**A novel method for computing the derivatives of the mean and amplitude of physiological variables with respect to the parameters of a cardiovascular system model**

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**Objectives**

While studying the cardiovascular system (CVS), it is frequent that only the mean and amplitude of physiological variables (pressures and volumes) are available. Computing the derivative of this discrete data with respect to the parameters of a CVS model is a necessary step to identify these parameters. Currently, such derivatives are computed through forward difference approximations, hence requiring two model simulations per derivative. In this work, we develop a method aiming to compute the derivatives along with the model simulation.

**Methods**

A lumped model of the left heart and systemic circulation [1] is used to test the method. The sensitivity equations are developed and solved [2]. First, taking the mean of the solution gives the derivative of the mean (of the state variables) with respect to the parameters. Second, the first Fourier coefficient of the solution allows approximating the derivative of the amplitude (of the state variables) with respect to the parameters. Computed derivatives are compared with forward difference approximations using a spacing of 1% of the parameter value.

**Results**

The derivatives of the mean of the state variables with respect to the parameters match the finite difference approximation with errors below 1%. The approximate derivative of the amplitude of the state variables with respect to the parameters gives errors up to 262% when compared to the finite difference approximation. Despite these large errors, the sign of the derivative is always correctly estimated (except in one case, for which the derivative is close to zero).

**Conclusion**

The method presented here has a large scope of applications for parameter identification. First, it allows computing approximate Jacobian and Hessian matrices, which can be used to accelerate parameter identification methods. In turn, this permits to perform a sensitivity analysis that can be used to select a subset of parameters to identify [3].

**References**

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