Coding for Shared Satellite Channel Communications

Philippa A. Martin, *Senior Member, IEEE*, Marcel A. Ambroze, *Member, IEEE*, Desmond P. Taylor, *Life Fellow, IEEE*, and Martin Tomlinson, *Senior Member, IEEE*

***Abstract*—This paper investigates the design and performance of a two-user satellite communication system. Each user is independently encoded using a structured Turbo code with identical symbol interleavers. This permits Turbo decoding to be performed using the combined component code trellises, which provides significant gains over independent decoding. The design of the code-matched symbol interleavers is described. Both ideal and imperfect knowledge of the relative phase shift between users is considered. Performance is compared to the single user case and the performance degradation due to a third unknown user is considered. These investigations show that the proposed system performs well in a variety of conditions.**

***Index Terms*—Turbo Code, satellite communications, interleaver design, iterative decoding, multiuser communications.**

I. Introduction

Wireless data communication systems often employ a star network architecture in which multiple remote terminals communicate with a central hub or base station using a shared channel. Transmission is in frames or packets and a fundamental problem is the efficient sharing of the channel [27], [19]. Typical of such systems are *very small aperture terminal (VSAT)* satellite systems [21], [1], [3]. These are packet radio systems and their communication characteristics are summarized in [2], [29].

One approach to mitigating the effect of multiple cochannel signals in satellite systems involves joint iterative interference cancellation and decoding [5]. In contrast, in this paper we utilize coding to mitigate some of the loss due to simultaneous transmission by users in the same channel (collisions). We code in such a manner that when only two users collide, we may with some minimal performance degradation recover both packets. In many systems two-user collisions are the dominant cause of packet loss [29] and the elimination or reduction of frame loss in this case leads to significant improvement in achievable throughput and usage of system resources.

We consider the situation when two users continuously transmit packets or frames simultaneously and independently through the shared memoryless *additive white Gaussian noise (AWGN)* channel and are jointly decoded. Their transmissions are assumed synchronized at frame and symbol levels, but are not phase locked. Both ideal and imperfect knowledge of the relative phase shift between users is considered. We assume the availability of pilot symbols to initialize the phase estimator in each joint receiver. Both users are assumed to have uplink power control, as used for example in the DVB-RCS standard [15].

In this work we investigate systems that do not use code, time or frequency division multiple access. Instead we focus on using error control coding (with code rates of $1/3$) to protect multiple users in a shared channel[1]. In [10] rate $1/16$ Turbo codes are used to independently encode each user's data, which is transmitted on a multiple-access adder AWGN channel. Each user is assigned a different power, which allows the decoding of high power users to converge and hence reduce interference so that the decoding of lower power users can then converge. Iterative decoding is used within each Turbo code and between Turbo codes of different users. Each user has a randomly generated coded bit interleaver after the Turbo encoder. In [28], [31] a *Reed Solomon (RS)* code is concatenated with a Turbo code and used for transmission over a *digital video broadcasting (DVB)* satellite system. However, only single user performance is considered.

Here we use Turbo codes to independently encode each user's data. We assume that the codes used by each user are known at the hub or receiver. Unlike [10], we avoid concatenating iterative decoders as they are suboptimal and extrinsic information tends to saturate. Instead, the Turbo codes of both users are jointly decoded using a combined trellis for each of the component convolutional codes and iterative decoding is performed between the combined trellis decoders. This exploits the fact that in Turbo codes the component convolutional codes have low complexity and as a consequence the combined trellis still has manageable complexity. The transmitted symbols are structured such that it is possible to use optimal *maximum a posteriori (MAP)* decoding of the combined component codes of both users' Turbo codes. This approach allows performance to be significantly improved compared to decoding the two users' codes independently, while still maintaining feasible decoding complexity. In addition, since we are only iterating between two MAP decoders instead of within and between two Turbo code decoders [10] we do not need to worry about scheduling

Paper approved by A. H. Banihashemi, the Editor for Coding and Communication Theory of the IEEE Communications Society. Manuscript received August 1, 2007; revised March 13, 2008 and August 19, 2008.

P. A. Martin and D. P. Taylor are with the Department of Electrical and Computer Engineering, University of Canterbury, Christchurch, New Zealand (e-mail: {p.martin, taylor}@elec.canterbury.ac.nz).

M. A. Ambroze and M. Tomlinson are with the School of Computing, Communications and Electronics, University of Plymouth, United Kingdom (e-mail: {m.ambroze, m.tomlinson}@plymouth.ac.uk).

Digital Object Identifier 10.1109/TCOMM.2009.08.070390

[1]We note that this could be argued to be a form of code division multiple access, but not in the traditional meaning of the term.

the iterative decoding [9].

Symbol-based decoding is used in order to improve combined decoder convergence. The decoders use metrics based on the composite constellation created by the transmitted symbols from both users. From the decoder's point of view this is equivalent to a higher order modulation. Turbo codes do not converge well with higher order modulation and it is found, similar to [16], that symbol interleaving improves convergence. As shown later, careful code-matched interleaver design is necessary to ensure that the distance properties of the Turbo codes result in a low error floor.

In Section II we describe the proposed communication system. This includes a description of the interleaver design. Section III presents extensive simulation results, which investigate the robustness of the proposed scheme under various conditions. Finally, conclusions are drawn in Section IV.

II. System Overview

We consider a two user satellite communication system and focus on the decoding of user A. A memoryless AWGN channel is considered. The vectors of M-ary constellation points transmitted by users A and B are denoted $\boldsymbol{s}^A = (s_1^A, \cdots, s_N^A)$ and $\boldsymbol{s}^B = (s_1^B, \cdots, s_N^B)$, respectively, where N is the frame length and M is the size of each users' constellation. The set of all possible points is denoted $\{c_i^A\}_{i=1}^M$ for user A and $\{c_i^B\}_{i=1}^M$ for user B. We assume both users simultaneously transmit codes with the same length, and have symbol and frame synchronization. We also assume that both users have uplink power control so that the relative power levels received at the satellite may be preset. The noise free received signal vector is then defined as

$$\boldsymbol{y} = \sqrt{E_s^A}\boldsymbol{s}^A + \sqrt{E_s^B}\boldsymbol{s}^B \exp(-j\boldsymbol{\phi}), \tag{1}$$

where E_s^A and E_s^B are the average symbol (constellation point) energies for user A and B, respectively, and $\boldsymbol{\phi} = (\phi_1, \cdots, \phi_N)$ denotes the phase difference between their signals. The resulting composite noise free received constellations for $E_s^A = 1$ and various values of E_s^B and $\boldsymbol{\phi}$ are shown in Fig. 1 for QPSK. For two users each transmitting QPSK the composite constellation varies between having 9 non-unique points to having 16 unique points (including 16-QAM and 16-APSK constellations). The phase shift, $\boldsymbol{\phi}$, is primarily caused by the relative motion between the satellite and user A and B's terminals on Earth. We assume we can track the motion of the satellite. However, short-term frequency instabilities in the two terminals will result in a time varying phase. As a result, we consider both ideal and imperfect knowledge of $\boldsymbol{\phi}$ at the receiver. However, the estimation and tracking of phase is beyond the scope of the present paper. The reference phase for the system is assumed to be that of user A. Unless otherwise stated we also assume the relative phase difference between the two transmitted signals varies linearly with time such that the phase difference accumulated over each block of N transmitted symbols from each user is a small multiple of 2π.

A. Encoder

The proposed two-user system is shown in Fig. 2. Each user employs a conventional Turbo code using *recursive systematic convolutional (RSC)* component codes [7] and identical symbol based interleavers. This allows the interleaved component codes to be decoded using a low complexity combined trellis. The modulated encoded data from both users is sent simultaneously and adds linearly on a symbol by symbol basis according to (1), where there are $log_2(M)$ bits per symbol/ constellation point. Gray mapping is used. Each M-ary symbol transports either data, parity 1 (parity from the non-interleaved component encoders) or parity 2 (parity from the interleaved component encoders), but not a combination of them. This structure simplifies the symbol-based decoding. Since we assume user A and B have frame synchronization, both users simultaneously transmit the same type of symbol (namely data, parity 1 or parity 2). We denote the binary data from user A and B as $\boldsymbol{d}^A = [d_1^A, \cdots, d_k^A]$ and $\boldsymbol{d}^B = [d_1^B, \cdots, d_k^B]$, respectively, where k is the number of information bits per frame for each user. The parity bits from the i^{th} component encoder of user A and B are denoted $\boldsymbol{p}_i^A = [p_{i,1}^A, \cdots, p_{i,m}^A]$ and $\boldsymbol{p}_i^B = [p_{i,1}^B, \cdots, p_{i,m}^B]$, respectively, where m is the number of parity bits from each encoder per frame. The rates of user A and B's Turbo codes are denoted $\mathcal{R}_{ecc}^A$ and $\mathcal{R}_{ecc}^B$, respectively.

In order to associate the decoded data with the correct user, it is necessary for each user to have a different signature. The simplest solution is to use different component codes. The component codes are encoded/ decoded using the tail biting method [8], [23] in order to avoid trellis termination

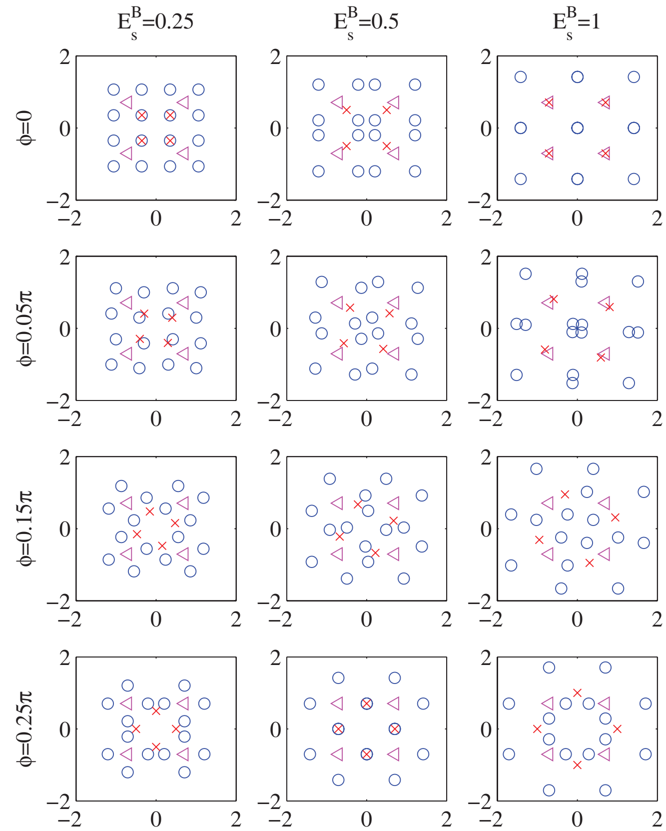

Fig. 1. Noise free received composite constellations for $E_s^A = 1$ and various values of E_s^B and ϕ. The transmitted constellations from user A and B are labelled by ◃ and ×, respectively. The resulting composite noise free received constellation points are labelled by ∘.

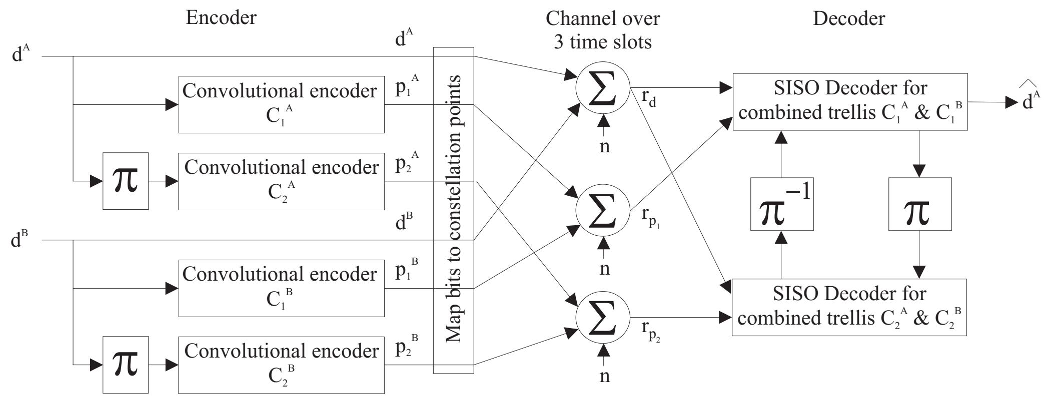

Fig. 2. Proposed two user communication system with symbol interleaver π. Note, pairs of bits from the same output stream (for example $\boldsymbol{d}^A$) are mapped to QPSK constellation points. Both users send either data, parity 1 or parity 2 information simultaneously. The three output types are then time multiplexed on a symbol basis. There is only one channel and noise source, but here we show the three time slots used to transmit the encoded data as three separate adders.

overheads. Two sections in each component code's trellis are combined to allow symbol based decoding. In the following we assume rate $1/2$ RSC component codes for ease of exposition. For each of users A and B we denote the current state in the i^{th} component trellis as $\mathcal{S}^{A,i}$ and $\mathcal{S}^{B,i}$ and the next state as $\mathcal{S}^{A,i/}$ and $\mathcal{S}^{B,i/}$, respectively. The transition from state $\mathcal{S}^{A,i}$ to $\mathcal{S}^{A,i/}$ is labelled by $(d_1^A, d_2^A, p_{i,1}^A, p_{i,2}^A)$ for user A and similarly for user B. Each pair of states $(\mathcal{S}^{A,i}, \mathcal{S}^{B,i})$, one from the i^{th} component code's trellis of each user, determine a state in the i^{th} component code's combined trellis. Therefore, for each transition in the component trellis of user A there are 4 transitions in the component code's combined trellis. The transition labels in the combined trellis are the union of the labels in the trellises of the two users. Therefore, the transition label for the transition from state $(\mathcal{S}^{A,i}, \mathcal{S}^{B,i})$ to $(\mathcal{S}^{A,i/}, \mathcal{S}^{B,i/})$ is $(d_1^A, d_2^A, p_{i,1}^A, p_{i,2}^A, d_1^B, d_2^B, p_{i,1}^B, p_{i,2}^B)$.

The combined trellis complexity grows exponentially with the number of users, but reduced state techniques can be used to reduce complexity. If one of the users is substantially different in power or if the *signal to noise ratio (SNR)* is large only a few of the states will have significant probabilities. Thus, states can be pruned without significant loss in performance.

B. Interleaver Design

Identical code matched symbol interleavers of length $L = k/\log_2(M)$ symbols are employed by each user. A symbol interleaver allows symbol probabilities to be exchanged during the iterative process, which gives improved convergence [14]. The design of the symbol interleaver has to take into account low weight Turbo codewords involving component code error events that have two 1's contained in one interleaver symbol. These low weight codewords are possible because a symbol interleaver does not separate the bits within a symbol. They are not characteristic of bit interleavers. Consequently, two constraints are imposed on the design of the symbol interleaver. Firstly, we use a (symbol-wise) S-random interleaver [12] of size $S \approx \sqrt{L/2}$. Secondly, low code weight error events are determined and as much as possible removed from the interleaver by symbol swaps in an iterative fashion.

The symbol interleaver design starts with a symbol based S-random interleaver, which provides a reasonable starting point for the iterative procedure described below. The procedure has two steps:

1) Determine the Turbo code weight spectrum up to the target design distance and identify the (symbol) interleaver entries that cause the low weight codewords.
2) Swap the selected interleaver entries with randomly chosen entries under an S-random constraint (the swapping should not result in a significant reduction of the S parameter). If there are no more codewords with weight lower than the design distance or a set number of iterations has been exceeded, then exit this design procedure. Otherwise, return to step 1.

For the two-user case, the weight spectra of the Turbo codes of both users has to be considered simultaneously. It has been found that this procedure reduces the number of error events discussed above, but does not completely remove them due to their large number. This is why their weight has to be maximized.

The first terms of the Hamming weight spectrum of the Turbo code are evaluated using a branch and bound search algorithm as introduced in [17]. The algorithm used for this work has been optimized to take advantage of the short constraint length of the component codes [24]. This results in a highly optimized algorithm which can compute not only the minimum distance of the code but also a significant number of higher weight terms.

C. Channel Metric

Since we consider a memoryless AWGN channel the received baseband signal, $\boldsymbol{r} = (r_1, \cdots, r_N)$, after matched filtering and sampling, can be written as

$$\boldsymbol{r} = \boldsymbol{y} + \boldsymbol{n} = \sqrt{E_s^A}\boldsymbol{s}^A + \sqrt{E_s^B}\boldsymbol{s}^B \exp(-j\boldsymbol{\phi}) + \boldsymbol{n}, \quad (2)$$

where $\boldsymbol{n} = (n_1, \cdots, n_N)$ is AWGN with variance

$$\sigma_n^2 = \frac{N_0}{2} = \frac{E_s^A}{2\log_2(M)\mathcal{R}_{ecc}^A 10^{0.1SNR}}, \quad (3)$$

and $SNR = 10\log_{10}(E_b^A/N_0)$ denotes the signal to noise ratio in terms of user A's data bit energy, E_b^A, and the noise spectral density, N_0.

A soft channel metric is calculated for each possible pair of symbols (s_t^A, s_t^B) at time t. We assume all possible symbols are equiprobable. Since we assume a memoryless AWGN channel, for each time t, we want to find

$$\begin{aligned}(\hat{s}_t^A, \hat{s}_t^B) &= \arg\max_{\{c_i^A, c_l^B\}} \left\{p(c_i^A, c_l^B | r_t, \phi_t)\right\} \\ &= \arg\max_{\{c_i^A, c_l^B\}} \left\{\frac{p(r_t | c_i^A, c_l^B, \phi_t) Pr(c_i^A) Pr(c_l^B)}{p(r_t|\phi_t)}\right\} \\ &= \arg\max_{\{c_i^A, c_l^B\}} \left\{p(r_t | c_i^A, c_l^B, \phi_t)\right\}, \ i, l = 1, \cdots, M.\end{aligned} \tag{4}$$

Assuming a Gaussian distribution this becomes

$$(\hat{s}_t^A, \hat{s}_t^B) = \arg\max_{\{c_i^A, c_l^B\}} \left\{ \exp\left(-\frac{|r_t - \sqrt{E_s^A} c_i^A - \sqrt{E_s^B} c_l^B \exp(-j\phi_t)|^2}{2\sigma_n^2}\right)\right\}. \tag{5}$$

Therefore, the normalized metric for the hypothesized user A and B symbols (c_i^A, c_l^B) at time t is defined as

$$\mathcal{M}_t^{i,l} = \frac{\Gamma_t^{i,l}}{\sum_{q=1}^M \sum_{p=1}^M \Gamma_t^{q,p}}, \quad i, l = 1, \cdots, M, \tag{6}$$

where

$$\Gamma_t^{q,p} = \exp\left(-\frac{|r_t - \sqrt{E_s^A} c_q^A - \sqrt{E_s^B} c_p^B \exp(-j\phi_t)|^2}{2\sigma_n^2}\right). \tag{7}$$

D. Decoder

Here, we assume that the decoder knows the codes of both users. The uninterleaved component codes for both users are jointly decoded using a combined trellis. The same length $k/\log_2(M)$ symbol interleaver is used by both users, which allows the interleaved component codes for both users to also be jointly decoded using a combined trellis. Soft information is passed between the combined decoders as shown in Fig. 2.

A symbol-based MAP decoder is used to obtain symbol-based extrinsic information, which is exchanged during the iterative process. For each iteration, the input to the i^{th} combined trellis decoder is the extrinsic information and channel symbol probability for: user A data symbol, d^A, user A parity symbol, p_i^A, user B data symbol, d^B, and user B parity symbol, p_i^B. The extrinsic information probabilities are provided by the other component decoder during each iteration and are initialized to uniform probabilities before the first iteration. For each trellis section t, the channel symbol probability is given by

$$P_C\{x^A = u, x^B = v | r_x\} = P_C\{s_t^A = c_i^A, s_t^B = c_l^B | r_x\} = \mathcal{M}_t^{i,l}, \tag{8}$$

where $\mathcal{M}_t^{i,l}$ is calculated using (6) and (7), $x \in \{d, p_i\}$, r_x is a received symbol and $u, v \in \{\{00\}, \{01\}, \{10\}, \{11\}\}$.

The MAP output for combined decoding of the i^{th} component code is given for each trellis section by (the section index is omitted for clarity):

$$\begin{aligned}P\{d^A = u, d^B = v | \mathbf{r}\} &= \lambda_1 P\{d^A = u, d^B = v | r_d\} \\ &\times \sum_{(\mathcal{S}^A, \mathcal{S}^{A/}): d^A = u} \ \sum_{(\mathcal{S}^B, \mathcal{S}^{B/}): d^B = v} \\ &\left\{\alpha(\mathcal{S}^A \mathcal{S}^B) \beta(\mathcal{S}^{A/} \mathcal{S}^{B/}) P\{p_i^A, p_i^B | r_{p_i}\}\right\},\end{aligned} \tag{9}$$

where $P\{d^A, d^B | r_d\}$ and $P\{p_i^A, p_i^B | r_{p_i}\}$ are the transition probabilities, in which r_d and r_{p_i} denote a noisy received data symbol and parity symbol, respectively. Note λ_1 is a multiplicative constant[2], and $\alpha(\mathcal{S}^A \mathcal{S}^B)$ and $\beta(\mathcal{S}^{A/} \mathcal{S}^{B/})$ result from the alpha and beta recursions of the MAP algorithm [4]. In the iterative decoder,

$$P\{d^A, d^B | r_d\} = \lambda_2 P_E'\{d^A, d^B | \mathbf{r}\} P_C\{d^A, d^B | r_d\} \tag{10}$$

is the extrinsic probability from the previous decoder times the channel probability calculated using (8) and λ_2 is again a multiplicative constant. The output extrinsic information for both users' data symbols is given by

$$\begin{aligned}P_E\{d^A = u, d^B = v | \mathbf{r}\} &= \lambda_3 \sum_{(\mathcal{S}^A, \mathcal{S}^{A/}): d^A = u} \ \sum_{(\mathcal{S}^B, \mathcal{S}^{B/}): d^B = v} \\ &\left\{\alpha(\mathcal{S}^A \mathcal{S}^B) \beta(\mathcal{S}^{A/} \mathcal{S}^{B/}) P\{p_i^A, p_i^B | r_{p_i}\}\right\},\end{aligned} \tag{11}$$

where λ_3 is a multiplicative constant.

The advantage of using this combined decoder can be illustrated by considering a scenario in which separate decoders are used. Now, the joint conditional probabilities are treated as if they were independent probabilities. This means we assume the inputs to each decoder are independent. Then $P\{d^A, d^B | r_d\} = P\{d^A | r_d\} P\{d^B | r_d\}$ and $P\{p_i^A, p_i^B | r_{p_i}\} = P\{p_i^A | r_{p_i}\} P\{p_i^B | r_{p_i}\}$. This corresponds to the user A decoder treating user B as independent interference. It can be shown that (9) then becomes[3]

$$\begin{aligned}P\{d^A = u, d^B = v | \mathbf{r}\} &= \lambda_1 \\ &\times P\{d^A = u | r_d\} \sum_{(\mathcal{S}^A, \mathcal{S}^{A/}): d^A = u} \alpha(\mathcal{S}^A) \beta(\mathcal{S}^{A/}) P\{p_i^A | r_{p_i}\} \\ &\times P\{d^B = v | r_d\} \sum_{(\mathcal{S}^B, \mathcal{S}^{B/}): d^B = v} \alpha(\mathcal{S}^B) \beta(\mathcal{S}^{B/}) P\{p_i^B | r_{p_i}\}.\end{aligned} \tag{12}$$

This can be split into two separate decoders with

$$\begin{aligned}P\{d^X = w\} &= \lambda_4 P\{d^X = w | r_d\} \\ &\times \sum_{(\mathcal{S}^X, \mathcal{S}^{X/}): d^X = w} \alpha(\mathcal{S}^X) \beta(\mathcal{S}^{X/}) P\{p_i^X | r_{p_i}\},\end{aligned} \tag{13}$$

where λ_4 is again a multiplicative constant, $X \in \{A, B\}$ and $w \in \{u, v\}$. This leads to separate MAP decoding which incorrectly assumes that the inputs to the two decoders are independent. The MAP summation in (13) is performed over the state space of only one user. The other user is treated as independent interference and this assumption destroys information available in the received signal, which could be used in combination with the trellis structure of the other user. In (9) the MAP summation is performed over the combined state

[2]Note λ_1, λ_2, λ_3 and λ_4 are multiplicative constants as in [4].

[3]The alpha and beta recursions were also analyzed to reach (12).

space of the two users. In this way, the knowledge available at the decoder about the structure of the encoded stream from user B is used to aid the decoding of user A.

E. Phase Errors

In practical systems the phase has to be calculated and tracked. In order to consider the impact of imperfect phase estimation on performance we model the phase error as a Gaussian random variable with variance $1/\gamma_{PE}$ [18]. The *probability density function (PDF)* of the phase error, θ, is then given by [18]

$$P(\theta) \approx \frac{\exp(-\theta^2\gamma_{PE}/2)}{\sqrt{2\pi/\gamma_{PE}}}, \tag{14}$$

where γ_{PE} is the effective loop SNR of the phase estimator. This model can be used when a phase lock loop with large loop SNR is considered. Note, that in the severe co-channel interference scenario considered here, it is likely that a more sophisticated phase estimator would be required. This is outside the scope of the current work. However, the simple model considered does allow us to investigate the robustness of the scheme to a random phase error. The channel metric is still calculated using (6), but now we define

$$\Gamma_t^{q,p} = \exp\left(-\left|\frac{r_t - \sqrt{E_s^A}s_q^A\exp(-j\theta_t^A)}{\sqrt{2}\sigma_n} - \frac{\sqrt{E_s^B}s_p^B\exp(-j(\phi_t+\theta_t^B))}{\sqrt{2}\sigma_n}\right|^2\right), \tag{15}$$

where θ_t^A and θ_t^B are the phase estimation errors for user A and B, respectively, based on the PDF given by (14).

III. Simulation Results

We now present simulation results for the proposed two user satellite communication system. Users A and B each transmit QPSK constellation points. A maximum of 50 decoding iterations[4] are used and results are based on at least 100 frame error events. All *bit error rate (BER)* and *frame error rate (FER)* results are presented from user A's perspective. However, the decoder can be used to obtain estimates of both users' data. We define SNR with respect to user A and set $E_s^A = 1$. Recall, unless otherwise stated, we assume the relative phase difference between the two transmitted signals varies linearly with time such that the phase difference accumulated over each block of N transmitted symbols from each user is a small multiple of 2π. See Section II for more details on the system setup.

A. Component code selection and interleaver design

Memory 2, 3 and 4 component codes were evaluated for the two user scenario. Memory 3 component codes have been found to provide the best compromise between convergence and error floor. The memory 3 component RSC codes in the Turbo codes are defined by the feed forward polynomial ff = 17 (octal) and feedback polynomials fb = 13 (octal) for user A and fb = 15 (octal) for user B. Each component RSC code has rate $\mathcal{R}_{cc} = 1/2$ giving an overall Turbo code rate of $\mathcal{R}_{ecc}^A = \mathcal{R}_{ecc}^B = 1/3$. Here, we will consider Turbo codes with $k = 1000$ (for each user).

The symbol interleaver is designed using the procedure outlined in Section II.B. Unless otherwise stated, we use a 2-bit symbol interleaver for $k = 1000$ bits. The S-random interleaver starts with $S = 14$ (symbols) and ends after the swaps with $S = 12$ (symbols). The weight spectra for weights of 34 or less is $A_{w\le 34}(x) = 6x^{22} + 14x^{25} + 8x^{26} + 5x^{27} + 8x^{28} + 25x^{29} + 46x^{30} + 22x^{31} + 3x^{32} + 102x^{33} + 76x^{34}$ before and $A_{w\le 34}(x) = 3x^{32} + 93x^{33} + 83x^{34}$ after the swaps, where the exponent of x denotes the weight of the error event and the multiplicative factors denote the multiplicity. As can be seen the final minimum distance is $d_{min} = 32$.

We now justify the selection of the memory 3 RSC component codes. The weight of the error events that cannot be removed by the symbol interleaver structure can be maximized by choosing the feed forward polynomial equal to ff = 17 (octal). This is due to the fact that most of these error events are caused by data sets that cancel the feedback, in user A's case the simplest error events are of the type $x^p(1+x+x^3)$, where p gives the position in the input stream of the error event. The parity output of the encoder is $y(x) = x^p \times \text{ff}(x)$ with maximum weight if all the coefficients of the feedforward polynomial $\text{ff}(x)$ are 1, meaning ff = 17 (octal). The case of user B is similar (the error events are of the type $x^p(1+x^2+x^3)$) resulting in the same choice of feed forward polynomial, ff = 17 (octal). Note that such error events are not generally allowed by an S-random *bit* interleaver as the first two bits of the error events would be interleaved away from each other. Note, the combined component code trellises have 64 states.

B. Two user system with combined user A and B decoding

Here, we consider the two user system with combined user A and B decoding. The effect of various values of E_s^B on BER and FER performance is shown in Fig. 3 and Fig. 4, respectively. When $E_s^B = 0$ we have the single user case. As E_s^B increases user B starts acting as interference and so degrades the performance of user A. The composite decoder can jointly decode the information for user A and B. As a result when the energy of user B, E_s^B, gets closer to that of user A, performance starts improving. In this case we inherently get co-operative decoding in the joint trellis decoders, and so information about both user's codes can be used to provide a better estimate of the data sent by each user. This can be seen in Fig. 5, where the loss due to $E_s^B > 0$ is shown (at a user A FER= 10^{-3}). It is interesting to note that the performance for $E_s^B = 0$ and $E_s^B = 2$, assuming combined decoding, is almost identical. Therefore, we want the other user to have either much smaller signal energy or at least as much signal energy as we do.

In the single user case the Turbo code considered has an error floor below BER 10^{-6} and FER 10^{-4} as shown in Fig. 3 and Fig. 4, respectively. This is already a low error floor due to careful design of the symbol interleaver. For $E_s^B = 1$ there is still no sign of an error floor at a FER of 10^{-5}. The

[4] A stopping rule is used similar to that described in [22].

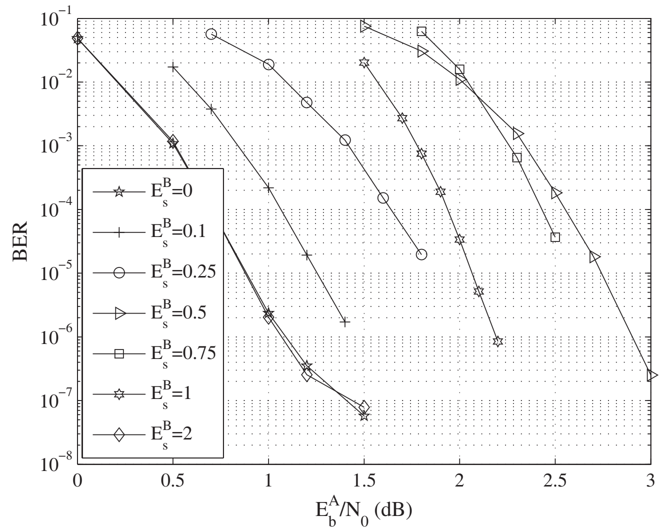

Fig. 3. Impact on BER of various values of E_s^B for $E_s^A = 1$.

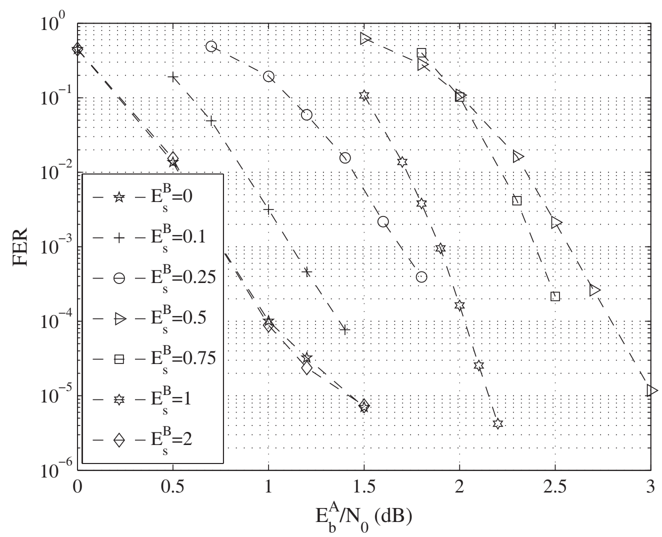

Fig. 4. Impact on FER of various values of E_s^B for $E_s^A = 1$.

performance for $E_s^B = 1$ is being determined not only by noise, but also by interference. If a lower single user error floor is needed, then several approaches could be used. An outer *Bose-Chaudhuri-Hocquenghem (BCH)* or RS code could be used [28], [31], [26] at the cost of BER performance in the waterfall region[5]. Alternatively, a serial or hybrid concatenated convolutional coding scheme could be used instead of the Turbo code [6], [13]. For a serial concatenated convolutional code a higher rate component code would be needed (larger number of transitions) in order to maintain an overall rate of $1/3$.

We now consider the two user case with $E_s^A = E_s^B = 1$, where the performances of user A and B are approximately the same. Overall this two user system transmits $2k = 2000$ data bits over $N = 1500$ symbol periods (using QPSK). A single user system with equivalent throughput and block length

[5]The outer BCH or RS code would result in a loss in energy efficiency (due to the reduced rate) of $10\log_{10}(1/R_{bch/rs})$dB [28], where $\mathcal{R}_{bch/rs}$ is the rate of the BCH or RS code.

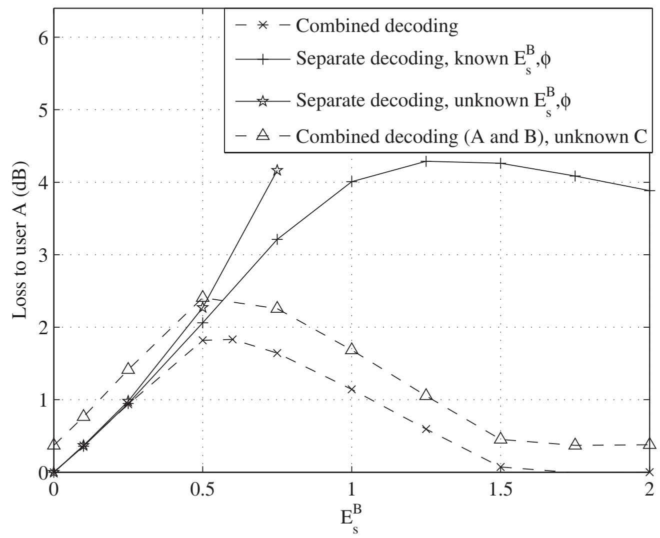

Fig. 5. Loss to user A compared to single user QPSK performance at $FER = 10^{-3}$ in the case of separate and combined decoding of user A and B. In the case of separate decoding we consider the case when E_s^B and ϕ are both known or unknown by the decoder (this alters the metric calculation). In addition, the performance of combined decoding of user A and B is considered in the presence of a third unknown user, C, with $E_s^C = 0.1$.

($N = 1500$) would need to transmit 16-QAM and use a Turbo code with $k = 2000$ data bits. Note, we assume that all users transmit using an average symbol energy of $E_s = 1$. If we were to use 16-QAM with an average symbol energy of $E_s = E_s^A + E_s^B = 2$, this would improve performance by 3dB. The single user 16-QAM system uses either bit interleaving with Gray mapping to 16-QAM or symbol interleaving with 4-bit symbol mapping to 16-QAM.

We found that the interleaver design was more difficult for the single user 16-QAM case. In order to maintain good convergence we had to perform symbol decoding with interleaver symbol size equal to the modulation symbol size (in this case, 4 bits). This requirement is illustrated in Fig. 6, where the single user 16-QAM system with a symbol based interleaver outperforms that with a bit based interleaver in terms of FER. This is in spite of the fact that the minimum distance of the Turbo code with the bit interleaver is $d_{min} = 51$ as opposed to $d_{min} = 32$ when using the symbol interleaver. In addition, the symbol interleaver case requires fewer iterations on average at a given BER/ FER as shown in Fig. 6. This shows the importance of using symbol decoding for Turbo codes with higher order modulation (also mentioned in [16]). However, this requirement reduces the interleaver design freedom. Overall the 4-bit symbol interleaver has size 500 symbols (2000 bits) and $S = 12$ (after swaps). While the bit interleaver has size 2000 bits and $S = 25$ (after swaps). One design freedom that has not been investigated to date is intra-symbol permutations.

In order to obtain good error floor performance we use a memory 5 component code with ff $= 45$ (octal) and fb $= 67$ (octal). The performance is compared to the two user performance in Fig. 6. As can be seen, the two user performance is only 0.2 dB away from the single user 16-QAM (symbol interleaver) performance at a FER of 10^{-2}. The single user QPSK performance is also shown in Fig. 6. It

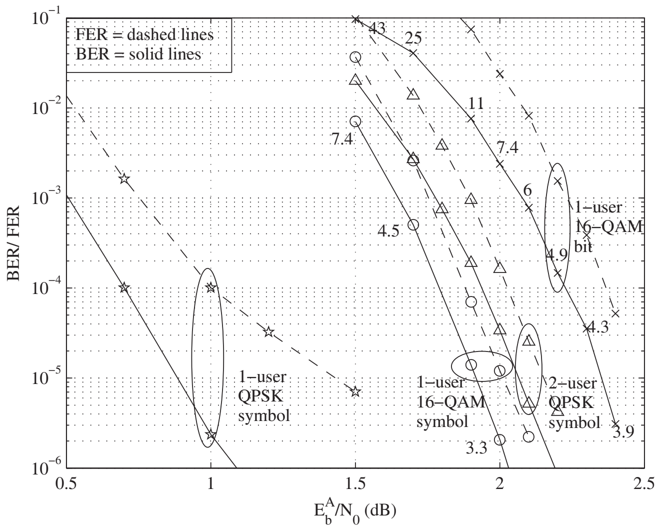

Fig. 6. Single user (16-QAM and QPSK performance) versus two user (QPSK) performance. Results use either a symbol or bit interleaver as labelled. The average number of iterations performed are shown for the single user 16-QAM BER curves.

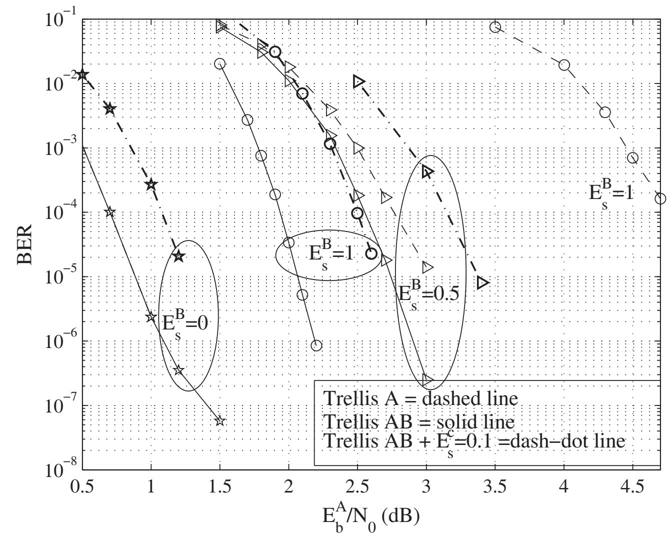

Fig. 7. BER performance when using a combined trellis for user A and B, trellis AB, and when using a trellis for only user A, trellis A. Both have the same performance for $E_s^B = 0$. In addition, the performance for trellis AB in the presence of an unknown third user, C, with $E_s^C = 0.1$ is shown.

provides the best performance, but only transmits $k = 1000$ data bits over $N = 1500$ symbol periods.

C. Two user system with independent user A decoding

We now consider a trellis decoder in a two user scenario where only the constraints of user A's Turbo code are used (no information about user B's code is used). Note that the soft input metric has knowledge of E_s^B and ϕ. It averages the metric over all possible user B constellation points. In this instance, for all $l = 1, \cdots, M$ (6) becomes

$$\mathcal{M}_t^{i,l} = \frac{\sum_{p=1}^{M} \Gamma_t^{i,p}}{\sum_{q=1}^{M} \sum_{p=1}^{M} \Gamma_t^{q,p}}, \quad ,i,q,p = 1, \cdots, M. \quad (16)$$

The BER and FER performance when the decoder uses only the constraints of user A's Turbo code are shown in Fig. 7. For $E_s^B = 0$ we have the single user QPSK performance, which is unchanged. For $E_s^B = 0.5$ we get a loss of approximately $0.25dB$ at a BER of 10^{-5} when using only user A's code constraints instead of the combined trellis. As can be seen, when $E_s^B = 1$ the loss increases to approximately $2.8dB$ at a BER of 10^{-4}. The loss to user A with increasing E_s^B, compared to single user QPSK performance, is shown in Fig. 5 for separate and combined decoding (at FER= 10^{-3}). For separate decoding the loss increases with E_s^B to a maximum of 4.29 dB at around $E_s^B = 1.25$ and then decreases. For combined decoding the loss increases to a maximum of 1.8 dB at around $E_s^B = 0.5$ and then decreases. Since E_s^B and ϕ are known by the receiver in both cases, once E_s^B is large enough the constellation improves sufficiently to allow improved performance. If we do not have information about E_s^B and ϕ and use only the user A trellis, then there is a further performance loss compared to the known E_s^B and ϕ case for $E_s^B \geq 0.5$, and this increases with E_s^B as seen in Fig. 5. In this case, we do not expect performance to improve for higher E_s^B values.

D. Two user system with an unknown interferer

We now consider the case of having a third unknown user/ interferer (user C). We assume that the receiver is unaware that user C exists (no knowledge of the code, signal energy or received phase). The new received signal is given by

$$\boldsymbol{r} = \sqrt{E_s^A}\boldsymbol{s}^A + \sqrt{E_s^B}\boldsymbol{s}^B \exp(-j\boldsymbol{\phi}) + \sqrt{E_s^C}\boldsymbol{s}^C \exp(-j\boldsymbol{\phi}_C) + \boldsymbol{n}, \quad (17)$$

where E_s^C is the signal energy of user C and $\boldsymbol{\phi}_C$ is the phase difference between user C and A. In this case we model $\boldsymbol{\phi}_C$ as a uniformly distributed random variable on $(0, 2\pi)$. User C is encoded using a rate $\mathcal{R}_{ecc}^C = 1/3$ Turbo code, however it could be random data. The receiver operates as if we had a two user (user A and B) system. BER performance is compared to the two user case in Fig. 7 for $E_s^C = 0.1$. As expected we lose performance compared to the two user case (or single user QPSK case with $E_s^B = 0$). However, successful decoding is still possible showing the robustness of the proposed system. The loss compared to the single user QPSK ($E_s^B = 0$) system is also shown in Fig. 5 for $E_s^C = 0.1$. The system can cope with larger values of E_s^C, but at the cost of further performance loss.

E. Impact of phase errors

The effect on FER of various phase differences between user A and B is shown in Fig. 8. When $\boldsymbol{\phi} = \mathbf{0}$ (or a multiple of $\pi/2$) and $E_s^A = E_s^B = 1$ the QPSK signals sent from user A and B have the same phase orientation and magnitude, and so some values cancel out when added together by the channel (see Fig. 1). This results in only 9 composite constellation points (ignoring AWGN) rather than the 16 unique points we would normally expect. This can be considered as a form of erasure channel. But as shown in Fig. 1 even a small value of $\boldsymbol{\phi}$ can result in 16 distinct points and hence in better performance as shown in Fig. 8. We looked at finding the optimal phase for each value of E_s^B, but as can be seen in Fig. 8 this provides

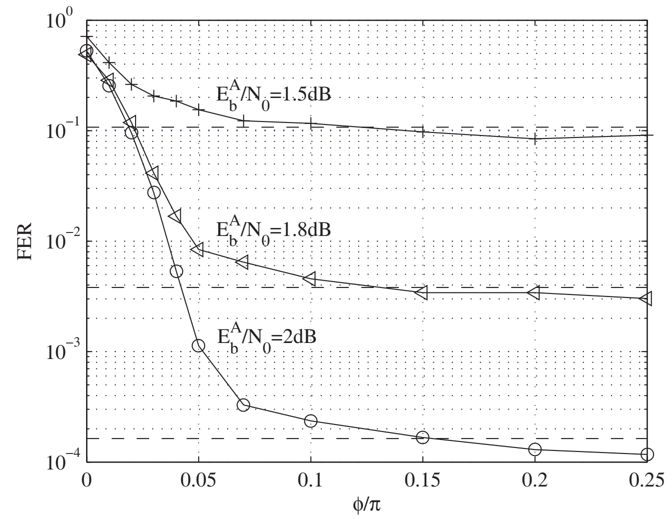

Fig. 8. Impact on FER of various fixed phase differences between user A and B, for $E_s^A = E_s^B = 1$. Dashed lines show result for a linearly time-varying phase difference between user A and B.

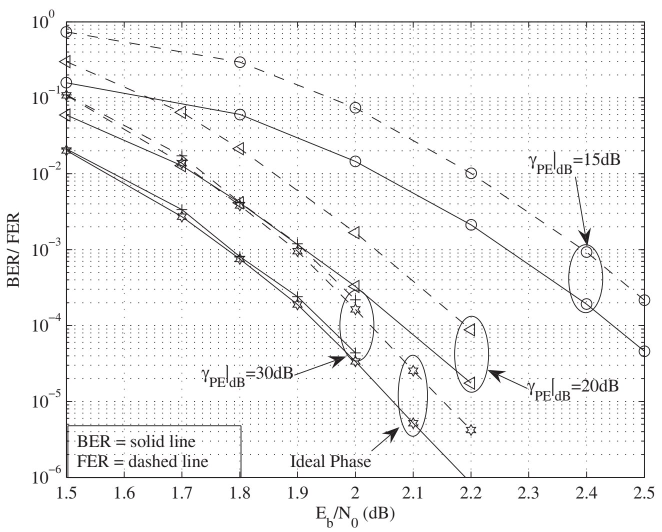

Fig. 9. Impact on BER and FER of phase estimation errors for $E_s^A = E_s^B = 1$ given a loop SNR of $\gamma_{PE}|_{dB}$.

little advantage over allowing the phase to vary linearly over the block. A time varying known phase is the more realistic situation in a satellite system.

We now look at the impact of imperfect phase estimation on performance. We model the phase error as a Gaussian random variable with variance $\frac{1}{\gamma_{PE}}$ and PDF given by (14). As a result, the channel metric of (15) is used. The BER and FER performance for $E_s^A = E_s^B = 1$ and various values of $\gamma_{PE}|_{dB}$ are shown in Fig. 9. As can be seen $\gamma_{PE}|_{dB} = 30dB$ results in similar performance to having no phase error, while at a BER of 10^{-3} we get a loss of approximately $0.13dB$ for $\gamma_{PE}|_{dB} = 20dB$ and approximately $0.48dB$ for $\gamma_{PE}|_{dB} = 15dB$. This shows the proposed system is robust to small random phase errors. Before practical implementation more sophisticated phase error models would need to be considered.

IV. Conclusions

A two-user satellite communication system has been described in which each user transmits a structured signal encoded with a Turbo code, which allows iterative joint decoding. The complexity of the proposed scheme is low due to the use of symbol based, common interleaver Turbo codes and iterative decoding. Using a combined user A and B trellis decoder to decode each of the component codes makes the decoding of the two user signals collaborative. The gain compared to the use of a single user trellis increases with increasing energy, E_s^B, of the second user. When the two users have equal power, the gain is 2.8 dB at a BER of 10^{-4}. The proposed approach can handle time-varying phase differences with negligible loss in performance compared to an optimized fixed phase offset between users. We have also investigated the impact of imperfect phase estimation on performance. The loss in performance is small if a phase estimator with good loop SNR is used (such as $\gamma_{PE}|_{dB} \geq 20dB$).

There are still several open research questions to address before any practical implementation of the proposed system. These include both symbol timing and carrier phase synchronization, issues involved with handling more than two users and protocols to ensure that all users obtain reasonable performance.

Acknowledgment

The authors wish to thank M. Z. Ahmed, D. M. Rankin and A. Rogers for useful discussions during this project. They also wish to thank the Editor and reviewers for their constructive comments.

References

[1] N. Abramson, “VSAT data networks,” *Proc. IEEE*, vol. 78, pp. 1267-1274, July 1990.
[2] N. Abramson, “Fundamentals of packet multiple access for satellite networks,” *IEEE J. Select. Areas Commun.*, vol. 10, pp. 309-316, Feb. 1992.
[3] N. Abramson, “Internet access using VSATs,” *IEEE Commun. Mag.*, pp. 60-68, July 2000.
[4] L. R. Bahl, J. Cocke, F. Jelenik, and J. Raviv, “Optimal decoding of linear codes for minimising symbol error rate,” *IEEE Trans. Inform. Theory*, vol. 20, pp. 284-287, Mar. 1974.
[5] B. F. Beidas, H. El Gamal, and S. Kay, “Iterative interference cancellation for high spectral efficiency satellite communications,” *IEEE Trans. Commun.*, vol. 50, no. 1, pp. 31-36, Jan. 2002.
[6] S. Benedetto, G. Montorsi, D. Divsalar, and F. Pollara, “Serial concatenation of interleaved codes: performance analysis, design and iterative decoding,” *JPL TDA Progress Report*, vol. 42-126, Aug. 15, 1996.
[7] C. Berrou, P. Thitimajshima, and A. Glavieux, “Near Shannon limit error correcting coding and decoding: turbo codes,” in *Proc. ICC*, pp. 1064-1070, Geneva, Switzerland, May 1993.
[8] C. Berrou and M. Jezequel, “Frame-oriented convolutional turbo codes,” *Electron. Lett.*, vol. 32, no. 15, pp. 1362-1364, 18 July 1996.
[9] F. Brannstrom, L. K. Rasmussen, and A. Grant, “Optimal scheduling for iterative decoding,” in *Proc. ISIT*, pp. 350, Yokohama, Japan, June 2003.
[10] N. Chayat and S. Shamai (Shitz), “Iterative soft onion peeling for multi-access and broadcast channels,” in *Proc. PIMRC*, pp. 1385-1390, 1998.
[11] T. Cover, “Some advances in broadcast channels,” *Advances in Communication Systems*, vol. 4, A. Viterbi, ed., Academic Press, 1975.
[12] D. Divsalar and F. Pollara, “Weight distributions for turbo codes using random and nonrandom permutations,” *JPL TDA Progress Report*, vol. 42-122, pp. 56-65, Aug. 1995.
[13] D. Divsalar and F. Pollara, “Hybrid concatenated codes and iterative decoding,” *JPL TDA Progress Report*, vol. 42-130, Aug. 1997.

[14] C. Douillard and C. Berrou, "Turbo codes with rate-m/(m+1) constituent convolutional codes," *IEEE Trans. Commun.*, vol. 53, no. 10, pp. 1630-1638, Oct. 2005.
[15] DVB-RCS ETSI Standard EN 301 790.
[16] C. Fragouli and R. Wesel, "Turbo-encoder design for symbol-interleaved parallel concatenation trellis-coded modulation," *IEEE Trans. Commun.*, vol. 49, no. 3, pp. 425-435, Mar. 2001.
[17] R. Garello, P. Pierleoni and S. Benedetto, "Computing the free distance of turbo codes and serially concatenated codes with interleavers: algorithms and applications," *IEEE J. Select. Areas Commun.*, vol. 19, no. 5, pp. 800-812, May 2001.
[18] M. C. Jeruchim, P. Balaban, and K. S. Shanmugan, *Simulation of Communication Systems*. Plenum Press, 1992.
[19] N. Mehravari, "TDMA in a random-access environment: an overview," *IEEE Commun. Mag.*, vol. 22, pp. 54-59, Nov. 1984.
[20] L. H. Ozarow, "The capacity of the white Gaussian multiple access channel with feedback," *IEEE Trans. Inform. Theory*, vol. IT-30, no. 4, pp. 623-629, July 1984.
[21] E. B. Parker, "Micro earth stations as personal computer accessories," *Proc. IEEE*, pp. 1526-1531, Nov. 1984.
[22] A. C. Reid, T. A. Gulliver, and D. P. Taylor, "Convergence and errors in turbo decoding," *IEEE Trans. Commun.*, vol. 49, no. 12, pp. 2045-2051, Dec. 2001.
[23] S. Riedel, "MAP decoding of convolutional codes using reciprocal dual codes," *IEEE Trans. Inform. Theory*, vol. 44, no. 3, pp. 1176-1187, May 1998.
[24] E. Rosnes and O. Ytrehus, "Improved algorithms for the determination of turbo-code weight distributions," *IEEE Trans. Commun.*, vol. 53, no. 1, pp. 20-26, Jan. 2005.
[25] C. E. Shannon, "Probability of error for optimal codes in a Gaussian channel," *Bell System Technical J.*, vol. 38, no. 3, pp. 611656, May 1959.
[26] O. Y. Takeshita, O. M. Collins, P. C. Massey, and D. J. Costello Jr., "On the frame error rate of concatenated turbo codes," *IEEE Trans. Commun.*, vol. 49, no. 4, pp. 602-608, Apr. 2001.
[27] F. A. Tobagi, R. Binder, and B. Leiner, "Packet radio and satellite networks," *IEEE Commun. Mag.*, vol. 22, pp. 24-40, Nov. 1984.
[28] M. C. Valenti, "Inserting turbo code technology into the DVB satellite broadcasting system," in *Proc. MILCOM*, 2000.
[29] C. J. Wolejsza, D. P. Taylor, M. Grossman, and W. P. Osborne, "Multiple access protocols for data communications via VSAT networks," *IEEE Commun. Mag.*, pp. 30-39, July 1987.
[30] A. D. Wyner, "Recent results in Shannon Theory," *IEEE Trans. Inform. Theory*, vol. IT-20, pp. 2-10, Jan. 1974.
[31] G. Zhou, T.-S. Lin, W. Wang, W. C. Lindsey, D. Lai, E. Chen, and J. Santoru, "On the concatenation of turbo codes and Reed-Solomon codes," in *Proc. ICC*, pp. 2134-2138, 2003.

Philippa A. Martin (S'95-M'01-SM'06) received the B.E. (Hons. 1) and Ph.D. degrees in electrical and electronic engineering from the University of Canterbury, Christchurch, New Zealand, in 1997 and 2001, respectively. From 2001 to 2004, she was a postdoctoral fellow, funded in part by the New Zealand Foundation for Research, Science and Technology (FRST), in the Department of Electrical and Computer Engineering at the University of Canterbury. In 2002, she spent five months as a visiting researcher in the Department of Electrical Engineering at the University of Hawaii at Manoa, Honolulu, Hawaii, U.S.A. Since 2004 she has been working at the University of Canterbury as a lecturer and then as a senior lecturer (since 2007). In 2007, she was awarded the University of Canterbury, College of Engineering young researcher award. She served as an Editor for the IEEE TRANSACTIONS ON WIRELESS COMMUNICATIONS 2005-2008 and regularly serves on technical program committees for IEEE conferences. Her current research interests include multilevel coding, error correction coding, iterative decoding and equalization, space-time coding and detection, and cooperative communications in particular for wireless communications.

Marcel A. Ambroze (M'01) received the BEng degree from Technical University of Cluj-Napoca, Romania in 1996 and the PhD in concatenated codes with iterative decoding for data transmission from the University of Plymouth, UK in 2000. He worked as a Research Fellow for two EPSRC projects in Digital Watermarking and Signal Processing for Small Satellite Earth Terminals at the University of Plymouth until 2003. Since 2003 he was employed as a Lecturer in Digital Communications Systems at the University of Plymouth. He is currently a member of the Fixed and Mobile Communications Research at the University of Plymouth, lead by Professor Martin Tomlinson. He has published over 30 papers in the fields of Error Correction Coding, Watermarking and Satellite Communications. His current research interests are in error correction coding and iterative decoding, signal processing and coding for wireless systems and watermarking as communications with side information at the transmitter.

Desmond P. Taylor (M'65-SM'90-F'94-LF'07) was born in Noranda, Quebec, Canada on July 5, 1941. He received the B.Sc.(Eng.) and M.Sc.(Eng.) degrees from Queen's University, Kingston, Ontario, Canada in 1963 and 1967 respectively, and the Ph.D. degree in 1972 in Electrical Engineering from McMaster University, Hamilton, Ontario, Canada. From July 1972 until June 1992, He was with the Communications Research Laboratory and Department of Electrical Engineering of McMaster University. Since July 1992, he has been with the University of Canterbury, Christchurch, New Zealand, where he is the Tait Professor of Communications. His research interests are centred on digital SISO and MIMO wireless communications systems with a primary focus on the development of robust, bandwidth-efficient modulation and coding techniques, and the development of iterative algorithms for joint equalisation and decoding. Secondary interests include problems in synchronisation, multiple access and networking. He is the author or co-author of approximately 225 published papers and holds two U.S. patents in spread spectrum communications. One paper won the S.O. Rice Award for the best Transactions paper in Communication Theory of 2001.

He is a Life Fellow of the IEEE, a Fellow of the Royal Society of New Zealand, and a Fellow of both the Engineering Institute of Canada and the Institute of Professional Engineers of New Zealand.

Martin Tomlinson (M'96-SM'08) received the BSc degree from the University of Birmingham, UK in 1967 and the PhD in adaptive equalisation for data trans-mission from the University of Loughboroughin 1970. He is best known for the invention of the Tomlinson-Harashima precoding technique.

He worked at Plessey Telecommunications Research Ltd, UK until 1975 in dig-ital communications and satellite transmission. He then spent seven years with the UK Ministry of Defence in the Satellite Communications Division of RSRE where he worked on the Skynet satellite, space and ground segments. He was project manager for the communications payload of the NATO IV satellite before joining the University of Plymouth as the Head of the Communication Engineering Department in 1982 where he became a Professor in 1987.

Dr Tomlinson is currently Head of Fixed and Mobile Communications Research at the University of Plymouth, leading research projects in satellite communications, coding, signal processing, wireless and watermarking which are his current research areas. He is a well known consultant to ESA, the UK Met Office and the satellite communications industry. In 2005 he was a Visiting Erskine Fellow at the University of Canterbury, NZ.

He has published over 200 papers in the fields of Digital Modulation and Coding, Signal Processing, Video Coding and Satellite Communications, as well as contributing towards various satellite and wireless standards. Professor Tomlinson has filed over 50 patents and is a member of the IET, a member of the IEEE Signal Processing Society, a member of the IEEE Information Theory Society, a member of the IEEE Communications Society and a member of the IEEE Satellite and Space Communications Society.