SHEAR WALLS

and the

APPLICATION OF PRESTRESS TO BRICKWORK.

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by

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1964.
FRONTSPÎŒCE Three walls on the test bed
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A. T. HINLEY
CHAPTER 1

INTRODUCTION

SYNOPSIS

In this chapter the function of a shear wall is described. Mention is made of the methods of analysing buildings under shear loading, and of experiments on the strength of shear walls. The work carried out in the project is described.

LIST OF SYMBOLS USED IN CHAPTER 1

\( K_{x_i} \) = stiffness of the member \( i \) in the \( x \) direction
\( K_{y_i} \) = stiffness of the member \( i \) in the \( y \) direction
\( x, y \) = co-ordinates of the centre of member \( i \) relative to the centre of rigidity
\( \delta_h \) = horizontal deflection of the top of a wall
\( V \) = horizontal force at the top of a wall
\( h \) = height of the applied force above the base of a wall
\( h \) = height of the centreline of the top loading beam above the base of a wall
\( E \) = Young's modulus of the material of the wall
\( \mu \) = Poisson's ratio
\( G \) = shear modulus of rigidity
\( A_w \) = cross-sectional area of a wall
\( I_{we} \) = the combined moment of inertia of a wall and the columns at the ends of the wall
\( V_{cr} \) = horizontal load causing initial cracking
\( l \) = length of a wall
\( d \) = distance between the centralines of the columns
\( t \) = thickness of a wall
\( A_{col} \) = area of a column including the transformed area of the steel
\( f_c' \) = tensile strength of a material
\( f_c' \) = compressive strength of a material
$h_p = \text{clear height of a panel}$

$l_p = \text{clear length of a panel}$

$c = \text{constant}$

$\beta = \frac{l_p}{h_p}$

$a_1 = \text{length of an opening in a wall}$

$b_1 = \text{height of an opening in a wall}$

$f' = \text{coefficient of internal friction in masonry}$

$s = \text{length dimension}$

$f_{e''} = \text{compressive strength of the infilling in the direction of the diagonal}$

$e = \text{strain in the infilling at the instant of failure}$
1.1 THE ACTION OF SHEAR WALLS

1.1-1 A shear wall is a vertical wall which is loaded horizontally at the top in its plane. The stiffness of the wall is such that the deflections due to bending and shear are of comparable magnitudes. Such loads are caused by earthquakes, wind, and bomb blast. In an earthquake the horizontal load is distributed along the wall, as it is dependent upon the inertia of the stories above the wall. Wind and bomb blast loads are transmitted from the face walls to the floors and hence to the shear walls.

1.1-2 The distribution of these shear loads to the frame of the building and to the shear walls depends upon the properties of the columns and the shear walls. Because of their large stiffnesses shear walls take such of the load. Interaction between columns, walls, floors and beams cause the behaviour to be complex. The foundation restraints are important.

1.1-3 Shear walls which are small may be considered part of a frame. Because there is zero moment at points of contraflexure in columns between two floors of a framed building, the floors may be considered to be hinged together at these points of contraflexure. A simple determination of the forces in the members around a floor in a framed building is therefore possible.

The positions of the points of contraflexure are assumed, and it is also assumed that a floor forms a diaphragm which does not compress in the horizontal direction, but allows rotation of joints of beams and columns. Load is apportioned to the resisting members in proportion to the stiffness of the members, and their distance from their centre of rigidity (see fig. 1-1). The rotation of joints at the roof and floor may be considered in the analysis by appropriate location of the points of contraflexure. Such an analysis is suited to buildings in which the load is taken mainly by the frame and in which the shear walls are small and continuous vertically.

1.1-4 The interaction of a shear wall and the surrounding floors and frame is complex. A simple analysis as indicated above may be carried out, however, if the load-deflection curves and the loads at which first cracking and failure occur are known. The shear wall may be an infill panel within a frame, or it may be a diaphragm of comparable dimensions and axial stiffness to the floors. Several tests on infill panels have been carried out, and will be summarized in this chapter. Tests on infill panels also
represent the action of shear walls in a box structure. The end walls of the box structure take the place of columns and with the floors above form a frame (see Fig. 1-2). A frame surrounding a wall prevents early failure of the wall caused by tension in the bottom of the wall.

A review of published research on infill panels

1.2-1 A considerable number of tests have been carried out by J. Benjamin and R. Williams (2 to 9). In their early work they tested reinforced concrete frames with solid brick and concrete infilling. To predict the behaviour of the frames they first used Haise's method. Haise assumes that the load-deflection relationship in the elastic range may be obtained from ordinary beam theory if it is assumed that bending is resisted by the beam and the wall, whereas shear is resisted by the wall only. For a single-storey as in figure 1-3

\[ \delta_h = \frac{Vh}{E} \left[ \frac{2(1 + \mu)}{\lambda_w} + \frac{h^2}{2I_{bc}} \right] \]  

(1-1)

After cracking, an empirical term \( R \) is introduced to match the predicted results to the experimental results.

\[ \delta_h = \frac{Vh}{EI} \left[ \frac{2(1 + \mu)}{\lambda_w} + \frac{h^2}{2I_{bc}} \right] \]  

(1-2)

After testing 5 concrete specimens and 5 brickwork walls, Benjamin and Williams found that equation (1-2) approximated the experimental results if \( R = 0.5 \) for the elastic behaviour of concrete, \( R = 0.06 \) up to ultimate failure of the concrete, \( R = 1 \) to 1.4 for elastic behaviour of brickwork, and \( R = 0.03 \) up to ultimate failure of the brickwork.

1.2-2 Prediction of failure loads by this method was unsatisfactory, so use was made of the lattice analogy method of analysis. This method of analysis is described in Chapter 2. Benjamin and Williams used the method to predict the load-deflection curve over the elastic range, and used it in conjunction with experimental observations to predict the load-deflection curve up to ultimate load. The theory was compared with the results of the tests previously mentioned and a further set of tests in which the frame
stiffness, the percentage of steel in the columns and the walls, and the
dimensions of the wall were varied.
1.2-3 Benjamin and Williams found that the failure of the reinforced
concrete frames with reinforced concrete panel walls occurred in the walls
provided that the amount of steel in the columns was not abnormally low,
that is, the amount of steel was greater than about 1%. The lattice in
figure 1-4 was used to derive the deflections, $\delta_h$, and the loads for the
first cracking, $V_{cr}$, when the horizontal load was uniformly distributed
as shown in the figure.

$$
\delta_h = \frac{V}{E} \left[ \frac{h^3}{12} \left( \frac{0.625}{\frac{A_{col}}{E} + \frac{2h}{1t}} \right) \right] \tag{1-5}
$$

$$
V_{cr} = \frac{ht f_y}{ht[\frac{0.25}{A_{col}} + \frac{0.3975}{A_{col}} + \frac{4t}{E}]} + 1.06 \tag{1-4}
$$

The deflections and loads predicted by the simple lattice analogy and by
simpler theories were compared. Deflections obtained from the lattice
analogy were nearer the experimental deflections. However the variations
in the experimental loads at first cracking was of the same order as the
differences in the values computed using the simple lattice analogy (1-4),
simple beam theory, and assuming the wall to be in pure shear.
1.2-4 Benjamin and Williams also found an empirical relationship for the
final failure when the compression column sheared off. Their work contains
detailed mathematical studies using the method of lattice analogy, but their
papers in the Journal of the American Society of Civil Engineers (7,8)
present only simple formulae, basically empirical. The formulae take into
account the sectional properties of the parts of the walls, the tensile
strength of concrete in reinforced concrete, and the bond-shear relationship
of brickwork (see section 4).
1.2-5 In the course of their experiments, Benjamin and Williams did not find
any scale effect between models and full size test specimens. A thin
reinforced concrete panel 58" x 33½" x 4" showed no signs of lateral instability.
Five model tests were carried out in which the shear wall was the web of a
tee and the base of the tee was a rigid foundation. These tests revealed that shear walls behaved similarly when the load was applied to the top of the end of the wall of the tee, and when the load was applied to the quarter points at the end of the flange of the tee.

1.2-6 S. Sachaniski (10) considered that analysis of the behavior of shear walls required the determination of the contact forces between the wall and the frame (11). This was done by considering the wall and the frame to be connected by pin joints along the perimeter of the wall. The redundant reactions were obtained by solving simultaneous equations. From the solution a stress function was adopted. Using the stress function the horizontal force to cause cracking and the diagonal shortening in the wall were derived.

\[
V_{cr} = \frac{32 \, t \, (\beta - 0.5 \beta^2)}{F}
\]

where

\[
F = 1 + \beta^2 \sqrt{\beta^2 - 0.3 \beta^2 + 1}
\]

\[
\delta_h = \frac{\phi_p \times \epsilon_y}{E \, t^2 (h_p^2 + l_p^2)}
\]

where \( \epsilon_y = 5.28 (\beta^4 - \frac{1}{\beta^4}) - 1.66 (\beta^2 - \frac{1}{\beta^2}) + 5.05 \, l_p \).

When there are openings in the wall \( V_{cr} \) and \( \delta_h \) are multiplied by the factor

\[
k_1 = 1 - \frac{2a_1}{5l} - \frac{3b_1}{5h}
\]

Sachaniski's formulas were applied to the results of tests on 64 specimens, reinforced concrete frames with infills, both scale models and full size. Sachaniski regards failure of the wall to occur at first cracking with the intent of preserving the usefulness of the wall. He found the experimental results from full-scale tests to be a little lower than those from models, and attributes this to a scale effect.

1.2-7 Rodinov (12) also presents equations for the horizontal load causing first cracking and for the deflection at first cracking.
\[ V_{cr} = 0.8 \frac{1}{\beta} \frac{f'_{c}}{f_{c}} \left( 1 - \frac{0.16}{0.75} \right) \beta \]  
(1-18)

where \( f' \) = coefficient of internal friction

= 0.7 for compact bricks

= 0 for lattice bricks.

\[ \delta_{h} = \frac{h_{p}}{\sqrt{h_{p}^2 + l_{p}^2}} \left( \frac{2v}{h_{p}} \right) \beta \]  
(1-19)

where \( B = 11.9 \times 10^6 \left[ \beta \left( \beta - 2 \right) + 2.93 \right] \).

1.2-8 The variability of the materials in panel walls is such that a simple approach will often yield results whose accuracy is comparable to that of a complicated theoretical analysis. As a design expedient, Holmes (13) considers the infilling material in a steel frame to be equivalent to a compression member in a frame. The area of the strut is \( \tau z / 3 \), where \( z \) is as shown in Figure 1-3. The horizontal force taken by the strut is

\[ V = \frac{h_{p}}{3} f'_{c} \cos \alpha \]  
(1-20)

and the horizontal deflection is

\[ \delta_{h} = \epsilon z \cos \alpha \]  
(1-21)

The values of \( f'_{c} \) and \( \epsilon \) for concrete may be readily found, but the values used for brickwork are arbitrary. The width \( z \) is also uncertain and varies with the stiffness of the frame.

1.2-9 Other experiments have been carried out on scale models of more than one panel. Apart from the tests on infill panels there is little theoretical work available on the interaction between the beams, columns, walls and floors of a building. In particular the transfer of loading from the wall to the beam is important. The analysis of a building containing shear walls must also take into account the rigidity of the foundations. The tests so far carried out on individual walls have resulted in formulae for the deflections of infill panels, suitable for a simple analysis. The redistribution of loading, to the author's knowledge, has not been comprehensively treated. It is probable that knowledge of this redistribution will allow savings in design.
In this project, single brick panels were prestressed vertically and loaded horizontally at the top through a steel loading beam. The bottom of the wall was also secured by a steel loading beam. The loading beams simulated the horizontal stiffness of floors. There was no strengthening of the ends of the walls other than prestressing. Bending effects in the beam at the top and bottom of the wall were considered.

The elastic behaviour of the walls was analysed by the method of lattice analogy (Chapter 2). Approximately six months was spent in developing computer programs to carry out the analyses. The analyses are presented in Chapter 5. A simple theory is given for the ultimate load in the wall (Chapter 5). Approximately ten months was spent on experimental work, which included tests on the materials used (Chapter 4). Seven brick walls were tested. The project requires the analysis of the interaction of a wall and the bases at the top and the bottom of the wall. The analysis is presented in detail in the hope that it will provide an introduction to further study in this field. Should the reader be interested in the wall tests only he is referred directly to Chapter 5.
At the centre of rigidity
\[ \sum K_{x_i} y_i = \sum K_{y_i} x_i = 0 \]

Section between two floors
Figure 1.1

Plan (a)

forces along an elevation (b) (c)

Figure 1.2
Failure stress occurs at X, and is the mean of the stresses in the tension column and at Y, the centre of the segment.
CHAPTER 2

ANALYSIS - PART I

SYNOPSIS

The methods of analysis of shear walls using the theory of elasticity are reviewed. The method of the lattice analogy is used in this project. A description of the analogy, and the derivation of equations for use in a computer program are presented.

### LIST OF SYMBOLS USED IN CHAPTER 2

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x,y$</td>
<td>mutually perpendicular axes</td>
</tr>
<tr>
<td>$\varphi$</td>
<td>Airy's stress function</td>
</tr>
<tr>
<td>$\nabla^4$</td>
<td>biharmonic operator</td>
</tr>
<tr>
<td>$\frac{\partial^4}{\partial x^4} + \frac{\partial^4}{\partial x^2 \partial y^2} + \frac{\partial^4}{\partial y^4}$</td>
<td></td>
</tr>
<tr>
<td>$\mu$</td>
<td>Poisson's ratio</td>
</tr>
<tr>
<td>$u,v$</td>
<td>components of displacement in the x and y directions</td>
</tr>
<tr>
<td>$z$</td>
<td>constant for a given value of $\mu$</td>
</tr>
<tr>
<td>$u_x,v_y$</td>
<td>horizontal and vertical displacements of the point $r$</td>
</tr>
<tr>
<td>$F_0,v_0$</td>
<td>horizontal and vertical forces applied at the point $0$</td>
</tr>
<tr>
<td>$E$</td>
<td>Young's modulus of both plate and material of framework</td>
</tr>
<tr>
<td>$A_0,A_1'$</td>
<td>cross-sectional areas of members of Roheeny's analogous frame</td>
</tr>
<tr>
<td>$A_0'A_1''A_2''$</td>
<td>cross-sectional areas of members of Kronmull's analogous frame</td>
</tr>
<tr>
<td>$l$</td>
<td>length of side</td>
</tr>
<tr>
<td>$t$</td>
<td>thickness</td>
</tr>
<tr>
<td>$F/A$</td>
<td>force in a side member of an elementary frame area (Benjamin and Williams' method)</td>
</tr>
<tr>
<td>$G/A$</td>
<td>component of force in the horizontal or vertical direction in a diagonal member divided by the area of the diagonal member (Benjamin and Williams' method)</td>
</tr>
</tbody>
</table>
\( a, \beta, \gamma \) = constants for a given value of \( \mu \)

\( f_1, f_2 \) = active and passive distribution coefficients

\( B_1, B_2 \) = applied forces

\( N_r \) = joint concentration of force at a joint \( r \) in a lattice

\( e_r \) = value of stress at joint \( r \)

\( e \) = elongation of a bar

\( \varepsilon \) = strain in a bar

\( \sigma \) = axial stress

\( \gamma_0, \gamma_A, \gamma_B, \gamma_C, \gamma_D \) = shear strains

\( \tau \) = shear stress

There is a further list of multiletter symbols following section 2.3-24.
2-1 INTRODUCTION

2.1-1 In this chapter a shear wall is considered to be a two-dimensional elastic plate. The methods of solution of problems of this type are reviewed, the equations used derived if they are original work, and the computer programs used to solve the problem are described.

2.2 REVIEW OF METHODS AVAILABLE

2.2-1 Exact mathematical solutions of problems in elasticity are confined to a few "special" cases of loading. The loadings used by the author are not "special" cases. Relaxation methods can be used. It is, however, more convenient to solve a two-dimensional plate problem by drawing an analogy between the deflections of a rectangular elastic plate element and the deflections of a pin jointed frame, of the same external dimensions and composed of elastic members. The analogy results in an iterative numerical process akin to relaxation. The method is termed lattice analogy.

APPLICATION OF RELAXATION METHODS TO THE EQUATIONS OF ELASTICITY

2.2-2 A shear wall, regarded as a thin isotropic elastic plate, is shown in figure 2.1. Solutions are possible for the case when the boundary conditions are forces (1), and for the case when the boundary conditions are displacements (2).

2.2-3 Where the boundary conditions are forces a solution may be obtained using the Airy stress function. The equation of strain compatibility, neglecting body forces, has the form

\[ \nabla^4 \varphi = 0, \]

where \( \nabla^4 \) is the biharmonic operator, and \( \varphi \) is the Airy stress function. The value of the Airy stress function at points lying on a "grid" network on the plate is to be determined. Integrations of the known boundary forces about points on the boundary are first carried out to give initial values of the Airy stress function at those points. Application of the biharmonic operator to successive points results in the determination of the values of \( \varphi \) over the area. The process converges only slowly. The
answer is in terms of the Airy stress function.

2.2-4. When the boundary conditions are given displacements the simultaneous
equations of displacement are considered. They are (2)

\[ \begin{align*}
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial x \partial y} + \frac{\partial^2 u}{\partial y^2} &= 0 \\
\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial x \partial y} + \frac{\partial^2 v}{\partial y^2} &= 0,
\end{align*} \]

(2.2)

where \( z = \frac{1-\mu}{1+\mu} \) for plane stress

or \( z = 1-2\mu \) for plane strain.

They are solved using a two variable technique (2). B. McKinsey uses these
same equations in another form to derive his displacement equations from
Taylor’s theorem (section 2.2-3). Solution of the equations gives the
displacements of points on the plate. Strains are the differences in the
displacements of points. Thus for accuracy in computation of stresses and
strains a high degree of convergence is required of the solution.

2.2-5 Both methods may be adapted to the case of mixed boundary conditions,
but this is a difficult and "unattractive" problem (2). Neither method was
used by the author.

**LATTICE ANALOGY METHODS**

2.2-6 In this method of solution an analogy is drawn between the deflections
of an elastic plate element and those of a pin-jointed framework of the same
external shape and dimensions. Equality of deflections, is possible for
cases of uniform axial or shear load, by defining the cross-sectional areas
of members of the framework. In derivation of the member areas it is
necessary and sufficient to consider the application of axial load in the \( x \)
and \( y \) directions (figures 2.2(a) and (b)), and the case of the shear loading
(figure 2.2(c)). With the areas of the framework members so defined the
deflections of an elastic body composed of rectangular elements, and the
deflections of a lattice composed of pin-jointed frames (figure 2.3) are
considered to be analogous. Because the lattice frame is highly indeter-
minate it is solved by iterative numerical methods rather than by solving
equations of indeterminate structures.

2.2-7 The analogy was drawn by A. Heilany (3), (4) and by A. Hronikoff (5), (6) about 1940. Recent work on the analogy has been carried out by J. Benjamin and H. Williams (7), (8), (9). This includes a method developed for more rapid convergence and an extensive coverage of the aspects of the analogy. The latter papers have not, unfortunately, been widely published. Analogies between plate segments and frames, and the available methods of solution of the lattice will be summarised in this section.

HEILANY'S METHOD

2.2-8 In Heilany's method the joints of the lattice are considered to be fixed in position, but not necessarily in equilibrium. When a joint is released it adopts an equilibrium position dependent on and relative to the positions of the eight joints surrounding it. By releasing and refixing the joints of the lattice an equilibrium position of the lattice is finally reached. From the displacements of joints, strains, and hence stresses, are calculated. The calculation throughout is in terms of displacements.

2.2-9 Heilany derives equations for the displacement of joints both by applying Taylor's theorem to the equations of displacement (2.2), and by using an analogous framework. This illustrates the fact that use of the lattice analogy is equivalent to a finite difference method. In the former derivation he adopts a Poisson's ratio of 1/3 to simplify the equations (2.2) to the form

\[
\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} + \frac{2\partial^2 u}{\partial x \partial y} = 0
\]

(2.3)

\[
\frac{\partial^2 v}{\partial y^2} + \frac{\partial^2 v}{\partial x^2} + \frac{2\partial^2 v}{\partial x \partial y} = 0
\]

The Taylor's theorem expansions for the displacements of surrounding joints, neglecting third and higher derivatives, are then multiplied by chosen integers. Summation of the expansions results in a cancelling out of the derivative terms, yielding equations of which (2.6) are typical. In the derivation from an analogous framework necessary conditions of analogous displacement require the following member areas (see figure 2.4(b)):
For axial loading

\[
\frac{M_o'}{t} = \frac{M_o}{2(1+\mu)}
\]

and \(\frac{M_1'}{\mu} = \frac{M_1\sqrt{2}}{1-\mu^2}\). (2.4)

For shear loading

\[
\frac{M_1'}{N_o} = \frac{M_1}{(1+\mu)\sqrt{2}}.
\] (2.5)

To use the same diagonal framework members for both axial and shear loading, it is necessary to define Poisson's ratio equal to \(1/3\).

2.2-10 The equations of displacement are given for four cases: internal, corner, and boundary joints and a joint on a re-entrant corner. Body forces may also be applied. The form of the sets of equations is similar. Only the internal equation will be given here, to illustrate that the coefficients of the displacement term are essentially integers. With the joints as shown in figure 2.4 (a)

\[
u_o = \frac{4}{9E}H_o + \frac{1}{12}((b_o + v_o + 4h_o + h_d - v_d + h_f + v_f + 4h_g + h_h + v_h)
\]

and \(v_o = \frac{4}{9E}V_o + \frac{1}{12}(4v_a + b_o + v_o - h_d + v_d + 4v_c + h_f + v_f - h_h + v_h),\) (2.6)

where \(H_o\) and \(V_o\) are forces applied at the point \(o\).

2.2-11 Because stress is proportional to the difference in displacement of two points, displacements must be found to a high degree of accuracy if stresses are to be accurate. Poisson's ratio has been taken as a third in deriving the equations. If a value of Poisson's ratio which is not a third is desired, equations of the form (2.6) could be derived from the Taylor's theorem expansions, but the coefficients of the terms would no longer be integers. There would be no analogous frame for both axial load and shear. Computation would be much more difficult. McHenry's method has the disadvantages that convergence is slow whilst the requirements of accuracy are high, and that a Poisson's ratio of one-third must be used.
2.2-12 This method, like McKelvey's, uses the successive releasing and refining of joints to arrive at an equilibrium position of the lattice frame. The forces in the members of the lattice are used in computation, thus the degree of convergence of the bar forces and hence the accuracy of the stresses is apparent.

2.2-13 By insertion of interior members into an elementary square framework (fig. 2.5(a)) Brezinkoff established the analogy for any value of Poisson's ratio. When the three conditions for identical displacement are applied, the following member areas are obtained.

\[
\begin{align*}
A_0 &= \frac{16}{2(1 + \mu)} \\
A_1 &= \frac{16}{\sqrt{2}(1 + \mu)} \\
A_2 &= \frac{3 - 1}{2(1 + \mu)(1 - 2\mu)} \quad \text{(2.7)}
\end{align*}
\]

2.2-14 When a joint is released the resultant of the bar forces on the joint must be zero. The cut of balance forces which were acting on the joint are distributed to the surrounding members. Equations for release of joints are derived in 2.3. It will be noted here, however, that when a joint is freed, there will be changes in the forces in all members connecting that joint to the surrounding joints, and also in the bar forces acting on some of the surrounding joints, but not on the freed joint, due to the action of the internal members of the lattice. The magnitude of the forces in the internal members is of little direct concern, whereas the evaluation of forces on the external joints of the frameworks forming the lattice constitutes the solution of the problem.

2.2-15 Stresses in the elastic plate are calculated from the bar forces obtained by an iterative numerical process. From the shortening of the frame members the deflected shape is deduced.

2.2-16 Brezinkoff's method is more versatile than McKelvey's method. Any value of Poisson's ratio may be used. In this respect it is superior also to the photoelastic analysis of a model in which Poisson's ratio is fixed for any given model material. Simple operators are used in the computation.
Both applied loads and displacements (section 2.3-32) may be handled. This method was chosen. The iterative numerical process and the evaluation of stresses and displacements was programmed to be carried out by an electronic digital computer.

**Benjamin and Williams' Method**

2.2-17 Benjamin and Williams (6), in order to reduce the time spent in the solution of the bar forces, assume trial values of bar forces which satisfy the requirements of equilibrium. From these assumed values they calculate the lack of fit for the bar members composing the lattice. The "lack of fit" or "strain incompatibility" is reduced to zero by the use of operators which satisfy the requirements of equilibrium.

2.2-18 The basic equation used in equating "lack of fit" is derived from an application of virtual work to the members of lattice segment. Each "strain incompatibility" (see Fig. 2.6(a)) is given as

\[
2 \sqrt{2} \sum \frac{E}{A} - \sum \frac{F}{A}
\]

Diagonals (Sides) \hspace{1cm} (2.9)

which for a perfect fit would be zero. Operators are applied to single segments, groups of four segments, and to diamond-shaped groups (see Fig. 2.6(b)) to reduce these calculated incompatibilities to zero or to nearly zero. Operators involving more than one segment are used first because this procedure speeds convergence.

2.2-19 The analogous frame used by Benjamin and Williams has the same shape as that of Lévy (5). When use of a value of Poisson's ratio which is not a third is desired, it is possible to find a lattice solution to the problem by using different areas of diagonal members for direct and shear loads.

Direct load results in equal diagonal bar forces. Shear load results in equal and opposite diagonal bar forces. In calculating incompatibility, the area associated with direct load is used with the sum of the diagonal bar forces, and the area associated with shear load used with the difference in diagonal bar forces.

2.2-20 Benjamin and Williams present their method as the most economical in time when computation is by hand, and recommend its use when an electronic digital computer is not available, or when the problem does not warrant solution by an electronic digital computer.
2.3 EQUATIONS FOR RELEASE OF JOINTS USING BREMERHOFF'S ANALOGOUS FRAME

MATHEMATICAL DERIVATION OF BAR AREAS AND DISTRIBUTION COEFFICIENTS

2.3-1 The derivation of bar areas for a simple framework with no secondary internal members may be found in reference (5). Bar area derivations for the general case when secondary members are used, may be found in appendix A-1. For movement in the direction of one of the axes of a joint of a single frame Bremerhoff derived distribution coefficients. These are defined and derived in appendix A-2, as the derivation will be referred to, and as far as is known has not been published. Distribution coefficients will be used in deriving the equations for the release of joints.


2.3-2 In the following sections use has been made of multiletter symbols, and of a method of subscripting forces, bar forces, and joints. These symbols and co-ordinates correspond to the symbols and co-ordinates used in the computer program. The program may be found in appendix B. The symbols, and some others also used in this chapter, will be listed here for convenience.
LIST OF MULTILETTER SYMBOLS

2.3-3

I = intercept of a joint on the I-axis (see figure 2.8)
K = intercept of a joint on the K-axis (see figure 2.8)
N = number of joints of a lattice column in the direction parallel to the I-axis
E = number of joints in a lattice row, in the direction parallel to the K-axis

M(I,K) = the force in the horizontal bar which is subscripted at the joint (I,K) (see figure 2.9 and section 2.3-8)

FV(I,K) = the force in the vertical bar which is subscripted at the joint (I,K) (see figure 2.9 and section 2.3-8)

S1(I,K) = the horizontal and vertical component of the diagonal bar force which is inclined at 45 degrees to the horizontal axis, and which is subscripted at the point (I,K) (see figure 2.9 and section 2.3-8)

S2(I,K) = the horizontal and vertical component of the diagonal bar force which is inclined at minus 45 degrees to the horizontal axis, and which is subscripted at the point (I,K) (see figure 2.9 and section 2.3-8)

P, P', Q, Q' = applied loads

HOUT = horizontal component of the out-of-balance force at a joint. It is positive if it acts in the direction of increasing K (see figure 2.8).

VOUT = vertical component of the out-of-balance force at a joint. It is sensed positive if it acts in the direction of decreasing I (see figure 2.8).

DELH = horizontal force applied at a joint which, acting in conjunction with DELFV, results in a horizontal applied force equal to HOUT. DELH is positive in the same direction as HOUT.

DELFV = vertical force applied at a joint which, acting in conjunction with DELH, results in a vertical applied force equal to VOUT. DELFV is positive in the same direction as VOUT.
$LH(I,K)$ = horizontal force which acts at the joint $(I,K)$ as a result of external loading. It is positive in the direction of increasing $K$.

$LV(I,K)$ = vertical force which acts at the joint $(I,K)$ as a result of external loading. It is positive in the direction of decreasing $I$.

$INC_{R1}$ $INC_{R2}$ $INC_{R3}$ $INC_{R4}$ $INC_{R5}$ $INC_{R6}$ = increments of force added to bar forces. They are due to the cancelling of the out-of-balance forces.

$AXIALH(I,K)$ = normal stress in the horizontal direction at the centre of the segment of which the top left-hand corner has the ordinate $(I,K)$

$AXIALV(I,K)$ = normal stress in the vertical direction at the centre of the segment of which the top left-hand corner has the ordinate $(I,K)$

$SHEAR(I,K)$ = shear stress at the centre of the segment of which the top left-hand corner has the ordinate $(I,K)$

$DELH(I,K)_{sh}$ = horizontal displacement of a joint of a lattice frame from the position it occupied when the lattice was unloaded.

$DELV(I,K)_{sv}$ = vertical displacement of a joint of a lattice frame from the position it occupied when the lattice was unloaded.
2.3-4 It will also be convenient at this stage to define other quantities used in the analysis.

\[
\begin{align*}
\mu &= \text{Poisson's ratio} \\
\alpha &= \frac{2(1-\mu)}{5-3\mu} \\
\beta &= \frac{1+\mu}{4(3-3\mu)} \\
\gamma &= \frac{3\mu-1}{4(5-3\mu)}
\end{align*}
\] (2.10)

For any given value of Poisson's ratio \( \alpha, \beta \) and \( \gamma \) are constants.

2.3-5 The lattice is composed of pin-jointed frames. If two bars lie alongside each other they may be regarded as a single bar. To avoid confusion the cross-sectional areas of the members will be restated (see figure 2.7).

\[
A_o = \frac{lt}{2(1+\mu)} = \text{cross-sectional area of a vertical or horizontal member lying on the perimeter of the lattice. It is a member of only one framework.}
\]

\[
A = \frac{lt}{1+\mu} = \text{cross-sectional area of a vertical or horizontal member, not lying on the perimeter of the lattice and thus shared by two frames. It is twice the area of } A_o
\]

\[
A_1 = \frac{lt}{\sqrt{2}(1+\mu)} = \text{cross-sectional area of the diagonal members of the frames.}
\]

\[
A_2 = \frac{3\mu-1}{2lt(1+\mu)/(1-2\mu)} = \text{cross-sectional area of the secondary internal horizontal and vertical members.}
\]

2.3-6 Before it is possible to derive the equations for release of joints it is necessary to define the directions of the axes, the direction in which a positive bar force acts on a joint, and the method of subscripting bar forces. The following conventions have been adopted.

2.3-7 Joint co-ordinates are referred to two axes \( I \) and \( K \) (see fig. 2.6). \( I \) and \( K \) values of joints increase by unit increments from 1 at the top left-hand corner to \( I \) at the bottom left-hand corner and \( K \) at the top
right-hand corner respectively. Thus each point has a unique \((I,K)\)
value, where \(I\) and \(K\) are integers. Normally \(I\) is a vertical co-ordinate
and \(K\) a horizontal co-ordinate.

2.3-8 Each bar force acts on two joints. In order to have only one
subscript for each force the following rules are stipulated.

(i) The lowest possible value of \(I\) for the two joints connected by the
bar is always to be used.

(ii) In the case of horizontal bar forces, where there is a choice of two
\(K\) values, the lower value of \(K\) is to be used.

2.3-9 The positive direction of a force must also be defined. The
direction of a force acting on a joint at one end of a member is the
opposite direction to that of the same force acting on the joint at the
other end. The force in the bar is to have the same sign. Accordingly
force directions are defined at that joint whose \((I,K)\) value they carry.
Positive directions are shown in figure 2.9, and the directions of positive
forces acting on an internal joint are shown in figure 2.10 to illustrate
the convention.

2.3-10 In the following text the terms "active" and "passive" have been
used. They will be explained here. When a joint is displaced along one
of the axes whilst the other joints remain fixed, one of the diagonals of
the frame "actively" opposes the movement. The other undergoes a change
of load due to the action of the internal members. Hrenikoff, in order
to distribute "out-of-balance" at a joint, derived distribution factors
(appendix A-2). A distribution factor is the ratio of the component
along the axis of the force caused in a diagonal member by displacement of
a joint along one of the axes to the bar force induced in the member lying
in the direction of movement. He described distribution factors as active
and passive. The component of the active diagonal and the force induced
in the member lying in the direction of movement counter the force causing
displacement of the joint. It will be convenient to use the terms active
and passive to imply the function of the diagonal.
2.3-11 When a joint is released the existing "out-of-balance" force at a joint becomes zero. This is equivalent to allowing the out-of-balance force to act on the joint and induce increments in the bar forces which oppose the out-of-balance. The application of forces causing purely horizontal and vertical displacement of joints is first considered. A typical corner joint is drawn in figure 2-11.

2.3-12 When a horizontal force \( P \) is applied to the corner joint \((I,K)\), and this joint only is permitted to move in the horizontal direction only, the joint will displace until \( P \) is equal to the horizontal components of the forces in \((I,K)\), \((I,K+1)\) and \((I,K)\) \((I+1,K+1)\). Resolving in the horizontal direction and remembering that \( S2(I,K) \) is the component of the force in the diagonal member:

\[
P = - \frac{6}{2}s2(I,K).
\]

The member \((I,K)\) \((I,K+1)\) lies along the boundary of the lattice and therefore its cross-sectional area \(A_o\) is half the area used in deriving the distribution coefficients (Appendix A-2). By doubling the distribution coefficient it is possible to write

\[
S2(I,K) = 2s2(I,K).
\]

Then

\[
P = - \frac{6}{2}s2(I,K) \left( 1 + 2 \frac{1 + \mu}{8(1 - \mu)} \right)
\]

\[
= - \frac{5 - 3\mu}{4(1 - \mu)} s2(I,K).
\]

\[
= - 2 s_\alpha P.
\]

\[
\therefore \quad s2(I,K) = \frac{4(1 - \mu)}{5 - 3\mu} P.
\] (2.11a)

Substituting,

\[
S2(I,K) = 2s2(I,K) \frac{6(1 + \mu)}{8(1 - \mu)} \cdot - \frac{4(1 - \mu)}{5 - 3\mu} P
\]

\[
= - \frac{1 + \mu}{5 - 3\mu} P
\]

\[
= - 4 \beta P
\] (2.11b)
2.3-13 There will also be a passive force induced in the member
\((I,K+1),(I+1,K)\). This force
\[ S_1(I,K+1) = 2P \frac{\delta H(I,K)}{\delta} \]
\[ = \frac{2(3\mu - 1)}{(1 - \mu)} \cdot - \frac{\delta(1 - \mu)}{5 - 3\mu} \cdot P \]
\[ = \frac{3\mu - 1}{5 - 3\mu} P \]
\[ = \frac{4 \gamma P}{\delta} \]  \hspace{2cm} (2.11a)

When distribution coefficients were derived there was no force in the
member perpendicular to the direction of displacement \(\text{Appendix A-2}\).
This is the condition that there be no vertical movement of the joint.
In order that this condition is maintained a vertical force \(P\) must also
be applied at \(I,K\), equal and opposite to the vertical component of the
force in the diagonal.

From equilibrium \(P' = 4\beta P\). \hspace{2cm} (2.11b)

2.3-14 Similar equations may be derived from figure 2-11(a) for the
application of a vertical load \(Q\) and an associated horizontal force \(Q'\)
to produce purely vertical movement of \((I,K)\). These equations are
\[ FV(I,K) = 2\alpha Q, \] \hspace{2cm} (2.12a)
\[ S_2(I,K) = 4\beta Q, \] \hspace{2cm} (2.12b)
\[ S_1(I,K+1) = -4\gamma Q, \] \hspace{2cm} (2.12c)
and \(Q' = -4\beta Q\). \hspace{2cm} (2.12d)

2.3-15 The components of the cut-of-balance force at a joint due to the
forces in the members at the joint in the horizontal and vertical direction
are \(H_{\text{OUT}}\) and \(V_{\text{OUT}}\) respectively. The equilibrium of a joint will be
affected by application of a set of forces equal to the cut-of-balance
forces. Forces \(\text{DELH}, 4\beta \text{DELH}, \text{DELV} \) and \(4\beta \text{DELV}\), which correspond to
the forces \(P, P', Q\) and \(Q'\) respectively may be applied, to result in the
application of \(H_{\text{OUT}}\) in the horizontal direction and of \(V_{\text{OUT}}\) in the
vertical direction. Resolving in the vertical and horizontal direction
\[ \text{DELH} + 4\beta \text{DELV} = H_{\text{OUT}} \]
and \[ \text{DELV} = 4\beta \text{DELH} = V_{\text{OUT}}. \] \hspace{2cm} (2.13)
Solving simultaneously

\[ \text{DELIM} = \frac{MIC \pm 4 \beta \text{PVIM}}{1 - 16 \beta^2} \]

and \[ \text{DELIV} = \frac{\text{PVIM} \pm 4 \beta \text{MIC}}{1 - 16 \beta^2} \]  

(2.14)

2.3-16 Equilibrium of the joint will be obtained by applying \( \text{DELIM} \), \( \text{DELIV} \), \( 4 \beta \text{DELIM} \) and \( 4 \beta \text{DELIV} \). The bar forces will be incremented proportionally to \( \text{DELIM} \) and \( \text{DELIV} \) as they would be to \( P \) and \( Q \). From equations 2.11 and 2.12 the increments are as follows

\[ A_{MI}(I,K) = -2 \alpha \text{DELIM} \]
\[ A_{IV}(I,K) = -2 \alpha \text{DELIV} \]
\[ A_{32}(I,K) = -4 \beta (\text{DELIM} - \text{DELIV}) \]
\[ A_{34}(I,K) = 4 \gamma (\text{DELIM} - \text{DELIV}) \]  

(2.15)

The values of \( \text{MIC} \) and \( \text{PVIM} \) may be found from the equations

\[ \text{MIC} = M(I,K) + 32(I,K) \]
\[ \text{PVIM} = F(V(I,K) - 32(I,K)). \]  

(2.16)

2.3-17 The treatment of the other three corners may be simplified considerably if the following concepts are understood. Firstly, the equations for calculating \( \text{MIC} \) and \( \text{PVIM} \) (2.16), the components of the cut-off-of-balance force, are a special case of general equations involving bar forces from the eight possible surrounding joints. Terms involving bar forces which do not exist are omitted (see section 2.3-20). Secondly, the magnitude of the coefficients for the increments are a function of the geometry of a corner. Although signs may change, \( 2 \alpha \), \( 4 \beta \) and \( 4 \gamma \) will be the magnitudes of the coefficients for all four corners. Finally, the signs of the increments are a result of the position of bars relative to the directions of the applied forces. It is shown in section 2.3-31 that increments may be added by a generalized set of equations similar to (2.15).

2.3-18 Forces \( \text{DELIM} \) and \( \text{DELIV} \) are obtained by solution of the simultaneous equations (2.14). The form these equations take is dependent on the position of the members relative to the direction and point of application of \( \text{DELIM} \) and \( \text{DELIV} \). The values of \( \text{DELIM} \) and \( \text{DELIV} \) are given for the four corners in table 2.1, from a consideration of figures 2.11(a) and 2.12.
<table>
<thead>
<tr>
<th>Joint</th>
<th>Axis</th>
<th>Applied Force ((P) or (Q)))</th>
<th>Secondary Force ((P') or (Q'))</th>
<th>Equilibrium Equation</th>
<th>Incremental Values from Solution of Equilibrium Equations</th>
</tr>
</thead>
<tbody>
<tr>
<td>1,1</td>
<td>Hor.</td>
<td>DELFH</td>
<td>(-4\beta \text{ DELFH})</td>
<td>(\text{PHOUT} = \text{DELFH} - 4\beta \text{ DELFH})</td>
<td>(\text{DELFH} = \frac{\text{PHOUT} + 4\beta \text{ PVOUT}}{1 - 16\beta^2})</td>
</tr>
<tr>
<td></td>
<td>Vert.</td>
<td>DELFV</td>
<td>(-4\beta \text{ DELFV})</td>
<td>(\text{PVOUT} = \text{DELFV} - 4\beta \text{ DELFH})</td>
<td>(\text{DELFV} = \frac{\text{PVOUT} + 4\beta \text{ PHOUT}}{1 - 16\beta^2})</td>
</tr>
<tr>
<td>1,N</td>
<td>Hor.</td>
<td>DELFH</td>
<td>(-4\beta \text{ DELFH})</td>
<td>(\text{PHOUT} = \text{DELFH} + 4\beta \text{ DELFH})</td>
<td>(\text{DELFH} = \frac{\text{PHOUT} + 4\beta \text{ PVOUT}}{1 - 16\beta^2})</td>
</tr>
<tr>
<td></td>
<td>Vert.</td>
<td>DELFV</td>
<td>(-4\beta \text{ DELFV})</td>
<td>(\text{PVOUT} = \text{DELFV} + 4\beta \text{ DELFH})</td>
<td>(\text{DELFV} = \frac{\text{PVOUT} + 4\beta \text{ PHOUT}}{1 - 16\beta^2})</td>
</tr>
<tr>
<td>N,1</td>
<td>Hor.</td>
<td>DELFH</td>
<td>(-4\beta \text{ DELFH})</td>
<td>(\text{PHOUT} = \text{DELFH} + 4\beta \text{ DELFH})</td>
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</tr>
<tr>
<td></td>
<td>Vert.</td>
<td>DELFV</td>
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<td>(\text{PVOUT} = \text{DELFV} + 4\beta \text{ DELFH})</td>
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</tr>
<tr>
<td>N,N</td>
<td>Hor.</td>
<td>DELFH</td>
<td>(-4\beta \text{ DELFH})</td>
<td>(\text{PHOUT} = \text{DELFH} - 4\beta \text{ DELFH})</td>
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<td>(\text{DELFV} = \frac{\text{PVOUT} + 4\beta \text{ PHOUT}}{1 - 16\beta^2})</td>
</tr>
</tbody>
</table>

Table 2.1
2.3-19 When forces $\Delta F_{EH}$ and $\Delta F_{LV}$ have acted there are no out-of-balance forces. This is equivalent to releasing the joint. There are four steps in releasing a corner joint. Firstly, the components of the out-of-balance forces are evaluated. In the computer program this is done using the overall equations (2.24). Secondly, the required applied forces $\Delta F_{EH}$ and $\Delta F_{LV}$ are evaluated. These equations are given in table 2-1. The third step is to calculate the magnitudes of the increments to the bar forces. Finally, the increments are added using the generalised set of equations (2.26). The total effect is the distribution of the "out-of-balance" force at the joint to the surrounding members.

**DISTRIBUTION PROCEDURE FOR A BOUNDARY JOINT**

2.3-20 A typical boundary joint is shown in figure 2-13. It lies on the top boundary of the lattice. The release of the joint will again be considered to be the application of the out-of-balance forces at the joint.

2.3-21 As with the case of a corner joint both a horizontal load $F$ and a vertical load $Q$ causing purely horizontal and vertical movement of the joint $(I,K)$ respectively are first considered. The joint $(I,K)$ is permitted to move, whilst the surrounding joints remain fixed. A significant difference between this case and the case of a corner joint is that no secondary forces $F'$ and $Q'$ are required because the components of diagonal forces perpendicular to the applied loads $F$ and $Q$ cancel each other. No calculation of $\Delta F_{EH}$ and $\Delta F_{LV}$ is required, as $F_{OUT}$ and $Q_{OUT}$ are in this case the loads applied.

2.3-22 The horizontal load is opposed by members $(I,K-1) (I,K)$, $(I,K)$ $(I,K+1)$, $(I+1,K-1) (I,K)$ and $(I,K) (I+1,K+1)$. The horizontal forces are equal in magnitude from symmetry, as are the diagonal forces. The two boundary members have an area $A_0$ because they lie on the perimeter of the lattice. Consequently the distribution factors are doubled when determining the components of the forces in the diagonal members. Resolving horizontally in figure 2-13(b).

$$P = \Delta F(I,K-1) - \Delta F(I,K) - \Delta F(I,K) - \Delta F(I+1,K)$$

Taking forces acting in the direction of $K$ increasing on joint $(I,K)$ as positive forces, and putting the force in horizontal members equal to $N$
\[ P = M(1 + 1 + 2f_a + 2f_a') \]
\[ = M(2 + \frac{1 + \mu}{\delta(1 - \mu)}) \]
\[ = \frac{5 - 3\mu}{2(1 - \mu)} B. \]

\[ \therefore M(I, K - 1) = - M(I, K) = \frac{2(1 - \mu)}{5 - 3\mu} P = a P, \]
\[ S_1(I, K) = - S_2(I, K) = -\frac{1 + \mu}{4(1 - \mu)} \cdot \frac{2(1 - \mu)}{5 - 3\mu} P = -2 \beta P, \]
and \[ S_2(I, K - 1) = S_1(I, K - 1) = \frac{3\mu - 1}{4(1 - \mu)} \cdot \frac{2(1 - \mu)}{5 - 3\mu} P = 2\gamma P. \]

2.3-23 The vertical load is opposed by the three members \((I, K)\), \((I+1, K-1)\), \((I, K)\) and \((I+1, K+1)\). The area of the vertical member is \(A\). Taking forces acting in the positive direction of \(I\) as positive forces, and putting the force in the vertical members equal to \(V\)
\[ Q = V(1 + f_a + f_a') \]
\[ = V(1 - \frac{2(1 + \mu)}{8(1 - \mu)}) \]
\[ = \frac{5 - 3\mu}{4(1 - \mu)} V. \]

Substituting to obtain the bar forces and components in terms of \(Q\)
\[ FV(I, K) = -\frac{\mu(1 - \mu)}{5 - 3\mu} Q = -2aQ, \]
\[ S_1(I, K) = - S_2(I, K) = -\frac{1 + \mu}{8(1 - \mu)} \cdot \frac{\mu(1 - \mu)}{5 - 3\mu} Q = -2\beta Q, \]
and \[ S_2(I, K - 1) = - S_2(I, K + 1) = \frac{3\mu - 1}{8(1 - \mu)} \cdot \frac{\mu(1 - \mu)}{5 - 3\mu} Q = 2\gamma Q. \]

2.3-24 Equilibrium is obtained by applying the forces \(\text{NOUT}\) and \(\text{FVOUT}\), where
\[ \text{NOUT} = L(I, K) - M(I, K - 1) + M(I, K) + S_1(I, K) + S_2(I, K) \]
and \[ \text{FVOUT} = LV(I, K) + FV(I, K) + S_1(I, K) - S_2(I, K). \]
The associated increments of force are
\[
\begin{align*}
\Delta E(I, K-1) &= a_{\text{NOUT}} \\
\Delta F(I, K) &= -a_{\text{FOUT}} \\
\Delta F(V, K) &= -2 a_{\text{FOUT}} \\
\Delta s_1(I, K) &= -2\beta (F_{\text{NOUT}} + F_{\text{FOUT}}) \\
\Delta s_2(I, K) &= -2\beta (F_{\text{NOUT}} - F_{\text{FOUT}}) \\
\Delta s_1(I, K+1) &= 2\gamma (F_{\text{NOUT}} + F_{\text{FOUT}}) \\
\text{and} \quad \Delta s_1(I, K-1) &= 2\gamma (F_{\text{NOUT}} - F_{\text{FOUT}}).
\end{align*}
\] (2.20)

2.3-25 The application of a generalized set of equations to all boundary joints is quite simple. As was the case with corner joints the "out-of-balance" equations (2.19) may be obtained from equation (2.24) by omitting terms if the bar does not exist. The magnitudes of the increments are again a result of the position of the members. The magnitudes of the increments to "active" diagonal components are
\[
2\beta (F_{\text{NOUT}} + F_{\text{FOUT}})
\]
and
\[
2\beta (F_{\text{NOUT}} - F_{\text{FOUT}})
\]
The magnitudes of the "passive" diagonal components are
\[
2\gamma (F_{\text{NOUT}} + F_{\text{FOUT}})
\]
and
\[
2\gamma (F_{\text{NOUT}} - F_{\text{FOUT}}).
\]
For the vertical or horizontal member normal to the boundary, the magnitude of the increment is
\[
2a \times \text{force for equilibrium normal to the boundary},
\]
whilst for those parallel to the boundary the increments are
\[
a \times \text{force for equilibrium parallel to the boundary}.
\]
As is the case with corner joints, a generalized set of equations (2.26) may be used to add the increments, as will be explained in section 2.3-31.

**DISTRIBUTION PROCEDURES FOR AN INTERIOR JOINT**

2.3-26 An interior joint (I, K) is sketched in figures 2-10 and 2-14. Treatment of release of a joint is once again allowing the out-of-balance force to act on the joint.

2.3-27 Application of a horizontal or vertical force is resisted by two bar forces, \( F \), in the line of action of that the applied force, equal
because of symmetry, and by four equal active diagonal components, 3.

Considering magnitudes alone

\[ P = (2 + 4f_a) P \]

\[ = \frac{5 - 3\mu}{2(1 - \mu)} P. \]

The components of the active bar forces acting at (I,K) perpendicular to the line of action of the applied force will cancel, thus the "applied" forces are again equal to \( F_{\text{HOUT}} \) and \( F_{\text{VOUT}} \) respectively.

2.3-28 The magnitudes of the bar forces lying in the line of action of an applied load are given by

\[ P = \frac{2(1 - \mu)}{3 - 3\mu} = aP. \]

The active components are given by

\[ 3 = \frac{1 + \mu}{8(1 - \mu)} \cdot \frac{2(1 - \mu)}{3 - 3\mu} P = \frac{1 + \mu}{4(5 - 3\mu)} P = \beta P, \]

and the passive components, \( T \), by

\[ T = \frac{3\mu - 1}{8(1 - \mu)} \cdot \frac{2(1 - \mu)}{3 - 3\mu} P = \frac{3\mu - 1}{4(5 - 3\mu)} P = \gamma P. \]

2.3-29 The sign of an increment to a bar force is only dependent on the direction of the positive bar force, at the joint at which force is applied, relative to the direction of the applied force at that joint. For example, the force \( P \) produces a positive increment in \( S_2(I-1,K-1) \) but a negative increment in \( S_2(I,K) \). Active and passive diagonal members will have increments for the application of both horizontal and vertical forces.

The magnitudes of increments are given in equations (2.21). Considering these relationships of sign, number of forces, and magnitude, in conjunction with figures 2-14, and substituting \( F_{\text{HOUT}} \) and \( F_{\text{VOUT}} \) for \( P \) and \( Q \), it is possible to write down by inspection the following increment equations.

\[ \Delta M(I,K-1) = aF_{\text{HOUT}} \]
\[ \Delta M(I,K) = -aF_{\text{HOUT}} \]
\[ \Delta FV(I-1,K) = aF_{\text{VOUT}} \]
\[ \Delta FV(I,K) = -aF_{\text{VOUT}} \]
\[ \Delta S(I-1,K+1) = \beta(F_{\text{HOUT}} + F_{\text{VOUT}}) \]
\[ \Delta S(I,K) = -\beta(F_{\text{HOUT}} + F_{\text{VOUT}}) \]  

(2.22)
\[ \begin{align*}
\Delta S2(I-1,K-1) &= \beta (\text{FOUT} - \text{FVOUT}) \\
\Delta S2(I,K) &= -\beta (\text{FOUT} - \text{FVOUT}) \\
\Delta S2(I,K-1) &= \gamma (\text{FOUT} + \text{FVOUT}) \\
\Delta S2(I-1,K) &= -\gamma (\text{FOUT} + \text{FVOUT}) \\
\Delta S1(I,K+1) &= \gamma (\text{FOUT} - \text{FVOUT}) \\
\Delta S1(I-1,K) &= -\gamma (\text{FOUT} - \text{FVOUT})
\end{align*} \]

From figure 2-10 the equations for FOUT and FVOUT are

\[ \text{FOUT} = \text{LV}(I,K) + \text{FH}(I,K) - \text{HH}(I,K-1) + S1(I,K) + S2(I,K) - S2(I-1,K-1) - S1(I-1,K+1) \]
and \[ \text{FVOUT} = \text{LV}(I,K) + \text{FV}(I,K) - \text{VV}(I-1,K) + S1(I,K) - S2(I,K) + S2(I-1,K-1) - S1(I-1,K+1). \]

\[(2.23)\]

Generalized Equations for Evaluation of the Forces to Bring a Joint into Equilibrium, and for the Addition of Increments Due to These Forces

2.3-30 Equations 2.23 were derived from figure 2-10. By omitting forces, the diagram can be made to represent the equilibrium of corners and boundaries. All the equations for FOUT and FVOUT may be derived from equations 2.23 by omitting terms. A term must represent a force in a member inside the lattice. A rectangular lattice has corners at the four points (1,1), (1,K), (K,1), (K,K). Giving the joint (I,K) drawn in figure 2-10 the (I,K) values of these corner joints in turn it is possible to rewrite the equations 2.23 and list below each term the I and K values for which the term does not apply.

\[ \begin{align*}
\text{FOUT} &= \text{LV}(I,K) + \text{FH}(I,K) - \text{HH}(I,K-1) + S1(I,K) + S2(I,K) - S2(I-1,K-1) \\
&\quad \text{not } I=1 \text{ not } K=1 \text{ not } I=K \text{ not } K=1 \\
&\quad \text{or } K=1 \text{ or } K=H \text{ or } K=1 \\
&\quad \quad \text{or } I=K \\
&\quad \quad \text{or } K=H \\
&\quad \quad \text{or } K=1 \quad \text{or } K=H \\
&\quad \quad \text{or } I=1 \\
&\quad = S1(I-1,K+1) \quad \text{or } \text{I-1} = 1 \quad \text{or } K=H
\end{align*} \]

\[(2.24)\]

\[ \begin{align*}
\text{FVOUT} &= \text{LV}(I,K) + \text{FV}(I,K) - \text{VV}(I-1,K) + S1(I,K) - S2(I,K) + S2(I-1,K-1) \\
&\quad \text{not } I=1 \text{ not } I=K \text{ not } I=K \text{ not } I=1 \\
&\quad \text{or } K=1 \text{ or } K=H \text{ or } K=1 \\
&\quad \quad \text{or } I=K \\
&\quad \quad \text{or } K=H \\
&\quad \quad \text{or } I=1 \\
&\quad \quad \text{or } K=H \\
&\quad = S1(I-1,K+1) \quad \text{or } \text{I-1} = 1 \quad \text{or } K=H
\end{align*} \]
2.3-34 The increments added to bar forces have six general patterns

\[
\begin{align*}
    \text{INC1} &= a_1 \alpha \text{ FH OUT} \\
    \text{INC2} &= a_2 \alpha \text{ FH OUT} \\
    \text{INC3} &= a_3 \beta (\text{ FH OUT } + \text{ FH OUT}) \\
    \text{INC4} &= a_4 \beta (\text{ FH OUT } - \text{ FH OUT}) \\
    \text{INC5} &= a_5 \gamma (\text{ FH OUT } + \text{ FH OUT}) \\
    \text{INC6} &= a_6 \gamma (\text{ FH OUT } - \text{ FH OUT}).
\end{align*}
\]

In the case of corner joints, DELFH and DELFV replace FH OUT and FH OUT. The magnitudes of the terms \(a_1, a_2, a_3, a_4, a_5, a_6\) depend only on the type of joint (corner, boundary, or interior), and in the case of boundary joints whether or not the bar force incremented is perpendicular or parallel to the boundary. The sign of the increment added to a bar force depends only on the direction of the positive bar force, at the joint at which the loads are applied, relative to the two directions of the applied loads.

The signs of increments have already been derived in section 2.3-29 from figure 2-4b. In the computer program FH OUT and FH OUT are first evaluated. Then the joint co-ordinate is examined to determine whether the joint lies on a corner, on a boundary, or internally. The appropriate values of INC1, INC2, ..., INC6 are calculated. The program then adds increments to bar forces, omitting equations, as it did terms in equations (2.25) if the bar number does not exist. The values of the increments as calculated are positive increments. They are added or subtracted according as to whether the sign on the right-hand side of the appropriate equation in 2.22 is positive or negative. The increment equations, and values for which they are omitted, for the rectangular lattice \((1,1), (1,2), (2,1), (2,2)\) are

\[
\begin{align*}
    \text{AFH(I,K-1)} &= \text{INC1} & \text{Exit if } K = 1 \\
    \text{AFH(I,K)} &= - \text{INC1} & I = 2 \\
    \text{AFV (I-1,K)} &= \text{INC2} & I = 1 \\
    \text{AFV (I,K)} &= - \text{INC2} & I = N \\
    \text{AS1(I-1,K-1)} &= \text{INC3} & I = 1 \text{ or } K = 2 \\
    \text{AS1(I,K)} &= - \text{INC3} & I = N \text{ or } K = 1 \\
    \text{AS2(I-1,K-1)} &= \text{INC4} & I = 1 \text{ or } K = 2 \\
    \text{AS2(I,K)} &= - \text{INC4} & I = N \text{ or } K = 1 \\
    \text{AS3(I-1,K)} &= \text{INC5} & I = 1 \text{ or } K = 2 \\
    \text{AS3(I,K)} &= - \text{INC5} & I = N \text{ or } K = 1 \\
    \text{AS4(I-1,K-1)} &= \text{INC6} & I = 1 \text{ or } K = 2 \\
    \text{AS4(I,K)} &= - \text{INC6} & I = N \text{ or } K = 1.
\end{align*}
\]
TREATISE OF BOUNDARY CONDITIONS

2.3-32 Boundary loads may be applied only at the joints of the lattice. A distributed load is apportioned to the joints so as to maintain equilibrium. The external forces applied must be in statical equilibrium. The application of loads is discussed in section 2.4.

2.3-33 When boundary displacements are given it is necessary to move joints to these positions and fix them there. In the undisplaced condition of the lattice there is zero force in the members at all joints. Movement of a joint results in forces in the members around the joint. The joint is then fixed so that there is no removal of out-of-balance forces at that joint. The relative position of the joints is preserved by changes in the lengths of members due to the forces from the applied displacement. As the distribution of out-of-balance forces proceeds, the lengths of members will vary, but the total change in displacement is unaffected.

2.3-34 The force required to produce an extension \( e \) in a member of cross-sectional area \( A \) is given by

\[
R = \frac{E h}{1 + \mu} e,
\]

where \( E \) is Young's modulus of both the material of the elastic element and the material of the framework members. Movement of corner and boundary joints has already been considered. The force in a corner joint due to a movement \( e \) is given by

\[
\frac{1}{2} \frac{E h}{1 + \mu} e,
\]

since \( A_p = \frac{1}{2} A \). The ratio of the force in the member lying in the direction of movement to the forces in the active and passive diagonals is

\[
2 a : 4 \beta : 4 \gamma = 1 : \frac{1 - \mu}{4(1 - \mu)} : \frac{3 \mu - 1}{4(1 - \mu)} \quad (2.27)
\]

For displacement of a joint normal to a boundary, the force in the member normal to the boundary is

\[
\frac{E h}{1 + \mu} e,
\]

and the ratio of the force in this member to the forces in the active and
passive diagonals is

\[ 2 \alpha : 2 \beta : 2 \gamma \]

\[ = 1 : \frac{1 + \mu}{2(1 - \mu)} : \frac{3 \mu - 1}{2(1 - \mu)} \quad (2.28) \]

Displacement parallel to the boundary of the joint requires a force in the two members lying on the boundary

\[ = \frac{1}{2} \frac{3 \mu - 1}{1 + \mu} e \quad . \]

The ratio of the force in these members to the forces in the active and passive diagonal members is

\[ a : 2 \beta : 2 \gamma \]

\[ = 1 : \frac{1 + \mu}{2(1 - \mu)} : \frac{3 \mu - 1}{2(1 - \mu)} \quad (2.29) \]

Boundary displacements may be set as forces. The only modification to the distribution procedure necessary is the omission of the release and bringing into equilibrium of the joints displaced.

2.3-35 When a crack occurs in a wall there can be no tensile force normal to the crack. A crack may close. Thus when cracking is considered modifications must be made accordingly to the distribution procedure. The location of cracks is limited to the boundaries between segments. No analyses of cracking were carried out, but by use of suitable logic statements in the program this is possible.

**PROGRAMS TO SOLVE THE LATTICE ANALOGY BY RELEASE OF JOINTS**

2.3-36 Two programs using the equations derived in this chapter to distribute the out-of-balance forces due to applied loading may be found in appendices B-1 and B-2. They are written in program language SEI for an I.B.M. 1620 electronic digital computer with 20,000 decimal digits of storage. The flow diagram is shown in figure 2-15. Both programs will allow stiffening of the top and the bottom of the wall, as is described in section 3.3.

In the program in appendix B-1 the bottom of the lattice is free to move. In the program in appendix B-2 the bottom joints of the lattice may not move vertically, enabling the solution of a lattice when the boundary conditions at the bottom are displacements. Both programs were used in the analysis of the shear wall tests conducted, as is described in section
3.4. Any value of Poisson’s ratio may be used.

2.3.57 The programs have work areas large enough to solve a rectangular lattice with 130 joints. The need to keep the work areas large influences the order of release of joints. The normal method of solving relaxation problems is to reduce to zero the largest residual, in this case the out-of-balance forces. Few logic statements would be required, but a storage area as large as a set of forces is necessary. Consequently a method of releasing joints from left to right in rows and working down in rows from the top downwards was adopted. As this method carries forwards and downwards the applied load on the top of the wall, it is reasonably efficient.

2.3.58 The operation of the programs in the computer is described in the appendices. Chapter 5 describes the uses made of these programs.
2.4 APPLICATION OF LOADS TO LATTICE JOINTS, AND INTERPRETATION OF THE SOLUTION

LIMITATIONS OF THE ANALOGY

2.4.1 The analogy has been derived for uniform loading on the sides of the framework (Appendix 1A). In general loads will not be uniform and the deflections of the lattice are approximately those of the plate. The accuracy of the approximation is unknown. This limitation of the analogy becomes evident when the application of loads to the lattice and the evaluation of stresses from the bar forces are considered. The smaller each segment, the more nearly uniform are the loading and stresses, and the more accurate is the solution.

APPORTIONING OF LOAD

2.4.2 Loads can only be applied to the lattice at joints. Requirements of equilibrium determine the values of applied loads. In Figure 2-18 both joint loads $P_1$ and $P_2$ and a varying distributed load $w$ acts on the boundary of an elastic plate segment. Isolating the boundary alone, and applying forces $P_1'$ and $P_2'$ to keep it in equilibrium, it is necessary for

$$z = 0$$

and

$$y = 0.$$ 

This uniquely determines $P_1'$ and $P_2'$. $P_1$ and $P_2$, equal and opposite loads to $P_1'$ and $P_2'$ are the loads applied to the two lattice joints corresponding in position with the ends of the plate element. The degree of agreement between the deflections of the plate element and those of the framework will depend on the uniformity of loading.

2.4.3 If internal loads are applied, consideration should be made of the location of the point of application and the load with respect to the four surrounding joints of the lattice framework. A point load as shown in Figure 2-19 should be shared among the four joints $(1,x)$, $(1,x+1)$, $(1+1,x+1)$, and $(1+1,x)$ in such a manner as to take into consideration the elastic shortening within the plate element.

2.4.4 The "analogous" lattice framework approximates the deformations of a plate segment for a finite size of frame member. Thought as to how to apportion plate loads to lattice joints can to some extent reduce errors in the solution of the lattice.
METHODS FOR EVALUATION OF STRESSES

2.4-5 In the evaluation of stresses the problem is to relate the bar forces in the lattice to the stresses in the analogous plate. It will be remembered that the lattice deflections are approximate because the load is not uniform. Two methods of evaluation of stresses are available. Both require further assumptions.

2.4-6 The first method to be described uses the displacements of the joints of the lattice framework. These approximate the displacement of points on the plate to a degree of accuracy dependent on the uniformity of loading. It is now further assumed (see Figure 2-20) that the strains in the centre of the segment of an elastic plate which has corners corresponding to the joints of an elementary frame may be obtained by an "averaging" of strains about the centre. The axial strain in the I direction at the centre of the segment shown in Figure 2-20 is taken as the average of the mean strains in the I direction at the two ends of the segment. For an interior segment

$$\varepsilon_I = \frac{F_H(I, K) + F_H(I + 1, K)}{2EI}$$  (2.30)

Similarly, remembering the $F_V(I, K)$ indicates compression,

$$\varepsilon_K = \frac{-F_V(I, K) + F_V(I + 1, K)}{2EI}$$  (2.31)

When calculating the shear distortion, the segment is assumed to have straight sides. The shear distortion at each of the four angles is averaged.

$$\gamma = \frac{1}{4} \left( 2 \angle BAC - \angle BCD + \angle ABC + \angle CDA - 2 \pi \right)$$  (2.32)

The method is considered more fully in paragraph 2.4-9.

2.4-7 The second method of evaluating stresses considers the equilibrium of forces when the lattice is cut by a section (Figure 2-21). The method assumes that the stress in the plate varies linearly between the stress values calculated at joints (Figure 2-22). At each joint alongside the section the cut members will cause concentrated forces at the joint normal and perpendicular to the section. These joint concentrations of force are assumed to result from the stress around the point. Apportioning the triangular stress loads as point loads at joints, as indicated in Figure 2-22, a set of simultaneous equations is found. The equation for
Joint r is typical. For normal stress on joint r
\[ N_r = \frac{4}{9} L \sigma_q + \frac{1}{2} L \sigma_y + \frac{1}{3} L \tau_y + \frac{1}{6} L \sigma_y. \] (2.33)

There is one such equation for each joint on the section. An iterative numerical method for the solution of these equations is discussed by Benjamin and Williams (7). Brezinkoff (6) first postulated the calculation of stresses from equilibrium, but his assumption that each joint concentration is the result of an equal triangular stress distribution either side of the point (figure 2-23) is incorrect. Joint concentrations are the resultants of stress. When the shear stress diagram is considered it is reasonable to assume that shear stress increases parabolically across the edge segments if the edge is free (figure 2-22(b)). If the edge is fixed a linear distribution will be a suitable form.

2.4-8 The choice between the two methods depends ultimately on the validity of the two assumptions. In the first method the assumption that the deflected shape of the lattice in that of the plate must be taken as an integral part of the method. The other assumption of this method, that the stresses and strains in the centre are the means of those at the edge is a good approximation. The second method deals only with loads, avoiding reference to the deflected shape, but there is no proof that this will lead to any greater accuracy than the first method. Without exploratory work on the relative accuracies of the methods the choice is simply between a short method and a lengthy method (solving the simultaneous equations of the form 2.32). The longer method has two advantages in that it yields one more value at each section, and gives boundary values. The shorter method was used.

CALCULATION OF STRESSES FROM THE SURROUNDING DISPLACEMENTS

2.4-9 The assumptions of this method have been discussed in the preceding sections. Axial stress equations may be quickly obtained from equations 2.30 and 2.31,
\[ \sigma_x = \frac{E}{1 - \mu} (\varepsilon_x + \mu \varepsilon_y) \]
\[ \sigma_y = \frac{E}{1 - \mu} (\varepsilon_x + \mu \varepsilon_y) \]
acting the stress in the centre of the segment by the \((x,y)\) co-ordinates.
of its top left-hand corner at the position (I,K), and denoting the axial stress in the I and K directions as AXIAH and AXIANV respectively, by substituting from 2.30 and 2.31

\[ AXIAH(I,K) = \frac{1}{2} \frac{1}{2} [FH(I,K) + FH(I+1,K) - \mu(FV(I,K) + FV(I,K+1))] \]

\[ = \frac{1}{2} \frac{1}{2} [FH(I,K) + FH(I+1,K) - \mu(FV(I,K) + FV(I,K+1))] \]

and

\[ AXIANV(I,K) = \frac{1}{2} \frac{1}{2} [FV(I,K) + FV(I+1,K) - \mu(FH(I,K) + FH(I,K+1))] \]

The strain in a boundary member for a given force is twice that of an internal member because its area is half as great. Forces in boundary members should be doubled when included in the above equations.

2.4-10 From the assumptions stated in section 2.4-6, equation 2.33 for shear strain was obtained.

\[ \gamma_B = \frac{1}{2} (\angle DAB + \angle BCD - \angle ABC - \angle CDAB - 2 \pi), \]

where A, B, C, and D are the joints in figure 2-30(c). Applying the cosine rule to triangle ABC

\[ \cos B = \frac{AB^2 + BC^2 - AC^2}{2.AB.BC}. \]

Angle B may be regarded as \( \frac{\pi}{2} \) less the shear strain at B. Similarly angle D is \( \frac{\pi}{2} \) less the shear strain at D. Angles A and B, however, are \( \frac{\pi}{2} \) plus twice the shear strains at A and C.

Thus \( B = 90 - \gamma_B \)

where \( \gamma \) denotes shear strain.

But \( \cos B = \sin \gamma_B \)

for small strains.

\[ \therefore \cos B = \gamma_B \]

\[ \therefore \gamma_B = \frac{AB^2 + BC^2 - AC^2}{2.AB.BC}. \]
The mean of the four shear strains at the four corners A, B, C and D is then

\[
\gamma_0 = \frac{1}{4} \left( \gamma_A + \gamma_B + \gamma_C + \gamma_D \right)
\]

\[
= \frac{1}{4.2} \left( \frac{AB^2 + BC^2 - AC^2}{AB \cdot AC} - \frac{CD^2 + DA^2 - BD^2}{CD \cdot DA} \right)
\]

\[
+ \frac{CD^2 + DA^2 - AC^2}{CD \cdot DA} - \frac{AB^2 + DA^2 - BD^2}{DA \cdot AB} \right).
\]

The element is square, and for small deflections

\[
AB = BC = CD = DA = 1
\]

\[
\Rightarrow \gamma_0 = \frac{1}{4} \left( \frac{2AB^2 - 2AC^2}{1} \right)
\]

\[
= \frac{1}{4} \left( (\sqrt{2} + e_{BD})^2 - (\sqrt{2} + e_{AC})^2 \right).
\]

Neglecting terms in \(e^2\) because \(e\) is very small

\[
\gamma_0 = \frac{1}{4} \left( 2\sqrt{2} e_{BD} - 2 \sqrt{2} e_{AC} \right).
\]

\[
= \frac{1}{4\sqrt{2}} \left( e_{AC} - e_{BD} \right).
\]

2.4.11 Then \(\mu = \frac{1}{2}\) the secondary internal members of area \(A_2\) do not exist. In this case it is possible to compare the result above with McHenry's stated equation (5), and Benjamin and Williams' work (7). For the case of pure shear alone (see appendix A-4)

\[
\gamma_0 = \frac{1}{\sqrt{2}} \times \sqrt{2} \times \frac{r_d}{M_1}
\]

\[
= \frac{2r_d}{M_1}
\]

where \(r_d\) is the force in the diagonal. McHenry's equation differs by a factor of \(\frac{1}{\sqrt{2}}\), but the author's equation agrees with that of Benjamin and Williams.

2.4.12 In the generalised lattice frame, because of the action of the internal members of area \(A_2\), the force along any diagonal between two joints, will in general not be uniform. Application of strain energy again
provides the solution. From table 2-2 and figure 2-20(a)

\[ \frac{\partial F}{\partial R_1} = \frac{(B_1 - B_2)1}{2A_2} + \frac{(B_1 - B_2 + 22)1}{\sqrt{2} A_4} \]

\[ = 0. \]

\[ \therefore \quad (B_2 - B_1) \frac{3(1 - \mu)(1 + \mu)}{2(1 + 2\mu)} = \frac{(B_1 - B_2 + 22)}{\sqrt{2} 18} \cdot \sqrt{2} (1 + \mu) \]

Putting the symbol

\[ r = \frac{3}{2(1 + 2\mu)} \]

\[ B_2 - B_1 = r(B_1 - B_2 + 22) \]

\[ B_1 = \frac{B_2(1 + r) - 22}{1 + 2r} \]

\[ \text{and} \quad F_{BD} = \frac{B_1(1 + r) - 22}{1 + 2r} \]

The elongation of the diagonal BD

\[ e_{BD} = \frac{1}{2}(B_2 + B_1) \times \frac{3(1 + \mu)}{8} \times \frac{2(1 + \mu)}{18} \]

\[ = \frac{(2 + 3r)B_2 - 22}{(1 + 2r)} \times \frac{1 + \mu}{8} \times \frac{2(1 + \mu)}{18}, \quad (2.35) \]

and the elongation of the diagonal AC

\[ e_{AC} = \frac{(2 + 3r)B_2 - 22}{(1 + 2r)} \times \frac{1 + \mu}{8} \times \frac{2(1 + \mu)}{18} \]

In section 2.1-10 the shear strain in the segment was found to be

\[ \gamma_0 = \frac{1}{\sqrt{2}} \left( e_{BD} - e_{AC} \right). \]
<table>
<thead>
<tr>
<th>Number</th>
<th>Force Due to ( B_2 ) and ( B_1 )</th>
<th>Force Due to ( R_1 )</th>
<th>( \frac{dF}{dR_1} )</th>
<th>( \frac{\sqrt{F}}{3E} \cdot \frac{1}{EA} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>FF</td>
<td>( \frac{B_2}{2} ) ( \frac{B_1}{2} )</td>
<td>( \frac{B_1}{\sqrt{2}} )</td>
<td>( \frac{1}{\sqrt{2}} )</td>
<td>( \frac{(B_1-B_2)1}{4MA_2} )</td>
</tr>
<tr>
<td>GG</td>
<td>( \frac{B_2}{2} ) ( \frac{B_1}{2} )</td>
<td>( \frac{B_1}{\sqrt{2}} )</td>
<td>( \frac{1}{\sqrt{2}} )</td>
<td>( \frac{(B_1-B_2)1}{4MA_2} )</td>
</tr>
<tr>
<td>HH</td>
<td>( \frac{B_2}{2} ) ( \frac{B_1}{2} )</td>
<td>( \frac{B_1}{\sqrt{2}} )</td>
<td>( \frac{1}{\sqrt{2}} )</td>
<td>( \frac{(B_1-B_2)1}{4MA_2} )</td>
</tr>
<tr>
<td>JJ</td>
<td>( \frac{B_2}{2} ) ( \frac{B_1}{2} )</td>
<td>( \frac{B_1}{\sqrt{2}} )</td>
<td>( \frac{1}{\sqrt{2}} )</td>
<td>( \frac{(B_1-B_2)1}{4MA_2} )</td>
</tr>
<tr>
<td>KK</td>
<td>( B_1-B_2 ) ( B_1 )</td>
<td>( B_1 )</td>
<td>( 1 )</td>
<td>( \frac{(B_1-B_2)1}{7EA_1} )</td>
</tr>
<tr>
<td>LL</td>
<td>( - ) ( B_1 )</td>
<td>( B_1 )</td>
<td>( 1 )</td>
<td>( \frac{B_11}{7EA_1} )</td>
</tr>
</tbody>
</table>

**Table 2-2**
Substituting for \( \sigma \) and \( \tau \),

\[
\gamma_0 = \frac{2(1+\mu)(B_2 - B_3)}{\sqrt{2} 2(1+\mu)} \frac{(1+\mu)}{E} \\
= \frac{1}{\sqrt{2}} \frac{(1+\mu)}{E} \left( B_2 - B_3 \right)
\]

Thus it appears that shear stress is calculated from the shear diagonal forces, as if there were no interior forces.

2.4.13 Referring to Figure 2-20, obeying the conventions, and remembering that \( s_1 \) and \( s_2 \) are the components of the diagonal forces,

\[
\gamma_0 = \frac{2(1+\mu)}{E} (2s_1(I,K+I) + 2s_1(I,K))
\]

But \( \gamma_0 = \frac{\tau_{ik}}{E} \frac{2(1+\mu)}{E} \)  

\[
\tau_{ik} = -\frac{2s_1(I,K+I) + 2s_1(I,K)}{E} \quad (2.35)
\]

Plotting the shear stress at the centre of the segment at which \( (I,K) \) is the top left-hand corner as \( \Sigma S_2 (I,K) \),

\[
\Sigma S_2 (I,K) = -\frac{2s_1(I,K+I) + 2s_1(I,K)}{E} \quad (2.36)
\]

**Accuracy of the Solution**

2.4.14 The accuracy of lattice analogy solutions increases as the size of the lattice elements decreases. It is not possible to say how accurate a solution is. By increasing the number of segments in a lattice and observing how the values change, an idea of the true value and hence the accuracy of the solutions for finite segment size is possible. This is due to the fact that the assumptions made, of deforability and of stress distribution, are becoming more nearly correct. Rabinow's displacement equations (2.6) were derived from the lattice analogy and from finite differences. For a grid of the same size, the accuracy of both a finite difference solution and a solution using the lattice analogy must be nearly the same.
The program used to evaluate stresses is listed in appendix B-3. The flow diagram is drawn in Figure 2-2. The card output from the program for evaluation of the forces in the members is loaded directly into this program. It uses equations (2.34) and (2.36) to evaluate stresses. Stresses are typed out.
2.5 DISTRIBUTION OF THE DEFLECTED STATE BY CONSIDERING THE MECHANICAL BEHAVIOR UNDER AXIAL LOAD OF THE BARS COMPOSING THE LATTICE

INTRODUCTION

2.5-1 The symbols \( h \) and \( v \) signify the horizontal and vertical displacements of a joint from its original position when the lattice was unloaded. In order to evaluate \( h \) and \( v \) values of joints, it is necessary to regard one joint as fixed, usually in its initial position. Also, because the deflected shape could rotate about this fixed joint, one other \( h \) or \( v \) value must be set. There will now be unique \( h \) and \( v \) values for each joint.

2.5-2 Very small deflections are assumed in deriving the theory of the lattice analogy. Thus at all times the bars composing the lattice may be assumed to be horizontal, vertical, or inclined at 45 degrees to the horizontal and vertical directions. With this assumption the horizontal displacement of a joint, relative to an adjacent joint in the same row, may be found by considering only the change in length of the connecting member. Similarly, the vertical deflections within a column and joints may be found by considering only the vertical connecting members. To derive the relationship for the relative position of the end joints of a diagonal member (section 2.5-3).

RELATIONS FOR CHANGES IN LENGTH OF HORIZONTAL AND VERTICAL MEMBERS

2.5-3 For a member under axial load the change in length is equal to the product of the strain in the bar and the length of the bar. \( E \delta \eta (I_{,R}) \) and \( E \delta v (I_{,R}) \) are the multiletter symbols containing subscripts which correspond to \( h \) and \( v \) respectively. In writing the equations for the shortening of horizontal and vertical members care must be taken to ensure that the correct cross-sectional areas are used. Members on the perimeter have half the cross-sectional area of internal members.

2.5-4 Defining a constant

\[
C = \frac{E(1+\mu)}{2}\]

(2.37)

for a member on the perimeter under force \( F \), the change in length
\[ \frac{3 \times 1}{\mu} = \frac{2(1 + \mu)}{E} \]
\[ = \frac{2(1 + \mu)}{E} \]
\[ = 6F. \]

For a member in the centre the change in length is \( \frac{1}{4}F \). The equations for relative joint displacements of members lying on the boundary of the lattice take the form

\[
\begin{align*}
\delta_{\text{DH}}(I,K) &= \delta_{\text{DH}}(I,K-1) + 6F_{I}(I,K-1) \\
\delta_{\text{DH}}(I,K) &= \delta_{\text{DH}}(I-1,K) + 6F_{V}(I-1,K)
\end{align*}
\]

The sign in front of the change in length term depends on whether or not the change in length produces an increase or a decrease of joint displacement in the positive direction of joint displacement. Thus, although in the member \((I,K)(I-1,K)\) a positive force \( F_{V}(I-1,K) \) is a compressive force, the displacement of the joint due to this positive force is upwards and therefore results in a decrease in \( \delta_{\text{DH}} \). Had \( \delta_{\text{DH}}(I,K) \) been evaluated from \( \delta_{\text{DH}}(I+1,K) \), a decrease in \( \delta_{\text{DH}} \) would occur. The equations for relative joint displacements between horizontal and vertical members inside the lattice take the form

\[
\begin{align*}
\delta_{\text{DH}}(I,K) &= \delta_{\text{DH}}(I,K-1) + \frac{6E}{1 + \mu} \delta_{\text{DH}}(I,K-1) \\
\delta_{\text{DH}}(I,K) &= \delta_{\text{DH}}(I-1,K) + \frac{6E}{1 + \mu} \delta_{\text{DH}}(I-1,K)
\end{align*}
\]

EQUATIONS FOR CHANGE IN LENGTH OF DIAGONAL MEMBERS

2.5-6 A typical diagonal member carrying a force whose components in the horizontal and vertical directions are \( F_{I}(I,K) \) is shown in Figure 2-25.

The symbols in the figure are used. Displacements are exaggerated for clarity.

\[ \angle \text{D} = 45^\circ - \theta \]

The change in length of the diagonal has been assumed to be very small, therefore the angle \( \theta \) subtended by chord AB is very small. The approximation

\[ \angle \text{D} = 45^\circ \]

therefore involves errors of a very small order.
The change in length of diagonal AB is given by

\[ \Delta s_1 = 2d \]

\[ = AB \cos (45^\circ - \varphi) \]

\[ = [\Delta r \sin \varphi + 2h \cos \varphi][\cos \varphi \cos 45^\circ + \sin \varphi \sin 45^\circ] \]

But \( \sin = \frac{\Delta r}{\sqrt{A h^2 + \Delta x^2}} \) and \( \cos = \frac{A h}{\sqrt{A h^2 + \Delta x^2}} \)

\[ \Delta s_1 = \frac{1}{\sqrt{2}} \left[ \frac{\Delta r^2 + \Delta h^2}{A h^2 + \Delta x^2} \right] \left[ \frac{A h + 2h}{A h^2 + \Delta x^2} \right] \]

\[ = \frac{\Delta h + \Delta x}{\sqrt{2}}, \]

\[ \Delta h + \Delta x = \sqrt{2} \Delta s_1 \] (2.40)
\[ \sqrt{2} a_{34} = -DELH(I, K) + DELH(I-1, K+1) - DELV(I, K) + DELV(I-1, K+1). \]

Isolating \( DELH(I, K) \) we have the equation

\[ \begin{align*}
DELH(I, K) &= DELH(I-1, K+1) - DELV(I, K) + DELV(I-1, K+1) \\
&\quad + C \times \left( 2 + \frac{2(I-1, K+1)}{1 + \frac{2}{r^2}} \right) \\
&= DELH(I, K) - DELH(I-1, K+1) + DELV(I, K) \\
&\quad - C \times \left( 2 + \frac{2(I-1, K+1)}{1 + \frac{2}{r^2}} \right)
\end{align*} \quad (2.42) \]

This will be referred to as the \( S1 \) equation, and is used to find \( DELH(I, K) \) from the other three displacements.

2.5-7 If it should be required to find the value of \( v_1 \) when \( h_2, v_2, \) and \( h_1 \) are known, the equation may be rearranged to take the form

\[ \begin{align*}
DELV(I-1, K+1) &= DELH(I, K) - DELH(I-1, K+1) + DELV(I, K) \\
&\quad - C \times \left( 2 + \frac{2(I-1, K+1)}{1 + \frac{2}{r^2}} \right)
\end{align*} \quad (2.43) \]

This will be referred to as the \( S123D \) equation.

ORDER OF EVALUATION OF JOINTS

2.5-8 In figure 2-27 a typical lattice has been drawn. At each joint there are \( h \) and \( v \) values. Alongside the \( h \) and \( v \) values there are reference numbers, referring to the step in which the \( h \) or \( v \) value is evaluated. The arrows indicate where either the \( S1 \) equation or the \( S123D \) equation is applied.

2.5-9 For convenience and clarity the order of evaluating displacements at joints is given as a series of steps.

1. Fix \( h_{1,1}, v_{1,1} \) and \( v_{1,2} \) as zero (see section 2.5-4).
2. From these values use equations (2.30) to evaluate the horizontal displacements of row 1, the vertical displacements of column 1, and equation (2.39a) to evaluate the vertical displacements of column 2.
3. Use the \( S1 \) equation (2.42) to evaluate \( h_{2,1} \).
4. Use equation (2.39a) to evaluate the horizontal displacements for row 2.
5. Use the \( S1 \) equation to evaluate \( h_{3,1} \), and equation (2.39a) to evaluate the horizontal displacements for row 3. Repeat this sequence for succeeding rows until all horizontal displacements have been evaluated.
6. Use the \( S123D \) equation (2.43) to evaluate \( v_{1,3} \).
7. Use equation (2.39b) to evaluate the vertical displacements for column 3.

8. Using the 812D equation and subsequently equation (2.39b), evaluate the vertical displacements in the remaining columns.

SHIFTING AND ROTATION OF THE CALCULATED DEFLECTED SHAPE

2.5-10 The relative position of joints is now known. The origin and orientation are, however, unconventional. The bottom left corner is a suitable new origin. The second condition imposed is that the bottom right corner shall be at the same height as the bottom left corner. The first condition is fulfilled simply, by subtracting the displacements of the bottom left corner from the displacements obtained at joints over the lattice. To fulfill the second condition the deflected shape must be rotated around the new origin.

2.5-11 Figure 2-28 shows a deflected shape which has been shifted to a new origin at the bottom left-hand corner. The lattice has N columns of joints, and N rows of joints. The displacements are now given by

\[
\text{DEW}(I,K) = \text{DEW}(N,1) \\
\text{and } \text{DEW}(I,K) = \text{DEW}(N,1).
\]

To bring the joint \((N,K)\) on to the same horizontal line as \((1,1)\) the deflected shape is now rotated anticlockwise through an angle \(\theta\), where

\[
\theta = - \frac{\text{DEW}(N,K) - \text{DEW}(1,1)}{(N-1)1}.
\]

For any joint with co-ordinates \((I,K)\), the new displacement can now be calculated. Let the distance of this point \((I,K)\) from \((1,1)\) be \(r\). A rotation \(\theta\) produces a horizontal displacement of the joint of value \(-r \theta \sin \theta\), and a vertical displacement of value \(r \theta \cos \theta\) (Figure 2-29). But

\[
\cos \theta = \frac{(N-I)}{N} \frac{1}{1} \\
\text{and } \sin \theta = \frac{(K-I)}{N} \frac{1}{1}
\]

Therefore the horizontal displacement at joint \(I,K\) due to rotation is

\[
- \left( \frac{\text{DEW}(N,K) - \text{DEW}(1,1)}{(N-1)1} \right) \times (K-1)1.
\]
and the vertical displacement is
\[ + \frac{N-1}{N} (\text{DEL}_V(N,1) - \text{DEL}_V(N,1)) \]

2.5-12 The displacements of any point, under the conditions that the bottom left joint \((N,1)\) shall occupy its initial position before the lattice was distorted, and that the bottom right joint \((N,N)\) shall lie on the same horizontal line as \((N,1)\), are as follows. The horizontal displacement is
\[ \text{DEL}_H(I,K) - \text{DEL}_H(N,1) - \frac{K-1}{K} (\text{DEL}_V(N,N) - \text{DEL}_V(N,1)) \]

The vertical displacement is
\[ \text{DEL}_V(I,K) - \text{DEL}_V(N,1) + \frac{K-1}{K} (\text{DEL}_V(N,N) - \text{DEL}_V(N,1)) \]

PROGRAM USED TO EVALUATE DISPLACEMENTS

2.5-13 The program used to evaluate displacements and then shift and rotate them, is listed in appendix B-4. The flow diagram is drawn in figure 2-29. Equations (2.2), (2.3), (2.4) and (2.5) are used to evaluate the displacements in the order described in section 2.5-2. Equations (2.4) and (2.5) are used to shift and rotate the deflected shape. Card input is the output from the program to determine the bar forces. Displacements are typed out.
Figure 2.1

Figure 2.2
Elastic body

Figure 2.3

Lattice frame

Figure 2.4

Figure 2.5
Figure 2.6

Figure 2.7
Figure 2.11

Figure 2.12
Figure 2.13
Figure 2.14
Type in $M, N$,

Read in $J$

Corner increments

Boundary increments

Central increments

Add increments

Direct to correct type of joint

Address bar forces

Increment $l, k$ values $l = N + 1$

Test $k$ value

Type in $M, N$

Read in bar forces

Set base fixity

$T = T + 1$

$T = 1$

$K = 1$

$J_{<i}$

Figure 2.15
### Figure 2.16

![Diagram of loads on boundary lattice joints](image)

Loads on boundary lattice joints

### Figure 2.17

![Diagram of loads at lattice joints](image)

### Figure 2.18

- (a) W_1, W_2, P'_1, P_1, P'_2, P_2
- Load distribution

### Figure 2.19

- (1, K) to (1, K+1)
- (1+1, K) to (1+1, K+1)

### Figure 2.20

- (1, K) to (1, K+1)
- (1, K) to (1+1, K+1)
- (1, K) to (1+1, K)
- (1+1, K) to (1+1, K+1)
- (1, K) to (1+1, K+1)

**Note:** The text in the image includes mathematical notations and symbols, which are not transcribed here.
Vertical section—exploded for clarity
Figure 2.21

Axial Stress (a)

Shear Stress (b)

Figure 2.22
Read in bar forces

Type in \( M, N, E, t, J, J_T, J_B \)

Modify edge forces

\[ T = 1 \]
\[ I = 1 \]
\[ K = 1 \]

I, K control

\[ T = 1 \]
\[ T = 2 \]
\[ T = 3 \]
\[ T = 4 \]

Calculate axial stresses

Print horizontal stresses

Print vertical stresses

Calculate shear stresses

Print shear stresses

Figure 2.24
Hrennikoff's assumed stress distribution

Figure 2.24

\[ \sigma_q = \frac{N_q}{l_t} \]

Figure 2.25

Figure 2.26
Figure 2.27

Figure 2.28
Read in bar forces

Type in
\{ M, N, E, t, \mu, J_T, J_B \}

Calculate constants

\( I = 1 \)
\( K = 1 \)

Increment
I, K values,
I, K control

HEQUN
Horizontal changes in
length

VEQUN
Vertical changes in
length

S1EQUN
Diagonal changes in
length

S12ND
Diagonal changes in
length

Addresses
of forces

Rotate the
deflected
shape

Type out
deflections

Figure 2.29
SYNOPSIS

The lattice analogy has been described in Chapter 2, and equations have been formulated for bringing joints into equilibrium. In this chapter the method is used to consider three simple loading configurations which might be taken as models of shear wall behaviour. It is used to analyse the stresses in a wall when the ends of the wall are prestressed. The method is extended to allow horizontal stiffening of the top and the bottom of the wall. The analysis of the walls tested, in which the bottom of the wall deflects under load, is presented.

LIST OF SYMBOLS USED IN CHAPTER 3

\[ \begin{align*}
\mu & = \text{Poisson's ratio} \\
\varepsilon & = \text{shortening of a member} \\
G & = \frac{2(1 + \mu)}{E} \\
\gamma & = \text{constant} \\
D_1, D_2 & = \text{forces (see Fig. 3-1)} \\
E & = \text{Young's modulus of the material of the members of the lattice} \\
\varepsilon & = \text{Young's modulus of the material of the wall} \\
t & = \text{thickness of the wall} \\
\alpha, \beta, \gamma & = \text{constants dependent on \( \mu \) (see section 2.2-4)} \\
I, K & = \text{co-ordinates of a joint} \\
F_{H}(I, K) & = \text{forces defined in section 2.2-3} \\
F_{V}(I, K) & = \text{forces defined in section 2.2-3} \\
S_{1}(I, K) & = \text{forces defined in section 2.2-3} \\
S_{2}(I, K) & = \text{forces defined in section 2.2-3} \\
F_{H} & = \text{forces defined in section 2.2-3} \\
F_{V} & = \text{forces defined in section 2.2-3} \\
D_{H} & = \text{forces defined in section 2.2-3} \\
D_{V} & = \text{forces defined in section 2.2-3} \\
s & = \text{shear stress} \\
p & = \text{normal stress}
\end{align*} \]
1 = length of the elementary frames of a lattice
f = axial stress
M = moment
Z = section modulus
k = constant
\( f_c \) = compressive stress of the material of a wall in the direction of the diagonal
\( \omega \) = angle between the base of a wall and a diagonal of the wall
V = horizontal load at the top of a wall
j = constant
\( A_o \) = area of an external horizontal or vertical member of an analogous frame
A = area of an internal vertical or horizontal member of an analogous frame
\( A_1 \) = area of a diagonal member of an analogous frame
F, F' = forces
\( f_a \) = active distribution factor
\( \lambda \, \nu, \rho \) = constants defined in text (section 1.3-5)
\( \delta_h \) = horizontal deflection
\( \delta_v \) = vertical deflection
\( N, T \) = normal and tangential components of the force at a joint when the lattice is cut by a section
L = length of a wall
H = height of a wall in the test frame
n = modular ratio
\( A_s \) = area of steel in a loading beam
\( A_b \) = area of brickwork in a loading beam
\( J_T \) = ratio of the area of the top loading beam to the area of an interior vertical or horizontal member of the lattice framework
\( J_B \) = ratio of the area of the bottom loading beam to the area of an interior vertical or horizontal member of the lattice framework
I_B = moment of inertia of the brickwork
\[ Q = \text{force} \]
\[ a, b = \text{length dimensions in fig. 3-14} \]
\[ y = \text{vertically downwards deflection} \]
\[ x = \text{distance along the bottom loading beam from the left-hand (tension) end} \]
\[ R = \text{reaction} \]
\[ M_1, M_2 = \text{moments at the end of that part of the bottom loading beam in contact with the wall} \]
\[ w = \text{non-uniform vertical load} \]
\[ w = \text{vertical point load} \]
\[ k = \text{constant} \]
\[ W(N,K) = \text{vertical point load at the joint (N,K)} \]
3.1 USE OF THE METHOD OF LATTICE ANALOGY AND THE PRESENTATION OF RESULTS

POISSON'S RATIO

3.1-1 Two values of Poisson's ratio were used. A value of 0.15 was used in the three preliminary analyses which may be found in section 3.2. This value is typical of concrete. The value was chosen in order to show Poisson's effect.

3.1-2 In the analysis of the tests, and in the determination of the stresses caused by prestressing the ends of a wall, a value typical of brickwork must be used. There is little published data on the value of Poisson's ratio for brickwork. Benjamin and Williams have found zero a suitable value. Poisson's ratio for building bricks varies from 0.01 to 0.12. Bricks form the greater part of brickwork. Because of the discontinuities in the bricks at mortar joints, and the possibility of shrinkage or other cracking in this joint, the low value of Poisson's ratio for building brick, 0.05, was adopted.

OPERATION OF THE COMPUTER PROGRAMS

3.1-3 The amount of shortening of the diagonal BB in figure 3-1,

\[ \varepsilon_{BB} = \frac{6}{2} \left[ \frac{(2 + 3r) D - w_2}{1 + 2r} \right] \]

where \( r = \frac{3(\mu - 1)}{2(1 - 2\mu)} \), and \( e = \frac{2(1 + \mu)}{Et} \).

It is necessary for conditions of strain compatibility that the diagonal AC "fit" the positions of the two pin joints A and C determined by calculations involving the diagonal BB. The condition of compatibility is very valuable for checking calculations, may be considered at almost any stage of the computation, and is particularly useful for testing programs.

3.1-4 Pure horizontal movement of the joint A in figure 3-1 under a unit horizontal force and the associated vertical force \( h_B \) induces forces in the member AB, in the diagonal members, and in the internal auxiliary members. Equation (2-35) expresses the shortening of the diagonal in terms of the diagonal forces acting on the pin joints. The shortening in AB is \( \frac{2\varepsilon}{\sqrt{2}} \), and the shortening in the diagonals AC and BD are \( \frac{2\varepsilon}{6} \) and zero.
Substituting for \( r \)
\[
\frac{2 \ v_{AC}}{\theta} = \frac{(1 + \mu) v_2 - (3\mu - 1) v_3}{2 \mu}
\]

Substituting for \( v_2 \) and \( v_3 \) in terms of \( S_1 \) and \( S_2 \), hence in terms of \( B \) and \( \gamma \) and finally substituting in terms of \( \mu \)

\[
\frac{\psi_2 \ v_{AC}}{\theta} = -\frac{(1 + \mu)(1 + \mu)}{5 - 3 \mu} + \frac{(3\mu - 1)(3\mu - 1)}{2 \mu}
\]

\[
= \frac{4(1 - 2\mu)}{5 - 3 \mu}
\]

\[
= 2 \ a_0.
\]

This proves the compatibility of the first diagonal for the movement of the joint \( A \). The change in length of the second diagonal is similarly obtained by substitution to be

\[
\frac{\psi_2 \ v_{BC}}{\theta} = -\frac{(1 + \mu) - (3\mu - 1) - (3\mu - 1) - (1 + \mu)}{5 - 3 \mu} - \frac{4(1 - 2\mu)}{2 \mu}
\]

\[
= 0.
\]

All joints contain segments of the type ABCD, and all movements can be separated into the sum of movements along the component axes. Therefore all movements will obey the laws of strain compatibility.

3.1-5 Any input bar forces must obey the compatibility relationship. Because joint movements to remove out-of-balance forces obey the compatibility relationship, any strain incompatibility in a segment as a result of the input forces will not be removed by joint movements. Even with the larger lattices used it was quicker to use all zero bar forces in the input data.

3.1-6 Joints of a lattice may be fixed by the boundary conditions (see section 2.3-36). Because these joints are not released the movement of the surrounding joints will result in a force at these joints. As the computation proceeds the value of this force approaches a limiting value. The joints act passively. In direct contrast an applied force at a joint actively affects the forces at the surrounding joints. A much faster convergence of the lattice occurs when the boundary conditions are forces
rather than displacements.

3.1-7 When the boundary conditions were forces approximately four 
hours of calculation by the IBM 1620 computer were required to obtain 
the bar forces of an 11 x 11 lattice to a sufficient degree of convergence. 
Two hundred cycles were carried out in this time. Each cycle took about 
one minute. In that minute, one hundred and twenty-one joints were 
brought into equilibrium. Evaluation of stresses and displacements 
required approximately forty minutes, mainly for the typing out of results.

PRESENTATION AND CHECKING OF RESULTS

3.1-8 The stresses and displacements were evaluated for the horizontal 
and vertical axes. The stresses are not principal stresses, nor is the 
shear stress the maximum shear stress. The study of brickwork shear 
walls is one of brick-mortar bond relationships, rather than principal 
stresses.

3.1-9 The stresses in the walls are shown as contours. These contours 
of stress draw are intended to show the distribution of stress rather 
than exact values. In general the value of the stress on a contour is 
half the value on the next highest contour, although the contours of zero 
stress and some others have been included. The contours of stress were 
obtained by graphical interpolation from the values of the stress at the 
centre of the segments. There is a certain inaccuracy in the assumption 
that the stress evaluated in a segment the mean stress at the centre of 
the segment (see section 2.4). On any individual section, however, it 
is possible to adjust the values of stresses by using the laws of statics 
which require the sum of the horizontal forces, the vertical forces, and 
the moments about any point to be zero.

3.1-10 Where a point load is applied there is a theoretical region of 
infinite stress. When stresses at the centre of an element are computed 
there is an actual peak. Contours of stress around point loads have 
been so drawn as to provide a closed contour if possible. There is a 
theoretical point on the free edge where the shear stress is not zero, 
under the load. As this is not a practical consideration, closed contours 
have been drawn. In practice the contours will converge under the load.

3.1-11 Stresses may be checked by taking sections and testing for
equilibrium. In checking the displacements, use may be made of the rule of strain compatibility. The lattice solution also may be checked by statics and compatibility.

3.1-12 In the analyses the horizontal point load was always applied at the top left-hand corner of the wall. The diagonal between this corner and the bottom right-hand corner will be referred to as the major diagonal, as this diagonal acts as a strut.

3.1-13 In considering the behaviour of walls, the brickwork is assumed to have the following properties. Its compressive strength is 3100 psi and its modulus of elasticity is $1.5 \times 10^6$ psi. The crushing strength of the mortar is 1900 psi; the tensile strength of the mortar is one-tenth this value. The brick has a crushing strength of 12000 psi, and a tensile strength of one-tenth that value. The brick-mortar bond has a tensile strength of 70 psi, and a compression-shear relationship given by

$$s = 390 + p$$  \hspace{1cm} (4-3)

where $p$ is greater than 80 psi (see section 4-4). The limit of this relationship is assumed to be when crushing of the brickwork occurs.

Only the tensile strength of the brick-mortar bond, the compression-shear relationship of the bond, and the crushing strength of the brick are important in ascertaining failure.

CONVERSION OF THE RESULTS OF THE ANALYSIS OF A WALL OF A GIVEN SHAPE TO ANALYSE A WALL OF THE SAME SHAPE, BUT DIFFERENT DIMENSIONS

3.1-14 A lattice analogy for a given value of Poisson's ratio is true for that value of Poisson's ratio only. It is possible, however, by simple proportion to modify the results of an analysis to apply them to a wall of the same shape but different dimensions, different thickness, and made of a material with a different modulus of elasticity.

3.1-15 In Chapter 2 it may be seen that stress is proportional to $\frac{1}{21(1-\mu)}$. Poisson's ratio may not be changed, but scaling of stresses may be effected because stress is inversely proportional to the length and to the thickness of the wall analysed. Similarly deflections may be scaled. In Chapter 2 it was found that deflection is proportional to $\frac{2(1-\mu)}{Es}$. It is interesting to note that stresses computed from the lattice solution
are not proportional to Young’s modulus, and deflections are not proportional to the length dimension of a wall.

3.2 CONSIDERATION OF THREE SIMPLE LOADING CONFIGURATIONS AS POSSIBLE MODELS OF A SHEAR WALL UNDER LOAD

3.2-1 When a horizontal point load is applied to the top of a vertical wall there are three simple ways in which it may be resisted. These are shown diagrammatically in figure 3-2. In figure 3-2(a) the wall acts as if it were a strut. The configuration of loading will be referred to as compression member action. In figure 3-2(b) the bottom left-hand corner of the wall is held down. This configuration of loading will be referred to as “tension restraint”. In the third configuration the base of the wall is rigidly fixed in both the vertical and the horizontal directions (fig. 3-2(c)). The wall is a vertical cantilever with a small span to depth ratio. This third case will be referred to as cantilever action.

3.2-2 These loading configurations were analysed by means of the lattice analogy. As they are only required for the purpose of comparison they have not been analysed on a fine grid. Unit thickness of the wall, a value of Young’s modulus of $1 \times 10^6$ units, and a length of side of 100 units have been used in this analysis for convenience.

STRESSES

3.2-3 The results of these analyses are shown in figures 3-3 to 3-40. The contours of vertical stress for compression member action of the wall are not given as both horizontal and vertical stresses are axial stresses caused by loads symmetrical about the major diagonal.

3.2-4 Where a point load is applied there is a theoretical region of infinite stress. Such areas, which have no physical existence, are apparent in all three analyses. The load in any practical situation will be spread over a finite area. In addition localized redistribution of stresses will reduce peak stresses.

3.2-5 The differences between the different conditions of loading are immediately apparent from the contours of stress. In compression member
action, axial stresses are compressive throughout. The tension restraint model is characterized by high vertical tensile stresses at the lower left-hand corner. Vertical stress is also the dominant feature in the cantilever action, the vertical stresses along the base being essentially those predicted by the equation:

\[ f = \frac{V}{2b} \]

3.2-6 The distribution of stresses within a wall is characterized in the case of the compression member action by concentration of axial stress about the major diagonal. The contours of direct stress appear as lobes which stretch obliquely out from the corner along the major diagonal. Stresses in the central area of the wall are highest about the diagonal. The contours of shear stress balloon outwards from a line of symmetry which is the major diagonal. In the top left-hand corner and in the bottom right-hand corner there are very high values of both horizontal, vertical and shear stress due to the application of point loads.

3.2-7 For both the tension restraint condition of loading and the cantilever action the stress conditions in the top of the wall are very similar. It is reasonable to conclude that differences in the restraint of the base have a minor effect on stresses at the top of the wall. The contours of horizontal stress again form lobes, but extend along the top of the wall rather than downwards.

3.2-8 The stresses in the centre of the wall under tension restraint and cantilever action are much lower than those when the wall is under compression member action. At the vertical edges of the wall the horizontal stress approaches zero, and the vertical stresses are highest. This is because bending, rather than compression and shear, is the dominant action.

3.2-9 The vertical point load providing tension restraint produces a region in which the vertical and shear stresses are high. Failure is caused by a combination of tension and shear. Tension and shear are low in regions of high vertical stress for cantilever action, suggesting that tensile strength alone might be used to compute failure loads.

### Probable Modes of Failure

3.2-10 The failure of a wall under compression member action is a function of the compression-shear relationship. This relationship was found to be,
for the bricks used (see section 4.4),

\[ s = 320 + p \]  \hspace{1cm} (b-3)

Benjamin and Williams obtained a similar relationship: \( s = 220 + 1.4p \). Computed failure from these relationships occurs when the calculated shear stress is exceeded. The analysis for compression member action was carried out on an 8 x 8 lattice. Stresses were computed at the centre of each lattice segment. Figure 3-1 shows these stresses, and the two quantities \( s-p \) and \( s-1.1p \). The maximum values of \( s-p \) and \( s-1.1p \) are underlined. It is interesting to note that failure according to the compression–shear relationship, assuming that the body is elastic, is predicted to occur in the vertical joints on the horizontal centreline of the wall, and in the horizontal joints on the vertical centreline of the wall. As none of the mortared joints is continuous, workmanship is important in determining the location of a central failure as poor workmanship will allow cracks to spread easily. The additional strength of brickwork when the mortar joints are not continuous was observed in the first two walls tested (see Chapter 5). In these tests vertical tensile stresses in the bottom left-hand corner caused cracking along the horizontal mortar joints. Stretcher bonded masonry failed at approximately half the load at which stack-bonded masonry failed. Cracks were first observed in the stack-bonded masonry at two-thirds the ultimate load, whereas in stretcher bonded masonry failure was sudden and complete.

3.2-11 In compression member action the compression–shear relationship also predicts failure in the vertical joints just below the top left-hand corner and above the bottom right-hand corner, and in the horizontal joints just in front of and behind these corners. This fact may explain the occurrence of some failure cracks in these locations.

3.2-12 Calculation of failure loads from the compression–shear relationship is undesirable because the difference of two large numbers is equated to a small number associated with the very variable tensile bond strength. For example, if a wall 100" square 4" thick, in which \( E = 10^6 \) psi, fails at a load of \( L \times 10^6 \) lbs, the failure loads may be calculated as follows.
If \( a = 390 + p \)
\[
k \times 654 = 390 \times 4
\]
\[
k = 2.360
\]
and failure occurs at a load of 106.4 tons.

If \( a = 220 + 1.4p \)
\[
k \times 600 = 220 \times 4
\]
\[
k = 1.568
\]
and failure occurs at a load of 65.6 tons. Holmes (1) considers a brick wall within a steel frame to be a strut which will fail a load of
\[
\frac{1}{\cos \omega} f_0 \cos \omega = \text{It } f_0
\]
He suggests from available test results a value of \( f'_0 \) of 450 psi. This value would give a failure load for a wall of the same dimensions of 80.4 tons.

3.2-13 The above discussion of the failure of a wall under compression member action has been based on the assumption that the "point loads" are applied in such a manner that premature failure due to crushing of the bricks beneath the load does not occur. However, the greatest weakness of the above argument is in the assumptions made in deriving the relationship

\[
a = 390 + p.
\]

It was assumed that the maximum shear stress causing failure in conjunction with the average compressive stress was 1.5 times the mean shear stress. This assumption is not proven (see section 4-4). Any differences in the ratio of \( a \) to \( p \) are important because the quantity \( a - p \) is taken. A simple rule such as that proposed by Holmes seems preferable.

3.2-14 The failure of a wall under tension restraint may not be separated from the method of applying the tensile restraint. Tension is critical, and it is impossible to affix a failure load without defining the method of restraint. The determination of the failure of the walls tested was in fact a determination of the tension restraint (see section 3.5).

3.2-15 The failure of a wall acting as a cantilever may be computed by means of simple beam theory, considering tension alone, since shear is comparatively small in regions of high vertical stress. Thus if failure occurs when a tensile stress of 70 psi is reached, the failure load of a
wall 100 inches long, 4 inches thick, and in which \( E = 1 \times 10^6 \) psi

\[
= 10^5 \times \frac{70}{6000} \times 4
= 2.34
\]

This is approximately one-fourtieth of the strength of a wall under compression member action.

**Applicability of the Three Simple Loading Configurations**

3.2-16 None of the three simple models is sufficient for analysis of the behaviour of a shear wall before cracking occurs. In shear walls, the tension in the lower left corner is the critical condition. The best of the three simple configurations, cantilever action, is limited by the assumption of infinite rigidity at the base. With the very great rigidity of the shear wall a very rigid base is impossible. Normally walls are supported by beams and floors. Support of the base of a wall is considered in section 3-4.

3.2-17 Compression member action may be used to consider the action of a wall after cracking has extended along the base of the wall if the top of the wall is held down. The frame around an infill panel may provide the holding down force. Prestressing steel used to delay tension cracking will also hold down the top of a wall.

3.2-18 As yet the actual manner in which the horizontal load is applied to the top of the wall has not been considered. It is reasonable to assume that the load will be applied by the floors above and below the wall. Although the stiffness of the floors may not be taken as infinitely greater than that of the wall, the condition of loading is more nearly one of uniform displacement than one of point loading. The distribution of loading is considered more fully in section 3-4. It may be noted in passing that from the two analyses conducted for tension restraint and cantilever action it appears that for square walls the method of application of the horizontal load is of secondary importance in evaluating failure stresses.
3.5 ANALYSIS OF A WALL WHICH IS HORIZONTALLY SHEARED BY FLOORS

LATTICE ANALOGY OF NON-UNIFORM PLATES

3.5-1 In appendix A-1 the cross-sectional areas of the members of a square pin-jointed framework of external side length 1, Young's modulus $E$ and Poisson's ratio $\mu$ were derived. The frames have the same deformability as an elastic body of the same external dimensions, modulus of elasticity, Poisson's ratio, and of thickness $t$ under conditions of uniform axial stress and uniform shear stress. A rectangular elastic body is considered as a lattice frame composed of these frameworks. The cross-sectional areas of the members are uniquely determined by the values of $l$, $t$, $E$ and $\mu$. The cross-sectional areas of the members of a frame corresponding to a plate segment of length $l$, thickness $t$, Poisson's ratio $\mu$, and Young's modulus $E$ may be similarly determined. Such a frame may be placed above the frame formerly described, connecting at two pin joints. The deflections of a lattice composed of frameworks with different deformabilities will be those of a plate composed of segments of different $E$, $l$, $\mu$ and $t$.

Equations for the bar forces resulting from the action of out-of-balance forces at the joints of such a lattice may be obtained by considering joint movements, as were the equations for a uniform plate.

3.5-2 For the pure horizontal movement of a joint connecting two or more frames with different deformabilities, a vertical force is necessary to balance the forces in the diagonal members at the joint. The release of a joint necessitates the solution of equations for the equivalent applied forces $BD_1$ and $BE_1$ in terms of the out-of-balance forces $PH_1$ and $PV_1$. This is the same general method as was used for the release of corner joints. A general equation may be derived for the release of a joint surrounded by four segments of different $E$, $l$, $\mu$ and $t$. The coefficients of the terms in this equation are a function of the member areas of the surrounding segments. These coefficients can be readily stored in a computer as a matrix.

3.5-3 The validity of the lattice analogy depends on the uniformity of loading on individual segments. Although a lattice may be composed of frameworks representing plate segments of differing thickness, modulus of elasticity, or Poisson's ratio, the validity of the analogy depends only
on the degree of uniformity of loading on each segment. Thus there is no greater inaccuracy when elements of differing thickness, modulus of elasticity or Poisson's ratio occur alongside each other, provided that the assumption of no stress gradient across the thickness is still appropriate.

3.3-4 The condition of analogous deflections is one of equal strains. The analogy increases in accuracy with increasing uniformity of loading. Two boundary members of adjacent frames may be considered to act as one member. Rectangular segments may occur next to square segments, provided that no joint of one frame lies within a member of another frame. In the limit a rectangular segment of zero height may be used to stiffen an exterior member of a frame. Such stiffening of the exterior members of a lattice is a simple and convenient way of analysing a shear wall in which a bounding beam or a floor stiffens the top or bottom of the wall in the horizontal direction.

EQUATIONS FOR THE RELEASE OF JOINTS ON A STIFFENED EDGE OF A LATTICE FRAME

3.3-5 Figure 3-13 shows a corner joint (I,K) of a framework in which the top member has been stiffened. The area of this member is \( A \), where \( A \) is the area of an internal vertical or horizontal member, and

\[
A = \frac{1}{1 + \mu}.
\]

The forces \( P \) and \( P' \) act horizontally and vertically as in the figure to produce only horizontal movement of the joint (I,K). The vertical member carries no load. If the displacement is such as to cause unit force in a member of area \( A \), from equilibrium

\[
P = - M_H (I,K) - S_2 (I,K)
\]

\[
= - \left( J + \frac{f_a}{A} \right),
\]

and

\[
P' = S_2 (I,K)
\]

\[
= \frac{f_a}{A},
\]

\[
P = - \left( J + \frac{f_a}{A} \right)
\]

\[
= - \left( J + \frac{\nu}{E(1-\nu)} \right)
\]

\[
= - \frac{E_2 + \mu + (1 - 2\mu)}{E(1 - \mu)} .
\]
\[ F_H(I, K) = -\frac{1}{\mu} \cdot P = -\frac{\mu - (1-\mu)}{3\mu + 1 + \mu(1-3\mu)} \cdot P \tag{3-1} \]

\[ S_2(I, K) = \frac{f}{P} \cdot P = -\frac{1 + \mu}{3\mu + 1 + \mu(1-3\mu)} \cdot P \]

and also
\[ S_1(I, K+1) = -\frac{f}{P} \cdot P = \frac{\mu - 1}{3\mu + 1 + \mu(1-3\mu)} \cdot P. \]

Defining the following quantities
\[ 2\lambda = \frac{3(1-\mu)}{3\mu + 1 + \mu(1-3\mu)} \tag{3-2} \]
\[ 4\nu = \frac{1 + \mu}{3\mu + 1 + \mu(1-3\mu)} \]
\[ 4\rho = \frac{3(1-\mu)}{3\mu + 1 + \mu(1-3\mu)} \]

the equations (3-1) may be rewritten in the form
\[ F_H(I, K) = -2\lambda P \]
\[ S_2(I, K) = -4\nu P \]
\[ S_1(I, K+1) = 4\rho P \tag{3-3} \]

In particular, if \( j \) is equal to a half we have the equations for the horizontal movement of a corner joint already derived in section 2.5
\[ F_H(I, K) = -\frac{2(1-\mu)}{3\mu} P = -2\alpha P \]
\[ S_2(I, K) = -\frac{4 + \mu}{3\mu} P = -4\beta P \tag{2-11} \]

and \[ S_1(I, K+1) = \frac{3\mu - 1}{3\mu} P = -4\gamma. \]

3.3-6 The application of theoretical force \( q \) and horizontal force \( q' \) causes vertical movement only of the joint \((I, K)\). There will be no force in the horizontal member \((I, K)\), \((I, K+1)\). The equations (2-12) derived in section 2.3-14 for a vertical movement of the joint are directly applicable.

3.3-7 Release of the joint \((I, K)\) at which the out-of-balance forces
$\text{FOUT}$ and $\text{FVOUT}$ act will result in changes in the forces in the members.

Pure horizontal movement requires a vertical force equal to $22(1,2)$ as well as the horizontal force $\text{DELH}$. Thus $\text{DELH}$ and a vertical force $4\, v\, \text{DELH}$ are required to produce horizontal movement of the joint.

Superimposed is the vertical movement of the joint produced by the vertical force $\text{DELFV}$ and the horizontal force $4\, \beta\, \text{DELFV}$. Equating horizontal and vertical forces

$$\text{DELH} + 4\, \beta\, \text{DELFV} = \text{FOUT} \quad (3-4)$$

$$\text{DELFV} + 4\, v\, \text{DELH} = \text{FVOUT}$$

*thence*

$$\text{DELH} = \frac{\text{FOUT} - 4\, \beta\, \text{FVOUT}}{1 + 16\, \beta\, v} \quad (3-5)$$

$$\text{DELFV} = \frac{\text{FVOUT} - 4\, v\, \text{FOUT}}{1 + 16\, \beta\, v}.$$

Similar equations may be obtained for the other four corners, as is done in Chapter 2. The changes in the coefficients caused by the stiffening of an exterior boundary are changes in the magnitudes of terms involving $\text{FOUT}$ and $\text{FVOUT}$, and not changes of sign. By appropriate substitution of coefficients the equations for the release of the other four joints may be written down by inspection from the equations in table 2.1.

3.3-6 A joint on a stiffened boundary may be considered as a joint between two adjacent corners. Components of the diagonal forces perpendicular to the joint will cancel, and no calculation of $\text{DELH}$ and $\text{DELFV}$ is required. The case is identical to that of a uniform lattice, except that the values of increments in the members for horizontal movement of the joint are changed. Indeed, by defining the ratios of the forces in the members caused by a horizontal force $P$ and a vertical force $P'$ as in equations (3-2) it is possible to obtain the increment equations directly by substituting the coefficients $\lambda, v$ and $P$ for the coefficients $\eta, \beta, \gamma$ when the action of horizontal movement is considered. The resulting increments for a corner are of the form

$$2\, \lambda\, \text{DELH}$$

$$2\, \lambda\, \text{DELFV}$$

$$4(\, v\, \text{DELH} + \beta\, \text{DELFV})$$

$$4(\, v\, \text{DELH} - \beta\, \text{DELFV})$$

$$4(\, \rho\, \text{DELH} + \gamma\, \text{DELFV})$$

$$4(\, \rho\, \text{DELH} - \gamma\, \text{DELFV}).$$
3.3-9 The equations for the release of central joints are unaffected by stiffening the boundaries.

**Compatibility in Members With a Stiffened Edge**

3.3-10 The changes in length of diagonal members have been shown to be a function of the forces in the diagonals and Poisson's ratio only (see the derivation of equation 2-35). Therefore the principle of strain compatibility is still valid. This is easily shown by substituting in the equation for diagonal shortening the forces induced from the pure horizontal movement of a boundary joint. In figure 3.1(b) the shortening in the diagonal AC

\[
\varepsilon_{AC} = \frac{2(1+\mu)}{3E_t} \left( \frac{(1+\mu)D_1 - (3\mu-1)D_2}{2\mu} \right)
\]

Substituting for \(D_1\) and \(D_2\), and transposing

\[
\sqrt{2} \varepsilon_{AC} = \frac{2(1+\mu)}{3E_t} \left[ \frac{(1+\mu)^2 + (3\mu-1)^2}{2\mu(3\mu + 1 + \mu(1-3\mu))} \right]
\]

The shortening in AB is given by

\[
\frac{1}{2} x \frac{1}{3} = \frac{2(1+\mu)}{3E_t} \times - \frac{6(1-\mu)}{3\mu + 1 + \mu(1-3\mu)}
\]

\[
= - \frac{2(1+\mu)}{3E_t} \times \frac{3(1-\mu)}{3\mu + 1 + \mu(1-3\mu)}
\]
3.3-11 The component of the horizontal shortening of AB is equal to the shortening of the diagonal AC. The shortening of the diagonal BD may be shown by substitution to be zero. Compatibility is preserved.

3.3-12 Thus the compatibility relationship may again be used in program testing. The changes in length of the horizontal and the vertical members are computed using the equations

\[ \delta v_{\text{AD}} = \frac{1}{2r} Fv_{\text{AB}} \]

\[ \delta h_{\text{AD}} = \frac{1}{2r} FH_{\text{AB}} \]  \hspace{1cm} (3-8)

The shortening of the diagonals is given by

\[ \sqrt{2} a_{\text{AC}} = \frac{(2+2r)S_A + r S_B}{1 + 2r} \]

and \[ \sqrt{2} a_{\text{BD}} = \frac{(2+2r)S_B + r S_A}{1 + 2r} \] \hspace{1cm} (3-9)

Giving \( h_A, v_A \) and \( v_B \) the value zero, \( h_D \) is given by the equation

\[ h_D = h_D + v_B - v_B - \sqrt{2} a_{\text{BD}} \] \hspace{1cm} (3-10)

\[ h_D = h_D + \delta h_{\text{BD}} \]

A second value of \( a_{\text{AC}} \) is given by

\[ \sqrt{2} a_{\text{AC}} = -h_A + v_A + h_C - v_C \] \hspace{1cm} (3-11)

If this value of \( \sqrt{2} a_{\text{AC}} \) is not equal to the value calculated from equation (3-9) there is no strain compatibility. Only if there is compatibility are the equations for the release of the four joints A, B, C and D working correctly. The correct functioning of the program is established by compatibility of strain in the four corners of the lattice frame. This fact enables a rapid and independent check of work. Checking by calculating the values by means of the equations is tedious and very slow.
FURTHER DEVELOPMENT OF THE METHOD OF LATTICE ANALOGY FOR THE SOLUTION
OF SHEAR WALL PROBLEMS

3.3-13 The method of handling an elastic plate made up of rectangular
segments of differing thickness, modulus of elasticity and Poisson's
ratio has been outlined. Provided that no pin joint shall lie on
another member, the size of elements may be varied at will. This
permits the analysis by lattice analogy of panel walls, and of multi-
storey shear walls. Proper control of the logic statements in a computer
program will allow the analysis of walls containing windows and doors.

3.4 ANALYSIS OF THE UNCRACKED BEHAVIOR OF THE WALLS TESTED

3.4-1 In section 3-2 three simple loading configurations were examined
as possible models of a shear wall under load. From these analyses it
was concluded that only infill panels may be accurately represented by
a simple model. As tension causes failure it is imperative that a test
of a shear wall should simulate the tension restraint of the base of the
wall.

3.4-2 In this project a type of test was chosen to represent a brickwork
shear wall bounded at the top and at the bottom by floors. There were,
however, no columns at the ends, and the brickwork did not form an infill.

DIMENSIONS AND DISSIPATION PROPERTIES OF THE TEST FRAME

3.4-3 Figure 3-12 shows the test frame in diagrammatic form. The
apparatus may be seen in plates 5-4, 5-5, and others. Two different wall
shapes were tested, one square and one rectangular. The dimensions of
the walls in the test frame are given with figure 3-12.

3.4-4 Steel channel sections formed beams at the top and the bottom of
the wall. The two 6" x 3" x 15.96 lb channel sections at the top of the
wall were axially loaded by a jack. Horizontal loading was transferred
from the steel sections to the wall by vertical shear connectors acting on
an intermediate cement layer. High tensile steel bolts between the
channels caused a high stress normal to the brick-concrete surface.
Friction was then sufficient to transfer the applied load to the wall.
from the intermediate concrete layer. The top shear connectors were made from steel angle sections welded vertically. There was no provision for transfer of vertical load from the steel to the concrete. It was assumed in the analysis that there was horizontal stiffness, but no vertical stiffness in the top loading beam.

3.4-5 The frame was bolted to the test bed at both ends of the bottom loading beam. Steel shear connectors on the beam transferred both horizontal and vertical force to the wall, again using an intermediate concrete layer. The shear connectors were two rows of boxes 3" x 4" and 1 3/8" deep formed by 3/8" steel plate. A vertical pair of these boxes was designed to take the full load.

3.4-6 When the walls were prestressed the loading beams had not been fitted. The wall area prestressed had the dimensions $L \times H + H_1 + H_2$. To analyze the uncracked behaviour of the walls it was assumed that the wall height was the distance $H$. This dimension was adopted as a suitable compromise between the unstiffened area of the wall and the centreline dimensions of the test frame. Only stresses in the unstiffened area of the wall were considered in calculating the failure loads.

3.4-7 Including the stiffening effect of the loading beams in an analysis using the lattice analogy is described in section 3-3. The values of $J_B$, the ratio of the actual axial stiffness of the top loading beam to the axial stiffness of a central horizontal or vertical member of area $A$, and $J_D$, the ratio for the bottom beam, were calculated as follows.

3.4-8 Bending effects due to eccentricity of horizontal loading about the central axis of the beams were ignored. The loading beam was considered to be in a state of axial compression. The area was taken to be that of the steel plus that of the brickwork outside of that row of bolts which was taken as the wall boundary. The modulus of elasticity of the brickwork obtained in tests was $1.5 \times 10^6$ psi. The steel area was multiplied by the modular ratio $n = \frac{E_T}{E_B} = 20$. Thus the area of the beam was given as

$$n A_B = A_B^*$$

The area of a lattice member of area $A$ is given by

$$A = \frac{I_T}{1 + \mu}.$$
Thus for the top loading beam

\[ J_T = \frac{n \times 2 \times h \times 62 + b \times 6}{(1 + \mu) h^3} \]

\[ = 52.9 \left( \frac{1 + \mu}{1} \right) . \]  \hspace{1cm} (3-12)

For the bottom loading beam

\[ J_B = \frac{n \times 2 \times 8.60 + b \times 10}{(1 + \mu) h^3} \]

\[ = 26 \left( \frac{1 + \mu}{1} \right) . \]  \hspace{1cm} (3-15)

The relative size of the boundary member is inversely proportional to the length of side of the individual elements of which the lattice is composed.

3.4-9 The value given to the area of the top beam was 211.6 in\(^2\), and that given to the bottom beam was 524 in\(^2\) of brickwork. To illustrate their significance let it be assumed that the base of a wall and the floor below act as a tee-beam. The breadth of the flange given in the British code of practice for a wall of thickness 4\(^\prime\) and length 62\(^\prime\) is 21\(^\prime\). For brickwork the top beam would be 10\(^\prime\) deep and the lower beam 21\(^\prime\). If concrete with a modulus of elasticity of 3 x 10\(^6\) formed the floors above and below the wall the slab thickness would be 5\(^\prime\) and 3\(^\prime\) respectively. The horizontal stiffening of the walls tested by the loading beams was representative of the action of floors above and below a shear wall.

**DISTRIBUTION OF HORIZONTAL LOADING BY THE LOADING BEAMS**

3.4-10 The action of a point load at one end of a shear wall results in an area of concentrated horizontal stress around the load. If the load is applied to a loading beam and thence to the wall, there will be a much greater spread of load, resulting in much lower horizontal stress and a more even distribution of shear stress. The effect of a loading beam may be instantly seen by comparison of figures 3-5 and 3-18.
intensity of horizontal stress is reduced to roughly one tenth.

3.4.11 Wind loads, bomb blast loads and settlement loads may be regarded as point loads. Earthquake loads result from the inertia of a structure and are distributed along the wall. For the same base fixity two analyses were conducted, one with a horizontal point load, the other for a uniformly distributed load. The deflections of the top of the beam with respect to the base were very similar. A uniform loading produced deflections ranging between 0.1123" at the two ends and 0.1122" in the middle, with an average deflection of 0.1125". A horizontal point load produced deflections ranging between 0.1123" nearest the load and 0.1101" at the unloaded end. The average deflection was 0.1130". The upward deflection of the point loaded end of 0.0589" was greater than the corresponding deflection of 0.0558" on the wall subjected to uniform load. Stresses were similar except near the points of application of the loads. As the vertical stresses due to base fixity cause initial cracking, and because the influence of these stresses of the type of horizontal loading at the top of the wall is very small, then there is a top loading beam no distinction need be made between distributed and point loads.

BASE FIXITY OF THE LOADING FRAME

3.4.12 Initial cracking of the walls was due to tension failure in the mortar joint at the base of the wall. The tensile stresses depended on the base fixity of the wall. Because the walls were very rigid, bending in the bottom loading beam determined these stresses. Bending of the beam was caused by loads from the wall and by the bolting of the loading beam to the test bed at a distance from the end of the wall. The forces along the base of a wall with a known deflected shape may be determined by the lattice analogy. The deflected shape of a beam under the action of these forces may be calculated. It is necessary to match the deflected shape of the bottom of the wall to the deflected shape of the beam.

3.4.13 Matching of the two deflected shapes is by trial and error. An initial shape of the bottom of the wall is first chosen. Section 2.3-32 describes the use of boundary displacements in the lattice analogy. The solution of the lattice determines the forces along the base of the wall.
The deflections of the loading beam due to these forces are added to
the deflections due to moment at the ends of the beam. The resulting
deflected shape is then compared to the deflected shape of the bottom of
the wall. For complete agreement the two shapes should be identical.

DEFLECTIONS OF THE BOTTOM LOADING BEAM

3,4-14 The deflected shape of a beam may be obtained from the equation

\[ y = - \int \int \frac{M}{EI} \, dx \, dx \]  

(3-14)

Figure 3-14(a) shows diagrammatically the loads on the beam. A horizontal
load \( V \) at the top of the wall is reacted by the bolt groups on the bottom
loading beam. This results in a moment \( V(H + 3.5) \), where \( H + 3.5 \) is the
vertical distance in inches between the centre lines of the top and bottom
loading beams. This moment is transmitted to the beam by vertical forces
along the beam. The moment on the beam is resisted by the vertical forces
\( Q \). These forces will produce moments at \( A \) and \( B \) equal to \( Qa \) and \( Qb \)
respectively. The problem may be broken into two parts as is illustrated
in figures 3-14(b) and (c) respectively. The forces introduced, \( R \), will
cancel on superposition of the two parts. The total deflection is the
sum of the deflections due to moment and to loading.

3.4-15 The equation for the deflections due to moment is

\[ EI \, y = \frac{M x^2}{2} - \frac{(M_1 + M_2)}{EL} x^3 - \frac{(2M_1 - M_2)}{8} \, l \times x \]  

(3-15)

The distance \( L + a + b \) was in all cases 105". The moment of inertia of
the bottom loading beam over the length \( AB \) was, for a modulus ratio of 20,

\[ 2 \times 160.33 \times 4 + \frac{4 \times 12^3}{12} \]

\[ = 7788.6 \, \text{in}^4 \] of brickwork.

Young's modulus was \( 1.5 \times 10^6 \) psi. All the calculations were carried
out for an applied load at the top of the wall of 100 kips.

3.4-16 For a "square" panel, of which the dimensions are given in figure 3-12,
equation 3-15 becomes
\[ y = 2.493 \times 10^{-5} \left( \frac{12\pi}{L} \right) + 1.231 \times 10^{-4} \left( \frac{12\pi}{L} \right)^2 - 1.469 \times 10^{-6} \left( \frac{12\pi}{L} \right)^3. \]  

(3-16)

for a rectangular panel, of which the dimensions are given in figure 3-12, equation 3-15 becomes

\[ y = -5.69 \times 10^{-5} \left( \frac{12\pi}{L} \right) + 1.312 \times 10^{-3} \left( \frac{12\pi}{L} \right)^2 - 7.00 \times 10^{-5} \left( \frac{12\pi}{L} \right)^3. \]  

(3.47)

Values of the deflection at points along the wall are given in tables 3-2 and 3-3.

3.4-17 To evaluate the deflections due to an irregular loading along the beam, the loading was considered to be concentrated at points along the length of the beam. In the analysis of a square panel the points were \( \frac{1}{10} \) apart. In the analysis of a rectangular panel the points were \( \frac{1}{12} \) apart. These points are in the same position as joints in the bottom of the lattice analogous to the panel wall.

3.4-18 The deflection due to the point load \( w \) in figure 3-14(a) is given by

\[ E I_3 Y = \frac{V L^3}{6} k (1-k)(2-k) \frac{3}{L^3} - \frac{(1-k) V L^3}{6} \left( \frac{3}{L^3} \right) + \frac{V L^3}{6} \left\{ \frac{3}{L} - k \right\}. \]  

(3-18)

The last term is omitted if \( \frac{3}{L} - k \) is negative.

**LATTICE ANALOGY SOLUTION OF A PANEL HORIZONTALLY STIFFENED AT THE TOP AND THE BOTTOM AND VERTICALLY FIXED ALONG THE BASE**

3.4-19 If the order of relaxation of joints is from left to right in rows from the top downwards, because the bottom of the wall is fixed and the joints therefore act passively (see section 3.1-6), approximately twenty-four hours of computational time would have been necessary for convergence. It was possible to reduce the time required for computation by modifying the method of computation.

3.4-20 It has already been noted that boundary conditions in the form of forces produce a very much more rapid solution than boundary conditions in the form of displacements. For this reason an initial analysis of a wall was carried out assuming that stress varied linearly along the base (see figure 3-15(a)). In this analysis the top and bottom of the wall were stiffened but all the joints were free. This analysis took about five hours.
3.4-21 The deflected shape of the bottom beam due to bending and that of the wall due to the linear variation of stress were computed. The mean deflections were taken as the first trial shape of the bottom of the wall. The output bar forces of the first distribution were then modified to result in a change in the vertical displacement of the joints along the bottom of the wall. The joints along the bottom of the wall were moved on to the first trial deflected shape. This was affected by changing the forces acting on the bottom row of joints (see section 2.3-32). The amended output was then used as the input for a program in which the bottom row of joints was fixed vertically. Because large out-of-balance forces now occurred at the bottom of the wall, it was not sufficient to release joints from the top downwards, as this procedure does not allow a rapid transfer of forces due to the out-of-balance forces at the bottom of the wall. The program was modified to allow reversal of the order of release of joints.

3.4-22 By using the output of the initial analysis (see section 3.4-20) a set of forces obeying the rules of compatibility and approximating the output forces of the new analysis were obtained. It had been necessary for forces to spread down from the top of the wall to a fixed base, a great deal more time would have been required. The first step in computation were to remove the large out-of-balance forces resulting from changes in the displacements in the bottom of the wall. Three sets of 50 cycles in which the joints were released from right to left, from the bottom row upwards, were sufficient. Sets of 50 cycles were then alternately from the top downwards and from the bottom upwards.

3.4-23 The rate of convergence was dependent on the geometrical shape of the lattice. The method of releasing joints in rows is better suited to a long wall than to a short wall. Convergence of a square 11 x 11 lattice of 121 joints took approximately twice as long as a rectangular lattice 13 x 9 of 117 joints. The rules of statics were used to assess the degree of convergence. For statical equilibrium both the sum of the vertical forces and the sum of the moments about any point must be zero. The output force at a joint on the bottom of the lattice which is vertically fixed is given by
\[ w(n,k) = w(n-1,k) + s1(n-1,k+1) - s2(n-1,k-1). \] (3-19)

\[ \text{exit if } k = n \quad \text{exit if } k = 1 \]

In computing moments it is convenient to regard the distance between two joints in the lattice as unit length. Table 3-1 gives a summary of the calculations carried out in an analysis to determine the degree of statical equilibrium. The average of the moments due to the forces \( w(n,k) \) about the two corners of the base was considered to be the degree of convergence. Vertical equilibrium was inadequate as a measure of the degree of convergence.

3.4-23. By using an amended input, convergence to approximately 85-90% was achieved in the first 100 cycles. However, convergence became much slower thereafter. An arbitrary limit of 95% convergence was set. At this stage of the computation, a definite ratio between the increments of the bar forces was discernible. The operator ratio in table 3-1 shows the tendency of increments to grow linearly from the centre of the wall outwards. Because of the prohibitive time required to obtain a degree of convergence substantially higher than 95%, the operator ratio obtained from the ratio of increments was used to amend the output forces \( w(n,k) \) to 100% convergence. The changes to the forces in the centre of the beam are small. This is advantageous because the forces in the centre of the bottom beam have a much greater influence than the forces at the ends on the deflected shape of the beam. The deflected shape is predictable to a high degree of accuracy.

3.4-25 The deflected shape of the beam obtained from the action of the end moment and the output forces from the lattice analogy was compared with the input deflected shape. A second deflected shape was chosen to lie between the two curves. The output forces were again amended to change the displacements along the bottom of the wall. The amended output became the input for a second lattice analogy. By successive trial and error the input deflected shape and the output deflected shape were brought into closer agreement.
3.4-26 Figure 3-16 shows the deflected shape of the bottom beam under the influence of moment only. There is downward deflection only due to the action of the two moments $M_1$ and $M_2$. Equation 3-16 gives the deflection in terms of the two moments. The deflection is zero at $x = 0$ and $x = L$.

3.4-27 If $y$ is to be positive at all values of $x$ the term involving $x$ must be positive, otherwise $y$ would be negative at small values of $x$.

Thus $M_2 > 2M_1$.

Substituting $M_2 = 2M_1$ in equation 3-15

$$M_1 y = \frac{M_2 x^2}{2} - \frac{M_1 x^3}{2E}$$

which is positive for $0 < x < L$.

In the test on the square wall

$$\frac{M_1}{M_2} = \frac{24}{12} = 2.58,$$

therefore only downwards deflection will occur.

3.4-28 The deflections obtained using a triangular loading along the base of the wall are also shown in figure 3-16. The predominance of moment action is apparent from the fact that the magnitude of the greatest deflection due to moment is approximately six times the greatest deflection caused by a triangular loading on the wall.

3.4-29 Figure 3-16 also shows the trial input and output shapes of the analysis used to compute the stresses and displacements in the wall.

Table 3-2 records the actual displacements. There is very good agreement. Differences between the input and output shapes are noticeable only at the compression end of the wall. This affects the forces at the tension end to a minor degree only. The discrepancy may have been caused by a lack of smoothness in the curvature of the input deflections at the compression end of the wall.

3.4-30 The final analysis was obtained by modifying the preliminary analysis three times. Approximately 30 hours of computer time were used. If output data were not utilized about 100 hours of computer time would have been required.
3.4-31 There was 95.6% convergence in the output forces. The stress contours in figures 3-13 to 3-20 will therefore be accurate to within 4.4%. Modification of the output forces gave the output forces to within 1%.

3.4-32 Initial cracking of the walls was caused by tensile stress in the bottom mortar layer. The vertical stresses along this layer, four inches above the base of the wall assumed in the analysis, are given in figure 3-17. The stresses obtained from the output forces were increased in the ratio of the assumed bar forces to the output bar forces (see table 3-1). Consideration of the moment required for equilibrium reveals a positive rate of increase in stress near the ends.

3.4-33 The effect of the moments at the ends of the beam is to carry the tension zone past the centre of the beam. It is interesting to note that the stress at the end of the wall is of the same order as the stress due to a linear variation of load as shown in figure 3-15. It is necessary to obtain the edge stresses by extrapolation. The value obtained for the edge tensile stress was 2500 psi. The corresponding stress for a linear variation in load is 2520 psi. The downward curvature of the beam would be expected to reduce stresses. However, at the edge curvature due to moment is overcome by the action of the wall, and stresses are high in the tension corner of the wall.

3.4-34 Table 3-4 gives the deflections of the top of the wall for different degrees of convergence. These deflections when divided by the degree of convergence (see section 3.4-23) give very nearly the same value. The magnitudes of the increments used to increase the output forces show an almost linear variation along the base of the wall. This indicates that convergence is increasing as the top of the wall bends over. The bending action is further substantiated by the fact that the deflections for a given degree of convergence when divided by that convergence give essentially the same value. This value may be regarded as the deflection for 100% convergence. It also indicates a variation in the degree of convergence down the wall. The mean and maximum deflections of the top of the wall were found to be 0.1234" and 0.1299" respectively, a variation of 5.3%.
RESULTS OF THE ANALYSIS OF THE RECTANGULAR WALL TEST

3.4-35 In this analysis the deflections due to moment on the beam and the triangular distribution of loading on the wall are of the same magnitude. An initial analysis and two analyses for trial sets of input deflections of the base of the wall were carried out. Agreement between the input and the output curves, see figure 3-21 and table 3-3, is quite close, as the true deflection curve lies between the input and output curves. The true curve will have a greater upwards deflection at the tension end of the beam. A distribution of force from the centre of the tension zone towards the centre and the end will increase the deflection whilst maintaining the moment lever arm. Thus the true curve will give a force distribution which is more concave on the tension side, and more convex on the compression side than the existing distribution. The differences will not, however, be large.

3.4-36 Stress contours are given in figures 3-23 to 3-25. They are drawn for 96.57% convergence. Deflections, when divided by the degree of convergence, gave a constant value. This again indicated a bending over of the top of the wall, and a variation in the degree of convergence from the top to the bottom of the wall. The maximum deflection of the top of the wall, 0.0652 in., is 14.1% greater than the mean deflection of 0.0569 in.

3.4-37 Vertical stresses along the mortar joint above the bottom loading beam are given in figure 3-22. Consideration of moment equilibrium of the base again reveals a concave slope at the ends of the curve plotting stress along the beam. Failure again occurred at a height of 4 in. above the bottom of the wall analysed. The stresses at this height were multiplied in the ratios of the amended forces to the output forces \( W(H,K) \) at 96.57% convergence.

3.4-38 Assuming a linear distribution of vertical stress along the base as in figure 3-15(a) the vertical stress in the tension corner would be 1220 psi for the applied load of 100 kips. The stress obtained in the analysis is 2100 psi. Vertical stresses over the central half of the wall may be neglected, and the stress distribution considered to be triangular over the end quarters of the wall (fig. 3-15(b)). This effect is caused by the action of the external moments, and by the comparatively low rigidity of the bottom loading beam.
THE VALIDITY OF THE WALL TESTS AS A MODEL OF SHEAR WALL BEHAVIOUR

3.4-39 The lattice analogy has been used to analyze the uncracked behaviour of the walls tested. They reveal that if a wall is not an infill panel, but is intermediate between two floors, the stiffening of the floors above and below substantially reduces the horizontal stresses, and that these stresses are unimportant. Because vertical tensile stresses cause failure, the most important factor in the analysis of these problems is the rigidity of the supporting beam. Because the rigidity of the beam is small compared with the rigidity of the wall, the action of fixed ends moments and the transfer of load from the wall must be considered in determining the behaviour of the wall up to first cracking. Further study of the bending of supporting beams and floors is essential before the action of shear walls supported by floors and beams may be fully understood.

3.4-40 Fixed ends and moments of the magnitude of the moments at the ends of the bottom beam in the test of the square wall are rarely encountered. The analyses carried out are sufficient to show, however, that fixed ends and moments and rigidity of the supporting beam determine the stresses in a wall. When considering the interaction of a wall and a beam, failure loads are determined by comparing material failure stresses with the stresses obtained by analysis. The behaviour of the wall is ultimately dependent upon the properties of the material of the wall. Therefore the tests carried out are representative of the action of a brickwork shear wall on an unsupported beam.

3.4-41 As a shear wall approaches its ultimate load, cracking of the mortar joint immediately above the bottom beam extends from left to right. The influence of the rigidity of the bottom beam on the wall becomes steadily less. The forces due to the anchorage of the prestressing wires act on the bottom loaded beam causing deflection. This deflection will, however, have a small effect only on the loads in the wires. The test frame is suitable for assessing the ultimate load of a prestressed wall because the top loading beam spreads the horizontal load, avoiding concentrations of stress at the top of the beam.
3.5 STRESSES INDUCED IN A WALL BY VERTICAL PRESTRESSING OF THE ENDS OF THE WALL

INTRODUCTION

3.5-1 Of the seven walls built and tested, two were unprestressed, three were uniformly prestressed in the vertical direction, and two were prestressed vertically at the ends only. It was assumed that when a wall was prestressed uniformly in the vertical direction that the mean vertical stress was the value of the intensity of vertical stress across any horizontal section not in the immediate vicinity of the prestressing anchorage. The method of lattice analogy was used to determine the stresses when the ends of the wall only were prestressed in the vertical direction.

LATTICE DIMENSIONS

3.5-2 The loading frame was fitted to the top and to the bottom of the walls after prestressing. The prestressing anchorage was at the top and at the bottom extremities of the wall, above the top loading beam and below the bottom loading beam. The dimensions of brickwork prestressed were the overall dimensions of the wall.

3.5-3 The dimensions of walls for tests on a square brickwork panel when prestressed were $L \times (H + H_1 + H_2) = 62'' \times 76''$. The height to length ratio was 1.226. An analogous lattice framework in which there were nine segments in the horizontal rows, and eleven segments in the vertical columns was used. The height to length ratio for the lattice was 1.222.

3.5-4 The dimensions of walls for tests on a rectangular brickwork panel when prestressed were $82'' \times 69''$. The length to height ratio was 1.206. A lattice with 13 horizontal joints and 11 vertical joints, giving a length to height ratio of 1.200, was used.

RESULTS OF THE ANALYSES

3.5-5 Contours of vertical, horizontal and shear stress have been drawn for both analyses. Since there is symmetry about the horizontal and the vertical central lines only the top left-hand segment has been drawn.

Figures 3-26, to 3-28 give stress contours for the effect of end prestressing
of walls used in tests of square brickwork panels. Figures 3-30 to 3-32 give stress contours for the effect of end prestressing the walls used for tests on rectangular brickwork panels. In the former case one prestressing wire at each end was used. In the latter two prestressing wires at each end were used.

3.5-6 In particular, the stresses along the horizontal mortar joint immediately above the bottom loading beam were required for analysis of the behaviour before first cracking. These stresses are given in figures 3-29 and 3-33. The mortar joint in which first cracking occurred was 0.196 of the total height above the base in the 62" x 76" walls, and 0.177 of the total height in the 82" x 60" walls.

3.5-7 During prestressing the load in the wires was increased to 10,000 lb. Theory predicts horizontal tensile stresses in the centre of the top and the bottom of the walls of 119 and 145 psi. If the tensile strength of brickwork is that given by cross-coupled tests, tensile failure would occur in these areas. Although some cracking noise was heard during prestressing, and a few vertical mortar joints appeared to have opened up, there was no overall failure. Had there been a continuous vertical mortar joint failure may have occurred. Mortar was packed into a vertical mortar joint with the edge of the mason's trowel. Horizontal mortar joints were made by spreading the mortar on the course below and tapping the top brick into place.

Accordingly the vertical joints were probably weaker than horizontal joints. The tensile stresses in the top and the bottom of the walls were unable to cause cracking once the loading beams were cemented and tightened in place over the areas of high tensile stress. Were the prestressing anchorage near the brickwork between the loading beams, these stresses would have required careful consideration.

3.5-8 Apart from the areas of high horizontal tensile stress described above, horizontal and shear stresses are of the order of one quarter or less than the maximum vertical compressive stress. Vertical stresses are high only near the ends of the wall. Referring to figure 3-29, along the bottom mortar joint the stress falls to half its maximum value at a distance of 0.196 and a quarter the maximum value at a distance 0.265. If for purposes of comparison, the load were assumed to be evenly spread out at 45° to the vertical, the load would in the former case be spread over 0.226
and in the latter case 0.265. The 45° dispersion gives a fair indication of where the stress is high when a point load is assumed.

3.5-9 The concentration of high vertical stress at the ends of the wall is an efficient way of resisting the tensile stresses due to bending. In both analyses the −400 psi contour runs almost vertically a short distance from the vertical load. The prestressing loads were applied at the centre of the bricks at the top and the bottom of the wall. In the case of uniform prestress of the wall, the same prestressing load was applied to the centre of every brick along the top and the bottom of the wall. The mean prestress for a 10,000 lb load at the centre of each brick was 275 psi. The vertical stress at the tension end of the mortar joint above the bottom loading beam resulting from the action of wires at the ends of the wall was of the order of 200 psi when one wire was used, and 260 psi when two wires were used. Assuming a tensile strength in the mortar of 70 psi the strength to first cracking of a wall prestressed by one wire at each end relative to that of a wall uniformly prestressed

\[ \frac{200 + 70}{260 + 70} \]

= 0.77.

The tensile strength of the wall prestressed by two wires at each end to that of uniformly prestressed wall

\[ \frac{260 + 70}{260 + 70} \]

= 0.94.

CONCLUSIONS

3.5-10 Two conclusions may be drawn. Firstly, there is a limit to the amount of vertical prestress which may be applied to the ends only of a wall. This limit is set by the ability of the vertical mortar joints to sustain cracking. Theoretical tensile stresses of the order of 110 psi were sustained without overall failure. Secondly, the efficiency of one wire and two wire prestressing systems approach the efficiency of a system of uniform prestressing. The nearer the anchorage is to the wall, the greater the compressive stress due to prestressing is, and the greater is the efficiency.
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<th>Output Cycles</th>
<th>Output Cycles</th>
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Total Vertical Force \( x 10^3 \): +0.01 +0.05 +0.01 +0.02 +0.01

Moment About 9.13 \( x 10^3 \): -920.25 -938.19 -950.00 -956.13 -998.33

Moment About 9.1 \( x 10^3 \): 920.05 937.77 950.40 956.35 998.48

Convergence %: 92.02 93.82 95.00 95.63 99.64

Typical Calculations to Determine the Degree of Convergence, and to Obtain the Output for Full Convergence — Analysis of the Square Wall Test — Third Trial

Table 3-1
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Comparison of Input and Output Deflections in the Analysis of the Square Wall Test

Table 3-2

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Comparison of Input and Output Deflections in the Analysis of the Rectangular Wall Test

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**Deflections of the top of the Wall in the Analysis of the Square Wall Test**

Table 3-4

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**Deflections of the Top of the Wall in the Analysis of the Rectangular Wall Test**

Table 3-5
Figure 3.1

Figure 3.2
"COMPRESSION MEMBER" ACTION

CONTOURS OF HORIZONTAL STRESS

Lattice 8x8, $\mu=0.15$, $t=1$, $E=10^6$

Figure 3.3
"COMPRESSION MEMBER" ACTION

CONTOURS OF SHEAR STRESS

Lattice 8x8, $\mu = 0.15$, $t = 1$, $E = 10^6$

Figure 3.4
TENSION RESTRAINT OF LOWER LEFT CORNER

CONTOURS OF HORIZONTAL STRESS

Lattice 6x6, μ=0.15, t=1, E=10^6

Figure 3.5
TENSION RESTRAINT OF LOWER LEFT CORNER

CONTOURS OF VERTICAL STRESS

Lattice 6x6, \( \mu = 0.15 \), \( t = 1 \), \( E = 10^6 \)

Figure 3·6
TENSION RESTRAINT OF LOWER LEFT CORNER

CONTOURS OF SHEAR STRESS

Lattice $6 \times 6$, $\mu = 0.15$, $t = 1$, $E = 10^6$

Figure 3.7
CANTILEVER ACTION

CONTOURS OF HORIZONTAL STRESS

Lattice 6x6, $\mu=0.15$, $t=1$, $E=10^6$

Figure 3.8
CANTILEVER ACTION

CONTOURS OF VERTICAL STRESS

Lattice 6x6, \( \mu = 0.15 \), \( t = 1 \), \( E = 10^6 \)

Figure 3.9
CANTILEVER ACTION

CONTOURS OF SHEAR STRESS

Lattice 6x6, \( \mu = 0.15 \), \( \tau = 1 \), \( E = 10^6 \).

Figure 3.10
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Horizontal and shear stresses
Compression member action

Figure 3.11
Direction of positive forces

Figure 3.13

Figure 3.14

Figure 3.15
TRIANGULAR LOADING ON WALL

MOMENT ONLY, ON BEAM

THIRD TRIAL

INPUT/OUTPUT

DEFLECTION CURVES FOR THE BOTTOM LOADING BEAM SQUARE WALL PANEL

Figure 3.16
VERTICAL STRESSES IN THE LOWEST MORTAR JOINT OF AN UNCRACKED SQUARE WALL, APPLIED LOAD 100KIPS

Figure 3-17
CONTOURS OF HORIZONTAL STRESS
LOADING BEAM ANALYSIS 1
Lattice 11x11, $J_T=8.85$, $J_B=16.28$, $\mu=0.05$, 
L=62, $t=4$, $E=1.5\times10^6$

Figure 3-18
CONTOURS OF VERTICAL STRESS LOADING BEAM ANALYSIS 1
Lattice 11x11, J_T = 8.85, J_B = 16.28, \( \mu = 0.05 \), L = 62, t = 4, \( E = 1.5 \times 10^6 \)

Figure 3-19
CONTOURS OF SHEAR STRESS
LOADING BEAM ANALYSIS 1
Lattice: 11x11, \(J_T = 8.85\), \(J_B = 16.28\), \(\mu = 0.05\),
\(L = 62\), \(t = 4\), \(E = 1.5 \times 10^6\)

Figure 3-20
DEFLECTION CURVES FOR THE BOTTOM LOADING BEAM RECTANGULAR WALL

Figure 3.21
VERTICAL STRESSES
IN THE LOWEST MORTAR JOINT OF AN
UNCRACKED RECTANGULAR PANEL, FOR AN
APPLIED LOAD OF 100 KIPS

Figure 9-22
CONTOURS OF HORIZONTAL STRESS
LOADING BEAM ANALYSIS 2
Lattice 13x9, L : H = 12 : 8, J_T = 8.12, J_B = 14.78.
L = 82, t = 4, E = 1.5 x 10^6, \mu = 0.05

Figure 3-23
CONTOURS OF VERTICAL STRESS LOADING BEAM ANALYSIS 2
Lattice 13x9, L/H = 12:8, J_T = 8·12, J_B = 14·28, L = 82, t = 4, E = 1·5 x 10^6, µ = 0·05

Figure 3-24
CONTOURS OF SHEAR STRESS
LOADING BEAM ANALYSIS 2
Lattice 13x9, L:H=12:8, J_T=8.12, J_B=14.28,
L=82, t=4, E=1.5x10^6, μ=0.05
Figure 3-25
VERTICAL PRESTRESSING OF BOTH ENDS - CONTOURS OF HORIZONTAL STRESS
Lattice 10x12, L:H=9:11, μ=0.05, t=4,
$E = 1.5 \times 10^6$

Figure 3.26
VERTICAL PRESTRESSING OF BOTH ENDS - CONTOURS OF VERTICAL STRESS
Lattice 10x12, L:H=9:11, \( \mu = 0.05 \), \( t = 4 \),
\( E = 1.5 \times 10^6 \)

Figure 3.27
VERTICAL PRESTRESSING
OF BOTH ENDS - CONTOURS
OF SHEAR STRESS
Lattice 10x12, L:H=9:11, \( \mu = 0.05 \), t=4,
\[ E = 1.5 \times 10^6 \]
Figure 3.28
VERTICAL STRESS p.s.i.

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<td>-12</td>
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<tr>
<td>5</td>
<td>-4</td>
</tr>
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VERTICAL STRESSES p.s.i.

SHEAR

HORIZONTAL

DISTANCE ALONG JOINT

STRESSES ALONG THE JOINT ABOVE THE BOTTOM LOADING BEAM — ONE WIRE AT EACH END AT 10,000 lb.

Figure 3.29
VERTICAL PRESTRESSING OF BOTH ENDS
CONTOURS OF HORIZONTAL STRESS
Lattice 13x11, L/H=12:10, µ=0.05, t=4,
E=1.5x10^6
Figure 3.30
VERTICAL PRESTRESSING OF BOTH ENDS

CONTOURS OF VERTICAL STRESS
Lattice 13x11, L:H = 12:10, μ = 0.05, t = 4,

\[ E = 1.5 \times 10^6 \]

Figure 3.31
VERTICAL PRESTRESSING
OF BOTH ENDS
CONTOURS OF SHEAR STRESS
Lattice 13x11, \( L : H = 12 : 10 \), \( \mu = 0.05 \), \( t = 4 \),
\( E = 1.5 \times 10^6 \)

Figure 3.32
STRESSES ALONG THE JOINT ABOVE THE BOTTOM LOADING BEAM — TWO WIRES AT EACH END AT 10,000 lb.

Figure 3.93
CHAPTER 4

PROPERTIES OF THE MATERIALS USED

SYNOPSIS

In sections 4-1 and 4-2 the general properties of mortar and bricks are discussed. In sections 4-3 and 4-4, the experimental determinations of the properties of brickwork which are particularly relevant, the compressive strength, the value of Young's modulus and the compression-shear relationship of brick-mortar bond are given. Sections 4-5 and 4-6 consider the measurement of load by strain gauges, and the amount of relaxation occurring in the wires.

LIST OF SYMBOLS USED IN CHAPTER 4

$F_c$ = compressive strength of brickwork
$\theta$ = the angle to the vertical of a mortar layer between two bricks in a couplet
$s$ = shear stress
$c$ = constant
$p$ = normal compressive stress
$tan \varphi$ = slope of the line denoting the relationship of shear stress to normal compressive stress
$E_p$ = the value obtained for Young's modulus using strain gauges to record strain
$E$ = Young's modulus obtained using Ewing's extensometer
$g$ = gauge calibration factor
4.1 MORTAR PROPERTIES

4.1-1 The term, mortar, is used to imply a cement, lime and sand mortar. It is beyond the scope of this thesis to present a detailed review of research carried out on mortars. However, a statement of the properties of the constituents of a mortar must be made before the steps taken to obtain brickwork of a high quality can be successfully described.

REQUIREMENTS OF A MORTAR

4.1-2 A good mortar should develop most of the possible brickwork compressive strength, and a reliable tensile strength. The mortar should be durable, and the joints should be weather tight. Bond between brick and mortar determines the tensile strength of brickwork.

4.1-3 Bad workmanship can destroy over half the strength of brickwork. For this reason it is very important that a mortar should be easily worked and travelled, encouraging rather than exacting good workmanship.

PROPERTIES OF THE CONSTITUENTS OF A MORTAR MASONRY

4.1-4 The proportion of cement in a mortar governs the strength of that mortar. The water-cement ratio of a mortar is determined by the amount of water required to make the mortar easily workable. The properties of Portland cement are well controlled as a result of its use in the concrete industry. Increasing the fineness of Portland cement particles increases the drying shrinkage of a mortar, which adversely affects the brick-mortar bond.

4.1-5 Several different types of limes are used in mortars. Considerable variations occur in the properties of these limes due to differences in geological origin, method of manufacture, and preparation for use. Wide variations may occur in lime supplied from the same source. Clay in a limestone which has been fired to produce quicklime will produce a percentage of materials similar to cement which cause the lime to set hard after slaking. Such limes are termed hydraulic limes. Lime may be obtained as hydraulic or non-hydraulic quicklime, or as dry hydrated lime.
4.1-6 Lime has two beneficial properties in a mortar. It retains
water in the mortar against brick suction, assisting bonding, and it
makes the mix "tatter" and more easily worked. However, when lime is
added to a mix of cement and sand the strength of the cement-lime-sand
mortar is less than that of the cement-sand mortar alone.
4.1-7 The grading of sand used is important in that a good grading produces
a mortar which is easily worked. Rounded particles make a sand more
workable. An excess of fine particles is associated with cracking of the
mortar.
4.1-8 Additives may be used in mortars, as they are in concrete, to improve
workability and to alter setting times. Their use in mortars has received
little study. Because there is little data available on the use of
additives in mortars and because the use of additives will vary with
locality, they were not used in this work.

METHODS OF TESTING PROPERTIES
4.1-9 The British Standard BS12:1958 was used to determine the compressive
strength of the mortar used. In the absence of suitable British standards
A.S.T.M. tests were used to measure the other properties.
4.1-10 Measurement of the area of bond can be made by allowing water contain-
ing a dye to pass through a joint, and then assessing the area of staining.
The area over which bond occurs is not necessarily proportional to the
tensile strength of the mortar joint. Tests of bond strength in tension
are unsatisfactory in that they always show a large spread in the values
of the tensile strength. (Pearson (1).) These tests put the mortar into
tension using either direct tension or bonding. Results have been obtained
for small specimens of two bricks, and for larger specimens such as beams.
The results of tensile tests vary greatly with the test procedure.
Correlation between tensile tests on two bricks and tests on large specimens
is poor. Because of this variability in tests of bond strength, bond
tension tests were not carried out. Values of tensile strength obtained
normally range from 30 to 70 psi. It was also felt that the tensile
strength of the mortar used could not be tested to any greater accuracy
than would be obtained by adoption of a value for tested tensile strength
of a mortar similar to that used.
ADOPTION OF A MORTAR MIX

4.1-11 The mortar adopted is regarded as a suitable mix to develop the greater part of the possible compressive strength of brickwork made of structural building bricks to MSS 366. These bricks are specified to have an average compressive strength of not less than 3000 psi. Brickwork strength varies approximately as the cube root of the strength of the mortar with which the bricks are joined. Typical results of brickwork tests may be found in the following references: (2),(3). From these results and others it is concluded that there is little advantage in the use of mortars of a very high strength. Indeed, as a high cement content promotes shrinkage cracking, it is best avoided. Tests conducted at the Building Research Station (3) indicate that there is little advantage in the use of mortars having strengths greater than 1000 psi with bricks of 3000 psi crushing strength, and in the use of mortars with strengths which are greater than 2500 psi with bricks of greater than 3000 psi crushing strength. A mortar having a minimum crushing strength of 1300 psi obtained with cement, lime and sand in the ratio 1:2:4 local by volume was selected.

4.1-12 The cement used complied with MSS 592. It was kept in airtight drums. Quicklime was obtained in sacks. The lime varied in its properties. When experiments began, a sack of lime was obtained. This slaked slowly. The properties of the lime in the sack deteriorated with time, and the lime was eventually discarded. A second sack was obtained from the same source. This lime was a little more active than the first and gave slightly better workability. A more rigid control of the standard of the lime, namely obtaining quicklime fresh from the manufacturer and storing it in airtight containers could have been carried out. The actual influence of the variations occurring in the lime is uncertain and probably unimportant.

4.1-13 The sand used had been dried and sieved, and was weighed batched in the proportions given in table 4-1. The grading was based on Connor's recommendations (4). In particular a large proportion of sand retained on the sieve 0.06 mm was included to improve workability by producing a "ball-bearing" action of the particles. Also on Connor's recommendations there was only a small percentage of fines, as a large proportion of fines is associated with cracking.
4.1-14. Much thought was given to the methods of promoting good bond. The tensile bond strength of brickwork is a much more variable quantity than the compressive strength. In the course of tests it was anticipated that it would be desirable to develop reliable tensile strength and compression-shear bond relationship (see section 4-1). For any particular brick there is a particular mortar which will develop the best bond. A theory explaining this was proposed by W. Voss (5). Voss observed a thin layer of material between the brick and the main body of the mortar, a "bond layer". He concluded that this was carbonated lime drawn to the interface initially by brick suction, and later by water movement. If water is extracted too slowly a layer of water forms at the interface. If extraction is too fast precipitation of the lime occurs. Palmer and Parsons (6) examined the bonding of bricks and mortars having different characteristics. Their experiments show that although overall rules may be deduced, each brick type is best suited by a particular mortar type. They conclude that in general higher cement contents give higher bond strengths, and that although a high lime content gave lower bond, the spread in the results was smaller.

4.1-15. Bond is affected primarily by the rate of absorption of water from the mortar by the bricks used, and the ability of the mortar to retain water against this action. For this reason a supply of bricks having an I.R.A. not greater than 25 gm was sought. Lime was present in the mix to retain water. A second important factor in the production of good bond is workmanship. Control of the sand grading and the presence of lime assisted workmanship. In addition, joint surfaces were fully "buttered" with a wiping motion and knocked down into line with the mason's trowel. (Pearson (1) records an increase of 50 to 100% in the bond strength when the top brick is pounded down into the mortar.) Mortar layers were approximately \( \frac{1}{8} \)" thick. A thinner layer will usually produce better bond. Because of irregularities in the bricks the thickness of layers in the walls was not precisely controllable. The amount of mortar prepared at any one time was limited to 100 lb. As a consequence the mortar was never more than three hours old at the time of use. It is noted in passing that the rough surface of the bricks used is considered as advantageous for tension bond by Fornicer and others (7).
TESTS ON THE ADOPTED MORTAR

4.1-16 The mortar mix of 1:1\frac{3}{4}:2 by volume of cement, lime, and sand was prepared and tested for flow. Sufficient extra water was added to make the mortar easily workable. These quantities by weight were then adopted and used in all future mixes. The quantities are given in table 4-1. Reworking the mortar with a little extra water was permitted during construction of the walls, as workmanship was considered more important than maintaining the exact composition of the mix. It is a limited generalization that a wetter mix gives a better bond. However, insufficient water may lead to a drastic reduction in bond strength.

4.1-17 Sand, cement and water were mixed in a horizontal pan mixer for a period of two minutes. Lime was then added, and mixing continued as long as necessary to obtain a uniform mass, but in no case less than six minutes.

4.1-18 Tests were taken on mortar mixes before adopting a standard mortar for use in all the experimental work. The properties of the mortar chosen were tested and recorded. These tests were not repeated with each mix for the following reasons. Firstly, the effect of variations in the mortar strength has a minor effect on the compressive strength of the brickwork. Secondly, it would not be possible to relate the bond strength of the brickwork to any variations that might occur in the workability and compressive strength of the mortar.

4.1-19 The compressive strength was found from six cubes to be 1720 psi. The maximum value was 1840 psi, and the minimum value was 1530 psi. These tests were carried out to BS 12:1936. Specimens were tested at an age of 28 days.

4.1-20 Mortar flow was tested to A.S.T.M. C67-58T. When fresh, the mortar had a flow of between 100 and 105%. This is the more variable half of the flow range described as the "desired consistency" in the standard. After remixing an hour later the flow was between 80 and 85%. These values are lower than those usually cited for mortar flow. This is because the test was conducted to A.S.T.M. C67-58, in which the flow table is subjected to ten blows. Many measurements of flow have used a standard in which 25 flows are specified. The numerical value of the flow becomes a theoretical abstraction when the time taken to build a wall is taken into consideration.
4.1-21 The mortar used was, on the basis of published data, sufficient to develop most of the compressive strength possible in brickwork made of structural bricks to N.Z.S.S. 366. The workability and ability to retain water against brick suction were consistent with the development of good bond. The mortar is of average strength. With the refinement that the sand grading is controlled, it was typical of a good mortar used in the building industry. It was in the control of the initial rate of absorption of the bricks, and in the supervision and care taken with workmanship that the work carried out in the laboratory was superior to similar construction carried out in the building industry.

4.2 BRICK PROPERTIES

4.2-1 In tests on brickwork two properties are important: the compressive strength of the brickwork, and the strength of the brick-mortar bond. As tension usually causes failure of brickwork, considerable care was taken to promote brick-mortar bonding. Good mortar bonding required control of the initial rate of absorption of the bricks as described in section 4.2-5. Compressive strength was regarded as a property of the material used, and not a subject of laboratory control.

4.2-2 The actual compressive strength of bricks is important only in so far as it affects the compressive strength of the brickwork. Tests on small brickwork specimens carried out to determine the value of Young’s modulus for the brickwork yielded indirectly the crushing strength of the brickwork (see section 4.3-15). The only compression tests carried out on the bricks themselves were those carried out by N.Z. P.A.C.S.A., in determining a source of supply of suitable bricks. Their tests to N.Z.S.S. M4749 on 12 specimens indicated a compressive strength of 12,460 psi with a standard deviation of 1,320 psi.

4.3-3 The initial rate of absorption of water from brickwork, the I.R.A., was measured using the test for initial rate of absorption A.S.T.M. C67. This I.R.A. is the weight of water in grams absorbed in 1 minute by 30 in² of brick immersed to a depth of ¾ in water. Initial rates of absorption
may vary between zero and 150 ga. Tests of bond strength for various I.R.A.'s (6) revealed that, in general, optimum bond is obtained with bricks having an I.R.A. of 30 ga. Values of the I.R.A. between 10 and 25 ga give good bond strengths (see fig. 4-1). With bricks of high initial rate of absorption it is desirable and customary to wet the bricks in order to lower the rate of absorption. The amount of wetting is important because saturated bricks give low bond strengths. The recommended condition for laying is "surface wet". This implies a wetted perimeter on which there is no free moisture.

4.2-4 In selecting bricks for the complete tests (see section 4.4), the initial rates of absorption of 183 bricks were tested. The results of these tests gave a mean I.R.A. of 25.4 ga. The maximum and minimum values were 36.6 ga and 1.4 ga. The standard deviation was 9.6 ga, giving a coefficient of variation of 37.8%. The results of these tests are listed in tables 4-2 and 4-3, and are shown in figure 4-2.

4.2-5 The initial rate of absorption is not far outside the desirable range. As a result of these tests, bricks were sprinkled with water until the surface ceased to draw off water rapidly. Tests on the bricks after this wetting showed improved I.R.A. values, although in a few cases a little below 10 ga. However, as bricks would stand in the atmosphere for some hours before use this procedure was adopted. I.R.A.'s of bricks after wetting are given in table 4-4.

4.2-6 The bricks supplied were stiff and side cut bricks fired in a circular kiln. More intense firing in parts of the kiln produces harder bricks with lower rates of absorption and a deeper colour. Bricks from the kiln were divided into groups by colour. Six specimens of five colour groups were tested for initial rate of absorption. The results of these tests are summarised in table 4-5. The bricks used were red bricks. Selection of bricks of I.R.A., between 10 and 25 ga by means of the colour of the bricks is unsuccessful. The bricks chosen 6s, however, have a moderately good I.R.A.
4.3 TESTS ON THREE BRICK SPECIMENS TO DETERMINE THE MODULUS OF ELASTICITY OF THE BRICKWORK

INTRODUCTION

4.3.1 By loading small specimens which were similar to a piece of brickwork taken from the walls tested a value of Young's modulus of elasticity was obtained for the brickwork of which the walls are made. This value is required for lattice analogy solutions of the test frames, and for evaluation of stresses from the lattice solution. A value for the compressive strength of the walls may also be calculated from the failure load of the specimens.

PREPARATION OF SPECIMENS

4.3.2 A rectangular piece of a stretcher bonded wall which is three bricks high may be so chosen that it has at the top and at the bottom a whole brick. If this piece of wall is compressed vertically in order that it gives a value of Young's modulus which is representative of the wall, each brick layer should have above it a layer of mortar. Thus there should be three layers of mortar in the piece of wall. If the piece of wall were compressed horizontally, there would be only one vertical mortar joint for each brick length. Thus it is necessary, in determining the value of the modulus of elasticity, to differentiate between the horizontal and vertical directions. Test specimens of three bricks, suitable for measuring the modulus of elasticity in the horizontal direction, are shown in plate 4.3. There is only one "mortar layer". Plate 4.2 shows a specimen suitable for measuring Young's modulus in the vertical direction. In this case there are three mortar layers. These specimens were prefixed H or V according as to whether they were tested with the bricks horizontal or vertical.

4.3.3 Bricks were rough sized in threes, and then one of each three bricks was split down the centre. The bricks were then moistened, and mortared together with the standard mortar described in section 4.1. All bricks were mortared together with the bricks horizontal. The bricks were tapped down into the mortar with the trowel to obtain good bond. The bricks were left uncovered and allowed to age in the laboratory air, as were the walls. After 27 days both ends of the specimens were capped with dental plaster. Care was taken to ensure that this capping had a plane surface. The layer
of capping plaster was normally $1/8$ to $3/16$" thick.

**TENSILE OF SPECIMENS**

4.3-4 All specimens were tested at an age of 28 days in an Instron testing machine. Stress was applied at the rate of 2000 psi per minute. Each test lasted approximately two minutes. The shortening of the specimen was measured on a Berar dial gauge. This dial gauge measured the position of the top compression plate relative to the position of the bottom plate. For every 500 psi increment in pressure the deflection was measured to $1/1000$".

4.3-5 After an initially large increment as the top plate made complete contact with the top of the specimen, increments increased at a steady rate. In many cases, cracking was heard before failure, from 50% of the final load onwards. In some cases this was associated with a change in the rate of straining or a sudden jump in the deflections. As the ultimate load was neared, there was a short period of increased rate of deflection followed by failure.

4.3-6 Failure of specimens in which the bricks were horizontal occurred when vertical cracks appeared in the end of the bricks (see plate 4-2). In two of these eleven specimens, small pieces spalled of the centre also. In three of the eleven specimens, horizontal cracks occurred in the mortar (see plate 4-1).

4.3-7 Failure of the specimens in which the bricks were vertical was also due to vertical cracking at the ends of the specimens (see plate 4-4). This was, in general, associated with spalling off of pieces of brickwork in the centre (see plate 4-5). Spalling was in some cases associated with cracking in the vertical mortar joints.

**RESULTS**

4.3-8 The load-deflection curve for each specimen was plotted. Typical load-deflection curves are shown in figure 4-3. The curves for H2 and W4 are typical curves. The curves for V2 and V3 show changes in deflection associated with audible cracking.

4.3-9 From the load-deflection curves the modulus of elasticity was obtained. Values were calculated using the actual height and cross-sectional area of the specimen from the slope of the line before any cracking occurred.
The experimentally obtained moduli of elasticity and the crushing strengths of the specimens may be found in tables 4-6 and 4-7.

4.3-10 As may be seen from these tables the values of the moduli of elasticity varied very widely. One very high reading occurred when the specimens were tested with the bricks horizontal and one when they were tested with the bricks vertical. In general, the values of Young's modulus were higher for the specimens in which the bricks were horizontal. All but the very high values (for specimens H2 and V3) may be included in the range $0.97 \times 10^6$ to $1.87 \times 10^6$. The mean values and standard deviations for both types of tests were of the same order.

4.3-11 The variations in the modulus of elasticity are considerable. The coefficient of variation is high, especially when it is remembered that a standard mortar and a selected range of bricks were used. Coefficients of variation of 30% or more occur in the complete test and tests of the initial rate of absorption, indicative of the variability of the material. Use of any value of the modulus of elasticity in calculating stresses is questionable because of the wide variation. Brickwork having a high modulus of elasticity alongside brickwork having a low modulus of elasticity, in order that the two specimens may have the same strains, will result in an internal self-straining system. It may be seen from the tests that a high modulus of elasticity is not always associated with a high failure stress. Thus premature failure of the weaker of these self-straining systems is possible if the criterion for failure is the compressive strength.

4.3-12 A value of Young's modulus of $1.5 \times 10^6$ psi in both the horizontal and vertical directions was adopted for use in the analysis, and for calculation of failure stresses and loads.

4.3-13 The failure stresses of both types of specimen showed a significantly smaller coefficient of variation. As was the case with the mean moduli of elasticity the mean failure stresses were in close agreement for both types of specimens. Ultimate failure in nearly all cases appears to have been failure of the bricks under induced shear stress. From the results of the tests it may be assumed that for the bricks used, crushing strength in both the vertical and horizontal directions is the same.

4.3-14 The variability of brickwork specimens is very significant. The tests were carried out on a restricted range of bricks, using a
standardized mortar with good properties and with close control over workmanship. Nevertheless the coefficients of variation are high. In normal brickwork much higher coefficients of variation may be expected. Assuming, for example, that the crushing strength of brickwork specimens obeys the normal frequency distribution and that a coefficient of variation of 4.0% is occurring in the compressive stress, in order to ensure a 99% probability that given masonry reaches a given strength it is necessary to have a mean strength of 1.734 times the desired given strength. For a criterion of minimum deflection, again assuming approximately twice the laboratory coefficient of variation, say 6.0%, and disregarding the fact that differences in the modulus of elasticity will tend to cancel, a value of Young's modulus 2.176 times the mean value of Young's modulus must be used for a 99% probability. These examples show the effect of the variability of brickwork. The probability of 99% is used purely as an example. There are about 650 bricks in a single skin 10 ft square, and any one of these bricks could be the first to fail.

CALCULATION OF THE COMPRESSIVE STRENGTH OF THE BRICKWORK IN THE TEST WALLS

4.3-15 The compressive strength of the brickwork in the walls tested may be estimated in two ways. The first method is to apply suitable strength correction factors (6) to the crushing strength obtained in the tests for Young's modulus. The second is to use general relationships between the compressive strength of bricks and that of the mortar cementing them.

4.3-16 Krafeld (6) carried out a series of tests on brickwork piers to determine a suitable size of specimen for testing the compressive strength of brickwork. The compressive strengths of the walls tested and the specimens used in determination of the value of Young's modulus may be related by means of the height to thickness ratio. Krafeld's specimens were piers of two cross-sections, 8" by 16" and 12" by 6". The ratio of length to depth of 2:1 is roughly that of the blocks used in determination of the value of Young's modulus (2.25:1). Krafeld found that a single curve was sufficient to represent the relationship between the compressive strength and the ratio of height to the least thickness dimension for all the brickwork specimens he tested. In the course of the wall tests it was found that when brickwork in the walls was subjected to its greatest
compressive stress, namely when cracking had extended well along the base of the wall (see Chapter 5), the ratio of length to breadth of the area under compression approached two to one. Thus the adoption of Krefeld's strength correction factors is justifiable.

4.3-17 Krefeld's strength correction factors are given in Table 4-6. The height to depth ratio of the specimens used to determine Young's modulus is 2.25 to 1. By interpolation a strength correction factor of 0.705 must be used to obtain that Krefeld refers to as the "normal" compressive strength of walls, that is, when the length to height ratio is six to one. The height to depth ratio of that part of the walls unrestrained by a loading beam may be taken as 55.5 to 1. The strength correction factor for a specimen of these dimensions is 1.00. Thus the compressive strength of the walls may be found from the compressive strength of the test specimen used in determination of Young's modulus by multiplying the latter by the factor

\[
\frac{0.705}{1.00} = 0.653.
\]

It may be argued that as the compressive strength in the wall is intense only over a small area, the ratio of the height to the least thickness dimension might well be reduced. Even if this ratio were halved it would amount to an increase of only 8%. When compared with the coefficient of variation such a correction is small.

4.3-18 Adopting the mean of the compressive strength tests in both the vertical and horizontal directions the compressive strength of the brickwork in the walls is given by

\[
f'_{oo} = 0.653 \times \left( \frac{4800 + 1520}{2} \right)
\]

\[
= 3000 \text{ psi}.
\]

4.3-19 Plummer (9) states that wall strength may be most accurately determined from tests on masonry prisms. If such data is not available he offers a method of calculation based on the compressive strength of the brick. The method is applicable to walls built under "inspected" workmanship, characterized by complete filling of all mortar joints and by smooth unfurred bed joints. After damp curing for 28 days, provided that the cement mortar has a two inch cube compressive strength of 2500 psi,
the wall strength will be approximately one-third the compressive strength of the brick when the compressive strength of the brick is greater than 4,500 psi. If all other factors remain constant, brick masonry strength is proportional to the cube root of the mortar strength. These rules are based on the results of tests on walls carried out by Stang, Parsons and McBurney (2).

4.3-20 Although the mortar was tested in 2.75 inch cubes and although the walls were not cured wet, workmanship was kept to as high a standard as possible. It is reasonable to apply Plummer's rules. The compressive strength of the bricks as found from the tests carried out by N.C. P.A.C.E.R.A., was 12,460 psi. Hence the compressive strength of the brick walls may be found to be

\[ 12,460 \times \frac{1}{3} \times \sqrt[3]{2900} \]

\[ = 3666 \text{ psi.} \]

It is of interest that Stang, Parsons and McBurney obtained compressive strengths for walls of length 6 ft, height approximately 9 ft and thickness 4" which were about five percent greater than the average crushing strengths obtained for walls of the same length and thickness but 8" and 12" thick, which had headers in every sixth course. This is an example of the statement made by Plummer (9) that "some tests indicate that the highest (compressive) strengths are obtained under central loading with masonry having no headers."

4.3-21 Krefeld's tests were on specimens having headers in every sixth course. It might be anticipated that the crushing stress calculated from the blocks tested by the author would have a higher compressive strength than that predicted from the compressive strength in the preceding paragraphs. This is not the case. The value calculated from the compressive strength of the bricks and based on the tests of Stang, Parsons and McBurney of 3666 psi is 10.6 percent greater than the calculated compressive strength of 3090 psi using Krefeld's strength correction factors.

4.3-22 Because of the large number of tests, the compressive strength of the three brick specimens modified by Krefeld's strength correction factors was adopted.
4.4 Tests on brick couples to determine the compression–shear relationship of the brick-mortar bond

Introduction

4.4.1 When considering the failure of a body made of homogeneous elastic material it is normal to compute the principal stresses in the body. This approach may be used successfully in considering the cracking of concrete, but is not applicable to brickwork. The properties of the mortar layers in brickwork determine the failure of brickwork. Analysis of brickwork stresses by the theory of elasticity results in a consideration of the stresses along the two bond layers which are, conventionally, the vertical and horizontal axes. Relationships of bond are not proportional to the crushing strength of the mortar. The crushing strength of brickwork may, however, be regarded as a special case of the bond relationships.

4.4.2 Brick couplet tests, to the best of the author's knowledge, were first used to determine the relationship of normal stress to shear stress in brickwork by Benjamin and Williams (10). Two bricks were mortared together, one directly over the top of the other. The couplet so formed when tested was loaded with a force through the centre of the mortar (see fig. 4.1). The plane of the mortar joint made an angle $\theta$ with the axis of loading. The axial force $P$ through the centre of the mortar joint produced a force on the brick-mortar interface and perpendicular to the interface which was equal to $P \sin \theta$. There was also a shear force on the brick-mortar interface and parallel to the interface which was equal to $P \cos \theta$.

Although the ratio of the normal to the shear force was $\tan \theta$ the loads at which forces in this ratio ceased the joint to fail were determined by the bond relationships. By varying the angle $\theta$ these relationships were found experimentally.

4.4.3 Benjamin and Williams carried out tests on couplets using an axial force $P$ to produce both tensile and compressive normal force (10). Let the angle $\theta$ be as shown in figure 4.1. Benjamin and Williams have published tests on three different mortars. For each mortar they tested as a general rule five specimens under each of the following conditions: direct tension, tension on the joint when $\theta$ was 45°, 30° and 15°.
direct shear, and compression on the joint when \( \theta \) was 15°, 30°, 45° and 60°. Their results are listed in table 4-12 and shown in figure 4-5.

4.4.4 Benjamin and Williams found that "tests involving tension are very critical with respect to test-procedure and give low magnitudes to measure," because of the variability and the uncertainty of the tensile strength only the compression-shear relationship of the mortar was determined in this project. It was felt that some reliance could be placed on such a relationship and that the results published by Benjamin and Williams were indicative but not conclusive of the relationship. Tests were carried out with a normal compressive stress on the joint when \( \theta \) was 15°, 30°, 45°, 40°, 45°, 50° and 60°.

PREPARATION AND TESTING OF SPECIMENS

4.4.5 Specimens were made from bricks of controlled absorption rate, cemented with the standard mortar, and tested after 28 days aging in the laboratory. Altogether 163 bricks were tested for their initial rate of absorption. The results of these tests are to be found in section 4.2.4. Bricks having an initial rate of absorption of less than 25 gm were mortared dry. Other bricks having an initial rate of absorption greater than 25 gm were wet before use, until water was no longer rapidly absorbed into the surface. Although bricks which are wet before use will not have exactly the same bonding characteristics as the dry bricks, both will be suitable for developing good bond. The bottom brick of each couplet was then liberally "buttered" with mortar and the top brick knocked down to the adopted bond width of \( \frac{1}{2} " \) by blows from the mason's trowel. Initially wooden spacers were used to determine the width of the mortar layer, but this practice was discontinued because of the difficulty in removing the wooden spacers without disturbing the bonded layers.

4.4.6 Couplets were allowed to age in the laboratory for 28 days and were neither covered nor moistened. These conditions were the same for all brickwork specimens including the walls tested. Although no elaborate measures were undertaken for curing, the brickwork was under fairly uniform conditions of temperature and moisture. Although the atmosphere in the laboratory had a fairly low humidity the brickwork was not exposed to extreme heat or cold.
At the end of 28 days aging the couplets were tested in hardwood blocks. These may be seen in plates 4-5 to 4-10. Plate 4-5 shows a couplet specimen ready for testing between the cross-members of the Baldwin test machine. Constant rates of loading were used on specimens with the same inclination to the vertical. These loading rates were intended to cause failure two minutes after loading began, and varied from 2500 lb/min to 40,000 lb/min. During testing, specimens showed a steadily increasing deformation under load, often yielded slightly just before failure, and usually failed abruptly.

RESULTS

The failure load of all couplets was divided by 35.5 in² to obtain the unit vertical pressure along the interface in pounds per square inch, except when wooden spacers had been used to fix the joint thickness at ½ when the factor was 31.5 in². These figures gave the average area of a brick over which bond may occur. The results of these tests are summarised in table 4-10, and shown in full in table 4-9. The different types of failure fall into three main categories: a clean bond failure as may be seen in the far specimen shown in plate 4-6, a central shearing through the mortar as may be seen in the central specimen in plate 4-6, and failure by cracking of the brick as may be seen in plates 4-9 and 4-10. In some cases more than one type of failure occurred at the same time. Thus in the summary of the tests given in table 4-10 the total of the different types of failure is greater than the number of specimens tested. Cracking of the bricks was usually vertical. There was, however, a tendency for specimens to crack along the centre, perpendicular to the mortar bond layer where there was a ⅜ diameter hole in the bricks, section of minimum area. Such cracking may be seen in plate 4-7. Vertical cracking in both the bricks and the mortar is probably due to principal tensile stresses.

Plates 4-6 to 4-10 illustrate the failure of couplets.

Estimation of the area of bond to the nearest 10% was carried out on twenty-two specimens. Multiplying the test results of these specimens to give 100% bond area gave a much wider spread of results than was encountered in the tests. No further attempts were made at correlation of the failure load and area of bonding. The initial rates of absorption of the two couplets in each of the same 22 specimens were recorded. Twelve
of these specimens failed along the side having the greater initial rate of absorption, 6 failed along the side having the smaller initial rate of absorption, and 4 results were inconclusive.

4.4-10 The average normal stress is plotted against the average shear stress for each angle θ in figure 4-5. Coefficients of variation of up to 42% occurred in the test results. Plotting only the average results for each angle θ a straight line may be used to represent the results for values of θ from 15° to 45° inclusive. The line may be represented by the equation

\[ s = c + \phi \tan \varphi \]  

(4.1)

This equation is also used to denote the shearing resistance of soils.

4.4-11 The maximum shear stress on the mortar joint in a complete test is unknown. In the absence of precise data on how the stresses vary along the joint, it is reasonable to assume that the normal stress is uniform and that the shear stress has a parabolic distribution. In this case the maximum shear stress would be 1.5 times the average shear stress. Benjamin and Williams also make these assumptions. The slope of the line in figure 4-5

\[ c = 0.662 \]

\[ \therefore \tan \varphi = 0.662 \]

\[ \varphi = 33.5° \]

If the maximum shear stress is assumed to be 1.5 times the average shear stress

\[ \tan \varphi = 0.993 \]

\[ \therefore \varphi = 45° \]

If \( \theta > \varphi \)

\[ s < \phi \]

\[ \therefore s < n \tan \varphi + \phi \]

and therefore there can be no shear failure obeying this relation if \( \theta \) is greater than \( \varphi \). The result for tests when \( \theta \) is equal to 50° diverges slightly from the line, and the result when \( \theta \) is equal to 60° is entirely separate. These results suggest a value of \( \tan \varphi \) a little greater than 45°. The actual stress distribution is unknown.

4.4-12 Benjamin and Williams, from the results of tests when \( \theta \) was equal to 15°, 30°, and 45°, obtain a similar line. This will be discussed more fully later in this section. It is interesting to note in passing their observation that tests in which \( \theta \) is greater than 50° "tend to be influenced by friction".
They do not, however, make any comment as to the validity of the individual tests.

4.4-13 Couplet test results may be considered to apply to bond failures only. Omitting the results for tests in which the brick failed, the results shown in table 4-11 are obtained. It is interesting to note that the same line fits these points as well as it fits points representing both modes of failure (see figure 4-5).

4.4-14 The line in figure 4-5 has the equation

\[ s = 260 + 0.662p \]  \hspace{1cm} (4-2)

Increasing the shear stresses by 50% to allow for a parabolic distribution the following equation is obtained.

\[ s = 390 + 0.993p \]

\[ s = 390 + p \]  \hspace{1cm} (4-3)

Benjamin and Williams found a similar equation

\[ s = 220 + 1.1p \]

Their results of tests on three different brick types could be represented by this line. One of these mortars was a 1\(\frac{1}{2}\)4\(\frac{1}{2}\) mortar which had the same proportions as the standard mortar used in the tests. Thus it appears that the value of tan \(\theta\) depends on the surface of the bricks, their roughness and absorption, or may be due to the method of testing.

4.4-15 Extrapolating the author's line to the axis along which the normal stress is zero would give a value for the strength of a mortar joint in shear alone. Extrapolating to the axis along which shear stress is zero gives a value for the tensile stress of mortar couples of 370 psi. This value does not occur in practice. Benjamin and Williams showed in their experiments that there is an abrupt change of slope when the compressive stress becomes small. Benjamin and Williams found a couplet tensile bond strength of approximately 40 psi. It is possible that the poor correlation experienced between tensile couplet tests and the large-scale experimental tests may be due to the fact that the latter is a tension-shear relationship and the former is not.

4.4-16 As was encountered with the initial rate of absorption tests and with the tests to determine Young's modulus, the coefficient of variation was of the order of 30%. As the angle \(\theta\) increased, the coefficient of variation at first increased and then decreased. The coefficient of
Variations were greatest for a value of $\theta$ equal to $35^\circ$ and $40^\circ$. Benjamin and Williams found, of the $15^\circ$, $30^\circ$ and $45^\circ$ tests reported in their paper (10), that a test with an angle corresponding to the author's $30^\circ$ "gives a good index of the quality of a particular couplet of masonry".

As the angle $\theta$ was increased the mode of failure of the test specimens changed. For tests in which $\theta$ was equal to and less than $35^\circ$ twenty-nine of the thirty-one specimens failed at the bond layer. If the tests in which $\theta$ was equal to and greater than $40^\circ$ twenty of the thirty-four specimens failed by cracking of the bricks. It may be significant that the value of $\theta$ in the equation:

$$s = a + p \tan \theta$$

where this represents the line for points of average shear and average normal stress, is $33.5^\circ$.

**Conclusions**

The compression-shear relationship of mortar to brick bending found by means of couplet test is given by the linear relationship

$$s = 390 + p$$

for the particular bricks and mortar used and for a high standard of workmanship. The values of shear stress should be modified to allow for poorer workmanship as appropriate. Couplet tests are most variable when $\theta = 35^\circ$. The couplet test is not applicable for values of $\theta$ greater than $45^\circ$.

When considering the ultimate failure of a wall the compressive and shear forces are concentrated over a small length of brickwork. It is more appropriate to apply the equation for the relationship between average shear and average compression

$$s = 260 + \frac{2}{3} p$$

(4-4)

over that length of brickwork.
4.5 THE USE OF STRAIN GAUGES TO PREDICT THE LOAD IN A WIRE FROM THE LOAD-STRAIN RELATIONSHIP

Introduction

4.5.1 A Philips FB8114 strain gauge was glued to each 0.276" diameter wire in the walls. These gauges had a resistance of 128.5 ± 0.5 Ω, a gauge constant of 2.02 ± 1.5% and a temperature coefficient of 31 ± 2 x 10^-6 Ω/°C per °C degree centigrade. Strains were measured on a Philips 4571 "null point" strain measuring bridge stated (11) to be accurate to a strain of 7 x 10^-6.

4.5.2 In the mill tests W4, W5, and W6, high tensile steel wire cut from a single coil was used. The steel from this coil will be referred to as steel 1. Lengths of wire cut from a second coil were used in walls W6 and W7. This steel will be referred to as steel 2. The high tensile steel wires used in the first prestressed wall, W3, appeared to be offcuts which had been lying around the supplier's yard. These wires were rusted and in some cases pitted. No standardization was possible for these wires. To evaluate the loads in these wires they have been regarded as steel 1. They will be referred to as W3 steel.

Tests of Steels 1 and 2 to Determine the Value of Young's Modulus of Elasticity and the Failure Loads of the Wires

4.5.3 The high tensile steel used had no definite limit of proportionality or yield point. The shape of the stress-strain curve, however, departed very little from a straight line below a stress of approximately 165,000 psi. Failure occurred at a stress of approximately 220,000 psi after a considerable period of yielding. In this test the term "modulus of elasticity" implies the slope of the straight line best fitting those values of stress and strain below the value at which the rate of straining begins to increase substantially. It is neither a secant nor a tangent modulus. The load in the wire when the rate of straining begins to increase substantially is approximately 10,000 lb, resulting in a stress of approximately 165,000 psi.

4.5.4 The values of Young's modulus were found using a Swaging extensometer to measure extensions and also by using strain gauges. Considerable variations occurred in the values obtained with the two methods of measuring. The values of Young's modulus obtained using a Swaging extensometer were adopted.
They were preferred because the values obtained were consistent and typical of the material and because the load-strain curve matched that of the manufacturer.

4.5-5 Two measurements of Young's modulus using the Eising extensometer were made for each type of steel. One of these was a direct loading to failure. Extensions were recorded for load increments of 1000 lb, up to a load of 12,000 lb when steel 1 was tested and up to a load of 11,000 lb when steel 2 was tested. Loading was applied at a rate of approximately 2000 lb/min. There were no strain gauges on these wires. The results are given in figures 4-6 and 4-7. The second test on each steel was carried out to determine whether or not the slope of the load-strain curve obtained on unloading had the same slope as the load-strain curve obtained on loading. Strain gauges were glued to the wires in order to determine whether or not substantial hysteresis occurred in the gauges. The results of these tests are given in tables 4-13 and 4-14, and are shown graphically in figures 4-8 and 4-9.

4.5-6 Considering only measurements with the Eising extensometer, when steel 1 was loaded directly to failure the value of Young's modulus obtained was $30.0 \times 10^6$ psi. The load-strain curve coincided exactly with the manufacturer's curve. The cross-sectional area of the circular wire was calculated in all the tests on the steels using a mean diameter which was the mean of eight diameters. In the test involving loading and unloading the value of Young's modulus obtained was $29.6 \times 10^6$ psi. The mean value of Young's modulus of the two tests on steel 1, $29.8 \times 10^6$ psi, was adopted as the value of the modulus of elasticity for steel 1. The values obtained from corresponding tests on steel 2 were $29.8 \times 10^6$ psi and $29.4 \times 10^6$ psi respectively, giving a mean value of $29.6 \times 10^6$ psi.

4.5-7 The load-strain curve obtained when the direction of loading was reversed was a straight line. Upon unloading there was an increase in the strain for a given load. This increase may be divided into two parts. The first is a constant increase dependent upon the intensity of loading. The second appears to be caused by a small increase in Young's modulus. The permanent deformation was the sum of the two effects. Steel 1 was loaded further than steel 2 beyond the value at which the rate of straining begins to increase substantially. The "load intensity set" was correspondingly
greater. The "load intensity set" for the load at which reversal in
the direction of loading takes place may be considered to be the difference
between that strain for the load obtained by extension of the straight
line used to calculate Young's modulus, and the actual value obtained in
the test. A straight line may be used to join all the values on the
graph obtained when the direction of loading was reversed, from the highest
value down. Backlash in the extensometer and hysteresis in the strain
gauge did not appear to be significant effects.

4.5-8 There appears to be little or no load intensity set below a load of
9000 lb. The load intensity set at 10,000 lb was of the order of $10 \times 10^{-5}$
for steel 1 and $5 \times 10^{-5}$ for steel 2. At a load of 11,000 lb the load
intensity set for steel 1 was $4.5 \times 10^{-5}$. The amount of load intensity set
was small, provided that the load applied to the wires did not exceed 10,000 lb.

4.5-9 The slope of the load-strain curve obtained on reversal of the
direction of loading changed by approximately two percent. The values of
Young's modulus obtained were $29.2 \times 10^6$ psi for steel 1 and $30.0 \times 10^6$ psi for
steel 2. This change in the value may have been due in part to a change
caused by cold working in the properties of the steel. The values on
reloading lay on or very near the same line joining the values for unloading.

CALIBRATION OF STRAIN GAUGES:

4.5-10 Considerable concern was caused by the difference between the values
obtained for Young's modulus using strain gauges, denoted by the symbol $E_p$,
and those values obtained using a direct extensometer $E$. Other strain
gauges used on wires in the walls showed even greater discrepancies in the
value of Young's modulus. Neglecting the results of the very variable W3
steel, the discrepancy between $E_p$ and $E$ varied between $-5.0\%$ and $+10.3\%$
(see tables 4-19 to 4-22). Only four of the twenty-one values of $E_p$ were
less than $E$. The mean discrepancy was $3.5\%$. In no case, however, did
the plotted values of load against strain lie on other than a straight line.
These results suggest the presence of a constant negative error in evaluating
the strain (an increase in strain for a given load results in a lower value
of Young's modulus) of $3.3\%$ and a variable error of $\pm 3\%$.

4.5-11 The gauges were mounted on a paper backing 1-11/16" long and 1/4" wide.
They were glued with a styroil-based glue and covered with a 1/8" layer of an
cyanlate-based water repellent. The glue and water repellent were supplied
in the lot set 53592.

4.5-12 Variations between $E_2$ and $E$ may be attributed to the tolerances in the gauge constants and to the fixing of the gauges. A fixed error of +1.5% may be accounted for in the following way. The gauge constant given by the manufacturer was $2.02 \pm 1.5\%$. Measurements on the bridge were made using a gauge constant of 2.00. The different gauge constant resulted in a decrease in the amount of strain recorded which appeared as an increase of 1% in the value of Young's modulus. A second factor causing a reduction in the strain recorded was the layer of waterproofing. E. Bace of the Department of Mechanical Engineering at the University of Canterbury found from eleven tests that the modulus of elasticity in compression of the waterproofing compound was $3.0 \pm 0.4 \times 10^5$ psi and that the compressive strength was 8700 psi. Using this value for the modulus of elasticity, the increase in cross-sectional area due to an eighth of an inch thick layer of waterproofing compound around half the perimeter of the wire was equal to 0.91% of steel area. Thus the water repellent may be considered to have increased the value of Young's modulus by 0.9%.

4.5-13 Variable discrepancies of +1.5% and -2.5% may be accounted for by the tolerance in the gauge constant $\pm 1.5\%$ and by a possible slant in fixing the gauge of $\frac{1}{16}$ over the length of 1-11/16" which would decrease the value of Young's modulus by 1%. The remaining discrepancies in the values of $E_2$ and $E$ must be attributed to the difficulty in gluing the gauges. It was necessary to bend the paper backing of the gauges around the wire. This probably resulted in incomplete bonding of the back of the gauges. Murray and Stein (12) have conducted a pertinent experiment on the effect of incomplete bonding. They obtained the load-strain curves for three strain gauges, one completely fixed over the whole length, one fixed at the ends only, and one fixed in the middle only. In all cases the load-strain relationship was a straight line. The slope of the load-strain relationship for conditions of tension were widely different. The strain gauge fixed at the two ends produced a line with a slope 25% greater than the slope of the line for the completely fixed gauge. The slope of the line for the gauge fixed in the middle was 10% less than the slope of the line for the completely fixed gauge. This change in slope but not in straightness, coupled with the method of fixing the gauges, suggests that incomplete bonding has caused the
additional variation between the values of $R_p$ and $E$.

4.5-15 It would have been impossible to calibrate every strain gauge in the testing machine because of the changes in slope on reversal of the direction of loading. The amount of time required to obtain consistent results for each gauge was prohibitive. No calibration of the region beyond the onset of substantial permanent set was possible. The strain gauge readings were not readings of the true strain. In order to determine the load in each wire for a given strain a simple system was used to calibrate the wires indirectly.

4.5-16 During the stressing of the wires, readings of the strains in the gauges were taken at load increments of 1000 lb. The loads were measured on the Bourdon gauge on the FSC monopole prestressing system jack, which had been calibrated on the Baldwin Southmark testing machine. To calibrate the Bourdon gauge a prestressing wire had its bottom anchorage bearing on the lower cross-head of the machine. The jack acted on the upper cross-head of the machine. As the wire was stressed the gauged load was "weighed". The Bourdon gauge was found to be surprisingly accurate.

4.5-17 The load-strain curves for wires in the walls were plotted using the corrected jack loads and recorded strains on the strain gauges. The slope of a load-strain curve so obtained may be compared with that of a standard gauge on a specimen of the same steel. The load-strain relationship of this standard gauge may be precisely determined on a testing machine up to the failure of the wire or of the gauge. It was assumed that the strains in the two wires for equal loads were in the same ratio as the slopes of the load-strain curves below the onset of substantial load intensity set. It is therefore possible to determine by simple proportion the load corresponding to any strain in the wire in the wall.

4.5-18 The results of the calibration of the prestressing jack gauge are summarized in table 4-17. These are a result of sixteen cycles of loading and unloading. Six of the cycles were used to calibrate the jack before prestressing the wires for test W3. Six of the cycles were used to calibrate the jack before stressing the wires in wall W5. A further four cycles were used to obtain average values for the jack loads. These average readings were used to adjust the jack readings for W5 and W7, and to amend the readings for test W5.
4.5-19 The Bourdon gauge could be read to an accuracy of 25 lb. The standard deviation in the calibrated loads was about 40 lb. The greatest difference between the calibrated and the gauge loads was 31 lb. The maximum possible error in the slope of the load-strain curve was therefore

$$\pm \frac{2 \times 106}{10,000} = \pm 2.1\%.$$  

The slope of this line was compared with the slope obtained with a standard gauge. The test machines used to calibrate the standard gauges were accurate to ± 0.5% over the range used. The loads in the wires are therefore known to within ± 1.4%, and are probably well within these limits. This accuracy is difficult to achieve even if the load-strain curve of the steel were perfectly known and the gauges were perfectly fixed because of variations in the cross-sectional area of the wires.

4.6-20 The above discussion has considered the linear part of the load-strain curve only. It may be argued that because of non-linearity at higher loads it would be better to assume a constant stress-strain relationship than to assume a constant load-strain relationship. Were it possible to measure the cross-section of the wires to a higher degree of accuracy this might be true. However, the load in the wires was measured more accurately over the linear range by use of a load-strain relationship.

When the rate of straining increased, the accuracy of the evaluated load increased because the difference in strains represented a smaller load. The calibration of the wires against the gauge on the prestressing jack, provided that assumptions made are true, gave the loads to within 5.1%.

**Standard Load-Strain Relationships**

4.5-21 It has been assumed that the strains in a given wire were directly proportional to the strains in a standard wire of the same steel for equal loads. The constant of proportionality, the gauge calibration factor \( c \), is the ratio of the value of Young's modulus of the given wire to the value of Young's modulus for a standard wire. The sequence of loading on the standard wire simulated the loading on the wires in the walls. The wires in the walls were loaded to 10,000 lb in increments of 1000 lb, the anchorages were fixed, and the jack load was released. The loads in the wires were decreased approximately 2000 lb when the jack was released.
The loads in the wires were then adjusted to a constant load of 9250 lb. After seven days the loads in the wires were measured and readjusted (see section 4.6). When the walls were tested the strain gauges were not read until the wires had ceased straining.

4.5-22 The standard wires were tested in an Olsen 30,000 lb testing machine. This machine was accurate to 0.2% over the range used. The same anchorages as were used in the walls were used, including the rubber insertion pads below the anchorage plates. A Hersey dial gauge measured the relative movement of the leading heads. A specimen in the Olsen testing machine is shown in plate 4-12.

4.5-23 The wires were loaded in increments of 1000 lb to 10,000 lb. The load was then decreased to 9250 lb. The difference between unloading to 8000 lb and then reloading to 9250 lb, and unloading directly to 9250 lb is negligible (see fig. 4-8). The load in the wires was at a constant load of 9250 lb ± 25 lb over a period of seven days to simulate the time between prestressing and testing a wall. It must be noted, however, that in the walls the wires were under a constant strain, whereas in the tests on standard wires a condition of constant load was used. The wires in the walls relaxed to loads of approximately 8600 lb. Although the fall in load is small the load in the wires decreased to below the load at which substantial load intensity set began.

4.5-24 The results of the tests are tabulated in tables 4-15 and 4-16. The load-strain curves after seven days are shown in figure 4-10. These are used to determine the loads in the wires during testing, using the gauge calibration factor $g$. For steel 1, $E_x = 27.4 \times 10^6$ psi. For steel 2, $E_x = 29.2 \times 10^6$ psi. The ratio of the two moduli is 1.09. At a load of 12,000 lb the ratio increased by approximately 2%. Provided that the strains recorded by the strain gauges are linearly proportional to the true strains this indicates that the load-strain curves of the two steels are different. The limit of the load-strain curves was set by failure of the gauges. For strain gauge readings at high loads it was necessary to wait up half an hour before the wire ceased straining at constant load.

4.5-25 When the wire of steel 1 was tested the strain gauge indicated an increase in strain of $5 \times 10^{-5}$ whereas the dial gauge recorded a movement equivalent to an increase in strain of $29 \times 10^{-5}$, that is, approximately 3%.
of the total strain in the wire. For steel 2 the strain gauge recorded a decrease in strain of $4 \times 10^{-5}$ whereas the dial gauge recorded a movement equivalent to an increase in strain of $50 \times 10^{-5}$, that is, approximately 10% of the total strain in the wire. Upon unloading, the values for the strains at 10,000 lb were not greatly different from those obtained when the wires were first stressed. The strain in the wire of steel 1 was 1% less and the strain in the wire of steel 2 was 2.2% greater.

4.5-26 The cause of the discrepancy between the strains measured on the strain gauge and on the dial gauge is uncertain. It was due in part to creep in the rubber pad below the anchorage assembly. Such creep would result in a decrease in load and strain, requiring readjustment of the load to 9250 lb, but not increasing strain in the wire at 9250 lb. It was possible that the extra cross-sectional area of the strain gauge glue and water repellent was sufficient to reduce the stress at the gauge sufficiently to prevent or decrease the creep at that section. Whereas the increase in the cross-sectional area was small, equivalent to 0.9% of the steel (see section 4.6-15), the steel stress was such that substantial yielding was beginning. Creep may have occurred in the wire of the strain gauge and in the glue cementing the gauge. The measuring bridge was also suspect. Although the variations in the readings with time may have been due to temperature differences in the circuit, the large and temporary drop in the recorded strain during the test of steel 2 may not be attributed to any such cause. The drop may have been caused by a change in the supply mains voltage, but the exact cause is unknown.

4.5-27 From the similarity of the values of strain before and after the period during which creeping occurred, it appears that the amount of relaxation in the wires is small, of the order of 2.5 to 4%. It is also apparent that the strain measuring equipment was inadequate for measuring strains over a long period of time. This is shown in particular by the fact that the strain gauge on the wire of steel 2 gave a decrease in strain for an increase in the extension at constant load. In the walls the wires are three times as long as the wires tested, therefore the influence of creep in the rubber insertion pad will be proportionally less, and the creep in the steel and the anchorages should be less than 5.2%.
4.6 MEASUREMENT OF THE DECREASE IN THE LOAD IN THE PRESTRESSING WIRES DUE TO RELAXATION

INTRODUCTION

4.6-1 The steel of the prestressing wires in the walls was prestressed to approximately two-thirds the ultimate tensile stress of the steel. At this stress, substantial load intensity set began to occur. The rubber pads beneath the anchorages were under a stress of approximately 1500 psi. In the region of the anchorages the brick was stressed to approximately one-quarter of the brick crushing stress. The mortar in the nearest mortar joint was under a stress of approximately one-quarter the compressive strength of the mortar. Ceramic materials show little or no creep at ordinary temperatures. Creep may have occurred in the mortar joints nearest the anchorages, but it is probable that the greatest part of the relaxation in load is due to creep in the steel of the prestressing wires and in the rubber below the anchorages.

4.6-2 The strains in the wires after stressing up to the time of testing were assumed to be those in the strain gauges attached. This ignores the possibility of creep in the glue fixing the gauges. This section gives an assessment of the results of such strain measurements, compares the loads in the wires predicted by the strain gauges with those actually found by lifting the wires, and gives the measurements of the relaxation which occurred.

EXPERIMENTAL PROCEDURE

4.6-3 The stressing of wires and the determination of the "modulus of elasticity", $E_p$, using strain gauges is described in section 4.5. After the loads in the wires had been adjusted to a uniform prestressing load readings of the strains in the wires were taken over the seven days before testing, frequently at first, but less frequently later on. The strain bridge variable-resistance arm was cleaned at regular intervals, normally each morning. At night the strain bridge was switched off. This was because the bridge became unsteady when left on for a period longer than one day.

4.6-4 At the end of seven days each anchorage was lifted (see plate 5-7).
The method of lifting is described fully in section 5.1. The principle behind the method is as follows. The wall and the prestressing wire formed a self-straining system. An upward load was applied to the prestressing wire above the top anchorage. This load was provided by a jack. The bottom of the jack pushed downwards on the wall, loading the region near the anchorage. Provided that the upwards load was not greater than the load in the wire below the anchorage, there was no change in the length of the self-straining system of the wall and the wire below the anchorage because there was no change in the forces of that system. The change was a localized change in the point of application. Load was transferred off the anchorage to the region around the anchorage. Then, however, the load above the anchorage became greater than the load originally below the anchorage the wire below the anchorage elongated. In elongating, the load in the wire increased to equal the load above the anchorage. A definite change in length and increase in strain occurred for the application of load greater than the load originally in the wire below the anchorage.

This abrupt change was easily detected by a strain gauge on the wire.

4.6-5 From the prestressing of the wires it was known that strain increases by approximately $55 \times 10^6$ for each 100 lb increment of load. In some cases partial bearing of the anchorage, for example one side bearing and the other side free, produced a noticeable change in the strain of the wire. However, the increase in strain became $55 \times 10^6$ for each 100 lb only when there was no bearing of the anchorage. Extrapolation of the straight line of this slope on the load-strain curve to the line of zero increase in strain gave the load originally in the wire below the anchorage.

4.6-6 This method gave a direct measurement of the load in the wire after relaxation. The loads so determined were compared with the loads predicted by the strain gauge readings, providing a valuable check on the working of the gauges.

RESULTS

4.6-7 Upon release of the load after prestressing, the strains in the wires normally showed a fairly rapid decrease in strain of the order $25 \times 10^{-6}$ over a period of five minutes. The rate of loss of strain decreased with time. Figure 4-11 contrasts two sets of strain gauge readings from the same
wall test with the strain gauge readings for a constant load of 3250 lb obtained from the test described in section 4.5. The shape of the two strain-time curves for wires in wall W, are typical of the results obtained for each of the wall tests. Temperature and possibly other influences cause the rate of decrease in strain to appear irregular. In contrast with the decreasing strain with time, the strain in the test on steel 1 gradually increased. Such a general increase with time would be anticipated, but the magnitude of the increase would be expected to be greater. Dial gauge readings indicated a much greater increase than indicated by the strain gauges for both steel 1 and steel 2. Indeed the curve for steel 2 is highly irregular in that it indicates a decrease in strain. This irregularity must be attributed to gauge and measuring bridge shortcomings.

4.6-9 It should be noticed that the curves for the two wires in wall W rise and fall together, as do the strain-time curves for the other wires in the wall. This indicates the presence of factors other than steady creep. Such effects are caused, at least in part, by temperature changes. A change of temperature causes differential expansion between the steel and the brick. Taking the coefficient of thermal expansion of masonry and steel as $4.0 \times 10^{-6}$ and $6.3 \times 10^{-6}$ per degree Fahrenheit respectively a change in temperature of 20°F would cause a differential strain of $45 \times 10^{-6}$. Also any temperature difference between the temperature of the gauge in the wall and the dummy gauge will cause a change in resistance which would alter the balancing of the bridge. Temperature differences may be caused by laboratory conditions or by heating of the resistances caused by the current passing through them.

4.6-10 The graphs shown in figure 4-11 for two of the wires in wall W, are typical of the complete set. The overall losses of load in the wires are summarized in tables 4-18 to 4-22.

4.6-10 The results for the wires in wall W must be considered separately from those for the other walls. These wires were loaded up to 14,500 lb and therefore have a much greater load intensity set. In addition, the wires varied in cross-sectional area and probably were different types of steel. The allowance for load intensity set of 650 lb is arbitrary as the load-strain characteristics of the different steels are not precisely known. In the course of taking measurements on wires in wall W, both
the bridge and the connection to the dummy gauge were found to be faulty. It was not possible to find the final strains in the wires because of differing scales in the two strain bridges. Subject to the uncertainty of the amount of load intensity set, however, the magnitude of the relaxation may be found. The average creep loss was approximately 400 lb.

4.6-11 Steel 1 was used in walls W4, W5 and W6, and steel 2 in walls W6 and W7. In fixing the bottom loading beam on to wall W5, gauge 1, on the tension end of the wall, was damaged and the gauge could not be used. In wall W7 the initial loads set in the wires were not equal as in the case of the other walls.

4.6-12 The method of lifting the wires should measure the loads in the wires to an accuracy of about ± 50 lb. A specimen set of results is given in figure 4.6-12. During pre-stressing the loads set were calculated from release of the jack and during the subsequent restraining from a linear load-strain relationship. By calculating the change in load for a change in strain relative to the strain at greatest load, the effect of load intensity set was essentially eliminated. Variations in the load in the wire resulting from friction along the ducting may have resulted in a difference between the mean stress and the stress at the strain gauge of about ± 20 lb. The calibration of the load-strain curve is accurate to ± 210 lb. Variations in the slopes of the load-strain curves on loading and unloading could result in a further variation of ± 10 lb. Thus if the strain gauges were working correctly there should be agreement between the results of the strain gauge and the jack determinations of the residual load to within ±290 and ±270 lb.

4.6-13 Nineteen of the twenty-one results for the walls W4, W5, W6 and W7 lie within these limits. Twelve of these lie within ± 50 lb. Seven strain gauge readings were higher than the measured values, the maximum difference being ±140 lb. Thirteen strain gauge readings were lower, the greatest difference being 330 lb. This tendency to give lower readings may be a result of creep in the strain gauge cement and in the gauge itself. It may be significant that the strains recorded by the strain gauges in the tests under constant load were lower than anticipated.

4.6-14 The accuracy of the strain gauges may also be stated in the following way. 57% of the gauges had a discrepancy of ±% or less. 94% had a
discrepancy of 2% or less. The greatest discrepancy was 3.3%. There
was no obvious correlation between a large discrepancy and a large difference
between the true modulus of elasticity and the apparent modulus of elasticity
calculated from the load strain relationship.

4.6-15 The average creep loss was approximately 400 lb. The creep loss
measured was as great as 800 lb. Creep did not appear to be greatest at
any particular locality in the wall. Creep loss is between 1/2 and 3% of
the maximum load of 10,000 lb, and is probably due to creep in the steel
and in the rubber insertion below the anchorage plates rather than in the
brickwork.
Proportions by Weight:

- cement 15.05 lb
- lime 8.35 lb
- sand 64.50 lb
- water 13.30 lb

Total 100.00 lb

Sand Grading:

<table>
<thead>
<tr>
<th>Sieve Size (B.S. Sieve)</th>
<th>Percentage Passing</th>
<th>Percentage Retained</th>
</tr>
</thead>
<tbody>
<tr>
<td>7</td>
<td>100</td>
<td>-</td>
</tr>
<tr>
<td>14</td>
<td>93</td>
<td>7</td>
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<tr>
<td>25</td>
<td>74</td>
<td>19</td>
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<td>52</td>
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<td>50</td>
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<tr>
<td>100</td>
<td>4</td>
<td>20</td>
</tr>
<tr>
<td>pen</td>
<td>-</td>
<td>4</td>
</tr>
</tbody>
</table>

Compressive Strength:

Tested to B.S., 12.1958  
Age 28 days

- Maximum value = 1840 psi
- Minimum value = 1590 psi
- Mean value of 6 tests = 1720 psi

Mortar Flow:

Tested to ASTM C807-93

- Fresh = 400-1050
- After 1 hr = 73-83%

Table 4-1
Properties of the Adopted Mortar
<table>
<thead>
<tr>
<th>IRA Number</th>
<th>IRA Number</th>
<th>IRA Number</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>4</td>
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<td>6</td>
</tr>
<tr>
<td>7</td>
<td>8</td>
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<td>18</td>
</tr>
<tr>
<td>19</td>
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Table 4-2
Initial Rates of Absorption of Bricks Tested

<table>
<thead>
<tr>
<th>Range of IRA in Range</th>
<th>Number</th>
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</thead>
<tbody>
<tr>
<td>0 - 4.5</td>
<td>1</td>
</tr>
<tr>
<td>4.5 - 9.5</td>
<td>4</td>
</tr>
<tr>
<td>9.5 - 14.5</td>
<td>12</td>
</tr>
<tr>
<td>14.5 - 19.5</td>
<td>34</td>
</tr>
<tr>
<td>19.5 - 24.5</td>
<td>40</td>
</tr>
<tr>
<td>24.5 - 29.5</td>
<td>40</td>
</tr>
<tr>
<td>29.5 - 34.5</td>
<td>19</td>
</tr>
<tr>
<td>34.5 - 39.5</td>
<td>14</td>
</tr>
<tr>
<td>39.5 - 44.5</td>
<td>12</td>
</tr>
<tr>
<td>44.5 - 49.5</td>
<td>3</td>
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<td>49.5 - 54.5</td>
<td>1</td>
</tr>
<tr>
<td>54.5 - 59.5</td>
<td>1</td>
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</table>

Mean = 25.4
Standard Deviation = 9.6
Coefficient of Variation = 37.8%

Table 4-2
Spread of the Initial Rates of Absorption
<table>
<thead>
<tr>
<th>IRA When Dry</th>
<th>IRA in Surface Wet Condition</th>
<th>IRA after Standing Approx. 2 hrs</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>44.6</td>
<td>13.1</td>
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</tr>
<tr>
<td>41.3</td>
<td>19.7</td>
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<td>40.8</td>
<td>16.9</td>
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</tr>
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<td>39.5</td>
<td>15.2</td>
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<tr>
<td>38.0</td>
<td>11.4</td>
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<td>36.7</td>
<td>11.2</td>
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<td>14.5</td>
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<td>10.8</td>
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<td>31.0</td>
<td>6.3</td>
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<td>9.0</td>
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<tr>
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<td>12.1</td>
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<td>12.7</td>
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<td>43.7</td>
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<tr>
<td>15.0</td>
<td>9.1</td>
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<td>Low</td>
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</table>

Table 4-4

Effect of Wetting on the Initial Rate of Absorption

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<thead>
<tr>
<th>Colour</th>
<th>Hardness</th>
<th>Mean IRA</th>
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</thead>
<tbody>
<tr>
<td>Salmon pink</td>
<td>soft</td>
<td>70</td>
</tr>
<tr>
<td>Reddy pink</td>
<td>ordinary</td>
<td>64.2</td>
</tr>
<tr>
<td>Pinky red</td>
<td>medium</td>
<td>54.5</td>
</tr>
<tr>
<td>Red</td>
<td>hard</td>
<td>23.9</td>
</tr>
<tr>
<td>Red brown</td>
<td>very hard</td>
<td>7.7</td>
</tr>
</tbody>
</table>

Table 4-5

Initial Rates of Absorption of Bricks Groups
Produced in the Manufacturer's Kiln
<table>
<thead>
<tr>
<th>Specimen</th>
<th>Young's Modulus x10^6 psi</th>
<th>Failure Stress psi</th>
<th>Remarks on Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>H2</td>
<td>2.76</td>
<td>5130</td>
<td>Vertical cracks at the ends. Spalling from centre.</td>
</tr>
<tr>
<td>H9</td>
<td>1.87</td>
<td>4640</td>
<td>Vertical cracks at the ends. Cracking heard at 2920 psi.</td>
</tr>
<tr>
<td>H8</td>
<td>1.76</td>
<td>4300</td>
<td>Vertical cracks at the ends.</td>
</tr>
<tr>
<td>H5</td>
<td>1.69</td>
<td>5090</td>
<td>Vertical cracks at the ends.</td>
</tr>
<tr>
<td>H14</td>
<td>1.65</td>
<td>5700</td>
<td>Vertical cracks at the ends. Horizontal crack in mortar.</td>
</tr>
<tr>
<td>H4</td>
<td>1.32</td>
<td>4350</td>
<td>Vertical cracks at the ends. Cracking heard at 2980 psi.</td>
</tr>
<tr>
<td>H10</td>
<td>1.45</td>
<td>4740</td>
<td>Vertical cracks at the ends. Horizontal crack in mortar. Cracking heard at 4390 psi.</td>
</tr>
<tr>
<td>H6</td>
<td>1.39</td>
<td>3990</td>
<td>Vertical cracks at the ends. Horizontal crack in mortar. Cracking heard at 3440 psi.</td>
</tr>
<tr>
<td>H3</td>
<td>1.30</td>
<td>5000</td>
<td>Vertical crack in one end only.</td>
</tr>
<tr>
<td>H4</td>
<td>1.01</td>
<td>3810</td>
<td>Vertical cracks at the ends. Spalling from centre on one side.</td>
</tr>
<tr>
<td>H7</td>
<td>0.97</td>
<td>3320</td>
<td>Vertical cracks at the ends.</td>
</tr>
</tbody>
</table>

**Mean** 1.57 4590

**Standard Deviation** 1.8 700

**Coefficient of Variation** 30.8% 15.3%

**Table 4-6**

Young's Modulus and Crushing Strength of Specimens when Brick Layers are Tested Horizontally
<table>
<thead>
<tr>
<th>Specimen</th>
<th>Young's Modulus x10^6 psi</th>
<th>Failure Stress psi</th>
<th>Remarks on Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>V3</td>
<td>2.78</td>
<td>3520</td>
<td>Vertical cracking. Cracking heard at 2910 psi.</td>
</tr>
<tr>
<td>V7</td>
<td>1.53</td>
<td>4220</td>
<td>Sides splayed out in the centre. Cracking heard at 3010 psi.</td>
</tr>
<tr>
<td>V6</td>
<td>1.44</td>
<td>5070</td>
<td>Vertical cracking at the ends, with spalling at the joints. Cracking heard at 2450 psi.</td>
</tr>
<tr>
<td>V5</td>
<td>1.26</td>
<td>5160</td>
<td>Vertical cracking at the ends. Cracking heard at 3600 psi.</td>
</tr>
<tr>
<td>V8</td>
<td>1.25</td>
<td>3720</td>
<td>Vertical cracking at the ends, and spalling in the centre.</td>
</tr>
<tr>
<td>V4</td>
<td>1.23</td>
<td>6320</td>
<td>Long vertical crack down the middle of one brick. Cracking heard at 4520 psi.</td>
</tr>
<tr>
<td>V2</td>
<td>1.05</td>
<td>5930</td>
<td>Horizontal cracking in the centre moved out to the edge.</td>
</tr>
</tbody>
</table>

**Mean** 1.52 4030

**Standard Deviation** .56 1020

**Coefficient of Variation** 37.0% 20.9%

---

Table 4-7
Young's Modulus and Crushing Strength of Specimens when Brick Layers are Tested Vertically
<table>
<thead>
<tr>
<th>Ratio of Height to Least Thickness Dimension</th>
<th>Strength Correction Factors</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>0.45</td>
</tr>
<tr>
<td>1.5</td>
<td>0.59</td>
</tr>
<tr>
<td>2.0</td>
<td>0.67</td>
</tr>
<tr>
<td>2.5</td>
<td>0.75</td>
</tr>
<tr>
<td>3.0</td>
<td>0.80</td>
</tr>
<tr>
<td>3.5</td>
<td>0.85</td>
</tr>
<tr>
<td>4.0</td>
<td>0.89</td>
</tr>
<tr>
<td>4.5</td>
<td>0.93</td>
</tr>
<tr>
<td>5.0</td>
<td>0.96</td>
</tr>
<tr>
<td>5.5</td>
<td>0.98</td>
</tr>
<tr>
<td>6.0</td>
<td>1.00</td>
</tr>
<tr>
<td>8.0</td>
<td>1.03</td>
</tr>
<tr>
<td>10.0</td>
<td>1.06</td>
</tr>
<tr>
<td>12.0</td>
<td>1.09</td>
</tr>
</tbody>
</table>

Table 4-3
Krefeld's Strength Correction Factors for Brick Masonry Prisms
<table>
<thead>
<tr>
<th>Angle of Loading</th>
<th>Mortar Rate</th>
<th>Pressure at Failure</th>
<th>Type of Joint</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>15°</td>
<td>2500</td>
<td>bond</td>
<td>bond</td>
<td>( \frac{1}{2} ) pt shear in mortar</td>
</tr>
<tr>
<td></td>
<td>293</td>
<td>bond</td>
<td>bond</td>
<td>( \frac{1}{2} ) pt shear in mortar</td>
</tr>
<tr>
<td></td>
<td>307</td>
<td>bond</td>
<td>bond</td>
<td>( \frac{1}{2} ) pt shear in mortar</td>
</tr>
<tr>
<td></td>
<td>310</td>
<td>bond</td>
<td>bond</td>
<td>( \frac{1}{2} ) pt shear in mortar</td>
</tr>
<tr>
<td></td>
<td>360</td>
<td>bond</td>
<td>bond</td>
<td>( \frac{1}{2} ) pt shear in mortar</td>
</tr>
<tr>
<td></td>
<td>400</td>
<td>bond</td>
<td>bond</td>
<td>( \frac{1}{2} ) pt shear in mortar</td>
</tr>
<tr>
<td></td>
<td>424</td>
<td>bond</td>
<td>mean = 313 psi, ( \sigma = 56.7 ) psi, ( v = 38.4% )</td>
<td></td>
</tr>
<tr>
<td>30°</td>
<td>10,000</td>
<td>bond and caf</td>
<td>bond</td>
<td>( \frac{1}{2} ) pt shear in mortar</td>
</tr>
<tr>
<td></td>
<td>334</td>
<td>bond</td>
<td>bond</td>
<td>( \frac{1}{2} ) pt shear in mortar</td>
</tr>
<tr>
<td></td>
<td>411</td>
<td>bond and caf</td>
<td>bond</td>
<td>( \frac{1}{2} ) pt shear in mortar</td>
</tr>
<tr>
<td></td>
<td>432</td>
<td>bond</td>
<td>bond</td>
<td>( \frac{1}{2} ) pt shear in mortar</td>
</tr>
<tr>
<td></td>
<td>479</td>
<td>bond</td>
<td>bond</td>
<td>( \frac{1}{2} ) pt shear in mortar</td>
</tr>
<tr>
<td></td>
<td>482</td>
<td>bond</td>
<td>bond</td>
<td>( \frac{1}{2} ) pt shear in mortar</td>
</tr>
<tr>
<td></td>
<td>515</td>
<td>caf</td>
<td>mean = 495 psi, ( \sigma = 110 ) psi, ( v = 22.0% )</td>
<td></td>
</tr>
<tr>
<td></td>
<td>524</td>
<td>bond</td>
<td>bond</td>
<td>( \frac{1}{2} ) pt shear in mortar</td>
</tr>
<tr>
<td></td>
<td>563</td>
<td>bond</td>
<td>bond</td>
<td>( \frac{1}{2} ) pt shear in mortar</td>
</tr>
<tr>
<td></td>
<td>599</td>
<td>bond</td>
<td>brick crack</td>
<td>mean = 579 psi, ( \sigma = 24.5 ) psi, ( v = 42.4% )</td>
</tr>
<tr>
<td>35°</td>
<td>10,000</td>
<td>bond and caf</td>
<td>bond</td>
<td>secondary brick cracking</td>
</tr>
<tr>
<td></td>
<td>303</td>
<td>bond and caf</td>
<td>bond</td>
<td>secondary brick cracking</td>
</tr>
<tr>
<td></td>
<td>334</td>
<td>bond</td>
<td>bond</td>
<td>secondary brick cracking</td>
</tr>
<tr>
<td></td>
<td>421</td>
<td>bond</td>
<td>bond</td>
<td>secondary brick cracking</td>
</tr>
<tr>
<td></td>
<td>434</td>
<td>bond</td>
<td>bond</td>
<td>secondary brick cracking</td>
</tr>
<tr>
<td></td>
<td>490</td>
<td>bond</td>
<td>bond</td>
<td>secondary brick cracking</td>
</tr>
<tr>
<td></td>
<td>527</td>
<td>bond</td>
<td>bond</td>
<td>secondary brick cracking</td>
</tr>
<tr>
<td></td>
<td>541</td>
<td>bond, brick and caf</td>
<td>bond</td>
<td>secondary brick cracking</td>
</tr>
<tr>
<td></td>
<td>760</td>
<td>bond</td>
<td>mortar shear 1&quot; from end</td>
<td></td>
</tr>
<tr>
<td></td>
<td>858</td>
<td>bond</td>
<td>mean = 579 psi, ( \sigma = 24.5 ) psi, ( v = 42.4% )</td>
<td></td>
</tr>
<tr>
<td>40°</td>
<td>10,000</td>
<td>bond and caf</td>
<td>bond</td>
<td>secondary brick cracking</td>
</tr>
<tr>
<td></td>
<td>1079</td>
<td>bond, brick</td>
<td>bond</td>
<td>secondary brick cracking</td>
</tr>
<tr>
<td></td>
<td>1079</td>
<td>bond</td>
<td>mean = 579 psi, ( \sigma = 24.5 ) psi, ( v = 42.4% )</td>
<td></td>
</tr>
<tr>
<td>Angle of Loading</td>
<td>Mortar Rate</td>
<td>Unit</td>
<td>Type of Failure</td>
<td>Remarks</td>
</tr>
<tr>
<td>------------------</td>
<td>-------------</td>
<td>------</td>
<td>-----------------</td>
<td>---------</td>
</tr>
<tr>
<td>Vertical</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>40°</td>
<td>20,000</td>
<td>366</td>
<td>brick</td>
<td>Side broke off</td>
</tr>
<tr>
<td></td>
<td></td>
<td>518</td>
<td>bond</td>
<td>Mortar shear 1&quot; from end</td>
</tr>
<tr>
<td></td>
<td></td>
<td>524</td>
<td>bond</td>
<td>Side broke off</td>
</tr>
<tr>
<td></td>
<td></td>
<td>668</td>
<td>brick</td>
<td>Mortar shear 1&quot; from end</td>
</tr>
<tr>
<td></td>
<td></td>
<td>676</td>
<td>bond</td>
<td>Central crack</td>
</tr>
<tr>
<td></td>
<td></td>
<td>769</td>
<td>bond</td>
<td>( \frac{1}{2} ) pt mortar shear</td>
</tr>
<tr>
<td></td>
<td></td>
<td>334</td>
<td>brick</td>
<td>Brick cracked, but final bond failure</td>
</tr>
<tr>
<td>45°</td>
<td>20,000</td>
<td>732</td>
<td>bond</td>
<td>( \sigma = 346 ) psi ( v = 44.1% )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>662</td>
<td>brick</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>685</td>
<td>brick</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>923</td>
<td>mortar</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>951</td>
<td>bond</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>954</td>
<td>brick</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1179</td>
<td>brick</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1562</td>
<td>bond</td>
<td>( \sigma = 1670 ) psi ( v = 32.1% )</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1854</td>
<td>brick</td>
<td></td>
</tr>
<tr>
<td>50°</td>
<td>20,000</td>
<td>1092</td>
<td>bond</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1140</td>
<td>bond</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1193</td>
<td>brick</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1468</td>
<td>brick</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1590</td>
<td>brick and mortar</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1582</td>
<td>bond</td>
<td>Secondary brick cracking</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1602</td>
<td>brick</td>
<td>Brick cracking before failure</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1760</td>
<td>bond</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1763</td>
<td>bond</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td>1780</td>
<td>brick</td>
<td>( \sigma = 1495 ) ( v = 17.4% )</td>
</tr>
<tr>
<td>Angle of Loading</td>
<td>Unit</td>
<td>Type of Failure</td>
<td>Remarks</td>
<td></td>
</tr>
<tr>
<td>-----------------</td>
<td>------</td>
<td>----------------</td>
<td>---------</td>
<td></td>
</tr>
<tr>
<td>Joint Vertical</td>
<td>psi</td>
<td>brick</td>
<td>Shear through the central hole of the bricks.</td>
<td></td>
</tr>
<tr>
<td>60°</td>
<td>40,000</td>
<td>620</td>
<td>mean = 1140 psi</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Mortar Rate</th>
<th>Pressure at Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>1lb/min.</td>
<td>1092</td>
</tr>
<tr>
<td></td>
<td>1172</td>
</tr>
<tr>
<td></td>
<td>1677</td>
</tr>
</tbody>
</table>

Types of Failure:
- **bond** = clean bond failure along one side
- **mortar shear** = near vertical failure through mortar
- **c.s.f.** = near vertical cracking through the mortar attributed to principal tensile stresses due to shear
- **brick** = near vertical cracking of brick attributed to principal tensile stresses due to shear stress
- *** = spacer blocks used**

Table 4-9
Individual Results of Shear Tests
<table>
<thead>
<tr>
<th>Angle of Vertical</th>
<th>Number of Mortar Layer to Tested Specimens</th>
<th>Mean Vertical Pressure psi</th>
<th>Standard Deviation psi</th>
<th>Coefficient of Variation percent</th>
<th>Average Shear Stress psi</th>
<th>Average Normal Stress psi</th>
<th>Bond Stress psi</th>
<th>Central Cracking Failure Brick in Mortar</th>
<th>Type of Failure</th>
</tr>
</thead>
<tbody>
<tr>
<td>15°</td>
<td>10</td>
<td>313</td>
<td>86.7</td>
<td>28.4</td>
<td>302</td>
<td>34</td>
<td>10</td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>30°</td>
<td>11</td>
<td>496</td>
<td>110</td>
<td>22.0</td>
<td>429</td>
<td>248</td>
<td>9</td>
<td>3</td>
<td>1</td>
</tr>
<tr>
<td>35°</td>
<td>10</td>
<td>579</td>
<td>245</td>
<td>42.4</td>
<td>473</td>
<td>332</td>
<td>10</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>40°</td>
<td>10</td>
<td>769</td>
<td>316</td>
<td>41.1</td>
<td>509</td>
<td>434</td>
<td>6</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>45°</td>
<td>10</td>
<td>1070</td>
<td>344</td>
<td>32.1</td>
<td>758</td>
<td>738</td>
<td>4</td>
<td>4</td>
<td>5</td>
</tr>
<tr>
<td>50°</td>
<td>10</td>
<td>1495</td>
<td>264</td>
<td>17.6</td>
<td>961</td>
<td>1146</td>
<td>5</td>
<td>1</td>
<td>6</td>
</tr>
<tr>
<td>60°</td>
<td>4</td>
<td>1140</td>
<td></td>
<td></td>
<td>570</td>
<td>988</td>
<td>4</td>
<td></td>
<td>4</td>
</tr>
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</table>

Table 4-10
Summary of the Results of the Couplet Tests
<table>
<thead>
<tr>
<th>Angle of Mortar to Vertical</th>
<th>Number of Specimens Tested</th>
<th>Mean Vertical Pressure, psi</th>
<th>Mean for all Types of Failure, psi</th>
<th>Average Shear Stress, psi</th>
<th>Average Normal Stress, psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>15°</td>
<td>10</td>
<td>313</td>
<td>313</td>
<td>302</td>
<td>81</td>
</tr>
<tr>
<td>30°</td>
<td>10</td>
<td>473</td>
<td>496</td>
<td>409</td>
<td>237</td>
</tr>
<tr>
<td>35°</td>
<td>10</td>
<td>579</td>
<td>579</td>
<td>473</td>
<td>332</td>
</tr>
<tr>
<td>40°</td>
<td>6</td>
<td>838</td>
<td>769</td>
<td>642</td>
<td>539</td>
</tr>
<tr>
<td>45°</td>
<td>5</td>
<td>934</td>
<td>1070</td>
<td>703</td>
<td>703</td>
</tr>
<tr>
<td>50°</td>
<td>5</td>
<td>1467</td>
<td>1495</td>
<td>943</td>
<td>1124</td>
</tr>
<tr>
<td>60°</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
</tbody>
</table>

Table 4-11
Summary of the Results of Couplet Tests when only Mortar Bond Failures are Considered
<table>
<thead>
<tr>
<th>Mortar Layer to the Vertical</th>
<th>Number of Mortar Specimens Tested</th>
<th>Average Shear Stress psi</th>
<th>Average Normal Stress psi</th>
</tr>
</thead>
<tbody>
<tr>
<td>1:3:5</td>
<td>90°</td>
<td>0</td>
<td>38.8 t</td>
</tr>
<tr>
<td></td>
<td>45°</td>
<td>26.3</td>
<td>26.3 t</td>
</tr>
<tr>
<td></td>
<td>30°</td>
<td>30.5</td>
<td>22.3 t</td>
</tr>
<tr>
<td></td>
<td>15°</td>
<td>59.0</td>
<td>15.8 t</td>
</tr>
<tr>
<td></td>
<td>0°</td>
<td>98.7</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>15°</td>
<td>265.3</td>
<td>64.2</td>
</tr>
<tr>
<td></td>
<td>30°</td>
<td>301</td>
<td>174</td>
</tr>
<tr>
<td></td>
<td>45°</td>
<td>618</td>
<td>618</td>
</tr>
<tr>
<td>1:8:4½</td>
<td>90°</td>
<td>0</td>
<td>32.0 t</td>
</tr>
<tr>
<td></td>
<td>45°</td>
<td>23.0</td>
<td>23.0 t</td>
</tr>
<tr>
<td></td>
<td>30°</td>
<td>31.5</td>
<td>18.2 t</td>
</tr>
<tr>
<td></td>
<td>15°</td>
<td>48.5</td>
<td>12.9 t</td>
</tr>
<tr>
<td></td>
<td>0°</td>
<td>72.6</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>15°</td>
<td>186.9</td>
<td>49.9</td>
</tr>
<tr>
<td></td>
<td>30°</td>
<td>243</td>
<td>145</td>
</tr>
<tr>
<td></td>
<td>45°</td>
<td>534</td>
<td>534</td>
</tr>
<tr>
<td>1:1:6</td>
<td>90°</td>
<td>0</td>
<td>30.7 t</td>
</tr>
<tr>
<td></td>
<td>45°</td>
<td>23.7</td>
<td>23.7 t</td>
</tr>
<tr>
<td></td>
<td>30°</td>
<td>42.5</td>
<td>21.5 t</td>
</tr>
<tr>
<td></td>
<td>15°</td>
<td>49.5</td>
<td>13.3 t</td>
</tr>
<tr>
<td></td>
<td>0°</td>
<td>78.6</td>
<td>0</td>
</tr>
<tr>
<td></td>
<td>15°</td>
<td>213</td>
<td>56.6</td>
</tr>
<tr>
<td></td>
<td>30°</td>
<td>252</td>
<td>151</td>
</tr>
<tr>
<td></td>
<td>45°</td>
<td>603</td>
<td>602</td>
</tr>
</tbody>
</table>

`t` denotes a tensile normal stress

Table 4-12

Results of Couplet Tests by Benjamin and Williams on Three Mortar Types
<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Extensometer Bridge (10^{-5})</th>
<th>Strain Bridge (10^{-6})</th>
<th>Load (lb)</th>
<th>Extensometer Bridge (10^{-5})</th>
<th>Strain Bridge (10^{-6})</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2000</td>
<td>170</td>
<td>2142</td>
</tr>
<tr>
<td>1000</td>
<td>52</td>
<td>607</td>
<td>3000</td>
<td>230</td>
<td>2706</td>
</tr>
<tr>
<td>2000</td>
<td>110</td>
<td>1219</td>
<td>4000</td>
<td>282</td>
<td>3256</td>
</tr>
<tr>
<td>3000</td>
<td>185</td>
<td>1818</td>
<td>5000</td>
<td>337</td>
<td>3874</td>
</tr>
<tr>
<td>4000</td>
<td>224</td>
<td>2466</td>
<td>6000</td>
<td>396</td>
<td>4443</td>
</tr>
<tr>
<td>5000</td>
<td>280</td>
<td>3067</td>
<td>7000</td>
<td>455</td>
<td>5089</td>
</tr>
<tr>
<td>6000</td>
<td>342</td>
<td>3735</td>
<td>8000</td>
<td>513</td>
<td>5581</td>
</tr>
<tr>
<td>7000</td>
<td>399</td>
<td>4392</td>
<td>9000</td>
<td>572</td>
<td>6164</td>
</tr>
<tr>
<td>8000</td>
<td>462</td>
<td>5007</td>
<td>10000</td>
<td>631</td>
<td>6754</td>
</tr>
<tr>
<td>9000</td>
<td>526</td>
<td>5636</td>
<td>11000</td>
<td>74.07</td>
<td></td>
</tr>
<tr>
<td>10000</td>
<td>593</td>
<td>6385</td>
<td>12000</td>
<td>9133</td>
<td></td>
</tr>
<tr>
<td>11000</td>
<td>681</td>
<td>7325</td>
<td>12500</td>
<td>11313</td>
<td></td>
</tr>
<tr>
<td>12000</td>
<td>634</td>
<td>6776</td>
<td>13500</td>
<td>Failure</td>
<td></td>
</tr>
</tbody>
</table>

Table 4-13

Steel 1 - Measurement of the Load Strain Relationship
<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Extensometer Strain x 10^-6</th>
<th>Recorded Strain x 10^-6</th>
<th>Load (lb)</th>
<th>Extensometer Strain x 10^-6</th>
<th>Recorded Strain x 10^-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>0</td>
<td>2000</td>
<td>140</td>
<td>931</td>
</tr>
<tr>
<td>1000</td>
<td>67</td>
<td>394</td>
<td>3000</td>
<td>195</td>
<td>1577</td>
</tr>
<tr>
<td>2000</td>
<td>123</td>
<td>881</td>
<td>4000</td>
<td>250</td>
<td>2021</td>
</tr>
<tr>
<td>5000</td>
<td>177</td>
<td>1418</td>
<td>5000</td>
<td>302</td>
<td>2587</td>
</tr>
<tr>
<td>4000</td>
<td>335</td>
<td>1956</td>
<td>6000</td>
<td>352</td>
<td>3130</td>
</tr>
<tr>
<td>5000</td>
<td>292</td>
<td>2492</td>
<td>7000</td>
<td>417</td>
<td>3687</td>
</tr>
<tr>
<td>6000</td>
<td>319</td>
<td>3056</td>
<td>8000</td>
<td>473</td>
<td>4220</td>
</tr>
<tr>
<td>7000</td>
<td>407</td>
<td>3615</td>
<td>9000</td>
<td>528</td>
<td>4800</td>
</tr>
<tr>
<td>8000</td>
<td>465</td>
<td>4160</td>
<td>10000</td>
<td>585</td>
<td>5342</td>
</tr>
<tr>
<td>9000</td>
<td>523</td>
<td>4714</td>
<td>11000</td>
<td>655</td>
<td>6057</td>
</tr>
<tr>
<td>10000</td>
<td>583</td>
<td>5314</td>
<td>12000</td>
<td>725</td>
<td>7313</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Extensometer Strain x 10^-6</th>
<th>Recorded Strain x 10^-6</th>
</tr>
</thead>
<tbody>
<tr>
<td>9000</td>
<td>531</td>
<td>4792</td>
</tr>
<tr>
<td>9000</td>
<td>477</td>
<td>4223</td>
</tr>
<tr>
<td>7000</td>
<td>448</td>
<td>3661</td>
</tr>
<tr>
<td>6000</td>
<td>368</td>
<td>3096</td>
</tr>
<tr>
<td>5000</td>
<td>325</td>
<td>2514</td>
</tr>
<tr>
<td>4000</td>
<td>292</td>
<td>1883</td>
</tr>
<tr>
<td>3000</td>
<td>215</td>
<td>1147</td>
</tr>
<tr>
<td>2000</td>
<td>140</td>
<td>919</td>
</tr>
<tr>
<td>1000</td>
<td>85</td>
<td>400</td>
</tr>
</tbody>
</table>

Table 4-14
Steel 2 - Measurement of the Load-Strain Relationship
Readings to Obtain Load-Strain Curve

<table>
<thead>
<tr>
<th>Load, lb</th>
<th>Strain, $\times 10^{-6}$</th>
<th>Load, lb</th>
<th>Strain, $\times 10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>6000</td>
<td>3.60</td>
</tr>
<tr>
<td>1000</td>
<td>500</td>
<td>7000</td>
<td>4.414</td>
</tr>
<tr>
<td>2000</td>
<td>1052</td>
<td>8000</td>
<td>4.775</td>
</tr>
<tr>
<td>3000</td>
<td>1649</td>
<td>9000</td>
<td>5.60</td>
</tr>
<tr>
<td>4000</td>
<td>2248</td>
<td>10000</td>
<td>6.230</td>
</tr>
<tr>
<td>5000</td>
<td>2846</td>
<td>9000</td>
<td>5.60</td>
</tr>
</tbody>
</table>

Variations with Time in Strain Gauge Readings at a Constant Load of 3250 lb

<table>
<thead>
<tr>
<th>Time Variation m/s</th>
<th>Strain Variation $\times 10^{-6}$</th>
<th>Time Variation m/s</th>
<th>Strain Variation $\times 10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>4.25</td>
<td>+15</td>
<td>51.8</td>
<td>+11</td>
</tr>
<tr>
<td>1.3</td>
<td>0</td>
<td>14.15</td>
<td>+30</td>
</tr>
<tr>
<td>3.2</td>
<td>-2</td>
<td>61.16</td>
<td>+40</td>
</tr>
<tr>
<td>3.7</td>
<td>-1</td>
<td>63.30</td>
<td>+30</td>
</tr>
<tr>
<td>3.3</td>
<td>+3</td>
<td>77.10</td>
<td>+56</td>
</tr>
<tr>
<td>2.6</td>
<td>0</td>
<td>82.00</td>
<td>+76</td>
</tr>
<tr>
<td>3.4</td>
<td>+3</td>
<td>87.30</td>
<td>+74</td>
</tr>
<tr>
<td>2.8</td>
<td>-1</td>
<td>90.30</td>
<td>+58</td>
</tr>
<tr>
<td>3.0</td>
<td>+2</td>
<td>95.30</td>
<td>+47</td>
</tr>
<tr>
<td>3.3</td>
<td>+3</td>
<td>95.50</td>
<td>+57</td>
</tr>
<tr>
<td>4.1</td>
<td>+1</td>
<td>97.80</td>
<td>+50</td>
</tr>
</tbody>
</table>

Strains Recorded on Loading to Failure

<table>
<thead>
<tr>
<th>Load, lb</th>
<th>Strain when there is no further creeping, $\times 10^{-6}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>634.3</td>
</tr>
<tr>
<td>10500</td>
<td>6635</td>
</tr>
<tr>
<td>11000</td>
<td>74.39</td>
</tr>
<tr>
<td>11500</td>
<td>843</td>
</tr>
<tr>
<td>12000</td>
<td>10030</td>
</tr>
<tr>
<td>12500</td>
<td>Bridge 2</td>
</tr>
<tr>
<td>13000</td>
<td>failure</td>
</tr>
<tr>
<td>13500</td>
<td>Box 3</td>
</tr>
</tbody>
</table>

Table 4-15

Strains in the Standard Wire Under Loading Simulated to the Tests - Steel 1
### Readings to Obtain Load-Strain Curve

<table>
<thead>
<tr>
<th>Load, lb</th>
<th>Strain, x10^{-6}</th>
<th>Load, lb</th>
<th>Strain, x10^{-6}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>6000</td>
<td>3265</td>
</tr>
<tr>
<td>1000</td>
<td>514</td>
<td>7000</td>
<td>3838</td>
</tr>
<tr>
<td>2000</td>
<td>1063</td>
<td>3000</td>
<td>4419</td>
</tr>
<tr>
<td>3000</td>
<td>1658</td>
<td>9000</td>
<td>5090</td>
</tr>
<tr>
<td>4000</td>
<td>2133</td>
<td>10000</td>
<td>5751</td>
</tr>
<tr>
<td>5000</td>
<td>2702</td>
<td>2250</td>
<td>5320</td>
</tr>
</tbody>
</table>

### Variations with Time in Strain Gauge Readings at a constant Load of 9250 lb

<table>
<thead>
<tr>
<th>Time (mins)</th>
<th>Strain Variation x10^{-6}</th>
<th>Time (mins)</th>
<th>Strain Variation x10^{-6}</th>
<th>Time (mins)</th>
<th>Strain Variation x10^{-6}</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0</td>
<td>10</td>
<td>+5</td>
<td>2</td>
<td>+4</td>
</tr>
<tr>
<td>20</td>
<td>+5</td>
<td>30</td>
<td>+6</td>
<td>60</td>
<td>+2</td>
</tr>
<tr>
<td>30</td>
<td>+6</td>
<td>60</td>
<td>+2</td>
<td>135</td>
<td>+5</td>
</tr>
<tr>
<td>60</td>
<td>+2</td>
<td>135</td>
<td>+5</td>
<td>180</td>
<td>+10</td>
</tr>
<tr>
<td>135</td>
<td>+5</td>
<td>180</td>
<td>+10</td>
<td>300</td>
<td>+11</td>
</tr>
<tr>
<td>180</td>
<td>+10</td>
<td>300</td>
<td>+11</td>
<td>390</td>
<td>+13</td>
</tr>
<tr>
<td>300</td>
<td>+11</td>
<td>390</td>
<td>+13</td>
<td>410</td>
<td>+8</td>
</tr>
<tr>
<td>390</td>
<td>+13</td>
<td>410</td>
<td>+8</td>
<td>530⁻</td>
<td>0</td>
</tr>
<tr>
<td>410</td>
<td>+8</td>
<td>530⁻</td>
<td>0</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

### Strains Recorded on Loading to Failure

<table>
<thead>
<tr>
<th>Load (lb)</th>
<th>Initial Strain, x10^{-6}</th>
<th>Strain when there is no further creeping, x10^{-6}</th>
<th>Bridge 1</th>
<th>Box 3</th>
</tr>
</thead>
<tbody>
<tr>
<td>10000</td>
<td>5707</td>
<td>7450</td>
<td></td>
<td></td>
</tr>
<tr>
<td>10500</td>
<td>6078</td>
<td>8917</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11000</td>
<td>6552</td>
<td>8917</td>
<td></td>
<td></td>
</tr>
<tr>
<td>11500</td>
<td>7312</td>
<td>8917</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12000</td>
<td>8332</td>
<td>8917</td>
<td></td>
<td></td>
</tr>
<tr>
<td>12500</td>
<td>11732</td>
<td>12582</td>
<td></td>
<td></td>
</tr>
<tr>
<td>13000</td>
<td>20500</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14050</td>
<td>failure</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

* indicates bridge turned off

### Table 4-16

**Standard Wire for Simulated Loading on Prestressing Wires of Steel 2**
<table>
<thead>
<tr>
<th>Load on Bourdon Testing Gauge Machine</th>
<th>Mean Load on 864</th>
<th>Standard Deviation 28.0</th>
<th>Difference of Greatest Reading 56</th>
<th>Difference of Least Reading +36</th>
</tr>
</thead>
<tbody>
<tr>
<td>1000</td>
<td>1000</td>
<td>22.5</td>
<td>85</td>
<td>+74</td>
</tr>
<tr>
<td>2000</td>
<td>2000</td>
<td>36.3</td>
<td>73</td>
<td>+53</td>
</tr>
<tr>
<td>3000</td>
<td>3000</td>
<td>29.2</td>
<td>58</td>
<td>+36</td>
</tr>
<tr>
<td>4000</td>
<td>4000</td>
<td>40.3</td>
<td>81</td>
<td>+48</td>
</tr>
<tr>
<td>5000</td>
<td>5000</td>
<td>28.3</td>
<td>77</td>
<td>+61</td>
</tr>
<tr>
<td>6000</td>
<td>6000</td>
<td>41.0</td>
<td>82</td>
<td>+52</td>
</tr>
<tr>
<td>7000</td>
<td>7000</td>
<td>24.6</td>
<td>89</td>
<td>+77</td>
</tr>
<tr>
<td>8000</td>
<td>8000</td>
<td>47.8</td>
<td>96</td>
<td>+72</td>
</tr>
<tr>
<td>9000</td>
<td>9000</td>
<td>36.5</td>
<td>73</td>
<td>+70</td>
</tr>
<tr>
<td>10000</td>
<td>10000</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Table 4-17

Calibration of the Bourdon Gauge on the Prestressing Jack
<table>
<thead>
<tr>
<th>Gauge No.</th>
<th>$E_0$ x10^6</th>
<th>$E_g$ x29.8 x10^6</th>
<th>Load Set in Wires</th>
<th>Load After Relaxation</th>
<th>Loss Load Measured</th>
<th>Intensity using the Set, lb</th>
<th>Jack, lb</th>
<th>lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>27.6</td>
<td>7.4</td>
<td>6600</td>
<td>2400</td>
<td>200</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>27.4</td>
<td>8.0</td>
<td>&quot;</td>
<td>7600</td>
<td>800</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30.7</td>
<td>3.0</td>
<td>&quot;</td>
<td>8700</td>
<td>-100</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>29.5</td>
<td>-4.0</td>
<td>&quot;</td>
<td>5200</td>
<td>400</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>32.5</td>
<td>9.1</td>
<td>&quot;</td>
<td>7700</td>
<td>900</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>32.6</td>
<td>9.4</td>
<td>&quot;</td>
<td>8300</td>
<td>300</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>34.0</td>
<td>44.1</td>
<td>&quot;</td>
<td>&quot;</td>
<td>&quot;</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Steel assumed to be steel 1.
Cross-sectional areas various.
Allowance for load intensity set is 650 lb.

Table 4-18
Summary of Results of Measuring the Decrease in the Load in the Prestressing Wires used in the Wall #3

<table>
<thead>
<tr>
<th>Gauge No.</th>
<th>$E_0$ x10^6</th>
<th>$E_g$ x29.8 x10^6</th>
<th>Load Set in Wires</th>
<th>Load After Relaxation as given by the Strain</th>
<th>Measured Value in Gauge, lb</th>
<th>Jack, lb</th>
<th>lb</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>31.7</td>
<td>8.4</td>
<td>9250</td>
<td>8960</td>
<td>3100</td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>29.7</td>
<td>-0.3</td>
<td>&quot;</td>
<td>8900</td>
<td>8900</td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>31.6</td>
<td>8.1</td>
<td>&quot;</td>
<td>8590</td>
<td>8600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>31.7</td>
<td>8.4</td>
<td>&quot;</td>
<td>8650</td>
<td>8600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>34.5</td>
<td>5.8</td>
<td>&quot;</td>
<td>8810</td>
<td>9000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>35.3</td>
<td>5.4</td>
<td>&quot;</td>
<td>8900</td>
<td>9000</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>34.3</td>
<td>5.1</td>
<td>&quot;</td>
<td>8700</td>
<td>9000</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Steel 1.
Cross-sectional area used 0.5595 in².

Table 4-19
Summary of Results of Measuring the Decrease in the Load in the Prestressing Wires used in the Wall #4.
<table>
<thead>
<tr>
<th>Gauge No.</th>
<th>(E_p) x10^6 psi</th>
<th>(E_p) x29.3x10^6 psi</th>
<th>Load Set in Relaxation</th>
<th>Load After Relaxation as given by Wire Strains using the Gauge, lb Jack</th>
<th>Load After Relaxation Measured</th>
<th>Load After Relaxation Jack Value (1) Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30.6</td>
<td>8.7</td>
<td>8250</td>
<td>8770</td>
<td>8750</td>
<td>500</td>
</tr>
<tr>
<td>2</td>
<td>29.9</td>
<td>3.3</td>
<td>8250</td>
<td>8770</td>
<td>8750</td>
<td>500</td>
</tr>
</tbody>
</table>

Steel 1.
Cross-sectional area used 0.0555 in^2

Connection to gauge 1 was damaged during the concreting of the bottom loading beam.

**Table 4-20**

Summary of the Results of Measuring of the Decrease in the Load in the Prestressing Wires used in the Wall W5

<table>
<thead>
<tr>
<th>Gauge No.</th>
<th>(E_p) x10^6 psi</th>
<th>(E_p) x29.3x10^6 psi</th>
<th>Load Set in Relaxation</th>
<th>Load After Relaxation as given by Wire Strains using the Gauge, lb Jack</th>
<th>Load After Relaxation Measured</th>
<th>Load After Relaxation Jack Value (1) Correct</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>28.3</td>
<td>-5.0</td>
<td>9250</td>
<td>8710</td>
<td>8750</td>
<td>500</td>
</tr>
<tr>
<td>2</td>
<td>30.5</td>
<td>2.4</td>
<td>8770</td>
<td>8750</td>
<td>8750</td>
<td>500</td>
</tr>
<tr>
<td>3</td>
<td>30.5</td>
<td>2.4</td>
<td>8730</td>
<td>8750</td>
<td>8750</td>
<td>500</td>
</tr>
<tr>
<td>4</td>
<td>30.4</td>
<td>2.0</td>
<td>8730</td>
<td>8750</td>
<td>8750</td>
<td>500</td>
</tr>
<tr>
<td>5</td>
<td>29.9</td>
<td>0.3</td>
<td>8730</td>
<td>8750</td>
<td>8750</td>
<td>500</td>
</tr>
<tr>
<td>6</td>
<td>29.3</td>
<td>-1.7</td>
<td>8730</td>
<td>8750</td>
<td>8750</td>
<td>500</td>
</tr>
<tr>
<td>7</td>
<td>29.2</td>
<td>-2.0</td>
<td>8730</td>
<td>8750</td>
<td>8750</td>
<td>500</td>
</tr>
<tr>
<td>8</td>
<td>30.9</td>
<td>3.7</td>
<td>8730</td>
<td>8750</td>
<td>8750</td>
<td>500</td>
</tr>
<tr>
<td>9</td>
<td>31.5</td>
<td>5.7</td>
<td>8730</td>
<td>8750</td>
<td>8750</td>
<td>500</td>
</tr>
</tbody>
</table>

Wires 1 to 6 of steel 1, cross-sectional area 0.0555 in^2
Wires 7 to 10 of steel 2, cross-sectional area 0.0600 in^2

**Table 4-21**

Summary of the Results of Measuring the Decrease in the Load in the Prestressing Wires used in the Wall W6
<table>
<thead>
<tr>
<th>Gauge No.</th>
<th>$E_t \times 10^6$</th>
<th>$E_p \times 10^6$</th>
<th>Load Set in Rel.</th>
<th>Load After Relaxation</th>
<th>Load After Relaxation Measured</th>
<th>Load After Relaxation Assuming Jack Value is Correct</th>
<th>Loss</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Wires, %</td>
<td>Gauge, lb</td>
<td>Jack, lb</td>
<td></td>
<td></td>
</tr>
<tr>
<td>1</td>
<td>32.7</td>
<td>+46.5</td>
<td>9300</td>
<td>9540</td>
<td>9400</td>
<td>400</td>
<td></td>
</tr>
<tr>
<td>2</td>
<td>31.7</td>
<td>+7.1</td>
<td>9600</td>
<td>9290</td>
<td>9225</td>
<td>375</td>
<td></td>
</tr>
<tr>
<td>3</td>
<td>30.8</td>
<td>+4.1</td>
<td>8600</td>
<td>8270</td>
<td>8600</td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td>31.9</td>
<td>+7.8</td>
<td>8600</td>
<td>8270</td>
<td>8575</td>
<td>225</td>
<td></td>
</tr>
</tbody>
</table>

Steel 2.
Cross-sectional area used 0.0600 in$^2$.
Allowance for load intensity set is 50 lb.

Table 4-22
Summary of the Results of Measuring the Decrease in the Load in the Prestressing Wires used in the Wall 67
TESTS ON THREE BRICK SPECIMENS
Figure 4.3

DEFLECTION in x 10^-5

STRESS p.s.i.

Figure 4.4
Figure 4.9

ALL COUPLETS
BOND FAILURES ONLY
TESTS ON A 1\:\frac{1}{2} : 4\frac{1}{4}
MORTAR BY BENJAMIN AND WILLIAMS
MEASUREMENT OF YOUNG'S MODULUS
USING A EWING EXTENSOMETER—STEEL 1

Figure 4-6
measurement of young's modulus using a ewing extensometer - steel 2

figure 4.7
LOAD-STRAIN RELATIONSHIP FOR A WIRE OF STEEL 2 WHEN LOADED AND UNLOADED

Figure 4.8
LOAD-STRAIN RELATIONSHIP FOR A WIRE OF STEEL 2 WHEN LOADED AND UNLOADED

Figure 4-9
LOAD - STRAIN RELATIONSHIP AFTER 7 DAYS AT 9250 lb. STRAIN GAUGE READINGS

Figure 4-10

NOMINAL AREAS

STEEL 1: 0.0595 in²
STEEL 2: 0.0600 in²

YOUNG'S MODULUS

STEEL 1: 27.4 x 10⁶
STEEL 2: 29.6 x 10⁶
PLATE 4-1 Horizontal bricks - horizontal cracks in the mortar

PLATE 4-2 Horizontal bricks - vertical cracking in the ends
PLATE 4-3  Vertical bricks - central spalling

PLATE 4-4  Vertical bricks - vertical cracking in the ends
PLATE 4-5  Couplet test

PLATE 4-6  Failure of 15 and 30° couplets
PLATE 4-11 Measurement of Young's modulus

PLATE 4-12 7-day test in the Olsen test machine
CHAPTER 5

TESTS ON UNPRESTRESSED AND PRESTRESSED BRICKWORK SHEAR WALLS

SYMBOLS

The preparation and testing of seven brickwork shear walls is described. Five of these walls were prestressed. The lattice analogy analysis of the behaviour before cracking is compared with the observed experimental behaviour. The ultimate failure loads and the type of ultimate failure are compared with the failure predicted from the properties of the materials.

LIST OF SYMBOLS USED IN CHAPTER 5

\[ P_c \] = force in the wire \( c \)
\[ P_u \] = uniform force in each wire used to prestress the walls
\[ P_1 \] = the additional force in a wire resulting from the application of load to the wall
\[ P_T \] = ultimate load which may be taken by a wire
\[ l \] = length of the wall
\[ l_c \] = length of the compression zone
\[ r \] = integer
\[ n \] = number of wires in a wall
\[ N \] = total vertical force in the wires
\[ f'_c \] = compressive strength of the brickwork
\[ V \] = horizontal load at the top of a wall
\[ h \] = the height of \( V \) above a mortar layer
\[ M \] = moment
\[ c \] = number of wires in the compression zone
\[ s \] = shear stress
\[ p \] = normal stress
\[ E \] = modulus of section
\[ L \] = length of a wall
\[ H \] = the vertical height of a wall
\[ W \] = the distance between the top row of bolts on the bottom loading beam and the bottom row of bolts on the top loading beam
$\delta$ = horizontal deflection of the centre of the top of a wall
$G$ = shear modulus of the material of the wall
$A$ = horizontal cross-sectional area of the wall
$E$ = Young's modulus of the material of the wall
$\mu$ = Poisson's ratio
5.1 CONSTRUCTION, PRESTRESSING AND PREPARATION FOR TESTING OF BRICK
FACTORY FALLS WITHOUT CENTERS

PREPARATION OF MATERIALS

5.1-1 The bricks used are described in section 4.2. As a result of the
tests of the initial rate of absorption described in that section, the
bricks were laid in a surface wet condition. Periodic resetting of the
bricks was carried out to maintain the surface wet condition.
5.1-2 The mortar used is described in section 4.1. Cement was taken
from airtight bins. Lime was slaked at least twenty-four hours before
use. Sand was taken from bins of graded sizes. Batches of mortar of
100 lb weight were prepared. This resulted in a mortar age of not greater
than three hours when the mortar was used.

PREPARATION OF THE PRESTRESSING WIRES

5.1-3 Steel wire was cut from coils of approximately seven feet diameter.
The coils had been exposed to the weather. Before the wires were included
in the construction of the wall the thin layer of surface rust was removed
by rubbing the wires with emery cloth. As curvature in the wire made the
construction of a plumb wall difficult, the greater part of this curvature
was bent out.
5.1-4 Strain gauges were attached about eight inches above the bottom
anchorage of the wires. Before attaching a gauge the surface of the steel
was thoroughly cleaned with emery paper. The cleaned surface was then
scratched with emery paper to assist bonding. The gauges were Philips
302804 strain gauges. They were glued and waterproofed using a Philips
30264 kit set. 12'9" leads of 10/0.040 wire were soldered to the gauge
connections. A layer of the waterproofing compound 3/4" thick was set over
the whole of the glued gauge (see Plate 5-1). A length of 1/4" diameter
plastic tubing about two feet long was used to protect the wire nearest the
gauge. The end of this tubing and the waterproofed strain gauge were then
covered with a six-inch length of 1/2" diameter tubing closed with insulating
tape.
5.1-5 The wire was then measurad with a heavy grease. Approximately three
thicknesses of greaseproof paper was wound around the wire. The grease-
proof paper was tied in position with linen thread. It provided a ducting
for post tensioning. A standard P.S.C. monowire 0.276 in. diameter wire
anchorage was fixed at the bottom end of the wire. Above it was a plate
5/16" x 2½" x 3" to spread the load. A rubber insertion pad 1/16" thick
was used to even out irregularities in the surface of the brick below the
anchorage plate. Plate 5-2 shows a wire ready to be included in a wall.

CONSTRUCTION OF THE WALLS

5.1-6 All the walls except W4 were built in position on timber blocks.
Bricks were laid by an experienced bricklayer who concentrated on obtaining
good brick-mortar bond. All the walls except W2 were stretch-bonded.
W2 was stack-bonded. The bottom row of bricks were set in position with
the prestressing wires passing through the central hole in the bricks.
The vertical mortar joints were filled. In order to fit the bottom loading
beam to the wall, two rows of 1" diameter high tensile steel bolts at 4½"
centres were used. To accommodate these bolts the two bottom mortar layers
were thickened. A timber template and greased timber dowels were used to
keep holes in the correct positions in these layers (see Plate 5-3). Mortar
was placed around the dowels and the bricks were tapped down into position.

5.1-7 When prestressing wires were built into the walls, care was taken to
allow movement of the prestressing wires in their ducting and to protect
the strain gauges. It was possible to pass the prestressing wires through
the 5/8" diameter hole in the centre of the bricks in every alternate layer.
These holes allowed free movement of the wires. When wires passed through
mortar they were moved slightly while the mortar was stiffening to provide
freedom of movement through the mortar. Although most of the curvature in
the wires had been removed, in some cases care was needed to keep the wires
in the middle of the wall. The bricks were not all the same length and
it was necessary to size, and in some cases to cut, bricks to fit them
between the wires.

5.1-8 Each wall took about 12 hours to build. Timber templates were also
used at the top of the wall to obtain holes for the two rows of 3/8" diameter
high tensile steel bolts at 6" centres which were required to assemble the
top loading beam. The timber dowels were removed when the walls were
finished. The prestressing wires were then moved up and down to ensure free movement in their ducts.

5.1-9 The walls were propped up and allowed to stand for 21 days before prestressing. They were tested 7 days later.

PRESTRESSING

5.1-10 The wires were stressed using standard P.S.C. monowire equipment. The hydraulic jack was hand operated. Its calibration is described in section 4-5. Strains were recorded on a Philips 4571 strain bridge. The dummy gauge used was cemented to a piece of wire, protected, and set in the centre of three bricks. Before use the variable resistance drum on the strain bridge was cleaned with acetone. Twenty minutes warming up was allowed before readings were taken.

5.1-11 The prestressing anchorages were standard P.S.C. monowire equipment. The bottom anchorage acted on a steel plate 5/16" x 2 1/2" x 3". Under the steel plate a piece of rubber insertion 1/16" thick was used to transfer the load to the uneven surface. A top anchorage may be seen in Plate 5-6. The anchorage was fitted on top of a 3/8" diameter bolt with a 5/16" hole drilled through the centre. The nut at the bottom of the bolt transferred the load to a steel plate 3/8" x 3" x 3". The bolt could be wound approximately one inch through a hole in this plate and the hole in the centre of the top brick. A rubber insertion 1/8" thick distributed the load from the plate to the wall.

5.1-12 Wires were stressed from the centre of the wall outwards. Strain gauge readings were taken for increments in loads of 1000 lb. The wires in wall W3 were loaded to 114,000 lb before the anchorage was driven home. The wires in the other walls were loaded to 10,000 lb and the anchorage driven home. The load in a wire after the jack was released was obtained from the strain gauge on the wire. During stressing the plotted values of load against strain up to a load of 9000 lb produced a straight line. In wall W3 the loads in the wires were obtained from the load-strain curve for loading. This resulted in an error due to load intensity set (see section 4-5) of approximately 650 lb. In the case of the other walls the effect of load intensity set on the strain gauge readings was eliminated by calculating the loads in the wires from a straight line, extending back
from the point of maximum load of 10,000 lb. This line had the same
slope as the line obtained when the wires were stressed.
5.1-13 The calculated load in a wire was adjusted by screwing or unscrewing
the bolt above the anchorage plate. The nut was turned around the bolt
to avoid torsion in the wire. The wire was always unloaded to the
desired value to standardize effects due to friction in the ducting. Loads
in the wires were adjusted to a standard load of 9250 lb.
5.1-14 The decreases in the loads in the wires due to relaxation are
discussed in section 4-6. These readings showed temperature effects on
the gauge readings. Strain bridges were left on continuously for several
days at first. The bridges became unstable if used for such a long time,
and therefore were left on only for sixteen to eighteen hours a day. Even
under these conditions the bridges were beginning to become unstable after
seven days.

PREPARATION OF WALLS FOR TESTING

5.1-15 The test frame was assembled around the walls, except in the case
of wall W4. Wall W4 was lifted into place in the test frame. Approximately
four days were required for two men to put the jack loading frame into
place, fit the top and bottom beams, pour the cement founda joint and fit
the dial gauges.
5.1-16 Normally the jack loading frame was set in place first. The end
supports of the bottom loading beam were then bolted in place. Plate 5-4
shows the test frame assembly partly ready for wall W4 to be lifted into
place. The square recesses for shear correction (see 3.4-3) were smeared
with a thin layer of grease and covered with greaseproof paper. These
may also be seen in Plate 5-4. Borax oil was used on the surface of the
top loading beam. The steel beams were so treated in order to prevent
adhesion of the concrete layer between the steel and the wall. It will
be remembered that load transfer from the shear connectors was to be by
bearing only.
5.1-17 The wall was at this stage supported on the bottom loading beam by
the 1" diameter high tensile steel bolts used to provide a force normal
to the face of the wall. The length of these bolts passing through the
wall was smeared with grease and wrapped in greaseproof paper to prevent
bonding to the concrete in the intermediate layers. The top loading beam was similarly assembled. The $\frac{3}{8}$" diameter bolts were greased.

5.1-18 Soft timber was used to form the area between the steel loading beams and the wall. Before pouring the cement fondu concrete layer between the steel beams and the walls, the surface of the brick was well moistened to prevent the bricks drying the concrete mix and decreasing its workability too much. The concrete mix had a water cement ratio of 0.45, and the cement fondu, sand, and $\frac{3}{16}$" to $\frac{5}{8}$" aggregate in the ratio 1:2:4. It developed 4,000 psi crushing strength at 20 hours, and 44,000 psi at 24 hours. A $\frac{1}{2}$" spatula vibrator was used to vibrate the concrete of the top loading beam. The bottom loading beam was more difficult to fill. A Kanghammer was used on the outside of the beam, and steel trowels were used to push the concrete down the narrow gap between the horizontal ribs of the shear connectors and the wall. Examination of the recesses when the wall was broken up after testing revealed that except for a few isolated cases the recesses were properly filled, and that the concrete had not crushed.

5.1-19 When the surface of the wall had dried, timber pieces were glued to the brickwork between the loading beams. These pieces provided two flat surfaces, one horizontal and one vertical, against which dial gauges could act. All the walls except W1 were whitewashed to make cracking more easily visible. When the cement fondu concrete was 20 hours old the high tensile steel bolts were tightened. Steel angles, 2" x 2" x 10" were then welded to the steel loading beams. These also provided surfaces against which dial gauges could act. The bearing surfaces of these steel pieces were filed and sanded flat. Plate 5-5 shows a wall assembled in the test frame. The wooden blocks used to support the wall during construction were removed from under the bottom loading beam. All the bolts of the loading frame and the test frame were tightened. Finally the dial gauges were set.

MEASUREMENT OF THE LOAD IN THE PRESTRESSING WIRE

5.1-20 To determine the load in the prestressing wires the anchorages were lifted with a hydraulic jack. Plate 5-7 shows the apparatus used to lift the anchorages. A length of pipe was placed outside the anchorage and the bolt. The prestressing wire passed through drilled plates and the centre of a hydraulic jack which was connected to the Riehle test machine.
used to test the walls. When the jack piston moved upwards it transmitted an upward load through a steel plate to an upper anchorage above the bolt anchorage. The downwards load from the jack acted on the bottom anchorage plate through the pipe without touching the bolt anchorage. As long as the upwards load above the bolt anchorage was less than the load below the bolt anchorage the downwards load on the wall at the bottom anchorage plate was unchanged. Load was transferred from the bolt anchorage to the pipe. There was no change in the force in the wire below the bolt anchorage or in the load on the wall. Only the load in the wire above the bolt anchorage was changed.

5.1-21 When the force in the wire above the bolt anchorage became greater than the force below the bolt anchorage the forces in the wall began to change. There was an increase in the downwards force at the bottom anchorage. The bolt anchorage was lifted and the strain in the wire below the bolt anchorage began to increase. This change in strain was easily detectable. If the full load in the wire below the bolt anchorage was taken by the wire above the bolt anchorage the slope of the load-strain curve was of the same order as that obtained when the wires were stressed. If the top anchorage was bearing unequally, resulting in a sharing of the downwards load between the anchorage and the wire above the anchorage, the slope of the load-strain curve was less. Extrapolating the line when the correct rate of straining occurred to the value of zero extra strain gave the residual load in the wire.

5.1-22 The measurements of the residual loads in the wires are given in section 4.6. The load measured by lifting the anchorage was adopted as the value of the load in the wire. These loads in the wires were also given by the strain gauges on the wires. The two sets of values obtained are compared in section 4.6. Preference was given to the values obtained by lifting the anchorages, firstly because of possible variations in the strain gauge readings, and secondly because the loads in the wires were then given in terms of the load used to break the walls.

5.1-23 The forces in the wires were reset to a uniform load. Small adjustments of less than 100 lb were not carried out. The loads in the wires were then considered to be constant and of the same magnitude. The strain in a wire at the start of the test loading was taken as the
strain for the uniform load set in the wires.

APPLICATION OF LOADING TO THE WALLS

5.1-24. The top loading beam was loaded at the left-hand end by a jack. Plate 5-8 shows the jack acting on the steel connecting member to which the steel beams on both sides of the wall were bolted. The loading end of the jack is curved and bears on the curved edge of a drilled hole in the plate. A thirty-ton jack was used to test all the walls except 86. A one-hundred ton jack was used to test wall 86. The jacks were pressurized by a Richele test machine.

5.1-25. The load was applied in increments. For each increment it required about a quarter of an hour to read the thirty-nine dial gauges and the strain gauges. As a result the load was applied to the wall gradually. The time of loading varied with the number of increments of load and the number of readings taken for each increment. Tests took five to nine hours. As the ultimate load of the walls was approached the prestressing wires began to show large strains for the applied increments of load. The load was kept constant until the wires had ceased to strain. This procedure probably gave lower failure loads than would occur under a faster rate of loading or under dynamic loading.

5.1-26. There was little or no noise associated with the appearance of cracking, provided that overall failure did not occur.

DIAL GAUGE READINGS

5.1-27. In the tests on square walls thirty-nine dial gauges were read. Plate 5-9 shows the position of these gauges. They divided the nominal dimensions of the walls into nine equal segments. There were another three gauge positions on the bottom loading beam. Horizontal and vertical displacements were measured at all of these positions. The thirty-ninth gauge measured the horizontal displacement below the jack.

5.1-28. In the tests on the rectangular walls the horizontal and vertical displacements were measured at nine evenly spaced positions along the centre lines of the top and the bottom loading beams. A thirty-seventh dial gauge measured the horizontal displacement below the top loading beam. 5.1-29 Dial gauges were fixed with the intention of obtaining deflections over the wall to be compared with the theoretical deflected shape, of
obtaining the vertical deflections under load of the bottom loading beam, and of measuring the horizontal shortenings in the beams due to the horizontal load along the beam. Readings were insufficiently accurate for any of these purposes. This was mainly caused by rotation of the wall. This rotation was caused by deflections due to bending in the length of the bottom loading beam which was not connected to the wall. Any movement of the bolts with which the bottom loading beam was fastened also would have resulted in rotation. The deflected shape obtained with the dial gauges was rotated back so that the two ends of the bottom of the wall were horizontal. The corrected deflections were approximately one-third the measured deflections. Errors in the dial gauge readings were therefore multiplied by three. The large horizontal displacement of a vertical dial gauge caused any unevenness in the surface on which the gauge acted to be recorded. Any deviation of the axis of the gauge from the vertical or horizontal was multiplied threefold. Torsion of the wall about the vertical axis due to eccentricity of the load was possible. Bending within the beams may have changed the position of the angle pieces welded to the beams.

5.1-30 In addition to these errors the gauges did not immediately show steady increments because of internal frictions. Horizontal gauges were usually more prone to this than vertical gauges. The gauge plungers were wiped clean with a clean dry rag before the gauges were mounted. After the gauges had been fitted on the design frame (see Plate 5-9) they and the bearing plates were lightly tapped. This was to remove any sticking in the gauges due to deflection of the frame and due to fixing of the gauges. Nevertheless there was still enough friction in the gauges to make measurement of small deflections inaccurate.

5.1-31 For each wall many sets of readings were taken. It was hoped that by considering the rate of change of increments that irregularities in the gauge readings could be removed. However, because of rotational effects and friction in the gauges when deflections were small, and because it would have been necessary to measure small differences in deflections accurately, none of the principal reasons for taking dial gauge readings were realized.

5.1-32 It was, however, possible to obtain a good load-deflection curve.
Most points on this curve were obtained from the average of the top beam horizontal deflections. For small loads the average of those gauges incrementing steadily was taken. For loads nearing ultimate failure the gauge below the jack was used. The deflection recorded by this gauge was multiplied by the ratio of the average deflections recorded by the gauges on the top beam to the deflection recorded on the lower gauge for the same load. The load-deflection curves do not record the deflection up to maximum load. This is because there were no replacements for any gauges which became damaged, and gauges were removed early to prevent possible damage. Subsequent experience showed that failure was rarely sudden and no damage to the gauges would have occurred.
5.2 Calculation of the Stresses at Which First Cracking Occurs and the Ultimate Failure Loads

First Cracking

5.2-1 First cracking was caused by vertical tensile stresses, and normally occurred along the mortar joint immediately above the bottom loading beam. In section 3.4 the analyses of square and rectangular walls for a load of 100 kips are presented. In section 3.5 the analyses of the effects of prestressing the walls with wires at the ends carrying loads of 10,000 lb are presented. In cases of uniform prestressing a 10,000 lb load in each wire was assumed to produce uniform vertical compressive stress of 282 psi. To calculate the first cracking stresses the results of these analyses were multiplied by appropriate factors and added. The vertical stress on the bottom mortar joint due to self weight of the wall and the top loading beam, assuming the brickwork had a density of 120 lb/cu.ft, was 6.4 psi for a 62" wall, and 5.7 psi for an 82" wall. All stresses at first cracking were calculated from the lattice analogy analyses.

Ultimate Failure Loads of a Uniformly Prestressed Wall Caused by Failure of the Steel

5.2-2 To calculate the ultimate failure loads it was necessary to make several assumptions. The first assumption is the distribution of compressive stress in the end of the beam. The steel wires extended very much further than the brickwork compressed. Compressive stress at failure was therefore confined to a small region at the end of the beam. It was assumed that the distribution of compressive stress in the brickwork was triangular and that the maximum compressive stress, $f_c'$, occurred in the right-hand fibre. The second assumption was that in the wires the variation in the loads additional to the initial uniform prestressing loads was linear. This is a conservative assumption since large strains for small increases in stress in wires highly stressed distributed loads to wires under lower stresses. In the tests on the square walls the downward deflection of the beam caused by the end moments on the beam increased the loads in the wires. In tests on the rectangular walls the influence of the end moments in the centre of the beam was small. In both cases the
deflections of the beam had only a small influence.

5.2-3 Figure 5-1 shows the assumed forces along the bottom of a wall which has a wire through the centre of each brick in the top and bottom courses. The load in the left-hand wire is the ultimate load of the steel wires. The force in the wire \( C \) is

\[
P_C = P_u + P_l \frac{M_c}{bd}
\]

where \( P_u \) is the uniform load in the wires from prestressing, and \( P_l \) is the additional load to the ultimate load of the wire \( P_T \).

\[
P_T = P_u + \frac{1 - 1_c - \frac{1}{2m} \frac{n - 1}{1 - 1_c - \frac{1}{2n}}}{1 - 1_c - \frac{2n}{1 - 1_c - 1}}
\]

\[
= P_T - P_l \frac{2m}{2n(1-1_c)-1}
\]

Therefore the total force in the area \( MN \), \( N \), is given by

\[
N = n P_T - P_l \left[ \frac{n(n-1)}{2n(1-1_c)-1} \right]
\]

\[
= n P_T - \frac{P_l n(n-1) l}{2n(1-1_c)-1}
\]

For a wall four inches thick:

\[
1_c = \frac{B}{2f_c}
\]

This leads to a quadratic equation for \( N \). Since \( 1_c \) is small compared with \( 1_p \) and \( P_l \) small with respect to \( P_T \), \( N \) is most quickly obtained by substituting a value for \( 1_c \) in equation 5-2 and comparing values with the value given in equation 5-3.

\[
1_c = \frac{n P_T + \frac{2}{3} P_l}{2f_c}
\]

is a suitable first trial value.
5.2-4 The applied load, \( V \), may be obtained by considering moments about any point on the crack at the bottom of the wall. If \( h \) is the height of the line of action of the applied load above the mortar layer

\[
V = \frac{N}{R}.
\] (5-5)

In figure 5-1 the moment of the load in wire \( i \) about the point \( B \),

\[
N = (P_T - \frac{P}{2n(\frac{1}{L} - \frac{1}{L} - 1)})(1 - \frac{(2n-1)}{2n}).
\]

The total moment about \( B \),

\[
M = \sum_{r=0}^{n-1} (P_T - \frac{P}{2n(\frac{1}{L} - \frac{1}{L} - 1)})(2n - 1 - 2r) - \frac{ML}{3}.
\]

\[
= \sum_{r=0}^{n-1} P_T \frac{(2n-1-2r)}{2n(\frac{1}{L} - \frac{1}{L} - 1)} - \frac{P}{2n(\frac{1}{L} - \frac{1}{L} - 1)} \cdot 2r
\]

\[
+ \sum_{r=0}^{n-1} \frac{P}{2n(\frac{1}{L} - \frac{1}{L} - 1)} \cdot 4r^2 - \frac{ML}{3}.
\]

\[
= \frac{1}{2} (M_T - \frac{P}{\frac{1}{L} - \frac{1}{L} - 1}) - \frac{ML}{3}.
\] (5-6)

5.2-5 The assumptions made in this method are conservative. The ultimate load was reached with probably at least two wires very near failure. In addition the assumption concerning the distribution of compressive stress affects the result. Localised yielding at points of maximum stress resulting in a greater seen stress may have increased the lever arm. The maximum compressive stress also determined the magnitude of the lever arm.

5.2-6 A second method may be used to compute the maximum possible load. The brickwork compressive stress is assumed to be uniformly the maximum compressive stress, and all the wires outside the compression zone are assumed to be at their failure load. The length under compression is

\[
l_c = \frac{N}{P_T - P_i}.
\] (5-7)

where \( N = n P_T - c P_i \). (5-8)
c is the number of wires within the compression zone. The moment is given by

\[
\frac{M}{1} = \frac{n}{2} P_T \left( \frac{2n-1}{2n} \right) - \frac{c}{2} P_1 \left( \frac{2n-1}{2n} \right) - \frac{Nl}{21}
\]

\[
= \frac{n}{2} P_T - \frac{c}{2} P_1 - \frac{Nl}{21}
\]  \hspace{1cm} (5-9)

5.2-7 In calculating the theoretical failure loads of the walls, the measured dimensions of the walls were used. Wires were assumed to be at the centre of evenly spaced bricks in the top and bottom courses.

**Ultimate Failure of Walls Prestressed at the Ends Only**

5.2-8 Ultimate failure was assumed to occur with the wire or wires at the cracked end of the wall at ultimate load. The other wire or wires carried the original prestressing load. The distribution of compressive stress was again triangular.

5.2-9 A maximum value can be obtained by considering the compressive stress to be uniformly the maximum compressive stress, and all the wires outside this area of compressive stress to be at ultimate load. There is little difference in the loads computed by the two methods.

**Failure of the Brickwork in the Compression Zone**

5.2-10 The compression-shear relationship obtained in the couplet tests was

\[
s = \frac{3}{8} p + 260, \hspace{1cm} (4-4)
\]

where the average shear stress and compressive stress are used. It was assumed that provided

\[
\frac{3}{8} p + 260 > s
\]

then no shear failure of the bottom right-hand corner would occur. In the calculations

\[
s = \frac{V}{41_0}
\]

\[
p = \frac{N}{41_0}
\]  \hspace{1cm} (5-10)

Equation \((4-4)\) was obtained in the couplet tests. The equation applies
to failure of the mortar. The equation was also found to be applicable when the brickwork failed in compression, up to an angle of 50° (see section 4.4). Accordingly the equation (4.4) will also predict cracking of the brickwork under compression.
5.3 TEST ON WALL W1

PARTICULARS OF THE WALL
5.3-1 Wall W1 was an un prestressed stretcher-bonded brick masonry wall. The dimensions L x H were 62" x 62". The surface of the wall was not whitewashed.

BEHAVIOUR UNDER LOADING
5.3-2 The wall was loaded to failure in small increments of approximately 250 lb (corresponding to an increase of 40 psi in the 30 ton jack used which had a cross-sectional area of 6.35 in²). At a load of approximately 1.4 kips opening up of shrinkage cracks was observed. These cracks did not grow. At a load of 2.5 kips the deflections of the wall began to increase at a faster rate and the dial gauges were removed. Failure did not occur until the load was 5.04 kips. The cause of failure was concentration of tensile stress at an incompletely filled mortar joint above the sixth course from the bottom (see plate 5-10). The lattice analogy value of the stress at this joint is 53 psi. The bottom of the wall sustained a tensile stress of 120 psi without failing. The dial gauges were not accurate enough to give a load-deflection curve.
PARTICULARS OF THE WALL
5.4-1 Wall W2 was an unproofoffied, stack-bonded brick masonry wall. The bottom of the wall was formed by two stretcher-bonded courses to allow the fitting of the high tensile steel bolts. The bottom of wall W2 may be seen in the background of plate 5-10. There were also two stretcher-bonded courses at the top of the wall. The dimensions L x H were 61\tfrac{1}{2}” x 62\tfrac{3}{4}”.

BEHAVIOUR UNDER LOADING
5.4-2 The top of the wall was loaded in increments of approximately 250 lb. The wall was loaded directly to failure over approximately seven hours. There was an initial opening up of shrinkage cracks at a load of approximately 2.2 kips. These cracks did not grow. From a load of approximately 5 kips onwards, deflections increased under constant load and then became steady. At a load of 8.5 kips, approximately two-thirds the ultimate load, the first cracks in the horizontal mortar joints due to vertical tensile stresses were observed. Figure 5-2 shows the cracking in the wall. The tensile stress at first cracking was 147 psi. Tensile forces then appeared to be redistributed by the vertical joints to the bricks. As the failure load was neared deflections began to increase more rapidly. Failure at the ultimate load of 12.4 kips was preceded by a short period of yielding.

5.4-3 Plate 5-11 and figure 5-2 show the failure of the wall which appears to have been a tensile failure in the vertical mortar joint at 0.056, leading to tensile and shear failure through the two bricks between 0.41” and 0.21”. Growth of the cracking due to the continued application of the load and release of strain energy resulted in the large diagonal crack. Had the large diagonal crack occurred first the cracking in the corner would probably not have occurred.

THE LOAD-DEFLECTION CURVE
5.4-4 The load deflection curve is shown in figure 5-3. Despite the scatter in the points an overall curvature is apparent. The slope of the
curve when loading was commenced indicates a deflection of the top of
the wall of \(0.8 \times 10^{-3}\) in/kip. For loads greater than 6 kips the wall
deflected \(2.9 \times 10^{-3}\) in/kip. The overall deflection at a load of
5.3 kips was \(1.34 \times 10^{-3}\) in/kip. In referring to the deflections of
the walls it will be convenient to use the term "flexure" to imply
the slope of the load-strain curve at a given load, in ina/kip.
5.5 **TEST ON WALL W3**

**PARTICULARS OF THE WALL**

5.5-1 Wall W3 was built of stretcher-bonded brick masonry. The dimensions L x H of the wall were 62½" x 63". Seven 0.276 in. diameter high-tensile steel wires were used to provide a uniform prestress in the wall. The wires were numbered 1 to 7 from the tension corner. The order of prestressing the wires was 6, 5, 3, 2, 6, 7, 1. There was no apparent damage from the prestressing. A frictional force of approximately 200 lb was encountered in the ducting when the wires were prestressed.

5.5-2 The measuring bridge used when the wires were prestressed was bridge No. 1. Between prestressing and testing the dummy gauge required repair. The strain measuring bridge became unfit for service and was replaced by bridge No. 3. It was then impossible to predict the loads in the wires from the strain gauge reading. All the wires except wire 7 were lifted to determine the residual load. From a consideration of the strain gauge readings on wire 7 and the other wires, the residual load in wire 7 was estimated to be 7400 lb. The average load in the other six wires was 8200 lb, a difference of 800 lb. The loads in the wires were set to a uniform value of 8200 lb. Friction in the ducting may have resulted in a difference of approximately 100 lb in the mean load.

**BEHAVIOUR UNDER TESTING**

5.5-3 The wall was loaded directly to failure in increments of approximately 1.3 kips. First cracking appeared silently in the mortar joint above the bottom loading beam at a load of between 14.4 and 15.7 kips. This corresponds to a tensile strength of between 123 and 156 psi. Cracking extended eleven inches along the wall. There was no discernible change in the rate of change of deflections. Cracking could not be observed over part of the base because it disappeared into the cement fondu layer. Figure 5-5 shows the observed growth of the crack along the bottom mortar joint. The cracking grew unevenly.

5.5-4 The load increased to a maximum load of 41.8 kips. Wires 1 and 2 had been yielding and wire 3 had begun to yield. Wire 2 failed first. The load dropped to 33.5 kips. Subsequent reloading to 42.6 kips was
possible, before wire 1 failed and the brickwork failed in compression. The wires failed low down in the walls at the height at which the horizontal cracking occurred. This may be seen for wire 1 in Plate 5-13 and in Plate 5-16, the test on #7.

LOAD-DEFLECTION CURVE

5.5-5 The load-deflection curve for #3 is shown in figure 5-4. The flexure increased with loading as in test #2, producing a smooth curve. The wall initially deflected 0.57 x 10^-3 in/kip. Over the range 6.5 to 31.1 kip the flexure was approximately 1.33 x 10^-3 in/kip. For a load of 15.7 kip the overall flexure was 1.32 in/kip. There was no abrupt change in the slope of the curve corresponding to initial cracking.

VARIATION IN THE LOADS IN THE WIRES

5.5-6 The variation of the loads in the wires is shown in figure 5-5. There was no significant change in the load before first cracking. Once cracking passed a wire the load in the wire increased fairly steadily until the wall was loaded to within a tenth of its ultimate load. At this juncture the rate of increase of the loads in the wires increased.

Wire 1 began yielding at 93.6% of the ultimate load, wire 2 at 75% of the ultimate load. However, the load in the wires was only 55% of the failure load of the wires when the wall was carrying 77% of its ultimate load.

5.5-7 The load in wire 6 changed little, and in wire 7 the load was always less than the original prestressing load. From the strain gauge readings, the amount of cracking at ultimate load was 0.7%. Cracking was observed to 0.22% from the end. This amount of cracking was visible at 72% of ultimate load. The positions of cracking indicated by the recorded strains were those observed.
5.6 TEST ON WALL 11.

PARTICULARS OF THE WALL

5.6-1 Wall 11 was built of stretcher-bonded brick masonry. The dimensions L x H were 64" x 62". The seven wires used to provide a uniform prestress were numbered 1 to 7 from the compression end. The order of prestressing was 4, 5, 3, 6, 2, 7, 1. Bridge No. 3 was used to measure strains.

5.6-2 The wall was built and prestressed off the test bed. Two days before it was tested, the wall was lifted into the test frame. Plate 5-12 shows the wall being moved. There were no changes in the strains in the prestressing wires.

5.6-3 Before testing, the loads in the wires were adjusted to 9000 lb. The friction in the ducting when the wires were first loaded was of the order of 90 lb or less.

BEHAVIOR UNDER LOADING

5.6-4 The wall was loaded at the top to a value of 22.2 kips in increments of approximately 1.3 kips. The load was then released to zero. In this test it was hoped to relate the appearance of the horizontal crack in the bottom mortar layer to the applied load. Unfortunately the first crack did not crack through the wall, but occurred in one side only. At 6.5 kips a hair crack was visible in the whitewashed face. At 7.8 kips the crack was confirmed. At 11.7 kips the crack had extended eighteen inches along the front of the wall, but was only just appearing at the back of the wall. At 14.35 kips the crack penetrated 1/4" into the wall on one side and 1/2" on the other. It was only at 19.55 kips that the front and rear cracks overlapped. Assuming zero tensile stress in the mortar, first cracking is predicted for a load of 10.4 kips. The early cracking must be attributed to eccentricity of the prestressing wire. Assuming that first cracking occurred at a tensile stress of 70 psi, the eccentricity for a linear stress gradient across the brick would be 0.63". Upon unloading at 13.0 kips much of the middle of the crack was open, at 6.3 kips the crack had closed to within 1/2" from the front of the wall, and at 2.6 kips the crack was completely closed.

5.6-5 To assess the performance of the prestressing wires and the frame
the loads in the wires were adjusted to different uniform loads. The top of the beam was then loaded. The loads were recorded when the crack was open at the sides only, and when the crack was fully open. Figure 5-7 shows graphically the results of altering the prestressing force in the wires. The theoretical load is nearer the load at which the ends only are open for small loads, but nearer the load for which the whole crack is open for high loads. Repeated opening and closing may have affected the cracking.

5.6-6 After opening and closing the crack for prestressing loads of 2000, 3000, 4000, 5000, 8000 and 9000 lb the wall was loaded to failure in increments of approximately 1.3 kips. The crack extended along the bottom mortar joint. At a load of 40.5 kips, 99% of the ultimate load, wire 1 began to yield. Failure was caused by the failure of wire 1 when the wall was under a load of 43.1 kips. The shock of the fracture shattered the bricks in the compression zone. This cracking spread back along the mortar joints. Ultimate failure is shown in plate 5-14. The crack along the bottom mortar joint is not prominent in this photograph.

LOAD-DEFLECTION CURVE

5.6-7 The load-deflection curve first obtained is shown in figure 5-6. There is again a gradual curving. The initial flexure was \(0.69 \times 10^{-5}\) in/kip. Between 14.4 and 16.3 kips the flexure was \(1.90 \times 10^{-5}\) in/kip. The overall flexures for loads 14.8, 13.1, and 14.4 kips were respectively \(0.90, 0.915, \) and \(0.92 \times 10^{-5}\) in/kip.

VARIATION OF THE LOADS IN THE WIRES

5.6-8 The variation of the loads in the wires with the applied load was very similar to that for wall W3 (see fig. 5-9). There was little change in the loads in the wires before cracking occurs. The increments in loads were similar to those for W3. Only 85.5% of the load in the wire was developed for 97% of the ultimate load of the wall. Prolonged creeping in wire 7 began when the applied load was 40.5 kips. This was a slightly greater load than for wall W3 (39.2 kips). There was a smaller compression zone at failure. The strain gauge readings gave 0.15, and measurements 0.15% for the length of the compression zone.
5.7 TEST ON WALL No.

PARTICULARS OF THE WALL

5.7-1 The 1 x H dimensions of this stretcher-bonded brick masonry wall were 63" x 63". One 0.276" prestressing wire acted at the centres of the end bricks in the top and bottom courses. Wire 1 was at the tension end of the wall, wire 2 at the compression end. Strain measurements were made on bridge No. 3. When the wires were prestressed wire 2 was loaded to 5000 lb first. This resulted in a stress of 52 psi assuming the formula f = \frac{N}{A} to be applicable. Wire 1 was then loaded to 10,000 lb. No damage was visible in the ends of the wall. However, some vertical mortar joints in the top and bottom centre wall had opened. Wire 2 was then loaded up to 10,000 lb, and both the wires were set to a uniform load of 9250 lb.

5.7-2 When the bottom loading beam was concreted to the wall, the lead to gauge 1 was damaged and no further readings could be obtained from the gauge. The residual load in wire 2 was found to be 8750 lb. It was assumed that this load also remained in wire 1.

BEHAVIOUR UNDER LOADING

5.7-3 Load was applied in increments of approximately 550 lb. Failure was sudden and cracking along the bottom mortar joint extended to within four inches of the compression end (see plate 5-15). The load in the jack was 14.4 kips, from which the vertical tensile stress of the mortar was calculated to be 181 psi. The load in the jack dropped when the mortar failed.

5.7-4 The wire at the compression end of the wall was then completely unloaded. There was no failure in the mortar due to shearing force. The load in the jack was then increased. The ultimate failure of the wall was caused by tensile failure of wire 1. The load in the wall was the same as the load at which first cracking occurred, 14.4 kips. The wire failed low down in the wall.

LOAD-DEFLECTION CURVE

5.7-5 The load-deflection curve showed a steady decrease in the stiffness (see fig. 5-10). The initial flexure was \(0.47 \times 10^{-3}\) in/kip, and the flexure at 10.5 kips was \(1.28 \times 10^{-5}\) in/kip. Overall flexures were \(0.77 \times 10^{-3}\) in/kip at 10.5 kips, and \(8.11 \times 10^{-3}\) in/kip at 11.1 kips.
5.8 TEST ON WALL W6

PARTICULARS OF THE WALL
5.8-1 Wall W6 was built of stretcher-bonded masonry. The lift dimensions were 62" x 55". The wall was uniformly prestressed by a wire through the centre of each of the nine bricks in the top and bottom courses. The wires were numbered from the tension end of the wall. Wires 1 to 6 were of steel 1. Wires 7 to 9 were of steel 2. The loads in the wires when the wall was tested were adjusted to 8000 lb.

BEHAVIOUR UNDER LOADING
5.8-2 The wall was loaded directly to failure. The increments of load were approximately 4.2 kips. The first crack appeared at a load of 20.6 kips in the mortar joint immediately above the bottom loading beam. The calculated vertical tensile stress was 172 psi. The crack grew slowly along the bottom of the wall as the applied load was increased. The ultimate failure of the wall occurred in the brickwork itself (see plate 5-1c) after wire 1 had begun to yield. Following cracking in the bottom right-hand corner, cracking occurred in the mortar joints up to the top of the wall. The ultimate failure load was 84.8 kips.

LOAD-DEFLECTION CURVE
5.8-3 The load-deflection curve is shown in figures 5-11 and 5-12. The initial flexure was 0.12 x 10^{-3} in/kip and the flexure at 21.2 kips was 7.5 x 10^{-3} in/kip. The overall deflections at loads of 17.0 and 21.2 kips were 3.12 and 3.65 x 10^{-3} in/kip respectively. There was no abrupt change in stiffness associated with initial cracking.

VARIATION OF THE LOADS IN THE WIRES
5.8-4 Before first cracking there was little change in the loads in the wires. The loads in the wires did not show a substantial increase until cracking had passed their position on the wall. Cracking was indicated to 0.12L from the compression end of the wall at 37.5% of the ultimate load where L is the length of the wall. The observed cracking was 0.14L. The brickwork finally failed at a distance of 0.11L from the end of the wall. Creeping began in the steel at 93% of the ultimate load. At 98.3% of the
ultimate load only 38% of the steel failure load had been developed in wire 1.

5.9 TEST ON WALL W7

PARTICULARS OF THE WALL
5.9-1 Wall W7, built of stretcher-bonded masonry had $L \times H$ dimensions of $80\frac{1}{2}$" x $62\frac{3}{4}$". There were ten wires at each end of the wall. When the wall was prestressed, wire 2 was first loaded to 5000 lb. Assuming $f = \frac{1}{2}$, this resulted in a tensile stress of 30 psi. Wire 3 was then loaded to 10,000 lb, wire 1 to 10,000 lb, wire 4 to 10,000 lb, and finally the load in wire 2 was increased to 10,000 lb. No failure occurred in the ends of the wall, although some vertical joints in the top and bottom opened. A uniform load of 9250 lb was set. Lifting of the wires showed a considerable variation in the values of residual load measured by the jack and the strain gauge. Recalculation of the loads set in the wires showed that they had not been uniform at 9250 lb. The loads in the wires were adjusted to a uniform load of 9000 lb before testing.

BEHAVIOUR UNDER LOADING
5.9-2 The wall was loaded in increments of 2.6 kips to failure. First cracking occurred at a load of 26.9 kips, corresponding to a tensile stress of 183 psi. The crack grew fairly rapidly along the bottom mortar joint (see figure 5-16). The two wires began to yield at a load of 57.9 kips, 94% of the ultimate load. Both wires yielded together, the crack widening from 3/16" at 85% ultimate load to over 1\frac{1}{2}" wide (see plate 5-17). Near failure all the compression was concentrated in the end 9" of the wall.

LOAD-DEFORMATION CURVE
5.9-3 W7 showed the same increase in the rate of deflection as was found in the other tests (see figs 5-14, and 5-15). Cracking caused a distinct change in the slope of the curve. The origin is at $-16.5 \times 10^{-3}$ in. The deflection was $0.33 \times 10^{-3}$ in/kip at the origin, and
0.33 \times 10^{-3} \text{ in/kip} at 20.9 kips when the initial cracking occurred. Overall stiffnesses were 0.39 \times 10^{-3} \text{ in/kip} at 18.3 kips, and 0.53 \times 10^{-3} \text{ in/kip} at 20.9 kips.

**VARIATION OF LOADS IN THE WIRES**

5.9.4. There was virtually no change in the loads in the wires before initial cracking. The increase in the loads in the wires is similar to those in the other tests. Both the strain gauges and visual observation showed the compression to be concentrated over 7% of the length of the wall at failure.
INITIAL CRACKING

5.10-1 Table 5-1 lists the flexure of each wall up to initial cracking. The vertical tensile stresses when initial cracking occurred were obtained from lattice analogy. The tensile stresses have a mean value of 169 psi, neglecting the results of the premature failure of W4, and the uncertain result of W2. This value is approximately twice to three times the value obtained by testing small specimens in tension. The workmanship in placing the bricks was good. Plate 5-15 shows how the mortar rather than the brick-mortar bond has failed. Benjamin and Williams (1, 2) found a similarly large ratio between tensile strengths developed in walls and those obtained by testing couples in tension. They attribute the difference "to the fact that failure is restricted in location and direction to the brick-mortar joints. Also the presence of friction and brick deformation in the complete testing procedure results in eccentricities, and consequently the couples fail at loads lower than those which could be sustained under pure tension". It has also been observed that a superimposed weight increases the bond strength considerably (3). The weight of the bricks laid above the bottom course was therefore a cause of the high tensile strength as was knocking the bricks in laying (4, 1-5).

5.10-2 In the analysis it was assumed that brickwork was a homogeneous elastic body. The experimental data does not support this assumption. The load-strain curve indicates increased deflection for higher stresses. This differs from the linear load-strain curve obtained when the three brick specimens were loaded (see section 4.3). It is probable that increased strain in areas under high stress decreases peak stresses before the material fails.

5.10-3 The interaction of the bottom loading beam, basically two steel sections, and the brickwork wall depends upon the ratio of Young's moduli of elasticity. When the value of the modulus of elasticity of the wall is lower than that used in the analysis the wall opposes the deflection of the beam less. The constant deflection of the bottom loading beam due to bending moment has a relatively smaller effect on the wall. In the analysis of the square wall tests, the end moments on the beam cause a
downwards deflection at the tension end of the wall (see fig. 3-46) transferring load away from the corner. Reducing the stiffness of the wall reduces the transfer of load. The fact that in the test on Wh (see fig. 5-7) the theoretical load for first cracking is nearer to the observed "partly-open crack" condition for small loads, and near the observed "fully-open crack" condition for large loads is congruent with this argument. In the analysis of the rectangular walls a lower stiffness in the wall decreases the concentration of stress at the ends of the wall.

5.10-4. The stiffness of the wall decreased with the applied load. Lattice analogy gives the flexure of the square walls as $1.23 \times 10^{-5}$ in/kip, and that of the rectangular walls as $0.569 \times 10^{-5}$ in/kip. Using the formula for shear deflection alone of a cantilever,

$$\delta = \frac{1.2 \pi H}{E A}, \quad (5-11)$$

where $E$ is the modulus of rigidity and $A$ is the cross-sectional area of the wall. The flexure of a square wall is $0.420 \times 10^{-5}$ in/kip and that of a rectangular wall $0.200 \times 10^{-5}$ in/kip. Adding to this the deflection of a cantilever due to bending the deflection

$$\delta = \frac{1.2 \pi H}{E A} + \frac{E H^3}{30 W}, \quad (5-12)$$

This gives the flexure of a square wall as $1.057 \times 10^{-5}$ in/kip and that of a rectangular wall as $0.477 \times 10^{-5}$ in/kip. Comparison of the three computed values shows that the overall deflections are most nearly those predicted by the formula (5-12) for shear and bending of a cantilever.

5.10-5 The severity of initial cracking is determined by the distribution of vertical stresses along the bottom morter joint. Then an area in tension fails there is a redistribution of forces so as to oppose the applied moment. The force in the wire nearest the tension end increases and the lever arm increases. As the applied load is increased the loads in the steel wires are increased, by this applied load and by load transfer from the tension zone of the brickwork. Considering the vertical stresses at first cracking sketched in Figure 5-17 and the number of wires in the walls, the mode of failure may be predicted. We had a small tension zone and several wires. The initial cracking should have produced a small crack which grew only slightly. We (and Wh) should have had a larger initial crack. The crack
should have grown slowly. After W7 cracked the crack should have grown slowly at first and then rapidly to the end. W5 should have cracked over most of its length, and since there was only one wire to take the tension little extra load should have been sustained before failure. These predictions were correct.

**ULTIMATE FAILURE**

5.10-6 In all tests except W6 ultimate failure was due to failure of the prestressing wires. The brickwork in wall W6 failed under compression just before the wires failed. The computation of the ultimate failure loads is described in section 5.2. They are listed in table 5-2 with the experimental failure loads. In tests W3, W4, and W6 the experimental load was between the two theoretical values, indicating that the loads in the wires were greater than the linear variation assumed in the first computation. In tests W5 and W7 the loads were greater than the two theoretical values. This is attributed to the fact that leading to failure was faster, and the tensile strength of the wires was therefore greater.

5.10-7 Examination of the loads in the wires reveals that at approximately 97% of the ultimate load of the wall the maximum load in the end wire was approximately 89% of the failure load of the wire. This suggests that either the gauges were not working correctly or there was a greater lever arm than assumed. All tests showed the same pattern, so the former alternative is eliminated. A greater lever arm was possible with a higher compressive stress and with distribution of load to the ends of the beam due to lack of rigidity in the centre of the beam.

5.10-8 When the wires began to show substantial yielding the jack began to develop an upward component of force (see Plate 5-6). For the 1 1/4" extension of the wires in test W5 the component may have been up to a tenth the load in the jack. This upward load broke the wire, but the ultimate load value was essentially unchanged. Wires failed near the bottom of the wall.

5.10-9 Whether or not a wall fails in shear in the compression zone at the bottom depends upon the loads in the prestressing wires and the geometry of the wall. Table 5-3 lists the normal and shear forces in
this zone and the calculated relationship for failure. Failure is predicted if

\[ s - \frac{3}{4} p \geq 260. \]  \hspace{1cm} (4.4)

The shear failure is predicted of walls W6 and W7, and of wall W5 when the prestressing wire at the compression end is unloaded. Failure of wall W6 was, however, due to compression failure of the brickwork. The relationship (4.4) was obtained mainly for bond failure, but was applicable to brickwork failure up to 50° also (see section 4.4). The angle of the load in the case of W6 is \( \tan^{-1} \frac{H}{V} = 53° \), so compression failure may be regarded as a particular case. The failure cracks are parallel to the resultant load (see Plate 5-16). Wall W7 did not fail in compression and shear. By dividing the theoretical normal and tangential forces by the area of uncracked brickwork observed in the test the values \( s - \frac{3}{4} p \) are less than 260 psi. This is, however, an arithmetical expedient, and the absence of shear failure may be due to the variability of the brickwork. The fact that W5 did not fail when the wire at the compression end was unloaded may be due to the variability of the brickwork, or may indicate that equation (4.4) is conservative.

5.10-10 The calculations for failure of the compression zone are interesting in that they show that failure of a brickwork shear wall is a function of the prestressing loads and of the geometry of the wall. With a longer wall the value of V increases, and as V increases so does the theoretical likelihood of a shear failure.
From the seven tests on the brick walls the following conclusions may be drawn:

1. Posttensioning of vertical wires in the brickwork shear walls will strengthen the walls. As the tensile strength of brickwork is very variable the posttensioning will make the wall behaviour much more reliable.

2. Prestressing of the ends of the walls is most efficient. There is a limit to the amount of prestressing force which may be applied at the ends only, due to tensile stresses in the centre of the top and the bottom of the wall and between the anchorages.

3. It is advantageous to have prestressing wires in the centre of the wall to limit the amount of initial cracking.

4. There is little or no permanent damage to walls after initial cracking and for small additional increments of load provided that there are wires in the centre of the wall to limit the extent of cracking. Two wires spaced at the end of the wall as in 9/7 are safe, whereas one wire only, 9/5, allows excessive cracking.

5. The load-strain curve before initial cracking is not linear, but shows increasing deflections at higher loads. This may be due to variations in the elastic constants or to yielding in the brickwork rendering elastic theory inapplicable.

6. The interaction of the wall and the top and bottom loading beams determines the stresses in the walls. Simple assumptions may be very inaccurate. This subject merits further study.

7. The lattice analogy analyses of the stresses in the walls before cracking indicate tensile stresses of 170 psi. The brickwork used is of a high quality. In ordinary prestressed brickwork an allowance of 50 psi for the tensile strength of the brickwork is reasonable, especially when there are prestressing wires in the centre of the wall to limit cracking.

8. The deflections at first cracking, when the brickwork is under a maximum tensile stress of approximately 170 psi and a maximum compressive stress of approximately 600 psi, are most nearly given by the formula
\[ s = \frac{1.2 \pi h}{6a} + \frac{W}{2Eh} \]  

(5.12)

\( E \) is the modulus of elasticity obtained in the tests on three brick specimens (see section 4.3), \( G \) is the shear modulus and \( \mu = 0.05 \).

9. The ultimate load is usually determined by the rise of the lever arm between the prestressing steel in tension and the brickwork in compression. The load is a function of the uniform prestressing loads and of the increases in the loads in the wires due to cracking. A conservative prediction may be made by assuming the increases in the loads in the wires to decrease linearly from the end wire which is at ultimate load, and that the brickwork has a triangular stress distribution up to failure stress at the compression end of the wall.

10. The deflections at ultimate load are determined by the extension of the steel. The wall may be considered to rotate about the compression corner.

11. Ultimate failure may also be caused by shear failure of the mortar joint or by cracking of the brickwork. This occurs in the compression corner. The relationship for mean normal and shear stress obtained in the coupon tests,

\[ s = \frac{2}{3} \sigma_{p} \]  

(4-4)

at failure, has not been fully tested, but appears to be a little conservative in the tests to date.

12. Failure due to the action of compression and shear and due to cracking is a function of the loads in the prestressing wires, the applied load and the geometry of the wall. Long walls appear to be susceptible to this type of failure.
<table>
<thead>
<tr>
<th>Test</th>
<th>First Cracking Load at kips</th>
<th>Tensile Stress at First Cracking psi</th>
<th>Plastic Strain $\times 10^{-3}$ in/kip</th>
<th>Initial</th>
<th>Final</th>
<th>Overall to First Cracking</th>
</tr>
</thead>
<tbody>
<tr>
<td>W1</td>
<td>5.04</td>
<td>53</td>
<td>-</td>
<td>-</td>
<td>-</td>
<td>-</td>
</tr>
<tr>
<td>W2</td>
<td>8.5</td>
<td>147</td>
<td>0.82</td>
<td>2.9</td>
<td>1.34</td>
<td></td>
</tr>
<tr>
<td>W3</td>
<td>15.7</td>
<td>123—156</td>
<td>0.57</td>
<td>1.3</td>
<td>1.13</td>
<td></td>
</tr>
<tr>
<td>W4</td>
<td>13.1</td>
<td>-</td>
<td>0.69</td>
<td>1.9</td>
<td>0.98</td>
<td></td>
</tr>
<tr>
<td>W5</td>
<td>14.4</td>
<td>181</td>
<td>0.47</td>
<td>1.24</td>
<td>0.81</td>
<td></td>
</tr>
<tr>
<td>W6</td>
<td>20.6</td>
<td>172</td>
<td>0.12</td>
<td>0.75</td>
<td>0.35</td>
<td></td>
</tr>
<tr>
<td>W7</td>
<td>20.9</td>
<td>183</td>
<td>0.33</td>
<td>1.93</td>
<td>0.53</td>
<td></td>
</tr>
</tbody>
</table>

* Joint was badly filled.
+ Sustained without failure

Wall behaviour up to First Cracking

Table 5-1
<table>
<thead>
<tr>
<th>Test</th>
<th>Load</th>
<th>Crack</th>
<th>Load</th>
<th>Crack</th>
<th>Load</th>
<th>Crack</th>
<th>Load</th>
<th>Crack</th>
<th>Load</th>
<th>Crack</th>
<th>All</th>
<th>Wires</th>
<th>Yield</th>
</tr>
</thead>
<tbody>
<tr>
<td>W3</td>
<td>41.8</td>
<td>0.22L</td>
<td>37.2</td>
<td>0.19L</td>
<td>42.6</td>
<td>0.14L</td>
<td>40.2</td>
<td>0.12L</td>
<td>44.9</td>
<td>0.05L</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W4</td>
<td>43.2</td>
<td>0.15L</td>
<td>39.8</td>
<td>0.21L</td>
<td>44.6</td>
<td>0.14L</td>
<td>45.6</td>
<td>0.08L</td>
<td>52.1</td>
<td>0.12L</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W5</td>
<td>44.4</td>
<td>0.05L</td>
<td>43.4</td>
<td>0.06L</td>
<td>43.6</td>
<td>0.06L</td>
<td>50.4</td>
<td>0.12L</td>
<td>51.3</td>
<td>0.06L</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W6</td>
<td>24.5</td>
<td>0.14L</td>
<td>70.6</td>
<td>0.21L</td>
<td>83.1</td>
<td>0.12L</td>
<td>47.8</td>
<td>0.09L</td>
<td>44.3</td>
<td>0.06L</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>W7</td>
<td>41.1</td>
<td>0.11L</td>
<td>40.7</td>
<td>0.09L</td>
<td>44.3</td>
<td>0.06L</td>
<td>44.3</td>
<td>0.06L</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

**Applied Loads for Ultimate Failure**

Table 5-2

<table>
<thead>
<tr>
<th>Test</th>
<th>Variation of the Loads in the Wires</th>
<th>Ic</th>
<th>I</th>
<th>V</th>
<th>3P-a</th>
</tr>
</thead>
<tbody>
<tr>
<td>W3</td>
<td>Linear</td>
<td>11.9</td>
<td>72.7</td>
<td>36.3</td>
<td>-350</td>
</tr>
<tr>
<td></td>
<td>All yield</td>
<td>7.99</td>
<td>97.7</td>
<td>42.4</td>
<td>-690</td>
</tr>
<tr>
<td>W4</td>
<td>Linear</td>
<td>12.7</td>
<td>78.4</td>
<td>37.3</td>
<td>-290</td>
</tr>
<tr>
<td></td>
<td>All yield</td>
<td>7.89</td>
<td>97.7</td>
<td>42.4</td>
<td>-690</td>
</tr>
<tr>
<td>W5</td>
<td>1 Yield</td>
<td>3.64</td>
<td>22.3</td>
<td>13.4</td>
<td>-100</td>
</tr>
<tr>
<td></td>
<td>1 Yield</td>
<td>2.34</td>
<td>27.4</td>
<td>13.6</td>
<td>-100</td>
</tr>
<tr>
<td></td>
<td>Experimental 1</td>
<td>4&quot;</td>
<td>13.5</td>
<td>14.4</td>
<td>-388</td>
</tr>
<tr>
<td>W6</td>
<td>Linear</td>
<td>17.2</td>
<td>106.8</td>
<td>70.6</td>
<td>-40</td>
</tr>
<tr>
<td></td>
<td>All yield</td>
<td>9.60</td>
<td>117.2</td>
<td>83.1</td>
<td>-277</td>
</tr>
<tr>
<td>W7</td>
<td>Linear</td>
<td>7.38</td>
<td>46.4</td>
<td>40.7</td>
<td>-310</td>
</tr>
<tr>
<td></td>
<td>All yield</td>
<td>4.30</td>
<td>51.2</td>
<td>41.3</td>
<td>-330</td>
</tr>
</tbody>
</table>

Computations to Determine Possible Brickwork Failure for the Theoretical Ultimate Failure Loads

Table 5-3
Figure 5.1

Figure 5.2
LOAD - DEFLECTION CURVE FOR W2

Figure 5.3
Figure 5.5

Wire loads and cracking for fractions of ultimate load for wall W3.

Load in wires (lb):
- 0.375 Initial cracking
- 0.5
- 0.6
- 0.7
- 0.8
- 0.9
- 0.97

Load in wires vs distance along beam (0 to 0.2L, 0.4L, 0.6L, 0.8L, L).

Crack growth over the range of loads and distances.
LOAD - DEFLECTION CURVE FOR THE FIRST LOADING OF W4

Figure 5.6
OPEN AT EDGES ONLY

FULLY OPEN

LOAD APPLIED TO THE TOP LOADING BEAM kip
OPENING UP OF THE FIRST CRACK - WALL W4

Figure 5.7
Figure 5.9

Wire loads for fractions of the ultimate failure load of the wall W4

Load in wires (lb)

Distance along beam

Failure load of wires

0.33 Initial cracking

0.5

0.6

0.7

0.8

0.9

0.97

7 6 5 4 3 2 1

0 0.2L 0.4L 0.6L 0.8L L
LOAD-DEFLECTION CURVE FOR W6

Figure 5:11
LOAD-DEFLECTION CURVE FOR W6

DEFLECTION \( \text{in} \times 10^{-4} \)

Figure 5-12
WIRE LOADS AND CRACKING FOR FRACTIONS OF THE ULTIMATE FAILURE LOAD OF THE WALL

WALL W6

Figure 5.13
LOAD-DEFLECTION CURVE FOR W7

Figure 5-14
LOAD-DEFLECTION CURVE FOR W7

Figure 5-15
Wire loads and cracking for fractions of the ultimate failure load of the wall

WALL W7

Figure 5.16
VERTICAL STRESSES AT INITIAL CRACKING

Figure 5.17
PLATE 5-1  Cemented strain gauge

PLATE 5-2  Protected gauge and wire ducting

PLATE 5-3  Construction of the bottom of W5
PLATE 5-4  Members of the loading frame

PLATE 5-5  Wall W4 in the loading frame
PLATE 5-7  Measurement of the residual loads

PLATE 5-6  P.S.C. Monowire jack on an adjustable anchorage
PLATE 5-8  Transfer of loading to the wall

PLATE 5-9  Dial gauges on a square wall
PLATE 5-14  Failure of W4

PLATE 5-15  Failure of W5
Wall W7 near failure

PLATE 5-18 Tension end of W7 after failure

PLATE 5-19 Cracks at the compression end
APPENDIX A1

DERIVATION OF THE CROSS-SECTIONAL AREAS OF THE MEMBERS OF AN ANALOGOUS FRAME

STATEMENT OF THE ANALOGY

A framework of rectangular external dimensions is to have the same deformability in all directions as a square plate, of the same external dimensions, of uniform thickness t, and made of elastic material. To do this the cross-sectional areas of the framework members must be defined in terms of the thickness of the plate, the length dimensions of the plate, and Poisson's ratio. Equivalence is possible for cases of uniform loading, both normal and shear. In cases of non-uniform loading the analogy is not strictly accurate, but is a valuable approximation.

CRITERIA OF EQUIVALENCE

The criteria of equivalence are most conveniently stated in terms of the following three conditions. Other formulations are possible.
1. A normal uniform load \( p \) per unit length is applied in the x direction of the plate. Point loads \( \frac{1}{2}p = \frac{1}{2}p_1 \) are applied at the joints of the framework in the x direction. In the y direction a normal load of \( p \) per unit length is applied to the plate (see Figure A1-1). At the joints of the frame point loads \( \frac{1}{2}p = \frac{1}{2}p_1 \) are applied in the y direction. \( l_1 \) and \( l_2 \) are the dimensions of the rectangle in the x and y directions respectively. Strains in a given direction in both the plate and the frame are to correspond

\[
\varepsilon_x = \frac{E(1-\nu^2)}{E} \]

and \( \varepsilon_y = 0 \)

2. Two similar equations are set up by reversing the axes on which the normal loads \( p \) and \( p \) per unit length act, whence

\[
\varepsilon_x = 0
\]

and \( \varepsilon_y = \frac{E(1-\nu^2)}{E} \).
The framework to be considered is symmetrical about the x and y axes, thus this condition is considered satisfied if condition 1 is satisfied.

3. A tangential load \( p \) per unit length is applied to the plate, and equivalent point loads are applied to the joints of the framework, resulting in shear deformation of both (see figure A4-2).

The deformation in both cases is to be

\[
\gamma_{xy} = \frac{2(1 + \mu)p}{Et}
\]

**THE FRAMEWORK**

The framework used is shown in figure A4-1. Joints A, B, C and D are the external joints of the frame, E, I, G and H are the internal joints. A, B, C, D, E, F, G and H are all pin joints. The framework is loaded at the external joints only. When composite action of more than one framework is considered, for equilibrium of the lattice only external joints such as A, B, C and D need to be considered. Areas and dimensions are as shown in the figure. The framework is square for simplicity, but relations defining the cross-sectional areas of rectangular frames where \( l_1 \) is not equal to \( l_2 \) are possible.

**AXIAL LOADING ON A SQUARE FRAMEWORK**

The point loads on the frame are

\[
\frac{p}{2} = \frac{1}{2}p_l
\]

and \( \mu \frac{p}{2} = \frac{1}{2}p_l \).

They act at the external joints of the framework. The strains are

\[
\begin{align*}
\varepsilon_x &= \frac{p}{E} (1 - \mu^2) \\
\varepsilon_y &= 0.
\end{align*}
\]
Since \( c_y = 0 \)
\[ F_{AB} = F_{BC} = 0 \]

\[ \therefore \frac{F_{BF}}{\sqrt{2}} = \frac{uF}{2} \quad \therefore 2y = 0 \text{ at } B \]

But \[ \frac{F_{BF}}{\sqrt{2}} + F_{BC} = \frac{P}{2} \quad \therefore x = 0 \text{ at } B \]

\[ \therefore F_{BC} = \frac{P}{2} - \frac{uF}{2} \]

\[ = \frac{(1-u)P}{2} \]

\[ \therefore F_{AE} = \frac{(1-u)P}{2} \]

\[ A_0 = \frac{(1-u)xt}{2E} = \frac{E}{p(1-u^2)} \]

\[ = \frac{lt}{2(1+u^2)} \quad \text{(A1-1)} \]

There is symmetry of the frame geometry and loading about both the \( x \) and \( y \) axes.

\[ \therefore F_{AE} = F_{BF} = F_{CC} = F_{DI} \]

\[ F_{EF} = F_{FG} = F_{CH} = F_{EP} \]

and \( F_{BE} = F_{HE} \).

The outward forces in the outer diagonal members \( AE, BE, CC, \) and \( DI \) cause joints 3, 6, 3 and 11 to move outwards. Let the distance moved outward be denoted by \( e \). The centres of the members, however, because of symmetry, still lie on the lines \( XX \) and \( YY \).

\[ \therefore \frac{c_{BF}}{2} = \frac{c}{2} \]

\[ \therefore \frac{c_{CH}}{2} = \frac{c}{2} \]

\[ \frac{c_{DI}}{2} = \frac{c_{BF}}{2} \]
$$\frac{F_{FH}}{EA_4} \times \frac{v_2}{2} = \frac{F_{EF}}{EA_2} \times \frac{1}{\sqrt{2}}$$

$$\frac{F_{FH}}{A_4} = \frac{F_{EF}}{A_2}$$

(M1-2)

Resolving parallel to BD at F:

$$F_{BF} = F_{FH} + \sqrt{2} F_{EF}$$

$$= F_{FH} \left(1 + \sqrt{2} \frac{A_2}{A_4}\right)$$

$$\therefore F_{FH} = \frac{A_4}{A_4 + \sqrt{2} A_2} F_{BF}$$

$$e_{BD} = \frac{a_0}{\sqrt{2}}$$

$$= \frac{a_0}{2E} (1 - \mu^2)$$.

$$\therefore \frac{1}{\sqrt{2}} \times \frac{w a_0}{2} (1 + \frac{A_4}{A_1 + \sqrt{2} A_2}) \times \frac{1}{EA_4} = \frac{2(1 - \mu^2)}{\sqrt{2} E}$$

Hence

$$A_2 = \frac{A_4 \left[(1 - \mu^2) a_0 - 2\sqrt{2} \mu a_0\right]}{2a_0 - \sqrt{2} (1 - \mu^2) A_4}.$$  (M1-5)

SHEAR LOADING ON A SQUARE FRAMEWORK

The uniform tangential load $p$ per unit length on the plate which produces shear loading is simulated on the framework by forces $\frac{F_p}{2}$ in both the $x$ and $y$ directions at each joint as shown in figure M1-2. Deflections are very small and are exaggerated for clarity in the figure.

Only the diagonal members carry load. Furthermore one diagonal is in compression whilst the other is in tension. The diagonal forces are equal and opposite.

$$F_{BF} = -F_{AE} = \frac{p}{\sqrt{2}}.$$
The elongation \( e = \frac{P}{E} \frac{a_1}{A_1} \)

\[ = \frac{e_1}{A_1} \] .

From triangle AOD:
\[ \tan (45^\circ - \frac{\gamma_{xy}}{2}) = \frac{e_1}{a_0} = \frac{1 - \frac{e}{2}}{1 + \frac{e}{2}} \]

But \( \tan (45^\circ - \frac{\gamma_{xy}}{2}) = \frac{\tan 45^\circ - \tan \frac{\gamma_{xy}}{2}}{1 + \tan 45^\circ \tan \frac{\gamma_{xy}}{2}} \)

\( \gamma_{xy} \) is very small

\[ \therefore \tan (45^\circ - \frac{\gamma_{xy}}{2}) \approx \frac{1 - \frac{\gamma_{xy}}{2}}{1 + \frac{\gamma_{xy}}{2}} \]

Thus \( \frac{\gamma_{xy}}{2} = \frac{e}{\sqrt{2}a_1} \)

\[ \therefore \gamma_{xy} = \frac{\sqrt{2}a_1}{1} = \sqrt{2} \frac{a_1}{E_1} \]

Equating to the strain of the plate
\( \frac{2(1-\mu) e}{E t} = \frac{\sqrt{2}a_1}{E_1} \)

\[ \therefore A_1 = \frac{lt}{\sqrt{2} (1+\mu)} \] (M1-4)

It is now possible to evaluate \( A_2 \) from equation (M1-3) by substituting the value of \( A_1 \).
\[ A_2 = \frac{A_1 \left(1 - \mu^2\right) A_1 - 2\sqrt{2} \, 1t}{21t - \sqrt{2} (1-\mu^2) A_1} \]

\[ = \frac{\frac{1t}{\sqrt{2(1+\mu)}} \frac{1-\mu^2}{\sqrt{2(1+\mu)}} \, 1t - 2\sqrt{2} \, 1t}{2 - \sqrt{2} (1-\mu^2) \frac{1}{\sqrt{2(1+\mu)}}} \]

\[ = \frac{\mu - \frac{3}{2}}{2(1+\mu)(1-2\mu)} \]

\[ \text{CONSIST ON THE BAR AREAS AND DERIVATIONS} \]

For values of \( \mu \) greater than \( \frac{1}{3} \) and less than \( \frac{1}{3} \), the area of \( A_2 \) is negative. \( A_2 \) then has only a mathematical significance. Kremikoff also mentions the instability of the internal frame EFGH in his paper (6). The framework is a mathematical and not a physical tool for the solution of problems.

A lattice representing an elastic plate is composed of frames analogous to plate segments. The analogy was derived for uniform loads. The stress in the plate to which the lattice corresponds will, in general, not be uniform but will vary continuously. Only if the frames were infinitely small would the loads on each corresponding to the product of stress by thickness over the appropriate length of plate be regarded as uniform. Thus it is important, when choosing a suitable lattice also, to consider the stress gradient in the elastic plate. The method of application of the external loading to the plate and the derivation of stresses are also affected by this limitation to the analogy.
**APPENDIX A2**

**DERIVATION OF THE DISTRIBUTION COEFFICIENTS**

When an external joint of a framework is displaced parallel to either the x or the y axis whilst the other external joints are fixed, the movement will induce forces in the member lying in the direction of movement and in the interior members of the frame. Of the forces in the interior members of the frame only the forces in the outer diagonal members will act on the external joints, and thus affect the distribution of out-of-balance forces within the lattice. The forces at each end of a diagonal are equal and opposite, as may be easily found from equilibrium of the internal frame. Forces in the external diagonal members are taken into consideration in the distribution process by the use of distribution coefficients. For movement in the x or y direction, a distribution coefficient is given by the ratio of the component in the x and y directions of the force in the diagonal member to the force in the member lying in the direction of movement.

There are two distribution coefficients, termed active and passive, signifying whether diagonal resists movement or transfers force from the internal members respectively. The distribution coefficients will now be derived.

Figure A2-1 shows a displacement as described above. Members shown as broken lines carry no load. AC is the "passive" diagonal, BD the "active" diagonal. $F_{AB}$ and $F_{DP}$ oppose the forces P and Q causing displacement. The problem may be separated into two parts, determination of the force required to elongate AB by a distance $\Delta A$, and determination of the force required to elongate the diagonal BD a distance $\Delta \sqrt{2}$. Assuming unit displacement, the force $RF$ may be found by considering the strain energy of the internal frame. $F_{MH}$ and $F_{DP}$ are replaced by forces $R_1$ and $R_2$, respectively, as shown in figure A2-2. Statics are used to obtain the forces in the members due to $R_1$ and $R_2$. The forces in the members and the tabulation to find the derivatives of strain energy with respect to $R_1$ and $R_2$ are given in table A2-4. In the direction PH joint D has been moved a distance $1/\sqrt{2}$. Applying the theorems of strain energy...
<table>
<thead>
<tr>
<th>Member to $R_1$</th>
<th>Force Due to $R_1$</th>
<th>Force Due to $R_2$</th>
<th>Total Force $L$</th>
<th>$A$</th>
<th>$\frac{\partial F}{\partial R_1}$</th>
<th>$\frac{\partial F}{\partial R_2}$</th>
<th>$\frac{\partial F}{\partial A}$</th>
<th>$\frac{\partial F}{\partial A}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>AB</td>
<td>$\frac{R_1}{2}$</td>
<td>$-\frac{R_2}{2}$</td>
<td>$\frac{R_1R_2}{2}$</td>
<td>$2\sqrt{2}$</td>
<td>$A_1$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{(R_1-R_2)}{2\sqrt{2}A_1}$</td>
</tr>
<tr>
<td>DH</td>
<td>$R_1$</td>
<td>$-R_2$</td>
<td>$\frac{R_1-R_2}{2}$</td>
<td>$2\sqrt{2}$</td>
<td>$A_1$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{(R_1-R_2)}{2\sqrt{2}A_1}$</td>
</tr>
<tr>
<td>CC</td>
<td>$\frac{R_1}{2}$</td>
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<td>$\frac{R_1-R_2}{2}$</td>
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<td>$\frac{(R_1-R_2)}{4A_2}$</td>
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<td>GH</td>
<td>&quot;</td>
<td>&quot;</td>
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<td>$2\sqrt{2}$</td>
<td>$A_1$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{(R_1-R_2)}{2\sqrt{2}A_1}$</td>
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<tr>
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<td>$2\sqrt{2}$</td>
<td>$A_1$</td>
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<td>$\frac{1}{2}$</td>
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<tr>
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<td>$\frac{R_2}{2}$</td>
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<td>$A_1$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{1}{2}$</td>
<td>$\frac{(R_1-R_2)}{2\sqrt{2}A_1}$</td>
</tr>
</tbody>
</table>
\[ \frac{\text{EF}}{\frac{\partial F}{\partial E}} \frac{L}{EA} = \frac{(3R_c - R_2)1}{2\sqrt{2} A_1 E} + \frac{(R_1 - R_2)1}{EA_2} = \frac{1}{42}, \quad (A2-1) \]

and \[ \frac{\text{EF}}{\frac{\partial F}{\partial R_2}} \frac{L}{EA} = \frac{(3R_c - R_1)1}{2\sqrt{2} EA_1} + \frac{R_1 - R_2}{EA_2} = 0, \quad (A2-2) \]

From (A2-2),
\[ B_2 = \frac{A_2 + 2\sqrt{2} A_1}{2\sqrt{2} A_1 + 3A_2} \cdot \frac{R_1}{\frac{A_2}{1}}, \quad (A2-3) \]

Substituting in (A2-1),
\[ R_1 = \frac{2\sqrt{2} A_1^2 + 3A_1 A_2}{4(A_2 + 2\sqrt{2} A_1)} \cdot \frac{R_1}{\frac{A_2}{1}}, \quad (A2-4) \]

In general the member \( AB \) will be an interior member. Assuming it to be an interior member of area \( A \), the bar member areas are as derived in appendix A1.
\[ A = 2A_0 = \frac{2t}{1+\mu} \]
\[ A_1 = \frac{2t}{\sqrt{2}(1+\mu)} \]
\[ A_2 = \frac{3\mu - 1}{2(1+\mu)(1-2\mu)} \]

The force in \( AB \) resulting from unit displacement is
\[ P_{AB} = \frac{EA}{L}, \quad (A2-5) \]

\[ \frac{P_{EF}}{P_{AB}} = \frac{R_1}{F_{26}} = \frac{2\sqrt{2} A_1^2 + 3A_1 A_2}{4(A_2 + 2\sqrt{2} A_1)} \cdot \frac{1}{EA}, \]

Substituting for the bar areas
\[ \frac{P_{EF}}{P_{AB}} = \frac{1+\mu}{4\sqrt{2}(1-\mu)}. \]
Thus the active coefficient is

\[
\frac{F_{AB}}{v_2} = \frac{1}{F_{AB}}
\]

\[
= \frac{1 - \frac{R_1}{R_2}}{8(1 - \mu)} \cdot \tag{A2-6}
\]

\[
\frac{F_{AE}}{F_{BC}} = \frac{R_1 - R_2}{2} \times \frac{1}{F_{BC}}
\]

\[
= \frac{R_1}{2} \left(1 - \frac{R_2}{R_1}\right) \cdot F_{BC}.
\]

Substituting for \( R_1 \) from (A2-4), \( \frac{R_2}{R_1} \) from (A2-3), and \( F_{BC} \) from (A2-5)

\[
\frac{F_{AE}}{F_{BC}} = \frac{2A_1A_2}{8(A_2 + \sqrt{2A_4}A)}
\]

Again substituting for member areas

\[
\frac{F_{AE}}{F_{BC}} = \frac{3\mu - 1}{4 \cdot 2(1 - \mu)} \cdot
\]

Thus the passive coefficient is

\[
\frac{3\mu - 1}{8(1 - \mu)} \cdot \tag{A2-7}
\]
Figure A1-1

Figure A1-2

Figure A2-1

Figure A2-2
APPENDIX B1

HUMPHREY'S ANALOGY - DISTRIBUTION PROGRAM

SCOPE OF THE PROGRAM

The program is written in SPS and requires 20,000 decimal digits of storage. It solves the lattice analogy by distributing the out-of-balance forces at joints to surrounding joints until equilibrium of the joints is obtained. The input and output of the program are in terms of the forces in the members of the lattice. The lattice must be square or rectangular. The number of joints must not exceed 150, and the number of joints in all but the bottom row and in all but the end column must not exceed 120. The top and bottom of the lattice may be horizontally stiffened. There are three options for the fixity of the base of the lattice. Any value of Poisson's ratio may be used for the material of the plate to which the lattice is analogous.

ENTRY OF DATA

The program is loaded into storage. When the START key is depressed
the typewriter writes

N NO COLUMNS, N NO ROWS, FIXED FT 2 DIGITS.

The number of columns and the number of rows must be entered at the typewriter as a four-digit number, and the release-start key depressed. For example, if there were three columns and eleven rows

0311

would be typed.

The forces in the lattice are automatically set to zero. Only non-zero forces need be entered, but all the zero forces preceding the non-zero force in storage must be entered. Forces are stored in rows of the lattice from the top downwards. Forces are entered on cards in the order FH, FV, St, S2, IH, and IV. All 50 positions on the input cards must be used. The end of each group of forces is marked by a record mark in position 1 on a card. With large lattices the overlap from the IV storage area may overlap on to and destroy part of the FH storage area. The data which would be so destroyed
should be included on the last card of the LV area and the record mark card.

Then all the cards have been read into storage the typewriter writes

**POISSON'S RATIO 57XXX00000**

indicating that Poisson's ratio must be entered at the typewriter as a floating point constant. Depression of the release-start key results in the calculation of constants involving Poisson's ratio. The typewriter then writes

**YOUR BEAM SIZE RATIOS, SIR.**

The ratios of the horizontal stiffnesses of the top and the bottom of the lattice to the stiffness of a central horizontal or vertical member of the lattice must then be entered as floating point numbers in that order. The release-start key is then depressed.

**BASE FIXITY**

The typewriter will then write

**SMB1 ON TO FIX BASE**

**SMB3 ON TO FIX CORNER.**

It is then necessary to set the sense switches for the desired base fixity. If sense switch 1 is on the base of the wall is fixed. If sense switch 1 is off and sense switch 3 is on the corner joint only is fixed. If sense switches 1 and 3 are both off no joints are fixed, and the applied forces must be in equilibrium.

**COMPUTATION AND OUTPUT**

Depression of the start key starts the computation. Joints are released in rows from the top downwards in each cycle. Bar forces are typed out after 1, 2, 3, 50 and multiples of 50 cycles. The typewriter message

**SMB4 ON TO FOCUS STRESSES**

followed by a halt in computation at the end of the type-out indicates how the output forces may be obtained on cards. Depression of the start key restarts the operation of the computer. The degree of convergence of the lattice solution must be assessed by the operator. The punched card output is used as the input for the program described in appendices B3 and B4.
<table>
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<tr>
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<th>DS</th>
<th>K</th>
<th>Value</th>
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<tr>
<td>21050FV</td>
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<td>K</td>
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<tr>
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<td>DS</td>
<td>K+1</td>
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<td>21080S12</td>
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<td>22120 DEND READ IN-12 02178</td>
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</table>
APENDIX B2

RUDINGOFF'S ANALOGY - DISTRIBUTION PROGRAM 2

SCOPE OF THE PROGRAM

The scope of the program is the same as that of distribution program 1 except in the base fixity of the bottom of the lattice. The joints along the bottom of the lattice may move horizontally but not vertically, allowing the vertical displacement of the bottom of the lattice to be entered as boundary conditions.

ENTRY OF DATA

As for distribution program 1.

BASE FIXITY

Base fixity is determined by the statements already in the program. In place of the statements about base fixity the typewriter writes

SSE2 IN IS TOP DOWN.

To speed convergence the direction of the release of the joints may be reversed by switching sense switch 2 on or off. With sense switch 2 on, the order of release is in rows from left to right from the top downwards.

SUMMATION AND OUTPUT

As for distribution program 1.
APPENDIX B3

HRETHKOFT'S ANALOGY - EVALUATION OF STRESSES

OPERATION OF THE PROGRAM

Then the start key is depressed after loading the program. The punched card output from either of the distribution programs is read into storage. The typewriter then writes

\[ N \times N \times \text{FIXED PT 2 DIGITS.} \]

The number of columns and the number of rows must then be entered at the typewriter as in the distribution programs. Depressing the release-start key results in a second typewritten message

\[ \text{LENGTH } L10, \text{THICK } F10. \]

The length of the lattice segments and the thickness of the plate to which the lattice is analogous must be entered as floating point numbers at the typewriter. Depressing the release-start key again results in a message

\[ \text{POISSON'S RATIO } F10. \]

requiring Poisson's ratio to be entered. The final constants to be entered are the ratios of the horizontal stiffnesses of the top and bottom of the lattice to the area of an internal vertical or horizontal member. These ratios, \( J_T \) and \( J_B \), must be entered at the typewriter as floating point numbers in response to the typewritten message

\[ J_T \text{ AND } J_B \text{ PLEASE.} \]

Depression of the release-start key starts the calculation. The program evaluates the stresses at the centre of the lattice segments and types them out in numbered rows. Upon completion of typing out the shear stresses, the program restarts itself.
APPENDIX 2A

BREZNIKOFF'S ANALOGY - EVALUATION OF DISPLACEMENTS

OPERATION OF THE PROGRAM

The program evaluates the displacements of the joints of the lattice assuming that the bottom left-hand corner does not move, and that the bottom right-hand corner is on the same horizontal plane as the bottom left-hand corner. When the program is loaded and started, the output from either of the distribution programs is read into storage. Typewritten messages ask for constants which are entered at the typewriter and the release-start key depressed. The messages are:

M XX, N XX;
requiring the number of columns and the number of rows to be entered as two-digit fixed point numbers.

E FL10, THICK FL10, B1 FL10;
requiring Young's modulus, the thickness of the plate and Poisson's ratio to be entered as fixed point quantities.

JT AND JB PLEASE FL10 FL10;
requiring the ratios of the horizontal stiffnesses of the top and the bottom of the wall to be entered as flagged floating point numbers.

The calculated values are typed out in numbered rows and the machine halts. Depression of the start key will restart the program.
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<thead>
<tr>
<th>Byte</th>
<th>Value</th>
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<td>010F</td>
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</tr>
</tbody>
</table>

**Notes:**
- The values represent hexadecimal numbers.
- The format indicates a sequence of bytes with values starting from 0102 to 0122.
- The values are likely related to a specific programming or data storage context.
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