Probable Loss Model and Spatial Distribution of Damage for Probabilistic Financial Risk Assessment of Structures

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ABSTRACT: Deficiencies in current seismic risk analysis procedures are assessed and measures to overcome these drawbacks proposed. A methodology for representing Lifetime Loss (LL), expressed in dollars, as a function of an exceedance probability is established. The methodology incorporates aleatoric variability and epistemic uncertainty in the relationship between intensity measures and engineering demand parameters, producing a probabilistic *demand model*. A continuous *probable loss model* is developed by integrating together fragility curves for damage states given demand and loss given damage state. The common form of seismic hazard relationship that specifies the annual frequency of hazard occurrence is reinterpreted in terms of probability of hazard exceedance in the service life of the structure in order to allow comparisons of seismic risk to be made based on whole life costs. Combining with the continuous probable loss model and seismic hazard model, the probabilistic *demand model* can be transformed to a probabilistic *loss hazard model*, which can be used to determine the Expected Lifetime Loss (ELL) and LL for a given exceedance probability. Recognising the fact that in MDOF systems demand parameters (which signify critical structural response) will be different for different degrees of freedom, a simplified method of evaluating ELL while giving due consideration to the spatial distribution of demand response parameters is presented. A ten-storey reinforced concrete building is used as a case study to illustrate the application of the ELL assessment procedure, the effect of spatial distribution of demand on ELL, and variation in LL with exceedance probability.

1 INTRODUCTION

The main aim of Performance Based Seismic Design is to ensure that the seismic performance of structures are satisfactory at various levels of seismic demand, and Performance Based Earthquake Engineering (PBEE) enables the assessment of seismic performance of structures and systems at different earthquake scenarios. Nevertheless, it has been identified that the decision making tools in this process should be based on more general interpretation of structural performance; such as risk and probable losses, which are more easily understood by different stakeholders of the structure. This requires not only an understanding of seismic risk and its inherent variability, but also the effective communication of the risk and all its associated uncertainties to decision makers such as owners, bankers and insurers. One useful tool that facilitates probabilistic seismic risk assessment addressing the aforementioned issues is the triple integral equation proposed by the Pacific Earthquake Engineering Research (PEER) Center (Deierlein et al. 2003). The PEER triple integral formula can be used to obtain the mean annual frequency of exceeding some decision variable (i.e. loss ratio):

$$\nu(lr) = \iint P\left(lr \middle| edp\right) \left| dG(edp|im) \right| \left| d\nu(im) \right| \tag{1}$$

in which *im* = intensity measure (e.g. peak ground acceleration, spectral acceleration); *edp* = engineering demand parameter (e.g. maximum interstorey drift); *lr* = loss ratio, the damage repair cost as a proportion of the initial cost of the structure; and $dG(x|y) = dP(x<X|Y=y)$ is the derivative of the conditional cumulative distribution function.

Equation 1 is slightly modified to that which is presented in some other literature (e.g. Deierlein, 2003) in that the (intermediate) damage measure (*dm*) variable (i.e. buckling, collapse) has been already incorporated into the loss-demand relationship. Equation 1 requires that relationships be defined between each of the above variables and can be broken into five subtasks: (i) assessment of seismic hazard ($\nu(im)$), which is covered in loading standards; (ii) analysis for structural response (*edp* vs. *im*), typically via Incremental Dynamic Analysis (IDA) (Vamvatsikos and Cornell, 2002); (iii) quantification of damage (*dm* vs. *edp*) and estimation of the loss ratio (*LR* vs. *dm*).

The Expected Annual Loss (EAL) can be obtained from Equation 1 by first converting the rate of exceedance to probability of exceedance (e.g. using the Poisson temporal model) and then integrating together different losses and there probability of occurrence:

$$EAL = \int lr \left| dP(lr) \right| \qquad (2)$$

Although there are many methods of quantifying loss (ATC 58-2, 2003), EAL provides a monetary value which is especially useful to decision makers for evaluating design solutions, retrofitting, and operational costs. The study presented in this paper has three main focuses. First attention is paid to the development of the loss-to-demand relationship (LR vs *edp*) via the use of damage states; secondly, the risk assessment is considered in terms of the Expected Lifetime Loss (ELL), which is obtained by considering Equations 1 and 2 in terms of the service life of the structure as opposed to the annual frequency. Finally, a simplified method of incorporating the spatial distribution of damage (due to variation in demands) is described.

2 DAMAGE AND LOSS MODELLING

Consideration of financial loss in structural risk analysis has often previously (Dhakal and Mander, 2006; Solberg et al., 2006) employed a discrete (deterministic) relationship between the Engineering Demand Parameter (EDP) and specified damage states (DS), which are then assigned a specified LR. The schematic diagram of the LR-EDP relationship shown in Figure 1b shows five damage states (DS1 – DS5) which are defined according to the recommendation of HAZUS (Mander and Basoz, 1999). This discrete nature of the damage model unnecessarily exaggerates the dependence of the EAL on the EDP and LR values at the DS boundaries (Robertson, 2006). By assessing the inadequacy of the current LR-EDP relationship it can be seen that a 'smoother' function is required to represent the expected LR for a given EDP, along with a probability distribution to represent the aforementioned sources of variation.

The current use of a deterministic and discrete loss model is the result of assuming that there is no uncertainty in the value of EDP at DS boundaries, and no uncertainty in the value of the LR in these DS. Figures 1a-1c present the progression from the deterministic and discrete loss model (Figure 1b) to a

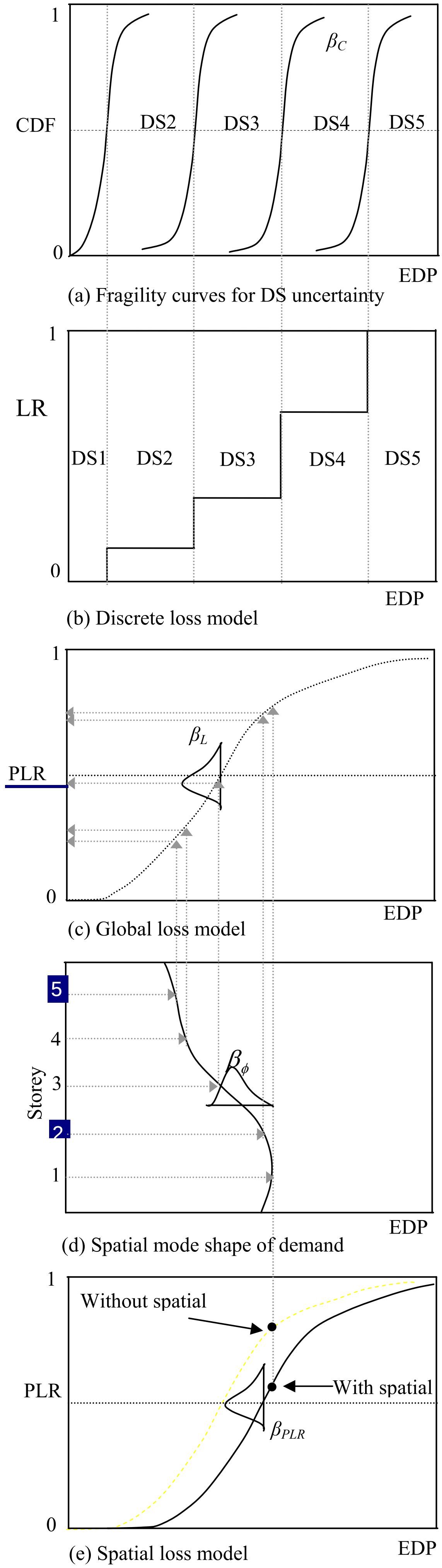


Figure 1: Probable loss model development

continuous probable loss model (Figure 1c), due to the consideration of the two aforementioned uncertainties. The approach adopted herein is to assume that onset of the various DS's and the magnitude of the LR's for these DS's are all probabilistically defined. Firstly, uncertainty in the value of EDP at DS boundaries is accounted for by the fragility curves shown in Figure 1a. As this uncertainty is related to the seismic capacity of the structure, the lognormal standard deviation (dispersion) of the fragility curves is represented by β_C. Note that β_C is comprised of epistemic uncertainty (with dispersion β_{UC}) in the response prediction modelling and the aleatoric variability (with dispersion β_{VC}) of the assumed material capacity. In order to quantify the likely variation of the capacity fragility curves as shown in Figure 1a, the two dispersions can be combined using the root-sum-squares method proposed by Kennedy et al. (1980).

$$\beta_C = \sqrt{\beta_{UC}^2 + \beta_{VC}^2} \qquad (2)$$

Secondly, the uncertainty in estimating the economic cost to repair the structure to fully recover from each DS can be accounted for by fragility curves similar to those shown in Figure 1a for the EDP values at the DS boundaries. This epistemic uncertainty related to loss estimation is represented by the dispersion, β_L.

The expected loss ratio (or Probable Loss Ratio, PLR) can be given as:

$$E[LR|EDP] = \sum_{i=1}^{5} E[LR|DS_i]P(DS_i|EDP) \qquad (3)$$

where $E[LR|DS_i]$ = mean loss for DS_i; $P(DS_i|EDP)$ = the probability of being in DS_i given EDP which can be found as the difference in the vertical height of the fragility curves for the value of EDP under consideration. It is assumed, as is done in the majority of the literature that the fragility curves are lognormally distributed. Furthermore, epistemic uncertainty in loss modelling results in variation in the value of loss ratio for a given EDP:

$$\sigma^2[LR|EDP] = E[LR^2|EDP] - \{E[LR|EDP]\}^2 \qquad (4)$$

Where $E[LR^2|EDP]$ is calculated in Equation 3 but for LR^2 as opposed to LR. This uncertainty is shown in Figure 1c.

3 SPATIAL DISTRIBUTION OF EDP

There are numerous parameters that can be used for the EDP; two common examples for multi-storey buildings are maximum interstorey drift and maximum floor acceleration which provide a reliable indication of structural and non-structural damage, respectively. Figure 1d shows an approximate profile of the maximum interstorey drifts occurring in different floors (for the typical deformed shape) of a multi-storey frame building. As the EDP's differ from storey to storey, the following question arises for such MDOF systems: Which EDP value should be used? The use of a single loss model based on an EDP such as the absolute maximum interstorey drift or maximum storey acceleration, which obviously does not occur over the entire system, will lead to conservative predictions of loss.

Two approaches are available to take into account the distribution of seismic demand and the associated damage over the structure. The first approach employs monitoring the maximum EDP occurring at various locations over the height of the structure (typically at each floor). Therefore, a vector of EDPs are monitored, and the risk assessment is carried out for each EDP separately, with consideration for correlations between EDPs. The second approach involves implicitly considering this spatial distribution of EDP and associated damage by introducing the idea of a 'demand mode shape' of the structure. The 'demand mode shape' is obtained by monitored EDP values at each degree-of-freedom (DOF) of the structure, and the maximum values of the EDP along different DOF's are normalised with respect to the maximum global EDP. While it is realized that the risk analysis could be carried out individually for each DOF and then aggregated, the proposed method allows the potential to use a simplified (i.e. Single-Degree-of-Freedom) structural model enabling a reduction in the onerous time constraints of IDA, which restricts the use of such assessments in engineering practice.

Figures 1c-1e depict the process of transforming the local loss model to the global loss via the demand mode shape. Figure 1c as explained earlier, gives a schematic illustration of the loss-edp relationship, herein referred to as the *probable loss model*. Similarly, Figure 1d schematically illustrates the "demand mode shape" typical for a MDOF frame building. The demand mode shape at a certain DOF, $\phi_{D,i}$ can be multiplied by the maximum global EDP to obtain the absolute EDP for the particular DOF, which in vector notation can be expressed as:

$$\{EDP\} = EDP_{\max}\{\phi_D\} \qquad (5)$$

The PLR for the particular DOF can then be directly obtained from the loss model relationship (Equation 4). Once the PLR is calculated for each DOF, the values are multiplied by the relative economic value represented by each DOF (i.e. weighting factor) and added to obtain the likely economic loss for the entire structure. For the case where the economic value is the same for each DOF, the global probable loss ratio can be represented as:

$$PLR_g = \frac{1}{n}\sum_{i=1}^{n} PLR_i \qquad (6)$$

where PLR_i is the probable loss ratio of DOF_i, and n is the number of DOF's.

The demand mode shape can be either approximated based on designed structural response or can be directly obtained from the IDA output. There will also be some uncertainty associated with the demand mode shape (due to higher modes etc.), which is denoted in Figure 1d by β_φ. When the mode shape is obtained from IDA output, one could calculate the dispersion of the IDA data from the mean demand mode shape. If rapid methods that do not employ IDA are used, then the dispersion could be estimated based on judgement and best practice. The additional uncertainty due to the variation in the demand mode shape, β_φ, and loss ratio, β_L, can be incorporated (assuming no correlation) using the root-sum-squares method to obtain:

$$\beta_{PLR} = \sqrt{\beta_L^2 + \beta_\phi^2} \qquad (7)$$

4 SEISMIC HAZARD RELATIONSHIP

In order to compute EAL, a relationship between hazard intensity (IM) and annual rate of occurrence (f_a) is required. This relationship, termed the hazard-recurrence relationship, can be obtained from Probabilistic Seismic Hazard Analysis. The hazard curve can be approximated by fitting a log-log linear curve through two known points, usually the events corresponding to 2% and 10% probability of occurrence in 50 years (Solberg et al, 2006). Due to the typical "concave downward" shape of a site-specific seismic hazard curve, the use of a linear log-log relationship, though conservative, is not strictly correct, particularly when $f_a >> 0.01$ (T < 100 years). It is to be noted that the rate of occurrence and probability of occurrence are numerically very similar for small probabilities, but diverge as the rate increases, since probabilities are bounded by the maximum value of 1. As a result, previous studies (Solberg et al, 2006) have disregarded damage incurred by earthquakes with annual frequencies above a certain threshold. The determination of this threshold frequency requires subjective judgement which may not always be trivial, and as EAL has been shown to be sensitive to damage associated with high frequency events (Robertson, 2006), the hazard recurrence model in its present form is not ideal. Issues have also arisen regarding the size of EAL values obtained, which are typically in the order of 0.001 ($1000 per million dollars of asset value) based on assessment of structural damage. Unless contribution of non-structural damage, downtime and death are quantified and included in the total loss, the EAL values may be too small for decision makers to grasp and appreciate their significance.

Therefore, this paper considers the hazard-recurrence model in terms of the occurrence in the service life of the structure, as opposed to the annual occurrence, which can be related by:

$$p = 1 - \left(1 - k_0 IM^{-k}\right)^T \qquad (8)$$

where p = the probability of occurrence over a lifetime of T years; T = the service life of the structure; and k_0, k = parameters from the conventional log-log hazard recurrence relationship. As mentioned by Der Kiureghian (2005), the implicit assumption adopted here that all random variables are ergodic is not strictly correct and results in minor errors for probabilities in excess of 0.01, with increasing error as probabilities increase. This minor error is neglected herein considering other sources of error introduced primarily by epistemic uncertainty which is difficult to quantify.

The modified seismic risk assessment methodology using the new hazard-recurrence interpretation would then yield an Expected Lifetime Loss (ELL) as opposed to EAL. ELL values can then be expressed as the financial risk to the structure over its service life.

5 COMPUTATIONAL IDA-ELL THEORY

The computational ELL estimation procedure is an enhanced version of methods being used to calculate EAL (Solberg et al, 2006; Robertson, 2006). The overall computational ELL assessment methodology is schematically shown in Figure 2. From the IDA data it is possible to produce IDA curves (IM vs. EDP plots) of constant exceedance probability, using a procedure called spectral reordering to sort data, assign confidence limits, and incorporate further sources of randomness and variability in the seismic demand (Solberg et al, 2006). The IM and EDP axes of this IDA probability plot can be transformed to service life frequency of occurrence and probable loss ratio, by respectively using the hazard-recurrence relationship (Equation 8 or Figure 2b) and probable loss model (Figure 2c). For a typical structure of service life 50 years (T=50) in a high seismic region of New Zealand, k_0 =0.00013 and k =3 (assuming PGA is the IM). Current best practice uses the 5% damped spectral acceleration at the period of the first translational mode of vibration (T_1) as the IM, which can be related to the PGA by S_A = PGA/T_1, assuming that it lies on the constant velocity region of the design response spectra. Thus, hazard loss curves for different exceedance probabilities can be obtained as shown in Figure 2d, the integration of which gives the Lifetime Loss (LL) with the prescribed exceedance probability. The ELL (equivalent to the mean LL) can then be calculated by integrating the volume enclosed beneath the percentile LL curves, i.e. Figure 2d where the 'out-of-the-page' axis corresponds to the exceedance probability of each percentile curve.

6 CASE STUDY: REINFORCED CONCRETE MOMENT-FRAME BUILDING

The generic methodology established so far is next applied to assess the financial implications of earthquake hazard exposure to a ten-storey reinforced concrete moment resisting frame building.

The ductile RC building in this case study is the well-known "Red Book" building (Bull and Brunsden, 1998), which serves as an example of multi-storey building frame design using capacity design principles of New Zealand concrete structures standard (Standards New Zealand, 1995). A full scale 3D computational model of the prototype building was conceptualised using the finite element program Ruaumoko3D (Carr, 2004). Detailed information on the case study building and modelling can be found elsewhere (Bradley et al, 2006).

A suite of 20 earthquake records obtained from the SAC steel project archive (SAC, 1995) were adopted for conducting the IDA. The suite of ground motions had a median source distance of 9.0 km, magnitude of 6.9, and spectral acceleration of 0.60g at the fundamental time period of the building. IDA was implemented using 3D time-history analyses with the scaled records applied to the buildings base in orthogonal directions. The IM selected for this case study was the 5% damped spectral acceleration at the fundamental time period of the model, which was 1.6 seconds, i.e. IM=S_A (T_1, 5%). The EDP was selected as the maximum interstorey drift (i.e. EDP=θ_{max}) for the analysis without spatial demand consideration. The interstorey drift for each floor

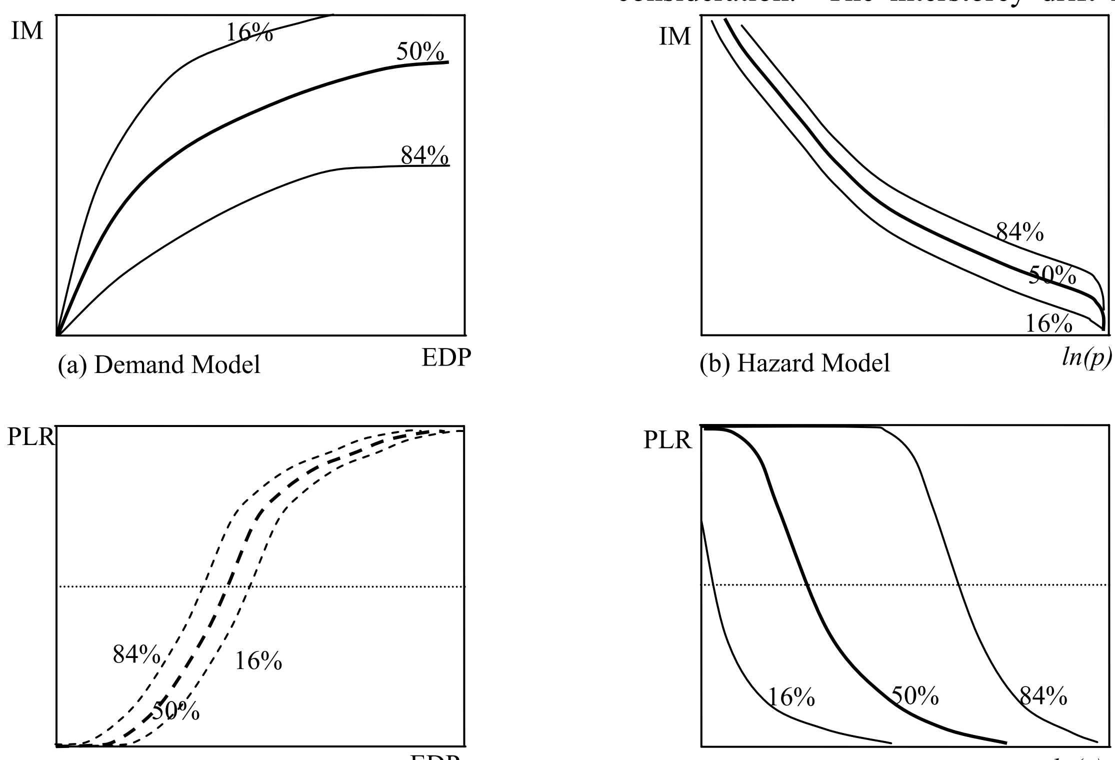


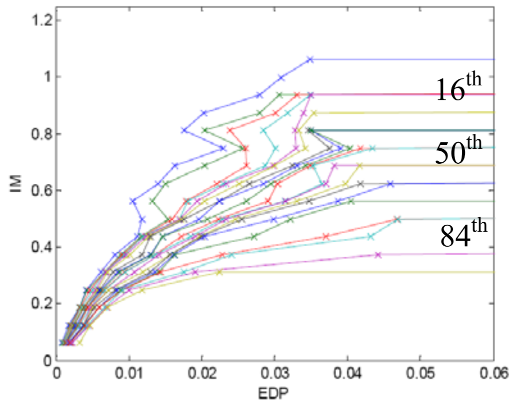
Figure 2: Compuational Risk Assessment Methodology

was used when the spatial variation was considered.

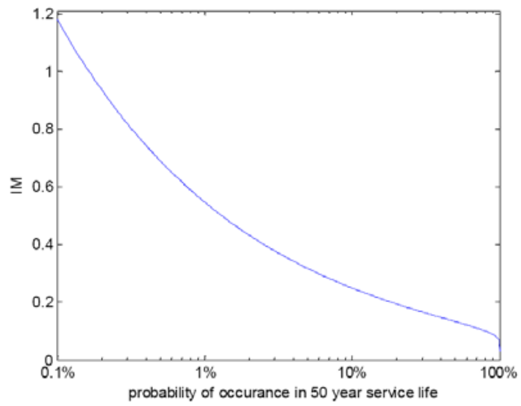
The steps presented in the probabilistic seismic financial risk assessment procedure outlined in the previous section were then conducted for the aforementioned ten-storey building. The following dispersion parameters were adopted based on the recommendations of FEMA 350 (2000), $\beta_{UC}=\beta_{UD}=0.18$ (i.e. $\sqrt{\beta_{UC}^2+\beta_{UD}^2}\approx$ 0.25); $\beta_{VC}=\beta_{VD}=0.14$ (i.e. $\sqrt{\beta_{VC}^2+\beta_{VD}^2}\approx$ 0.2). Uncertainty in the adopted LR values at the DS boundaries (β_L) was rationally taken as 0.1, while as previously mentioned β_{VH}, β_φ are obtained directly from IDA data and the uncertainty in the hazard-recurrence relationship was omitted as loading standards are silent on the variation of the given relationships and no other basis exists to assume a rational value of the dispersion of this relationship.

The procedures without considering spatial distribution are shown in Figure 3. First the IDA curves were spectrally re-ordered and re-scaled to produce the percentile IDA curves shown in Figure 3a. Next, assuming that the building was designed for a high seismic zone in New Zealand, the conventional seismic hazard-recurrence relationship based on annual frequency of seismic hazards was modified in terms of probability of hazard occurrence in the lifetime of the building; i.e. 50 years. This can be represented by Equation 8, which is plotted in Figure 3b. Then, the probable loss model for the building was formed by combining the discrete loss model with the uncertainties in identifying damage states and assigning corresponding losses, as explained by Equations 3 and 4 and illustrated in Figure 1. Thus the obtained final probable loss model for the building is shown in Figure 3c. Finally, the modified hazard-recurrence relationship and probable loss model were used to transform the percentile IDA curves to percentile loss-hazard curves depicted in Figure 3d.

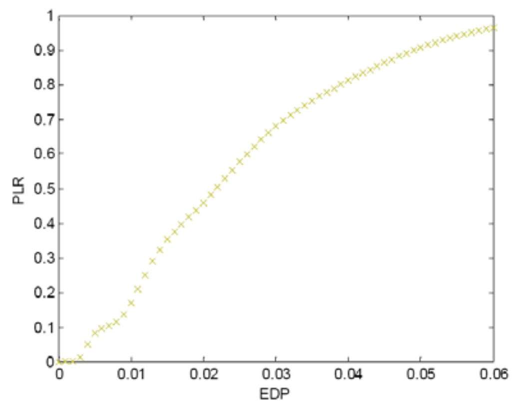
When spatial distribution was considered and the EDP was monitored at each DOF, ten IDA curves for the ten pairs of orthogonal ground motion records, similar to those in Figure 3a, could be plotted for each DOF. Nevertheless, the variation of the IDA curves in the ten floors could be completely described by the maximum global IDA curves of Figure 3a combined with the demand mode shape shown in Figure 4. Each data point on the demand mode shape refers to the ratio of the EDP at a specified floor (DOF) with respect to the maximum EDP for one ground motion record. Hence, the 400 data points for each DOF correspond to the 20 ground motion records used at 20 different IM's. The continuous curve joins the median value of the demand mode shape at each ordinate. The associated dispersion of the data points about the median curve (β_φ) was found to follow a lognormal distribution based on the Kolmogorov-Smirnov test (Benjamin and Cornell, 1970) with a beta value of 0.10. The global



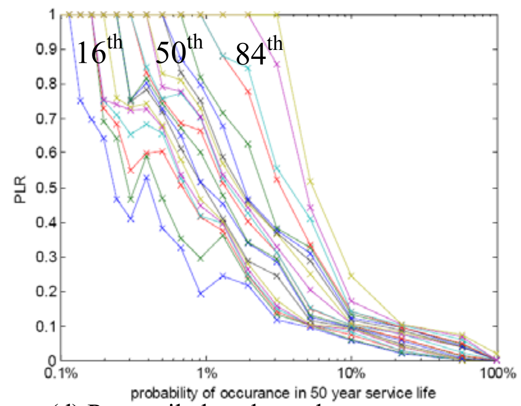
(a) Spectrally reordered and rescaled IDA curves



(b) Hazard recurrence model



(c) Probable Loss model



(d) Percentile loss-hazard curves

Figure 3: Case study risk assessment results

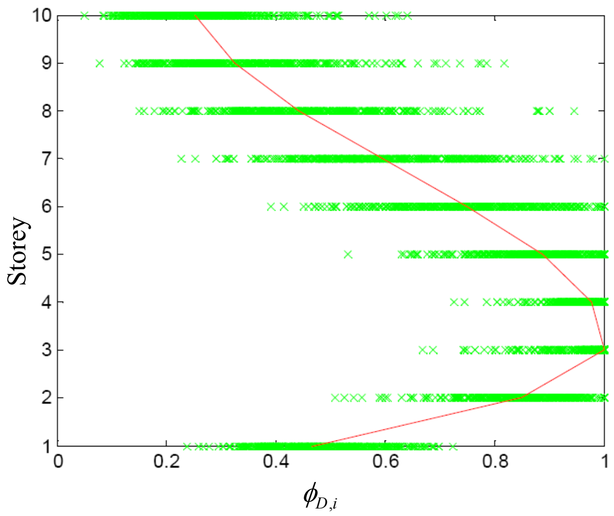


Figure 4: Demand mode shape from IDA

LR for each line in the re-ordered and re-scaled IDA curves of Figure 3a (these lines correspond to a contour of survival probability) was obtained from Equation 6 using the aforementioned method of combining the LR's at each DOF. This consideration of spatial distribution applied to the ten IDA curves produced a percentile LR vs. IM relationship, which was then converted to loss-hazard percentile curves, as shown in Figure 5 via the hazard-recurrence relationship (Equation 8). As expected, it can be seen that the loss-hazard curves when considering spatial distribution are shifted to the left compared to those without considering spatial distribution. This distinct change in the shape of the percentile loss-hazard curves with spatial distribution compared to the curves obtained without considering spatial distribution (Figure 3d) can be attributed to global collapse.

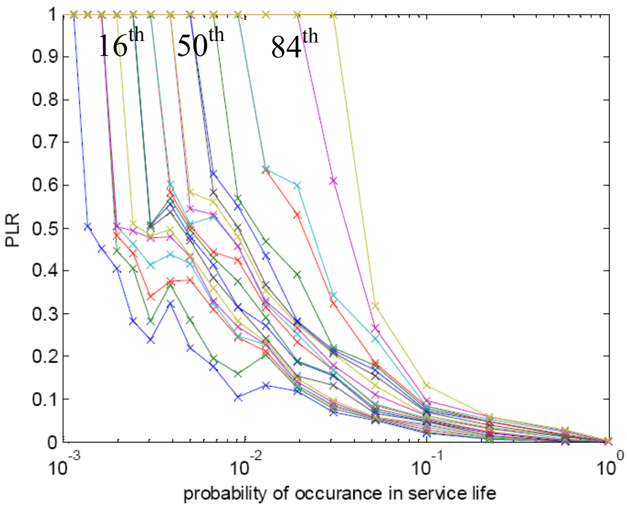


Figure 5: Loss-hazard curves considering spatial variations

Integration of these curves results in LL values corresponding to different exceedance probabilities. Figure 6 compares the plots of the probabilistic LL values with and without considering spatial distribution. As the x-axis scale is logarithmic, even though the two curves appear close the corresponding median LL values are quite different; i.e. 2.9% and 5.7% of the economic value of the building, with and without considering spatial distribution, respectively. This difference causes a 49% reduction in the median LL when considering spatial distribution. With reference to Figure 6 it can be seen that LL can be determined for any non-exceedance probability. For example, it can be stated that when considering spatial distribution there is a 90 percent non-exceedance probability that the ELL will not exceed 6.5%. The authors believe that defining the LL in terms of a non-exceedance probability is advantageous i.e. it allows decision makers such as insurers to assign competitive premiums while ensuring that based on probability theory they will on average make a profit to ELL only. Finally, loss hazard curves were integrated to calculate ELL, which yielded 3.5% and 6.4% loss (a reduction of 45%) for the building with and without considering spatial distribution, respectively. Dividing these values by the service life of the building (i.e. 50 years) allowed approximate values of EAL to be estimated as

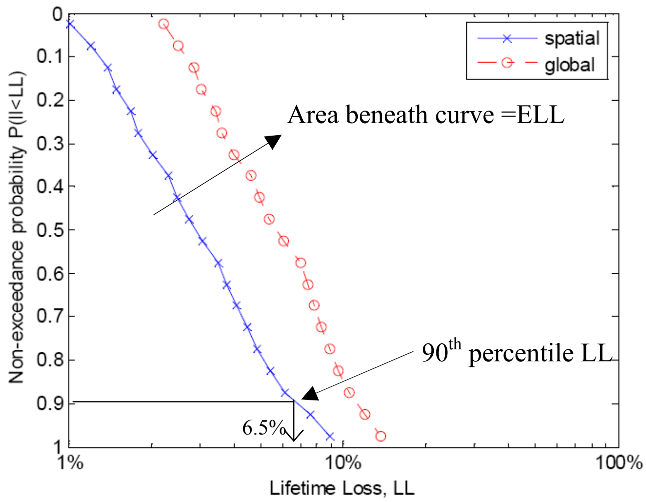


Figure 6: Loss-hazard curves considering spatial variations

\$700/\$1 million and \$1300/\$1 million with and without considering spatial distribution, respectively.

7 CONCLUSIONS

Based on the findings of this research, the following conclusions can be drawn:

1. An approach to define Lifetime Loss (LL) for different non-exceedance probabilities has been established.

2. A continuous probable loss model, which provides a probabilistic formulation that is more consistent than the existing discrete damage and loss models, has been developed by taking into account the probabilistic variations from numerous sources.
3. A method of estimating seismic risk considering effects of spatial distribution of damage has been established for application in MDOF systems. For the ten-storey RC ductile building chosen for the case study, the reduction in ELL was shown to be 45% when spatial distribution of demand was taken into account.

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