Article

Journal of Vibration and Control
2022, Vol. 0(0) 1–12

DOI: 10.1177/10775463221130925
journals.sagepub.com/home/jvc
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Characteristics of passive vibration control for exponential non-viscous damping system: Vibration isolator and absorber

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Abstract

Materials or structures made of polymers or composites often exhibit non-viscous damping behavior. The non-viscous damping forces that depend on the history velocity can be expressed by the exponential non-viscous damping model. Some polymer or composite materials can be used as damping materials for kinetic energy absorption in the dynamic systems. Vibration isolator and absorber are usually considered for shock absorption. In this study, transfer ratios of vibration isolator and absorber with exponential non-viscous damping system are derived by using the Laplace transform. The dimensionless amplitude of vibration absorber with exponential non-viscous damping is derived too. Compared to viscous damping system, transfer ratio and dimensionless amplitude of exponential non-viscous damping system are influenced by the ratio of the relaxation parameter and natural frequency or the frequency of the external load. With the non-viscous damping material used in vibration control, the ratio is therefore a non-negligible factor which should be considered in analysis.

Keywords

non-viscous damping, vibration isolator, vibration absorber, relaxation parameter

1. Introduction

Dynamic analysis has important applications in civil and mechanical engineering. Damping force is crucial for the dynamic analysis in structures. Viscous damping model is widely used in dynamic analysis for its simplicity and mathematical convenience. Mathematical studies related to the viscous damping model dynamics system also do some help for the application of viscous damping model in engineering. The viscous damping model could not represent real engineering materials accurately. Whether the calculation of the response or the vibration control of a system both need the precise damping model to represent the materials and the structures of a system (Lee, 2020a). Dynamic analysis needs a suitable damping model (Lee, 2020b; Wu et al., 2019). Several researchers (Bandstra, 1983; Papoulia and Kelly, 1997; Palmeri and Muscolino, 2011) may use different damping models such as the ideal hysteretic model (Crandall, 1970), fractional derivatives model (Gaul et al., 1991) and frequency-dependent model (Naylor, 1970) in different engineering fields. The convolution damping model is the most generalized model in the scope of linear models (Woodhouse, 1998). The non-viscous damping model can be expressed as:

$$f_d(t) = c\int_0^t g(t-\tau)\cdot\dot{x}(\tau)d\tau \tag{1}$$

In this equation, c represents the damping coefficient, $g(t)$ represents the damping kernel function, and $\dot{x}(\tau)$ represents the velocity. The kernel function $g(t)$ is a weighting function. Non-viscous damping model is generalized within the scope of linear models that can describe the local time or the non-local time damping mechanism by choosing different kernel functions. The non-viscous damping force is expressed by equation (1). The fractional derivatives of

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Received: 7 April 2022; accepted: 15 September 2022

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displacement damping model also can be obtained by choosing properly kernel function (Li et al., 2020). The exponential kernel function is the promising and widely studied by many researchers. The damping force with exponential kernel function is expressed as:

$$f_d(t) = \sum\nolimits_{k=1}^{n} c_k \int_0^t \mu_k e^{-\mu_k(t-\tau)} \cdot \dot{x}(\tau)d\tau \tag{2}$$

Several researchers have done research on the system with exponential kernel function. Many methods are proposed for the calculation of the responses of the non-viscous damping system such as state-space method(Li and Hu, 2016; Wagner and Adhikari, 2003; Wang and Wang, 2018), direct integration method(Cortés et al., 2009; Liu, 2014; Shen et al., 2019; Puthanpurayil et al., 2014), and weak form Galerkin method(Shen et al., 2021). Some researchers do some research on the calculation of the eigenvalue of the non-viscous damping system by using iterative method(Lin and Ng, 2019), perturbation method(Adhikari, 2001), and damping sensitivity(Lázaro, 2016).

In addition, critical damping(Adhikari, 2005; Lázaro, 2019a, 2019b), closed-form solution for free vibration(García-Barruetabeña et al., 2012), and other dynamic characteristics of the non-viscous damping linear(Adhikari, 2008) and non-linear(Sieber et al., 2008) system are analyzed by researchers in recent years. Some researchers proposed identification methods such as linear least square method(Adhikari and Woodhouse, 2001), parameter iterative method(Pan and Wang, 2015), and Kalamn filtering method(Reggio et al., 2013) for non-viscous damping system in frequency domain(Su et al., 2019) and in time domain(Shen et al., 2020).

Some materials exhibit non-viscous damping behavior such as polymer materials and composite materials structures (Liu, 2018). The use of the polymer materials and composite materials in engineering is much greater than before (Wu et al., 2019). The composite damping materials are used in some engineering fields such as navigation (Mouritz et al., 2001), vehicle transport (Fan et al., 2009), aerospace (Ghiringhelli et al., 2013), and electronics industries (Rao, 2003; Zhou et al., 2016). The non-viscous damping takes the complete velocity history into account through a convolution over an exponentially decaying kernel function. Non-viscous damping model is more likely to have a better match with experimental data of composite damping materials for the non-local character of time (Li et al., 2014, 2015).

Composite damping materials can be used as the vibration isolator material and the vibration absorber material. Thus, the passive vibration control for the system with the exponential non-viscous damping is discussed in this paper. The vibration isolator and absorber are widely used in engineering. The transfer ratio of the vibration isolator system and the amplitude of the vibration absorber are discussed in detail

2. Background

2.1. Vibration isolator for system with viscous damping

It is well known that the vibration transfer ratio of the system with viscous damping shown in Figure 1 can be expressed as follows:

$$T_{vis} = \left| \frac{k + i\omega c}{k - \omega^2 m + i\omega c} \right| = \sqrt{\frac{1 + (2\zeta\lambda)^2}{\left(1 - \lambda^2\right)^2 + (2\zeta\lambda)^2}} \tag{3}$$

The system is under harmonic load $F = F_0 e^{i\omega t}$. The natural frequency of the system is $\omega_n = \sqrt{\frac{k}{m}}$. The frequency ratio λ is equal to $\frac{\omega}{\omega_n}$. The viscous damping ratio ζ is equal to $\frac{c}{2m\omega_n}$. It is well known when the frequency ratio λ is equal to 1, and the damping ratio is the only factor which influences the vibration transfer ratio.

2.2. Vibration absorber for system with viscous damping

Figure 2 shows the primary system of the vibration absorber. The mass spring system m_2 reduces or eliminates the vibration on the vibration object m_1 that is under harmonic external load.

The system is under harmonic load $F = F_0 e^{i\omega t}$. The amplitude of the main system with viscous damping shown in the Figure 2 can be expressed as follows:

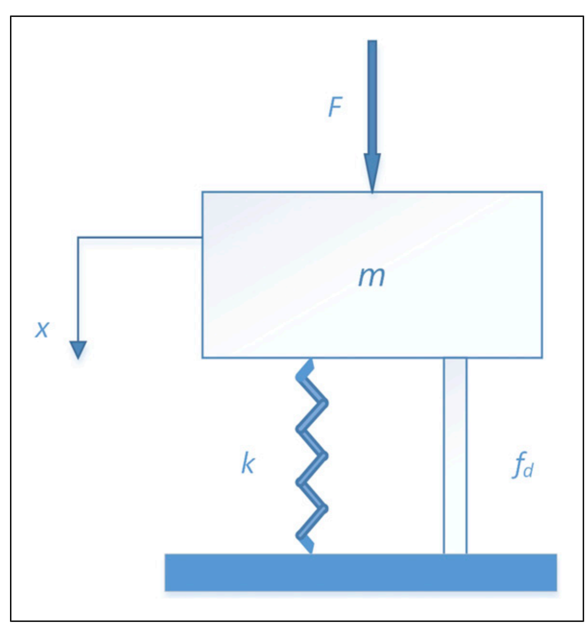

Figure 1. Vibration isolator of the dynamic system.

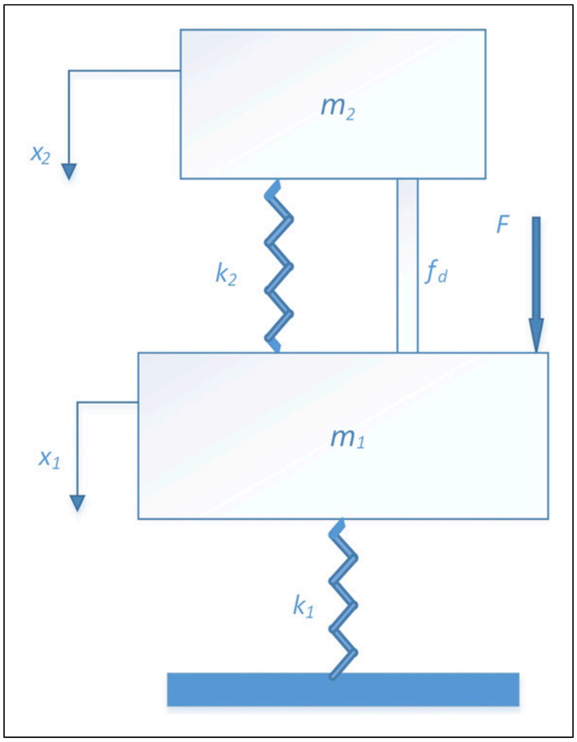

Figure 2. Vibration absorber of the dynamic system.

$$A_1 = F_0\left[\left(k_2 - m_2\omega^2\right)^2 + (c\omega)^2\right]^{\frac{1}{2}} \times \left\{\left[\left(k_1 - m_1\omega^2\right)\left(k_2 - m_2\omega^2\right) - k_2 m_2\omega^2\right]^2 + \left[c\omega\left(k_1 - m_1\omega^2 - m_2\omega^2\right)\right]^2\right\}^{\frac{-1}{2}} \quad (4)$$

The dimensionless amplitude $\overline{A_1}$ can be rewritten as follows:

$$\overline{A_1} = \left[\left(\gamma^2 - \alpha^2\right)^2 + (2\zeta\gamma)^2\right]^{\frac{1}{2}} \times \left\{\left[\left(\gamma^2 - 1\right)\left(\gamma^2 - \alpha^2\right) - \overline{\mu}\gamma^2\alpha^2\right]^2 + \left[(2\zeta\gamma)^2\left(\gamma^2 - 1 + \overline{\mu}\gamma^2\right)\right]^2\right\}^{\frac{-1}{2}} \quad (5)$$

The natural frequency of the system, respectively, are: $\omega_1 = \sqrt{\frac{k_1}{m_1}}$ and $\omega_2 = \sqrt{\frac{k_2}{m_2}}$. The frequency ratio γ is equal to $\frac{\omega}{\omega_1}$. The parameter α is equal to $\frac{\omega_2}{\omega_1}$. The mass ratio $\overline{\mu}$ is equal to $\frac{m_2}{m_1}$. The damping ratio ζ is equal to $\frac{c}{2m_2\omega_1}$.

The tuned condition can be expressed as: $\gamma = \frac{1}{1+\overline{\mu}}$. With the condition, the maximum dimensionless amplitude is equal to $\sqrt{\frac{2+\overline{\mu}}{\overline{\mu}}}$. The corresponding frequency ratios are: $\frac{1}{1+\overline{\mu}} \mp \frac{1}{1+\overline{\mu}}\sqrt{\frac{\overline{\mu}}{2+\overline{\mu}}}$. The damping ratio condition is: $\sqrt{\frac{3\overline{\mu}}{8(1+\overline{\mu})^3}}$.

3. Vibration isolator for exponential non-viscous damping system

In Figure 1, when the damping model is viscous damping, the damping force can be expressed as: $f_d = cv$. When the damping model is exponential non-viscous damping, the damping force can be expressed as equation (2).

Although researchers proposed analytical and numerical methods for non-viscous damping system, some fundamental dynamics characteristics of non-viscous damping system cannot be obtained just from the point of view of viscous damping system. Adhikari (2008) addressed dynamic response characteristics including critical damping factor, frequency response function, and response amplitude of non-viscous damping system by considering two parameters non-viscous model with $k = 1$ in equation (2). In this paper, authors address characteristics of passive vibration control for exponential non-viscous damping system also based on two parameters non-viscous model for the following reasons. Two parameters model is the simple and fundamental model of exponential kernel function. Basic characteristics of passive vibration control for exponential non-viscous damping system can be obtained through the study of two parameters model. In addition, with two parameters exponential non-viscous model considered in the identification procedure, damping parameters of the cantilever beam can be well estimated with a small error (Shen et al., 2022). Two parameters exponential non-viscous model is able to embodies damping mechanism of the simple structure. Two parameters non-viscous damping model is therefore considered in this paper.

The equation of motion with two parameters exponential non-viscous damping system first considered by Adhikari (2008) can be expressed as follows:

$$m\ddot{x} + c\int_0^t \mu e^{-\mu(t-\tau)} \cdot \dot{x}(\tau)d\tau + kx = f \quad (6)$$

Transforming equation (6) into the Laplace domain, one obtains:

$$\left[ms^2 + c\frac{\mu s}{s+\mu} + k\right]x(s) = f(s) \quad (7)$$

The foundation bears the impact force which can be expressed as follows:

$$p = c\int_0^t \mu e^{-\mu(t-\tau)} \cdot \dot{x}(\tau)d\tau + kx \quad (8)$$

The Laplace transform of equation (7) is shown as follows:

$$p(s) = \left[c\frac{\mu s}{s+\mu} + k\right]x(s) \quad (9)$$

3.1. Result of vibration transfer ratio

The vibration transfer ratio of the system with non-viscous damping can be obtained by using equations (7) and (9).

$$T_{non} = \left|\frac{p}{f}\right| = \left|\frac{(c\mu + k)s + k\mu}{ms^3 + \mu ms^2 + (c\mu + k)s + k\mu}\right| = \left|\frac{\left(\frac{c}{k}\mu + 1\right)s + \mu}{\frac{m}{k}s^3 + \mu\frac{m}{k}s^2 + \left(\frac{c}{k}\mu + 1\right)s + \mu}\right| \tag{10}$$

In equation (10), the Laplace variable s is equal to $i\theta$. In addition, the natural frequency $\omega_n = \sqrt{\frac{k}{m}}$, the damping ratio $\zeta = \frac{c}{2m\omega_n}$, frequency ratio $\beta = \frac{\theta}{\omega_n}$, and the ratio of the relaxation parameter μ and natural frequency ω_n, $v_{iso} = \frac{\mu}{\omega_n}$ are introduced. The unit of the parameter μ is the *rad/s*. The unit of the natural frequency ω_n is *rad/s*. The v_{iso} is a dimensionless ratio. Equation (10) can be rewritten as:

$$T_{non} = \left|\frac{\left(\frac{2\zeta}{\omega_n}\mu + 1\right)i\theta + \mu}{\frac{-1}{\omega_n^2}i\theta^3 + \mu\frac{-1}{\omega_n^2}\theta^2 + \left(\frac{2\zeta}{\omega_n}\mu + 1\right)i\theta + \mu}\right| = \left|\frac{(2\zeta v_{iso} + 1)i\beta + v_{iso}}{-i\beta^3 - v_{iso}\beta^2 + (2\zeta v_{iso} + 1)i\beta + v_{iso}}\right| = \sqrt{\frac{((2\zeta v_{iso} + 1)\beta)^2 + v_{iso}^2}{\left(2\zeta v_{iso}\beta + \beta - \beta^3\right)^2 + \left(v_{iso} - v_{iso}\beta^2\right)^2}} \tag{11}$$

3.2. Discussion on vibration transfer ratio

The transfer ratio of the system with viscous damping can be plotted in the Figure 3. When the frequency ratio is less than $\sqrt{2}$, the vibration isolator will magnify the amplitude of vibration. Especially, with the frequency ratio is identical to 1, the system is resonance and the damping ratio is the only factor that affects the transfer ratio. To decrease the amplitude of vibration, the frequency ratio must be more than $\sqrt{2}$ (Inman, 1994).The frequency ratio β usually ranges from 2.5 to 5 in engineering application (Wu, 2008). In order to increase the performance of the vibration isolation, the damping ratio could not be too large when the β ranges from 2.5 to 5.

In contrast, when the non-viscous damping system is under the resonance state, the vibration transfer ratio is influenced by damping ratio ζ and the ratio of the relaxation parameter and natural frequency v_{iso}. Equation (11) can be rewritten as in this situation ($\beta = 1$):

$$T_{non} = \sqrt{\frac{\left(2\zeta v_{iso} + 1\right)^2 + v_{iso}^2}{\left(2\zeta v_{iso}\right)^2}} \tag{12}$$

It is known from the Figure 4 that the transfer ratio of the resonant system with non-viscous damping decreases with the increase of the ratio v. Thus, the vibration transfer ratio is significantly influenced by the ratio v when the ratio v is small. In the non-viscous damping system, it is necessary to avoid using the small ratio v material or structure. The ratio $v_{iso} = \frac{\mu}{\omega_n}$ is determined by the relaxation parameter μ and the natural frequency ω_n. The relaxation parameter is mainly influenced by the characteristics of the materials. The natural frequency is influenced by the mass and stiffness. When the β ranges from 2.5 to 5, the ratio v_{iso} only has a slight influence on the vibration shown in the Figure 5 and Figure 6.

It can be concluded from the discussion above that when the system with non-viscous damping is in the resonance state, the transfer ratio is influenced heavily by the ratio v_{iso} and could be reduced to less 1. Thus, when the non-viscous damping material is used in vibration isolator, the ratio v_{iso} is a non-negligible factor which should be considered in analysis.

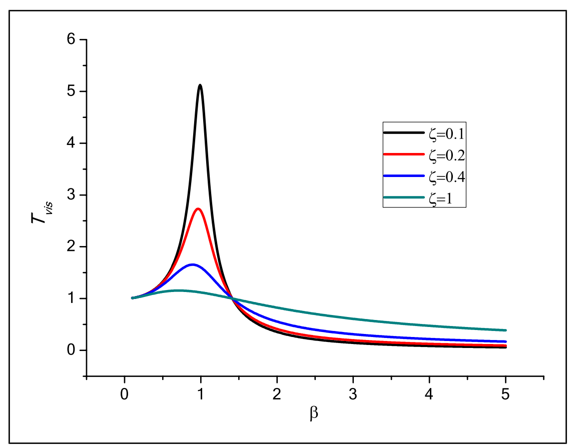

Figure 3. Transfer ratio of the system with viscous damping.

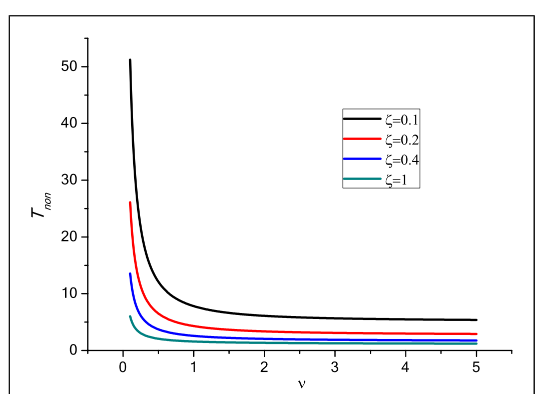

Figure 4. Transfer ratio of the system with non-viscous damping.

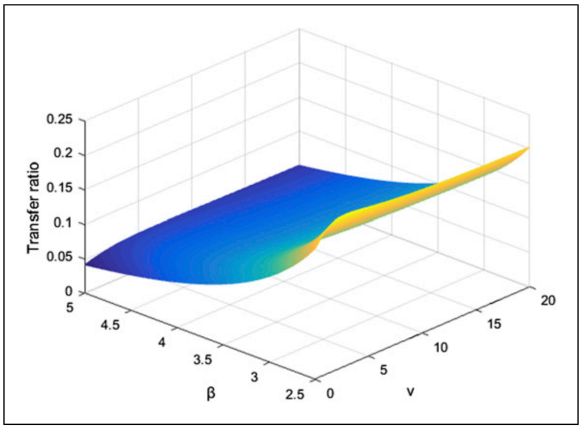

Figure 5. Transfer ratio of the system with non-viscous damping (damping ratio $\zeta = 0.05$).

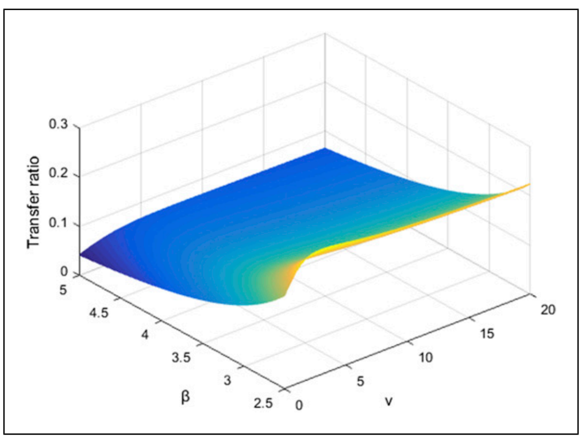

Figure 6. Transfer ratio of the system with non-viscous damping (damping ratio $\zeta = 0.1$).

4. Vibration absorber for exponential non-viscous damping system

In Figure 2, when the damping model is viscous damping, the damping force can be expressed as: $f_d = cv$. When the damping model is exponential non-viscous damping, the damping force can be expressed as equation (2).

The kinetic equation of the exponential non-viscous damping system in Figure 2 can be expressed as follows:

$$m_1\ddot{x}_1 + c\int_0^t \mu e^{-\mu(t-\tau)} \cdot \left(\dot{x}_1(\tau) - \dot{x}_2(\tau)\right)d\tau + k_2(x_1 - x_2) + k_1x_1 = fe^{i\omega t} \tag{13}$$

$$m_2\ddot{x}_2 + c\int_0^t \mu e^{-\mu(t-\tau)} \cdot \left(\dot{x}_2(\tau) - \dot{x}_1(\tau)\right)d\tau + k_2(x_2 - x_1) = 0 \tag{14}$$

Equations (13) and (14) can be rewritten in one equation as follows:

$$\begin{bmatrix} m_1 & 0 \\ 0 & m_2 \end{bmatrix}\begin{bmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{bmatrix} + \begin{bmatrix} c & -c \\ -c & c \end{bmatrix}\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} + \begin{bmatrix} k_1+k_2 & -k_2 \\ -k_2 & k_2 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} fe^{i\omega t} \\ 0 \end{bmatrix} \tag{15}$$

In equation (15), the y_1 is equal to $\int_0^t g(t-\tau)\cdot\dot{x}_1(\tau)d\tau$ and y_2 is equal to $\int_0^t g(t-\tau)\cdot\dot{x}_2(\tau)d\tau$.

4.1. Result of dimensionless amplitude of vibration absorber

Transforming equation (15) into the Laplace domain and the Laplace variable s is equal to $i\theta$, one obtains:

$$\begin{bmatrix} k_1 + k_2 - \theta^2 m_1 + \dfrac{c\mu i\theta}{i\theta+\mu} & -k_2 - \dfrac{c\mu i\theta}{i\theta+\mu} \\ -k_2 - \dfrac{c\mu i\theta}{i\theta+\mu} & k_2 - \theta^2 m_2 + \dfrac{c\mu i\theta}{i\theta+\mu} \end{bmatrix} \times \begin{bmatrix} \overline{A_1} \\ \overline{A_2} \end{bmatrix} = \begin{bmatrix} f \\ 0 \end{bmatrix} \tag{16}$$

In equation (16), the $\overline{A_1}$ and $\overline{A_2}$ are respective the complex amplitudes of response of the first degree of freedom and the second degree of freedom.

The complex amplitudes of response of the first degree of freedom and the second degree of freedom can be expressed as:

$$\begin{bmatrix} \overline{A_1} \\ \overline{A_2} \end{bmatrix} = \frac{f}{\nabla}\begin{bmatrix} k_2 - \theta^2 m_2 + \dfrac{c\mu i\theta}{i\theta+\mu} \\ k_2 + \dfrac{c\mu i\theta}{i\theta+\mu} \end{bmatrix} \tag{17}$$

where the ∇ can be expressed as:

$$\nabla = \left(k_1 + k_2 - \theta^2 m_1 + \frac{c\mu i\theta}{i\theta+\mu}\right)\left(k_2 - \theta^2 m_2 + D\frac{c\mu i\theta}{i\theta+\mu}\right) - \left(-k_2 - \frac{c\mu i\theta}{i\theta+\mu}\right)\left(-k_2 - \frac{c\mu i\theta}{i\theta+\mu}\right) \tag{18}$$

The complex amplitude of the main system can be expressed as:

$$\overline{A_1} = \frac{f}{\nabla}\times\left(k_2 - \theta^2 m_2 + \frac{c\mu\theta^2}{\mu^2+\theta^2} + \frac{c\mu^2\theta}{\mu^2+\theta^2}i\right) \tag{19}$$

The amplitude of the main system can be expressed as:

$$A_1=\left|\overline{A_1}\right|=f\times\left[\left(\left(k_2-\theta^2m_2\right)\left(\mu^2+\theta^2\right)+c\mu\theta^2\right)^2+\left(c\mu^2\theta\right)^2\right]^{\frac{1}{2}}$$
$$\times\left\{\left[\left(k_1k_2-k_1m_2\theta^2-k_2m_1\theta^2\right.\right.\right.$$
$$\left.-k_2m_2\theta^2+m_1m_2\theta^4\right)\left(\mu^2+\theta^2\right)$$
$$\left.+\left(k_1-m_1\omega^2-m_2\omega^2\right)c\mu\theta^2\right]^2$$
$$\left.+\left[\left(k_1-m_1\omega^2-m_2\omega^2\right)c\mu^2\theta\right]^2\right\}^{\frac{-1}{2}} \tag{20}$$

The natural frequency of the system respective are: $\omega_1=\sqrt{\frac{k_1}{m_1}}$ and $\omega_2=\sqrt{\frac{k_2}{m_2}}$. The frequency ratio γ is equal to $\frac{\theta}{\omega_1}$. The parameter α is equal to $\frac{\omega_2}{\omega_1}$. The mass ratio $\overline{\mu}$ is equal to $\frac{m_2}{m_1}$. The damping ratio ζ is equal to $\frac{c}{2m_2\omega_2}$.

The dimensionless amplitude can be rewritten as follows:

$$\frac{A_1}{\frac{f}{k_1}}=\left[\left(\left(\gamma^2-\alpha^2\right)\left(\mu^2+\theta^2\right)+2\zeta\gamma\mu\theta\right)^2+\left(2\zeta\gamma\mu^2\right)^2\right]^{\frac{1}{2}}$$
$$\times\left\{\left[\left(\left(\gamma^2-1\right)\left(\gamma^2-\alpha^2\right)-\overline{\mu}\gamma^2\alpha^2\right)\left(\mu^2+\theta^2\right)\right.\right.$$
$$\left.+2\zeta\gamma\mu\theta\left(\gamma^2-1+\overline{\mu}\gamma^2\right)\right]^2$$
$$\left.+\left[\left(\gamma^2-1+\overline{\mu}\gamma^2\right)2\zeta\gamma\mu^2\right]^2\right\}^{\frac{-1}{2}} \tag{21}$$

The ratio of the relaxation parameter μ and the frequency of the external load θ, $v_{abs}=\frac{\mu}{\theta}$ is introduced. The unit of the parameter μ is the *rad/s*. The unit of the frequency of the external load θ is *rad/s*. The v_{abs} is a dimensionless ratio. Equation (20) can be rewritten as follows:

$$\frac{A_1}{\frac{f}{k_1}}=\left[\left(\left(\gamma^2-\alpha^2\right)\left(v_{abs}^2+1\right)+2\zeta\gamma v_{abs}\right)^2+\left(2\zeta\gamma v_{abs}^2\right)^2\right]^{\frac{1}{2}}$$
$$\times\left\{\left[\left(\left(\gamma^2-1\right)\left(\gamma^2-\alpha^2\right)-\overline{\mu}\gamma^2\alpha^2\right)\left(v_{abs}^2+1\right)\right.\right.$$
$$\left.+2\zeta\gamma v_{abs}\left(\gamma^2-1+\overline{\mu}\gamma^2\right)\right]^2$$
$$\left.+\left[\left(\gamma^2-1+\overline{\mu}\gamma^2\right)2\zeta\gamma v_{abs}^2\right]^2\right\}^{\frac{-1}{2}} \tag{22}$$

In the non-viscous damping system, there are two fixed points which have nothing to do with the damping ratio. The fixed points can be determined with the condition $\zeta=\infty$ and $\zeta=0$.

$$\left.\frac{A_1}{\frac{f}{k_1}}\right|_{\zeta=\infty}=\left.\frac{A_1}{\frac{f}{k_1}}\right|_{\zeta=0} \tag{23}$$

where the $\left.\frac{A_1}{\frac{f}{k_1}}\right|_{\zeta=\infty}$ and $\left.\frac{A_1}{\frac{f}{k_1}}\right|_{\zeta=0}$ can be expressed as:

$$\left.\frac{A_1}{\frac{f}{k_1}}\right|_{\zeta=\infty}=\frac{1}{\gamma^2-1+\overline{\mu}\gamma^2} \tag{24}$$

$$\left.\frac{A_1}{\frac{f}{k_1}}\right|_{\zeta=0}=\frac{\left(\gamma^2-\alpha^2\right)}{\left(\gamma^2-1\right)\left(\gamma^2-\alpha^2\right)-\overline{\mu}\gamma^2\alpha^2} \tag{25}$$

Equation (23) can be rewritten in detail as:

$$\frac{\left(\gamma^2-\alpha^2\right)}{\left(\gamma^2-1\right)\left(\gamma^2-\alpha^2\right)-\overline{\mu}\gamma^2\alpha^2}=\frac{1}{\gamma^2-1+\overline{\mu}\gamma^2} \tag{26}$$

4.2. Discussion on dimensionless amplitude of vibration absorber

Equations (24), and (25) and (26) do not consist the ratio v_{abs} that is related to the relaxation parameter. Thus, the dimensionless amplitude of the two fixed points is the same as the viscous damping system. In addition, the corresponding frequency ratios and the tuned condition that both can be obtained from equations (24), (25), and (26) are the same as the viscous damping system.

With the tuned condition $\alpha=\frac{1}{1+\overline{\mu}}$, the amplitude expressed by equation (22) can be plotted in figures. In Figure 7(a) and (b), the mass ratio is equal to 0.1.

Figure 7(a) and (b) show the dimensionless amplitude of the system when the ratio v in different ranges. It can be known from Figure 7 that ratio v_{abs} has a significant influence on the amplitude when the ratio v_{abs} is small. In contrast, when the ratio v_{abs} is greater than 10, the amplitude levels off. In Figure 8(a) and (b), the mass ratio is equal to 0.05. In Figure 9 (a) and (b), the mass ratio is equal to 0.01. The law of the amplitude in Figure 8(a) and (b), Figure 9 (a) and (b) is similar to the amplitude in Figure 7(a) and (b).

In Figure 7(a) and (b), the damping ratio ζ is equal to 0.16 which satisfies the condition $\zeta=\sqrt{\frac{3\overline{\mu}}{8(1+\overline{\mu})^3}}$.

Figure 10, Figure 11, Figure 12, Figure 13 and Figure 14 show the amplitudes when the damping ratio ζ are, respectively, equal to 0.1527, 0.06, 0.04, 0.03, and 0.02. With the decrease of damping ratio, the amplitudes of vibration absorber increase.

It can be concluded from the discussion above that the dimensionless amplitude of non-viscous damping system is highly influenced by the ratio v_{abs} which is the ratio of the relaxation parameter μ and the frequency of the external load θ. The tuned condition still exists in the non-viscous damping vibration absorber. It is necessary to avoid the small ratio v_{abs} because the dimensionless amplitude is large when the ratio v_{abs} is small. The ratio v_{abs} is a non-negligible

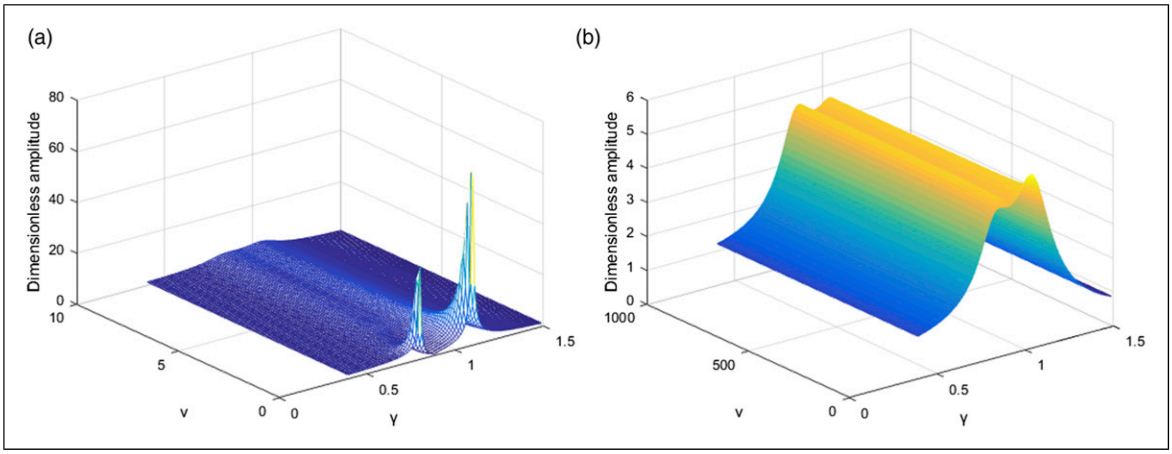

Figure 7. (a) Dimensionless amplitude of the system with non-viscous damping (ratio $0.1 \leq v_{abs} \leq 10$) (b) Dimensionless amplitude of the system with non-viscous damping (ratio $10 \leq v_{abs} \leq 1000$).

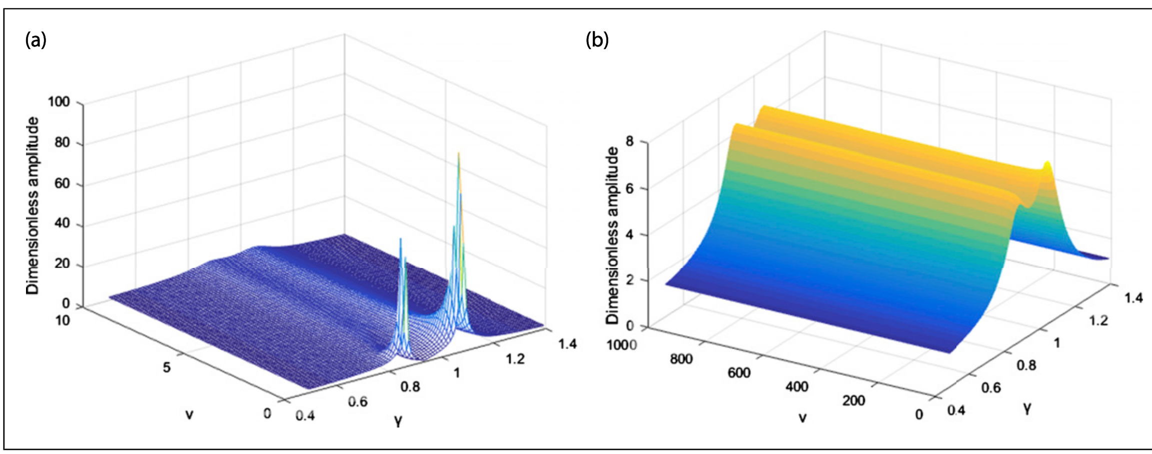

Figure 8. (a) Dimensionless amplitude of the system with non-viscous damping (ratio $0.1 \leq v_{abs} \leq 10$) (b) Dimensionless amplitude of the system with non-viscous damping (ratio $10 \leq v_{abs} \leq 1000$).

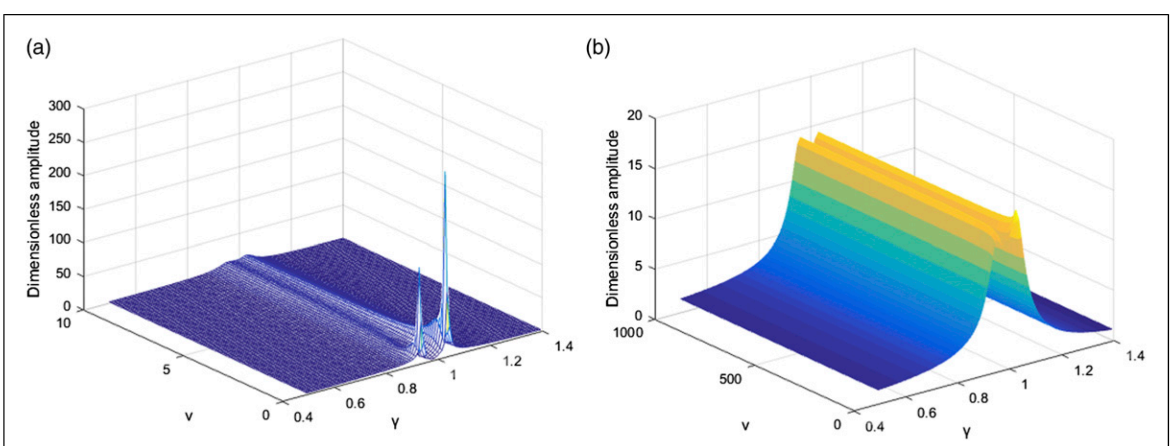

Figure 9. (a) Dimensionless amplitude of the system with non-viscous damping (ratio $0.1 \leq v_{abs} \leq 10$) (b) Dimensionless amplitude of the system with non-viscous damping (ratio $10 \leq v_{abs} \leq 1000$).

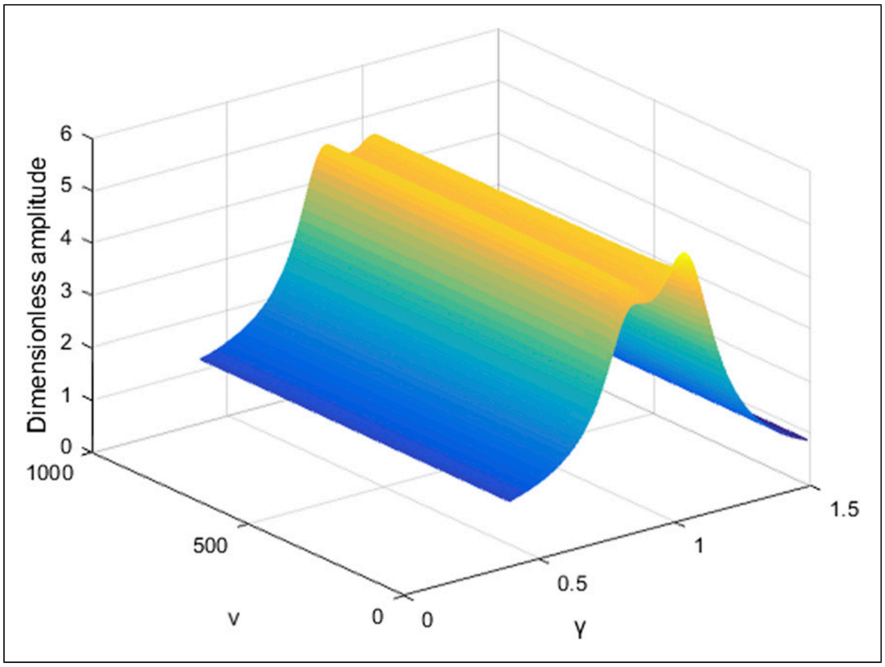

Figure 10. Dimensionless amplitude of the system with non-viscous damping (damping ratio $\zeta = 0.1527$).

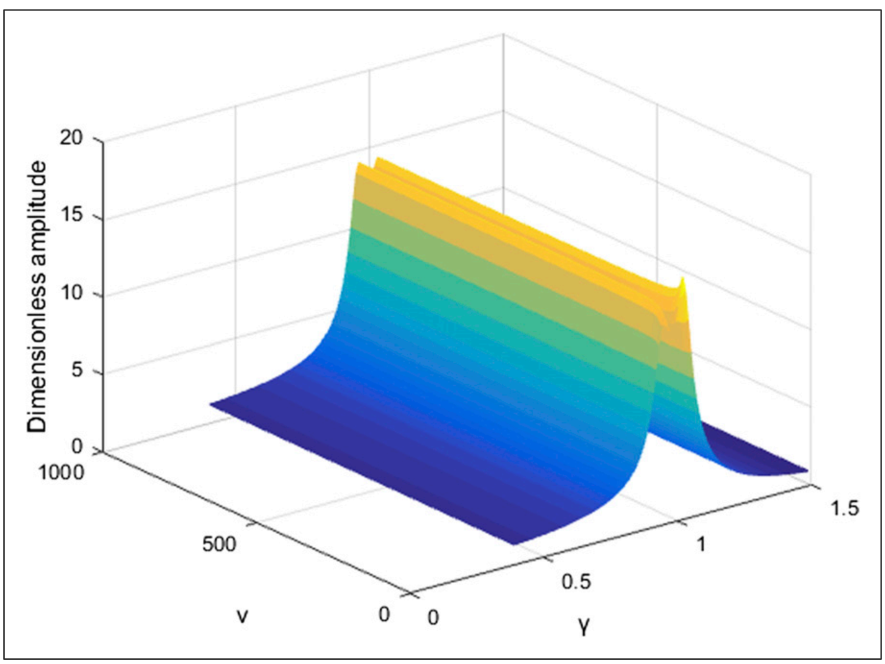

Figure 11. Dimensionless amplitude of the system with non-viscous damping (damping ratio $\zeta = 0.06$).

factor which should be considered in analysis of vibration absorber.

The damping parameters influence the dimensionless amplitude. It is difficult to distinguish from the three-dimensional figures. The large v_{abs} that is assumed to be 1000. The dimensionless amplitude of vibration absorber can be plotted in a plane figure. The dimensionless amplitudes can be obtained by equation and plotted in Figure 15 with different damping ratios. It can be seen from Figure 15 that the damping parameters influence the

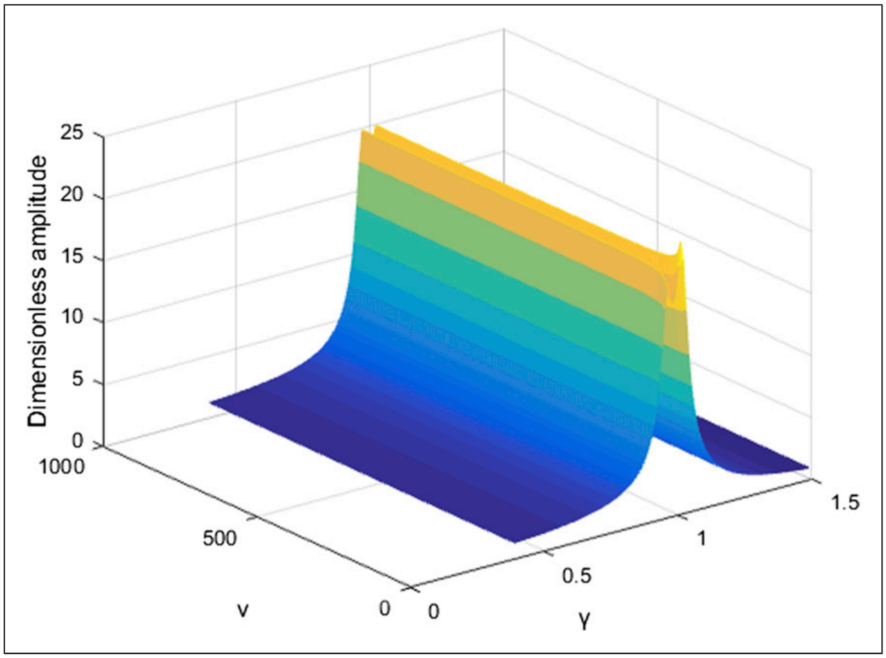

Figure 12. Dimensionless amplitude of the system with non-viscous damping (damping ratio $\zeta = 0.04$).

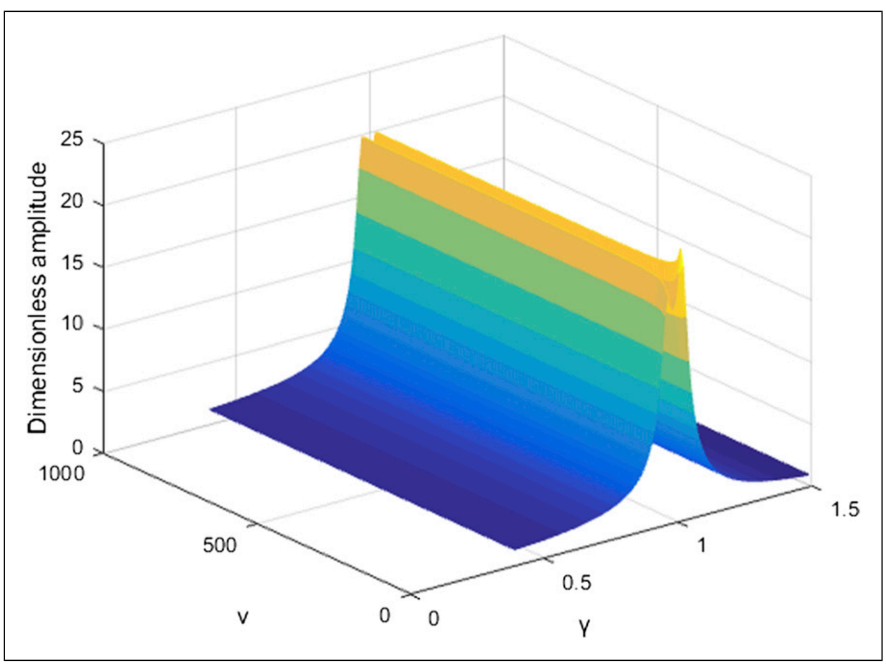

Figure 13. Dimensionless amplitude of the system with non-viscous damping (damping ratio $\zeta = 0.03$).

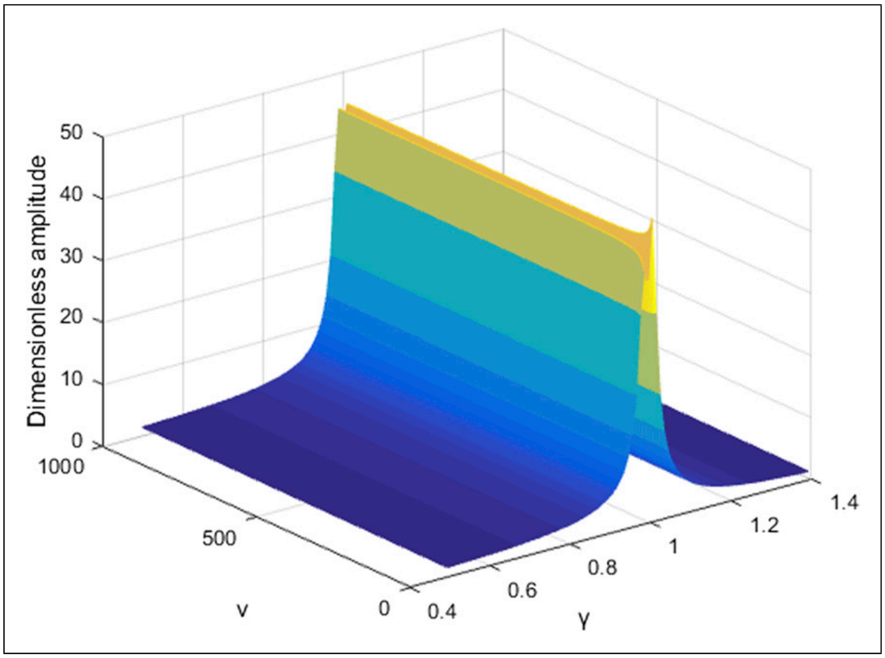

Figure 14. Dimensionless amplitude of the system with non-viscous damping (damping ratio $\zeta = 0.02$).

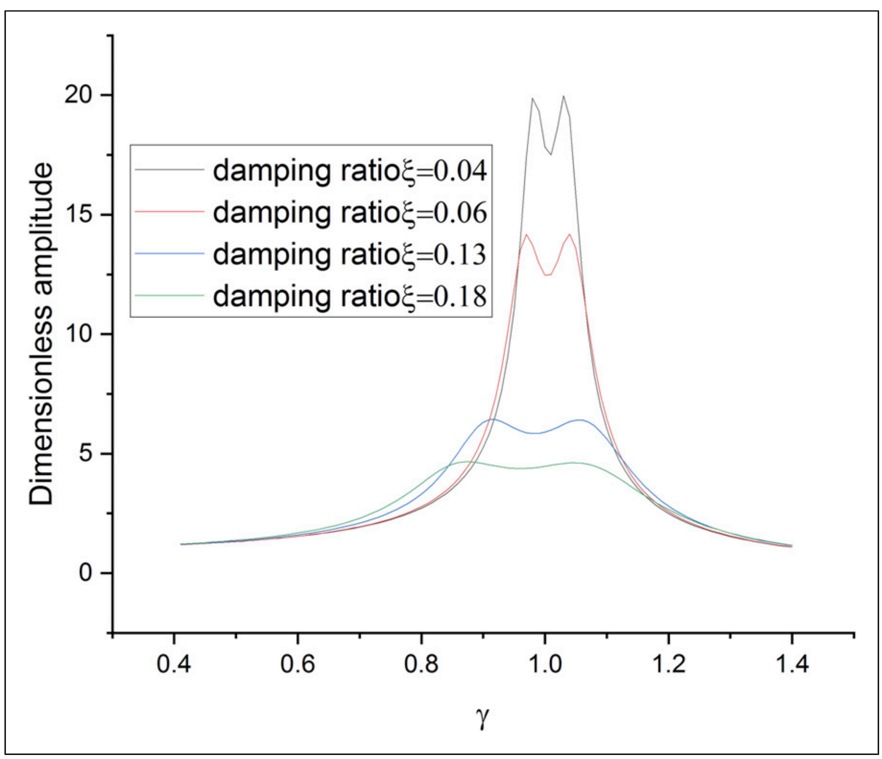

Figure 15. Dimensionless amplitude of vibration absorber with different damping ratios.

dimensionless amplitude of the exponential non-viscous damping model vibration absorber.

5. Conclusions

The vibration transfer ratio formula of the vibration isolator of the system with non-viscous damping is derived based on the Laplace transform. The vibration transfer ratio is influenced by the damping ratio, frequency ratio, and the ratio of the relaxation parameter and the natural frequency. When the system is under the resonance state, the vibration transfer ratio is significantly influenced by the ratio of the relaxation parameter and the natural frequency. The ratio is a non-negligible factor which should be considered in analysis of vibration isolator.

The dimensionless amplitude formula of the vibration absorber of system with non-viscous damping is derived based on the Laplace transform. The tuned condition still exists in the non-viscous damping system. The damping ratio also satisfies the optimized condition. The ratio of the relaxation parameter and the frequency of the external load has a significant influence on the amplitude when the ratio is small. It is necessary to avoid the small ratio. The ratio is a non-negligible factor for non-viscous damping system and it should be considered in analysis of vibration absorber.

Experimental testing of the vibration isolator and vibration absorber with non-viscous damping materials will be investigated further.

Acknowledgments

The authors would like to thank the reviewers and the instructor for their constructive comments.

Declaration of Conflicting Interests

The author(s) declared no potential conflicts of interest with respect to the research, authorship, and/or publication of this article.

Funding

The author(s) disclosed receipt of the following financial support for the research, authorship, and/or publication of this article: This work is supported by Fundamental Research Funds for the Central Universities (Grant No. B220204002), the National Natural Science Foundation of China (Grant No. 51679075) and China Scholarships Council (No. 201906710172).

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