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Discussion of "Principal Volatility Component Analysis" by Yu-Pin Hu and Ruey Tsay

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Abstract: This note discusses some aspects of the paper by Hu and Tsay (2014), "Principal Volatility Component Analysis". The key issues are considered, and are also related to existing conditional covariance and correlation models. Some caveats are given about multivariate models of time-varying conditional covariance and correlation models.

Keywords: Principal Component Analysis, Principal Volatility Component Analysis, Vector time-varying conditional heteroskedasticity, BEKK, DCC, asymptotic properties.

JEL Classifications: C32, C55, C58, F37.

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1. Discussion

It is a pleasure to provide some comments on the excellent and topical paper by Hu and Tsay (2014) on “Principal Volatility Component Analysis”.

The paper extends Principal Component Analysis (PCA) to Principal Volatility Component Analysis (PVCA), and should prove to be an invaluable addition to the existing multivariate models for dynamic covariances and correlations that are essential for sensible risk and portfolio management, including dynamic hedging.

As one of the key obstacles to developing multivariate covariance and correlation models is the “curse of dimensionality”, namely the number of underlying parameters to be estimated, the paper is concerned with dimension reduction through the use of PCA, which is possible if there are some common volatility components in the time series.

In particular, the method searches for linear combinations of a vector time series for which there are no time-varying conditional variances or covariances, and hence no time-varying conditional correlations.

The authors extend PCA to PCVA in a clear, appealing and practical manner. Specifically, they use a spectral analysis of a cumulative generalized kurtosis matrix to summarize the volatility dependence of multivariate time series and define the principal volatility components for dimension reduction.

The technical part of the paper starts in Section 2 with a vectorization of the volatility matrix, and a connection to the BEKK model of Engle and Kroner (1995).

However, as a primary purpose of PCVA is to search for the absence of multivariate time-varying conditional in vector time series, it would have been helpful to see how PCVA might be connected to the conditional covariances arising from various specializations of BEKK (for further details, see below).

Theorem 1 assumes the existence of fourth moments of a weakly stationary vector time series, but Theorems 2 and 3 assume the existence of sixth moments. The latter two Theorems beg the question as to whether the assumption is testable.

Interestingly, in the simulation study, the four simple ARCH models considered are understood to "not satisfy the moment condition of Theorems 2 and 3", with a reference to Box and Tiao (1977) that traditional PCA works well in finite samples for non-stationary time series.

However, the purported connection between PCA for non-stationary time series, on the one hand, and time-varying conditional covariances and correlations for a weakly stationary vector time series, on the other, is not entirely clear.

The empirical analysis considers a data set of weekly log returns of seven exchange rates against the U.S. dollar from March 2000 to October 2011, giving 605 observations, which would be considered a relatively small number of observations for purposes of estimating dynamic vector covariance and correlation matrices.

The simple GARCH(1,1) model is used to estimate the conditional volatility models for the first six principal volatility components. It would have been useful to compare the GARCH estimates with the univariate asymmetric GJR and (possibly) leverage-based EGARCH alternatives.

A simple comparison is made of the PVCA results with the varying conditional correlation (VCC) model of Tse and Tsui (2002), though VCC is referred to as a "dynamic conditional correlation (DCC) model".

As the effect of "news" in the VCC model has an estimated coefficient of 0.013 and a standard error of 0.004, it is stated that the model "works reasonably well for the 6-dimensional series".

It is worth emphasizing some aspects of time-varying multivariate conditional covariance and conditional correlation models before any reasonable comparison can be made with the empirical results arising from PVCA.

In particular, it should be emphasized that only the scalar and diagonal BEKK models of the full BEKK model of Engle and Kroner (2005) can lead to statistical properties, other than by assumption (specifically, the multivariate log-moment condition for consistency (see Jeantheau (1988)) and the (untestable) 8'th moment condition for asymptotic normality (see Comte and Lieberman (2003)).

McAleer et al. (2008) established the consistency and asymptotic normality of scalar and diagonal BEKK, and argued that the Hadamard and full BEKK models that are widely estimated could not be derived from a vector random coefficient autoregressive process, so that any stated or claimed statistical properties were suspect.

They also developed the Generalized Autoregressive Conditional Correlation (GARCC) model, which has testable regularity conditions and standard asymptotic properties.

Caporin and McAleer (2008, 2012) showed the connection between the DCC and scalar BEKK models, and also questioned whether DCC had any statistical properties.

Caporin and McAleer (2013) referred to DCC as a "representation" rather than a "model", and raised a number of problematic issues with DCC, namely that DCC:

(1) does not directly yield conditional correlations;
(2) is stated rather than derived;
(3) has no moments;
(4) does not have testable regularity conditions;
(5) yields inconsistent two step estimators;
(6) has no desirable asymptotic properties, if any at all;

(7) is not a special case of GARCC;

(8) is not dynamic empirically and the impact of variance misspecification is not known;

(9) cannot be distinguished empirically from diagonal BEKK in small systems;

(10) may be a useful filter or a diagnostic check, but it is not a model.

Similar reservations apply to the varying conditional correlation (VCC) model of Tse and Tsui (2002), which is used by Hu and Tsay (2014) in their empirical analysis.

For the reasons given above, the empirical results arising from PVCA should be compared only with the indirect conditional correlations arising from the scalar and diagonal BEKK models (see Caporin and McAleer (2008)).

2. Summary

The paper by Hu and Tsay (2014) is sure to become a widely used tool by practitioners in risk and portfolio management, at the very least as an accompaniment to the existing popular BEKK and DCC models, regardless of the statistical properties (or lack thereof) of the latter two models.

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