harmonics of desired level is often an expensive task due to the high cost of the required measuring equipment. HSE is used to estimate harmonic levels based on measured values and system information. Several techniques have been discussed in research literature that can facilitate HSE for over-determined, determined and under-determined systems. Further details of HSE will be discussed in the Chapter 2.

1.1.4 Unison Networks Limited (UNL)

Unison Networks Limited (UNL) is a power distribution company operating in Hawkye's Bay region of New Zealand. UNL is one of the leading companies in acquiring new concepts and practice for smart power distribution. Several expensive and sophisticated power quality measuring instruments have been installed at various grid exit points (GXP) and zone substation (ZS) levels, and numerous others need to be installed at distribution transformer (DTX) levels. UNL has concerns in power quality assurance for its commitment towards consumer satisfaction and compliance with acts and standards of the regulators. UNL has been planning for the development of Power Quality Management System. Therefore, it requires monitoring of different significant electrical parameters, analysis of the acquired measurement data and evaluation of power quality indices. It is also required to find strategic locations and numbers of such instruments for achieving its goals in power quality.

1.2 RESEARCH MOTIVATION

While operating a smart grid, compliance with power quality regulations must be ensured. Therefore, power distribution industries are making huge investment on infrastructure for meeting quality standards. UNL is one such distribution company in New Zealand. It is sincerely concerned with the power quality issues because of its requirements to meet consumer guarantees act, support asset management policy, optimize network design and operation, and respond to new technologies on DG's. This is also true for other power distribution companies. Therefore, it is required to look for reliable methods and technologies that can evaluate, analyse and estimate the system-wide power quality indices with high accuracy and efficiency. The queries that motivate for this research are:

- How can the Power Quality Management System be made more effective for smart electricity distribution system?
- Have the power system state estimation techniques, relevant to the power quality monitoring and assessment, been practically helpful for the industries?
- Are the measuring instruments in industries adequate for making estimation?
- Have the errors between measurements and estimation been analysed ?
- Have the measuring instruments been optimally located?
- Are there alternatives to the existing analytical methods that can be pragmatically implemented in this process of power quality assessment?

There is an increasing concern over the rise of harmonic levels in the modern distribution systems. This can be caused by increasing integration of new technologies like photovoltaic generations, electric vehicles and use of power electronics based equipment. Other sources of harmonics such as transformer saturation are small compared to the harmonics from consumer loads. This necessitates robust monitoring and instrumentation prior to employing control measures for restraining harmonics to their limits. One straight forward solution would be to install numerous Power Quality (PQ) monitors at all the locations of concern. However, this is understandably not an economic solution as these monitors are costly. This requires a smart solution which can be provided by Harmonic State Estimation (HSE).
In the past distribution systems were radial and served passive loads. However, today the distribution networks are undergoing radical changes, with a large increase in the deployment of distributed generators, battery storage systems as well more automation and control. Hence the distribution system is no longer passive. These changes heighten the need for state estimation techniques, and as these changes progress, the need will be greater so that proper control and optimisation of resources can occur. HSE has been performed before in transmission systems but not applied to actual distribution system networks. Hence, application of HSE to distribution systems as a smart solution for the system-wide information on harmonics is the major motivation behind this research.

1.3 PROBLEM FORMULATION

“How the data measured through costly power quality monitoring equipment can be utilized, analysed and processed with higher accuracy and efficiency for the system-wide evaluation of harmonic levels in distribution networks, and how the cost effective methods for optimal location can be explored for measuring equipment” are the key research questions.

1.4 OBJECTIVES, SCOPES AND METHODOLOGY

1.4.1 Objective

The main objective of this research is to carry out error analysis in Harmonic State Estimation and determine the optimal location of power quality monitoring equipment in a power distribution network. In order to fulfil this main objective, the following specific objectives need to be fulfilled.

1. Investigate on Power Quality State Estimation algorithms for Harmonic State Estimation.
2. Investigate on the errors in the estimates.
3. Investigate for optimal placements of measuring equipment.
4. Develop and implement an algorithm for data extraction and power quality assessment of an existing electricity distribution network, i.e. Unison Networks Limited.
5. Develop an algorithm and a model for visualisation of power quality performance indicators, their trend and analysis.

1.4.2 Scope of Study

This research is primarily based on the existing distribution network of Unison Networks Limited (UNL) located in Hawkes Bay, New Zealand, and utilizes the data collected from the existing power quality monitoring equipment installed at various locations of UNL. Harmonics are monitored along a distribution system feeder as a test system and the data is utilized for making HSE and comparison of the results.

1.4.3 Methodology

In this Section, the methodology used for this research is discussed. The methods used for data collection, the software tools in use, and the techniques used for the analyses are introduced.

1.4.3.1 Data Collection

Power quality data are recorded by the power quality monitors which have been installed on the distribution network at the low voltage (LV) side of the distribution transformers with voltage rating 11/0.415 kV. The data are communicated and stored which can be accessed programmatically using PI data-link and Matrix Laboratory (MATLAB). The current and voltage harmonics data are logged in the power quality analysers which are installed at the LV side of all the distribution transformers in one of the feeders. These data are used for the verification of HSE.

1.4.3.2 Analytical Tools

MATLAB is used as the main software to write programs for data processing and analysis. Open Distribution System Simulator (OpenDSS)[Dugan 2010] is used for simulation and generation of data for testing of the HSE algorithms programmed in MATLAB.

1.4.3.3 Power Quality Assessment

Power Quality at various distribution transformers in Unison Networks Limited are analysed based on the data collected using the power quality monitors. The Power Quality Guidelines developed by Electricity Engineers' Association (EEA) are followed to evaluate power quality indices. These Guidelines are followed in New Zealand and Australia, and are based on international standards. Power quality indices are evaluated and compared with the standard limits.

1.4.3.4 System Modelling

The distribution test feeder is modelled in OpenDSS and MATLAB. Admittance matrix models of the components in distribution system are used to create a compound admittance matrix that represents the system. Harmonic loads are considered as fixed harmonics current sources. MATLAB is used for system modelling and calculations.

1.4.3.5 Harmonic State Estimation (HSE)

Harmonics are estimated using the HSE technique based on Singular Value Decomposition (SVD) method. A background literature related to HSE is discussed in Chapter 2 and its detailed algorithm is described further in Chapter 3.

1.4.3.6 Visualisation of Power Quality Indices

A power quality assessment tool based on MATLAB is developed for the evaluation of power quality indices. The indices are visualised using Graphical User Interface Development Environment (GUIDE) in MATLAB. The detailed discussion about this tool is included in Chapter 5.

1.4.3.7 Simulation model and Optimal Location for Measuring Equipment

A simulation model is created in Open Distribution Simulation Software (OpenDSS) and a harmonic penetration analysis is performed to generate measured data regarded as actual values for HSE in MATLAB. The number and locations of measuring equipment are varied and the analysis is repeated. The locations that can provide maximum number of observable states with maximum accuracy in HSE are considered as optimum locations for measurements.

1.4.4 Thesis Contribution

This thesis makes the following contributions.

- Application of Harmonic State Estimation (HSE) to distribution system: Application of HSE to a distribution system is a novel approach which has not been used before for practical distribution systems though this method has been used for transmission systems. This application has been researched because harmonics are more prevalent in the distribution systems and carry more importance for the safety of electrical appliances. This research has provided evidence with practical data on how HSE, based on SVD, can be applied to power distribution networks.
- Evaluation of number and selection of the locations for measurements: Since the power quality analysers are costly, power quality assessment by direct measurement is not the preferred alternative. HSE provides Estimation as an alternative to calculate harmonics and avoids capital cost on the equipment. This research demonstrates an algorithm for the selection of adequate (minimum) number of measuring equipment, and the identification of the placement for the equipment to make HSE reasonably accurate.
- A new and simple method of adding new equation to the measurement equation for HSE of the non-triplen odd harmonics has been implemented in this research. This is based on the fact that the sum of the phase currents for the non-triplen odd harmonics is zero at the neutral point on the star side of the the Dyn11 tranformers. This helps to improve the observability of the system states.
- A Power Quality Assessment and Visualisation tool has been developed for Unison Networks Limited which can be helpful for the Power Quality Management System of the company.

1.4.5 Thesis Outline

There are six chapters in this thesis. They are briefly introduced as follows:
Chapter '1' introduces the thesis. The background of the thesis is discussed. The motivation behind the research is presented. The research problem is clarified and the objectives, scopes and methodology for the research are described. The contribution made by the research and the outline of the thesis are also stated.
Chapter '2' describes the HSE in details. The background review of the literature related to HSE is explained. The techniques for modelling components are discussed, and the HSE problem is defined. Existing techniques for HSE are also discussed.
Chapter '3' is dedicated for the description and validation of the HSE algorithm applied in this research. It presents the evidence of the accuracy of estimation using the simulated results obtained from harmonic penetration study in OpenDSS and MATLAB environment.
Chapter '4' presents the application of HSE to the selected distribution feeder in the network using the actual measured data. Detailed discussion on the analysis of the error in estimation

is also presented. The method and example of the optimal location of power quality monitors are presented.

Chapter ‘5’ discusses in details the tool developed for the Power Quality Assessment and Visualisation. Features of the interface are briefly introduced for each type of Power Quality Index that the developed tool can analyse.

Chapter ‘6’ presents the conclusion and the future works related to the research area of the Harmonic State Estimation.

Chapter 2

HARMONIC STATE ESTIMATION

Harmonic State Estimation is a type of Power Quality State Estimation where the harmonic states at various buses are estimated based on the partial measurement of harmonics. The states could be either bus voltages or line currents, and measurement could be either voltage or current. Knowledge of harmonic states can provide complete information necessary for the calculation of harmonic power-flow and the total harmonic distortion. State estimation problems are classified as over-determined, completely determined and under-determined based on whether the number of measurements is greater than, equal to, or less than the number of state variables, respectively. Only the completely determined problems can be solved uniquely but not the under-determined and over-determined ones. Usually, HSE for a real power system is under-determined as it uses partial measurements [Du *et al.* 1996].

2.1 BACKGROUND

HSE is a type of PQSE which helps to estimate the harmonic levels and location of harmonic sources. Heydt [Heydt 1989], and Najjar and Heydt [Najjar and Heydt 1991] mentioned and employed a reverse power flow procedure to identify the harmonic signal sources throughout the network. They used the measured voltages and currents, made harmonic active and reactive power estimates with least squares estimators, and identified the sites injecting energy into the network as the sources of harmonics. However, inaccuracies were introduced due to reactive voltampere losses, active power losses and estimation errors.

Beides and Heydt used Kalman filter to achieve optimal estimate of the power system harmonics [Beides and Heydt 1991]. This was a dynamic state estimation and had a capability to track harmonic content versus time. It was used to estimate and track the effect of load variation on harmonics for a period of one day.

Meliopoulos, Zhang and Zelingher suggested power system state estimation for multiphase, multi-frequency model using current and voltage measurements with emphasis on the effect of modelling and measurement on harmonic state estimation [Meliopoulos *et al.* 1994]. They performed sensitivity analysis to demonstrate the impact of asymmetry and imbalance on the accuracy of state estimations. Du et al. developed and extended their proposed HSE to find the location and type of the sources of harmonics with a few measurements [Du *et al.* 1996]. Suspicious harmonic source has been interpreted as Norton equivalent circuit at the respective harmonic frequency. They used only a few synchronized, partial and asymmetric measurements across the system and lines (distant from the harmonic source). They presented an algorithm to model the system for harmonic state estimation and explained how an under-determined model could be modified to an over-determined one. This method could be used for continuous estimation of harmonics. Matair et al. presented a new HSE algorithm based on Singular Value Decomposition (SVD) method which could give solution even for a partially observable network [Matair *et al.* 2000]. They also introduced an algorithm for HSE, including reduction method, to convert from under-determined HSE into an over-determined one, thereby simplifying the required efforts [Du *et al.* 1996]. Meliopoulos et al. showed the biases present in conventional state estimation and explained the impact of modelling errors and system imbalance on the estimates [Fardanesh and Zelingher 2001]. They even proposed three-phase state estimation and checked the overall accuracy and performance using the concept of confidence level, thereby avoiding those biases.

Watson et al. used SVD for three-phase HSE of partially observable systems and demonstrated the superiority of SVD for reduction of errors in estimation [Yu and Watson 2004]. They also pointed out that observability analysis was no longer required if SVD were to be used. Kanao et al. performed HSE utilizing the field data to remove uncertainty in transmission line parameters [Kanao *et al.* 2005]. Liao formulated HSE as a constrained sparsity maximization problem based on L1-norm minimization, and solved HSE efficiently in order to convert under-determined system into a determined one [Liao 2007]. Nguyen and Yang used an approach utilizing basic circuit analysis and compared the results with SVD for making HSE [Nguyen *et al.* 2010]. This method formed sensitivity indices to relate the change in real harmonic load power of a given node to that of the injected harmonic current from each source, and assessed dominance of particular harmonic source to harmonic real power flows in loads. Bahabadi et al. proposed an optimal placement of Phasor Measurement Units based on genetic algorithm for harmonic state estimation in unbalanced distribution system [Bahabadi *et al.* 2011]. They used a hybrid genetic algorithm and simulated annealing to improve the algorithm efficiency. They used SVD for solving HSE and identifying unobservable buses. Rad and Karimi proposed a sequential algorithm to identify observable buses in each step, and remove them from the rest of the buses at which a meter could be installed at other steps [Rad *et al.* 2012]. This would optimize the measurements and minimize estimation errors. Rakpenthai et al. solved HSE of power system with uncertain network parameters [Rakpenthai *et al.* 2013]. They presented algorithm to adjust weights of Weighted Least Square (WLS) taking network parameter deviations into consideration to tune covariance matrix. Although HSE has been proposed before, it has been applied mostly to transmission systems [Kanao *et al.* 2005]. However, this technique is even more significant for the distribution systems. Use of SVD for HSE in an IEEE test feeder was presented in [Breda *et al.* 2016]. HSE based on Phasor Measurement Unit using parallel processing for real time applications was discussed in [de Melo *et al.* 2016]. SVD was used for medium voltage distribution system in [Bahabadi *et al.* 2011].

2.2 COMPONENT MODELS FOR HSE

2.2.1 Line Resistance

The AC resistance of cable is directly obtained using (2.1) and (2.2) [Nasser 2008].

$$R_{(ac)} = R_{(dc)}[1 + \gamma(k_S + k_P)]\Omega/km \tag{2.1}$$

$$R_{(dc)} = \frac{1000\rho}{A}[1 + \alpha_{20}(T - 20)]\Omega/km \tag{2.2}$$

where $\gamma = 1$ for single-core, two-core and three-core cables. 'k_S' is skin factor and 'k_P' is proximity factor. 'ρ' is the conductor resistivity in 'Ωm' , 'A' is the area of cross-section , 'T' is the conductor temperature in '$^\circ C$', and 'α_{20}' is temperature constant at $20^\circ C$. Skin-effect factor is calculated using (2.3).

$$k_S = \begin{cases} \dfrac{z^4}{0.8z^4 + 192}, (0 < z \leq 2.8), \\ 0.0563z^2 - 0.0177z - 0.136, (2.8 < z \leq 3.8), \\ 0.354z - 0.773, (z > 3.8) \end{cases} \tag{2.3}$$

where

$$z = \sqrt{\frac{8\pi f a_z}{10^4 R_{dc}}} \tag{2.4}$$

$$k_P = \begin{cases} 2.9 \times F(p)\left(\dfrac{d_c}{S}\right)^2, \text{ for two-core and two single-core cables} \\ F(p)\left(\dfrac{d_c}{S}\right)^2\left[0.312\left(\dfrac{d_c}{S}\right)^2 + \dfrac{1.18}{F(p) + 0.27}\right], \text{ for three-core and three single-core cables} \end{cases} \tag{2.5}$$

where

$$\left.\begin{aligned} F(p) &= \frac{p^4}{0.8 \times p^4 + 192} \\ p &= \sqrt{\frac{8\pi f a_p}{10^4 R_{dc}}} \end{aligned}\right\} \tag{2.6}$$

Du and Watson have presented an algorithm and a practical example of HSE modelling and performance [Du *et al.* 1996]. They have used oriented graph method to establish relationship among the line currents and bus voltages. Further, they have identified and classified buses in such a way that they could eventually convert the under-determined system to a determined one.

2.2.2 Distribution Line

Distribution lines are modelled using the compound component concept [Arrillaga *et al.* 2000]. This may or may not include the cases of mutual coupling. Hence, the element admittance matrix representation for the short line between the busbars 'i' and 'k' in Fig. 2.1 can be represented by (2.7) if there is no mutual coupling [Arrillaga and Watson 2001].

$$\begin{bmatrix}[I_i]\\ [I_k]\end{bmatrix} = \begin{bmatrix}[Z]^{-1}+[\frac{Y}{2}] & -[Z]^{-1}\\ -[Z]^{-1} & [Z]^{-1}+[\frac{Y}{2}]\end{bmatrix}\begin{bmatrix}[V_i]\\ [V_k]\end{bmatrix} \tag{2.7}$$

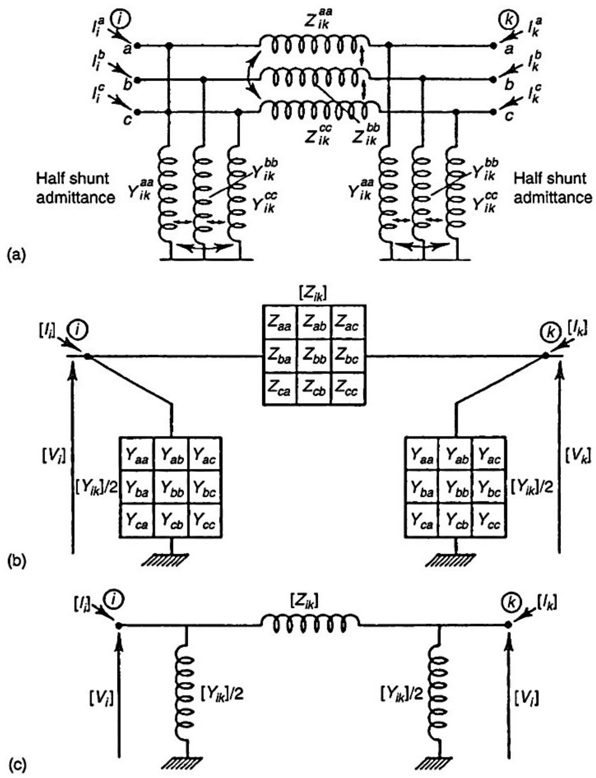


Figure 2.1: Three phase model of transmission line [Arrillaga and Watson 2001]

$$\begin{bmatrix}I_A\\ I_B\\ I_C\\ I_D\end{bmatrix} = \begin{bmatrix}Y_{11}+Y_{33} & Y_{12}+Y_{34} & -Y_{11}-Y_{12} & \\ Y_{12}^T+Y_{34}^T & Y_{22}+Y_{44} & -Y_{12}^T & -Y_{22}\\ -Y_{11} & -Y_{12} & Y_{11}+Y_{55} & Y_{11}+Y_{56}\\ -Y_{22}^T & -Y_{22} & Y_{12}^T+Y_{56}^T & Y_{22}+Y_{66}\end{bmatrix}\begin{bmatrix}V_A\\ V_B\\ V_C\\ V_D\end{bmatrix} \tag{2.8}$$

It can take the form as in (2.8) if mutual coupling is also included [Arrillaga and Watson 2001]. This can be succinctly expressed as in (2.9) where the symbols have usual meanings as mentioned in [Arrillaga 1997].

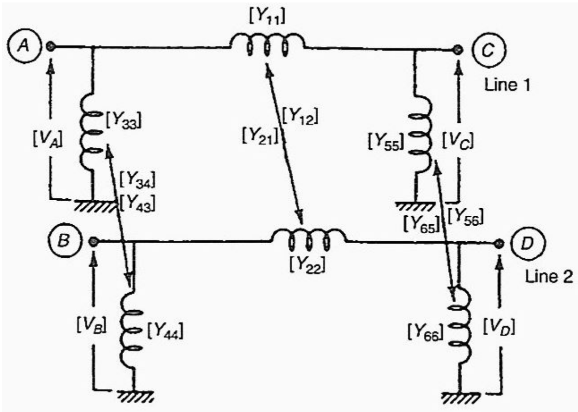


Figure 2.2: Compound admittance model of three phase lines with mutual coupling [Arrillaga and Watson 2001]

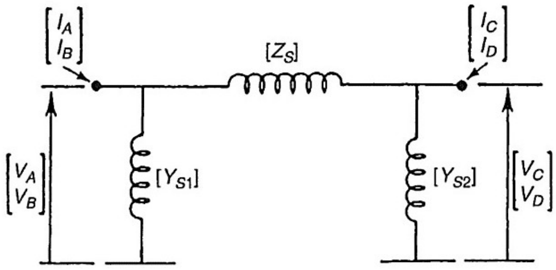


Figure 2.3: Simple representation of three phase lines [Arrillaga and Watson 2001]

$$\begin{bmatrix} \begin{bmatrix} I_A \\ I_B \end{bmatrix} \\ \begin{bmatrix} I_C \\ I_D \end{bmatrix} \end{bmatrix} = \begin{bmatrix} [Z_S]^{-1} + [\frac{Y_{S1}}{2}] & -[Z_S]^{-1} \\ \\ -[Z_S]^{-1} & [Z_S]^{-1} + [\frac{Y_{S1}}{2}] \end{bmatrix} \begin{bmatrix} \begin{bmatrix} V_A \\ V_B \end{bmatrix} \\ \begin{bmatrix} V_C \\ V_D \end{bmatrix} \end{bmatrix} \tag{2.9}$$

While lumped PI model is alright for shorter lines and lower frequency range, it will not be the exact representative of the real behaviour of the longer lines or the shorter lines for harmonic penetration studies. Hence, the equivalent PI model of distributed line parameters need to be used for harmonic studies. Therefore, this requires the evaluation of correction factors on series and shunt components of equivalent PI circuit [Bowman and McNamee 1964].

2.2.3 Shunt Elements

Shunt elements are represented as shown in Fig. 2.4. The admittance matrix is usually diagonal and consists of self-admittance of the particular bus [Arrillaga 1997].

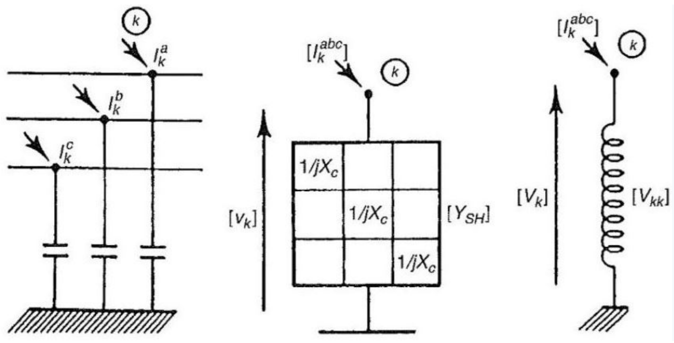


Figure 2.4: Model for shunt elements [Arrillaga and Watson 2001]

2.2.4 Series Elements

Series elements are represented as in Fig. 2.5. They are connected directly between two buses. When uncoupled, the admittance matrix of such elements are diagonal [Arrillaga 1997].

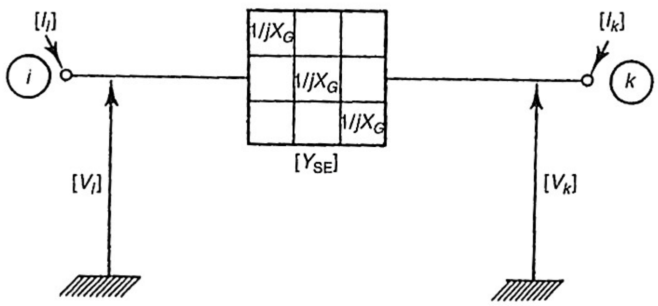


Figure 2.5: Model for series elements [Arrillaga and Watson 2001]

2.2.5 Transformers

Primitive admittance matrix for two-winding three-phase-transformer can, in general, be represented by (2.10) [Arrillaga 1997]. This equation should be modified in accordance with the change brought in the connection and taps of the transformer.

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \\ I_5 \\ I_6 \end{bmatrix} = \begin{bmatrix} Y_p & Y_m' & Y_m' & -Y_m & Y_m'' & Y_m'' \\ Y_m' & Y_p & Y_m' & Y_m'' & -Y_m & Y_m'' \\ Y_m' & Y_m' & Y_p & Y_m'' & Y_m'' & -Y_m \\ -Y_m & Y_m'' & Y_m'' & Y_s & Y_m''' & Y_m''' \\ Y_m'' & -Y_m & Y_m'' & Y_m''' & Y_s & Y_m''' \\ Y_m'' & Y_m'' & -Y_m & Y_m''' & Y_m''' & Y_s \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \\ V_5 \\ V_6 \end{bmatrix} \tag{2.10}$$

Where Y_m' is the mutual admittance between primary coils.

Y''_m is the mutual admittance between primary and secondary coils.
Y'''_m is the mutual admittance between secondary coils.

2.2.6 Load Modelling

A very general representation of load is not available for harmonic analysis. Three basic representations for loads have been suggested [Arrillaga and Watson 2001].

1. Passive loads can be represented by

$$Z_r(\omega) = R_r\sqrt{h} + jX_r h \tag{2.11}$$

   Where R_r =load resistance at the fundamental frequency,
   X_r=load resistance at the fundamental frequency,
   h=harmonic orders (ω/ω_1)
   In harmonic studies involving transmission lines, the loads are treated as equivalent parts of the distribution network, specified by active and reactive power consumptions, and a parallel model is used.

$$Y_L(\omega) = \frac{1}{R_p} + j\frac{1}{hX_p} \tag{2.12}$$

   Where R_p=load resistance at the fundamental frequency
   X_p=load reactance at the fundamental frequency
   h=harmonic orders (ω/ω_1)

$$X_P = \frac{V^2}{Q} \tag{2.13}$$

$$R = \frac{V^2}{P} \tag{2.14}$$

$$X = j\frac{V^2}{(0.1h + 0.9)Q} \tag{2.15}$$

$$X_P = \frac{V^2}{(0.1h + 0.9)P} \tag{2.16}$$

2. Various motive loads have been suggested using combination of resistive and reactive equivalents which are more closely defined by the system boundaries.
3. Power electronic loads do not have a fixed R, L, C configuration and do not possess any linear harmonic equivalent model. They are usually kept open circuited during evaluation of harmonic impedances but need to be included when the power ratings are relatively high [Arrillaga and Watson 2001].

2.3 HARMONIC STATE ESTIMATION TECHNIQUES

2.3.1 The HSE Problem

The task of HSE is to generate the best estimate of voltage and current harmonics across the system from limited measured harmonic data, corrupted with measurement noise [Watson 2010]. Harmonic flow in the system can be determined from knowledge of the system states. System states are those variables which completely specify the system [Watson 2010]. Some of these variables are measured while others need to be evaluated using mathematical methods. System variables are related to each other and the system parameters through basic circuit laws. This relationship can be represented using nodal equations in the form of:

$$[Y][V] = [I] \tag{2.17}$$

where Y = system admittance matrix (complex)
V = Bus voltages (phasor)
I = Injected Currents (phasor)
The system equations in (2.17) can now be rearranged into the measurement matrix equation (2.18).

$$[Z] = [H][X] \tag{2.18}$$

These new equations consist of elements from one or more of the matrices 'Y', 'V' or 'I'. 'X' is the vector of state variables and includes all the unknown quantities. The measurement matrix 'H' includes known parameters like system parameters while the measurement vector 'Z' includes the measured values. Hence, the aim is to find 'X' which can be obtained by

$$[X] = [H]^{-1}[Z] \tag{2.19}$$

The above equations give unique solutions only if the inverse of 'H' can be calculated. This is possible if 'H' has equal number of rows and columns. However, there can be different scenarios in reality.

- More number of rows than the columns: This scenario appears when there are more number of measurements causing the known variables to exceed the number of system equations. This is called an over-determined case.
- More number of columns than the rows: This scenario appears when there are less number of measurements causing the number of system equations to exceed the known variables. This is called an under-determined case.

If all the state variables can be evaluated using the information of 'Z' and 'H', the system is said to be observable. If the state variables can be estimated partially, the system is said to be partially observable; otherwise, it is said to be 'unobservable'.
In reality, only a few measurements of harmonics are financially viable. This causes the system to

be under-determined. Hence, it is required to have mathematical techniques which can estimate solutions for system states of over-determined cases as well as under-determined ones. Such techniques used for the estimation of harmonics throughout the systems are harmonic state estimation techniques. In addition, the measurements can also have errors, and are, therefore, included in (2.18). The new equation to be used will now be (2.20).

$$[Z] = [H][X] + [E] \tag{2.20}$$

The measurement error vector 'E' is assumed to be made of independent random variables with Gaussian distribution.

2.3.2 Existing HSE Techniques

2.3.2.1 The Basic Method

The basic method is applicable for over-determined and determined cases. Exact solution to (2.20) for over-determined cases is not possible to achieve. A solution needs to be sought which has least squared errors. The error 'E' can be obtained from (2.21)

$$[E] = [Z] - [H][X] \tag{2.21}$$

In other words, the aim is to minimize the squared error 'J' defined by

$$J = E^T E \tag{2.22}$$

Taking its derivative and equating to zero gives

$$\frac{dE}{dX} = \frac{d(HX - Z)^T (HX - Z)}{dX} = 0 \tag{2.23}$$

$$[X] = [H^T H]^{-1}[H^T][Z] \tag{2.24}$$

This solution will provide the required estimates [Arrillaga *et al.* 2000].

2.3.2.2 Weighted Least Square (WLS)

Weighted Least Square uses the weights, based on the standard deviation of the measurement error. They are arranged in a diagonal matrix 'R'. Matrix 'R' is a covariance matrix. This allows applying higher weightings to measurements that are known to be more accurate. However, the covariances of the measurements are usually unknown. Hence, 'R' is replaced by the identity matrix in such cases. Hence,

$$J(X) = [HX - Z]^T R^{-1} [HX - Z] \tag{2.25}$$

Equation to be solved for minimum 'J' is,

$$[H^T R^{-1} H]X = H^T R^{-1} Z \tag{2.26}$$

For over-determined systems, 'H' will have more rows than columns but $H^T R^{-1} H$ is square, and therefore, its solutions are achievable using techniques like LU decomposition and back-substitution, Cholesky decomposition or Gauss-Jordan elimination [Yu and Watson 2004].

2.3.2.3 Singular Value Decomposition (SVD)

Although the standard traditional methods used to get solutions for over-determined systems are helpful, they are of not much help in HSE. This is because the systems for HSE are usually under-determined. Limited number of measuring equipment are used for measurement of harmonics because of their high cost. This may result in making the system unobservable. Hence, a robust method is required to get solutions for such under-determined systems. SVD is a technique to deal with such systems. It can help in getting the solutions for singular or near singular (ill-conditioned) matrices. It inherently possesses the least square solutions for the state variables and provides a null space vector for every singularity. The null space vector is used to determine the observable and non-observable islands within the system [Yu and Watson 2004]. In this technique, any matrix can be decomposed into three significant matrices, $[U]$, $[W]$, $[V]$ such that

$$[A] = [U][W][V]^T \tag{2.27}$$

Where $[U]$ is column-orthogonal while $[V]$ is both square and row-orthogonal. SVD creates special orthonormal bases for the null space and range of a matrix. $[U]$ is the orthonormal eigenvector matrix of $[A][A]^T$, and the columns of $[V]$ are orthonormal eigenvectors of $[A]^T[A]$. $[W][W]^T$ is a diagonal matrix of eigenvalues in descending order. $[W]$ is the diagonal matrix of the singular values which are the square roots of the eigenvalues of $[A]^T[A]$. The columns of $[U]$ corresponding to the non-zero singular values are the orthonormal set of basis vectors that span the range of $[A]$. The columns of $[V]$ corresponding to the zero singular values are the orthonormal sets of basis vectors that span the null space.

$$[X] = [A]^+[B] \tag{2.28}$$

$$[A]^+ = [V][W]^{-1}[U]^T \tag{2.29}$$

$$[X] = [X_p] + \sum_{i=1}^{NoS} k_i[X_{ni}] \tag{2.30}$$

The particular solution with least square error for the state variables is then obtained from the pseudo-inverse ($[A^+]$) of $[A]$ using equations from (2.28) to (2.30) [Arrillaga *et al.* 2000].

Here, $[X_p]$ is particular solution, k_i is a constant, and $[X_{ni}]$ is null space vector. 'NoS' is the number of zero singular values. Null space vector is given by the columns of $[V]$ corresponding to the zero singular values. If there are zeros in the rows of null-space vectors, the corresponding variables in $[X]$ will have unique solutions, and are therefore, considered observable. Otherwise, they will be unobservable. Yu and Watson have discussed and shown the use of SVD for harmonic systems for partially observable systems [Yu and Watson 2004]. Another outstanding feature of SVD is that it does not require observability analysis before estimation ([Arrillaga *et al.* 2000],[Liao 2007],[Yu and Watson 2004]), as it can identify partially observable systems. The $[W]$ and $[V]$ matrices are used to analyse observability. The normal equation approach works well with the overdetermined systems but cannot solve the underdetermined systems. However, due to the lack of PQ monitors, the systems that HSE is applied to are the underdetermined systems, therefore SVD is used.

2.3.3 Sources of Errors in Harmonic State Estimation

Accuracy in estimation only can make any harmonic state estimation technique more relevant. Only a few works ([Yu *et al.* 2005], [Castillo *et al.* 2012]) contributing to the analysis of errors in harmonic state estimation has been found. There can be three major sources of errors viz. modelling, estimation techniques and the measurements during estimation. Errors can be 'Gross errors' or 'Measurement noise' errors [Yu *et al.* 2005]. Gross errors arise due to equipment failure or data communication failure and measurement noise errors arise, most likely, in all the measurements.

2.3.3.1 Modelling

Power transmission system is not perfectly symmetric [Meliopoulos *et al.* 2001]. Impedance of all the phases are not exactly equal. Further, line parameters may change with environmental conditions. Therefore, erroneous models will eventually lead to errors during estimation. They have studied the effect of the transmission line models and measurement schemes by computing the confidence level, and the sensitivity indices. Kanao et al. [Kanao *et al.* 2005] have used measured data to confirm line parameters and avoid modelling errors. Harmonic state estimation of power system with uncertain network parameters varied within known bounds was analysed in [Rakpenthai *et al.* 2013]. Temperature of transmission line was considered as the parameter with uncertainty. HSE problem as a parametric interval linear system of equations was formulated. The solutions thus obtained gave the outer bounds of the state variables. A technique was proposed to adjust the weights used in WLS criterion considering both measurement and network parameter variations.

2.3.3.2 Estimation Technique

Errors could be introduced because of the estimation methods used [Arrillaga *et al.* 2000]. The weighted least square has been less promising for HSE. So far, the SVD method, has been considered more efficient [Yu and Watson 2004],[Nguyen *et al.* 2010], [Moghadasian *et al.* 2010]. Efficiency of different estimation methods in estimating the states needs to be investigated. Castillo, Bretas and London [Castillo *et al.* 2012] investigated on parameter errors and gross errors. An approach was proposed for the detection, identification and correction of network branch parameter even if they occur simultaneously.

2.3.3.3 Measurement Error

Erroneous measurement has been the major focus of bad data analysis in previous works [Arrillaga *et al.* 2000]. Major works have been observed for detection and removal of fundamental frequency than for harmonics [Yu *et al.* 2005]. The measurement errors that are likely to occur need to be detected and removed. Measurement error has been modelled as random noise generated with Gaussian distribution [Du *et al.* 1999]. Analytical methods for estimation should thus be able to detect and remove such measurement errors to increase the accuracy of estimation.

Chapter 3

ALGORITHM AND VALIDATION

This chapter presents details of the admittance matrix model as well as evaluation of the harmonic estimates. Examples are used to demonstrate the accuracy of estimation to present evidence for the validation of the algorithm which will be applied to a practical system later in Chapter 4.

3.1 DEVELOPMENT OF SYSTEM ADMITTANCE MATRIX MODEL

A system nodal admittance matrix model of a distribution system feeder is constructed by aggregating the compound admittances of all the elements in the distribution system [Arrillaga and Watson 2001]. The sum of self-admittance matrices of all the components connected to a particular busbar gives the self-admittance at that busbar. The self-admittance sub-matrices of the components already incorporate the admittance coupling the terminal busbars (for transmission lines, cables and transformers). Likewise, the sum of mutual admittance sub-matrices of the components between any two busbars represents the mutual admittance matrix between those two busbars. Sections 3.1.1 to 3.1.3 discuss the admittance matrix for the components of the test feeder.

3.1.1 Harmonic Source Model

Loads that are harmonic sources are considered as fixed harmonic current sources. Hence, measured load currents are regarded as sources. The loads are unbalanced, and their values are randomly selected for simulation purposes. These harmonic currents are considered as injected currents into the system.

3.1.2 Transformer Model

Transformer is modelled by constructing a nodal admittance matrix. The primitive admittance matrix model is converted into a 6×6 nodal admittance matrix which is of the form as presented in (3.1). The model presented in (3.1) is for a Dyn11 transformer.

$$\begin{bmatrix} 2Y_l/3 & -Y_l/3 & -Y_l/3 & -Y_d & 0 & Y_d \\ -Y_l/3 & 2Y_l/3 & -Y_l/3 & Y_d & -Y_d & 0 \\ -Y_l/3 & -Y_l/3 & 2Y_l/3 & 0 & Y_d & -Y_d \\ -Y_d & Y_d & 0 & Y_l/a^2 & 0 & 0 \\ 0 & -Y_d & Y_d & 0 & Y_l/a^2 & 0 \\ Y_d & 0 & -Y_d & 0 & 0 & Y_l/a^2 \end{bmatrix} \tag{3.1}$$

where Y_l = leakage admittance , a=Tap Ratio, and Y_d=$Y_l/a\sqrt{3}$.

3.1.3 Distribution Line Model

The distribution line model is represented by the 6×6 compound nodal admittance matrix built from the series impedance and shunt admittance of the lines. The physical parameters and other constructional details of cables and overhead lines used in the network are obtained from the data sheet of the cables and lines. Equations described in [Nasser 2008] are used for the evaluation of the resistances, self-impedance and the mutual impedances using the physical parameters of the conductors and geometry. The AC resistance of the cable is obtained using (2.1), (2.2), (2.3) and (2.5). The self-impedance of a core conductor with earth return is evaluated from (3.2).

$$\left.\begin{aligned} Z_{cc} = R_{c(ac)} + \pi^2 10^{-4} f + j4\pi 10^{-4} f \Big[\frac{\mu_c}{4} \times f_n(r_{oc}, r_{ic}) + log_e\Big(\frac{D_{erc}}{r_{oc}}\Big)\Big]\Omega/km \\ f_n(r_{oc}, r_{ic}) = 1 - \frac{2r_{ic}^2}{(r_{oc}^2 - r_{ic}^2)} + \frac{4r_{ic}^4}{(r_{oc}^2 - r_{ic}^2)} log_e\Big(\frac{r_{oc}}{r_{ic}}\Big) \end{aligned}\right\} \tag{3.2}$$

Similarly, the self-impedance of sheath with earth return is evaluated from (3.3).

$$\left.\begin{aligned} Z_{ss} = R_{s(ac)} + \pi^2 10^{-4} f + j4\pi 10^{-4} f \Big[\frac{\mu_s}{4} \times f_n(r_{os}, r_{is}) + log_e\Big(\frac{D_{erc}}{r_{os}}\Big)\Big]\Omega/km \\ f_n(r_{os}, r_{is}) = 1 - \frac{2r_{is}^2}{(r_{os}^2 - r_{is}^2)} + \frac{4r_{is}^4}{(r_{os}^2 - r_{is}^2)} log_e\Big(\frac{r_{os}}{r_{is}}\Big) \end{aligned}\right\} \tag{3.3}$$

Likewise, the self-impedance of armour with earth return is evaluated from (3.4)

$$\left.\begin{aligned} Z_{aa} = R_{a(ac)} + \pi^2 10^{-4} f + j4\pi 10^{-4} f \Big[\frac{\mu_a}{4} \times f_n(r_{oa}, r_{ia}) + log_e\Big(\frac{D_{erc}}{r_{oa}}\Big)\Big]\Omega/km \\ f_n(r_{oa}, r_{ia}) = 1 - \frac{2r_{ia}^2}{(r_{oa}^2 - r_{ia}^2)} + \frac{4r_{ia}^4}{(r_{oa}^2 - r_{ia}^2)} log_e\Big(\frac{r_{oa}}{r_{ia}}\Big) \end{aligned}\right\} \tag{3.4}$$

The mutual impedance between a core or a sheath or an armour 'i' and another core or sheath or armour 'j', with earth return is evaluated from (3.5)

$$Z_{mm} = \pi^2 10^{-4} f + j4\pi 10^{-4} f \times log_e\Big(\frac{D_{erc}}{S_{ij}}\Big)\Omega/km \tag{3.5}$$

where 'S_{ij}' is the distance between the centres of cables 'i' and 'j' if the conductors belong to different cables. If the conductors belong to the same cable, 'S_{ij}' is the geometric mean distance between the two conductors [Nasser 2008]. Using these equations for self-impedances and mutual impedances, the impedance matrix of the format in (3.6) is formed for each line depending upon its construction.

$$\begin{bmatrix} Z_{c1c1} & Z_{c1c2} & Z_{c1c3} & Z_{c1s1} & Z_{c1s2} & Z_{c1s3} & Z_{c1A} \\ Z_{c2c1} & Z_{c2c2} & Z_{c2c3} & Z_{c2s1} & Z_{c2s2} & Z_{c2s3} & Z_{c2A} \\ Z_{c3c1} & Z_{c3c2} & Z_{c3c3} & Z_{c3s1} & Z_{c3s2} & Z_{c3s3} & Z_{c3A} \\ Z_{s1c1} & Z_{s1c2} & Z_{s1c3} & Z_{s1s1} & Z_{s1s2} & Z_{s1s3} & Z_{s1A} \\ Z_{s2c1} & Z_{s2c2} & Z_{s2c3} & Z_{s2s1} & Z_{s2s2} & Z_{s2s3} & Z_{s2A} \\ Z_{s3c1} & Z_{s3c2} & Z_{s3c3} & Z_{s3s1} & Z_{s3s2} & Z_{s3s3} & Z_{s3A} \\ Z_{Ac1} & Z_{Ac2} & Z_{Ac3} & Z_{As1} & Z_{As2} & Z_{As3} & Z_{AA} \end{bmatrix} \tag{3.6}$$

Since the armour and the sheaths in the cables used in the feeder are solidly bonded together, the rows and columns corresponding to the sheath and the armour in the matrix shown in (3.6) can be reduced. The reduction is performed by applying Kron's reduction method which results in a new 3×3 series impedance matrix. The shunt admittance model of the lines is constructed using the susceptance obtained using (3.7). This model is represented by a 3×3 admittance matrix.

$$B_{cs} = \frac{2\pi f \times 2\pi\varepsilon_{\mathrm{r}} \times 8.85 \times 10^{-12}}{log(\frac{R_{out}}{R_{in}})} \tag{3.7}$$

Once the three phase series impedance and the shunt admittance matrix models are formed, the 6×6 compound nodal admittance matrix model of the lines are constructed using (3.8).

$$\begin{bmatrix} [Z_{epm}]^{-1} + [\frac{Y_{epm}}{2}] & -[Z_{epm}]^{-1} \\ -[Z_{epm}]^{-1} & [Z_{epm}]^{-1} + [\frac{Y_{epm}}{2}] \end{bmatrix} \tag{3.8}$$

The matrices Z_{epm} and Y_{epm} used in (3.8) are the equivalent π series impedance and shunt admittance of the π model [Bowman and McNamee 1964] and [Arrillaga and Watson 2001]. They are calculated using (3.9), (3.10) and (3.11) where l is the distribution line length and [M] is the matrix of normalized eigenvectors and γ_j is the j^{th} eigenvalue for $j/3$ mutually coupled circuits.

$$[Z_{epm}] = l[Z'][M]\left[\frac{sinh(\gamma l)}{\gamma l}\right][M]^{-1} \tag{3.9}$$

$$\frac{1}{2}[Y_{epm}] = \frac{1}{2}l[M]\left[\frac{tanh(\gamma l/2)}{\gamma l/2}\right][M]^{-1}[Y'] \tag{3.10}$$

Since the *sinh* and *tanh* of the matrix need to be calculated in (3.9) and (3.10) but cannot be obtained directly, modal analysis [Wedepohl and Wasley 1966] is used to determine these values.

Calculation of *sinh* is performed using (3.11). Similary, the *tanh* of the matrix is also obtained.

$$\left[\frac{sinh(\gamma l)}{\gamma l}\right] = \begin{bmatrix} \frac{sinh(\gamma_1 l)}{\gamma_1 l} & 0 & \dots & 0 \\ 0 & \frac{sinh(\gamma_2 l)}{\gamma_2 l} & \dots & 0 \\ \vdots & \vdots & & \vdots \\ 0 & 0 & \dots & \frac{sinh(\gamma_j l)}{\gamma_j l} \end{bmatrix} \tag{3.11}$$

3.1.4 Load Model

The harmonic loads are modelled as current sources. Since the load currents are directly measured by the PQ monitors at the low voltage terminals of all the transformers, they are considered as injected currents. The sign convention for current flowing into the node is positive. Therefore, these harmonic load currents will have a negative sign when considered as injected currents as they are moving from nodes to the loads. For the simulations using OpenDSS also, the harmonics to be generated are exclusively defined and ensured that the order of harmonics of interest are included for analysis.

3.2 HARMONIC STATE ESTIMATION ALGORITHM

In order to perform HSE, firstly a measurement equation (3.12) needs to be formed [Yu and Watson 2004].

$$[Z] = [H][X] \tag{3.12}$$

Formation of measurement equation is discussed next.

3.2.1 Formation of Measurement Equation

- Measurement Vector ([Z])
  The known phase voltages at all the buses are taken as a column vector $[V_k]$. The known phase currents at all the buses are also taken as a column vector $[I_k]$. Hence, the measurement vector will have the form :

$$\left[Z\right] = \begin{bmatrix} [V_k] \\ [I_k] \end{bmatrix} \tag{3.13}$$

- State Vector ([X])
  Column vector of all the phase harmonic voltages are taken as state variables and represented by $[V_x]$ which is [X] in (3.12).
- Measurement Matrix ([H])
  The three-phase system admittance matrix ($[Y_{sys}]$) is partitioned horizontally into two sub-matrices. The upper one, $[Y_{sysH}]$ contains all the elements of the rows of $[Y_{sys}]$ which

pertain to the buses with known injected currents; and the lower one, $[Y_{sysUK}]$ contains all the elements of the rows of $[Y_{sys}]$ which pertain to the buses with unknown injected currents. Matrix $[Y_{sysUK}]$ is excluded but $[Y_{sysH}]$ is included in the measurement matrix. The measurement matrix also includes submatrix $[C_H]$ which relates the measured voltages $[V_k]$ to the state vector $[V_x]$ and contains rows of 0's and 1's such that

$$\begin{bmatrix} V_k \end{bmatrix} = \begin{bmatrix} [C_H][V_x] \end{bmatrix} \tag{3.14}$$

$[C_H]$ has the same number of rows as $[V_k]$ and the same number of columns as $[V_x]$. Thus (3.12) can be written as (3.15).

$$\begin{bmatrix} [V_k] \\ [I_k] \end{bmatrix} = \begin{bmatrix} [C_H] \\ [Y_{sysH}] \end{bmatrix} \begin{bmatrix} V_x \end{bmatrix} \tag{3.15}$$

where

$$\left.\begin{aligned} \begin{bmatrix} Z \end{bmatrix} &= \begin{bmatrix} [V_k] \\ [I_k] \end{bmatrix} \\ \begin{bmatrix} H \end{bmatrix} &= \begin{bmatrix} [C_H] \\ [Y_{sysH}] \end{bmatrix} \\ \begin{bmatrix} X \end{bmatrix} &= \begin{bmatrix} V_x \end{bmatrix} \end{aligned}\right\} \tag{3.16}$$

- **Including new equations to measurement equation**
  New equations can be included to the measurement equation (3.15) if other information is available besides the direct measurements. Hence, (3.15) can be modified to (3.17) if (3.18) is true. This case is discussed further in Section 3.2.2.

$$\begin{bmatrix} [V_k] \\ [I_k] \\ [Z_{add}] \end{bmatrix} = \begin{bmatrix} [C_H] \\ [Y_{sysH}] \\ [H_{add}] \end{bmatrix} \begin{bmatrix} V_x \end{bmatrix} \tag{3.17}$$

$$[Z_{add}] = [H_{add}][V_x] \tag{3.18}$$

3.2.2 Inclusion of New Rows to the Measurement Equation

Since the transformers used are of Dyn11 type, the triplen harmonics (which are zero sequence in nature) are trapped in the delta winding. The low voltage side zero sequence currents in all the phases can flow from the neutral to the ground. However, the non-triplen odd harmonics are either positive or negative sequence in nature which means there is no current flowing in the neutral. Hence, the sum of the phase currents on the star side of the delta-star transformers will be zero for these harmonics. Based on this concept, a new equation is formed and added to (3.17) using (3.18). Those buses, which are connected to the low voltage sides of the transformers and have no known low voltage side currents, are selected . For each bus, three equations for phase currents (one equation per phase) are taken. Sum of the currents in three phases, represented by '$[Z_{add}]$', will be zero. The corresponding rows of '$[Y_{sys}]$' are added together columnwise to produce '$[H_{add}]$'. This is also shown by (3.19) and (3.20) for the currents in the n^{th} row in the current injection vector.

$$\begin{bmatrix} I_{na} \\ I_{nb} \\ I_{nc} \end{bmatrix} = \begin{bmatrix} Y_{na1} & Y_{na2} & Y_{na3} & Y_{na4} & .... & ... \\ Y_{nb1} & Y_{nb2} & Y_{nb3} & Y_{nb4} & .... & ... \\ Y_{nc1} & Y_{nc2} & Y_{nc3} & Y_{nc4} & .... & ... \end{bmatrix} \begin{bmatrix} V_x \end{bmatrix} \tag{3.19}$$

$$\begin{bmatrix} I_{na} + I_{nb} + I_{nc} \end{bmatrix} = \begin{bmatrix} [Y_{na1} + Y_{nb1} + Y_{nc1}] & [Y_{na2} + Y_{nb2} + Y_{nc2}] & [.... + ... + ...] & .... \end{bmatrix} \begin{bmatrix} V_x \end{bmatrix} \tag{3.20}$$

If there are more than one transformer, a new row per transformer is added in (3.17) in a similar manner. New rows are included to improve the observability of the state variables.

3.2.3 Application of Singular Value Decomposition Method (SVD)

Once the measurement equation is formed, it can now be solved for the state variable vector ([X]). As discussed in Chapter 2, SVD is very robust in estimating the harmonic state variables. Hence, SVD is applied, and the pseudo-inverse of 'H' is obtained using (2.28), (2.29) and(2.30) which gives the solution for '$[X]$'. The solution obtained using pseudo-inverse has the least square errors.

3.2.4 Observability

Observability of the state variables can be assessed by observing the null-space of the measurement matrix. Once the measurement matrix is decomposed using SVD, the diagonal matrix of singular values '$[W]$' is checked, and the rows containing 'zero' singular values are identified. If there are not any zeros in its diagonal, then the state variables are observable. However, if there are any rows with zero singular values, their row numbers are identified; and the corresponding columns of 'V' are noted which constitute the null-space. Now, the rows of the null-space are checked. If any rows in the null-space have one or more non-zero values, the corresponding state variables will be unobservable. However, if any rows in these columns contain zeros only, then the corresponding state variables will be observable.

3.3 VALIDATION OF THE HSE ALGORITHM

In order to validate the algorithm, HSE is performed on a test feeder simulated in OpenDSS. Some of the OpenDSS results obtained from harmonic penetration study are taken as measured values and passed to the HSE algorithm [Bhujel *et al.* 2017]. The HSE estimates for the unmeasured locations are then compared to the simulation results. The detailed description of the test system and the measurement locations are presented in the following sections. Finally, the algorithm is validated by comparing the estimates and the simulated values.

3.3.1 Test Feeder

The test feeder considered for the validation of the algorithm is a medium voltage (11 kV) three-phase distribution system feeder named 'Massey' in Hastings, New Zealand. It is typical of many other feeders in the distribution network of Unison Networks Limited. This feeder has twenty-one buses and seven step-down (11/0.415 kV) transformers feeding the consumers. Its single-line diagram is shown in Fig. 3.1.

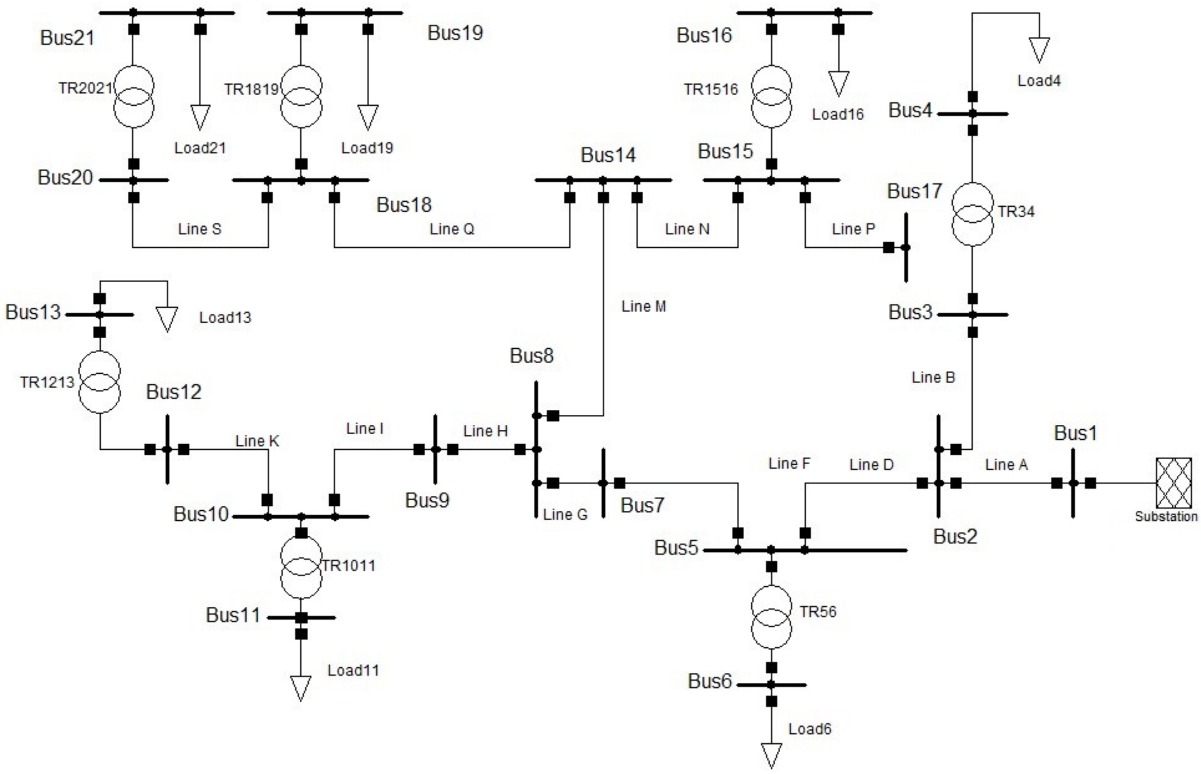


Figure 3.1: Single-line Diagram of Distribution System

3.3.1.1 Transformers

The details of the transformers used are presented in Table 3.1. The transformers are two-winding three-phase with delta-star (Dyn11) connection and have tap-ratio '1' for simplicity.

Table 3.1: Transformer Details

Name	KVA	HV Rating (kV)	LV Rating (kV)	LoadLoss (%)	No Load Loss (%)
TR34	300	11	0.415	1.43	0.07
TR56	300	11	0.415	1.43	0.07
TR1011	300	11	0.415	1.43	0.07
TR1213	100	11	0.415	1.385	0.222
TR1516	300	11	0.415	1.0413	0.1487
TR1819	300	11	0.415	1.43	0.07
TR2021	200	11	0.415	1.43	0.105

3.3.1.2 Distribution Lines

The selected distribution feeder comprises of mainly the three-phase three-core cables and a short overhead line. The distribution line details are given in Table 3.2 and Table 3.3.

Table 3.2: Distribution Line Details

Name	Type	Length (m)	Name	Type	Length (m)
A	1	231	K	2	292
B	3	11	M	1	276
D	1	351	N	1	125
F	1	6	P	1	217
G	5	290	Q	1	69
H	5	101	S	1	258
I	4	2			

The types of lines used and their parameters are shown in Table 3.3.

Table 3.3: Distribution Line Parameters

Type	R1 (Ω/km)	X1 (Ω/km)	R0 (Ω/km)	X0 (Ω/km)	C1 (nF/km)	C0 (nF/km)
1	0.198	0.081	1.342	1.171	165.05	165.05
2	1.043	0.096	1.963	1.824	132.95	132.95
3	1.377	0.132	1.9	1.433	158.48	158.48
4	1.377	0.131	1.901	1.431	159.77	159.77
5	1.27	0.407	1.418	1.691	0	0

3.3.1.3 Harmonic Sources

The currents drawn by the arbitrarily selected three-phase unbalanced load are taken as the harmonic sources. Loads shown in Table 3.4 are the arbitrarily selected loads that are assumed to be connected at the low voltage side of the transformers.

Table 3.4: Loads Connected to the LV Side of Transformers

Bus	Phase 'A'		Phase 'B'		Phase 'C'	
	P(kW)	Q(kVAR)	P(kW)	Q(kVAR)	P(kW)	Q(kVAR)
4	27	0.4	1	0.4	1	0.4
6	43	0.4	1	0.4	4	0.4
11	10	3.4	8	0.4	1	0.4
13	7	0.4	25	4.0	1	0.4
16	8	1.4	3	0.4	5	0.4
19	1	0.4	6	0.4	15	0.4
21	0	0.4	1	0.4	8	0.4

3.3.2 Measurement Locations

The low voltage side of all the transformers i.e. buses '4', '6', '11', '13', '16', '19', '21' and the head of the feeder, Bus '1' in Fig. 3.1 are selected as locations for measuring current and voltage harmonics. This is because the available PQ monitors are not rated for direct connection to the 11 kV system. Hence, most of the actual measurements are made at the 415 V level, and the 11 kV busbar condition determined using them. In practice, direct connection of PQ monitors to busbars '2', '7', '8', '9', '14' is not possible. However, the values at all the buses are known when the simulations are performed using OpenDSS. Hence, comparison is possible at any bus in the system for the simulated case.

3.3.3 Simulation of model

The system components are defined and modelled in OpenDSS which can be interfaced with MATLAB. Therefore, MATLAB is used to drive OpenDSS for the desired harmonic penetration simulations. All the system information and results are extracted in the MATLAB environment. HSE is also performed in MATLAB.

3.3.4 Comparison of Results

The errors between the estimates and the simulated results are displayed in Figs. 3.2 to 3.12 and discussed in Sections 3.3.4.1 and 3.3.4.2. The errors in the magnitude of phase voltages are expressed in per unit (based on transformer nominal voltages), and the errors in the angles in degrees. The plots shown are only for phase 'A' as similar results are obtained for the other phases.

3.3.4.1 Estimation without Measurement Noise

Out of the eight locations (including the feeder head), seven are taken as the locations with measurements available (obtained from simulation) while one location is assumed to have no measurement. Estimation is performed for this 8^{th} location. This procedure is repeated so that each location has a turn at being the unmonitored location and is estimated. This is carried out for all the odd harmonics from '1' to '17'. Differences between the estimated and the simulated values are called as errors hereafter in this chapter. The magnitude of the differences between the phasor of the estimates and the phasor of the simulated values as defined in (3.21) are shown in Fig. 3.2. This shows the extremely good accuracy that can be achieved under ideal conditions i.e. no measurement noise.

$$Error_1 = |\textit{Estimated Value - Simulated Value}| \tag{3.21}$$

The second measure is the difference in the absolute magnitude as defined in (3.22). The error in magnitudes are shown in Fig. 3.3, and the error in angles are shown in Fig. 3.4.

$$Error_2 = |\textit{Estimated Value}| - |\textit{Simulated Value}| \tag{3.22}$$

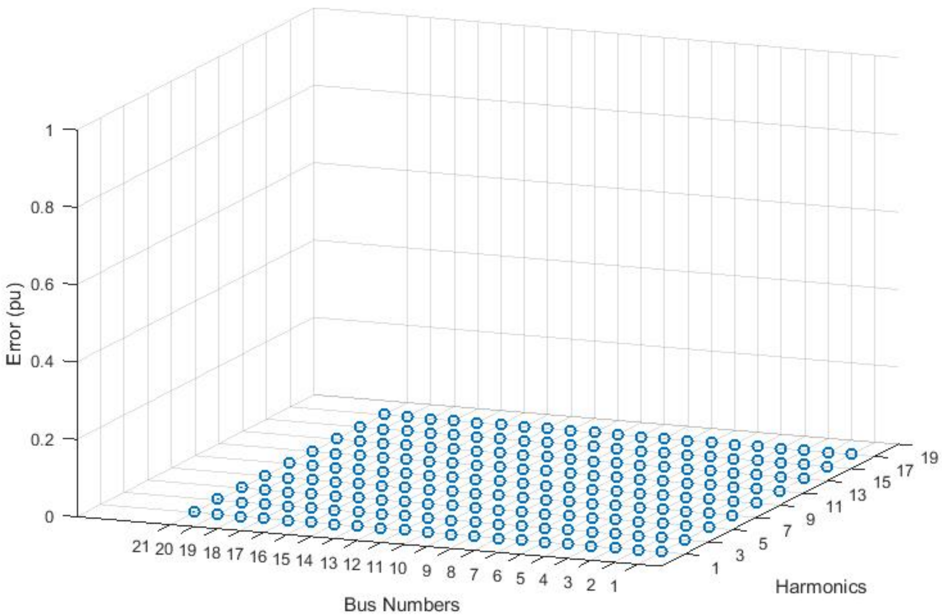


Figure 3.2: Magnitude of the Estimation Errors

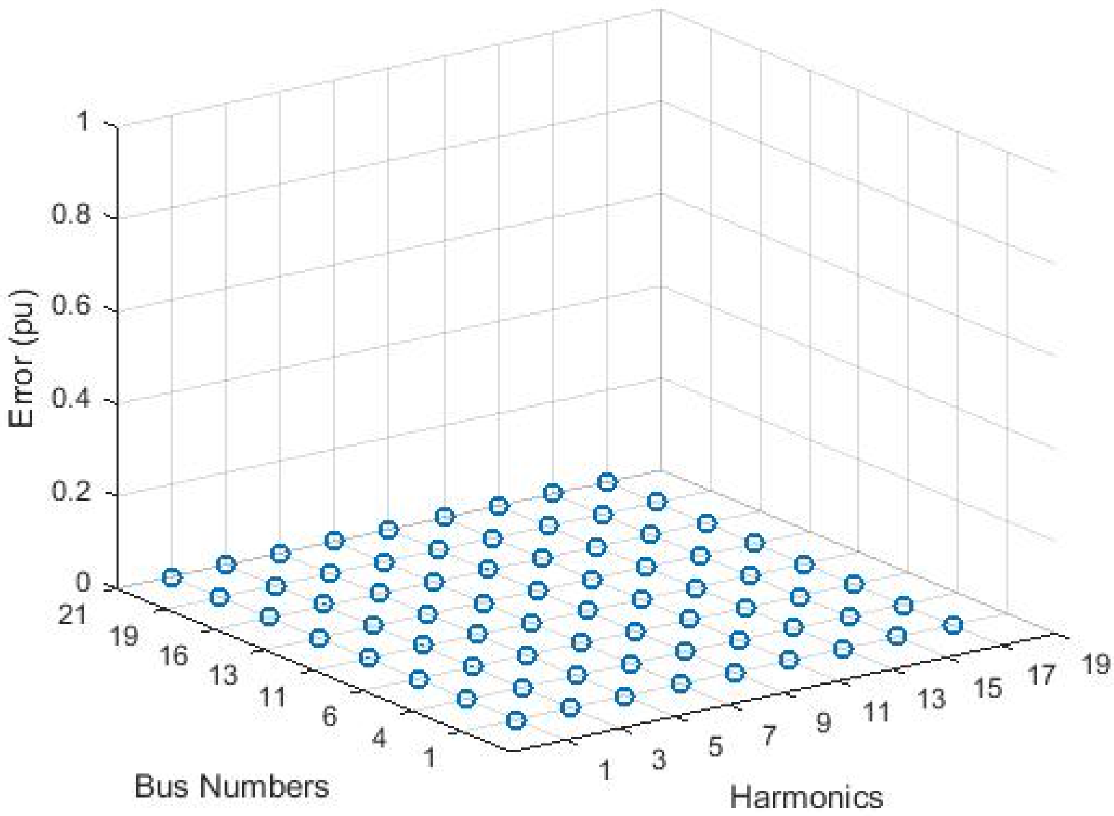


Figure 3.3: Estimation Error in the Magnitudes

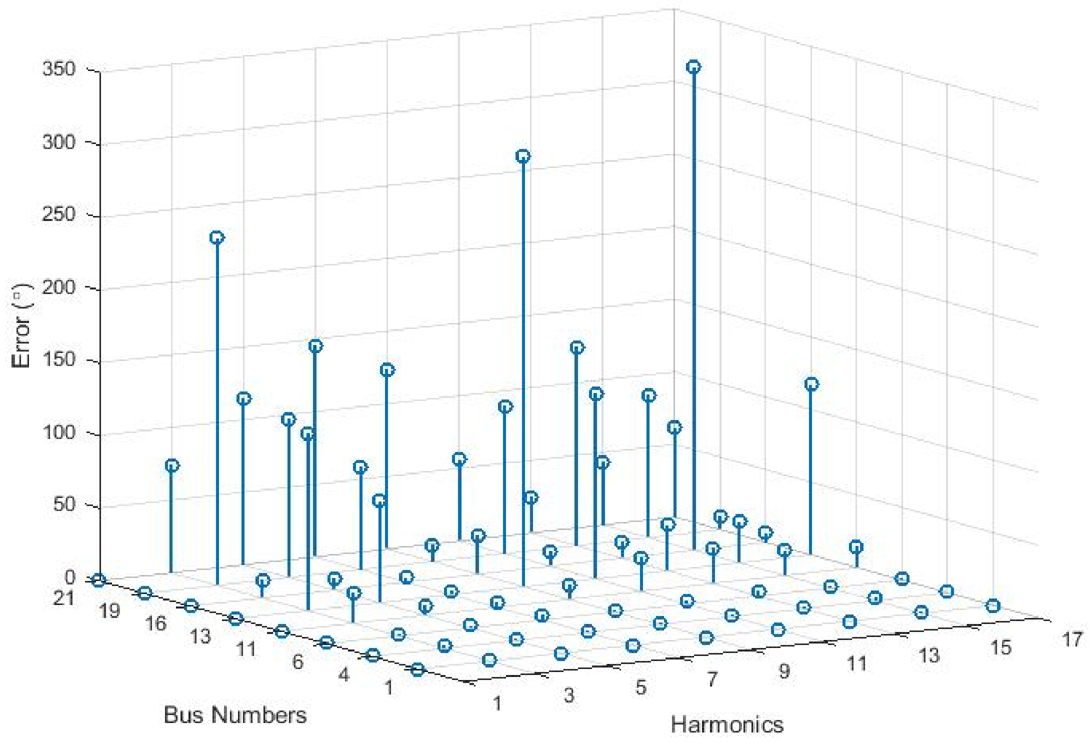


Figure 3.4: Estimation Error in the Angles

These graphs show clearly how small are the errors. Besides these, residues are also calculated using (3.23) which is derived from the measurement equation.

$$[R] = [H][X] - [Z] \tag{3.23}$$

For instance, the values of 'Z', 'HX' and the residues 'R' in (3.23), calculated for one particular case (fifth harmonic and seven measurements considered available), are shown in Appendix A (Table 1). Residues are very close to zero signifying that the estimates are reasonably closer to the correct values which shows the accuracy of HSE. By using OpenDSS to generate the simulated harmonics allows the errors between the estimates and the simulated values to be calculated, something that is difficult with measured data. It also allows this to be performed for busbars which it is not feasible to physically measure.

Table 3.5 shows the absolute value of the differences between the estimated and the simulated values. It also shows the differences between the angle of the estimated and the simulated values. It can be seen that the magnitude of the difference between the estimates and the actual values are very small which means that the differences between the corresponding angles of the estimated and the simulated values are negligible even though they look comparatively greater in values. Some values of the difference in angles in Fig. 3.4 look to be higher in values, for example 300^0. However, it is actually 60^0 only. Similar comparison results performed for different order of harmonics are shown in Appendix A (Table 2).

Table 3.5: Comparison of Magnitudes and Angles without Measurement Noise

Bus	Measurements	Harmonics			
		(1)	(3)	(5)	(7)
1	Absolute Difference (Volts)	0.00	0.00	0.00	0.00
	Difference in Angles (degrees)	0.00	0.15	0.02	0.00
4	Absolute Difference (Volts)	1.48	2.37	0.75	0.53
	Difference in Angles (degrees)	0.35	1.91	-1.22	-0.96
6	Absolute Difference (Volts)	2.29	3.99	1.14	0.79
	Difference in Angles (degrees)	0.54	0.89	1.59	-3.28
11	Absolute Difference (Volts)	0.64	1.51	0.12	0.35
	Difference in Angles (degrees)	0.05	-21.20	6.65	3.06
13	Absolute Difference (Volts)	3.59	8.01	2.21	0.87
	Difference in Angles (degrees)	-0.53	122.18	-69.81	-2.15
16	Absolute Difference (Volts)	0.22	1.34	0.18	0.06
	Difference in Angles (degrees)	0.05	-12.11	8.16	2.98
19	Absolute Difference (Volts)	0.67	1.82	0.36	0.28
	Difference in Angles (degrees)	-0.11	239.03	109.38	70.64
21	Absolute Difference (Volts)	0.60	1.08	0.31	0.28
	Difference in Angles (degrees)	-0.07	74.06	114.49	-145.49

Table 3.6 presents the magnitude and Table 3.7 presents the angle of both the estimates, and the simulated values at the buses where measurements are assumed to be taken. This comparison has been performed for all the odd harmonics from '1' to '17' but only the 1^{st}, 3^{rd}, 5^{th} and 7^{th} are shown in Figs. 3.5 to 3.11. More details are given in Appendix A (Tables 1, 3 and 4).

Table 3.6: Estimated and Simulated Harmonic Magnitudes without Measurement Noise

Bus	Measurements	Magnitude of Harmonics (Volts)			
		(1)	(3)	(5)	(7)
1	Estimated	6350.75	0.47	0.28	0.18
	Simulated	6350.75	0.47	0.28	0.18
4	Estimated	238.88	4.46	1.51	1.04
	Simulated	238.54	6.82	2.26	1.57
6	Estimated	238.27	6.84	2.44	1.68
	Simulated	237.9	10.83	3.58	2.47
11	Estimated	239.15	1.33	0.85	0.27
	Simulated	238.54	2.67	0.9	0.62
13	Estimated	241.36	3.8	2.07	2.1
	Simulated	238.52	5.31	1.76	1.23
16	Estimated	239.06	0.74	0.55	0.44
	Simulated	239.03	2.06	0.7	0.49
19	Estimated	238.86	1.65	0.31	0.29
	Simulated	239.36	0.28	0.11	0.07
21	Estimated	238.84	1.11	0.27	0.26
	Simulated	239.36	0.15	0.07	0.02

The estimates for the harmonic voltage magnitudes are good, and are better for some busbars than others. The triplen (3^{rd} harmonic) shows the greatest discrepancies. The harmonic voltage angles estimation (Table 3.7) shows reasonable accuracy and again some busbars (e.g. busbar '13') shows greater discrepancy. Busbar '6' shows good angle estimates while the magnitude estimates are not as good.

Table 3.7: Estimated and Simulated Harmonic Angles without Measurement Noise

Bus	Measurements	Angle of the Harmonics (degrees)			
		(1)	(3)	(5)	(7)
1	Estimated	-0.01	-24.04	43.78	55.97
	Simulated	-0.01	-24.13	43.76	55.97
4	Estimated	29.26	-11.09	41.22	86.16
	Simulated	28.92	-13.01	42.43	87.12
6	Estimated	28.83	-15.01	39.25	77.22
	Simulated	28.28	-15.9	37.66	80.5
11	Estimated	29.66	-83.26	-31.93	-22.55
	Simulated	29.61	-62.06	-38.58	-25.61
13	Estimated	28.63	103.54	-37.31	71.42
	Simulated	29.15	-18.61	32.5	73.57
16	Estimated	29.73	-46.51	13.61	38.77
	Simulated	29.68	-34.4	5.46	35.79
19	Estimated	29.85	171.91	68.19	44.14
	Simulated	29.95	-67.11	-41.19	-26.5
21	Estimated	29.93	153.91	92.56	23.08
	Simulated	29.99	79.84	-21.93	168.57

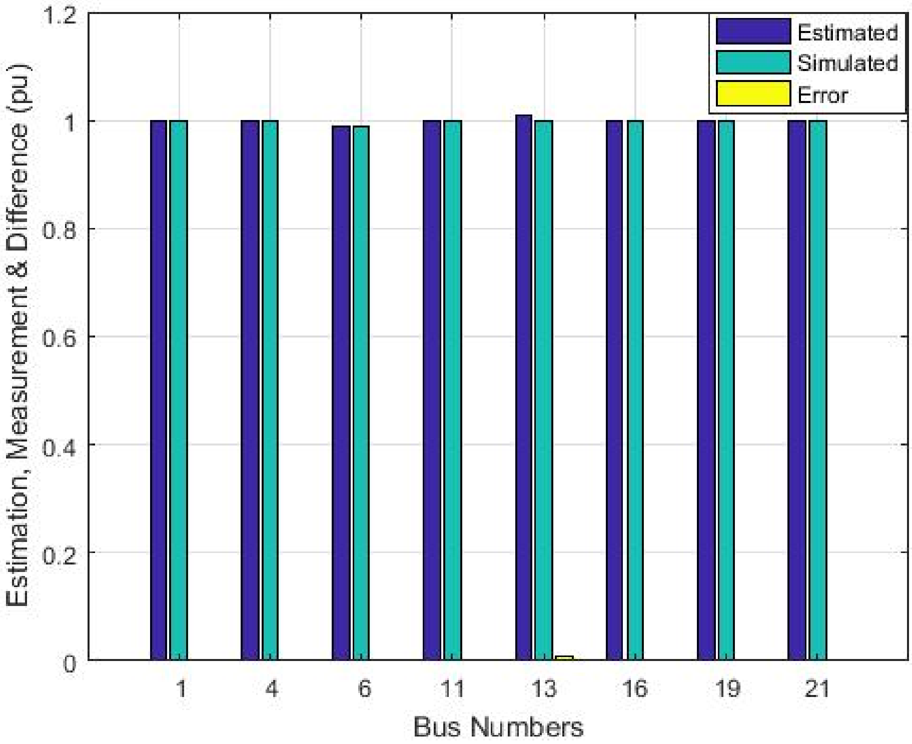


Figure 3.5: Estimation Error in the Magnitudes of the Fundamental

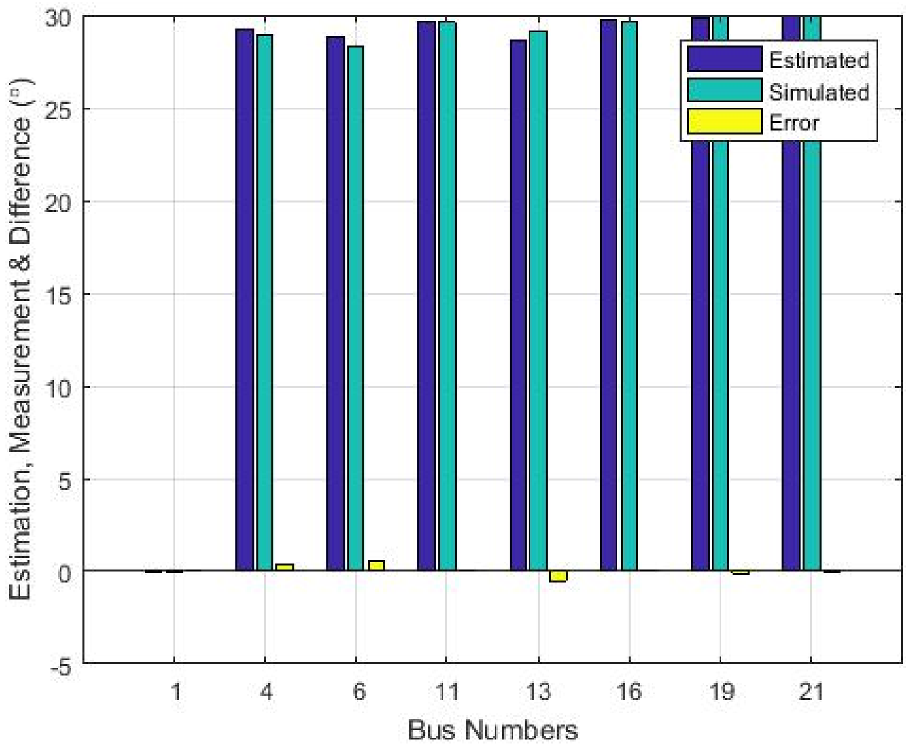


Figure 3.6: Estimation Error in the Angles of the Fundamental

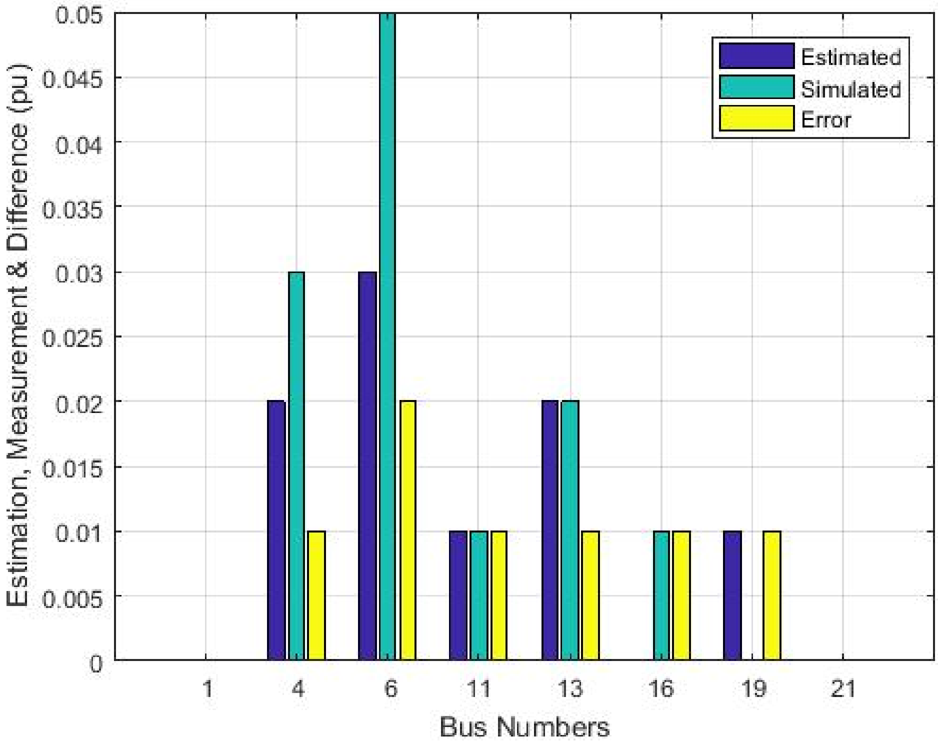


Figure 3.7: Estimation Error in the Magnitudes of the 3^{rd} harmonic

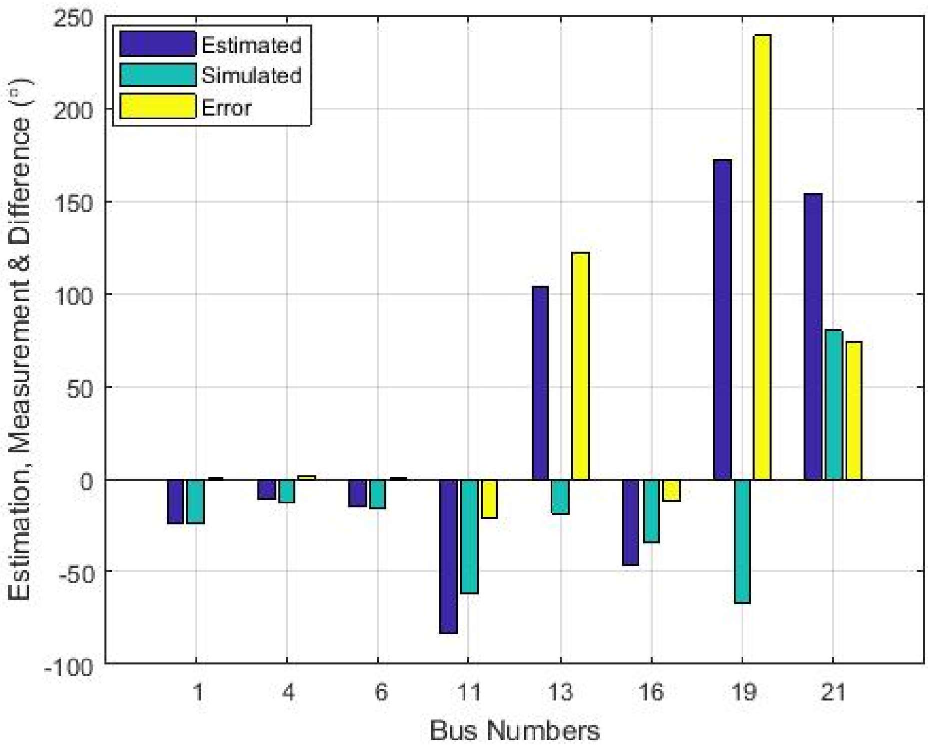


Figure 3.8: Estimation Error in the Angles of the 3^{rd} harmonic

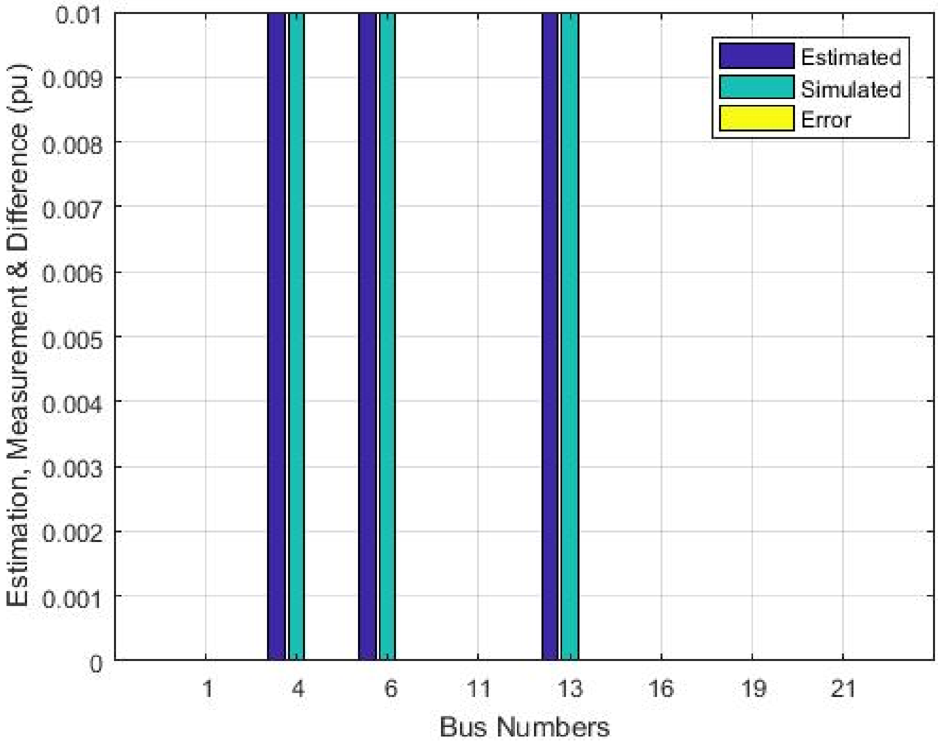


Figure 3.9: Estimation Error in the Magnitudes of the 5^{th} harmonic

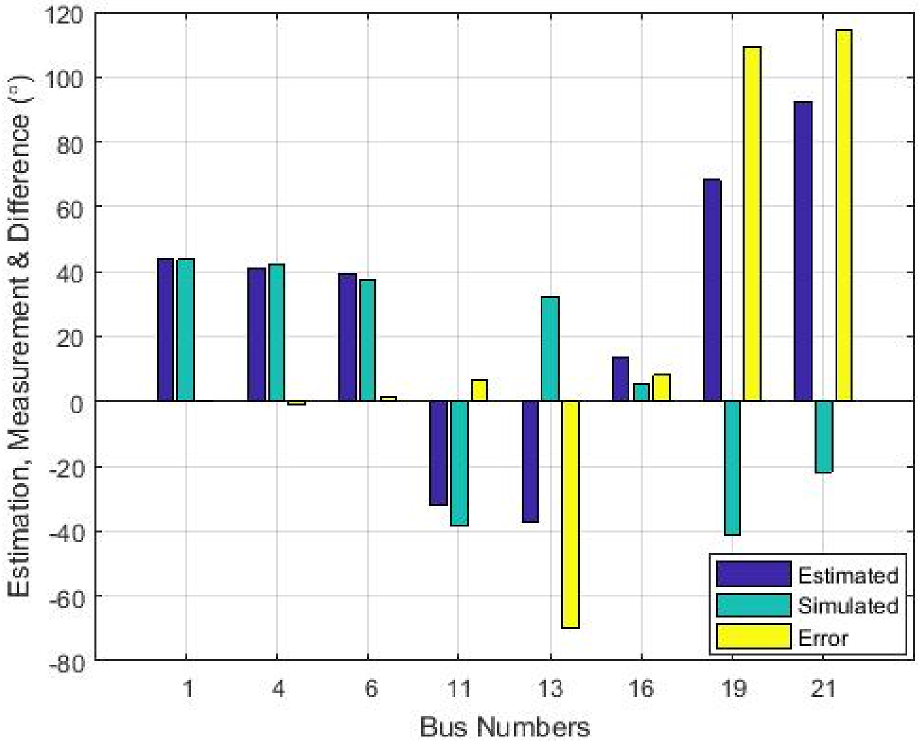


Figure 3.10: Estimation Error in the Angles of the 5^{th} harmonic

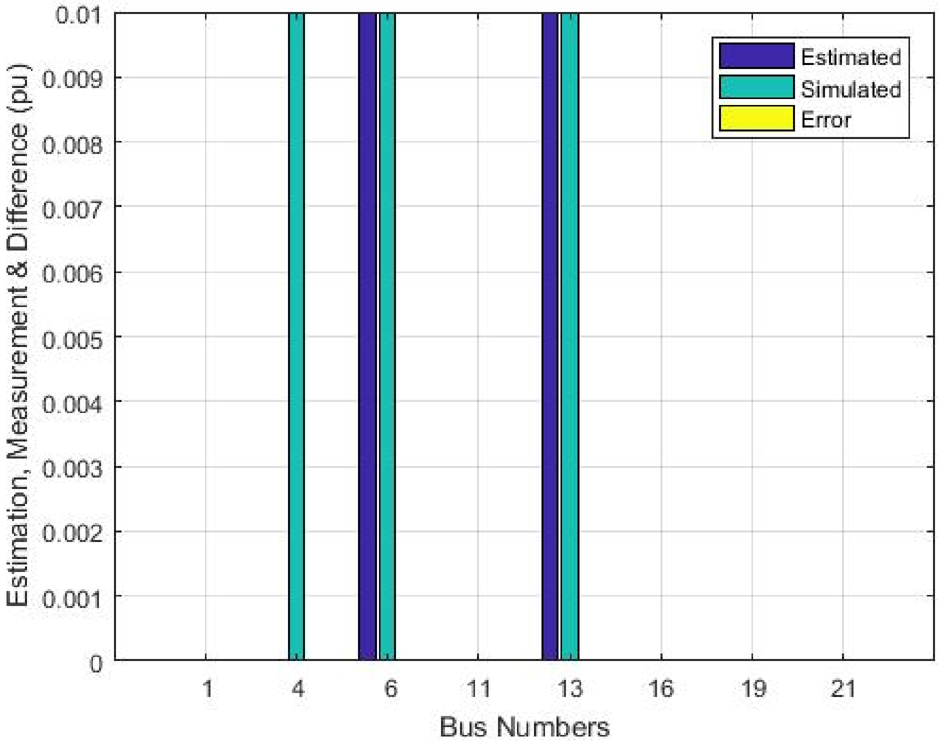


Figure 3.11: Estimation Error in the Magnitudes of the 7^{th} harmonic

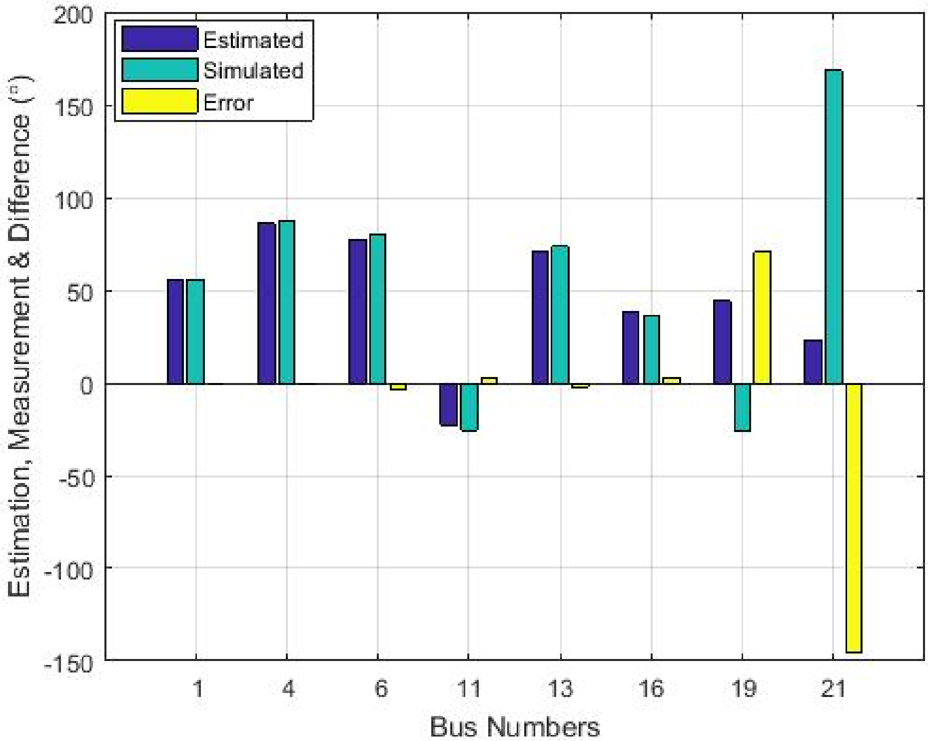


Figure 3.12: Estimation Error in the Angles of the 7^{th} harmonic

3.3.4.2 Estimation in the Presence of Noise

Similar to the analysis presented in Section 3.3.4.1, an analysis is performed where noise is included in the voltage measurements. The estimates are obtained using the simulated data mixed with noise which is generated by (3.24).

$$noise = 0.01 \times voltage\ harmonics \times random\ number \tag{3.24}$$

The estimates are compared with the simulated values (with noise) considered as actual. The performance of HSE in the presence of noise is summarised in Table 3.8. This table shows the magnitude of the errors and the difference between the angle of the estimates and the simulated values (with noise) .

Table 3.8: Comparison of Magnitudes and Angles (with Noise)

Bus	Measurements	Harmonics			
		(1)	(3)	(5)	(7)
1	Absolute Difference (Volts)	9.36	0.19	0.05	0.01
	Difference in Angles (degrees)	0.00	3.99	10.23	-0.08
4	Absolute Difference (Volts)	2.94	2.32	0.75	0.56
	Difference in Angles (degrees)	0.51	1.94	-1.16	-1.22
6	Absolute Difference (Volts)	3.35	3.96	1.15	0.82
	Difference in Angles (degrees)	0.66	0.89	1.57	-3.59
11	Absolute Difference (Volts)	1.91	1.52	0.11	0.36
	Difference in Angles (degrees)	0.05	-22.01	6.95	7.08
13	Absolute Difference (Volts)	4.99	8.05	2.20	0.87
	Difference in Angles (degrees)	-0.90	122.80	-70.44	-1.86
16	Absolute Difference (Volts)	1.28	1.36	0.18	0.05
	Difference in Angles (degrees)	0.03	-13.21	8.37	4.06
19	Absolute Difference (Volts)	0.97	1.83	0.36	0.29
	Difference in Angles (degrees)	-0.19	238.97	109.62	71.68
21	Absolute Difference (Volts)	0.88	1.10	0.31	0.28
	Difference in Angles (degrees)	-0.17	74.61	115.00	-142.72

Table 3.9 shows the magnitudes of the estimates and the simulated values of harmonics.

Table 3.9: Estimates and Simulated Harmonic Magnitudes (with Noise)

Bus	Measurements	Harmonic Magnitudes (Volts)			
		(1)	(3)	(5)	(7)
1	Estimated	6360.11	0.28	0.28	0.19
	Simulated	6350.75	0.47	0.28	0.18
4	Estimated	240.56	4.51	1.51	1.01
	Simulated	238.54	6.82	2.26	1.57
6	Estimated	239.81	6.87	2.43	1.66
	Simulated	237.9	10.83	3.58	2.47
11	Estimated	240.44	1.33	0.86	0.26
	Simulated	238.54	2.67	0.9	0.62
13	Estimated	241.79	3.82	2.04	2.1
	Simulated	238.52	5.31	1.76	1.23
16	Estimated	240.3	0.73	0.55	0.45
	Simulated	239.03	2.06	0.7	0.49
19	Estimated	239.92	1.67	0.31	0.3
	Simulated	239.36	0.28	0.11	0.07
21	Estimated	239.87	1.13	0.27	0.26
	Simulated	239.36	0.15	0.07	0.02

Table 3.10 shows the angles of the estimates and the simulated values of harmonics.

Table 3.10: Estimated and Simulated Harmonic Angles (with Noise)

Bus	Measurements	Harmonic Angles (degrees)			
		(1)	(3)	(5)	(7)
1	Estimated	-0.01	-20.14	53.99	55.89
	Simulated	-0.01	-24.13	43.76	55.97
4	Estimated	29.43	-11.07	41.27	85.9
	Simulated	28.92	-13.01	42.43	87.12
6	Estimated	28.94	-15.01	39.23	76.91
	Simulated	28.28	-15.9	37.66	80.5
11	Estimated	29.66	-84.07	-31.63	-18.53
	Simulated	29.61	-62.06	-38.58	-25.61
13	Estimated	28.25	104.19	-37.94	71.71
	Simulated	29.15	-18.61	32.5	73.57
16	Estimated	29.71	-47.61	13.83	39.85
	Simulated	29.68	-34.4	5.46	35.79
19	Estimated	29.76	171.86	68.43	45.18
	Simulated	29.95	-67.11	-41.19	-26.5
21	Estimated	29.82	154.46	93.07	25.85
	Simulated	29.99	79.85	-21.93	168.57

3.3.4.3 Observability

Observability of the system is analysed to identify the observable buses. Observability is checked to identify the maximum number of observable buses for each combination. The maximum number of observable buses (for instance 5^{th} harmonic) in one such combination for each set of measurements is shown in Table 3.11. Fig. 3.13 shows that Bus '1' is critical to measure as the number of observable buses decline greatly when measurements are not taken at this bus. This is true because Bus '1' is an 11kV bus at the feeder head. If no measurements are taken at this bus, many other 11kV buses become unobservable too.

Table 3.11: Maximum Number of Observable Buses Including Phases

	Number of Observable Buses Including Phases					
No. of Monitors →	(7)	(6)	(5)	(4)	(3)	(2)
Bus (as in Fig. 3.1) with No Monitor						
1	21	21	18	15	12	9
4	63	63	45	39	39	25
6	63	60	36	36	39	29
11	63	60	30	30	27	9
13	63	60	30	30	27	9
16	63	21	18	18	12	24
19	63	63	33	37	37	6
21	63	33	33	36	36	6

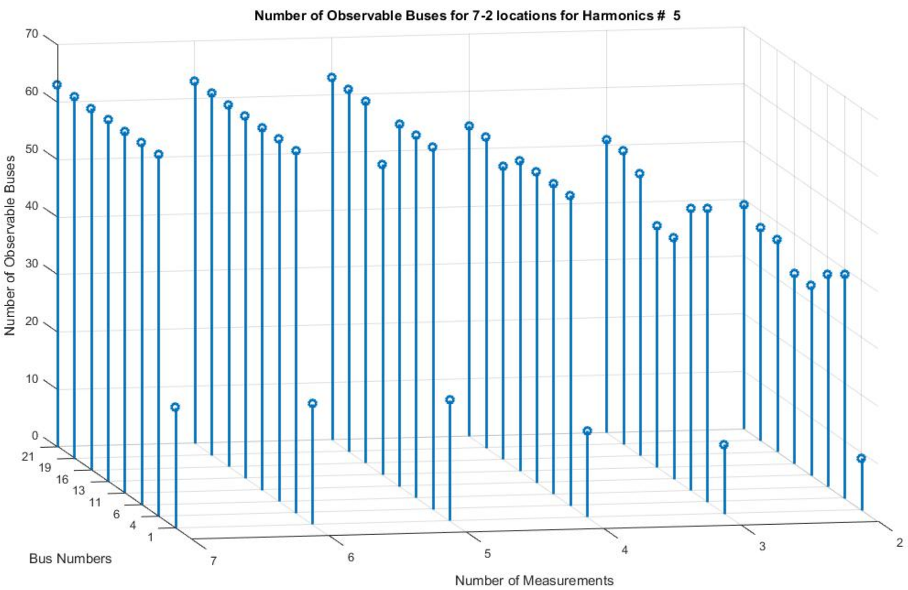


Figure 3.13: Number of Observable Nodes for Different Measurement Locations (without Noise)

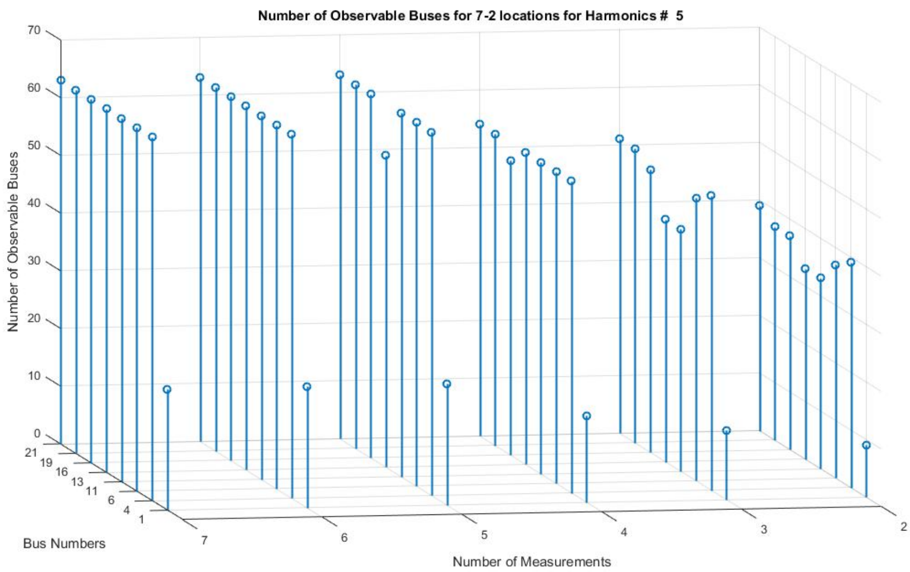


Figure 3.14: Number of Observable Buses for Different Measurement Locations (with Noise)

3.3.5 Optimal Placement of Measurements

Placement of a minimum number of measurements giving the maximum number of observable buses is taken as optimal placement of measurements since the purpose is to reduce the investment cost on measuring equipment without compromise on the accuracy of estimation. Observability is analysed and the errors are evaluated by varying the number of available measurements. The comparison is performed with sets of two to seven measurements. In each set, different possible combinations of measurement locations are considered. The combinations with maximum observable buses are identified. Out of these combinations, the combination, which gives results with the least error, is selected. The placement of measurements in this combination is considered as the optimal location of the measurements. The errors for this combination are shown in Figs. 3.15, 3.16, 3.17 and 3.18 respectively.

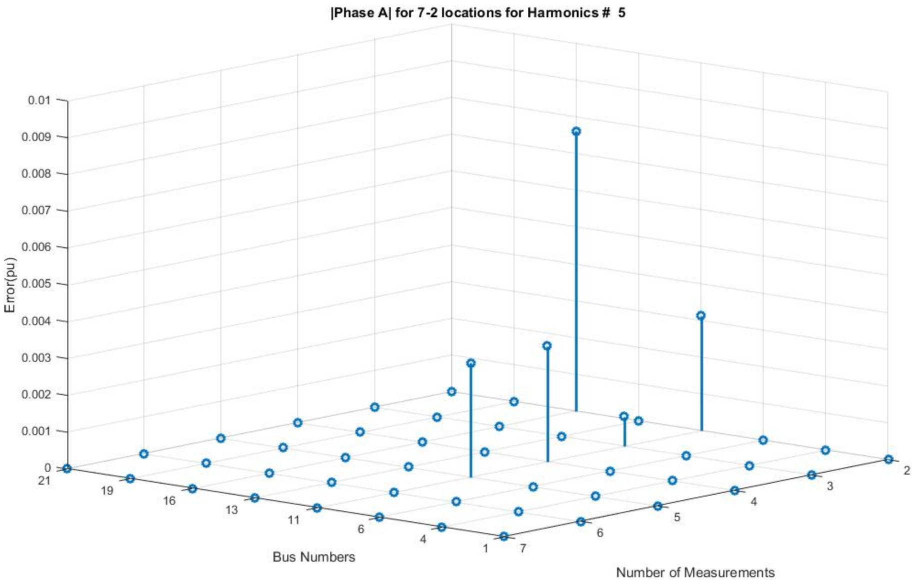


Figure 3.15: Magnitude of the Estimation Errors (without Noise)

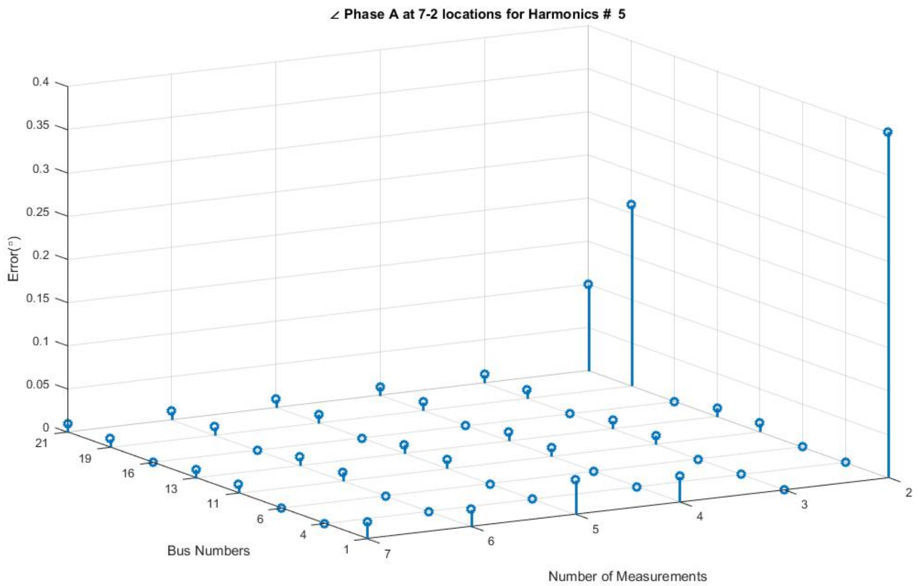


Figure 3.16: Error in the Angles (without Noise)

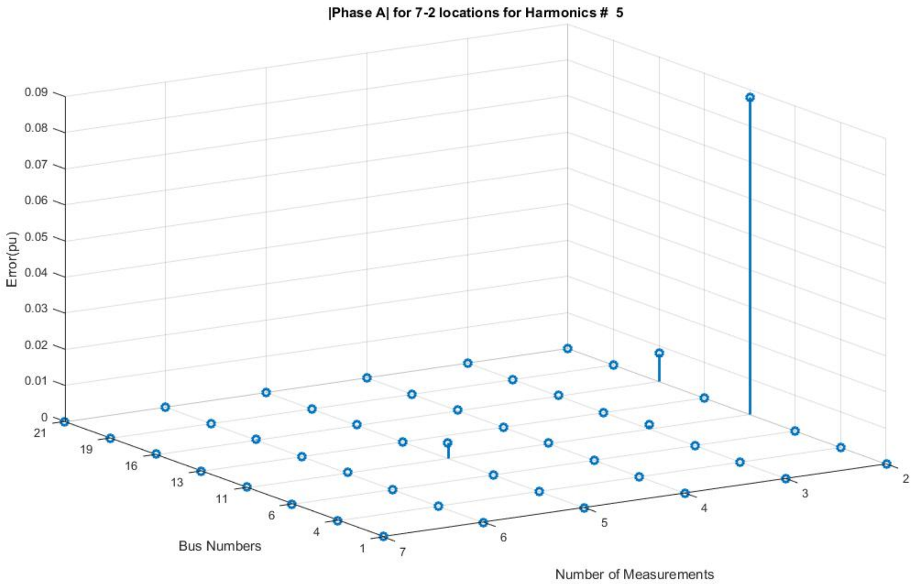


Figure 3.17: Magnitude of the Estimation Errors (with Noise)

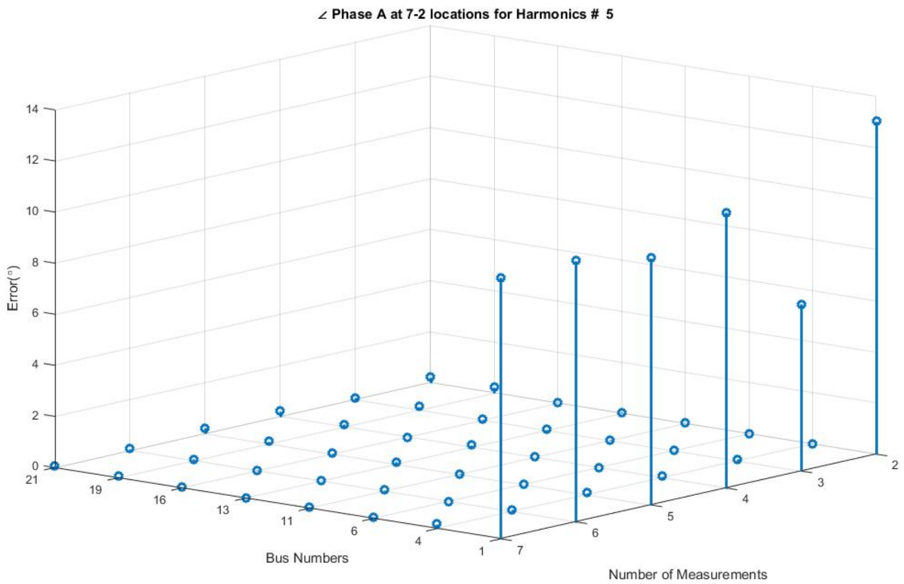


Figure 3.18: Error in the Angles (with Noise)

Table 3.12: Number of Observable Buses Including Phases for 5^{th} Harmonic

	Number of Observable Buses Including Phases					
No. of Monitors →	(7)	(6)	(5)	(4)	(3)	(2)
Bus (as in Fig. 3.1) with No Monitor						
1	21	21	15	12	12	9
4	63	24	48	42	30	21
6	63	30	48	48	27	21
11	63	33	30	30	9	9
13	63	54	30	12	9	6
16	63	51	32	33	32	6
19	63	55	30	30	39	6
21	63	54	51	42	33	6

Chapter 4

APPLICATION OF HARMONIC STATE ESTIMATION

This chapter demonstrates the application of harmonic state estimation to a practical distribution system feeder. An 11kV feeder 'Massey' connected to Hasting substation of Unison Networks has been selected as a test feeder for this purpose. The instrumentation for the measurement of harmonics, data acquisition and processing of collected data are discussed. Samples of measurement data are also shown. The comparison of the harmonic estimates and the measurements are presented. The estimation errors are also illustrated further through figures and tables.

4.1 MEASUREMENT OF HARMONICS AND DATA PROCESSING

Eight PQ data analysers of four different types were installed to log the harmonic data at the eight locations on the test feeder shown in Fig. 4.1. The loggers and the transformers are indicated by diamond symbols and transformer ID's respectively in this figure. They are also shown in Table 4.1. The loggers are connected to the LV side of all the transformers whereas the logger at the feeder head is connected to the output terminals of the instrument transformers (Voltage and Current transformers) already installed in the substation for the measurement of voltage and current at the 11kV side. Each logger is able to log the voltage and current harmonics from all the phases. Their installations are discussed in Section 4.1.1.

Table 4.1: Location of Data Loggers

Data Loggers	Bus	Transformer ID
PQ_Box200	1	Feeder Head
PM45	4	3474
Hioki	6	1909
PM45	11	1693
PQ_Box200	13	5913
PM45	16	3790
Hioki	19	395
PMI	21	3438

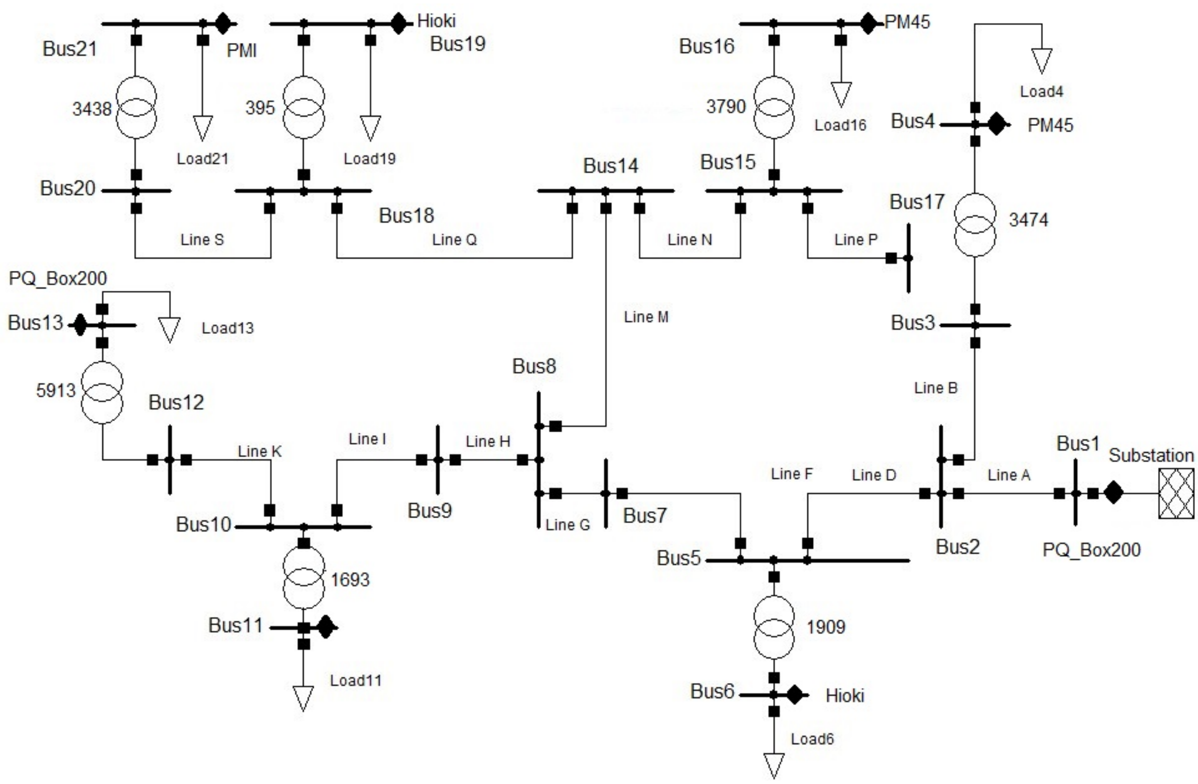


Figure 4.1: Single-line Diagram of Distribution System Feeder

4.1.1 Installation of Data Loggers

The following PQ monitors were installed for taking the measurements.

- PQ-Box 200
- Powermonic Analyser (PM45)
- HIOKI (PW3198)
- PMI Revolution

Now, the details related to the installation of these PQ monitors are presented.

- PQ-Box 200

  PQ-Box 200, as of date, is the latest high performance, portable network-analyzer and transient recorder from Aberle company. Two PQ-Box 200 were installed. One of them was installed at the feeder head to measure the voltage and current harmonics. The voltage probes of the logger were connected to the output terminals of the voltage transformer (VT) at the substation while the current transformers (CT) from the logger were clamped on the secondary of the CT's at the substation. The logger was set as per the instruction manual to take the measurements directly as in Fig. 4.2.

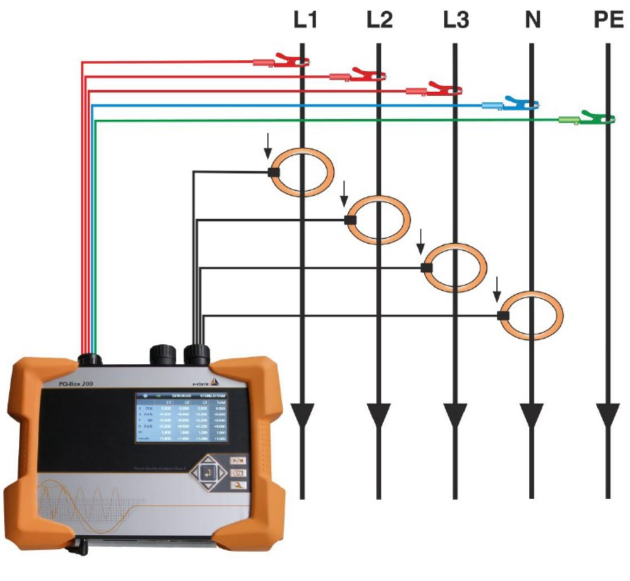


Figure 4.2: PQ-Box 200 Connection Diagram

  The other logger was installed at the low voltage (LV) side terminals of the transformer '5913' directly following the same connection procedure as discussed.

- Powermonic Analyser (PM45)

  Powermonic Analyser (PM45) is the IEC 61010, CAT IV 600V category power quality analyser of CHK Power Quality company. Three 'PM45' loggers were installed at the LV side terminals of the transformers '3474', '1693' and '3790' at the buses '4', '11' and '16' respectively as shown in Fig. 4.1 . The connection was made according to Fig. 4.3.

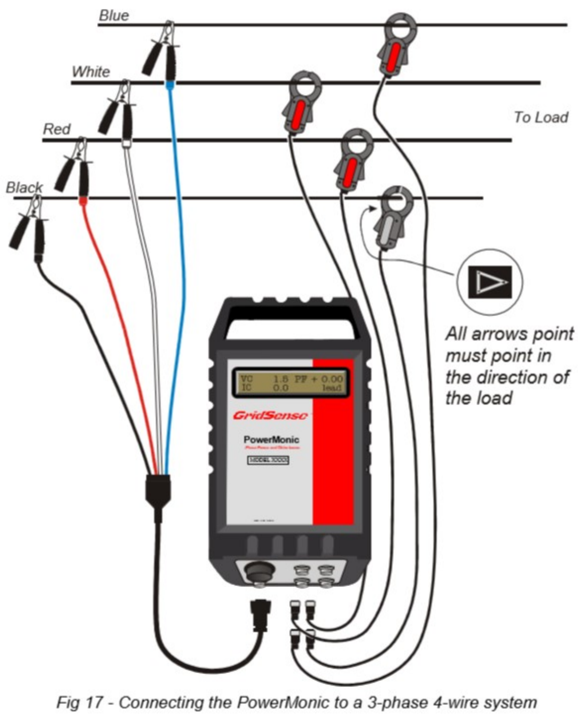


Figure 4.3: PM45 Connection Diagram

- HIOKI(PW3198)

  Two HIOKI (PW3198) power quality analysers were used. They were installed at the LV side terminals of the transformers ‘1909’ and ‘395’ installed at Bus ‘6’ and Bus ‘19’ respectively. They were connected as indicated in Fig. 4.4.

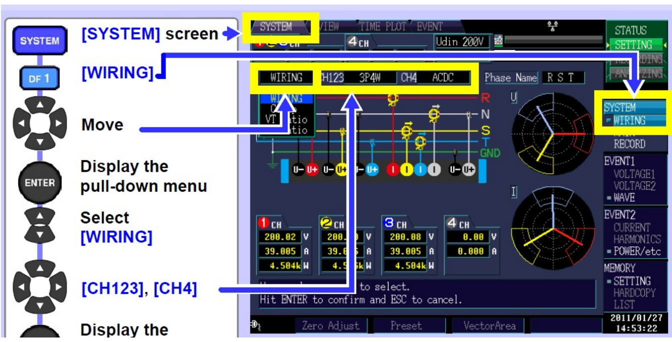


Figure 4.4: Hioki Connection Diagram

- PMI Revolution

  PMI Revolution (Fig. 4.5) is a power quality logger manufactured by Power Monitors Inc. One PMI Revolution logger was installed at the LV side terminal of the transformer ‘3438’ at Bus ‘21’.

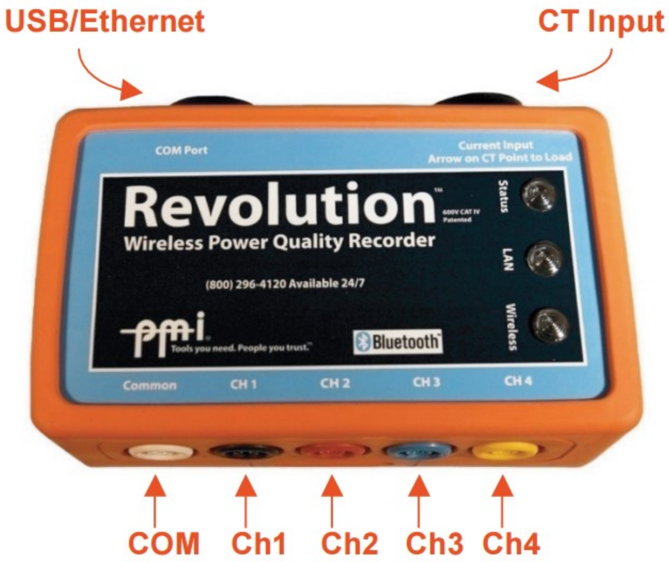


Figure 4.5: PMI Revolution

4.1.2 Data Logging and Acquisition

The current and voltage harmonic magnitudes and angles were logged by the loggers. Odd harmonics upto the order of '37' were measured. They were synchronized to the same personal computer clock time and preset to start logging from the same instant of time. Data was logged for thirteen hours. After logging, it was connected to the computer using data-cable on site, and the data was downloaded using the dedicated software for the respective logger.

4.1.3 Processing of Data

Since the data had its own format and file-type depending on the type of logger, the data was first imported to the MATLAB environment and processed. Some of the required parameters were not directly recorded by the loggers, and therefore, they had to be derived after processing. The loggers had been set for different data aggregation time intervals based on their ability. Hence, some calculations were required to obtain all the data for the same time interval. The data aggregation time interval was six seconds for five loggers while it was three seconds for the other three. Therefore, the three-second data was aggregated later for six seconds by calculating the root mean square values of the two consecutive three-second data. Other corrections applied to the data are discussed from Section 4.1.3.1 to 4.1.3.4.

4.1.3.1 Aberle PQ_Box 200

The current and voltage harmonics logged in this logger did not have the fundamental current angles. They had to be derived using other measurements. The fundamental current angles are derived from the fundamental voltage angles and the power phase angles using (4.1).

$$\textit{Angle of fundamental current} = \textit{Angle of fundamental voltage} - \textit{Power phase angle} \tag{4.1}$$

Harmonic angles are referred to their respective fundamental voltages. Hence, the correction mentioned in (4.2) is performed so that all the measurements are referred to the same reference i.e the fundamental voltage of the first phase 'A'.

$$\begin{aligned} \textit{Harmonic voltage or current angle} &= \textit{Measured harmonic angle} \\ + \textit{Fundamental phase voltage angle} \times \textit{Order of harmonics} \; &- \; \textit{30} \times \textit{Order of harmonics} \end{aligned} \tag{4.2}$$

The last term in (4.2) is for the correction required to adjust the lag in the phase angle on delta (the high voltage (HV) side) as compared to the star (the low voltage (LV) side) of the distribution transformers with Dyn11 connection. All the loggers are installed at the LV side where the reference angle is zero. When this reference angle is referred to 11 kV side, it becomes 30° less. To match this relative angle, the angle measured by PQ_Box 200 at the feeder head must be reduced by 30°.

4.1.3.2 PowerMonic (PM45) Analyser

Since all the angles are measured with respect to the common reference i.e. the phase angle of the fundamental of the phase 'A' voltage, no correction is required to the angles of the harmonics measured by these loggers. Data aggregration time interval selected in these loggers is also 6 sec. Hence, no further manipulation is required for the data.

4.1.3.3 Hioki (PW3198)

In these Hioki loggers, angles are measured with respect to the common reference. So, no correction on the angles is required. However, the data aggregation time needs to be modified. The three-second values are logged and converted to six-second values later. This is calculated by taking the root mean square values of the two consecutive three-second values. Two Hioki loggers are installed at the LV terminals of the two transformers '1909' and '395' at Bus '6' and Bus '19' respectively.

4.1.3.4 PMI Revolution

For this PQ monitor, the following corrections are performed.

- Voltage Harmonics
  No correction is required on the voltage harmonics as all the angles of the voltage harmonics are referred to the fundamental voltage on phase 'A'.

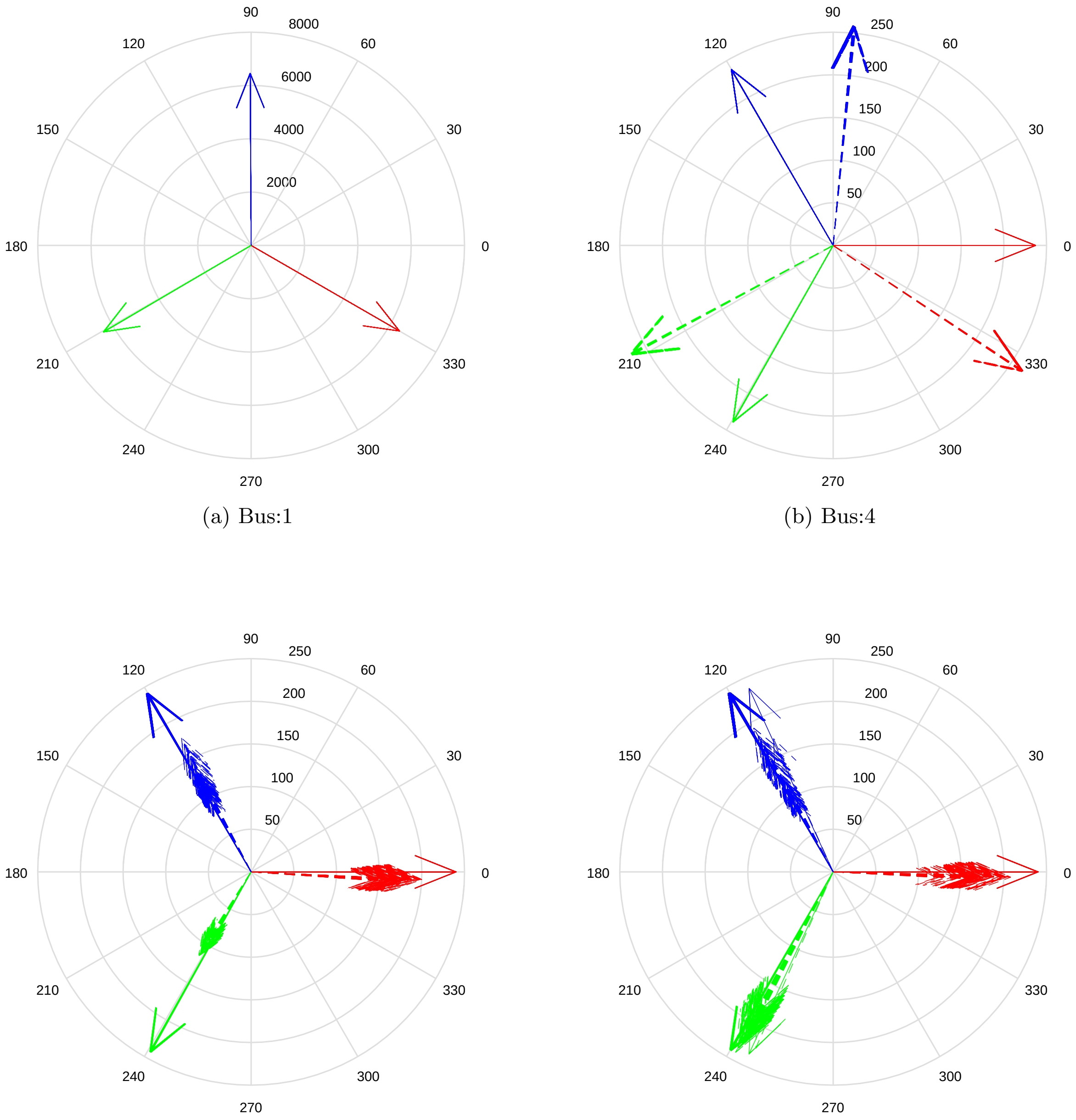
- Current Harmonics
  The angles of current harmonics are referred to the fundamental voltage of the corresponding phase. Hence, the current harmonic angles are corrected as in (4.3).

$$\begin{aligned} \textit{Current harmonic angle} &= \textit{Measured current harmonic angle} \\ &+ \textit{Angle of fundamental voltage of the respective phase} \end{aligned} \tag{4.3}$$

- Data Aggregation Interval
  The data for magnitudes and angles was aggregated for three seconds while logging. Hence, it's also modified to six-second rms values as explained in the case of Hioki logger mentioned in Section 4.1.3.3.

4.2 INPUT DATA TO HSE

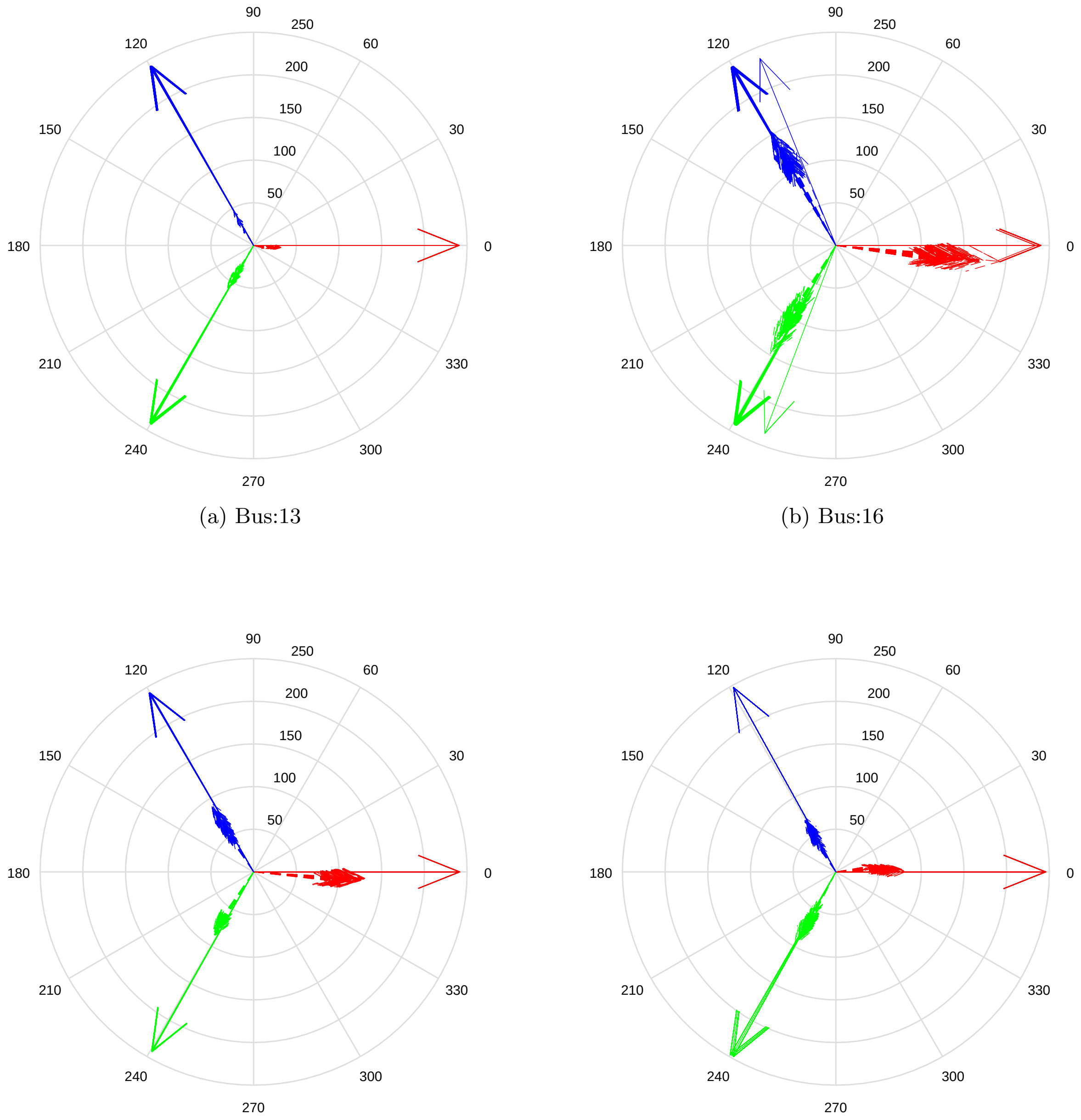
The input data consists of the voltage and current harmonic magnitudes and angles besides the system parameters. The phasor plots of some of the collected data are shown in Figs. 4.6, 4.7, 4.8 and 4.9 where the colors red, green and blue represent phases 'A', 'B' and 'C' respectively. Dotted arrows represent the current phasors whereas the solid arrows represent the voltage phasors. Only the measurements taken for 100 intervals, each of 6 seconds, have been visualised to get a realization of the correctness of the processing of the measured data. The phasors have been demonstrated to clarify the phase sequence and the magnitude of various harmonics. The fundamental and the seventh harmonic phase voltages have positive phase sequence and angular difference of 120^0 approximately. The fifth harmonic phase voltages have negative phase sequence and angular difference of 120^0 approximately. However, most of the third harmonic phase voltages have angular difference of 0^0 approximately. Figs. 4.6 and 4.7 demonstrate the fundamental voltage and current phasors at the measurement locations. Since this feeder is supplying power to the residential area, the fundamental current phasors have angles very close to those of the fundamental voltage phasors. However, the load at Bus '4' is inductive because of the inductive loads connected here. Hence, the current phasors are lagging the voltage phasors. Figs 4.8 and 4.9 show the input phase voltage and current harmonic (1^{st}, 3^{rd}, 5^{th} and 7^{th}) phasors for the Buses '11' and '16'. A sample of input data (for the 60^{th} interval) is presented for example in Appendix B (Table 9 to Table 17). Data collected for 13 hours starting from 6:00 pm, 3^{rd} November to 7:00 am, 4^{th} November 2016 have been used for the analysis.



(a) Bus:1 (b) Bus:4

(c) Bus:6 (d) Bus:11

Figure 4.6: Fundamental Input Voltage and Current Phasors at Buses (1, 4, 6, 11) for 100 Intervals (6 sec)



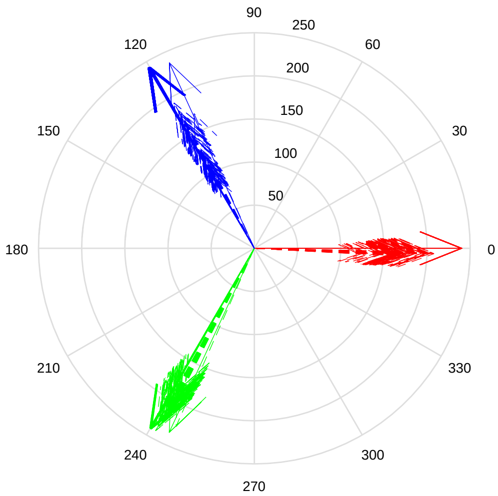
(a) Bus:13

(b) Bus:16

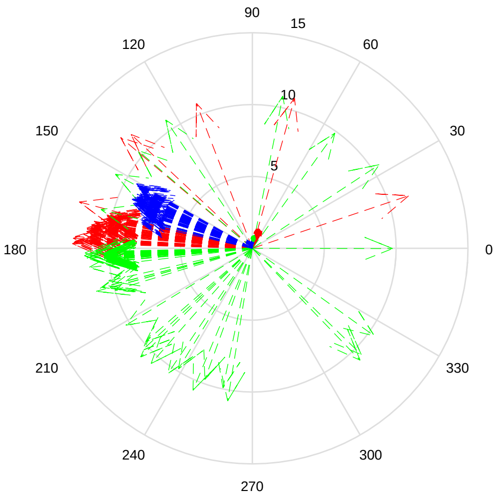
(c) Bus:19

(d) Bus:21

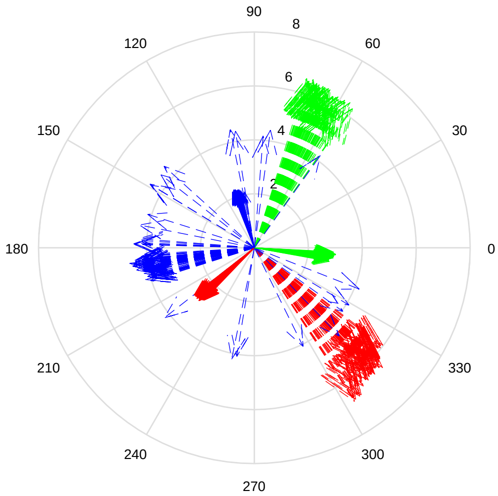
Figure 4.7: Fundamental Input Voltage and Current Phasors at Buses (13, 16, 19, 21) for 100 Intervals (6 sec)



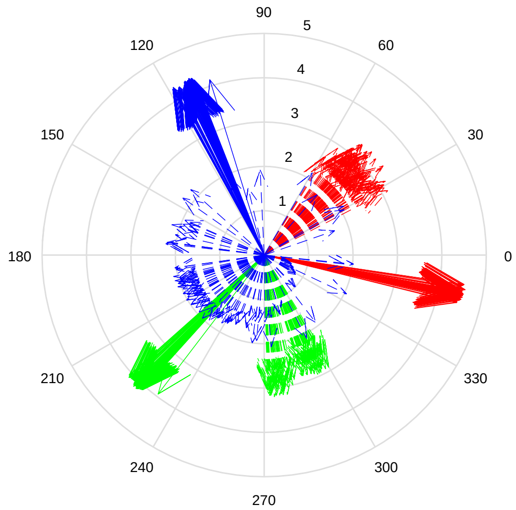
(a) Harmonic Order: 1



(b) Harmonic Order: 3

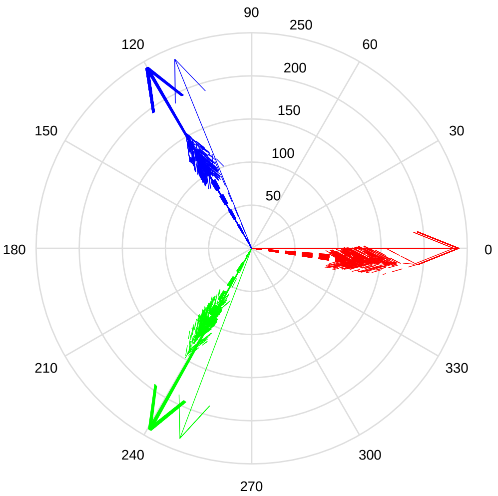


(c) Harmonic Order: 5

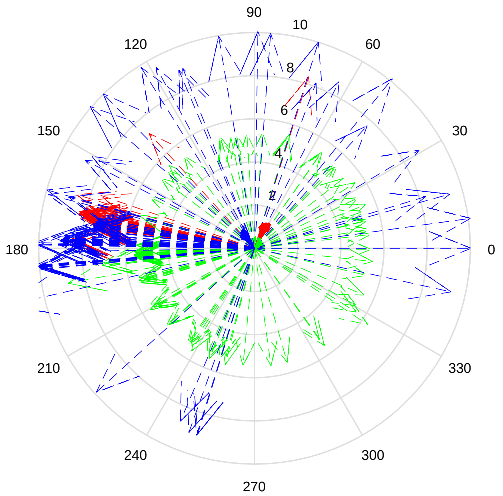


(d) Harmonic Order: 7

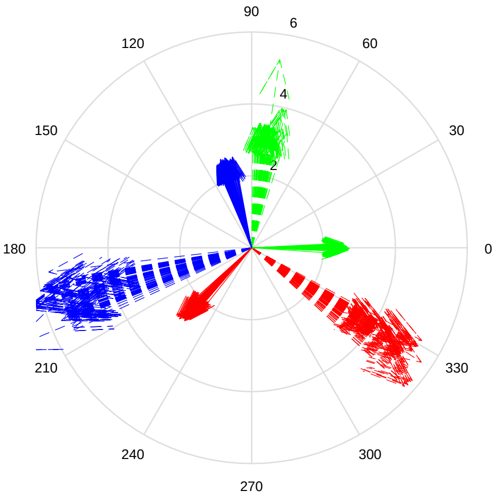
Figure 4.8: Input Voltage and Current for 100 Intervals (6 sec) at Bus ‘11’



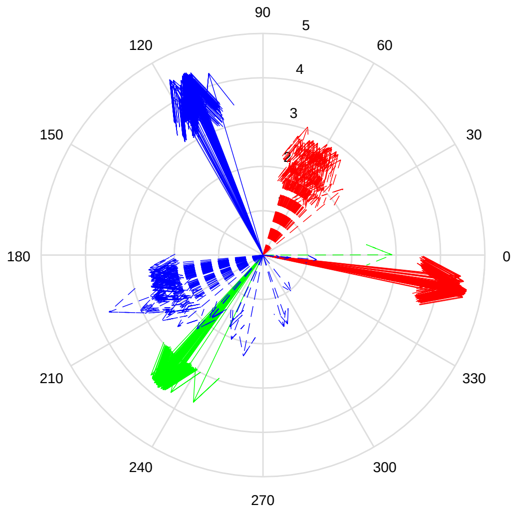
(a) Harmonic Order: 1



(b) Harmonic Order: 3



(c) Harmonic Order: 5



(d) Harmonic Order: 7

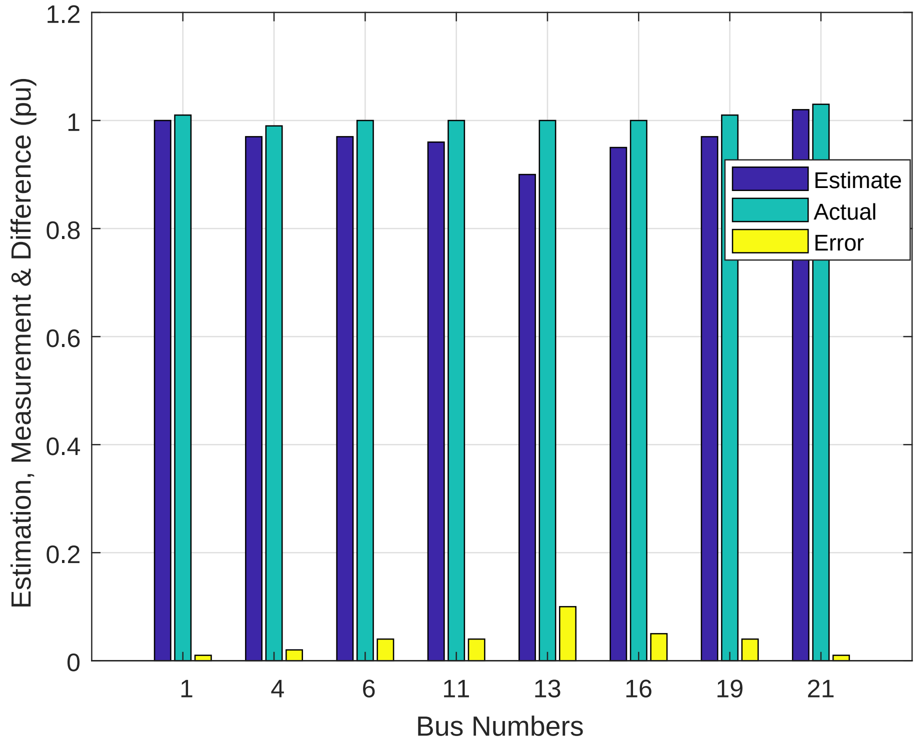
Figure 4.9: Input Voltage and Current for 100 Intervals (6 sec) at Bus '16'

4.3 COMPARISON OF ESTIMATED AND MEASURED HARMONICS

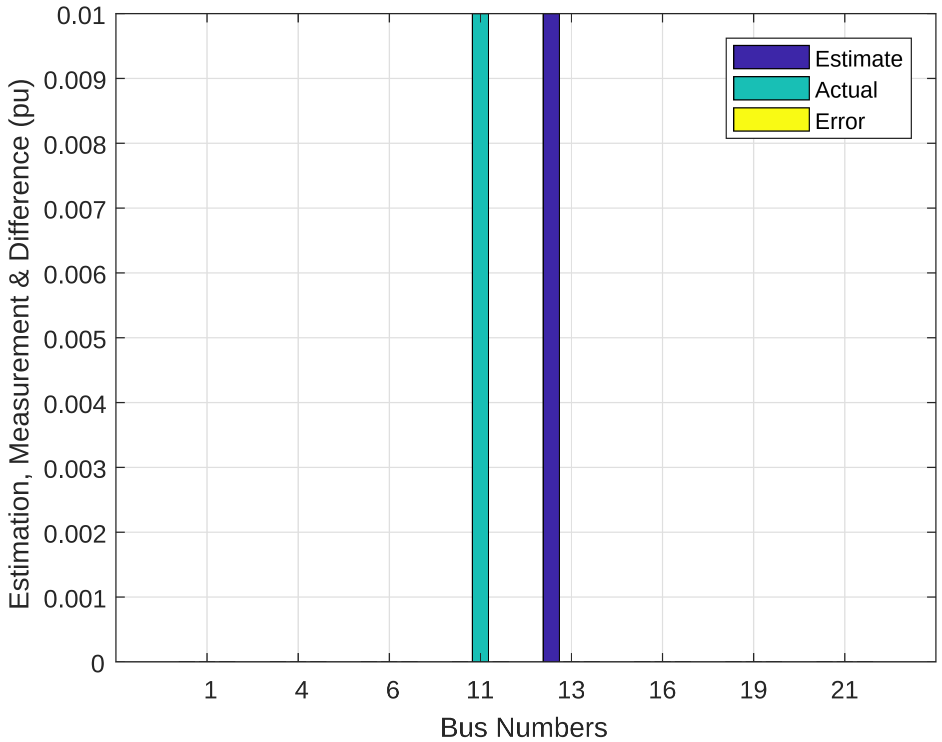
In this section, the results of the comparison between the estimated and the measured harmonics are demonstrated with illustrative graphs and tables. In the real world, the measurements usually have noise and the system parameters are uncertain. In the absence of measurements, there is no other direct alternative to confirm any estimate as accurate but it should be agreed that they are reasonable. Hence, the method followed for estimation has to be robust enough to rely upon its results. The opportunity for making a comparison between the estimates and the measurements is utilised to validate the method so that it can be reliably applied for estimation in future. The maximum number of available measurements in this research is eight. Firstly, the comparison using seven measurements to estimate the 8^{th} position is presented. Then, the comparison between the estimates and the actuals is given considering various harmonics and different time intervals. Similarly, the results obtained using combinations of varying number of measurements are also demonstrated. Variation of the estimates with different aggregation time intervals is also presented. Detailed discussions are presented next.

4.3.1 Comparison Taking Seven Measurements Available

In this case, the number of available measurements is seven, and the measurements at one of the eight transformers is assumed to be unavailable. Therefore, HSE is performed to find the estimated value at this location. The estimates obtained are then compared with the measured values. The results are plotted and shown in Figs. 4.10 to 4.13. Although harmonics have been logged for many six-second intervals, one of the intervals (the 60^{th} interval) is selected, and the data logged during this period are plotted. This is repeated for all the buses where measurements are taken. The results show that the estimates are very close to the measured values.

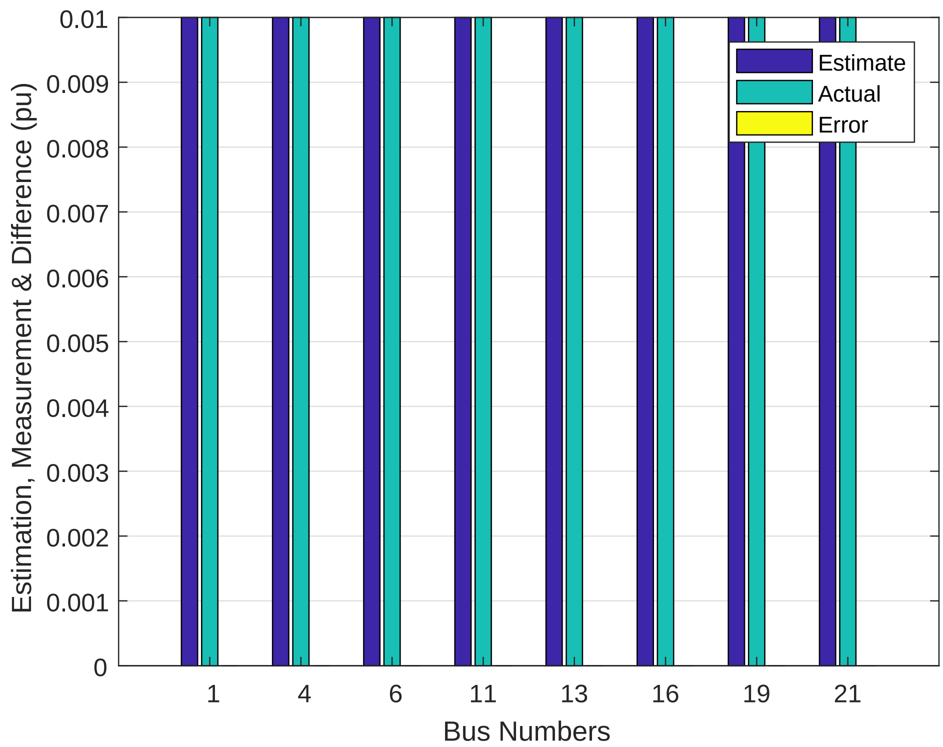


(a) Harmonic Order: 1

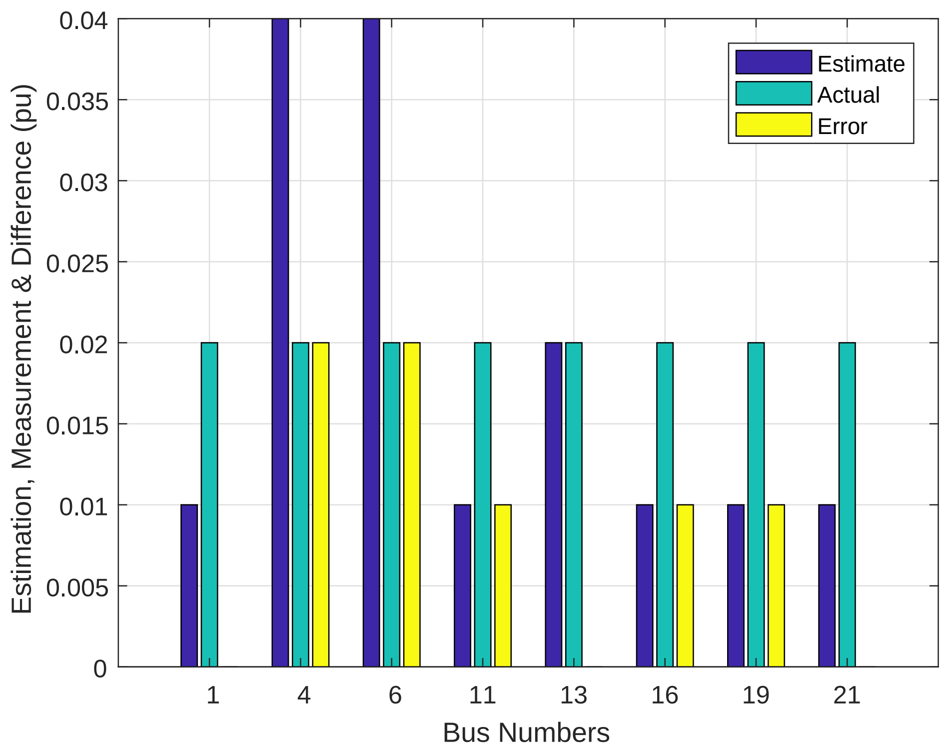


(b) Harmonic Order: 3

Figure 4.10: Magnitude Comparison of Estimated and Measured Values of 1^{st} (a) and 3^{rd} (b) Harmonic

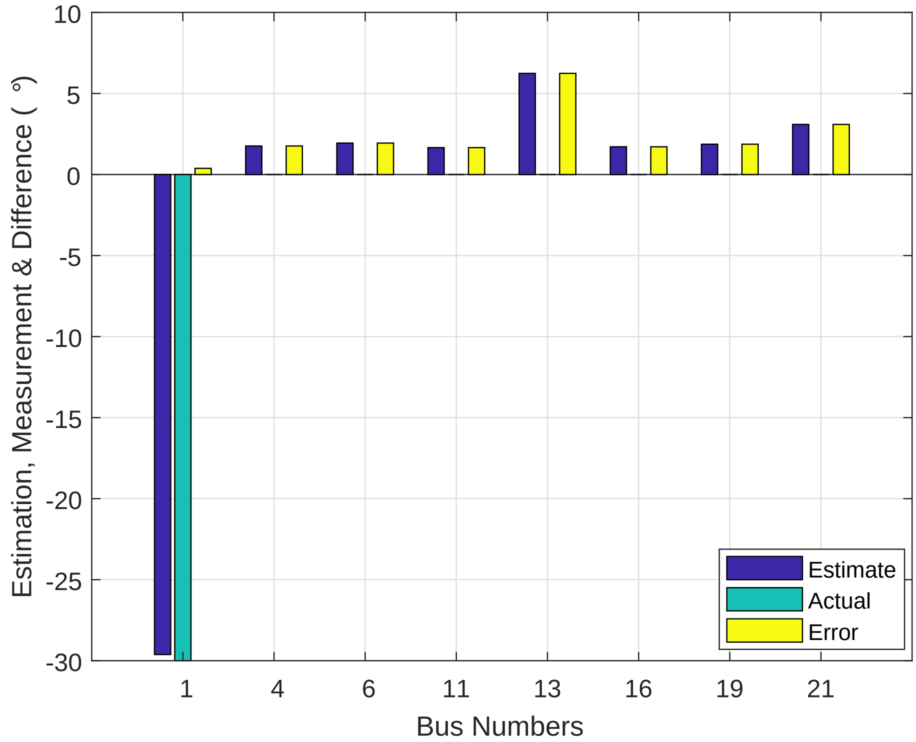


(a) Harmonic Order: 5

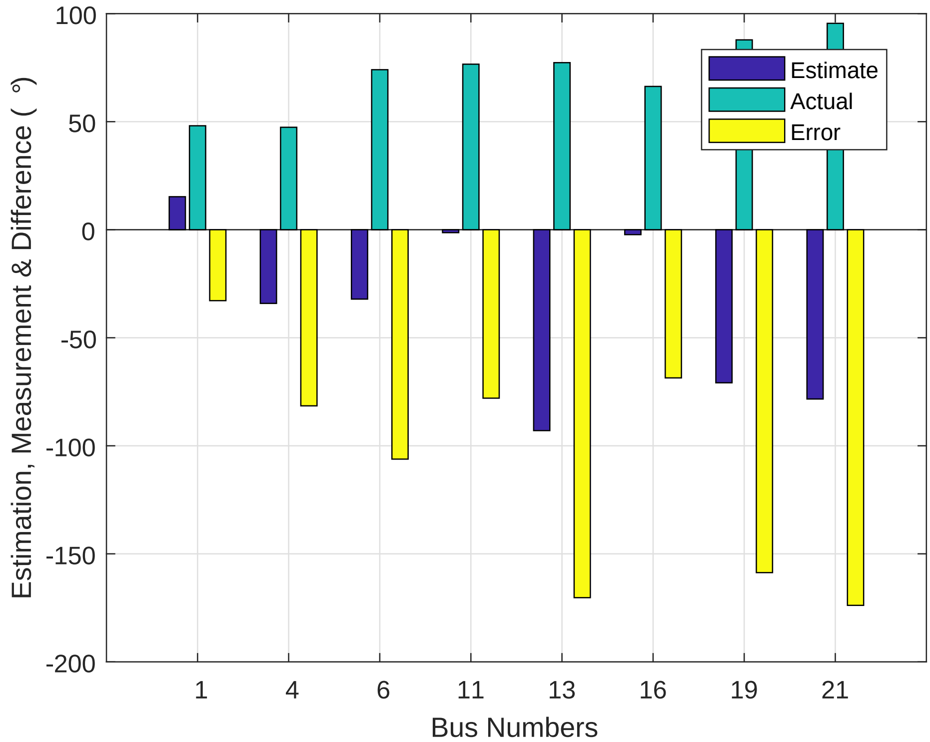


(b) Harmonic Order: 7

Figure 4.11: Magnitude Comparison of Estimated and Measured Values of 5^{th} (a) and 7^{th} (b) Harmonic

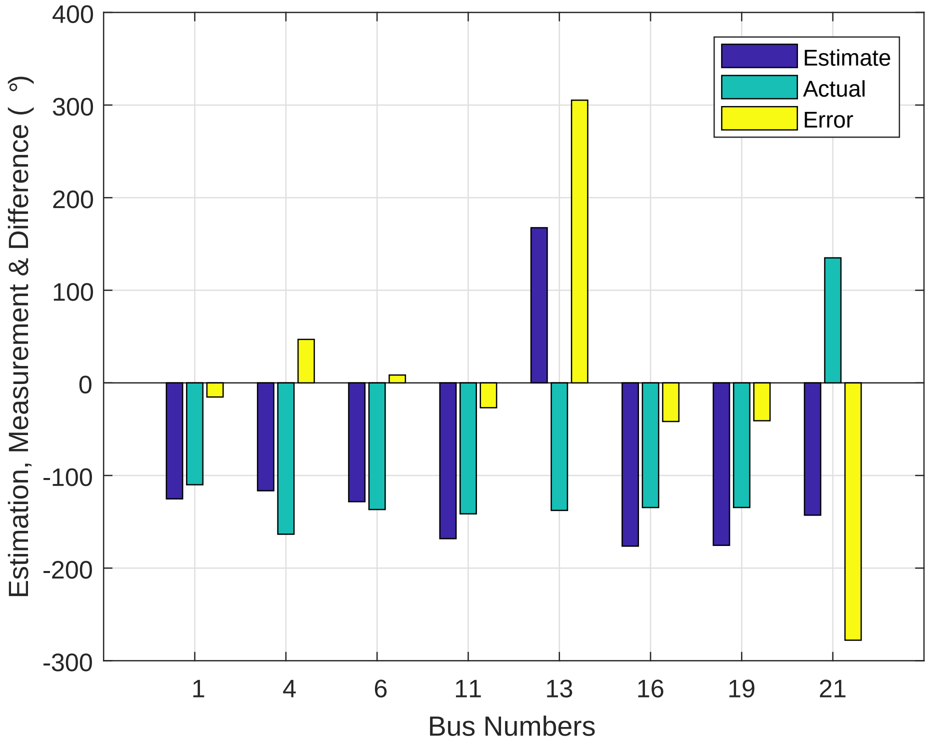


(a) Harmonic Order: 1

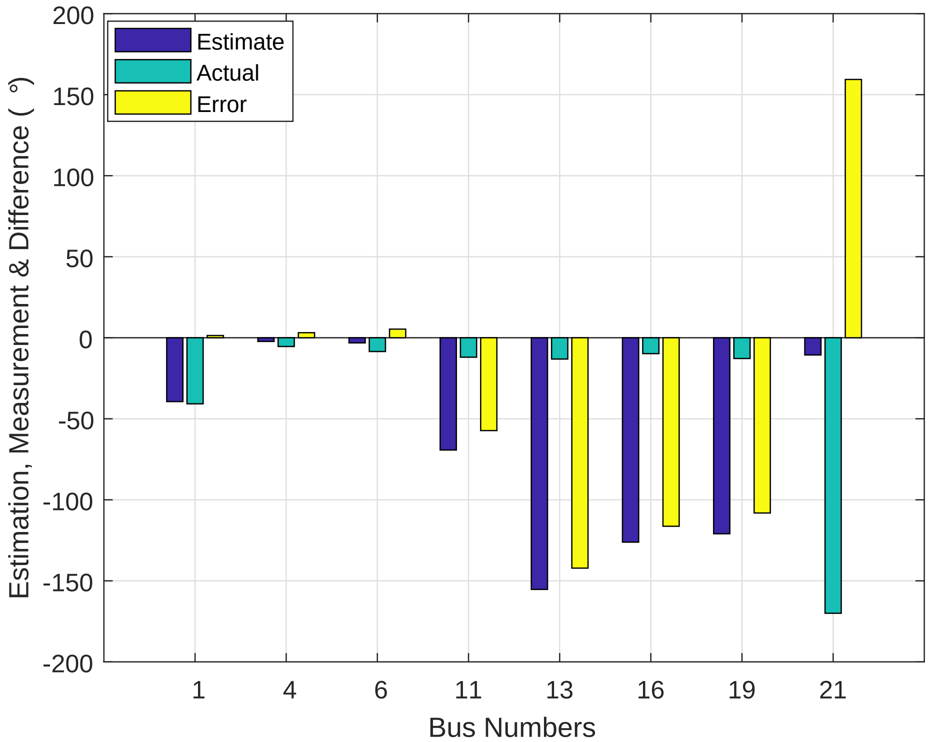


(b) Harmonic Order: 3

Figure 4.12: Angle Comparison of Estimated and Measured Values of 1^{st} (a) and 3^{rd} (b) Harmonic



(a) Harmonic Order: 5



(b) Harmonic Order: 7

Figure 4.13: Angle Comparison of Estimated and Measured Values of 5^{th} (a) and 7^{th} (b) Harmonic

Further to this analysis, the estimates and the actual measured values of all the odd harmonics from '1' to '17' for a selected interval (the 60^{th} six-second interval) are compared. The absolute magnitude of the errors in the estimates and the simulated values, as defined in (4.4), is shown in Fig. 4.14. It shows that the errors are very small. The difference in the magnitudes of the estimates and the actual measurements, as defined in (4.5), is shown in Fig. 4.15, whereas the error in the angles is shown in Fig. 4.16. The error in the estimation of angles looks to be a bit larger but the magnitude of the errors is very small. Hence, the error in the estimation of angles is not very important. This indicates that the estimation is appreciably close to the measured values.

$$\text{Error} = |\text{Phasor of Estimated Value - Phasor of Measured Value}| \tag{4.4}$$

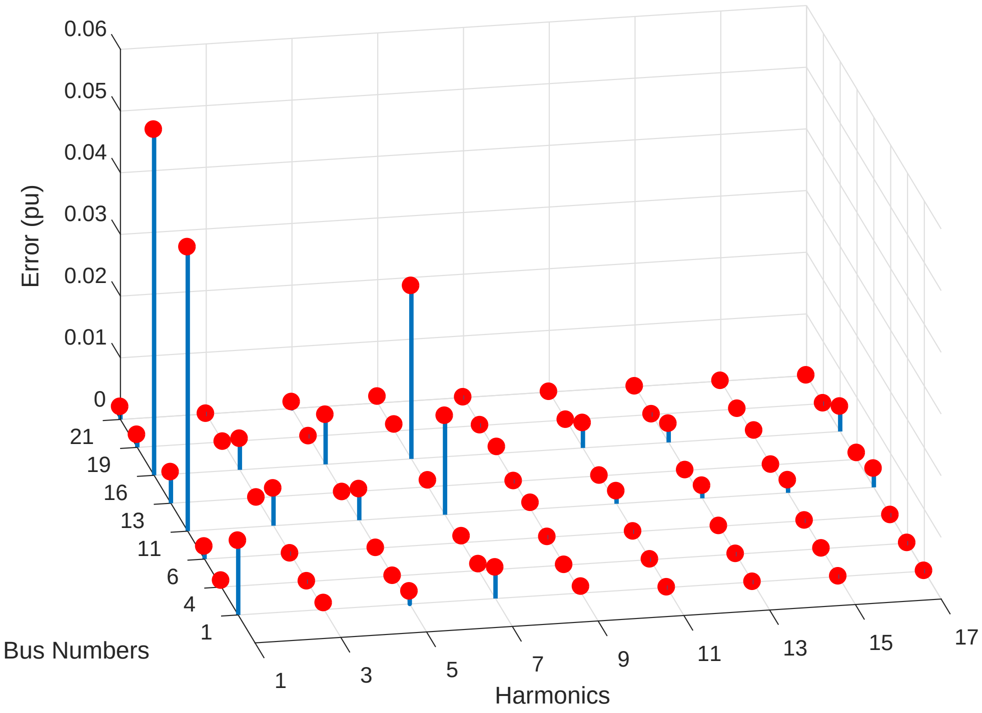


Figure 4.14: Magnitude of the Estimation Errors

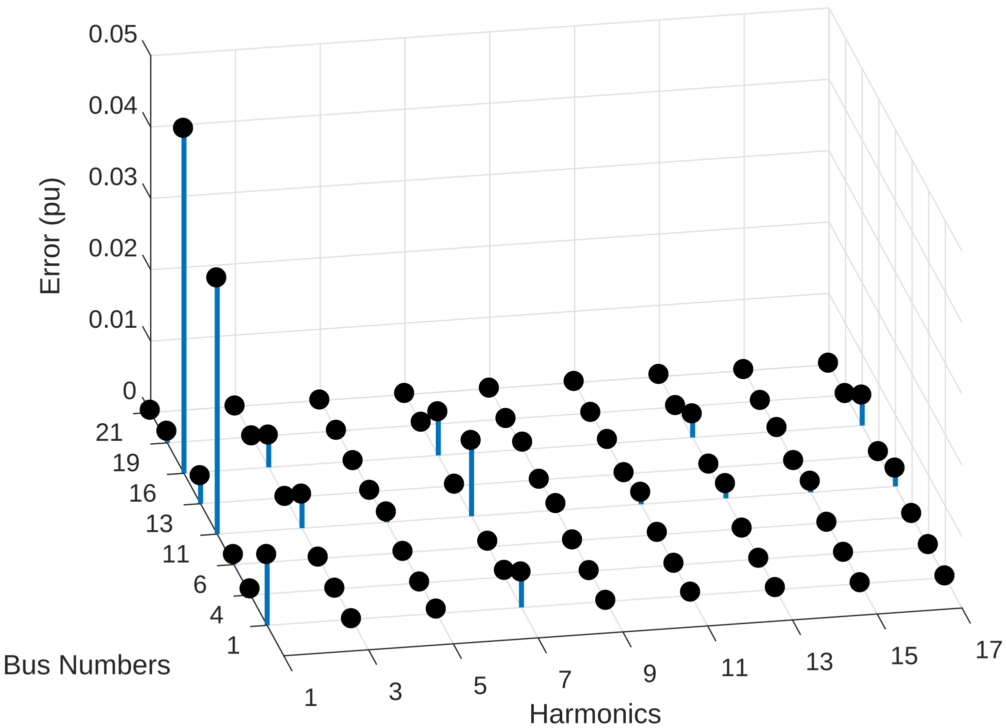


Figure 4.15: Estimation Error in the Magnitudes

$$\text{Error} = |\text{Estimated Value}| - |\text{Measured Value}| \tag{4.5}$$

where, both 'Estimated Value' and 'Measured Value' are the phasor values.

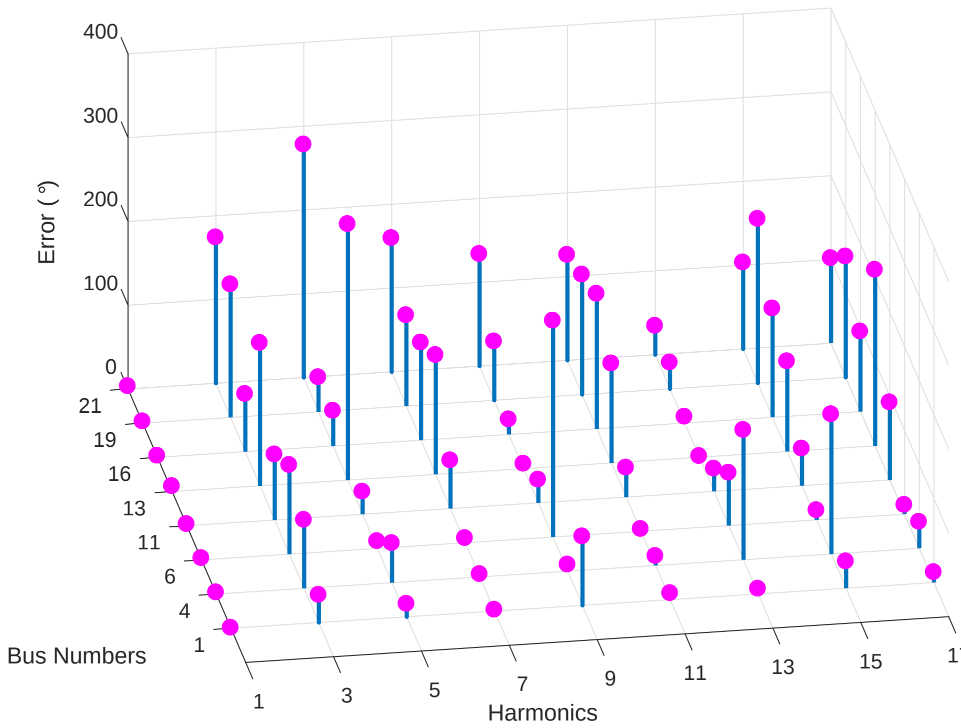
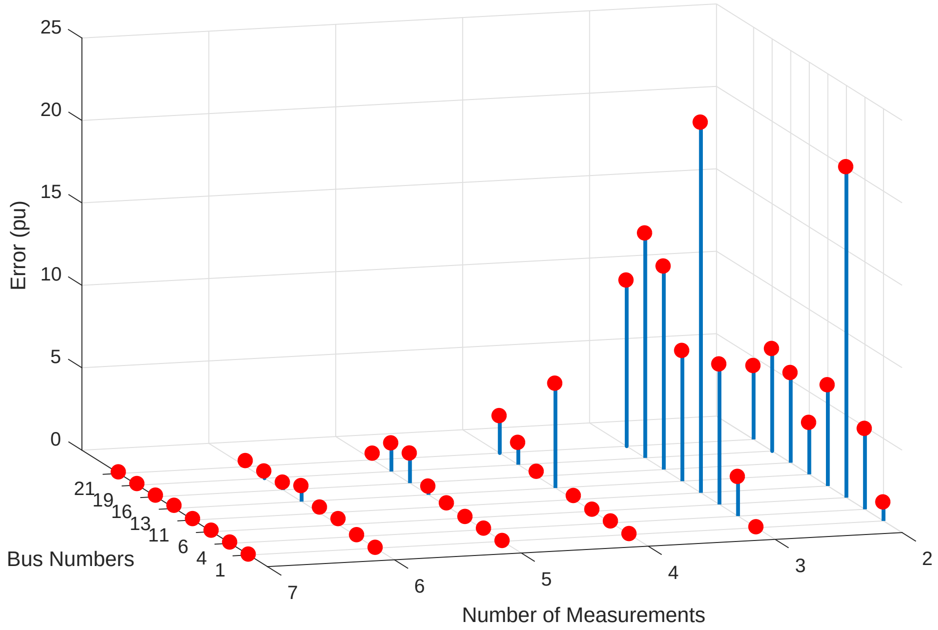


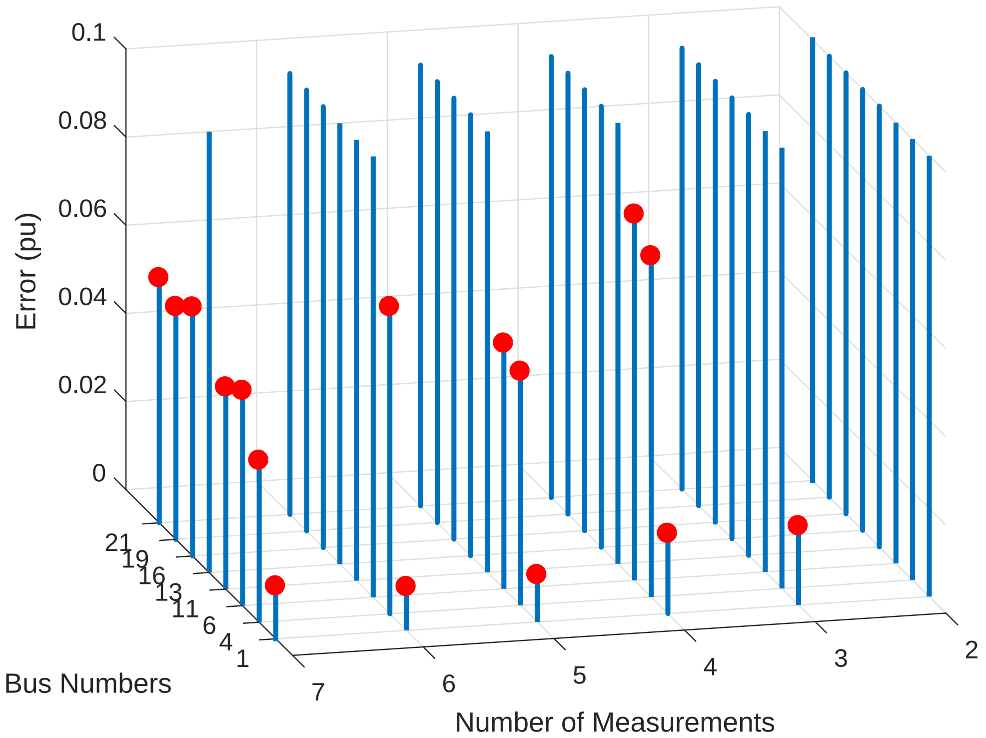
Figure 4.16: Estimation Error in the Angles

4.3.2 Comparison Taking Different Number of Measurements

Comparison is performed by varying the number of measurements available. Two to seven measurements are taken systematically and the estimation is carried out. The estimates and the measured values are compared which are presented in Figs. 4.17a to 4.21b. Fig. 4.17a demonstrates the absolute value of the errors between the estimates and the measured values at different locations. Fig. 4.17b is just the magnified view to show the errors clearly for the analysis when the number of measuring equipment is higher. Fig. 4.18a shows the observability. Fig. 4.18b shows the errors in angles. It can be seen that when the observability is higher, the errors are small. There seems to be a greater difference in the angles than in the magnitudes. However, magnitude of the differences is very small which means that the difference in angles can be ignored. But, in the case when only a few measurements are available, observability is poorer, and the error in the magnitudes and angles is more. It is also likely, as seen in Figs. 4.17a and 4.17b, that there might be marginal improvement of errors in changing the number of measurements. In such a case, the obvious choice is choosing fewer measurements if the observability is also equally good. Knowing the state variables of the buses near to a selected bus with unknown variables is significant in achieving an accurate estimate of those unknown state variables.

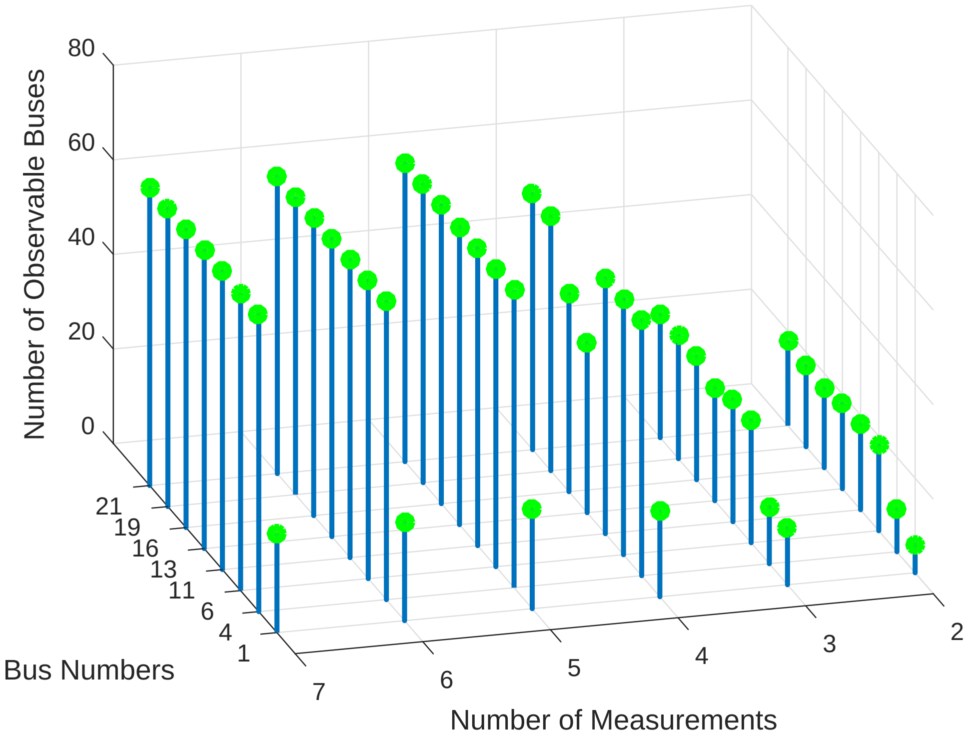


(a) Magnitude of the Errors in Fundamental Voltage

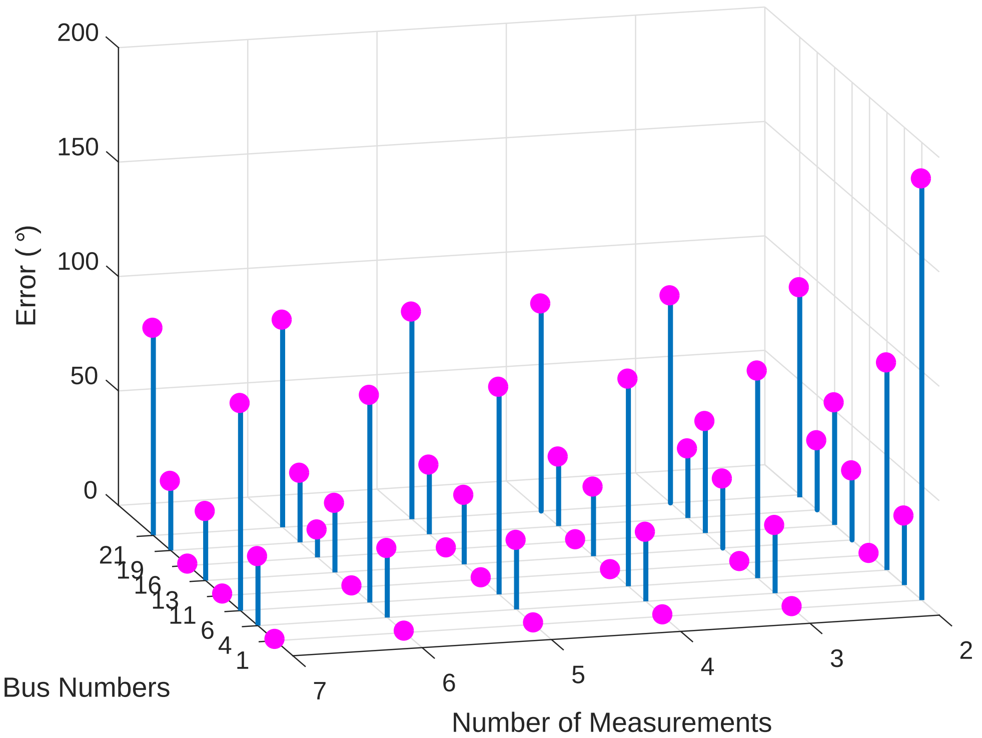


(b) Magnitude of the Errors in Fundamental Voltage (zoomed for clarity)

Figure 4.17: Comparison of Estimated and Measured Values (Fundamental) with Different Number of Measurements

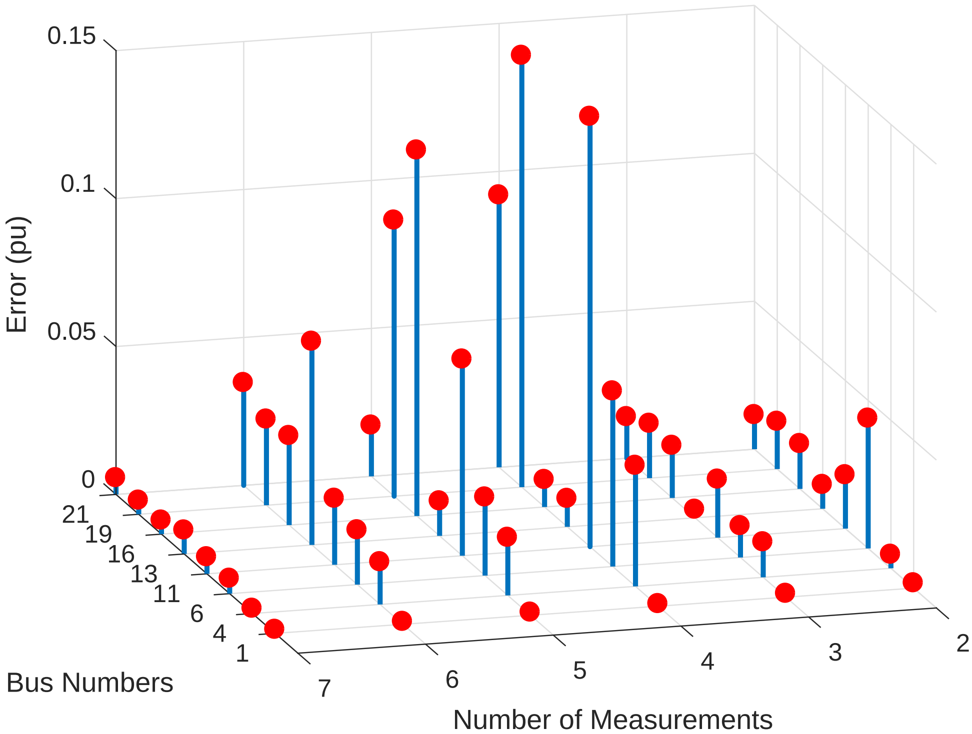


(a) Observable Buses

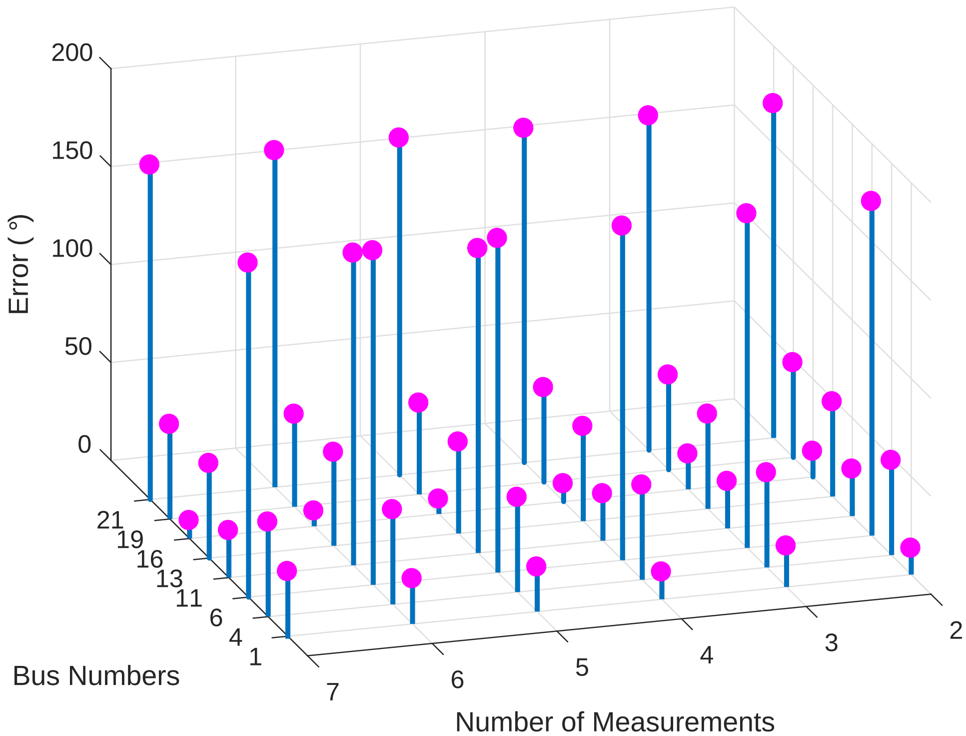


(b) Errors in Angles

Figure 4.18: Comparison of Estimated and Measured Values (Fundamental) with Different Number of Measurements

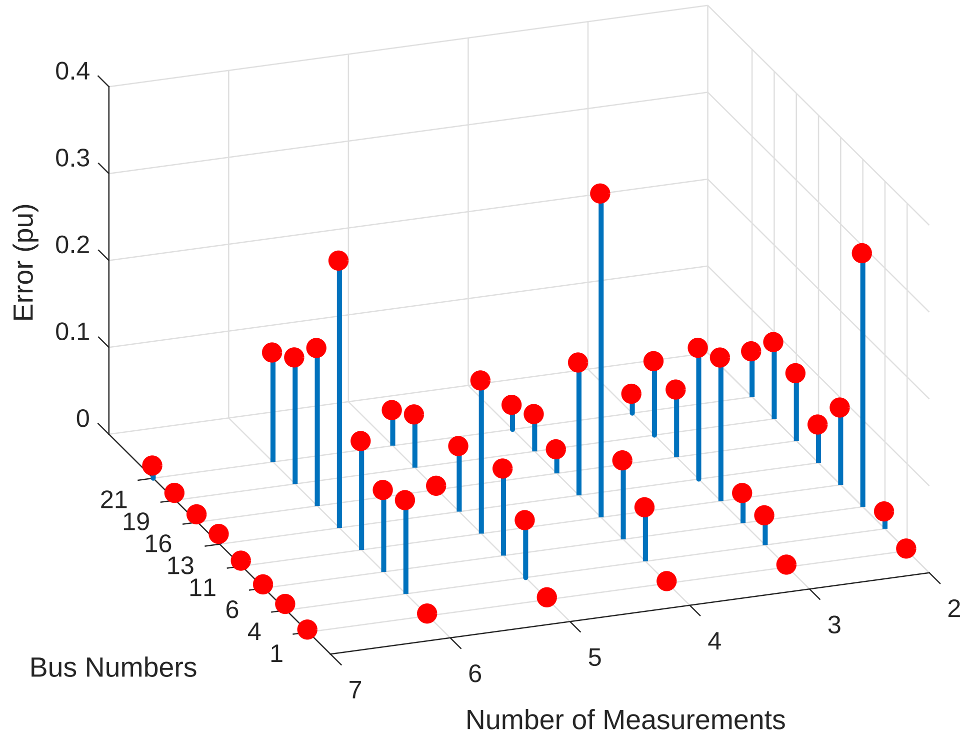


(a) Magnitude of the Errors

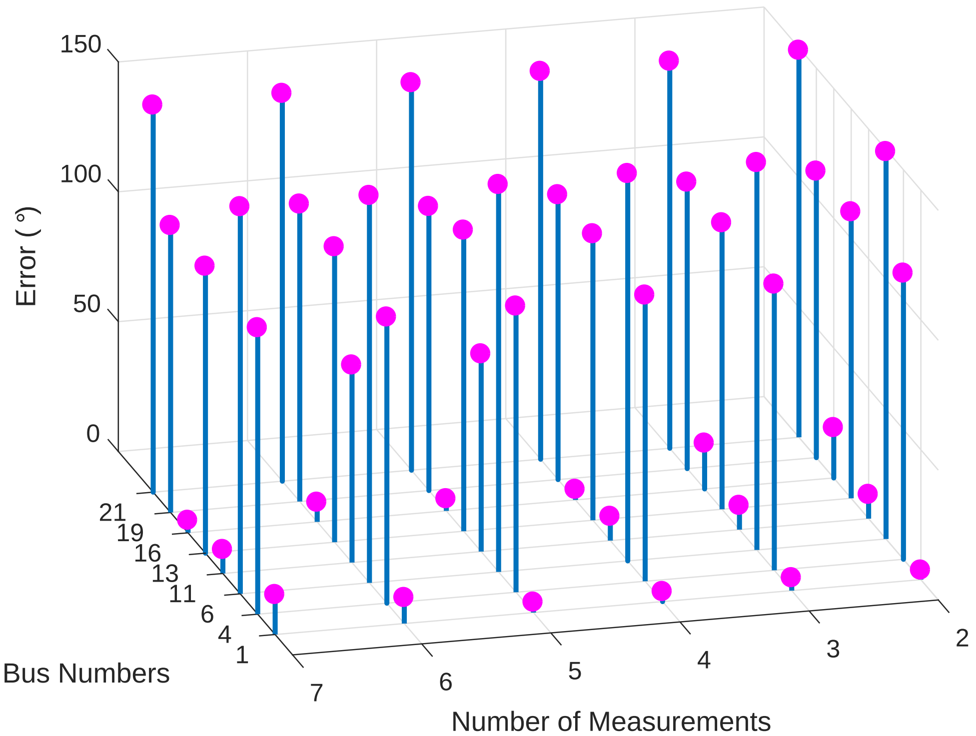


(b) Error in the Angles

Figure 4.19: Comparison of Estimated and Measured Values (3^{rd} harmonic) with Different Number of Measurements

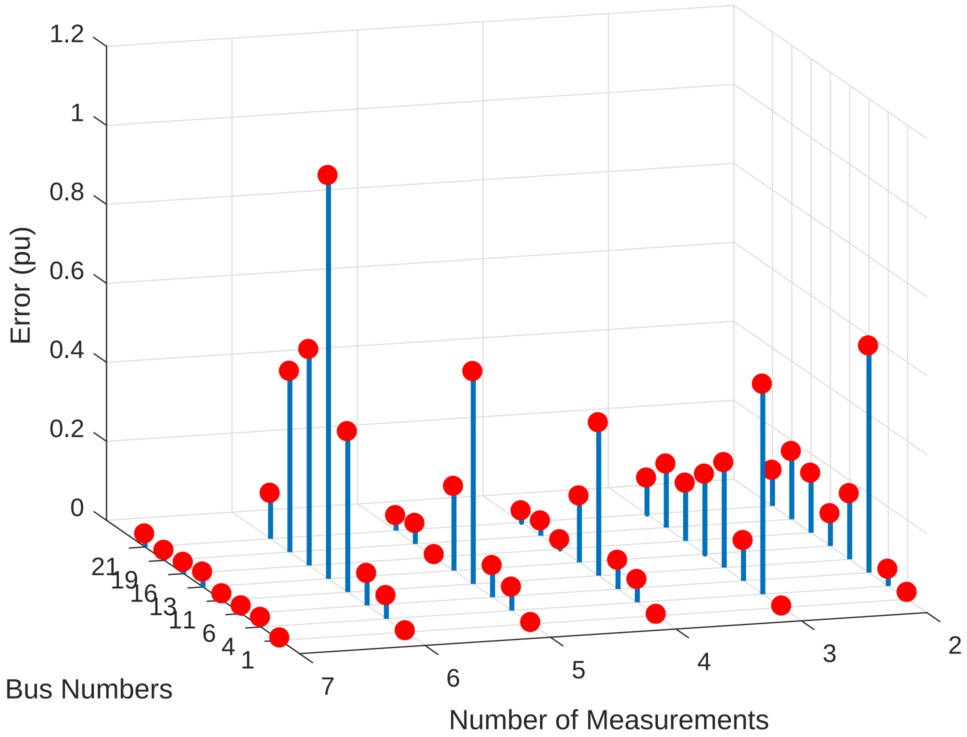


(a) Magnitude of the Errors

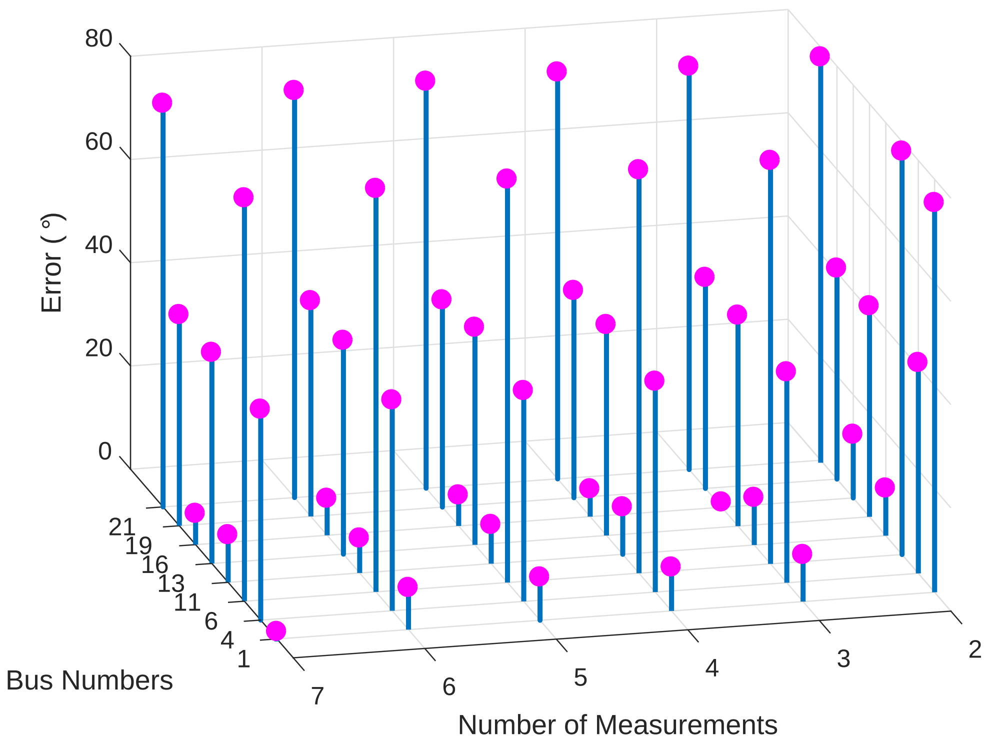


(b) Error in the Angles

Figure 4.20: Comparison of Estimated and Measured Values (5^{th} harmonic) with Different Number of Measurements



(a) Magnitude of the Errors

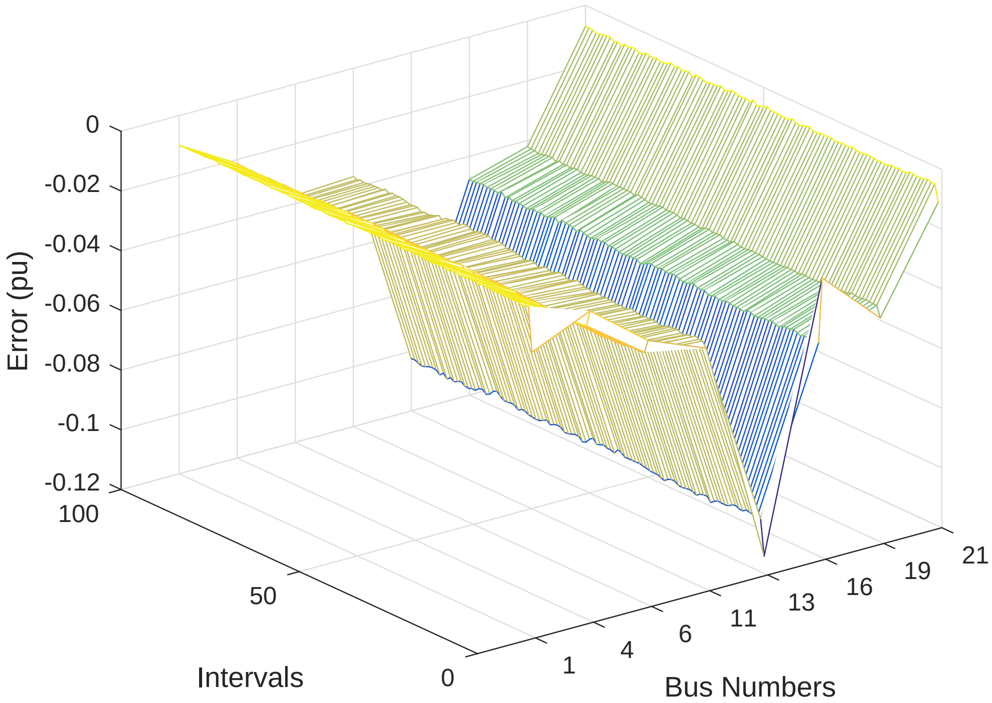


(b) Error in the Angles

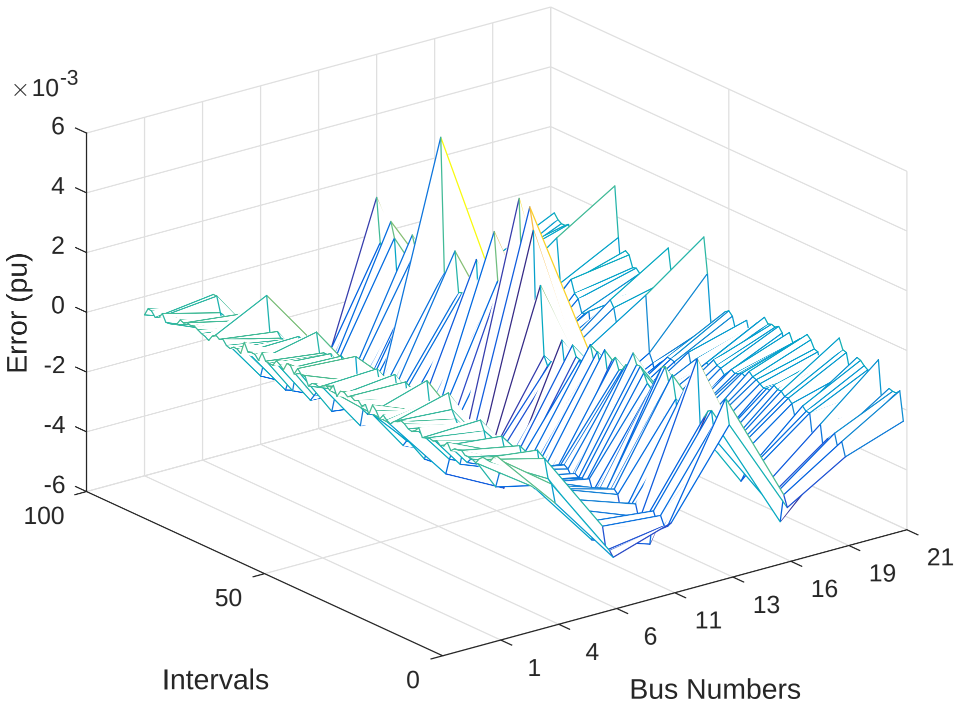
Figure 4.21: Comparison of Estimated and Measured Values (7^{th} Harmonic) with Different Number of Measurements

4.3.3 Variation of Error in Magnitude and Angle with Time Intervals

In this section, the variation of estimation error with time has been shown. Although the 6-second snapshots for 7800 intervals are available, only the first 100 intervals have been selected in this section for demonstration purpose. Figs. 4.22, 4.23, 4.24 and 4.25 demonstrate the error in the magnitudes and the angles. It is observed that the estimates are consistently close to the actual measured values. Even though different sets of harmonic values are obtained for different time intervals for a given set of buses with known harmonics, HSE is providing reliable estimates. This consistent closeness of the estimates to the measurements over many intervals is actually evidence of the robustness of the method used for estimation.

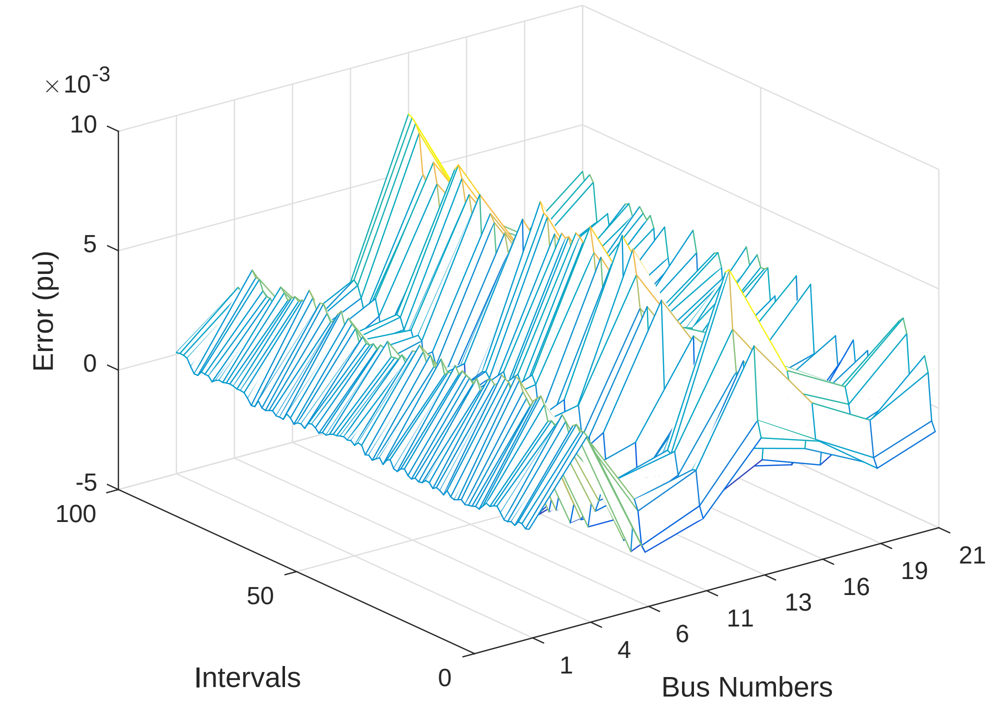


(a) Harmonic Order: 1

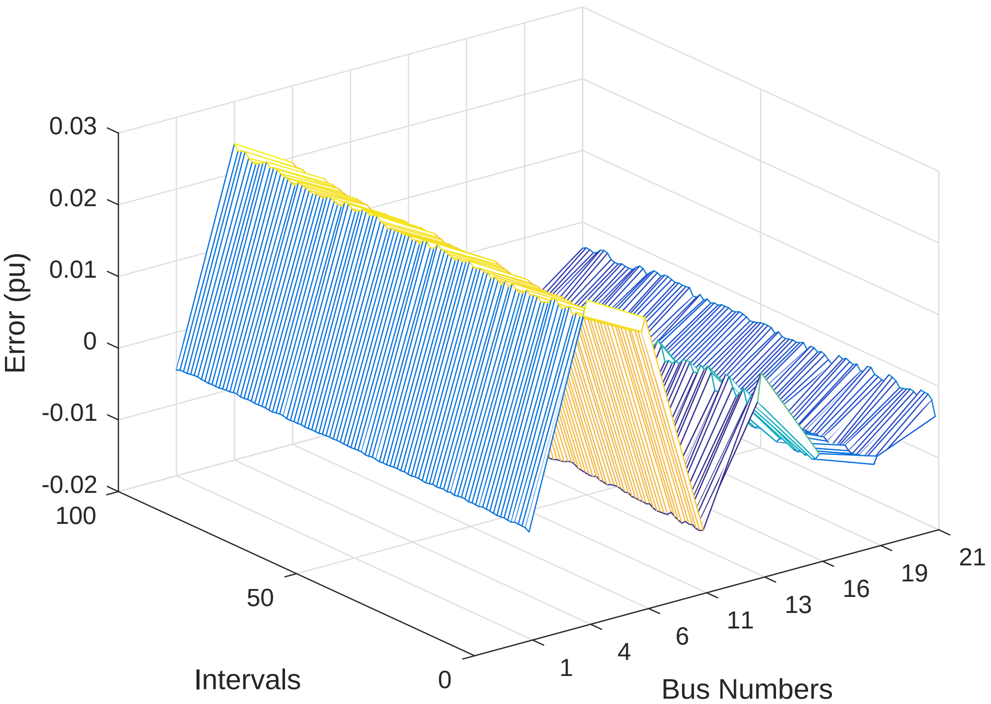


(b) Harmonic Order: 3

Figure 4.22: Variation of Magnitude Error (for 1^{st} and 3^{rd} Harmonic) for 100 Intervals (6 seconds each)

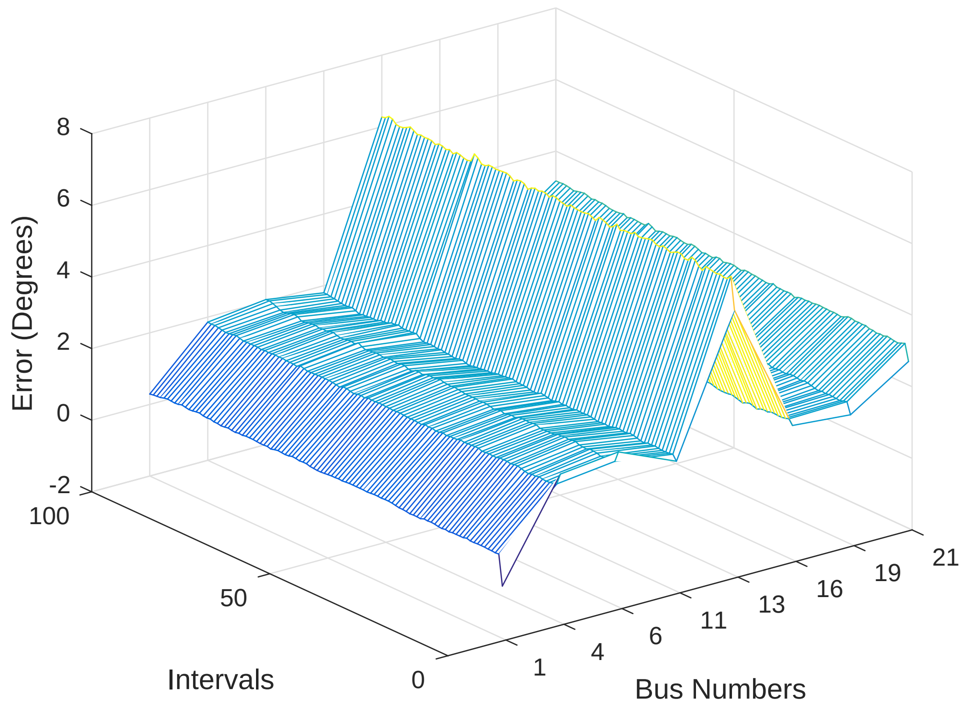


(a) Harmonic Order: 5

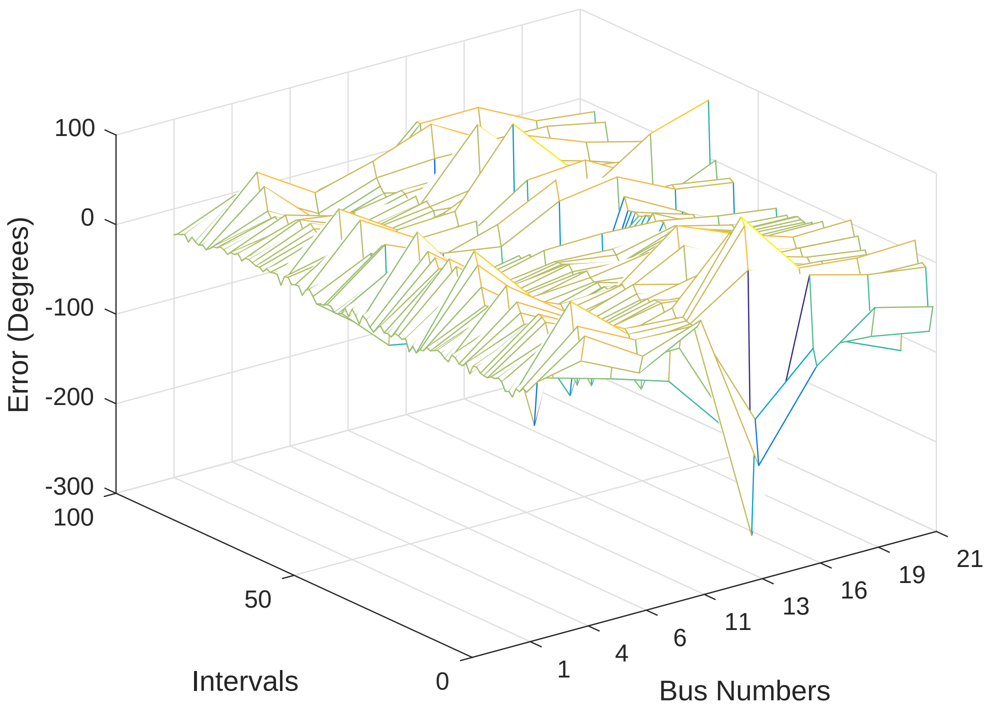


(b) Harmonic Order: 7

Figure 4.23: Variation of Magnitude Error (for 5^{th} and 7^{th} Harmonic) for 100 Intervals (6 seconds each)

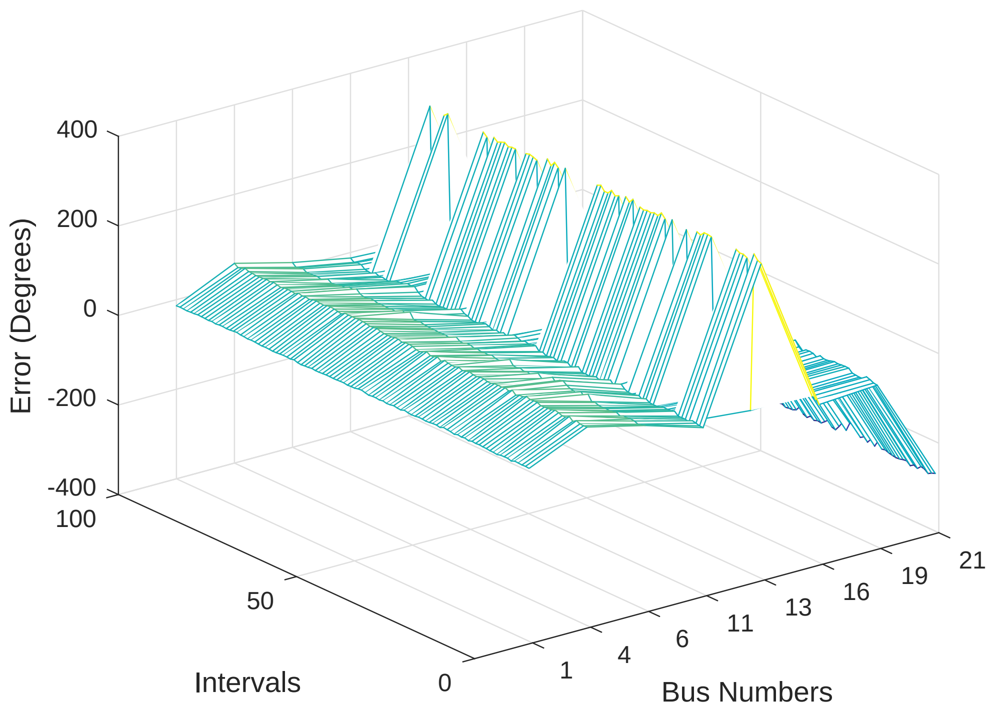


(a) Harmonic Order: 1

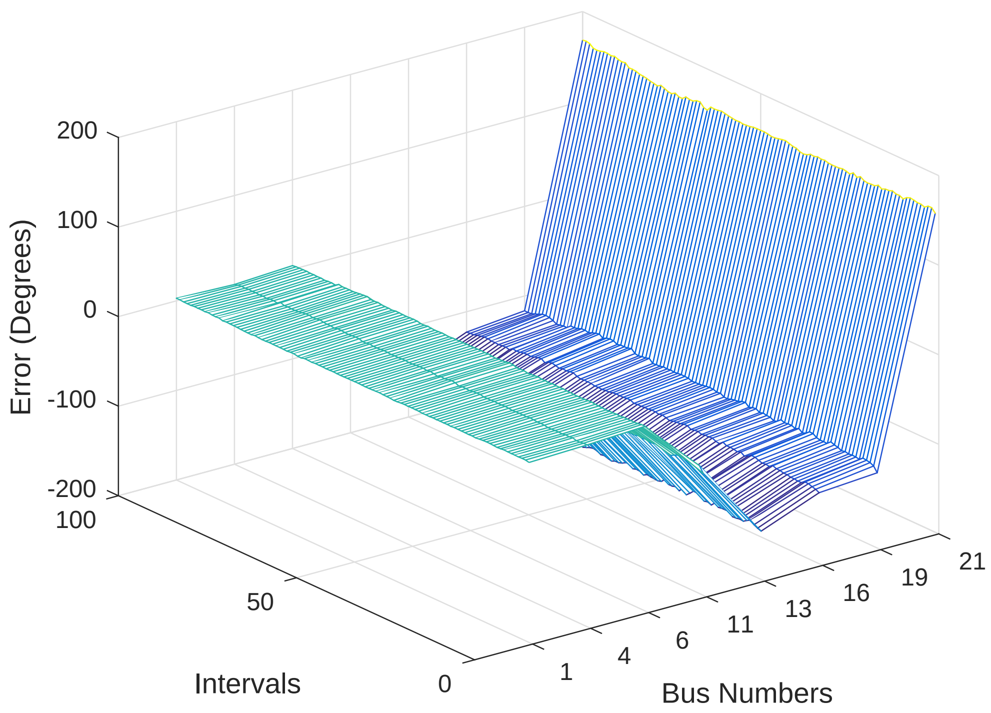


(b) Harmonic Order: 3

Figure 4.24: Variation of Angle Error (for 1^{st} and 3^{rd} Harmonic) for 100 Intervals (6 seconds each)



(a) Harmonic Order: 5

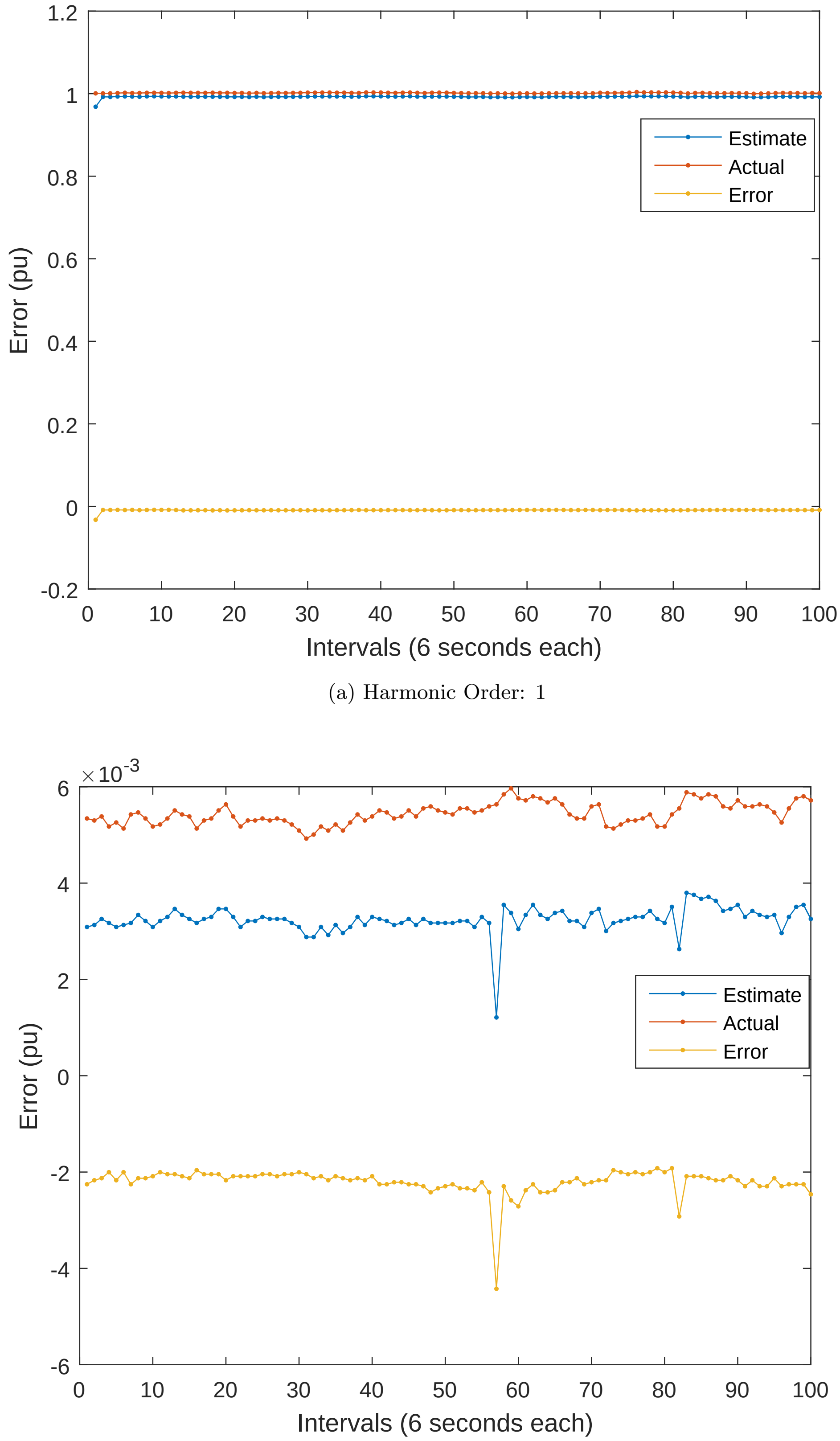


(b) Harmonic Order: 7

Figure 4.25: Variation of Angle Error (for 5^{th} and 7^{th} Harmonic) for 100 Intervals (6 seconds each)

4.3.3.1 Magnitude and Angle Error Variation at Bus '11'

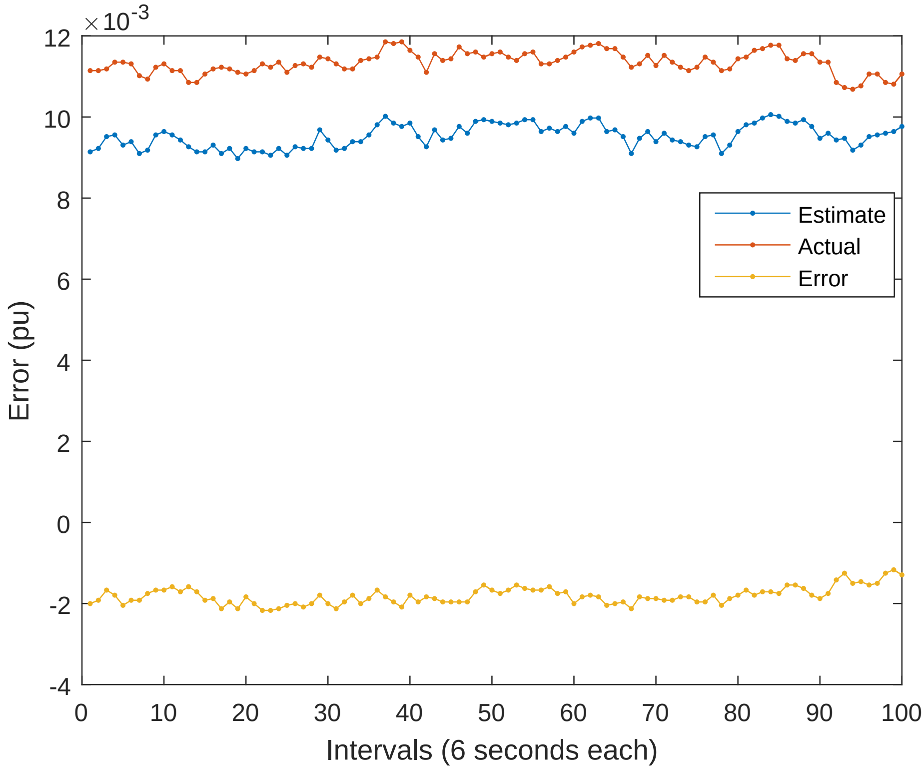
The variation of errors for the intervals is also shown for one of the buses i.e. Bus '11' as a sample. Figs. 4.26 and 4.27 show how the error in magnitudes are varying. Similarly, Figs. 4.28 and 4.29 show the variation of error in the angles. Fig. 4.30 shows the magnitude of the errors between the phasors of the estimates and the measured values for 100 intervals at Bus '11'. This figure demonstrates the closeness of the estimates with the measurements for several observations.



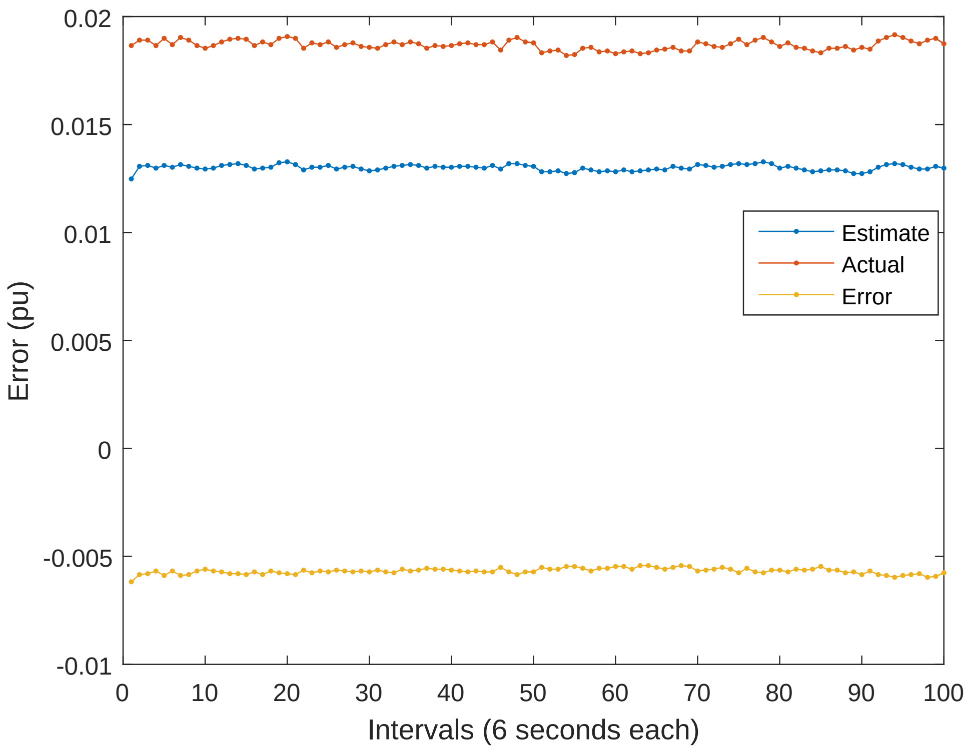
(a) Harmonic Order: 1

(b) Harmonic Order: 3

Figure 4.26: Variation of Magnitude Error (for 1^{st} and 3^{rd} Harmonic) at Bus '11'

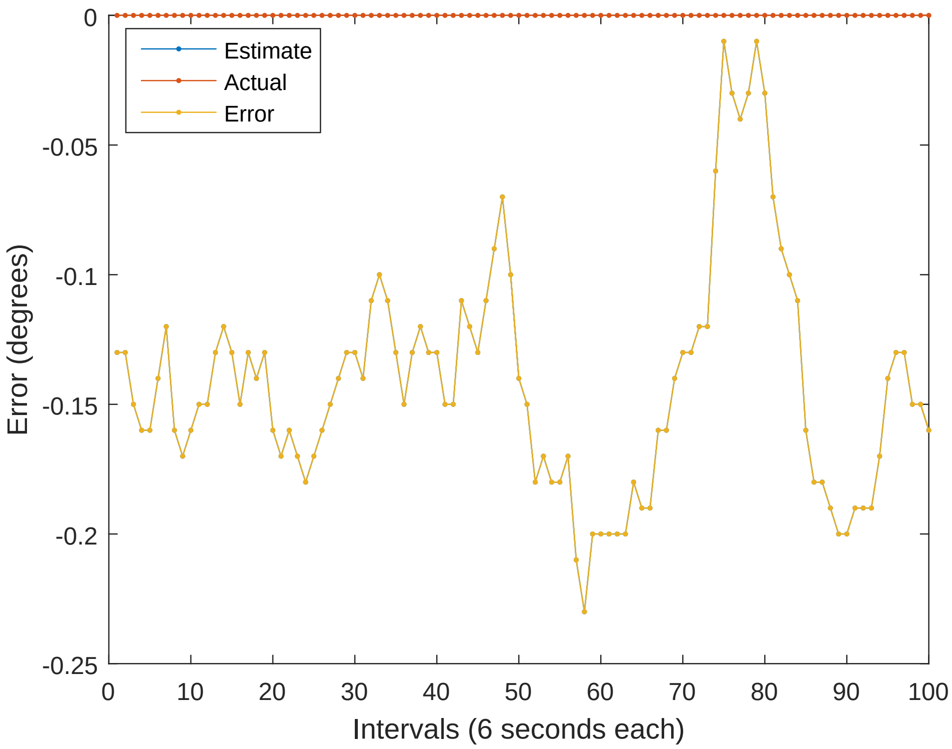


(a) Harmonic Order: 5

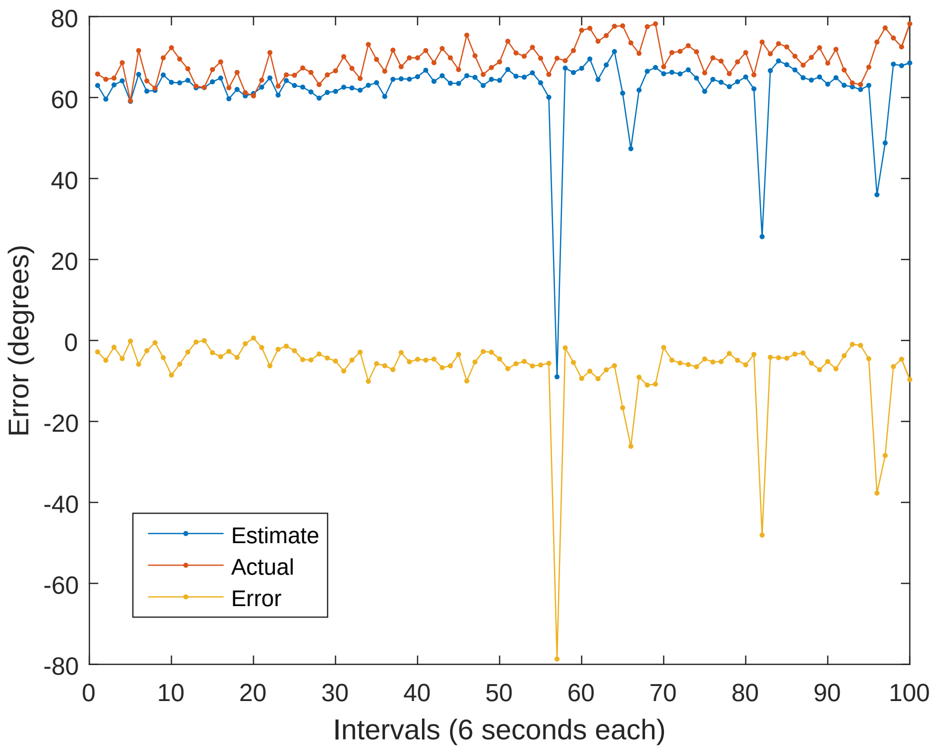


(b) Harmonic Order: 7

Figure 4.27: Variation of Magnitude Error (for 5^{th} and 7^{th} Harmonic) at Bus '11'

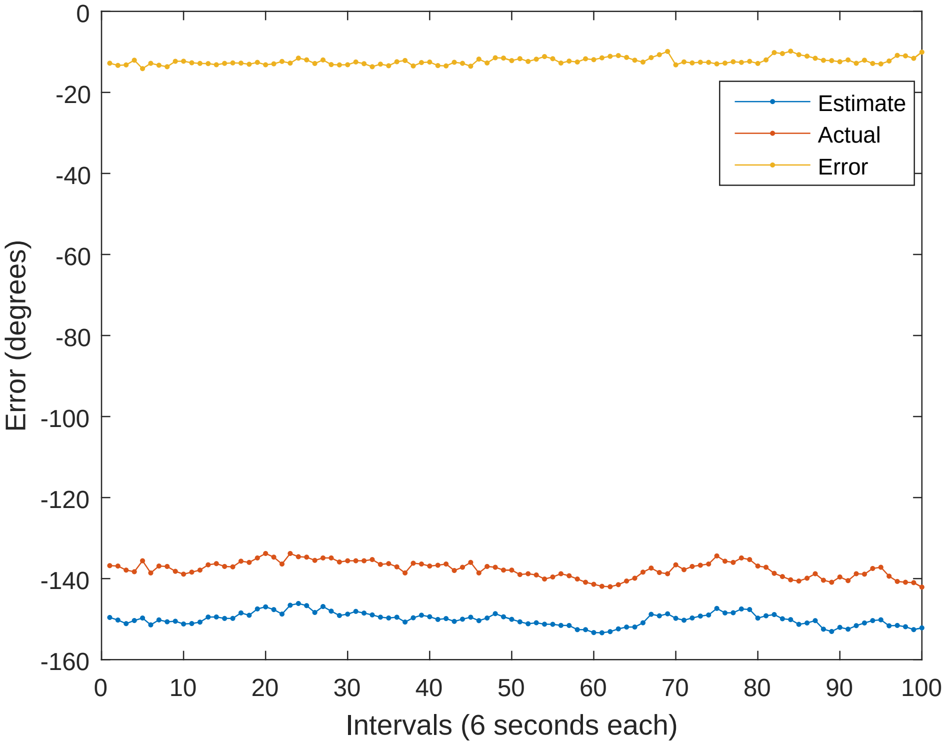


(a) Harmonic Order: 1

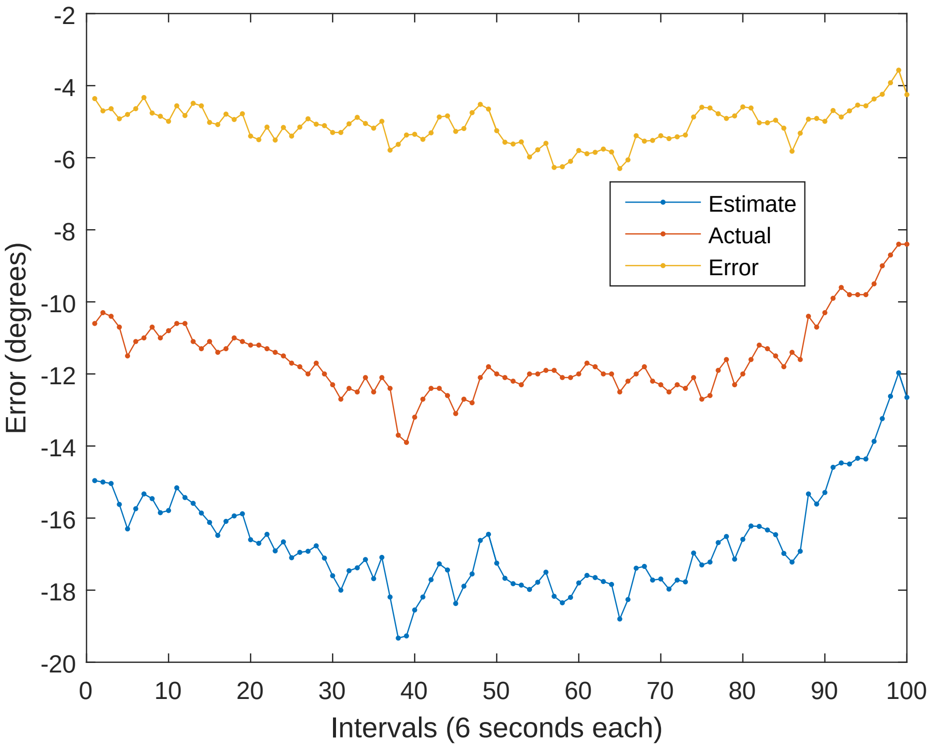


(b) Harmonic Order: 3

Figure 4.28: Variation of Angle Error (for 1^{st} and 3^{rd} Harmonic) at Bus '11'



(a) Harmonic Order: 5



(b) Harmonic Order: 7

Figure 4.29: Variation of Angle Error (for 5^{th} and 7^{th} Harmonic) at Bus '11'

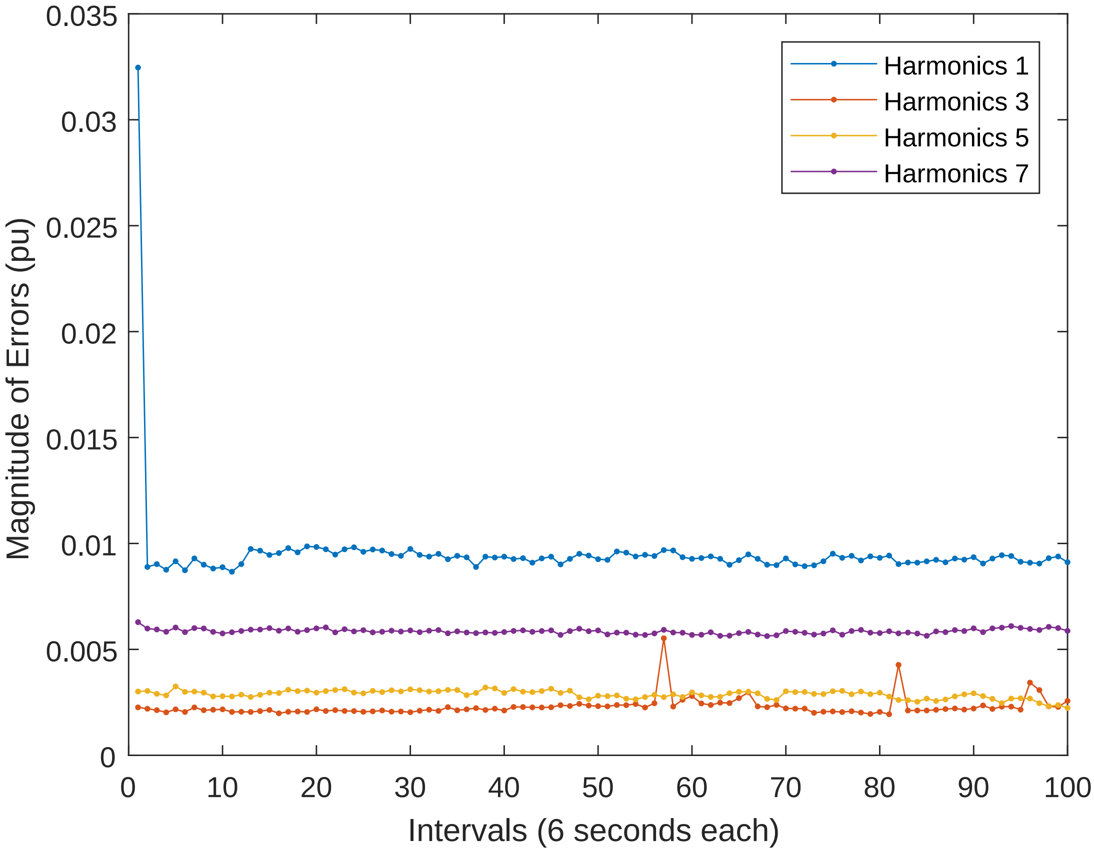
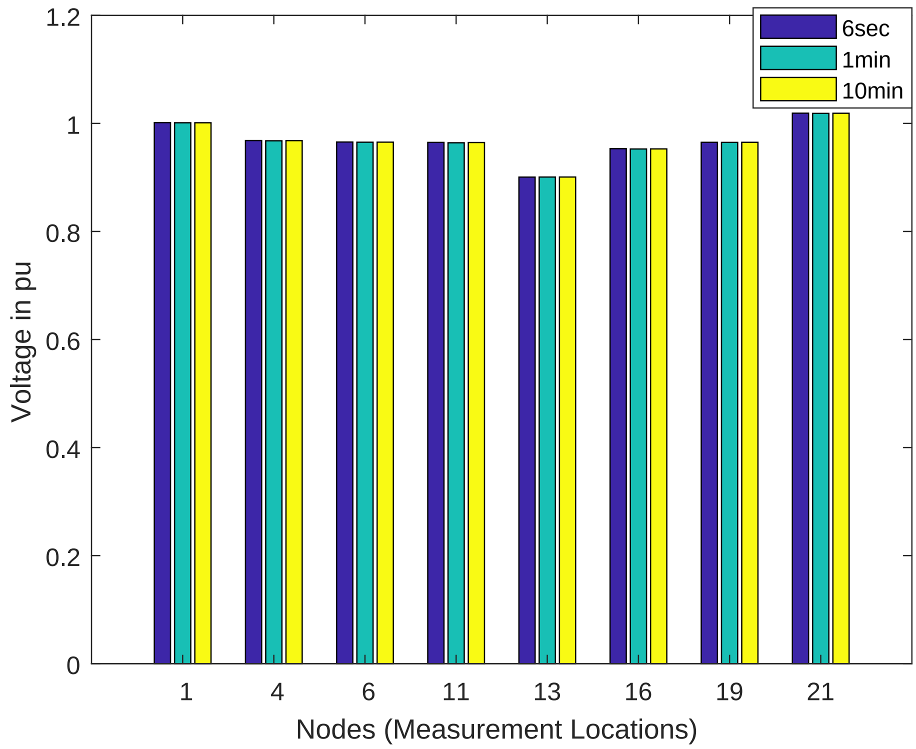


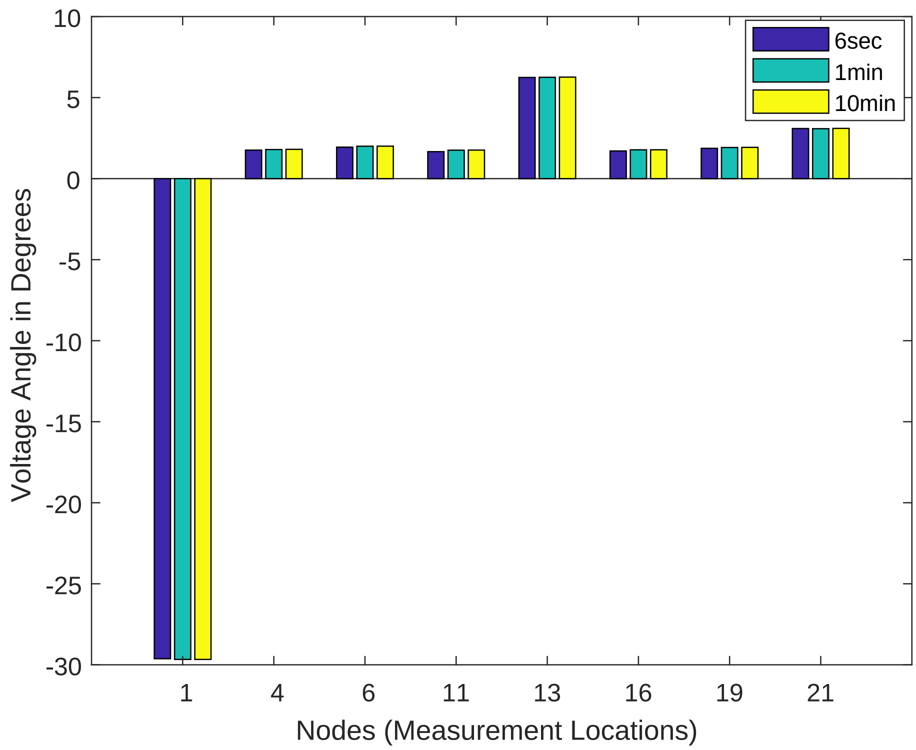
Figure 4.30: Magnitude of Errors at Bus '11'

4.3.4 Effect of Data Aggregation Intervals

Figures 4.31a to 4.38b demonstrate the estimates when the data aggregation time is varied. Three data aggregation time intervals '6 seconds', '1 minute' and '10 minutes' have been selected. The same data that were logged by PQ data loggers for either '3 seconds' or '6 seconds' (mentioned in Section 4.1.1) are used. These data are aggregated for the three time intervals '6 seconds', '1 minute' and '10 minutes'. Using these data as measurements, the harmonic state estimates for each case are obtained and compared. It is seen that there is no significant variation in the magnitudes.

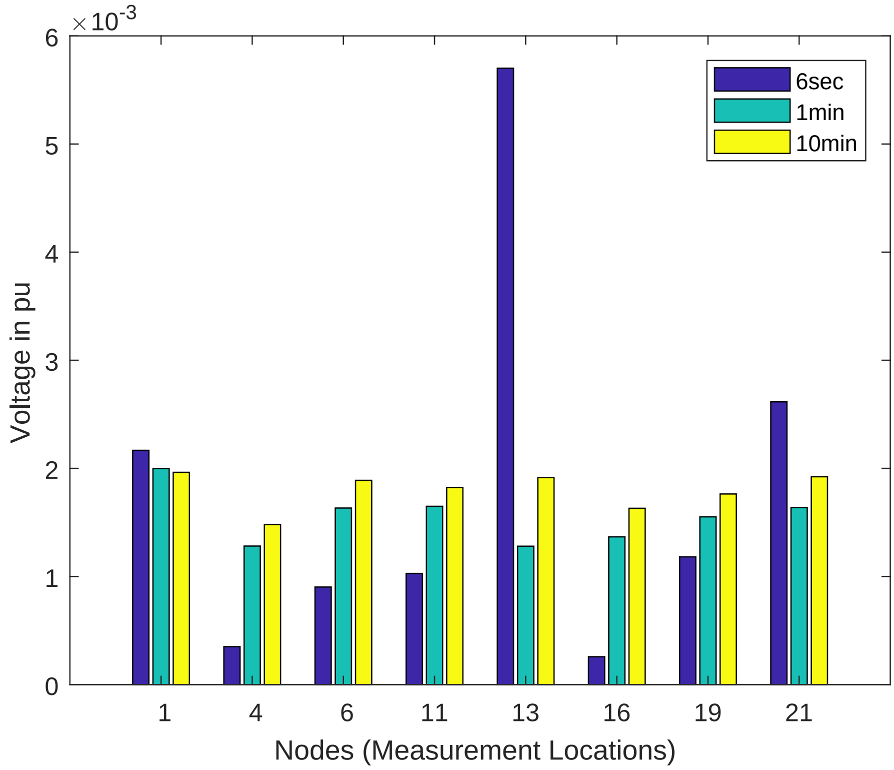


(a)

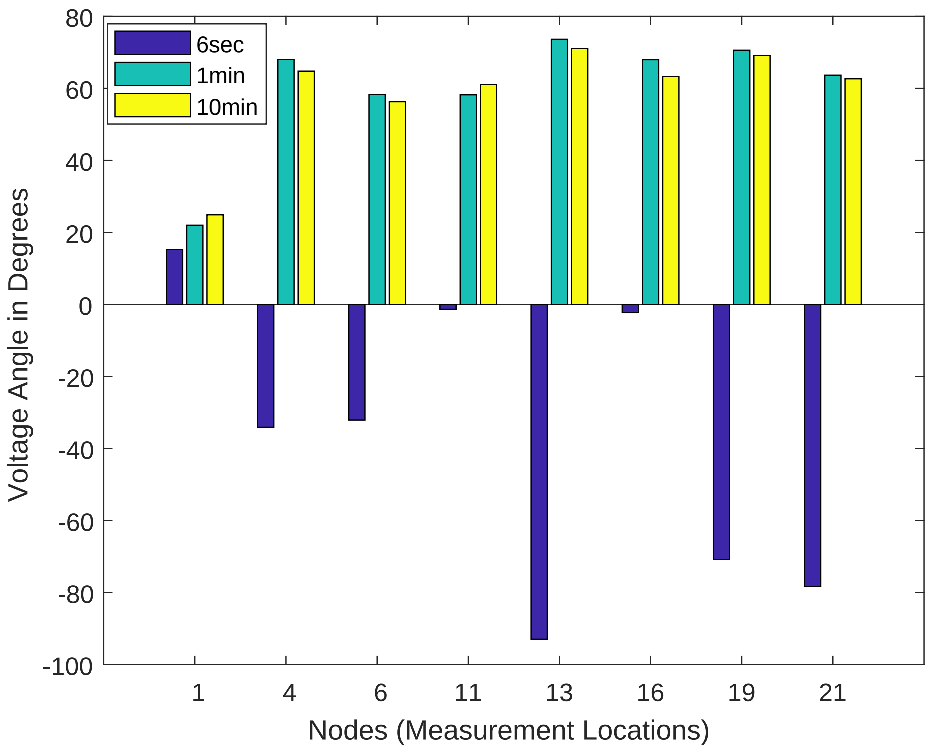


(b)

Figure 4.31: Estimated Values with Different Aggregation Intervals for Fundamental

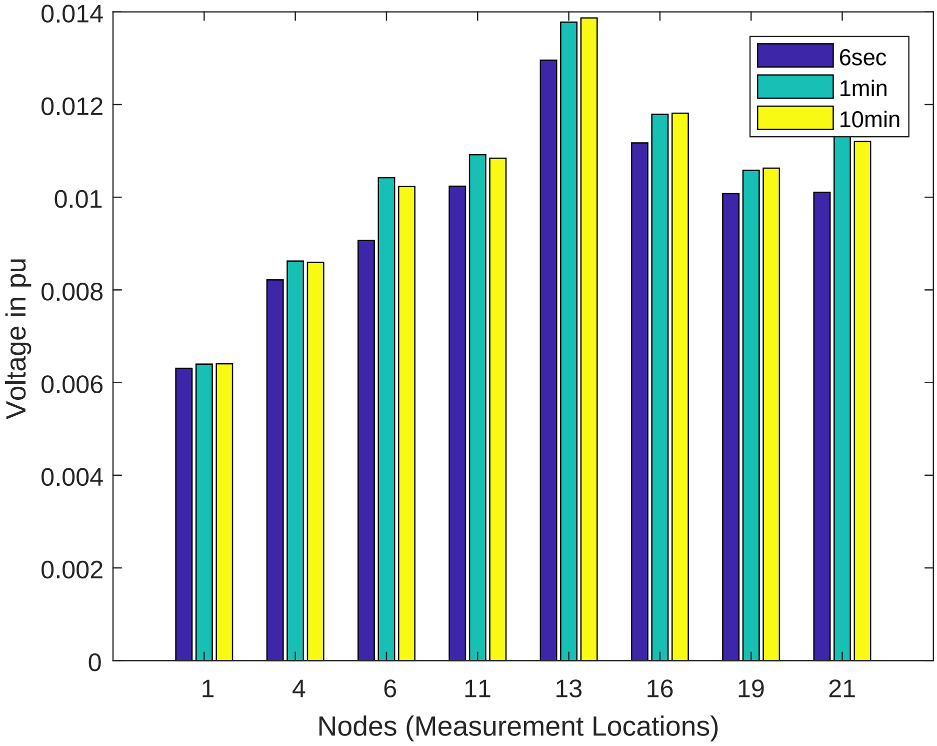


(a) Magnitude of Errors

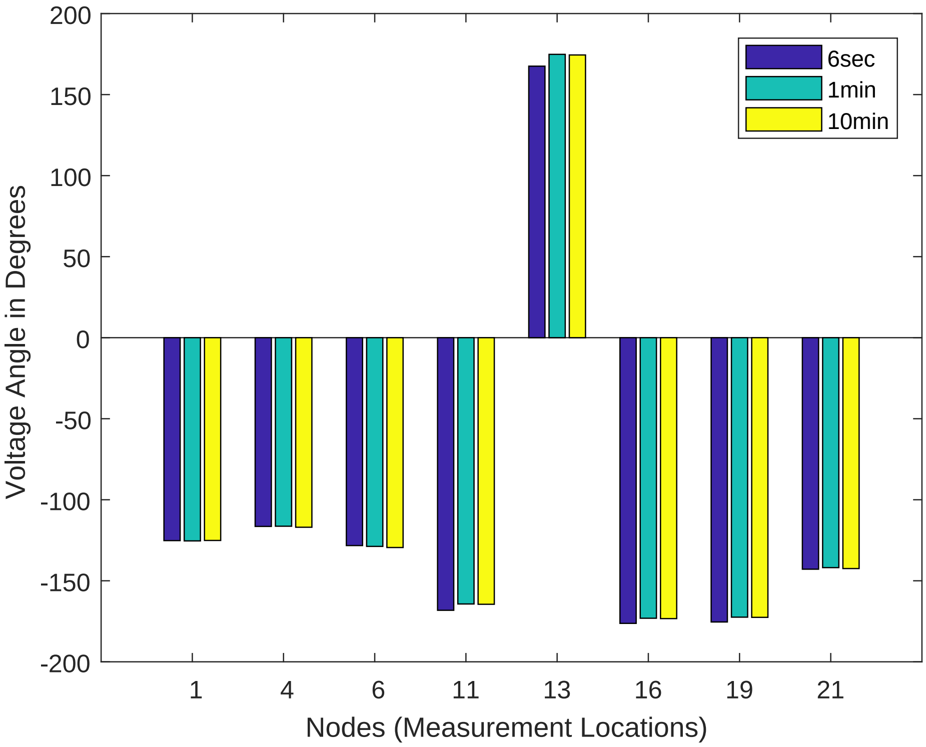


(b) Errors in Angles

Figure 4.32: Estimated Values with Different Aggregation Intervals for 3^{rd} Harmonics



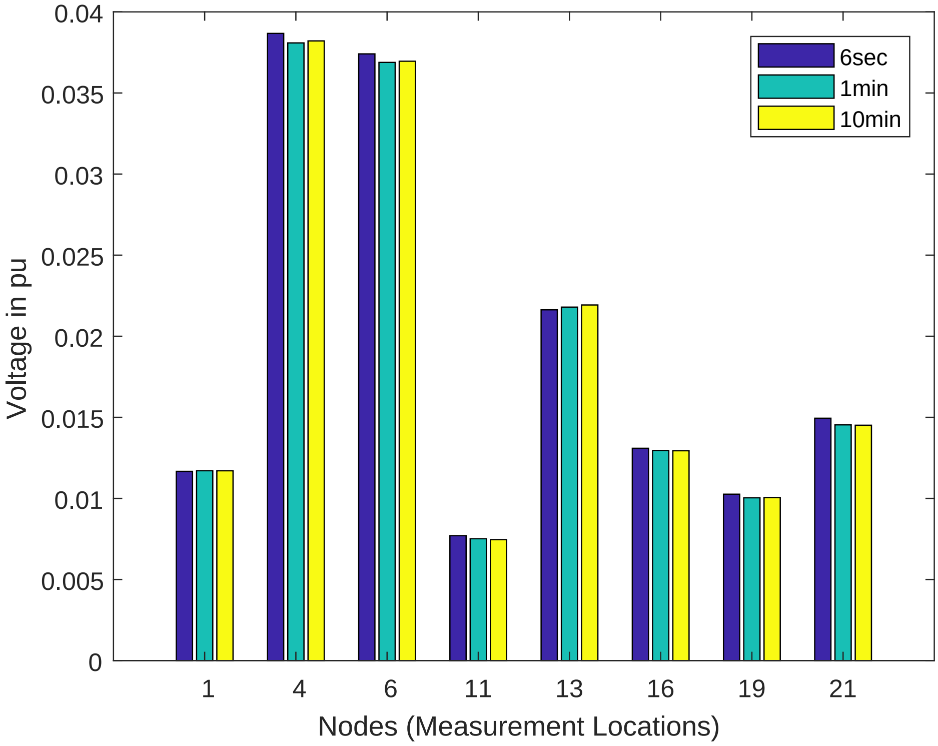
(a) Magnitude of Errors



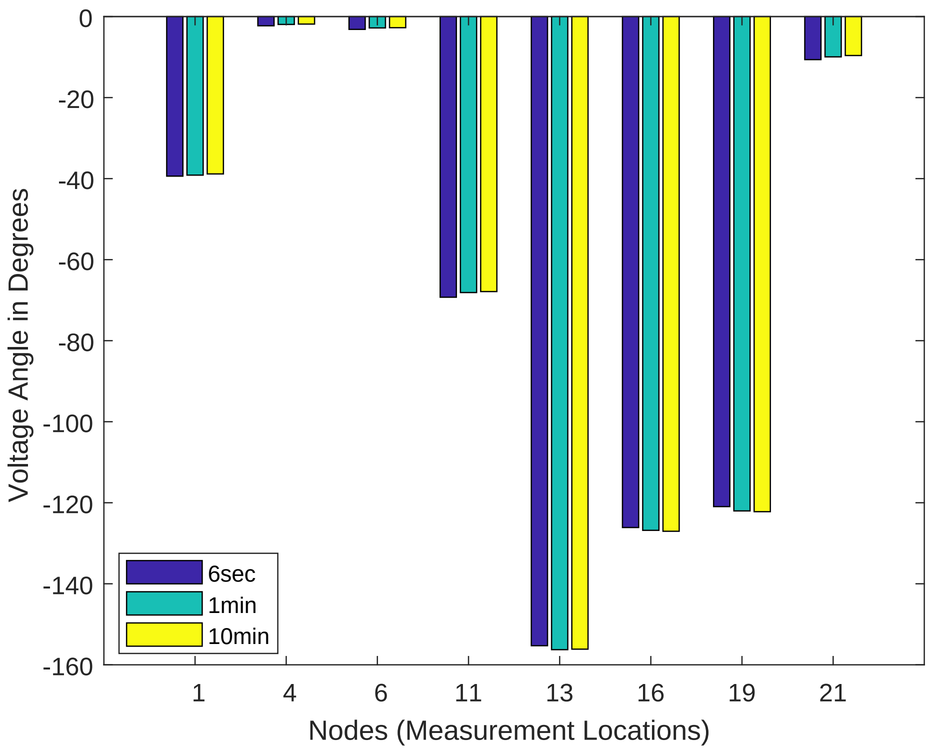
0

(b) Errors in Angles

Figure 4.33: Estimated Values with Different Aggregation Intervals for 5^{th} Harmonics



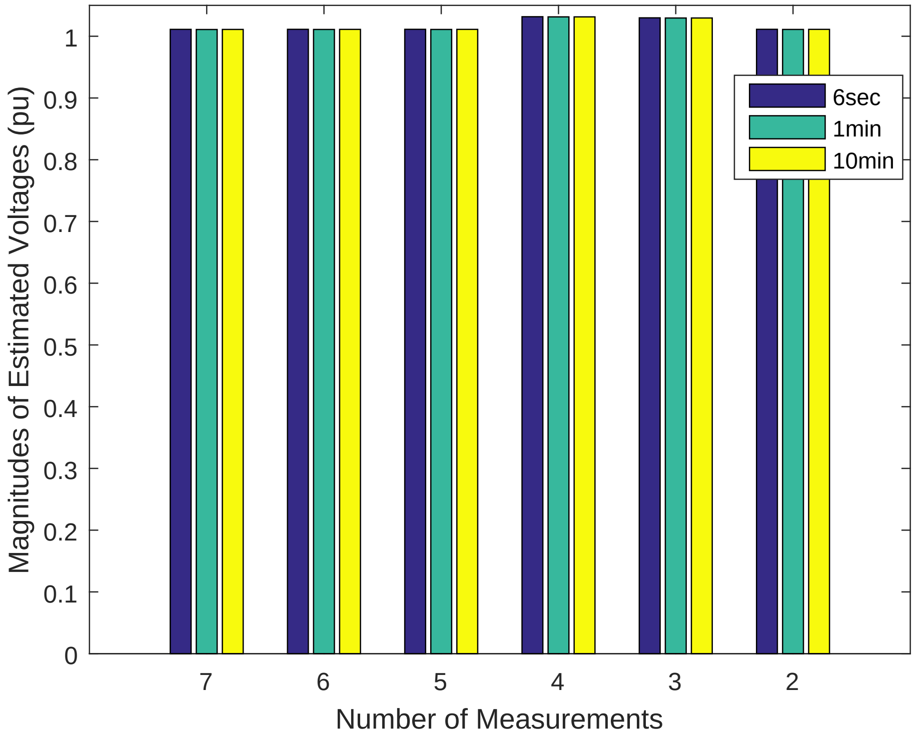
(a) Magnitude of Errors



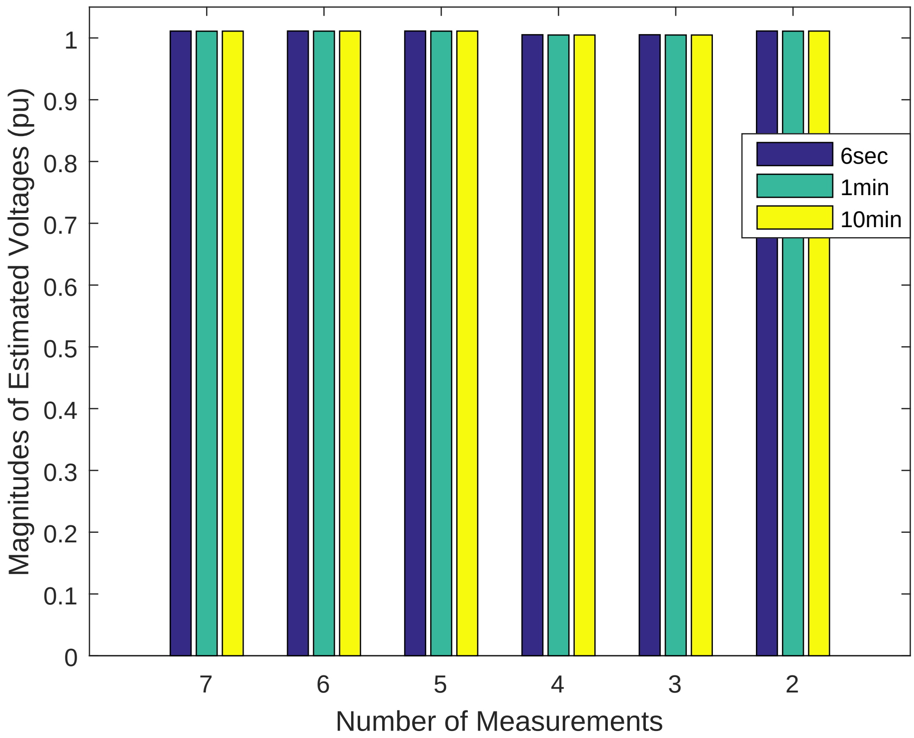
(b) Errors in Angles

Figure 4.34: Estimated Values with Different Aggregation Intervals for 7^{th} Harmonics

Comparison of the estimates is also repeated for the cases when different number of measurements are available. Seven to two measurement locations are taken systematically, and the estimates are obtained for the three data aggregation time intervals. They are shown for the odd harmonics '1', '3', '5' and '7' at two buses '11' and '16' via Figs. 4.35a to 4.38b. Small differences in the estimates are observed but there are no significant variations.

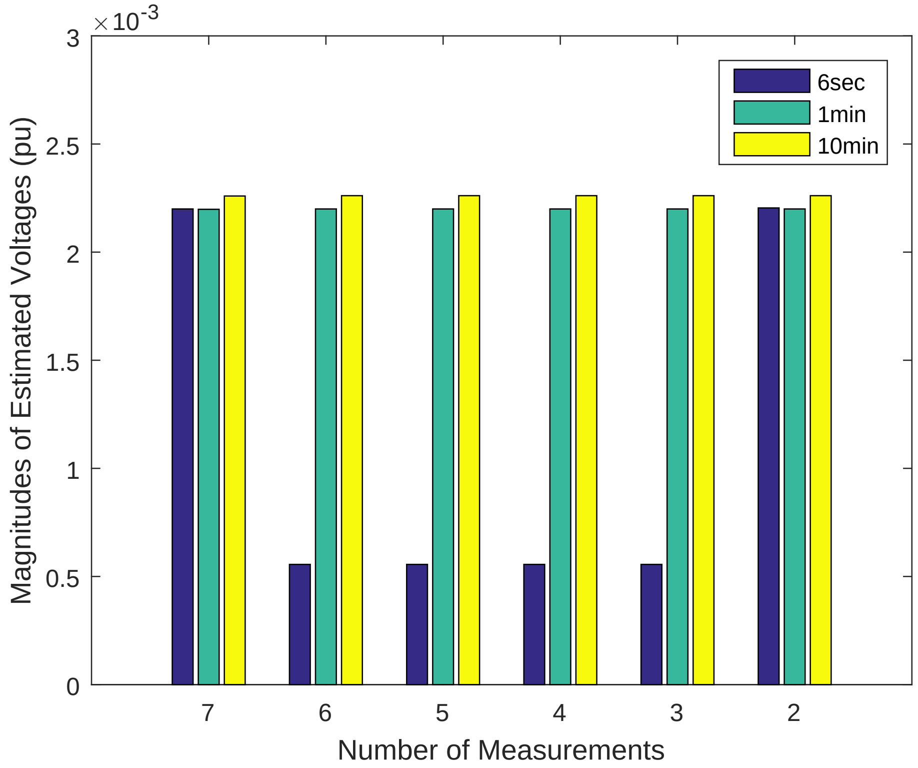


(a) Bus 11

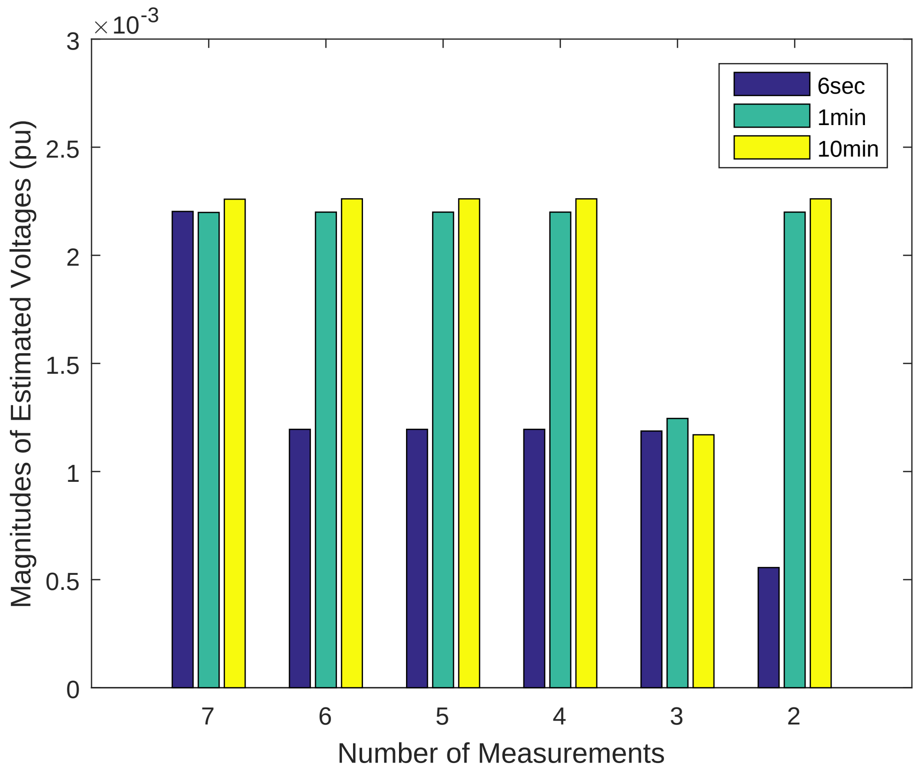


(b) Bus 16

Figure 4.35: Estimated Fundamental Voltages for Different Aggregation Time and Number of Measurements

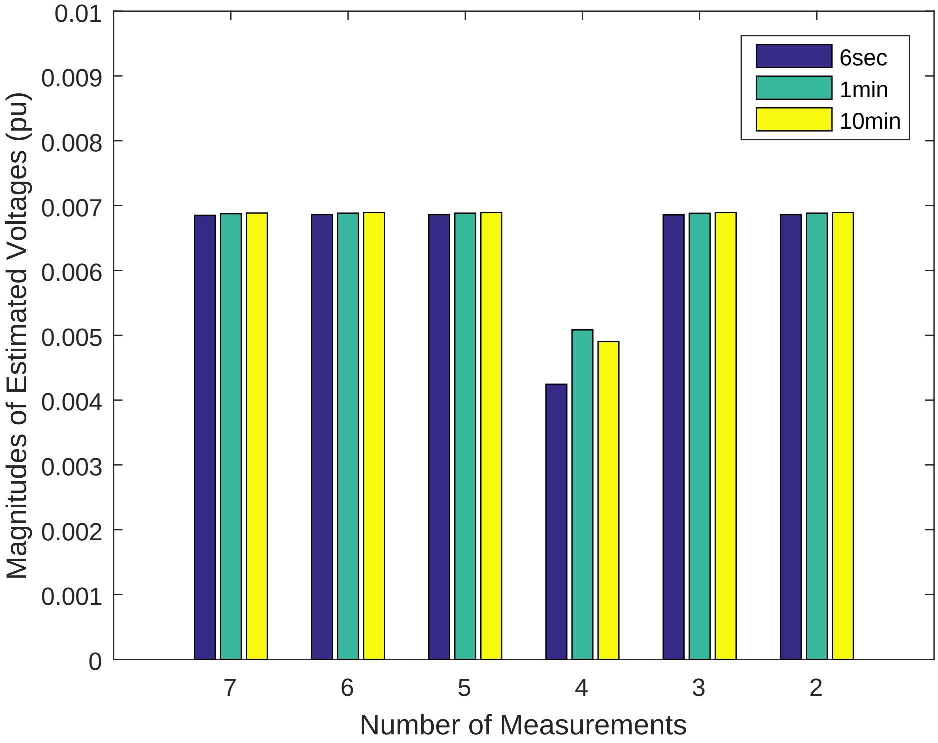


(a) Bus 11

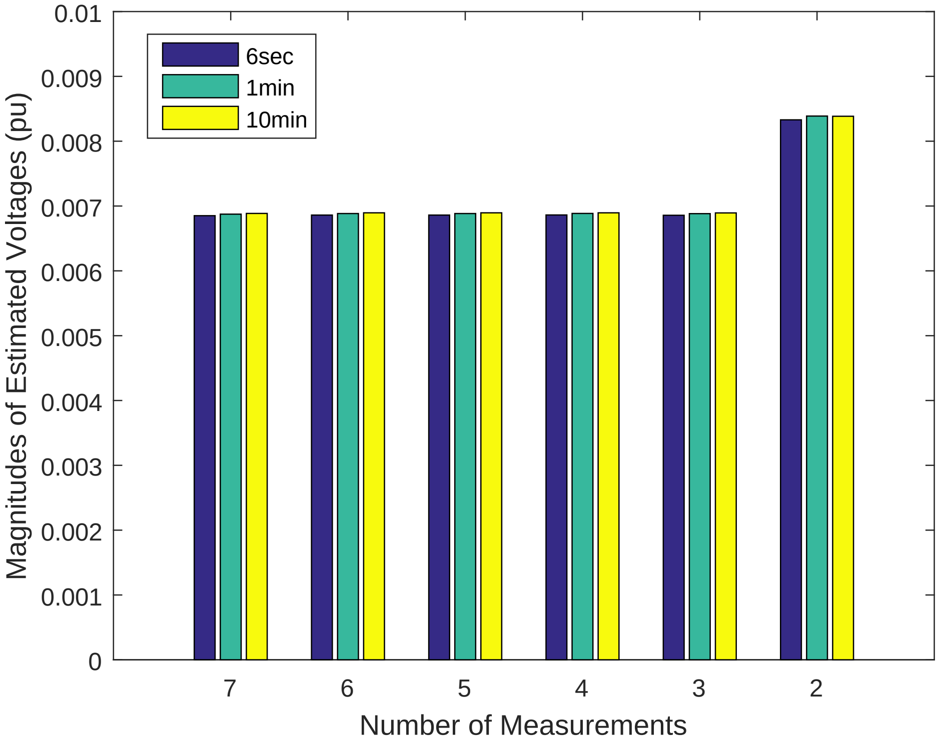


(b) Bus 16

Figure 4.36: Estimated 3^{rd} Harmonic Voltages for Different Aggregation Time and Number of Measurements

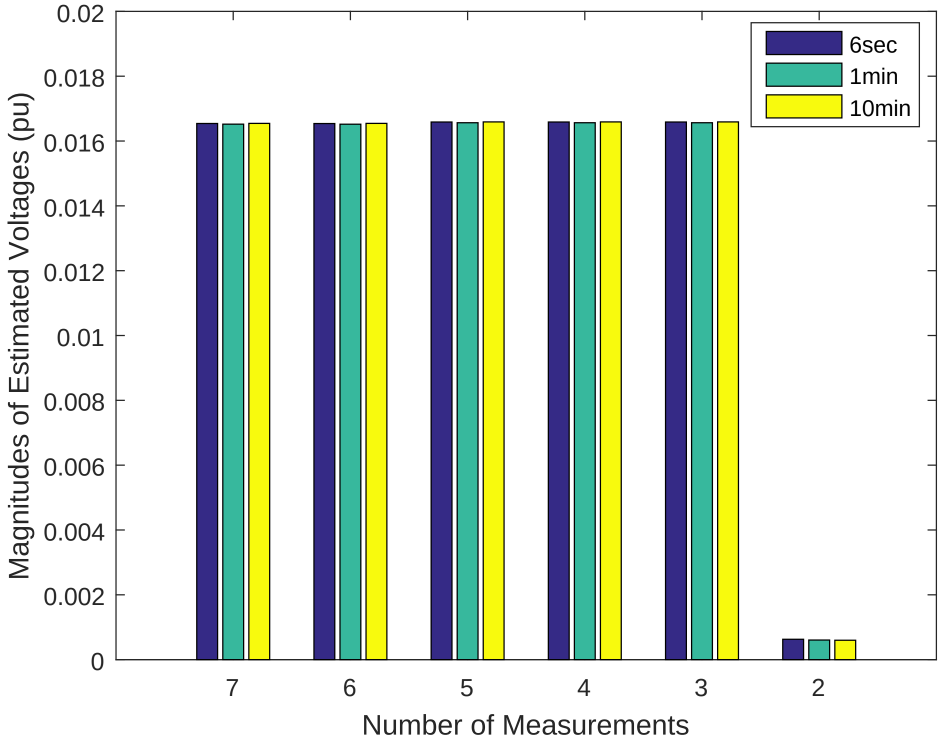


(a) Bus 11

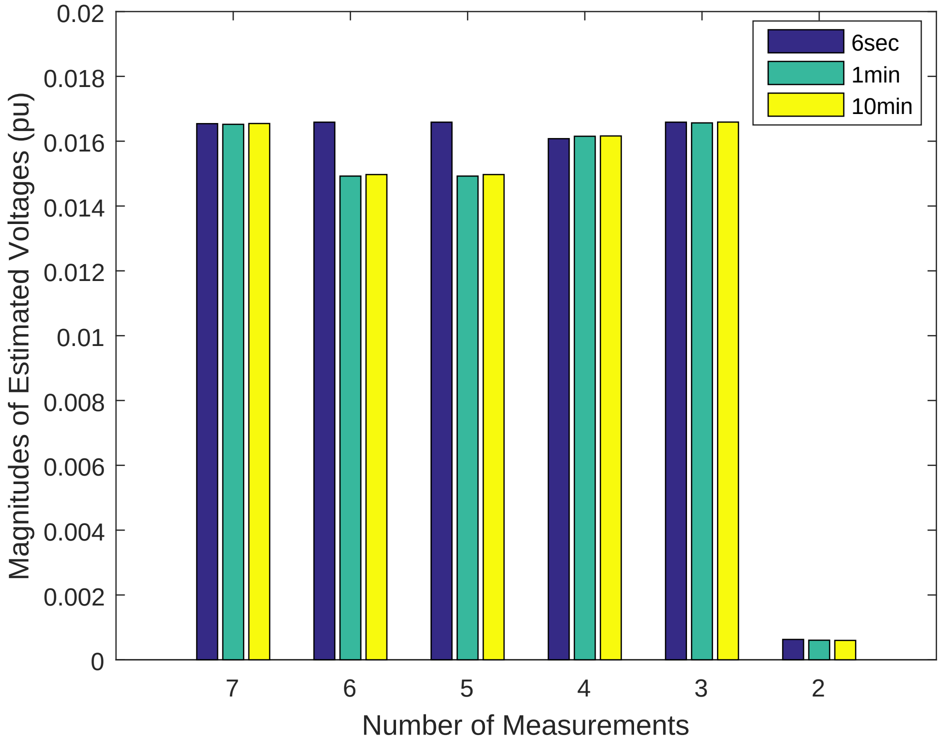


(b) Bus 16

Figure 4.37: Estimated 5^{th} Harmonic Voltages for Different Aggregation Time and Number of Measurements



(a) Bus 11



(b) Bus 16

Figure 4.38: Estimated 7^{th} Harmonic Voltages for Different Aggregation Time and Number of Measurements

4.4 OPTIMAL LOCATION OF MEASUREMENTS

Since measuring equipment is costly, it is always desired that a minimum number of PQ monitors are used for measurements without compromising on the accuracy of estimation. Therefore, it is necessary to find the minimum number of locations for the placement of measuring equipment which can make all the buses in the system observable. Complete harmonic profile of the line can be obtained only if the buses are completely observable. There can be more than one combination of measurements for which the number of observable nodes is same. However, those combinations which can give minimum error in the estimates should be selected as the optimal locations for measurements. Fig. 4.18a shows the number of observable nodes when the number of available measurements is varied between seven to two. Figs. 4.17a, 4.17b and 4.18 (mentioned in Section 4.3.2) show the magnitudes, angles and observability when there are different numbers of available measurements. For the same scenario, Table 4.2 presents the number of observable nodes and Table 4.3 presents the errors (magnitude only). It can be seen that the minimum number of measurements that could give maximum observable nodes is five. This is shown in the third column in Table 4.2. Corresponding to the column '3', the minimum error observed in Table 4.3 is the second row. This is the case when the measurements are placed in the buses '1', '6', '11', '16' and '19'. The other option could be '1', '4', '11', '16' and '19'. However, the error obtained in the latter combination is slightly more than that in the former. Although the errors look minimum in the case when the measurements at Bus '1' are absent, this case cannot be considered for the selection of optimal locations. This is because the observability is very poor and the harmonic estimates obtained for the unobservable buses cannot be relied upon.

Table 4.2: Number of Observable Buses with Different Number of Monitors

	Number of Observable Buses Including Phases					
Number of Monitors →	7	6	5	4	3	2
Bus Number (as in Fig. 4.1) with No Monitor						
1	21	21	21	18	12	6
4	63	63	63	54	12	9
6	63	63	63	54	26	18
11	63	63	63	54	26	18
13	63	63	63	36	24	18
16	63	63	63	42	26	17
19	63	63	63	54	26	17
21	63	63	63	54	26	18

Table 4.3: Error Between the Estimates and the Measurements for Different Measurement Combinations

	Magnitude of the Errors (pu)					
Number of Monitors →	7	6	5	4	3	2
Bus Number(as in Fig. 4.1) with No Monitor						
1	0.012	0.010	0.011	0.018	0.018	1.111
4	0.037	0.070	0.053	0.077	2.366	4.866
6	0.049	0.338	0.056	0.083	8.482	20.032
11	0.046	0.334	0.174	0.207	22.440	6.101
13	0.146	0.929	0.487	6.314	7.887	3.110
16	0.056	0.433	1.786	0.263	12.301	5.429
19	0.053	0.404	1.694	1.312	13.601	6.180
21	0.056	0.327	0.364	2.228	10.038	4.441

Chapter 5

POWER QUALITY ASSESSMENT

This chapter presents and describes the tool developed for power quality assessment. The graphical user interfaces developed for the calculation of power quality indices are also shown with some examples. Comparison of the calculated power quality indices with the power quality standards, based on the Power Quality Guidelines, is also demonstrated. The data collected from Unison Networks Limited is described and the Power Quality Guidelines, taken as standards, are briefed.

5.1 DATA COLLECTION

The power quality data is logged by 176 EDMI meters installed at the low voltage sides of the various distribution transformers in four different regions viz. Hastings, Napier, Rotorua and Taupo of Unison Networks Limited. These regions are referred to as region 'A', 'B', 'C' and 'D' respectively from here onwards. There are 47 different parameters over which information is collected from each meter. There are 48, 46, 53 and 29 EDMI meters in Hastings, Napier, Rotorua and Taupo respectively. Data is initially stored in PI data server at Unison. The ten minute r.m.s data for voltages and currents has been extracted for a year and saved in '.mat' files using MATLAB. This has been repeated for all the meters of all the mentioned locations of Unison. These '.mat' files have been used as the source files for further processing and assessment.

5.2 THE POWER QUALITY GUIDELINES

The EEA Power Quality Guidelines [Watson *et al.* 2013] have been prepared by Electricity Engineers Association of New Zealand in association with University of Canterbury, New Zealand and University of Wollongong, Australia to provide guidance on power quality in general for public AC power systems to meet the requirements of Electricity (Safety) Regulations 2010 of New Zealand. It has followed various Acts and Regulations of New Zealand besides various other series of relevant standards and documents covered in International Electrotechnical Commission (IEC), Australian Standards (AS)/ New Zealand Standards (NZS), European and

IEEE Standards. It has mentioned different types of PQ disturbances and defined the procedures for PQ assessment. The basic aim is to set the general reference Guidelines for the management of PQ disturbances. The PQ disturbances, which are covered in the Guidelines, are overvoltage, undervoltage, voltage unbalance, harmonics, interharmonics, voltage fluctuations and flicker, voltage dips or sags, transients, voltage swells, and frequency deviations. The Guidelines have mentioned the definitions of Power Quality and the associated terminologies, the types of disturbances and the assessment procedure. The sources, effects, limits and the emission assessment have been documented. The mitigation measures have also been mentioned briefly for each kind of PQ disturbance discussed. Following the assessment procedures mentioned in the Guidelines, various PQ performance indices have to be calculated using the formulae mentioned in it. The values have to be compared with the compliance margins to ensure whether the performance are in acceptable window of values. The Guidelines for compliance checking can be summarized in Table 5.1.

Table 5.1: Summary of Guidelines for compliance check[Watson *et al.* 2013]

Index	Emission Assessment	Limits for LV
Steady-state-voltage	99^{th} percentile of 10 minute rms over a week	230V+6% of 230V
	1^{st} percentile of 10 minute rms over a week	230V-6% of 230V
Voltage Unbalance	95^{th} percentile of 10 minute rms over a week	1% (NZECP), 2% (IEC)
Voltage Harmonics	95^{th} percentile of 10 minute rms over a week	Chart for Compatibility Level (CL) and Planning Levels (90% of CL)
Voltage Total Harmonic Distortion	95^{th} percentile of 10 minute rms over a week	7.2% for planning (90% of compatibility level)
Current Harmonics	Maximum value of 10 minute rms over a week	Limits allocated based on calculations
Current Total Harmonic Distortion	95^{th} percentile of 10 minute rms values over a week	Limits allocated based on calculations
Voltage Dip/Sag	Trigger Level set to 90% of the nominal voltage.	Event should fall in the allowable region of ITIC curve.
Voltage Swells	Trigger Level set to 110% of the nominal voltage.	Event should fall in the allowable region of ITIC curve.

5.3 DEVELOPMENT OF POWER QUALITY ASSESSMENT TOOL

An assessment tool (as shown in Fig. 5.1) has been developed for the assessment of power quality in MATLAB environment.

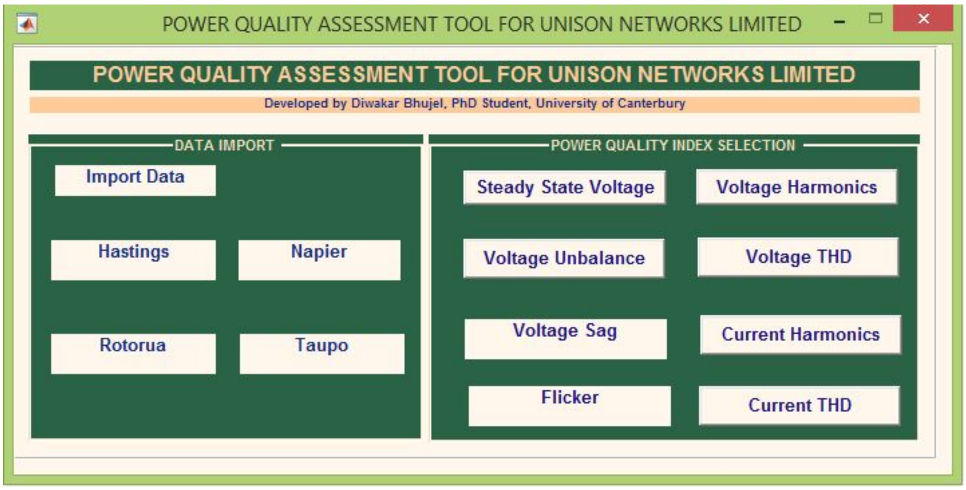


Figure 5.1: Main Graphical User Interface Window

Programs have been developed in Graphical User Interface Development Environment (GUIDE) of MATLAB for power quality assessment and visualization. It gives a flexibility for user to select and analyse the following power quality indices .

- Steady-state-voltage
- Voltage Harmonics
- Current Harmonics
- Voltage Total Harmonics Distortion
- Current Total Harmonics Distortion
- Voltage Unbalance Factor

5.3.1 Steady-state-voltage

A Graphical User Interface for the assessment of Steady-state-voltage is shown in Fig. 5.2. Its features and method of handling are described now.

Site Selection:

Users can select any of the four regions from the drop-down list. Any of the meters can be selected from the list, and the logged data for any phase can be analysed. Data is accessed from the source files and user is prompted for the selection of the start and end date and time. User can also select the weeks over which the steady-state-voltage assessment has to be performed.

Percentile Analysis:

The 99^{th} and the 1^{st} percentile values of the 10 minute r.m.s values for all the selected numbers of weeks are evaluated and compared. The maximum of the 99^{th} percentile values and the minimum of the 1^{st} percentile values for all the weeks are taken as indices and compared with the reference values. The upper limit taken for comparison is 6% higher and the lower limit taken is 6% lower than the nominal voltage. The evaluated 99^{th} percentile and 1^{st} percentile values are displayed together with the status on whether the limit has been crossed or not.

Tables and values:

Tables of all the 10 minute r.m.s values over all the weeks can be viewed. All the sites crossing the 99^{th} percentile and the 1^{st} percentile limits can be identified and the respective values viewed.

Graphical Plots:

The weekly data, and their 99^{th} and 1^{st} percentile values are plotted. Absolute Voltage Deviation, which is the absolute value of the difference between the actual voltage and the nominal voltage (mean of the upper and lower limits), has also been calculated. AVD can also be viewed on the graphs. The 95^{th} percentile value of AVD is obtained and the maximum value is taken as index. The cumulative distribution function and histogram plots are also available for user selection.

Comparison:

Comparison of the steady-state-voltage indices can be made for various locations in a single graph and their data saved in '.mat' files. Steady-state-voltage index values for any selected location for all three phases can be evaluated and displayed too. The Steady-state-voltage assessment was performed for a one-year period which has been shown in the Table 5.2.

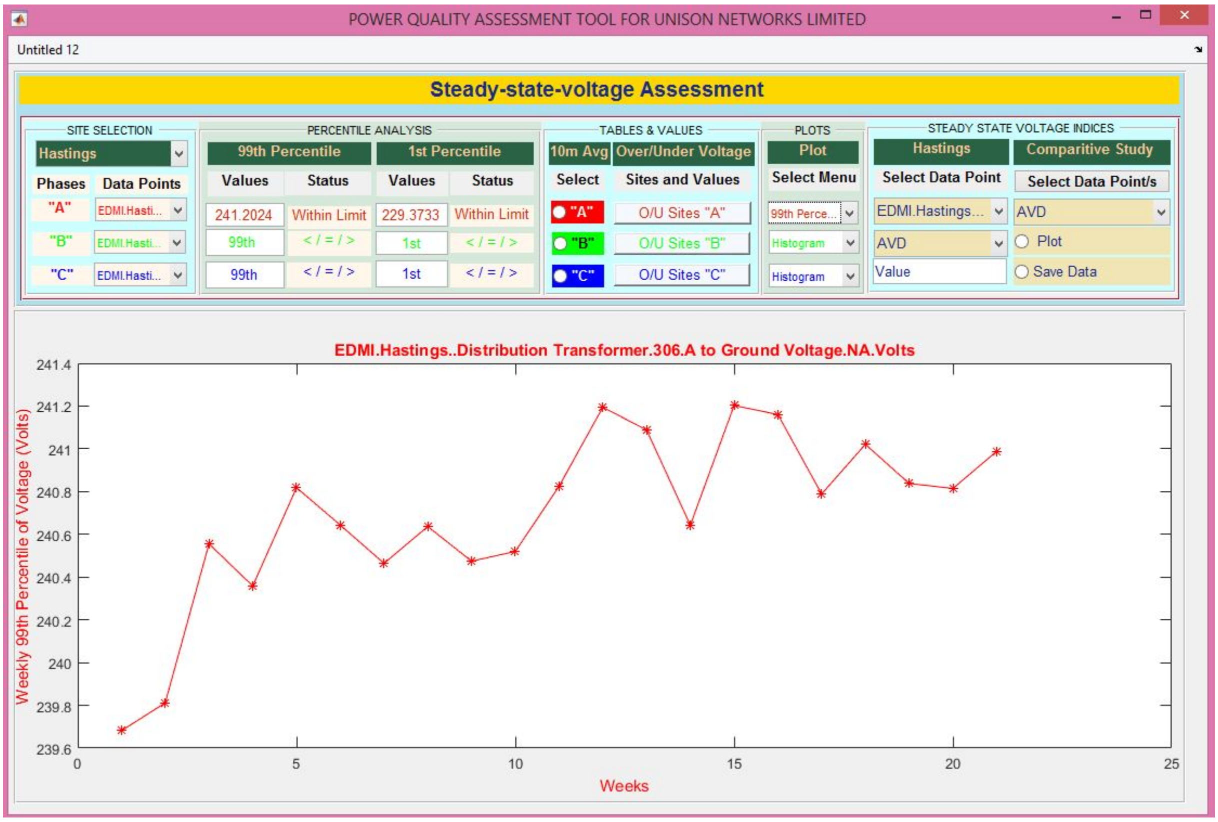


Figure 5.2: GUI for Steady-state-voltage

Table 5.2: Steady-state-voltage cases in different regions.

Region	Total Sites	Overvoltage (w.r.t. upper limit)		Undervoltage (w.r.t. lower limit)	
		Site (%)	Maximum(%)	Site (%)	Maximum (%)
A	47	12.8	2.2	8.5	-50.01
B	47	23.4	3.2	4.3	-84.23
C	53	50	2.9	1.9	-14.68
D	29	38	1.0	0	NA

The maximum values of undervoltage and overvoltage presented in this table are the difference between the actual values and the limits as the percentage of the limits. The negative sign indicates that the actual values are lower than the limits, and the positive sign indicates that the actual values are greater than the limits. Hence, this table shows that the maximum value of the overvoltage was 3.2% higher than the upper limit and was found in 23.4% of the sites in region 'B'. Similarly, the minimum undervoltage found was 84.23% lower than the limit and was found in 4.3% sites in region 'B'.

5.3.2 Voltage Harmonics

The GUI for Voltage Harmonics is shown in Fig. 5.3. Three voltage harmonics namely 3^{rd}, 5^{th} and 7^{th} are measured by EDMI meters. Many features of the interface for Steady-state-voltage index are also present in the interface for Voltage Harmonics index as well. Therefore, the detailed description of these features for Voltage Harmonics is avoided.

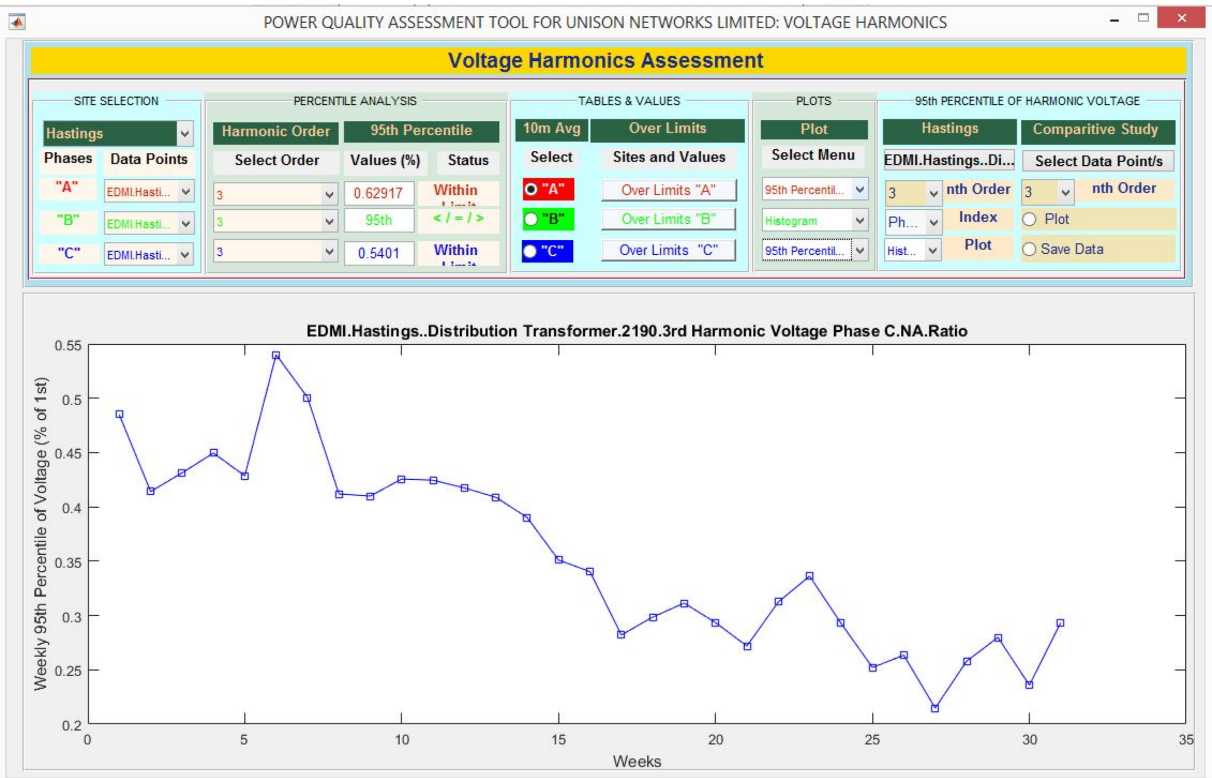


Figure 5.3: GUI for Voltage Harmonics

Users can analyse the data related to any phase from any of the meters installed in any of the four regions. Respective values can be observed in graphs and tables. The sites, where limits are exceeded, and also the percentage values of the harmonics at these sites are tabulated. The 95^{th} percentile values for each week are calculated, and their maximum is retained as the index. For a particular harmonic order and location, the maximum of the 95^{th} percentile values of all the phases is taken as the index. The indices for different orders at one or more locations can be seen in a single graph for comparison. Calculated indices can be saved in '.mat' files. A summary of assessment of Voltage Harmonics is presented in Table 5.3.

Table 5.3: Voltage Harmonics Assessment in different regions

Region	Total Sites	Total Harmonic Distortion		Sites and Values crossing Harmonic Limits					
				Order 3		Order 5		Order 7	
		No. of Sites (%)	Max. Values w.r.t Limits (%)	Sites (%)	Max.Values (%)	Sites (%)	Max.Values (%)	Sites (%)	Max. Values (%)
A	47	0	-51	0	-78	0	-42	0	-31
B	47	0	-45.97	Data Not Available					
C	53	1.9	24.23	Data Not Available					
D	29	0	-46.6	0	-78.5	0	-37.7	0	27.1

5.3.3 Current Harmonics

The interface developed for Current Harmonics is similar to that for Voltage Harmonics and is shown in Fig. 5.4. Current harmonics of 3^{rd}, 5^{th} and 7^{th} order are measured by EDMI meters. The weekly maximum of 10 minute r.m.s values of harmonics are compared, and the largest value is taken as index. This value is compared with the allocation limit. At present, values for current harmonics limits have not been calculated and allocated. However, flexibility has been provided for user to select and fix one particular value. Incorporation of calculation of these limits in the interface is left for future development. Tabulation, visualization and saving of data is performed in a similar way to that used for Voltage Harmonics.

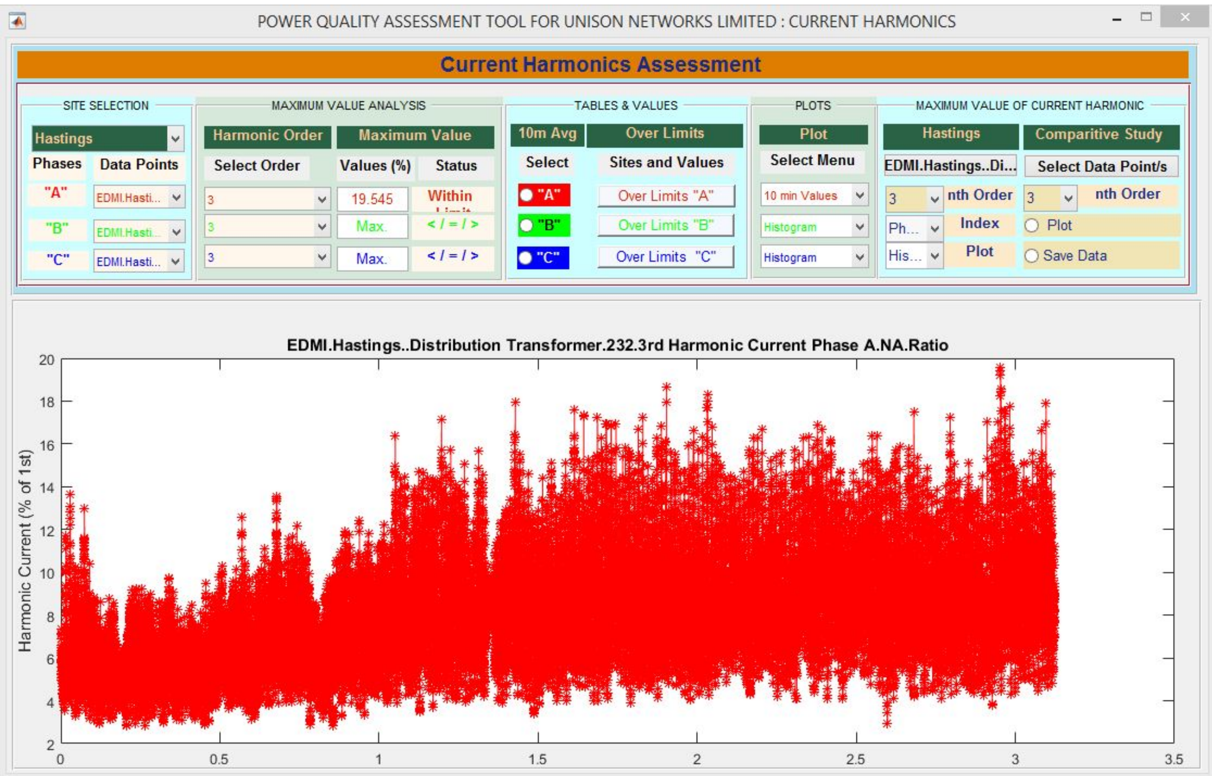


Figure 5.4: GUI for Current Harmonics

5.3.4 Total Harmonic Distortion (THD)

THD for both current and voltage is measured by EDMI meters. The weekly 95^{th} percentile values are compared with the limits. The largest value of weekly 95^{th} percentile values is taken as THD index. The graphical user interface, shown in Fig. 5.5, can be used to show the graphs, tables and values of the indices for the user-selected phases and sites.

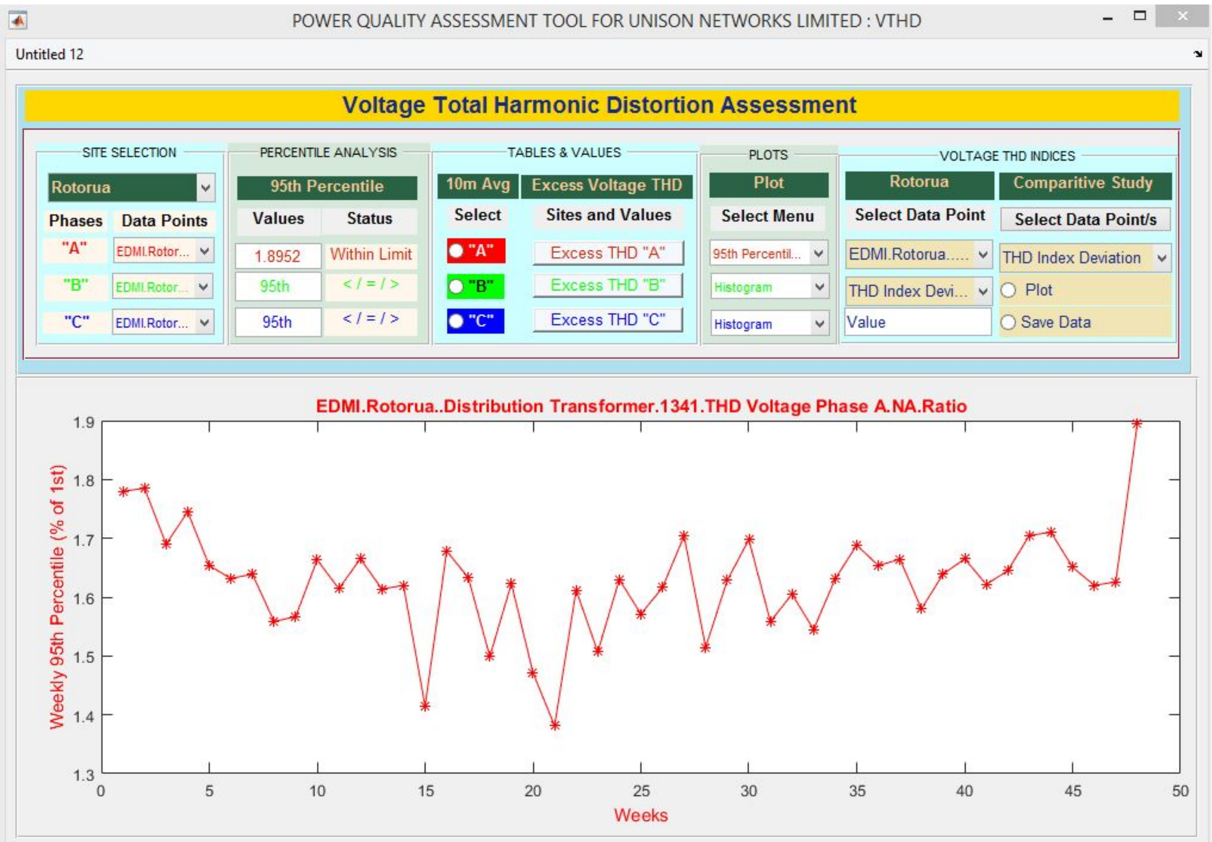


Figure 5.5: Voltage Total Harmonic Distortion

5.3.5 Voltage Unbalance Factor (VUF)

The GUI developed for VUF is shown in Fig. 5.6. VUF is calculated based on the information from EDMI meters. Since these meters measure only the magnitudes of the phase voltages but not their angles, the phase voltage angles need to be evaluated. For this, it is assumed that the sum of the phase voltages is zero. The phase voltage angles are calculated using the phase voltage magnitudes using the cosine law of triangles. Once the phase voltages are known, the sequence components are evaluated. Then the VUF is also calculated using (5.1).

$$VUF = \frac{V_N}{V_P} \frac{(NegativeSequenceComponent)}{(PositiveSequenceComponent)} \tag{5.1}$$

The weekly 95^{th} percentile values of VUF are evaluated. It can be checked for any meter in any region for the number of weeks selected by user. The values and status are displayed on the interface window. The values can be visualized in tables, and the sites crossing the limits can be identified. The graphical plots are shown in various forms. Histogram, cumulative distribution function, weekly variation, variation of the 95^{th} percentile values and variation of VUF index can be plotted. VUF index is the ratio of the VUF and the allocated limit. VUF can be visualized, tabulated and saved for all the sites. It is also possible to demonstrate variation of VUF of the selected sites on the same graph for comparison.

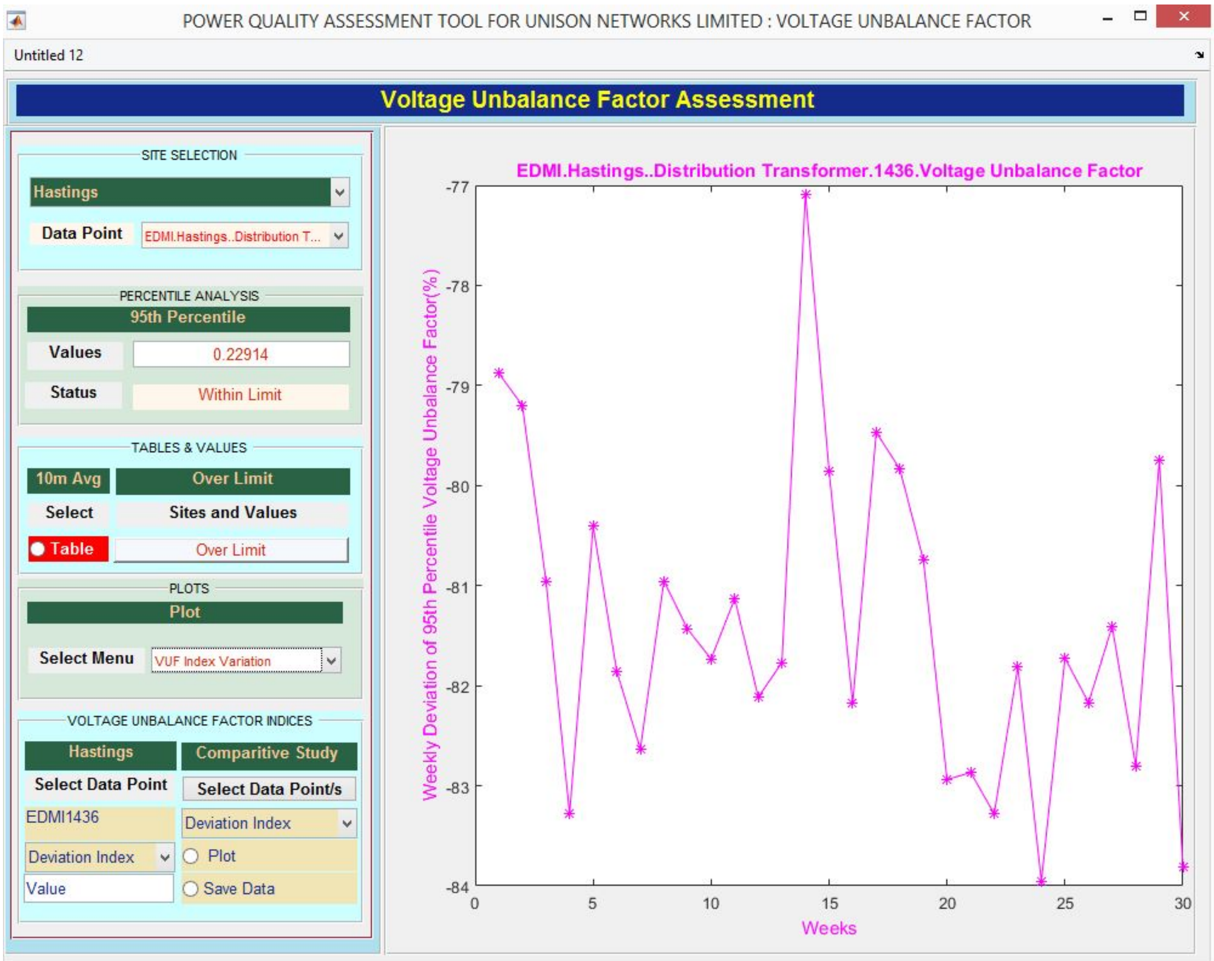


Figure 5.6: Voltage Unbalance Factor

Table 5.4: Voltage Unbalance Factor Assessment in different regions

Region	Total Sites	Sites and Values Exceeding Voltage Unbalance Limit (w.r.t. upper limit)	
		Sites (%)	Maximum (%)
A	47	0	-30
B	47	0	-46.14
C	53	1.9	10.68
D	29	0	-37.64

Chapter 6

CONCLUSIONS AND FUTURE WORK

This chapter concludes the thesis by highlighting the major findings of this research. It also presents recommendations for future work.

6.1 CONCLUSION

This research has demonstrated the application of HSE, based on SVD, to an actual distribution system using actual on-site measurement data for the first time. Previous works have used simulation studies to represent the actual system. However, application of HSE in the real world is always of paramount importance as the estimation results have to be relied upon the measurements which may have noise, and the parameters do have uncertainties. This is where the superiority of SVD method lies as it is a robust method, and can give solutions with least square errors. This research has proved that this method can also be applied to practical distribution systems, and an optimal number of locations for the installment of measuring equipment can be found to minimise the cost of the measuring equipment for harmonics. The main achievements of this research are:

1. This research has shown that calculation of electrical parameters of components like cables, lines and transformers using their geometry and test parameters are accurate enough to build an HSE model of an actual distribution system.
2. The first step in application of HSE to an actual system is to validate the HSE algorithm. This is best achieved by using simulated data to represent actual measurements. When this validation has been achieved, then the measurements from the actual system can be used. One measurement is held back from the estimation, and this is used to test the HSE accuracy. Harmonic penetration study was simulated in OpenDSS, driven by MATLAB. The harmonic data generated was used and the Harmonic State Estimation performed using SVD method. The algorithm used was then validated by establishing that the estimates and the simulated values were adequately close to each other.
3. The harmonic state estimates are the least square solutions obtained using a pseudo-inverse, calculated by applying the SVD method. The algorithm was applied to one feeder of a practical distribution system, and the harmonic state estimates were obtained

using the actual system data and the measurements. Comparison of the estimates with the measured values demonstrated that they were close to each other but the accuracy of the estimates depended on the location and number of the measurements.

4. A method of improving the observability of the system was introduced and applied to the non-triplen odd harmonics based on the fact that the positive and negative sequence currents are absent in the neutral of a three-phase delta-star transformer. This helped to add new equations to the measurement equation, and hence improve observability.
5. This work also found the optimal placement of the measuring equipment in the feeder. Observability was checked and the placements, which could give maximum observable nodes with the least errors in the estimates, were considered optimal. This would require only a minimum number of measuring equipment and the overall cost on the measurements would be reduced.
6. The tool developed for the assessment of power quality and visualization of power quality indices for Unison Networks Limited was presented. This graphical user interface tool is capable of analysing the power quality data obtained from the EDMI meters installed at several locations of the distribution network of Unison Networks Limited.

6.2 FUTURE WORK

Since it has been found that the measurements at the head of the feeder is crucial, appropriate instrumentation is required for the measurements at 11kV voltage level. Most of the transducers are designed only for the low voltage measurements. So, cheaper and accurate instruments capable of measuring harmonics at 11kV voltage level need to be used. Such a measurement allows verification of the transfer of harmonics from 415V/11kV voltage level, and possibly also refine the transformer model for HSE. This research has demonstrated Harmonic State Estimation on only one actual feeder. However, the true advantage of the application can be realised when it is applied to an entire distribution system. Therefore, the future work should consider applying this method to a complete distribution system. Distribution network is even larger than transmission network and the need for state estimation is even more. Hence, finding optimal location and number of equipment for an entire distribution system is even more helpful in determining the complete harmonic profile of the entire network. This application can be extended further to obtain estimates on a real-time basis.

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APPENDICES

A ANALYSIS OF RESIDUES AND ERRORS

Table 1: Residue Comparison (without Noise)

$\|Z\|$	$\|HX\|$	$\|Z\| - \|HX\|$	$\angle Z$	$\angle HX$	$\angle Z - \angle HX$
0.2781	0.2781	0	43.7614	43.7615	-0.0001
0.2596	0.2596	0	-175.715	-175.715	-0.0002
0.1824	0.1824	0	-71.4701	-71.4723	-0.0022
2.2626	2.2626	0	42.4344	42.4357	-0.0013
0.0927	0.0927	0	74.6041	74.5995	0.0046
0.0975	0.0975	0	-166.278	-166.282	-0.0035
3.5764	3.5764	0	37.6579	37.6565	0.0014
0.0955	0.0955	0	76.2395	76.2392	0.0003
0.3499	0.3499	0	-93.6033	-93.5989	0.0044
0.9037	0.9037	0	-38.578	-38.5774	0.0006
0.6917	0.6917	0	158.3493	158.3476	0.0017
0.1095	0.1095	0	-160.089	-160.094	-0.0043
1.7641	1.7641	0	32.4977	32.4987	-0.001
5.9575	5.9575	0	106.3106	106.3138	-0.0032
0.2901	0.2901	0	-167.753	-167.754	-0.0017
0.703	0.703	0	5.456	5.4547	0.0013
0.2803	0.2803	0	136.8891	136.8913	-0.0022
0.4438	0.4438	0	-88.4727	-88.4745	-0.0018
0.1091	0.1091	0	-41.1905	-41.1935	-0.003
0.5256	0.5256	0	154.1807	154.1806	0.0001
1.2745	1.2745	0	-76.8576	-76.8576	0
0.9466	0.9466	0	136.62	136.6202	-0.0002
0.8834	0.8834	0	-82.85	-82.8504	-0.0004
0.6209	0.6209	0	21.39	21.3895	0.0005
0	0	0	0	-155.607	-155.607

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Table 1 – continued from previous page

$\lvert Z\rvert$	$\lvert HX\rvert$	$\lvert Z\rvert - \lvert HX\rvert$	$\angle Z$	$\angle HX$	$\angle Z - \angle HX$
0	0	0	0	150.7466	-150.747
0	0	0	0	-63.2101	-63.2101
0	0	0	0	-155.244	-155.244
0	0	0	0	150.6582	-150.658
0	0	0	0	-63.2179	-63.2179
11.1842	11.1842	0	-45.07	-45.0701	-0.0001
0.4494	0.4494	0	-19.34	-19.3398	0.0002
0.4493	0.4493	0	100.64	100.6401	-0.0001
0	0	0	0	-158.888	-158.888
0	0	0	0	156.5331	-156.533
0	0	0	0	-68.1113	-68.1113
17.6992	17.6992	0	-49.88	-49.8799	0.0001
0.4495	0.4495	0	-19.34	-19.34	0
1.6728	1.6728	0	179.89	179.8898	0.0002
0	0	0	0	-158.9	-158.9
0	0	0	0	156.6006	-156.601
0	0	0	0	-68.2172	-68.2172
0	0	0	0	-162.54	-162.54
0	0	0	0	-163.839	-163.839
0	0	0	0	-164.506	-164.506
0	0	0	0	-161.515	-161.515
0	0	0	0	-117.665	-117.665
0	0	0	0	171.6093	-171.609
0	0	0	0	-161.495	-161.495
0	0	0	0	-117.077	-117.077
0	0	0	0	171.6047	-171.605
4.3806	4.3806	0	-127.13	-127.13	0
3.3303	3.3303	0	72.49	72.4901	-0.0001
0.4496	0.4496	0	100.62	100.6202	-0.0002
0	0	0	0	-157.905	-157.905
0	0	0	0	-51.3551	-51.3551
0	0	0	0	172.1481	-172.148
2.8924	2.8924	0	-54.63	-54.6303	-0.0003
9.8224	9.8224	0	18.56	18.5588	0.0012
0.4491	0.4491	0	100.02	100.0207	-0.0007
0	0	0	0	-163.645	-163.645

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Table 1 – continued from previous page

$\|Z\|$	$\|HX\|$	$\|Z\| - \|HX\|$	$\angle Z$	$\angle HX$	$\angle Z - \angle HX$
0	0	0	0	-163.528	-163.528
0	0	0	0	-163.659	-163.659
0	0	0	0	-164.225	-164.225
0	0	0	0	-163.241	-163.241
0	0	0	0	-163.368	-163.368
3.3676	3.3676	0	-82.55	-82.5499	0.0001
1.2602	1.2602	0	50.86	50.8599	0.0001
2.0867	2.0867	0	-174.87	-174.87	0.0001
0	0	0	0	-164.224	-164.224
0	0	0	0	-163.241	-163.241
0	0	0	0	-163.367	-163.367
0	0	0	0	-163.61	-163.61
0	0	0	0	-163.61	-163.61
0	0	0	0	-163.61	-163.61
0.4496	0.4496	0	-139.41	-139.41	0.0001
2.501	2.5011	-0.0001	68.54	68.54	0
6.2318	6.2318	0	-163.67	-163.67	0
0	0	0	0	-163.61	-163.61
0	0	0	0	-163.61	-163.61
0	0	0	0	-163.61	-163.61
0	0	0	0	24.426	-24.426

Table 2: Magnitude of the Errors and the Difference in Angles (without Noise)

Bus	Measurement	Harmonics								
		(1)	(3)	(5)	(7)	(9)	(11)	(13)	(15)	(17)
1	Absolute Difference (Volts)	0	0	0	0	0	0	0	0	0
	Difference in Angles (degrees)	0	0.15	0.02	0	0.09	0.11	0.1	-0.48	-0.12
4	Absolute Difference (Volts)	0	0	0	0	0	0	0	0	0
	Difference in Angles (degrees)	0.34	1.91	-1.21	-0.96	0.39	0.54	1	-2.39	0.73
6	Absolute Difference (Volts)	0	0	0	0	0	0	0	0	0
	Difference in Angles (degrees)	0.55	0.88	1.59	-3.28	1.16	2.34	-3.04	0.82	1.03
11	Absolute Difference (Volts)	0	0	0	0	0	0	0	0	
	Difference in Angles (degrees)	0.05	-21.2	6.65	3.06	-9.37	23.16	-24.3	16.97	-14.38
13	Absolute Difference (Volts)	0	0	0	0	0	0	0	0	0
	Difference in Angles (degrees)	-0.52	122.18	-69.81	-2.15	-296.15	-127.67	-31.6	28.22	117
16	Absolute Difference (Volts)	0	0	0	0	0	0	0	0	0
	Difference in Angles (degrees)	0.05	-12.11	8.15	2.98	-26.06	9.86	10.92	332.41	6.37
19	Absolute Difference (Volts)	0	0	0	0	0	0	0	0	0
	Difference in Angles (degrees)	-0.1	239.02	109.38	70.64	-11.39	-101.33	-136.66	97.66	8.7
21	Absolute Difference (Volts)	0	0	0	0	0	0	0	0	0
	Difference in Angles (degrees)	0.6	1.08	0.31	0.28	0.21	0.16	0.08	0.19	0.12

Table 3: Magnitude of Estimated and Simulated Harmonics (without Noise)

Bus	Measurement	Harmonics Magnitudes (Volts)								
		(1)	(3)	(5)	(7)	(9)	(11)	(13)	(15)	(17)
1	Estimated	6350.75	0.47	0.28	0.18	0.08	0.09	0.06	0.05	0.08
	Simulated	6350.75	0.47	0.28	0.18	0.08	0.09	0.06	0.05	0.08
4	Estimated	238.88	4.46	1.51	1.04	1.11	0.63	0.37	0.85	0.49
	Simulated	238.54	6.82	2.26	1.57	1.6	0.97	0.56	1.28	0.71
6	Estimated	238.27	6.84	2.44	1.68	1.61	1.03	0.57	1.24	0.76
	Simulated	237.9	10.83	3.58	2.47	2.49	1.49	0.86	1.93	1.07
11	Estimated	239.15	1.33	0.85	0.27	0.53	0.24	0.16	0.42	0.15
	Simulated	238.54	2.67	0.9	0.62	0.63	0.38	0.22	0.53	0.3
13	Estimated	241.36	3.8	2.07	2.1	1.5	0.43	0.67	1.56	0.34
	Simulated	238.52	5.31	1.76	1.23	1.23	0.75	0.44	1	0.56
16	Estimated	239.06	0.74	0.55	0.44	0.23	0.2	0.16	0.24	0.13
	Simulated	239.03	2.06	0.7	0.49	0.49	0.3	0.18	0.4	0.22
19	Estimated	238.86	1.65	0.31	0.29	0.46	0.13	0.1	0.33	0.12
	Simulated	239.36	0.28	0.11	0.07	0.06	0.04	0.02	0.06	0.04
21	Estimated	238.84	1.11	0.27	0.26	0.19	0.17	0.09	0.21	0.12
	Simulated	239.36	0.15	0.07	0.02	0.04	0.02	0.02	0.03	0.02

Table 4: Angle of Estimated and Simulated Harmonics (without Noise)

Bus	Measurement	Harmonics Angles (degrees)								
		(1)	(3)	(5)	(7)	(9)	(11)	(13)	(15)	(17)
1	Estimated	-0.01	-23.98	43.78	55.97	119.66	-156.61	-132.16	-59.71	17.04
	Simulated	-0.01	-24.13	43.76	55.97	119.57	-156.72	-132.26	-59.23	17.16
4	Estimated	29.26	-11.1	41.22	86.16	144.13	-160.34	-103.53	-51.32	7.33
	Simulated	28.92	-13.01	42.43	87.12	143.74	-160.88	-104.53	-48.93	6.6
6	Estimated	28.83	-15.02	39.25	77.22	136.48	-168.67	-119.34	-61.45	-7.17
	Simulated	28.28	-15.9	37.66	80.5	135.32	-171.01	-116.3	-62.27	-8.2
11	Estimated	29.66	-83.26	-31.93	-22.55	-12.97	42.26	18.17	81.14	71.62
	Simulated	29.61	-62.06	-38.58	-25.61	-3.6	19.1	42.47	64.17	86
13	Estimated	28.63	103.57	-37.31	71.42	-169.06	51.25	-159.9	-47.58	93.49
	Simulated	29.15	-18.61	32.5	73.57	127.09	178.92	-128.3	-75.8	-23.51
16	Estimated	29.73	-46.51	13.61	38.77	51.09	127.92	170.92	171.77	-113.38
	Simulated	29.68	-34.4	5.46	35.79	77.15	118.06	160	-160.64	-119.75
19	Estimated	29.85	171.91	68.19	44.14	-36.52	-110.26	-133.19	121.98	49.11
	Simulated	29.95	-67.11	-41.19	-26.5	-25.13	-8.93	3.47	24.32	40.41
21	Estimated	29.93	153.91	92.56	23.08	-40.04	-101.88	-177.83	123.13	39.59
	Simulated	29.99	79.85	-21.93	168.57	82.91	-46.06	-153.47	79.58	-21.71

Table 5: Residue Comparison in Presence of Noise

$\lvert Z\rvert$	$\lvert HX\rvert$	$\lvert Z\rvert - \lvert HX\rvert$	$\angle Z$	$\angle HX$	$\angle Z - \angle HX$
0.2796	0.2795	0.0001	43.7614	43.8337	-0.0723
0.261	0.2608	0.0002	-175.715	-175.845	-0.13
0.1834	0.1842	-0.0008	-71.4701	-71.4974	-0.0273
2.2822	2.2634	0.0188	42.4344	42.4353	-0.0009
0.0935	0.0928	0.0007	74.6041	74.6271	-0.023
0.0984	0.0976	0.0008	-166.278	-166.249	0.0297
3.5406	3.575	-0.0344	37.6579	37.6563	0.0016
0.0945	0.0955	-0.001	76.2395	76.2693	-0.0298
0.3464	0.3498	-0.0034	-93.6033	-93.5986	0.0047
0.8947	0.9034	-0.0087	-38.578	-38.5754	0.0026
0.6848	0.6916	-0.0068	158.3493	158.3376	0.0117
0.1084	0.1095	-0.0011	-160.089	-160.012	0.0775
1.7769	1.7675	0.0094	32.4977	32.4981	-0.0004
6.0007	5.9692	0.0315	106.3106	106.3139	-0.0033
0.2922	0.2907	0.0015	-167.753	-167.729	0.0233
0.7015	0.7029	-0.0014	5.456	5.4552	0.0008
0.2797	0.2804	-0.0007	136.8891	136.8767	0.0124
0.4429	0.4439	-0.001	-88.4727	-88.4721	0.0006
0.1102	0.1091	0.0011	-41.1905	-41.1764	0.0141
0.5308	0.5259	0.0049	154.1807	154.1681	0.0126
1.2873	1.2752	0.0121	-76.8576	-76.8582	-0.0006
0.9466	0.9465	0.0001	136.62	136.6436	-0.0236
0.8834	0.8837	-0.0003	-82.85	-82.8793	-0.0293
0.6209	0.6217	-0.0008	21.39	21.4204	-0.0304
0	0.0004	-0.0004	0	-126.175	-126.175
0	0.0004	-0.0004	0	-144.245	-144.245
0	0.0008	-0.0008	0	44.1486	-44.1486
0	0.0004	-0.0004	0	-125.481	-125.481
0	0.0004	-0.0004	0	-144.534	-144.534
0	0.0008	-0.0008	0	44.1658	-44.1658
11.1842	11.188	-0.0038	-45.07	-45.0701	-0.0001
0.4494	0.4495	-0.0001	-19.34	-19.3419	-0.0019
0.4493	0.4495	-0.0002	100.64	100.6381	0.0019
0	0.0003	-0.0003	0	-142.438	-142.438
0	0.0003	-0.0003	0	-128.046	-128.046

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Table 5 – continued from previous page

$\lvert Z\rvert$	$\lvert HX\rvert$	$\lvert Z\rvert - \lvert HX\rvert$	$\angle Z$	$\angle HX$	$\angle Z - \angle HX$
0	0.0007	-0.0007	0	44.9021	-44.9021
17.6992	17.6923	0.0069	-49.88	-49.8799	0.0001
0.4495	0.4493	0.0002	-19.34	-19.3472	-0.0072
1.6728	1.6721	0.0007	179.89	179.8898	0.0002
0	0.0003	-0.0003	0	-142.416	-142.416
0	0.0003	-0.0003	0	-128.02	-128.02
0	0.0007	-0.0007	0	44.9265	-44.9265
0	0	0	0	-138.424	-138.424
0	0	0	0	-104.427	-104.427
0	0	0	0	56.3972	-56.3972
0	0	0	0	-10.1504	-10.1504
0	0.0001	-0.0001	0	36.7798	-36.7798
0	0.0001	-0.0001	0	-153.497	-153.497
0	0	0	0	-8.5672	-8.5672
0	0.0001	-0.0001	0	37.2937	-37.2937
0	0.0001	-0.0001	0	-152.837	-152.837
4.3806	4.3789	0.0017	-127.13	-127.131	-0.0005
3.3303	3.329	0.0013	72.49	72.4912	-0.0012
0.4496	0.4494	0.0002	100.62	100.611	0.009
0	0.0001	-0.0001	0	1.4661	-1.4661
0	0.0003	-0.0003	0	82.8703	-82.8703
0	0.0003	-0.0003	0	-117.31	-117.31
2.8924	2.8981	-0.0057	-54.63	-54.6314	-0.0014
9.8224	9.8414	-0.019	18.56	18.5589	0.0011
0.4491	0.45	-0.0009	100.02	100.0209	-0.0009
0	0	0	0	-148.222	-148.222
0	0	0	0	-116.264	-116.264
0	0	0	0	62.7349	-62.7349
0	0	0	0	-157.618	-157.618
0	0	0	0	-103.896	-103.896
0	0	0	0	59.2903	-59.2903
3.3676	3.3673	0.0003	-82.55	-82.5499	0.0001
1.2602	1.26	0.0002	50.86	50.8608	-0.0008
2.0867	2.0864	0.0003	-174.87	-174.87	0.0002
0	0	0	0	-157.618	-157.618
0	0	0	0	-103.897	-103.897

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Table 5 – continued from previous page

$\|Z\|$	$\|HX\|$	$\|Z\| - \|HX\|$	$\angle Z$	$\angle HX$	$\angle Z - \angle HX$
0	0	0	0	59.2905	-59.2905
0	0	0	0	-163.61	-163.61
0	0	0	0	-163.61	-163.61
0	0	0	0	-163.61	-163.61
0.4496	0.4498	-0.0002	-139.41	-139.406	0.0041
2.501	2.502	-0.001	68.54	68.5397	0.0003
6.2318	6.2342	-0.0024	-163.67	-163.67	-0.0003
0	0	0	0	-163.61	-163.61
0	0	0	0	-163.61	-163.61
0	0	0	0	-163.61	-163.61
0	0	0	0	104.8919	-104.892

Table 6: Magnitude of the Errors and the Differences in Angles in Presence of Noise

Buses	Measurements	Harmonics								
		(1)	(3)	(5)	(7)	(9)	(11)	(13)	(15)	(17)
1	Absolute Difference (Volts)	9.36	0.19	0.05	0.01	0.03	0.022218	0.001708	0.03211	0.016358
	Difference in Angles (degrees)	0.00	3.99	10.23	-0.08	0.06	12	-1.63	-20.85	9.92
4	Absolute Difference (Volts)	2.94	2.32	0.75	0.56	0.480805	0.330202	0.202504	0.440988	0.210057
	Difference in Angles (degrees)	0.51	1.94	-1.16	-1.22	-1.19	-0.84	4.05	-1.63	-0.47
6	Absolute Difference (Volts)	3.35	3.96	1.15	0.82	0.880047	0.461349	0.310929	0.720587	0.320071
	Difference in Angles (degrees)	0.66	0.89	1.57	-3.59	0.26	1.63	-2	1.09	0.43
11	Absolute Difference (Volts)	1.91	1.52	0.11	0.36	0.147999	0.177164	0.105905	0.1721	0.147935
	Difference in Angles (degrees)	0.05	-22.01	6.95	7.08	-10.83	22.51	-25.26	16.12	-12.52
13	Absolute Difference (Volts)	4.99	8.05	2.20	0.87	1.484052	1.054112	0.395936	0.80544	0.766268
	Difference in Angles (degrees)	-0.90	122.80	-70.44	-1.86	-295.57	-130.32	-31.2	26.24	118.48
16	Absolute Difference (Volts)	1.28	1.36	0.18	0.05	0.314007	0.110774	0.04266	0.207918	0.091564
	Difference in Angles (degrees)	0.03	-13.21	8.37	4.06	-28.25	11.16	13.61	332.29	5.71
19	Absolute Difference (Volts)	0.97	1.83	0.36	0.29	0.391726	0.142336	0.125689	0.333971	0.080879
	Difference in Angles (degrees)	-0.19	238.97	109.62	71.68	-12.83	-99.7	-138.49	98.25	9.84
21	Absolute Difference (Volts)	0.88	1.10	0.31	0.28	0.226042	0.169154	0.081457	0.20021	0.122573
	Difference in Angles (degrees)	-0.17	74.61	115.00	-142.72	-126.35	-54.42	-19.74	45.7	64.02

Table 7: Magnitude of Estimated and Simulated Harmonics in Presence of Noise

Buses	Measurements	Harmonics Magnitudes (Volts)								
		(1)	(3)	(5)	(7)	(9)	(11)	(13)	(15)	(17)
1	Estimated	6360.11	0.28	0.28	0.19	0.05	0.1	0.06	0.02	0.07
	Simulated	6350.75	0.47	0.28	0.18	0.08	0.09	0.06	0.05	0.08
4	Estimated	240.56	4.51	1.51	1.01	1.12	0.64	0.36	0.84	0.5
	Simulated	238.54	6.82	2.26	1.57	1.6	0.97	0.56	1.28	0.71
6	Estimated	239.81	6.87	2.43	1.66	1.61	1.03	0.55	1.21	0.75
	Simulated	237.9	10.83	3.58	2.47	2.49	1.49	0.86	1.93	1.07
11	Estimated	240.44	1.33	0.86	0.26	0.53	0.25	0.15	0.42	0.16
	Simulated	238.54	2.67	0.9	0.62	0.63	0.38	0.22	0.53	0.3
13	Estimated	241.79	3.82	2.04	2.1	1.52	0.4	0.7	1.57	0.32
	Simulated	238.52	5.31	1.76	1.23	1.23	0.75	0.44	1	0.56
16	Estimated	240.3	0.73	0.55	0.45	0.22	0.2	0.17	0.26	0.13
	Simulated	239.03	2.06	0.7	0.49	0.49	0.3	0.18	0.4	0.22
19	Estimated	239.92	1.67	0.31	0.3	0.45	0.13	0.11	0.32	0.12
	Simulated	239.36	0.28	0.11	0.07	0.06	0.04	0.02	0.06	0.04
21	Estimated	239.87	1.13	0.27	0.26	0.2	0.18	0.1	0.22	0.13
	Simulated	239.36	0.15	0.07	0.02	0.04	0.02	0.02	0.03	0.02

Table 8: Angle of Estimated and Simulated Harmonics in Presence of Noise

Buses	Measurements	Harmonics Angles (degrees)								
		(1)	(3)	(5)	(7)	(9)	(11)	(13)	(15)	(17)
1	Estimated	-0.01	-20.14	53.99	55.89	119.63	-144.72	-133.89	-80.08	27.08
	Simulated	-0.01	-24.13	43.76	55.97	119.57	-156.72	-132.26	-59.23	17.16
4	Estimated	29.43	-11.07	41.27	85.9	142.55	-161.72	-100.48	-50.56	6.13
	Simulated	28.92	-13.01	42.43	87.12	143.74	-160.88	-104.53	-48.93	6.6
6	Estimated	28.94	-15.01	39.23	76.91	135.58	-169.38	-118.3	-61.18	-7.77
	Simulated	28.28	-15.9	37.66	80.5	135.32	-171.01	-116.3	-62.27	-8.2
11	Estimated	29.66	-84.07	-31.63	-18.53	-14.43	41.61	17.21	80.29	73.48
	Simulated	29.61	-62.06	-38.58	-25.61	-3.6	19.1	42.47	64.17	86
13	Estimated	28.25	104.19	-37.94	71.71	-168.48	48.6	-159.5	-49.56	94.97
	Simulated	29.15	-18.61	32.5	73.57	127.09	178.92	-128.3	-75.8	-23.51
16	Estimated	29.71	-47.61	13.83	39.85	48.9	129.22	173.61	171.65	-114.04
	Simulated	29.68	-34.4	5.46	35.79	77.15	118.06	160	-160.64	-119.75
19	Estimated	29.76	171.86	68.43	45.18	-37.96	-108.63	-135.02	122.57	50.25
	Simulated	29.95	-67.11	-41.19	-26.5	-25.13	-8.93	3.47	24.32	40.41
21	Estimated	29.82	154.46	93.07	25.85	-43.44	-100.48	-173.21	125.28	42.31
	Simulated	29.99	79.85	-21.93	168.57	82.91	-46.06	-153.47	79.58	-21.71

B HARMONICS INPUT DATA

Table 9: Input Data for Fundamental Harmonic for 60^{th} Interval

	Phase ‘A’ Voltage		Phase ‘B’ Voltage		Phase ‘C’ Voltage		Phase ‘A’ Current		Phase ‘B’ Current		Phase ‘C’ Current	
Bus	(Volts)	(degrees)	(Volts)	(degrees)	(Volts)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)
1	6422.25	-30	6406.56	-149.65	6449.13	90.33	32.61	-35.84	33.85	-159.12	31.43	79.92
4	237.04	0	237.81	-119.5	238.14	120.1	265.49	-33.7	268.38	-151.5	258.71	84.7
6	239.89	0	241.45	-119.11	242.11	120.4	179.55	-2.97	92.83	-121.82	114.08	118.58
11	239.77	0	240.7	-119.8	242.55	120.4	193.96	-1.7	218.94	-117.8	116.88	119.2
13	240.38	0	240.94	-120.04	241.83	119.69	19.96	-8.46	42	-121.16	37.35	120.88
16	239.97	0	240.72	-119.7	241.87	119.9	133.06	-5.2	148.66	-121.2	150.63	119.6
19	241.24	0	241.52	-119.46	242.49	120.25	119.7	-3.68	75.75	-122.94	70.53	122.38
21	245.75	0	247.6	-119	246.7	119	49.52	7	70.52	-118	66.07	120.48

Table 10: Input Data for 3^{rd} Harmonic for 60^{th} Interval

	Phase ‘A’ Voltage		Phase ‘B’ Voltage		Phase ‘C’ Voltage		Phase ‘A’ Current		Phase ‘B’ Current		Phase ‘C’ Current	
Bus	(Volts)	(degrees)	(Volts)	(degrees)	(Volts)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)
1	14	48.1	2.57	-103.52	10.43	170.31	0.24	-138.87	0.09	17.93	0.17	51.01
4	0.4	47.4	0	0	0	0	1.81	-64.9	0	0	3.41	80.7
6	1.14	74.05	0.52	70.7	0.6	133.05	9.93	176.49	7.75	-171.73	7.94	-176.09
11	1.38	76.6	0.82	79.4	0.51	115	12.33	177.7	10.58	-44.3	8.86	154.8
13	0.55	77.31	0.31	66.48	0.48	164.4	1.3	-150.9	1.94	-165.37	1.58	-167.91
16	1.13	66.3	0.5	61.7	0.88	116	7.2	172.1	5.41	-0.5	10.13	50.9
19	0.85	87.85	0.43	77.57	0.55	164.5	7.65	-159.33	7.25	-171.92	4.5	-166.72
21	0.7	95.5	0.6	23	0.4	105	6	-10.5	3	-10	6	-7

Table 11: Input Data for 5^{th} Harmonic for 60^{th} Interval

	Phase ‘A’ Voltage		Phase ‘B’ Voltage		Phase ‘C’ Voltage		Phase ‘A’ Current		Phase ‘B’ Current		Phase ‘C’ Current	
Bus	(Volts)	(degrees)	(Volts)	(degrees)	(Volts)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)
1	43.57	-109.97	57.77	16.46	46.35	149.12	0.87	-36.12	0.89	90.42	0.79	-150.81
4	1.42	-163.4	1.4	-20.9	1.07	78.2	7.96	25.1	8.41	-57.5	8.52	65.1
6	2.85	-136.76	2.82	-0.88	2.09	114.89	6.28	-44.93	5.69	87.81	4.19	-153.57
11	2.78	-141.4	2.92	-7.1	2.15	107.9	6.62	-56.7	6.32	62.7	3.99	-172.4
13	2.18	-137.72	2.24	-4.81	1.86	111.12	0.52	-8.47	0.35	83.4	0.91	-150.69
16	2.67	-134.6	2.55	-2.2	2.5	108.1	5.1	-31.3	3.32	72.9	6.01	-168.5
19	2.6	-134.56	2.74	-4.96	1.87	113.04	4.8	-24.06	4.79	85.35	2.75	-146.94
21	2.5	135	2	-110.5	2.8	4	3	14	0	0	3	159

Table 12: Input Data for 7^{th} Harmonic for 60^{th} Interval

	Phase ‘A’ Voltage		Phase ‘B’ Voltage		Phase ‘C’ Voltage		Phase ‘A’ Current		Phase ‘B’ Current		Phase ‘C’ Current	
Bus	(Volts)	(degrees)	(Volts)	(degrees)	(Volts)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)
1	105.34	-40.77	102.61	-165.6	96.4	78.08	0.56	49.03	0.39	-85.2	0.39	-176.37
4	3.71	-5.4	3.14	-122.1	3.64	121.2	2.55	-118.5	4.26	102.7	3.19	-37.8
6	4.52	-8.5	4	-125.47	4.07	116.62	2.99	68.68	2	-8.37	1.55	-157.51
11	4.38	-12	4.05	-134.7	4.1	114.9	3.07	30.1	2.94	-79.4	0	0
13	4.12	-13.14	3.66	-135.08	3.81	110.09	0.28	51.61	0.22	-132.35	0.2	127.96
16	4.59	-9.8	3.71	-131.4	4.31	112.1	2.78	65.4	0	0	2.44	-171.5
19	4.53	-12.81	4.03	-128.92	4	114.05	3.34	56.72	1.9	-15.06	1.18	-153.74
21	4.2	-170	4	67	3.9	-49	0	0	0	0	0	0

Table 13: Input Data for 9^{th} Harmonic for 60^{th} Interval

	Phase 'A' Voltage		Phase 'B' Voltage		Phase 'C' Voltage		Phase 'A' Current		Phase 'B' Current		Phase 'C' Current	
Bus	(Volts)	(degrees)	(Volts)	(degrees)	(Volts)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)
1	2.82	-29.62	1.92	-108.57	1.38	-170.74	0.01	35.54	0.03	-68.07	0.03	124.51
4	0	0	0	0	0	0	0	0	0	0	0	0
6	0.35	-164.46	0.46	-129.97	0.39	-139.01	1.69	-70.37	1.91	-42.27	1.5	-35.97
11	0	0	0	0	0	0	0	0	0	0	0	0
13	0.23	-74.41	0.33	-121.72	0.16	-114.55	0.4	1.17	0.52	-32.06	0.3	8.15
16	0	0	0	0	0.5	-171.2	0	0	0	0	1.8	-49.6
19	0.29	-107.53	0.45	-125.2	0.26	-137.64	1.52	-23.35	1.89	-34.17	1.04	-21.17
21	0.2	89	0.3	135	0.3	121	0	0	0	0	0	0

Table 14: Input Data for 11^{th} Harmonic for 60^{th} Interval

	Phase 'A' Voltage		Phase 'B' Voltage		Phase 'C' Voltage		Phase 'A' Current		Phase 'B' Current		Phase 'C' Current	
Bus	(Volts)	(degrees)	(Volts)	(degrees)	(Volts)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)
1	9.64	35.62	8.36	134.8	11.51	-101.51	0.15	114.76	0.14	-122.93	0.14	-10.01
4	0.73	0.4	0.69	100.6	0.81	-123.2	0	0	0	0	1.53	-21.8
6	0.39	-6.88	0.52	123.42	0.61	-110.44	0.48	43.24	0.53	-114.35	0.63	-19.89
11	0	0	0.26	28.9	0.5	-122.2	0	0	0	0	0	0
13	0.32	28.56	0.4	138.06	0.46	-110.23	0.21	-174.19	0.33	-61.95	0.11	43.08
16	0.53	-7.6	0.3	44.8	0.59	-129.3	0	0	0	0	0	0
19	0.46	-4.5	0.5	128.48	0.54	-105.45	0.69	69.7	0.78	-98.75	0.52	21.62
21	0.3	151	0.5	-72.5	0.4	63.5	0	0	0	0	0	0

Table 15: Input Data for 13^{th} Harmonic for 60^{th} Interval

	Phase 'A' Voltage		Phase 'B' Voltage		Phase 'C' Voltage		Phase 'A' Current		Phase 'B' Current		Phase 'C' Current	
Bus	(Volts)	(degrees)	(Volts)	(degrees)	(Volts)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)
1	4.3	-1.74	3.12	-123.37	3.64	136.82	0.13	78.04	0.13	-63.45	0.08	-177.77
4	0	0	0	0	0	0	0	0	0	0	0	0
6	0.54	38.41	0.27	-55.25	0.29	179.67	1.19	129.42	0.44	49.34	0.39	-78.7
11	0.51	21	0	0	0	0	0	0	0	0	0	0
13	0.21	4.59	0.23	-126.29	0.16	137.56	0.1	60.1	0.21	-63.45	0.08	148.63
16	0.3	9.8	0	0	0.47	143.5	0	0	0	0	0	0
19	0.32	22.59	0.25	-55.64	0.17	146.8	0.62	109.61	0.42	55.16	0.19	165.93
21	0.2	-28.5	0.2	-153	0.2	74.5	0	0	0	0	0	0

Table 16: Input Data for 15^{th} Harmonic for 60^{th} Interval

	Phase 'A' Voltage		Phase 'B' Voltage		Phase 'C' Voltage		Phase 'A' Current		Phase 'B' Current		Phase 'C' Current	
Bus	(Volts)	(degrees)	(Volts)	(degrees)	(Volts)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)
1	1.01	102.27	0.9	-35.32	0.28	105.47	0.03	-175.67	0.04	35.67	0.02	-106.01
4	0	0	0	0	0	0	0	0	0	0	0	0
6	0.44	-164.78	0.21	-117.28	0.31	-110.25	1.05	-68.15	0.57	-31.47	0.75	-21.46
11	0	0	0	0	0	0	0	0	0	0	0	0
13	0.1	-84.22	0.22	-108.86	0.1	-73.72	0.14	18.63	0.22	-21.66	0.09	19.34
16	0.12	-11.1	0	0	0.45	-152.3	0	0	0	0	0	0
19	0.31	-164.87	0.42	-104.34	0.16	-112.36	0.83	-62.61	1.16	-13.86	0.4	-19.78
21	0.1	-70.5	0.2	-64	0.2	-63.5	0	0	0	0	0	0

Table 17: Input Data for 17^{th} Harmonic for 60^{th} Interval

	Phase 'A' Voltage		Phase 'B' Voltage		Phase 'C' Voltage		Phase 'A' Current		Phase 'B' Current		Phase 'C' Current	
Bus	(Volts)	(degrees)	(Volts)	(degrees)	(Volts)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)	(Amps)	(degrees)
1	6.41	-23.73	6.34	93.9	6.83	-145.54	0.13	90.83	0.07	-142.83	0.11	-59.88
4	0.99	-27.9	0.79	59.6	1.2	-167.5	0.4	2.7	0	0	2.67	-77.6
6	0.38	-47.34	0.26	101.45	0.27	-154.59	0.35	30.66	0.23	-90.63	0.04	1.25
11	0	0	0	0	0	0	0	0	0	0	0	0
13	0.09	-90.46	0.04	119.39	0.23	-178.31	0.17	-119.08	0.19	-17.54	0.04	143.86
16	0	0	0	0	0	0	0	0	0	0	0	0
19	0.19	-30.95	0.18	79.26	0.23	-159.06	0.22	-166.51	0.18	-35	0.12	77.59
21	0.2	15	0.1	127.5	0.1	-74.5	0	0	0	0	0	0

C COMPARISON OF ESTIMATED VALUES

Table 18: Comparison of the 1^{st} Harmonic for One Selected Interval (60^{th})

	Phase 'A'						Phase 'B'						Phase 'C'					
	Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)		
Bus	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff
1	1.001	1.011	-0.01	-29.62	-30	0.38	0.998	1.009	-0.011	-29.62	-30	0.38	1.006	1.015	-0.01	-29.62	-30	0.38
4	0.968	0.989	-0.021	1.76	0	1.76	0.969	0.993	-0.023	1.76	0	1.76	0.957	0.994	-0.037	1.76	0	1.76
6	0.966	1.001	-0.036	1.94	0	1.94	0.968	1.008	-0.04	1.94	0	1.94	0.954	1.01	-0.056	1.94	0	1.94
11	0.965	1.001	-0.036	1.66	0	1.66	0.964	1.005	-0.04	1.66	0	1.66	0.95	1.012	-0.062	1.66	0	1.66
13	0.901	1.003	-0.103	6.24	0	6.24	0.893	1.006	-0.112	6.24	0	6.24	0.85	1.009	-0.16	6.24	0	6.24
16	0.953	1.002	-0.048	1.71	0	1.71	0.958	1.005	-0.047	1.71	0	1.71	0.939	1.009	-0.071	1.71	0	1.71
19	0.965	1.007	-0.042	1.87	0	1.87	0.966	1.008	-0.042	1.87	0	1.87	0.954	1.012	-0.058	1.87	0	1.87
21	1.019	1.026	-0.007	3.09	0	3.09	1.018	1.033	-0.015	3.09	0	3.09	0.994	1.03	-0.036	3.09	0	3.09

Table 19: Comparison of the 3^{rd} Harmonic for One Selected Interval (60^{th})

	Phase 'A'						Phase 'B'						Phase 'C'					
	Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)		
Bus	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff
1	0.002	0.002	0	15.27	48.1	-32.83	0.001	0	0.001	15.27	48.1	-32.83	0.001	0.002	-0.001	15.27	48.1	-32.83
4	0	0.002	-0.001	-34.11	47.4	-81.51	0.002	0	0.002	-34.11	47.4	-81.51	0.002	0	0.002	-34.11	47.4	-81.51
6	0.001	0.005	-0.004	-32.11	74.05	-106.16	0.002	0.002	0	-32.11	74.05	-106.16	0.003	0.003	0.001	-32.11	74.05	-106.16
11	0.001	0.006	-0.005	-1.36	76.6	-77.96	0.001	0.003	-0.002	-1.36	76.6	-77.96	0.002	0.002	0	-1.36	76.6	-77.96
13	0.006	0.002	0.003	-92.97	77.31	-170.28	0.008	0.001	0.006	-92.97	77.31	-170.28	0.005	0.002	0.003	-92.97	77.31	-170.28
16	0	0.005	-0.004	-2.28	66.3	-68.58	0.002	0.002	0	-2.28	66.3	-68.58	0.003	0.004	-0.001	-2.28	66.3	-68.58
19	0.001	0.004	-0.002	-70.84	87.85	-158.69	0.003	0.002	0.001	-70.84	87.85	-158.69	0.003	0.002	0.001	-70.84	87.85	-158.69
21	0.003	0.003	0	-78.35	95.5	-173.85	0.005	0.003	0.002	-78.35	95.5	-173.85	0.004	0.002	0.002	-78.35	95.5	-173.85

[1]

[1] Estm=Estimated; Msrd=Measured Values; Diff=Difference (Applies from Table 18 to Table 26)

Table 20: Comparison of the 5^{th} Harmonic for One Selected Interval (60^{th})

	Phase 'A'						Phase 'B'						Phase 'C'					
	Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)		
Bus	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff
1	0.006	0.007	-0.001	-125.22	-109.97	-15.25	0.008	0.009	-0.001	-125.22	-109.97	-15.25	0.006	0.007	-0.001	-125.22	-109.97	-15.25
4	0.008	0.006	0.002	-116.46	-163.4	46.94	0.011	0.006	0.005	-116.46	-163.4	46.94	0.009	0.004	0.004	-116.46	-163.4	46.94
6	0.009	0.012	-0.003	-128.27	-136.76	8.48	0.014	0.012	0.003	-128.27	-136.76	8.48	0.011	0.009	0.002	-128.27	-136.76	8.48
11	0.01	0.012	-0.001	-168.21	-141.4	-26.81	0.012	0.012	0	-168.21	-141.4	-26.81	0.007	0.009	-0.002	-168.21	-141.4	-26.81
13	0.013	0.009	0.004	167.54	-137.72	305.26	0.016	0.009	0.006	167.54	-137.72	305.26	0.007	0.008	-0.001	167.54	-137.72	305.26
16	0.011	0.011	0	-176.29	-134.6	-41.69	0.012	0.011	0.001	-176.29	-134.6	-41.69	0.008	0.01	-0.003	-176.29	-134.6	-41.69
19	0.01	0.011	-0.001	-175.42	-134.56	-40.86	0.011	0.011	-0.001	-175.42	-134.56	-40.86	0.007	0.008	-0.001	-175.42	-134.56	-40.86
21	0.01	0.01	0	-142.85	135	-277.85	0.014	0.008	0.005	-142.85	135	-277.85	0.01	0.012	-0.002	-142.85	135	-277.85

Table 21: Comparison of the 7^{th} Harmonic for One Selected Interval (60^{th})

	Phase 'A'						Phase 'B'						Phase 'C'					
	Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)		
Bus	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff
1	0.012	0.017	-0.005	-39.39	-40.77	1.38	0.012	0.016	-0.004	-39.39	-40.77	1.38	0.011	0.015	-0.004	-39.39	-40.77	1.38
4	0.039	0.015	0.023	-2.28	-5.4	3.12	0.039	0.013	0.026	-2.28	-5.4	3.12	0.041	0.015	0.026	-2.28	-5.4	3.12
6	0.037	0.019	0.019	-3.17	-8.5	5.33	0.039	0.017	0.022	-3.17	-8.5	5.33	0.041	0.017	0.024	-3.17	-8.5	5.33
11	0.008	0.018	-0.011	-69.28	-12	-57.28	0.007	0.017	-0.01	-69.28	-12	-57.28	0.009	0.017	-0.008	-69.28	-12	-57.28
13	0.022	0.017	0.004	-155.29	-13.14	-142.15	0.014	0.015	-0.001	-155.29	-13.14	-142.15	0.018	0.016	0.002	-155.29	-13.14	-142.15
16	0.013	0.019	-0.006	-126.12	-9.8	-116.32	0.01	0.015	-0.006	-126.12	-9.8	-116.32	0.012	0.018	-0.006	-126.12	-9.8	-116.32
19	0.01	0.019	-0.009	-120.95	-12.81	-108.14	0.008	0.017	-0.009	-120.95	-12.81	-108.14	0.01	0.017	-0.007	-120.95	-12.81	-108.14
21	0.015	0.018	-0.003	-10.63	-170	159.37	0.017	0.017	0	-10.63	-170	159.37	0.019	0.016	0.003	-10.63	-170	159.37

Table 22: Comparison of the 9^{th} Harmonic for One Selected Interval (60^{th})

	Phase 'A'						Phase 'B'						Phase 'C'					
	Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)		
Bus	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff
1	0.001	0	0	52.43	-29.62	82.05	0	0	0	52.43	-29.62	82.05	0.001	0	0	52.43	-29.62	82.05
4	0	0	0	-7.73	0	-7.73	0.001	0	0.001	-7.73	0	-7.73	0.001	0	0.001	-7.73	0	-7.73
6	0	0.001	-0.001	93.49	-164.46	257.95	0.001	0.002	-0.001	93.49	-164.46	257.95	0	0.002	-0.001	93.49	-164.46	257.95
11	0.001	0	0.001	-27.37	0	-27.37	0.001	0	0.001	-27.37	0	-27.37	0.001	0	0.001	-27.37	0	-27.37
13	0.002	0.001	0.001	-68.77	-74.41	5.64	0.002	0.001	0	-68.77	-74.41	5.64	0.002	0.001	0.001	-68.77	-74.41	5.64
16	0.001	0	0.001	-17.78	0	-17.78	0.001	0	0.001	-17.78	0	-17.78	0.001	0.002	-0.001	-17.78	0	-17.78
19	0.001	0.001	0	-37.49	-107.53	70.04	0.001	0.002	-0.001	-37.49	-107.53	70.04	0.001	0.001	0	-37.49	-107.53	70.04
21	0.001	0.001	0	-44.62	89	-133.62	0.001	0.001	-0.001	-44.62	89	-133.62	0.001	0.001	0	-44.62	89	-133.62

Table 23: Comparison of the 11^{th} Harmonic for One Selected Interval (60^{th})

	Phase 'A'						Phase 'B'						Phase 'C'					
	Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)		
Bus	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff
1	0.002	0.002	0	28.17	35.62	-7.45	0.001	0.001	-0.001	28.17	35.62	-7.45	0.001	0.002	0	28.17	35.62	-7.45
4	0.008	0.003	0.005	-10.66	0.4	-11.06	0.012	0.003	0.009	-10.66	0.4	-11.06	0.009	0.003	0.006	-10.66	0.4	-11.06
6	0.009	0.002	0.007	-9.35	-6.88	-2.48	0.012	0.002	0.01	-9.35	-6.88	-2.48	0.009	0.003	0.007	-9.35	-6.88	-2.48
11	0.002	0	0.002	35.04	0	35.04	0.001	0.001	-0.001	35.04	0	35.04	0.001	0.002	-0.001	35.04	0	35.04
13	0.004	0.001	0.003	147.1	28.56	118.54	0.007	0.002	0.005	147.1	28.56	118.54	0.003	0.002	0.001	147.1	28.56	118.54
16	0.002	0.002	-0.001	153.31	-7.6	160.91	0.003	0.001	0.001	153.31	-7.6	160.91	0.002	0.002	-0.001	153.31	-7.6	160.91
19	0.001	0.002	-0.001	138.54	-4.5	143.04	0.003	0.002	0.001	138.54	-4.5	143.04	0.002	0.002	0	138.54	-4.5	143.04
21	0.004	0.001	0.003	25.34	151	-125.66	0.003	0.002	0.001	25.34	151	-125.66	0.006	0.002	0.004	25.34	151	-125.66

Table 24: Comparison of the 13^{th} Harmonic for One Selected Interval (60^{th})

	Phase 'A'						Phase 'B'						Phase 'C'					
	Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)		
Bus	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff
1	0.001	0.001	0	-7.75	-1.74	-6.01	0.001	0	0	-7.75	-1.74	-6.01	0	0.001	0	-7.75	-1.74	-6.01
4	0.002	0	0.002	154.71	0	154.71	0.003	0	0.003	154.71	0	154.71	0.001	0	0.001	154.71	0	154.71
6	0.001	0.002	-0.001	101.25	38.41	62.84	0.003	0.001	0.002	101.25	38.41	62.84	0.002	0.001	0.001	101.25	38.41	62.84
11	0.004	0.002	0.002	-5.87	21	-26.87	0.003	0	0.003	-5.87	21	-26.87	0.004	0	0.004	-5.87	21	-26.87
13	0.013	0.001	0.013	3.46	4.59	-1.13	0.01	0.001	0.009	3.46	4.59	-1.13	0.006	0.001	0.005	3.46	4.59	-1.13
16	0.005	0.001	0.003	2.48	9.8	-7.32	0.004	0	0.004	2.48	9.8	-7.32	0.003	0.002	0.001	2.48	9.8	-7.32
19	0.005	0.001	0.004	-8.65	22.59	-31.24	0.004	0.001	0.003	-8.65	22.59	-31.24	0.004	0.001	0.003	-8.65	22.59	-31.24
21	0.008	0.001	0.007	5.71	-28.5	34.21	0.004	0.001	0.003	5.71	-28.5	34.21	0.006	0.001	0.005	5.71	-28.5	34.21

Table 25: Comparison of the 15^{th} Harmonic for One Selected Interval (60^{th})

	Phase 'A'						Phase 'B'						Phase 'C'					
	Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)		
Bus	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff
1	0	0	0	134.11	102.27	31.84	0	0	0	134.11	102.27	31.84	0	0	0	134.11	102.27	31.84
4	0.002	0	0.002	-166.58	0	-166.58	0	0	0	-166.58	0	-166.58	0.002	0	0.002	-166.58	0	-166.58
6	0.002	0.002	0	-176.08	-164.78	-11.3	0	0.001	-0.001	-176.08	-164.78	-11.3	0.002	0.001	0.001	-176.08	-164.78	-11.3
11	0.002	0	0.002	44.02	0	44.02	0.001	0	0.001	44.02	0	44.02	0.002	0	0.002	44.02	0	44.02
13	0.004	0	0.003	23.16	-84.22	107.38	0.002	0.001	0.001	23.16	-84.22	107.38	0.005	0	0.005	23.16	-84.22	107.38
16	0.001	0.001	0.001	118.59	-11.1	129.69	0.001	0	0.001	118.59	-11.1	129.69	0	0.002	-0.002	118.59	-11.1	129.69
19	0.002	0.001	0.001	30.88	-164.87	195.75	0.001	0.002	-0.001	30.88	-164.87	195.75	0.003	0.001	0.002	30.88	-164.87	195.75
21	0.003	0	0.003	32.53	-70.5	103.03	0.002	0.001	0.001	32.53	-70.5	103.03	0.004	0.001	0.004	32.53	-70.5	103.03

Table 26: Comparison of the 17^{th} Harmonic for One Selected Interval (60^{th})

	Phase 'A'						Phase 'B'						Phase 'C'					
	Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)			Magnitude (pu)			Angle (degrees)		
Bus	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff	Estm	Msrd	Diff
1	0.001	0.001	0	-11.68	-23.73	12.05	0.001	0.001	0	-11.68	-23.73	12.05	0.001	0.001	0	-11.68	-23.73	12.05
4	0.012	0.004	0.008	-59.31	-27.9	-31.41	0.014	0.003	0.01	-59.31	-27.9	-31.41	0.014	0.005	0.009	-59.31	-27.9	-31.41
6	0.012	0.002	0.011	-58.15	-47.34	-10.81	0.013	0.001	0.012	-58.15	-47.34	-10.81	0.013	0.001	0.012	-58.15	-47.34	-10.81
11	0.003	0	0.003	92.4	0	92.4	0.003	0	0.003	92.4	0	92.4	0.002	0	0.002	92.4	0	92.4
13	0.015	0	0.015	119.29	-90.46	209.75	0.016	0	0.016	119.29	-90.46	209.75	0.016	0.001	0.015	119.29	-90.46	209.75
16	0.004	0	0.004	95.42	0	95.42	0.005	0	0.005	95.42	0	95.42	0.004	0	0.004	95.42	0	95.42
19	0.005	0.001	0.004	113.14	-30.95	144.08	0.006	0.001	0.005	113.14	-30.95	144.08	0.006	0.001	0.005	113.14	-30.95	144.08
21	0.005	0.001	0.004	116.33	15	101.33	0.005	0	0.005	116.33	15	101.33	0.005	0	0.005	116.33	15	101.33

Table 27: Comparison of the 1_{st} Harmonic for Different Number of Measurements for One Selected Interval (60^{th})

	Magnitude of Errors ($\|Estimated - Measured\|$)(pu)						Difference ($\angle Estimated$-$\angle Measured$) of Angles (degrees)					
No. of Equipment →	7	6	5	4	3	2	7	6	5	4	3	2
Bus	0.38	0.44	0.5	0.47	0.51	183.87	0.38	0.44	0.5	0.47	0.51	183.87
1	-30	-30	-30	-30	-29.41	-30	-30	-30	-30	-30	-29.41	-30
4	90.33	90.33	90.33	90.33	90.33	90.33	90.33	90.33	90.33	90.33	90.33	90.33
11	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49	-0.49
13	-30	-30.01	-29.96	-30.01	-30.01	-30.01	-30	-30.01	-29.96	-30.01	-30.01	-30.01
16	-0.42	11.8	-0.42	-0.42	48.55	53.17	-0.42	11.8	-0.42	-0.42	48.55	53.17
19	-30.01	-30.01	-30.02	-30.01	-30.01	-30.02	-30.01	-30.01	-30.02	-30.01	-30.01	-30.02
21	90.33	90.33	90.32	90.32	90.32	90.32	90.33	90.33	90.32	90.32	90.32	90.32

Table 28: Comparison of the 3^{rd} Harmonic for Different Number of Measurements for One Selected Interval (60^{th})

	Magnitude of Errors ($\|Estimated - Measured\|$)(pu)						Difference ($\angle Estimated$-$\angle Measured$) of Angles (degrees)					
No. of Equipment →	7	6	5	4	3	2	7	6	5	4	3	2
Bus	-32.83	-22.89	-22.57	-13.74	-20.71	-13.29	-32.83	-22.89	-22.57	-13.74	-20.71	-13.29
1	48.13	48.15	48.16	48.16	48.15	48.15	48.13	48.15	48.16	48.16	48.15	48.15
4	170.35	170.35	170.36	170.36	170.36	170.36	170.35	170.35	170.36	170.36	170.36	170.36
11	-23.85	159.18	155.22	-23.77	-23.77	-23.77	-23.85	159.18	155.22	-23.77	-23.77	-23.77
13	48.13	47.53	46.47	48.16	48.16	48.16	48.13	47.53	46.47	48.16	48.16	48.16
16	-8.99	-7.59	-7.39	-8.95	17.84	12.92	-8.99	-7.59	-7.39	-8.95	17.84	12.92
19	48.12	47.01	46.4	48.03	48.16	48.18	48.12	47.01	46.4	48.03	48.16	48.18
21	170.55	171.61	171.73	170.41	170.49	170.45	170.55	171.61	171.73	170.41	170.49	170.45

Table 29: Comparison of the 5^{th} Harmonic for Different Number of Measurements for One Selected Interval (60^{th})

	Magnitude of Errors ($\|Estimated - Measured\|$)(pu)						Difference ($\angle Estimated$-$\angle Measured$) of Angles (degrees)					
No. of Equipment →	7	6	5	4	3	2	7	6	5	4	3	2
Bus	-15.25	-9.88	3.88	3.74	4.83	3.55	-15.25	-9.88	3.88	3.74	4.83	3.55
1	-110.16	-110.05	-110.05	-110.05	-110.05	-110.05	-110.16	-110.05	-110.05	-110.05	-110.05	-110.05
4	148.93	149.02	149.02	149.02	149.02	149.02	148.93	149.02	149.02	149.02	149.02	149.02
11	9.1	75.92	75.98	9.2	9.19	9.19	9.1	75.92	75.98	9.2	9.19	9.19
13	-110.35	-113.61	-115.79	-110.16	-110.16	-110.17	-110.35	-113.61	-115.79	-110.16	-110.16	-110.17
16	-4.68	-7.42	-4.53	-3.93	-17.51	19.25	-4.68	-7.42	-4.53	-3.93	-17.51	19.25
19	-110.37	-114.39	-109.25	-109.53	-110.16	-110.18	-110.37	-114.39	-109.25	-109.53	-110.16	-110.18
21	148.91	149.17	149.06	149.19	148.9	148.89	148.91	149.17	149.06	149.19	148.9	148.89

Table 30: Comparison of the 7^{th} Harmonic for Different Number of Measurements for One Selected Interval (60^{th})

	Magnitude of Errors ($\|Estimated - Measured\|$)(pu)						Difference ($\angle Estimated$-$\angle Measured$) of Angles (degrees)					
No. of Equipment →	7	6	5	4	3	2	7	6	5	4	3	2
Bus	1.38	8.09	8.32	8.44	9.06	-75.43	1.38	8.09	8.32	8.44	9.06	-75.43
1	-40.79	-40.78	-40.78	-40.78	-40.79	-40.79	-40.79	-40.78	-40.78	-40.78	-40.79	-40.79
4	78.08	78.09	78.09	78.09	78.09	78.09	78.08	78.09	78.09	78.09	78.09	78.09
11	-9.16	6.71	7.53	-9.14	-9.16	-9.16	-9.16	6.71	7.53	-9.14	-9.16	-9.16
13	-40.83	-41.35	-42.08	-40.8	-40.81	-40.81	-40.83	-41.35	-42.08	-40.8	-40.81	-40.81
16	-5.99	-7.1	-5.94	-5.32	-0.96	12.27	-5.99	-7.1	-5.94	-5.32	-0.96	12.27
19	-40.84	-41.73	-40.08	-40.09	-40.81	-40.81	-40.84	-41.73	-40.08	-40.09	-40.81	-40.81
21	78.13	78.79	78.78	78.76	78.07	78.07	78.13	78.79	78.78	78.76	78.07	78.07

Table 31: Comparison of the 9^{th} Harmonic for Different Number of Measurements for One Selected Interval (60^{th})

	Magnitude of Errors ($\|Estimated - Measured\|$)(pu)						Difference ($\angle Estimated$-$\angle Measured$) of Angles (degrees)					
No. of Equipment →	7	6	5	4	3	2	7	6	5	4	3	2
Bus	82.05	55.01	12.19	-34.28	-130.77	29.62	82.05	55.01	12.19	-34.28	-130.77	29.62
1	-29.71	-29.66	-29.66	-29.66	-29.67	-29.67	-29.71	-29.66	-29.66	-29.66	-29.67	-29.67
4	-170.69	-170.67	-170.68	-170.68	-170.68	-170.68	-170.69	-170.67	-170.68	-170.68	-170.68	-170.68
11	-65.47	20.11	12.82	-65.16	-65.16	-65.16	-65.47	20.11	12.82	-65.16	-65.16	-65.16
13	-29.8	-30.91	-30.19	-29.72	-29.72	-29.72	-29.8	-30.91	-30.19	-29.72	-29.72	-29.72
16	1.09	1.76	1.02	0.41	-12.89	186.37	1.09	1.76	1.02	0.41	-12.89	186.37
19	-29.79	-30.63	-30.45	-32.23	-29.75	-29.76	-29.79	-30.63	-30.45	-32.23	-29.75	-29.76
21	-170.82	-172.28	-170.32	-166.63	-170.73	-170.6	-170.82	-172.28	-170.32	-166.63	-170.73	-170.6

Table 32: Comparison of the 11^{th} Harmonic for Different Number of Measurements for One Selected Interval (60^{th})

	Magnitude of Errors ($\|Estimated - Measured\|$)(pu)						Difference ($\angle Estimated$-$\angle Measured$) of Angles (degrees)					
No. of Equipment →	7	6	5	4	3	2	7	6	5	4	3	2
Bus	-7.45	-6.41	14.65	16.03	-5.74	12.49	-7.45	-6.41	14.65	16.03	-5.74	12.49
1	35.58	35.54	35.54	35.54	35.53	35.53	35.58	35.54	35.54	35.54	35.53	35.53
4	-101.62	-101.55	-101.54	-101.55	-101.55	-101.55	-101.62	-101.55	-101.54	-101.55	-101.55	-101.55
11	1.12	8.42	13.89	1.09	1.09	1.09	1.12	8.42	13.89	1.09	1.09	1.09
13	35.47	36.98	36.07	35.37	35.37	35.37	35.47	36.98	36.07	35.37	35.37	35.37
16	-3.2	-0.96	-3.26	-0.58	-53.12	15.39	-3.2	-0.96	-3.26	-0.58	-53.12	15.39
19	35.48	36.98	36.22	38.82	35.41	35.37	35.48	36.98	36.22	38.82	35.41	35.37
21	-101.68	-103.11	-104.92	-103.38	-101.51	-101.54	-101.68	-103.11	-104.92	-103.38	-101.51	-101.54

Table 33: Comparison of the 13^{th} Harmonic for Different Number of Measurements for One Selected Interval (60^{th})

	Magnitude of Errors ($\|Estimated - Measured\|$)(pu)						Difference ($\angle Estimated$-$\angle Measured$) of Angles (degrees)					
No. of Equipment →	7	6	5	4	3	2	7	6	5	4	3	2
Bus	-6.01	30.79	37.07	30.57	28.06	28.23	-6.01	30.79	37.07	30.57	28.06	28.23
1	-1.93	-1.86	-1.87	-1.88	-1.93	-1.93	-1.93	-1.86	-1.87	-1.88	-1.93	-1.93
4	136.42	136.55	136.55	136.55	136.55	136.55	136.42	136.55	136.55	136.55	136.55	136.55
11	-80.07	15.99	10.68	-79.91	-79.91	-79.91	-80.07	15.99	10.68	-79.91	-79.91	-79.91
13	-2.27	-3.26	-6.78	-2.2	-2.21	-2.2	-2.27	-3.26	-6.78	-2.2	-2.21	-2.2
16	-8.94	-9.84	-8.86	-9.36	140.46	-9.71	-8.94	-9.84	-8.86	-9.36	140.46	-9.71
19	-2.44	-4.9	-2.45	1.95	-2.22	-2.19	-2.44	-4.9	-2.45	1.95	-2.22	-2.19
21	136.19	136.37	136.23	134.46	136.06	136.13	136.19	136.37	136.23	134.46	136.06	136.13

Table 34: Comparison of the 15^{th} Harmonic for Different Number of Measurements for One Selected Interval (60^{th})

	Magnitude of Errors ($\|Estimated - Measured\|$)(pu)						Difference ($\angle Estimated$-$\angle Measured$) of Angles (degrees)					
No. of Equipment →	7	6	5	4	3	2	7	6	5	4	3	2
Bus	31.84	24.63	-47.44	53.38	-56.48	-102.27	31.84	24.63	-47.44	53.38	-56.48	-102.27
1	101.83	102.19	102.23	102.2	102.14	102.06	101.83	102.19	102.23	102.2	102.14	102.06
4	107.31	106.17	106.18	106.17	106.17	106.17	107.31	106.17	106.18	106.17	106.17	106.17
11	-44.4	164.38	-171.18	-44.68	-44.68	-44.68	-44.4	164.38	-171.18	-44.68	-44.68	-44.68
13	99.85	88.73	83.85	101.75	101.7	101.71	99.85	88.73	83.85	101.75	101.7	101.71
16	-7.57	-8.1	-7.45	-6.58	342.59	292.89	-7.57	-8.1	-7.45	-6.58	342.59	292.89
19	99.03	82.59	71.82	104.39	101.71	101.69	99.03	82.59	71.82	104.39	101.71	101.69
21	113.47	140.98	168.21	86.37	106.73	107.31	113.47	140.98	168.21	86.37	106.73	107.31

Table 35: Comparison of the 17^{th} Harmonic for Different Number of Measurements for One Selected Interval (60^{th})

	Magnitude of Errors ($\|Estimated - Measured\|$)(pu)						Difference ($\angle Estimated$-$\angle Measured$) of Angles (degrees)					
No. of Equipment →	7	6	5	4	3	2	7	6	5	4	3	2
Bus	12.05	12.9	8.94	7.8	26.17	-2.16	12.05	12.9	8.94	7.8	26.17	-2.16
1	-23.59	-23.64	-23.65	-23.64	-23.66	-23.66	-23.59	-23.64	-23.65	-23.64	-23.66	-23.66
4	-145.58	-145.6	-145.61	-145.61	-145.61	-145.61	-145.58	-145.6	-145.61	-145.61	-145.61	-145.61
11	3.08	-6.16	-8.08	2.98	2.98	2.98	3.08	-6.16	-8.08	2.98	2.98	2.98
13	-23.19	-20.78	-19.87	-23.54	-23.54	-23.54	-23.19	-20.78	-19.87	-23.54	-23.54	-23.54
16	-8.72	-6.67	-8.95	-7.92	-17.81	0.03	-8.72	-6.67	-8.95	-7.92	-17.81	0.03
19	-23.23	-20.99	-22.42	-22.26	-23.52	-23.53	-23.23	-20.99	-22.42	-22.26	-23.52	-23.53
21	-145.5	-144.9	-145.29	-145.15	-145.64	-145.63	-145.5	-144.9	-145.29	-145.15	-145.64	-145.63

D COMPARISON OF ERRORS FOR DIFFERENT TIME INTERVALS

Table 36: Variation of Error in Magnitude and Angle (1^{st} Harmonic) with Time (1^{st} to 100^{th})

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus →	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
1	-0.02489	-0.01632	-0.03147	-0.03927	-0.11423	-0.02634	-0.04503	-0.01148	-0.54	2.15	2.33	1.63	5.41	1.75	1.61	2.66
2	-0.01005	-0.02187	-0.03598	-0.03748	-0.1023	-0.04854	-0.04132	-0.00593	0.31	1.83	2.02	1.78	6.31	1.89	1.93	3.12
3	-0.01032	-0.02162	-0.03631	-0.03744	-0.1023	-0.04858	-0.04136	-0.00588	0.3	1.8	2.03	1.76	6.2	1.86	1.9	3.08
4	-0.0106	-0.02099	-0.03581	-0.03694	-0.10184	-0.04787	-0.04128	-0.00547	0.31	1.78	2.01	1.75	6.19	1.84	1.87	3.04
5	-0.01018	-0.02204	-0.03614	-0.03781	-0.10192	-0.04862	-0.04107	-0.0058	0.31	1.77	2	1.75	6.23	1.84	1.88	3.05
6	-0.01019	-0.02154	-0.03623	-0.0376	-0.10154	-0.04841	-0.04132	-0.00618	0.32	1.78	1.98	1.76	6.2	1.85	1.89	3.07
7	-0.01027	-0.02183	-0.03648	-0.03819	-0.10238	-0.04908	-0.04153	-0.00609	0.32	1.8	2.02	1.78	6.22	1.85	1.9	3.09
8	-0.01012	-0.02195	-0.03685	-0.03811	-0.1023	-0.04858	-0.04136	-0.00597	0.32	1.77	2.03	1.76	6.24	1.81	1.88	3.05
9	-0.01031	-0.02133	-0.03677	-0.03748	-0.10171	-0.048	-0.04107	-0.00584	0.31	1.78	2.04	1.77	6.26	1.82	1.9	3.06
10	-0.01028	-0.02162	-0.03698	-0.03765	-0.10196	-0.04846	-0.04165	-0.00655	0.31	1.75	2	1.73	6.11	1.8	1.87	3.04
11	-0.01011	-0.02179	-0.03689	-0.03769	-0.10292	-0.04837	-0.04186	-0.00668	0.34	1.77	2.01	1.74	6.18	1.75	1.89	3.07
12	-0.01009	-0.02212	-0.03681	-0.03794	-0.10309	-0.04862	-0.04186	-0.00626	0.31	1.8	2.02	1.77	6.3	1.79	1.92	3.07
13	-0.00995	-0.02233	-0.03702	-0.03848	-0.10305	-0.04937	-0.04207	-0.00659	0.31	1.83	2.07	1.8	6.34	1.82	1.95	3.11
14	-0.01003	-0.02225	-0.03719	-0.03869	-0.10288	-0.04908	-0.04236	-0.00639	0.31	1.82	2.06	1.81	6.21	1.79	1.96	3.1
15	-0.00997	-0.02174	-0.03689	-0.03823	-0.10359	-0.04854	-0.0424	-0.00684	0.31	1.8	2.04	1.78	6.21	1.74	1.93	3.11
16	-0.0102	-0.02191	-0.03702	-0.03806	-0.10451	-0.04887	-0.0429	-0.0071	0.29	1.82	2.06	1.78	6.3	1.77	1.95	3.1
17	-0.00982	-0.02229	-0.03669	-0.0384	-0.10296	-0.04891	-0.04224	-0.00676	0.31	1.82	2.04	1.8	6.3	1.8	1.97	3.13
18	-0.01002	-0.02191	-0.03689	-0.03811	-0.10309	-0.04912	-0.04232	-0.00697	0.29	1.82	2.04	1.79	6.25	1.83	1.96	3.13
19	-0.00983	-0.02237	-0.03685	-0.03856	-0.10338	-0.04942	-0.0424	-0.00676	0.3	1.79	2	1.76	6.2	1.81	1.93	3.08
20	-0.01012	-0.02199	-0.03677	-0.03831	-0.1043	-0.0495	-0.04253	-0.00664	0.32	1.79	1.99	1.73	6.25	1.8	1.93	3.04
21	-0.01014	-0.02187	-0.03669	-0.03802	-0.10363	-0.04896	-0.04253	-0.00647	0.33	1.8	1.99	1.75	6.29	1.8	1.95	3.06
22	-0.01025	-0.02129	-0.03639	-0.0376	-0.10346	-0.04862	-0.04236	-0.0068	0.32	1.79	1.98	1.74	6.22	1.81	1.93	3.07
23	-0.0099	-0.02199	-0.03669	-0.03831	-0.10367	-0.04962	-0.04245	-0.00689	0.33	1.78	1.98	1.73	6.24	1.84	1.92	3.06
24	-0.01001	-0.02237	-0.03694	-0.03848	-0.10388	-0.04942	-0.04274	-0.00684	0.29	1.82	2.03	1.75	6.31	1.87	1.97	3.07
25	-0.01026	-0.02162	-0.03677	-0.03823	-0.10384	-0.04937	-0.04253	-0.00639	0.3	1.82	2.03	1.77	6.3	1.87	1.97	3.07

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Table 36 – continued from previous page ($1^{st} Harmonic$)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
26	-0.00993	-0.02212	-0.03698	-0.03844	-0.10292	-0.04942	-0.04257	-0.00672	0.3	1.81	2.01	1.78	6.27	1.88	1.96	3.08
27	-0.01011	-0.02174	-0.0366	-0.03802	-0.10288	-0.04912	-0.04236	-0.0068	0.29	1.81	2.02	1.77	6.24	1.86	1.95	3.07
28	-0.00988	-0.02191	-0.0366	-0.03802	-0.10334	-0.04891	-0.0427	-0.00655	0.31	1.8	2.02	1.77	6.23	1.82	1.95	3.05
29	-0.01025	-0.02166	-0.03652	-0.03785	-0.10438	-0.04946	-0.04286	-0.00676	0.32	1.82	2.01	1.78	6.3	1.88	1.96	3.06
30	-0.01014	-0.02195	-0.03689	-0.03819	-0.10388	-0.04954	-0.04286	-0.00697	0.32	1.79	1.96	1.75	6.22	1.81	1.92	3.04
31	-0.01011	-0.02162	-0.03669	-0.03765	-0.10334	-0.049	-0.04286	-0.00651	0.34	1.78	1.98	1.75	6.22	1.78	1.92	3.05
32	-0.00998	-0.0217	-0.03614	-0.0379	-0.10338	-0.04937	-0.04299	-0.00668	0.36	1.78	1.96	1.77	6.22	1.78	1.92	3.03
33	-0.00979	-0.02191	-0.03593	-0.03823	-0.10334	-0.04967	-0.04282	-0.00668	0.37	1.76	1.93	1.75	6.17	1.76	1.88	2.98
34	-0.00998	-0.02129	-0.03606	-0.03748	-0.10296	-0.04858	-0.04261	-0.00659	0.37	1.79	1.97	1.78	6.3	1.77	1.91	3.04
35	-0.00986	-0.02145	-0.03598	-0.03756	-0.1028	-0.04891	-0.04245	-0.00697	0.36	1.76	1.95	1.75	6.19	1.74	1.88	3.04
36	-0.00981	-0.02179	-0.03593	-0.03719	-0.10238	-0.04896	-0.04249	-0.00701	0.37	1.75	1.92	1.72	6.13	1.75	1.87	3.03
37	-0.01015	-0.02074	-0.0356	-0.03639	-0.10255	-0.04804	-0.04228	-0.00618	0.37	1.78	1.95	1.75	6.2	1.75	1.88	3.04
38	-0.00985	-0.02137	-0.03593	-0.0371	-0.1025	-0.04846	-0.04249	-0.00643	0.36	1.77	1.97	1.77	6.24	1.73	1.91	3.04
39	-0.0098	-0.02166	-0.03598	-0.03752	-0.10255	-0.04904	-0.04274	-0.00672	0.35	1.79	1.98	1.79	6.26	1.77	1.94	3.09
40	-0.00982	-0.0217	-0.03614	-0.0374	-0.10259	-0.04908	-0.0427	-0.00718	0.36	1.75	1.96	1.74	6.13	1.72	1.89	3.04
41	-0.00973	-0.02149	-0.0361	-0.03685	-0.10171	-0.04866	-0.04245	-0.00689	0.36	1.76	1.98	1.73	6.18	1.72	1.91	3.03
42	-0.00997	-0.02116	-0.03531	-0.03652	-0.10108	-0.04808	-0.04194	-0.00609	0.36	1.77	1.98	1.74	6.23	1.73	1.92	3.05
43	-0.0101	-0.02108	-0.03598	-0.03702	-0.1028	-0.04812	-0.04265	-0.00684	0.36	1.77	1.98	1.77	6.2	1.7	1.92	3.06
44	-0.00972	-0.02174	-0.03627	-0.03765	-0.10234	-0.04846	-0.04265	-0.00684	0.36	1.77	1.98	1.78	6.19	1.68	1.92	3.08
45	-0.00966	-0.02212	-0.03614	-0.03752	-0.1023	-0.04846	-0.04249	-0.0068	0.35	1.78	2	1.78	6.2	1.69	1.91	3.08
46	-0.00978	-0.02104	-0.03568	-0.03689	-0.10175	-0.04787	-0.0419	-0.00668	0.34	1.79	2.01	1.79	6.21	1.71	1.91	3.1
47	-0.0097	-0.02154	-0.03593	-0.03748	-0.10221	-0.04841	-0.04207	-0.00664	0.35	1.79	2.03	1.82	6.24	1.71	1.92	3.1
48	-0.00976	-0.02204	-0.03644	-0.03802	-0.10263	-0.0485	-0.04224	-0.00647	0.34	1.78	2.01	1.8	6.16	1.66	1.89	3.05
49	-0.00971	-0.02158	-0.03602	-0.03719	-0.10113	-0.04795	-0.04174	-0.00601	0.34	1.79	2.02	1.79	6.19	1.69	1.91	3.07
50	-0.0099	-0.02154	-0.0361	-0.03694	-0.10204	-0.04846	-0.04186	-0.00651	0.33	1.79	2.03	1.77	6.21	1.71	1.91	3.1
51	-0.00994	-0.02166	-0.03656	-0.03752	-0.10355	-0.04908	-0.0424	-0.00684	0.32	1.8	2.06	1.77	6.26	1.73	1.92	3.1
52	-0.01004	-0.02191	-0.03664	-0.03781	-0.10401	-0.04958	-0.04261	-0.00701	0.31	1.8	2.04	1.75	6.27	1.74	1.92	3.08

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Table 36 – continued from previous page ($1^{st} Harmonic$)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
53	-0.01028	-0.02116	-0.03589	-0.0371	-0.10275	-0.04871	-0.0422	-0.00584	0.31	1.79	2.02	1.72	6.19	1.73	1.9	3.05
54	-0.00977	-0.02174	-0.03656	-0.03702	-0.10263	-0.04879	-0.04245	-0.0071	0.32	1.79	2.03	1.72	6.25	1.71	1.92	3.11
55	-0.01004	-0.0212	-0.03564	-0.0366	-0.10309	-0.04879	-0.04207	-0.00634	0.33	1.8	2.02	1.71	6.26	1.73	1.91	3.1
56	-0.00995	-0.02133	-0.03581	-0.03664	-0.10342	-0.04896	-0.04199	-0.00664	0.36	1.76	1.97	1.69	6.22	1.73	1.87	3.04
57	-0.01012	-0.02166	-0.03635	-0.03664	-0.10359	-0.04917	-0.04236	-0.00659	0.36	1.78	2	1.68	6.25	1.75	1.89	3.05
58	-0.01017	-0.02112	-0.03573	-0.03585	-0.1025	-0.049	-0.0419	-0.00614	0.35	1.77	1.99	1.67	6.21	1.76	1.88	3.07
59	-0.01014	-0.02066	-0.03531	-0.03564	-0.10225	-0.04854	-0.04165	-0.00605	0.37	1.74	1.93	1.64	6.13	1.72	1.84	3.04
60	-0.00987	-0.02099	-0.0356	-0.03585	-0.10259	-0.04829	-0.04178	-0.00672	0.38	1.76	1.94	1.66	6.24	1.71	1.87	3.09
61	-0.01003	-0.0212	-0.03627	-0.03593	-0.10321	-0.04862	-0.04186	-0.00697	0.35	1.79	1.99	1.7	6.27	1.75	1.87	3.13
62	-0.01031	-0.02045	-0.03548	-0.03531	-0.10221	-0.04795	-0.04144	-0.00609	0.34	1.79	1.99	1.69	6.25	1.77	1.86	3.14
63	-0.01014	-0.0207	-0.03577	-0.03535	-0.10284	-0.04833	-0.04165	-0.00668	0.35	1.74	1.96	1.66	6.18	1.73	1.83	3.09
64	-0.01	-0.02083	-0.03552	-0.03535	-0.10346	-0.04787	-0.04194	-0.0068	0.36	1.76	1.97	1.67	6.27	1.72	1.85	3.09
65	-0.00997	-0.02108	-0.03606	-0.03552	-0.10363	-0.04837	-0.04186	-0.00672	0.34	1.78	1.99	1.69	6.31	1.77	1.86	3.11
66	-0.00985	-0.02179	-0.03644	-0.03606	-0.10342	-0.04883	-0.04215	-0.00726	0.31	1.79	1.98	1.69	6.3	1.78	1.87	3.14
67	-0.01001	-0.02137	-0.03585	-0.03573	-0.10326	-0.04854	-0.04165	-0.0068	0.34	1.79	1.95	1.72	6.29	1.77	1.88	3.13
68	-0.00988	-0.02087	-0.03581	-0.03518	-0.10259	-0.04804	-0.04149	-0.00643	0.33	1.79	1.97	1.7	6.28	1.76	1.89	3.1
69	-0.00996	-0.02041	-0.03543	-0.0351	-0.10242	-0.04754	-0.04107	-0.00634	0.33	1.79	1.98	1.72	6.3	1.74	1.9	3.11
70	-0.00963	-0.02191	-0.03644	-0.03639	-0.1033	-0.04862	-0.0419	-0.00743	0.34	1.78	1.96	1.73	6.3	1.72	1.89	3.12
71	-0.00984	-0.02116	-0.0356	-0.03568	-0.10217	-0.04783	-0.04132	-0.0063	0.35	1.78	1.95	1.72	6.24	1.72	1.89	3.09
72	-0.00981	-0.02099	-0.03539	-0.03552	-0.102	-0.04762	-0.04132	-0.00659	0.35	1.77	1.93	1.71	6.24	1.73	1.88	3.05
73	-0.00979	-0.02091	-0.03568	-0.03585	-0.10225	-0.04754	-0.04128	-0.0068	0.35	1.8	1.97	1.76	6.36	1.78	1.9	3.12
74	-0.0101	-0.02074	-0.03573	-0.03598	-0.10221	-0.04712	-0.04128	-0.00626	0.33	1.83	2.03	1.85	6.43	1.77	1.93	3.17
75	-0.00968	-0.02162	-0.03614	-0.03715	-0.10142	-0.04854	-0.04149	-0.00684	0.34	1.77	1.96	1.83	6.19	1.78	1.86	3.08
76	-0.00993	-0.02087	-0.03585	-0.0361	-0.10021	-0.0472	-0.04119	-0.00609	0.32	1.79	1.98	1.83	6.16	1.8	1.9	3.08
77	-0.00974	-0.02154	-0.03631	-0.03673	-0.10083	-0.04779	-0.04128	-0.00684	0.33	1.79	1.97	1.82	6.18	1.81	1.91	3.09
78	-0.00965	-0.02191	-0.03623	-0.03715	-0.1023	-0.04821	-0.04174	-0.00714	0.35	1.79	1.95	1.82	6.22	1.79	1.92	3.1
79	-0.00982	-0.02191	-0.03631	-0.03748	-0.10292	-0.04841	-0.04228	-0.00735	0.33	1.81	1.94	1.86	6.24	1.8	1.92	3.1

Continued on next page

Table 36 – continued from previous page ($1^{st} Harmonic$)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
80	-0.00988	-0.02145	-0.03631	-0.03727	-0.10184	-0.04787	-0.04182	-0.00676	0.32	1.8	1.96	1.83	6.16	1.77	1.9	3.06
81	-0.00959	-0.02216	-0.03689	-0.03756	-0.10209	-0.04829	-0.04182	-0.00722	0.34	1.81	2	1.83	6.23	1.77	1.93	3.12
82	-0.01	-0.02137	-0.03648	-0.03673	-0.10321	-0.04783	-0.04186	-0.00697	0.34	1.8	1.98	1.78	6.19	1.75	1.92	3.09
83	-0.00979	-0.02133	-0.03602	-0.03669	-0.10321	-0.04837	-0.04224	-0.00672	0.37	1.79	1.95	1.76	6.24	1.76	1.91	3.08
84	-0.00973	-0.02187	-0.03656	-0.03694	-0.10363	-0.04875	-0.04253	-0.00743	0.36	1.8	1.96	1.76	6.25	1.78	1.92	3.08
85	-0.00968	-0.02166	-0.03652	-0.03656	-0.10342	-0.04837	-0.04249	-0.00764	0.37	1.78	1.94	1.71	6.19	1.7	1.92	3.09
86	-0.00962	-0.02162	-0.03627	-0.03619	-0.1025	-0.04812	-0.04199	-0.00697	0.4	1.8	1.97	1.71	6.25	1.72	1.95	3.12
87	-0.00981	-0.02133	-0.03606	-0.03589	-0.103	-0.04795	-0.04211	-0.00684	0.39	1.8	1.96	1.71	6.26	1.73	1.96	3.14
88	-0.00981	-0.02154	-0.03664	-0.0361	-0.10367	-0.04825	-0.04265	-0.00743	0.37	1.8	1.95	1.69	6.25	1.73	1.96	3.13
89	-0.00976	-0.02154	-0.03627	-0.03581	-0.10284	-0.04758	-0.04215	-0.00701	0.36	1.8	1.92	1.69	6.27	1.72	1.96	3.15
90	-0.00944	-0.02229	-0.03685	-0.03631	-0.1038	-0.04837	-0.04253	-0.00801	0.38	1.79	1.91	1.66	6.24	1.68	1.96	3.12
91	-0.00956	-0.02137	-0.03619	-0.0356	-0.10242	-0.04691	-0.04194	-0.00726	0.4	1.82	1.98	1.71	6.3	1.69	2	3.16
92	-0.00965	-0.02183	-0.03719	-0.03627	-0.10359	-0.04787	-0.04278	-0.00776	0.37	1.85	2	1.75	6.36	1.75	2.01	3.19
93	-0.00983	-0.02149	-0.03677	-0.03614	-0.1025	-0.04758	-0.0419	-0.00684	0.36	1.83	1.98	1.73	6.3	1.76	1.96	3.17
94	-0.00977	-0.02174	-0.0366	-0.03623	-0.1023	-0.04758	-0.04203	-0.00689	0.37	1.81	1.95	1.72	6.25	1.73	1.97	3.15
95	-0.00966	-0.0217	-0.03664	-0.03627	-0.10246	-0.04766	-0.0422	-0.00714	0.38	1.8	1.92	1.73	6.23	1.73	1.96	3.12
96	-0.00987	-0.02154	-0.03652	-0.03644	-0.10296	-0.04729	-0.04249	-0.00751	0.36	1.82	1.94	1.75	6.25	1.73	1.97	3.15
97	-0.00984	-0.02179	-0.03702	-0.0366	-0.10342	-0.0475	-0.0427	-0.00764	0.33	1.84	1.99	1.77	6.35	1.73	2.01	3.17
98	-0.0098	-0.02166	-0.03702	-0.03635	-0.10292	-0.04762	-0.04232	-0.00718	0.32	1.86	2.03	1.78	6.38	1.78	2.02	3.18
99	-0.00989	-0.02208	-0.03735	-0.03673	-0.10242	-0.04787	-0.04228	-0.00693	0.31	1.85	2.06	1.8	6.3	1.81	2.02	3.17
100	-0.00997	-0.02129	-0.03731	-0.03627	-0.1025	-0.04775	-0.04207	-0.00705	0.29	1.86	2.05	1.8	6.27	1.84	2	3.17

Table 37: Variation of Error in Magnitude and Angle (3^{rd} Harmonic) with Time (1^{st} to 100^{th})

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
1	0.0001	-0.0020	-0.0043	-0.0037	-0.0010	-0.0043	-0.0031	-0.0024	-23.32	21.69	-18.26	4.14	-175.50	-81.97	-34.29	-50.90
2	0.00	-0.0014	-0.0042	-0.0039	-0.0001	-0.0039	-0.0026	-0.0015	-21.33	-7.87	-38.84	-4.69	-125.76	-63.88	-68.33	-80.12
3	-0.0002	-0.0013	-0.0042	-0.0039	-0.0014	-0.0048	-0.0030	-0.0016	-26.44	28.96	-3.77	-5.67	-257.11	16.18	-1.21	-10.24
4	-0.0002	-0.0007	-0.0035	-0.0036	-0.0008	-0.0039	-0.0025	-0.0016	-25.16	16.25	-11.51	-5.34	37.14	8.07	-5.79	-7.30
5	-0.0002	-0.0013	-0.0036	-0.0039	-0.0017	-0.0045	-0.0028	-0.0015	-35.93	53.01	-1.23	-3.95	84.90	-3.19	-6.96	-14.25
6	-0.0003	-0.0005	-0.0034	-0.0036	-0.0008	-0.0042	-0.0028	-0.0019	-31.74	36.93	3.25	-1.25	92.90	18.45	11.90	14.09
7	-0.0005	-0.0018	-0.0041	-0.0048	-0.0008	-0.0041	-0.0033	-0.0022	-33.40	10.73	-38.35	-37.91	-154.62	-1.60	-44.86	-75.74
8	-0.0004	-0.0010	-0.0032	-0.0039	-0.0011	-0.0040	-0.0024	-0.0008	-24.54	13.84	-21.67	-5.61	-34.25	-8.03	-23.71	-35.49
9	-0.0004	-0.0008	-0.0032	-0.0042	-0.0018	-0.0039	-0.0028	-0.0019	-21.97	6.66	-14.59	3.33	70.41	-4.86	-6.44	-15.27
10	-0.0004	-0.0012	-0.0036	-0.0041	0.0008	-0.0038	-0.0032	-0.0019	-24.50	-41.59	-60.04	-80.38	-151.69	-13.25	-88.15	-116.37
11	-0.0003	-0.0006	-0.0030	-0.0036	-0.0010	-0.0036	-0.0022	-0.0013	-24.47	0.24	-21.71	-8.14	-11.20	-8.84	-18.08	-35.80
12	-0.0003	-0.0008	-0.0032	-0.0037	-0.0010	-0.0036	-0.0022	-0.0012	-27.36	16.91	-9.58	3.18	-4.43	-1.62	-13.25	-24.44
13	-0.0003	-0.0013	-0.0033	-0.0037	-0.0003	-0.0038	-0.0020	-0.0010	-27.00	20.95	-14.46	3.04	-20.02	1.55	-17.82	-30.03
14	-0.0004	-0.0011	-0.0033	-0.0040	-0.0012	-0.0041	-0.0023	-0.0010	-25.98	22.40	-11.70	3.25	-16.30	-4.56	-16.63	-27.30
15	-0.0002	-0.0011	-0.0035	-0.0042	-0.0020	-0.0035	-0.0026	-0.0023	-21.19	9.96	-15.99	-2.50	-7.87	-1.40	-15.76	-31.23
16	-0.0004	-0.0006	-0.0029	-0.0036	-0.0009	-0.0036	-0.0021	-0.0016	-21.74	5.02	-16.86	0.64	13.56	-4.13	-9.92	-18.29
17	-0.0004	-0.0008	-0.0029	-0.0035	-0.0002	-0.0034	-0.0020	-0.0010	-31.89	6.19	-32.06	-21.12	-58.44	-3.78	-36.77	-60.85
18	-0.0003	-0.0008	-0.0029	-0.0035	-0.0005	-0.0035	-0.0020	-0.0010	-22.32	3.55	-19.10	-4.24	-12.74	-0.10	-17.45	-31.52
19	-0.0004	-0.0009	-0.0029	-0.0035	0.00	-0.0035	-0.0019	-0.0006	-26.64	14.63	-18.46	0.71	-19.31	1.55	-19.02	-35.00
20	-0.0003	-0.0012	-0.0032	-0.0039	-0.0001	-0.0035	-0.0019	-0.0012	-26.65	24.94	-12.31	9.27	-7.75	6.52	-12.74	-22.46
21	-0.0004	-0.0007	-0.0029	-0.0037	-0.0004	-0.0034	-0.0018	-0.0013	-21.75	8.17	-17.85	-0.43	-16.61	0.65	-18.75	-29.60
22	-0.0004	-0.0004	-0.0030	-0.0038	-0.0010	-0.0035	-0.0020	-0.0017	-20.33	6.24	-14.99	-1.25	31.51	-3.25	-6.16	-10.53
23	-0.0002	-0.0010	-0.0033	-0.0039	-0.0010	-0.0035	-0.0020	-0.0018	-29.56	37.67	-8.36	3.56	14.44	5.89	-8.58	-13.30
24	-0.0003	-0.0008	-0.0031	-0.0035	-0.0008	-0.0038	-0.0020	-0.0020	-27.88	25.44	-2.58	0.14	50.32	23.37	2.24	3.69
25	-0.0004	-0.0009	-0.0031	-0.0037	-0.0005	-0.0034	-0.0020	-0.0014	-24.13	8.54	-18.59	-0.99	-15.41	6.06	-17.05	-23.67

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Table 37 – continued from previous page ($3^{rd} Harmonic$)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
26	-0.0004	-0.0008	-0.0032	-0.0036	-0.0004	-0.0032	-0.0019	-0.0014	-22.35	4.26	-21.48	-6.29	-16.73	1.22	-19.89	-27.55
27	-0.0003	-0.0008	-0.0032	-0.0036	-0.0004	-0.0033	-0.0018	-0.0011	-24.47	0.61	-26.36	-8.98	-20.56	-2.25	-23.29	-33.35
28	-0.0002	-0.0010	-0.0031	-0.0035	0.00	-0.0034	-0.0017	-0.0010	-26.71	5.27	-28.19	-7.90	-26.22	-2.75	-25.24	-38.55
29	-0.0003	-0.0007	-0.0031	-0.0037	-0.0006	-0.0035	-0.0020	-0.0014	-24.78	4.38	-31.92	-10.96	-35.25	-6.41	-26.76	-38.99
30	-0.0003	-0.0005	-0.0030	-0.0035	-0.0005	-0.0033	-0.0019	-0.0013	-30.08	23.36	-24.92	-6.69	-24.86	-4.12	-22.31	-30.73
31	-0.0005	0.00	-0.0031	-0.0034	-0.0008	-0.0032	-0.0020	-0.0014	-30.98	46.12	-16.67	-7.96	-15.92	-3.05	-19.39	-27.39
32	-0.0004	-0.0001	-0.0032	-0.0035	-0.0008	-0.0034	-0.0020	-0.0015	-36.30	60.02	-9.14	-4.88	5.22	9.99	-11.89	-12.57
33	-0.0003	-0.0008	-0.0033	-0.0036	-0.0005	-0.0034	-0.0019	-0.0012	-30.64	34.05	-14.08	0.64	-18.77	3.99	-19.14	-23.52
34	-0.0005	-0.0008	-0.0038	-0.0043	-0.0006	-0.0033	-0.0029	-0.0021	-38.49	24.51	-40.92	-48.19	-150.49	-7.61	-51.06	-104.43
35	-0.0002	-0.0008	-0.0033	-0.0038	-0.0013	-0.0037	-0.0022	-0.0013	-25.74	19.02	-12.83	-4.64	-11.08	-5.42	-17.46	-19.07
36	-0.0001	-0.0009	-0.0032	-0.0037	-0.0004	-0.0035	-0.0020	-0.0012	-28.57	-0.24	-38.32	-29.47	-71.92	-3.95	-43.04	-58.30
37	-0.0005	0.0001	-0.0028	-0.0037	-0.0011	-0.0033	-0.0020	-0.0013	-26.33	33.40	-17.07	-7.51	-12.49	-3.05	-18.12	-22.62
38	-0.0002	-0.0008	-0.0029	-0.0037	-0.0005	-0.0032	-0.0019	-0.0012	-25.50	19.56	-15.71	-1.46	-16.78	2.83	-18.23	-21.48
39	-0.0003	-0.0005	-0.0031	-0.0038	-0.0009	-0.0034	-0.0021	-0.0013	-31.29	38.19	-11.26	-3.15	-17.68	1.01	-17.93	-19.89
40	-0.0002	-0.0008	-0.0030	-0.0036	-0.0004	-0.0031	-0.0021	-0.0011	-29.37	23.52	-15.92	-4.76	-21.48	1.06	-20.07	-21.98
41	-0.0002	-0.0007	-0.0033	-0.0040	-0.0013	-0.0036	-0.0024	-0.0013	-30.39	24.12	-16.64	-4.37	-13.68	1.14	-15.41	-34.24
42	-0.0002	-0.0011	-0.0034	-0.0043	-0.0017	-0.0038	-0.0024	-0.0014	-24.74	21.49	-14.30	-5.48	-8.97	-5.61	-18.67	-23.84
43	-0.0002	-0.0003	-0.0030	-0.0038	-0.0011	-0.0031	-0.0021	-0.0015	-28.40	32.15	-19.87	-9.51	-22.09	-9.57	-19.50	-30.56
44	-0.0001	-0.0013	-0.0036	-0.0043	-0.0010	-0.0033	-0.0026	-0.0020	-35.10	13.91	-38.00	-32.48	-125.92	-0.30	-45.84	-82.08
45	-0.0001	-0.0009	-0.0031	-0.0040	-0.0007	-0.0033	-0.0020	-0.0015	-31.34	35.53	-14.05	1.60	2.88	-0.27	-12.58	-21.61
46	-0.0004	-0.0002	-0.0032	-0.0040	-0.0011	-0.0032	-0.0024	-0.0016	-25.54	20.44	-34.93	-31.72	-84.85	-5.49	-37.59	-63.15
47	-0.0001	-0.0013	-0.0036	-0.0044	-0.0018	-0.0035	-0.0025	-0.0020	-22.18	6.16	-29.25	-16.04	-85.20	4.42	-27.90	-56.09
48	-0.0003	-0.0019	-0.0041	-0.0051	-0.0007	-0.0038	-0.0030	-0.0020	-33.10	51.60	-27.48	-29.52	-172.28	-2.45	-43.66	-103.12
49	-0.0002	-0.0013	-0.0035	-0.0046	-0.0018	-0.0038	-0.0025	-0.0015	-26.13	24.61	-22.44	-10.36	-116.87	-1.61	-27.56	-48.46
50	-0.0002	-0.0008	-0.0031	-0.0043	-0.0016	-0.0038	-0.0025	-0.0014	-20.43	5.80	-26.28	-17.20	-69.12	-4.87	-29.71	-49.75
51	-0.0003	-0.0003	-0.0030	-0.0041	-0.0016	-0.0038	-0.0025	-0.0015	-30.36	11.32	-28.53	-20.53	-63.10	-6.19	-28.13	-48.53
52	-0.0003	-0.0010	-0.0035	-0.0046	-0.0018	-0.0039	-0.0028	-0.0022	-22.89	2.76	-29.33	-17.05	-179.12	-3.68	-23.92	-52.66

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Table 37 – continued from previous page ($3^{rd} Harmonic$)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
53	-0.0003	-0.0011	-0.0035	-0.0046	-0.0015	-0.0038	-0.0028	-0.0021	-30.70	1.96	-31.23	-24.31	-154.82	-4.67	-34.95	-72.28
54	-0.0005	-0.0013	-0.0040	-0.0052	0.0008	-0.0038	-0.0036	-0.0021	-33.20	3.34	-36.75	-80.83	-194.40	-5.36	-210.29	-191.08
55	-0.0002	-0.0009	-0.0034	-0.0041	-0.0019	-0.0039	-0.0029	-0.0022	-33.39	-3.22	-33.56	-32.15	-169.84	8.34	-26.18	-70.63
56	-0.0002	-0.0017	-0.0037	-0.0051	0.0025	-0.0038	-0.0026	-0.0005	-28.07	-89.29	-101.59	-70.82	-150.75	-43.54	-127.34	-156.73
57	0.00	-0.0002	-0.0020	-0.0040	0.0033	-0.0020	-0.0008	0.0007	-28.09	21.41	-5.97	-16.17	17.11	26.56	-4.65	-14.75
58	-0.0003	-0.0007	-0.0029	-0.0041	-0.0004	-0.0038	-0.0020	-0.0013	-30.58	17.72	-12.25	6.80	38.52	-1.57	-1.71	-11.95
59	-0.0001	-0.0015	-0.0036	-0.0048	0.0001	-0.0040	-0.0030	-0.0018	-31.05	-20.64	-57.30	-57.96	-162.03	5.40	-85.39	-139.65
60	0.00	-0.0013	-0.0038	-0.0047	0.0034	-0.0045	-0.0023	-0.0003	-32.83	-81.51	-106.16	-77.96	-170.28	-68.58	-158.69	-173.85
61	-0.0003	-0.0004	-0.0030	-0.0044	-0.0020	-0.0033	-0.0026	-0.0019	-26.41	4.16	-29.09	-15.06	5.82	-12.37	-22.63	-47.71
62	-0.0002	-0.0007	-0.0028	-0.0038	-0.0004	-0.0031	-0.0018	-0.0013	-20.48	-4.07	-24.79	-17.06	-13.69	-16.73	-27.47	0.70
63	-0.0001	-0.0009	-0.0033	-0.0044	-0.0014	-0.0038	-0.0028	-0.0023	-22.93	-12.32	-38.00	-39.33	-183.16	-1.33	-36.77	-90.44
64	-0.0003	-0.0005	-0.0035	-0.0043	0.0008	-0.0042	-0.0025	-0.0019	-30.54	35.87	-2.67	-0.55	-245.85	2.42	44.44	64.24
65	-0.0003	-0.0005	-0.0030	-0.0043	-0.0013	-0.0033	-0.0023	-0.0017	-22.18	-14.71	-41.91	-36.51	-97.78	-7.40	-45.53	-80.52
66	-0.0003	-0.0005	-0.0028	-0.0038	0.0003	-0.0030	-0.0017	-0.0012	-22.44	17.56	-13.00	-19.04	24.01	28.93	-6.40	-16.13
67	-0.0001	-0.0008	-0.0027	-0.0038	0.0019	-0.0030	-0.0015	-0.0003	-24.06	-30.91	-64.45	-42.83	-92.76	-24.55	-72.32	-103.34
68	-0.0004	0.00	-0.0025	-0.0036	0.0002	-0.0030	-0.0017	-0.0010	-17.80	-7.79	-43.32	-27.74	-67.98	-23.41	-47.22	-80.78
69	-0.0002	-0.0003	-0.0031	-0.0036	-0.0003	-0.0037	-0.0025	-0.0018	-19.13	-4.58	-48.33	-14.63	-138.62	-70.07	-64.85	-114.87
70	-0.0002	-0.0010	-0.0033	-0.0040	-0.0010	-0.0041	-0.0025	-0.0019	-30.21	37.28	-3.67	-0.50	79.42	12.49	2.98	0.43
71	-0.0003	-0.0010	-0.0031	-0.0042	-0.0014	-0.0033	-0.0022	-0.0018	-20.31	-4.13	-29.64	-12.97	-26.56	-0.87	-24.94	-54.75
72	0.00	-0.0013	-0.0038	-0.0040	0.0007	-0.0044	-0.0033	-0.0023	-20.28	-14.83	-46.31	-21.51	-186.48	-54.71	-111.15	-181.46
73	-0.0004	-0.0004	-0.0029	-0.0037	-0.0014	-0.0033	-0.0022	-0.0020	-21.42	-6.19	-20.34	-7.73	21.92	-8.52	-12.08	-24.73
74	-0.0003	-0.0008	-0.0030	-0.0039	-0.0017	-0.0035	-0.0023	-0.0021	-19.14	-16.86	-33.74	-30.76	-79.69	-6.21	-34.21	-61.03
75	-0.0003	-0.0010	-0.0032	-0.0039	-0.0010	-0.0036	-0.0023	-0.0015	-24.30	-2.58	-36.51	-27.86	-78.51	-7.64	-41.04	-68.77
76	-0.0004	-0.0007	-0.0029	-0.0035	-0.0007	-0.0037	-0.0019	-0.0014	-21.18	-1.46	-20.63	-10.83	-17.41	-10.87	-21.20	-38.92
77	-0.0004	-0.0007	-0.0030	-0.0035	-0.0007	-0.0038	-0.0018	-0.0015	-21.77	2.11	-16.10	-8.18	0.95	-0.74	-15.62	-31.27
78	-0.0002	-0.0007	-0.0030	-0.0033	0.0006	-0.0034	-0.0014	-0.0011	-19.86	6.24	-18.99	-3.85	-14.43	-0.08	-20.02	-37.72
79	-0.0003	-0.0005	-0.0032	-0.0033	-0.0001	-0.0035	-0.0016	-0.0010	-18.21	0.77	-19.06	-7.82	-12.56	-4.32	-20.24	-38.86

Continued on next page)

Table 37 – continued from previous page ($3^{rd} Harmonic$)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
80	-0.0002	-0.0010	-0.0038	-0.0040	-0.0023	-0.0035	-0.0026	-0.0021	-18.66	-6.14	-21.47	-12.95	60.10	-4.73	-18.62	-45.94
81	-0.0003	-0.0008	-0.0035	-0.0035	-0.0003	-0.0038	-0.0017	-0.0007	-23.20	4.59	-19.13	-3.95	-8.28	1.25	-17.20	-39.02
82	-0.0005	0.0005	-0.0020	-0.0033	0.0042	-0.0017	-0.0002	0.0010	-17.10	2.88	-6.66	-20.25	22.18	26.92	1.95	-14.74
83	-0.0004	-0.0006	-0.0032	-0.0037	0.00	-0.0037	-0.0015	-0.0010	-19.83	-6.20	-26.88	-12.04	-22.82	-0.51	-23.22	-49.03
84	-0.0004	-0.0006	-0.0033	-0.0038	-0.0006	-0.0037	-0.0018	-0.0013	-19.57	-8.30	-23.86	-6.79	-2.33	-2.75	-15.17	-36.37
85	-0.0003	-0.0008	-0.0035	-0.0038	-0.0006	-0.0037	-0.0018	-0.0009	-23.33	2.96	-15.72	-4.63	-3.43	1.01	-14.24	-32.62
86	-0.0002	-0.0011	-0.0038	-0.0038	-0.0002	-0.0035	-0.0016	-0.0014	-20.10	-2.62	-27.48	-13.00	-33.06	6.50	-26.83	-51.93
87	-0.0003	-0.0010	-0.0036	-0.0040	-0.0004	-0.0035	-0.0020	-0.0014	-18.78	-7.71	-34.24	-17.63	-45.79	2.43	-30.38	-61.19
88	-0.0002	-0.0007	-0.0030	-0.0038	-0.0008	-0.0033	-0.0019	-0.0014	-21.46	-4.50	-38.32	-20.92	-50.55	-0.63	-33.26	-63.23
89	-0.0003	-0.0005	-0.0028	-0.0035	0.00	-0.0032	-0.0015	-0.0009	-22.27	-8.12	-35.52	-20.86	-36.62	-2.60	-31.61	-55.79
90	-0.0002	-0.0008	-0.0030	-0.0035	0.0005	-0.0033	-0.0015	-0.0007	-26.13	-1.72	-36.94	-16.82	-32.63	-0.26	-31.27	-54.69
91	-0.0004	-0.0010	-0.0037	-0.0046	-0.0019	-0.0033	-0.0027	-0.0025	-29.22	23.75	-34.78	-10.97	-212.05	-8.70	-16.63	-70.42
92	-0.0005	-0.0010	-0.0034	-0.0040	-0.0011	-0.0034	-0.0021	-0.0017	-25.58	9.43	-25.66	-3.47	0.43	-0.15	-14.15	-37.80
93	-0.0005	-0.0013	-0.0038	-0.0044	-0.0012	-0.0040	-0.0023	-0.0014	-27.74	36.07	-4.30	13.34	36.98	1.27	-0.43	-13.50
94	-0.0005	-0.0013	-0.0037	-0.0044	-0.0016	-0.0040	-0.0023	-0.0016	-25.86	28.21	-16.11	3.14	14.51	0.49	-11.80	-30.84
95	-0.0003	-0.0008	-0.0029	-0.0036	0.0001	-0.0033	-0.0016	-0.0010	-22.72	-1.54	-35.91	-17.62	-38.14	-0.46	-31.17	-57.53
96	-0.0005	-0.0003	-0.0028	-0.0036	0.0006	-0.0025	-0.0015	-0.0011	-29.61	12.14	-8.24	-35.02	32.25	32.66	0.23	-7.18
97	-0.0005	-0.0003	-0.0029	-0.0038	-0.0003	-0.0033	-0.0019	-0.0011	-23.14	5.31	-13.70	-27.79	32.26	24.25	-3.85	-18.32
98	-0.0004	-0.0008	-0.0032	-0.0040	-0.0006	-0.0035	-0.0021	-0.0010	-25.19	-9.74	-34.02	-25.56	-59.44	-6.66	-36.51	-62.22
99	-0.0003	-0.0010	-0.0032	-0.0039	-0.0003	-0.0036	-0.0018	-0.0008	-26.33	0.17	-26.55	-11.48	-26.07	-1.78	-24.49	-42.31
100	-0.0006	-0.0013	-0.0037	-0.0047	0.0012	-0.0040	-0.0027	-0.0013	-28.90	-37.97	-70.59	-90.41	-155.62	-20.63	-111.14	-127.91

Table 38: Variation of Error in Magnitude and Angle (5^{th} Harmonic) with Time(1^{st} to 100^{th})

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
1	-0.0005	0.0027	-0.0028	-0.0020	0.0007	-0.0001	-0.0019	-0.0010	-14.04	43.13	11.46	-28.48	303.14	-47.18	-38.96	-271.31
2	-0.0006	0.0028	-0.0026	-0.0016	0.0013	-0.0001	-0.0015	-0.0007	-13.06	42.29	9.32	-28.16	306.66	-39.53	-37.94	-274.23
3	-0.0004	0.0028	-0.0012	-0.0002	0.0044	0.0013	0.00	0.0013	-13.67	42.86	7.04	-25.74	317.00	-35.57	-33.95	-278.79
4	-0.0004	0.0030	-0.0008	-0.0002	0.0033	0.0009	-0.0003	0.0019	-13.02	46.56	10.50	-23.99	-36.33	-32.11	-31.41	-272.57
5	-0.0006	0.0029	-0.0030	-0.0020	0.00	-0.0007	-0.0019	-0.0014	-12.49	41.34	11.46	-30.44	301.40	-42.74	-38.88	-270.56
6	-0.0005	0.0024	-0.0025	-0.0013	0.0035	0.0005	-0.0008	-0.0001	-15.92	48.52	10.21	-28.02	305.38	-39.92	-39.64	-278.46
7	-0.0006	0.0029	-0.0027	-0.0018	0.0008	-0.0004	-0.0016	-0.0009	-13.28	43.66	10.66	-28.16	304.46	-40.89	-38.60	-274.37
8	-0.0006	0.0031	-0.0018	-0.0007	0.0030	0.0008	-0.0007	0.0002	-13.51	42.61	6.91	-28.43	309.01	-39.77	-37.67	-279.60
9	-0.0004	0.0028	-0.0003	0.0002	0.0047	0.0014	0.0002	0.0025	-13.79	45.80	11.41	-20.98	-32.74	-30.17	-30.30	-272.48
10	-0.0002	0.0025	-0.0008	0.00	0.0071	0.0022	0.0009	0.0031	-15.90	48.13	9.81	-23.40	-31.08	-29.70	-30.06	-273.44
11	-0.0002	0.0027	-0.0006	0.0001	0.0069	0.0023	0.0008	0.0029	-14.82	44.91	7.96	-24.33	-33.65	-31.39	-30.59	-275.73
12	-0.0003	0.0026	-0.0008	-0.0001	0.0051	0.0017	0.0002	0.0024	-14.71	44.66	8.88	-23.51	-37.62	-33.38	-31.73	-276.42
13	-0.0003	0.0032	-0.0009	-0.0004	0.0046	0.0016	-0.0002	0.0023	-13.66	43.64	7.88	-25.83	-33.41	-32.53	-30.42	-275.07
14	-0.0004	0.0035	-0.0015	-0.0010	0.0029	0.0010	-0.0008	0.0010	-13.35	43.27	8.56	-28.40	-40.05	-35.56	-32.71	-273.23
15	-0.0006	0.0033	-0.0020	-0.0015	0.0002	-0.0004	-0.0016	0.00	-12.43	46.94	11.55	-28.46	311.46	-37.65	-34.94	-270.13
16	-0.0006	0.0032	-0.0017	-0.0008	0.0020	0.0003	-0.0010	0.0008	-13.17	46.85	9.98	-27.98	312.84	-37.71	-35.08	-271.87
17	-0.0007	0.0029	-0.0028	-0.0028	-0.0033	-0.0020	-0.0029	-0.0014	-12.13	46.03	22.66	-22.84	312.98	-35.46	-32.02	-257.14
18	-0.0007	0.0031	-0.0023	-0.0014	0.0017	-0.0001	-0.0015	-0.0003	-14.47	49.13	12.31	-28.63	304.36	-40.25	-38.61	-272.56
19	-0.0007	0.0033	-0.0027	-0.0028	-0.0024	-0.0018	-0.0029	-0.0014	-12.13	44.69	20.82	-22.87	307.71	-37.85	-34.02	-260.68
20	-0.0006	0.0038	-0.0009	-0.0002	0.0036	0.0011	-0.0006	0.0010	-13.62	44.56	9.55	-25.07	-40.63	-35.15	-32.72	-270.96
21	-0.0007	0.0038	-0.0022	-0.0025	-0.0005	-0.0008	-0.0022	-0.0002	-12.05	41.04	15.30	-26.53	-41.67	-36.20	-33.54	-262.67
22	-0.0007	0.0029	-0.0030	-0.0026	0.0010	-0.0009	-0.0020	-0.0008	-14.72	48.68	16.75	-26.65	301.30	-41.86	-40.06	-267.85
23	-0.0007	0.0035	-0.0023	-0.0029	-0.0011	-0.0013	-0.0025	-0.0004	-12.62	43.04	20.70	-23.07	-44.06	-37.46	-33.98	-254.99
24	-0.0005	0.0035	-0.0008	-0.0022	-0.0007	-0.0015	-0.0023	0.0012	-11.91	46.63	26.78	-15.71	-8.83	-23.10	-22.56	-244.92
25	-0.0007	0.0036	-0.0019	-0.0028	-0.0041	-0.0028	-0.0034	-0.0003	-10.55	44.53	24.19	-20.47	-28.80	-32.01	-29.10	-247.19

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Table 38 – continued from previous page (5^{th} Harmonic)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
26	-0.0006	0.0032	-0.0028	-0.0023	0.0013	-0.0010	-0.0019	0.0001	-14.24	45.04	15.06	-26.86	308.84	-40.30	-37.70	-266.83
27	-0.0006	0.0031	-0.0021	-0.0024	-0.0013	-0.0019	-0.0025	0.0002	-12.42	44.63	23.83	-17.41	-32.56	-32.66	-29.87	-252.56
28	-0.0007	0.0033	-0.0030	-0.0021	0.00	-0.0014	-0.0022	-0.0009	-12.74	41.78	14.98	-26.89	304.21	-42.02	-38.10	-265.33
29	-0.0005	0.0031	-0.0019	-0.0004	0.0045	0.0007	-0.0004	0.0010	-13.96	42.09	7.21	-26.94	313.32	-39.18	-36.88	-276.45
30	-0.0005	0.0026	-0.0030	-0.0015	0.0030	-0.0003	-0.0013	-0.0005	-14.56	44.40	9.94	-29.43	303.69	-42.99	-40.35	-273.91
31	-0.0006	0.0026	-0.0033	-0.0025	-0.0004	-0.0017	-0.0023	-0.0014	-13.66	44.58	18.16	-25.07	306.16	-39.97	-37.54	-261.96
32	-0.0007	0.0030	-0.0030	-0.0020	0.0006	-0.0011	-0.0020	-0.0015	-13.23	42.40	12.62	-27.40	302.93	-41.65	-38.87	-269.15
33	-0.0006	0.0027	-0.0026	-0.0008	0.0040	0.0003	-0.0008	-0.0003	-14.59	42.61	7.01	-30.40	304.20	-43.18	-40.58	-275.93
34	-0.0007	0.0029	-0.0028	-0.0014	0.0031	-0.0002	-0.0011	-0.0007	-15.15	45.52	10.03	-29.41	301.62	-42.64	-41.51	-274.35
35	-0.0006	0.0029	-0.0024	-0.0010	0.0038	0.0005	-0.0008	-0.0001	-14.21	43.42	7.57	-29.68	304.54	-41.97	-40.36	-277.48
36	-0.0004	0.0030	-0.0002	0.0003	0.0051	0.0013	0.00	0.0027	-12.39	41.03	11.10	-20.86	-32.83	-31.24	-30.54	-271.53
37	-0.0005	0.0028	-0.0014	-0.0001	0.0061	0.0014	0.0002	0.0017	-15.49	44.71	8.45	-24.97	314.33	-37.31	-36.71	-278.19
38	-0.0006	0.0031	-0.0025	-0.0011	0.0037	0.0003	-0.0009	-0.0005	-14.76	41.76	9.40	-29.34	304.61	-43.37	-40.21	-274.46
39	-0.0006	0.0028	-0.0030	-0.0024	-0.0005	-0.0015	-0.0022	-0.0011	-13.19	41.74	19.61	-22.77	313.09	-38.54	-34.16	-261.21
40	-0.0003	0.0027	-0.0008	0.0003	0.0065	0.0015	0.0005	0.0028	-14.88	43.00	9.25	-22.59	-37.31	-34.79	-32.49	-272.13
41	-0.0004	0.0025	-0.0028	-0.0010	0.0041	0.0004	-0.0007	0.0002	-15.47	45.24	8.73	-28.44	307.68	-42.22	-39.03	-275.61
42	-0.0006	0.0031	-0.0024	-0.0012	0.0024	0.0001	-0.0012	-0.0003	-13.54	40.07	6.62	-29.38	303.80	-43.80	-39.97	-276.13
43	-0.0004	0.0026	-0.0019	-0.0005	0.0051	0.0009	-0.0003	0.0013	-15.19	45.79	7.65	-25.97	312.20	-39.55	-37.83	-277.53
44	-0.0005	0.0027	-0.0031	-0.0019	0.0015	-0.0006	-0.0017	-0.0010	-13.93	42.73	10.84	-27.64	303.07	-43.15	-39.25	-273.52
45	-0.0007	0.0031	-0.0025	-0.0013	0.0027	-0.0001	-0.0013	-0.0004	-14.12	41.28	7.84	-29.85	302.84	-44.13	-40.78	-274.91
46	-0.0005	0.0027	-0.0017	-0.0006	0.0051	0.0007	-0.0003	0.0015	-15.49	46.44	8.84	-24.72	312.75	-37.88	-37.02	-276.91
47	-0.0006	0.0031	-0.0018	-0.0009	0.0037	0.0004	-0.0009	0.0007	-13.62	42.76	7.56	-27.17	309.33	-41.02	-38.65	-276.79
48	-0.0002	0.0033	-0.0001	0.0003	0.0052	0.0013	0.00	0.0026	-11.46	39.18	9.90	-17.83	-26.02	-28.86	-26.88	-273.14
49	-0.0002	0.0030	-0.0004	0.0007	0.0062	0.0017	0.0005	0.0025	-12.41	39.98	8.48	-19.06	-29.61	-29.73	-28.35	-276.44
50	-0.0004	0.0029	-0.0006	0.0002	0.0053	0.0010	0.0001	0.0025	-14.25	43.76	11.20	-21.18	-36.11	-31.68	-31.77	-273.42
51	-0.0002	0.0023	-0.0008	0.00	0.0054	0.0008	0.0002	0.0029	-14.25	44.88	11.99	-18.87	-30.58	-28.38	-29.69	-272.43
52	-0.0003	0.0023	-0.0012	0.00	0.0056	0.0010	0.0001	0.0021	-14.92	44.64	8.07	-22.17	-36.93	-31.95	-32.78	-277.50

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Table 38 – continued from previous page (5^{th} Harmonic)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
53	-0.0002	0.0025	-0.0006	0.0004	0.0057	0.0012	0.0004	0.0023	-13.51	42.20	8.77	-18.79	-28.64	-27.72	-28.41	-276.85
54	-0.0003	0.0023	-0.0005	0.00	0.0048	0.0006	0.00	0.0030	-13.70	45.25	14.70	-15.85	-25.38	-25.22	-26.87	-271.76
55	-0.0003	0.0023	-0.0008	0.00	0.0054	0.0008	0.0002	0.0027	-14.35	45.43	11.03	-19.99	-33.61	-29.99	-30.74	-274.42
56	-0.0003	0.0023	-0.0020	-0.0003	0.0053	0.0009	0.00	0.0010	-14.59	43.05	6.06	-26.51	313.88	-37.23	-37.02	-279.44
57	-0.0004	0.0028	-0.0008	0.0003	0.0054	0.0012	0.0002	0.0019	-14.12	42.82	8.33	-21.87	-37.15	-32.94	-32.76	-276.07
58	-0.0005	0.0025	-0.0028	-0.0011	0.0030	-0.0002	-0.0010	-0.0004	-14.12	43.09	7.89	-26.45	308.19	-39.33	-38.78	-277.59
59	-0.0005	0.0025	-0.0014	-0.0002	0.0052	0.0010	0.00	0.0016	-14.66	44.48	7.39	-23.44	316.31	-35.61	-36.11	-277.26
60	-0.0006	0.0023	-0.0028	-0.0014	0.0038	0.00	-0.0008	-0.0003	-15.25	46.94	8.48	-26.81	305.26	-41.69	-40.86	-277.85
61	-0.0004	0.0025	-0.0014	-0.0003	0.0058	0.0012	0.0003	0.0017	-15.29	45.62	7.90	-22.85	316.63	-36.49	-35.98	-277.92
62	-0.0003	0.0023	-0.0010	-0.0001	0.0059	0.0014	0.0003	0.0023	-14.22	44.82	8.67	-20.37	-37.45	-33.60	-33.12	-276.23
63	-0.0004	0.0023	-0.0013	-0.0002	0.0063	0.0013	0.0003	0.0022	-15.55	46.79	9.31	-21.21	-41.25	-34.46	-34.35	-276.33
64	-0.0006	0.0025	-0.0030	-0.0017	0.0033	-0.0004	-0.0011	-0.0005	-16.01	47.92	11.50	-26.18	302.95	-41.71	-40.69	-275.06
65	-0.0005	0.0023	-0.0028	-0.0012	0.0047	0.0001	-0.0005	0.0003	-16.96	48.69	9.76	-26.87	306.31	-42.00	-39.28	-276.10
66	-0.0005	0.0028	-0.0022	-0.0011	0.0041	0.0002	-0.0006	0.0005	-15.87	48.07	10.06	-26.53	308.62	-41.44	-37.69	-274.81
67	-0.0007	0.0029	-0.0030	-0.0030	-0.0017	-0.0022	-0.0029	-0.0016	-13.22	47.86	22.84	-21.61	306.56	-38.85	-35.68	-258.84
68	-0.0005	0.0030	-0.0007	-0.0002	0.0052	0.0008	-0.0002	0.0018	-15.27	47.82	10.14	-19.39	-36.55	-32.37	-31.05	-273.30
69	-0.0004	0.0027	-0.0001	-0.0003	0.0048	0.0005	-0.0003	0.0027	-14.43	48.38	18.49	-12.94	-23.42	-25.67	-24.88	-263.67
70	-0.0007	0.0032	-0.0020	-0.0009	0.0037	0.0004	-0.0007	0.00	-14.28	43.00	7.87	-28.92	305.99	-43.67	-38.50	-276.35
71	-0.0006	0.0030	-0.0022	-0.0011	0.0036	0.0002	-0.0008	0.0002	-14.95	44.88	9.42	-27.66	305.88	-42.65	-38.30	-276.36
72	-0.0007	0.0027	-0.0027	-0.0013	0.0035	-0.0002	-0.0010	-0.0001	-15.66	46.50	10.83	-28.79	303.60	-42.96	-39.64	-274.46
73	-0.0005	0.0025	-0.0021	-0.0008	0.0043	0.0002	-0.0007	0.0013	-16.04	47.58	8.92	-27.15	309.55	-41.06	-37.64	-272.40
74	-0.0006	0.0028	-0.0022	-0.0011	0.0031	0.0002	-0.0011	0.0008	-15.23	46.77	9.79	-27.42	306.65	-42.05	-37.78	-273.63
75	-0.0006	0.0032	-0.0025	-0.0016	0.0023	0.00	-0.0013	0.0001	-14.81	44.31	10.24	-28.92	302.87	-44.58	-38.77	-270.72
76	-0.0006	0.0031	-0.0016	-0.0008	0.0045	0.0009	-0.0005	0.0011	-15.46	44.96	9.08	-27.42	309.68	-40.48	-37.14	-272.12
77	-0.0004	0.0028	-0.0007	-0.0003	0.0048	0.0012	-0.0003	0.0023	-14.74	46.23	11.98	-22.21	-37.01	-34.17	-31.26	-270.06
78	-0.0007	0.0032	-0.0025	-0.0020	0.0013	-0.0005	-0.0020	-0.0006	-13.89	45.22	12.30	-27.62	303.21	-42.44	-38.28	-271.64
79	-0.0007	0.0035	-0.0018	-0.0013	0.0019	0.00	-0.0014	-0.0004	-13.42	42.29	9.74	-25.58	310.68	-39.01	-35.17	-275.37

Continued on next page

Table 38 – continued from previous page (5^{th} Harmonic)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
80	-0.0005	0.0028	-0.0007	0.00	0.0054	0.0015	0.0001	0.0020	-14.43	44.41	9.39	-24.39	-40.77	-35.86	-32.60	-276.06
81	-0.0003	0.0029	0.00	0.0001	0.0050	0.0015	0.00	0.0025	-13.10	42.75	12.14	-19.17	-29.22	-30.03	-27.06	-273.30
82	-0.0003	0.0028	0.0002	-0.0003	0.0043	0.0009	-0.0004	0.0023	-12.83	45.38	19.47	-10.21	-13.70	-19.80	-19.85	-265.46
83	-0.0003	0.0029	0.0004	0.0002	0.0052	0.0017	0.0001	0.0025	-10.95	40.93	14.84	-12.18	-16.04	-22.69	-21.09	-268.08
84	-0.0002	0.0028	0.0003	0.00	0.0048	0.0013	-0.0002	0.0027	-11.78	42.21	15.91	-10.92	-15.42	-21.93	-20.52	-269.17
85	-0.0002	0.0026	-0.0002	0.0001	0.0058	0.0018	0.00	0.0021	-12.80	42.56	9.87	-16.47	-27.06	-28.59	-26.31	-276.61
86	-0.0002	0.0029	0.00	0.0005	0.0063	0.0022	0.0004	0.0021	-13.20	42.16	11.22	-15.97	-26.08	-27.80	-25.21	-274.20
87	-0.0003	0.0030	0.0002	0.0006	0.0060	0.0022	0.0005	0.0026	-12.23	39.80	9.58	-16.58	-24.19	-28.00	-24.27	-273.75
88	-0.0003	0.0026	-0.0006	0.00	0.0051	0.0016	0.00	0.0025	-12.52	41.68	7.55	-20.22	-32.61	-32.37	-28.79	-278.18
89	-0.0003	0.0022	-0.0021	-0.0008	0.0040	0.0010	-0.0006	0.0011	-13.82	43.87	6.30	-24.49	314.83	-38.98	-34.58	-280.46
90	-0.0005	0.0024	-0.0032	-0.0023	0.00	-0.0006	-0.0021	-0.0011	-12.19	41.66	11.89	-24.87	307.59	-41.70	-35.56	-272.93
91	-0.0003	0.0024	-0.0020	-0.0008	0.0037	0.0009	-0.0007	0.0003	-13.42	42.97	6.46	-22.49	-40.72	-34.91	-32.59	-280.92
92	-0.0003	0.0025	-0.0013	0.0002	0.0051	0.0017	0.00	0.0007	-12.66	41.53	4.68	-23.64	-38.34	-34.52	-31.72	-282.08
93	-0.0003	0.0030	-0.0004	0.0008	0.0059	0.0022	0.0004	0.0018	-12.28	41.24	7.08	-19.95	-30.95	-30.54	-27.96	-279.18
94	-0.0005	0.0030	-0.0022	-0.0007	0.0027	0.0007	-0.0010	-0.0009	-11.84	40.64	6.38	-25.75	313.46	-37.97	-33.96	-281.09
95	-0.0005	0.0032	-0.0010	0.0002	0.0049	0.0017	-0.0001	0.0006	-13.05	42.21	5.81	-24.35	-40.25	-34.97	-32.58	-280.18
96	-0.0002	0.0023	-0.0015	0.00	0.0052	0.0015	0.00	0.0012	-13.22	42.29	5.00	-22.92	-37.23	-33.23	-31.40	-283.60
97	0.00	0.0020	-0.0006	0.0006	0.0068	0.0022	0.0007	0.0028	-14.17	45.83	9.57	-16.16	-25.11	-26.89	-25.34	-278.01
98	0.0001	0.0021	-0.0004	0.0010	0.0072	0.0025	0.0009	0.0030	-13.59	42.10	8.45	-15.96	-22.20	-26.53	-24.02	-277.08
99	0.0001	0.0022	-0.0007	0.0012	0.0073	0.0028	0.0010	0.0023	-13.01	40.21	5.70	-18.35	-25.95	-28.56	-26.03	-282.27
100	0.0001	0.0020	-0.0002	0.0010	0.0074	0.0025	0.0010	0.0030	-13.74	45.98	12.59	-12.42	-19.11	-21.87	-21.82	-276.33

Table 39: Variation of Error in Magnitude and Angle (7^{th} Harmonic) with Time (1^{st} to 100^{th})

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
1	-0.0052	0.0250	0.0203	-0.0115	0.0083	-0.0052	-0.0078	-0.0044	0.94	2.98	6.17	-63.82	-146.00	-120.70	-116.04	156.19
2	-0.0048	0.0225	0.0182	-0.0118	0.0040	-0.0062	-0.0091	-0.0023	1.81	2.96	6.26	-57.28	-144.52	-118.69	-110.29	160.62
3	-0.0048	0.0222	0.0179	-0.0117	0.0033	-0.0064	-0.0091	-0.0018	1.92	2.39	5.59	-56.62	-142.53	-117.62	-108.30	160.50
4	-0.0048	0.0226	0.0181	-0.0114	0.0042	-0.0058	-0.0086	-0.0018	1.63	2.19	5.08	-57.39	-139.27	-116.11	-107.56	157.56
5	-0.0047	0.0220	0.0177	-0.0116	0.0030	-0.0063	-0.0089	-0.0017	1.81	3.49	5.22	-56.52	-141.48	-116.12	-106.94	159.00
6	-0.0049	0.0230	0.0182	-0.0114	0.0049	-0.0058	-0.0087	-0.0027	1.75	2.87	5.14	-57.79	-144.10	-118.62	-110.22	158.88
7	-0.0049	0.0226	0.0180	-0.0119	0.0041	-0.0063	-0.0090	-0.0023	1.84	3.15	5.13	-56.34	-143.54	-118.65	-109.43	159.89
8	-0.0048	0.0222	0.0178	-0.0117	0.0034	-0.0063	-0.0090	-0.0022	2.07	2.42	4.70	-56.52	-144.36	-118.13	-109.13	160.48
9	-0.0049	0.0226	0.0179	-0.0114	0.0044	-0.0061	-0.0088	-0.0024	1.62	2.71	5.00	-58.20	-144.65	-118.05	-109.73	159.03
10	-0.0049	0.0231	0.0183	-0.0111	0.0057	-0.0057	-0.0083	-0.0026	1.26	2.54	4.75	-60.37	-140.43	-117.10	-108.86	155.72
11	-0.0049	0.0229	0.0181	-0.0116	0.0057	-0.0051	-0.0086	-0.0027	1.63	2.32	4.62	-57.38	-140.74	-117.07	-109.75	157.65
12	-0.0048	0.0221	0.0176	-0.0117	0.0043	-0.0056	-0.0090	-0.0020	1.98	2.13	4.32	-56.47	-140.81	-116.09	-107.88	157.56
13	-0.0047	0.0220	0.0176	-0.0117	0.0028	-0.0063	-0.0094	-0.0017	1.99	2.99	4.73	-55.52	-141.49	-115.96	-106.92	159.16
14	-0.0047	0.0218	0.0175	-0.0118	0.0028	-0.0061	-0.0093	-0.0016	1.75	3.22	5.02	-54.47	-138.49	-114.33	-105.28	157.75
15	-0.0048	0.0223	0.0177	-0.0116	0.0051	-0.0053	-0.0089	-0.0028	1.42	2.92	5.09	-56.15	-142.51	-117.04	-109.18	157.57
16	-0.0048	0.0222	0.0176	-0.0113	0.0058	-0.0049	-0.0085	-0.0028	0.86	3.49	5.68	-56.67	-140.89	-116.36	-108.73	155.75
17	-0.0048	0.0220	0.0175	-0.0115	0.0051	-0.0054	-0.0088	-0.0025	1.26	3.79	5.46	-55.10	-141.84	-117.00	-108.95	158.09
18	-0.0048	0.0225	0.0178	-0.0114	0.0054	-0.0053	-0.0086	-0.0025	1.06	3.27	5.17	-56.24	-138.86	-115.72	-107.98	155.77
19	-0.0047	0.0219	0.0175	-0.0114	0.0041	-0.0057	-0.0089	-0.0017	1.25	3.39	4.88	-54.89	-136.50	-114.12	-105.31	155.70
20	-0.0047	0.0217	0.0174	-0.0114	0.0040	-0.0057	-0.0088	-0.0018	1.26	3.39	5.02	-56.33	-136.27	-113.91	-105.10	155.16
21	-0.0047	0.0219	0.0174	-0.0114	0.0052	-0.0053	-0.0088	-0.0027	1.31	3.31	4.97	-56.51	-140.36	-116.48	-108.38	155.97
22	-0.0048	0.0225	0.0178	-0.0111	0.0052	-0.0053	-0.0089	-0.0029	1.45	2.94	5.39	-56.47	-143.92	-118.33	-110.00	158.58
23	-0.0047	0.0218	0.0173	-0.0112	0.0051	-0.0054	-0.0085	-0.0022	1.04	3.54	5.38	-56.00	-138.34	-115.59	-106.90	156.02
24	-0.0048	0.0225	0.0179	-0.0111	0.0043	-0.0059	-0.0091	-0.0025	1.34	3.83	5.85	-56.80	-143.04	-117.09	-108.85	159.15
25	-0.0047	0.0216	0.0171	-0.0109	0.0044	-0.0067	-0.0088	-0.0023	1.17	3.54	5.66	-58.09	-140.40	-116.17	-107.17	156.05

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Table 39 – continued from previous page(7^{th} Harmonic)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
26	-0.0048	0.0220	0.0173	-0.0110	0.0038	-0.0066	-0.0089	-0.0020	1.31	3.49	5.65	-56.16	-141.52	-115.84	-107.23	157.40
27	-0.0048	0.0222	0.0175	-0.0111	0.0040	-0.0065	-0.0089	-0.0024	1.13	3.84	6.09	-56.43	-141.55	-115.91	-107.84	158.13
28	-0.0047	0.0219	0.0174	-0.0114	0.0036	-0.0066	-0.0088	-0.0021	1.29	3.38	5.36	-55.38	-137.45	-113.73	-106.31	157.27
29	-0.0049	0.0224	0.0177	-0.0112	0.0056	-0.0060	-0.0085	-0.0035	0.94	3.47	5.78	-57.29	-141.86	-117.22	-110.00	157.09
30	-0.0049	0.0223	0.0175	-0.0112	0.0062	-0.0056	-0.0084	-0.0038	1.10	3.34	5.50	-58.52	-142.98	-116.51	-111.24	157.01
31	-0.0049	0.0225	0.0178	-0.0109	0.0051	-0.0057	-0.0086	-0.0034	0.95	4.33	6.29	-56.88	-142.53	-114.67	-109.06	157.28
32	-0.0048	0.0223	0.0178	-0.0111	0.0045	-0.0061	-0.0088	-0.0029	1.44	4.11	5.79	-56.84	-142.26	-115.19	-108.88	158.47
33	-0.0048	0.0223	0.0178	-0.0111	0.0044	-0.0062	-0.0089	-0.0027	1.46	3.87	5.41	-56.36	-141.49	-115.13	-107.99	157.99
34	-0.0049	0.0229	0.0181	-0.0111	0.0046	-0.0060	-0.0091	-0.0030	1.61	3.07	5.12	-56.86	-143.87	-116.10	-109.28	158.69
35	-0.0049	0.0225	0.0180	-0.0110	0.0041	-0.0061	-0.0090	-0.0026	1.36	3.58	5.87	-55.82	-141.54	-114.63	-106.86	157.37
36	-0.0048	0.0224	0.0179	-0.0109	0.0040	-0.0062	-0.0090	-0.0027	1.60	3.64	5.76	-55.77	-141.18	-115.09	-107.35	157.82
37	-0.0049	0.0225	0.0179	-0.0106	0.0053	-0.0061	-0.0085	-0.0033	1.35	2.68	5.55	-60.23	-142.28	-116.03	-108.89	155.60
38	-0.0048	0.0221	0.0177	-0.0104	0.0044	-0.0059	-0.0085	-0.0026	0.84	4.58	6.79	-56.67	-138.51	-113.67	-105.24	154.59
39	-0.0048	0.0223	0.0179	-0.0106	0.0048	-0.0059	-0.0087	-0.0028	0.66	5.10	7.29	-56.03	-139.20	-113.76	-106.02	155.26
40	-0.0049	0.0225	0.0180	-0.0109	0.0046	-0.0060	-0.0088	-0.0030	1.24	4.28	6.42	-56.48	-141.51	-115.10	-107.66	157.32
41	-0.0050	0.0227	0.0182	-0.0110	0.0043	-0.0061	-0.0091	-0.0029	1.35	3.66	5.94	-56.67	-142.89	-115.30	-108.23	157.71
42	-0.0049	0.0227	0.0183	-0.0111	0.0040	-0.0063	-0.0092	-0.0030	1.48	3.93	5.90	-56.78	-142.98	-116.08	-109.01	158.90
43	-0.0050	0.0232	0.0186	-0.0113	0.0044	-0.0061	-0.0093	-0.0032	1.85	2.74	5.26	-56.63	-144.71	-117.15	-110.50	160.27
44	-0.0049	0.0224	0.0180	-0.0114	0.0045	-0.0059	-0.0091	-0.0031	1.63	3.57	5.61	-55.69	-143.69	-115.81	-110.28	159.68
45	-0.0048	0.0222	0.0178	-0.0111	0.0039	-0.0059	-0.0090	-0.0027	1.38	4.42	6.32	-55.93	-142.38	-114.59	-107.89	158.73
46	-0.0049	0.0230	0.0183	-0.0108	0.0048	-0.0056	-0.0088	-0.0031	1.27	3.57	6.05	-56.26	-143.06	-115.54	-108.50	157.90
47	-0.0049	0.0228	0.0183	-0.0112	0.0042	-0.0059	-0.0090	-0.0026	1.19	4.36	6.32	-54.74	-141.02	-114.68	-107.15	157.62
48	-0.0048	0.0221	0.0179	-0.0113	0.0036	-0.0061	-0.0091	-0.0025	1.37	4.21	6.13	-55.08	-139.94	-114.54	-106.92	158.61
49	-0.0048	0.0223	0.0180	-0.0111	0.0035	-0.0062	-0.0091	-0.0025	1.53	3.88	5.96	-55.02	-140.70	-115.31	-107.41	159.15
50	-0.0048	0.0222	0.0178	-0.0111	0.0039	-0.0063	-0.0091	-0.0026	1.50	3.49	5.89	-56.96	-142.47	-115.31	-108.14	159.00
51	-0.0048	0.0225	0.0180	-0.0108	0.0043	-0.0059	-0.0089	-0.0028	1.37	3.30	6.09	-57.05	-143.95	-115.82	-108.67	158.94
52	-0.0048	0.0225	0.0179	-0.0108	0.0043	-0.0060	-0.0089	-0.0029	1.35	3.75	5.95	-57.00	-142.95	-115.68	-108.79	158.68

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Table 39 – continued from previous page(7^{th} Harmonic)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
53	-0.0048	0.0224	0.0179	-0.0108	0.0050	-0.0058	-0.0087	-0.0030	1.18	3.65	5.82	-56.58	-141.78	-116.23	-109.11	157.70
54	-0.0047	0.0224	0.0179	-0.0107	0.0044	-0.0056	-0.0086	-0.0027	1.13	3.36	5.80	-57.00	-140.72	-114.33	-107.64	157.28
55	-0.0047	0.0224	0.0179	-0.0107	0.0047	-0.0057	-0.0086	-0.0025	1.25	3.44	5.73	-56.34	-140.47	-114.30	-107.36	156.93
56	-0.0048	0.0224	0.0180	-0.0110	0.0042	-0.0061	-0.0087	-0.0024	1.52	3.11	5.23	-56.62	-140.79	-114.93	-107.20	157.63
57	-0.0047	0.0221	0.0177	-0.0109	0.0044	-0.0060	-0.0085	-0.0027	1.34	3.59	5.55	-58.11	-140.58	-115.01	-107.70	157.28
58	-0.0048	0.0225	0.0181	-0.0107	0.0043	-0.0061	-0.0086	-0.0028	1.16	3.83	5.89	-58.24	-141.50	-114.88	-108.06	157.90
59	-0.0048	0.0227	0.0182	-0.0107	0.0040	-0.0061	-0.0086	-0.0025	1.44	3.25	5.52	-57.25	-141.71	-115.40	-107.60	158.54
60	-0.0049	0.0232	0.0185	-0.0106	0.0044	-0.0061	-0.0086	-0.0026	1.38	3.12	5.33	-57.28	-142.15	-116.32	-108.14	159.37
61	-0.0049	0.0231	0.0183	-0.0107	0.0040	-0.0062	-0.0086	-0.0026	1.53	3.05	5.60	-57.00	-142.95	-116.48	-107.71	160.29
62	-0.0049	0.0228	0.0182	-0.0107	0.0049	-0.0060	-0.0084	-0.0029	1.01	3.77	6.07	-57.18	-142.28	-117.10	-108.84	159.33
63	-0.0049	0.0231	0.0184	-0.0106	0.0051	-0.0059	-0.0086	-0.0028	1.31	3.00	5.48	-57.00	-142.51	-116.78	-108.93	158.99
64	-0.0049	0.0232	0.0184	-0.0106	0.0053	-0.0058	-0.0087	-0.0030	1.50	2.83	5.36	-57.79	-143.30	-117.03	-109.46	159.27
65	-0.0048	0.0227	0.0180	-0.0101	0.0057	-0.0059	-0.0082	-0.0027	0.36	4.17	6.35	-58.67	-138.53	-116.01	-105.55	154.01
66	-0.0049	0.0225	0.0179	-0.0104	0.0056	-0.0062	-0.0083	-0.0031	0.55	4.29	6.54	-59.23	-140.52	-116.01	-106.94	155.23
67	-0.0049	0.0226	0.0181	-0.0107	0.0038	-0.0065	-0.0093	-0.0025	1.32	4.01	6.30	-56.02	-143.00	-115.89	-107.22	159.54
68	-0.0050	0.0231	0.0183	-0.0105	0.0045	-0.0061	-0.0091	-0.0031	1.27	3.51	6.02	-57.08	-143.31	-116.46	-108.34	159.16
69	-0.0049	0.0232	0.0183	-0.0105	0.0053	-0.0055	-0.0088	-0.0032	0.70	3.84	6.20	-56.80	-140.28	-115.35	-107.50	156.86
70	-0.0048	0.0223	0.0179	-0.0108	0.0037	-0.0060	-0.0088	-0.0023	0.95	4.27	6.39	-55.40	-137.19	-113.36	-103.74	156.81
71	-0.0047	0.0223	0.0179	-0.0109	0.0037	-0.0059	-0.0090	-0.0023	1.04	4.32	6.51	-55.58	-138.77	-113.79	-105.05	157.85
72	-0.0048	0.0225	0.0178	-0.0104	0.0050	-0.0056	-0.0084	-0.0024	1.02	3.35	5.70	-57.87	-137.14	-114.77	-105.40	154.19
73	-0.0049	0.0228	0.0180	-0.0105	0.0049	-0.0057	-0.0086	-0.0023	1.10	4.02	6.04	-57.07	-138.74	-114.27	-105.36	154.30
74	-0.0049	0.0227	0.0181	-0.0111	0.0044	-0.0054	-0.0090	-0.0025	1.17	3.91	6.09	-56.41	-140.08	-115.45	-106.54	156.56
75	-0.0048	0.0223	0.0178	-0.0112	0.0050	-0.0055	-0.0083	-0.0023	0.83	4.37	6.24	-56.41	-136.17	-114.33	-104.39	154.13
76	-0.0048	0.0223	0.0177	-0.0110	0.0047	-0.0052	-0.0087	-0.0021	0.75	5.13	6.98	-53.81	-136.10	-113.33	-104.04	154.51
77	-0.0048	0.0224	0.0178	-0.0113	0.0043	-0.0054	-0.0090	-0.0021	1.05	4.40	6.52	-53.35	-137.23	-114.44	-104.92	156.24
78	-0.0049	0.0222	0.0178	-0.0114	0.0042	-0.0059	-0.0093	-0.0025	1.54	3.97	6.58	-54.52	-140.72	-116.74	-107.04	157.83
79	-0.0048	0.0224	0.0182	-0.0111	0.0042	-0.0053	-0.0089	-0.0023	0.79	4.84	8.10	-53.51	-135.70	-113.88	-104.00	155.48

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Table 39 – continued from previous page(7^{th} Harmonic)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
80	-0.0048	0.0225	0.0182	-0.0111	0.0046	-0.0052	-0.0087	-0.0022	0.85	4.51	7.65	-54.07	-136.63	-114.67	-104.98	156.46
81	-0.0049	0.0227	0.0182	-0.0113	0.0050	-0.0053	-0.0088	-0.0027	1.01	4.00	7.35	-54.85	-138.11	-115.81	-106.29	157.10
82	-0.0049	0.0229	0.0184	-0.0111	0.0052	-0.0051	-0.0088	-0.0032	1.20	3.28	6.76	-56.84	-139.84	-117.01	-108.10	158.25
83	-0.0047	0.0222	0.0179	-0.0111	0.0040	-0.0057	-0.0092	-0.0026	1.56	2.81	6.34	-55.36	-139.61	-116.43	-107.13	159.03
84	-0.0047	0.0224	0.0182	-0.0111	0.0040	-0.0055	-0.0093	-0.0025	1.71	2.80	6.41	-55.02	-140.20	-116.81	-107.31	159.37
85	-0.0049	0.0226	0.0182	-0.0109	0.0043	-0.0055	-0.0095	-0.0029	1.79	3.08	6.42	-55.04	-143.49	-117.83	-108.94	160.78
86	-0.0049	0.0227	0.0184	-0.0111	0.0044	-0.0054	-0.0094	-0.0034	2.02	2.74	6.18	-57.07	-144.17	-118.29	-109.88	161.22
87	-0.0049	0.0229	0.0186	-0.0111	0.0046	-0.0057	-0.0094	-0.0034	1.44	4.23	6.96	-56.58	-143.68	-117.73	-109.64	161.03
88	-0.0050	0.0232	0.0186	-0.0116	0.0045	-0.0060	-0.0099	-0.0034	2.76	1.98	4.50	-57.76	-148.26	-120.43	-113.69	164.87
89	-0.0050	0.0233	0.0188	-0.0113	0.0047	-0.0057	-0.0097	-0.0033	2.36	2.72	5.08	-57.43	-147.13	-119.34	-113.40	164.04
90	-0.0050	0.0231	0.0187	-0.0114	0.0043	-0.0058	-0.0096	-0.0035	2.51	2.59	4.88	-58.31	-147.45	-119.79	-114.19	164.83
91	-0.0050	0.0237	0.0191	-0.0115	0.0043	-0.0055	-0.0099	-0.0035	3.13	1.78	4.34	-56.76	-149.18	-120.19	-115.10	166.43
92	-0.0050	0.0232	0.0187	-0.0117	0.0042	-0.0057	-0.0097	-0.0034	2.60	2.35	4.94	-56.16	-146.63	-119.87	-112.85	165.34
93	-0.0050	0.0230	0.0185	-0.0117	0.0033	-0.0063	-0.01	-0.0027	2.85	1.93	4.71	-54.79	-146.64	-119.96	-111.35	166.08
94	-0.0048	0.0227	0.0183	-0.0118	0.0026	-0.0064	-0.0102	-0.0026	2.78	2.47	4.95	-53.96	-145.31	-119.26	-110.77	166.24
95	-0.0049	0.0227	0.0184	-0.0117	0.0033	-0.0061	-0.01	-0.0028	2.88	2.15	4.78	-54.46	-145.59	-119.12	-111.19	164.84
96	-0.0049	0.0230	0.0185	-0.0117	0.0045	-0.0056	-0.0096	-0.0033	2.36	2.58	5.15	-55.41	-145.67	-120.20	-113.12	164.12
97	-0.0050	0.0237	0.0189	-0.0117	0.0051	-0.0053	-0.0096	-0.0036	2.42	2.12	5.07	-56.24	-146.13	-120.87	-114.54	164.63
98	-0.0050	0.0236	0.0189	-0.0121	0.0042	-0.0061	-0.0101	-0.0034	2.46	2.31	5.00	-54.79	-148.60	-121.99	-116.16	167.83
99	-0.0050	0.0233	0.0189	-0.0123	0.0040	-0.0064	-0.0102	-0.0033	2.93	1.80	4.73	-55.23	-148.85	-122.30	-116.67	168.82
100	-0.0052	0.0241	0.0193	-0.0119	0.0047	-0.0062	-0.0098	-0.0036	2.89	0.71	4.18	-56.51	-149.38	-122.78	-116.51	168.18

Table 40: Variation of Error in Magnitude and Angle (9^{th} Harmonic) with Time (1^{st} to 100^{th})

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
1	0.0004	-0.0002	-0.0011	0.0013	0.0015	0.0007	0.0002	0.0006	61.24	-145.66	-1.79	-39.89	26.96	-5.98	82.80	-120.04
2	0.00	0.0006	-0.0012	0.0008	0.00	0.0011	-0.0003	-0.0002	123.77	-54.53	118.79	-4.65	78.27	-9.06	120.35	-49.06
3	0.0001	0.00	-0.0013	0.0010	0.0007	0.0011	-0.0002	0.0001	102.70	-22.65	267.42	-10.41	65.06	-10.26	109.92	-70.38
4	0.0002	0.0012	-0.0006	0.0004	-0.0007	0.0008	-0.0011	-0.0002	142.01	-9.26	169.15	2.48	131.94	-30.91	136.16	3.24
5	0.0002	0.0006	-0.0013	0.0009	0.0006	0.0011	-0.0004	0.0003	122.99	-77.80	79.59	9.65	115.58	-14.18	138.51	-35.53
6	0.0002	0.0006	-0.0013	0.0008	0.00	0.0011	-0.0007	-0.0002	118.37	-64.98	100.85	2.60	110.35	-13.12	133.08	-22.68
7	0.0002	0.0007	-0.0012	0.0009	0.0006	0.0009	-0.0003	0.0003	124.97	-78.68	75.22	10.49	115.76	-5.71	137.07	-52.59
8	0.0002	0.0005	-0.0014	0.0009	0.0004	0.0009	-0.0003	0.0002	113.23	-79.24	67.93	0.61	102.27	-3.29	128.07	-56.38
9	0.0002	0.0010	-0.0009	0.0005	-0.0006	0.0009	-0.0010	-0.0006	125.74	-36.87	132.67	-5.82	109.56	-10.71	129.67	7.86
10	0.0001	0.0011	-0.0003	0.0006	-0.0004	0.0010	-0.0009	-0.0003	136.45	10.42	186.92	12.17	125.55	-14.11	135.46	13.45
11	0.0001	0.0006	-0.0009	0.0005	-0.0003	0.0010	-0.0009	-0.0005	143.42	5.41	199.81	3.35	100.00	-6.21	130.38	24.36
12	0.00	0.0006	-0.0012	0.0008	0.0002	0.0009	-0.0008	0.0003	125.53	-44.46	141.51	17.17	123.57	-15.32	149.53	-4.50
13	0.00	0.0004	-0.0014	0.0009	0.0005	0.0009	-0.0006	0.0004	110.28	-87.04	44.43	9.66	106.08	-12.02	137.19	-42.16
14	0.00	0.0002	-0.0011	0.0010	0.0006	0.0010	-0.0005	0.0006	95.10	110.95	266.93	-1.53	91.31	-22.20	122.88	-57.59
15	-0.0001	0.0010	-0.0008	0.0005	-0.0006	0.0008	-0.0011	-0.0005	143.95	-23.49	151.34	6.48	142.19	-4.63	154.28	31.48
16	0.00	0.0010	-0.0008	0.0005	-0.0004	0.0009	-0.0010	-0.0003	156.52	-16.39	158.52	16.28	152.45	-14.47	158.93	28.38
17	0.0001	0.0015	-0.0001	0.0007	-0.0007	0.0011	-0.0010	-0.0004	145.31	-3.75	165.97	4.45	131.00	-21.59	133.67	10.64
18	0.0001	0.0012	-0.0004	0.0008	0.0003	0.0010	-0.0008	-0.0004	67.56	10.60	186.02	-23.65	1.76	-26.31	74.08	-101.70
19	0.0003	0.0005	-0.0008	0.0010	0.0005	0.0012	-0.0003	0.0002	88.01	53.10	239.44	-1.52	40.34	-12.19	104.25	-68.38
20	0.0003	0.0006	-0.0006	0.0011	0.0011	0.0012	-0.0001	0.0006	69.94	72.58	250.36	-19.61	20.96	-17.24	86.62	-103.57
21	0.0003	0.0003	-0.0010	0.0010	0.0008	0.0012	0.00	0.0002	82.34	51.91	246.76	-6.98	48.85	-5.44	108.60	-76.24
22	0.0003	0.0005	-0.0013	0.0010	0.0005	0.0012	-0.0002	0.0002	108.92	-39.81	165.86	16.39	90.83	-3.78	139.08	-35.32
23	0.0003	0.0003	-0.0013	0.0015	0.0017	0.0011	0.0004	0.0009	106.63	-130.76	303.04	20.72	96.70	-6.98	142.09	-42.87
24	0.0003	0.0005	-0.0012	0.0013	0.0011	0.0013	0.0001	0.0007	104.27	-41.68	173.51	23.39	94.37	-10.59	145.52	-33.35
25	0.0003	0.0009	-0.0006	0.0009	0.0002	0.0013	-0.0005	-0.0002	97.89	9.55	193.36	11.45	65.68	-5.46	117.26	-20.52

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Table 40 – continued from previous page (9^{th} Harmonic)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
26	0.0004	0.0003	-0.0010	0.0014	0.0016	0.0013	0.0003	0.0010	79.38	152.18	288.27	0.53	65.01	-7.59	107.44	-64.59
27	0.0003	0.0003	-0.0011	0.0015	0.0017	0.0013	0.0005	0.0010	78.98	-153.78	320.63	2.15	64.82	-0.36	109.91	-72.29
28	0.0003	0.0006	-0.0011	0.0018	0.0021	0.0011	0.0007	0.0015	92.95	-118.74	12.82	25.02	102.11	0.43	136.13	-48.97
29	0.0001	0.0009	-0.0008	0.0014	0.0011	0.0010	0.0003	0.0010	90.12	-67.80	111.90	13.51	93.35	-3.77	124.41	-68.66
30	0.0002	0.0015	-0.0003	0.0010	0.0005	0.0010	-0.0003	0.0001	115.18	-76.54	88.29	17.39	106.01	-1.79	139.64	-52.38
31	0.00	0.0005	-0.0012	0.0010	0.0004	0.0010	-0.0003	-0.0002	114.72	-96.48	54.02	5.43	64.64	5.46	115.30	-98.48
32	0.00	0.0006	-0.0011	0.0010	0.0006	0.0009	-0.0002	0.00	108.69	-102.26	49.86	5.67	73.59	5.22	120.84	-96.35
33	0.00	0.0005	-0.0012	0.0012	0.0007	0.0009	0.00	0.0001	85.18	-90.34	73.25	-3.89	56.23	0.90	107.23	-105.12
34	0.0002	0.0001	-0.0013	0.0013	0.0010	0.0010	0.0001	0.0005	73.72	-87.89	246.66	-7.75	38.82	-11.77	96.86	-97.99
35	0.0002	0.0003	-0.0013	0.0013	0.0010	0.0011	0.0001	0.0005	84.74	-86.21	70.06	-7.48	40.61	-5.82	99.61	-100.73
36	0.0002	0.0003	-0.0013	0.0011	0.0008	0.0010	0.00	0.0002	98.36	-48.43	174.93	-2.66	46.33	1.06	106.87	-99.04
37	0.0001	0.0011	-0.0004	0.0009	0.00	0.0009	-0.0005	-0.0003	90.11	-48.68	122.15	-9.82	22.20	-9.80	96.20	-101.93
38	0.0001	0.0017	-0.0001	0.0015	0.0017	0.0009	0.0003	0.0008	120.30	-79.48	84.36	29.07	120.24	9.37	151.12	-55.73
39	0.00	0.0008	-0.0008	0.0014	0.0013	0.0010	0.0001	0.0005	108.22	-100.79	57.47	12.69	88.19	-0.62	127.29	-85.74
40	0.0002	0.0006	-0.0010	0.0015	0.0015	0.0011	0.0003	0.0011	84.86	-143.32	-10.90	11.55	84.34	-13.89	122.72	-76.02
41	0.0002	0.0008	-0.0008	0.0014	0.0015	0.0010	0.0002	0.0011	87.31	-125.42	25.37	0.12	67.96	-4.53	111.64	-97.72
42	0.0001	0.0008	-0.0008	0.0014	0.0013	0.0010	0.0002	0.0010	98.32	-110.07	42.90	-2.70	70.16	-1.90	110.82	-103.04
43	-0.0001	0.0010	-0.0008	0.0013	0.0010	0.0009	0.0001	0.0009	100.91	-89.36	68.87	-1.27	76.37	3.13	113.25	-95.83
44	0.00	0.0013	-0.0005	0.0011	0.0007	0.0008	-0.0002	0.0005	109.14	-98.63	57.26	1.04	85.57	9.27	121.35	-92.56
45	0.0001	0.0007	-0.0010	0.0013	0.0007	0.0009	0.00	0.0007	88.96	-96.09	57.09	-11.27	54.40	-12.54	95.06	-102.98
46	0.0001	0.0003	-0.0013	0.0012	0.0008	0.0010	-0.0001	0.0005	83.17	-91.01	43.25	-14.43	33.01	-12.12	88.41	-111.42
47	0.0002	0.0002	-0.0012	0.0011	0.0008	0.0010	0.00	0.0004	80.97	-80.72	249.03	-10.17	28.18	-5.00	87.10	-117.88
48	0.0003	0.0003	-0.0012	0.0009	0.0004	0.0009	-0.0003	0.0001	90.12	-102.18	-1.72	-3.51	44.35	5.83	98.86	-117.06
49	0.0004	0.0003	-0.0011	0.0008	0.0002	0.0009	-0.0004	-0.0001	78.48	-22.30	205.71	-9.44	16.48	-5.41	91.06	-133.65
50	0.0002	0.0002	-0.0012	0.0009	0.0005	0.0010	-0.0003	0.0001	89.39	-21.04	221.25	-15.38	22.00	-7.06	86.06	-132.40
51	0.0002	0.0001	-0.0013	0.0008	0.0003	0.0010	-0.0004	-0.0001	89.21	-104.48	297.01	-17.83	11.64	-11.76	79.45	-127.76
52	0.0001	0.0003	-0.0013	0.0007	-0.0003	0.0008	-0.0006	-0.0003	109.33	-101.88	5.86	-4.25	48.58	-14.96	105.77	-95.75

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Table 40 – continued from previous page (9^{th} Harmonic)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
53	0.00	0.0003	-0.0013	0.0007	0.00	0.0006	-0.0006	0.00	109.50	-106.50	11.78	-26.42	22.11	-10.18	82.72	-141.67
54	-0.0002	0.0013	-0.0006	0.0005	-0.0007	0.0005	-0.0011	-0.0006	142.17	-64.78	89.10	0.30	194.95	-7.40	153.95	-47.49
55	-0.0001	0.0015	-0.0005	0.0005	-0.0003	0.0006	-0.0010	-0.0008	157.08	-50.11	109.05	25.72	222.73	2.55	193.24	9.16
56	0.00	0.0008	-0.0011	0.0005	-0.0006	0.0008	-0.0008	-0.0003	133.38	-66.73	90.40	26.89	141.53	-4.13	156.25	-13.39
57	0.0001	0.0003	-0.0011	0.0008	0.0002	0.0009	-0.0004	0.00	108.84	-174.66	321.11	-6.44	45.10	-8.22	101.53	-118.52
58	0.0001	0.0003	-0.0012	0.0008	0.0002	0.0008	-0.0004	0.00	98.32	-146.07	335.60	-21.65	20.12	-9.26	84.93	-141.80
59	0.00	0.00	-0.0013	0.0008	0.0005	0.0009	-0.0003	0.0001	82.35	8.72	273.02	-26.53	5.80	-16.52	73.82	-141.49
60	0.0001	0.0001	-0.0013	0.0009	0.0006	0.0010	-0.0003	0.0001	82.05	-7.73	257.95	-27.37	5.64	-17.78	70.04	-133.62
61	0.0002	0.0004	-0.0012	0.0009	0.0005	0.0010	-0.0004	0.0001	86.40	-9.75	193.52	-28.40	-3.17	-20.62	66.64	-137.56
62	0.0002	0.0005	-0.0011	0.0008	0.0002	0.0010	-0.0005	-0.0002	100.95	-15.00	172.07	-20.83	-5.45	-13.25	73.47	-133.02
63	0.0002	0.0007	-0.0010	0.0007	0.00	0.0011	-0.0007	-0.0007	105.83	-25.75	154.15	-14.29	-6.12	-9.02	84.15	-143.51
64	0.00	0.0010	-0.0006	0.0007	0.00	0.0010	-0.0008	-0.0006	98.78	-35.47	130.33	-25.65	-32.45	-18.84	68.47	-165.48
65	0.00	0.0018	0.00	0.0007	-0.0008	0.0008	-0.0013	-0.0009	123.99	-43.02	116.85	-9.65	-54.82	-4.66	134.31	-177.64
66	0.0001	0.0017	-0.0001	0.0007	-0.0008	0.0011	-0.0011	-0.0005	123.44	-56.17	105.01	8.31	131.63	-16.16	162.34	6.95
67	0.0002	0.0012	-0.0006	0.0006	-0.0005	0.0010	-0.0011	-0.0007	124.90	-81.58	72.87	-4.58	60.27	-8.03	130.50	-103.30
68	0.00	0.0003	-0.0013	0.0009	0.0005	0.0010	-0.0007	0.0001	106.49	-89.79	18.46	-26.97	15.81	-18.42	87.38	-156.12
69	0.00	0.0006	-0.0010	0.0010	0.0003	0.0010	-0.0007	0.0001	103.98	-47.32	108.85	-23.92	3.59	-19.83	84.54	-149.99
70	0.00	0.0017	0.00	0.0005	-0.0001	0.0010	-0.0014	-0.0003	129.24	-23.09	140.27	-21.16	-62.68	-15.39	30.30	-241.40
71	-0.0001	0.0014	-0.0003	0.0007	-0.0008	0.0008	-0.0010	-0.0005	135.40	-27.29	140.70	12.59	162.55	-11.94	151.60	-22.65
72	0.00	0.0010	-0.0008	0.0013	0.0008	0.0011	-0.0003	0.0005	123.18	-43.96	130.83	24.87	113.26	-3.00	150.63	-20.42
73	0.0001	0.0005	-0.0009	0.0013	0.0006	0.0012	-0.0003	0.0008	85.54	16.20	209.38	-3.82	48.75	-14.14	105.71	-47.53
74	0.0001	0.0004	-0.0009	0.0014	0.0009	0.0013	-0.0002	0.0010	87.85	20.65	213.06	-2.43	50.26	-15.07	109.02	-38.71
75	0.0002	0.0025	0.0010	0.0008	-0.0007	0.0013	-0.0013	0.0002	149.14	-3.81	163.83	22.49	223.99	-21.88	180.49	77.70
76	0.0001	0.0011	-0.0007	0.0015	0.0012	0.0013	0.00	0.0009	124.14	-67.41	102.70	22.94	107.08	-12.63	149.40	-16.43
77	0.0003	0.0015	-0.0002	0.0011	0.0003	0.0014	-0.0005	0.0009	124.41	-35.74	136.48	29.91	128.48	-19.90	163.49	19.59
78	0.0002	0.0003	-0.0013	0.0014	0.0009	0.0015	-0.0001	0.0009	107.81	-21.44	221.75	7.72	73.63	-20.23	118.94	-40.28
79	0.0001	0.0012	-0.0007	0.0012	0.00	0.0010	-0.0005	0.0005	113.34	-33.40	153.94	14.92	97.82	-25.94	137.57	-54.30

Continued on next page

Table 40 – continued from previous page (9^{th} Harmonic)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
80	0.0001	0.0009	-0.0008	0.0010	-0.0003	0.0010	-0.0008	-0.0003	89.64	6.43	209.40	6.47	-0.32	-14.37	115.51	-90.78
81	0.0003	0.0011	-0.0005	0.0009	0.0009	0.0010	-0.0006	-0.0003	56.30	32.97	231.82	4.89	-20.08	-17.73	87.96	-131.61
82	0.0003	0.0009	-0.0006	0.0010	0.0007	0.0012	-0.0004	-0.0003	68.06	33.19	235.02	-8.16	-16.93	-20.82	80.43	-130.60
83	0.0003	0.0014	-0.0001	0.0010	0.0015	0.0013	-0.0003	-0.0001	48.73	45.37	237.97	-11.12	-21.93	-28.16	71.72	-149.32
84	0.00	0.0015	-0.0003	0.0007	0.0006	0.0011	-0.0008	0.0001	83.73	15.58	210.25	-24.17	-71.63	-19.68	45.05	-259.87
85	0.0002	0.0009	-0.0007	0.0008	0.0008	0.0009	-0.0007	-0.0005	31.24	31.88	236.47	-6.81	-33.65	-15.34	80.88	-165.63
86	0.0006	0.0012	-0.0004	0.0008	0.0014	0.0012	-0.0006	-0.0004	59.18	35.91	229.56	1.61	-6.02	-26.55	78.38	-143.29
87	0.0004	0.0007	-0.0008	0.0014	0.0016	0.0013	0.00	0.0007	65.31	54.53	251.66	-6.14	40.07	-19.81	99.45	-112.63
88	0.0003	0.0005	-0.0006	0.0014	0.0015	0.0013	0.00	0.0008	59.83	60.66	247.42	3.05	31.35	-19.67	109.00	-99.11
89	0.0002	0.00	-0.0011	0.0013	0.0011	0.0013	-0.0002	0.0005	96.55	-29.29	267.02	4.74	55.42	-10.46	117.22	-82.18
90	0.0002	0.0002	-0.0012	0.0014	0.0014	0.0013	0.00	0.0009	110.34	-118.94	289.87	12.57	78.60	-5.65	122.98	-77.54
91	0.0001	0.0002	-0.0011	0.0011	0.0007	0.0011	-0.0003	0.0003	70.47	16.50	245.48	-10.54	12.39	-10.63	88.64	-129.73
92	0.0001	0.0003	-0.0009	0.0012	0.0008	0.0012	-0.0003	0.0003	73.94	51.84	255.19	-10.26	23.43	-7.79	89.05	-127.30
93	0.0002	0.0007	-0.0005	0.0010	0.0010	0.0012	-0.0003	0.0001	55.67	63.77	253.43	-8.99	2.34	-14.36	82.64	-126.60
94	0.0004	0.0013	0.0001	0.0010	0.0020	0.0013	-0.0002	0.0003	50.98	77.49	257.90	-12.88	-2.32	-23.52	73.03	-142.10
95	0.00	0.0010	-0.0003	0.0009	0.00	0.0009	-0.0007	-0.0004	42.25	32.79	226.55	-0.12	-27.00	-17.94	90.93	-111.08
96	0.0001	0.0006	-0.0006	0.0010	0.0006	0.0009	-0.0005	0.00	63.20	27.32	226.11	-14.32	-0.09	-11.29	81.88	-145.65
97	0.0002	0.0008	-0.0003	0.0013	0.0013	0.0012	-0.0002	0.0007	53.07	51.21	237.04	-2.79	13.83	-20.11	92.37	-116.01
98	0.0002	0.0015	0.0004	0.0014	0.0018	0.0013	0.0002	0.0008	7.53	59.11	241.65	-9.56	-23.01	-22.62	72.32	-126.11
99	0.0002	0.0009	-0.0001	0.0014	0.0017	0.0015	0.0003	0.0006	40.76	76.30	254.84	-5.87	7.12	-13.22	81.93	-117.79
100	0.0002	0.0006	-0.0004	0.0013	0.0007	0.0013	0.00	0.0004	65.15	43.15	238.08	-0.90	24.74	-17.66	92.57	-94.79

Table 41: Variation of Error in Magnitude and Angle (11^{th} Harmonic) with Time (1^{st} to 100^{th})

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
1	0.0003	0.0039	0.0058	0.0013	0.0039	-0.0011	-0.0006	0.0026	-11.93	5.66	10.62	-1.90	-169.02	-168.87	-102.28	-169.84
2	0.0003	0.0038	0.0058	0.0021	0.0021	-0.0011	-0.0012	0.0031	-14.47	-3.83	4.15	37.19	-217.12	180.12	119.13	-126.18
3	0.0003	0.0038	0.0056	0.0022	0.0020	-0.0011	-0.0013	0.0033	-14.09	-4.14	3.89	34.34	-206.30	-169.76	107.33	-131.17
4	0.0003	0.0040	0.0058	0.0020	0.0020	-0.0010	-0.0014	0.0029	-13.68	-2.74	5.51	34.32	-196.21	-160.49	146.82	-137.39
5	0.0003	0.0042	0.0060	0.0016	0.0030	-0.0008	-0.0011	0.0022	-14.30	4.64	10.79	26.94	-190.88	-159.69	-159.29	-146.75
6	0.0003	0.0040	0.0060	0.0021	0.0023	-0.0009	-0.0013	0.0030	-11.36	-8.47	4.19	33.77	-203.41	-166.55	126.78	-131.10
7	0.0003	0.0046	0.0066	0.0020	0.0030	-0.0010	-0.0013	0.0028	-9.44	-1.12	10.19	30.52	-206.45	-173.44	162.25	-146.26
8	0.0003	0.0045	0.0065	0.0021	0.0030	-0.0010	-0.0012	0.0029	-8.86	-2.79	10.35	32.46	-205.10	-172.05	151.92	-144.95
9	0.0003	0.0045	0.0064	0.0021	0.0031	-0.0008	-0.0010	0.0031	-9.94	-8.18	4.46	33.90	139.05	181.02	130.49	-128.50
10	0.0003	0.0049	0.0066	0.0018	0.0036	-0.0006	-0.0007	0.0028	-7.33	-13.94	0.26	37.93	122.51	171.63	135.34	-122.28
11	0.0004	0.0039	0.0058	0.0020	0.0024	-0.0010	-0.0009	0.0035	-10.44	-15.88	-2.42	35.29	102.92	159.32	107.26	-122.11
12	0.0003	0.0047	0.0066	0.0019	0.0035	-0.0008	-0.0010	0.0028	-10.28	-7.16	3.21	31.95	132.94	172.62	140.86	-134.99
13	0.0003	0.0040	0.0062	0.0020	0.0027	-0.0009	-0.0016	0.0030	-9.96	2.80	12.71	20.53	-198.62	-172.19	-141.54	-158.46
14	0.0003	0.0041	0.0063	0.0021	0.0020	-0.0010	-0.0018	0.0032	-6.98	3.33	12.23	27.50	-195.49	182.97	-169.20	-155.24
15	0.0004	0.0041	0.0061	0.0020	0.0013	-0.0012	-0.0014	0.0031	-7.08	-3.90	6.05	35.05	-214.74	171.29	134.84	-137.67
16	0.0002	0.0047	0.0065	0.0018	0.0038	-0.0007	-0.0012	0.0023	-10.20	1.67	10.70	28.14	-190.25	-163.26	-164.93	-143.22
17	0.0002	0.0045	0.0063	0.0019	0.0037	-0.0007	-0.0013	0.0024	-11.27	7.45	13.31	23.99	-184.64	-163.90	-147.64	-144.65
18	0.0003	0.0053	0.0072	0.0019	0.0039	-0.0005	-0.0010	0.0024	-8.61	-3.21	8.09	33.92	146.34	173.42	158.58	-132.25
19	0.0003	0.0048	0.0068	0.0020	0.0032	-0.0010	-0.0015	0.0028	-11.95	4.00	13.63	27.53	-186.71	-167.48	-153.03	-153.26
20	0.0003	0.0050	0.0070	0.0020	0.0031	-0.0008	-0.0015	0.0028	-10.74	3.95	11.78	32.65	-192.95	181.26	-172.40	-151.42
21	0.0003	0.0053	0.0073	0.0020	0.0038	-0.0005	-0.0013	0.0027	-8.26	-0.20	7.90	34.83	-206.42	176.77	169.43	-142.23
22	0.0003	0.0049	0.0066	0.0020	0.0024	-0.0005	-0.0016	0.0035	-7.13	-9.91	0.03	31.30	126.79	160.84	125.22	-131.04
23	0.0002	0.0041	0.0059	0.0020	0.0020	-0.0008	-0.0020	0.0039	-8.92	0.17	7.41	19.47	-199.89	-172.35	-111.65	-143.99
24	0.0002	0.0047	0.0065	0.0019	0.0034	-0.0005	-0.0016	0.0030	-7.52	2.69	10.93	20.38	-187.62	-165.07	-139.26	-143.21
25	0.0002	0.0045	0.0065	0.0017	0.0018	-0.0007	-0.0015	0.0032	-5.16	-0.52	9.42	28.13	-195.92	182.26	-171.59	-150.39

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Table 41 – continued from previous page (11^{th} Harmonic)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
26	0.0002	0.0044	0.0063	0.0017	0.0016	-0.0008	-0.0015	0.0032	-5.08	1.37	10.67	29.54	-196.13	183.69	-168.77	-146.37
27	0.00	0.0052	0.0071	0.0013	0.0032	-0.0006	-0.0010	0.0023	-4.02	4.08	13.12	29.17	-190.55	184.53	-156.76	-147.56
28	0.00	0.0052	0.0073	0.0012	0.0038	-0.0006	-0.0007	0.0023	-5.74	6.83	17.60	14.54	-180.78	186.44	-137.10	-167.78
29	0.00	0.0053	0.0073	0.0013	0.0032	-0.0006	-0.0009	0.0023	-4.99	4.29	13.55	28.52	-189.49	173.93	-153.35	-154.23
30	0.00	0.0058	0.0076	0.0012	0.0046	-0.0003	-0.0006	0.0018	-2.06	3.16	12.40	26.13	-187.71	-168.54	-156.32	-147.96
31	0.0002	0.0056	0.0073	0.0012	0.0029	-0.0004	-0.0008	0.0018	-3.13	-0.62	7.47	40.30	-202.29	181.28	181.30	-142.39
32	0.00	0.0051	0.0069	0.0015	0.0041	-0.0002	-0.0011	0.0024	-5.18	6.77	12.82	26.11	-177.42	-161.29	-143.67	-150.74
33	0.00	0.0043	0.0061	0.0015	0.0035	-0.0005	-0.0011	0.0030	-7.89	10.46	16.62	18.32	-159.02	-148.53	-107.23	-152.38
34	0.0001	0.0051	0.0068	0.0014	0.0042	-0.0002	-0.0009	0.0018	-3.82	1.94	12.04	34.84	-184.59	-167.80	-162.07	-141.99
35	0.0002	0.0051	0.0071	0.0015	0.0036	-0.0005	-0.0010	0.0022	-5.32	1.05	11.43	37.94	-183.25	-167.46	-171.78	-140.46
36	0.0002	0.0050	0.0071	0.0016	0.0034	-0.0005	-0.0009	0.0023	-7.60	-0.11	10.35	44.41	-187.36	-164.17	174.62	-137.24
37	0.0002	0.0055	0.0073	0.0013	0.0038	-0.0004	-0.0006	0.0018	-2.77	-3.59	7.20	41.12	-202.80	183.60	180.30	-132.11
38	0.00	0.0055	0.0073	0.0007	0.0051	-0.0006	0.00	0.0010	-8.07	11.83	18.43	22.00	-169.49	-148.51	-139.06	-168.07
39	0.00	0.0063	0.0081	0.0004	0.0061	-0.0001	0.0005	0.0007	-10.18	12.82	19.35	11.18	-175.30	-156.47	-141.52	-186.41
40	-0.0001	0.0059	0.0076	0.0005	0.0061	-0.0003	0.0003	0.0010	-8.17	12.38	18.71	21.14	-172.79	-159.80	-141.95	-180.32
41	0.00	0.0062	0.0082	0.0007	0.0054	-0.0003	0.0001	0.0008	-7.50	6.26	15.45	31.33	-189.90	-170.67	-154.47	-167.91
42	-0.0001	0.0056	0.0076	0.0010	0.0054	-0.0005	-0.0001	0.0017	-7.36	12.25	17.89	8.46	-176.85	-164.70	-131.60	-180.28
43	0.0001	0.0063	0.0081	0.0006	0.0048	-0.0004	0.0001	0.0006	-3.48	2.60	11.66	43.76	-194.52	-171.29	-157.78	-158.11
44	-0.0001	0.0054	0.0072	0.0009	0.0043	-0.0005	-0.0003	0.0019	-5.62	9.09	15.58	1.92	-179.58	-164.21	-133.15	-167.73
45	-0.0001	0.0057	0.0076	0.0008	0.0055	-0.0001	-0.0001	0.0015	-4.55	9.40	15.44	10.19	-178.39	-156.10	-139.16	-166.38
46	0.0002	0.0050	0.0067	0.0013	0.0028	-0.0003	-0.0010	0.0023	-1.38	-3.61	8.05	28.57	-195.95	184.80	-168.44	-145.90
47	0.0001	0.0055	0.0074	0.0013	0.0043	-0.0003	-0.0007	0.0020	-4.56	4.25	13.83	24.46	-182.57	-159.66	-147.89	-160.03
48	0.0002	0.0051	0.0072	0.0015	0.0038	-0.0006	-0.0009	0.0025	-7.96	8.72	18.75	21.14	-162.76	-149.64	-126.29	-167.13
49	0.0002	0.0046	0.0068	0.0018	0.0032	-0.0006	-0.0012	0.0030	-7.29	8.52	19.96	21.40	-161.30	-156.03	-111.00	-164.26
50	0.0002	0.0047	0.0067	0.0017	0.0036	-0.0007	-0.0010	0.0029	-7.11	7.14	14.92	13.26	-167.50	-150.03	-111.58	-166.89
51	0.0003	0.0047	0.0065	0.0015	0.0010	-0.0010	-0.0014	0.0030	-4.62	-3.41	5.43	22.91	-191.69	188.49	-163.49	-148.23
52	0.0002	0.0051	0.0070	0.0017	0.0029	-0.0007	-0.0012	0.0027	-5.23	0.42	9.62	22.82	-188.07	-163.90	-152.47	-153.65

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Table 41 – continued from previous page (11^{th} Harmonic)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
53	0.0002	0.0049	0.0068	0.0017	0.0029	-0.0008	-0.0013	0.0026	-7.74	2.22	9.48	26.11	-185.64	-163.36	-152.14	-153.87
54	0.0003	0.0039	0.0058	0.0017	0.0002	-0.0011	-0.0018	0.0035	-5.70	-3.78	4.65	23.81	-196.73	179.43	-165.51	-148.51
55	0.0003	0.0045	0.0064	0.0017	0.0016	-0.0009	-0.0015	0.0028	-5.12	-3.97	4.90	27.06	-203.84	183.65	180.88	-146.06
56	0.0003	0.0042	0.0062	0.0019	0.0012	-0.0010	-0.0018	0.0033	-6.53	-0.75	7.70	25.64	-205.13	176.30	-178.00	-147.97
57	0.0003	0.0040	0.0062	0.0020	0.0010	-0.0012	-0.0018	0.0035	-6.29	-0.86	7.68	22.68	-199.80	177.53	-53.37	-150.59
58	0.0003	0.0044	0.0066	0.0020	0.0014	-0.0010	-0.0017	0.0032	-4.84	-0.29	8.83	27.63	-206.95	173.93	162.24	-147.05
59	0.0004	0.0045	0.0066	0.0019	0.0011	-0.0009	-0.0017	0.0033	-6.83	-3.52	4.35	25.56	135.68	169.26	149.59	-142.66
60	0.0004	0.0053	0.0070	0.0016	0.0029	-0.0005	-0.0009	0.0028	-7.45	-11.06	-2.48	35.04	118.54	160.91	143.04	-125.66
61	0.0003	0.0055	0.0073	0.0016	0.0034	-0.0003	-0.0009	0.0025	-5.72	-8.60	0.91	33.53	134.29	167.25	155.30	-126.92
62	0.0002	0.0054	0.0073	0.0019	0.0038	-0.0004	-0.0013	0.0027	-9.51	-0.73	7.13	27.48	-196.29	183.29	-168.88	-141.55
63	0.0003	0.0054	0.0072	0.0018	0.0039	-0.0002	-0.0010	0.0026	-6.93	-9.32	-0.02	31.96	127.92	164.84	148.55	-126.08
64	0.0003	0.0051	0.0067	0.0017	0.0030	-0.0004	-0.0010	0.0031	-6.85	-14.65	-3.50	34.95	105.20	152.44	129.71	-123.74
65	0.0003	0.0048	0.0065	0.0018	0.0032	-0.0003	-0.0012	0.0032	-6.40	-12.94	-2.95	31.48	115.37	157.96	126.04	-127.96
66	0.0003	0.0048	0.0066	0.0018	0.0022	-0.0006	-0.0015	0.0030	-6.88	-4.08	4.24	25.06	138.09	171.30	162.90	-144.23
67	0.0003	0.0041	0.0062	0.0022	0.0018	-0.0007	-0.0017	0.0035	-5.69	-2.18	8.86	27.82	142.28	167.95	91.07	-144.41
68	0.0003	0.0044	0.0063	0.0019	0.0013	-0.0003	-0.0014	0.0032	-3.84	-6.45	5.89	37.24	121.55	158.06	117.88	-138.59
69	0.0003	0.0050	0.0068	0.0019	0.0023	-0.0002	-0.0013	0.0029	-4.95	-7.92	3.66	37.52	121.31	157.71	126.51	-135.37
70	0.0003	0.0043	0.0063	0.0019	0.0010	-0.0008	-0.0016	0.0032	-4.66	-2.00	7.16	35.18	131.82	162.83	133.41	-142.34
71	0.0004	0.0045	0.0064	0.0018	0.0013	-0.0008	-0.0013	0.0029	-5.68	-3.88	5.04	41.30	126.66	159.48	132.51	-136.38
72	0.0003	0.0056	0.0072	0.0013	0.0034	-0.0004	-0.0007	0.0020	-4.74	-6.80	1.39	50.27	136.07	174.03	160.19	-131.06
73	0.0003	0.0062	0.0078	0.0010	0.0044	-0.0001	-0.0003	0.0016	-4.31	-6.74	2.15	46.83	137.49	181.72	172.18	-140.83
74	0.0003	0.0028	0.0046	0.0022	-0.0004	-0.0010	-0.0013	0.0015	-9.27	-5.85	2.46	23.21	133.59	26.39	8.70	27.36
75	0.0001	0.0056	0.0072	0.0009	0.0030	-0.0009	-0.0005	0.0018	-6.79	6.59	12.96	12.84	-188.24	-170.42	-146.06	-173.64
76	0.0002	0.0043	0.0061	0.0013	0.0011	-0.0010	-0.0013	0.0028	-4.81	2.11	9.93	21.83	-183.26	-172.41	-144.13	-164.68
77	0.0002	0.0014	0.0034	0.0027	0.0010	-0.0012	0.0001	0.0037	-10.30	12.69	19.24	3.70	-81.36	-32.15	-30.20	68.78
78	0.0003	0.0033	0.0054	0.0020	-0.0005	-0.0014	-0.0015	0.0041	-6.39	2.35	9.23	19.58	-142.66	185.82	-41.52	-167.68
79	0.0003	0.0034	0.0053	0.0021	-0.0011	-0.0014	-0.0018	0.0043	-6.48	-0.31	4.39	26.15	-171.92	167.73	3.51	-154.20

Continued on next page

Table 41 – continued from previous page (11^{th} Harmonic)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
80	0.0003	0.0042	0.0060	0.0020	0.0003	-0.0009	-0.0018	0.0037	-3.87	-4.36	-0.30	34.80	142.86	157.61	105.17	-145.57
81	0.0004	0.0045	0.0064	0.0018	0.0009	-0.0010	-0.0014	0.0030	-6.59	-2.92	2.00	40.35	141.24	162.95	137.15	-140.95
82	0.0003	0.0043	0.0062	0.0021	0.0003	-0.0008	-0.0015	0.0035	-2.30	-6.05	2.32	37.00	126.53	157.58	98.76	-135.44
83	0.0003	0.0049	0.0068	0.0020	0.0013	-0.0007	-0.0015	0.0033	-1.66	-4.17	2.89	37.24	142.12	162.50	132.11	-140.64
84	0.0003	0.0047	0.0065	0.0019	0.0002	-0.0008	-0.0018	0.0037	-3.46	-2.16	0.49	33.88	150.70	169.75	138.21	-148.00
85	0.0003	0.0042	0.0061	0.0019	-0.0003	-0.0010	-0.0020	0.0037	-3.43	-2.82	2.51	30.46	-195.12	175.03	130.05	-148.72
86	0.0003	0.0045	0.0065	0.0019	0.0009	-0.0007	-0.0015	0.0034	-1.92	-4.73	3.12	35.32	142.52	167.72	131.79	-142.49
87	0.0001	0.0054	0.0074	0.0012	0.0028	-0.0005	-0.0008	0.0019	-0.91	-0.38	6.90	31.50	-196.22	174.78	-169.99	-152.89
88	0.00	0.0048	0.0067	0.0010	0.0025	-0.0010	-0.0006	0.0022	-4.99	6.09	18.32	3.53	-173.22	-165.10	-131.34	-176.76
89	0.00	0.0053	0.0072	0.0008	0.0029	-0.0007	-0.0004	0.0018	-5.03	7.78	17.19	1.28	-179.85	188.73	-141.07	-174.92
90	-0.0001	0.0053	0.0072	0.0011	0.0045	-0.0004	-0.0003	0.0021	-7.17	9.95	19.53	-1.17	-174.67	-163.12	-131.02	-182.29
91	0.0001	0.0051	0.0069	0.0009	0.0032	-0.0006	-0.0003	0.0017	-6.54	7.41	16.77	-6.11	-173.94	-167.47	-133.32	-180.83
92	0.0002	0.0046	0.0068	0.0013	0.0026	-0.0006	-0.0010	0.0020	-2.61	1.23	13.27	24.38	-193.48	189.57	-157.66	-151.59
93	0.0002	0.0040	0.0063	0.0018	0.0026	-0.0008	-0.0013	0.0030	-6.49	5.14	19.82	22.38	-170.62	-161.40	-117.96	-155.79
94	0.0002	0.0043	0.0066	0.0020	0.0023	-0.0007	-0.0016	0.0032	-3.50	1.51	15.42	26.89	-185.20	187.30	-166.72	-147.28
95	0.0004	0.0035	0.0058	0.0020	-0.0005	-0.0012	-0.0017	0.0039	-3.63	-3.07	11.57	28.06	135.79	172.80	70.34	-149.88
96	0.0002	0.0047	0.0067	0.0013	0.0018	-0.0005	-0.0011	0.0024	-2.62	-1.11	12.62	25.65	-195.02	190.78	-164.48	-157.19
97	0.0002	0.0043	0.0063	0.0013	0.0014	-0.0010	-0.0012	0.0025	-3.65	-0.27	13.16	21.23	-188.26	-167.50	-153.29	-160.93
98	0.0001	0.0053	0.0073	0.0009	0.0024	-0.0009	-0.0005	0.0017	-4.60	4.89	17.43	11.14	-178.15	185.32	-142.25	-173.49
99	0.0002	0.0045	0.0066	0.0012	0.0018	-0.0008	-0.0006	0.0028	-10.26	10.63	22.63	-1.59	-156.17	-165.33	-117.02	-171.66
100	0.0001	0.0049	0.0070	0.0010	0.0019	-0.0011	-0.0007	0.0023	-6.56	7.35	20.39	12.02	-169.55	187.86	-133.58	-162.54

Table 42: Variation of Error in Magnitude and Angle (13^{th} Harmonic) with Time (1^{st} to 100^{th})

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
1	0.0001	0.0032	0.0020	0.0015	0.0096	0.0029	0.0034	0.0055	-19.81	92.21	31.12	-49.34	-38.38	-0.23	-66.86	36.77
2	0.0001	0.0025	0.0011	0.0011	0.0088	0.0032	0.0026	0.0051	-11.14	90.96	28.15	-26.92	-14.35	-4.36	-46.91	63.90
3	0.0001	0.0022	0.0008	0.0010	0.0093	0.0039	0.0024	0.0048	-8.76	87.17	24.05	-25.51	-8.86	0.81	-45.72	62.99
4	0.0001	0.0025	0.0010	0.0011	0.0093	0.0033	0.0025	0.0051	-9.90	87.71	26.00	-30.02	-12.08	-5.32	-50.03	67.83
5	0.0001	0.0024	0.0010	0.0010	0.0093	0.0039	0.0025	0.0046	-9.08	87.59	26.93	-26.56	-11.91	-4.93	-49.71	72.92
6	0.0001	0.0021	0.0007	0.0014	0.0087	0.0033	0.0025	0.0049	-7.48	90.23	26.69	-29.65	-7.81	-4.44	-43.94	63.38
7	0.0001	0.0027	0.0013	0.0010	0.0094	0.0033	0.0025	0.0048	-12.63	86.80	27.35	-24.43	-22.04	-5.01	-47.71	60.92
8	0.00	0.0032	0.0018	0.0008	0.0091	0.0034	0.0024	0.0043	-18.68	86.16	28.49	-28.35	-24.29	-5.97	-55.45	55.25
9	-0.0002	0.0039	0.0028	0.0001	0.0046	0.0032	0.0012	0.0018	-18.04	47.97	3.41	-43.67	-23.50	-6.89	-73.97	55.46
10	-0.0001	0.0031	0.0020	0.0003	0.0048	0.0032	0.0007	0.0027	-4.31	37.38	-4.04	-26.05	-2.35	-2.78	-55.20	85.78
11	0.00	0.0020	0.0008	0.0012	0.0085	0.0033	0.0023	0.0047	-9.28	79.31	20.93	-30.91	-5.73	0.04	-41.97	60.57
12	0.00	0.0020	0.0008	0.0009	0.0080	0.0034	0.0020	0.0045	-5.79	71.95	15.35	-22.62	-10.01	0.82	-41.94	68.01
13	0.0001	0.0029	0.0013	0.0013	0.0108	0.0030	0.0030	0.0058	-15.03	102.21	38.76	-21.69	-23.60	-11.83	-42.94	69.37
14	0.0002	0.0035	0.0014	0.0015	0.0133	0.0034	0.0036	0.0067	-15.86	120.75	53.81	-21.80	-20.27	-11.54	-41.49	74.33
15	0.0001	0.0030	0.0013	0.0013	0.0112	0.0034	0.0031	0.0058	-15.57	106.10	41.96	-25.32	-21.18	-9.70	-41.96	60.21
16	0.00	0.0026	0.0011	0.0017	0.0081	0.0025	0.0028	0.0054	-12.26	94.84	30.37	-24.02	-21.26	-11.86	-41.95	64.77
17	0.00	0.0037	0.0022	0.0014	0.0083	0.0022	0.0027	0.0047	-22.89	88.26	30.60	-28.76	-30.59	-22.61	-53.55	60.17
18	-0.0001	0.0040	0.0028	0.0003	0.0079	0.0039	0.0018	0.0029	-22.84	68.12	17.11	-36.45	-34.68	-6.05	-65.67	45.71
19	0.0002	0.0035	0.0014	0.0014	0.0140	0.0038	0.0035	0.0066	-15.22	122.32	56.27	-24.84	-21.48	-7.30	-42.42	66.02
20	0.0003	0.0039	0.0014	0.0017	0.0154	0.0036	0.0042	0.0074	-14.05	134.05	68.72	-24.10	-21.28	-14.79	-38.94	71.35
21	0.0002	0.0040	0.0018	0.0016	0.0144	0.0039	0.0039	0.0067	-17.99	123.78	60.28	-26.94	-20.95	-15.24	-39.99	63.31
22	-0.0001	0.0034	0.0020	0.0013	0.0072	0.0025	0.0025	0.0044	-18.17	80.53	24.23	-36.48	-27.30	-13.91	-51.16	55.65

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Table 42 – continued from previous page (13^{th} Harmonic)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
23	0.00	0.0034	0.0018	0.0016	0.0097	0.0028	0.0033	0.0056	-18.44	102.34	40.09	-27.13	-33.02	-21.05	-45.85	63.51
24	0.00	0.0032	0.0015	0.0017	0.0090	0.0024	0.0031	0.0057	-18.58	101.16	37.44	-30.24	-26.25	-21.83	-46.60	63.07
25	0.00	0.0019	0.0004	0.0012	0.0088	0.0026	0.0024	0.0048	-9.04	101.10	28.16	-19.96	-13.54	-14.83	-36.64	71.58
26	0.00	0.0023	0.0007	0.0015	0.0094	0.0021	0.0026	0.0056	-14.48	104.99	34.30	-16.03	-21.06	-30.54	-36.81	71.99
27	0.0001	0.0026	0.0006	0.0012	0.0122	0.0030	0.0030	0.0059	-14.01	119.33	48.96	-15.89	-21.85	-23.43	-36.68	60.90
28	0.0002	0.0027	0.0001	0.0018	0.0144	0.0030	0.0038	0.0070	-10.51	146.58	72.65	-11.96	-14.68	-25.78	-27.85	63.95
29	0.0001	0.0019	0.00	0.0014	0.0112	0.0024	0.0030	0.0060	-11.60	122.98	43.43	-12.99	-17.77	-26.10	-30.43	55.79
30	0.00	0.0018	0.0006	0.0013	0.0085	0.0032	0.0022	0.0040	-9.26	78.75	18.02	-15.27	-15.03	-10.51	-37.62	45.40
31	0.00	0.0022	0.0011	0.0015	0.0074	0.0035	0.0019	0.0034	-9.82	68.23	14.16	-18.25	-18.24	-9.06	-42.80	52.41
32	0.0001	0.0008	-0.0003	0.0013	0.0087	0.0026	0.0023	0.0051	-1.88	86.98	10.78	-15.58	-2.06	-9.00	-28.24	55.09
33	0.00	0.0010	0.00	0.0011	0.0086	0.0023	0.0020	0.0050	-3.66	64.82	4.75	-16.12	-3.13	-9.09	-29.83	39.49
34	0.00	0.0012	0.0003	0.0010	0.0079	0.0036	0.0017	0.0035	-0.35	34.29	-6.15	-25.34	4.65	-1.98	-29.84	34.04
35	0.0001	0.0013	-0.0003	0.0024	0.0095	0.0032	0.0031	0.0056	-3.38	110.95	30.24	-15.46	-8.39	-6.52	-30.17	26.83
36	0.0001	0.0014	-0.0002	0.0020	0.0103	0.0030	0.0032	0.0059	-7.15	112.34	32.25	-23.76	-4.79	-6.34	-30.79	32.71
37	0.00	0.0008	-0.0005	0.0014	0.0101	0.0028	0.0027	0.0051	-3.69	111.11	16.44	-23.23	-0.64	-10.53	-26.74	32.55
38	0.00	0.0007	-0.0004	0.0012	0.0096	0.0032	0.0023	0.0049	-1.58	94.10	11.35	-14.41	0.04	-3.86	-25.34	39.36
39	0.0001	0.0011	-0.0003	0.0015	0.0093	0.0030	0.0026	0.0049	-2.04	110.35	25.96	-13.16	-3.71	-10.06	-28.11	52.13
40	0.0001	0.0006	-0.0006	0.0013	0.0096	0.0033	0.0024	0.0047	-1.05	108.81	11.54	-16.60	4.67	-0.72	-25.22	40.06
41	0.00	0.0009	0.00	0.0009	0.0084	0.0033	0.0019	0.0041	-1.39	54.89	1.59	-19.31	0.89	-0.25	-30.15	51.76
42	0.0001	0.0010	-0.0004	0.0014	0.0103	0.0030	0.0027	0.0055	-3.88	110.92	21.53	-17.62	0.55	-9.74	-27.55	60.96
43	0.0001	0.0005	-0.0007	0.0014	0.01	0.0026	0.0025	0.0057	-1.89	102.63	6.64	-17.21	4.16	-10.36	-21.76	54.48
44	0.0001	0.0008	-0.0005	0.0014	0.0102	0.0030	0.0026	0.0053	-2.44	110.55	15.87	-16.64	-0.28	-5.76	-26.04	44.11
45	0.0001	0.0008	-0.0010	0.0015	0.0109	0.0033	0.0028	0.0054	-1.40	153.14	19.61	-17.13	4.03	-6.57	-23.42	39.88
46	0.00	0.0004	-0.0005	0.0011	0.0093	0.0035	0.0021	0.0044	0.20	23.18	-9.61	-24.34	8.68	-0.38	-24.25	41.68
47	0.0001	0.0009	-0.0006	0.0017	0.0104	0.0028	0.0032	0.0059	-4.83	119.89	24.90	-24.97	-0.39	-5.30	-27.98	39.46
48	0.0002	0.0013	-0.0012	0.0023	0.0122	0.0023	0.0041	0.0076	-4.23	163.48	62.49	-18.21	-1.93	-20.35	-20.93	47.55
49	0.0002	0.0022	-0.0010	0.0026	0.0127	0.0024	0.0044	0.0080	-4.91	168.41	87.97	-16.93	-3.98	-22.46	-20.66	45.87

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Table 42 – continued from previous page (13^{th} Harmonic)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
50	0.0003	0.0021	-0.0012	0.0025	0.0130	0.0022	0.0043	0.0082	-4.18	173.82	91.55	-17.04	2.78	-22.04	-20.08	56.23
51	0.0001	0.0011	-0.0013	0.0019	0.0113	0.0023	0.0033	0.0064	-5.64	170.84	39.03	-22.71	6.47	-22.50	-21.51	45.75
52	0.0003	0.0025	-0.0010	0.0025	0.0134	0.0022	0.0044	0.0081	-2.23	-176.04	116.32	-14.75	4.58	-23.82	-16.80	47.82
53	0.0003	0.0024	-0.0008	0.0025	0.0138	0.0022	0.0044	0.0083	-4.56	176.00	102.64	-13.14	2.36	-27.30	-20.21	54.00
54	0.0001	0.0011	-0.0011	0.0018	0.0116	0.0020	0.0035	0.0066	-7.18	158.17	38.82	-22.64	2.20	-23.03	-26.27	50.86
55	0.0002	0.0011	-0.0010	0.0019	0.0114	0.0021	0.0034	0.0066	-6.45	155.77	39.52	-19.53	0.63	-22.04	-26.32	50.17
56	0.0003	0.0026	-0.0008	0.0025	0.0141	0.0021	0.0046	0.0084	-2.63	-179.49	112.60	-12.48	0.06	-23.23	-21.94	59.41
57	0.0003	0.0026	-0.0007	0.0025	0.0140	0.0023	0.0046	0.0083	-3.29	178.04	109.16	-14.29	0.09	-24.38	-22.88	62.11
58	0.0003	0.0025	-0.0008	0.0024	0.0141	0.0023	0.0046	0.0082	-3.92	178.17	107.67	-16.17	-0.27	-21.81	-22.30	57.93
59	0.0003	0.0023	-0.0012	0.0023	0.0135	0.0024	0.0044	0.0079	-3.53	-177.61	109.59	-16.97	2.20	-23.12	-22.51	52.71
60	0.0002	0.0017	-0.0008	0.0020	0.0126	0.0033	0.0040	0.0067	-6.01	154.71	62.84	-26.87	-1.13	-7.32	-31.24	34.21
61	0.0002	0.0016	-0.0018	0.0024	0.0123	0.0025	0.0042	0.0075	-1.75	-171.97	75.27	-22.18	7.79	-13.45	-22.31	40.35
62	0.0001	0.0010	-0.0012	0.0023	0.0104	0.0019	0.0039	0.0071	-7.15	158.09	31.47	-18.35	-1.63	-25.25	-22.65	43.93
63	0.0002	0.0018	-0.0012	0.0024	0.0124	0.0025	0.0042	0.0076	-5.32	168.53	72.83	-20.12	-3.12	-21.52	-20.12	40.08
64	0.0003	0.0023	-0.0012	0.0024	0.0139	0.0029	0.0044	0.0078	-2.67	-177.26	108.88	-17.76	1.32	-19.76	-17.46	37.97
65	0.0001	0.0008	-0.0015	0.0014	0.0114	0.0025	0.0030	0.0058	-3.28	-169.99	12.71	-18.32	5.16	-23.41	-13.83	49.30
66	0.0002	0.0015	-0.0010	0.0018	0.0123	0.0024	0.0037	0.0067	-5.39	162.20	60.30	-22.06	-1.52	-20.56	-19.74	51.62
67	0.0001	0.0015	-0.0004	0.0015	0.0116	0.0026	0.0033	0.0060	-9.02	130.58	41.70	-27.66	-4.34	-19.43	-26.19	57.13
68	0.0001	0.0004	-0.0012	0.0013	0.0110	0.0026	0.0028	0.0061	0.76	-155.45	-6.31	-23.68	5.27	-15.09	-13.86	63.59
69	0.00	0.0004	-0.0006	0.0014	0.0097	0.0027	0.0027	0.0051	-3.78	98.24	4.49	-25.85	0.19	-17.52	-21.30	59.34
70	0.0003	0.0028	0.0001	0.0021	0.0139	0.0029	0.0043	0.0076	-12.23	149.48	77.31	-23.79	-11.42	-21.44	-25.71	53.03
71	0.0001	0.0015	-0.0004	0.0018	0.0110	0.0023	0.0034	0.0063	-11.82	135.65	45.31	-23.18	-4.43	-22.19	-23.54	54.09
72	0.0001	0.0006	-0.0013	0.0016	0.0101	0.0019	0.0028	0.0062	-0.33	-172.38	9.76	-13.97	6.31	-25.22	-11.21	76.28
73	0.00	0.0001	-0.0009	0.0010	0.0096	0.0016	0.0021	0.0062	1.68	-157.86	-3.16	-16.22	8.17	-11.68	-10.39	93.94
74	0.00	0.0013	0.0003	0.0006	0.0089	0.0025	0.0018	0.0048	-4.37	73.13	14.39	-23.60	1.79	-3.57	-27.67	94.28
75	0.00	0.0005	-0.0005	0.0010	0.0093	0.0018	0.0023	0.0056	-1.67	97.70	10.21	-13.01	5.75	-19.35	-16.46	86.22
76	0.00	0.0009	-0.0002	0.0009	0.0098	0.0024	0.0022	0.0053	-4.35	95.20	18.79	-18.44	3.73	-12.18	-21.00	82.79

Continued on next page

Table 42 – continued from previous page (13^{th} Harmonic)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
77	0.0001	0.0020	0.0005	0.0012	0.0104	0.0025	0.0028	0.0058	-10.32	107.84	40.45	-29.27	-5.52	-8.38	-32.64	80.33
78	0.0002	0.0023	0.00	0.0018	0.0119	0.0025	0.0036	0.0072	-7.19	145.28	74.73	-19.09	-4.70	-17.66	-24.26	90.10
79	0.0002	0.0022	0.0003	0.0017	0.0118	0.0032	0.0035	0.0065	-9.43	134.25	66.85	-27.42	-3.61	-8.02	-25.96	67.42
80	0.0002	0.0015	-0.0002	0.0016	0.0112	0.0038	0.0031	0.0060	-7.02	126.75	47.78	-26.95	-2.95	1.25	-26.84	56.59
81	0.0002	0.0020	-0.0005	0.0023	0.0121	0.0028	0.0041	0.0073	-6.89	153.11	75.71	-20.46	-6.69	-12.43	-19.49	52.94
82	0.0002	0.0014	-0.0010	0.0022	0.0117	0.0029	0.0039	0.0071	-3.73	159.00	62.22	-25.79	0.14	-12.41	-16.76	43.10
83	0.0003	0.0023	-0.0005	0.0024	0.0131	0.0030	0.0044	0.0078	-6.98	165.43	97.89	-21.29	-2.47	-15.87	-15.90	43.74
84	0.0003	0.0022	-0.0011	0.0023	0.0139	0.0030	0.0044	0.0080	-3.37	-179.17	115.08	-20.93	4.51	-17.55	-12.00	58.79
85	0.0003	0.0020	-0.0013	0.0022	0.0135	0.0033	0.0042	0.0076	-0.37	-177.86	109.64	-21.94	3.05	-10.44	-12.83	50.63
86	0.0001	0.0007	-0.0008	0.0015	0.0108	0.0028	0.0032	0.0058	-4.69	132.68	21.91	-24.31	1.47	-14.73	-16.68	28.45
87	0.0001	0.0003	-0.0010	0.0015	0.0105	0.0030	0.0029	0.0058	2.45	-132.90	-8.59	-14.93	7.34	-7.75	-10.53	37.39
88	0.0001	0.0003	-0.0014	0.0018	0.0114	0.0030	0.0031	0.0064	4.39	-127.43	-23.51	-11.83	6.41	-12.49	-4.65	53.60
89	0.00	0.0008	0.00	0.0012	0.0093	0.0033	0.0022	0.0050	0.28	24.81	-6.89	-15.78	0.47	-6.24	-14.50	42.27
90	0.0001	0.0003	-0.0010	0.0014	0.0113	0.0031	0.0028	0.0060	1.30	171.04	1.58	-11.99	0.11	-9.17	-13.17	48.72
91	0.0001	0.0006	-0.0006	0.0016	0.0112	0.0032	0.0030	0.0057	-3.24	110.76	11.69	-18.89	-0.72	-15.33	-20.72	45.40
92	0.0003	0.0019	-0.0018	0.0024	0.0136	0.0028	0.0043	0.0082	4.08	-164.81	122.66	-13.91	4.09	-15.59	-12.87	38.90
93	0.0002	0.0013	-0.0012	0.0025	0.0128	0.0026	0.0043	0.0076	-1.28	167.85	68.43	-11.91	-6.39	-22.34	-15.48	34.98
94	0.0002	0.0020	-0.0012	0.0024	0.0133	0.0025	0.0043	0.0083	-2.71	171.00	83.10	-11.09	-7.51	-19.66	-14.96	35.23
95	0.0003	0.0020	-0.0010	0.0025	0.0133	0.0028	0.0043	0.0081	-3.68	165.85	82.51	-15.27	-3.93	-21.00	-17.22	35.23
96	0.0002	0.0017	-0.0008	0.0020	0.0130	0.0029	0.0038	0.0071	-6.54	153.15	60.09	-19.05	-3.43	-21.60	-20.68	44.98
97	0.0002	0.0010	-0.0018	0.0015	0.0124	0.0027	0.0032	0.0069	0.74	-154.34	-0.53	-11.27	4.01	-23.38	-11.49	67.35
98	0.0001	0.0008	-0.0014	0.0015	0.0121	0.0027	0.0030	0.0069	0.65	-175.79	10.83	-10.45	0.05	-20.71	-12.95	66.59
99	0.00	0.0005	-0.0004	0.0015	0.0101	0.0023	0.0028	0.0064	-2.93	78.10	4.95	-11.12	-3.02	-22.58	-14.37	47.85
100	0.0001	0.0007	-0.0001	0.0015	0.0097	0.0021	0.0027	0.0059	-3.44	51.98	0.77	-19.83	0.36	-25.65	-19.33	51.45

Table 43: Variation of Error in Magnitude and Angle (15^{th} Harmonic) with Time (1^{st} to 100^{th})

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
	Magnitude of the Errors (pu)								Error in the Angles (degrees)								
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
1	0.00	0.0021	0.0008	0.0013	0.0045	0.0001	0.0011	0.0029	31.72	156.33	328.92	-22.50	46.46	132.70	142.50	75.41
2	0.00	0.0018	0.0004	0.0016	0.0038	0.0003	0.0010	0.0031	17.73	-162.39	-4.98	44.12	101.03	134.65	193.46	129.64
3	0.00	0.0016	0.0001	0.0016	0.0037	0.0008	0.0010	0.0029	19.90	-144.53	8.44	53.27	113.03	125.89	206.71	130.73
4	0.00	0.0018	0.0003	0.0015	0.0038	0.0005	0.0010	0.0029	19.66	-154.38	0.62	47.98	110.42	121.92	201.04	135.29
5	-0.0001	0.0020	0.0006	0.0015	0.0042	0.0003	0.0012	0.0032	27.64	-167.20	-11.04	39.17	101.49	131.91	193.38	146.45
6	0.00	0.0016	0.0002	0.0014	0.0035	0.0004	0.0009	0.0026	33.55	-159.91	-5.94	47.30	105.46	130.57	199.30	139.79
7	-0.0001	0.0011	-0.0004	0.0012	0.0029	0.0008	0.0005	0.0019	31.27	-150.16	-1.59	59.83	112.94	123.39	205.63	138.19
8	0.00	0.0019	0.0004	0.0015	0.0038	0.0004	0.0011	0.0026	36.37	-164.20	-8.86	44.43	105.33	138.77	197.86	122.90
9	0.00	0.0010	-0.0006	0.0012	0.0026	0.0008	0.0004	0.0020	36.24	-127.65	11.42	73.19	128.51	122.58	220.52	155.91
10	0.0001	0.0017	-0.0009	0.0012	0.0011	0.0009	-0.0003	0.0017	55.33	-10.52	147.92	124.81	192.65	130.98	288.09	240.09
11	0.00	0.0008	-0.0008	0.0012	0.0027	0.0009	0.0004	0.0022	37.27	-148.44	-6.64	60.60	111.99	129.89	213.17	155.24
12	0.00	0.0011	-0.0005	0.0014	0.0030	0.0011	0.0006	0.0022	29.44	-132.13	9.70	62.98	115.05	119.08	219.27	140.80
13	0.00	0.0007	-0.0008	0.0011	0.0025	0.0009	0.0002	0.0015	44.27	-152.85	-6.72	59.20	107.72	135.86	209.91	146.52
14	-0.0001	0.0007	-0.0008	0.0010	0.0025	0.0009	0.0002	0.0015	41.87	-154.19	-7.61	59.63	110.69	139.20	209.14	150.11
15	0.00	0.0006	-0.0011	0.0011	0.0025	0.0010	0.0002	0.0016	43.66	-121.06	8.60	71.13	126.97	140.06	219.51	148.23
16	0.00	0.0008	-0.0008	0.0012	0.0028	0.0009	0.0003	0.0021	41.07	-147.21	-3.58	63.63	116.22	128.10	213.64	146.64
17	0.00	0.0008	-0.0009	0.0011	0.0023	0.0010	0.0002	0.0020	37.04	-120.55	12.25	72.81	124.30	134.22	223.46	167.42
18	0.00	0.0005	-0.0015	0.0010	0.0015	0.0008	-0.0003	0.0016	45.15	-58.32	34.67	88.45	132.41	152.69	237.08	169.18
19	0.00	0.0009	-0.0013	0.0011	0.0016	0.0009	-0.0002	0.0016	36.58	-58.96	62.25	87.78	144.77	151.36	246.54	179.12
20	0.00	0.0006	-0.0010	0.0012	0.0021	0.0008	0.00	0.0015	37.76	-139.14	-1.52	56.69	108.62	138.51	214.78	149.37
21	0.0001	0.0010	-0.0014	0.0011	0.0015	0.0008	-0.0002	0.0015	43.13	-42.69	95.31	87.74	153.56	143.81	250.81	178.09
22	0.00	0.0011	-0.0003	0.0012	0.0027	0.0002	0.0004	0.0022	28.64	-174.62	-17.18	40.26	101.89	172.15	194.08	133.39

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Table 43 – continued from previous page (15^{th} Harmonics)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
23	0.00	0.0010	-0.0005	0.0011	0.0024	0.0004	0.0001	0.0021	36.32	-170.56	-15.30	44.29	104.99	170.20	198.14	149.92
24	0.00	0.0020	0.0005	0.0013	0.0033	-0.0002	0.0007	0.0024	35.02	173.95	336.41	24.68	90.83	191.57	182.37	123.85
25	0.0001	0.0011	-0.0013	0.0006	0.00	0.0005	-0.0010	0.0010	53.94	13.70	196.90	118.10	191.56	-162.46	285.99	203.86
26	0.00	0.0017	0.0002	0.0011	0.0027	-0.0003	0.0004	0.0020	39.04	163.54	327.95	23.79	89.97	208.38	174.78	117.72
27	-0.0001	0.0023	0.0009	0.0013	0.0033	-0.0004	0.0008	0.0021	31.41	156.00	320.69	13.55	78.16	-107.42	164.55	108.55
28	0.00	0.0014	-0.0001	0.0012	0.0025	0.0002	0.0004	0.0018	23.69	-171.56	-15.91	53.37	111.22	152.32	201.86	132.52
29	0.00	0.0012	-0.0002	0.0009	0.0020	-0.0004	0.00	0.0009	35.16	169.11	329.77	46.68	102.19	167.80	188.97	116.75
30	0.00	0.0016	0.0001	0.0014	0.0031	0.0002	0.0007	0.0023	17.88	-152.35	1.76	51.05	114.95	121.74	206.03	130.97
31	0.00	0.0018	0.0003	0.0013	0.0033	-0.0001	0.0008	0.0023	6.95	179.12	338.88	34.90	96.97	-159.53	184.01	134.37
32	-0.0001	0.0025	0.0011	0.0015	0.0040	-0.0005	0.0011	0.0025	5.86	175.28	336.46	26.14	91.89	161.70	177.13	113.80
33	0.00	0.0018	0.0003	0.0015	0.0035	0.0003	0.0009	0.0026	5.38	-159.56	-5.00	47.76	111.94	130.73	197.76	128.91
34	0.00	0.0012	-0.0004	0.0012	0.0024	0.0008	0.0003	0.0019	14.53	-133.61	12.34	66.28	126.01	126.85	214.87	137.19
35	0.0001	0.0006	-0.0017	0.0008	0.0005	0.0003	-0.0007	0.0012	48.75	-9.53	179.91	106.48	156.20	142.23	254.50	185.77
36	0.00	0.0002	-0.0012	0.0008	0.0012	0.0002	-0.0003	0.0012	39.22	-112.15	341.85	79.45	124.20	172.38	217.87	163.96
37	0.0001	0.0004	-0.0012	0.0010	0.0016	0.0004	-0.0001	0.0016	30.48	-99.27	5.75	82.73	135.34	123.68	228.10	157.66
38	0.0001	0.0006	-0.0010	0.0005	0.0005	0.0002	-0.0008	0.0001	65.09	75.99	275.87	86.12	93.10	134.85	197.15	178.49
39	0.0001	0.0009	-0.0006	0.0003	0.0006	0.0002	-0.0009	-0.0001	67.84	79.83	267.91	88.43	64.94	144.50	162.48	189.28
40	0.0001	0.0007	-0.0010	0.0005	0.00	0.0008	-0.0011	0.0003	68.81	56.73	261.25	110.43	124.55	129.38	233.65	203.26
41	0.0001	0.0011	-0.0008	0.0005	-0.0002	0.0004	-0.0013	0.0004	-294.14	53.51	242.91	130.74	9.89	134.41	324.50	223.98
42	0.0001	0.0011	-0.0008	0.0007	-0.0001	0.0007	-0.0010	0.0008	65.49	36.49	225.57	129.78	235.25	124.74	290.57	238.33
43	0.0001	0.0011	-0.0006	0.0005	0.0001	0.0003	-0.0012	0.0004	54.67	61.91	252.05	123.10	89.29	89.53	180.80	214.55
44	0.00	0.0009	-0.0008	0.0013	0.0023	0.0007	0.0003	0.0019	25.24	-109.43	22.73	78.08	140.46	107.83	227.26	144.60
45	0.00	0.0011	-0.0003	0.0012	0.0027	0.0006	0.0004	0.0019	19.40	-161.69	-9.99	50.54	111.59	113.84	200.16	116.81
46	0.00	0.0010	-0.0004	0.0011	0.0023	0.0006	0.0003	0.0017	33.14	-160.52	-9.57	53.01	110.55	121.71	202.16	116.32
47	0.00	0.0001	-0.0013	0.0009	0.0012	0.0002	-0.0003	0.0010	42.11	-121.92	339.37	80.60	119.94	140.05	220.62	167.75
48	0.0001	0.0005	-0.0016	0.0010	0.0009	0.0006	-0.0003	0.0015	43.68	-40.94	44.75	105.92	162.97	107.87	256.71	203.41
49	0.0001	0.0005	-0.0017	0.0009	0.0008	0.0005	-0.0004	0.0013	46.19	-30.84	31.67	105.58	159.07	121.28	257.05	201.28

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Table 43 – continued from previous page (15^{th} Harmonics)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
50	0.0001	0.0002	-0.0013	0.0007	0.0005	0.0003	-0.0006	0.0005	68.58	64.73	300.31	94.60	136.47	135.40	228.76	186.10
51	-0.0001	0.0020	0.0006	0.0013	0.0032	0.0002	0.0009	0.0022	23.86	-175.90	340.96	39.19	100.50	138.97	190.30	99.97
52	0.00	0.0006	-0.0015	0.0010	0.0013	0.0007	-0.0001	0.0013	55.70	-57.50	38.80	95.39	158.07	122.06	247.45	176.71
53	0.00	0.0014	0.00	0.0006	0.0015	-0.0005	-0.0003	0.00	64.68	133.86	303.94	39.05	82.89	144.74	166.53	122.14
54	0.00	0.0010	-0.0003	0.0010	0.0020	-0.0002	0.00	0.0008	51.99	164.34	324.39	57.76	110.56	116.71	196.75	136.91
55	0.00	0.0008	-0.0005	0.0010	0.0016	-0.0002	0.00	0.0010	56.76	174.10	329.48	61.62	107.26	126.54	202.66	141.14
56	0.00	0.0018	0.0004	0.0004	0.0016	-0.0005	-0.0003	-0.0003	-286.86	113.39	285.26	15.94	54.68	153.73	138.75	72.20
57	0.00	0.0011	-0.0003	0.0006	0.0011	-0.0002	-0.0005	0.00	71.62	116.41	289.69	56.90	86.12	114.17	174.86	158.27
58	0.0001	0.0011	-0.0005	0.0005	0.0004	0.00	-0.0009	0.00	78.06	82.21	265.56	79.22	80.62	116.18	185.17	191.01
59	0.0001	0.0004	-0.0012	0.0007	0.0009	0.00	-0.0006	0.0007	66.96	75.50	284.91	79.64	125.33	110.24	214.61	170.16
60	-0.0001	0.0016	0.0003	0.0015	0.0035	0.0005	0.0009	0.0026	31.84	-166.58	-11.30	44.02	107.38	129.69	195.75	103.03
61	-0.0001	0.0021	0.0007	0.0016	0.0039	0.0005	0.0012	0.0028	29.66	-169.95	-11.94	40.99	109.45	130.90	194.14	94.45
62	0.00	0.0016	0.0003	0.0011	0.0028	0.00	0.0005	0.0018	40.73	163.81	325.08	38.23	99.25	170.93	183.52	79.61
63	-0.0001	0.0025	0.0010	0.0010	0.0031	-0.0004	0.0005	0.0018	45.96	138.07	304.71	9.71	74.61	194.08	158.12	62.57
64	-0.0001	0.0023	0.0009	0.0015	0.0036	0.00	0.0009	0.0025	35.05	165.50	326.00	27.19	91.46	168.40	178.11	80.19
65	0.0001	0.0029	0.0009	0.0008	0.0020	-0.0006	-0.0001	0.0018	-279.10	59.12	229.93	-146.14	-35.55	236.51	54.21	-71.84
66	0.00	0.0021	0.0005	0.0004	0.0020	-0.0010	-0.0002	0.0002	65.66	90.96	265.17	-66.01	12.73	252.97	108.67	-14.16
67	-0.0001	0.0024	0.0012	0.0014	0.0036	-0.0002	0.0008	0.0023	25.37	165.35	327.45	21.09	84.02	176.87	176.54	103.47
68	-0.0001	0.0023	0.0010	0.0013	0.0035	-0.0002	0.0008	0.0021	28.45	162.25	324.52	17.18	76.30	174.14	173.41	70.02
69	-0.0001	0.0021	0.0008	0.0015	0.0035	0.00	0.0008	0.0026	19.77	-179.92	338.79	30.44	91.07	162.05	186.34	110.79
70	0.00	0.0011	-0.0003	0.0013	0.0025	0.0004	0.0003	0.0019	34.49	-142.87	2.18	58.34	116.31	140.81	215.02	144.77
71	0.00	0.0009	-0.0004	0.0011	0.0021	0.0004	0.0001	0.0015	32.20	-159.74	-10.92	55.82	112.24	142.47	208.23	145.08
72	0.00	0.0025	0.0008	0.0005	0.0021	-0.0012	-0.0002	0.0005	59.70	81.03	256.59	-80.92	17.67	283.40	97.31	-11.33
73	0.00	0.0021	0.0002	0.0004	0.0005	0.00	-0.0010	0.0007	50.04	43.25	220.17	168.19	-29.19	264.42	29.48	-2.60
74	0.00	0.0011	-0.0003	0.0013	0.0030	0.0006	0.0004	0.0025	21.22	-150.51	-3.71	46.26	110.18	174.92	205.96	137.15
75	-0.0001	0.0013	0.00	0.0011	0.0030	0.0002	0.0004	0.0026	26.07	179.38	338.50	29.78	98.75	190.59	188.47	111.71
76	-0.0001	0.0019	0.0004	0.0009	0.0030	-0.0004	0.0003	0.0021	46.20	116.27	289.44	-24.27	57.88	244.44	140.43	66.88

Continued on next page

Table 43 – continued from previous page (15^{th} Harmonics)

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
77	-0.0001	0.0025	0.0009	0.0009	0.0031	-0.0007	0.0003	0.0017	64.58	94.58	268.18	-51.82	38.11	274.00	114.57	55.60
78	-0.0001	0.0024	0.0010	0.0012	0.0039	-0.0003	0.0007	0.0025	54.31	123.42	297.19	-23.86	56.38	-103.87	139.13	93.04
79	-0.0001	0.0026	0.0013	0.0012	0.0039	-0.0009	0.0008	0.0018	45.66	123.10	297.77	-29.41	59.01	257.52	137.63	57.82
80	-0.0001	0.0026	0.0013	0.0015	0.0042	-0.0005	0.0012	0.0025	23.38	164.47	331.64	12.21	84.93	198.77	173.42	91.67
81	-0.0001	0.0013	-0.0002	0.0013	0.0028	0.0007	0.0004	0.0018	18.88	-154.31	-0.57	51.41	116.22	122.79	205.01	130.85
82	0.00	0.0013	-0.0001	0.0014	0.0029	0.0008	0.0005	0.0020	14.32	-151.36	2.21	55.74	123.05	119.42	207.74	117.36
83	-0.0001	0.0012	-0.0003	0.0013	0.0026	0.0010	0.0004	0.0019	19.86	-139.56	7.52	62.24	126.86	130.67	215.11	123.79
84	-0.0001	0.0023	0.0010	0.0015	0.0038	-0.0002	0.0010	0.0021	14.17	177.20	341.91	31.57	102.61	154.87	187.24	114.87
85	-0.0001	0.0021	0.0008	0.0017	0.0040	0.0002	0.0010	0.0025	9.69	-174.01	348.86	42.55	111.10	128.22	197.48	116.34
86	0.00	0.0010	-0.0005	0.0015	0.0029	0.0011	0.0005	0.0020	18.86	-135.97	8.65	66.63	133.43	112.70	220.34	121.58
87	0.00	0.0009	-0.0007	0.0014	0.0025	0.0011	0.0004	0.0019	21.00	-120.79	18.23	68.93	136.92	122.03	224.15	125.30
88	-0.0001	0.0013	-0.0001	0.0014	0.0029	0.0005	0.0005	0.0021	8.80	-160.66	-8.23	54.28	119.45	136.95	206.59	133.39
89	0.00	0.0008	-0.0005	0.0011	0.0020	0.0005	0.00	0.0014	17.45	-176.21	335.21	57.87	115.27	145.83	203.90	134.00
90	0.00	0.0006	-0.0006	0.0012	0.0022	0.0004	0.0001	0.0015	19.67	-167.32	336.62	63.03	121.31	127.71	206.53	142.66
91	0.00	0.0011	-0.0003	0.0015	0.0029	0.0006	0.0005	0.0020	7.53	-153.07	-7.25	55.07	121.76	114.67	204.16	134.11
92	-0.0001	0.0015	0.0002	0.0015	0.0034	0.0006	0.0007	0.0022	-12.54	-161.11	-8.17	45.64	115.44	113.33	195.79	105.50
93	-0.0001	0.0015	0.0003	0.0013	0.0033	0.0003	0.0006	0.0019	-5.90	-177.91	340.55	37.24	104.15	136.69	186.06	113.18
94	-0.0001	0.0015	0.0003	0.0014	0.0031	0.0003	0.0006	0.0018	-5.73	178.60	336.86	37.78	100.16	133.80	185.80	111.13
95	0.00	0.0011	-0.0002	0.0013	0.0025	0.0007	0.0004	0.0018	11.14	-154.68	-7.02	53.01	118.53	141.87	201.64	115.95
96	0.00	0.0013	-0.0002	0.0014	0.0028	0.0009	0.0005	0.0021	11.52	-132.36	11.54	59.49	128.37	125.87	211.00	132.18
97	-0.0001	0.0008	-0.0004	0.0011	0.0021	0.0005	0.00	0.0015	20.21	-179.59	333.90	47.27	102.89	178.93	193.34	137.42
98	-0.0001	0.0007	-0.0005	0.0011	0.0019	0.0006	0.00	0.0015	24.61	-157.52	-14.41	56.87	106.88	154.19	203.29	134.52
99	-0.0001	0.0007	-0.0008	0.0012	0.0018	0.0010	0.0001	0.0018	24.04	-120.10	7.70	66.48	123.42	149.62	214.02	120.19
100	-0.0001	0.0022	0.0010	0.0015	0.0038	-0.0004	0.0010	0.0024	7.41	177.51	337.65	26.80	93.59	196.19	176.42	93.76

Table 44: Variation of Error in Magnitude and Angle (17^{th} Harmonic) with Time (1^{st} to 100^{th})

	Magnitude of the Errors (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
3	0.0001	0.0086	0.0103	0.0032	0.0171	0.0053	0.0056	0.0050	17.46	-33.13	-11.53	98.57	198.28	100.93	145.20	84.41
4	0.0002	0.0086	0.0110	0.0027	0.0154	0.0048	0.0051	0.0041	13.53	-32.50	-11.30	92.05	202.26	95.73	144.43	73.71
5	0.0002	0.0081	0.0106	0.0024	0.0149	0.0043	0.0047	0.0045	11.34	-29.02	-9.31	95.51	212.22	98.81	146.05	71.01
6	0.0002	0.0086	0.0111	0.0025	0.0151	0.0045	0.0048	0.0042	13.56	-32.26	-10.94	92.73	202.57	98.27	144.38	72.69
7	0.0001	0.0086	0.0105	0.0031	0.0165	0.0051	0.0056	0.0049	15.24	-30.05	-7.82	102.93	203.34	103.99	148.18	87.58
8	0.0002	0.0085	0.0105	0.0030	0.0164	0.0050	0.0053	0.0049	15.75	-31.84	-9.52	96.01	203.36	99.60	145.83	93.18
9	0.0002	0.0084	0.0108	0.0025	0.0149	0.0043	0.0048	0.0044	12.92	-30.25	-8.83	99.14	206.65	100.03	145.29	83.09
10	0.0002	0.0081	0.0109	0.0020	0.0139	0.0038	0.0042	0.0040	9.74	-30.15	-9.01	94.08	199.03	99.93	146.98	79.31
11	0.0002	0.0082	0.0106	0.0025	0.0146	0.0043	0.0045	0.0042	13.59	-32.72	-8.15	87.97	192.32	94.41	138.47	74.45
12	0.0002	0.0085	0.0107	0.0027	0.0151	0.0047	0.0048	0.0042	13.36	-31.80	-10.00	95.58	209.94	95.45	136.67	87.63
13	0.0001	0.0083	0.0102	0.0029	0.0160	0.0050	0.0051	0.0049	15.06	-30.47	-9.09	98.21	210.91	101.19	140.37	91.48
14	0.0001	0.0083	0.0101	0.0031	0.0167	0.0051	0.0053	0.0054	15.41	-31.24	-11.09	102.04	198.85	104.61	143.76	89.47
15	0.0002	0.0083	0.0103	0.0029	0.0161	0.0048	0.0051	0.0051	16.23	-30.74	-6.44	97.88	202.57	102.57	142.69	96.57
16	0.0002	0.0085	0.0110	0.0024	0.0150	0.0044	0.0048	0.0042	12.17	-31.41	-9.36	94.68	214.10	98.74	142.83	74.40
17	0.0002	0.0081	0.0109	0.0022	0.0142	0.0040	0.0044	0.0042	9.17	-29.63	-9.59	93.83	214.56	98.71	142.64	68.23
18	0.0002	0.0085	0.0107	0.0028	0.0157	0.0047	0.0051	0.0047	13.98	-29.45	-6.61	99.47	209.73	104.40	143.71	85.66
19	0.0001	0.0086	0.0106	0.0032	0.0165	0.0052	0.0055	0.0051	16.14	-30.99	-9.31	99.69	212.73	103.53	144.47	93.57
20	0.0001	0.0086	0.0105	0.0034	0.0166	0.0052	0.0055	0.0054	16.65	-30.65	-11.16	104.01	217.84	102.70	144.53	94.00
21	0.0001	0.0087	0.0106	0.0033	0.0165	0.0052	0.0055	0.0052	17.76	-31.41	-11.20	99.58	206.54	102.91	146.86	93.45
22	0.0001	0.0085	0.0107	0.0028	0.0152	0.0045	0.0048	0.0048	15.49	-30.68	-9.44	96.52	207.28	100.93	146.28	75.46
23	0.0002	0.0087	0.0115	0.0025	0.0145	0.0042	0.0046	0.0044	10.74	-30.29	-9.39	100.71	219.74	101.51	149.58	63.24
24	0.0002	0.0087	0.0111	0.0028	0.0153	0.0045	0.0050	0.0046	14.25	-30.43	-8.91	98.25	216.96	101.25	150.87	68.90
25	0.0001	0.0089	0.0109	0.0032	0.0164	0.0049	0.0055	0.0053	16.65	-28.81	-8.07	106.81	214.25	108.31	155.39	85.77
26	0.0002	0.0085	0.0109	0.0028	0.0153	0.0045	0.0051	0.0048	13.70	-29.25	-6.72	103.99	214.66	106.65	153.77	72.24
27	0.0001	0.0086	0.0109	0.0031	0.0158	0.0048	0.0053	0.0051	14.90	-28.31	-4.89	105.96	224.41	107.67	152.47	85.29

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Table 44 – continued from previous page (17^{th} Harmonic)

	Magnitude of the Errors (pu) (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
28	0.0002	0.0077	0.0097	0.0030	0.0148	0.0040	0.0047	0.0042	16.63	-36.86	-15.53	85.75	213.29	122.40	139.37	77.14
29	0.0002	0.0081	0.0103	0.0032	0.0154	0.0045	0.0051	0.0046	17.00	-34.77	-14.35	88.60	207.86	100.51	142.31	95.86
30	0.0001	0.0082	0.0110	0.0023	0.0141	0.0037	0.0045	0.0048	9.94	-26.27	-8.32	104.53	223.82	99.70	152.72	102.01
31	0.0002	0.0082	0.0108	0.0027	0.0146	0.0045	0.0048	0.0045	13.15	-28.69	-11.48	96.35	216.57	99.75	144.26	93.46
32	0.0002	0.0080	0.0106	0.0026	0.0142	0.0042	0.0045	0.0044	13.52	-29.63	-12.43	93.15	205.45	97.82	145.79	107.96
33	0.0002	0.0079	0.0103	0.0027	0.0143	0.0042	0.0045	0.0046	12.75	-29.35	-11.42	93.44	209.20	96.50	144.13	93.78
34	0.0002	0.0076	0.0103	0.0021	0.0131	0.0035	0.0039	0.0041	10.18	-28.21	-10.91	95.71	211.46	100.61	147.99	90.81
35	0.0002	0.0080	0.0103	0.0028	0.0144	0.0045	0.0046	0.0043	15.22	-32.33	-13.86	92.01	207.14	94.70	139.99	104.15
36	0.0002	0.0080	0.0104	0.0029	0.0150	0.0045	0.0048	0.0048	15.96	-29.71	-10.67	97.67	205.68	102.84	148.60	111.63
37	0.0002	0.0076	0.0104	0.0024	0.0135	0.0039	0.0042	0.0040	12.38	-31.15	-12.88	95.05	218.06	97.48	143.84	101.85
38	0.0002	0.0073	0.0101	0.0022	0.0132	0.0036	0.0039	0.0043	9.92	-28.56	-12.01	96.88	225.92	98.61	144.86	101.67
39	0.0002	0.0078	0.0102	0.0028	0.0145	0.0043	0.0046	0.0048	15.20	-29.00	-11.64	96.13	214.34	100.09	141.98	110.78
40	0.0002	0.0077	0.0102	0.0026	0.0143	0.0042	0.0045	0.0045	12.32	-29.49	-11.96	94.25	206.63	97.77	141.09	96.07
41	0.0002	0.0079	0.0102	0.0028	0.0146	0.0044	0.0047	0.0046	12.98	-28.33	-7.83	97.71	211.90	100.36	144.09	110.93
42	0.0001	0.0078	0.0099	0.0030	0.0153	0.0046	0.0048	0.0051	15.76	-29.87	-11.93	97.99	205.69	100.95	145.76	114.77
43	0.0002	0.0078	0.0104	0.0027	0.0143	0.0042	0.0045	0.0043	11.70	-29.87	-10.08	94.85	206.32	98.69	150.66	112.24
44	0.0002	0.0078	0.0106	0.0023	0.0136	0.0037	0.0042	0.0040	9.74	-29.53	-10.64	94.37	208.62	92.95	149.99	110.47
45	0.0002	0.0077	0.0101	0.0027	0.0144	0.0043	0.0045	0.0043	11.55	-29.99	-11.85	92.60	212.33	94.38	144.74	97.45
46	0.0002	0.0076	0.0104	0.0023	0.0134	0.0038	0.0041	0.0038	11.18	-32.64	-10.73	89.91	210.06	93.59	143.04	106.85
47	0.0002	0.0081	0.0106	0.0028	0.0146	0.0044	0.0047	0.0044	12.44	-30.29	-8.36	95.57	208.87	98.87	147.21	115.17
48	0.0001	0.0076	0.0096	0.0028	0.0153	0.0043	0.0047	0.0052	14.40	-29.16	-7.45	101.43	211.34	103.32	152.49	106.19
49	0.0001	0.0073	0.0092	0.0029	0.0154	0.0041	0.0047	0.0049	15.26	-33.69	-12.31	95.66	208.66	105.25	147.85	92.46
50	0.0002	0.0081	0.0102	0.0029	0.0155	0.0046	0.0049	0.0048	15.38	-30.79	-9.05	97.95	204.99	101.00	150.40	112.25
51	0.0002	0.0079	0.0107	0.0021	0.0134	0.0036	0.0040	0.0038	9.96	-31.10	-11.55	94.06	210.02	97.20	147.84	113.11
52	0.0002	0.0081	0.0106	0.0025	0.0145	0.0041	0.0045	0.0041	13.63	-30.83	-10.91	94.39	210.10	96.67	148.14	115.91
53	0.0002	0.0081	0.0107	0.0025	0.0140	0.0041	0.0043	0.0039	12.48	-31.41	-14.77	90.81	208.04	92.86	141.52	119.59
54	0.0002	0.0081	0.0106	0.0024	0.0140	0.0040	0.0043	0.0039	12.62	-32.02	-13.14	92.56	214.38	94.55	143.37	114.31

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Table 44 – continued from previous page (17^{th} Harmonic)

	Magnitude of the Errors (pu) (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
55	0.0002	0.0081	0.0107	0.0025	0.0141	0.0040	0.0043	0.0040	13.05	-32.01	-11.45	91.98	209.37	94.62	143.69	116.67
56	0.0002	0.0082	0.0105	0.0028	0.0152	0.0046	0.0047	0.0045	14.09	-31.55	-11.76	93.42	204.89	94.73	140.99	119.87
57	0.0001	0.0086	0.0105	0.0032	0.0167	0.0051	0.0054	0.0054	15.83	-28.82	-9.41	103.89	210.55	106.40	152.22	112.17
58	0.0001	0.0083	0.0103	0.0031	0.0160	0.0049	0.0051	0.0049	16.31	-32.75	-12.27	95.45	206.37	96.31	143.52	117.44
59	0.0002	0.0083	0.0104	0.0030	0.0156	0.0047	0.0049	0.0047	16.45	-32.23	-10.99	94.68	205.85	96.60	145.00	115.48
60	0.0002	0.0082	0.0108	0.0025	0.0145	0.0042	0.0045	0.0041	12.05	-31.41	-10.81	92.40	209.75	95.42	144.08	101.33
61	0.0002	0.0081	0.0105	0.0027	0.0149	0.0045	0.0046	0.0043	14.35	-32.66	-11.85	90.73	205.49	93.95	141.50	99.27
62	0.0002	0.0081	0.0106	0.0026	0.0146	0.0043	0.0045	0.0043	12.46	-31.21	-9.24	94.89	207.66	95.24	145.59	107.02
63	0.0002	0.0081	0.0108	0.0025	0.0143	0.0042	0.0043	0.0043	11.06	-32.23	-11.90	93.73	208.28	92.54	138.59	104.16
64	0.0001	0.0087	0.0111	0.0028	0.0159	0.0045	0.0048	0.0048	13.34	-29.47	-7.73	96.58	218.72	98.99	140.43	105.47
65	0.0001	0.0086	0.0111	0.0028	0.0160	0.0047	0.0048	0.0048	13.83	-30.62	-10.24	98.05	217.22	99.55	138.08	110.14
66	0.0001	0.0086	0.0112	0.0028	0.0155	0.0046	0.0048	0.0048	12.77	-29.31	-10.41	100.71	217.17	99.62	139.37	119.13
67	0.0001	0.0085	0.0106	0.0032	0.0168	0.0050	0.0051	0.0057	16.12	-29.60	-9.17	103.30	213.39	102.60	141.20	107.29
68	0.0001	0.0081	0.0098	0.0030	0.0162	0.0048	0.0048	0.0052	16.90	-33.02	-12.98	98.77	210.27	95.95	137.64	100.54
69	0.0001	0.0081	0.0097	0.0031	0.0164	0.0049	0.0049	0.0053	17.70	-33.44	-13.41	101.26	214.12	97.91	137.26	102.22
70	0.0001	0.0083	0.0102	0.0034	0.0167	0.0053	0.0051	0.0056	17.38	-33.19	-12.21	98.39	211.71	95.70	131.08	109.67
71	0.0002	0.0082	0.0104	0.0031	0.0157	0.0050	0.0048	0.0049	16.09	-33.92	-13.97	92.81	214.59	91.70	129.64	114.59
72	0.0001	0.0087	0.0112	0.0028	0.0157	0.0047	0.0048	0.0048	14.35	-30.16	-10.59	101.25	227.96	102.97	138.49	111.64
73	0.0001	0.0092	0.0120	0.0029	0.0154	0.0051	0.0051	0.0039	14.02	-33.61	-13.16	95.88	220.81	97.58	134.97	77.67
74	0.0001	0.0090	0.0113	0.0033	0.0166	0.0055	0.0054	0.0049	17.19	-32.93	-11.35	96.87	217.31	98.90	135.02	78.67
75	0.0001	0.0094	0.0118	0.0034	0.0164	0.0055	0.0055	0.0049	16.35	-31.23	-11.98	98.15	207.43	100.31	133.79	78.08
76	0.0002	0.0091	0.0115	0.0032	0.0162	0.0053	0.0054	0.0048	16.09	-31.72	-10.07	96.21	201.98	100.76	137.16	81.13
77	0.0002	0.0091	0.0119	0.0029	0.0154	0.0048	0.0051	0.0046	12.84	-31.34	-10.27	98.04	203.34	102.89	140.58	85.01
78	0.0001	0.0085	0.0106	0.0033	0.0162	0.0050	0.0050	0.0054	17.52	-32.52	-15.26	94.96	201.93	99.06	134.70	67.18
79	0.0002	0.0083	0.0107	0.0030	0.0153	0.0047	0.0047	0.0050	16.66	-32.65	-21.70	89.25	200.30	94.55	132.69	97.35
80	0.0002	0.0087	0.0116	0.0028	0.0150	0.0046	0.0046	0.0045	14.62	-33.57	-19.18	89.19	200.05	94.20	132.84	111.75
81	0.0002	0.0080	0.0103	0.0032	0.0154	0.0049	0.0048	0.0048	16.51	-34.39	-19.23	86.30	197.16	88.32	126.64	103.44

Continued on next page

Table 44 – continued from previous page (17^{th} Harmonic)

	Magnitude of the Errors (pu) (pu)								Error in the Angles (degrees)							
Bus→	1	4	6	11	13	16	19	21	1	4	6	11	13	16	19	21
Interval																
82	0.0001	0.0081	0.0098	0.0032	0.0161	0.0048	0.0049	0.0052	18.62	-32.46	-19.12	98.42	201.50	100.72	138.03	106.36
83	0.0001	0.0079	0.01	0.0032	0.0157	0.0048	0.0047	0.0052	18.06	-33.92	-21.17	90.14	197.13	92.36	129.68	100.24
84	0.0002	0.0080	0.0104	0.0028	0.0144	0.0043	0.0042	0.0047	15.59	-32.26	-20.65	86.64	195.93	90.91	131.03	107.19
85	0.0002	0.0077	0.0101	0.0029	0.0146	0.0045	0.0043	0.0047	15.64	-33.55	-21.65	85.99	194.67	87.83	129.04	105.61
86	0.0001	0.0076	0.0095	0.0032	0.0159	0.0048	0.0048	0.0053	16.97	-32.10	-20.89	97.22	203.11	98.50	132.55	110.19
87	0.0002	0.0076	0.0097	0.0032	0.0156	0.0048	0.0047	0.0052	17.31	-33.40	-18.45	93.92	197.55	95.71	129.15	108.29
88	0.0002	0.0080	0.0104	0.0029	0.0152	0.0045	0.0047	0.0051	14.37	-29.42	-10.43	96.93	199.33	101.43	134.87	114.24
89	0.0002	0.0077	0.0102	0.0028	0.0146	0.0043	0.0043	0.0045	13.38	-29.49	-12.66	89.65	204.37	92.88	130.09	114.87
90	0.0002	0.0081	0.0108	0.0029	0.0151	0.0046	0.0048	0.0046	13.63	-31.04	-13.71	92.14	210.26	94.50	130.38	117.82
91	0.0002	0.0076	0.0102	0.0029	0.0147	0.0044	0.0045	0.0046	13.41	-32.48	-13.50	88.58	196.54	90.73	131.08	105.14
92	0.0001	0.0078	0.0098	0.0031	0.0158	0.0046	0.0049	0.0052	14.00	-30.13	-9.35	98.30	201.47	100.02	136.23	113.30
93	0.0001	0.0078	0.0098	0.0033	0.0162	0.0050	0.0051	0.0052	15.32	-32.31	-11.17	95.17	200.23	96.83	134.64	120.38
94	0.0001	0.0081	0.0102	0.0034	0.0165	0.0052	0.0053	0.0053	15.38	-32.98	-10.07	95.50	198.25	97.33	133.99	119.04
95	0.0001	0.0084	0.0104	0.0035	0.0168	0.0053	0.0054	0.0054	16.56	-30.68	-8.54	98.60	193.06	99.77	137.59	106.77
96	0.0002	0.0077	0.0101	0.0029	0.0150	0.0044	0.0046	0.0048	13.98	-33.46	-9.74	89.68	199.35	91.67	133.29	102.19
97	0.0001	0.0085	0.0112	0.0027	0.0154	0.0044	0.0049	0.0049	11.36	-28.63	-5.90	101.65	209.37	104.79	142.08	107.95
98	0.0001	0.0084	0.0108	0.0032	0.0162	0.0051	0.0053	0.0050	15.25	-32.85	-7.85	93.09	203.48	94.62	135.66	102.54
99	0.0001	0.0086	0.0106	0.0035	0.0173	0.0055	0.0058	0.0054	16.61	-30.70	-6.71	100.44	202.31	101.58	142.76	106.21
100	0.0001	0.0086	0.0106	0.0034	0.0168	0.0053	0.0056	0.0053	16.74	-33.20	-7.38	97.79	205.95	97.96	139.54	114.39

E EFFECT OF AGGREGATION TIME

Table 45: Comparison of the 1^{st} Harmonic Estimate in Different Aggregation Time

	Magnitude (pu)			Angle (degrees)		
Aggregation Time →	6 sec	1 min	10 min	6 sec	1 min	10 min
Bus						
1	1.002	1.001	1.001	-29.624	-29.668	-29.668
4	0.968	0.968	0.968	1.759	1.793	1.806
6	0.966	0.965	0.965	1.941	1.999	2.002
11	0.965	0.964	0.965	1.663	1.754	1.76
13	0.901	0.901	0.901	6.241	6.251	6.264
16	0.953	0.953	0.953	1.707	1.776	1.778
19	0.965	0.965	0.965	1.869	1.92	1.927
21	1.019	1.019	1.019	3.091	3.08	3.099

Table 46: Comparison of the 3^{rd} Harmonic Estimate in Different Aggregation Time

	Magnitude (pu)			Angle (degrees)		
Aggregation Time →	6 sec	1 min	10 min	6 sec	1 min	10 min
Bus						
1	0.002	0.002	0.002	15.268	21.991	24.883
4	0	0.001	0.001	-34.113	68.034	64.774
6	0.001	0.002	0.002	-32.111	58.261	56.285
11	0.001	0.002	0.002	-1.358	58.202	61.079
13	0.006	0.001	0.002	-92.967	73.63	71.031
16	0	0.001	0.002	-2.283	67.953	63.282
19	0.001	0.002	0.002	-70.844	70.586	69.148
21	0.003	0.002	0.002	-78.349	63.652	62.645

Table 47: Comparison of the 5^{th} Harmonic Estimate in Different Aggregation Time

	Magnitude (pu)			Angle (degrees)		
Aggregation Time →	6 sec	1 min	10 min	6 sec	1 min	10 min
Bus						
1	0.006	0.006	0.006	-125.216	-125.398	-125.127
4	0.008	0.009	0.009	-116.465	-116.34	-116.971
6	0.009	0.01	0.01	-128.271	-128.794	-129.482
11	0.01	0.011	0.011	-168.213	-164.315	-164.503
13	0.013	0.014	0.014	167.539	174.82	174.44
16	0.011	0.012	0.012	-176.286	-173.116	-173.337
19	0.01	0.011	0.011	-175.422	-172.449	-172.57
21	0.01	0.011	0.011	-142.854	-141.879	-142.458

Table 48: Comparison of the 7^{th} Harmonic Estimate in Different Aggregation Time

	Magnitude (pu)			Angle (degrees)		
Aggregation Time →	6 sec	1 min	10 min	6 sec	1 min	10 min
Bus						
1	0.012	0.012	0.012	-39.389	-39.139	-38.845
4	0.039	0.038	0.038	-2.279	-1.948	-1.857
6	0.037	0.037	0.037	-3.169	-2.806	-2.75
11	0.008	0.008	0.007	-69.275	-68.121	-67.879
13	0.022	0.022	0.022	-155.285	-156.262	-156.134
16	0.013	0.013	0.013	-126.116	-126.813	-127.038
19	0.01	0.01	0.01	-120.954	-122.018	-122.206
21	0.015	0.015	0.015	-10.631	-9.952	-9.618

Table 49: Comparison of the 9^{th} Harmonic Estimate in Different Aggregation Time

	Magnitude (pu)			Angle (degrees)		
Aggregation Time →	6 sec	1 min	10 min	6 sec	1 min	10 min
Bus						
1	0.001	0	0	52.433	69.537	51.297
4	0	0.001	0.001	-7.727	-61.39	-34.835
6	0	0.001	0	93.494	-53.608	-9.415
11	0.001	0.001	0.001	-27.374	3.549	0.003
13	0.002	0.001	0.001	-68.771	-6.349	-30.903
16	0.001	0.001	0.001	-17.781	-14.634	-16.412
19	0.001	0.001	0.001	-37.49	6.711	-3.492
21	0.001	0.001	0.001	-44.62	18.555	2.135

Table 50: Comparison of the 11^{th} Harmonic Estimate in Different Aggregation Time

	Magnitude (pu)			Angle (degrees)		
Aggregation Time →	6 sec	1 min	10 min	6 sec	1 min	10 min
Bus						
1	0.002	0.002	0.002	28.167	31.32	31.394
4	0.008	0.008	0.008	-10.657	0.842	0.796
6	0.009	0.008	0.008	-9.351	1.199	0.939
11	0.002	0.002	0.002	35.041	25.761	22.483
13	0.004	0.005	0.004	147.097	-169.038	-166.417
16	0.002	0.002	0.002	153.305	-179.805	175.294
19	0.001	0.001	0.001	138.544	-172.787	-159.781
21	0.004	0.004	0.004	25.344	11.884	8.53

Table 51: Comparison of the 13^{th} Harmonic Estimate in Different Aggregation Time

	Magnitude (pu)			Angle (degrees)		
Aggregation Time →	6 sec	1 min	10 min	6 sec	1 min	10 min
Bus						
1	0.001	0.001	0.001	-7.747	-10.235	-10.289
4	0.002	0.002	0.002	154.71	112.306	125.402
6	0.001	0.003	0.002	101.25	79.391	86.5
11	0.004	0.004	0.004	-5.872	-4.475	-2.987
13	0.013	0.011	0.012	3.458	-3.929	-3.93
16	0.005	0.004	0.004	2.48	-3.708	-3.085
19	0.005	0.005	0.005	-8.646	-14.857	-12.547
21	0.008	0.007	0.007	5.708	6.26	6.61

Table 52: Comparison of the 15^{th} Harmonic Estimate in Different Aggregation Time

	Magnitude (pu)			Angle (degrees)		
Aggregation Time →	6 sec	1 min	10 min	6 sec	1 min	10 min
Bus						
1	0	0	0	134.107	143.497	137.968
4	0.002	0.001	0.001	-166.583	-157.585	164.241
6	0.002	0.002	0.002	-176.081	-169.448	164.505
11	0.002	0.001	0.001	44.017	58.103	31.918
13	0.004	0.003	0.003	23.157	29.692	3.935
16	0.001	0.001	0.001	118.587	126.225	132.873
19	0.002	0.002	0.002	30.881	38.442	11.851
21	0.003	0.002	0.002	32.529	54.026	24.385

Table 53: Comparison of the 17^{th} Harmonic Estimate in Different Aggregation Time

	Magnitude (pu)			Angle (degrees)		
Aggregation Time →	6 sec	1 min	10 min	6 sec	1 min	10 min
Bus						
1	0.001	0.001	0.001	-11.677	-10.741	-10.233
4	0.012	0.012	0.012	-59.309	-59.455	-60.952
6	0.012	0.012	0.012	-58.153	-58.103	-59.854
11	0.002	0.003	0.003	92.399	95.495	92.118
13	0.015	0.015	0.015	119.286	119.146	118.544
16	0.004	0.004	0.005	95.417	97.526	97.15
19	0.005	0.006	0.005	113.137	113.587	111.144
21	0.005	0.005	0.005	116.334	117.56	115.27

F LIST OF PUBLICATIONS

The following papers were published during this thesis work.

- Bhujel, D., Watson, N.R. and Jalal, T.S. (2017), 'Application of harmonic state estimation to a distribution system', In *2017 IEEE Manchester PowerTech*, June, pp. 1-6.
- Bhujel, D., Watson, N.R. and Jalal, T.S. (2016), 'Harmonic State Estimation on an 11 kV Distribution System', *EEA Conference*, 22-24 Jun 2016.