NEW ZEALAND INSTITUTE FOR THE STUDY
OF COMPETITION AND REGULATION INC.

Report Prepared for the New Zealand
Ministry of Agriculture and Forestry

Forest and Forest Land Valuation: How to Value Forests and Forest Land to Include Carbon Costs and Benefits

Richard Meade
Gabriel Fiuza
Andrea Lu

With Glenn Boyle and Lewis Evans

August 2008

Contents Page

Exchange rates as at 30 June 2008:

1 NZD = 0.79 AUD = 0.48 EUR = 0.38 STG = 0.76 USD = 80.98 YEN

Abbreviations

BROA	Bootstrapping Real Options Analysis
CO_2	Carbon Dioxide
CP1	First Commitment Period under the Kyoto Protocol, being 2008-12 inclusive
DCF	Discounted Cash Flow
ETS	New Zealand Emissions trading Scheme
EU ETS	European Union Greenhouse Gas Emissions Trading Scheme
GBM	Geometric Brownian Motion (a form of statistical process)
MAF	Ministry of Agriculture and Forestry
MCS	Monte Carlo Simulation
MR	Mean reversion
NPV	Net Present Value
NZU	New Zealand Unit
OU	Ornstein-Uhlenbeck (a form of mean-reverting statistical process)
RMA	Resource Management Act
ROA	Real Options Analysis
SO_x	Sulphur Oxide
tCO_2	Tonnes of CO_2
TRV	Total Recoverable Volume
TSV	Total Stand Volume

Descriptions of Key Variables

See Appendices B.1, B.2 and D for further variable definitions and details.

Alpha	Parameter that determines the height/profile of a posited forest or forest land use decision boundary, and whose optimal value is determined using "bootstrapping" Monte Carlo Simulation
Beta	Additional parameter (if required) determining the height/profile of a posited forest or forest land use decision boundary, and whose optimal value is jointly determined (along with Alpha's) using "bootstrapping" Monte Carlo Simulation
T*(i)	First date in BROA simulation trial i in which a forest or forest land use decision is crystallised
T**(i)	Second date in BROA simulation trial i in which a forest or forest land use decision is crystallised, with T**(i) = T*(i) + 27 years assumed for illustrative purposes
T_{max}	Assumed latest feasible forest harvest date
T_{min}	Assumed earliest feasible forest harvest date

Acknowledgements

The authors gratefully acknowledge the support and assistance of a number of MAF personnel. Particular thanks go to John Eyre, Alice Marfell-Jones, Rod Forbes, Loretta Dobbs, Robyn Hampton, Peter Nichols, Chris Pugsley and Tracy MacDonald.

Other organisations and individuals have also provided helpful data and discussions. They are, in alphabetical order of organisation: Luke van Velthooven of Abel Properties, Eli Grace-Webb of Castalia, Paul Morgan of Federation of Maori Authorities, Stuart Frazer of Frazer Lindstrom, Ola Carlen of the Journal of Forest Economics, Steve Wakelin of Scion Research, Euan Mason of the New Zealand School of Forestry, University of Canterbury, and Graeme Guthrie of Victoria University of Wellington.

Helpful comments on a draft of the report were received from Gerard Horgan of MAF.

Our apologies for any worthy mentions inadvertently not made. As always, any errors or omissions in this report are not the responsibility of these parties.

Author Contacts

Richard Meade	New Zealand Institute for the Study of Competition and Regulation Victoria University of Wellington Cognitus Advisory Services Limited *Correspondence to*: richard.meade@cognitus.co.nz
Gabriel Fiuza	New Zealand Institute for the Study of Competition and Regulation Victoria University of Wellington
Andrea Lu	New Zealand Institute for the Study of Competition and Regulation Victoria University of Wellington
Glenn Boyle	New Zealand Institute for the Study of Competition and Regulation Victoria University of Wellington
Lewis Evans	New Zealand Institute for the Study of Competition and Regulation Victoria University of Wellington

Important Notice

In preparing this report the authors have relied upon public information, and information provided by MAF and others, the accuracy or completeness of which has not been verified. No warranty or indemnity, express or implied, is given by the authors or their related institutions, to any party, as to the accuracy or completeness of this report, and no liability is accepted by the authors or their related institutions for any losses or damages incurred by any party relying on this report. The views expressed in this report should not be taken to represent those of the institutions to which the authors are affiliated, or of any clients for whom either of the authors have acted or are currently acting that have an interest in any of the matters addressed in this report. Nor does this report purport to offer any advice to any party as to the merits or otherwise of investing in a forest, forest land or any other asset or investment (in respect of such investment decisions independent advice should be sought from a competent party).

1. Introduction and Summary

1.1 Purpose of the Report

New Zealand has introduced legislation to implement the world's first "all sectors, all gases" emissions trading scheme (ETS) as a way to reduce the country's greenhouse gas emissions. The Scheme is to retrospectively introduce a price for carbon emissions in forestry from 1 January 2008, and will phase in other sectors over time (notably, agriculture from 2013). It is therefore timely to consider what impact the price of carbon might have on forest and forest land value. The New Zealand Ministry of Agriculture and Forestry (MAF) has commissioned the New Zealand Institute for the Study of Competition and Regulation (ISCR) to develop a methodology for assessing this impact.

For illustration purposes ISCR has developed such a valuation methodology, for the following:

- The main commercial exotic forest species in New Zealand – *pinus radiata*;
- A representative one hectare forest of a single age class;
- Dairy land conversion as a potential alternative to retaining forest land in forestry;
- A valuation date of 2 January 2008 (being one day after the anticipated retrospective introduction of forestry into the ETS).[1]

The methodology that has been developed is a "partial equilibrium" approach, in that it does not account for changes in the general economy arising from changes in forest and forest land management predicted by our approach. Nor does it seek to account for all changes in the general economy arising under carbon pricing which may indirectly affect forest or forest land management.

Finally, this report is intended for audiences familiar with existing forest and forest land valuation practices. For example, we assume readers are familiar with existing valuation techniques such as comparable sales analysis, and discounted cash flow (DCF) analysis. We also assume that readers are familiar with the key features of the ETS, and so leave a summary of its key features in respect of forestry to Appendix A.

---

1 The choice of 2 January instead of 1 January is to avoid issues of whether or not a one-off free allocation of emission rights to forest landowners under the ETS affect forest or forest land value. Purchasers of certain ("pre-1990") forest land who bought their land after ETS policies were announced but before 1 January 2008 may have factored into their purchase price some allowance for such an allocation (analogous to an investor buying a share "cum dividend"). However, since any such entitlement is attributable to the owner of the relevant land as at 1 January 2008, that entitlement is no longer attributable to purchasers from 2 January.

1.2 The Valuation Problem

In this report a methodology is proposed for valuing both post-1989 and pre-1990 forests and forest land. Post-1989 forests are those first planted after 31 December 1989, in which case the ETS provides that owners of such forests can apply to earn tradable property rights called New Zealand Units (NZUs) as their forests sequester carbon from 1 January 2008. This adds a new source of potential revenue to such forests. Conversely, if they enter their forests into the ETS, such foresters must also surrender NZUs, each of which represents an entitlement to emit one tonne of carbon dioxide (tCO_2), following harvest or other cause of carbon loss (e.g. fire). If they do not already possess such NZUs, then they must acquire them (or equivalent units under the Scheme) at the prevailing market price, which introduces a cost of carbon emissions from the activity of harvest. If the owner of a post-1989 forest does not elect to enter their forest into the ETS, then they can manage their forest as if they faced no carbon price, save for possible increases in compliance and monitoring costs, and also likely increases in fuel and fertiliser costs through the ETS' impacts on other sectors. Furthermore, forest value is unlikely to remain the same as that without an ETS, if only because the ETS will affect the forestry sector in general, and as other countries also implement such schemes the international economics of forestry may alter.

Pre-1990 forests are those that were first planted in forests before 1990 and were not deforested before 2008. Under the ETS such forests do not qualify to earn NZUs as they sequester carbon, but nor does harvest trigger an obligation to surrender NZUs. However, owners of pre-1990 forest land will face a liability to surrender the requisite amount of NZUs if they convert their land into a non-forestry use (i.e. deforest their land), which affects the value of land use options attaching to such land.

The ETS is being introduced at a time when there is considerable uncertainty regarding the future course of NZU prices, and in the detail of international rules that are to succeed the Kyoto Protocol from 2013. Such uncertainty may have an impact on the optimal decisions of forest (land) owners to plant or harvest their forests, convert their forest land into alternative uses, or enter their post-1989 forest into the ETS.

1.3 Summary of Possible Valuation Approaches

Three potential valuation methodologies are available for assessing the impact of carbon prices on forest and forest land value:

- **Comparable sales analysis** – using transaction data to infer the "market's" assessment of carbon pricing on value. Due to limited and heterogeneous sales evidence, and because other techniques are commonly required (at least to value forests) even without the complication of carbon pricing, this approach is likely to be of limited use. With the passage of time, and longer sales series and sales data benefitting from the resolution of key ETS-related uncertainties, this approach may be of more use in the future.

- **Discounted Cash Flow (DCF) analysis** – involves discounting expected cash flows at an appropriate discount rate. This is a common valuation approach used when comparable sales data cannot be relied upon, or to complement comparable sales analysis. It has the merit that it is relatively simple to implement, though this simplicity in part reflects strong assumptions made under the approach regarding the treatment of uncertainty in key state variables. An important limitation of the approach, however, is that it makes strong assumptions about how the forest being valued will be managed over time. In particular, no allowance is made for flexibility to alter forest management in light of new information arising over time. Where decisions such as harvest are irreversible, and key state variables are uncertain, such flexibility adds value which static DCF analysis ignores. With NZU prices introducing new and uncertain returns/costs to forestry – on top of the uncertainty foresters already face through volatile log prices – ignoring such flexibility could seriously bias value estimates (downwards). Additionally, to value ETS impacts on pre-1990 forestry requires assessment of the Scheme's impact on land use change options, to which DCF analysis is ill-suited.

- **Real Options Analysis (ROA)** – extends DCF analysis by explicitly accounting for the value of "real options" afforded to asset owners when they have flexibility to defer/time irreversible investment decisions (such as harvesting a tree) in the face of uncertain state variables (i.e. log, NZU and land conversion values). By doing so ROA explicitly accounts for uncertainties that DCF overlooks. In principle this is the preferable approach to valuing forests and forest land, even without the added uncertainties created by the ETS.

In practice, however, implementing ROA can be challenging even for relatively simple managerial flexibility and few uncertain state variables. This necessitates a trade-off between model sophistication and solution complexity. In this report we present an approach which we have called Bootstrapping Real Options Analysis (BROA). This approach adapts the powerful Monte Carlo Simulation solution technique that is often used where other techniques are not tractable. The "bootstrapping" aspect of the method is that simulation is used to determine the optimal solution by trial and error.[2] The resulting valuations are approximate only, meaning that in general they will not

---

2 The definition of "bootstrap" in the tenth (1999) edition of *The Concise Oxford Dictionary* includes an explanation of the expression to "pull oneself up by one's bootstraps", namely to "improve one's position by one's own efforts". The trial and error nature of the BROA approach means it might also be regarded to be "heuristic", meaning "proceeding to a solution by trial and error or by rules that are only loosely defined." Our use of the term "bootstrapping" is to be contrasted with the conventional statistical use of the term, which refers to improving estimates by using repeated sampling.

be the “true” (unknown) valuations. They will likely, however, present a lower bound on value (provided they represent a feasible solution), and improve on a static DCF analysis by allowing for the value of real options in forestry.

1.4 Outline of Preferred Valuation Approach

BROA involves positing an optimal management rule for the forest (land) to be valued. That management rule will be parameterised by one or more parameters (in this report called “alpha” and “beta”). Simulation is then used, given assumptions about the way in which key state variables evolve randomly over time, to determine the optimal value of the forest given the posited management rule and given values of “alpha” and “beta”. The simulation model is then re-run for different values of those two parameters until the highest achieved valuation is produced, at which point the optimal values of those parameters are determined. This then identifies the BROA forest (land) valuation. The bootstrapping approach can also be implemented by trialling different forms of management rules, in addition to values of the parameters applying to those decision rules.

1.5 Implementation of Preferred Valuation Approach

The BROA approach has been implemented using illustrative assumptions for a one hectare *pinus radiata* forest. While we have endeavoured to implement the approach using generally realistic assumptions, the valuation of any given forest or forest land asset will necessitate the use of actual data. The approach has been implemented using free open-source software called “R”. This is useful for model development purposes, and to communicate the approach to readers. However, in practise the BROA approach is likely to be most easily and efficiently implemented in most instances using commercial spreadsheet software such as Microsoft Excel, and commonly used simulation add-ins for such spreadsheets like Palisade’s @RISK or Oracle’s Crystal Ball.

1.6 Results of Preferred Valuation Approach

The BROA approach appears to perform as expected. It was tested against a simple option pricing problem with a known solution and produces valuations consistent with that solution. In respect of forest and forest land valuation the model suggests:

- If pre-1990 forests have no conversion potential, then their value is invariant to average NZU prices;

- As average NZU prices rise, the value of a post-1989 forest rises also;

- Where pre-1990 forest land does have alternative use potential, the value of that potential falls as average NZU prices rise, and rises as the value of alternative land use rises.

- The probability of a pre-1990 or post-1989 forest being harvested where there are no land conversion options is 95-98% or more in almost all scenarios modelled, except for post-1989 forests when average NZU prices are $50/tCO_2$ or more, in which case the harvest probability can fall to almost 25% (indicating either "carbon farming" or forest abandonment being preferable at such average NZU prices);

- The probability of a pre-1990 forest being harvested and converted declines as average NZU price rises or as the value of alternative land use falls;

- Average harvest age for a pre-1990 forest declines as the value of land conversion rises.

1.7 Areas for Further Research

The BROA model presented in this report is a significant step in applying real options technology to valuing forests and forest land under carbon pricing. It combines a number of building blocks that can be adapted to valuing a variety of forest or forest land permutations. Other future refinements include:

- Valuing the option to enter Post-1989 forests into the ETS;
- Modelling alternative dairy conversion approaches;
- Modelling NZU trading/hedging strategies;
- Modelling a reforestation option following land conversion;
- Modelling alternative conversion counterfactuals;
- Modelling alternative uses of biomass;
- Extension to mixed-age forests;
- Modelling the impact of policy uncertainty; and
- Modelling subsequent land-use decisions using analytical models.

Each of these, and more, is discussed in Section 7.

1.8 Conclusions

The BROA approach complements existing comparable sale and DCF approaches to valuation. It captures the value of real options in forestry, including that part of those real options attributable to carbon pricing while retaining as much tractability as possible, given the complexity of the valuation problem. It likely produces a lower bound estimate on option value, and improves on less sophisticated techniques, while leaving room for future work to produce more accurate valuation results.

2. The Valuation Problem

2.1 Existing Valuation Approaches

Common practise is to value forest land by reference to comparable land sales data where available. Implicitly such data includes premiums for the ability to convert land into alternative land uses, or for other forms of flexibility in managing the relevant land (e.g. to plant a forest and decide when best to harvest). Where market transactions involving forest land are conducted in an efficient market between knowledgeable parties aware of potential ETS-related imposts or new revenue streams there should be some expectation that the agreed sales prices reflect such imposts or revenue streams. This of course begs the question as to how those imposts or revenue streams are evaluated by the vendor and buyer. Without some explicit decision framework it is possible that the agreed prices simply reflect a measure of ignorance about ETS-related impacts. Furthermore, such evidence becomes of limited use if ETS-related rules change in a way not anticipated at the time previous sales transactions were completed.

An additional complication is that comparable land sales transactions may be few and far between where forest land sales involve heterogeneous land blocks. This is particularly relevant when attempting to value forests (rather than forest land) using sales evidence. Forests can differ markedly in terms of key variables such as age, silvicultural regime and species, meaning sales evidence can be of limited comparability, and other techniques are required.

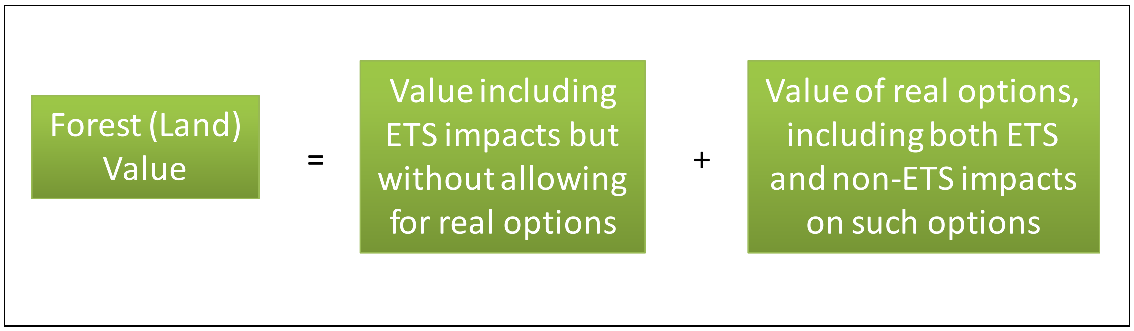
Where comparable sales evidence is unavailable or simply unreliable, the technique most commonly used to value either forests or forest land is discounted cash flow (DCF) analysis. This technique involves projecting expected net cash flows from the forest or forest land, and discounting those cash flows at some suitable discount rate (reflecting the riskiness and time profile of those cash flows). Often comparable sales evidence is used to complement DCF valuation, providing an alternative check on value,

One limitation of this approach is that – unlike comparable sales analysis – the DCF approach does not inherently allow for any ability to convert land into alternative land uses, or for flexibility in forest or land use (such as flexibility to choose when to harvest a forest in response to information available at the future decision date). Moreover, it can be difficult to model such flexibility within the DCF framework, since it is based around expected cash flows and does not explicitly account for variability in future cash flows, or possible future changes in forest and land use decisions in response to new future information.

Even without an ETS or alternative form of carbon pricing, any forest or forest land valuation approach would desirably allow for the value impact of any flexibility in forest or land use management – so-called "real options". With the introduction of the ETS or alternative form of carbon pricing, being able to model such impacts becomes particularly important. This is particularly so for pre-1990 forests (see below), in which case extensions of the standard valuations approach should be considered. The most common such approach – Real Options Analysis (ROA) – is discussed further in Section 3. The key feature of this approach is that it extends DCF analysis by explicitly allowing for the value impacts of managerial flexibility when key state variables (such as log and NZU prices) are uncertain. Such flexibility is valuable when forest or forest land use decisions are not costlessly reversible, which is clearly the case for forest harvest, and also for forest land use change where land conversion and/or deforestation costs are sunk. These ideas are illustrated schematically in Figure 2.1.

Figure 2.1

Schematic of Forest (Land) Valuation – Real Options and Carbon Pricing



The balance of Section 2 sets out more of the relevant valuation context, which then informs the discussion of alternative valuation approaches in Section 3.

2.2 The New Zealand Emissions Trading Scheme (ETS)

2.2.1 General Provisions

At the time of writing the Climate Change (Emissions Trading and Renewables Preference) Bill 187-1 (2007) (ETS Bill) remains before the New Zealand Parliament, and may be passed into law in 2008. It is described as the world's first "all sectors all gases" cap and trade scheme for reducing greenhouse gases such as carbon dioxide. The ETS Bill proposes that various economic sectors be introduced into the scheme between 2008 and 2013, with forestry to be introduced retroactively from 1 January 2008, liquid fuels to be introduced subsequently (revised proposals have delayed the fuels sector's introduction by a year), and agriculture to be introduced from 1 January 2013. Many key details of the New Zealand ETS are still to be settled, particularly for agriculture, but the forestry sector details – at least at a high level – have been determined.[3]

[3] For a general discussion of the ETS' impacts on various sectors, including forestry, see Insley and Meade (2008).

Being a cap and trade system, the primary new property right created under the ETS Bill is a New Zealand Unit (NZU). The owner of such a unit is entitled to release the equivalent of one tonne of carbon dioxide. Being a tradable property right, NZUs can be bought or sold, either bilaterally, via brokers, over the counter, or through organised exchanges (as and when they develop or as existing exchanges expand their product lines to include carbon instruments like NZUs). For further details regarding NZUs see Appendix A.

2.2.2 Provisions Affecting Pre-1990 Forests

If and when the ETS Bill becomes law, owners of forest land that was planted in exotic (but not indigenous) forest prior to 1 January 1990 and which was not deforested before 2008 will be liable for the carbon deemed released if and when they change the land use into some non-forestry use (i.e. deforest their land) on 1 January 2008 or later.[4] This means that if they deforest their land, they will be required to surrender the requisite number of NZUs to cover the amount of carbon deemed released. If they do not already possess such units, they will have to purchase them (or equivalent internationally-traded instruments recognised under the ETS Bill) at the prevailing market price.

Importantly, a pre-1990 forest land owner will not be liable for merely harvesting their forest, provided they replant it or allow it to naturally regenerate. Nor will they be liable for any carbon loss arising from natural losses such as fire or windthrow, or qualify for NZU allocations in respect of any carbon sequestration. They will, however, face additional annual costs for carbon verification and administration/compliance, whether or not they deforest.

In recognition of the adverse impact of the ETS on pre-1990 forest land values, the ETS Bill provides for a one-off free allocation of NZUs to be made to pre-1990 forest land owners. While the ETS Bill does not state this, it is expected that any free allocation would be to the owner of the land as at 1 January 2008. Since NZUs are property rights aside from forest land, the assumed valuation date is 2 January 2008, and because the free allocation entitlement does not pass to subsequent owners of the pre-1990 forest land (unless the vendor agrees to sell them their NZU allocation along with the land), this report treats the free allocation of NZUs as a matter not relevant to forest or forest land valuation.

---

4 An exemption to this liability can be applied for where pre-1990 forest land owners have less than 50 hectares of relevant land. Notably, under both the Kyoto Protocol and the ETS the amount of carbon deemed released on deforestation is the total carbon stored in the relevant forest at the date of deforestation. This clearly makes no allowance for any delayed actual carbon release into the atmosphere, for example where the wood from the forest is used to make durable products. Should future international agreements change this treatment it could materially affect the ETS's impact on forest and forest land value.

2.2.3 Provisions Affecting Post-1989 Forests

The ETS Bill provides owners of forests (either exotic or indigenous) first planted after 1989 with the option – but not the obligation (unlike for pre-1990 forest land owners) – to enter their forests into the ETS. Should they do so, they would then be entitled to apply for annual allocations of NZUs in line with verified carbon sequestration from their forestry activities from 1 January 2008 (but ignoring previous sequestration). Simultaneously, however, they would also become liable to surrender NZUs for any carbon deemed released from their forestry activities, either from harvest, deforestation (since this involves harvest), or even natural carbon losses (e.g. from fire or windthrow).[5] Entering their forest into the ETS would require them to incur additional annual costs, both for carbon verification, and also administration/compliance costs (more so than pre-1990 foresters). Post-1989 foresters, whether or not they enter their forests into the ETS, will also face the additional transportation and fertiliser costs arising from the phased introduction of the liquid fuels and agriculture sectors into the ETS.

If the post-1989 forest owner does not elect to enter their forest into the ETS, they are entitled under the ETS to conduct their forestry activities as they would have had the Scheme not been implemented. This means, for example, that they would not incur additional annual ETS-related costs, but nor would they derive NZUs or liabilities for carbon loss.[6] They continue to have the option, however, of entering their forest into the Scheme at a later date. For a more detailed summary of the ETS Bill's post-1989 forestry provisions, see Appendix A.

In this report we focus on valuing a post-1989 forest that has been assumed to be entered into the ETS. Section 7 discusses how to evaluate the decision as to whether or not to enter a post-1989 forest into the Scheme, and when best to so enter.

---

5 As for pre-1990 deforestation liabilities, such deemed immediate release of carbon makes no allowance for the actual rate of carbon release, and any future changes to this treatment could materially affect the ETS's impact on forest and forest land value. It should also be noted that the ETS Bill provides that a post-1989 forest owner will not be liable to surrender more NZUs than they have been allocated in respect of their forest, which caps their liability to a degree.

6 If such foresters do not enter their forests into the ETS the question arises as to whether they could secure carbon value by some other means. In short there is some potential for them to sell their carbon on "grey markets", at prices reflecting the institutional soundness or otherwise of those markets (e.g. reflecting any lack of verification or "additionality" in their sequestration). Moreover the New Zealand government will, under the Kyoto Protocol, be accounting for any carbon sequestered from post-1989 forests in New Zealand whether or not those forests have been entered into the Scheme (entry simply determines whether the government passes on the relevant assets and liabilities to the forest owner). Hence any "grey market" carbon sales in respect of non-entered post-1989 forests would in effect involve the double counting of that carbon sequestration, with the New Zealand government claiming credits (and liabilities) under the Kyoto Protocol, and the "grey market" purchaser paying again for the same sequestered carbon.

2.2.4 Key Institutional Uncertainties

The timing of passage of the ETS Bill, and its final form, are currently unknown. For the purposes of this report the ETS provisions as set out in the ETS Bill are assumed. Even if the ETS Bill does not become law, it is likely that some similar arrangements will be implemented, given the government's obligations in respect of forestry under the Kyoto Protocol.

Additional institutional uncertainties remain, however. Foremost is the question whether and how developing nations will be covered by the successor to the Kyoto Protocol (which applies only in respect of CP1 – being 2008 through 2012 inclusive – and which places no binding emissions caps on such countries). Also of significance will be whether the current Kyoto Protocol forestry provisions will be retained, or changed. How each of these is resolved could fundamentally alter the international supply and demand for emissions units and hence their price. They could also fundamentally affect any liabilities arising from harvest, deforestation or natural carbon losses from forestry, and the potential for additional revenue streams in respect of post-1989 forestry.

Of secondary importance will be the speed at and extent to which ETS-like schemes are rolled out internationally, particularly as they apply to forestry. This could result in a fundamental shift in international forestry economics, with implications for the New Zealand forestry industry.

This report acknowledges these institutional uncertainties, and that there are many more besides. For the purpose of the present analysis, however, it is assumed that the ETS as proposed will be an enduring regime beyond 2013, and that until further information is available regarding the main institutional questions, at best this could be reflected in valuation analysis using illustrative scenarios to assess the possible impact of different outcomes. These matters are taken up further in Section 7.

2.3 Carbon Prices

2.3.1 Appropriate Pricing Points

At present there is no established market for NZUs, not least because the instruments are yet to be created by the passage of the ETS Bill. While limited forward trading of NZUs may be arising in anticipation of the ETS being implemented as proposed, such prices would involve possible discounts for the risk of non-delivery in the event that the ETS Bill is not enacted (although the need for NZUs would likely be diminished in that case, meaning non-delivery risk is at least partially hedged from the purchaser's perspective).

While NZUs can be owned by any party, they are currently not fungible for similar instruments overseas (e.g. emission units in the European ETS), and hence there is currently no natural international demand for NZUs, with domestic buyers being the main source of likely demand (e.g. oil companies anticipating the introduction of liquid fuels into the ETS). The converse, however, is not true. The ETS Bill provides that certain types of internationally traded emission units (such as Certified Emissions Reductions, or CERs) will be acceptable instruments for liable parties under the ETS to surrender in place of NZUs. Hence, while NZU prices will not be perfectly correlated with such overseas units, given the domestic idiosyncracies affecting the supply and demand of NZUs, the prices of such fungible alternatives should provide some measure of cap on NZU prices, and NZU prices should track these international prices to some degree.

In this regard the New Zealand Treasury has provided a monthly time series of emission unit prices beginning in March 2005 for the purposes of valuing the New Zealand government's fiscal liability under the Kyoto Protocol.[7] Castalia (2007), however, argue that the price series used by Treasury reflects the primary CER market price, which is lower than the secondary CER market price. Primary CERs are those issued in respect of approved international projects to reduce CO_2 emissions; secondary CERs are emission rights that have already been issued and are traded on secondary markets. The price difference arises because primary CERs are more risky than secondary CERs, since there is risk that the primary CER projects will not be certified and emission rights issued in a timely fashion, whereas secondary CERs arise where the underlying projects have already received certification and the resulting emission rights issued, and hence do not share this risk.

Governments under the Kyoto Protocol have a five year settlement horizon and consolidated buying power, meaning they have some ability to purchase their required emission units via the more risky primary CER market. Conversely, under the ETS liable parties will have to settle their obligations on an annual basis. The argument is thus that such liable parties will need to buy either NZUs or secondary CERs in order to meet their ETS obligations on a timely basis, and hence the NZU price should more closely approximate the secondary CER price, not primary CER prices (which the Treasury price series reflects).[8]

For the purposes of this report, it is acknowledged that the relevant pricing point for NZUs is yet to clearly emerge. Hence the analysis has been based on secondary CER prices, with the Treasury price series used for comparison purposes.

---

7 See: http://www.treasury.govt.nz/government/liabilities/kyoto.

8 Indeed, if true, this argument suggests there is a possible case for either a government or private sector buying group to be formed to enable purchases of lower-priced international emission units, as a way of reducing the cost of the ETS.

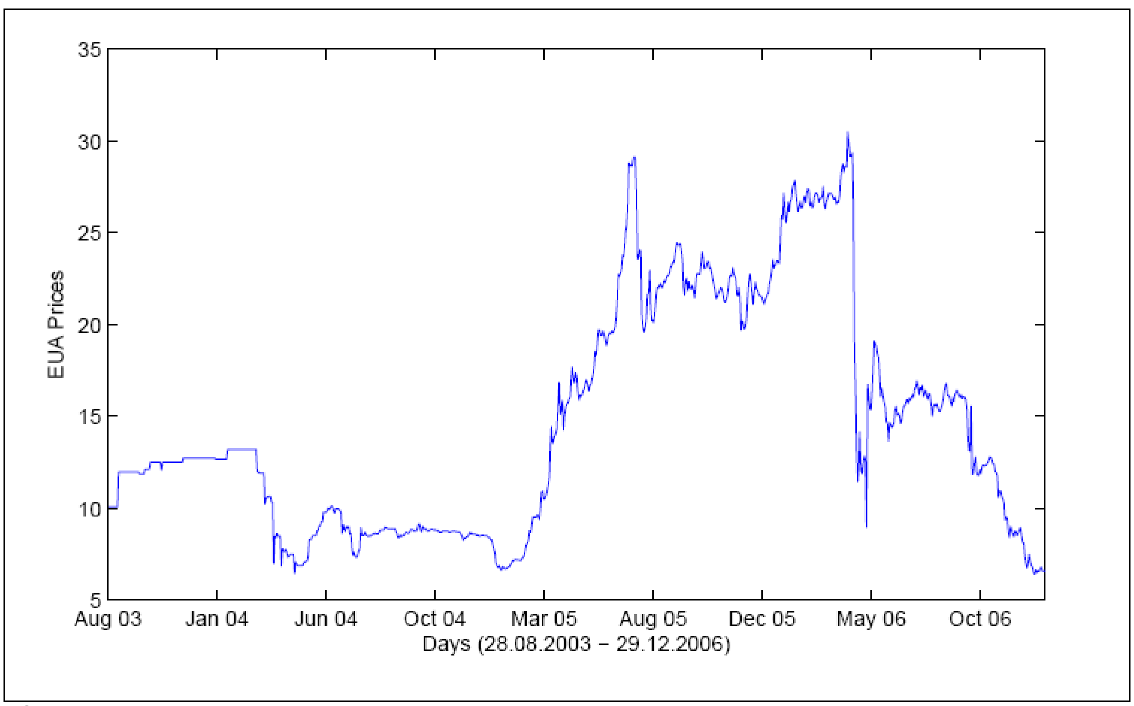
2.3.2 Key Uncertainties

As mentioned in Section 2.1.4, there are key institutional uncertainties regarding the successor to the Kyoto Protocol, as well as to the timing and final shape of the ETS. The resolution of these uncertainties could be expected to result in either step jumps or migration to possibly quite different emission unit price levels (from current prices) as traders of emission units become aware of that resolution. Additionally, political decisions – both internationally and domestically – can be expected to have profound impacts on NZU prices. Indeed, Benz and Truck (2008) categorise the main factors driving carbon prices as being policy and regulatory issues, and market fundamentals affecting the production of emissions and thus the demand and supply of emission units. Paolella and Taschini (2006) further argue that political uncertainties severely complicate the modelling of emission unit prices using standard modelling approaches. Furthermore, Burtraw et al. (2001) investigate the impact of differing emission right allocation schemes in a national electricity system model with emission trading, finding that carbon price levels are affected by the allocation scheme adopted.

Figure 2.2 illustrates the substantial fall experienced in EU ETS prices in 2006 following revelations that liable emitters had been over-allocated free emission units under the EU ETS.

Figure 2.2

EU ETS Prices



Source: Benz and Truck (2008), Figure 1, p. 11.

One consequence of these revelations is that these emitters now face much tighter emissions caps and diminished free allocations of emission units in the second phase of the EU ETS, meaning that emission unit prices have rebounded accordingly. Such examples merely highlight how volatile emission unit prices are likely to be internationally, which will have flow-on effects for NZU prices. Indeed, while writing this report New Zealand government decisions were announced delaying the entry of the liquid fuels sector into the ETS from the timeline originally proposed. Such decisions affect the balance between NZU supply and demand, at least over time, and hence should be expected to affect the level, evolution and volatility of NZU prices.[9]

In this report, our proposed valuation methodology incorporates the best that can currently be inferred about NZU prices. Future, and better, information will allow further refinement of these price inputs – and hence more accurate estimates of forest and forest land values – while leaving our proposed methodology unchanged.

2.3.3 Price Dynamics

Incorporating NZU prices in the valuation of forest or forest land would be straightforward if NZU prices were constant and non-volatile (or, trivially, if forest and forest land values were independent of NZU prices). Under the ETS, however, NZU prices are likely to be highly volatile, thus complicating their impact on forest and forest land value. Moreover, this volatility should persist at least in the short- to medium-term, as key institutional uncertainties are resolved, and as governments, administrators and asset owners adjust to the costs, benefits, risks and opportunities of ETS-like schemes and emission unit trading.

Given the likely volatility of NZU prices, and the likely importance of NZU prices to forest and forest land values under the ETS, the question naturally arises as to how the evolution of NZU prices should be modelled. This is particularly important in the case of forestry, since the investment horizon for forestry is typically measured in decades, and involves consideration of values beyond mere future harvest of a forest. The nature of the assumed price process affects not just the expected level of future prices, but also the way in which prices change over time. Irrespective of the valuation method used (see Section 3), such considerations will remain important.

To date the longest and most liquid market for carbon-dioxide related emission units is that for the EU ETS which was introduced in January 2005. Seifert et al. (2006) argue that there was then too short a trading history on which to estimate emission right price dynamics. Consistent with earlier research, they find that EU ETS emission right prices do not exhibit mean reversion. Taking into account features specific to the EU ETS, they find that emission right prices are not seasonal, and

9 In turn this should affect the feasibility and profitability of, and demand for, instruments to hedge NZU price risk. While this is a topic beyond the scope of this report, Section 7 briefly discusses NZU hedging/trading strategies.

that discounted penalty costs arising under the EU scheme impose a strict upper bound on prices in some cases. Furthermore, emission right price volatility increases as the end of the trading period is neared, and reaches zero when prices approach price bounds (and strongly decreases when prices approach the risk-neutral price limit if market participants are risk-averse). In another early EU ETS study, Paolella and Taschini (2006) empirically examine emission right prices, suggesting a mixed-normal GARCH model would be appropriate.

More recent studies of the EU ETS include that of Benz and Truck (2008) who investigate the short-term behaviour of CO_2 emission rights in the EU ETS, observing skewness, excess kurtosis and episodic changes in volatility. They suggest that Markov-switching and AR-GARCH models are appropriate for stochastic modelling. Also, Daskalakis et al. (2008) provide a comprehensive econometric analysis of EU ETS emission right spot and inter-phase futures prices. They argue that the inability of emission rights to be "banked" between different phases in the EU ETS has affected inter-phase price dynamics, and resulted in raised derivative prices. In modelling spot and inter-phase prices they suggest that a jump diffusion model should be used for the underlying process, and a stochastic, mean reverting convenience yield.

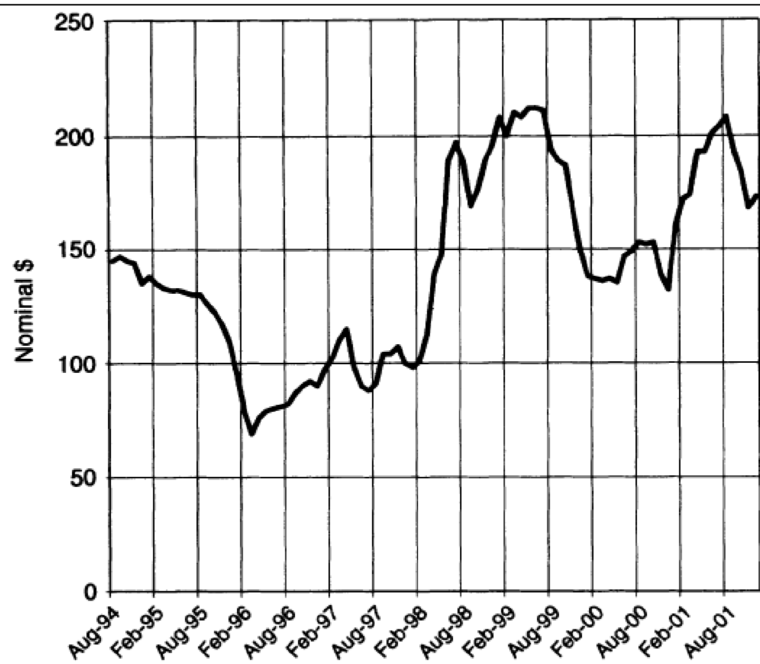
Prior to the EU ETS, the market for sulphur oxide (SO_2) emissions trading under the 1991 US Clean Air Act was the leading example of a cap and trade system for environmental emission rights. This scheme also gives insight into emission right price dynamics, though for a different market. Montero and Ellerman (1998) survey explanations for the observed low price of SO_2 emission rights, concluding that the main reason for such rights' prices undershooting expected prices was an unanticipated fall in emissions due to increased use of low-sulphur coal (i.e. fuel switching), augmented by the impact of irreversible investments made in sulphur scrubbers when expected prices were high. They thus stress the role of expectational errors and investment decision irreversibility in affecting the course of emission right prices. Insley (2003) observes that firms making decisions in the early days of the US Clean Air Act would have had no historical information on which to base their pollution control investment decisions. She argues that they would reasonably have assumed emission right prices to be volatile, with perhaps some form of upward long-run drift as economic growth lead to increased demand for electricity from thermal generation. Insley thus models SO_2 emission right prices as geometric Brownian motion (GBM). A price history from the US SO_2 scheme is illustrated in Figure 2.3 overleaf.

For the purposes of this report a commonly employed mean-reverting process has been assumed for NZU prices (see Section 4). While there is evidence from the EU ETS that allowance prices are not mean-reverting, the particular process chosen has among its merits the fact that it does not allow for dramatic increases in the overall level of or variability in prices over longer timeframes, while explicitly allowing for both volatility and path dependency in prices. Clearly, as more data and research becomes available about the nature of emission price units internationally, and for NZUs in particular, then better assumptions can be made for modelling the future course of such prices.

Among other things, the evidence emerging from the EU ETS of non-constant variance in allowance prices is a feature that should be explored when sufficient New Zealand data becomes available.

Figure 2.3

SO_2 Allowance Prices under the US Clean Air Act



Source: Insley (2003), Figure 1, p. 865.

2.4 Impact of the ETS on Forest and Forest Land Management

2.4.1 Impact of Emissions Pricing on Wood Prices

As argued by Insley and Meade (2008), devolving deforestation liabilities to owners of pre-1990 forests should encourage the retention of a greater proportion of such forests in forestry, rather than see them converted into non-forestry uses. At the same time, new forest plantings (i.e. afforestation) should be encouraged by providing owners of post-1989 forests with the possibility of earning NZUs on their forests. In the latter case, however, it cannot be assumed that increased plantings will translate into higher future wood supply, since some of the plantings may be intended to or end up in long-term carbon sequestration ("carbon farming"). The likely net impact of the ETS on future wood prices is unclear, although increased future wood supply relative to the situation where no ETS-like scheme was introduced is possible. In part the outcome will hinge on the future course of NZU prices, which will affect not only harvest/deforestation costs, but also transportation and fertiliser costs. Additionally, uncertainty in NZU prices may also induce higher retention of non-forestry land in that non-forestry use, also affecting wood supply. Other market fundamentals and institutional credibility will naturally continue to play an important role in determining the viability of New Zealand forestry.

Modelling by van Kooten et al. (1995) – assuming a post-1989 ETS-like regime – predicts that timber supply increases overall under carbon pricing. Sohngen and Mendelsohn (2003) also develop a model of forest carbon sequestration, finding that the two most important factors in carbon sequestration are land-use change and increases in forest rotation (including forests being held for conservation rather than harvest purposes). They predict that timber supply globally would initially decline with carbon pricing, as forest rotations increase. Eventually, however, increased afforestation and longer rotations should lead to increased timber supply and reduced long-run global timber prices.

2.4.2 Pre-1990 Forests

From a valuation perspective the introduction of the ETS will, in principle, decrease pre-1990 forest land value by at least the present value of post-tax ETS-related annual costs, relative to a world in which there is no carbon pricing, since this is the minimum ETS impost even where no change in land use is possible. Indeed, the forest land value should also reflect expected additional fuel and fertiliser costs because of the phased introduction of the liquid fuels and agriculture sectors into the ETS under the ETS Bill. These additional annual, transportation and fertiliser costs should be directly reflected in reduced forest value on pre-1990 forest land.

However, where deforestation of pre-1990 land – for example by conversion into pastoral farming (i.e. sheep, beef and/or dairy farming) – is feasible, the ETS imposes potentially significant deforestation charges in addition. Depending on the amount of carbon deemed released by deforestation, the prevailing price of NZUs and the profitability of non-forestry land uses, this deforestation charge will either reduce the profitability of such conversion, or make it economically unviable. In such cases the value of pre-1990 forest land should in principle fall by more than just the present value of additional ETS-related annual, transportation and fertiliser costs. The fall should also reflect the lost value of conversion options available to the land owner, and hence be reflected directly in the pre-1990 forest land value. It is likely that ETS-related deforestation charges will be of a much higher order of magnitude than other ETS-related forestry cost increases, but the relative value impact of the deforestation charges will depend on the probability and profitability of conversion, whereas the other charges will be incurred wherever these productive inputs are used.[10]

---

[10] In principle some of the value decrease caused by the ETS may be shared between land buyers and sellers, in much the same way that conventional demand and supply analysis suggests the burden of a tax will be shared in proportion to buyers' and seller's relative absolute price elasticities. However, if it is assumed that the supply of investment capital for forestry land is perfectly price elastic (which appears reasonable in an efficient capital market), then the entire ETS-related value fall should fall on landowners.

It should be noted that the profitability of converting pre-1990 forest land into alternative uses will also be affected by the ETS. If pastoral conversion is the leading alternative land use to forestry, then the introduction of agriculture into the ETS, given also increases in fuel (and electricity) costs, will diminish the value of such pastoral activities. Thus the ETS' impact on pre-1990 forest land value will come from at least two sources – the lost conversion potential due to deforestation charges, but also the reduced profitability of conversion alternatives.

Where pre-1990 forest land cannot be converted into non-forestry uses, the ETS's impacts on forest and forest land values will be confined to its impacts on transportation and fertiliser costs. Examples of possible constraints on land use changes include nitrates emission rules in the Lake Taupo catchment under the Resource Management Act (RMA), and the possible introduction of similar rules around Rotorua. Other examples include forests with committed wood flows, or replanting requirements for soil conservation purposes. Wider industry impacts might also come into play even in such cases, for example if rising transportation and energy costs fundamentally affect the viability of wood processing and pulp and paper manufacturing in New Zealand. As above, the ETS may also affect wood prices through changing overall supply and demand for forest biomass.

While optimal forest rotation lengths are commonly predicted to rise under carbon pricing (see below), for pre-1990 forests this is not obviously to be expected. Since the ETS does not fundamentally alter the returns to forestry itself for such forests (except via flow-on impacts on certain forestry costs, and perhaps by changing the overall balance of wood supply and demand), optimal rotation lengths are unlikely to change to the same degree as for post-1989 forests. Hence, aside from reduced conversion options, pre-1990 forestry economics may be largely unaffected by the ETS.

2.4.3 Post-1989 Forests

From a valuation perspective the possibility of entering a post-1989 forest into the ETS means the forest-owner could either augment or fundamentally alter the revenue streams produced by their forest. This is because the rapid growth in biomass for a young forest could produce potentially significant allocations of NZUs which could then be on-sold at the prevailing market price, thereby producing revenues well ahead of any ultimate harvest. They would also incur additional ongoing ETS-related forestry costs, and possibly significant harvest (or deforestation) liabilities.

Another consequence of possible carbon sequestration value for post-1989 forests is that optimal harvest ages and silviculture regimes may change under the ETS. This is because foresters might benefit from deferring harvest and continuing to sequester carbon to generate NZU value (relative to optimal harvest dates determined solely by reference to wood returns). Also, the costs of intensive silviculture will need to take into account not just improved timber log grade out-turns, but also changed carbon sequestration. For example, modelling by Guthrie and Kumareswaran (2007) predicts that providing carbon credits based on annual carbon increments induces later harvest dates. Conversely, van Kooten et al. (1995) predict that optimal forest rotation with carbon benefits increases only marginally compared with conventional rotations, but under some carbon regimes harvest may never be optimal. At a carbon price of US$20/tonne they find rotation ages for North American forests increase by around 20% over the level where there are no carbon costs or benefits.

Modelling by Stainback and Alavalapati (2005) predicts that with positive carbon prices forest rotations will increase and more intensive forest management (e.g. use of fertiliser to encourage biomass growth) is expected under various forest management regimes, relative to no carbon pricing. However, shorter rotations are predicted for their most intensive assumed forest management regime. Murray (2003) similarly predicts increased rotation lengths under carbon pricing.

As for pre-1990 forestry, the ETS may have other direct or wider industry impacts on post-1989 forestry.

2.5 Key Requirements of Alternative Methodologies

Based on this discussion the following factors are suggested as important in formulating any approach for valuing forests or forest land under carbon pricing (i.e. New Zealand's ETS):

- Pre-1990 forests and forest land:
    - Explicit recognition of deforestation costs is required if a change in land use is contemplated or simply possible, affecting land value in particular.
    - The timing and likelihood of such land use change is also relevant, and complicates the assessment of the land conversion option value.
    - ETS impacts on the profitability of non-forestry alternative land uses must also be considered (e.g. the impacts on dairy land values of agriculture being phased into the ETS from 2013 in some shape or form).
- Post-1989 forests and forest land:
    - Whether or not, and if so when, to enter a post-1989 forest into the ETS is a preliminary question.

- In all cases:

    - Consideration of carbon sequestration and applicable NZU prices at the relevant times is required, since this affects either harvest/deforestation liabilities, or carbon sequestration revenues, and thereby both forest and forest land values.

    - Consideration of other ETS impacts – such as on transportation and fertiliser costs – may also be required, which will directly affect forest values, and hence flow-on to forest land values (particularly where the land has no alternative uses, but also to some degree even where land conversion options exist).

Naturally, these factors stand in addition to those normally considered for any forest or forest land valuation (i.e. log prices, forestry costs, etc). Additional factors might also be considered, such as the impact of the ETS on land conversion options for post-1989 forestry, since such land may enjoy competition from non-forestry uses. While modelling this complexity may be desirable, it is likely to be of lesser importance than it is for pre-1990 forestry, since the additional value potential for post-1989 forestry from carbon sequestration should reduce the likelihood of conversion into non-forestry land uses.

The next section sets out criteria for selecting a valuation approach to take account of these factors, and assesses possible valuation approaches against these criteria.

3. Summary of Possible Valuation Approaches

3.1 Criteria for Selection of Preferred Approach

The following criteria are important in selecting a valuation methodology to account for the ETS-related factors relevant to forest and forest land value identified in Section 2:

- At a minimum, sufficient sophistication to accommodate *expected* harvest/deforestation liabilities and/or carbon sequestration revenues in some shape or form;

- Preferably, an ability to model both expected future prices (e.g. for logs, or NZUs) and the dynamics (e.g. volatility and inter-temporal correlations) of those prices;

- Very desirably, the sophistication to model managerial decision-making flexibility in response to volatile future state variables (such as log, land and NZU prices), such as harvest timing and land use change decisions; and

- Critically, the methodology should be able to be implemented using readily available techniques and by parties with only a reasonable degree of technical sophistication.

The first criterion follows from the discussion in Section 2. The second and third criteria are also suggested by the discussion in Section 2, but their importance will become clearer with the discussion below, particularly regarding real options analysis. The fourth criterion is suggested for practical purposes, in that developing a sophisticated methodology that defies application by parties commonly involved in valuations will be of limited utility (though it should be recognised that more complex problems may naturally require more complex solutions).

This section begins by discussing the suitability of comparable sales analysis for assessing the impact of carbon pricing on forest and forest land value. Conventional discounted cash flow analysis is then appraised, followed by a discussion of real options analysis. The final subsection assesses each methodology against the criteria stated above and suggests a preferred approach.

3.2 Comparable Sales Analysis

As argued in Section 2, the ETS should have a depressing effect on pre-1990 forest land values, and also reduce the supply of pre-1990 forest land available for conversion into non-forestry uses. On the other hand, by potentially making post-1989 forestry more attractive, the demand for land not planted in pre-1990 forests for forestry purposes should be expected to increase, all other things being equal. For example, Murray (2003) derives a simplified model of bare forest land value and optimal rotation for various North American species under carbon pricing. As expected, with rising carbon price the value of bare forest land per hectare rises, as does optimal rotation length, for the three species considered.

Titman (1985) sets out real options arguments for the evaluation of land under uncertainty. He argues that where land prices are uncertain, the option to select different land use types becomes more valuable, and hence the decision to commit the land to any particular land use now becomes less attractive. He further highlights how policy uncertainty can also increase the value of the option to defer irreversible land use decisions. For example, until the ETS policy package was announced in September 2007, preceding government policy was that deforestation liabilities might be devolved to pre-1990 forest land owners, in some unspecified way, if deforestation rates in CP1 exceeded a pre-set level (based on 10% of the historical deforestation rate). In respect of post-1989 forests, the former policy was that government would retain all benefits and costs of carbon sequestration, rather than devolve them to forest owners. This suggests that uncertainty regarding future ETS or post-2013 international provisions may cause both pre-1990 and post-1989 forest land use decisions to be deferred longer than they would otherwise be. In turn this could dampen demand for forest land while such uncertainty persists.[11]

Once details of the ETS and long-term international institutional arrangements become more settled, impacts such as these should be increasingly reflected in traded land sales data. Distant historical land sales data is unlikely to reflect ETS impacts to any significant degree, however, particularly where its key features were not anticipated.

These considerations suggest that there is likely to be a transition period in which forest land values begin to reflect the new policy environment and whatever long-term expectations buyers and sellers collectively form about ETS impacts on land values. Until this transition has been completed, however, it is questionable whether observed land sales data will offer much insight into the ETS's impacts on value, except perhaps for the latest available data. Such data will quickly become dated should the institutional arrangements for the ETS and Kyoto Protocol successor differ from previous expectations. Moreover, it will be of limited use in assessing different future policy or institutional scenarios, since it will reflect only the expectations of the parties to the transactions, not alternative hypothetical scenarios. This suggests that explicit techniques for evaluating ETS impacts on land values will be necessary, if only for a time.

---

[11] Boyle and Guthrie (2003) model a situation where uncertainty (regarding the availability of future investment financing) may in fact cause investment decisions to be accelerated rather than deferred. Depending on the particular investment decision, therefore, uncertainty may have offsetting effects on optimal irreversible investment timing.

These considerations apply equally in respect of comparable sales analysis for forests rather than forest land. Since forest sales data is so heterogeneous (i.e. differing according to forest maturity, silviculture regime, site quality, etc) it should be expected that the transitional issues are even more pronounced than for forest land. Accordingly, as already discussed, it is likely that comparable sales techniques will be of little use for assessing any ETS impacts on forest (land) value. Instead, other methods such as discounted cash flow (DCF) analysis will have to be used, and it is these to which we now turn.

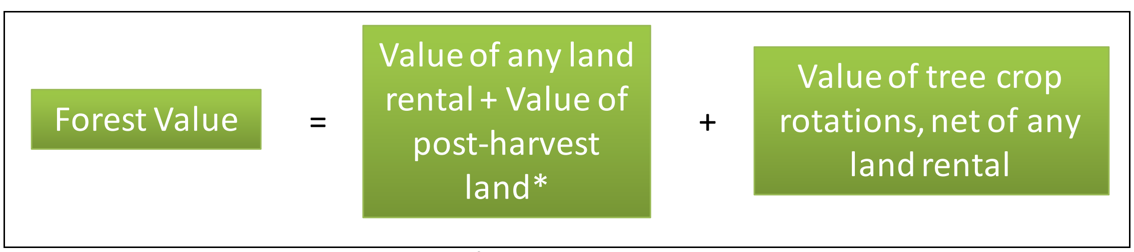
3.3 Conventional Discounted Cash Flow (DCF) Analysis

Conventional discounted cash flow (DCF) analysis is commonly used in valuing forests, and also in valuing forest land. It is particularly used when there is inadequate comparable sales data, or adjustments are required (e.g. to bare land values, when forest land is encumbered with forestry rights or licences, such as Crown Forest Licences (CFLs) under the Crown Forest Assets Act 1989).

Figure 3.1 sets out a schematic approach to valuing forests and forest land. Where the land and trees are jointly owned, the forest value equals the value of tree crop rotations plus the value of post-harvest land. Where the land and trees are separately owned, the value accruing to the landowner is the first term on the right hand side (i.e. the value of the forest land), while the value accruing to the forest owner is the second term on the right hand side (i.e. the value of the tree crops). The total value to be shared between the land owner and the forest owner is constrained by the productivity of forest land use while the land is used for forestry, but can then factor in the value arising from potentially higher alternative land uses once the land is no longer committed to forestry.

Figure 3.1

Schematic of Forest Valuation



** Including any post-harvest conversion value net of relevant conversion costs.*

The DCF approach requires expected values of future harvest yields at some “optimal” harvest date, forestry costs between the valuation and assumed harvest dates, and future log prices and harvest costs (including transportation costs) as at harvest date.[12] The net present value (NPV) of the resulting expected forestry costs and harvest revenues, net of expected harvest costs (producing "stumpage") – net of tax as applicable – can then be produced by discounting these cash flows at a suitable forestry discount rate reflecting the relative investment risk of forestry. Where the land beneath the forest is rented, this NPV will also reflect the deduction of post-tax costs of any ongoing rental, stumpage-based rental, combination thereof or whatever alternative rental arrangements are in place. The resulting NPV is the discounted cash flow value of that particular stand of trees. Where subsequent rotations are assumed, costs and revenues can be projected beyond the assumed harvest date. Alternatively, some forest models assume the relevant land is sold following harvest, at some assumed future land value. Making these refinements then produces a value of that forest stand beyond merely its initial harvest returns.

In principle the post-tax rental stream can also be discounted, to estimate the value of forest land, if that land is not owned jointly with the trees. As for the forest stand itself, however, the value of the land will reflect more than just the rental stream up to initial harvest date. If subsequent rotations are allowed for, then the land value should reflect the NPV of ongoing rentals in subsequent rotations. Alternatively, the land value now could be estimated as the NPV of post-tax rentals up until the date at which the land is assumed to be either sold or put to some other land use. If the land is assumed sold following harvest, then – as for forest valuation – assumptions are required as to what that future land value might be. In principle, it might include the value of alternative land uses such as pastoral farming (where land is suitable for that use). Thus the land value will reflect the initial forestry use, and then the choice between ongoing forestry use and land conversion following initial harvest (if such alternative land use options are feasible).

Under carbon pricing, of course, if the land is pre-1990 forest land then expected deforestation liabilities would also need to be accommodated. This requires projections of not just timber yields at harvest date, but also carbon sequestered at that time and the NZU price at that time. In principle this can be accommodated in much the same way as a normal forest valuation. The complication arising under the ETS, however, is that if any assessment of alternative land use is to be made, the value of forest replanting following harvest must be contrasted with land conversion at that date, net of conversion costs and expected deforestation costs.

While a simple comparison of the NPVs of these two alternatives might be made – with the better of the two alternatives being assumed in the final NPV – this requires strong assumptions about how the future land use decision will be made. In particular, this valuation approach assumes that the forest (or land) owner will commit at the valuation date to forestry up to the assumed future

---

[12] The forestry "golden rule" attributed to Faustmann (1849), derived under the assumption of a constant log price under static DCF analysis, is that harvest arises optimally (i.e. maximises forest value) when the proportional growth rate in harvest value at any given time equals the relevant discount rate. For a survey of the optimal harvest rule literature see Newman (2002).

harvest date, and then to either replanting or converting to the assumed alternative land use at that harvest date, given only the information available today. It does not recognise that the forest (or land) owner will have the ability to change his or her mind regarding when and if to harvest, and what to do with the land following harvest, closer to the decision date, and with the benefit of the information available then (such as to relative log and NZU prices, and stumpage returns relative to net conversion profitability, etc).

This highlights the "static" nature of DCF analysis. It makes no allowance for the managerial flexibility often enjoyed by foresters – e.g. to accelerate (or delay) harvest if future stumpage returns are better (worse) then originally expected.[13] Hence, while the approach is relatively easy to implement, and has a widely understood and accepted basis, it potentially underestimates true forest (land) value, since it ignores the value of any options available to the forest (land) owner – which would rationally be exercised only if they improve the forester's returns. In other words, the true forest (land) value should not just be the NPV of assumed activities, but the sum of that NPV and the value of options the forest (land) owner possesses to change how the forest (land) is managed in the future.

Similar criticisms can be made of the DCF approach when applied to valuing post-1989 forests (or forest land). Setting aside questions of possible future land conversion (which can apply equally for post-1989 forests as they do for pre-1990 forests), in principle the valuation of a post-1989 forest proceeds much along the lines of an ordinary forest valuation, but with additional carbon revenues and costs. Thus, as for pre-1990 forestry, projections are required of carbon sequestered over the forest's life – in this case not just at the assumed harvest date, but also in the years preceding (and possibly also following) harvest in which NZUs can be earned in respect of carbon sequestered by a growing forest. These amounts must then be converted into assumed carbon revenues or costs by multiplying by the assumed NZU price at each future relevant date (i.e. not just at the assumed harvest date), and in the case of post-1989 forests (but not pre-1990 forests), by also deducting tax at the applicable rate.[14]

Once again, however, the static nature of the analysis could lead to the value of other managerial flexibility being overlooked – namely the flexibility foresters will have to alter future harvest dates in light of relative log and carbon prices at harvest date (given that harvest liabilities under the ETS need to be accounted for in this case). Indeed, even if current NZU prices are not high enough to suggest this, future NZU prices may be sufficiently high that the forester decides in future to

13 These arguments are also explored in Meade (2006). Arrow and Fisher (1974) provide early arguments in favour of deferring irreversible harvest decisions when faced with uncertainty – i.e. suggesting there is a valuable option to defer harvest (i.e. wait) under such circumstances. Section 3.4 surveys numerous articles predicting significant option value arising from a forester's ability to change harvest timing in response to price variability.

14 Deforestation liabilities for pre-1990 forests under the ETS Bill are treated as capital items, whereas carbon revenues and costs for post-1989 forests are treated as revenue items for tax purposes. See Appendix A for more details.

abandon harvest and simply continue sequestering carbon (i.e. "carbon farming" as opposed to conventional forestry).

Hence, once again, the conventional DCF approach could be adapted to incorporate carbon revenues and costs. However, it can only do so in a static way, and it ignores potentially valuable options for forest (land) owners to change their future forest management decisions in light of information available then. Given the considerable uncertainties and hence variability that should be expected in NZU prices, neglecting to consider this managerial flexibility may significantly bias the resulting DCF-based NPV estimates.[15] The real options analysis technique discussed below attempts to address such questions.

3.4 Real Options Analysis (ROA)

3.4.1 General Approach

Real options analysis (ROA) extends DCF analysis by attempting to value flexibility in managerial decision making when managers make decisions that are at least partially irreversible (such as is the case with forest harvest – the trees can't be un-harvested) and where key state variables (e.g. future prices) are uncertain.[16] In a world where all investments were costlessly reversible or all future state variables were known with certainty, real options would not arise. In that world, managers could either costlessly reverse their decisions in light of new information so as to maximise asset values, or they could predict with perfect foresight their optimal future decisions. Since neither is the case, particularly for forestry, there is a strong argument in principle that real options must be considered when valuing forest assets – and this argument arises with or without carbon pricing.

Indeed, there is a well-developed literature on "reserve pricing strategies" for forest assets using real options analysis and other techniques. These strategies recognise that managing the harvest decision in the light of future price information can significantly increase forest value. Typically they produce a reserve pricing threshold, as a function of forest age, such that the forest should only be harvested at any given age if the log price exceeds the non-zero (i.e. positive) threshold.

---

15 Based on a consideration of historical New Zealand log price dynamics, Niquidet and Manley (2007) question the extent of the value to delay or advance harvest in response to new price information, and suggest standard DCF-based techniques for forest valuation should be adequate. However, the appropriateness of this approach for valuing forests under the ETS and with volatile NZU prices was not discussed, and the authors acknowledge that analysis using a longer time series could lead to different conclusions.

16 The commonly cited reference on real options is Dixit and Pindyck (1994), or a shorter forerunner, Pindyck (1991). Another text on real options is Copeland and Antikarov (2001), and an edited collection of the major early contributions to the literature is provided in Schwartz and Trigeorgis (2001). For an approachable introduction to the subject see Boyle and Irwin (2004).

Commonly this threshold is a declining function of age (see examples in Section 4.1.4). For example, Brazee and Mendelsohn (1988) find a reservation price that declines with forest age, that reservation prices are higher than historical average prices, and that the option value arising from harvest flexibility is proportionately greater on lower site quality lands (i.e. marginal lands). Other examples include Haight and Holmes (1991), Thomson (1992), Gong (1999), Yin (2001), Insley (2002) and Gong and Lofgren (2007).[17]

Indeed, as Gong and Lofgren (2007, p. 218) put it: "The reservation price strategy (RPS) is a conceptually superior and practically more attractive alternative to the Faustmann rule (FR) [i.e. conventional optimum harvest rule] for forest harvest decisions." They too derive an optimal reserve pricing strategy that is positive and declining in forest age. As the forest ages, the stock of timber increases while the growth in timber stock slows, meaning the opportunity cost of harvest increases, and expected value increment falls, as forest age rises.

One caution offered by some of these authors, however, is that the widespread adoption of reserve price strategies may serve to dampen variability in log prices, and hence diminish the option value from the harvest timing decision. For example, Gong and Lofgren (2007) argue that if foresters change from a Faustmann rule regime to one based on reservation prices, this will have the effect of reducing timber supply and thus increasing timber price. Furthermore, the reserve pricing strategy reduces the short-run variation in timber prices (as foresters increase supply in response to positive price variation and vice versa), while both the mean and variance of timber prices stabilise in the long run. Brazee and Mendelsohn (1988) offer similar arguments. Such an argument, however, rests in part on an assumption that historical log prices were not already determined by reserve pricing strategies being applied by foresters.

Another caution is that real option value in forest management can only arise where there is indeed the flexibility to change harvest dates in response to new information. Where forests are subject to supply commitments, for example, the forester may be required to harvest particular stands at particular stand ages, irrespective of whether log prices are favourable or otherwise at harvest date. Where foresters have contracted log prices instead of basing their harvest decisions on volatile spot market prices this too would reduce the value of harvest flexibility. Finally, practicalities such as long lead-times in booking harvesting contractors may also reduce the ability of foresters to increase forest value through timing decisions in response to new information. Similarly, whether or not forest land is capable of conversion into non-forestry uses might also be constrained, for example due to environmental or resource management regulations. Accordingly,

---

17 Similar modelling is provided by Clark and Reed (1989) – whose decision barrier compares forest size and age, rather than producing a reservation price as such – and Malchow-Moller and Thorsen (2003). Morck et al. (1989) provide a real options analysis of forestry where both prices and inventories are uncertain (i.e. stochastic), though their model applies only to a ten year forestry lease.

the extent to which ROA is usefully applied in respect of any given forest or piece of forest land should be considered in the light of the actual flexibility available to the forest (land) owner.

Probably the most relevant contribution to the ROA forest valuation literature for present purposes is that of Guthrie and Kumareswaran (2007), who provide a useful example of the application of real options techniques to assess the impact of carbon pricing on post-1989 forest management and valuation using New Zealand data. They examine three carbon credit allocation scenarios, the first of which ("flows method") closely approximates the regime proposed under New Zealand's ETS. Forest abandonment is provided for in the form of the forest land being converted into pastoral use at a constant assumed conversion value if either harvest followed by replanting, or harvest followed by conversion into pastoral land use, are not viable.

The key insight of these authors is that, given the irreversibility of the harvest decision, a forest will not be harvested as soon as the return from doing so is positive, if log prices are uncertain. Instead there is value to deferring harvest in order to gather further information, and the more uncertain are prices the more valuable this option to wait becomes. Under their approach they derive price boundaries governing their forester's decision for a given forest age. For prices below one threshold the forester should harvest and convert into the alternative land use; above another the decision should be to harvest and replant. For all intermediate prices the optimal decision is to defer making any harvest and subsequent land-use decision. Figure 3.2 overleaf illustrates the resulting decision boundaries.

Using their model and representative New Zealand forest data these authors find that providing carbon credits based on annual carbon increments induces later harvest ages than in the absence of carbon pricing, and lower conversion into alternative land use. While no explicit carbon prices are given, their model suggests forest values of around $3,500/ha for "greenfields" post-1989 forestry, and around $27,000/ha for a "brownfields" 20 year old forest (which would miss qualifying for carbon credits under the ETS due to having been planted before 1990).[18]

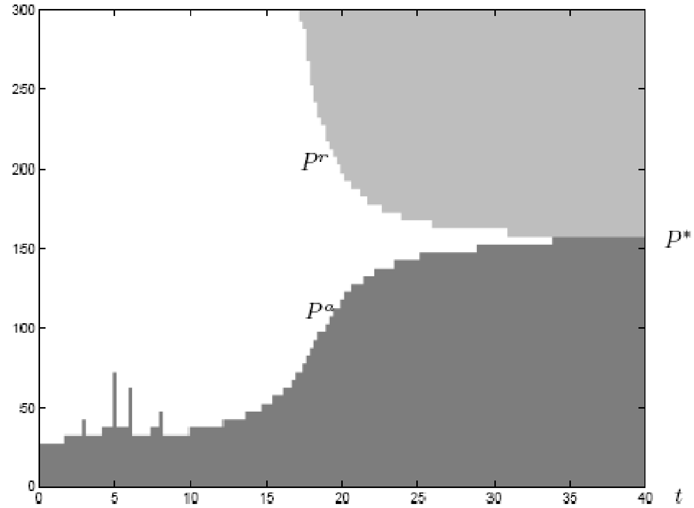
The merit of the approach in Guthrie and Kumareswaran (2007) is that it highlights a number of the key decisions confronting a post-1989 forester under the ETS, and shows how these decisions can be modelled and evaluated using real options analysis. Despite having only one uncertain state variable (a single stochastic log grade, with both carbon price and conversion land value constant), their model is sufficiently complex that it cannot be solved analytically, and instead requires more involved numerical solution techniques.

---

18 "Greenfields" refers to a brand new forestry investment (i.e. one involving the planting of a new forest); "brownfields" to an existing forestry investment (i.e. where the trees have already been planted).

Figure 3.2

Guthrie and Kumaresaran (2007) Price Thresholds for Optimal Forest Management



Notes: This figure represents the replanting and abandonment decision thresholds for the owner of a representative *P. radiata* forest, who faces uncertainty in timber prices. Within the lightly-shaded region it is optimal to harvest and then immediately replant. Within the darkly-shaded region it is optimal to harvest and then abandon. In the clear region it is optimal to defer harvest. At any point in time, P^r is the minimum price that would induce replanting and P^a is the maximum price that would induce abandonment. P^* is the trigger price required to persuade the owner to switch from a replanting to a conversion decision.

Source: Guthrie and Kumareswaran (2007), Figure 1, p. 11.

Hence, while in principle the real options approach has merit for valuing forests and forest land under carbon pricing, its practical use will be limited unless the associated solution techniques are implementable without too great a degree of sophistication. This then brings us to a discussion of solution techniques that are commonly used to produce real options valuations.[19]

3.4.2 Analytical Solutions

For relatively simple straightforward real options analysis problems it is sometimes possible to value managerial flexibility using closed form analytical solutions. The classic such formula in the context of financial (as opposed to real) options is the Black-Scholes-Merton option pricing model, which provides an exact solution to the pricing of an option that can be exercised just once to purchase (or sell) a given asset, at a given exercise price, at a given future date. This formula can be calculated using a small set of readily estimable parameters, and can be calculated by calculator or spreadsheet. A widely used corporate finance text, Brealey et al. (2006), illustrates

19 See Hull (2005) for further background on these techniques.

the use of such a model for real options problems. Another relatively simple financial option model for which there is an analytical solution appears in Margrabe (1978) which values the option to exchange one asset for another, with a zero exercise price. The seminal analytical formula for valuing a real option is that of McDonald and Siegel (1986), which values the ability to determine the timing of an irreversible investment decision when the value of the investment project is uncertain.

If such models can conveniently be spliced into a discounted cash flow analysis to reflect managerial discretions then it is relatively straightforward to estimate the value of managerial flexibility. In practise, however, real options problems are sufficiently complex that this approach is not feasible, and more demanding solution techniques are required.

3.4.3 Numerical Techniques

Even for relatively simple real options problems it is often not possible to derive convenient closed form analytical solutions to value managerial flexibility. This is particularly so where those managerial discretions can be exercised at more than just one date, and the payoff to a decision on any given date depends on the course of each key state variable – so-called “path-dependent American-style options”. Instead numerical techniques are often applied, involving methods such as binomial or trinomial trees, dynamic programming, or finite difference methods to solve differential equations. None of these approaches is trivial to implement, and they are practical only for problems with three or less uncertain state variables, as their computational requirements rise exponentially with each additional such variable.

Accordingly, while such techniques are suitable for some real options problems, the likely complexity of pre-1990 forest (land) valuations – if uncertainty is to be allowed for in NZU prices, alternative land use values (e.g. dairy land conversion values) as well as one or more log grade prices – means these techniques will not be suitable. As mentioned above, even the relatively simple model derived by Guthrie and Kumareswaran (2007) for valuing post-1989 forestry required numerical solution techniques, and this with only one uncertain state variable (a single log grade price). If uncertainty is to be allowed in other key state variables – notably NZU prices – then alternative techniques need to be considered.

3.4.4 Monte Carlo Simulation

Monte Carlo Simulation (MCS) is a very powerful solution technique commonly used where other solution techniques are infeasible.[20] First suggested for the valuation of options by Boyle (1977), it

---

20 As Tilley (1993) puts it: “Mathematicians seem to resort to simulation models to analyse a problem only when all other methods fail to yield a solution.”

involves the use of computer simulation to generate forward-looking sample paths for each uncertain state variable, and then applies assumed decision rules to determine what possible future payoffs might be for the modelled managerial discretions. By repeatedly generating such sample paths for assumed statistical distributions or processes for each state variable, and computing the resulting decision value for each path, it is then possible to estimate the expected NPV of the real option problem being modelled (i.e. by taking the simple average of the resulting decision values over the repeated sample runs).

The chief merit of MCS is that it is relatively simple to implement (e.g. in a spreadsheet, with a free or proprietary simulation add-in), and can accommodate an arbitrary number of uncertain state variables – with any given statistical process for each – with ease (since computational requirements rise only linearly with each such variable, not exponentially as for other techniques). Additionally, at least when modelling one-off managerial decisions, MCS can easily model arbitrary payoff structures and complex decision rules based on the assumed state variables. Furthermore, the method can be augmented to provide confidence bounds on estimated NPVs (e.g. see Broadie et al. (1997)).

Complications arise, however, when using MCS to value American-style options, especially if these are path-dependent. The method inherently operates in a "forward recursive" manner, meaning that it begins with near decisions and proceeds to successively evaluate later decisions. Problems arise where decisions at any point in time require information about later decisions, a situation that more naturally lends itself to "backward recursive" techniques (such as dynamic programming). In principle simulation may be adapted to accommodate such dependencies, particularly when there are few intermediate decisions to evaluate, for example by nesting simulations within simulations. The computational requirements of such an approach quickly grow however, diminishing one of the key advantages of MCS over other techniques.

However, Tilley (1993) observes that MCS for a long time was considered unsuitable for valuing such options, but demonstrates that it can be modified to accommodate at least simple path-dependent American options (e.g. for a single uncertain state variable). Since then a number of authors have demonstrated how to modify MCS to allow for these types of problems, with examples including Broadie et al. (1997), Broadie and Glasserman (1997) and Tsitsiklis and van Roy (2001). Grant et al. (1997) combine MCS with dynamic programming to value American options. Longstaff and Schwartz (2001) show how least squares estimation and MCS can be combined to value American-style options, though this technique (like that of Grant et al.) becomes computationally complex, especially for a larger number of state variables or possible decision dates. Andersen and Broadie (2004) extend Broadie and Glasserman (1997) using nested MCS (i.e. requiring multiple sample paths to be simulated at any given decision date along a given sample path, thus giving rise to both computational complexity and exponential computational requirements).

A number of other authors have demonstrated that MCS can be adapted to value even high-dimensional American-style options (i.e. potentially with up to hundreds of uncertain state variables). For example, Barraquand and Martineau (1995) develop a numerical technique involving MCS to value American-style options with up to 400 risk sources. They combine simulation with a method of partitioning the underlying asset space which they term "stratified state aggregation", which in effect aggregates multi-dimensional problems into a single-dimensional problem that can be easily solved. Their technique is by no means straightforward to implement, however, and as Wan (2002) observes, its accuracy hinges on whether the optimal exercise policy can be summarised by a single payoff variable. Like Barraquand and Martineau, Wan proposes a model in which multiple stochastic variables for a basket-style option with fixed asset weights are summarised into a single variable which then facilitates American option evaluation. Since neither stumpage returns nor forest land conversion options can be assumed to involve fixed weights, however, this approach is likely to be of limited assistance for valuing forests under carbon pricing.

In summary, MCS is a powerful and relatively simple valuation technique that can be used to value managerial flexibility, and possibly even certain classes of complex high-dimensional American-style option problems. This offers some encouragement for the valuation of forests and forest land under carbon pricing, since in practice forest (land) owners will face multiple possible land use decision dates, in the light of multiple uncertain state variables (i.e. NZU prices, land conversion payoffs (e.g. dairy land values, if dairy conversion is a possibility), and multiple log grade prices).[21] This immediately raises dimensionality problems, and forces consideration of MCS-based valuation methods if managerial flexibility in the light of irreversible harvest decisions and uncertainty in key state variables is to be valued under carbon pricing.

The less encouraging news is that higher-dimensional American-style options can remain difficult to value even using MCS. This means further adaptation to MCS is required if the complexity of the forest (land) valuation problem under carbon pricing is to be accommodated in a way that remains practical to implement. We propose such an adaptation – what we call Bootstrapping Real Options Analysis (BROA) – in Section 4.

3.4.5 Risk-Neutral Valuation

An important feature of option valuation techniques in general, and also in ROA, is the use of so-called risk-neutral valuation. Under this approach advantage is taken of the fact that the valuation

[21] While it might be suggested (as do Guthrie and Kumareswaran (2007)) that multiple log grade prices might be summarised in a single, weighted-average log price, in practise this is unlikely to be feasible. This is because the relevant weights – the proportion of a tree that converts into each relevant log grade – will depend not just on time, but also the age of the tree. Hence, while it would be highly desirable to reduce the number of uncertain state variables in the forest (land) valuation exercise by using a weighted-average log price series, for practical purposes multiple log grade prices should be allowed for.

problem can be couched in terms where investor risk preferences can be eliminated, meaning option cash flows can be discounted using risk-free interest rates (though under this approach investors are not themselves assumed to be risk-neutral). While this simplifies the valuation problem, it comes at the cost of requiring cash flows to be expressed in correspondingly risk-neutral terms. In practise, this requires applying “risk-adjusted drift rates” in assumed price processes for key state variables, meaning that instead of using actual drift rates (i.e. average rates of return) based on historical data, drift is estimated to arise at a rate equal to the risk-free interest rate less an allowance for so-called “convenience yield”.[22]

Such an approach overcomes the problem that option cash flow risk is often a function of underlying uncertain variables such as asset prices. Because of the way in which such prices affect option value, this means risk-adjusted discount rates cannot be assumed to be constant, instead changing in a systematic way as underlying asset values change.[23]

Accordingly, one approach is to apply actual drift rates in price processes, with risk-adjusted discount rates applied despite complications in estimating such rates for option valuation problems. Alternatively, risk-adjusted drift rates are assumed in price processes (requiring estimates of convenience yields), and the resulting cash flows discounted at the risk-free interest rate. This report applies the latter approach, based on indicative convenience yield estimates.

3.5 Appraisal of Possible Approaches and Selection of Preferred Approach

Based on this discussion and the criteria set out in Section 3.1, we draw the following conclusions regarding the techniques available for valuing forest (land) under carbon pricing:

- **Comparable sales analysis** – in time sales evidence for pre-1990 and post-1989 forest land should shed some light on how buyers and sellers of such land value the impacts of carbon pricing. Given ongoing international and domestic uncertainty regarding carbon pricing for forestry and more generally, buyers and sellers will face some difficulty in assessing the value impacts of carbon pricing, and certainly historical sales data (predating ETS announcements) will offer little insight into the relevant impacts. For forest

22 Convenience yield is the benefit accruing to physically owning an asset rather than just a claim over such an asset. For further details on this approach see Hull (2006), Morck et al. (1989) or Schwartz (1997).

23 See Copeland and Weston (1988) for a derivation of the relationship between the systematic risk of a call option and underlying asset price. Additionally, Gjolberg and Guttormsen (2000) derive a real options model for forest harvest assuming mean-reverting log price. With this they are able to provide a possible explanation for the apparently irrational use of low discount rates often observed in forestry. If log prices are mean-reverting then price risk is possibly not as high as is suggested with the use of risk-adjusted discount rates over long compounding periods, or with alternative price models.

sales data these problems are exacerbated by the general paucity of sales data, and the considerable heterogeneity among forests sold (which reduces comparability). Accordingly, any reliance on land sales data will likely require supplementation using valuation techniques such as DCF analysis at the very least to assess carbon-pricing impacts as the institutional rules develop and key prices (such as NZU prices) evolve over time. For forest valuations this is clearly the case.

- **DCF analysis** – should provide a useful starting point in assessing the impact of carbon pricing on forest (land) value. It has the important merit that is it relatively easy to implement. In principle each key state variable can be estimated as at the relevant assumed decision (i.e. harvest and/or land conversion) date, including NZU prices with corresponding carbon value or cost implications. This can then enable a static assessment of the value arising from various assumed land use decisions, and summarised in a DCF-based NPV. The chief limitation of this approach is that it ignores managerial flexibility in the light of irreversible harvest decisions (and possibly irreversible land use change decisions as well), and likely volatility in key future state variables such as NZU prices. As such, DCF analysis should produce downward biased estimates of true forest (land) value, since it ignores the option value arising from managerial flexibility in forest and forest land management. In principle DCF analysis could be run for multiple possible valuation and pricing scenarios to gain a better impression of achievable value, but if this is to be done with any degree of completeness, then the implementation requirements begin to become sufficiently demanding that more sophisticated approaches such as real options analysis should be considered.

- **Real options analysis (ROA)** – in principle is conceptually the most sound approach to valuing forests and forest land, with or without carbon pricing, given foresters' discretion in varying harvest timing to suit pricing conditions on the day. The chief limitation of ROA is that it can become extremely difficult to implement for even relatively simple valuation problems. The cause for optimism is that using MCS to value real options problems has developed considerably, even for high-dimensional American-style options, making the valuation of complex real options more tractable. The lingering cause for pessimism is that even MCS may be difficult to implement for problems as complex as the valuation of forest (land) under carbon pricing, given the number of uncertain state variables concerned. As shown in Section 4, however, further adaptation of MCS – so-called Bootstrapping Real Options Analysis (BROA) – can provide a suitable balance in the trade-off between sophistication and ease of use.

4. Outline of Preferred Valuation Approach

4.1 Key Features of Bootstrapping Real Options Analysis (BROA)

4.1.1 Summary of the Approach

As discussed in Section 3, if account is to be taken of a forester's ability to choose the time of irreversible harvest and land use changes in response to the realisation of key state variables such as log prices, NZU prices and land conversion values, then some form of real options analysis (ROA) is required. While the complexity of the decision problem is such that an analytical solution is unlikely to be feasible, Monte Carlo simulation (MCS) is potentially feasible, even given multiple possible decision dates. However, to keep even a MCS-based ROA approach tractable, further adaptation of the technique is required. We suggest an approach which we call Bootstrapping Real Options Analysis (BROA) - due to it using "repeated trial and error" in order to identify an optimal decision rule.[24]

The key feature of this approach is that it uses insights from the ROA literature to posit a parameterised decision rule and then employs MCS to determine the optimal form of this rule and its valuation implications. That is, instead of deriving a formal ROA model, which typically produces both a valuation and an optimal decision rule simultaneously (along the lines of the reserve pricing strategy threshold discussed in Section 3.4.1), BROA *specifies* (rather than derives) the general form of an assumed decision rule and then estimates the value produced by this rule. The optimal form of the rule (and its associated payoff) is then found by identifying the combination of decision rule parameters (which we call "alpha" and "beta") that maximises the simulated payoff.

The key ROA insight incorporated in this approach is that irreversible decisions such as harvest or conversion into non-forestry land uses should only occur when the returns to doing so are "sufficiently positive". In other words, if irreversible decisions are made under uncertainty there can be value in waiting, in order to gain further information before committing to such a decision: we call this the value of the option to "wait" (i.e. to defer making an irreversible decision). This option is lost when the irreversible decision is made, so the decision should only be made when the value from doing so exceeds the value of the lost deferral option. Committing to an irreversible decision when the value from doing so is non-negative – as a simple DCF-based model would specify – is only optimal when there is no valuable option to defer making that irreversible decision.

Because the BROA approach numerically searches for the optimal policy based on an assumed decision rule, it is likely to provide a lower bound on true value (rather than an unbiased estimate).

---

[24] Meade (2006) also provides a general summary of this approach.

From a valuation perspective, this shortcoming may in reality be a virtue, insofar as a globally optimal decision rule is unlikely to be exactly implemented in practice.[25]

Another desirable feature of the BROA approach is that it accounts for decisions made by the forest (land) owner beyond the first possible harvest date. This is particularly important for a pre-1990 forest, since at the assumed 2 January 2008 valuation date a pre-1990 forest will be at least 18 years of age. As harvest of *pinus radiata* in New Zealand typically occurs at around age 27, this means the initial harvest decision is very "near" in present value terms, and hence the impact of post-harvest decisions on forest (land) NPV will be considerable. By allowing for multiple possible initial harvest dates and subsequent land use decisions (e.g. either immediate land use change, or replanting in forestry followed by subsequent replanting, conversion into alternative land use, harvest abandonment in favour of carbon farming or outright abandonment), the valuation impact of such decisions is directly estimated.[26]

This is accommodated within the BROA framework by being "smart then dumb", i.e.:

- BROA is used to explicitly model the initial harvest decision, based on not just the immediate harvest returns, but also the returns flowing from subsequent land use decisions; and

- Those subsequent land use decisions are modelled using DCF-based techniques.

In this sense the BROA approach uses MCS to model initial managerial flexibility with a fair measure of sophistication, but to preserve tractability it employs the simpler DCF approach when assessing the value of multiple subsequent land use decisions.[27] To the extent that land use decisions beyond the initial harvest date occur far more distantly into the future, any value overlooked by this approach (which in the main should comprise neglected option value from subsequent land use decisions) will be diminished through discounting at a positive discount rate. While the approach will in general not produce the "true" forest (land) value, it captures many of

---

25 Moreover, by re-running the BROA model for different forms of posited optimal decision rule it is possible to gain some sense of how sensitive the valuation results are to possible mis-specification of the rule. The approach thus involves "bootstrapping" in the sense that both the form of optimal decision rule, and the optimal parameterisation of that rule, is unknown, and derived by simulation-based "trial and error".

26 While abandonment may be a statistically unlikely forest or forest land use decision, it may arise under some scenarios, and to a greater degree with carbon pricing. For example, if forestry returns are poor and NZU prices high, then both replanting and deforestation may become unviable for pre-1990 forests. Similarly, if harvest becomes uneconomic by virtue of either low log prices or high NZU prices then harvest abandonment in favour of carbon farming, or outright harvest abandonment, may be optimal for post-1989 forests. Indeed, sufficiently high NZU prices could justify outright carbon farming with no prospect of harvest returns, which may be the case for certain types of post-1989 forest reversion investments (e.g. involving indigenous species such as manuka or kanuka).

27 A variation on this approach can be found in Dixit and Pindyck (1994).

the essential elements of the valuation problem, and does so in a way that remains relatively straightforward to implement.

In summary, the BROA represents a significant improvement on existing valuation techniques, and is sufficiently flexible to allow easy incorporation of future new information.

The following sub-sections set out in more detail the BROA approach, both in general and for specific forest and forest land permutations arising under the ETS. Specific derivations of the approach are provided in Appendix B. As noted earlier in the report, the post-1989 valuation approach assumes the relevant forest has already been entered into the ETS. It is left to Section 7 to evaluate the decision as to if, and when, to enter such a forest into the Scheme.

4.1.2 Key Decisions to be Modelled

We have implemented two different "base case" versions of the BROA approach (see Appendix B for precise details, and Appendix E for "R" code):

- **Non-conversion model for a greenfields forest as at 2 January 2008** – this simpler version of the technique models the minimum sufficient level of complexity to incorporate carbon pricing into a post-1989 forest valuation, while also allowing for flexibility in harvest timing. It can also be used to value pre-1990 forestry for the case in which the relevant land has no deforestation options, in which case carbon pricing becomes irrelevant, but the value of managerial flexibility from volatility in log prices can be modelled. Unlike previous research, this approach allows for six uncertain log prices, and one uncertain carbon price, and also for correlation among these seven stochastic variables.

- **Conversion model for an 18 year old forest as at 2 January 2008** – this more sophisticated version of the technique extends the pre-1990 non-conversion model by allowing for the possible conversion of the forest land following either initial or subsequent harvest into dairy land (by way of example). It thus captures the additional essential feature of carbon pricing for pre-1990 forests (i.e. the impact of possible deforestation charges).[28]

Each is described in turn below.

---

[28] Appendix B.2 sets out the modifications required of the pre-1990 conversion model to enable the valuation of a post-1989 forest with land conversion options.

Non-Conversion Model

For the non-conversion model the key decisions to be modelled are:

- Starting at the soonest possible decision date t in a predefined decision window {T_{min}, T_{max}} – the forester chooses from among the following decisions:[29]

    - **Harvest** the trees at t and replant if the post-tax net harvest return H(t) <u>is</u> "sufficiently positive" (see below), producing a NPV at t of H(t) plus the negative present value R(t) as at t of post-tax forestry costs from t to assumed subsequent harvest date t + 27 years, with this NPV at t replicated to infinity assuming no real growth in forest costs or harvest returns to capture the value of later rotations;

    - For so long as t is still less than T_{max}, **defer** any harvest decision until t+1 years if H(t) is <u>not</u> "sufficiently positive" (see below); and

    - If t = T_{max} <u>and</u> H(t) is not "sufficiently positive" (see below) to warrant harvest, then choose to EITHER **abandon** the forest for assumed NPV at t of 0, OR **continue carbon sequestration** (i.e. "carbon farm", in the case of a post-1989 forest) with NPV at t based on expected NZU prices from t onwards given the "known" (i.e. simulated) NZU price at t and the assumed statistical process for NZU prices, whichever is the more valuable;[30]

- Whether or not the returns from harvest and replanting, or harvest and either abandonment or ongoing carbon sequestration, are "sufficiently positive" at time t is determined relative to the posited form of reserve pricing threshold – or "decision barrier" B(t) – applicable for a forest of a given age at time t. Figure 4.1 overleaf illustrates three representative reserve pricing threshold profiles arising from earlier studies – the essential feature being the shape of and decline in the reserve pricing threshold as a function of forest age.

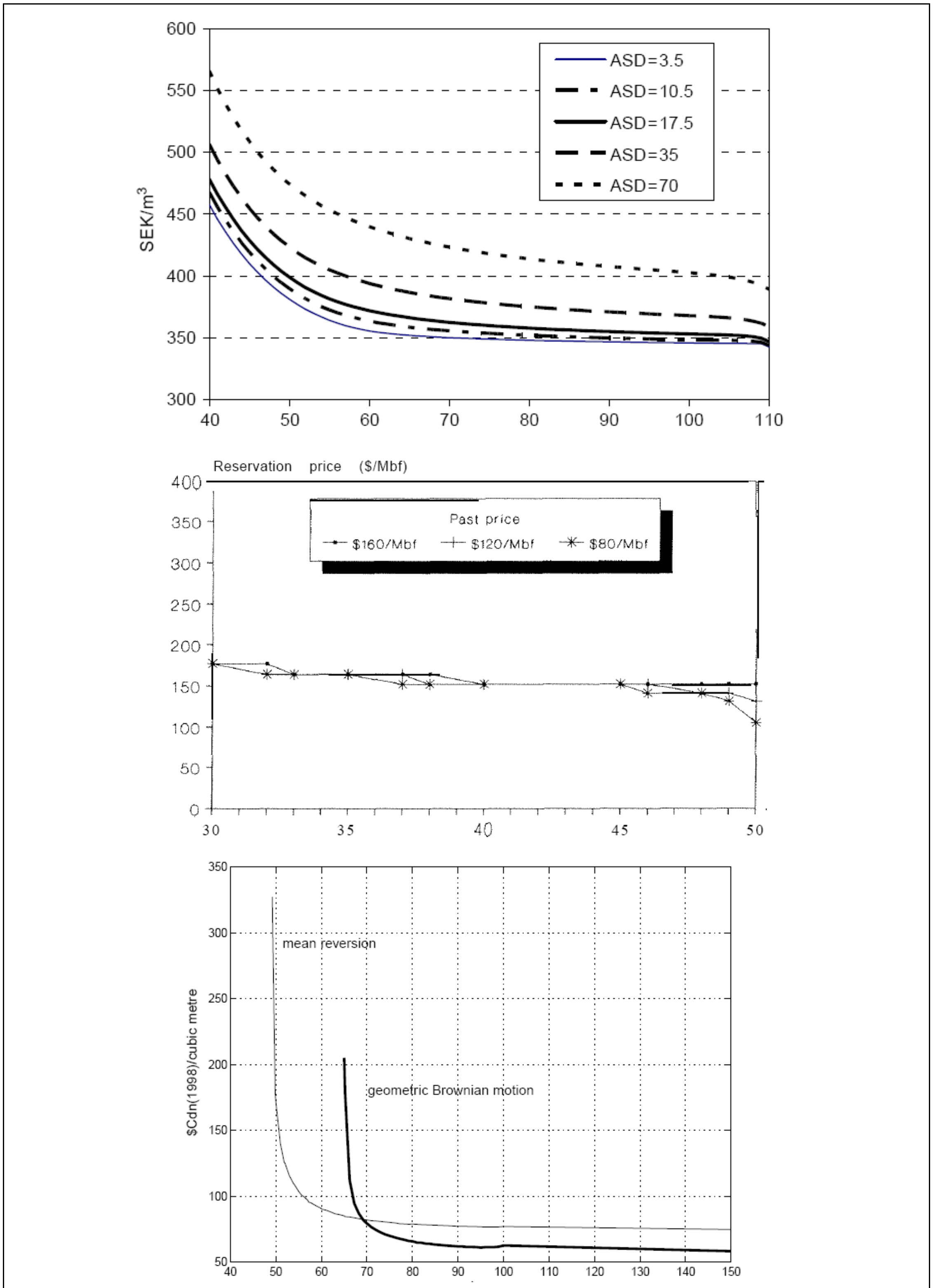
---

29 T_{min} is chosen to reflect the minimum feasible harvest age of the forest (which could be zero years), while T_{max} is set to reflect the possibility that once a forest grows beyond a certain age, it may not be economically harvested (e.g. due to log diameters exceeding mill capacity).

30 Our model assumes harvest is intended from the outset but might be abandoned in favour of ongoing carbon farming. An alternative approach is to assume carbon farming is intended from the outset but allow for the option to harvest. One possible merit of the former is that assuming harvest from the outset means that silvicultural activities to preserve harvest value in the future can be accommodated, whereas under the alternative approach thinning or pruning might not be undertaken, meaning only lower timber value might be realized under any harvest decision. Either way the optimal silvicultural regime for a post-1989 forest in the ETS will need to reflect both timber and carbon value, and may differ from traditional optimal forest management based only on timber value considerations.

Figure 4.1

Representative Reserve Pricing Threshold Profiles vs Stand Age from Previous Studies



Sources: Gong and Lofgren (2007) – cf Gong (1999) and Yin (2001); Haight and Holmes (1991); Insley (2002). SEK/m^3 means Swedish Krone per m^3; $/Mbf means US$ per thousand board feet and $Cdn(1998)/cubic metre means 1998 Canadian dollars per m^3.

To replicate these decision barrier profiles and test for the value of alternative profiles, three simple proxy reserve pricing threshold profiles were implemented in the non-conversion model:

- The Gong and Lofgren (2007) "S-shaped" B(t) profile was replicated using the mathematical arcos(x) function, parameterised by a single variable which we call "**alpha**", and assuming the ratio of the endpoint of the threshold to the starting point is approximately as in the Gong and Lofgren graph;[31]

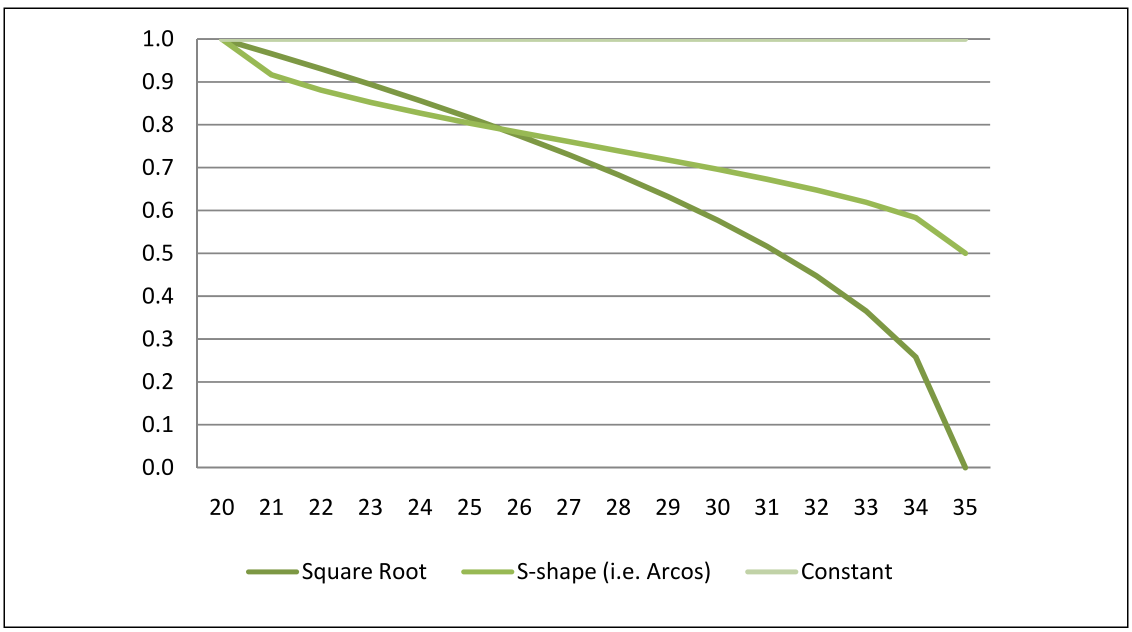
- A flat B(t) profile was replicated using a constant threshold having value "**alpha**" over the decision window $\{T_{min}, T_{max}\}$; and

- To test the sensitivity of NPV to alternative threshold specifications, a simple declining "square root" B(t) function "**alpha**" x $\sqrt{(T_{max} - t)}$ was modelled.[32]

Figure 4.2 illustrates these three profiles on a unit-free basis (i.e. all are scaled so that the maximum value of B(t) is one).

Figure 4.2

Three Decision Barriers Implemented in the Non-Conversion Model vs Forest Age



Note: T_{min} = 20 years and T_{max} = 35 years assumed

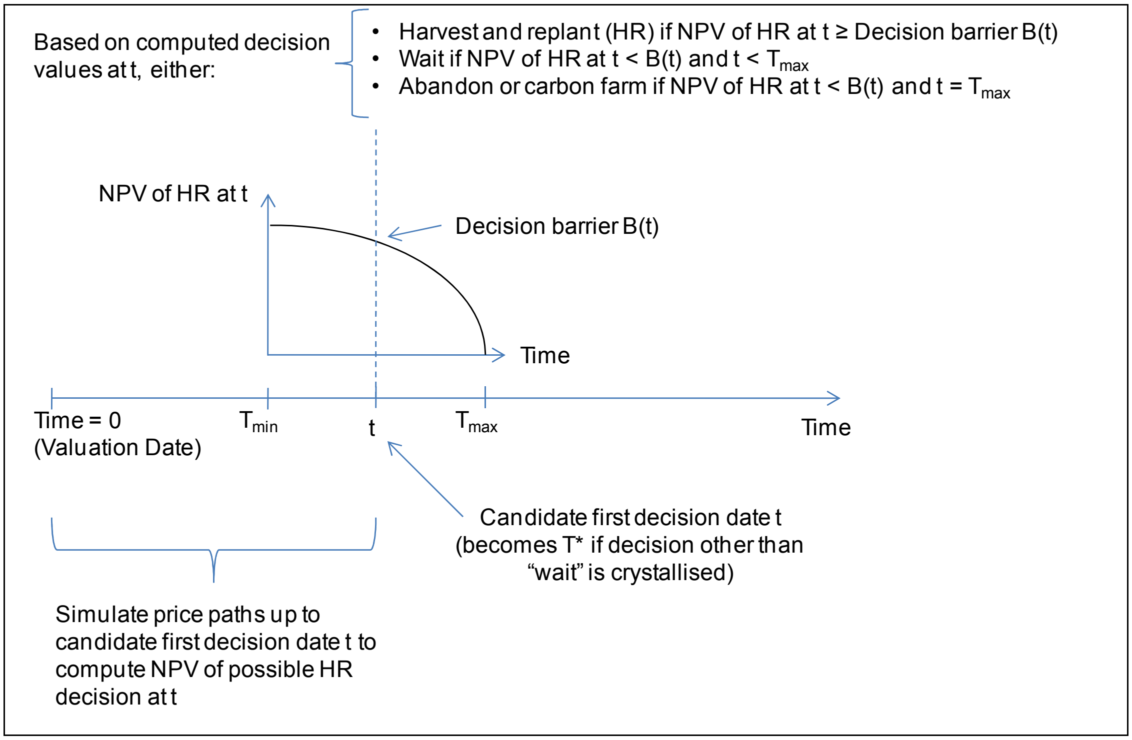
31 See their Figure 1, p. 230, in which the ratio of the highest point of the reserve price graph to its lowest point is around 600 to 350 . Arcos(x) is a real number for $-1 \leq x \leq 1$, taking values between π and 0 on this range. It can be computed in Microsoft Excel using the ACOS function, and scaled to produce a profile of the desired height for the desired range of input values.

32 Like the Gong and Lofgren (2007) profile, the square root profile suggests that the value of deferring harvest "runs out" as the latest feasible harvest date is approached.

Given an assumed candidate value of alpha, in each simulation path the first decision date t at which the NPV from harvest and replanting (replicated in perpetuity) exceeds the value of the relevant alpha-parameterised B(t) threshold at time t is the actual decision date T* at which a harvest and replanting decision is crystallised. The value of the decision at time t is calculated based on simulated values of the key state variables up to time t, and hence a different optimal decision value is produced for each simulated sample path. The NPV of this decision at time 0 is then computed for this simulation trial, including the NPV of any revenues or costs incurred up to T*. This basic approach is illustrated in Figure 4.3.

Figure 4.3

Schematic of BROA approach for a given Simulation Trial – Non-Conversion Model



Computing this decision value for (e.g.) 5,000 simulated sample paths then enables the average resulting NPV at time 0 (i.e. the forest value as at 2 January 2008) to be computed. This NPV(0) is the forest's estimated value, *given* the assumed candidate value of alpha.

Since each reserve pricing threshold is parameterised by the single unknown variable alpha, the simulation process (i.e. runs of 5,000 sample paths) is re-run for different values of alpha, with "optimal" alpha being that which produces the highest overall value of NPV(0). It is this maximised value of NPV(0) that is the BROA estimate of forest value. Similarly, the average value of T* produced for the 5,000 simulated sample paths generated using the optimal value of alpha provides an estimate of the optimal harvest date suggested by the BROA approach (which may differ to the conventional Faustmann date).

Conversion Model

For the pre-1990 forest conversion model, the following alternative decisions are modelled, similar to those in Guthrie and Kumareswaran (2007):

- At some possible decision date t, in a predefined decision window $\{T_{min}, T_{max}\})$[33] – the forester chooses from among the following decisions:

    - **Harvest the forest and convert it into an alternative land use** (for a simulated land value realised then)[34] if the NPV generated at t by doing so, net of assumed conversion and simulated deforestation costs, is "sufficiently greater" (see below) than the NPV at time t of harvesting and replanting the forest at time t, with the NPV of the latter decision including the NPV from choosing the better of harvesting and replanting, harvesting and converting, harvesting and abandoning, or outright abandoning the forest at time t + 27 years (based on expected prices at that time given "known" (i.e. simulated) prices at t) to capture the value of subsequent decisions;[35]

    - **Harvest the forest and replant it** at time t, with the NPV of this decision at t also including the NPV of choosing the better of harvesting and replanting, harvesting and converting, harvesting and abandoning, or outright abandoning the forest at time t + 27 years (based on expected prices at that time given "known" (i.e. simulated) prices at t), if the NPV generated by doing so is "sufficiently greater (see below) than the NPV at time t generated by harvesting the forest at that time and converting it then into an alternative land use (for a simulated land value realised at time t);

---

[33] Note that in principle different values of T_{min} and T_{max} might apply for differing land use decisions. For example, while there may be a minimum and maximum feasible forest age for harvest purposes, this need not necessarily restrict the forester's ability to convert the forest land into some alternative use outside of this decision window. For simplicity, however, the same window has been assumed to apply to all decisions in this version of the model, with extensions discussed in Section 7.

[34] In fact the model could be augmented to reflect the possibility that the landowner could possibly reforest the land following any initial deforestation. For ease of illustration this feature has not been explicitly provided for, but the approach taken to modeling the post-harvest decisions in the case that the forest is harvested and replanted could equally be adapted to allow for possible reforestation options. The added difficulty in that case, however, would be that reforestation need not wait 27 years to occur – in principle it could occur at any time.

[35] Modeling the forester's decision at t + 27 years is all the more important for an existing pre-1990 forest since that forest will be at least 18 years of age at the assumed 2 January 2008 valuation date. This means that the initial harvest decision (if any) will occur quite "near" the valuation date in present value terms, and hence the contribution of post-harvest value to total value will be significant (i.e. discounting over a long time period cannot be assumed to reduce the significance of post-harvest value).

- For so long as t is still less than T_{max}, **defer** any harvest and replant or harvest and convert decision if neither of the above tests is satisfied; and

- If t = T_{max} and neither of the returns from harvest and conversion or harvest and replanting is "sufficiently greater" (see below) than the other to warrant either decision being made, then choose to EITHER **abandon** the forest for assumed NPV at t of 0, OR **harvest and abandon** the forest then generating post-tax net harvest revenue based on the "known" (i.e. simulated) log prices at that date, whichever is the more valuable.

- Whether or not returns from harvest and conversion or harvest and replanting is "sufficiently greater" than the other to warrant either decision being made at time t is determined relative to the posited form of a "decision map" applicable for a forest of a given age at time t. This decision map generalises the reserve pricing threshold approach used in the simpler non-conversion model, now that conversion must also be allowed for. Figure 4.4 overleaf illustrates two such posited decision maps.

Note that in these decision maps two unknown parameters now require optimisation through the BROA approach:

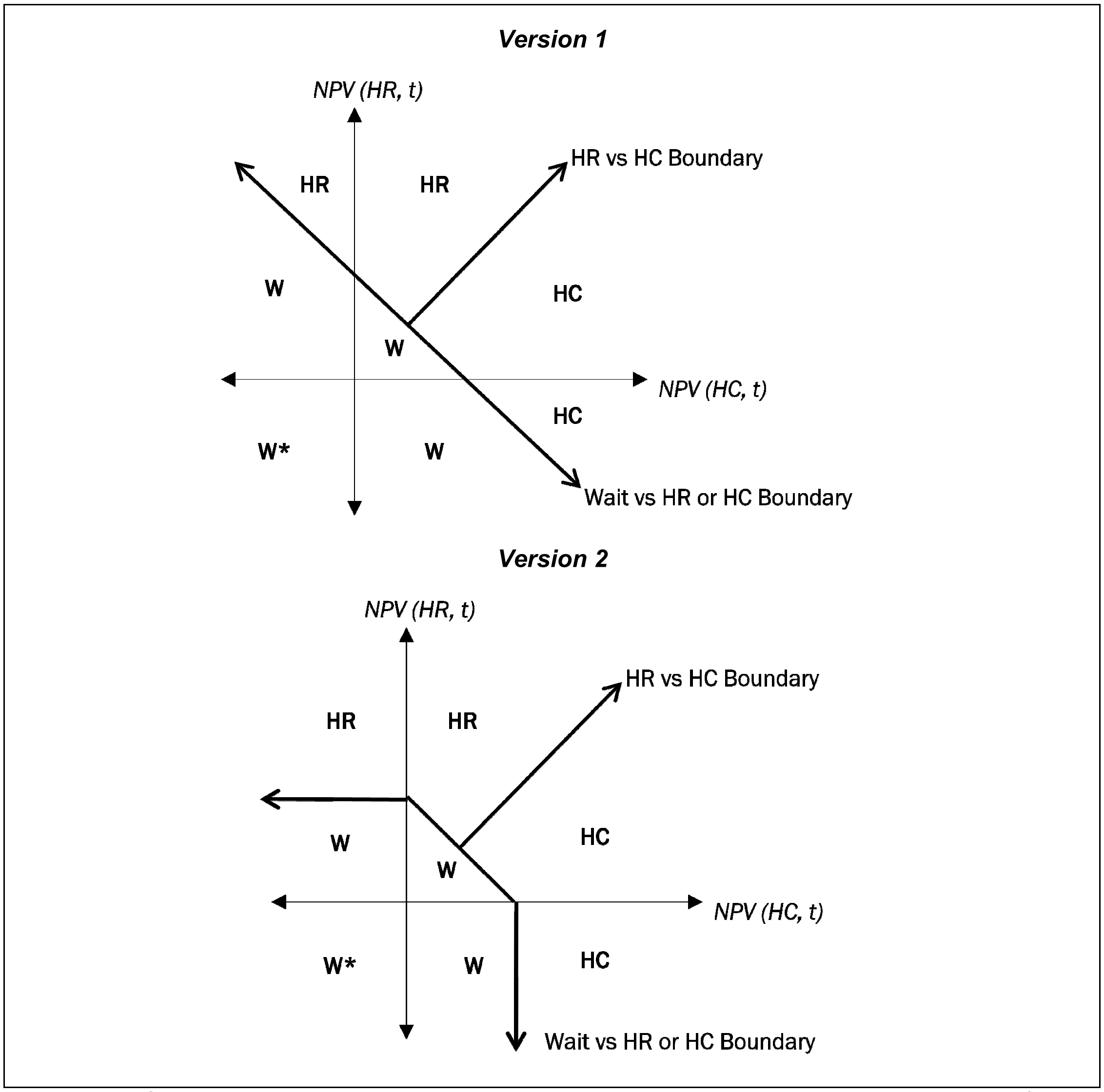
- **Alpha** – as before, parameterising the intercept on the vertical axis, reflecting the value to waiting in respect of the decision to harvest and replant. For the conversion model the square root reserve pricing threshold profile was assumed to apply to this intercept, meaning that as t advances from T_{min} to T_{max}, the vertical intercept in the conversion decision map becomes successively smaller until it reaches 0 when t = T_{max};

- **Beta** – a new parameter, setting the horizontal intercept in the decision map, reflecting the value of waiting in respect of the decision to harvest and convert. For the conversion model it is assumed that beta is constant – i.e. that the value of deferring harvest and conversion does not diminish with time (unlike harvest, no latest land conversion date need necessarily be imposed).[36]

[36] Indeed, the surge in deforestation in favour of dairy land conversion occurring in New Zealand prior to the 1 January 2008 start date of the ETS illustrates how beta might be lower in certain instances. In that case the value to deferring conversion was lower than that applicable once the ETS commencement date was reached, since waiting until 1 January 2008 would only result in liabilities that could be avoided by accelerating conversion. Such effects might arise again in the future, for example if stricter deforestation liabilities for pre-1990 foresters should be announced with a future commencement date.

Figure 4.4

Two Possible Decision Maps for Pre-1990 Conversion Model for t in {T_{min}, T_{max}}



Notes: HR (HC) = decision to harvest and replant (harvest and convert) at time t. The corresponding NPVs of those decisions as at time t are NPV(HR,t) and NPV(HC,t). W = decision to defer taking any decision at time t (i.e. to wait). W means defer taking any decision if t < T_{max}, otherwise choose either outright abandonment or harvest and then abandonment at time t, whichever is the more valuable.*

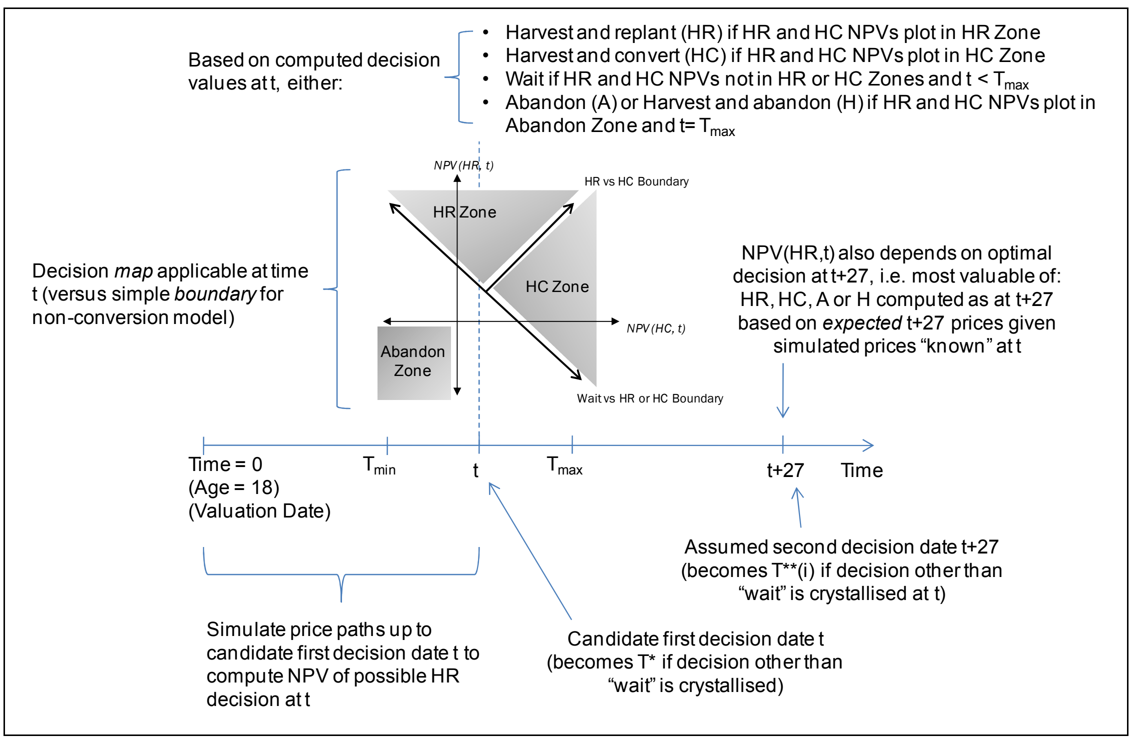
These decision maps are posited on the basis that they each require the NPVs from either harvest and replanting or harvest and conversion at the relevant possible decision date to be both sufficiently positive *and* greater than the other before they are taken as the optimal decision. The first version is posited as a simple and easily implemented decision map, capturing this intuition. The second version refines the first by assuming a less restrictive barrier for one decision if the NPV of the other at the relevant possible decision date is negative. As for the non-conversion model, if the NPV of either decision falls into the zone in the decision map where it exceeds the relevant thresholds (plural now), then that possible decision becomes the actual decision at that time t, and t is fixed as T*. In the harvest and replant case the corresponding NPV also includes the expected NPV of the greater of harvest and replanting, harvest and conversion, outright abandonment, or harvest and abandonment at time T** = T* + 27 years.[37]

Also as for the non-conversion model, in the conversion model the optimal values of alpha and beta are determined using "bootstrapping", by successively running the simulation model for 5,000 sample paths for each of various combinations of alpha and beta, and then choosing the optimal combination as that producing the highest average simulated NPV. The additional bootstrapping element is the choice of decision map, with either version 1 or version 2 being chosen based on whichever produces the highest average NPV for the corresponding optimal alpha-beta pair (which may differ depending on which version of decision map is used).

Figure 4.5 illustrates the approach for any given simulation trial in the BROA conversion model.

Figure 4.5

Schematic of BROA approach for a given Simulation Trial – Conversion Model



---

[37] Where the expected NPV of harvest and replanting at T** also allows for subsequent rotations by assuming the relevant NPV is replicated indefinitely with nil real growth. A useful refinement would be to set the gap between T* and T** based on the optimal harvest age determined by the BROA simulations, rather than just assume a conventional "optimal" harvest age. Fixing this age at 27 years has been for the sake of convenience, and given the impact of discounting over a 27 year period, varying this assumption by some small amount should not materially change overall value.

4.1.3 Modelling of State Variables

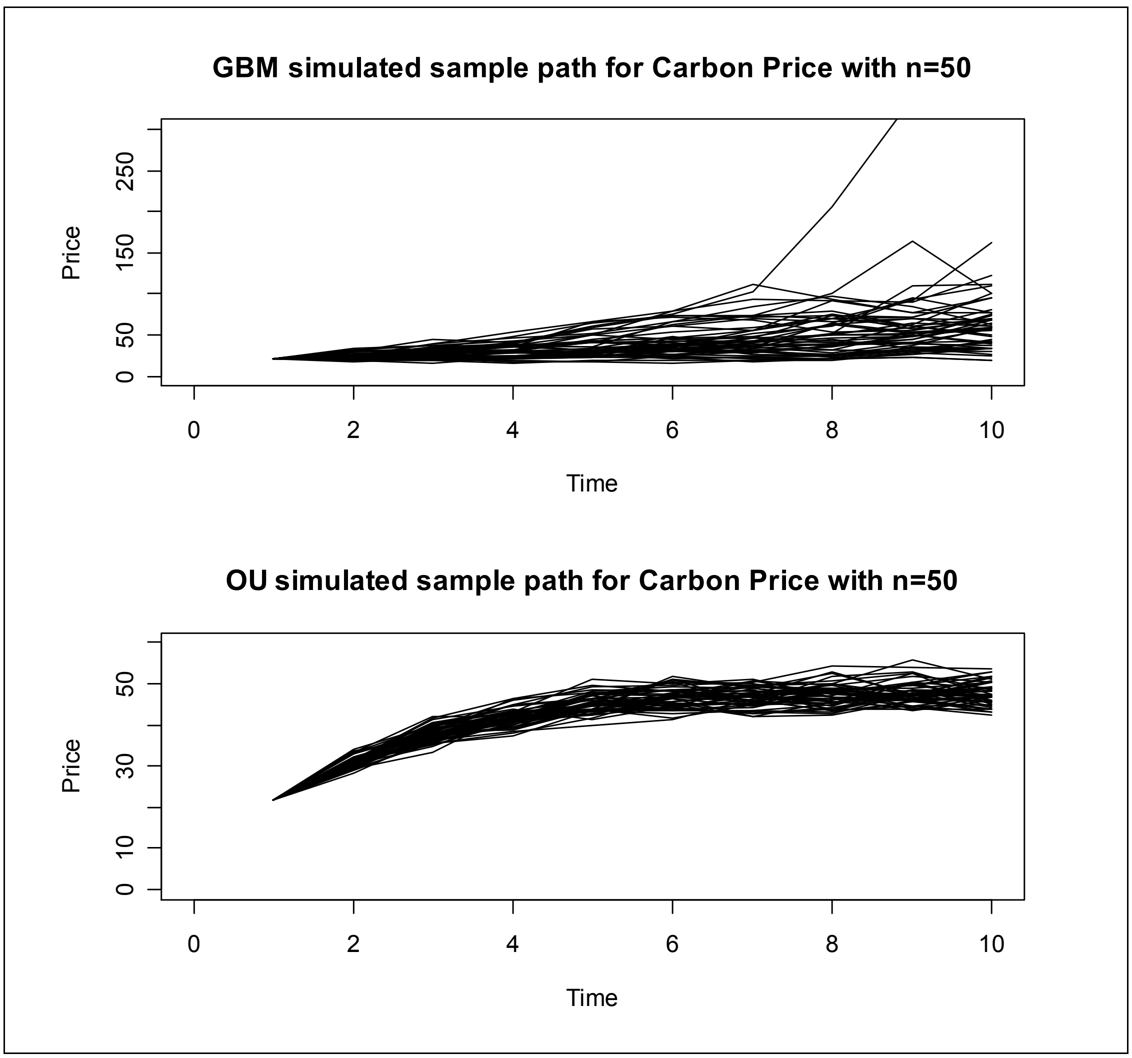
In option valuation analysis two commonly assumed statistical price processes are:

- Non-Stationary Geometric Brownian Motion (GBM); and

- Stationary Mean-Reversion (MR, such as the Ornstein-Uhlenbeck (OU) process).

Figure 4.6 illustrates how these two price processes evolve over time using 50 simulated price paths over 10 time periods. In the case of MR – in this case with the mean to which prices revert being higher than the initial price – prices fluctuate from one period to the next but tend to converge on that mean over time. Conversely under GBM prices fluctuate randomly around a fixed growth path.

Figure 4.6
Simulated Geometric Brownian Motion (GBM) and Ornstein-Uhlenbeck (OU) Prices



The chief merit of GBM is that it is easy to implement, and lends itself to the derivation of closed-form analytical solutions, at least for option problems that are sufficiently simple. An important possible limitation of GBM is that it is unbounded in both level and variability over time – i.e. over longer time periods the expected GBM price level increases indefinitely, as does the variance of the resulting prices. Both of these features have been criticised, and various authors instead prefer to assume some form of MR process, especially where commodity prices are being modelled (e.g. see Schwartz (1997), Insley (2002) and Guthrie and Kumareswaran (2007)). The chief rationale underpinning MR over GBM for commodities is that supply-side reactions to rising or falling prices should serve to dampen major price deviations, and over time prices (at least in competitive markets) should be expected to reflect underlying production costs.

Whether or not NZU and log prices follow GBM or MR is a matter of debate. Research was cited in Section 2 suggesting EU ETS allowance prices did not exhibit mean reversion, and did exhibit instability in variances that suggests models other than MR. While Niquidet and Manley (2007) reject mean reversion for New Zealand log prices, they – like Unterschultz et al. (1998) – acknowledge that the length of data series can be important, since it can be difficult to distinguish GBM from MR even with decades of data. Also, Yin and Newman (1995) find evidence supporting MR in log prices in 14 southern US markets. On a different tack, Metcalf and Hassett (1995) suggest there is little practical difference between GBM and MR in terms of investment decisions, at least for smaller price variability. Conversely, Insley (2002) finds that the choice of pricing process makes a considerable difference for reserve pricing thresholds in forestry.

It is beyond the scope of this report to resolve such debates. Instead, MR is assumed as a pragmatic and conservative approach, in that unbounded rises (or falls) in NZU or log prices seem in principle untenable, particularly given the relatively long time-horizons modelled in forestry valuations.[38] The OU process used has the merit shared with GBM that its future expected value is easily computed given initial prices, and it is relatively straightforward to both estimate its parameters and simulate its future values. Many other processes could have been assumed, and when a good time series of NZU price data comes to hand the BROA lends itself to easily accommodating any price process changes.

One complication with the use of MR price processes is that they commonly assume prices converge over time to some historical mean level. While this may be appropriate for price series with a stable level over time, this is less so where there are long-term trends in prices. Various possible responses are to allow for a trend in the mean to which the MR process converges (e.g.

---

[38] Arguments similar to those mounted for mean reversion in log and other commodity prices can be mounted for NZU prices. As NZU prices rise successively more expensive methods of mitigating emissions will become economically viable, thus eliciting a supply-side response that serves to dampen such rises. Conversely, as NZU prices fall, some emission mitigation methods will become uneconomic and be retired, thus reducing the supply of emission reductions and creating upward pressure on the price of emission rights.

see Ewald and Yang (2007)), or to fit logistic-type curves to trending data (which causes long-term predicted values to either plateau or bottom out). For this report the pragmatic compromise has been struck of using latest available prices as the long-term mean, rather than historical averages, where there are price trends in the data.

4.2 Valuing Forests and Forest Land when the Land is Owned Separately from the Trees

As discussed in Section 3.3, it is a relatively straightforward matter to incorporate some form of rental stream into a DCF-based forest valuation model. In the BROA model this is similarly so, but it is important to recognise any constraints that land tenure arrangements might place on managerial flexibility in forestry. For example, foresters who rent land will not, in general, also have an unfettered ability to change land use, or if they do so, their rental rate might also change. Moreover, there are two features of the ETS worth noting where the forest and forest land are separately owned:

- Pre-1990 forests – liability for deforestation falls to the land owner in general, unless the land owner can demonstrate that the decision to deforest the land was in the hands of some other party (e.g. perhaps through forestry right requirements that land be returned to pasture following harvest).

- Post-1989 forests – the ETS Bill contemplates bargaining between land owners and foresters as to whom should get what share of carbon credits and liabilities where the land is separately owned from the trees.

Both of these details would need to be reflected in any BROA-based (or DCF-based) valuation model of the trees in such cases. For example, the holder of a forestry right over pre-1990 forest land who does not have the right to determine land use following harvest should value their trees net of any agreed rental stream, but also ignoring any deforestation liability or conversion option (e.g. using the non-conversion model). Conversely, the owner of such land would receive a rental stream and retain the value of any land conversion option, but this option would have to be valued net of any conversion and deforestation costs. The timing of the possible exercise of their land conversion option would be constrained by the anticipated date by which the forest right holder vacates the land. Where the right does not have a fixed termination date, but instead depends on when harvest occurs, the land owner could use the BROA valuation approach for the associated trees to predict when harvest is likely to occur, and implement the land conversion aspects of the conversion model to value their own land conversion options (i.e. net of any deforestation costs, and ignoring any harvest returns). If their rental was tied to stumpage on harvest, the BROA forest valuation model could again be used to predict what the likely net stumpage will be.

An important consideration when assessing the value of conversion options for pre-1990 forest land is the date when deforestation is assumed to occur. Under the ETS Bill, the level of deforestation liability is generally based on the carbon stored in the trees at the time of deforestation. However, under clause 162 of the ETS Bill the liability will instead be based on the carbon stored in the preceding crop if deforestation occurs for trees that are eight years or younger. This presents both risks and opportunities for pre-1990 forest land owners. One possibility is that land owners choose to allow reversion long enough for any tree regrowth to reach eight years of age, and then deforest the land. This would involve trading off the opportunity cost and direct costs of letting the land remain unproductive for that period in order to face a reduced deforestation liability (since it will be based on the lower biomass, assuming NZU prices are not predicted to rise dramatically over the relevant time frame to eliminate the advantage). This possibility is modelled as part of the sensitivity analysis in Section 6.

Conversely, if pre-1990 forest land encumbered by some form of forestry right owned by a third party is valued on the basis that it will be sold following the expiration of that right, the value that can be expected to be realised, net of conversion and deforestation costs, will depend on whether or not buyers of convertible pre-1990 forest land are prepared to allow regeneration to occur for a time before deforesting.[39] If not, then buyers may discount their bids by the value of any deforestation liability they incur based on the previous tree crop, rather than the present value of the smaller deforestation liability, net of holding costs, from deforesting based on the deemed nine year old trees.

Where post-1989 forest land is owned separately from trees that have been entered into the ETS, the approach to determining land value will depend to some degree on how carbon credit revenues and liabilities are agreed to be shared between the parties. Obviously, if the landowner receives just a fixed rental and no share of carbon revenues or liabilities, then the BROA model might only be usefully applied in modelling the land conversion options once the land has been vacated (which, again, could be predicted using the non-conversion BROA model), with the initial rental stream valued in the usual manner. Conversely, where the land owner receives a share of carbon revenues and costs, the non-conversion model could be used to predict their share of the returns and costs, as well as the predicted land vacation date (at which point a variation on the conversion BROA model could be used to value their land use options). The essential point is that the building blocks used to create the conversion and non-conversion models, and the underlying

---

[39] Indeed, an important practical consideration for buyers of such land is to ascertain what level of carbon liability they will face if they deforest before any reversion or replanting reaches eight years of age. If they buy bare forest land from a seller, they will not necessarily know exactly how old any reversion or replanting is, or what the total carbon sequestered was in the previous tree crop. In fact, even the vendor of the land might not know this, if the land was encumbered by a third party's forestry right (about which the vendor may have only limited forestry data). While the ETS provides for standard carbon tables which will create default liabilities, there are likely to remain some important practical considerations to be resolved and addressed by potential buyers of cut-over or regenerating pre-1990 forest land.

approaches to predicting values of key future state variables, can be adopted and adapted in either simple or sophisticated approaches to valuing the land separately from the trees.

4.3 Summary

To meaningfully accommodate some of the key complexities of the ETS in forest and forest land valuation it is necessary to use a valuation approach that recognises possible flexibility to change land use. Indeed, without this flexibility, valuing a pre-1990 forest or forest land would be trivial, in that the ETS could easily be accommodated within existing valuation approaches such as DCF analysis. However, once the value of flexibility is recognised, the ETS potentially introduces significant new costs for pre-1990 land owners in making their land use decisions. Indeed, even for post-1989 forests variability in log and NZU prices will possibly create valuable management options even where land use change is not contemplated, simply in terms of harvest timing (or abandonment in favour of carbon farming) decisions.

The BROA approach described in this section offers an easily-implementable mechanism for accommodating ETS-related valuation impacts. Importantly, the building blocks and modelling tools underlying the BROA approach can be adopted and adapted to value alternative scenarios, such as when the land and trees are separately owned.

In the next section we describe how the BROA approach was implemented to illustrate its efficacy for valuing forests owned along with forest land. Section 6 then provides results from “proof of principle” calibration tests to show the approaches’ efficacy in solving a simple known American-style option pricing problem, and then provides results from the models’ implementation using representative New Zealand forestry and other data.

5. Implementation of Preferred Valuation Approach

5.1 Data Sources

To illustrate the application of the Bootstrapping Real Options Analysis (BROA) approach to the valuation of pre-1990 and post-1989 forests we have used representative assumptions for *pinus radiata* forests in New Zealand from a variety of sources (see Appendix C for figures):

- **Total biomass yields (m^3/ha)** – proxied by Total Stand Volume (TSV) per hectare for a forest aged between 0 and 60 years, using the "Initial growth model" and "Growth and yield model for *Pinus radiata* in the Central North Island region of New Zealand" available from the New Zealand School of Forestry website at University of Canterbury. For simplicity 100% of these yields are assumed harvestable – a refinement would be to model losses (e.g. due to fire or windthrow);[40]

- **Recoverable log yields by grade (m^3/ha)** – Total Recoverable Volume (TRV) approximated as 85% of TSV, with yields for grades P1, P2, S1S2, S3&L3, L1&L2, and Pulp for forest ages 20 through 35 as per indicative ratios provided by the School of Forestry[41] (and indicatively extrapolated thereafter to forest age 60);

- **Log prices by grade ($/$m^3$)** – produced using the log price history on the MAF website, from September 1994 to December 2007, with conversion to price per cubic metre where necessary;

- **Carbon sequestration (tCO_2/m^3, and tCO_2/m^3/yr)** – total carbon sequestered simplistically assumed to be 0.77 x TSV,[42] with annual carbon sequestration estimated as annual change in total sequestered carbon;

- **Forestry costs ($/ha for establishment and other one-off costs, $/ha/yr for ongoing costs, $/m3 for harvest costs)** – based on representative figures contained in the New Zealand Institute of Forestry 2005 *Forestry Handbook*, including thinning and pruning costs;

- **NZU prices ($/t$CO_2$)** – proxied by prices posted on the Treasury website for the purposes of estimating the New Zealand government Kyoto Protocol liability, and separately by daily

---

40 Stems per hectare of 950 thinned to 700 at age 5, then 300 at year 7. Calculated site index of 34.7.

41 Personal communication, Euan Mason, New Zealand School of Forestry.

42 Personal communication, Steve Wakelin, Scion Research.

Reuters bid and ask prices posted by major brokers in the secondary CER market from 9 March 2007 to 9 May 2008 (converted into New Zealand dollars);

- **Dairy land prices ($/ha)** – Quotable Value average sales price per hectare for dairy land, 1989 through 2007;

- **Land conversion cost ($/ha)** – assumed $7,500 per hectare to convert forest land into dairy land;

- **Price growth rates and convenience (cash) yield for mean reverting price processes** – following conventional practise, the BROA valuations have been conducted under the assumption of "risk neutrality", in which case the growth (or drift) rates in all assumed price processes are set equal to the risk-free interest rate less an assumed or estimated cash (or convenience) yield. For illustrative purposes a convenience yield of 2.5% has been assumed for all price processes;

- **Discount rates** – consistent with the "risk neutral" valuation approach discussed in Section 3.4.5, the risk-free interest rate is used as discount rate (10 year New Zealand government stock yield, in real terms). Where comparative DCF valuations are produced, a post-tax real discount rate of 8.5% is assumed, consistent with figures currently applied in New Zealand;[43] and

- **Correlations between price variables for price simulations** – have been estimated statistically using price data to the extent it is available for the same time frame as the other price series to which it is being correlated (e.g. for NZU prices, only a limited data history is available with which to do so).

These figures have been applied to value a representative hectare of forest, i.e. all valuations are per hectare. All financial figures were assumed to be in December 2007 dollars, or converted into December 2007 dollars using the Consumers Price Index for dairy land values, or Producers Price Index (Outputs, All Industries) for other series, with both series available from Statistics New Zealand.[44]

A number of these assumptions have been deliberately simplified for convenience, and could be refined for any actual valuation, or as better information comes to hand. For example:

---

[43] For a recent survey of discount rates applied in the New Zealand forestry sector, see Manley (2007).

[44] In principle the Consumers Price Index series should be used to deflate all financial variables, since investors will be concerned with real lifetime consumption opportunities from investment returns.

- The assumed carbon sequestration figures abstract from soil carbon and carbon in tree roots and leaves (etc): as more refined methodologies emerge for measuring total carbon these could easily be incorporated;

- Deterministic biomass growth rates have been assumed, and no impact of climate change itself has been assumed on growth rates. However, if average temperatures rise, rainfall conditions change, and/or droughts, storm events or new pests and diseases arise as a consequence of climate change then future and historical growth rates may differ. As above, a particular advantage of simulation methods is that any such complications can be explicitly modelled, either deterministically or stochastically, and readily incorporated in the BROA model;

- NZU prices have been proxied using available short-term price series. As clearer pricing points emerge and more data becomes available – particularly as institutional uncertainties regarding the ETS and international arrangements are resolved –better price estimates can be used in the valuations;

- Convenience (cash) yield assumptions will affect the simulated price paths for each uncertain price variable in the BROA model. More refined estimates of these yields could be obtained, for example for NZUs as and when better NZU price data becomes available.

- Dairy land conversion has been assumed as the relevant conversion counterfactual for forest land. Clearly not all forest land is capable of conversion into dairy land (e.g. land of an unsuitable gradient). Steep forest land might, for example, instead be convertible into pastoral uses such as sheep farming. Dairy conversion has been modelled as it has recently been one of the more profitable alternative uses of forest land, and so should provide a relevant demonstration of the BROA valuation model's performance. Alternative conversion counterfactuals may be equally valid, particularly over time, but have not been implemented. This is discussed further in Section 7.

5.2 Key Parameter Estimates

Appendix C sets out the forestry assumptions used in the BROA implementation. Table 5.1 (overleaf) summarises the key parameter estimates also used. Appendix D sets out the pricing models used in the BROA.

For illustration purposes it is assumed that harvest of a forest can only occur between forest ages 20 through 35 (i.e. T_{min} = forest age of 20 years, and T_{max} = forest age of 35 years).

Table 5.1

Key Parameter Estimates (See Appendix D for Models)

			Log Grade				Dairy	Proxy NZU Price	
	P1	P2	S1S2	S3L3	L1L2	Pulp	Land	Treasury	Reuters
Price Process Assumptions:									
Latest Price	139	108	84	73	67	49	19,879	22	35
Mean Price (OU)	204	157	114	82	85	53	11,333	13	29
OU Mean (logarithm)	5.32	5.06	4.74	4.41	4.44	3.96	9.34	2.60	3.37
OU Variance	0.01	0.03	0.02	0.04	0.03	0.03	0.08	0.16	0.09
OU Rho	0.88	0.68	0.74	0.35	0.57	0.42	0.79	0.38	0.04
OU Kappa	0.13	0.39	0.30	1.06	0.57	0.86	0.24	0.95	3.13
GBM Mean	-0.06	-0.04	-0.04	-0.01	-0.02	-0.01	0.10	0.37	0.41
GBM Standard Deviation	0.10	0.17	0.13	0.20	0.17	0.17	0.26	0.38	0.31

			Log Grade				Dairy	NZU -
	P1	P2	S1S2	S3L3	L1L2	Pulp	Land	Treasury
Correlations (Between logarithms of all variables):								
P1	1.00							
P2	0.93	1.00						
S1S2	0.88	0.87	1.00					
S3L3	0.72	0.68	0.84	1.00				
L1L2	0.83	0.82	0.87	0.74	1.00			
Pulp	0.59	0.52	0.67	0.70	0.69	1.00		
Dairy land	-0.46	-0.37	-0.32	-0.30	-0.20	-0.15	1.00	
NZU – Treasury	-0.73	-0.71	-0.22	0.52	0.25	0.16	0.00	1.00

Notes: OU = Ornstein-Uhlenbeck mean reversion; GBM = geometric Brownian motion. Treasury NZU proxy prices used to construct correlation table because Reuters data series is too short. Zero correlation between logarithm of carbon price and logarithm of dairy land value due to absence of data. Logarithms of prices used to avoid negative prices being generated.

Source: Authors' estimates.

5.3 Software

The BROA model has been implemented using the statistical language “R” for the following reasons:

- It is freely available open-source software;
- Its functionality simplifies the process of implementing changes to model structure, which was desirable for model development purposes;
- Unlike a spreadsheet, producing code in a language like R makes all calculations explicit, aiding in communicating the approach to users; and
- R has a number of convenient pre-programmed features that simplify implementation.

Appendix E contains sample code for both the non-conversion (pre-1990 and post-1989 forests) and conversion (pre-1990 forests) variants of the BROA approach.

In practice it may be more convenient for most users to implement the approach in a commercial spreadsheet package like Microsoft Excel, using commercially available add-ins like Palisade’s @RISK, or Oracle’s Crystal Ball (software which some valuers already have and with which they are familiar).

Having set out these implementation details, Section 6 presents results from the BROA implementation, and compares those results with simple DCF-based forest valuations.

6. Results of Preferred Valuation Approach

6.1 Proof of Principal Calibration Test

In order to demonstrate that the basic Bootstrapping Real Options Analysis (BROA) approach can value at least a simple American-style option we have applied it to the valuation of a simple investment timing problem – an exact analytical solution for which has been provided by McDonald and Siegel (1986) (as summarised in Boyle and Guthrie (2003)). This allows us to evaluate the BROA solution in a situation where the true solution is known. In the McDonald and Sigel model the option (in this case to make an investment) is optimally exercised only if the exercise value from doing so exceeds the investment cost by a specified ratio greater than 1. This is analogous to the BROA presumption that a forest should be harvested only if the net harvest proceeds exceed zero by more than a specified amount.

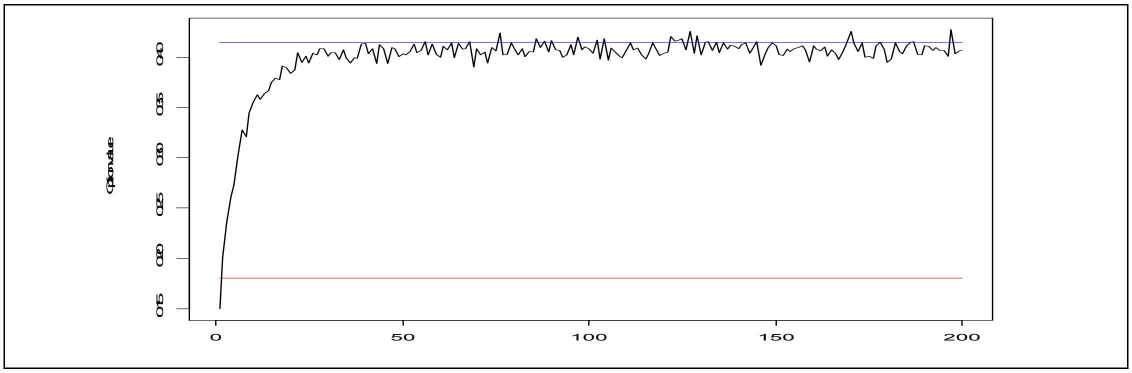
The calibration thus requires setting up an exercise threshold (called alpha in our terminology) and simulating the resulting present value of option exercise for a number (e.g. 10,000) sample paths, assuming the same form of price process as in McDonald and Siegel's paper (GBM). The simulation is then re-run for various values of alpha until the value of alpha producing the highest average NPV is identified – this then produces the optimal alpha, and the BROA estimate of option value. Both of these can be compared with the McDonald and Siegel results (i.e. the specified ratio by which investment value must exceed investment cost, and corresponding option value). If there was agreement between the two methodologies then this offers some reassurance that the BROA approach – which, like any simulation model, can suffer from appearing like a "black box" – is performing as expected.

As shown in Figure 6.1 overleaf (value on the vertical axis), as the time period over which the BROA simulation is run increases, the BROA optimal valuation indeed converges, and quickly so, to very close to the exact option value (flat blue line at top). Note that the McDonald and Siegel model assumes the option can be exercised over an infinite time horizon, whereas the simulation is necessarily run over a shorter timeframe.

The flat red line (lower down) shows the option value if only a single fixed exercise date is assumed, which option can never be worth more than the infinitely lived American-style option, and if early exercise is ever optimal, then it should be worth less. The unevenness in the black line representing optimal BROA option value is a consequence of randomness in the simulations. The closeness and speed of the convergence produced by the BROA approach suggests that it is indeed a reasonable approximation method.

Figure 6.1

Convergence of BROA Solution on Exact McDonald and Siegel Solution



6.2 Bootstrapping Real Options Analysis Valuation Results

6.2.1 Pre-1990 Forest with No Conversion Options

Figure 6.2 overleaf illustrates NPVs produced by the BROA approach for a forest planted on 2 January 2008 on pre-1990 forest land assuming no land conversion options.[45] Aside from ETS-related impacts on fertiliser and transport costs, under this scenario forest value should be insensitive to changes in NZU prices, since no deforestation liabilities would arise. Aside from minor random variation in NPVs as a consequence of simulation, this is what has been produced by the BROA approach.

Two particular features of note from this figure are:

- There is very little variation in value arising from the use of different decision barrier shapes – whether the decision barrier is flat, follows a square root function, or is s-shaped (i.e. using the arcos function to mimic Gong and Lofgren (2007)), the same level of forest NPV is produced; and

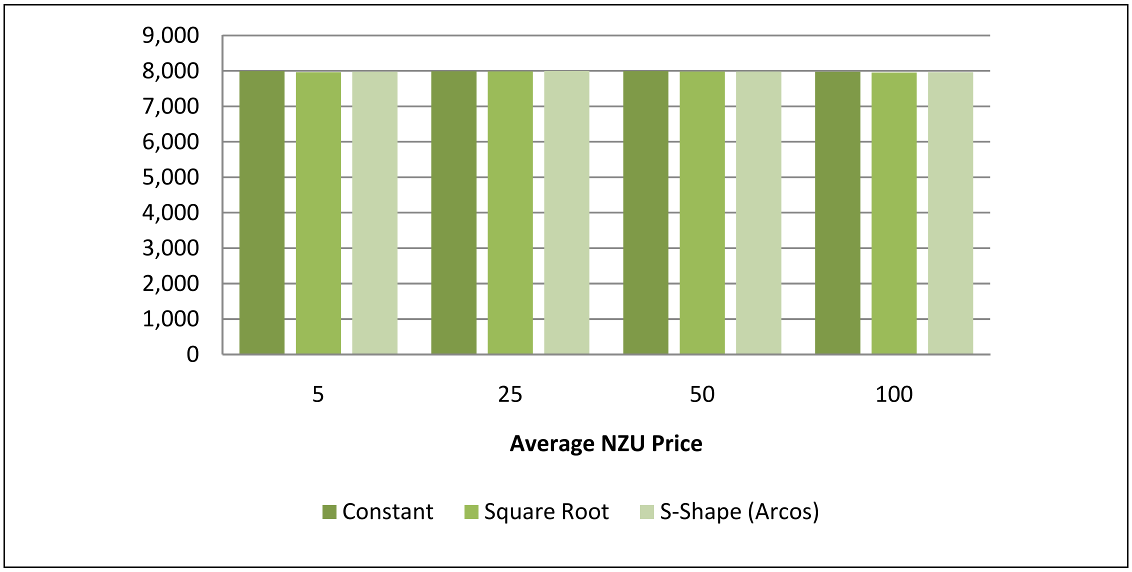
- The overall level of forest value per hectare is significantly higher than that predicted using single rotation DCF analysis (see, for example, the pre-1990 value of around $1,600/ha in Figure 6.9 later in this section).

---

45 Assuming any given piece of land has no conversion options is clearly a strong assumption, give that technology changes over the life of a forest can (and historically has) made the difference between convertibility and non-convertibility. There are, however, other reasons why a piece of land may not have conversion options – e.g. regulatory constraints. This variant of the model is presented for illustration purposes and extended subsequently. In assuming the valuation proceeds without explicit regard to conversion potential this variant is comparable to most other DCF forest valuations.

Figure 6.2

Pre-1990 Forest NPV ($/ha) versus Average NZU Price ($/t$CO_2$) – No Land Conversion Options



There are a number of possible explanations for the latter. First, the BROA approach values perpetual land use, not a single forest crop, which tends to increase value. Second, the use of risk-neutral valuation, involving a risk-free interest rate of around 5% p.a. instead of a risk-adjusted discount rate of 8.5% p.a. has a significant impact on value. Re-running the BROA model with a discount rate of 8.5% p.a. produces a pre-1990 forest value without land conversion options of $1,920/ha, which is more in line with values commonly arising for such forests. In principle, with all parameters correctly estimated, the risk-neutral valuation approach (using risk-neutral drift rates and a risk-free discount rate) should produce the same result as that using risk-adjusted discount rates and raw price process drift rates.

An additional explanation relates to the assumed level of convenience yield. It is possible the flat 2.5% rate assumed in our analysis understates the true level of convenience yield, which has the effect (under risk-neutral valuation) of increasing the drift rate of the assumed price processes. In this case that would tend to increase forest values, as it would involve a higher rate of growth in log prices. For example, assuming a convenience yield equal to the risk-free interest rate (i.e. a nil rate of drift in log prices) reduces the BROA pre-1990 forest value to around $6,900/ha, though this still appears relatively high. A negative risk-adjusted drift rate could be justified, however, as was assumed in Guthrie and Kumareswaran (2007) who allow for a 7.5% convenience yield.

Finally, some of the value difference between the BROA approach and commonly arising values for new pre-1990 forests might be attributable to the fact that the BROA approach allows for the value of harvest timing flexibility. By fixing $T_{min} = T_{max} = 27$ it is possible to eliminate this additional value. Doing so, and setting convenience yield equal to 7.5% (implying a negative risk-adjusted drift in log prices) results in a lower, but still relatively high, forest value of $5,400/ha.

Also of note in the pre-1990 BROA valuations without conversion options are the following:

- **Harvest age** – the constant harvest decision barrier suggests an optimal harvest age of around 24 years, while the other two barrier types suggest an optimal harvest age of around 27 years (consistent with current practise);

- **Harvest probability** – irrespective of the type of harvest decision barrier assumed, first rotation harvest probabilities are predicted to be of the order of 98% or more, and 100% over subsequent rotations; and

- **Optimal alpha** – at all average NZU prices the optimal value of alpha produced by the BROA approach is $5,000/ha for the constant harvest decision barrier, and $6,000/ha for the other two barrier shapes.

6.2.2 Post-1989 Forest with No Conversion Options

Figure 6.3 overleaf illustrates NPVs produced by the BROA approach for a post-1989 forest planted on 2 January 2008 assuming no land conversion options.[46] In addition to ETS-related impacts on fertiliser and transport costs, under this scenario forest value should be positively related to in the level of NZU prices, provided the present value of NZUs earned from sequestration exceeds that of NZUs that must be retired following any harvest decision (which decision is modelled as being contingent upon positive harvest returns net of NZU costs). Aside from minor random variation in NPVs as a consequence of simulation, this is what has been produced by the BROA approach, with higher average NZU prices resulting in higher forest value.

Two features of note in this figure are:

- Once again the overall level of forest value per hectare at low average NZU prices (which should add little to overall value from carbon sequestration) is high relative to the value

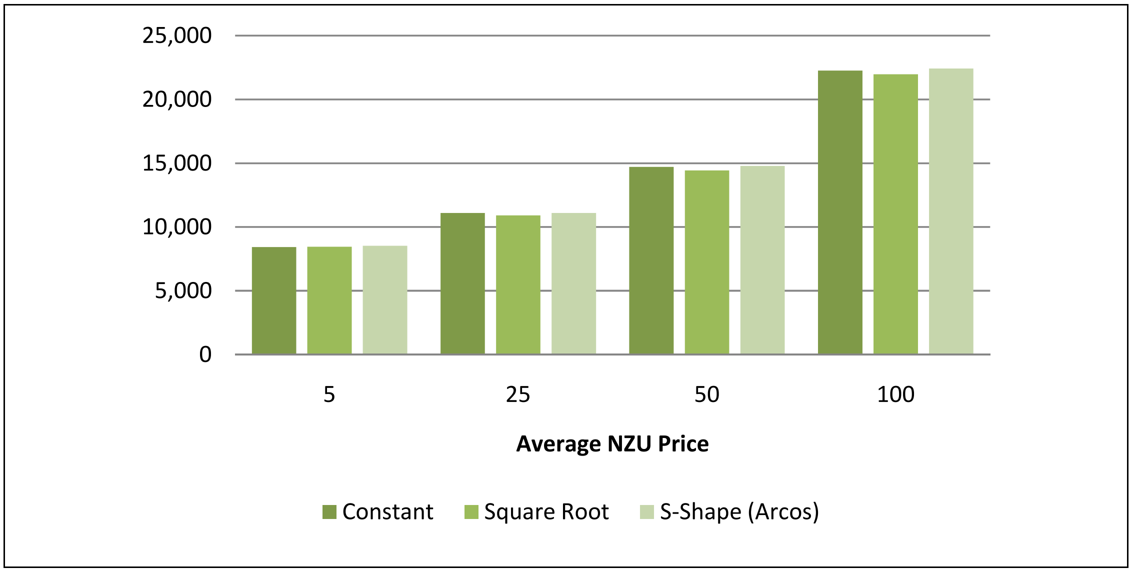
[46] As noted for the pre-1990 discussion above assuming no conversion options, land conversion optionality can change over the life of a forestry investment, though regulatory or other barriers may impede land use changes even where fundamental economics do not. Post-1989 forest land may be more likely than pre-1990 forest land to be capable of some other land use, since it was already in some non-forestry use prior to 1990. However, many areas of post-1989 forestry compliant land include marginal farm land, either due to topography and/or location (i.e. distance to port), in which case such land may have no economic conversion options prior to forest establishment. As noted before, in making no explicit allowance of land use change this variant of the model is no different to DCF models commonly used in forestry.

level commonly arising for forest-only land. The explanations offered above as to why this might be the case continue to apply here.

- Forest value is sensitive to average NZU prices, and continues to increase as average NZU prices increase. Since the BROA approach assumes harvest arises only if the returns from doing so exceed (by some margin) the total cost of harvest including any ETS-related harvest liabilities, this rising value indicates that the value of carbon sequestration, even with volatile NZU prices, can more than compensate for such a liability.

Figure 6.3

Post-1989 Forest NPV ($/ha) versus Average NZU Price ($/t$CO_2$) – No Land Conversion Options



Also of note in the post-1989 BROA valuations without conversion options are the following:

- **Harvest age** – the constant harvest decision barrier suggests an optimal harvest age of around 23-28 years, while the other two barrier types suggest an optimal harvest age of around 29-33 years (higher than current practise, and consistent with research cited earlier predicting longer rotation lengths when carbon sequestration is valuable); and

- **Harvest probability** – irrespective of the type of harvest decision barrier assumed, first rotation harvest probabilities are predicted to be of the order of 96% or more, and 100% over subsequent rotations, but only for lower average NZU prices. When average NZU prices are $50 or more the probability of harvest can fall dramatically – e.g. the first rotation probability at an average price of $100/t$CO_2$ can be as low as 94%, with the probability over subsequent rotations falling to as low as 26-40%. In such cases the high average NZU price should discourage harvest (since it entails a high ETS harvest liability) and instead encourage either ongoing carbon sequestration instead of harvest (i.e. "carbon farming") or much less likely, forest abandonment (where neither harvest nor carbon farming are viable); and

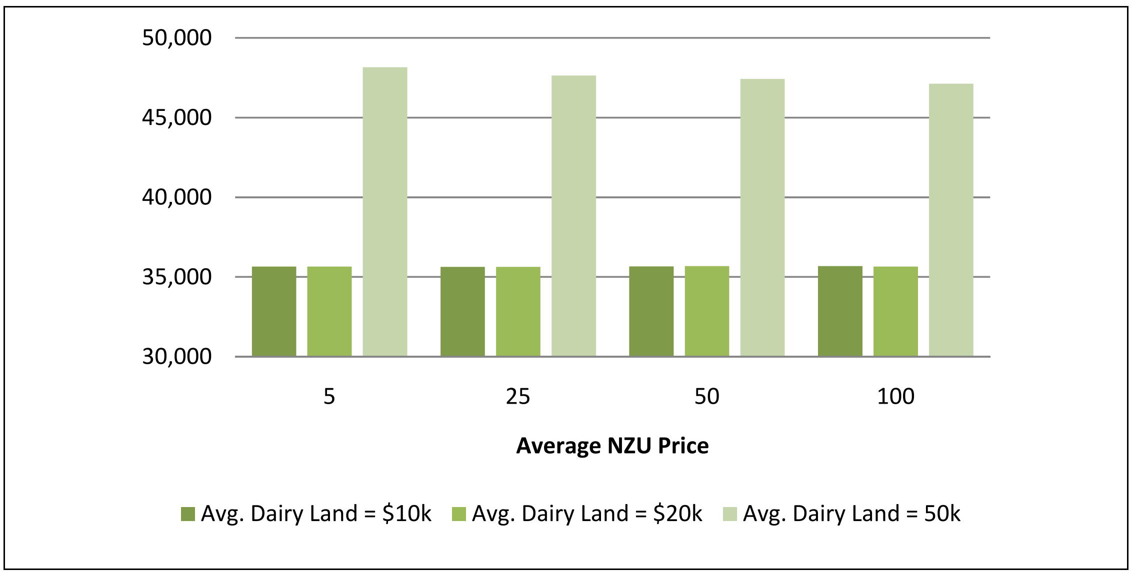
- **Optimal alpha** – the optimal value of alpha produced by the BROA approach continues to be lower for the constant harvest decision barrier than for the other two barrier shapes, and both this spread and the overall level tend to rise as average NZU price rises. For an average NZU price of $5/t$CO_2$ the optimal alpha range is $6,000-7,000/ha, whereas for an average NZU price of $100/t$CO_2$ the range is $18,000-30,000/ha.

6.2.3 Pre-1990 Forest with Dairy Conversion Options

Figure 6.4 illustrates the NPVs produced by the BROA approach assuming conversion into dairy land is possible for a pre-1990 forest.[47] The values are plotted against average NZU prices of $5, $25, $50 and $100 per t$CO_2$. Three scenarios are shown for each carbon price, corresponding to average dairy land values of $10,000, $20,000 and $50,000 per hectare.

Figure 6.4

Pre-1990 Forest NPV ($/ha) versus Average NZU Price ($/tCO$_2$) for Three Average Dairy Land Value Scenarios ($/ha)



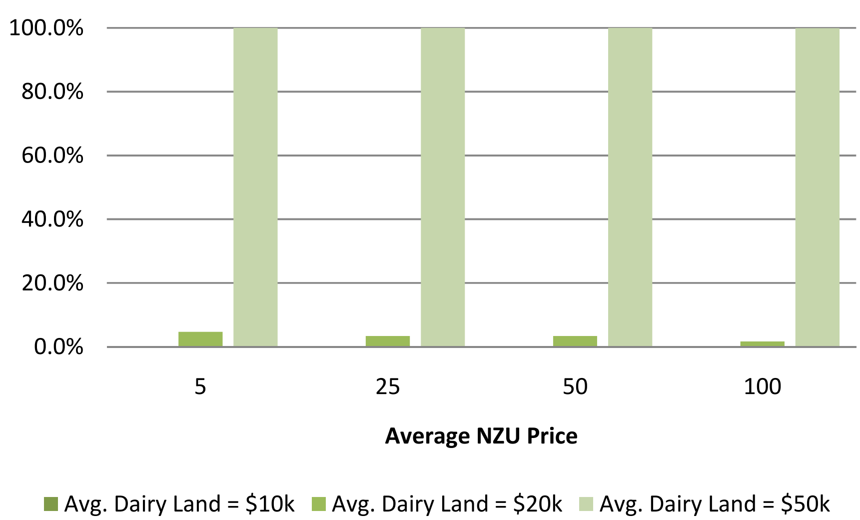
47 Note that the ETS's likely impact on dairy land values has not been modeled and instead left to future work (see Section 7).

As can be seen, for average dairy land values of $10,000/ha or $20,000/ha, forest NPV is fairly invariant to the level of average NZU price. This is explicable in terms of the probability of harvest and conversion being 0% for all four NZU prices considered (see Figure 6.5 overleaf), meaning that forest value is driven by harvest and replanting value only. For an average dairy land value of $50,000/ha, however, the picture changes dramatically. In this case the probability of harvest and conversion is exactly 100% for all but a NZU price of $100/t$CO_2$ (see below), meaning much of the forest NPV derives from the dairy land value conversion, with only small decreases in forest NPV as average NZU prices rise (i.e. the return from conversion is so strong with dairy land averaging $50,000/ha that the deforestation cost is a proportionately lesser burden on value than when dairy land value is lower.

Figure 6.5 plots the probability of harvest and conversion versus the selected average NZU prices for the three average dairy land values considered.

Figure 6.5

Pre-1990 Forest Harvest and Conversion Probability (%) versus Average NZU Price ($/t$CO_2$) for Three Average Dairy Land Value Scenarios ($/ha)



Note that the probability of abandonment is nil in all cases, so the probability of harvest and replanting is one minus the probabilities shown. As can be seen, for a relatively low average dairy land value per hectare there is no prospect of economic dairy land conversion even at relatively low average NZU prices. Conversely, for a relatively high average dairy land value per hectare the reverse is true – dairy land conversion is economic even at very high average NZU prices, with the value gained from conversion dominating the value lost from deforestation charges at high NZU prices. For an intermediate level of average dairy land value there is a small probability of harvest and conversion occurring, and that probability declines as average NZU price increases – in this case from 4.7% at a NZU price of $5/tCO$_2$, through to 1.7% at a price of $100/tCO$_2$. At this average dairy land value conversion is economic in a small proportion of cases, despite the deforestation charge, but that proportion declines as rising NZU prices increase the level of deforestation charge.

Table 6.1 illustrates the optimal alpha and beta figures produced by the BROA model for the three average dairy land value scenarios, which appeared to be fairly insensitive to average NZU prices. However, while increasing average dairy land value does not affect the optimal level of beta, it does increase the optimal alpha. Thus to maximise NPV when dairy land values are high the model predicts that the harvest barrier should be higher, therefore biasing towards choosing harvest and conversion over harvest and replanting.

Table 6.1

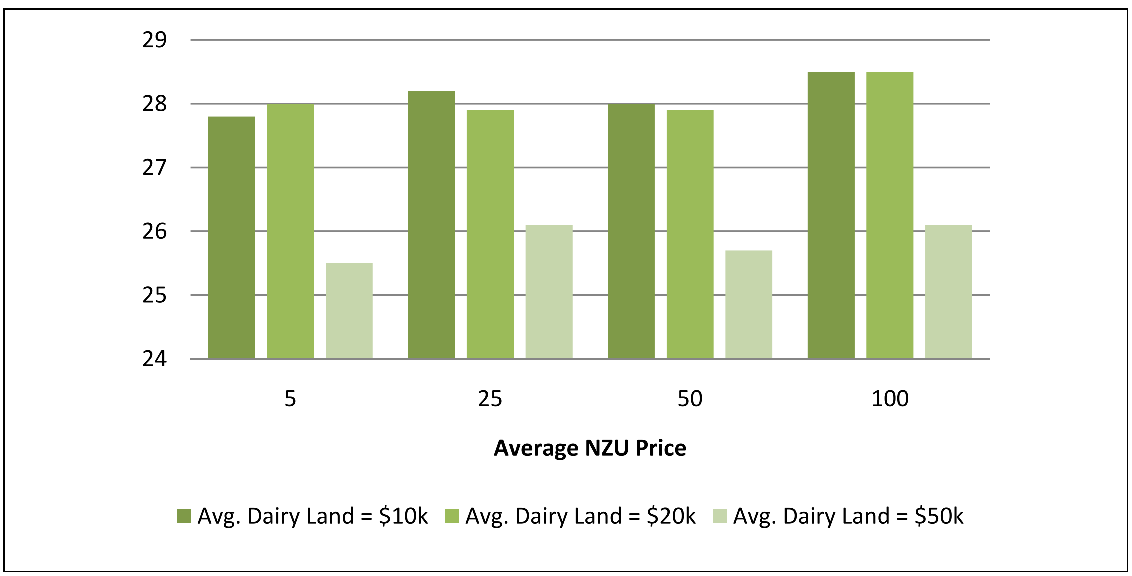
Optimal BROA Alpha and Beta for Three Average Dairy Land Value Scenarios ($/ha)

Parameter	Average Dairy Land Value		
	$10,000	$20,000	$50,000
Optimal Alpha	50,000 – 55,000	60,000 – 65,000	80,000 – 90,000
Optimal Beta	90,000 – 100,000	95,000 – 100,000	90,000 – 100,000

Finally, Figure 6.6 illustrates optimal harvest age predicted by the BROA model.

Figure 6.6

Optimal Harvest Age (Years) versus Average NZU Price ($/tCO2) for Three Average Dairy Land Value Scenarios ($/ha)



While the optimal harvest age appears to generally display no systematic relationship to average NZU price, average harvest age exceeds 27 years in all cases, and for an average NZU price of \$100/tCO_2 it exceeds the predicted age for lower NZU prices. Conversely, for an average dairy land value of \$50,000/ha there is no clear relationship between optimal harvest age and average NZU price, however optimal harvest age is lower than both 27 years and the optimum suggested by the model for lower dairy land values. At this higher level of dairy land value presumably the value of the option to convert into dairying begins to dominate the value of deferring harvest.

6.3 Sensitivity Analysis

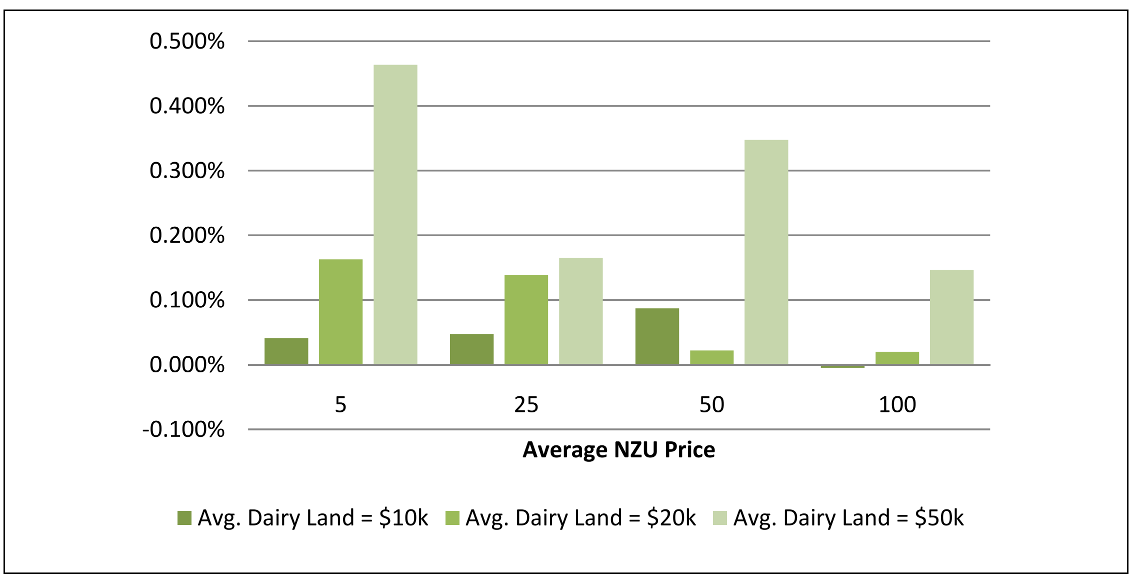
6.3.1 Alternative Decision Boundary Specifications

As illustrated in Figures 6.2 and 6.3 above for the non-conversion BROA model, overall forest value for both pre-1990 and post-1989 forests is fairly insensitive to the assumed decision boundary shape. This suggests that the error from using an approximate decision boundary shape instead of the true theoretical boundary may not be material in practice.

Figure 6.7 shows the percentage change in value, from the pre-1990 conversion model results summarised in Figure 6.4, which arises by implementing the Version 2 decision map presented in Figure 4.4 instead of the Version 1 decision map.

Figure 6.7

Percentage Change in Pre-1990 Conversion Value from Changing to Version 2 Decision Map in Figure 4.4



As can be seen, in all cases the value variation from this alternative map is trivial, further suggesting that the form of decision map posited may not be particularly important for value (even if the parameterisation of that form – i.e. the choice of parameter values for alpha, beta, etc – is important).

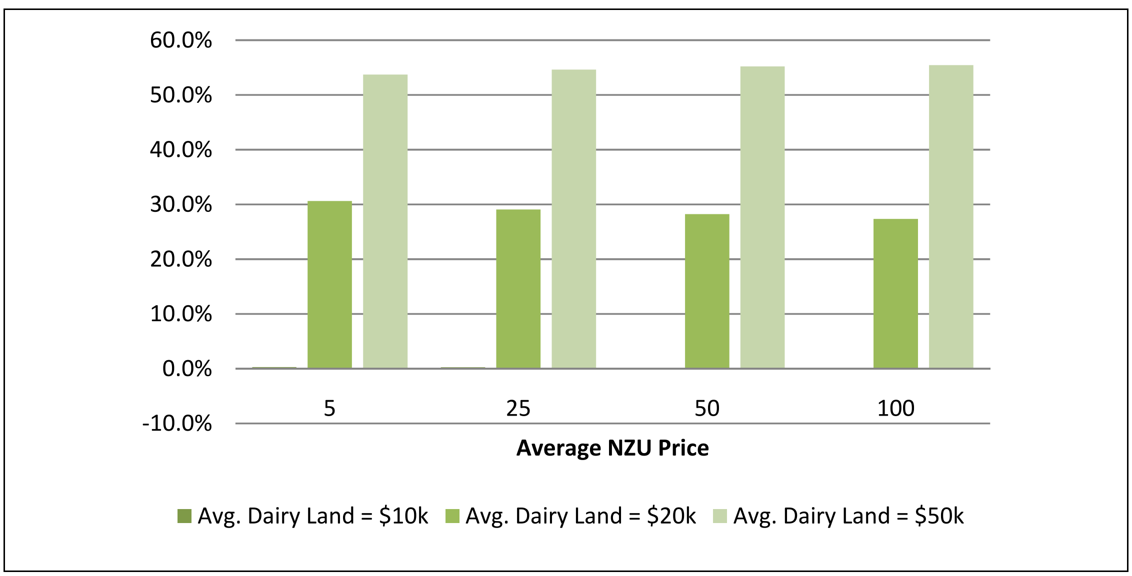
6.3.2 Conversion Delayed Eight Years After Harvest

As discussed in Section 4.2, the ETS Bill provides special rules for determining the size of deforestation liability when forests of less than eight years of age are deforested. In that case the deforestation liability is based not on the carbon stored in the young trees, but instead on the carbon stored in the previous crop of trees (or on an assumed nine year old forest, if that previous crop was also of less than eight years age). Where a pre-1990 forest is deforested just after eight years following harvest, however, the deforestation liability is based on the carbon stock of the immature regrowth, not of the previously harvested forest. This could well give rise to a significantly lower deforestation liability than immediate land conversion following harvest.

Figure 6.8 illustrates the percentage change in value, from the pre-1990 conversion model results summarised in Figure 6.4, which arises by assuming land conversion is delayed by eight years after harvest.

Figure 6.8

Percentage Change in Pre-1990 Conversion Value from Delaying Conversion until Eight Years after Harvest (versus Immediate Conversion after Harvest)



Additional land conversion costs of $500/ha are assumed for clearing regrowth, and the relevant decision map variable NPV(HC,t) must now be based on *expected* deforestation costs (i.e. using expected NZU prices eight years after harvest) as well as *expected* land conversion value eight years after harvest rather than simulated prices as at the harvest decision date (t).[48]For higher average dairy land values there is potentially a large value gain to be had by deferring any dairy conversion until eight years after harvest rather than immediately converting after harvest (where harvest and conversion is sufficiently more valuable than harvest and replanting, that is). For an average dairy land value of $20,000/ha this strategy produces a forest NPV almost 30% higher irrespective of average NZU price, and a probability of harvest and (delayed) conversion of 100%. For an average land value of $50,000/ha the value gain rises to around 55%, with the same predicted certainty of harvest and (delayed) conversion. For a lower average dairy land value, however, the value is essentially unchanged from before. The probability of harvest and (delayed) conversion is 22% at an average NZU price of $5/tCO_2$, falling to 7% at $100/tCO_2$, and forest value is driven by harvest and replanting value more than it is by conversion value (delayed or otherwise).

These results suggest that the pre-1990 conversion model with any land conversion deferred by eight years following harvest may be preferable to the version based on any conversion occurring immediately following harvest, particularly where the probability of harvest and conversion is significant.

6.4 Comparison with Discounted Cash Flow Analysis

6.4.1 DCF Forest Valuation with No Dairy Conversion Option

As shown in Figure 6.9 overleaf, a static DCF greenfields forest valuation assuming a constant real NZU price and no dairy conversion option performs as expected.[49] For higher levels of the constant assumed NZU price the NPV of a post-1989 forest rises linearly. This is because the harvest liability in tCO_2 is just the sum of all previous annual carbon sequestration increments. Since those increments generate a revenue stream at the constant NZU price in the years preceding the realisation of the harvest liability at the same NZU price, with a positive discount rate the post-1989 forest value gains by the time value of money through being able to earn carbon revenues.[50] Given the forest value generated from non-carbon values is assumed unaffected by

---

48 Appendix B.3 sets out the assumed model variations in detail.

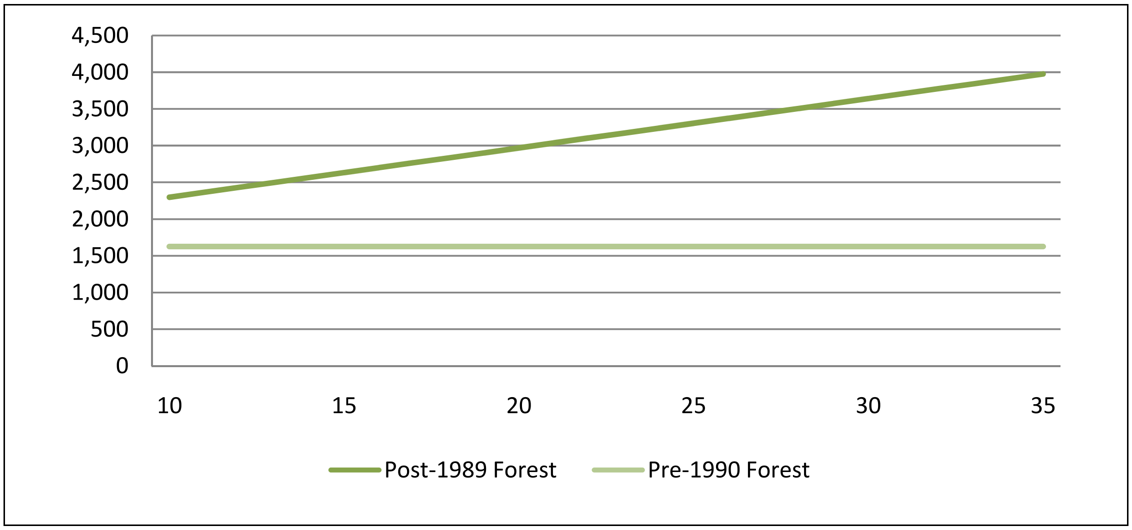
49 Note that these DCF valuations assume constant expected log and NZU prices, and make no allowance for real options arising from flexibility in harvest timing.

50 Indeed, since most if not all of any NZUs earned on a forest must eventually be repaid if the forest is harvested or the carbon stored in the forest is otherwise released (e.g. due to natural losses), any value from carbon sequestration in post-1989 forestry is essentially based on the time value of money, any favourable expectation

NZU prices, and since the only variable changing here is the NZU price, the post-1989 forest NPV is linear in NZU price.

Figure 6.9

Forest NPV ($/ha) versus NZU price ($/tCO$_2$) – Greenfields, No Dairy Conversion Option



Also not surprising is that the pre-1990 forest NPV is lower (it enjoys no carbon value), and is invariant to NZU price level (since no deforestation liability arises when no land conversion option is available).

Indeed, it is possible to show that even with growing NZU prices post-1989 forest value will be a positively increasing function of initial NZU price for growth rates less than a certain level.[51] At a NZU price of zero the post-1989 forest value will end up being slightly lower than the pre-1990 forest value due to the assumption that post-1989 foresters will face slightly higher ETS-related monitoring and compliance costs, but otherwise the two forest values would be identical.

6.4.2 DCF Forest Valuation with Dairy Conversion Option – Pre-1990 Only

To demonstrate the application of a static DCF analysis of pre-1990 forest value with a conversion option, the following simple approach is suggested by way of example:[52]

---

regarding the course of NZU prices (i.e. high when NZUs are earned, low when NZUs must be surrendered) and/or the option value from optimally timing or even abandoning harvest.

51 Trivially, and setting aside the benefit of caps on emission liabilities as provided in Clause 168 of the ETS Bill, if g is the annual growth rate in NZU price, r is the annual discount rate, and T is the tax rate, then post-1989 forest NPV will rise linearly with increasing initial NZU price provided $(1+g)(1-T) < r$.

52 Refinements might include computing the NPVs described at different possible harvest and conversion dates (i.e. other than just at forest maturity, for example, allowing immediate conversion as a possibility).

- Compute the NPV of the forest as if it was harvested at maturity (e.g. age 27 years) and the land then converted into dairy land, net of deforestation and conversion costs;

- Compute the NPV of the forest as if it was harvested at maturity and then replanted, with the resulting value assumed replicated indefinitely to value of future rotations;

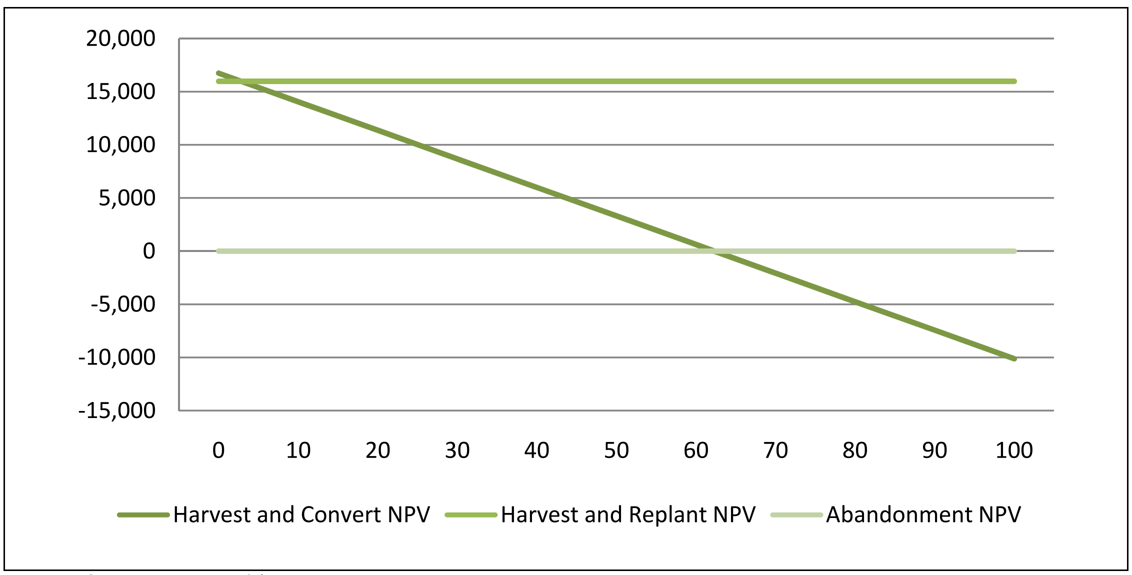
- Compute the NPV of immediately abandoning the forest (e.g. nil value); and

- Finally compute the NPV of the forest by taking the maximum of these three NPVs at the relevant assumed NZU price.

Figure 6.10 illustrates the value of each of these three NPV components for differing carbon prices.

Figure 6.10

Pre-1990 Forest NPV Components ($/ha) vs NZU Price ($/tCO$_2$) – 18 Year Old Forest, Dairy Conversion Option



Notes: Conversion cost of $10,000/ha assumed in this example.

As can be seen, only for very low NZU prices would the optimal strategy be to lock in the decision to harvest and convert as at the valuation date. In that case the NPV from harvest and conversion exceeds that for harvest and replanting (and both exceed the NPV of abandonment). For a lower level of conversion cost the breakeven level of constant NZU price would be commensurately higher. Under the assumed forest valuation parameters immediate abandonment is never optimal, being always dominated by the harvest and replant NPV. The harvest and convert NPV declines with rising NZU prices due to the increase this causes in deforestation costs. The harvest and replant NPV has been modelled as invariant to NZU prices, though if NZU price-impacts on fuel and fertiliser costs were accounted for some slight value decline would arise for increasing NZU prices. In this example the forest NPV is around $16,744/ha for a NZU price of less than approximately $5/t$CO_2$, and $15,967/ha for all higher NZU prices.[53]

[53] Note that these values per hectare are higher than for the non-conversion model principally because an 18 year old forest is assumed in the conversion model, whereas the non-conversion model assumed a greenfields forest. For NZU prices less than around $5/t$CO_2$ the conversion model also produces a higher value due to the value gain assumed possible from dairy conversion at that level of NZU price.

7. Areas for Further Research

7.1 Valuing the Option to Enter Post-1989 Forests into the ETS

We have developed a methodology for valuing post-1989 forests assuming they have been entered into the ETS, but we have not valued the option of entering into the ETS in the first place. In principle the option to enter the ETS might be deferred if doing so involved irreversible decisions with sunk costs and there was uncertainty about future returns from the investment. In fact this risk may possibly be relevant for marginal greenfields post-1989 forest investments, if future carbon credit returns are the difference between viability and non-viability. With all the institutional and political risks surrounding both the ETS and international arrangements beyond 2012, there may be a positive rationale for deferring entry into the ETS until some of these uncertainties have been resolved. Furthermore, risks to post-1989 forest owners from natural carbon losses (e.g. due to windthrow or fire) may also diminish the value of entering the Scheme.

One way to approach this question is to run the BROA model for a finite number of ETS entry dates, perhaps with scenarios for how future institutional arrangements might evolve (e.g. with a step increase in NZU prices or NZU price volatility with some probability, and step decrease with some other probability). The entry date indicating the highest BROA NPV across such scenarios could be taken to be an indication of the optimal entry date.

7.2 Alternative Dairy Conversion Approaches

7.2.1 Conversion Prior to Forest Harvest Window

To avoid unnecessary complexity in the description of our proposed methodology, we have constrained dairy conversion to occur only within the assumed forest harvest window {T_{min} to T_{max}}. In fact dairy conversion could occur for an immature forest, and might be more economic than deforesting a mature forest since the carbon liability will be smaller, and the cost of clearing the trees will also be much lower (e.g. the costs of bulldozing immature stands versus the cost of chain-sawing a mature forest and then removing tree stumps). This would require forest age-dependent land conversion costs to be modelled, as opposed to flat conversion costs independent of forest age, as well as perhaps different harvest revenue assumptions. Once such costs (and harvest revenues) were provided for, however, it would be relatively straightforward to model pre-harvest conversion simply by setting $T_{min} = 0$.

7.2.2 Modelling Dairy Conversion Explicitly

For the sake of simplicity we have modelled dairy conversion by utilising modelled dairy land prices based on historical sale price parameters. As discussed in Section 3.2, there is every reason to believe that historical dairy land sale prices – except perhaps for the most recent prices – do not reflect expectations as to ETS-related impacts on dairy land values. With time such sales data may provide a more informed guide as to the market evaluation of ETS impacts on dairy land values.

However, an alternative approach is to model the value of dairy land conversion explicitly, perhaps treating the annual cooperative milk-fat price as the relevant uncertain state variable to be simulated. Advantages of this approach include being able to value policy scenarios not being contemplated by buyers and sellers of dairy land, and also to explicitly factor in ETS-related impacts on dairying value (e.g. some form of charge on methane emissions from ruminant animals from 2013) where sales evidence is unreliable.

7.3 Modelling NZU Trading/Hedging Strategies

Not modelled here is the impact on post-1989 forest value of warehousing some or all of the NZUs earned as a forest grows to use for offsetting any subsequent harvest liability. Based on the static DCF analysis presented in Section 6.4, it might be predicted that doing so would reduce forest NPV, for the simple reason that it foregoes the time value of money advantage gained by selling NZUs as they are earned, and buying them as they are required for harvest (assuming constant NZU prices). Indeed, in the trivial case where all of the NZUs are simply warehoused, then the forester gains and loses nothing, with forest value being the same as that arising as if the ETS had not been entered (actually, a little worse due to higher compliance costs).

It would be relatively straightforward to extend the BROA valuation approach to account for some or all of the NZUs earned on a post-1989 forest being warehoused until harvest date. The advantage of going so is that the effects of randomness (particularly in NZU prices) and harvest timing flexibility could be taken into account in determining the value impacts. Indeed, if there are significant costs associated with harvest abandonment (as opposed to our assumption that abandonment has a payoff of zero), then simulating the consequences of warehousing on abandonment probability could be useful, and the optimal warehousing level (if any) – i.e. that maximising forest NPV – could be identified.

7.4 Modelling Reforestation Option following Conversion

We have assumed that any conversion of forest land is permanent (except, perhaps, to the extent that modelled dairy land sales prices already encapsulate the option that dairy land owners have to convert their land into non-dairy uses in the future). An alternative approach would be to explicitly allow for dairy land to be converted again – post its initial conversion – either back into forestry (i.e. reforestation), or some other land use (which conversions would not involve a deforestation charge).

Indeed, under the ETS Bill it is possible for pre-1990 land to be "disinfected" by being deforested, in which case it could then be entered into the ETS as post-1989 forest land, and qualify to earn NZUs on carbon sequestered (but with a liability now on harvest, rather than subsequent deforestation). Reforestation might therefore be an economic option post initial deforestation. It could even be considered without fully converting the land into an alternative land use, but simply ensuring deforestation occurs in a cost-minimising way (e.g. that way the expense of dairy land conversion could be avoided).

In principle some of the building blocks to model these possibilities have already been developed. These are provided for in the way that the NPV of harvest and replanting at any given time t is calculated by also allowing for the value of subsequent decisions at time t + 27 years (i.e., as in Section 4.1.2, the value contribution from the later decision is taken to be the higher of the values from either later harvest and replanting, harvest and conversion, harvest and abandonment, or outright abandonment). The complication is that there is no strong reason to constrain reforestation or other land use beyond the initial deforestation to occur at the second harvest date. Instead such reforestation or other conversion could arise, in principle, at any time.

One approach to capture this possibility is to model those reforestation or other conversion options for a range of possible decision dates subsequent to the initial deforestation decision, and take the maximum of all the corresponding NPVs. This simply generalises the approach taken for the harvest and replanting NPV, which assumes just one future decision date. While this would involve multiple calculations, they are in principle relatively straightforward to implement, since only NPVs based on expected future values are required, and the fundamental structure of the BROA approach remains the same.

7.5 Alternative Conversion Counterfactuals

As discussed in Section 5.1, dairy land conversion is only one of many possible non-forestry land uses that could be contemplated for forest land. Alternatives include sheep farming, mixed sheep and beef farming, or possible lifestyle block conversions. In principle it is a trivial matter to model any one of these instead of dairy land conversion as the relevant conversion counterfactual. All that is required is that the relevant conversion cost assumption be modified, and the land value process simulated for conversion land values be updated to reflect the alternative counter-factual.

It would be a more complicated matter to try to model multiple conversion options simultaneously. In principle this can be achieved by generalising the "decision map" approach set out in Section 4.2 to a "decision space" approach. In other words, an n-dimensional decision space could be modelled with the relevant conversion decision being taken when the vector of payoffs from each possible decision at any given time t falls within the relevant part of the decision space. Once again it would be possible to allow for the boundaries in this decision space to be time dependent, though obviously the modelling complexity of this approach would be greater.

7.6 Alternative Uses of Biomass

Not modelled here is the possibility of different forest uses arising over time, for example forest biomass being used for non-timber purposes. An important possible example is the use of biomass for bio-fuels (or bio-plastics) production. Importantly, switching the use of the biomass would not constitute deforestation, since land use would remain in forestry in this case. Instead, the payoff function from "harvest" could be taken to be the greater of the returns from log sales for timber production, and the returns from log sales for biomass production – i.e. the BROA payoff function would simply choose the better of the two alternatives, and compare that with "harvest" and conversion for determining whether deforestation should occur.[54] Different costs may be involved in either case (e.g. less care may be needed in harvesting a tree destined for bio-fuels or bio-plastics), and the possibility of such multiple uses of existing biomass may change the incentives for silviculture. For example, if the returns from bio-fuels or bio-plastics use outstrip those from timber with some significant probability (which would be revealed by the simulations under a BROA approach), then less intensive silviculture (with lower silviculture costs) may be warranted, with wood quality being traded off in favour of greater biomass growth. In principle it should be relatively straightforward to model such an approach.

It should also be mentioned that modifying the BROA approach to allow for multiple simultaneous land uses should also be straightforward (e.g. forestry plus eco-tourism, for example). The basic structure of the BROA approach remains the same, and all that is required is a modification to the payoff functions assumed for each land use decision.

---

[54] In this case the NPV at the relevant time t from harvest and replanting would also need to be modified, with the NPV from the subsequent decision being modified to reflect the possibility that "harvest" of subsequent rotations might also involve using the biomass for (e.g.) biofuels instead of timber.

7.7 Extension to Mixed-Age Forests

The BROA approach has been modelled assuming a forest with a single age class. If a mixed age-class forest is to be valued, in principle this could be accommodated by running the BROA model independently for each age class and then aggregating the resulting values – in effect valuing a portfolio of options. More complicated would be modelling any interdependencies or constraints on the management of multiple age-classes (i.e. valuing an option over an integrated forest portfolio). One approach would be to run the BROA model with multiple parallel streams, each relating to a given age class, with modifications to the decision map framework. For example, if the forest estate is subject to supply commitments, any decision to harvest and replant could be made subject to the suggested BROA decision rules if the supply commitment constraint is not binding, or forced if the constraint is binding. Conversely, the decision to harvest and convert could similarly be made subject to the BROA rules, provided this did not breach any downstream supply obligations. The basic BROA decision map might stay the same, but how it is used to determine land use decisions could be made subject to other such constraints.

7.8 Modelling Subsequent Land-Use Decisions using Analytical Models

The BROA approach is, as described in Section 4.1.1, "smart then dumb". The "dumb" aspect relates to how the land use decisions following an initial harvest and replanting decision are modelled, with the later decision's contribution to NPV being simply the highest expected NPV of the available alternatives. Since the subsequent decision is assumed to arise 27 years after the initial decision, the value consequence of shortcomings in this approach is diminished by virtue of 27 years of discounting at a positive discount rate (as discussed in Section 4.1.1, this approach should bias value downwards, but due to 27 years of discounting, perhaps not by much).

A more sophisticated approach would be to try to capture the option value available to the forest (land) owner at the subsequent decision based on price information available at the first decision date. This could be done by using an analytical option valuation formula parameterised by current prices, instead of picking from among expected future NPVs. At best, taking the maximum of the expected NPVs only provides a lower bound on the true value of the subsequent decision, because the forest (land) owner will be making his or her land use decision at that date based on price information available then, not expected NPVs produced in the past based on historical prices prevailing then.

One possibility is to adapt the analytical solution derived by Margrabe (1978) to valuing an option to exchange one asset for another. Since the payoff function for the Margrabe formula is simply the maximum of two future asset values, in principle this is well-suited to the evaluation of the land use decision to be taken following an initial harvest and replant decision under the BROA approach. A complication in implementing such an approach is in conveniently specifying the "assets" which the Margrabe option chooses among. While conceptually the assets might be defined as "harvest and replanting" and "harvest and converting", in practice it is difficult to define a return series on which to estimate the relevant option pricing parameters. In part this stems from the problem that the returns to either decision involve harvest returns, and harvest returns are forest age-specific as well as time dependent. If future research was able to identify a solution to this complication, then it should be relatively simple to implement the refinement. Indeed, if preferred, the Margrabe model, which assumes asset values evolve according to GBM instead of MR, could also be modified to allow for mean-reverting prices. Possible approaches include those in Deng (2000), Deng et al. (2001), Carmona and Durrleman (2003) and Maribu et al. (2007).

7.9 Modelling Effects of Policy Uncertainty

Due to the flexibility afforded by using simulation, the BROA approach enables the modelling of a great number of uncertainties. An important class of such uncertainty is policy uncertainty, and the approach can be used to determine the extent to which policy uncertainties affect forest (land) value and optimal forest (land) management decisions (e.g. harvest deferral, or deforestation acceleration). As mentioned previously, while a general theme emerging from the real options literature is that uncertainty creates a valuable option to wait when making irreversible investment decisions, some forms of uncertainty can create an offsetting incentive to accelerate such decisions. By attaching varying probabilities to key future policy alternatives it is possible, using the approach, to gain a sense of whether greater or lesser policy certainty will contribute to increased (reduced) forest and forest land values, or increased (reduced) activities such as harvest or deforestation. This could then provide policymakers with an improved tool for assessing the desirability of greater or lesser policy clarity around forestry-related (and dairy-related) ETS and other policies.

7.10 Other Refinements

Other possible refinements to the BROA approach include:

- Improving the assumption about how much carbon is sequestered as the forest grows, and released as a forest is harvested. Explicit account could be taken to carbon sequestration and release in soil, leaves, roots, etc. So too could differing carbon sequestration rates for forests of different ages (e.g. with greater sequestration for immature forests).

- Explicitly fitting price processes to NZU, log and land price data, particularly as longer time series become available, as institutional uncertainties are resolved, as carbon markets mature, as comparable land sales data better reflect carbon pricing, and as the impacts of carbon pricing on international forestry economics are revealed.

- Biomass growth could be better modelled, perhaps stochastically, and also taking into account the expected effects (if any) of climate change itself on forestry.

8. Conclusions

Valuing forests and forest land in New Zealand has always presented challenges under existing valuation methodologies. Capturing the value of land use change options, and of flexibility in forest management decisions in response to multiple uncertain state variables, has always been a challenge poorly met by existing approaches such as discounted cash flow analysis. In principle, real options analysis – where tractable - is the preferred valuation approach in such situations.
Valuing forests and forest land under the ETS further suggests the importance of real-options based valuation for post-1989 forestry. Should a forest owner enter their compliant forest into the ETS then they will have to optimise their forest management decisions over both carbon and timber value possibilities, and potentially others besides (e.g. bio-ethanol, bio-plastics, eco-tourism). In the case of pre-1990 forests, however, the need for a valuation technique that can formally accommodate land use change decisions – both current and future – becomes all the more pressing, further suggesting a role for real options analysis.

The bootstrapping real options analysis (BROA) models presented in this report are a first step to meeting these valuation requirements while retaining as much tractability as possible. Any practitioner of real options analysis will readily acknowledge that the techniques required to value even relatively simple real options can quickly become daunting, often to the point of being beyond all except the most specialised practitioners. This is why the BROA approach has been formulated – providing sufficient complexity that it can value the most important features of both the ETS and forestry, while using a modelling technique (i.e. Monte Carlo simulation) that can be implemented without the need for obscure software or a high level of technical expertise.

Another challenge in developing and implementing real options valuation models is that they often involve "black box" elements that make them both mysterious and possibly unpredictable. This raises the importance of sensitivity analysis and "stress testing", to ascertain whether the model performs as one might predict based on principle, or where it doesn't, to force inquiry as to why. The BROA model presented in this report appears to satisfy basic tests of robustness and "sensibility". When used to value a problem with a known solution it converges quickly on that known solution. When key input parameters are changed, values change in the direction that might be expected. Whether the level of BROA valuations and the size of their change in response to input changes are reasonable are matters that will finally be determined in "the field" instead of our "laboratory" setting. The ultimate usability of the approach will hinge not just on the technical requirements of its implementation, but also the confidence and acceptance of users as to its outputs. We hope that the discussion in this report presents sufficient material for the process of refining, developing and proving the approach to begin.

* * *

References

Alesii 2003

Alesii, G. Rules of thumb in real options analysis. *Universitá degli Studi--L'Aquila, Facoltá,* **2003**

Andersen and Broadie 2004

Andersen, L. & Broadie, M. Primal-dual simulation algorithm for pricing. Multidimensional American options. *Management Science,* **2004***, 50*, 1222-1234

Arrow and Fisher 1974

Arrow, K. & Fisher, A. Environmental preservation, uncertainty, and irreversibility. *The Quarterly Journal of Economics, JSTOR,* **1974***, 88*, 312-319

Bahn 1999

Bahn, O.; Kypreos, S.; Bueler, B. & Luthi, H. Modelling an international market of CO 2 emission permits. *International Journal of Global Energy Issues, Inderscience Publishers Ltd,* **1999***, 12*, 283-291

Barraquand and Martineau 1995

Barraquand, J. & Martineau, D. Numerical valuation of high dimensional multivariate American securities. *The Journal of Financial and Quantitative Analysis, JSTOR,* **1995***, 30*, 383-405

Bates 2003

Bates, D. Empirical option pricing: A retrospection. *Journal of Econometrics, Elsevier,* **2003***, 116*, 387-404

Benz and Truck 2008

Benz, E. & Trueck, S.

Modeling the price dynamics of CO2 emission allowances

Bhattacharya 1978

Bhattacharya, S. Project valuation with mean-reverting cash flow streams, *The Journal of Finance*, **1978**, 33(5), 1317-1331

Boyle 1977

Boyle, P. Options: A monte carlo approach. *Journal of Financial Economics,* **1977***, 4*, 323-338

Boyle and Guthrie 2003

Boyle, G. & Guthrie, G. Investment, uncertainty, and liquidity, *The Journal of Finance, Blackwell Synergy,* **2003***, 58*, 2143-2166

Boyle and Irwin 2004

Boyle, G. & Irwin, T. A primer on real options. *INFINZ Journal,* **2004***, 1*, 44-59

Brazee and Mendelsohn 1988

Brazee, R. & Mendelsohn, R. Timber harvesting with fluctuating prices. *Forest science, Society of American Foresters,* **1988***, 34*, 359-372

Brealey et al. 2006

Brealey, R. A., Myers, S. C. and F. Allen, Corporate finance, 8 ed., International student edition, McGraw-Hill Irwin, **2006**

Brennan 1985

Brennan, M. & Schwartz, E. Evaluating natural resource investments. *Journal of Business, University of Chicago Press,* **1985***, 58*, 135.

Broadie and Glasserman 1997

Broadie, M. & Glasserman, P. Pricing American-style securities using simulation. *Journal of Economic Dynamics and Control, Elsevier,* **1997***, 21*, 1323-1352

Broadie, Glasserman and Jain 1997

Broadie, M.; Glasserman, P. & Jain, G. Enhanced monte carlo estimates for American option prices. *Journal of Derivatives,* **1997***, 5*, 25-44

Burtraw 2001

Burtraw, D.; Palmer, K.; Bharvirkar, R. & Paul, A. The effect of allowance allocation on the cost of carbon emission trading. *Resources for the Future Discussion Paper,* **2001**, 01-30

Carmona and Durrleman 2003

Carmona, R. & Durrleman, V. Pricing and hedging spread options. *SIAM Review, SIAM,* **2003***, 45*, 627-685

Castalia 2007

The New Zealand emissions trading scheme: How do we make it work?, November, **2007**

Chesney and Taschini 2008

Chesney, M. & Taschini, L. The endogenous price dynamics of the emission allowances: An application to CO2 option pricing

Clarke and Reed 1989

Clarke, H. & Reed, W. The tree-cutting problem in a stochastic environment. The case of age-dependent growth. *Journal of Economic Dynamics and Control,* **1989***, 13*, 569-595

Copeland and Antikarov 2001

Copeland, T. & Antikarov, V. Real options: A practitioner's guide. *New York: Texere,* **2001**

Copeland and Weston 1988

Copeland, S. and J. Weston, *Financial theory and corporate policy*, 3 ed., Addison Wesley, **1988**

Cox 1975

Cox, J. & Ross, S. The valuation of options for alternative stochastic processes. *Graduate School of Business, Stanford University,* **1975**

Cunningham 2006

Cunningham, C. House price uncertainty, timing of development, and vacant land prices: Evidence for real options in Seattle. *Journal of Urban Economics, Elsevier,* **2006***, 59*, 1-31

Daskalakis et al. 2008

Daskalakis, G.; Psychoyios, D. & Markellos, R., Modeling CO2 emission allowance prices and derivatives: Evidence from the European trading scheme, **2008**

Deng 1999

Deng, S. Stochastic models of energy commodity prices and their applications: Mean-reversion with jumps and spikes. *POWER Working Paper,* **1999**

Deng 2001

Deng, S.; Johnson, B. & Sogomonian, A. Exotic electricity options and the valuation of electricity generation and transmission assets. *Decision Support Systems, Elsevier,* **2001***, 30*, 383-392

Diaz-Balteiro and Romero 2003

Diaz-Balteiro, L. & Romero, C. Carbon captured as a new instrument in forest management: Some implications, *Carbon,* **2003**, 103-114

Dixit 1994

Dixit, A. & Pindyck, R. Investment under uncertainty. *Princeton University Press Princeton, NJ,* **1994**

Ewald and Yang 2007
Ewald, C. & Yang, Z. Geometric mean reversion: Formulas for the equilibrium density and analytic moment matching, **2007**

Faustmann 1849
Faustmann, M. Calculation of the value which forest land and immature stands possess for forestry. *Journal of Forest Economics,* **1995***, 1*, 7-44

Fehr and Hinz 2006
Fehr, M. & Hinz, J. A quantitative approach to carbon price risk modeling
Zürich. Working Paper, ETH Zentrum, **2006**

Fleten et al. 2007
Fleten, S.; Maribu, K. & Wangensteen, I. Optimal investment strategies in decentralized renewable power generation under uncertainty. *Energy, Elsevier,* **2007***, 32*, 803-815

Gjolberg and Guttormsen 2000
Gjølberg, O. & Guttormsen, A. Forest valuation when prices are mean reverting

Gong 1999
Gong, P. Optimal harvest policy with first-order autoregressive price process. *Journal of Forest Economics,* **1999***, 5*, 413-439

Gong and Lofgren 2007
Gong, P. & Löfgren, K. Market and welfare implications of the reservation price strategy for forest harvest decisions. *Journal of Forest Economics, Elsevier,* **2007***, 13*, 217-243

Grant 1997
Grant, D.; Vora, G. & Weeks, D. Path-dependent options: Extending the monte carlo simulation Approach. *Management Science, JSTOR,* **1997***, 43*, 1589-1602

Guthrie and Kumareswaran 2007
Guthrie, G. & Kumareswaran, D., Carbon subsidies, taxes and optimal forest management

Haight and Holmes 1991
Haight, R. & Holmes, T. Stochastic price models and optimal tree cutting: Results for loblolly pine. *Natural Resource Modeling,* **1991***, 5*, 423-443

Henry 1974

Henry, C. Option values in the economics of irreplaceable assets. *The Review of Economic Studies, JSTOR,* **1974***, 41*, 89-104

Hull 2006

Hull, J. Options, futures, and other derivatives. *Prentice Hall Upper Saddle River, 6 ed.,* **2005**

Insley and Meade (2008)

Insley, C. and R. Meade, *Maori impacts from the emissions trading scheme: Detailed analysis and conclusions*, report prepared for the Ministry for the Environment, January, **2008**

Insley 2002

Insley, M. A real options approach to the valuation of a forestry investment. *Journal of Environmental Economics and Management, Elsevier,* **2002***, 44*, 471-492

Insley 2003

Insley, M. On the option to invest in pollution control under a regime of tradable emissions allowances. *Canadian Journal of Economics, Blackwell Synergy,* **2003***, 36*, 860-883

Insley and Fox

Insley, M. & Fox, G. The economics of intensive forest management a stand level analysis for the Romeo Malette forest in Ontario a report prepared for Tembec Inc., the Ontario Ministry of Natural Resources, and ULERN, by Margaret Insley, Glenn Fox and Kimberly Rollins

Johnson 1987

Johnson, H. Options on the maximum or the minimum of several assets. *Journal of Financial and Quantitative Analysis, JSTOR,* **1987***, 22*, 277-283

Krawczyk and Windsor 1997

Krawczyk, J. & Windsor, A. An approximated solution to continuous-time stochastic optimal control problems through markov decision chains. *Research Papers in Economics,* **1997**

Longstaff and Schwartz 2001

Longstaff, F. & Schwartz, E. Valuing American options by simulation: A simple least-squares approach. *Review of Financial Studies, Soc Financial Studies,* **2001***, 14*, 113-147

Manley 2007

Manley, B. Discount rates used for forest valuation – Results of 2007 survey, *New Zealand Journal of Forestry*, **2007**, 52(3), 21-27

Maribu 2007
Maribu, K.; Galli, A. & Armstrong, M. Valuation of spark-spread options with mean reversion and stochastic volatility. *International Journal of Electronic Business management*, 5(3), 173-181, **2007**

Meade 2006
Meade, R. Valuing the impact of climate change policies on forestry, *New Zealand Journal of Forestry*, Vol. 51, No. 1, May **2006**, 14 – 18.

Metcalf and Hassett 1995
Metcalf, G. & Hassett, K. Investment under alternative return assumptions Comparing random walks and mean reversion. *Journal of Economic Dynamics and Control, Elsevier,* **1995***, 19*, 1471-1488

Malchow-Moller and Thorsen 2003
Malchow-Møller, N. & Thorsen, B. Optimal stopping with two exclusive real options. *CEBR Discussion Paper 2003-33, CEBR, Copenhagen,* **2003**

Malchow-Moller, Strange and Thorsen 2004
Malchow-Møller, N.; Strange, N. & Thorsen, B. Real-options aspects of adjacency constraints. *Forest Policy and Economics, Elsevier,* **2004***, 6*, 261-270

Mayhew 2002
Mayhew, S. Security price dynamics and simulation in financial engineering
Simulation Conference, 2002. Proceedings of the Winter, **2002***, 2*

Margrabe 1978
Margrabe, W. The value of an option to exchange one asset for another. *Journal of Finance, JSTOR,* **1978***, 33*, 177-186

Merton 1973
Merton, R. Theory of rational option pricing. *Bell Journal of Economics and Management Science, JSTOR,* **1973***, 4*, 141-183

McDonald and Siegel 1986
McDonald, R. & Siegel, D. The value of waiting to invest. *The Quarterly Journal of Economics, JSTOR,* **1986***, 101*, 707-728

Montero and Ellerman 1998
Montero, J. & Ellerman, A. Explaining low sulfur dioxide allowance prices: The effect of

expectation errors and irreversibility. *MIT Center for Energy and Environmental Policy Research, WP,* **1998**, 98-011

Morck, Schwartz and Stangeland 1989
Morck, R.; Schwartz, E. & Stangeland, D. The valuation of forestry resources under stochastic prices and inventories. *The Journal of Financial and Quantitative Analysis, JSTOR,* **1989***, 24*, 473-487.

Murray 2003
Murray, B. Chapter 13 Economics of forest carbon sequestration. *Forests in a Market Economy, Springer,* **2003**

Newman 2002
Newman, D. Forestry's golden rule and the development of the optimal forest rotation literature. *Journal of Forest Economics, Elsevier,* **2002***, 8*, 5-27

Niquidet and Manley 2007
Niquidet, K. & Manley, B. Price dynamics in the New Zealand log market. *New Zealand Journal of Forestry,* **2007***, 52*(4)

Paolella and Taschini 2006
Paolella, M. & Taschini, L., An econometric analysis of emission trading allowances

Pindyck 1991
Pindyck, R. Irreversibility, uncertainty, and investment. *Journal of Economic Literature, JSTOR,* **1991***, 29*, 1110-1148

Reed 1993
Reed, W. The decision to conserve or harvest old-growth forest. *Ecological Economics,* **1993***, 8*, 45-69

Romero, Ros and Daz-Balteiro 1998
Romero, C.; Ros, V. & Daz-Balteiro, L. Optimal forest rotation age when carbon captured is considered: Theory and applications. *Journal of the Operational Research Society,* **1998***, 49*, 121-131

Samuelson 1976
Samuelson, P. Economics of forestry in an evolving society . *Economic Inquiry, Blackwell Synergy,* **1976***, 14*, 466-492

Seifert et al. 2006
Seifert, J.; Uhrig-Homburg, M. & Wagner, M. Dynamic behavior of CO 2 spot prices-a stochastic equilibrium model. *Working Paper, University of Karlsruhe,* **2006**

Schwartz 1997
Schwartz, E. The stochastic behavior of commodity prices: Implications for valuation and hedging. *The Journal of Finance, JSTOR,* **1997***, 52*, 923-973

Schwartz and Trigeorgis 2001
Schwartz, E. & Trigeorgis, L. Real options and investment under uncertainty. *MIT Press Cambridge, Mass,* **2001**

Sohngen and Mendelsohn 2003
Sohngen, B. & Mendelsohn, R. An optimal control model of forest carbon sequestration. *American Journal of Agricultural Economics, Blackwell Synergy,* **2003***, 85*, 448-457

Stainback and Alavalapati 2005
Stainback, G. & Alavalapati, J. Effects of carbon markets on the optimal management of slash pine (Pinus elliottii) plantations. *Southern Journal of Applied Forestry, Society of American Foresters Bethesda, USA,* **2005***, 29*, 27-32

Sundaresan 2000
Sundaresan, S. Continuous-time methods in finance: A review and an assessment. *The Journal of Finance, Blackwell Synergy,* **2000***, 55*, 1569-1622

Thomson 1992
Thomson, T. Optimal forest rotation when stumpage prices follow a diffusion process. *Land Economics, JSTOR,* **1992***, 68*, 329-342

Tilley 1993
Tilley, J. Valuing American options in a path simulation model. *Transactions of the Society of Actuaries,* **1993***, 45*, 83-104

Titman 1985
Titman, S. Urban land prices under uncertainty. *The American Economic Review, JSTOR,* **1985***, 75*, 505-514

Tsitsiklis and van Roy 2001
Tsitsiklis, J.; Van Roy, B. & Mit, C. Regression methods for pricing complex American-style options. *Neural Networks, IEEE Transactions on,* **2001***, 12*, 694-703

Unterschultz 1998

Unterschultz, J.; Novak, F. & Koontz, S. Design and evaluation of long term commodity pricing contracts. *NCR-134 Conference Applied Commodity Price Analysis, Forecasting, and Market Risk Management,* **1998**, 20-21

Van Kooten 2004

van Kooten. Chapter 14, Economics of forest and agricultural carbon sinks. *Resource Economics and Policy Analysis Research Group working paper, 2004*

Van Kooten et al. 1995

Van Kooten, G.; Binkley, C. & Delcourt, G. Effect of carbon taxes and subsidies on optimal forest rotation age and supply of carbon services. *American Journal of Agricultural Economics, JSTOR,* **1995***, 77*, 365-374

Wan 2000

Wan, H. Pricing American-style basket options by implied binomial tree. *Manuscript, University of Carlifornia at Berkely,* **2000**

Woollons Brown and Manley 2005

Woollons, R.; Brown, J.; Manley, B. & Zealand, N. Estimation of errors associated with the calculation of carbon in kyoto forests

Yamaguchi 1995

Yamaguchi, H. The real option premium in Japanese land prices. *International Management Research Institute, International University of Japan,* **1995**

Yin 2001

Yin, R. Combining forest-level analysis with options valuation approach--a new framework for assessing forestry investment. *Forest Science,* **2001***, 47*, 475-483

Yin and Newman 1996

Yin, R. & Newman, D. Are markets for stumpage informationally efficient? *Canadian Journal of Forest Research (Print), National Research Council of Canada,* **1996***, 26*, 1032-1039

Appendix A – Summary of Proposed Emissions Trading Scheme Forestry Provisions

Note - Section references are to the Climate Change (Emissions Trading and Renewable Preference) Bill 187-1 (2007).

Key Definitions

deforest, in relation to forest land, means to convert forest land to non-forest land [though see s164 re deemed deforestation]

forest land—

(a) means an area of land of at least 1 hectare that has, or will at maturity have, tree crown cover (or equivalent stocking level) of more than 30% in each hectare and in which—

(i) the trees are forest species; and

(ii) the forest consists of—

(A) closed forest formations where trees of various storeys and undergrowth cover a high proportion of the ground; or

(B) open forest; and

(b) includes an area normally forming part of a forest that is temporarily unstocked as a result of human intervention or natural causes but that is expected to revert to forest; but

(c) does not include—

(i) a shelter belt where the tree crown cover at maturity has, or is expected to have, an average width of less than 30 metres; or

(ii) an area of land where the tree crown cover at maturity has, or is expected to have, an average width of less than 30 metres, unless the area is contiguous with other forest land

forest species means tree species capable of reaching at least 5 metres in height at maturity in the place where it is located

post-1989 forest land means forest land that—

(a) was not forest land on 31 December 1989; or

(b) was forest land on 31 December 1989 but was deforested between 1 January 1990 and 31 December 2007; or

(c) was pre-1990 forest land—

(i) that was deforested on or after 1 January 2008; and

(ii) in respect of which any liability to surrender units arising in relation to an activity listed in Part 1 of Schedule 3 [i.e. deforestation] has been satisfied

pre-1990 forest land means forest land—

(a) that was forest land on 31 December 1989; and

(b) that remained as forest land on 31 December 2007; and

(c) where the forest species on the forest land consisted of exotic forest species

registered forestry right means a forestry right registered under the Forestry Rights Registration Act 1983

registered lease,—

(a) in relation to a lease in respect of land registered under the Land Transfer Act 1952,—

(i) means a lease registered under that Act; and

(ii) includes a lease registered under the Land Transfer (Computer Registers and Electronic Lodgement) Amendment Act 2002:

(b) in relation to a lease in respect of land that is not registered under the Land Transfer Act 1952, means a lease registered under the Deeds Registration Act 1908

removals, in relation to a removal activity, means carbon dioxide equivalent greenhouse gases that are, as a result of the removal activity,—

(a) removed from the atmosphere; or

(b) not released into the atmosphere

surrender means the transfer of a unit to a surrender account in the Registry with the effect specified in section 18CA(2) or (4)

1) s6: <u>unit definition</u> – unit means a Kyoto unit, a New Zealand unit, or an approved overseas unit, where Kyoto unit means AAUs, CERs ERUs, long term CERs, RUs, and temporary CERs (i.e. units defined under the Kyoto Protocol); New Zealand unit means a unit issued by the registrar and designated as a New Zealand unit (i.e. property rights created in New Zealand and tied to Kyoto units); an approved overseas unit means a unit other than a Kyoto unit issued by an overseas registry and prescribed as a unit that may be transferred to accounts in the registry (i.e. a placeholder).

<u>Pre-1990 Forests</u> – relevant "activity" is deforestation of such land

2) s56: <u>Requirement to register as ETS participant</u> – from 1 January 2008 any person who DEFORESTS more than 2 ha of pre-1990 forest land (that is not exempt land) within CP1 or in any subsequent five year period MUST register as an ETS participant within 20 working days of

that deforestation (no registration fee is provided for), though 163(2) provides that notice may be given at any time up to 31 January 2009 in respect of any deforestation in 2008.

3) s60: exemptions – not interesting for MAF project purposes (since forest valuation reduces to ordinary forest valuation problem, and hence is just trivial special case of valuation model).

4) s158(1): Who is liable for deforestation liability – landowner, unless they can prove the deforestation decision was controlled by others (e.g. forestry right holders in some cases).

5) s61: holding accounts – participants must open a holding account for the surrendering of units and receiving units to which they become entitled (for pre-1990 forests the only units to which participants become entitled are free allocations of NZUs to offset value loss from ETS introduction).

6) s62: monitoring of deforestation emissions – each year participants MUST collect prescribed data and information (which may be required to be verified by recognised persons/organisations), calculate deforestation emissions using prescribed methodologies, (if required to by regulation) have these calculations verified by recognised persons/organisations, and keep records of these data, information and calculations

7) s63: liability to surrender units to cover deforestation emissions – participant must transfer one unit for each tonne of emissions arising from deforestation as calculated under the Act (see also s162), and at the times required under the Act (see s65) – requires them to apply to transfer the required number of units from their holding account to the designated surrender account.

8) s65: annual emission returns – between 1 January and 31 March each year participant must submit annual emissions return (cf income tax returns) regarding deforestation activities in the immediately preceding year, possibly with a fee, and BY 30 APRIL must surrender the self-assessed (but possibly verified) number of emission units listed in their annual emissions return.

9) s171: transitional provisions – first annual emission return – first emission return is not required in respect of year ended 31 December 2008, but must be submitted for period 1 January 2008 to 31 December 2009, and may not be submitted before 1 January 2010.

10) s162: methodology for trees on deforested land that are 8 years or younger – for purposes of s65, must apply methodology as if the trees harvested were those last harvested, or if the trees last harvested were also 8 years or younger, as if the trees harvested when deforesting were nine years old: and surrender units on this basis.

11) s66: records – must retain records for at least 20 years after the year to which they relate.

12) s69 (plus s73 and s74): allocation of free NZUs to pre-1990 forestry – Act simply provides for total allocation in different periods, and for allocation plans to be issued by OIC made on recommendation of Minister (no timeframe stated). Discussion with MAF and consultation documents indicate owners of pre-1990 forests as at 1 January 2008 will be entitled to receive an allocation of three tranches of free NZUs – those available for use in CP1, those post-dated for use in the subsequent five year period, and those post-dated for use in the five year period following that (all post-dated NZUs will be tradable, but not operative until the relevant period – with tranches targeted to coincide with expected harvest date (i.e. while allocation will occur all at once, not all landowners will receive NZUs operative in CP1; rather they will have NZU allocations for each five year period tied to their forest age (see pp 28-29 of *Forestry in a New Zealand Emissions Trading Scheme*)). MAF advises that the legal owner of the land as at 1 January 2008 is the person entitled to apply for a free allocation of NZUs – any subsequent owner would have to negotiate with that owner to require them to apply for the free allocation, and agree a price for acquiring the NZUs from them, if they wish to acquire both the land and the NZU allocation that the initial landowner is entitled to.

13) s164: deemed deforestation – a hectare of pre-1990 forest land is treated as deforested if s164(1)(a): within four years of felling that hectare has not been replanted in forest species or naturally established a significant covering of forest species; or s164(1)(b) or (c): exotic (indigenous) forest species are growing but that hectare does not have crown cover of 30%+ from trees of 5+ metres in height within 10 (20) years of felling.

Post-1989 Forests – relevant "activity" is ownership of post-1989 forest land or being holder of registered forestry right or registered lease over such land.

14) s57: Ability to register as ETS participant – from 1 January 2008 any person who OWNS post-1989 forest land, or any person holding a registered forestry right or being the leaseholder under a registered lease of post-1989 forest land, MAY apply to be registered as an ETS participant in respect of some or all of their land/right/lease (s165 and s166(1)(b)) – registration fees are possible (s57(2)(b)(ii))

15) s165: Restrictions on participant registration – person that is the landowner can only register as a participant if there is no registered forestry right or registered lease over their land, or they have the written agreement of the right/lease-holder if one exists; or a person that is such a right/lease-holder can only register as a participant if they have the written agreement of the landowner → the Bill explicitly envisages bargaining between landowner and right/lease-holders over right to generate emission units where land and tress are not owned by the same person.

16) s58: removal from schedule of participants – participant can apply to be removed from schedule of participants at any time (possibly for a fee), and if they are to cease to own post-1989 forest land, or to hold a registered forestry right or to be the leaseholder under a registered lease of post-1989 forest land, then they MUST notify the registrar that they have ceased or will cease to carry out the designated activity.

17) s166(1): By when must such registration occur – before 1 January 2010, or within 18 months of the beginning of each subsequent commitment period (etc).

18) s61: holding accounts – participants must open a holding account for the surrendering of units and receiving units to which they become entitled.

19) s62: monitoring of emissions and removals – each year participants MUST collect prescribed data and information (which may be required to be verified by recognised persons/organisations), calculate emissions and removals (i.e. carbon sequestration) using prescribed methodologies, (if required to by regulation) have these calculations verified by recognised persons/organisations, and keep records of these data, information and calculations

20) s63: liability to surrender units to cover emissions – participant must transfer one unit for each tonne of emissions arising from harvest, deforestation, or other natural or unnatural carbon losses as calculated under the Act, and at the times required under the Act (see s167 and s168 below) – requires them to apply to transfer the required number of units from their holding account to the designated surrender account.

21) s168(2): cap on emission liability/wrap-up calculation – participant not required to surrender a greater number of units than the number of units they have received for removals from that land. Where a participant is removed from the register or cases to carry out the activity on more than one hectare of land, they must file an emissions return within 20 WORKING DAYS of the removal/cessation, setting out total number of units issues and surrendered, with net of units issued less units surrendered required to be surrendered.

22) s64: entitlement to receive NZUs for removal activities – participant is entitled to receive one NZU for each tonne of carbon sequestered, as calculated under the Act – requiring registrar to transfer the unit to the participant’s holding account (there may be some time lag between sequestration being calculated and notified, and ultimate transfer).

23) s167: emission returns for post-1989 forestry (modifies s65 position for post-1989 forests) – annual emission returns are NOT required, but participant MAY submit returns in respect of any period of at least 12 months, though they MUST submit a return by 31 March 2013 (for period ending 31 December 2012) and at any other time after then prescribed in regulations. The commencement date must be later of 1 January 2008 or the date at which the activity

commenced, or the day after the end of the period covered by the participant's last return. Emission returns can include an assessment of the participant's NET liability to surrender units (which surrender MUST be made WITHIN 20 WORKING DAYS) or entitlement to receive NZUs (note asymmetry in liability and entitlement unit of currency), and may elect to surrender/receive the NET number of units.

→ if participant wants to claim NZUs for carbon sequestered, they can do so no more frequently than annually, but may enjoy a timing advantage in respect of when they file an emissions return for emission liabilities (e.g. be able to defer recognition for at least some time). Also, no claim for NZUs can be made in respect of carbon sequestered prior to 2008.

24) s172 – transitional provisions - first emission return – a post-1989 forest emission return may not be filed before 1 January 2009.

25) s66: records – must retain records for at least 20 years after the year to which they relate.

26) s169: transfer – participant may, for a possible fee, apply to transfer their registration to a party to which they intend to transfer their land or registered forestry right/lease, with the transferee treated as the same participant as the transferor from the date at which the registration transfer application is accepted. Furthermore, the wrap up surrender calculation under s168 is not required if the transferee is themselves a registered participant.

* * *

Appendix B.1 – BROA Valuation Details: Non-Conversion Model (Post-1989 and Pre-1990)

Post-1989

Define the following:

Alpha	Parameter setting height of assumed harvest barrier (cf reserve price threshold in reserve pricing strategy models) (December 2007 dollars per ha)
B(t)	Posited harvest barrier at time t, parameterised by alpha, e.g. B(t) = alpha x √ (T_{max} – t), or B(t) = alpha, or B(t) = arcos function parameterised by t, alpha, and assumed ratio of minimum to maximum barrier value over T_{min} to T_{max} range
C(t)	Real forestry costs (other than harvest, carbon liability or land conversion costs) incurred in year t (December 2007 dollars per ha)
CL(t)	Carbon liability from harvest at time t, being S(t) x P(C,t)
DF(j)	Discount factor for j time periods, being $e^{-r \times j}$ when using risk-neutral valuation
HC(t)	Real harvest costs at time t (December 2007 dollars per m^3)
NPV(0,T*(i))	Net present value produced by simulation sample path i (i = 1, …, 5,000)
NPV(0)	Net present value of the forest as at the valuation date, for a given set of decision barrier/map parameters. The actual forest value is determined by optimising NPV(0) with respect to that parameter set by repeated simulation-based "trial and error" ("bootstrapping")
P(C,t)	Real price of a NZU (i.e. "Carbon") at time t (December 2007 dollars per tCO_2)
P(L_j,t)	Real price of log grade j at time t (December 2007 dollars per m^3)
r	Real risk-free interest rate, continuously compounded
∆S(t)	Change in carbon dioxide sequestered in the forest between times t-1 and t, being S(t) – S(t-1) (in tCO_2 per ha)

$S(t)$	Carbon dioxide sequestered in the forest at time t, being $\Delta S(1) + \Delta S(2) + \ldots + \Delta S(t)$ (in tCO_2 per ha)
$T^*(i)$	First year in simulation sample path i (i = 1, …, 5,000) that harvest is the optimal decision, based on B(t)
T_c	Company tax rate
T_{min}	Assumed earliest harvest date
T_{max}	Assumed latest harvest date
$y(j,t)$	Forest yield for log grade j if harvest occurs at time t (m^3 per ha; j = 1, …, k)

The BROA approach for post-1989 forest valuation with no land conversion options is as follows. For sample path i (i = 1, 2, … 5,000 simulation trials) and candidate harvest year t in harvest window (T_{min}, T_{max}), assuming some candidate value for alpha:

1) Simulate values of P(C,0) to P(C,t) and $P(L_j,t)$ for j = 1, …, k;

2) Compute value of decision (i.e. harvest) barrier B(t) based on the candidate value of alpha at time t;

3) Given simulated values of P(C,t) and $P(L_j,t)$, compute post-tax net harvest return at time t, net of post-tax carbon liability from harvest (CL(t)), being:

   $P(L_1,t) \times y(1,t) + \ldots + P(L_k,t) \times y(k,t) \} \times (1 - T_c)$ post-tax gross harvest revenue

   $- HC(t) \times \{ y(1,t) + \ldots + y(k,t) \} \times (1 - T_c)$ post-tax harvest costs

   $- C(t) \times (1 - T_c)$ post-tax forestry costs

   $- CL(t) \times (1 - T_c)$ post-tax carbon liability from harvest

4) If amount calculated in (3) exceeds B(t), then crystallise the harvest decision (H) in year t (meaning t becomes T*(i)), with NPV(0,T*(i)) assumed to be the NPV produced up to T, replicated to infinity without real price or cost growth assuming fixed harvest age of 27 years (to account for subsequent rotation value), namely:

   Let $NPV_H(T^*(i))$ = NPV at time 0 of { Value from (3) above + post-tax carbon sequestration revenue from time 0 to time T*(i) – post-tax forestry costs C(.) from time 0 to time T*(i)-1 }

= NPV at time 0 of { Value from (3) above

+ [$P(C,1) \times \Delta S(1) + \ldots + P(C,T^*(i)) \times \Delta S(T^*(i))$] x $(1 - T_c)$

– [$C(0) + C(1) + \ldots + C(T^*(i)–1)$] x $(1 - T_c)$ }

then:

$$NPV(0,T^*(i)) = NPV_H(T^*(i)) / [\ 1 - 1 / (1 + r)^{27}\ ]$$

5) If amount calculated in (3) does not exceed B(t), and $t = T_{max}$, then either abandon the forest in favour of carbon farming (CF), or simply abandon (A) the forest altogether, depending on whichever is the greater, in which case a decision is crystallised, t becomes T*(i), and NPV(0,T*(i)) is calculated as:

   Let $NPV_{A\ or\ CF}(T^*(i))$ = max { 0, NPV at time T*(i) of post-tax carbon sequestration revenue from time T*(i) onwards – post-tax forestry costs C(.) from time T*(i) onwards }

   = max { 0, NPV at time T*(i) of [$P(C,T^*(i)) \times \Delta S(T^*(i)) + P(C,T^*(i)+1) \times \Delta S(T^*(i)+1) + \ldots$] x $(1 - T_c)$ – NPV at time T*(i) of [$C(T^*(i)) + C(T^*(i)+1) + \ldots$] x $(1 - T_c)$

   then:

   $NPV(0,T^*(i))$ = NPV at time 0 of { $NPV_{A\ or\ CF}(T^*(i))$

   + [$P(C,1) \times \Delta S(1) + \ldots + P(C,T^*(i)) \times \Delta S(T^*(i))$] x $(1 - T_c)$

   – [$C(0) + C(1) + \ldots + C(T^*(i)–1)$] x $(1 - T_c)$ }

6) If amount calculated in (3) does not exceed B(t), and $t < T_{max}$, then wait (W) – i.e. continue simulating sample path i to time t+1, and repeat these tests to see if harvest, abandonment or carbon farming, or waiting are optimal for time t+1.

Once all 5,000 sample paths have been simulated, producing T*(i) and NPV(0,T*(i)) for each sample path, compute forest value using the candidate value of alpha as the simple average of all 5,000 values of NPV(0,T*(i)), i.e.:

$$NPV(0) = 1/5{,}000 \times \{ NPV(0,T^*(1)) + NPV(0,T^*(2)) + \ldots + NPV(0,T^*(5{,}000)) \}$$

Repeat the above procedure for different values of alpha until NPV(0) is maximised, thus producing the estimated forest value and optimal value of alpha.

Pre-1990

Proceed as above, except:

1) Amend cost series (C(.)) to be based on pre-1990 ETS compliance costs rather than post-1989 ETS compliance costs; and

2) Delete all references to NZU prices (P(C(.)), change in carbon dioxide sequestered (ΔS(.)) and carbon dioxide sequestered (S(.)).

Appendix B.2 – BROA Valuation Details: Conversion Model (Pre-1990 and Post-1989)

Pre-1990

Use same definitions as in Appendix B.1 except for the following:

ψ(t)	Replaces B(t), and is assumed to equal alpha x √ (T_{max} – t) for illustrative purposes, being the time-varying vertical intercept in Version 1 of Figure 4.4, which delimits the harvest and replant (HR) boundary
Beta	Parameter for delimiting the harvest and convert (HC) boundary
Φ(t)	The possibly time-varying horizontal intercept in Version 1 of Figure 4.4, delimiting the harvest and convert (HC) boundary, and assumed equal to beta (i.e. a constant) for illustrative purposes (a more general formulation involving beta and t could be assumed)
CC(t)	Real cost of converting one hectare of forest land into dairy land at time t (excluding harvest costs or any deforestation-related carbon liability) – assumed non-tax deductible
DL(t)	Replaces CL(t), being the real cost of deforesting one hectare of forest land at time t, assumed to equal S(t) x P(C,t) (i.e. same as CL(t)) and further assumed to be non-tax deductible (as opposed to the treatment for post-1989 forests)
NPV(HR,t)	NPV at time t of harvesting and replanting at time t (calculations below)
NPV(HC,t)	NPV at time t of harvesting and immediate conversion into dairy land at time t (calculations below)
P(D,t)	Real price of dairy land at time t December 2007 dollars per hectare)
R(t)	NPV at time t of post-tax real costs of replanting a forest at time t up to an assumed subsequent harvest date of time t + 27 years (excluding both harvest revenues and costs)

Instead of using decision barrier B(t), we now have two barriers to apply in our decision map:

1) The downward sloping part of Version 1 in Figure 4.4, having equation:

NPV(HR,t) = ψ(t) - ψ(t)/Φ(t) x NPV(HC,t); and

2) The upward sloping part of Version 1 in Figure 4.4, for simplicity assumed to have the equation (i.e. a 45 degree line):

NPV(HR,t) = NPC(HC,t)

These barriers – <u>parameterised by candidate values of alpha and beta</u> – now define three <u>zones</u>:

1) Harvest and replant zone (HR zone), where:

NPV(HR,t) ≥ ψ(t) - ψ(t)/Φ(t) x NPV(HC,t) <u>AND</u> NPV(HR,t) ≥ NPC(HC,t)

2) Harvest and convert zone (HC zone), where:

NPV(HR,t) ≥ ψ(t) - ψ(t)/Φ(t) x NPV(HC,t) <u>AND</u> NPV(HR,t) < NPC(HC,t)

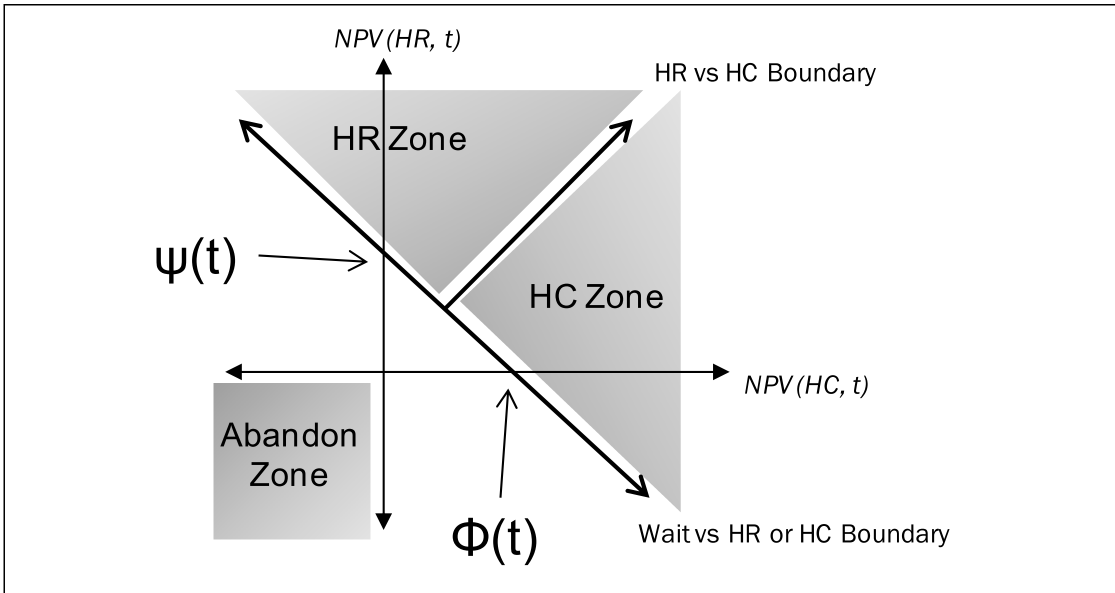
3) Abandon zone (A zone), where:

NPV(HR,t) < 0 <u>AND</u> NPC(HC,t) < 0 <u>AND</u> $t = T_{max}$.

These zones and intercepts are illustrated in Figure B.1.

Figure B.1

Conversion Model Decision Map Details (cf Version 1 in Figure 4.4)



The BROA approach for pre-1990 forest valuation with a dairy land conversion option is as follows. For sample path i (i = 1, 2, … 5,000 simulation trials) and candidate harvest year t in harvest window (T_{min}, T_{max}), <u>assuming candidate values of both alpha and beta</u>:

1) Simulate values of P(C,0) to P(C,t), P(D,t) and P(L_j,t) for j = 1, …, k;

2) Compute values for the HR and HC barriers at time t – <u>given the candidate values of alpha and beta</u> – as above;

3) Given simulated values of P(C,t), P(D,t) and P(L_j,t), compute the NPV of harvest and conversion at time t, NPV(HC,t), being:

   P(L_1,t) x y(1,t) + … + P(L_k,t) x y(k,t) } x (1 – T_c) post-tax gross harvest revenue

   – HC(t) x { y(1,t) + … + y(k,t) } x (1 – T_c) post-tax harvest costs

   – C(t) x (1 – T_c) post-tax forestry costs

   – DL(t) ETS deforestation liability

   – CC(t) land conversion costs

   + P(D(t) value of dairy land resulting from conversion

4) Given simulated values of P(C,t), P(D,t) and P(L_j,t), compute the NPV of harvest and replanting at time t, NPV(HC,t), being:

   P(L_1,t) x y(1,t) + … + P(L_k,t) x y(k,t) } x (1 – T_c) post-tax gross harvest revenue

   – HC(t) x { y(1,t) + … + y(k,t) } x (1 – T_c) post-tax harvest costs

   – C(t) x (1 – T_c) post-tax forestry costs

   – R(t) NPV at time t of post-tax forest replanting costs from candidate decision date t and second assumed decision date t+27

   +Z(t) see below

   Where:

   Z(t) = NPV <u>at time t</u> of the greater of the expected payoff at time t + 27 years from abandonment, harvest and conversion, or harvest and replanting

   = NPV <u>at time t</u> of max { 0, E(NPV(HC,t+27)), E(NPV(HR,t+27)) }

And E(NPV(HC,t+27)) is computed as:

H(t+27) = Expected post-tax harvest revenue at time t+27 given log yields and expected log prices at time t+27 (using assumed log price processes and prices known at time t) calculated analogously as above

– Expected dairy land conversion costs at t+27 (E[CC(t+27)])

– Expected deforestation liability at t+27 given expected NZU price at time t+27 (E[DL(t+27)], using assumed NZU price process and price known at time t)

+ Expected dairy land value resulting from conversion at time t+27 (E(P(D,t+27)], using assumed dairy land price process and land price known at time t)

And furthermore E(NPV(HR,t+27)) is computed as:

H(t+27) + E(R(t+27)), replicated repeatedly (to account for the value from future rotations beyond t+27, i.e.:

$$E(NPV(HR,t+27)) = \{ H(t+27) + E(R(t+27) \} / [1 - 1 / (1 + r)^{27}]$$

5) Given NPV(HR,t), NPV(HC,t) and the HR, HC and W boundaries:

If the { NPV(HR,t), NPV(HC,t) } pair plots in the HR zone, then crystallise the harvest and replanting (HR) decision producing NPV(HR,t) at that time t, (meaning t becomes T*(i)), with NPV(0,T*(i)) then computed as:

NPV(0,T*(i)) = NPV at time 0 of NPV(HR(t)) – NPV at time 0 of post-tax forestry costs from time 0 to T*(i)

Otherwise if the { NPV(HR,t), NPV(HC,t) } pair plots in the HC zone, then crystallise the harvest and conversion (HC) decision producing NPV(HC,t) at that time t, (meaning t becomes T*(i)), with NPV(0,T*(i)) then computed as:

NPV(0,T*(i)) = NPV at time 0 of NPV(HC,t) – NPV at time 0 of post-tax forestry costs from time 0 to T*(i)

Otherwise if t = T_{max} and both NPV(HR,t) and NPV(HC,t) are negative then crystallise and abandonment (A) decision at that time t (meaning t becomes T*(i)), with NPV(0,T*(i)) then computed as:

NPV(0,T*(i)) = NPV at time 0 of max { 0 (i.e. assumed abandonment return), post-tax harvest return plus 0 (for abandonment) } – NPV at time 0 of post-tax forestry costs from time 0 to T*(i)

7) If none of the HR, HC or A decisions are crystallised in (6) then wait (W) – i.e. continue simulating sample path i to time t+1, and repeat these tests to see if HR, HC, A or waiting are optimal for time t+1.

Once all 5,000 sample paths have been simulated, producing T*(i) and NPV(0,T*(i)) for each sample path, compute forest value – based on the candidate values of alpha and beta – as the simple average of all 5,000 values of NPV(0,T*(i)), i.e.:

NPV(0) = 1/5,000 x { NPV(0,T*(1)) + NPV(0,T*(2)) + … + NPV(0,T*(5,000)) }

Repeat the above procedure for different values of alpha and beta until NPV(0) is maximised, thus producing the estimated forest value and optimal values of alpha and beta.

Post-1989

Proceed as above, except:

1) From t = 0 up to harvest (whether HR or HC is crystallised), and after any replanting (HR decision only), post-tax carbon sequestration revenue is earned (see post-1989 non-conversion model for calculation);

2) Upon any HC decision being crystallised, a tax-deductible harvest liability is also crystallised (being equal in size to DL(.) less tax) instead of a deforestation liability (DL(.));

3) When computing NPV(HR,t):

    a. recognise that the forest generates carbon sequestration value between candidate decision date t and second assumed decision date t+27, in addition to just forest replanting costs, the value of which is computed using expected future NZU prices based on the assumed price process and NZU price known at T_{max}; and

b. when computing Z(t) allowance must also be made for the possibility that carbon farming beyond t+27 might be superior to either abandonment, HR or HC at t+27 (i.e. incurring ongoing post-tax forestry costs, but continuing to earn post-tax carbon sequestration revenues, the value of which is computed using expected future NZU prices based on the assumed price process and NZU price known at T_{max}); and

4) If neither HC or HR decisions are crystallised by T_{max}, then instead of simply abandoning the forest for nil return, the forester chooses the better (i.e. greater) of either:

a. outright abandonment (for nil return); or

b. the present value at T_{max} of ongoing carbon farming (i.e. incurring ongoing post-tax forestry costs, but continuing to earn post-tax carbon sequestration revenues, the value of which is computed using expected future NZU prices based on the assumed price process and NZU price known at T_{max}).

Appendix B.3 – BROA Valuation Details: Conversion Model (Pre-1990 with Conversion Delayed Eight Years)

The pre-1990 conversion BROA model specified in Appendix B.2 assumes that in the event that a forest is harvested, it is either replanted immediately, or its land immediately converted into non-forestry (i.e. dairy) use. Under the proposed ETS rules any forest that is deforested before its tree have reached at least eight years of age is deemed to produce a deforestation liability equal to that in the previous crop (provided that crop was more than eight years old). However, if the land is deforested immediately after the trees reach eight years of age then the deforestation liability is based on the carbon stock in the young trees, not any prior mature crop. This raises the possibility that the option of harvest and conversion might better be specified with conversion (i.e. deforestation) occurring eight years after harvest rather than immediately.

In terms of modifying the pre-1990 conversion model structure, this modification can be implemented by changing the calculation of NPV(HC,t) while leaving the remainder of the model as in Appendix B.2. This implies the following changes (assuming the same notation as in Appendices B.1 and B.2):

1) When computing NPV(HC,t), the costs and returns from deforestation are assumed to arise at t+8, based on expected values generated as at t (i.e. using expected NZU price P(C,t+8) and expected dairy land price P(D,t+8), generated using the assumed price processes and NZU and dairy land prices known at t);

2) In the intervening period, rather than assuming replanting costs, only annual forest overhead costs need be assumed (and any other costs to ensure deforestation is not deemed to arise sooner – e.g. ensuring grazing does not occur) – for illustration purposes we assumed $105/ha/year (December 2007 dollars);

3) Since the naturally regenerating forest land will require additional clearance when deforestation occurs, an additional conversion cost could be assumed (e.g. $500/ha in December 2007 dollars);

4) In principle the forester retains the option to not deforest at t+8, but instead could replant then, or abandon the forest, etc – for simplicity our implementation assumes that the deforestation proceeds at t+8 as planned at the preceding harvest date t;[55] and

---

[55] Similarly, NPV(HR,t) could be modified to compute the possibility of harvest and conversion at time t+27 along similar lines (i.e. delaying conversion 8 years after harvest at t+27) using expected NZU and dairy land values at t+27+8 based on assumed price processes and prices known at t. For simplicity this refinement was not implemented.

5) The deforestation liability at t+8 will be based on a much lower carbon stock than S(t) assuming the forest is relatively mature at t – we assumed S(t)/10 for illustrative purposes.

Thus the modified NPV(HC,t), denoted NPV*(HC,t), can be stated as:

NPV*(HC,t) = Post-tax harvest revenue at time t

+ NPV as at t of expected dairy land value at t+8

- NPV as at t of expected conversion costs at t+8

- NPV as at t of expected deforestation liability at t+8

- NPV as at t of expected post-tax holding costs from t to t+8

The remainder of the pre-1990 conversion model implementation then proceeds as before.

Appendix C – Forestry Assumptions

Real harvest costs (HC(t)): 45 (12/07 $/m3) - note: not forest age-specific (i.e. this cost applies in whatever year harvest is assumed)

* Note that yields are available only for forest age 20+. For illustration purposes Tmin is assumed to be no less than this age
** Note that these costs assume no rental cost - i.e. that land and trees owned together

					Log Yields by Grade* (y(i,t), i = 1 … 6))						
Forest Age	Total Stand Volume (TSV)	Total Recoverable Volume (TRV)	Total Carbon Sequestered (S(t))	Incremental Carbon Sequestered (?S(t))	P1	P2	S1S2	S3&L3	L1&L2	Pulp	Real Forestry Costs (C(t))**
		0.85	0.77								
(Years)	(m3/Ha)	(m3/Ha)	(tC02/m3)	(tC02/m3/year)	(m3/ha)	(m3/ha)	(m3/ha)	(m3/ha)	(m3/ha)	(m3/ha)	(12/07 $/ha)
0	0	0.0	0.0		n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	905
1	6	5.1	4.6	4.6	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
2	12	10.2	9.2	4.6	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
3	18	15.3	13.9	4.6	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
4	24	20.4	18.5	4.6	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
5	23	19.6	17.7	-0.8	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	930
6	44	37.4	33.9	16.2	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	435
7	38	32.3	29.3	-4.6	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	800
8	59	50.2	45.4	16.2	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
9	83	70.6	63.9	18.5	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
10	112	95.2	86.2	22.3	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
11	143	121.6	110.1	23.9	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
12	177	150.5	136.3	26.2	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
13	213	181.1	164.0	27.7	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
14	250	212.5	192.5	28.5	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
15	288	244.8	221.8	29.3	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
16	327	278.0	251.8	30.0	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
17	366	311.1	281.8	30.0	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
18	405	344.3	311.9	30.0	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
19	444	377.4	341.9	30.0	n.a.	n.a.	n.a.	n.a.	n.a.	n.a.	105
20	482	409.7	371.1	29.3	4.0	144.6	64.3	120.5	32.1	44.2	105
21	520	442.0	400.4	29.3	13.3	145.9	75.1	128.2	39.8	39.8	105
22	557	473.5	428.9	28.5	28.1	140.6	84.4	131.3	46.9	42.2	105
23	593	504.1	456.6	27.7	35.3	141.1	100.8	131.1	55.4	40.3	105
24	628	533.8	483.6	27.0	42.7	144.1	112.1	128.1	69.4	37.4	105
25	662	562.7	509.7	26.2	73.2	118.2	135.0	112.5	84.4	39.4	105
26	695	590.8	535.2	25.4	99.4	99.4	140.4	122.8	93.6	35.1	105
27	727	618.0	559.8	24.6	112.4	93.6	162.3	106.1	112.4	31.2	105
28	758	644.3	583.7	23.9	121.2	95.7	165.9	108.4	121.2	31.9	105
29	788	669.8	606.8	23.1	134.0	87.1	167.5	113.9	134.0	33.5	105
30	817	694.5	629.1	22.3	145.8	83.3	180.6	104.2	145.8	34.7	105
31	844	717.4	649.9	20.8	143.5	86.1	186.5	107.6	157.8	35.9	105
32	871	740.4	670.7	20.8	155.5	81.4	192.5	111.1	162.9	37.0	105
33	896	761.6	689.9	19.3	184.6	46.2	215.4	92.3	184.6	38.5	105
34	921	782.9	709.2	19.3	195.7	47.0	219.2	93.9	187.9	39.1	105
35	944	802.4	726.9	17.7	198.6	47.7	222.4	95.3	198.6	39.7	105
36	966	821.1	743.8	16.9	200.9	48.2	227.6	97.6	206.1	40.6	105
37	988	839.8	760.8	16.9	203.1	48.8	232.8	99.8	213.7	41.6	105
38	1009	857.7	776.9	16.2	205.0	49.2	237.8	101.9	221.3	42.5	105
39	1028	873.8	791.6	14.6	206.4	49.5	242.2	103.8	228.5	43.3	105
40	1047	890.0	806.2	14.6	207.7	49.9	246.7	105.7	235.9	44.1	105
41	1065	905.3	820.1	13.9	208.7	50.1	251.0	107.6	243.1	44.8	105
42	1082	919.7	833.1	13.1	209.5	50.3	255.0	109.3	250.2	45.5	105
43	1099	934.2	846.2	13.1	210.1	50.4	259.0	111.0	257.4	46.2	105
44	1114	946.9	857.8	11.6	210.3	50.5	262.5	112.5	264.2	46.9	105
45	1129	959.7	869.3	11.6	210.5	50.5	266.0	114.0	271.1	47.5	105
46	1144	972.4	880.9	11.6	210.5	50.5	269.6	115.5	278.1	48.1	105
47	1158	984.3	891.7	10.8	210.3	50.5	272.9	116.9	285.0	48.7	105
48	1171	995.4	901.7	10.0	209.9	50.4	275.9	118.3	291.6	49.3	105
49	1183	1005.6	910.9	9.2	209.2	50.2	278.8	119.5	298.1	49.8	105

[redacted]

Appendix D – Pricing Models

GBM simulation

$$P_{t+1} = P_t \cdot \exp(((r-\delta) - \frac{\hat{\sigma}^2}{2})dt + \hat{\sigma} \cdot Z)$$

Where Z~N(0,1)

P_{t+1} is the logarithm of the Price at t+1

P_t is the logarithm of the price at time t

r is the risk free discount rate

δ is the yield

$\hat{\sigma}^2$ is the GBM variance

OU simulation

$$P_{t+1} = P_t \cdot \exp(-\kappa \cdot dt) + (\mu - \frac{\lambda - r}{\kappa}) \cdot (1 - \exp(-\kappa \cdot dt)) + \sigma \cdot \sqrt{\frac{1-\exp(-2\kappa \cdot dt)}{2\kappa}} \cdot Z$$

Where Z~N(0,1)

P_{t+1} is the logarithm of the Price at t+1

P_t is the logarithm of the price at time t

κ is the velocity of mean reverting in OU

μ is the OU mean

σ is the OU standard deviation

λ is the total return.

Note1: all the prices here are the logarithm of the actual prices.
And this is the formula of Dixit and Pindyck (1994). The derivation is from:

http://www.puc-rio.br/marco.ind/sim_stoc_proc.html#discretization

Note2: The convenience yield in risk neutral adjusted OU is $\delta = r - \kappa\left(mu - P_t - \frac{(\lambda - r)}{\kappa}\right)$

That is, the convenience yield is not constant and depends on the distance from the average, the mean reverting velocity parameter and risk premium ($\lambda - r$). If $mu = P_t$ then $\delta = 2r - \lambda$. That is if we consider r=5% and total return of 7.5% our convenience yield would be 2.5%.

For OU process:

Mean Price (OU) = $\frac{1}{n}\sum_{i=1}^{n} Pt$

OU Mean (logarithm) = $\hat{\mu}$ = $\frac{1}{n}\sum_{t=1}^{n}\log(P_t)$

OU Rho (Quarterly) = Correlation coefficient between log(Pt) and log(Pt-1)

$$= \hat{\rho} = Corr(\log(P_t), \log(P_{t-1})) = \frac{Cov(\log(P_t), \log(P_{t-1}))}{\sigma_{Pt}\sigma_{pt-1}}$$

OU Rho = Correlation coefficient between log(Pt) and log(Pt-1)

$$= \tilde{\rho} = \frac{1}{dt}\hat{\rho} = \frac{1}{dt}\frac{Cov(\log(P_t), \log(P_{t-1}))}{\sigma_{Pt}\sigma_{pt-1}}$$

OU standard error squared (Quarterly) = $\hat{\sigma}_{\varepsilon}^{2}$ = $\frac{1}{n}\sum_{t=2}^{n}(\log(P_t) - \hat{\mu} - \hat{\rho}(\log(P_{t-1}) - \hat{\mu}))^2$

OU kappa = Speed of mean reverting

$$= \kappa = -\frac{1}{dt}\log(\rho)$$

OU Variance = $\hat{\sigma}^2 = \frac{2\kappa.\hat{\sigma}_{\varepsilon}^{2}}{1-\hat{\rho}^2}$

For GBM Process:

GBM Mean (Quarterly) = $\hat{\mu}$ = $\frac{1}{n}\sum_{i=1}^{n}\log(\frac{Pt}{P_{t-1}}) + 0.5\ \frac{1}{n}\sum_{i=2}^{n}[\log(\frac{P_t}{P_{t-1}}) - \frac{1}{n}\sum_{i=2}^{n}(\frac{P_t}{P_{t-1}})]^2$

GBM Variance (Quarterly) = $\hat{\sigma}^2$ = $\frac{1}{n}\sum_{t=2}^{n}[\log(\frac{P_t}{P_{t-1}}) - \frac{1}{n}\sum_{i=2}^{n}(\frac{P_t}{P_{t-1}})]^2$

GBM Mean = $\tilde{\mu} = \frac{1}{dt}\hat{\mu}$

GBM Variance = $\tilde{\sigma}^2 = \frac{1}{dt}\hat{\sigma}^2$

Corr(X,Y) = $\frac{Cov(X,Y)}{\sigma_X\sigma_Y} = \frac{E((X-\mu_X)(Y-\mu_Y))}{\sigma_X\sigma_Y}$

Appendix E.1 – BROA Code Using “R”: Non-Conversion Model (Pre-1990 and Post-1989)

```
rm(list=ls(all=TRUE))

data<-read.csv("data.csv")


OU.broa<-function(n,alfa,PCmu=log(13.44427)){

########Objective

##########Develop post-1989 and pre-1990 forest model with no conversion


# Stochastic process:

#OU with trend and deterministic cost function

# Ornstein - Uhlenbeck without trend Parameters (risk neutral exact formula)
# advantage: we can use big values for delta.t
# disadvantage: we need delta.t > -log(2*kappa)/2*kappa (that is we cannot use kappa smaller than 0.3 for delta.t=1)

# Note load package "mnormt" to run normal multivariate random sample
# Use function 'rmnorm(n = 1, mean = rep(0, d), varcov)' to generate random mnormal series

## Monte carlo Parameters

g<- 60
delta.t<-1


## Bootstrapping Real Options Analysis Parameter

Tmin<-20
Tmax<-35

Tmin <- Tmin+1
Tmax <- Tmax+1

###### Model Parameters

r <- 0.05

sigma1 <-sqrt(0.011657)
sigma2 <-sqrt(0.031556)
sigma3 <-sqrt(0.016532)
sigma4 <-sqrt(0.043397)
sigma5 <-sqrt(0.029713)
```

```
sigma6 <-sqrt(0.032138)
sigmaC <-sqrt(0.1596643)
sigmaD <-sqrt(0.0760058)

lambda1 <- r+0.075
lambda2 <- r+0.075
lambda3 <- r+0.075
lambda4 <- r+0.075
lambda5 <- r+0.075
lambda6 <- r+0.075
lambdaC <- r+0.075
lambdaD <- r+0.075

kappa1 <- 0.3
kappa2 <- 0.388956
kappa3 <- 0.300546
kappa4 <- 1.057335
kappa5 <- 0.567348
kappa6 <- 0.858477
kappaC <- 0.9549317
kappaD <- 0.3


P10 <- 4.931171
P20 <- 4.685213
P30 <- 4.435271
P40 <- 4.289317
P50 <- 4.199705
P60 <- 3.89097
PC0 <- 3.084598905
PD0 <- 9.897421

P1mu <- 5.31734
P2mu <- 5.055295
P3mu <- 4.737019
P4mu <- 4.412626
P5mu <- 4.436973
P6mu <- 3.964214

PDmu <- 9.335498

varcov <- diag(1,8,8)

varcov[1,2]<-0.932138 #corr(1,2)
varcov[1,3]<-0.883531 #corr(1,3)
varcov[1,4]<-0.718127 #corr(1,4)
varcov[1,5]<-0.829232 #corr(1,5)
varcov[1,6]<-0.585644 #corr(1,6)
varcov[1,7]<- -0.72651 #corr(1,C)
varcov[1,8]<- -0.45611 #corr(1,D)
```

```
varcov[2,3]<-0.86808 #corr(2,3)
varcov[2,4]<-0.677265 #corr(2,4)
varcov[2,5]<-0.815388 # ...
varcov[2,6]<-0.522891
varcov[2,7]<- -0.71127
varcov[2,8]<- -0.37441
varcov[3,4]<-0.840848
varcov[3,5]<-0.868466
varcov[3,6]<-0.672502
varcov[3,7]<- -0.21608
varcov[3,8]<- -0.32254
varcov[4,5]<-0.740378
varcov[4,6]<-0.704376
varcov[4,7]<-0.522423
varcov[4,8]<- -0.30201
varcov[5,6]<-0.692611
varcov[5,7]<-0.247694
varcov[5,8]<- -0.19653
varcov[6,7]<-0.157482
varcov[6,8]<- -0.15357
varcov[7,8]<-0 #

for (i in 1:ncol(varcov)){
for (j in 1:nrow(varcov)){
varcov[i,j]<-varcov[j,i]
}
}

Y1<-as.double(data[,5])
Y2<-as.double(data[,6])
Y3<-as.double(data[,7])
Y4<-as.double(data[,8])
Y5<-as.double(data[,9])
Y6<-as.double(data[,10])

S<-as.double(data[,4])

delta.S <-diff(S)
delta.S <- c(0,delta.S)

D<-1 # Dairy Whatever


C<-matrix(0,n,g/delta.t+1)
cc<-as.double(data[,11])


HC<- 45

HC<- HC*(Y1+Y2+Y3+Y4+Y5+Y6)
```

```
t <- 0.3 # corporate tax


######### Price Processes

P1<-matrix(0,n,g/delta.t)
P1<-cbind(P10,P1)
P2<-matrix(0,n,g/delta.t)
P2<-cbind(P20,P2)
P3<-matrix(0,n,g/delta.t)
P3<-cbind(P30,P3)
P4<-matrix(0,n,g/delta.t)
P4<-cbind(P40,P4)
P5<-matrix(0,n,g/delta.t)
P5<-cbind(P50,P5)
P6<-matrix(0,n,g/delta.t)
P6<-cbind(P60,P6)
PC<-matrix(0,n,g/delta.t)
PC<-cbind(PC0,PC)
PD<-matrix(0,n,g/delta.t)
PD<-cbind(PD0,PD)

for (i in seq(1,g/delta.t,1)){

mz <- rmnorm(n,mean=rep(0,ncol(varcov)),varcov)

P1[,i+1] <- P1[,i]*exp(-kappa1*delta.t)+ (P1mu -((lambda1-r)/kappa1))*(1-exp(-kappa1*delta.t))+sigma1*sqrt((1-exp(-
2*kappa1*delta.t))/(2*kappa1))*mz[,1]
P2[,i+1] <- P2[,i]*exp(-kappa2*delta.t)+ (P2mu -((lambda2-r)/kappa2))*(1-exp(-kappa2*delta.t))+sigma2*sqrt((1-exp(-
2*kappa2*delta.t))/(2*kappa2))*mz[,2]
P3[,i+1] <- P3[,i]*exp(-kappa3*delta.t)+ (P3mu -((lambda3-r)/kappa3))*(1-exp(-kappa3*delta.t))+sigma3*sqrt((1-exp(-
2*kappa3*delta.t))/(2*kappa3))*mz[,3]
P4[,i+1] <- P4[,i]*exp(-kappa4*delta.t)+ (P4mu -((lambda4-r)/kappa4))*(1-exp(-kappa4*delta.t))+sigma4*sqrt((1-exp(-
2*kappa4*delta.t))/(2*kappa4))*mz[,4]
P5[,i+1] <- P5[,i]*exp(-kappa5*delta.t)+ (P5mu -((lambda5-r)/kappa5))*(1-exp(-kappa5*delta.t))+sigma5*sqrt((1-exp(-
2*kappa5*delta.t))/(2*kappa5))*mz[,5]
P6[,i+1] <- P6[,i]*exp(-kappa6*delta.t)+ (P6mu -((lambda6-r)/kappa6))*(1-exp(-kappa6*delta.t))+sigma6*sqrt((1-exp(-
2*kappa6*delta.t))/(2*kappa6))*mz[,6]
PC[,i+1] <- PC[,i]*exp(-kappaC*delta.t)+ (PCmu -((lambdaC-r)/kappaC))*(1-exp(-kappaC*delta.t))+sigmaC*sqrt((1-exp(-
2*kappaC*delta.t))/(2*kappaC))*mz[,7]

PD[,i+1] <- PD[,i]*exp(-kappaLV*delta.t)+ (PDmu -((lambdaD-r)/kappaD))*(1-exp(-kappaD*delta.t))+sigmaD*sqrt((1-exp(-
2*kappaD*delta.t))/(2*kappaD))*mz[,8]
}

R<-matrix(0,n,g/delta.t+1)

for(i in 1:n){
```

```
R[i,]<-exp(P1[i,])*Y1+exp(P2[i,])*Y2+exp(P3[i,])*Y3+exp(P4[i,])*Y4+exp(P5[i,])*Y5+exp(P6[i,])*Y6

}


#Present value vector of ongoing variables

#On going Costs

for(i in 1:n){

   C[i,]<-cc

}

for (j in seq(1,g/delta.t+1,1)){

   C[,j]<-C[,j]*(1-t)*exp(-r*(j-1)*delta.t)

}

for(i in 1:n){

   C[i,]<-cumsum(C[i,])

}


# Carbon revenue

CR<-matrix(0,n,g/delta.t+1)
CRe<-matrix(0,n,g/delta.t+1)

for (j in Tmax:g/delta.t+1){

   CRe[,j]<-PC[,Tmax]*exp(-kappaC*(j-Tmax))+PCmu*(1-exp(-kappaC*(j-Tmax))) #OU expectation formula

   }


for(i in 1:n){

CR[i,]<-exp(PC[i,])*delta.S
CRe[i,]<-exp(CRe[i,])*delta.S

}

for (j in seq(1,g/delta.t+1,1)){
```

```
    CR[,j]<-CR[,j]*(1-t)*exp(-r*(j-1)*delta.t)
    CRe[,j]<-CRe[,j]*(1-t)*exp(-r*(j-1)*delta.t)

}


for(i in 1:n){

    CR[i,]<-cumsum(CR[i,])
    CRe[i,]<-cumsum(CRe[i,])



}


#pre 1990

Rr<-matrix(0,n,g/delta.t+1)

for(i in 1:n){

Rr[i,] <- R[i,] - HC #Not considering yet ongoing costs - considered later in the NPV calculation

}

## Post 89

CL<-matrix(0,n,g/delta.t+1)

for(i in 1:n){

CL[i,]<-exp(PC[i,])*S

}

Rd<-Rr-CL #not considering yet ongoing costs and carbon revenues - considered later in the NPV calculation



############################################## Barriers Functions
########################################


B1<-matrix(0,1,length(P1[1,]))
B2<-matrix(0,1,length(P1[1,]))
B3<-matrix(0,1,length(P1[1,]))

# Function B1 -> constant (value to wait proxy)
```

```
B1 <- rep(1,length(P1[1,]))

# Function B2 <- squared root (value to wait proxy)

for (i in seq(1,Tmax,1)){

B2[1,i] <- sqrt(Tmax-i)/sqrt(Tmax-Tmin)
}

# Function B3 < - arcos(x) -> calibrated to 350-600 (value to wait proxy)

B3[1,]  <-c(rep(sqrt(Tmax-Tmin),20),c(sqrt(Tmax-Tmin),3.5509,3.4121,  3.3013,  3.2039,  3.1141,  3.0288,  2.9458,  2.8636,
2.7806, 2.6952, 2.6054, 2.5080, 2.3973, 2.2584, sqrt(Tmax-Tmin)/2),rep(sqrt(Tmax-Tmin)/2,25))/sqrt(Tmax-Tmin)


########################                                Present                                Value
Calculation##########################################################

wm1r<-matrix(g/delta.t+1,n,1)
wm2r<-matrix(g/delta.t+1,n,1)
wm3r<-matrix(g/delta.t+1,n,1)
wm1d<-matrix(g/delta.t+1,n,1)
wm2d<-matrix(g/delta.t+1,n,1)
wm3d<-matrix(g/delta.t+1,n,1)


CPV1r<-matrix(0,n,g/delta.t+1)
CPV2r<-matrix(0,n,g/delta.t+1)
CPV3r<-matrix(0,n,g/delta.t+1)
CPV1d<-matrix(0,n,g/delta.t+1)
CPV2d<-matrix(0,n,g/delta.t+1)
CPV3d<-matrix(0,n,g/delta.t+1)

NPV1r<-matrix(0,n,1)
NPV2r<-matrix(0,n,1)
NPV3r<-matrix(0,n,1)

NPV1d<-matrix(0,n,1)
NPV2d<-matrix(0,n,1)
NPV3d<-matrix(0,n,1)

mNPV1r<-matrix(0,n,1)
mNPV2r<-matrix(0,n,1)
mNPV3r<-matrix(0,n,1)

mNPV1d<-matrix(0,n,1)
mNPV2d<-matrix(0,n,1)
mNPV3d<-matrix(0,n,1)
```

```
iNPV1r<-matrix(0,n,1)
iNPV2r<-matrix(0,n,1)
iNPV3r<-matrix(0,n,1)

iNPV1d<-matrix(0,n,1)
iNPV2d<-matrix(0,n,1)
iNPV3d<-matrix(0,n,1)


##### Pre 1990 Calculations

# B1

for (j in seq(1,g/delta.t+1,1)){

   CPV1r[,j]<-Rr[,j]*(1-t)*exp(-r*(j-1)*delta.t)-C[,j]#At harvest decision
}


for (i in seq(1,n,1)){

x1r <- which(CPV1r[i,]>B1*alfa)

for (j in 1:length(x1r)){
x1r[j]<-ifelse(x1r[j]<Tmin|x1r[j]>Tmax|is.na(x1r[j]),g/delta.t+1,x1r[j])
}

wm1r[i,1]<- ifelse(is.integer(min(x1r))|is.double(min(x1r)),min(x1r),g/delta.t+1)

}


for (i in seq(1,n,1)){

   NPV1r[i,1]<- ifelse(wm1r[i,1]>Tmax,0,CPV1r[i,wm1r[i,1]])
   mNPV1r[i,1]<- ifelse(wm1r[i,1]>Tmax,CPV1r[i,Tmax],CPV1r[i,wm1r[i,1]])


  }


value1r<-sum(NPV1r,na.rm=TRUE)/n



# B2


for (j in seq(1,g/delta.t+1,1)){
```

```
  CPV2r[,j]<-Rr[,j]*(1-t)*exp(-r*(j-1)*delta.t)-C[,j]#At harvest decision
}


for (i in seq(1,n,1)){

x2r <- which(CPV2r[i,]>B2*alfa)

for (j in 1:length(x2r)){
x2r[j]<-ifelse(x2r[j]<Tmin|x2r[j]>Tmax|is.na(x2r[j]),g/delta.t+1,x2r[j])
}

wm2r[i,1]<- ifelse(is.integer(min(x2r))|is.double(min(x2r)),min(x2r),g/delta.t+1)

}


for (i in seq(1,n,1)){

  NPV2r[i,1]<- ifelse(wm2r[i,1]>Tmax,0,CPV2r[i,wm2r[i,1]])
  mNPV2r[i,1]<- ifelse(wm2r[i,1]>Tmax,CPV2r[i,Tmax],CPV2r[i,wm2r[i,1]])

 }


value2r<-sum(NPV2r,na.rm=TRUE)/n

# B3

for (j in seq(1,g/delta.t+1,1)){

  CPV3r[,j]<-Rr[,j]*(1-t)*exp(-r*(j-1)*delta.t)-C[,j]#At harvest decision
}


for (i in seq(1,n,1)){

x3r <- which(CPV3r[i,]>B3*alfa)

for (j in 1:length(x3r)){
x3r[j]<-ifelse(x3r[j]<Tmin|x3r[j]>Tmax|is.na(x3r[j]),g/delta.t+1,x3r[j])
}

wm3r[i,1]<- ifelse(is.integer(min(x3r))|is.double(min(x3r)),min(x3r),g/delta.t+1)

}


for (i in seq(1,n,1)){
```

```
    NPV3r[i,1]<- ifelse(wm3r[i,1]>Tmax,0,CPV3r[i,wm3r[i,1]])
    mNPV3r[i,1]<- ifelse(wm3r[i,1]>Tmax,CPV3r[i,Tmax],CPV3r[i,wm3r[i,1]])


  }


value3r<-sum(NPV3r,na.rm=TRUE)/n




##### Post 1989 Calculations

# B1

for (j in seq(1,g/delta.t+1,1)){

    CPV1d[,j]<-Rd[,j]*(1-t)*exp(-r*(j-1)*delta.t)-C[,j]+CR[,j]#At harvest decision
}


for (i in seq(1,n,1)){

x1d <- which(CPV1d[i,]>B1*alfa)

for (j in 1:length(x1d)){
x1d[j]<-ifelse(x1d[j]<Tmin|x1d[j]>Tmax|is.na(x1d[j]),g/delta.t+1,x1d[j])
}

wm1d[i,1]<- ifelse(is.integer(min(x1d))|is.double(min(x1d)),min(x1d),g/delta.t+1)

}


for (i in seq(1,n,1)){

    NPV1d[i,1]<- ifelse(wm1d[i,1]>Tmax,0,CPV1d[i,wm1d[i,1]])
    mNPV1d[i,1]<- ifelse(wm1d[i,1]>Tmax,CPV1d[i,Tmax],CPV1d[i,wm1d[i,1]])

  }


value1d<-sum(NPV1d,na.rm=TRUE)/n



# B2
```

```
for (j in seq(1,g/delta.t+1,1)){

   CPV2d[,j]<-Rd[,j]*(1-t)*exp(-r*(j-1)*delta.t)-C[,j]+CR[,j]#At harvest decision
}


for (i in seq(1,n,1)){

x2d <- which(CPV2d[i,]>B2*alfa)

for (j in 1:length(x2d)){
x2d[j]<-ifelse(x2d[j]<Tmin|x2d[j]>Tmax|is.na(x2d[j]),g/delta.t+1,x2d[j])
}

wm2d[i,1]<- ifelse(is.integer(min(x2d))|is.double(min(x2d)),min(x2d),g/delta.t+1)

}


for (i in seq(1,n,1)){

   NPV2d[i,1]<- ifelse(wm2d[i,1]>Tmax,0,CPV2d[i,wm2d[i,1]])
   mNPV2d[i,1]<- ifelse(wm2d[i,1]>Tmax,CPV2d[i,Tmax],CPV2d[i,wm2d[i,1]])

 }


value2d<-sum(NPV2d,na.rm=TRUE)/n

# B3

for (j in seq(1,g/delta.t+1,1)){

   CPV3d[,j]<-Rd[,j]*(1-t)*exp(-r*(j-1)*delta.t)-C[,j]+CR[,j]#At harvest decision
}


for (i in seq(1,n,1)){

x3d <- which(CPV3d[i,]>B3*alfa)

for (j in 1:length(x3d)){
x3d[j]<-ifelse(x3d[j]<Tmin|x3d[j]>Tmax|is.na(x3d[j]),g/delta.t+1,x3d[j])
}

wm3d[i,1]<- ifelse(is.integer(min(x3d))|is.double(min(x3d)),min(x3d),g/delta.t+1)

}
```

```
for (i in seq(1,n,1)){

   NPV3d[i,1]<- ifelse(wm3d[i,1]>Tmax,0,CPV3d[i,wm3d[i,1]])
   mNPV3d[i,1]<- ifelse(wm3d[i,1]>Tmax,CPV3d[i,Tmax],CPV3d[i,wm3d[i,1]])
 }


value3d<-sum(NPV3d,na.rm=TRUE)/n



#######NPV to the infinity... (Valuing the whole strategy)

for(i in 1:n){

wm1r[i]<-ifelse(NPV1r[i,1]>0,wm1r[i],Tmax)
wm2r[i]<-ifelse(NPV2r[i,1]>0,wm2r[i],Tmax)
wm3r[i]<-ifelse(NPV3r[i,1]>0,wm3r[i],Tmax)

wm1d[i]<-ifelse(NPV1d[i,1]>0,wm1d[i],Tmax)
wm2d[i]<-ifelse(NPV2d[i,1]>0,wm2d[i],Tmax)
wm3d[i]<-ifelse(NPV3d[i,1]>0,wm3d[i],Tmax)

#t*

avt1r<-mean(wm1r,na.rm=TRUE)
avt2r<-mean(wm2r,na.rm=TRUE)
avt3r<-mean(wm3r,na.rm=TRUE)

avt1d<-mean(wm1d,na.rm=TRUE)
avt2d<-mean(wm2d,na.rm=TRUE)
avt3d<-mean(wm3d,na.rm=TRUE)

}


for(i in 1:n){

iNPV1r[i,1]<-ifelse(mNPV1r[i,1]>0,mNPV1r[i,1]+(1/((1+r)^wm1r[i]))*mNPV1r[i,1]*1/(1-1/((1+r)^wm1r[i])),0)
iNPV2r[i,1]<-ifelse(mNPV2r[i,1]>0,mNPV2r[i,1]+(1/((1+r)^wm2r[i]))*mNPV2r[i,1]*1/(1-1/((1+r)^wm2r[i])),0)
iNPV3r[i,1]<-ifelse(mNPV3r[i,1]>0,mNPV3r[i,1]+(1/((1+r)^wm3r[i]))*mNPV3r[i,1]*1/(1-1/((1+r)^wm3r[i])),0)

iNPV1d[i,1]<-ifelse(mNPV1d[i,1]>CR[i,Tmax]-C[i,Tmax],mNPV1d[i,1]+(1/((1+r)^wm1d[i]))*mNPV1d[i,1]*1/(1-
1/((1+r)^wm1d[i])),CR[i,Tmax]-C[i,Tmax] + max(0,(CRe[i,g/delta.t+1]-CRe[i,Tmax])-(C[i,g/delta.t+1]-C[i,Tmax])))
iNPV2d[i,1]<-ifelse(mNPV2d[i,1]>CR[i,Tmax]-C[i,Tmax],mNPV2d[i,1]+(1/((1+r)^wm2d[i]))*mNPV2d[i,1]*1/(1-
1/((1+r)^wm2d[i])),CR[i,Tmax]-C[i,Tmax] + max(0,(CRe[i,g/delta.t+1]-CRe[i,Tmax])-(C[i,g/delta.t+1]-C[i,Tmax])))
iNPV3d[i,1]<-ifelse(mNPV3d[i,1]>CR[i,Tmax]-C[i,Tmax],mNPV3d[i,1]+(1/((1+r)^wm3d[i]))*mNPV3d[i,1]*1/(1-
1/((1+r)^wm3d[i])),CR[i,Tmax]-C[i,Tmax] + max(0,(CRe[i,g/delta.t+1]-CRe[i,Tmax])-(C[i,g/delta.t+1]-C[i,Tmax])))

}
```

```
ivalue1r<-sum(iNPV1r,na.rm=TRUE)/n
ivalue2r<-sum(iNPV2r,na.rm=TRUE)/n
ivalue3r<-sum(iNPV3r,na.rm=TRUE)/n

ivalue1d<-sum(iNPV1d,na.rm=TRUE)/n
ivalue2d<-sum(iNPV2d,na.rm=TRUE)/n
ivalue3d<-sum(iNPV3d,na.rm=TRUE)/n


#Proportion that choose to harvest in T<max

j1r<-1-length(subset(wm1r,wm1r[]>=Tmax))/n
j2r<-1-length(subset(wm2r,wm2r[]>=Tmax))/n
j3r<-1-length(subset(wm3r,wm3r[]>=Tmax))/n

j1d<-1-length(subset(wm1d,wm1d[]>=Tmax))/n
j2d<-1-length(subset(wm2d,wm2d[]>=Tmax))/n
j3d<-1-length(subset(wm3d,wm3d[]>=Tmax))/n

#Proportion that choose to harvest

k1r<-length(subset(mNPV1r[,1],mNPV1r[,1]>0))/n
k2r<-length(subset(mNPV2r[,1],mNPV2r[,1]>0))/n
k3r<-length(subset(mNPV3r[,1],mNPV3r[,1]>0))/n

k1d<-length(subset(mNPV1d[,1],mNPV1d[,1]>CR[,Tmax]-C[,Tmax]))/n
k2d<-length(subset(mNPV2d[,1],mNPV2d[,1]>CR[,Tmax]-C[,Tmax]))/n
k3d<-length(subset(mNPV3d[,1],mNPV3d[,1]>CR[,Tmax]-C[,Tmax]))/n


#Proportion that abandon the forest and decide to not sequester carbon after Tmax

l1r<-length(subset(iNPV1r,iNPV1r[,1]==0))/n
l2r<-length(subset(iNPV2r,iNPV2r[,1]==0))/n
l3r<-length(subset(iNPV3r,iNPV3r[,1]==0))/n

l1d<-length(subset(iNPV1d,iNPV1d[,1]==CR[,Tmax]-C[,Tmax]))/n
l2d<-length(subset(iNPV2d,iNPV2d[,1]==CR[,Tmax]-C[,Tmax]))/n
l3d<-length(subset(iNPV3d,iNPV3d[,1]==CR[,Tmax]-C[,Tmax]))/n



c(value1r,value2r,value3r,value1d,value2d,value3d,ivalue1r,ivalue2r,ivalue3r,ivalue1d,ivalue2d,ivalue3d,j1r,j2r,j3r,j1d,j2d,j3
d,k1r,k2r,k3r,k1d,k2d,k3d,l1r,l2r,l3r,l1d,l2d,l3d,avt1r,avt2r,avt3r,avt1d,avt2d,avt3d)

}

### The Bootstrapping Real Options Analysis
```

```
alfa <- seq(0,100000,1000) # alfa parameters

PCmu <- log(13.44427)#log(c(5,13.44427,25,50,100))# carbon prices


n<- 5000

########


ALFA <- matrix(0,length(alfa),36)
tm<-matrix(0,1,36)
AMaximum<-matrix(0,length(PCmu),36)
aMaximum<-matrix(0,length(PCmu),36)
prob1<-matrix(0,length(PCmu),36)
prob2<-matrix(0,length(PCmu),36)
prob3<-matrix(0,length(PCmu),36)
tstar<-matrix(0,length(PCmu),36)

for(j in 1:length(PCmu)){


for (i in 1:length(alfa)){

ALFA[i,]<-OU.broa(n,alfa[i],PCmu[j])

}

#Maximum value choice

for(i in 1:36){
tm[1,i] <- which.max(ALFA[,i])
AMaximum[j,i]<-ALFA[tm[1,i],i]
aMaximum[j,i]<-alfa[tm[1,i]]
}

for(i in 13:18){

prob1[j,i]<-ALFA[tm[1,i-12],i]

}

for(i in 19:24){

prob2[j,i]<-ALFA[tm[1,i-18],i]

}

for(i in 25:30){
```

```
prob3[j,i]<-ALFA[tm[1,i-24],i]

}

for(i in 31:36){

tstar[j,i]<-ALFA[tm[1,i-30],i]

}

}

NPV<-AMaximum[,1:12]
ALFAopt<-aMaximum[,1:12]

p1<-prob1[,13:18]
p2<-prob2[,19:24]
p3<-prob3[,25:30]
tstar<-tstar[,31:36]
```

Appendix E.2 – BROA Code using “R”: Conversion Model (Pre-1990)

```
rm(list=ls(all=TRUE))

data<-read.csv("data.csv")

OU.broa<-function(n,alfa=1,beta=1, CC=10000, PCmu=13.44427, LVmu=11333.27, h=45){

########Objective

##########Develop pre-1990 forest model with conversion


# Stochastic process:

#OU with trend and deterministic cost function

# Ornstein - Uhlenbeck without trend Parameters (risk neutral exact formula)
# advantage: we can use big values for delta.t
# disadvantage: we need delta.t > -log(2*kappa)/2*kappa (that is we cannot use kappa smaller than 0.3 for delta.t=1)

# Note load package "mnormt" to run normal multivariate random sample
# Use function 'rmnorm(n = 1, mean = rep(0, d), varcov)' to generate random mnormal series

## Montecarlo Parameters

g<- 60
delta.t<-1


## Bootstrapping Real Options Analysis Parameter

Tmin<-2
Tmax<-17

Tmin <- Tmin
Tmax <- Tmax

###### Model Parameters

r <- 0.05

sigma1 <-sqrt(0.011657)
sigma2 <-sqrt(0.031556)
sigma3 <-sqrt(0.016532)
sigma4 <-sqrt(0.043397)
sigma5 <-sqrt(0.029713)
sigma6 <-sqrt(0.032138)
```

```
sigmaC <-sqrt(0.1596643)
sigmaLV <-sqrt(0.0760058)

lambda1 <- r+0.025
lambda2 <- r+0.025
lambda3 <- r+0.025
lambda4 <- r+0.025
lambda5 <- r+0.025
lambda6 <- r+0.025
lambdaC <- r+0.025
lambdaLV <- r+0.025

kappa1 <- 0.3
kappa2 <- 0.388956
kappa3 <- 0.300546
kappa4 <- 1.057335
kappa5 <- 0.567348
kappa6 <- 0.858477
kappaC <- 0.9549317
kappaLV <- 0.3


P10 <- 4.931171
P20 <- 4.685213
P30 <- 4.435271
P40 <- 4.289317
P50 <- 4.199705
P60 <- 3.89097
PC0 <- 3.084598905
LV0 <- 9.897421

P1mu <- 5.31734
P2mu <- 5.055295
P3mu <- 4.737019
P4mu <- 4.412626
P5mu <- 4.436973
P6mu <- 3.964214
PCmu <- 2.598553
LVmu <- 9.335498

varcov <- diag(1,8,8)

varcov[1,2]<-0.932138 #corr(1,2)
varcov[1,3]<-0.883531 #corr(1,3)
varcov[1,4]<-0.718127 #corr(1,4)
varcov[1,5]<-0.829232 #corr(1,5)
varcov[1,6]<-0.585644 #corr(1,6)
varcov[1,7]<- -0.72651 #corr(1,C)
varcov[1,8]<- -0.45611 #corr(1,LV)
varcov[2,3]<-0.86808 #corr(2,3)
```

```
varcov[2,4]<-0.677265 #corr(2,4)
varcov[2,5]<-0.815388
varcov[2,6]<-0.522891
varcov[2,7]<- -0.71127
varcov[2,8]<- -0.37441
varcov[3,4]<-0.840848
varcov[3,5]<-0.868466
varcov[3,6]<-0.672502
varcov[3,7]<- -0.21608
varcov[3,8]<- -0.32254
varcov[4,5]<-0.740378
varcov[4,6]<-0.704376
varcov[4,7]<-0.522423
varcov[4,8]<- -0.30201
varcov[5,6]<-0.692611
varcov[5,7]<-0.247694
varcov[5,8]<- -0.19653
varcov[6,7]<-0.157482
varcov[6,8]<- -0.15357
varcov[7,8]<-0

for (i in 1:ncol(varcov)){
for (j in 1:nrow(varcov)){
varcov[i,j]<-varcov[j,i]
}
}

Y1<-as.double(data[,5])

Y2<-as.double(data[,6])

Y3<-as.double(data[,7])

Y4<-as.double(data[,8])

Y5<-as.double(data[,9])

Y6<-as.double(data[,10])

S<-as.double(data[,4])

delta.S <-diff(S)
delta.S <- c(0,delta.S)


### Ongoing Cost

C1<-matrix(0,n,g/delta.t+1)

C<-matrix(0,n,g/delta.t+1-18)
```

```
cc<-as.double(data[,11])

Ch<- h*(Y1+Y2+Y3+Y4+Y5+Y6)## Harvest costs

t <- 0.3 # corporate tax


######### Price Process

P1<-matrix(0,n,g/delta.t)
P1<-cbind(P10,P1)
P2<-matrix(0,n,g/delta.t)
P2<-cbind(P20,P2)
P3<-matrix(0,n,g/delta.t)
P3<-cbind(P30,P3)
P4<-matrix(0,n,g/delta.t)
P4<-cbind(P40,P4)
P5<-matrix(0,n,g/delta.t)
P5<-cbind(P50,P5)
P6<-matrix(0,n,g/delta.t)
P6<-cbind(P60,P6)
PC<-matrix(0,n,g/delta.t)
PC<-cbind(PC0,PC)
LV<-matrix(0,n,g/delta.t)
LV<-cbind(LV0,LV)

for (i in seq(1,g/delta.t,1)){

mz <- rmnorm(n,mean=rep(0,ncol(varcov)),varcov)

P1[,i+1] <- P1[,i]*exp(-kappa1*delta.t)+ (P1mu -((lambda1-r)/kappa1))*(1-exp(-kappa1*delta.t))+sigma1*sqrt((1-exp(-
2*kappa1*delta.t))/(2*kappa1))*mz[,1]
P2[,i+1] <- P2[,i]*exp(-kappa2*delta.t)+ (P2mu -((lambda2-r)/kappa2))*(1-exp(-kappa2*delta.t))+sigma2*sqrt((1-exp(-
2*kappa2*delta.t))/(2*kappa2))*mz[,2]
P3[,i+1] <- P3[,i]*exp(-kappa3*delta.t)+ (P3mu -((lambda3-r)/kappa3))*(1-exp(-kappa3*delta.t))+sigma3*sqrt((1-exp(-
2*kappa3*delta.t))/(2*kappa3))*mz[,3]
P4[,i+1] <- P4[,i]*exp(-kappa4*delta.t)+ (P4mu -((lambda4-r)/kappa4))*(1-exp(-kappa4*delta.t))+sigma4*sqrt((1-exp(-
2*kappa4*delta.t))/(2*kappa4))*mz[,4]
P5[,i+1] <- P5[,i]*exp(-kappa5*delta.t)+ (P5mu -((lambda5-r)/kappa5))*(1-exp(-kappa5*delta.t))+sigma5*sqrt((1-exp(-
2*kappa5*delta.t))/(2*kappa5))*mz[,5]
P6[,i+1] <- P6[,i]*exp(-kappa6*delta.t)+ (P6mu -((lambda6-r)/kappa6))*(1-exp(-kappa6*delta.t))+sigma6*sqrt((1-exp(-
2*kappa6*delta.t))/(2*kappa6))*mz[,6]
PC[,i+1] <- PC[,i]*exp(-kappaC*delta.t)+ (PCmu -((lambdaC-r)/kappaC))*(1-exp(-kappaC*delta.t))+sigmaC*sqrt((1-exp(-
2*kappaC*delta.t))/(2*kappaC))*mz[,7]

LV[,i+1] <- LV[,i]*exp(-kappaLV*delta.t)+ (LVmu -((lambdaLV-r)/kappaLV))*(1-exp(-kappaLV*delta.t))+sigmaLV*sqrt((1-
exp(-2*kappaLV*delta.t))/(2*kappaLV))*mz[,8]

}
```

```
##### Harvest returns

#### Since the beginning of the forest

R1<-matrix(0,n,g/delta.t+1)


for(i in 1:n){

R1[i,]<-exp(P1[i,])*Y1+exp(P2[i,])*Y2+exp(P3[i,])*Y3+exp(P4[i,])*Y4+exp(P5[i,])*Y5+exp(P6[i,])*Y6

}


##### From the 18th year


R<-matrix(0,n,g/delta.t+1-18)

for(i in 1:n){

R[i,]<-exp(P1[i,1:(g/delta.t+1-18)])*Y1[18:g/delta.t+1]+exp(P2[i,1:(g/delta.t+1-
18)])*Y2[18:g/delta.t+1]+exp(P3[i,1:(g/delta.t+1-18)])*Y3[18:g/delta.t+1]+exp(P4[i,1:(g/delta.t+1-
18)])*Y4[18:g/delta.t+1]+exp(P5[i,1:(g/delta.t+1-18)])*Y5[18:g/delta.t+1]+exp(P6[i,1:(g/delta.t+1-18)])*Y6[18:g/delta.t+1]

}


#Present value vector of ongoing variables

#On going Costs



for(i in 1:n){

   C1[i,]<-cc
   C[i,]<-cc[18:g/delta.t+1]

}


### From the beginning


for (j in seq(1,g/delta.t+1,1)){

   C1[,j]<-C1[,j]*(1-t)*exp(-r*(j-1)*delta.t)
```

```
}

### from the 18th year

for (j in seq(1,g/delta.t+1-18,1)){

   C[,j]<-C[,j]*(1-t)*exp(-r*(j-1)*delta.t)

}

for(i in 1:n){

   C1[i,]<-cumsum(C1[i,])
   C[i,]<-cumsum(C[i,])

}

########## Net harvest Revenue (without ongoing costs)

Hr<-matrix(0,n,g/delta.t+1-18)

for(i in 1:n){

Hr[i,] <- R[i,] - Ch[18:g/delta.t+1] # Net harvest Revenue (without ongoing costs)

}

Rd<-Hr #not considering yet ongoing costs and carbon revenues - considered later in the NPV calculation

############################################## Barrier and Trade off Functions
########################################

# Function B1 -> "value to wait" proxy function(square root)

B1<-matrix(0,1,length(P1[1,]))

for (i in seq(1,Tmax,1)){

B1[1,i] <- sqrt(Tmax-i)/sqrt(Tmax-Tmin)
}

# Function B2 -> trade off function between HRt and HCt (linear)
```

```
B2<-matrix(0,1,length(P1[1,]))

B2<-rep(1,length(P1[1,]))

#######################                    Present                    Value
Calculation########################################################

wm1d<-matrix(g/delta.t+1,n,1)

CPV1d<-matrix(0,n,g/delta.t+1-18)
Conv<-matrix(0,n,g/delta.t+1-18)


NPV1d<-matrix(0,n,1)
NPV2d<-matrix(0,n,1)


mNPV1d<-matrix(0,n,1)
mNPV2d<-matrix(0,n,1)

iNPV1d<-matrix(0,n,1)
iNPV2d<-matrix(0,n,1)


##### Action Outcomes


# Post tax Harvest net revenue

for (j in seq(1,g/delta.t+1-18,1)){

  CPV1d[,j]<-Rd[,j]*(1-t)*exp(-r*(j-1)*delta.t)-C[,j]# harvest decision

}


# Conversion Strategy

for (j in seq(1,g/delta.t+1-18,1)){

  Conv[,j]<-(exp(LV[,j])-PC[,j]*S[j+18]-CC)*exp(-r*(j-1)*delta.t)

}


##  Replant Strategy


###### replant and Harvest (t*+27)
```

```
Re<-matrix(0,n,(g/delta.t+1-18))

P1e<-matrix(0,n,(g/delta.t+1-18))

P2e<-matrix(0,n,(g/delta.t+1-18))

P3e<-matrix(0,n,(g/delta.t+1-18))

P4e<-matrix(0,n,(g/delta.t+1-18))

P5e<-matrix(0,n,(g/delta.t+1-18))

P6e<-matrix(0,n,(g/delta.t+1-18))

PCe<-matrix(0,n,(g/delta.t+1-18))

LVe<-matrix(0,n,(g/delta.t+1-18))

Rep<-matrix(0,n,(g/delta.t+1-18))


### Revenues

for (i in 1:n){

   P1e[i,]<-P1[i,1:(g/delta.t+1-18)]*exp(-kappa1*27)+P1mu*(1-exp(-kappa1*27)) # OU expectation formula

   P2e[i,]<-P2[i,1:(g/delta.t+1-18)]*exp(-kappa2*27)+P2mu*(1-exp(-kappa2*27))

   P3e[i,]<-P3[i,1:(g/delta.t+1-18)]*exp(-kappa3*27)+P3mu*(1-exp(-kappa3*27))

   P4e[i,]<-P4[i,1:(g/delta.t+1-18)]*exp(-kappa4*27)+P4mu*(1-exp(-kappa4*27))

   P5e[i,]<-P5[i,1:(g/delta.t+1-18)]*exp(-kappa5*27)+P5mu*(1-exp(-kappa5*27))

   P6e[i,]<-P6[i,1:(g/delta.t+1-18)]*exp(-kappa6*27)+P6mu*(1-exp(-kappa6*27))

   PCe[i,]<-PC[i,1:(g/delta.t+1-18)]*exp(-kappaC*27)+PCmu*(1-exp(-kappaC*27))

   LVe[i,]<-LV[i,1:(g/delta.t+1-18)]*exp(-kappaLV*27)+LVmu*(1-exp(-kappaLV*27))


   Re[i,]<-
exp(P1e[i,])*Y1[28]+exp(P2e[i,])*Y2[28]+exp(P3e[i,])*Y3[28]+exp(P4e[i,])*Y4[28]+exp(P5e[i,])*Y5[28]+exp(P6e[i,])*Y6[28]

   Rep[i,] <- (Re[i,]-h*(Y1[28]+Y2[28]+Y3[28]+Y4[28]+Y5[28]+Y6[28])-C1[i,28])/(1-1/((1+r)^27))*exp(-r*27)

}
```

```
# Coversion in t*+27

HC2star<-matrix(0,n,(g/delta.t+1-18))
Z2star<-matrix(0,n,(g/delta.t+1-18))

for (i in 1:n){

HC2star[i,]<-(Rep[i,]*(1-1/((1+r)^27))+(LV[i,28]-PC[i,28]*S[28]-CC))*exp(-r*27)

}

for (j in 1:(g/delta.t+1-18)){

HC2star[i,]<-HC2star[i,]*exp(-r*(j-1)*delta.t)

}

for (i in 1:n){

Z2star[i,] <-max(Rep[i,],HC2star[i,],0)

}

################### Decision Rules (until t*)


H<-CPV1d
C<-Conv
R<-Z2star


HC<-H+C
HR<-H+R


for (i in seq(1,n,1)){

x1d <- which(HR[i,]>=alfa*B1-HC[i,]*alfa*B1/(beta*B2+0.0000001))

for (j in 1:length(x1d)){
x1d[j]<-ifelse(x1d[j]<Tmin|x1d[j]>Tmax|is.na(x1d[j]),Tmax,x1d[j])
}

wm1d[i,1]<- ifelse(is.integer(min(x1d))|is.double(min(x1d)),min(x1d),Tmax)

}

for (i in seq(1,n,1)){

  NPV1d[i,1]<- ifelse(HR[i,wm1d[i,1]]>HC[i,wm1d[i,1]],HR[i,wm1d[i,1]],0)
```

```
    NPV2d[i,1]<- ifelse(HR[i,wm1d[i,1]]<HC[i,wm1d[i,1]],HC[i,wm1d[i,1]],0)
    mNPV1d[i,1]<- ifelse(NPV1d[i,1]+NPV2d[i,1]<H[i,wm1d[i,1]],H[i,wm1d[i,1]],NPV1d[i,1]+NPV2d[i,1])
    mNPV1d[i,1]<- ifelse(mNPV1d[i,1]<0,0,mNPV1d[i,1])
  }


value1d<-mean(mNPV1d,na.rm=TRUE)


# Average t*

avt<-mean(wm1d,na.rm=TRUE)

#Proportion of Harvest and Replant

j1d<-length(subset(mNPV1d[,1],mNPV1d[,1]==NPV1d[,1]))/n

#Proportion of Harvest and Conversion in Dairy

k1d<-length(subset(mNPV1d[,1],mNPV1d[,1]==NPV2d[,1]))/n

#Proportion that Harvest and Abandon the forest

l1d<-length(subset(mNPV1d[i,1],mNPV1d[i,1]==H))/n

#Proportion that Abandon the forest

m1d<-length(subset(mNPV1d[i,1],mNPV1d[i,1]==0))/n

c(value1d,j1d,k1d,l1d,m1d,avt)

}

### Bootstrapping Real Options Analysis, Monte Carlo Simulation

alfa <- seq(0,100000,5000) # alfa parameters
beta <- seq(0,100000,5000) # beta parameters
PCmu <- 13.33#c(0, 25, 50, 75, 100)#Carbon Prices
LVmu <- c(10000,20000,50000) #Land Values

n<- 10

########


PAR <- matrix(0,1,10)

for (l in 1:length(LVmu)){

for (k in 1:length(PCmu)){
```

```
for(j in 1:length(beta)){

for (i in 1:length(alfa)){

PAR<-rbind(PAR,c(LVmu[l],PCmu[k],beta[j],alfa[i],OU.broa(n,alfa[i],beta[j],0,PCmu[k], LVmu[l])))

}
}
}
}

PAR<-PAR[-1,]

#Maximum values choices

carb <- 13.33
land <- 20000

X<-which(PAR[,1]==land&PAR[,2]==carb)
x<-which.max(PAR[X,5])
MAX<-PAR[x+min(X)-1,]
```

Appendix E.3 – BROA Code Using “R”: Conversion Model (Pre-1990 with Alternative Decision Map)

```
rm(list=ls(all=TRUE))

data<-read.csv("data.csv")

OU.broa<-function(n,alfa=1,beta=1, CC=10000, PCmu=13.44427, LVmu=11333.27, h=45){

########Objective

##########Develop Pre-1990 forest model with Conversion (Alternative map)


# Stochastic process:

#OU with trend and deterministic cost function

# Ornstein - Uhlenbeck without trend Parameters (risk neutral exact formula)
# advantage: we can use big values for delta.t
# disadvantage: we need delta.t > -log(2*kappa)/2*kappa (that is we cannot use kappa smaller than 0.3 for delta.t=1)

# Note load package "mnormt" to run normal multivariate random sample
# Use function 'rmnorm(n = 1, mean = rep(0, d), varcov)' to generate random mnormal series

## Montecarlo Parameters

g<- 60
delta.t<-1


## Bootstrapping Real Options Analysis Parameter

Tmin<-2
Tmax<-17

Tmin <- Tmin
Tmax <- Tmax

###### Model Parameters

r <- 0.05

sigma1 <-sqrt(0.011657)
sigma2 <-sqrt(0.031556)
sigma3 <-sqrt(0.016532)
sigma4 <-sqrt(0.043397)
sigma5 <-sqrt(0.029713)
sigma6 <-sqrt(0.032138)
```

```
sigmaC <-sqrt(0.1596643)
sigmaLV <-sqrt(0.0760058)

lambda1 <- r+0.025
lambda2 <- r+0.025
lambda3 <- r+0.025
lambda4 <- r+0.025
lambda5 <- r+0.025
lambda6 <- r+0.025
lambdaC <- r+0.025
lambdaLV <- r+0.025

kappa1 <- 0.3
kappa2 <- 0.388956
kappa3 <- 0.300546
kappa4 <- 1.057335
kappa5 <- 0.567348
kappa6 <- 0.858477
kappaC <- 0.9549317
kappaLV <- 0.3


P10 <- 4.931171
P20 <- 4.685213
P30 <- 4.435271
P40 <- 4.289317
P50 <- 4.199705
P60 <- 3.89097
PC0 <- 3.084598905
LV0 <- 9.897421

P1mu <- 5.31734
P2mu <- 5.055295
P3mu <- 4.737019
P4mu <- 4.412626
P5mu <- 4.436973
P6mu <- 3.964214
PCmu <- log(PCmu)
LVmu <- log(LVmu)

varcov <- diag(1,8,8)

varcov[1,2]<-0.932138 #corr(1,2)
varcov[1,3]<-0.883531 #corr(1,3)
varcov[1,4]<-0.718127 #corr(1,4)
varcov[1,5]<-0.829232 #corr(1,5)
varcov[1,6]<-0.585644 #corr(1,6)
varcov[1,7]<- -0.72651 #corr(1,C)
varcov[1,8]<- -0.45611 #corr(1,LV)
varcov[2,3]<-0.86808 #corr(2,3)
```

```
varcov[2,4]<-0.677265 #corr(2,4)
varcov[2,5]<-0.815388
varcov[2,6]<-0.522891
varcov[2,7]<- -0.71127
varcov[2,8]<- -0.37441
varcov[3,4]<-0.840848
varcov[3,5]<-0.868466
varcov[3,6]<-0.672502
varcov[3,7]<- -0.21608
varcov[3,8]<- -0.32254
varcov[4,5]<-0.740378
varcov[4,6]<-0.704376
varcov[4,7]<-0.522423
varcov[4,8]<- -0.30201
varcov[5,6]<-0.692611
varcov[5,7]<-0.247694
varcov[5,8]<- -0.19653
varcov[6,7]<-0.157482
varcov[6,8]<- -0.15357
varcov[7,8]<-0

for (i in 1:ncol(varcov)){
for (j in 1:nrow(varcov)){
varcov[i,j]<-varcov[j,i]
}
}

Y1<-as.double(data[,5])

Y2<-as.double(data[,6])

Y3<-as.double(data[,7])

Y4<-as.double(data[,8])

Y5<-as.double(data[,9])

Y6<-as.double(data[,10])

S<-as.double(data[,4])

delta.S <-diff(S)
delta.S <- c(0,delta.S)


### Ongoing Cost

C1<-matrix(0,n,g/delta.t+1)

C<-matrix(0,n,g/delta.t+1-18)
```

```
cc<-as.double(data[,11])

Ch<- h*(Y1+Y2+Y3+Y4+Y5+Y6)## Harvest costs

t <- 0.3 # corporate tax


######### Price Processes

P1<-matrix(0,n,g/delta.t)
P1<-cbind(P10,P1)
P2<-matrix(0,n,g/delta.t)
P2<-cbind(P20,P2)
P3<-matrix(0,n,g/delta.t)
P3<-cbind(P30,P3)
P4<-matrix(0,n,g/delta.t)
P4<-cbind(P40,P4)
P5<-matrix(0,n,g/delta.t)
P5<-cbind(P50,P5)
P6<-matrix(0,n,g/delta.t)
P6<-cbind(P60,P6)
PC<-matrix(0,n,g/delta.t)
PC<-cbind(PC0,PC)
LV<-matrix(0,n,g/delta.t)
LV<-cbind(LV0,LV)

for (i in seq(1,g/delta.t,1)){

mz <- rmnorm(n,mean=rep(0,ncol(varcov)),varcov)

P1[,i+1] <- P1[,i]*exp(-kappa1*delta.t)+ (P1mu -((lambda1-r)/kappa1))*(1-exp(-kappa1*delta.t))+sigma1*sqrt((1-exp(-
2*kappa1*delta.t))/(2*kappa1))*mz[,1]
P2[,i+1] <- P2[,i]*exp(-kappa2*delta.t)+ (P2mu -((lambda2-r)/kappa2))*(1-exp(-kappa2*delta.t))+sigma2*sqrt((1-exp(-
2*kappa2*delta.t))/(2*kappa2))*mz[,2]
P3[,i+1] <- P3[,i]*exp(-kappa3*delta.t)+ (P3mu -((lambda3-r)/kappa3))*(1-exp(-kappa3*delta.t))+sigma3*sqrt((1-exp(-
2*kappa3*delta.t))/(2*kappa3))*mz[,3]
P4[,i+1] <- P4[,i]*exp(-kappa4*delta.t)+ (P4mu -((lambda4-r)/kappa4))*(1-exp(-kappa4*delta.t))+sigma4*sqrt((1-exp(-
2*kappa4*delta.t))/(2*kappa4))*mz[,4]
P5[,i+1] <- P5[,i]*exp(-kappa5*delta.t)+ (P5mu -((lambda5-r)/kappa5))*(1-exp(-kappa5*delta.t))+sigma5*sqrt((1-exp(-
2*kappa5*delta.t))/(2*kappa5))*mz[,5]
P6[,i+1] <- P6[,i]*exp(-kappa6*delta.t)+ (P6mu -((lambda6-r)/kappa6))*(1-exp(-kappa6*delta.t))+sigma6*sqrt((1-exp(-
2*kappa6*delta.t))/(2*kappa6))*mz[,6]
PC[,i+1] <- PC[,i]*exp(-kappaC*delta.t)+ (PCmu -((lambdaC-r)/kappaC))*(1-exp(-kappaC*delta.t))+sigmaC*sqrt((1-exp(-
2*kappaC*delta.t))/(2*kappaC))*mz[,7]

LV[,i+1] <- LV[,i]*exp(-kappaLV*delta.t)+ (LVmu -((lambdaLV-r)/kappaLV))*(1-exp(-kappaLV*delta.t))+sigmaLV*sqrt((1-
exp(-2*kappaLV*delta.t))/(2*kappaLV))*mz[,8]
}
```

```
##### Harvest returns

#### Since the beginning of the forest

R1<-matrix(0,n,g/delta.t+1)


for(i in 1:n){

R1[i,]<-exp(P1[i,])*Y1+exp(P2[i,])*Y2+exp(P3[i,])*Y3+exp(P4[i,])*Y4+exp(P5[i,])*Y5+exp(P6[i,])*Y6

}


##### From the 18th year


R<-matrix(0,n,g/delta.t+1-18)

for(i in 1:n){

R[i,]<-exp(P1[i,1:(g/delta.t+1-18)])*Y1[18:g/delta.t+1]+exp(P2[i,1:(g/delta.t+1-
18)])*Y2[18:g/delta.t+1]+exp(P3[i,1:(g/delta.t+1-18)])*Y3[18:g/delta.t+1]+exp(P4[i,1:(g/delta.t+1-
18)])*Y4[18:g/delta.t+1]+exp(P5[i,1:(g/delta.t+1-18)])*Y5[18:g/delta.t+1]+exp(P6[i,1:(g/delta.t+1-18)])*Y6[18:g/delta.t+1]

}


#Present value vector of ongoing variables

#On going Costs



for(i in 1:n){

  C1[i,]<-cc
  C[i,]<-cc[18:g/delta.t+1]

}


### From the beginning


for (j in seq(1,g/delta.t+1,1)){

  C1[,j]<-C1[,j]*(1-t)*exp(-r*(j-1)*delta.t)
```

```
}

### from the 18th year

for (j in seq(1,g/delta.t+1-18,1)){

   C[,j]<-C[,j]*(1-t)*exp(-r*(j-1)*delta.t)

}

for(i in 1:n){

   C1[i,]<-cumsum(C1[i,])
   C[i,]<-cumsum(C[i,])

}

########## Net harvest Revenue (without ongoing costs)

Hr<-matrix(0,n,g/delta.t+1-18)

for(i in 1:n){

Hr[i,] <- R[i,] - Ch[18:g/delta.t+1] # Net harvest Revenue (without ongoing costs)

}

Rd<-Hr #not considering yet ongoing costs and carbon revenues - considered later in the NPV calculation

##############################################     Barrier     and     Trade     off     Functions
##########################################

# Function B1 -> "value to wait" proxy function(square root)

B1<-matrix(0,1,length(P1[1,]))

for (i in seq(Tmin,Tmax,1)){

B1[1,i] <- sqrt(Tmax-i)/sqrt(Tmax-Tmin)
}

# Function B2 -> trade off function between HRt and HCt (linear)

B2<-matrix(0,1,length(P1[1,]))
```

```
B2<-rep(1,length(P1[1,]))

######################## Present Value
Calculation##########################################################

wm1d1<-matrix(g/delta.t+1,n,1)
wm1d2<-matrix(g/delta.t+1,n,1)
wm1d3<-matrix(g/delta.t+1,n,1)
wm1d<-matrix(g/delta.t+1,n,1)

CPV1d<-matrix(0,n,g/delta.t+1-18)
Conv<-matrix(0,n,g/delta.t+1-18)


NPV1d<-matrix(0,n,1)
NPV2d<-matrix(0,n,1)


mNPV1d<-matrix(0,n,1)
mNPV2d<-matrix(0,n,1)

iNPV1d<-matrix(0,n,1)
iNPV2d<-matrix(0,n,1)


##### Action Outcomes


# Post tax Harvest net revenue

for (j in seq(1,g/delta.t+1-18,1)){

  CPV1d[,j]<-Rd[,j]*(1-t)*exp(-r*(j-1)*delta.t)-C[,j]# harvest decision

}


# Conversion Strategy

for (j in seq(1,g/delta.t+1-18,1)){

  Conv[,j]<-(exp(LV[,j])-PC[,j]*S[j+18]-CC)*exp(-r*(j-1)*delta.t)

}


##  Replant Strategy
```

```
###### replant and Harvest (t*+27)

Re<-matrix(0,n,(g/delta.t+1-18))

P1e<-matrix(0,n,(g/delta.t+1-18))

P2e<-matrix(0,n,(g/delta.t+1-18))

P3e<-matrix(0,n,(g/delta.t+1-18))

P4e<-matrix(0,n,(g/delta.t+1-18))

P5e<-matrix(0,n,(g/delta.t+1-18))

P6e<-matrix(0,n,(g/delta.t+1-18))

PCe<-matrix(0,n,(g/delta.t+1-18))

LVe<-matrix(0,n,(g/delta.t+1-18))

Rep<-matrix(0,n,(g/delta.t+1-18))


### Revenues

for (i in 1:n){

  P1e[i,]<-P1[i,1:(g/delta.t+1-18)]*exp(-kappa1*27)+P1mu*(1-exp(-kappa1*27)) # OU expectation formula

  P2e[i,]<-P2[i,1:(g/delta.t+1-18)]*exp(-kappa2*27)+P2mu*(1-exp(-kappa2*27))

  P3e[i,]<-P3[i,1:(g/delta.t+1-18)]*exp(-kappa3*27)+P3mu*(1-exp(-kappa3*27))

  P4e[i,]<-P4[i,1:(g/delta.t+1-18)]*exp(-kappa4*27)+P4mu*(1-exp(-kappa4*27))

  P5e[i,]<-P5[i,1:(g/delta.t+1-18)]*exp(-kappa5*27)+P5mu*(1-exp(-kappa5*27))

  P6e[i,]<-P6[i,1:(g/delta.t+1-18)]*exp(-kappa6*27)+P6mu*(1-exp(-kappa6*27))

  PCe[i,]<-PC[i,1:(g/delta.t+1-18)]*exp(-kappaC*27)+PCmu*(1-exp(-kappaC*27))

  LVe[i,]<-LV[i,1:(g/delta.t+1-18)]*exp(-kappaLV*27)+LVmu*(1-exp(-kappaLV*27))


  Re[i,]<-
exp(P1e[i,])*Y1[28]+exp(P2e[i,])*Y2[28]+exp(P3e[i,])*Y3[28]+exp(P4e[i,])*Y4[28]+exp(P5e[i,])*Y5[28]+exp(P6e[i,])*Y6[28]

  Rep[i,] <- (Re[i,]-h*(Y1[28]+Y2[28]+Y3[28]+Y4[28]+Y5[28]+Y6[28])-C1[i,28])/(1-1/((1+r)^27))*exp(-r*27)

}
```

```
# Coversion in t*+27

HC2star<-matrix(0,n,(g/delta.t+1-18))
Z2star<-matrix(0,n,(g/delta.t+1-18))

for (i in 1:n){

HC2star[i,]<-(Rep[i,]*(1-1/((1+r)^27))+(LV[i,28]-PC[i,28]*S[28]-CC))*exp(-r*27)

}

for (j in 1:(g/delta.t+1-18)){

HC2star[i,]<-HC2star[i,]*exp(-r*(j-1)*delta.t)

}

for (i in 1:n){

Z2star[i,] <-max(Rep[i,],HC2star[i,],0)

}

################## Decision Rules (until t*)


H<-CPV1d
C<-Conv
R<-Z2star


HC<-H+C
HR<-H+R

### HRa

for (i in seq(1,n,1)){

x1d1<-which((HR[i,]>=alfa*B1))

for (j in 1:length(x1d1)){
x1d1[j]<-ifelse(x1d1[j]<Tmin|x1d1[j]>Tmax|is.na(x1d1[j]),Tmax,x1d1[j])
}

wm1d1[i,1]<- ifelse(is.integer(min(x1d1))|is.double(min(x1d1)),min(x1d1),Tmax)

}
```

```
### HRb or HCb

for (i in seq(1,n,1)){

x1d2<-which(((HC[i,]>=0)&(HR[i,]>=alfa*B1-HC[i,]*alfa*B1/(beta*B2+0.1))))

for (j in 1:length(x1d2)){
x1d2[j]<-ifelse(x1d2[j]<Tmin|x1d2[j]>Tmax|is.na(x1d2[j]),Tmax,x1d2[j])
}

wm1d2[i,1]<- ifelse(is.integer(min(x1d2))|is.double(min(x1d2)),min(x1d2),Tmax)

}

### HCa

for (i in seq(1,n,1)){

x1d3<-which(HC[i,]>=beta)

for (j in 1:length(x1d3)){
x1d3[j]<-ifelse(x1d3[j]<Tmin|x1d3[j]>Tmax|is.na(x1d3[j]),Tmax,x1d3[j])
}

wm1d3[i,1]<- ifelse(is.integer(min(x1d3))|is.double(min(x1d3)),min(x1d3),Tmax)

}

wm1d<-pmin(wm1d1,wm1d2,wm1d3)


for (i in seq(1,n,1)){

   NPV1d[i,1]<- ifelse(HR[i,wm1d[i,1]]>HC[i,wm1d[i,1]],HR[i,wm1d[i,1]],0)
   NPV2d[i,1]<- ifelse(HR[i,wm1d[i,1]]<HC[i,wm1d[i,1]],HC[i,wm1d[i,1]],0)
   mNPV1d[i,1]<- ifelse(NPV1d[i,1]+NPV2d[i,1]<H[i,wm1d[i,1]],H[i,wm1d[i,1]],NPV1d[i,1]+NPV2d[i,1])
   mNPV1d[i,1]<- ifelse(mNPV1d[i,1]<0,0,mNPV1d[i,1])
  }



value1d<-mean(mNPV1d,na.rm=TRUE)


# Average t*

avt<-mean(wm1d,na.rm=TRUE)

#Proportion of Harvest and Replant
```

```
j1d<-length(subset(mNPV1d[,1],mNPV1d[,1]==NPV1d[,1]))/n

#Proportion of Harvest and Conversion in Dairy

k1d<-length(subset(mNPV1d[,1],mNPV1d[,1]==NPV2d[,1]))/n

#Proportion that Harvest and Abandon the forest

l1d<-length(subset(mNPV1d[i,1],mNPV1d[i,1]==H))/n

#Proportion that Abandon the forest

m1d<-length(subset(mNPV1d[i,1],mNPV1d[i,1]==0))/n

c(value1d,j1d,k1d,l1d,m1d,avt)

}

### Bootstrapping Real Options Analysis Monte Carlo Simulation

alfa <- seq(0,100000,5000) # alfa parameters
beta <- seq(0,100000,5000) # beta parameters
PCmu <- c(0, 25, 50, 75, 100)#Carbon Prices
LVmu <- c(10000,20000,50000) #Land Values

n<- 5

########


PAR <- matrix(0,1,10)

for (l in 1:length(LVmu)){

for (k in 1:length(PCmu)){

for(j in 1:length(beta)){

for (i in 1:length(alfa)){

PAR<-rbind(PAR,c(LVmu[l],PCmu[k],beta[j],alfa[i],OU.broa(n,alfa[i],beta[j],0,PCmu[k], LVmu[l])))

}
}
}
}

PAR<-PAR[-1,]
```

```
#Maximum values choices

carb <- 25
land <- 10000

X<-which(PAR[,1]==land&PAR[,2]==carb)
x<-which.max(PAR[X,5])
MAX<-PAR[x+min(X)-1,]
```

Appendix E.4 – BROA Code Using "R": Conversion Model (Pre-1990 with Conversion Delayed Eight Years)

```
rm(list=ls(all=TRUE))

data<-read.csv("data.csv")

OU.broa<-function(n,alfa=1,beta=1, CC=10000, PCmu=13.44427, LVmu=11333.27, h=45){


########Objective

##########Develop pre-1990 forest model with conversion (8 years between the conversion decision and operationalization of decision)


# Stochastic process:

#OU with trend and deterministic cost function

# Ornstein - Uhlenbeck without trend Parameters (risk neutral exact formula)
# advantage: we can use big values for delta.t
# disadvantage: we need delta.t > -log(2*kappa)/2*kappa (that is we cannot use kappa smaller than 0.3 for delta.t=1)

# Note load package "mnormt" to run normal multivariate random sample
# Use function 'rmnorm(n = 1, mean = rep(0, d), varcov)' to generate random mnormal series

## Montecarlo Parameters

g<- 60
delta.t<-1


## Bootstrapping Real Options Analysis Parameter

Tmin<-2
Tmax<-17

Tmin <- Tmin
Tmax <- Tmax

###### Model Parameters

r <- 0.05

sigma1 <-sqrt(0.011657)
sigma2 <-sqrt(0.031556)
sigma3 <-sqrt(0.016532)
sigma4 <-sqrt(0.043397)
```

```
sigma5 <-sqrt(0.029713)
sigma6 <-sqrt(0.032138)
sigmaC <-sqrt(0.1596643)
sigmaLV <-sqrt(0.0760058)

lambda1 <- r+0.025
lambda2 <- r+0.025
lambda3 <- r+0.025
lambda4 <- r+0.025
lambda5 <- r+0.025
lambda6 <- r+0.025
lambdaC <- r+0.025
lambdaLV <- r+0.025

kappa1 <- 0.3
kappa2 <- 0.388956
kappa3 <- 0.300546
kappa4 <- 1.057335
kappa5 <- 0.567348
kappa6 <- 0.858477
kappaC <- 0.9549317
kappaLV <- 0.3


P10 <- 4.931171
P20 <- 4.685213
P30 <- 4.435271
P40 <- 4.289317
P50 <- 4.199705
P60 <- 3.89097
PC0 <- 3.084598905
LV0 <- 9.897421

P1mu <- 5.31734
P2mu <- 5.055295
P3mu <- 4.737019
P4mu <- 4.412626
P5mu <- 4.436973
P6mu <- 3.964214
PCmu<-log(PCmu)
LVmu<-log(LVmu)



varcov <- diag(1,8,8)

varcov[1,2]<-0.932138 #corr(1,2)
varcov[1,3]<-0.883531 #corr(1,3)
varcov[1,4]<-0.718127 #corr(1,4)
varcov[1,5]<-0.829232 #corr(1,5)
```

```
varcov[1,6]<-0.585644 #corr(1,6)
varcov[1,7]<- -0.72651 #corr(1,C)
varcov[1,8]<- -0.45611 #corr(1,LV)
varcov[2,3]<-0.86808 #corr(2,3)
varcov[2,4]<-0.677265 #corr(2,4)
varcov[2,5]<-0.815388
varcov[2,6]<-0.522891
varcov[2,7]<- -0.71127
varcov[2,8]<- -0.37441
varcov[3,4]<-0.840848
varcov[3,5]<-0.868466
varcov[3,6]<-0.672502
varcov[3,7]<- -0.21608
varcov[3,8]<- -0.32254
varcov[4,5]<-0.740378
varcov[4,6]<-0.704376
varcov[4,7]<-0.522423
varcov[4,8]<- -0.30201
varcov[5,6]<-0.692611
varcov[5,7]<-0.247694
varcov[5,8]<- -0.19653
varcov[6,7]<-0.157482
varcov[6,8]<- -0.15357
varcov[7,8]<-0

for (i in 1:ncol(varcov)){
for (j in 1:nrow(varcov)){
varcov[i,j]<-varcov[j,i]
}
}

Y1<-as.double(data[,5])

Y2<-as.double(data[,6])

Y3<-as.double(data[,7])

Y4<-as.double(data[,8])

Y5<-as.double(data[,9])

Y6<-as.double(data[,10])

S<-as.double(data[,4])

delta.S <-diff(S)
delta.S <- c(0,delta.S)


### Ongoing Cost
```

```
C1<-matrix(0,n,g/delta.t+1)

C<-matrix(0,n,g/delta.t+1-18)

cc<-as.double(data[,11])

Ch<- h*(Y1+Y2+Y3+Y4+Y5+Y6)## Harvest costs

t <- 0.3 # corporate tax


######### Price Process

P1<-matrix(0,n,g/delta.t)
P1<-cbind(P10,P1)
P2<-matrix(0,n,g/delta.t)
P2<-cbind(P20,P2)
P3<-matrix(0,n,g/delta.t)
P3<-cbind(P30,P3)
P4<-matrix(0,n,g/delta.t)
P4<-cbind(P40,P4)
P5<-matrix(0,n,g/delta.t)
P5<-cbind(P50,P5)
P6<-matrix(0,n,g/delta.t)
P6<-cbind(P60,P6)
PC<-matrix(0,n,g/delta.t)
PC<-cbind(PC0,PC)
LV<-matrix(0,n,g/delta.t)
LV<-cbind(LV0,LV)

for (i in seq(1,g/delta.t,1)){

mz <- rmnorm(n,mean=rep(0,ncol(varcov)),varcov)

P1[,i+1] <- P1[,i]*exp(-kappa1*delta.t)+ (P1mu -((lambda1-r)/kappa1))*(1-exp(-kappa1*delta.t))+sigma1*sqrt((1-exp(-
2*kappa1*delta.t))/(2*kappa1))*mz[,1]
P2[,i+1] <- P2[,i]*exp(-kappa2*delta.t)+ (P2mu -((lambda2-r)/kappa2))*(1-exp(-kappa2*delta.t))+sigma2*sqrt((1-exp(-
2*kappa2*delta.t))/(2*kappa2))*mz[,2]
P3[,i+1] <- P3[,i]*exp(-kappa3*delta.t)+ (P3mu -((lambda3-r)/kappa3))*(1-exp(-kappa3*delta.t))+sigma3*sqrt((1-exp(-
2*kappa3*delta.t))/(2*kappa3))*mz[,3]
P4[,i+1] <- P4[,i]*exp(-kappa4*delta.t)+ (P4mu -((lambda4-r)/kappa4))*(1-exp(-kappa4*delta.t))+sigma4*sqrt((1-exp(-
2*kappa4*delta.t))/(2*kappa4))*mz[,4]
P5[,i+1] <- P5[,i]*exp(-kappa5*delta.t)+ (P5mu -((lambda5-r)/kappa5))*(1-exp(-kappa5*delta.t))+sigma5*sqrt((1-exp(-
2*kappa5*delta.t))/(2*kappa5))*mz[,5]
P6[,i+1] <- P6[,i]*exp(-kappa6*delta.t)+ (P6mu -((lambda6-r)/kappa6))*(1-exp(-kappa6*delta.t))+sigma6*sqrt((1-exp(-
2*kappa6*delta.t))/(2*kappa6))*mz[,6]
PC[,i+1] <- PC[,i]*exp(-kappaC*delta.t)+ (PCmu -((lambdaC-r)/kappaC))*(1-exp(-kappaC*delta.t))+sigmaC*sqrt((1-exp(-
2*kappaC*delta.t))/(2*kappaC))*mz[,7]
```

```
LV[,i+1]   <-   LV[,i]*exp(-kappaLV*delta.t)+   (LVmu   -((lambdaLV-r)/kappaLV))*(1-exp(-kappaLV*delta.t))+sigmaLV*sqrt((1-exp(-2*kappaLV*delta.t))/(2*kappaLV))*mz[,8]
}


##### Harvest returns

#### Since the beginning of the florest

R1<-matrix(0,n,g/delta.t+1)


for(i in 1:n){

R1[i,]<-exp(P1[i,])*Y1+exp(P2[i,])*Y2+exp(P3[i,])*Y3+exp(P4[i,])*Y4+exp(P5[i,])*Y5+exp(P6[i,])*Y6

}


##### From the 18th year


R<-matrix(0,n,g/delta.t+1-18)

for(i in 1:n){

R[i,]<-exp(P1[i,1:(g/delta.t+1-18)])*Y1[18:g/delta.t+1]+exp(P2[i,1:(g/delta.t+1-18)])*Y2[18:g/delta.t+1]+exp(P3[i,1:(g/delta.t+1-18)])*Y3[18:g/delta.t+1]+exp(P4[i,1:(g/delta.t+1-18)])*Y4[18:g/delta.t+1]+exp(P5[i,1:(g/delta.t+1-18)])*Y5[18:g/delta.t+1]+exp(P6[i,1:(g/delta.t+1-18)])*Y6[18:g/delta.t+1]

}


#Present value vector of ongoing variables

#On going Costs



for(i in 1:n){

   C1[i,]<-cc
   C[i,]<-cc[18:g/delta.t+1]


}


### From the beginning
```

```
for (j in seq(1,g/delta.t+1,1)){

  C1[,j]<-C1[,j]*(1-t)*exp(-r*(j-1)*delta.t)

}

### from the 18th year

for (j in seq(1,g/delta.t+1-18,1)){

  C[,j]<-C[,j]*(1-t)*exp(-r*(j-1)*delta.t)

}

for(i in 1:n){

  C1[i,]<-cumsum(C1[i,])
  C[i,]<-cumsum(C[i,])

}

########## Net harvest Reveue (without ongoing costs)

Hr<-matrix(0,n,g/delta.t+1-18)

for(i in 1:n){

Hr[i,] <- R[i,] - Ch[18:g/delta.t+1] # Net harvest Revenue (without ongoing costs)

}

Rd<-Hr #not considering yet ongoing costs and carbon revenues - considered later in the NPV calculation

##############################################    Barrier    and    Trade    off    Functions
########################################

# Function B1 -> "value to wait" proxy function(square root)

B1<-matrix(0,1,length(P1[1,]))

for (i in seq(1,Tmax,1)){

B1[1,i] <- sqrt(Tmax-i)/sqrt(Tmax-Tmin)
```

```
}

# Function B2 -> trade off function between HRt and HCt (linear)

B2<-matrix(0,1,length(P1[1,]))

B2<-rep(1,length(P1[1,]))

######################## Present Value Calculation##########################################################

wm1d<-matrix(g/delta.t+1,n,1)

CPV1d<-matrix(0,n,g/delta.t+1-18)
Conv<-matrix(0,n,g/delta.t+1-18)

NPV1d<-matrix(0,n,1)
NPV2d<-matrix(0,n,1)


mNPV1d<-matrix(0,n,1)
mNPV2d<-matrix(0,n,1)

iNPV1d<-matrix(0,n,1)
iNPV2d<-matrix(0,n,1)


##### Action Outcomes


# Post tax Harvest net revenue

for (j in seq(1,g/delta.t+1-18,1)){

  CPV1d[,j]<-Rd[,j]*(1-t)*exp(-r*(j-1)*delta.t)-C[,j]# harvest decision

}


# Conversion Strategy

for (j in seq(1,g/delta.t+1-18,1)){

  Conv[,j]<-(exp(LV[,j])-PC[,j]*S[j+18]-CC)*exp(-r*(j-1)*delta.t)

}



##  Replant Strategy
```

```
###### replant and Harvest (t*+27)

Re<-matrix(0,n,(g/delta.t+1-18))

P1e<-matrix(0,n,(g/delta.t+1-18))

P2e<-matrix(0,n,(g/delta.t+1-18))

P3e<-matrix(0,n,(g/delta.t+1-18))

P4e<-matrix(0,n,(g/delta.t+1-18))

P5e<-matrix(0,n,(g/delta.t+1-18))

P6e<-matrix(0,n,(g/delta.t+1-18))

PCe<-matrix(0,n,(g/delta.t+1-18))

LVe<-matrix(0,n,(g/delta.t+1-18))

Rep<-matrix(0,n,(g/delta.t+1-18))


### Revenues

for (i in 1:n){

  P1e[i,]<-P1[i,1:(g/delta.t+1-18)]*exp(-kappa1*27)+P1mu*(1-exp(-kappa1*27)) # OU expectation formula

  P2e[i,]<-P2[i,1:(g/delta.t+1-18)]*exp(-kappa2*27)+P2mu*(1-exp(-kappa2*27))

  P3e[i,]<-P3[i,1:(g/delta.t+1-18)]*exp(-kappa3*27)+P3mu*(1-exp(-kappa3*27))

  P4e[i,]<-P4[i,1:(g/delta.t+1-18)]*exp(-kappa4*27)+P4mu*(1-exp(-kappa4*27))

  P5e[i,]<-P5[i,1:(g/delta.t+1-18)]*exp(-kappa5*27)+P5mu*(1-exp(-kappa5*27))

  P6e[i,]<-P6[i,1:(g/delta.t+1-18)]*exp(-kappa6*27)+P6mu*(1-exp(-kappa6*27))

  PCe[i,]<-PC[i,1:(g/delta.t+1-18)]*exp(-kappaC*27)+PCmu*(1-exp(-kappaC*27))

  LVe[i,]<-LV[i,1:(g/delta.t+1-18)]*exp(-kappaLV*27)+LVmu*(1-exp(-kappaLV*27))


  Re[i,]<-
exp(P1e[i,])*Y1[28]+exp(P2e[i,])*Y2[28]+exp(P3e[i,])*Y3[28]+exp(P4e[i,])*Y4[28]+exp(P5e[i,])*Y5[28]+exp(P6e[i,])*Y6[28]

  Rep[i,] <- (Re[i,]-h*(Y1[28]+Y2[28]+Y3[28]+Y4[28]+Y5[28]+Y6[28])-C1[i,28])/(1-1/((1+r)^27))*exp(-r*27)
```

```
}

# PCe2

PCe2<-matrix(0,n,(g/delta.t+1-18+8))

for (i in 1:n){

  PCe2[i,]<-PC[i,1:(g/delta.t+1-18+8)]*exp(-kappaC*8)+PCmu*(1-exp(-kappaC*8))

}


# Coversion in t*+27

HC2star<-matrix(0,n,(g/delta.t+1-18))
Z2star<-matrix(0,n,(g/delta.t+1-18))

for (i in 1:n){

HC2star[i,]<-(Rep[i,]*(1-1/((1+r)^27))+(LV[i,28]-PC[i,28]*S[28]-CC))*exp(-r*27)

}

for (j in 1:(g/delta.t+1-18)){

HC2star[i,]<-HC2star[i,]*exp(-r*(j-1)*delta.t)

}

for (i in 1:n){

Z2star[i,] <-max(Rep[i,],HC2star[i,],0)

}

################## Decision Rules (until t*)


H<-CPV1d
C<-Conv
R<-Z2star


# Conversion Strategy 8 years ahead

Conv1<-matrix(0,n,g/delta.t+1-18)

for (j in seq(1,g/delta.t+1-18,1)){
```

```
  Conv1[,j]<-C[,j]+(C1[,9]-C1[,1])+(exp(LV[,j+8])-exp(PCe2[,j+8])*(S[8]/10)-(CC+500))*exp(-r*(8-1)*delta.t)

}


HC<-H+Conv1
HR<-H+R


for (i in seq(1,n,1)){

x1d <- which(HR[i,]>=alfa*B1-HC[i,]*alfa*B1/(beta*B2+0.0000001))

for (j in 1:length(x1d)){
x1d[j]<-ifelse(x1d[j]<Tmin|x1d[j]>Tmax|is.na(x1d[j]),Tmax,x1d[j])
}

wm1d[i,1]<- ifelse(is.integer(min(x1d))|is.double(min(x1d)),min(x1d),Tmax)

}

for (i in seq(1,n,1)){

   NPV1d[i,1]<- ifelse(HR[i,wm1d[i,1]]>HC[i,wm1d[i,1]],HR[i,wm1d[i,1]],0)
   NPV2d[i,1]<- ifelse(HR[i,wm1d[i,1]]<HC[i,wm1d[i,1]],HC[i,wm1d[i,1]],0)
   mNPV1d[i,1]<- ifelse(NPV1d[i,1]+NPV2d[i,1]<H[i,wm1d[i,1]],H[i,wm1d[i,1]],NPV1d[i,1]+NPV2d[i,1])
   mNPV1d[i,1]<- ifelse(mNPV1d[i,1]<0,0,mNPV1d[i,1])
 }


value1d<-mean(mNPV1d,na.rm=TRUE)


# Average t*

avt<-mean(wm1d,na.rm=TRUE)

#Proportion of Harvest and Replant

j1d<-length(subset(mNPV1d[,1],mNPV1d[,1]==NPV1d[,1]))/n

#Proportion of Harvest and Conversion in Dairy

k1d<-length(subset(mNPV1d[,1],mNPV1d[,1]==NPV2d[,1]))/n

#Proportion that Harvest and Abandon the forest

l1d<-length(subset(mNPV1d[i,1],mNPV1d[i,1]==H))/n
```

```
#Proportion that Abandon the forest

m1d<-length(subset(mNPV1d[i,1],mNPV1d[i,1]==0))/n

c(value1d,j1d,k1d,l1d,m1d,avt)

}

### The Bootstrapping Real Options Analysis  Monte Carlo Simulation

alfa <- seq(0,100000,5000) # alfa parameters
beta <- seq(0,100000,5000) # beta parameters
PCmu <- c(5, 25, 50, 100) #Carbon Prices
LVmu <- c(10000,20000,50000) #Land Values

n<- 5000

########


PAR <- matrix(0,1,10)

for (l in 1:length(LVmu)){

for (k in 1:length(PCmu)){

for(j in 1:length(beta)){

for (i in 1:length(alfa)){

PAR<-rbind(PAR,c(LVmu[l],PCmu[k],beta[j],alfa[i],OU.broa(n,alfa[i],beta[j],0,PCmuk], LVmu[l])))

}
}
}
}

PAR<-PAR[-1,]

#Maximum values choices

carb <- 5
land <- 10000

X<-which(PAR[,1]==land&PAR[,2]==carb)
x<-which.max(PAR[X,5])
MAX<-PAR[x+min(X)-1,]
MAX
```

* * *