# Financial Seismic Risk Assessment of RC Bridge Piers using a Distribution-Free Approach

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ABSTRACT: Expected annual loss (EAL), which can be expressed in dollars, is an effective way of communicating the seismic vulnerability of constructed facilities to owners and decision makers. A concise method for computing EAL without the inherent bias of requiring a specific analytical probability distribution is presented. The relationships between intensity measures and engineering demand parameters resulting from an Incremental Dynamic Analysis are sorted into fractal intervals by way of spectral reordering and modified to incorporate additional sources of uncertainty and randomness. Damage measures are defined to determine thresholds for damage states. Damage is quantified by loss ratios defined as repair cost divided by replacement cost. The results are numerically integrated to give EAL. An example illustrating the method is performed, comparing the seismic vulnerability of two highway bridge piers; one pier traditionally designed for ductility, and the other designed for damage avoidance. The damage avoidance pier has a clear advantage over the conventional pier, with an EAL some 85% less than its ductile counterpart. This is shown to be primarily due to its inherent damage-free behaviour for almost all ground motions, except for very rare events that could potentially lead to toppling.

# 1 INTRODUCTION

One primary aim of *performance based earthquake engineering* (PBEE) is to predict, with a certain level of confidence, the seismic performance of structures at various levels of excitation, or seismic demand. This requires the engineer to understand seismic risk and its inherent randomness and modelling uncertainty. As an adjunct to conventional design it is desirable that the engineer be able to communicate that risk in a way easily understood by decision makers such as owners, bankers, and insurers. One primary development is the *Pacific Earthquake Engineering Research* (PEER) *Centre's* triple integral framework formula, which can be used to arrive at a mean annual frequency of a decision variable (Deierlein et al. 2003):

$$\lambda(dv) = \iiint G(dv \mid dm) \mid dG(dm \mid edp) \parallel dG(edp \mid im) \parallel d\lambda(im) \mid (1)$$

where im = intensity measure (i.e. peak ground acceleration, spectral acceleration); edp = engineering demand parameter (i.e. maximum interstory drift); dm = damage measure (i.e. maximum drift without damage); dv = decision variable (i.e. repair cost, downtime); and G(x/y) = P(x < X/Y = y); the conditional complementary cumulative distribution function. Equation 1 can be broken into four subtasks: (i) assessment of seismic hazard; (ii) analysis for structural response; (iii) quantification of damage; and (iv) estimation of the decision variable.

Investigations of equation 1 (Der Kiureghian 2005) have hinted on the possibility of performing a fourth integration, thereby arriving at the mean cumulative value of a *decision variable* (DV) given one year in time. In the context of financial risk assessment, this could come in the form of *expected annual loss* (EAL). EAL is an effective communication tool. It incorporates a range of seismic scenarios, return rate, and expected damage into a single median dollar loss. This is especially useful to decision makers for financial analysis of design alternatives or seismic retrofit and general inclusion into operating budgets.

A primary step within PBEE is defining a relationship between specified demand levels and a hazard environment. This relationship, termed the demand model, has gained a lot of attention in the past decade. Vamvatsikos and Cornell (2002) investigated the feasibility of using an *Incremental Dynamic Analysis* (IDA) as a means of relating an *engineering demand parameter* (EDP) to an *intensity measure* (IM). An IDA basically consists of performing a series of time-history analyses to arrive at a set of EDP's, obtained by scaling the IM to various intensities over a suite of earthquake records. It is similar (though far superior) to a static pushover in that it encompasses the entire range of likely behaviour, from pre-yield to collapse. Cornell et al (2002) proposed data generated from IDA can be approximated by a median curve:

$$EDP = a(IM)^{\flat} \tag{2}$$

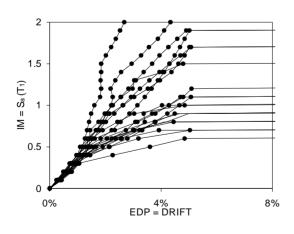
where *a* and *b* are empirical constants. Assuming the above relationship will conform to a lognormal distribution, a lognormal standard deviation,  $\beta$ , can be used with equation 2 to fully define the demand model. This method has gained considerable acceptance since its inclusion in Appendix A of FEMA 350 (2000). Furthermore, when combined with a similar hazard model, the integrations can be solved in closed form, greatly simplifying the process.

As an alternative to the above method, this study will set out to develop a new way of processing IDA data in the context of financial seismic risk assessment. Rather than assuming a probability distribution, this method, termed the distribution-free method, will reorder data by way of spectral reordering. It will go on to incorporate various forms of uncertainty and randomness, and finally numerically integrate to arrive at EAL. A case study of two different bridge piers will illustrate the effectiveness of the approach and its potential for loss estimation and decision making.

## 2 INCREMENTAL DYNAMIC ANALYSIS

The basic concept of IDA has been well researched (Vamvatsikos and Cornell 2002) and is not a focus of this study, but rather the data processing that follows. However, a brief description of the analysis technique follows.

Conducting an IDA consists of running a series of inelastic dynamic time history analyses at various levels of excitement, over a suite of earthquake records. This results in a matrix of data from which probabilistic studies can be conducted. Figure 1 illustrates an example IDA for a single degree of freedom system, where the IM is scaled at increments of 0.1g. Choosing an appropriate IM is an important step, since it can have significant effects on the scatter of data. A current best practice suggests using the 5% damped spectral acceleration at the fundamental period of the structure. This IM was adopted for this study, as illustrated in Figure 2.



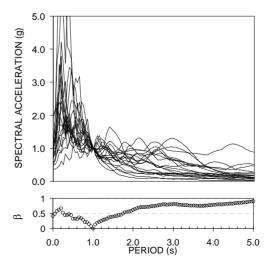


Figure 1. An example IDA for a single degree of freedom system. Each dot is a separate analysis.

Figure 2. The 5 % damped spectral acceleration scaled to 1g at 1 second and the lognormal standard deviation.

Performing an IDA can be a complicated process. An inelastic structural model must be prepared, a suite of appropriate earthquake records must be selected, and an efficient way of performing a series of dynamic analyses must be developed. Though this process may seem time consuming, being computational, it can largely be automated.

## **3 THE DISTRIBUTION-FREE METHOD**

#### 3.1 Spectral re-ordering and accounting for sources of variation

The resulting data points are analysed using a procedure called spectral reordering to organize data and assign confidence bounds. This is accomplished by sorting the EDP's at each IM in descending order and defining survival probability, *S*, by the following formula:

$$S_i = 1 - \frac{i - 0.5}{n} \tag{3}$$

where i = the rank of EDP's (i.e. n to 1) in descending order and n = number of earthquake records used for IDA. No curve fitting is performed, as is typical (and necessary) to arrive at closed form solutions for integration. Though this method does not lend itself to an easily quantifiable curve (i.e. a simple median and dispersion) it is a more 'exact' representation of the data presented, though the size and relevance of the earthquake suite can dictate the accuracy of results.

The variability of results apparent in Figure 1 comes solely from the input motion; however other sources of uncertainty and randomness must be considered. Throughout this process numerous approximations are made regarding damping, material strengths, modelling simplifications, etc. These approximations can be grouped into epistemic uncertainty, where further investigation may lead to an increase in accuracy, and aleotoric variability (randomness), which cannot be reduced by its random nature. An example of the former would be uncertainty in analytical modelling, and of the later would be the inherent record-to-record randomness of earthquake ground motions.

Knowing that the data will likely conform to a lognormal distribution ( $\beta$ ), it is possible to modify the data to incorporate broader variation. As part of the present research, a convenient explicit form of a cumulative probability function representing the lognormal distribution, before and after scaling, can be shown to be:

$$P = \frac{1}{1 + \left(\frac{x_s}{\widetilde{x}}\right)^{\frac{18}{\overline{\rho}/D}}} = \frac{1}{1 + \left(\frac{x_u}{\widetilde{x}}\right)^{\frac{18}{\overline{\rho}_D}}}$$
(4)

in which  $\tilde{x} =$  median;  $x_u$  = unscaled data points with group dispersion of  $\beta_D$  that arises from record-torecord randomness only; and  $x_s$  = rescaled data that accounts for all sources of aleotoric randomness and epistemic uncertainty,  $\beta_{CD}$ . A proof of equation (4) is given elsewhere (Solberg 2006). For consistent probabilities before and after spectral scaling, rearranging equation 4 gives:

$$x_{s} = \widetilde{x} \cdot \left| \frac{x_{u}}{\widetilde{x}} \right|^{\frac{\beta_{C/D}}{\beta_{D}}} = \widetilde{x} \cdot \left| \frac{x_{u}}{\widetilde{x}} \right|^{\sqrt{1 + \frac{\beta_{C}^{2} + \beta_{U}^{2}}{\beta_{D}^{2}}}}$$
(5)

To determine  $\beta_{C/D}$ , variations are combined by the root-sum-squares method (Kennedy et al. 1980):

$$\beta_{C/D} = \sqrt{\beta_D^2 + \beta_C^2 + \beta_U^2}$$
(6)

where  $\beta_c$  = the aleotoric randomness in structural capacity; and  $\beta_U$  = epistemic modelling uncertainty. In this study, recommendations of FEMA 350 (2000) have been adopted, thus  $\beta_c = 0.2$  and  $\beta_U = 0.25$ . The reordered curves, incorporating this randomness and uncertainty, are illustrated in Figure 3, which also gives median response.

## 3.2 Assign damage limit states and loss ratios

This study will adopt *damage states* (DS) as a damage measure as defined in *HAZUS* (Mander and Basoz 1999) and summarized in Table 1. These DS's were developed for bridge piers; applicable to the case study to follow.

For each damage state it is possible to associate it with a *loss ratio* (LR), defined as the cost of repair to full function divided by the cost of complete replacement of the structure. The LR will be used as the decision variable in this study.

Damage State		Failure Mechanism	Loss Ratio; Range (duct / DAD)	Drift Limit (duct / DAD)
DS1	None	Pre-Yield	0	0.6% / 3.0%
DS2	Minor / Slight	Post-Yield / Spalling	0.03/0.01 ; 0.01-0.03	2.2% / -
DS3	Moderate	Post-Spalling, Bar Buckling	0.08/- ; 0.02-0.15	3.6% / -
DS4	Major / Extensive	Degrading of Strength	0.25/- ; 0.10-0.40	4.9% / 10.0%
DS5	Complete	Collapse	1.00/1.00 ; 0.30-1.00	

Table 1. Damage states definitions, loss ratios and damage limits adopted for the case study.

#### 3.3 Define an earthquake recurrence relationship

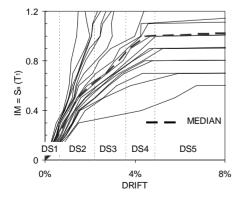
To arrive at an EAL, it is necessary to define a relationship between an IM and annual frequency  $(f_a)$ . It is possible to approximate the hazard curve by fitting a log-log linear curve through two known points:

$$f_a(IM) = k_a(IM)^{-k} \tag{7}$$

where  $k_o$  and k are empirical constants. Using the 1 second spectral acceleration as the IM, plotted in Figure 4 is equation 7 which can be written as follows for a high seismic zone in New Zealand:

$$f_a = \frac{1}{475} \left( \frac{F_v S_1}{0.4} \right)^{-3} \tag{8}$$

As noted by Der Kiureghian (2005), earthquakes are discrete, rather than continuous events, and should be modelled as a Poisson process. In this case, the hazard relation curve formulated above, though conservative, is perhaps not strictly correct when  $f_a > 0.01$  (T < 100 years). In this study, the aforementioned deficiency is rectified by disregarding any damage potential below a certain threshold. This threshold is defined by 90% probability of not sustaining damage. In other words, this is the intersection of the 90<sup>th</sup> percentile curve and the DS1-2 limit.



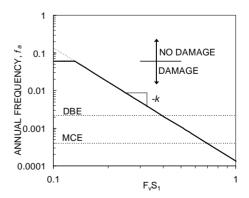


Figure 3. Re-ordered IDA curve incorporating uncertainty and randomness

Figure 4. Earthquake recurrence relationship for a high seismic zone in New Zealand

## 3.4 Calculate EAL

For a given DM, the corresponding IM at each confidence interval can be found by linear interpolation of the re-ordered data. Given the hazard recurrence relationship, this can be expressed by a hazard survival curve, where the probability of *not* exceeding a damage state is plotted against a range of annual frequencies. Given the set of data points from the spectrally modified IDA, EAL can be computed by numerically integrating the following:

$$EAL = \sum_{i=1}^{n} \sum_{j=1}^{m} f_{a_i} LR_j \frac{1}{n}$$
(9)

where n = total number of earthquake records; m = number of damage states; and*i*corresponds to each confidence interval calculated in equation (4). This formula is the volume of the 3D plot given in Figure 5.

# 4 CASE STUDY

To illustrate the effectiveness of the method presented, a case study will be conducted to compare the performance of two highway bridges with substantially different design attributes; the first being a conventional pier designed for ductility, the second a rocking pier designed for damage avoidance.

## 4.1 **Prototype design details**

Consider the reinforced concrete bridge pier shown in Figure 6. The pier is typical of modern design, with 40m spans, 10m transverse width, and 7m height. The seismic weight of the super-structure is assumed to be 7,000kN and it is located in a high seismic, firm soil site in New Zealand. The peak ground acceleration of the design basis earthquake (10%/50yrs) is 0.4g. Two structural design alternatives were considered. One alternative being a conventional ductile system, detailed according to the concrete design standard of New Zealand (NZS3101 1995), the second designed according to the principles of *damage avoidance design* (DAD). To avoid damage, the latter implements the steel armouring techniques developed by Mander and Cheng (1997). The bridge column is post-tensioned to the foundation to provide strength and stiffness and supplementary dampers are provided to increase energy dissipation. Both piers were designed for the same moment capacity.

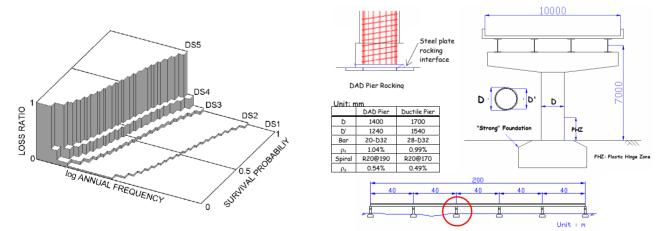


Figure 5. A hazard-survival plot with loss ratios. The volume of this plot will give an approximate EAL.

Figure 6. Prototype bridge design details used in the case study.

#### 4.2 Perform Incremental Dynamic Analysis

To perform the IDA, a non-linear structural model was developed. A pier most likely to be critical to the structure was idealized as a SDOF system; i.e. a lumped mass centreline column with rotational springs at its base. The hysteretic properties of the springs were calibrated based on experimental testing. The ductile pier was modelled using a Takeda hysteretic loop and strength degradation after

ductility of 8. Using the multi-spring principles of modelling, as given in the original DRAIN-2DX software for this class of precast concrete structure (Prakash et al. 1992), the DAD pier was modelled using two springs; one spring representing the bi-linear elastic behaviour inherent in post-tensioned rocking systems and a second elasto-plastic spring representing energy dissipation. Thus, Ruaumoko2D, an inelastic dynamic time-history analysis program (Carr 2004), was used to conduct the analyses. Elastic Rayleigh damping was taken as 5%. Soil-structure interaction was not considered (assumed firm soil site), nor was torsion and slab interaction.

The 20 records used by Vamvatsikos and Cornell (2004) were adopted for this study. These records range in magnitude between 6.5 and 6.9, have moderate epi-central distance, and are recorded on firm soil. The natural period for both designs was approximately  $T_n = 1.0$  second, thus  $S_a(T_n=1\sec,5\%)$  is taken as IM. The twenty records were scaled twenty times, from 0.1 - 2.0g, resulting in 400 separate analyses. The resulting IDA plots are given in Figure 7a, along with the re-ordered curves in Figure 7b.

# 4.3 Computation of EAL by distribution-free method

EDP limits and LR's were established as shown in Table 1. The DS1-2 limit for the DAD pier was calculated based on yielding of the tendons, while for the ductile pier this was calculated for yield of the longitudinal steel. Following the theory presented in Section 3, hazard-survival curves can be formed. These curves are numerically integrated, as illustrated in Figure 5, for EAL. The results are of stark contrast for the piers. EAL was calculated to be \$990 and \$140 per \$1 million of asset value for the ductile and DAD piers, respectively.

# 5 **DISCUSSION**

# 5.1 Comparison of structural systems

The resulting EAL for the ductile and DAD systems were of stark contrast, even though both systems had similar ultimate capacities. The ductile system's vulnerability was dominated by minor damage occurring at relatively low drifts and moderate ground shaking. The DAD pier, on the other hand, was dominated by the ultimate failure condition, resulting in an annual risk of only 15% that of the ductile system. Such an improvement of performance is attributed to the rocking concept, which is relatively new in earthquake engineering. Findings from this study suggest current design practices are deficient in protecting the structure from minor yet costly damage. Improvement in this area, possibly through further development of DAD, is a necessary next step for earthquake engineers.

The findings of this and further DAD system studies may be the evidence needed for DAD to become feasible in a highly competitive construction industry. Although this study was limited to a bridge pier, further studies at the University of Canterbury are investigating the methodology applied to large moment-resisting frame systems. Results should further clarify the advantages of the system and where improvements can be made.

Not considered in this study is the significant additional risk associated with non-structural damage, downtime, and loss of life. Bridges are especially vital for the flow of goods and people; closure of key transportation arteries can have severe economic consequences. DAD, by rocking, will likely incur little to no residual displacement, allowing full operation following even large events. Non-structural damage will need to be considered in buildings, where it is likely to govern global loss. Study in these areas is in its infancy, and further work regarding such risk is necessary to fully address the viability of damage avoidance design.

# 5.2 **The distribution-free method**

Goals of this study were to develop a simple and effective method to quantify the seismic vulnerability of a structure by calculation of its EAL. A 'distribution free' approach was proposed, based largely on concepts born from the PEER triple integral formula, expanded to include the dimension of time.

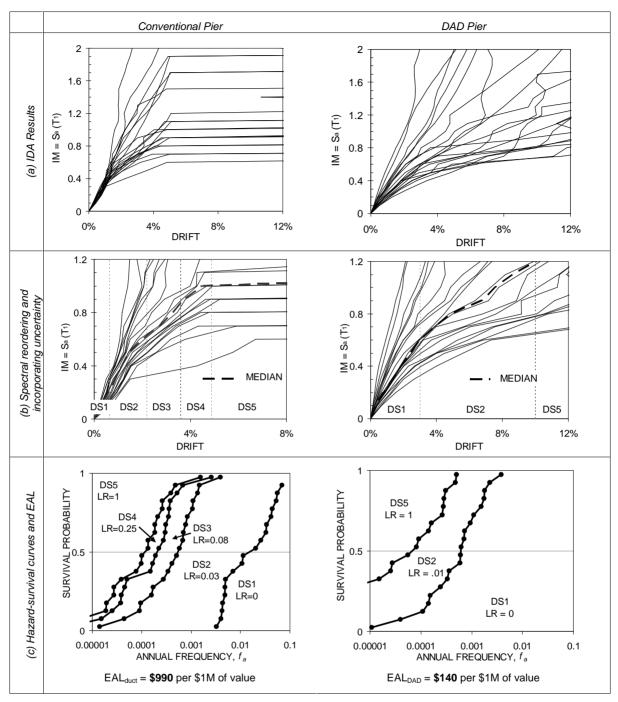


Figure 7. Results of a distribution-free financial risk assessment of two different bridge piers.

The proposed approach can be an effective tool in seismic risk assessment and decision making for engineers, owners, and insurers. The relative simplicity of the method comes from easy numerical integration, which can be incorporated into a spreadsheet and automatically calculated given a set of data points generated from IDA. Most effort should focus on selection on an appropriate suite of records and building the analytical model.

The authors recognize the analysis required to perform an IDA is time consuming and may not be practical for consulting engineers. Research into efficient approximations is currently underway and the outcome looks promising. Furthermore, this method needs additional development by extending it to other types of loss such as non-structural damage, downtime, and loss of life. MDOF steel and concrete structures need to be investigated and further sensitivity studies regarding approximations used in the procedure need to be addressed for a wide range of earthquake suites, building types, and EDP's.

## 6 CONCLUSIONS

From this study, the following general conclusions can be drawn:

- IDA is an attractive method for relating seismic demand to capacity of constructed facilities in a probabilistic context. IDA generates an IM vs. EDP relationship which serves as one of the four major sub-tasks of the EAL assessment process.
- Many methods have been proposed for processing the results of IDA. This study has advanced the use of a distribution-free approach whereby data is reorganized without any preconceived probability distribution function. Other sources of uncertainty and randomness are combined resulting in hazard-survival functions. When combined with loss ratios and numerically integrated, EAL can be assessed.
- Through the use of the proposed financial risk assessment method, the seismic vulnerability of two bridge piers with very different detailing was examined. One pier, designed to avoid all forms of damage except from toppling, was shown to have an EAL approximately 85% less than a conventional ductile pier.
- Recognizing the time consuming nature of IDA, further research is currently underway to develop rapid loss estimation methods more suitable for office practice.

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