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THE K-SERIES EMISSION SPECTRUM OF NEON

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The writer is now on the staff of the Dominion Physical Laboratory (D.S.I.R.) and wishes to thank the Director for the opportunity of preparing this report.
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SUMMARY

Experiments are reported in which the inert gases argon and neon were subjected to a high-voltage electrodeless discharge. The characteristics of the discharge were studied, and a theory of the mechanism of origin of the emitted X-radiation is proposed.

The construction of a composite X-ray tube, incorporating both orthodox electrodes and an electrodeless discharge tube, is described. This involved the building of a vacuum evaporating unit.

Details are given of the construction of a photographically recording bent crystal vacuum spectrograph. Using a mica crystal, the working range of the instrument is about 5-19 kX in the first order of reflection, the inverse linear dispersion being about 37 X/μm at 15 kX.

A spectroscopic analysis of the X-radiation emitted by the inert gas neon is described. Twenty-one K-series lines were recorded, many of them for the first time. The majority of the lines have been identified and the wavelengths measured in all cases. The shapes of the main neon lines were obtained on an intensity scale, and their half-width values, uncorrected for possible diffraction breadth, were determined.

As the result of these investigations certain conclusions are drawn.
GENERAL INTRODUCTION

In an earlier report (Moore 1949) the results were given of a preliminary study of the electrodeless discharge as a method for exciting X-radiation in gaseous elements. Briefly, it was found that if the output from a spark-controlled Tesla Coil was applied to a pyrex glass discharge tube containing argon gas at low pressure, X-radiation was emitted from the tube. The latter was about 30 cm in length, having a circular cross-section of 2 cm diameter. A side-arm at about 13 cm from one end of the tube was connected to the associated vacuum system. The tube end, remote from the side-arm, was sealed with a hemi-spherical termination, while the other was equipped with a thin (0.25 mm) beryllium window. The closed end of the tube was placed against the spherical high-voltage terminal of the Tesla Coil, the other external electrode being a brass ring connected to ground, and sliding on the tube at the window end.

For peak Tesla Coil voltages of up to 80 kV, and with gas pressures about 0.01 mm Hg, various experiments were performed in which the radiation passing out of the beryllium window was photographically detected. A stage in these investigations was reached when, the suitability of the method of excitation as a spectroscopic source being accepted, the need for a spectrometer became evident. After some further work with the original equipment, this latter requirement has been realised with the

# Any further references to this report will be by way of the page number concerned.
construction of a bent crystal vacuum spectrograph. This has enabled the discharge source to be utilised in a special X-ray tube for a spectroscopic study in the region of soft X-rays. In the present report, results are given of wavelength and line-shape measurements in the K-series emission spectrum of the inert gas neon (Z=10).

The use of the electrodeless discharge as an X-ray source was pioneered by Groven (1939, 1947) as already discussed (Moore p.6). In the interim, reports of other work have appeared. These are concerned with the excitation of X-radiation from elements in the vapour phase. Groven (1949) has studied the L-series of mercury vapour and the K-series of the elements in the atomic number range 30-36; Morlet (1949), the K-series of selenium; Groven and Lagasse (1949), the L-series of mercury; Coens (1950), radiation from arsenic vapour; Morlet (1950), the K-series of bromine; and Groven and Morlet (1951) the K-series of the elements in the atomic number range 30-36. The characteristic radiations examined by these various workers were all in the region of 1 keV. Their spectrometric techniques were accordingly different from those used in the present work, and need not be discussed.

In presenting this report under four main headings, recognition has been given to a natural division of the subject matter rather than to a strictly chronological treatment.
PART I

THE ELECTRODELESS DISCHARGE

AS A

SOURCE OF X-RADIATION
The phenomenon of the emission of x-radiation from a rarefied gas, subjected to an electrodeless discharge, was first utilised as a source for spectroscopic analysis by Groven (1947). The characteristics of such a gaseous x-ray source have not been specifically studied, although following Groven several workers have contributed observations on certain features. These contributions are now reviewed prior to a description, in following sections, of further investigations of the subject.

Probably the most striking feature of the X-ray source is the annular distribution of intensity when the tube is regarded axially, a fact first noted by Groven (1949), and independently by Moore (p.162). Groven used a linear tube of circular cross-section on which slid two external electrodes. Excitation was by means of a damped wave generator. He maintains that a single-pinhole photograph revealed a feebly emitting column of gas on the tube axis, surrounded by a sleeve-like region of intense emission not extending as far as the inner wall of the tube, and that it was the gas and not the tube walls which emitted. It is questionable (Moore p.173) whether Groven’s deduction of a limited gas source, without wall radiation, is possible from a single-pinhole exposure. As to the cause of the annular shape of the source, Groven puts forward the explanation that, since his tube was immersed in oil maintained at a uniform temperature, there would arise an annular distribution of temperature and pressure in the gas, giving rise
to the observed annular structure of the discharge. This explanation may be criticised for two reasons. In the first place, since the kinetic mean free path of a gas atom at the pressure concerned (about 0.01 mm Hg) is of the same order of magnitude as the tube diameter, the suggested temperature and pressure gradient could hardly occur. Again, the sheath-like source distribution has been observed in a tube operated in air (Moore p.162) and, as shown in Section 1.9, in a tube of non-circular cross-section immersed in an oil bath in which the temperature was certainly not uniform.

Using a similar experimental arrangement to Groven (1949), Groven and Lagasse (1949) and Morlet (1949), all observed the typical annular discharge. The latter apparently credits the circular electrodes as having some effect on the source distribution. With selenium vapour in the tube, Morlet observed the deposition of a metallic film on the inner glass wall between the electrodes. The deposit was thickest towards the electrodes, but did not extend right up to them. Morlet takes this as indicating a "potential hole" near the electrodes which is gradually "filled up" as the central section of the tube is approached. Under the influence of a radial field the positive ions migrate outwards to give the metallic deposit on the walls. It is difficult to assess the value of Morlet's argument on the information given. Certainly it can be said that the phenomenon of the annular source is not simply a function of the two ring electrodes employed by Morlet, since it has been observed for the sphere and ring electrode system of the present work.
Goens (1950) used a tube of similar dimensions to the other workers cited, but produced the excitation with a pulsed microwave generator. The individual waves would not, therefore, be damped. By means of a single-pinhole camera he obtained photographs indicating that the annular source is not a continuous sleeve, but rather is localised into two series of annula, the outer and more intense of which are opposite the electrodes. The second pair of annula are less intense and do not appear on all plates exposed, while their position relative to the electrodes is not stated. Here again, the possibility of unjustifiable deductions from a single pinhole exposure, of the actual dispositions in the tube of the pairs of annula, must be suggested. Goens puts forward no explanation of the phenomena he observed.

The present writer (Moore p. 176) previously established the annular nature of the source by means of the double-pinhole camera. The linear distribution of the source had not, however, been certainly established, nor had the effect on the emission intensity of varying pressure, applied voltage, or the electrode spacing. The original discharge tube, described in the General Introduction, was modified to the extent of re-attaching the side-arm immediately adjacent to the window. This allowed a continuous protecting sleeve to be placed over the tube, and gave the ring electrode freedom of movement along the tube length. The tube was mounted to facilitate the various measurements to be described. It is noted that gas pressures are usually given, on a relative laboratory scale, in 'gauge microns.' However, in one or two instances where the actual pressure has been required, the
Laboratory scale was calibrated against a McLeod gauge.

**SECTION I-2  THE DISTRIBUTION OF INTENSITY IN THE SOURCE**

In the first experiment a paper-covered film strip was placed parallel to the tube, and, with the double-pinhole camera in position, simultaneous exposures were taken of the linear and axial source distributions. The blackening on the developed film was determined and plotted as a function of tube length, while measurements on the double-pinhole image, together with a knowledge of the experimental geometry, led to the reconstruction of Fig.1. The density of the photographic blackening is taken to be a measure of the intensity of the incident radiation. Clearly, both exposures are in excellent agreement as to the location of the region of maximum intensity in an annular source extending over most of the inter-electrode region.

Next, the linear distribution was determined for various electrode spacings \( L \). In this case, care was taken to process and densitometer the films under comparable conditions. The results, plotted in Fig. 2, establish that at least over the range of \( L, \) 12-22 cm, the linear distribution is independent of electrode spacing.

The curves of Fig. 3 show the results of the final experiment in which, for a fixed electrode separation, the linear distribution was measured as a function of applied voltage. The latter was controlled by altering the setting of the quenched spark gap in the primary Tesla circuit (Moore p.82). The variation in intensity is similar in each case, the rapid decrease with reduced voltage
being consistent with the less penetrating nature of the radiation.

SECTION I-5

THE TUBE VOLTAGE

To study the potential drop across the discharge tube, under various conditions of excitation, a voltmeter was required. In view of the magnitude, frequency, and wave-form of the voltage involved, the sphere-gap meter was preferred to other methods. Further, the electrode system enabled the measuring device to be realised simply, by bringing an earthed sphere close to the spherical terminal of the Tesla Coil. In the experiments to be described in later sections, it will be seen that it is the change in tube voltage rather than its actual value which is important. However, since the breakdown voltage of a sphere-gap is not necessarily a linear function of gap length, a calibration curve was constructed. Values of gap length and corresponding breakdown voltage for various sized spheres are plentiful at power frequencies, but apart from the work of Reukema (1927), there is little information available at the higher frequencies. The situation in a gap under these conditions is complicated by field-distorting space charges of positive and negative ions which accumulate in accordance with their mobilities and rates of diffusion. Reukema's curve for the breakdown voltage of 6.25 cm diameter spheres, over the frequency range 60,000-425,000 c/s, may be extrapolated to higher frequencies,
provided that the frequency does not become comparable with the
time of transit of electrons across the gap. This fact is
the basis of the derived calibration curve shown in Fig. 4,
Reukema's values having been suitably adjusted for use with
spheres of 7 cm diameter, one being the Tesla Coil termination.

With argon in the tube, measurements of the potential
drop across the tube were made as the gas pressure was varied.
In the first case the electrode spacing $L$, measured between
the spherical termination and the rear edge of the earthed
ring, had a fixed value, readings being taken for the various
settings of the spark generator. Some representative values
are plotted as the graphs of Fig. 5. They indicate a sensibly
linear relationship between pressure and tube potential drop,
the latter decreasing with increase in pressure.

In the second experiment, potential readings were taken
with a fixed generator setting for various values of $L$. Such
a set of measurements is portrayed by the graphs of Fig. 6.
They indicate that, over the $L$ range 10-22 cm, the potential
drop along the tube is independent of the position of the
earthed ring. Measurements for $L<10$ cm were not possible,
due to spark-over occurring along the outer tube surface.
SECTION I-1

THE TUBE CURRENT

The experimental arrangement did not favour the accurate determination of the discharge current flowing in the tube. However, an idea of the manner in which it varied under differing discharge conditions was obtained by placing a meter in the earth lead of the ring electrode. Because of the damped and intermittent nature of the spark generator, the meter consisted of a thermo-couple converter and galvanometer. It was calibrated directly, in r.m.s. milliamperes, with direct current. Despite the earthed ring, the discharge was seen to extend beyond and pass down the tube's pumping line. This, and the fact that a corona current flowed on the outer surface of the tube for small electrode separations, meant that the meter reading did not show the true tube current. It would, nevertheless, be proportional to this current, and since the measurements were concerned with the nature of its variation only, it is conveniently referred to as tube current.

In the first experiment the electrode spacing was constant. The pressure of argon in the tube was varied, and the tube current recorded for each of three settings of the generator. A typical set of values is plotted as the graphs of Fig. 7. The current is seen to rise sharply with pressure at first, pass through a double-humped maximum, and decrease less rapidly as
the pressure further increases. Assuming the identity of peaks on the three curves, it is noted that the secondary peak is relatively more prominent for the lower applied voltages.

In a second series of measurements, the generator setting was constant. The change of tube current with pressure was recorded for various electrode spacings L. The graphs of Fig. 8 portray some representative values. The manner of variation of current with pressure is shown to be independent of the value of L, the curves being similar in form to those of the preceding experiment. Because of such complicating factors as the corona current previously mentioned, no emphasis is placed on the relative magnitudes of the three curves at any given pressure. Particularly is this so at low pressures, when, due to the higher voltage across the tube, the corona current becomes excessive.

SECTION I-5 \textbf{AN IONISATION CHAMBER}

Incomplete evidence of a variation of the emission intensity, with gas pressure, had been obtained earlier (Moore p.169). To extend this work in a quantitative manner, it was decided to construct an ionisation chamber, this being the most convenient and well-established device for giving a continuous indication
of an X-ray beam intensity. Relative ionisation currents, observed with a chamber, give true relative powers in two beams of different wavelength, since the energy absorbed per ion-pair produced in a gas is independent of wavelength.

Although it was not intended to measure absolute intensities, sufficient care was taken in the design and building of the ionisation chamber. Granting the correct geometry, stability of construction leading to reliable and reproducible results was aimed at. A sectional diagram of the chamber is given in Fig. 9, while the assembled chamber is shown in Plate I. It contains air at atmospheric pressure.

Referring to Fig. 9, the chamber is essentially a concentric electrode system completely enclosed by an earthed cylinder. The collecting electrode A is an aluminium rod of radius 0.6 cm, being connected to the measuring instrument B with shielded co-axial cable. C is a Pye-type connector. The outer electrode D is an aluminium cylinder of radius 3.2 cm, and is connected through shielded co-axial cable to negative high voltage. The use of low atomic number aluminium for the electrodes decreased the possible effects of secondary radiations. Both electrodes are located and held secure by the thick perspex discs E, which slide neatly in the outer brass cylinder F, giving an effective chamber length of 13 cm. Radiation enters the chamber by way of the collimator G placed centrally between the electrodes. It consists of two steel washers H, aligned axially 1 in. apart, and having apertures of 0.25 in. diameter. The electrical continuity of the outer cylinder, which is effectively an extended
guard-ring, is ensured by the thin aluminium foil window I, fitted to the collimator.

The chamber was designed in accordance with certain theoretical requirements now summarised.

The radial dimensions are sufficient to ensure that the majority of the photo-electrons released in the sensitive volume come to the end of their ionising range without striking the electrodes.

The length should be sufficient to enable the primary radiation to be completely absorbed. This was not possible, as the chamber length was determined by the limited space in which it had to operate.

The ion collecting time of the chamber is much less than the recovery time of the external measuring circuit.

The strength and uniformity of the collecting field is everywhere sufficient to offset the recombination of the ions.

Upon completion, the chamber was tested by the establishment and examination of its characteristic curve. That is, for a constant flux of radiation entering the chamber, the variation of ion current with chamber voltage was investigated. The chamber was rigidly mounted inside the screening box surrounding the spark generator; the direct current amplifier used as measuring instrument, and the chamber voltage supply being outside and separately screened. The collimator of the chamber was aligned coaxially with the tube, and immediately opposite the tube window. Any error in alignment was ascertained by allowing radiation to enter the chamber with no voltage applied to it. If the adjustment were incorrect, radiation would strike the electrodes, giving rise to a detectable photo-electric current.

The discharge tube was operated with a constant argon pressure
and the ion current noted as the chamber voltage was applied in increasing steps. The readings, which could consistently be repeated, are plotted as the graph of Fig. 10. This curve is characteristic of a correctly functioning ionisation chamber. A definite plateau is present at about 50 volts, for a saturation current of the order of 0 µA. Provided then that sufficient voltage is applied to produce saturation at the highest radiation flux to be measured, the chamber can be relied upon to give an ion current proportional to the radiation intensity. In this respect a chamber voltage of 135 volts was found adequate, and used in the work detailed in the next sections.

SECTION I-4 THE TOTAL INTENSITY OF THE SOURCE

Using the previously described ionisation chamber, a study was made of the intensity of radiation passing through the window of the discharge tube under various conditions.

In the first experiment, with a fixed electrode spacing, the pressure of argon was varied for each of three settings of the generator, the intensity (ion current) being noted. Typical of the results obtained are those plotted as the curves of Fig. 11. The dependence of intensity on pressure is clear, the curves rising sharply at low pressures, passing through a maximum region as the pressure is increased, and then falling less rapidly at higher pressures. The value of the intensity at any pressure increases with the applied voltage, while the curves show a tendency to double peak at their maxima.

The intensity was next measured as a function of pressure,
at a fixed generator setting, for various electrode spacings L.
From the results obtained, it could not be concluded that the
intensity depended on L over the range examined. The values for
an average case are plotted in Fig. 12.

As it was noted that the variation of intensity with pressure
is similar to the variation of tube current with pressure,
simultaneous measurements of intensity and tube current were
made. The typical values plotted, as the curves of Fig. 13,
indicate a direct correlation between tube current and radiation
intensity.

SECTION I-7

THE FIRST NEON SOURCE

Up to this stage in the investigations of the discharge
characteristics, all pinhole exposures had indicated the annular
nature of the source, with little or no emission occurring on
the tube axis. However, it was assumed that such a complete
absence of emission from the axial part of the tube was unlikely.
The lack of blackening in the photographs was considered as being
more probably due to the absorbing effect of the beryllium window.
Thus it was always expected that, if the tube were equipped with
a window offering a low absorption path to the wavelengths
concerned, it would be found to emit radiation of sufficient
intensity from its axial region. It was with this expectation,
and at this point in the work being described, that the composite
X-ray tube detailed in Part II was designed. In fact, the
original external electrode tube (Section II-4) was similar in
dimensions to the previously used discharge tube which it was
now to supersede.

When first placed in position, the composite X-ray tube (and hence, external electrode tube) was equipped with a thin (0.25 mm) beryllium window. As it was not possible to introduce the ionisation chamber opposite the tube window, the pressure corresponding to maximum emission intensity was determined photographically. The gas used in this and all subsequent work was neon, the method of excitation and electrode arrangement being as before. Six exposures of ten minutes duration were taken at various pressures on a film placed against the tube window. The densities of the resultant spots of blackening were measured, and plotted against the corresponding pressures to give the curve of Fig. 14. It was concluded from this graph that a pressure of about 14 gm would be the best at which to attempt future experiments with neon.

The beryllium window having been replaced by a low absorption window of the kind to be described in Section II-7, several pinhole exposures were taken of the source. It was intended that these should confirm the expectation, earlier mentioned, that useful intensity was originating in the body of the discharge. They did not do so, the blackening produced in that region of the circular image corresponding to the tube axis being negligible. Now, the source under discussion was intended to be used, in conjunction with the spectrometer of Part III, to study neon radiation with wavelengths about 14.5 k\(\lambda\). But geometrical considerations showed that, if radiation of these wavelengths were to originate in the limited region of the annular source,
it could not possibly be Bragg-reflected in the spectrometer to
form a spectrum. On realising this fact, attempts were made to
alter the source shape. They are now described.

SECTION I-2  AN ATTEMPT TO ALTER THE SOURCE DISTRIBUTION

The first attempt to alter the annular distribution of intensity
was based on certain speculations, since rejected, concerning the
disposition of the electric field in the tube. The supposed
consequence was an outward deflection of the accelerated electrons
to produce maximum ionisation near the tube periphery. But it
was considered that such a diverging beam of electrons could be
returned toward the tube axis by the application of an axial
magnetic field. The situation envisaged is analogous to the
magnetic focusing employed in cathode-ray tubes.

Making reasonable assumptions as to the electric field
strengths and electron deviations involved, the magnitude of an
axial magnetic field, sufficient to constrain the electrons in a
helical spiral of small radius, was estimated to be 250 oersted.s.
Magnetic fields of this strength and greater, roughly uniform
over limited regions of the tube, were produced with horseshoe
magnets. To continuously observe the effect on the source
distribution of introducing the magnetic field, the tube was
fitted with the beryllium window and pinhole. A fluorescent
screen was placed opposite the latter, and, although not intense,
the circular image of the source was clearly visible on the
screen in a darkened room. An attempt to scan the fluorescent
image with a collimated photomultiplier tube was unsuccessful.
Various experiments were performed with the magnetic fields, and, when required, the screen would be replaced by a photographic plate. It was found possible to destroy the image completely, destroy it partially, reduce its intensity, but not possible to suitably alter its shape. The focusing idea was, therefore, abandoned.

**SECTION I-9**

**THE FINAL NEON SOURCE**

The next approach to the problem of the annular source distribution was through a consideration of the tube geometry. Presumably, if the tube diameter were reduced, radiation would still originate in a sheath near the walls. For a sufficiently small diameter, therefore, the circular source should degenerate into a spot on the tube axis. But, it was calculated, such a spot would correspond to too limited a wavelength range to be useful spectroscopically. The source required extension in a horizontal sense, that is, several overlapping spot sources. An equivalent effect was aimed at by the fabrication of a tube having an approximately rectangular cross-section. The longer side of the rectangular section was in a horizontal plane.

The replacement of the circular external electrode tube by that of rectangular cross-section is described in Part II. For the present, it is sufficient to note the dimensions of the new tube given in Fig. 15, while a reproduction of a pinhole exposure of the resultant source is shown in Plate II. This exposure was made through a thin window of the kind described in Section II-7. The intensity distribution achieved with the tube of rectangular
cross-section, was a major factor in the success of the spectroscopic study given in Part IV.

SECTION I-10 THE MECHANISM OF X-RAY EXCITATION IN THE ELECTRODELESS DISCHARGE

The experiments which have been described in Sections I-2 to I-6 were carried out to aid in the development of a suitable source of X-radiation. At the same time, they provide information which enables a possible explanation of the discharge phenomena to be put forward. Since the subject of electrical discharges is in general very complex, no attempt will be made to enter into detailed discussion of the various mechanisms involved. Recent developments in these matters are given in the review article by Llewellyn Jones (1953).

It is postulated that, for the discharge tube conditions of the present work, an equilibrium stage is reached wherein a positive space charge forms in the region of the earthed ring electrode. It extends about half-way along the discharge tube. There will be little field intensity or potential gradient in the body of this space charge. The evidence for its presence (we are not here concerned with its mode of origin) is given in Section I-3, where the independence of the voltage drop along the tube, and the position of the ring electrode over the L range 10-23 cm, is described. Likewise, the independence of tube current, and the position of the ring electrode over the L range 12-22 cm, described in Section I-4, is in agreement with the postulate of negligible potential drop (due to space charge) along about half the length of the tube, measured from the earthed ring.
It might be argued that the apparent constancy of the tube voltage, as the ring electrode is moved towards the sphere, would be explained if the internal impedance of the exciting generator were much less than the effective impedance of the discharge tube. In Figs. 5 and 6 of Section 1-3, if we may regard the gas pressure as being a measure of the tube impedance, the tube voltage is equal to the emf of the generator at zero pressure (infinite impedance). From the manner in which the tube voltage falls off with increasing pressure (decreasing impedance), it may be inferred that the generator impedance is, at least, not negligible.

In Fig. 16 (a,b) are given graphs depicting the idealised variation along the discharge tube of electric field and potential considered to result from the presence of the positive space charge. When the spherical electrode has a positive potential with respect to the grounded ring, the situation is as in (a), and similarly in (b), when the sphere potential is negative. The graphs, in which the field has been plotted as positive when it forces a positive charge towards the sphere, are to be regarded as indicating variations rather than actual magnitudes.

Consider the position when the sphere is positive. In the immediate neighbourhood of the earthed ring there will be a strong negative field, extending between the ring and the space charge, and falling to zero as the space charge is entered. The field remains at zero value throughout the space charge, and then assumes a negative value as the space charge terminates near the middle of the tube. This negative field increases steadily as the sphere is approached. The potential curve is derived from that
for the field, remembering that electric field intensity is equal to the space rate of decrease of potential.

When the sphere has a negative potential with respect to the grounded ring, there will be no electric field anywhere in the space charge region between the ring and the middle of the tube. The field intensity rises to a large positive value as the space charge falls off, and then steadily increases as the sphere is approached. The potential curve follows as before.

The space charge is regarded as producing an asymmetry in the discharge condition. Thus, when the sphere is positive, the space charge effectively decreases the field intensity in the region S to C (Fig. 16), whereas it has the opposite effect when the sphere is negative. When the sphere is positive, a number of the electrons accelerated in the region C to S will strike the closed end of the tube at S with energy in excess of the 40 electron volts known as sufficient to eject electrons from the glass surface (Gill and von Engel, 1948). There will have been no release of X-radiation in the region R to C (Fig. 16) during the positive cycle, due to the initial absence of free electrons. The electrons accelerated between C and S will have ionising collisions in the gas, but, for the present, it is their ability to establish a concentration of electrons in the vicinity of the tube wall at S which is emphasised.

When the field is reversed, the sphere becoming negative, the electrons concentrated at S fall through a substantial difference of potential, between S and C, and can experience ionising collisions with emission of radiation throughout the
entire tube length. It is the radiation given off during this negative cycle of operation which determines the character of the linear source distribution discussed in Section I-2 (Fig. 1, 2, and 3).

The intensity of radiation $I_1$, emitted from an elemental length of gas $dl$, at distance $l$ from the tube end $S$, depends primarily on two things. First, the probability of an ionising collision occurring in the length $dl$, and secondly, the total energy released as the result of such collisions. Now, the probability of a collision occurring may be taken as equal to the product of $N$, the number of electrons which have travelled the distance $l$ without colliding, and $\omega$, the collision frequency per unit distance. If there were originally $N_0$ electrons at $S$, it may readily be shown that

$$N = N_0 \exp\left(-\frac{l}{\gamma}\right),$$

where $\gamma$ is the average free path between collisions. The collision frequency is the reciprocal of $\gamma$, and independent of $l$.

The radiation penetrating the tube walls, and detected in the experiments of Section I-2, would be mainly from the continuous spectrum. In any elemental length $dl$, the total energy liberated as continuous radiation would be proportional to the square of the potential through which the colliding electrons had fallen. Referring to the curve of Fig. 16, b, it is assumed that the potential is a linear function of distance $l$, between $S$ and $C$, and thereafter remains constant. It follows that the total energy released as radiation in the element $dl$, at distance $l$ from $S$, is proportional to $l^2$, for $l \leq 11$ cm (say), and thereafter
retains a constant value. The intensity of radiation $I_1$, may therefore be expressed as

$$I_1 = \begin{cases} \frac{k}{1^2} \exp(-1/\gamma) \quad l \geq 11 \text{ cm} \\ \frac{k121}{11} \exp(-1/\gamma) \quad l > 11 \text{ cm} \end{cases},$$

where $k$ is a constant of proportionality.

If the proposed function $I_1$, is to be consistent with the experimental results, it must possess its maximum value when $l$ is about 5 cm, as shown in Fig. 17 where the curve of Fig. 1 has been replotted. The condition that $I_1$ be a maximum is found, by differentiation, to be that

$$l_{\text{max}} = 2 \gamma \quad (l < 11 \text{ cm})$$

In other words, the electron mean free path $\gamma$, in argon under the conditions of the experiment, should have been about $\frac{1}{2} l_{\text{max}}$, or 2.5 cm.

The concept of mean free path involves the definition of what is meant by a collision. In the present case, a collision implies that an electron penetrates the extra-nuclear field sufficiently to suffer a change in its energy. The cross-section for such a collision will be different to that involved in the derivation of electron mean free paths by kinetic theory. It will probably be greater, but of the same order of magnitude. We therefore assume that the previously defined $\gamma$ will be of the same order of magnitude, if somewhat greater than the kinetic electron mean free path, under the same experimental conditions.

The kinetic electron mean free path (itself an indefinite quantity, being a complex function of the electron velocity) has a value of about 1.4 cm at 20°C and 0.03 mm Hg. The latter is
the pressure at which the discharge tube was operated in the experiment of Fig. 1 (17). \( \gamma \) is thus of the order of 1.4 cm at 20°C. Since we may take the mean free path to be approximately proportional to absolute temperature, it is clear that a quite possible discharge temperature of about 200°C would correspond to a value for \( \gamma \) of some 2.5 cm. But this is the value for \( \gamma \), previously shown to be required, for the peak in the theoretical intensity distribution to agree with that found experimentally. We are therefore justified in writing that

\[
I_1 = \begin{cases} \frac{k l^2 \exp(-l/2.5)}{1 \times 11 \text{ cm}} \\ \frac{k 121 \exp(-l/2.5)}{1 \times 11 \text{ cm}} \end{cases}
\]

Having explained the peak in the intensity distribution, we may proceed to consider the mechanism of excitation in more detail. The \( I_1 \) function has been derived on the assumption that when an electron suffers a collision it makes no further contribution to the excitation of the radiation. This is not necessarily true. For, it is estimated that an electron may have sufficient energy left after a first collision to proceed and have further collisions. Again, even if an electron is brought to rest at its first collision, it may be re-accelerated to ionising velocity during the period the field is present. The nett results of these two effects must be to raise the level of the emitted intensity, particularly as \( l \) increases beyond the value of 5 cm corresponding to the maximum. (The maximum itself may be shifted in position, but not significantly).

The processes of excitation, so far discussed, occur when the sphere has a negative potential with respect to the earthed
ring. As mentioned earlier, some radiation must also be liberated in the region from C to S (but not from R to C), during the positive cycle. To avoid burdening the experimental results with too precise an interpretation, and since the chief characteristic of the intensity distribution has been reasonably explained, it is sufficient to assume that the average result of combining this positive cycle process with those of the preceding paragraph, is a roughly uniform increase in the emission intensity throughout the whole tube length.

We therefore finally express the theoretical intensity distribution by

\[ I_1 = \begin{cases} \exp(-l/2.5) & (l<11\ cm) \\ -121 \exp(-l/2.5) & (l>11\ cm) \end{cases} + K.\]

The constants k and K were found, on an arbitrary scale, by fitting the graph of the \( I_1 \) function to the experimental curve of Fig. 17 at the two points where \( l=5\ cm \) and \( l=15\ cm \). The \( I_2 \) function was then plotted as shown. The good agreement obtained between the theoretical and experimental curves indicates that the proposed explanation of the linear distribution of intensity is correct, at least in its broad features.

The effect of varying the pressure, on the discharge conditions, must be considered. According to the previous arguments, the peak in the linear intensity distribution should alter its position as the pressure (mean free path) is changed. Presumably, the postulated space charge would be destroyed, for extreme values of pressure. Because the linear intensity distribution was not recorded at pressures other than 0.03 mm Hg, the effect of pressure
variations will be discussed in terms of the total intensity leaving the tube window. For, if the proposed explanation of the linear intensity distribution is correct, the underlying concepts should be equally applicable in explaining the characteristics of the total intensity, described in Section I-5, (Figs. 11 and 12).

Consider the ionisation chamber of Section I-5 to be in position opposite the tube window. We idealise the physical situation, and estimate that the chamber's "centre of sensitivity" is at a distance $D = 40$ cm from the closed end of the tube. We assume that most of the radiation recorded is emitted during the negative cycle of operation, and this alone will be considered. Take an elemental cross-section of the gas, of thickness $dl$, at distance $l$ from the closed tube end. Then this element is at a distance $(D - l)$ from the recording chamber. When the mean free path has a value $\gamma$, let the intensity of radiation, originating at any point in the elemental length $dl$, contribute an amount $dI_\gamma$ to the chamber reading. The contribution $dI_\gamma$ will be proportional to $\exp(-l/\gamma)$, the "number" of electrons leaving the closed end and reaching the elemental section; to $\omega$, the collision frequency per unit distance; to $l^2$, the "energy" of the colliding electrons (we assume a constant potential gradient over the whole tube length); and inversely proportional to $(D - l)^2$, due to the operation of the inverse square law. Thus,

$$dI_\gamma = k \omega (l^2/(D - l)^2) \exp(-l/\gamma) \, dl,$$

where $k$ is a constant of proportionality.

Previously, we had that $\omega = 1/\gamma$, while $\gamma$ is itself inversely
proportional to pressure \( p \). Writing \( \gamma = b/p \), where \( b \) is a constant to be determined, we have that
\[
dI_p = (k/b)p \left( \frac{1^2}{(D-1)^2} \right) \exp(-lp/b).
\]
The reading of the ionisation chamber \( I_p \), corresponding to a discharge tube pressure \( p \), is the resultant of all the contributions \( dI_p \) summed over the \( l \) range \( 0-L \), where \( L \) is the tube length. Thus,
\[
I_p = \int_0^L (k/b)p \left( \frac{1^2}{(D-1)^2} \right) \exp(-lp/b) \, dl.
\]
In the treatment of the linear intensity distribution, the mean free path \( \gamma \) was assigned a value of 2.5 cm at a pressure of 30 microns (30\( \mu \) = 0.03 mm Hg). This gives \( b(=p\gamma) \) a value of 75\( \mu \) cm. Remembering that \( D=40 \) cm, and \( L=30 \) cm, the final expression for \( I_p \) is
\[
I_p = K \int_0^{30} p \left( \frac{1}{(d-1)^2} \right) \exp(-pl/75) \, dl.
\]
It is difficult to perform this last integration by usual methods. It has therefore been evaluated numerically, using Simpson's rule, for several values of \( p \). The relative values of \( I_p \) thus obtained are plotted in the graph of Fig. 18. On comparing this theoretical curve with the experimental curves of Figs. 11 and 12, we find good agreement as regards their general form. Thus, the theoretical curve has the required sharp rise as the pressure first increases from zero. It then passes through a maximum value and falls off less rapidly as the pressure is further increased.

The fact that the theoretical curve has its maximum at a pressure lower than the corresponding pressures for the experimental curves, and exhibits no structure, is not necessarily a discrepancy.
Obviously, the physical model taken involves many simplifications. It is noted that the laboratory pressure scale, used in the plotting of Figs. 11 and 12, was not very reliable for pressures lower than 5\(\mu\). Inability to differentiate the expression derived for \(I_p\) prohibits any worthwhile discussion of the factors which determine the value of \(p\), for which \(I_p\) is a maximum.

In Section I-1, the explanation proposed by Groven (1949) for the annular distribution of intensity when the tube is viewed axially, was rejected. An alternative explanation, which is consistent with all the known observations of this aspect of the discharge, is now proposed. It is simply that, at the frequencies concerned, the electrons are concentrated towards the outer boundary of the gas column (regarded as a conductor) by the inductance phenomenon commonly known as skin-effect. The majority of the accelerated electrons thereby have their ionising collisions in the gas near the tube walls, giving rise to the observed distribution of intensity. This skin-effect should occur irrespective of the cross-sectional shape of the tube; which is borne out by the pinhole exposure of Plate II, Section I-9.
PART II

THE COMPOSITE X-RAY TUBE
The work described in Part I led to the adoption of the electrodeless discharge tube (external electrode tube) as a suitable spectroscopic source. It was intended to measure wavelengths in the spectra of the gases studied by comparison with known wavelengths of lines from a solid emitter. Accordingly, it was necessary to provide for the excitation of the latter. To this end a composite X-ray tube was decided upon. It is, effectively, a hot filament high vacuum tube, into whose envelope is sealed a discharge tube. The arrangement is such that the "calibration" radiation from the internal electrode tube is emitted in the same direction as the "gaseous" radiation from the external electrode tube to fall on the analysing crystal.

As may be seen in the general arrangements of Figs. 19 and 20, the composite X-ray tube is attached to the vacuum chamber containing the spectrometer by means of a mounting tube. Power supplies for both the internal and external sets of electrodes are provided, together with the necessary vacuum, gas control and cooling systems. The design, assembly, and function of these various components are treated in detail in the following sections.

SECTION II-2 THE INTERNAL ELECTRODE TUBE

The envelope of the internal electrode tube, detailed in Fig. 21 (a, b), is constructed in pyrex glass. It was thoroughly annealed in a temperature-controlled oven. The double-walled
water-cooled nose A fits over a mounting tube (Section II-6) projecting horizontally from the spectrograph case. Its ground face then matching a similar one on the mounting tube, it is vacuum sealed in position with black wax. The vertical tubes B and C hold the cathode and anticathode respectively, the ground flanges facilitating the making of wax vacuum seals. The larger side-arm D is the pumping line, and is sealed directly into a cold trap. It has a tap and cone (not shown) let into it to allow the introduction of an ion gauge. Directly opposite, the other side-arm E is connected to the gas monitoring system of Section II-9. Finally, in the same horizontal axis as the tube nose, there is the tubular stub F, upon which is mounted the external electrode tube (Section II-4).

The anticathode structure is heavy, and, for this reason, is independently supported when adjusted in position and sealed into the tube envelope. Its details are shown in Fig. 22. The replaceable, 60° angled anticathode block A, is screwed to a base-block B. The latter is water-cooled internally by means of the concentric tube-system C, to which it is attached. This tube-system passes through a hole in the supporting disc D, and is fitted externally with a screw-thread E, and control nuts F. Since the bridge G is rigidly attached to the supporting disc, the screw mechanism allows the anticathode to be driven in or out. The vacuum is maintained by the inclusion of the bronze bellows H which yield an overall movement of 1 in. This enables the anticathode to be completely withdrawn when the external electrode tube is operating. The split sleeve I ensures
reproducible positioning of the whole structure with respect to the tube envelope. As a step towards avoiding unknown impurity lines in the spectra to be recorded, only copper was used in constructing those parts of the anticathode exposed to possible electron bombardment.

The cathode structure is shown in detail in Fig. 23. The base-block A is supported and water-cooled internally by the concentric tube system B. The latter passes through a sleeve let in the supporting disc C, allowing the block to be correctly adjusted, whereupon a soft-solder vacuum seal is effected at the sleeve end D. Two glass-insulated nickel leads E pass through the block-holes F and the corresponding sleeves G let in the supporting disc. Wax vacuum seals are made at the sleeve ends H. Base of attachment for the single-spiral tungsten filament I is obtained with the nickel collar and screw J; while the mounting of a focusing device has been allowed for by the provision of the hole-pierced ridge K. Reproducible positioning of the all-copper structure is ensured by the split sleeve L.

The cooling systems of the cathode and anticathode are connected in series, a single water supply being used. Since the anticathode is at positive high potential, with respect to the earthed cathode, its insulation is protected by the inclusion of sufficient glass tubing in the water leads.

SECTION II-3  ELECTRICAL ACCESSORIES - INTERNAL ELECTRODE TUBE

Power for the internal electrode tube is drawn from a high tension supply (McIntosh 1951). It employs a standard full-wave
rectifier circuit, and is capable of delivering 50 mA at potentials up to 5 kV. During operation, the anticathode is at positive high voltage with respect to the earthed cathode. At other times, the anticathode structure is made safe by the application of an earthing probe.

One side of the tungsten filament is connected to the cathode structure. Filament current is supplied from a 12 volt, 50 watt constant voltage transformer (Advance). It is controlled by two rheostats in series, the first being of 16 ohms (coarse), and the second of 3 ohms (fine).

SECTION II-4 THE EXTERNAL ELECTRODE TUBE

The external electrode tube was, originally, a thick-walled pyrex tube of circular cross-section, continuous with the envelope of the internal electrode tube. The reason for the replacement of this tube by one of rectangular cross-section has been discussed in Section I-9.

The construction of the final tube is shown in Fig. 15. It was first made in thickened pyrex, but, even when cooled by oil immersion, was unable to withstand the high-power valve excitation described in the next section. Silica was therefore resorted to and has been found successful, if oil-cooled. The tube A is provided with a cylindrical nose B which fits over the stub C in the envelope of the internal electrode tube. It is then vacuum sealed in position with wax D.

The sphere and ring electrode system of Part I is retained, the former being adjacent to the specially thickened end of the
tube, and the latter sliding on the circular nose.

The external electrode tube was immersed in an oil-bath for two reasons. In the first place, if power from the valve generator is supplied to the tube in air, sparking occurs near the closed end. The conductivity of glass increases rapidly with temperature, and, after a short time, the local heating is sufficient for sparks to penetrate the tube end. The dielectric strength of condenser oil is such as to prevent the initial sparking. Secondly, the majority of the power dissipated in the discharge tube appears as heat, and, since the thermal equivalent of the tube is small, a large and probably non-uniform rise in temperature results. Strains develop and cracking finally occurs. Even with oil-cooling, the useful lifetime of a pyrex tube, under such conditions, is short and unpredictable. On the other hand, provided the oil is circulated and cooled, a silica tube will withstand the operating conditions.

The arrangement of the cooling system is shown in Fig. 24. The oil-bath A is fabricated from sheet perspex, and measures 4 x 8 x 14 in. The bath is sitting on skids B, and when it is pushed forward the external electrode tube passes through a circular hole C in the front plate D. This hole is concentric with, and has clearance from, the nose of the tube so that the filled bath transmits no weight to the X-ray tube. The oil seal is effected by means of the neoprene O-ring E which is stretched and rolled onto the tube nose prior to positioning the bath. The O-ring is then pressed against the front plate by the large brass washer F which is secured on three studs G fixed to the plate.
The hot surface oil leaves the bath through the outlet H and passes to the heat exchanger I. In this, the oil goes through a copper u-tube, to which copper fins are attached, and about which water is circulated. The pump J then forces the cooled oil back into the bath through the inlet K. The rate of oil circulation is adjusted to its optimum value by controlling the pump speed. As the dielectric strength of condenser oil is very dependent on its water content, only well dessicated oil is used.

SECTION II-5  ELECTRICAL ACCESSORIES - EXTERNAL ELECTRODE TUBE

The intensity of X-ray emission from the external electrode tube is a function of the electrical energy delivered to it. It was found necessary, when making a spectroscopic examination of the radiation from neon, to increase the generator output.

As discussed in the previous report (Moore p. 84), the spark generator supplies power at the average rate of \( \frac{1}{2} \) NCV^2 Joules per second; \( N \) being the pulse frequency, \( C \) the storage capacity, and \( V \) the peak voltage to which the condenser is charged. Frequency requirements fixed the value of \( C \), while further increase in \( V \) was at variance with the condenser ratings. It was therefore decided to increase the pulse rate \( N \), by using a motor alternator to supply current to the charging circuit. Three such machines were experimented with, the spark generator being rebuilt to accommodate them.

With pulse rates up to 1600 per second, the required power output was obtained. However, the condensers became overheated
under the increased load placed on them, the estimated discharge currents being of the order of 100 A r.m.s. Various methods of cooling the condensers were attempted, but the spark generator was finally abandoned in favour of a valve-controlled generator.

A standard circuit is employed in the valve generator. Its three stages include a triode oscillator, followed by a tetrode isolating amplifier driving a power amplifier. The final tube is self-rectifying and operates Class C. With reference to Fig. 25, voltage amplification is still obtained by using a Tesla Coil A, which is inductively coupled by a simple link B to the tank coil of the valve generator C. The Tesla Coil is suspended vertically so that its spherical termination is immersed in the oil-bath in contact with the end of the discharge tube. It has been wound for an operating frequency of 1.7 Mc/s, a heavier gauge wire being used than in the former coils. This ensures that the coil may safely carry the normal operating power, estimated to be 135 W r.m.s., with a peak value of 675 W. Using the sphere-gap, the output voltage of the coil in air has been measured as 40 kV peak.

The inclusion of an ion trap between the external electrode tube and the thin window (Section II-7) was found necessary. Irrespective of the pressure, there are always some ions with free paths as long as the X-ray tube. If such an ion is accelerated in the body of the discharge, it may travel parallel with the tube axis and strike the window with considerable energy. This leads to discolouration, and finally destruction, of the aluminium coating on the window. To prevent this, any positive
ions or electrons that are accelerated beyond the ring electrode are removed by deflecting fields.

The trap consists of a magnetic field, to remove electrons, arranged horizontally and at right-angles to an electric field, to remove positive ions. The polarities of the fields are oriented to act in the same sense. In Fig. 25 the trap is shown in relation to the external electrodes. The electric field \( E \) was created by connecting a 400 V battery \( D \) between the cathode structure \( F \) and the anticathode structure \( G \) of the internal electrode tube. A limiting resistance was included in the battery circuit. For the magnetic field \( H \), a permanent magnetron magnet of 800 oersteds was placed with its poles on either side of the nose of the external electrode tube, adjacent to the oil-seal.

Although the improvised trap was not perfect, its incorporation greatly increased the working life of the X-ray tube window.

SECTION II-6 THE MOUNTING TUBE AND VARIABLE APERTURE

The construction of the mounting tube is shown in Fig. 26. It is essentially a copper tube \( A \) passing through, and attached to, a brass disc \( B \). The tube on one side of the disc is slit \( C \) in a manner which enables it to slide firmly inside the nose of the internal electrode tube. The other end of the tube is introduced through a clearance hole \( E \) in the wall of the spectrograph case (Part III). Four studs \( F \) in the disc pass through corresponding holes \( G \) in the case wall and bolt the mounting tube in such a position that its axis passes through the vertical pole of the spectrometer crystal. The two faces of the disc are ground to
match similar surfaces on the case wall and the nose of the X-ray tube, a vacuum seal being made with a fillet of black wax H. Two small pins I in the tube end towards the spectrometer, are to locate the X-ray tube window of the next section.

A further function of the mounting tube is to support the variable aperture now described. This aperture was provided, for reasons discussed in Section III-2, to allow limitation of the effective radiation source in both a horizontal and vertical sense. With reference to Fig. 27, two pairs of aperture blades A slide in guide rails B at right angles to each other on opposite sides of the face plate C. Each blade is independently controlled by a spring loaded adjusting screw D. The face plate is attached to the sleeve E which fits over the spectrometer end of the mounting tube, and is correctly oriented by means of a key entering the vee-shaped channel F. The material is brass.

**SECTION II-7 THE X-RAY TUBE WINDOW**

A window is required to separate the composite X-ray tube from the spectrograph case. Its primary function is to contain the gas in the external electrode tube at the correct pressure during the time of operation. Because of this, and the position it occupies between the sources of radiation and the spectrometer, it must possess the following properties:

(a) Freedom from leaks, so that the spectrograph vacuum will not deteriorate at the expense of the gas pressure.

(b) Ability to withstand the gas pressure over the working range experienced.
(c) Total aperture large enough for the source to fill the spectrometer crystal.

(d) Opacity to the intense light from the visible discharge, to prevent fogging of the photographic plates.

(e) Transparency to X-radiation in the region of 15 kK, to enable photographic recording of line spectra.

The last requirement is the most stringent, and immediately limits the possible window materials to elements (or compounds of elements) of low atomic number, which may exist as thin sheets. Those considered in this work were the metals beryllium and aluminium, the complex borate known as Lindemann glass, and the organic compounds celluloid and Formvar. They are now discussed with reference to the window properties listed. The expression percentage transmission $T$ will be used. It is defined by $T = 100 \left( \frac{I}{I_0} \right)$, where $I_0$ and $I$ are the intensity before and after penetrating the window thickness concerned.

Beryllium ($Z=4$) has a mass absorption coefficient at 15 kK of 1122 (Victoreen 1949). It has been prepared in air-tight sheets down to a thickness of 0.001 cm (The Davey Faraday Laboratory 1951), and is readily available with a thickness of 0.01 in. (Machlett Laboratories 1950). For the least thickness of 0.001 cm its percentage transmission $T$ would be 13. Apart from the considerable technical difficulties involved in its production, it was considered doubtful whether a window of sufficient strength over the required aperture could be made from beryllium in this form.

The same conclusion was reached with regard to constructing the window from aluminium foil. With a mass absorption coefficient
at 15 kX of 1739 (Victoreen 1942), the percentage transmission $T$ would be 10 for a window 0.0006 cm thick, which is the thickness at which holes are expected to appear. It is noted that both beryllium and aluminium have the desirable property of being opaque to visible light.

Lindemann glass, which contains boron, lithium and beryllium, instead of silicon, sodium and calcium, as in ordinary glass, is commonly used for X-ray tube windows and specimen tubes. A strong dome-shaped window may be made by blowing a thin bubble of the molten glass, and picking off the section required. Although no values for its absorption coefficient are known at 15 kX, some given at shorter wavelengths by Nachlett (1942) indicate that a negligible percentage transmission could be expected for a window thickness of the order of 0.001 cm.

Rogers (1952) has successfully used small aperture celluloid films with a thickness of about $10^{-5}$ cm. The mass absorption coefficient of this substance, measured by Neufeldt (1931), is 87,000 at 17.5 kX; the K-absorption edges for its constituents occurring at longer wavelengths. It is thus sufficient to assume that the absorption coefficient varies as the wavelength cubed (Victoreen 1942), in order to estimate the percentage transmission $T$ at 15 kX. This gives $T$ a value of 46 for a film of thickness $10^{-5}$ cm, which is much better than the values possible with the other materials discussed. However, the mechanical properties of celluloid in this form are not good. Rogers found that, even when used to cover a fine slit, they have a tendency to develop holes. Their use over a larger aperture was therefore not
attempted.

A material combining the low absorption property of celluloid with excellent durability and resistance to abrasion, was found in the polyvinyl formal resin known as "Formvar." (Kindly supplied by Canadian Resins and Chemicals Ltd., Montreal, together with information on its chemical and mechanical properties.) O'Bryan (1940) states that a Formvar film $2 \times 10^{-6}$ cm thick, has a percentage transmission of about 50 for a wavelength of 300 kx. This is in good agreement with the value for a similar thickness of celluloid at 300 kx, calculated from the results of O'Bryan (1932). Since no other measurements of absorption in Formvar are known, this agreement at 300 kx, together with the fact that the constituent elements of celluloid and Formvar are similar, led to the assumption that their percentage transmissions at shorter wavelengths would be of the same order. Accordingly, the values of the mass absorption coefficient for celluloid, given by Neufeldt, have been used for Formvar.

Being colourless, Formvar did not fulfill the requirement of opacity to visible light. This was remedied by coating the Formvar film with evaporated aluminium. From the experience of electron diffraction workers (Edwards & Co. 1951), it was expected that a layer $10^{-5}$ cm thick would be opaque to light. At 15 kx the percentage transmission of this thickness of aluminium is estimated to be 95, so that the absorbing effect of the aluminium is small in comparison with the Formvar. Aluminium was preferred to beryllium in this connection because of its relative ease of evaporation.
To prepare the Formvar films, a rigorously cleaned glass plate is momentarily placed with one face in contact with the surface of an 0.5 per cent solution of Formvar in chloroform. The plate is then suspended vertically to drain, and left to dry in air. It is then held for a few seconds in the vapour above a beaker of boiling water before being introduced at an angle beneath a clean water surface. With perhaps a little preliminary encouragement at its leading edge, the film will float off onto the water surface, free from wrinkles. If the film is not given the preliminary steam wetting it will adhere tenaciously to the glass. With this method, films seven centimetres square, of thickness about 1.5 x 10^{-5} cm, have readily been made.

The mounting of the film is illustrated in Fig. 28. A smooth metal scoop A, containing a circular hole of larger diameter than the required window, is used to lift the film B off the water surface. The scoop is reversed C and carefully lowered over the window mount D so that the latter passes through the hole and receives the dry side of the stretched film on its upper surface. The window mount E, which is a brass washer having the indicated dimensions, has a little of the original Formvar solution smeared on its surface before receiving the film. The mounted film is dried in a vacuum dessicator. Experience shows that the longer a Formvar film is in contact with water, the more likely it is to finally deteriorate. Therefore, the above operations, starting from the floating of the film onto the water surface, are carried through as quickly as possible.

When dry, the mounted film is taut, and exhibits interference
colours which allow an estimation of the thickness. A colour-thickness scale for Formvar has been calculated at the Dominion Physical Laboratory (N.Z.) with reference to an article by Kubota (1950). This indicates that the films made have a probable thickness range of \((1-2)\times 10^{-5}\) cm.

The aluminising of the window, to render it opaque to light, is performed in the vacuum evaporating chamber described in the next section. The required opacity is determined by looking through the window at the white-hot evaporating source, and coating until extinction occurs. The corresponding thickness of aluminium is found by weighing. One of the batch of windows being made is selected as a control, and secured at the centre of a 7 cm diameter mica disc. This has previously been cleaned and thoroughly dried in the vacuum dessicator. The combination is accurately weighed before introduction to the evaporating chamber, where the requisite coating of aluminium is received. The localised source of aluminium being at about 20 cm from the disc, the assumption is made that the latter is uniformly coated over its whole area. A determination of the increase in weight of the combination allows the aluminium thickness on the window to be calculated. The mica disc is used to obtain a weight increase large enough to be accurately measured. It was found that the source filament was not visible through an aluminium layer of thickness \((8-2)\times 10^{-6}\) cm.

The finished window is located on the spectrometer end of the mounting tube by means of two small pins which pass through corresponding holes in the window mount. A vacuum seal is then
made by running a fillet of shellac D around the window edge, as indicated in Fig. 26. During operation of the external electrode tube on long exposures, it was found that the brilliant visible discharge did cause some fogging of the plates. This was completely remedied by placing a second window in series with that making the vacuum seal. Such a combination is estimated to have a transmission value of \( T = 0 \) at 15 keV.

SECTION II-5  A VACUUM EVAPORATING UNIT

A vacuum evaporating unit was built to enable the X-ray tube windows to be aluminised. Apart from this, the design was kept sufficiently general for the unit to have other applications. Useful discussions on the theory and technique of vacuum evaporating are given by Strong (1938) and Holland (1948).

The evaporating chamber is shown detailed in Fig. 29. It is a steel cylinder A having identical flanged ends B. The latter are grooved for the O-rings C, which make vacuum seals when the thick glass windows D are in position. The windows are carried in aluminium frames E, and are held by three studs and knurled nuts F. An O-ring sealed flange G allows the chamber to be coupled directly to a type 102 (Edwards & Co.) oil diffusion pump. Four symmetrically placed ports (H, I, J and K) hold the object being coated, the evaporating source, and any controls such as pressure gauge and screens. These various accessories are constructed so as to be interchangeable between the identical O-ring sealed ports.

For aluminising the Formvar windows, the source was an
aluminium-coated tungsten filament, drawing about 30 A at 5 V r.m.s. Three 5.5 in. lengths of 20 mil tungsten wire were twisted together, and wound into a fine coil filament about 1 in. long. The filament was then mounted on 2 mm tungsten rods which pass through glass-metal seals in a supporting disc. The latter vacuum-seals on the O-ring of the chamber port being used. Short lengths of aluminium wire were crimped around each turn of the filament. When the filament temperature reaches the melting point of aluminium, the latter is drawn into the interstices of the multiple winding by surface tension, and an efficient source of aluminium vapour results.

The windows to be coated were attached over holes in a mica disc. This was introduced into the chamber and clamped to a rotating arm operated through one of the vertical ports. Rotation of the windows in the vapour stream favours complete coverage, as does the bevelled edge of the window mount. It was found that aluminium will not adhere to the Formvar film where there are localised surface imperfections, such as drying stains. The windows were therefore reversed and coated on both sides to reduce the possibility of completely transparent spots. The filament was fitted through the other vertical port, while the side ports carried a pressure gauge and a rotating shutter. This shutter was interposed between source and window being coated, during the initial fusion of the aluminium, and collected any impurities expelled from the source during that time.

The assembled evaporating unit is shown in Plate III.
SECTION II-9  THE PUMPING AND GAS MONITORING SYSTEM

The arrangements for evacuating the composite X-ray tube, together with those for introducing gas to the external electrode tube, are not independent of the pumping system for the spectrograph case. Accordingly, they are described in terms of Fig. 30 which shows the complete vacuum system.

The X-ray tube A is pumped by a type 03B (W. Edwards & Co., London) oil diffusion pump which is backed by a type 1S150 (W. Edwards & Co.) rotary oil pump. A baffled cold trap B, of a kind commonly used in this laboratory (adapted from More, Humphreys, and Watson, 1937), is included on the high-vacuum side of the diffusion pump. It is cooled with a mixture of acetone and solid carbon dioxide, and, by removing condensable vapours, reduces contamination of the anticathode and allows a better ultimate vacuum to be attained. The steel tap C offers a low pumping impedance when open, and enables the X-ray tube to be isolated from its pumps. Between the diffusion and backing pumps is a junction-box of large volume D, which, besides being a means of joining the various pumping lines which converge at that place, allows the backing pump to be turned off for short periods without affecting the diffusion pump operation. This has often been found convenient.

Pressures in the X-ray tube are measured by the Pirani gauge E (W.G. Pye & Co., Cambridge) and the ionisation gauge F (Pye & Co.). When not in use, the ionisation gauge is isolated by the tap G. This avoids the need for frequent degassing of the gauge elements,
with a consequent increase in working life. If required during leak testing, a gauge may be sealed to the junction-box through the cone fitting H. This connection also allows the independent use of the backing pump for pumping auxiliary equipment, such as the vacuum dessicator mentioned in Section II-7. The discharge tube I allows a quick check on the backing system pressure during pumping operations. Its colour sensitivity to different gases has been helpful in the present work.

The monitoring system, for introducing the gas under study to the external electrode tube, is constructed in glass. The gas is supplied, at atmospheric pressure, in a one litre flask J. This is sealed to an arrangement of taps, separated by various volumes, which allows the gas to be expanded and tapped off at lower pressures. The sequence of operations is varied, according to the gas pressure available in the flask, and that required in the tube. A drying chamber L is included, while the by-pass line M avoids the high impedance of the small taps N during the initial evacuation. If the gas pressure in the external electrode tube is too high, for any reason, it may be reduced by pumping through the small fine-control tap 0 which by-passes the closed tap C.

It is noted that during operation of the external electrode tube, the gas occupies a volume, which includes the cold trap, of about 1.5 litres.

The spectrograph case P (Part III) is pumped by a type 203 (W. Edwards & Co., London) oil diffusion pump, equipped with a baffle valve Q. This pump is connected directly, with an O-ring seal, to the underside of the case, and shares the backing system
already described.

Now, if the case and tube are both evacuated from atmospheric pressure through their individual pumping systems, the volumes and pumping speeds are such as to cause a large pressure difference, directed from case to tube. Conversely, if air is admitted through the backing inlet R, when the apparatus is evacuated, a large pressure difference is established, directed from tube to case. In either eventuality, the X-ray tube window S would be destroyed. To avoid this, a separate vacuum line is used for the initial evacuation from atmospheric pressure, and for the final re-admission of air. It is shown in broken outline in the figure. The tap T is fitted with a scale, marked in time settings. It was calibrated, for both evacuation and admission of air, with a differential manometer connected across the tube window. Thus, at no time during the discontinuous opening or closing of this tap, does an intolerable pressure difference appear across the window. During evacuation, and upon reaching a sufficiently low pressure, taps T, U and V are closed, and pumping is continued, by the previously heated diffusion pumps, through taps C and Q. An advantage of the extra vacuum line is that no time is lost waiting for the diffusion pumps to cool before air can be re-admitted. A second discharge tube W, allowing quick pressure checks, is conveniently placed in the line.

The actual arrangement of the complete vacuum system may be seen in Plate IV. The working pressure reached in the spectrograph case is about $10^{-4}$ mm Hg, and in the internal electrode tube is about $10^{-5}$ mm Hg.
SECTION II-10  THE ALIGNMENT OF THE COMPOSITE X-RAY TUBE

A description of the principal adjustments involved in setting up the composite X-ray tube may be followed in Figs. 19 and 20, and in Plate IV.

A telescope was arranged to swing in a horizontal arc, so that, with the spectrograph case open, it was focused on both the centre of the spectrometer crystal and the anticathode block seen through the glass envelope. The block was then adjusted in position by the screw mechanism until the vertical projection of its 60° angled face was seen to symmetrically cover the crystal area. With a pinhole attached over the tube window, a photograph was taken of the anticathode face during the operation of the internal electrode tube. For this, the cathode structure was given a temporary setting, with its filament horizontal, and parallel to the tube axis. It was thus found possible, without recourse to focusing the electron beam, to obtain sufficiently uniform emission from the majority of the anticathode face. Further, the projected source area adequately filled the crystal aperture. The cathode setting was thereupon made permanent, and the screw mechanism on the anticathode structure calibrated, enabling the anticathode block to be withdrawn and returned at will.

The alignment of the external electrode tube, in a horizontal plane through the crystal centre, was checked with the telescope of the previous paragraph. As explained in Part III, the spectrometer arm may be set to be identical with the X-ray tube axis. This was done, and, by referring to the arm, a straight
edge was established parallel to the X-ray tube axis and extending alongside the external electrode tube. The latter was then adjusted into the X-ray tube axis by referring to the straight edge. Finally, the external electrode tube was rotated about the X-ray tube axis, bringing its rectangular section into the horizontal plane, as indicated by a spirit level. When these alignments were satisfactory, the external electrode tube was sealed to its mounting stub with black wax.
PART III

THE BENT CRYSTAL VACUUM SPECTROGRAPH
There are, in practice, two mechanisms generally used for the diffraction of X-radiation, both of which are employed in spectrometers for the production of spectral lines. The first is the naturally periodic structure of a crystal, and the second is the periodicity artificially imprinted on a ruled grating. Since the diffraction constant of a crystal is much smaller than that of a grating, the dispersion in crystal spectrometers is always superior to that of the grating instruments. However, crystal spectroscopy at present has an effective upper wavelength limit of about 20 kX, this being twice the lattice constant of mica. There are several long chain compounds with larger X-ray spacings than mica (Bisset and Ibll 1954), but their use for measuring wavelengths presents difficulties. For work below 20 kX therefore, a crystal spectrometer is to be preferred, while for the longer wavelengths a grating spectrometer is necessary. Now, the primary purpose of the present spectrometer was to enable wavelength measurements, at about 15 kX, to be carried out in the K-emission spectrum of neon. Alternatively, if this proved unsuccessful, it was to be suitable for studies of the L-spectra of certain metals in the same wavelength region. A crystal spectrometer was therefore decided upon.

The choice of a particular type of crystal spectrometer is not independent of the radiation source, nor of the method of detection with which it is to be associated. In the present case it was the nature of the source which effectively determined
what was required of the spectrometer and recorder, in order to form a combination capable of attaining the experimental end. Absorption at the wavelengths to be examined was such as to require the spectrometer to be of a reflection type, and to be enclosed in a vacuum chamber. The discharge source was known to be extended, and probably the amount of energy emitted as line radiation would be small. A large aperture spectrometer, having focal properties, and preferably without narrow slits, was therefore indicated. Also, the weak source favoured the use of a detector with the ability to integrate over any length of time. An instrument which fulfills all of these requirements is the photographically recording bent crystal spectrometer of the reflecting type. It is discussed in the next section. It is noted that the photographic method of detection has the further desirable properties of providing permanent records, and of being able to record an extended spectral region. Its simplicity is deceptive, since the recorded information can only be made available through the use of a precision comparator or microphotometer.

SECTION III-2 THE JOHANN APPROXIMATION TO AN EXACT FOCUS

Methods of focusing the diffracted rays from a monochromatic divergent beam in order to gain intensity, have been considered from the earliest days of X-ray spectroscopy (e.g. M. de Broglie 1914). An exact theoretical solution of the problem was first conceived by J.W.M. Dumond in 1927, for both the transmission and
reflection types of bent crystal spectrometer. However, he considered that its practical realisation would prove too difficult, in the case of a single crystal, and so applied the theory to the multiple crystal spectrometer instead (Dumond and Kirkpatrick 1930). Johann (1931) built the first approximate form of Dumond's ideal reflection spectrometer, while the first approximate form of the ideal transmission type was achieved by Cauchois (1932). Finally, an exact form of the bent crystal reflection spectrometer was built by Johansson (1932).

It is convenient in crystal diffraction work, when the main interest is in the diffracted beam forming the intensity maximum, to formulate the discussion in terms of the reflection concept due to Bragg (1912). Thus, reflection occurs in accordance with Bragg's Laws:

\[ n\lambda = 2d \sin \beta, \]
\[ \beta = \beta'. \]

These are a special interpretation of the condition for constructive interference of the scattered waves from a regular space structure.

Consider the two-dimensional reflector PQ shown in Fig. 31 (a), where S is a point source of radiation, and I is a point image of the wavelength \( \lambda \) in the spectrum of source S. The requirement is to find a curve, say \( y = f(x) \), such that a flexible crystal constrained thereto, would Bragg reflect X-radiation from S and focus it selectively at I. But Bragg reflection imposes two conditions at every point of the surface PQ, which cannot be met by any single equation of the form \( y = f(x) \), viz.,

(i) at all points on the surface, the angle of deviation \( 2\beta \)
of the reflected beam must be constant, and equal to
\[ 2\theta = 2 \sin^{-1} \left( n \lambda / 2D \right), \]
and,

(ii) at all points on the surface, the angles of incidence and reflection referred to the atomic planes must be equal, i.e.,
\[ \theta = \theta'. \]

The first condition is satisfied only if the curve is a circular arc through S and I of radius \( \frac{3I}{2 \sin 2\theta} \), whereas the second condition is not satisfied by this solution. But, in fact, the two conditions are not imposed on the same surface since it is not essential that the atomic reflecting planes be parallel to the crystal boundary. Thus, if matters are arranged as in Fig. 31 (b), the ideal problem is completely solved. Here, the reflecting boundary of the crystal coincides with part of the circle of centre 0, while the atomic reflecting planes are bent to coincide with concentric circles centred on the circumference of circle 0, at the point 0. For all points of the surface condition (i) is fulfilled, since the arc SPI measures twice the angle of deviation of the beam; while condition (ii) is fulfilled, since the equal arcs SC and IC now measure the complements of the angles of incidence and reflection on the atomic planes, whose normal is PC. The locus of the focused spectrum is evidently the circle of centre 0.

The above solution is the one used by Johansson, and it involves the difficult operation of grinding or polishing the crystal to the required profile. This may be avoided if advantage is taken of the fact that condition (i) fixing the crystal boundary is less stringent than condition (ii) fixing the direction of the
atomic reflecting planes. It follows that if the arc of the focal circle covered by the crystal is not too large, the focal error introduced can be kept sufficiently small for practical purposes. This is the approximation introduced by Johann, and is the basis of the instrument used in the present work.

Let Fig. 32 depict a section parallel to the focal plane of the spectrometer, there being no focusing in the vertical sense. The bent crystal UV, of aperture \( \delta \), has a radius of curvature \( R \) and is centred at \( P \). Its vertical pole touches the focal circle of centre \( O \) at the point \( T \), on the focal diameter \( PT \). Radiation from the source point \( S \) is Bragg reflected with angle \( \phi \) at \( T \), to cut the focal circle in \( I \). Now, the arcs \( SP \) and \( IP \) subtend peripheral angles all of equal value \( \theta = (90^\circ - \phi) \). The Bragg angle is therefore formed again between \( SU' \) or \( IU' \) and the perpendicular to \( FU' \) at \( U' \). But, since the crystal is not ground, there is no reflecting surface at \( U' \). The ray path is accordingly shifted (approximately parallel) a small amount to use the reflecting element at \( U \); and similarly for \( V \) on the remote side of \( T \). Consequently, the line produced at \( I \) is broadened in the same sense in both cases, that is, towards short wavelengths. The long wavelength edge remains well defined. This broadening is the focal defect \( b' \) first calculated by Johann. For the instrument described in the next section, where the image is produced on a photographic plate set perpendicular to the reflected beam, it is given by

\[
b' = \left( \frac{\delta^2}{8R} \right) \cos \phi.
\]

A second aberration, not considered by Johann, is due to the
finite height of the radiation source. This introduces the possibility of Bragg reflections occurring for rays not travelling in the horizontal or focal plane of the crystal. A broadening of the image named the height defect results, this time in the direction of longer wavelengths. For a source of height \( h \) at distance \( a \) from the crystal, Sandström (1933) gives an expression for the height defect \( b'' \), in the form \( b'' = \left( \frac{h^2}{4(R+a)} \right) \tan \phi \). He gives no derivation, and it has not been found possible to reconcile his expression with the instrumental geometry involved. The derivation of an alternative expression is given in Appendix I. It involves the crystal to image distance \( d \), and gives the height defect as

\[
b'' = \left( \frac{h^2d}{2(a+d)^2} \right) \tan \phi
\]

Another possible defect in the recorded image, due to the oblique passage of the rays through the photographic emulsion, is avoided by the perpendicular mounting employed (Section III-3).

When designing the spectrometer, the requirements of intensity were considered equally with those of dispersion and resolution. The luminosity of the instrument, or its ability to utilise the available intensity, is a function of the same geometry as that determining the magnitudes of the focal and height aberrations. The condition that there be maximum luminosity for minimum aberration breadth was derived as shown in Appendix II. It may be written in the form

\[
\delta = 2h\left( \frac{R \tan \phi}{(a+R \sin \phi)} \right)
\]

\[
b' = b''
\]

This result indicates that for a given crystal radius \( R \), there is
an optimum value for the ratio \( \delta/2h \) of crystal aperture to source height, at each reflection angle \( \phi \) \( \sin^{-1}(\lambda/2D) \).

In Appendix II the total "instrument intensity" \( I \) is expressed by the relationship

\[
I = k \left( \frac{6l}{R^2} \sin^2 \phi \cos \phi \right),
\]

which indicates that \( R \) should be of small value. But this is opposed to the requirement, for accuracy in measuring wavelengths, that the inverse linear dispersion \( (\partial \lambda/\partial x)_x \) be small. For the latter is later shown (cf. Section IV-6) to be inversely proportional to \( R \), which should therefore be large. Bearing these facts in mind, and having regard to the crystal radii employed by other workers, a compromise value for \( R \) of 50 cm was chosen.

Having fixed the value of \( R \), it was considered desirable to have means of controlling the magnitudes of the aberrations \( b' \) and \( b'' \). For they should be negligibly small, in comparison to natural line width (Part IV), at all wavelengths. This was done by providing the variable aperture of Section II-6 and the knife-edge to be described in the next section. These allow the effective values of \( 2h, \delta, \) and \( l \) to be altered, with the possibility of retaining the previously mentioned optimum value for \( (6/2h) \).

For the wavelength settings employed in the spectroscopic work of Part IV, \( b' \) and \( b'' \) had the sufficiently small values 0.0015 cm and 0.0005 cm respectively.

**SECTION III-3 THE SPECTROMETER**

The incorporation of the previously outlined method of
focusing into a spectrometer may be achieved in various ways. Two main types of spectrometer mounting are recognised, depending on whether a fixed or rotating crystal is employed. Since the former method involves both source and detector having equivalent rotations about the crystal pole, in order to give the required variation in reflection angle, it was rejected in favour of the rotating crystal method. In this, either the source or the detector may remain fixed.

Whether to move the source or the detector was next considered. If an orthodox X-ray tube only were involved, the method of the rotating source, as used by Haglund (1942), has many obvious advantages. It allows the use of a small vacuum chamber and facilitates a rigid construction. However, its adoption in the present case was rendered impracticable by the nature of the source and its accessories. Consequently, the mounting chosen for the spectrometer was that of the rotating crystal with fixed source and moveable detector. In this case the focal circle revolves about the crystal pole which is on its circumference, according to the angle of reflection.

The photographic detector may take the form of either a curved film holder or a plane plate holder. The former, as in the case of Shearer (1935), would have half the crystal radius of curvature and pivot about the crystal pole. Its principal virtue is the ability to record an extended range of "in focus" wavelengths. This is offset by the difficulty of generating its surface with exactly double the curvature of the bent crystal, its relatively complex positional adjustment, and the undesirable
vacuum properties of film. On the other hand, a plane plate holder sliding at right-angles on an arm which rotates about the crystal pole is readily adjusted into position. Further, it allows the use of photographic plates which are easily manipulated and possess good vacuum properties. The latter method of detection was therefore chosen.

The components of the best crystal spectrometer are shown in Plate V, while Plate VI is a view of the assembled instrument. Figs. 19 and 20 indicate its operating position with respect to the vacuum case and composite X-ray tube. In building the spectrometer emphasis was placed on the achievement of certain desirable features. Thus, without sacrificing mechanical stability, the construction is independent of the rest of the apparatus. This allows the spectrometer to be easily removed from the vacuum case, while means are provided for its exact location when returned. Further, being optically self-consistent, the operating controls are simple and placed in readily accessible positions. In brief, the instrument consists of a photographic plate holder revolving on an arm about an axis identical with that of a rotating crystal mounting.

Details of the rotating mechanism are shown in Fig. 33. The steel base plate A supports a rigid cylindrical steel column B whose interior and exterior surfaces have been accurately turned about a common axis. An accurately machined cylindrical brass rod C, carrying the crystal table D, is located axially inside the column by means of the pair of precision tapered roller bearings E (SKF 30302). The figure indicates how these bearings
are held in place with the lock ring F, after the rod is introduced through a hole in the base plate, and before the crystal table is secured with screw and taper pin. Another pair of precision bearings G (SKF 30210) allows the substantial brass cylinder H, whose interior is accurately turned, to be placed outside and concentric with the supporting column. This outer bearing is constrained by the lock ring I. The internal boring of the brass cylinder was done with reference to a previously machined flat J on its outer surface. This flat thus became a tangent plane accurately parallel to the instrument axis, and upon it is bolted the arm K.

It is believed that the use of tapered roller bearings is an innovation for spectrometers of this type at least. The alternative and common practice would have been a double cone bearing. Since the components of a cone bearing must be lapped together, it is doubtful whether a double cone bearing could have been realised with a unique axis perpendicular to the plane of rotation of the arm. Further, the operation of the instrument involves constant use of the bearings, particularly the arm bearing. A greased cone would inevitably require attention for satisfactory performance, an adjustment possibly involving the dismantling of the instrument. On the other hand, the precision bearings used have been accurately fitted to the other components by means of cylindrical machining alone.

Both the arm and the crystal table may be rotated to any desired setting and locked in place by the controls shown in Fig. 33. For the arm, the outer brass cylinder carries a slip
ring L riding on the supporting disc N. When the hand screw 0 is turned it forces a brake pad P against the brass cylinder causing the slip ring to grip it firmly. Fine control and locking of the arm is then possible with the two hand screws Qa and Qb carried by supports Ra and Rb fixed to the base-plate, and working against the control post S. If several exposures are being taken at the same arm setting, it is necessary to rotate the arm to the case front to reload the plate camera. To avoid resetting the arm each time, the support Ra (which is secured by a pin and hand screw not shown) is easily removed, allowing the arm to be rotated and then returned exactly with the control post hard against the set hand screw Qb.

Control of the crystal table is effected in a similar manner. The central rod carries a slip ring T riding on a supporting disc U. A brake pad V is driven by the hand screw W turning in the control post X. The latter is positioned and locked by the fine control screws Y carried by the supports Z.

As indicated in Fig. 33, the fixed cylindrical column B carries a circular brass disc. Its outer edge is bevelled at a 45° angle to match the similarly bevelled edges of the crystal table and the outer arm cylinder. The clearances between the fixed and rotating parts of the conical frustrum thus formed are small, so that a common scale could be engraved on the frustrum surface. This allows the angular position of the arm and crystal table to be determined with reference to the fixed central scale. Now, in the spectrometer mounting employed, the direction of the incident radiation is fixed and passes through the axis of rotation. Thus, if the reflecting face of the crystal is set at a Bragg angle $\theta$ with respect to the incident direction, the rotating arm must be set at an angle $2\theta$ with respect to that
direction. This fact is the basis of the manner in which the scales have been engraved and oriented. Details are given in Fig. 34. The central fixed scale is a vernier facing the front of the spectrometer case. It has a $19^\circ$ arc divided into 20 equal parts allowing the rotating scales to be read to $(1/20)^\circ$. The crystal $(0)$ scale is graduated in degrees from $0^\circ$ to $90^\circ$, while the arm $(2\theta)$ scale is graduated in degrees from $0^\circ$ to $180^\circ$. The angles are numerically indicated every ten degrees.

The rotating arm which carries the plate camera is a steel fabricated T-sectioned cantilever. Details are given in Fig. 35. The face of the butt A is a smooth plane accurately at right angles to the surface of the cantilever B. Four $\frac{1}{4}$ in. bolts passing through the clearance holes C attach the arm to the tangent plane on the outer cylinder. When adjusted so that the surface of the cantilever is in a plane at right angles to the instrument axis, the bolts are tightened, stability being ensured by two taper pins driven through the butt into the cylinder. These taper pins (one of which is visible in Plate VI) are fitted with withdrawal nuts, and enable the arm to be removed and accurately replaced in position. The surface and one edge D of the arm are smooth planes, accurately at right angles to each other. They determine the position of the plate camera which bears upon them. The central region of the surface has been relieved to avoid the difficulty of hand-scraping over a large area. A $45^\circ$ bevel E on the other edge of the arm will be discussed later in terms of the plate camera. Along this edge also, is engraved a linear scale F reading from 10 cm to 50 cm,
with the 10 cm mark at a distance of 14.032 cm from the instrument axis. The scale faces the front of the spectrometer case and indicates the distance from crystal to photographic plate. With the arm in place, the axis of rotation was identified with the bit of an engraving machine which then drew a fine centre-line along the arm surface. It is referred to in the next section.

Simplicity in the construction of the plate camera was demanded by the need to load it in a darkened room. It is made in brass and shown detailed in Fig. 36. The carriage A slides on the rotating arm which passes through a channel B. A 45° angled wedge C, driven by two screws D against a corresponding bevel on the arm, pulls the smooth underside of the carriage against the arm bearing surfaces previously discussed. The face-plate E, having a rectangular aperture F, is attached to the carriage so that the face remote from the crystal is at right angles to both bearing surfaces of the arm. Two guide rails G and H locate the photographic plate whose emulsion is pressed against the face-plate by two phosphor bronze springs I. These springs are mounted on a rod J passing through bearings K and fitted with a knob L. When the rod is turned sufficiently to press the plate into position, the compression spring M causes a pin N attached to the rod to locate itself in a slot in the bearing O. The photographic plate is released by pulling the knob to clear the pin from its slot, and then rotating outwards. A vernier P attached to the carriage allows the arm scale to be read to 0.01 cm. When the camera was assembled the distance from the vernier zero to the near surface of the face-plate was measured as 4.032 cm on a precision travelling microscope.
For this reason the 10 cm mark on the arm scale was placed at 14.032 cm from the instrument axis, enabling the distance from crystal pole to photographic emulsion to be read off directly.

The assembled crystal holder is shown in Fig. 37. The crystal is constrained to the correct curvature by being clamped between two steel blocks A and B having a concave cylindrical and convex cylindrical profile respectively. The profiling of the blocks was carried out according to the method described by Dumond, Lind, and Cohen (1947). Considerable care was taken over this operation, particularly in the final stage where the blocks are lapped to exactly the same radius of curvature. It follows that, when the blocks are separated by the crystal thickness, it is actually the convex block which determines the crystal curvature. For the concave block will touch at the crystal extremities first and provide a symmetrical bending moment about the pole of the convex surface. A special steel (Joseph Gillett & Sons, Sheffield) of a kind used in gauges, dies, etc., was chosen for the crystal blocks. All machine work on the blocks, except the actual profiling, was carried out before the steel was hardened by the recommended heat treatment.

Two steel brackets C attached to the convex block carry screws D whose hemispherical ends bear on the steel plate E. The latter is separated from the back of the concave block by two ball bearings F seated in conical holes. This device enables the convex and concave surfaces to be pressed together with an even distribution of pressure over the crystal area. To the base of the concave block is attached a steel foot G whose smooth
undersurface sits on the correspondingly plane surface of the brass base plate H. The position of the block assembly on the base plate is determined by two pairs of screws at right angles to each other. One pair I are balanced by the heavy phosphor-bronze leaf spring J being held against a block K. The other two screws L are opposed to each other at opposite ends of the blocks. A heavy phosphor-bronze leaf spring M bearing on the upper surface of the block foot ensures positive contact between the block assembly and base plate.

Radiation reaches the crystal through a horizontal channel N cut in the convex block. It permits a minimum angle of reflection of about 14°, the crystal area exposed being a square of side about 1 cm. A combined knife-edge and screen O is provided to limit the crystal aperture and check scattered radiation. It fits in a dove-tail slide in the block K attached to the base plate, and is locked in position at right angles to the crystal by the screw P.

Three screws Q pass through clearance holes in the base plate and firmly attach the crystal holder to the rotating crystal table R. Their specially shaped heads sit in spherical cavities in the base plate, and, working against the compression springs S, give fine control in the adjustment of the horizontal plane of the crystal assembly.

A rigorous analysis of the mechanics of the operations involved in generating the profile of the cylindrical block yields a value for its radius of curvature of 50.03 cm with an error not greater than ± 0.05 cm. This has accordingly been accepted
as being the radius $R$ of the crystal when bent between the blocks. Thus,

$$R \text{(Radius of bent crystal)} = 50.03 \pm 0.05 \text{ cm}.$$  

SECTION III-A  THE ADJUSTMENT OF THE SPECTROMETER

Certain spatial relationships are assumed to have been established between some of the spectrometer components by the precision of the construction. They are:

(a) That the crystal table and arm bearings are concentric with a common axis of rotation.

(b) That the surface of the arm moves in a plane at right angles to the axis of rotation.

(c) That the centre-line engraved on the arm is a radius of rotation.

(d) That the emulsion plane of the plate is always parallel to the axis of rotation and moves in a direction parallel to the arm centre-line.

The remaining adjustment was to place the bent crystal in its holder at the correct position on the crystal table. This adjustment was achieved in the five steps (i-v) now described with reference to Fig. 38.

(i) Axis of rotation in true vertical.

The spectrometer was mounted on a heavy iron base $A$ equipped with four levelling screws $B$. A sensitive level $C$ was placed on the arm which was rotated, the levelling screws being adjusted until the level gave a constant reading.

(ii) Crystal mounting base plate in horizontal plane.

The sensitive level was set to read true when viewed with an eye-glass. Readings were taken in two settings at right angles with respect to the base plate adjusting screws $D$ as shown. The
level was reversed "end-for-end" in each position until no change in reading was detectable.

(iii) Crystal centre in axis of rotation.

A fine vertical wire E was mounted on the crystal base plate and viewed with a short objective microscope $M_1$. By rotating the table into positions at right angles to each other, the wire was brought into the axis of rotation, the microscope being focused on its edges with the cross-hair at its centre. A telescope $T_1$ provided with a horizontal scale was focused with the scale centre on the vertical wire. The wire was replaced by the crystal in its mounting blocks F. By means of its horizontal positioning screws G, the crystal surface, dusted with Lycopodium powder, was brought into focus in $M_1$. Then the table was rotated through 90° and the crystal aperture adjusted symmetrically about the rotation axis by comparison with the scale in $T_1$.

(iv) Crystal oriented with angular scales, and crystal pole in a vertical plane through axis of rotation.

The arm and crystal scales were locked at 0° and 270° respectively. A microscope $M_2$ was focused on the vee-shaped nick made by the engraved centre-line at the arm extremity H. A fine silk plumb-line was suspended at H to coincide with the nick. An extension J was fitted to the plate camera which was moved towards H until the plumb-line was contacted at K where it stuck to a thin smear of previously applied shellac. The distance x was measured with a travelling microscope and the plumb-line shortened to clear the surface of the arm. The plate
camera was then shifted to a reading (R-x) on the arm scale calculated to place the plumb-line at a point L on the centre-line at distance R cm from the rotation axis. The plumb-line was adequately illuminated and regarded as an object for reflection in the cylindrical crystal mirror. The latter was rotated about its pole by its horizontal adjusting screws N until the image coincided with the object as viewed through a microscope M₃. The image appeared to be in focus with the object; to show "no parallax" with the object; and to be linear with a well-defined and apparently normal type of diffraction pattern.

The crystal scale was then released and rotated to displace the image slightly from the object. The distance between object and image was measured with the vernier traverse on M₃ at sixteen places over the whole length of the image. There was no deviation from parallelism outside the error in measurement.

(v) Pole of crystal identical with rotation axis.

With the arm and crystal scales locked at 0° and 270° respectively, gauge blocks O and P of identical height were placed on the arm as shown. They were suitably marked with chalk and illuminated from above so that on looking through the telescope T₂ (infinity) the edges of O and P, and the image Q of O in the crystal, appeared as three parallel lines. Slight adjustment of the crystal base plate screw S brought these lines, and therefore the crystal and rotation axes, into coincidence.

Steps (iii), (iv) and (v) were repeated in that order until the adjustment was stable. Summarising, steps (i) and (ii) established the crystal mounting base plate in a convenient
horizontal reference plane at right angles to a convenient vertical reference axis which is the axis of rotation. Steps (iii), (iv) and (v) then used this reference system to set up the spectrometer as an optically self-consistent unit.

The value found for $R$ in step (iv) is, in fact, an experimental determination of the crystal radius of curvature. It is $R = 50.00$ cm. An analysis of the errors of measurement (focal depths of microscopes, etc.), indicates that this value should have an error not greater than $\pm 0.35$ cm. This confirms the accepted value of $R = 50.03 \pm 0.05$ cm determined by the method of profiling.

section III-8 

THE VACUUM CASE

As may be seen in Figs. 19 and 20, and Plate IV, the spectrograph case is of an irregular construction. It is a compromise between the desire to utilise the full angular range of the spectrometer and the intensity requirement of placing the X-ray tube as close as possible to the crystal. The case is an externally braced welded steel fabrication standing on three large bevelling screws. It has a volume of about 120 litres, the case interior being tin plated to improve its vacuum properties. The internal height is $9\frac{1}{2}$ in., the wall thickness varying from $\frac{1}{4}$ to $\frac{1}{2}$ in. Apart from the holes for the mounting tube and diffusion pump previously mentioned, access to the case is through two $16 \times 11$ in. hatches forming the front wall. The inch-wide surface of contact between case and hatches is a hand-scraped plane, the vacuum seal being perfected by a fillet of plasticine around the outer edge.
Locating studs on the inner surface of the hatches allow them to be positioned in a darkened room, clamps being provided to hold them while the plasticine is applied. More recently, another set of hatches incorporating O-ring seals has been built and found satisfactory.

The spectrometer base plate is pushed into position on smooth runners in a pencil-box slide attached to the interior case bottom. One side of the pencil-box and its back stop are straight edges at right angles, against which the base plate is correctly located. It is secured by screws in the overhanging sides of the pencil-box bearing down on its upper surface. The straight edges are placed so that the horizontal X-ray tube axis intersects the vertical spectrometer axis at the crystal centre.

SECTION III-6

THE CHOICE OF A MICA CRYSTAL

AND AN INITIAL TEST OF THE SPECTROMETER

The flexible character of a thin sheet of mica makes it particularly suitable for use in either a transmission or reflection bent crystal spectrometer. Apart from its ease in bending, it may be assumed that, since it does not cleave in a "step" fashion, it will have less tendency to false curvature than more brittle crystals. In discussing the mechanism of bent crystals, Carlsson (1933) indicates that any deformations introduced would be more critical in the transmission setting. Thus Watson (1937), using a transmission mica crystal, states that: "In no case did we obtain spectral lines which were straight, and in many cases our lines showed well-defined breaks along their length." On the
other hand, Stephenson (1939) evinces surprise at Watson's experience since he obtained spectra which were sharp and straight with half widths indicating that mica is very satisfactory. It is usual to first examine the crystal surface optically both in the flat and curved condition. But Farineau (1938), who rejected all samples used in his reflection mica instrument if they did not give a sharp image of a light source parallel to the axis, states that bad spectra may still be obtained. It was clear, therefore, when the crystal for the present instrument was being considered, that while certain precautions could be taken, the perfection of any given specimen would finally be decided by the spectra it produced.

The crystals were obtained from Mica & Micanite Ltd., London, who had previously supplied those used successfully by Baglind (1942). They are in the form of plates measuring 25 x 50 x 0.30mm, the mica variety being muscovite, of specially selected "Clear Ruby" grading. Before bending, the specimen used was subjected to a thorough optical examination, which did not reveal any surface imperfection. This surface perfection was retained on bending as evidenced by the excellent image produced by an illuminated fibre placed near its centre of curvature (Section III-4).

Finally, the crystal spectrometer was tested by setting it to record the L-spectrum of Copper at about 13 kV. An exposure of 200 mA. min. at 4 kV yielded the $L\alpha_{1,2}$ and $L\beta_1$ lines with photometerable intensity on a process plate. The lines were straight and free from breaks of any kind. Their general
shape and separations being in agreement with the results of other workers (e.g., Saur 1936), it was concluded that the spectrometer was in correct adjustment, a conclusion substantiated by the final results.
PART IV

A SPECTROSCOPIC STUDY OF THE K-EMISSION SPECTRUM

OF

THE INERT GAS NEON (Z = 10)
The spectroscopic study of the K-emission spectrum of neon is intended as the beginning of a systematic examination of the X-spectra of the inert gases. The importance of obtaining further information about this series of elements is emphasised by the unique position they hold in the periodic table. This is due to the spherical symmetry of their electronic configurations, a fact of some importance in the theoretical interpretation of their spectra. Related to this is the certainty of their existing as free atomic species, thus avoiding the complications of outer level modification introduced in the molecular, and particularly, the solid state.

The only known examination of the K-spectrum of neon at the time of commencing the present work (1950), was that of Backovsky (1936). Using as source a special low voltage ion tube, he obtained the diagram line $K\alpha_1,2$ in the first and second orders with a plane grating. The low dispersion available, apart from the presence of impurity oxygen and aluminium lines on the plates, presumably prevented the discernment of any structure in the spectrum. He gives a value of 14.61 Å for neon $K\alpha_1,2$. In the closing sentence of his paper, Backovsky remarked that Kunz1 was then studying the neon K-series with a focusing mica crystal.

The results of Kunz1's investigation carried out at Prague were published in Czechoslovakia in 1946 (Kunz1 and Svobodova-Joanelli 1946), but were not known by the writer until 1952 (Physics Abstracts) when this study was well advanced.

The low voltage ion tube used as source by Kunz1 and
Svobodova-Joanelli was a modification of that previously employed by Kunzl (1936). It was operated with a current of 100-200 mA at 1800-2000 V. Their spectrometer was based on a method of focusing previously described by Kunzl (1935). It involves a bent mica crystal which gives a focused spectrum over a limited wavelength region when the crystal is rotated as in the method of Bragg and de Broglie. Each wavelength region examined requires a different radius of crystal curvature. Kunzl and Svobodova-Joanelli give no details of the dispersion of their spectrometer nor any discussion of the exactness of its focal properties. A comparison with the present instrument is not therefore possible.

The results obtained by Kunzl and Svobodova-Joanelli show a marked improvement on those of Backovsk, for, besides the diagram line \( \text{K}\alpha \), they obtained several satellite lines. Their work is summarised in the following table and will be discussed in terms of the writer's findings in Section IV-11.

<table>
<thead>
<tr>
<th>Neon K-series</th>
<th>Wavelength (( \text{km} ))</th>
<th>Remarks</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \alpha_{1,2} )</td>
<td>Relative value</td>
<td>14.587 (14.618 ( \AA ))</td>
</tr>
<tr>
<td>( \alpha_{1} )</td>
<td>Absolute calculated value</td>
<td>14.550</td>
</tr>
<tr>
<td>( \alpha_{3,4} )</td>
<td>Long wavelength limit</td>
<td>14.526</td>
</tr>
<tr>
<td>( \beta_{1} )</td>
<td></td>
<td>14.485 ( (14.470) )</td>
</tr>
<tr>
<td>( \alpha_{8-9} )</td>
<td>( \beta_{9-10} )</td>
<td>14.435</td>
</tr>
</tbody>
</table>

N.B.: (i) Identification of lines was by Noursey diagram comparison with the spectra of Na, Mg, and Al (Kunzl 1936). (ii) Ne K\( \alpha_{1,2} \) measured with respect to \( \text{Si} \) K\( \alpha_{1,2} \). (iii) Ne satellites measured with respect to Ne K\( \alpha_{1,2} \).
SECTION IV-2  THE CHOICE OF WAVELENGTH CALIBRATION LINES

The comparison method of wavelength measurement requires the use of known calibration lines, the number depending on the particular method employed. Calibration in terms of a vacuum spark spectrum not having been possible, a search was made for suitable X-ray lines whose wavelengths had been accurately established by reputable workers. For reasons in connection with the variation of the diffraction characteristics of a crystal with reflection order (Section IV-8), it was decided to use first order lines only. Guided by the tables of Cauchois and Hulubei (1947), an examination of the literature indicated that the two nickel lines Lα1,2 and Lβ1 as measured by Tyren (1936) were the most suitable.

Tyren's measurements were accomplished with a concave grating in terms of hydrogen-like spark lines. By using a 480 cm radius grating and a recording plate at right angles to the beam, he obtained good dispersion without sacrifice of intensity, leading to precise measurements. The wavelengths of the nickel lines were calculated by linear interpolation between the hydrogen-like lines, and then corrected for deviation from linear dispersion. Tyren's values for nickel Lα1,2 and Lβ1 are given as the arithmetical means of several determinations in various orders as follows.

<table>
<thead>
<tr>
<th>Nickel Lα1,2</th>
<th>Order</th>
<th>Nickel Lβ1</th>
<th>Order</th>
</tr>
</thead>
<tbody>
<tr>
<td>14.571 Å</td>
<td>2</td>
<td>14.283 Å</td>
<td>2</td>
</tr>
<tr>
<td>14.566 &quot;</td>
<td>2</td>
<td>14.280 &quot;</td>
<td>2</td>
</tr>
<tr>
<td>14.568 &quot;</td>
<td>3</td>
<td>14.278 &quot;</td>
<td>3</td>
</tr>
<tr>
<td>14.564 &quot;</td>
<td>3</td>
<td>14.277 &quot;</td>
<td>3</td>
</tr>
<tr>
<td>14.565 &quot;</td>
<td>5</td>
<td></td>
<td></td>
</tr>
<tr>
<td>14.565 &quot;</td>
<td>5</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Mean \( \lambda = 14.566 \) Å  
Mean \( \lambda = 14.279 \) Å
It is noted that the writer used the same form of anticathode material (metallic nickel) as did Tyren, thus avoiding the possibility of any change in the shape or positions of the lines due to chemical combination.

SECTION IV-3 THE EXPERIMENTAL PROCEDURE

The spectrograph is situated in a room which can be darkened at will, allowing the safe introduction and withdrawal of the photographic plates. A simple gravity shutter attached to the crystal side of the camera allows three exposures to be taken on each plate. The shutter, which is an addition to the previously described camera, has a horizontal slot and is dropped in guide rails across the plate face by the withdrawal of supporting pins. These are pulled in any desired sequence by a nylon cord on a windlass manually operated by a flexible drive passing through a shaft seal in one hatch of the case. The exposures may be made to overlap, or may be kept separate as required, the total exposed area measuring 2 x 7.5 cm.

The spectra were recorded with Ilford Q-emulsions coated on glass plates. Skinner (1938) notes that the Q-emulsion "is very even in its properties and therefore is quite suitable for accurate photometry, including the determination of intensities." Two grades, Q1 and Q2, were used, the latter being the faster but having the larger grain size. Following exposure the plates were developed, fixed, and washed according to the manufacturers' recommendations. They were dried in air and suitable precautions were taken to avoid dust or scratch damage to the emulsion during their subsequent examination.
As discussed in Section I-7, it had been found that a neon gas pressure of about 14 g\textsuperscript{a} corresponded to a maximum in the total emission intensity. The first attempts to record the K-series lines of neon were accordingly made at that pressure. They were not successful, and, in fact, the optimum pressure for the emission of line radiation near 14.5 kK was subsequently found by trial to be about 100 g\textsuperscript{a}. This discrepancy is probably simply explained in terms of each wavelength having a similar response curve to that shown in Fig. 14, but, with the maximum having a magnitude and occurring at a pressure determined by the excitation voltage concerned. The "envelope" of these various curves would then correspond to Fig. 14.

The exposure times employed for recording both the neon and nickel spectra varied in accordance with the requirements and the experimental conditions. One factor which made it difficult to systematise the control of photographic blackening by means of exposure times, was the X-ray tube window. This was replaced from time to time after accidental destruction, whereupon it was found that thickness variations between different windows had a definite effect on the recorded intensity. Care has therefore been taken to make intensity comparisons only between plates exposed through the same window. Under favourable circumstances it has been possible to record the stronger lines in the neon K-series with visual intensity on a Q1 plate in two hours, while a five-hour exposure on a Q2 plate has been sufficient to over-expose Ne K\alpha\textsuperscript{1,2}. In this respect it is noted that the Ilford Q-emulsions have an upper density limit of about 2.
Reproductions of some of the photographic records are shown in Plates VII to XII. In checking the reality of the lines in the neon spectrum, it was found that two broadish bands present in the neon spectrum were also recorded in identical positions when the neon was replaced by air. The distance between the two bands was found to be constant and independent of focus, whereas the spacing between the bands and the authentic neon lines as a whole, varied with the wavelength in focus. These facts are completely explained in terms of background radiation from the gas and tube walls.

Thus, in the central source region there is a fairly uniform continuous radiation due to the gas, with perhaps a small contribution of continuous radiation from the closed tube end. As the walls are approached, the intensity of the background must rise. For, with increase in the source angle subtended by the plate, radiation is recorded from the tube wall and vicinity as one proceeds from the closed end towards the window end of the tube. At the same time, the wall radiation will be comparatively more effective than the adjacent gas background because of the higher atomic number of the emitter. After reaching a maximum value, this composite background must fall off fairly rapidly due to non-penetration of the wall by the exciting electrons, coupled with the probability of little radiation being excited towards the window end of the tube as shown in Section I-2.

The above interpretation of these "edge effects" is corroborated by two important facts. Firstly, the separation of the two bands is as expected for a tube of the internal breadth
of that employed. Secondly, altering the wavelength in focus has a peculiar effect on the recorded spectra. Thus, if the focus is shifted towards short wavelengths, the short wavelength edge occupies a position suitable for Bragg reflection of the $K\alpha_1,2$ line of silicon in the second order ($\approx 2 \times 7.112 \text{ kX}$). Since the tube is of pure silica the expected line was observed with high intensity. On the other hand, if the focus is shifted towards long wavelengths, none of the line radiation can be Bragg reflected, only the edge effects being recorded. Exhaustive tests proved that, with the exception of the second order of $Si \ K\alpha_1,2$, no line radiation was recorded when the external electrode tube was operated with air under conditions identical to those for which the lines ascribed to the neon K-series were recorded. These facts are illustrated clearly in the microdensitometer records (Figs. 39 and 40) now to be discussed.

Following visual examination, the useful plates were surveyed with the microdensitometer described by Tait and Chalklin (1952). This instrument yields microammeter readings which are directly proportional to the difference in density of the region being examined and that of an arbitrarily chosen reference point. Thus,

$$M(x) = k(D(x) - D_0),$$

where $M(x)$ is the meter reading corresponding to the position on the plate of density $D(x)$, the reference density being $D_0$.

The large grain size of the Q-emulsions, together with the small vertical height of the microdensitometer slit (about 1 mm), produced large random fluctuations in the microdensitometer records.
if too narrow a slit were employed. Slit widths of 0.02 mm and 0.05 mm for the Q1 and Q2 emulsions respectively, were the narrowest tolerable. A slit width of 0.05 mm, with the dispersion employed, corresponds to about 0.1 V at a line voltage of 850 V (14.5 kX) which is satisfactory from the viewpoint of resolution.

Given that the microdensitometer is in operating condition, only one critical adjustment is involved when surveying a plate. That is the alignment of the instrument slit with the lines of the spectrum being examined. The effect of errors in this adjustment is discussed in the next section.

Originally, the microdensitometer was manually operated, the precision screw-thread driving the plate carriage being turned by hand and the meter readings being recorded after each displacement. The readings were then plotted on graph paper. It was usually necessary to survey each plate several times, in different places, to avoid the remnant grain effect from the photographic emulsion. The several sets of readings thus obtained were plotted as individual curves on the same graph paper and compared. If they were found shifted relative to one another, due to lack of parallelism between the vertical shift mechanism on the microdensitometer and the recorded lines, the necessary adjustments were made and the curves then averaged.

To avoid the considerable labour involved in the above-described method, the microdensitometer was modified during the present work to allow automatic recording. The plate carriage is now motor driven in synchronism with a strip chart recorder, the relative rates of displacement of plate and chart being
accurately known and able to be varied as required. The individual records or graphs are readily averaged by placing them in pairs on a special 5 x 1 ft. illuminated viewing screen and drawing in the visual mean on a superimposed blank chart. This method has proved capable of producing the required records quickly and consistently.

Examples of some of the microdensitometer records, reproduced on a reduced scale, are given in Figs. 39 and 40. Fig. 39 shows the effect on the recorded neon spectrum and edge effects of altering the focal position, while Fig. 40 compares the plate records of a neon and an air exposure taken under similar conditions.

SECTION IV-4 THE NATURE OF THE EXPERIMENTAL DATA

It is recalled that no X-ray line has a unique frequency, but rather a certain intensity distribution I (0). This distribution about a central maximum intensity is called the shape of the natural line, and it is the shape of the natural line and its frequency separation from the other natural lines in the spectrum which is required. But any experiment can only provide a spectrum which includes the effects of source, diffracting crystal, and detector (microdensitometer). The nature and magnitude of these effects must accordingly be known if the necessary corrections are to be applied.

The two main source broadenings of a natural line are due to temperature (Doppler) and pressure. The temperature effect has been calculated as negligible in the present case, while the effect of pressure may be assumed small in a low density source (Neitler
1944).

Consider next the effect of the microdensitometer slit. It concerns the errors introduced, particularly to line shape, due to the finite slit width, and also, from lack of parallelism between slit and line. References to the more recent work on this subject are given by Brodersen (1954) in a general treatment of the effects of slit widths upon emission lines, absorption lines, and continuous absorption spectra. A paper by Gokhale (1950) is directly concerned with the present problem, but he assumes parallelism of slit and line. In Appendix XI a derivation is given of the effect on line shape of a microdensitometer slit of width $2S$, length $2l$, and whose axis is inclined at angle $\theta$ to the line under survey.

It is shown that the microdensitometer as here used has negligible effect on the shape of the recorded lines.

In anticipation, it may be said that the line shapes reported here have not been corrected for any effects due to crystal diffraction. The reason for this decision is contained in the following outline of the phenomenon of the diffraction of X-rays by crystals. In the earlier sections it has been sufficient to accept the idea that crystals reflect X-radiation according to Bragg's laws. But, in fact, the radiation scattered by a crystal is produced or influenced by a number of processes, either within the crystal or at its surface. Thus, as summarised by Lonsdale (1943), the secondary radiation emerging from an irradiated crystal may consist of: (i) fluorescent radiation, (ii) Compton scattered radiation, (iii) Laue or Bragg reflected radiation, (iv) Faren-Waller scattered radiation, (v) Debye-Jauncey scattered radiation.
Although the present concern is primarily with the intense Bragg reflections it is noted that the other types of scattering may contribute, particularly to the background pattern.

In the derivation of Bragg's interpretation of Laue's equations for a three dimensional geometrical lattice, such as given by Compton and Allison (1935), it is concluded that the direction of maximum reflected intensity is at a glancing angle \( \theta = \sin^{-1}(n\lambda/2D) \). But such a (wave kinematical) treatment involves the assumption that the incident beam is not affected by the crystal medium, which is untenable, since absorption processes must occur. Two conditions are recognised, that of normal absorption due to photo-electric ejection and Compton scattering, and the additional absorption called extinction due to the large energy transfer to the diffracted waves when the Bragg equation is satisfied. In a general (dynamical) theory of X-ray diffraction account must be taken not only of absorption, but also of the interaction between incident and scattered radiation. This involves satisfaction of the electromagnetic field equations for the resultant internal wave field, and it is here that the concept of the dielectric constant (refractive index squared) as a positional function, with the periodicity of the crystal lattice, is introduced (Zachariasen 1945). The outcome of such a dynamical treatment is to show that the Bragg diffraction pattern centre does not correspond to a glancing angle \( \theta_B = \sin^{-1}(n\lambda/2D) \). Further discussion of this aspect of the situation is postponed until the section (IV-8) on wavelength measurement.

Besides the direction of its central maximum, the intensity
distribution of the Bragg diffraction pattern for an ideal crystal is completely given by the dynamical theory. But its geometry and magnitude are critically dependent on the assumptions made concerning absorption, extinction, etc. Again, although in a few cases there is quantitative agreement between observed and calculated diffraction patterns, this is not so for the great majority of real crystals. For these, the total intensity and the width of observed patterns are always many times greater than the ideal crystal theory predicts. In general, good agreement between theory and observation is possible only if real crystals are regarded as being imperfect and the ideal crystal theory modified accordingly. Two types of imperfections, called displacement disorders and substitution disorders, are recognised. They are discussed by Wilson (1946) who prefers to call them "mistakes."

In real crystals, disorder is associated with the so-called mosaic structure, a concept due to Darwin (1922), wherein a crystal is made up of blocks, each block being in itself an ideal crystal, but adjacent blocks not accurately fitting together. A useful survey of real and ideal crystals from the point of view of the theory of crystal growth, is given by Buckley (1951).

Experimentally, diffraction broadening has been best studied with the double crystal spectrometer, an instrument which gives a diffraction pattern independent of the intensity distribution in the incident beam. Allison (1932) and Parratt (1932) were thus able to find, after careful examination of many specimens, a set of calcite crystals in nearly perfect agreement with ideal crystal theory. Such a case is exceptional, the variation in
diffraction pattern found within a given crystal species being usually large. Thus, Renninger (1934) found a variation in the diffraction pattern half width from 5 to 900 seconds of arc in the case of rock salt. This variation was a function of the history of the cleavage face and is only explicable in terms of the mosaic theory. But the converse is not true. For, since one has no knowledge a priori regarding the actual condition of a given crystal, it follows that no realistic prediction can be made as to the width and form of its diffraction pattern. The situation may well be summarised in the conclusion of Hendricks (1940) in a paper on layer-silicate (mica) lattices, where he says that, "While line width, line shape, and intensities are affected, the phenomenon (of the mosaic crystal) has not been very susceptible of study, and the distribution of sizes of the mosaic volumes are unknown."

Finally, it is noted that even if the present crystal had been studied in a double crystal spectrometer, there is every likelihood that its subsequent bending would radically alter its diffracting character. Probably the only successful theoretical consideration of the diffraction of X-rays by a bent three-dimensional ideal crystal is that due to Borovskii and Bezirganyan (1953). Their treatment is essentially wave kinematical and hence, as previously indicated, is not applicable to a real bent crystal.

SECTION IV-5 THE THEORY OF THE MEASUREMENT OF INTENSITY

Before an analysis into component structure of a photograph-
ically recorded spectrum is possible, it is necessary to establish a relationship between density of photographic blackening and intensity of incident radiation. Apart from the ratio-slit method of Chalklin (1949), which avoids the Bunsen-Roscoe reciprocity law, it is more usual to assume the validity of this law for soft X-rays in order to set up the so-called blackening curve. This was done, for instance, by Hill (1937) for the determination of absorption coefficients in the wavelength region 13–24 Å, and by Skinner (1946) for the examination of emission spectra in the wavelength region 40–500 Å.

Now Webster (1929) showed that the reciprocity law is obeyed at 4 Å (over an intensity range of 1 – 3) while Mulbach (1937) found it to be valid at 4 Å and 7 Å. The assumption made in the present work, that the reciprocity law is applicable at about 15 Å, would therefore not appear to be a serious one. Against this supposition must be cited the fact that the intensity variation in the K-emission band of graphite (44 Å) determined by Chalklin with his ratio-slit method, is not in agreement with that found by Skinner when assuming the reciprocity law, although here the discrepancy may be due to extraneous causes. Finally, the fact that this is the first occasion on which an attempt has been made to obtain the intensity distribution in the neon K-spectrum, is sufficient justification for the method of plate calibration employed.

Any plate for which a blackening curve was required, was exposed in different regions by means of the camera shutter, for
times \(1 \times t\) and \(n \times t\), say, with constant source intensity \(I\).

The respective exposures \((E)\) may be written as

\[
E_1 = I (1 \times t),
\]

and

\[
E_n = I (n \times t).
\]

Assuming the reciprocity law, the latter expression becomes

\[
E_n = (n \times I) t,
\]

so that the two exposures \(E_1\) and \(E_n\) are effectively due to the

respective intensities \(1(I)\) and \(n(I)\) operating for time \(t\).

Let the densities of blackening in equivalent parts of the

recorded spectrum due to \(1I\) and \(nI\) be \(D^1\) and \(D^n\) respectively.

Then, as explained in Section IV-3, the corresponding micro-

densitometer readings are

\[
M^1(x) = k(D^1(x) - D_0),
\]

and

\[
M^n(x) = k(D^n(x) - D_0).
\]

The plates were photometered so that the reference level of

density \(D_0\) was zero for incident X-radiation giving, therefore,

that

\[
M^1(x) = kD^1(x),
\]

and

\[
M^n(x) = kD^n(x).
\]

This assumes that chemical fog, etc., may be regarded as uniform

in value over the area of plate emulsion examined.

The blackening curve was then constructed in the usual manner

by letting any small value of \(M^1(x) (kD^1(x))\) correspond to an

intensity \(I = 1\) on an arbitrary scale and extrapolating over the

required density range. The curve must pass through the common

origin of the \(M\) and \(I\) scales. Examples of blackening curves

thus established are shown in Figs. 41 to 44. The shape of any
given curve has been found to be independent of its arbitrarily chosen initial point, while the curves derived from two separate examinations of any given plate were found to be in excellent agreement.

SECTION IV-6 THE SHAPE OF THE NEON LINE Kα1,2

Two plates, Q1/7 and Q1/9, reproduced as Plates VII and IX, were exposed to neon radiation under selected conditions. For Q1/7, the two exposures required for the blackening curve were recorded on the upper region of the plate; the two exposures were of 2 and 5 hours; the knife-edge was in a position limiting the useful area of the crystal to about one-quarter of its unscreened value. For Q1/9, the two exposures required for the blackening curve were recorded on the lower region of the plate; the two exposures were of 4 and 10 hours; the knife-edge was absent.

The resultant blackening curves for Q1/7 and Q1/9 are shown in Figs. 41 and 42 respectively. By means of these curves the microdensitometer records, in the region of the diagram line Kα1,2 on the corresponding plates, were converted to intensity scales and plotted as in Fig. 45. The line peaks have been adjusted to the same value by means of a scale factor.

The agreement between the two experimental curves is excellent. This justifies the method of establishing the intensity scale, since the blackening curves of Figs. 41 and 42 were derived, as far as was possible, from extreme variations of the experimental assumptions involved. The fact that the shape of the experimental line is independent of whether the knife-edge is present or absent
supports the contention of Section III-2 that the focal aberration b' is negligible. For, its magnitude being proportional to the square of the crystal aperture, it would be sixteen times larger for plate Q1/9 than for Q1/7, and, if contributing to the line width, would be detectable.

Plates Q1/7 and 9 were exposed through a pair of windows of good opacity so that there is no contribution of visible light to their backgrounds. Again, the plates were focused as in Fig. 39(b), which ensured that the long wavelength edge effect was removed from line Kα1,2. It was consequently assumed that, in the immediate region of the relatively intense line Kα1,2, the effects of background, edge effect, and short wavelength satellites were sufficiently small to be neglected in any consideration of the shape of the doublet.

The distribution of intensity in, or shape of, an emission line is, in quantum theory, usually expressed according to the treatment of Weisskopf and Wigner (1930). Although their expression for the natural line shape may be written in a form which is similar to that of classical radiation theory (Heitler 1944) such as

$$I(\nu) = \frac{I_0}{2\pi \left( \nu - \nu_o \right)^2 + \nu_o^2/4},$$

the concepts involved are radically different. In particular, the line breadth at half maximum intensity $\nu_o$ is, in the quantum theory, a measure of the total transition probability per unit time for the states concerned. Provided that the width of the natural line is, on a wavelength ($\lambda$) scale, small compared to the
wavelength of its central maximum, the natural line shape may also be expressed in the form

$$I(\lambda) = \frac{w_\lambda}{2\pi} \frac{I_o}{(\lambda - \lambda_o)^2 + w_\lambda^2/4}$$

It will be shown that the inverse linear dispersion ($\partial\lambda/\partial x$) is constant for distances (x) of the order of the widths of the recorded lines. It follows that wavelength is a linear function of plate distance over the width of a line. Assuming that the experimentally established line shapes may be expressed in terms of the Weisskopf-Wigner theory, the equivalent expression on a linear scale for the distribution of intensity in the natural line is

$$I(x) = \frac{w_x}{2\pi} \frac{I_o}{(x - x_o)^2 + w_x^2/4}$$

Here, $I_o$ is the total line intensity, $x_o$ is the position of the central maximum, and $w_x$ is the full width of the line at half maximum intensity (half width). In practice, simplification of the theoretical expression is convenient, and the form

$$I(x) = \frac{S}{(x - x_o)^2 + w_x^2/4},$$

where $S = w_x I_o / 2\pi$ is treated simply as a scale factor, has been used.

If an axis is drawn through the peak of the experimental shape of $K\alpha 1,2$ in Fig. 45, the line is found to be asymmetrical, being broader on the long wavelength side. It cannot, therefore, be fitted by the above expression $I(x)$ which is a symmetrical
function in \( x \). The most probable cause of the long wavelength broadening is considered to be partial separation of the doublet components \( K\alpha_1 \) and \( K\alpha_2 \), and the experimental shape has been analysed on this basis.

Let \( E(x) \) be the function representing the experimental line shape and \( I(x) \) the contribution to this shape made by the \( K\alpha_1 \) line. Then the \( K\alpha_2 \) line contribution is evidently \( kI(x-s) \) where \( k \) and \( s \) are the intensity ratio and plate distance separation of \( K\alpha_1 \) and \( K\alpha_2 \). Now \( k \) is known to have a value \( 1/2 \) while examination of the measured separations for the \( K\alpha_1,2 \) doublet, listed by Cauchois and Hulubel for elements down to \( Z = 13 \), leads to an estimated value of \( s \approx 0.05 \text{ mm} \) \((2X)\). Since the doublet components have equal half widths, this gives the following expression for \( E(x) \).

\[
E(x) = I(x) + \frac{1}{2}I(x-0.05)
\]

\[= \frac{S}{(x-x_0)^2 + w_x^2/4} + \frac{1}{2} \frac{S}{(x-0.05-x_0)^2 + w_x^2/4}\]

Although a generalised method of decomposing experimental line shapes has been given by Dumond (1933), it was sufficient in the present case to determine by trial the values of \( S \) and \( w_x \) required for best fit. The result is given in Fig. 45 where the full line is the envelope of the two partially separated components \( K\alpha_1 \) and \( K\alpha_2 \) shown in broken line. The agreement between the theoretical and experimental doublet shape is seen to be good from the peak down to where the line has a breadth about twice its half width. The departure of the theoretical shape from the experimental shape as one proceeds further from the peak is not
greater than might be expected, as then the assumptions regarding contribution of satellite and edge effect become more critical, also the Weisskopf-Wigner theory itself becomes more approximate as one departs further from the natural line peak.

For reasons given in Section IV-4, the experimental curve is not corrected for diffraction broadening. The success of the analysis despite this indicates either that the diffraction pattern is of small breadth, or that it has a form similar to the Weisskopf-Wigner line. The half width value of both the \( \alpha 1 \) and \( \alpha 2 \) components of the \( K\alpha 1,2 \) doublet found from this analysis is \( w_x = 0.40 \) mm. The corresponding values, \( w_\lambda \) on a wavelength scale, and \( w_E \) on an energy scale, are determined as follows.

\[
w_\lambda = w_x \left( \frac{\partial \lambda}{\partial x} \right)
\]

To derive \( \left( \frac{\partial \lambda}{\partial x} \right) \) it is convenient to take the following expression for \( \lambda_x \) given, with reference to Section IV-8, in Appendix III.

\[
\lambda_x = \lambda + \left( \frac{x}{L} \right) (2D \cos \phi) - \left( \frac{x^2}{L^2} \right)(D \sin \phi) - \left( \frac{x^3}{L^3} \right)(D \cos \phi) + \ldots
\]

On differentiating, and writing \( L = R \sin \phi \), one obtains

\[
\left( \frac{\partial \lambda}{\partial x} \right) = (2D/R) \cot \phi \left( 1 - \left( \frac{x}{R} \right) \sec \phi - \left( \frac{3x^2}{2R^2} \right) \cosec^2 \phi + \ldots \right)
\]

Now, as will be discussed in detail in Section IV-9, the values of \( x \) and \( \phi \) to be used in the present case are 1.29 mm and 47°4.009, respectively, while \( D = 9927.58 \) X and \( R = 500.3 \) mm. It follows, on evaluating, that \( \left( \frac{\partial \lambda}{\partial x} \right) = 36.8 \) X/mm.

\[
\therefore \quad w_\lambda = 0.40 \times 36.8 = 14.7 \text{ X}
\]

If the wavelength \( \lambda \) corresponds to a quantum energy \( E \), measured in electron volts, we have that

\[
w_E = w_\lambda \left( \frac{\partial E}{\partial \lambda} \right)
\]
A recent determination of the relationship between $E$ and $\lambda$, when
the latter is measured in keV on the crystal scale, is given by
Felt, Harris, and Dumond (1953) as

$$E \lambda = 12,370$$

$$\Rightarrow (\partial E/\partial \lambda) = -12,370/\lambda^2.$$  

The value of $\lambda$ (Ne Kα1,2) is shown in Section IV-9 to be 14.585 keV,
which gives a magnitude for $(\partial E/\partial \lambda)$ of 58.15 eV/keV.

$$\therefore w_E = 0.0147 \times 58.15$$

$$= 0.85 \text{ eV.}$$

The various half width values for neon Kα1,2, uncorrected
for diffraction breadth, are collected in the following table.

**PLATES Q1/7,9**

<table>
<thead>
<tr>
<th>LINE</th>
<th>HALF-WIDTH VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$w_x$</td>
</tr>
<tr>
<td>K{</td>
<td>\alpha1</td>
</tr>
<tr>
<td>{</td>
<td>\alpha2</td>
</tr>
</tbody>
</table>

**SECTION IV-7  ANALYSES OF SOME NEON K-SERIES SATELLITE STRUCTURE**

The satellite line structure on the short wavelength side
of the diagram line Kα1,2 has been studied in terms of the records
obtained from two plates, Q2/20 and Q2/36. They will be discussed
separately.

Plate Q2/20 is reproduced in Plate X. The two nickel
exposures provided density values for the construction of the
blackening curve of Fig. 43. Only the region of the nickel
spectra near the line L[\beta_1] is suitable for this purpose, the
result being that the blackening curve does not cover the density range corresponding to the upper part of Ne Kα1,2. The average of three manually recorded microdensitometer runs on the neon exposure (cf. Fig. 39(b)) was converted to the intensity scale and plotted as indicated in Fig. 46. The peak of Kα1,2 must occupy the same position on either an intensity or density plot, and has been located from the latter.

Analysis of the satellite structure into component lines is not easy. A decision must first be made as to the form of the non-satellite background. In the present case it was assumed that the contribution of continuous radiation to the background can be neglected in comparison to that of the short wavelength base of line Kα1,2. Although, as noted in the previous section, the base of line Kα1,2 is not perfectly described by the Weisskopf-Wigner theory, it at least seems consistent to retain for it an expression of the form

\[ E(x) = \frac{S}{(x-x_0)^2 + \nu_x^2/4} + \frac{1}{2} \frac{S}{(x-s-x_0)^2 + \nu_x^2/4} \]

\[ = \frac{S}{(x-x_0)^2 + (0.4)^2/4} + \frac{1}{2} \frac{S}{(x-0.05-x_0)^2 + (0.40)^2/4} \]

in the satellite region. The intensity value of the peak not being known, the scale factor S was determined by fitting the expression E(x) at two points on the short wavelength side of the line. The resultant Kα1,2 line shape is shown drawn in Fig. 46.

* The analysis was not found to improve when various arbitrary values of continuous background were inserted.
It is noted that Parratt (1936), who has done considerable work on X-ray satellites, when dealing with a situation exactly similar to the above, gives three alternatives for the non-satellite background. He then draws between them what he believes to be a "reasonable" background.

The next step in the analysis was to decide, from the variation of the experimental envelope of the satellite group, the minimum number of component lines present. Lines II, III and V of Fig. 46, all of which are visually resolved in the original plate, were thus predicted. Starting with the strongest of these III, which will be least affected by the others, the experimental envelope was systematically decomposed. As the analysis proceeded it was necessary to introduce the component lines I and IV in order to reasonably fit the experimental contour. The result is shown in Fig. 46.

The component lines are assumed to be of the Weisskopf-Wigner form \( I(x) = \frac{S}{(x-x_0)^2 + \frac{w^2}{4}} \). The analysis was repeated several times, making suitable adjustments in the parameter values, before the final form was accepted. It is contended that the analysis is unique, within the experimental uncertainty, in terms of the number of component lines postulated. Although a perfect fit could be obtained by the introduction of further components of the required intensity and half width, there is no certain experimental justification for so doing.

The following are the half widths of the various component lines, calculated in a manner similar to that previously described for line \( K\alpha 1,2 \).
Plate Q2/36 is reproduced in Plate XII, while a microdensitometer record of one of the neon exposures is given in Fig. 40(a). The spectrometer was set for a shorter wavelength to be in focus than in the case of plate Q2/20. This resulted in the discovery of a relatively weak band of close lines which had previously been obscured by the short wavelength edge effect. This band of close lines will be considered apart from the rest of the spectrum.

The blackening curve derived from plate Q2/36 is given in Fig. 44. With this, the average of four automatically recorded microdensitometer runs over the long wavelength region of the spectrum was converted to an intensity scale as shown in Fig. 47. In decomposing this intensity envelope into its component lines, it is necessary to assume the presence of background radiation. Some of this would be expected from the now close proximity of the long wavelength edge effect. Guided by the variation in background shown by the curve of Fig. 40(b), in the case of an air exposure, a continuous background was drawn in as indicated. Next, the line Ko1,2 was taken to have its previously determined
half width value \( w_x \) of 0.40 mm, and expressed in the Weisskopf-Wigner form

\[
R_x = \frac{S}{(x-x_0)^2 + (0.40)^2/4} + \frac{1}{2} \frac{S}{(x-0.05-x_0)^2 + (0.40)^2/4}
\]

The scale factor \( S \) was found by fitting the theoretical and experimental curves at the line peak. The agreement obtained is seen in Fig. 47 to be fair, particularly on the short wavelength side of the line. The discrepancy on the long wavelength side is not unexpected in view of the presence of the edge effect, a factor which cannot be accurately allowed for. It should not affect the analysis on the short wavelength side of \( K\alpha_{1,2} \). This proceeded in a manner similar to that for plate Q2/30, and is shown on an enlarged scale in Fig. 48.

As for plate Q2/30, the lines I, II, III, IV and V must be postulated in order to obtain reasonable agreement between the theoretical and experimental contours. On comparing Figs. 46 and 48 it is immediately perceived that the intensity relationships within the line group \( K\alpha_{1,2} \) (I-V) are different for the two plates. This is due to the non-uniform nature of the discharge source and prevents any reliable estimate of relative intensities. It is recalled that the blackening curve depends only on source constancy so that the actual analyses of the intensity plots are in no way affected by the fact of a non-uniform source.

The following are the half width values of the lines I-V according to the foregoing analysis of the record of plate Q2/36.
Towards shorter wavelengths on plate Q2/36 there is a weak but definite band, probably consisting of many close lines. At least five lines are visible to the eye on the original plate, some of which have survived reproduction and may be seen in Plate XII. The plate region between line III and the short wavelength edge effect was carefully surveyed with the microdensitometer, the average of eight displaced runs being given in Fig. 49. There appears to be no alternative explanation of the maxima in this density contour, other than that they correspond to the peak positions of several partially resolved emission lines.

The microdensitometer record under discussion is not shown converted to an intensity scale as it was decided not to attempt an analysis of its structure. For it was considered that the amount of additional information resulting from such an analysis would be small in comparison to the labour involved. The line group is of low intensity and hence sensitive to the effects of any error in the intensity scale or in the method of decomposition. The latter would involve allowing for the influence of the much over-exposed second order of silicon Kα, the shape of whose base

<table>
<thead>
<tr>
<th>LINE</th>
<th>( w_x (mm) )</th>
<th>( w_\lambda (X) )</th>
<th>( w_E (eV) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>0.33</td>
<td>12.3</td>
<td>0.72</td>
</tr>
<tr>
<td>II</td>
<td>0.47</td>
<td>17.6</td>
<td>1.03</td>
</tr>
<tr>
<td>III</td>
<td>0.40</td>
<td>15.0</td>
<td>0.88</td>
</tr>
<tr>
<td>IV</td>
<td>0.23</td>
<td>8.6</td>
<td>0.51</td>
</tr>
<tr>
<td>V</td>
<td>0.40</td>
<td>15.0</td>
<td>0.89</td>
</tr>
</tbody>
</table>
is not known.

On the other hand, at least a minimum number of line components can be decided from the contour maxima. Judging from the previous analyses it is not expected that these are much displaced from the actual line peaks. They have, accordingly, been labelled as lines (VI-XXI).

**SECTION IV-B**  
**THE THEORY OF THE MEASUREMENT OF WAVELENGTH**

The wavelengths of the lines in the neon spectrum have been determined with reference to the wavelength values of the \( L\alpha_1,2 \) and \( L\beta_1 \) lines of nickel. An analysis of the sources of error in the method of determination has been carried out so that if the values at present accepted for the nickel lines are at a future date altered, the neon wavelengths may be adjusted accordingly.

Two methods of wavelength determination have been considered; a "single line" method and a "multiple line" method. The single line method assumes a perfect spectrometer in the sense that the only errors present are those whose causes are known and whose effects may be calculated. On the other hand, the multiple line method implies that, in addition to the known errors, there may be others whose presence is not suspected. The use of more than one known line is a prudent safeguard designed to reduce the effect of unknown errors. In the limit, an infinite number of known lines would allow wavelengths to be determined on an instrument having any kind of unknown error, provided the lines were identifiable.
Derivations of the wavelength functions for use in the single line and multiple line methods of measurement are given in Appendices III and IV respectively. A comparison of the possible accuracies of the two methods, in the case of a perfect spectrometer, is outlined in Appendix V. In Appendices VI and VII respectively, the significance of certain errors in the spectrometer axis of rotation and in the angular setting of the spectrometer are considered.

The measurement of wavelength is dependent on the use of the Bragg equation. In Section IV-4 it is indicated that the centre of the diffraction pattern (which is either at or very near the maximum) does not correspond to a Bragg glancing angle of diffraction \( \phi_B = \sin^{-1}(n\lambda/2D) \), but to \( \phi_C \), given by Zachariasen (1945) as

\[
\phi_C = \phi_B + \frac{1-b}{2b \sin 2\phi_B} \psi_0^*.
\]

This relationship between \( \phi_B \) and \( \phi_C \), in which \( b \) is approximately the ratio of the direction cosines in reciprocal lattice space of the incident and diffracted wave, and \( \psi_0^* \) is a function of the refractive index periodic with the crystal lattice is applicable, strictly to the Bragg method in which the glancing angle is varied for a fixed wavelength. Now in this case it may be shown that \( b \) is negative so that \( \phi_C \) is always greater than \( \phi_B \) and a Bragg optical (specular) reflection is not possible. This is not so in the Laue method, where for a fixed direction of incidence the wavelength is varied, for here \( b \) may take the value +1 whereupon the centre of the diffraction pattern \( \phi_C \) coincides with the Bragg
angle $\varphi_B = \sin^{-1}(n \lambda/2D)$, and an ideal Bragg specular reflection occurs.

The present measurements involve the Bragg method in which it is the glancing angle of diffraction $\varphi_C$ that is determined. The actual wavelength $\lambda$ corresponding to this direction is required. It is the value given by $n \lambda = 2D \sin \varphi_C$ when

$$\varphi_C = (1-b) \psi_o / 2b \sin 2\varphi_B$$

is substituted for $\varphi_B$. Thus,

$$n \lambda = 2D \sin(\varphi_C - \frac{1-b \psi_o}{2b \sin 2\varphi_B}).$$

Since $(1-b) \psi_o / 2b \sin 2\varphi_B$ is a small quantity, this may be written as

$$n \lambda = 2D \sin \varphi_C \cos \frac{1-b \psi_o}{4b \sin \varphi_B \cos \varphi_B},$$

$$= 2D \sin \varphi_C \left(1 - \frac{1-b \psi_o}{4b \sin^2 \varphi_C}\right),$$

for we may put $\varphi_B = \varphi_C$ in the term concerned.

Alternatively, if the correctly measured angle $\varphi_C$ is substituted in an equation $n \lambda = 2D \sin \varphi_C$, an incorrect wavelength $\lambda'$ is obtained. But, since

$$n \lambda = 2D \sin \varphi_C \left(1 - \frac{1-b \psi_o}{4b \sin^2 \varphi_C}\right),$$

it follows that the true wavelength may be found by writing

$$\lambda = \lambda' \left(1 - \frac{1-b \psi_o}{4b \sin^2 \varphi_C}\right),$$

where the term in brackets is to be regarded as a wavelength correction factor. The last equation may be re-arranged in the
form
\[ \lambda = \lambda^1 \left\{ 1 - \frac{D}{n^2} \left( 1 - k_n \right) \psi_0 \right\} \]
\[ = \lambda^1 (1 - k_n), \]

where the correction factor \((1 - k_n)\) refers to a measurement in the \(n\)th order. As shown by Siegbahn (1925), a good approximate value may be estimated for \((1 - k_n)\). For muscovite mica in the first order, \(k_n\) is calculated to have a value of 0.0015.

In practice it is convenient and usual to correct the Bragg equation as though the error is due to the lattice constant. Consider the last expression
\[ \lambda = \lambda^1 (1 - k_n). \]
\[ \therefore n \lambda = 2D \sin \theta_c (1 - k_n) \]
\[ = 2 \left\{ D(1 - k_n) \right\} \sin \theta_c \]
\[ = 2D_n \sin \theta_c. \]

Here, \(D_n = D \omega (1 - k_n)\) is an effective lattice constant which, when substituted in the simple Bragg equation, gives the correct value of wavelength for a measurement in the \(n\)th order.

The unit of wavelength for measurements with crystal spectrometers is the X-unit. It is an artificial unit, nearly equal to \(10^{-14}\) cm, and defined in terms of an assumed value for the lattice spacing of calcite. Thus, Siegbahn (1943) states that, "All wavelengths in the X-ray spectra determined by crystal measurements are based on the value \(D_1 = 3029.04\) X-units for the lattice constant of calcite at 18°C and the corresponding values of \(D_2, D_3, \ldots \) for the higher orders. As secondary standards for
the longer wavelengths, quartz \( D_1 = 4244.92 \), gypsum \( D_1 = 7579.07 \),
and mica \( D_1 = 9927.58 \) X-units have been used. Rock salt, which
was used in the very first days of X-ray spectroscopy, was
abandoned because the spectral lines obtained with this crystal
did not have sufficiently good definition to allow the desired
precision in the measurements." The values here quoted were
first given by Siegbahn (1931) and that for mica \( D_1 = 9927.58 \) X
has been used, for example, in the researches of Shearer (1935)
and Haglund (1942). It must be assumed that the value
\( D_1 = 9927.58 \) X, which gives the spacing of the \((001)\) or cleavage
planes, applies strictly to muscovite mica. The references
given are not explicit on this point. However, since the \( D (001) \)
spacings of muscovite and biotite are known to differ significantly,
being given by Bragg (1937) as \( 9.94 \) Å and \( 10.03 \) Å respectively,
the assumption is justified.

It was concluded in Section IV-4 that, due to the unknown
nature of mosaic-like imperfections in real crystals, no worth-
while estimate can be made of the width of the diffraction pattern
of the mica crystal used. By the same token, the validity of
the application of a lattice constant determined in terms of one
crystal specimen to any other specimen of the same mineral must
be examined.

In a study of the L-spectra of nickel and copper Cauchois
(1953) rejects the wavelength determinations of Shearer (1935)
for the reasons stated as follows. "This author \( i.e., \) Shearer
made use of a bent mica crystal. Reference lines of high order
were registered close to Ni L bands in the first order. The
Bragg angles were then calculated with the help of wavelengths and grating constants from Siegbahn's "Spektroskopie der Rontgenstrahlen." But wavelength values have since been modified; and moreover mica crystals may have widely different 'constants', depending on the nature of the mica. This may introduce errors in the final results."

As it is given, Cauchois' censure of the use of a mica crystal cannot be accepted. In the first place, it does not specify the variety of mica concerned. Thus, while it is readily admitted that the lattice constants of the various varieties of mica (muscovite, biotite, lepidolite, etc.,) have different values, it is not admitted that the constants of the variety muscovite vary within the species itself. Secondly, no evidence in favour of the latter situation, which is what is presumably implied, is given by Cauchois. But it is not impossible to present evidence for the contrary assertion, here made, that muscovite mica has a constant structure insofar as X-ray diffraction is concerned.

Hendricks (1940) has studied the presence of variable structure in layer silicate lattices by an examination of the continuous X-ray scattering which appears when the lattice periodicity normal to the cleavage planes is destroyed. Crystals from one hundred different mica specimens were examined. Eighty of the specimens were similar in structure to biotite and gave the continuous scattering effect with but few exceptions. None of the twenty crystals examined having the muscovite structure showed observable continuous scattering. Thus muscovite, in contrast to biotite, is invariant in structure among the twenty specimens examined.
Now Hendricks shows that, in the case of biotite, the top silicate layer in a mica sequence can have any of three orientations, differing by 60° in rotation about the layer normal, without affecting the relationship of neighbouring oxygen ions in the last adjacent surface since they are arranged in an hexagonal array. The repetition of the arrangements resulting from these various orientations can produce six different lattice structures, all of which have been observed for biotite micas (Hendricks 1939).

The presence of a particular (hkl) reflection in the diffraction pattern of muscovite which is not present for biotite in Hendricks' work, is taken by him to indicate a distortion of the silicate layer in muscovite compared with the ideal structure described by Pauling (cf. Bragg 1937). Irrespective of the explanation of the distortion given by Hendricks, its magnitude is sufficient to make different orientations of the silicate layer non-equivalent and thus leads to a unique structure. Although this may appear contrary to first expectations it is, as Hendricks remarks, equivalent to stating that regular blocks (biotite) can be stacked in many more ways to fill space than can irregular ones (muscovite). Recalling that muscovite crystals showed no observable continuous scattering under experimental conditions giving pronounced effects with biotite, the conclusion is that muscovite has a unique invariant structure, and therefore, a unique invariant lattice constant Dn.

This conclusion would appear to be corroborated by the fact that Siegbahn (1943), as previously quoted, retains mica (muscovite) as a secondary standard together with gypsum and quartz, although
he rejects rock salt because of the imperfection of the diffraction lines it produces. Finally, it must be noted that the work described by Canchois, in the paper in which she criticises Shearer's use of mica, is performed in part with mica crystals.

SECTION IV-2 WAVELENGTH MEASUREMENTS OF THE K-SERIES LINES OF NEON

Most of the neon lines whose wavelengths have been determined are relatively low intensity satellites. These occurred too close to the strong nickel calibration lines to allow a direct comparison by overlapping the two spectra. This was possible, however, for the intense neon diagram line Kα1,2. The neon satellite lines were then measured with reference to neon Kα1,2 on plates not having the nickel spectrum present.

Four plates, Q2/22 (Plate XI), Q1/7 (Plate VII), Q1/8 (Plate VIII), and Q1/9 (Plate IX), were considered in determining the wavelength of neon Kα1,2. The average of three microdensitometer records taken in the overlap region of the nickel and neon spectra was obtained in the case of each plate. The resultant density curve from Q2/22, for example, is given in Fig. 50. The linear separations of the nickel lines Lβ1-Lα1,2 (=q), and nickel Lα1,2-neon Kα1,2 (= p), were measured between the peaks on these curves. The following table collects certain information to be referred to later.
<table>
<thead>
<tr>
<th>PLATE</th>
<th>KNEF-EDGE</th>
<th>DENSITY $Lp - L\alpha$ (q mm)</th>
<th>$L\alpha - K\alpha$ (pmm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Q1/7</td>
<td>In</td>
<td>0.2/0.3</td>
<td>7.55</td>
</tr>
<tr>
<td>Q1/8</td>
<td>Out</td>
<td>0.4/0.4</td>
<td>7.60</td>
</tr>
<tr>
<td>Q1/9</td>
<td>Out</td>
<td>0.8/0.2</td>
<td>7.58</td>
</tr>
<tr>
<td>Q2/9</td>
<td>In</td>
<td>0.7/0.6</td>
<td>7.60</td>
</tr>
</tbody>
</table>

Several factors must be considered when estimating the accuracy of the measured values of $p$ or $q$. The possibility that the peak positions of the close lines $L\alpha$ and $K\alpha$ might be altered by mutual interaction was considered (cf. Archold 1954). At least it can be said that there is no correlation between the values of $p$, and either the actual or the relative peak density values of these lines as given in the table.

The nickel anticathode and the effective neon source are at quite different distances from the crystal. If the latter were thereby illuminated in a differential fashion by the two sources, and the instrumental aberrations of Section III-2 were significant, it is possible that the density centres, and hence the peaks of the recorded lines, could be displaced. Particularly if the plate were too far from the exact focal position for the assumptions of Appendix III (Section IV-2) to be valid. The absence of correlation, as shown in the table, between the values of $p$ or $q$ and the position of the knife-edge (its presence limits the crystal aperture to one-quarter of its normal value) indicates that no such effect was occurring.

In general, the peak to peak separation $x$ of any two lines
determined from a microdensitometer record, may possibly be in error by an amount $\Delta x = (\delta x' + \delta x'' + \delta x''' + \ldots)$ where the contributory errors $\delta x'$, etc., are independent.

$\delta x'$ is the error, due to the finite slit displacement (cf. footnote to Appendix XI), and on the averaged curve has a value $\pm 0.020$ mm.

$\delta x''$ is the error, due to the lead-screw on the microdensitometer plate carriage, and has a value $\pm 0.005$ mm.

$\delta x'''$ is the error, due to inherent and observational uncertainty in the microdensitometer readings (or automatic chart recorder) which, together with any error due to remnant grain effect, is estimated to have a value $\pm 0.005$ mm.

It follows that the possible error in $x$ is $\Delta x \pm 0.03$ mm which is in accord with the 0.05 mm range covered by the values for $p$ or $q$ measured on the four plates. Taking the best value of $p$ or $q$ to be the arithmetical mean of the four separate measurements, a Gaussian probable error was also assigned to the measurements. Finally then,

$$\begin{align*}
p &= 1.29 \text{ mm} \\
q &= 7.58 \text{ mm} \} \text{ Best values} \\
(\Delta p = \Delta q) &= \pm 0.01 \text{ mm (Probable error)} \\
(\Delta p = \Delta q) &= \pm 0.03 \text{ mm (Possible error)}.
\end{align*}$$

The wavelength of neon $K \alpha_{1,2}$ is evaluated by writing $c=p$ and $d=q$ in Case III of the multiple line method of Appendix IV, the other quantities involved being as follows

$$\begin{align*}
\lambda (\text{Ni } L \beta_1) &= 14.27900 \ldots \lambda = 14.250499 \ldots \text{kX} \\
\lambda (\text{Ni } L \alpha_{1,2}) &= 14.56600 \ldots \lambda = 14.536926 \ldots \text{kX} \\
(\text{Conversion Factor } \frac{\lambda}{\lambda_0} = 1.00200 \ldots \text{cf. Wood 1947}) \\
D &= 9.9275800 \ldots \text{kX}
\end{align*}$$
This yields a value for \( \lambda (\text{Ne K}\alpha 1,2) \) of 14.585044...kX. Since
the method of measurement is a relative one, both the reference
wavelengths and the crystal lattice constant are assumed to be
accurately known. Any error in the measured wavelength is there-
fore due to the errors \( \Delta p \) and \( \Delta q \) alone. It is calculated in
Appendix VIII, and we may accordingly write that

\[
\lambda (\text{Ne K}\alpha 1,2) = 14.5850 \pm \{0.0004 \text{ (Probable)} \} + 0.001 \text{ (Possible)} \text{kX}.
\]

The wavelengths of the satellite lines (I-V), found by the
analysis of the intensity record of plate Q2/20 described in
Section IV-7, were determined with reference to \( \lambda (\text{Ne K}\alpha 1,2) \).
The spectrometer setting for this plate was the same as for
Q1/7,8,9 and Q2/22, i.e., for \( \lambda (\text{Ni L}\alpha 1,2) \) to be in focus.
The distance \( u \), from the peak position of \( K\alpha 1,2 \) on plate Q2/20
to the wavelength position in focus, was thus known to be
\( u(=p)=1.29 \text{ mm} \). If the various separations of the lines (I-V)
from \( K\alpha 1,2 \), as measured on the analysed record of Q2/20, are
denoted by \( v \), and we write \( c=v \) and \( d=u \), Case II of the multiple
line method of Appendix IV is applicable, and the wavelengths
may be evaluated. The other quantities involved are

\[
\lambda (\text{Ne K}\alpha 1,2) = 14.585044...\text{kX}
\]

and

\[
D = 9.92758...\text{kX}.
\]

The possible error \( \Delta \lambda \), in any of the satellite wavelengths
\( \lambda_3 (I-V) \) thus determined, is due to the possible errors
\( \Delta \lambda (K\alpha 1,2) \), \( \Delta v \), and \( \Delta u \), the latter being given by
\( \Delta u (= \Delta p) = 0.03 \text{ mm} \). Since we are considering the record from
a single plate the error $\Delta v$ is contributed to by $\delta x' + \delta 0.020 \text{ mm}$, $\delta x'' + \delta 0.005 \text{ mm}$, and $\delta x''' + \delta 0.005 \text{ mm}$ as explained previously in the measurement of $\lambda(\text{Ne K}\alpha 1,2)$. There is now an additional error contribution to $\Delta v$ arising from the structural analysis of the plate record. Thus, it is considered that the peak positions of any of the postulated component lines (I-V) could be in error by an amount $\delta x'''' + \delta 0.010 \text{ mm}$ without destroying the uniqueness of the agreement obtained. It follows that the possible error in any measurement of $v$ is $\Delta v + \delta 0.04 \text{ mm}$. The value of $\Delta \lambda_3(\text{I-V})$ is calculated in Appendix IX, and we may accordingly list the following wavelengths determined from plate 02/20.

### PLATE 02/20

<table>
<thead>
<tr>
<th>LINE</th>
<th>WAVELENGTH (kX)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>14.538 $\pm$ 0.003</td>
</tr>
<tr>
<td>II</td>
<td>14.527</td>
</tr>
<tr>
<td>III</td>
<td>14.435</td>
</tr>
<tr>
<td>IV</td>
<td>14.475</td>
</tr>
<tr>
<td>V</td>
<td>14.465</td>
</tr>
</tbody>
</table>

Satellite wavelengths were also measured with reference to $\lambda(\text{Ne K}\alpha 1,2)$ on plate 02/36, the spectrometer setting having been such that the wavelength in exact focus corresponded to a Bragg angle of $46^0 \pm 0.000^\circ$. In Section IV-7 an analysis of the intensity record of that region of the plate containing lines (I-V) was given. The remainder of the satellite lines (VI-XXI) were identified from the density contour itself. If the measured peak to peak separation of Ne K$\alpha 1,2$ and any satellite line is called $n$, and the estimated distance from Ne K$\alpha 1,2$ to the wave-
length position in exact focus is called m, then the satellite
wavelengths \( \lambda_s (I-XXI) \) may be evaluated by writing \( c=n \) and \( d=m \)
in Case II of the multiple line method of Appendix IV. The value
of m is calculated as in Appendix X, the other quantities involved
being

\[ \lambda (Ne K\alpha_1, 2) = 14.585044... \text{kX} \]

and

\[ D = 9.2758... \text{kX}. \]

The possible error \( \Delta \lambda_s (I-XXI) \) in the wavelengths thus
determined is due to the possible errors \( \Delta \lambda (K\alpha_1, 2), \Delta m, \) and
\( \Delta n. \) In the case of lines (I-V) the contributions to the error
\( \Delta n \) are the same \( (\delta x', \text{ etc.}) \), as defined in the measurements on
plate Q2/20. Their magnitudes are altered, however, by the fact
that the microdensitometer runs over Plate Q2/36 were automatically
recorded. This results in the values \( \delta x' \pm 0.013 \text{ mm}, \)
\( \delta x'' \pm 0.005 \text{ mm}, \delta x''' \pm 0.003 \text{ mm}, \) and \( \delta x'''' \pm 0.006 \text{ mm}, \)
which add up to a value of \( \Delta n \pm 0.03 \text{ mm}. \) For the remaining
lines (VI-XXI), n has similar contributions of \( \delta x' \pm 0.013 \text{ mm}, \)
\( \delta x'' \pm 0.005 \text{ mm}, \) and \( \delta x''' \pm 0.006 \text{ mm}, \) but it is more
difficult to assign a value to the contribution \( \delta x''''. \) This is
regarded as being the amount by which the peaks in the density
curve might be displaced from the actual line peaks they have been
assumed to represent. It is argued that \( \delta x''' \) retains a value
\( \pm 0.003 \text{ mm} \) since, if the plate record had been analysed in this
region, the density peaks would merely be converted to intensity
peaks with identical separations. But this assumes that the
supposed resultant component lines (VI-XXI) would be of similar
breadths, relative intensities, and spacings as the actual
component lines (I-V). A value of ε × ′′′′ ‡ ± 0.005 mm has accordingly been associated with all of the lines (I-XXI), for which we therefore write Δ n ≥ ± 0.03 mm. The value of Δ λ g (I-XXI) is calculated in Appendix X and attached to the following wavelengths determined from plate 92/36.

PLATE 92/36

<table>
<thead>
<tr>
<th>LINE</th>
<th>WAVELENGTH (kX)</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>14.541 ± 0.0025</td>
</tr>
<tr>
<td>II</td>
<td>14.529</td>
</tr>
<tr>
<td>III</td>
<td>14.486</td>
</tr>
<tr>
<td>IV</td>
<td>14.477</td>
</tr>
<tr>
<td>V</td>
<td>14.468</td>
</tr>
<tr>
<td>VI</td>
<td>14.450</td>
</tr>
<tr>
<td>VII</td>
<td>14.407</td>
</tr>
<tr>
<td>VIII</td>
<td>14.402</td>
</tr>
<tr>
<td>IX</td>
<td>14.394</td>
</tr>
<tr>
<td>X</td>
<td>14.387</td>
</tr>
<tr>
<td>XI</td>
<td>14.383</td>
</tr>
<tr>
<td>XII</td>
<td>14.377</td>
</tr>
<tr>
<td>XIII</td>
<td>14.369</td>
</tr>
<tr>
<td>XIV</td>
<td>14.363</td>
</tr>
<tr>
<td>XV</td>
<td>14.359</td>
</tr>
<tr>
<td>XVI</td>
<td>14.352</td>
</tr>
<tr>
<td>XVII</td>
<td>14.343</td>
</tr>
<tr>
<td>XVIII</td>
<td>14.335</td>
</tr>
<tr>
<td>XIX</td>
<td>14.327</td>
</tr>
<tr>
<td>XX</td>
<td>14.315</td>
</tr>
<tr>
<td>XXI</td>
<td>14.279</td>
</tr>
</tbody>
</table>

SECTION IV-10 IDENTIFICATION OF THE K-SERIES LINES OF NEON

It is not possible, in a report of this nature, to review the considerable literature on X-ray satellite lines. Reference may be made to the summary articles of Richtmyer (1937), Hirsh (1942), and Cauchois (1944), while various experimental papers particularly concerned with K-series satellites will be subsequently referred to.
The identity of any X-ray line might be decided on a theoretical basis, i.e., by calculating the wavelengths and intensities of the various lines possible for a given atom according to the assumed mechanism of origin. Such calculations are, in the case of satellite lines, few in number and usually involve simplifying approximations. They will not be discussed here. From another viewpoint, lines may be identified or classified on a purely descriptive basis, by comparison with the experimentally established data from adjacent elements.

The most commonly employed device for comparing X-ray spectra is the so-called Moseley diagram. Moseley (1913) found that the square root of the frequency of a typical line is a linear function of the atomic number. It is now conventional, in a Moseley diagram, to plot \((\sqrt{\nu}/R)^{1/2}\) against atomic number \(Z\), where \(\nu\) is the wave-number (or reciprocal wavelength), and \(R\) is the Rydberg constant for an atom of infinite mass. Such a diagram has been constructed for the \(K\alpha\) and \(K\beta\)-series lines of the elements in the atomic number range 10-18 and is reproduced as Fig. 51. The \((\sqrt{\nu}/R)^{1/2}\) values were taken from the tables of Cauchois and Hulubei (1947) except in the case of \(A(18)\), which are due to Nilsson and Slatis (1944), and in the case of \(Ne(10)\) where the present determinations, collected in the next section, were used. The plot, part of which is enlarged in Fig. 52, illustrates clearly the "linear" relationships existing within both the \(K\alpha 1,2\) and \(K\beta 1\)-series of diagram lines. (The line \(K\beta 1\) would not normally occur in the neon \(K\)-series, a matter which will be discussed later). However, since it is impractical to plot on
a scale sufficient to resolve them, this method of portrayal is of little value in establishing relationships between the satellite lines.

As early discovered by Richtmyer (1928), a semi-Moseley law does exist between typical satellite lines. In this case a plot of \( (\Delta (v/R))^{1/3} \) against atomic number \( Z \) is expected to be linear if \( \Delta (v/R) = (v/R_{\text{Satellite}} - v/R_{\text{Parent}}) \). The nearest strong diagram line is then to be regarded as the parent line of the satellites concerned. As evidenced by the literature, there is a certain amount of confusion as to the significance of the parent line in the satellite semi-Moseley diagram. Thus, Deodhar (1931) showed that if certain satellite lines, normally thought to be in relation to a parent line \( K\beta 1 \), were plotted as if in relation to a parent line \( K\alpha 1 \), much improved linearity resulted. His work was criticised by Hirsh (1932), whereas Ford (1932), who was aware of Hirsh's criticisms, points out that the improvement in the semi-Moseley diagram of the \( \beta \) -satellites is not limited to the lines selected by Deodhar, but extends to all the \( \beta \) -satellites. The conclusion most consistent with the theoretical and experimental information available would appear to be that, although it is most plausible (in terms of mechanism of origin) that the nearest strong diagram line to a group of satellites is the parent line of that group, both the \( K\beta 1 \) and \( K\alpha 1,2 \) groups of satellites are more amenable to empirical classification (in terms of the semi-Moseley diagram) if referred to \( K\alpha 1,2 \). It makes little difference whether \( K\alpha 1 \) or \( K\alpha 2 \) is chosen for the reference line. This implies that the relationships portrayed in the semi-Moseley
The semi-Moseley diagram of Fig. 53 was constructed by plotting values of \( \Delta (\mathcal{V}/R) \) \( \frac{1}{3} \) against atomic number \( Z \) over the range 10-18, where \( \Delta (\mathcal{V}/R) \) = \( \mathcal{V}/R \text{Satellite} - \mathcal{V}/R \text{K}\alpha 1,2(\text{K}\alpha 1 \text{ if resolved}) \). This diagram, part of which is shown enlarged in Fig. 54, is the main basis on which certain of the neon satellite lines recorded in the present work have been identified.

Identification of lines is complicated, in the region of neon, by the overlapping of the K\( \beta \) and K\( \alpha \) groups of satellites. This raises the question, previously referred to, of the likelihood of obtaining K\( \beta \)-series lines in the spectrum of neon when the M level is unoccupied. In fact, however, the diagram line K\( \beta 1(K \rightarrow M_{III}) \) has been reported in both Na and Mg although here too the M level is normally vacant. The explanation is that for these elements the K\( \beta 1 \) line is "semi-optical" (Backlin, Siegbahn and Thoraeus 1925) in that an electron is previously raised to an outer unoccupied (optical) level by the absorption of energy, and then falls into a subsequently created inner vacancy with the emission of energy (Maitra 1937). A weaker component K\( \beta 3(K \rightarrow M_{II}) \) may also exist but has not been resolved at these low atomic numbers. Having thus established the possibility of its occurrence, the neon line labelled VI was identified as the semi-optical diagram line K\( \beta 1 \) by extrapolation of the Moseley plot of Fig. 51, and has also been indicated in the semi-Moseley plot of Fig. 53.

There is now no sufficient reason for denying the K\( \beta 1 \) parentage of some of the neon satellites. A simplifying
restriction is introduced by considering that the satellite lines occur only on the short wavelength side of their parent lines. Thus, only $\alpha$-satellites may occur between the positions of $K\alpha^1,2$ and $K\beta^1$ shown in Fig. 53.

There seems little doubt that lines II, III and V are to be identified as $\alpha^1$, $\alpha^3$ and $\alpha^4$ respectively.

The $\alpha^3,4$ group of satellites is possibly more complex than the known data plotted in the graphs indicates. If the two components $\alpha^3'$ and $\alpha^4'$ recorded by Deodhar (1931) for Si(14) are accepted, it is probable that they exist for the other elements but have not been resolved. It has therefore been preferred to identify line IV as $\alpha^3',\alpha^4'$.

Above the position of ($K\beta^1$) on the diagram, both $\beta$ and $\alpha$ satellites are possible. Lines IX, XIII, XVI and XIX are doubtless to be identified as $\alpha^8,\alpha^5,\alpha^7$ and $\alpha^6$ respectively; line XX is probably $\beta x$ (a line consistently present up to $Z=17$); line VII is possibly $\beta 0$; while line XXI is equally likely to be $\alpha 9$ or $\beta v (\alpha^9, \beta v)$.

Any lines present in the neon spectrum corresponding to $\alpha$ or $\beta$ satellites above the diagram position of $\alpha 9, \beta v$ would have been obliterated by the edge-effect (Section IV-3).

Lines VIII, X, XI, XII, XIV, XV, XVII and XVIII have no obvious correlation with the known lines from the elements of higher atomic number. They could well be semi-optical $\beta$-satellites whose probability of occurrence has been favoured by the conditions in the discharge source where many atoms will be excited into optical states.
There remains to be identified line I. It would appear to be the satellite $K\alpha''$ reported by Deodhar (1931) for Si(14) and P(15). However, Deodhar himself expressed some doubt as to the origin of his $\alpha''$ lines (he regarded $K\alpha'$, $\alpha''$ as being a doublet), particularly in the case of silicon. The specimen of this substance worked with was the same as previously used by Backlin (1925), who recorded $\alpha'$ alone. The matter has been referred to by Ford (1932) who, incidentally, errs in thinking that Deodhar found $\alpha'$ to be a doublet in silicon alone. Working with a greater dispersion than Deodhar, Ford did not find any trace of the doublet nature of $\alpha'$ between Mg(12) and Ge(32).

Parratt (1936) reports an extensive experimental examination of a satellite line named $K\alpha''$ which occurs on the short wavelength side, near the base, of the $K\alpha$ diagram line with elements S(16) to V(23). It is important to note that in these elements the line $\alpha'$ is well separated from Parratt's $\alpha''$, being closer to the $\alpha_{3,4}$ satellite group. He gives a discussion on the confusion which exists in regard to the designation of X-ray satellites, and illustrates his remarks by reference to the use which has been made of the symbol $\alpha''$. He refers to Deodhar's work on silicon and phosphorus. It is concluded, after reading Parratt's paper, that the line he labels $K\alpha''$ for the elements in the atomic number range 16-23, is probably not to be classified with the lines $K\alpha''$ reported by Deodhar for silicon and phosphorus. The positions of Parratt's $K\alpha''$ for S(16) and Cl(17) are noted on the diagram. Finally, the uncertainty which is present in
regard to the reality of line I found in the present work does not warrant its use as an argument in favour of the doublet nature of $K\alpha'$. It seems in order, nevertheless, to temporarily designate it as $K\alpha''$.

SECTION IV-11 THE EXPERIMENTAL RESULTS SUMMARISED AND COMPARED WITH THOSE OF KUNZL AND SVOBODOVA-JOANETTI

The measurements and determinations detailed in this report are summarised in the following two tables. The value of $\lambda(\alpha'1,2)$ is from plates Q1/7,8,9; Q2/22 (Section IV-9). The values of $\lambda(I-V)$ are the means of the measurements on plates Q2/20,36 (Section IV-9). The values of $\lambda(VI-XXI)$ are from plate Q2/36 (Section IV-9). To calculate the values of $\mathcal{V}/R$, $\mathcal{V}$ was expressed in reciprocal X-units and the generally accepted value of $R=109,737$ cm$^{-1}$ used. The value of $w_{\lambda,E}(\alpha'1,2)$ is from plates Q1/7,9 (Section IV-6). The values of $w_{\lambda,E}(I-V)$ are the means of the measurements on plates Q2/20,36 (Section IV-7).

**K-SERIES LINES OF NEON**

FULL WIDTHS AT HALF MAXIMUM INTENSITY
(Uncorrected for possible diffraction breadth)

<table>
<thead>
<tr>
<th>LINE</th>
<th>$w_{\lambda}(X)$</th>
<th>$w_{E}(ev)$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha'1,2$</td>
<td>14.7</td>
<td>0.85</td>
</tr>
<tr>
<td>I</td>
<td>$\alpha''$</td>
<td>12.5</td>
</tr>
<tr>
<td>II</td>
<td>$\alpha'$</td>
<td>16.9</td>
</tr>
<tr>
<td>III</td>
<td>$\alpha'$</td>
<td>14.1</td>
</tr>
<tr>
<td>IV</td>
<td>$\alpha_3',\alpha_4'$</td>
<td>9.5</td>
</tr>
<tr>
<td>V</td>
<td>$\alpha_4$</td>
<td>14.1</td>
</tr>
</tbody>
</table>
### K-Series Lines of Neon

**Wavelengths Etc.**

<table>
<thead>
<tr>
<th>Line</th>
<th>(λx)</th>
<th>υ/R</th>
<th>(υ/R)^2</th>
<th>(Δ(υ/R))^2</th>
</tr>
</thead>
<tbody>
<tr>
<td>α1,2</td>
<td>14.5850</td>
<td>62.480</td>
<td>7.9045</td>
<td>-</td>
</tr>
<tr>
<td>I</td>
<td>14.539</td>
<td>62.68</td>
<td>7.917</td>
<td>0.447</td>
</tr>
<tr>
<td>II</td>
<td>14.528</td>
<td>62.73</td>
<td>7.920</td>
<td>0.500</td>
</tr>
<tr>
<td>III</td>
<td>14.486</td>
<td>62.91</td>
<td>7.932</td>
<td>0.656</td>
</tr>
<tr>
<td>IV</td>
<td>14.476</td>
<td>62.95</td>
<td>7.934</td>
<td>0.686</td>
</tr>
<tr>
<td>V</td>
<td>14.466</td>
<td>62.99</td>
<td>7.937</td>
<td>0.714</td>
</tr>
<tr>
<td>VI</td>
<td>14.430</td>
<td>63.15</td>
<td>7.947</td>
<td>0.819</td>
</tr>
<tr>
<td>VII</td>
<td>14.407</td>
<td>63.25</td>
<td>7.953</td>
<td>0.878</td>
</tr>
<tr>
<td>VIII</td>
<td>14.402</td>
<td>63.27</td>
<td>7.954</td>
<td>0.889</td>
</tr>
<tr>
<td>IX</td>
<td>14.394</td>
<td>63.31</td>
<td>7.957</td>
<td>0.911</td>
</tr>
<tr>
<td>X</td>
<td>14.387</td>
<td>63.34</td>
<td>7.959</td>
<td>0.927</td>
</tr>
<tr>
<td>XI</td>
<td>14.383</td>
<td>63.36</td>
<td>7.960</td>
<td>0.938</td>
</tr>
<tr>
<td>XII</td>
<td>14.377</td>
<td>63.38</td>
<td>7.961</td>
<td>0.949</td>
</tr>
<tr>
<td>XIII</td>
<td>14.369</td>
<td>63.42</td>
<td>7.964</td>
<td>0.970</td>
</tr>
<tr>
<td>XIV</td>
<td>14.363</td>
<td>63.45</td>
<td>7.966</td>
<td>0.985</td>
</tr>
<tr>
<td>XV</td>
<td>14.359</td>
<td>63.46</td>
<td>7.966</td>
<td>0.990</td>
</tr>
<tr>
<td>XVI</td>
<td>14.352</td>
<td>63.49</td>
<td>7.968</td>
<td>1.005</td>
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<tr>
<td>XVII</td>
<td>14.343</td>
<td>63.53</td>
<td>7.971</td>
<td>1.025</td>
</tr>
<tr>
<td>XVIII</td>
<td>14.335</td>
<td>63.57</td>
<td>7.973</td>
<td>1.044</td>
</tr>
<tr>
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<td>14.327</td>
<td>63.61</td>
<td>7.976</td>
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<tr>
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<td>14.315</td>
<td>63.66</td>
<td>7.979</td>
<td>1.086</td>
</tr>
<tr>
<td>XXI</td>
<td>14.279</td>
<td>63.82</td>
<td>7.989</td>
<td>1.158</td>
</tr>
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</table>

Some comparisons may now be made with the earlier work on neon, carried out by Kunzl and Svobodova-Joanelli (1946), and summarised previously in Section IV-1. Although they give no estimate of the accuracy of their wavelength measurements it is probably correct to say that their values for lines α1,2, α′, α′′, α3, α4, β0, β1, etc., are in good agreement with the present results.
$\alpha'3$ and $P1$ are in reasonable agreement with the present ones. Now, Kunzl and Svobodova-Joanelli apparently examined their plates with a microscope only and did not obtain the actual intensity contour of the spectrum. This lead them to regard the line $\alpha'$ as being broadened towards long wavelengths, and they accordingly gave a value for its long wavelength limit. Contrary to this interpretation, the present analysis of the intensity contour is in favour of the presence of the line $\alpha''$, rather than an unexplained asymmetry in $\alpha'$. Again, their microscope examination did not allow them to discern the line $\alpha'3'$, $\alpha'4'$ reported here, although they suspected its presence. It explains why their wavelength value for line $\alpha'4$ is further from the expected value (Moseley diagram value) than the present one. Finally, the diffuse band reported by Kunzl and Svobodova-Joanelli in the region of $K(\alpha'8-\alpha'9)$ has, in the present work, been shown to consist of several lines, some of which have been identified, and to all of which wavelength values have been ascribed.

The present study of the shapes of the principal neon lines, together with the measurements of their half width values, is the first to be reported.
GENERAL CONCLUSION

It has been demonstrated that the high voltage electrodeless discharge is a useful source for the purposes of X-ray spectroscopy. The effectiveness of the source could be improved by increasing the power delivered to it, and possibly by raising the frequency of operation. It is desirable that the breadth of the external electrode tube be increased so as to eliminate the edge effects from the recorded spectra.

The results obtained show that the spectrograph is capable of providing spectral records from gases in the region of soft X-rays. With a few minor improvements of a technical nature the apparatus will be completely reliable and present no difficulties in its operation.

The wavelength and half width values reported for the K-series of neon are considered to be the best at present available.
REFERENCES

BACKLIN, E., SIEGBAHN, M., and THORAEUS, R., 1925, Phil. Mag., 42, 513.
CAUCHOIS, Y., 1953, Phil. Mag., 44, 173.
DARWIN, C.G., 1922, Phil. Mag., 43, 800.
DUMOND, J.W.M., 1933, Rev. Mod. Phys., 1, 5.
HIRSH, F.R., 1942, Rev. Mod. Phys., 14, 45.
KUNZL, V., 1936, Z. Phys., 22, 482.
KUNZL, V., and SOBOLOVA-JOANELLI, M., 1946, Acad. Int.
Tchéque Sci., 47, 91.
(London: Physical Society).
MOSELEY, H.G.J., 1913, Phil. Mag., 26, 1024; 1914, Ibid., 27, 703.
REININGER, M., 1934, Z. Krist., 93, 544.
RICHTMYER, F.K., 1928, Phil. Mag., 6, 64.
SHEARER, J., 1935, Phil. Mag., 20, 504.
SIEGBAHN, M., 1931, Spektroskopie der Rontgenstrahlen (Berlin: Springer).
APPENDICES I - XI
APPENDIX I

Image Aberration due to an Extended Source.

This broadening of the image, called the height defect $b''$, arises from the fact that monochromatic radiation other than that travelling in the horizontal focal plane of the spectrometer can be Bragg reflected by the crystal to fall in the image region. In fact, for an extended source, each point in an elemental vertical strip of the crystal reflects rays which are generators of a right circular cone that intersects the plate to give a parabolic trace. The trace from each reflecting point is displaced in a vertical sense from its neighbours and results in what is effectively a line, of breadth $b''$. The magnitude of $b''$ found from the present simple derivation is identical with that obtained in a mathematically rigorous treatment.

It may be shown that the broadening on the focal circle is approximately equal to that which occurs if the recording plate is parallel to the tangent plane through the crystal pole. The figure is drawn accordingly.
Let the source extend a distance $h$ above and below the central focal plane STP.

For broadening to occur at $P$, an arc $PK$ must extend downwards as far as the line STP. This is possible only if there is actual source on the arc $SJ$ above line STP.

As $SS_i$ increases, i.e., as $P_i$ departs from $P$, the magnitude of the arc displacement $PK$, which gives the broadening, also increases. Evidently it has its maximum value when the line $KT_iJ$ resides with $J$ at the source extremity.

We have that

$$JF/KG = \frac{(SH-SS_i)}{(SH-SiH)}$$
$$= \frac{(h-SS_i)}{(h-(h-SS_i))}$$
$$= \frac{(h-TT_i)}{TT_i}$$
$$= \frac{h}{TT_i} - 1$$

Also

$$JF/KG = \frac{FT_i}{GT_i}$$
$$= \frac{(SiTi-S_iF)}{(TiP_i-GP_i)}$$
$$= \frac{SiTi}{TT_i} \quad [\text{since } SiTi \gg SiF \text{ etc.,}]$$
$$= \frac{ST}{TP}$$
$$= \frac{P_i}{P_2}, \text{ say.}$$

Therefore

$$\frac{h}{TT_i} - 1 = \frac{P_i}{P_2}$$

$$h/TT_i = (P_i + P_2)/P_2$$

$$TT_i = hP_2/(P_1 + P_2)$$
Further,

\[(TK + KP)^2 = TG^2\]
\[= TK^2 + KG^2\]
\[\therefore RTK \cdot KP = KG^2 - KP^2\]
\[= KG^2 \quad [\text{since } KG \gg KP]\]
\[\therefore KP = KG^2/2TK\]
\[= TT_i^2/2(\text{TP} - KP)\]
\[= TT_i^2/2\text{TP} \quad [\text{since } TP \gg KP]\]
\[= TT_i^2/2p_2\]

Previously we had that
\[TT_i = h\rho_2/(\rho_1 + \rho_2)\]

Whence
\[KP = h^2\rho_2/2(\rho_1 + \rho_2)^2\]

Now, KP is the broadening b'', and if the source and plate are at distances a and d respectively from the crystal, the Bragg angle being \(\phi\), we have that

\[\rho_1 = a \cos \phi\]
\[\text{and } \rho_2 = d \cos \phi\]

Consequently
\[b'' = h^2d \cos \phi/[2(a+d)^2 \cos^2 \phi]\]
\[= h^2d/[2(a+d)^2 \cos \phi]\]

For a recording plate at right angles to the reflected beam the last expression must be multiplied by a factor \(\sin \phi\) giving, finally, that
\[b'' = [h^2d/[2(a+d)^2]] \tan \phi.\]
With reference to the figure, suppose that the spectrometer allows a monochromatic source of vertical extent $2h$ at a distance $a$ from the crystal to contribute a total "instrument intensity" $I$ to a line image formed at a distance $d$ from the crystal.

In the vertical sense there is no focusing and the intensity contribution $I_v$ may be expressed by

$$I_v = k_v \frac{2h}{(a+d) \cos \phi}$$

$$= k_v \frac{l}{d \cos \phi}.$$ 

In the horizontal sense there is focusing and the intensity contribution $I_h$ may be expressed by

$$I_h = k_h \frac{s}{d}.$$ 

It follows that the total instrument intensity $I$ is given by

$$I = I_v \times I_h.$$
\[ I = I_v \times I_n \]
\[ = k' \frac{6l}{d^2 \cos \phi} \left[ = k' \frac{6l}{R \sin \phi \cos \phi} \right. \text{, cf. Section III-2} \]
\[ = k \delta l \]
since \( d \) and \( \phi \) are constant for a given wavelength, and where \( k \) is a constant of proportionality.

The derivative of \( I \) may be written as
\[ \Delta I = \Delta \delta (\partial I/\partial \delta) + \Delta l (\partial I/\partial l) \]
\[ = k l \Delta \delta + k \delta \Delta l \]
and this is zero when \( I \) has its maximum value.

\[ |\Delta \delta + \delta \Delta l| = 0 , \]
or, since \( l \) is proportional to \( h \),
\[ h \Delta \delta + \delta \Delta h = 0 . \]

The aberration breadth of the image (cf. Section III-2) is \((b'+b'')\) given by
\[ (b'+b'') = \frac{8^2}{8R} \cos \phi + [h'd/(2(a+d)^3)] \tan \phi . \]
The derivative of \((b'+b'')\) may be written for present purposes as
\[ \Delta (b'+b'') = \Delta \delta \left[ \partial (b'+b'')/\partial \delta \right] + \Delta h \left[ \partial (b'+b'')/\partial h \right] \]
\[ = \left( \frac{8}{4R} \right) \cos \phi \Delta \delta + \left[ \frac{hd}{(a+d)^3} \right] \tan \phi \Delta h , \]
and this is zero when \((b'+b'')\) has a minimum value.
\[
\frac{(s/4R) \cos \phi \, \Delta s + [hd/(a+d)^2] \tan \phi \, \Delta h}{\Delta s/\Delta h = 0}
\]

\[
\Delta s + (4R/\cos \phi) \left[ hR \sin \phi / (a+d)^2 \right] \tan \phi \, \Delta h = 0
\]

\[
\Delta s + [4R^2 \tan^2 \phi / (a+d)^3] \, h \, \Delta h = 0
\]

\[
\Delta s/\Delta h = - (h/8) \left[ 4R^2 \tan^2 \phi / (a+d)^3 \right]
\]

Previously we had that

\[
h \Delta s + s \, \Delta h = 0
\]

\[
\therefore \Delta s/\Delta h = - \frac{s}{h},
\]

and equating the two expressions for \(\Delta s/\Delta h\) gives that

\[
s/h = (h/8) \left[ 4R^2 \tan^2 \phi / (a+d)^3 \right]
\]

\[
\therefore s^2 = h^2 \left[ 4R^2 \tan^2 \phi / (a+d)^3 \right]
\]

\[
\therefore s = [2R \tan \phi / (a+d)] \, h.
\]

\[
= [2R \tan \phi / (a+R \sin \phi)] \, h.
\]

or,

\[
s/2h = [R \tan \phi / (a+R \sin \phi)]
\]

This optimum relationship between crystal aperture and source height also implies that the two breadth aberrations are of equal magnitude. We have that

\[
s^2 = 4h^2 RR \tan \phi \tan \phi / (a+d)^3
\]

\[
= 8R h^2 d \tan \phi \tan \phi / 2 \sin \phi (a+d)^2
\]

\[
= 8R h^2 d \tan \phi / 2 \cos \phi (a+d)^2
\]

\[
\therefore (s'/8R) \cos \phi = [h^2 d / 2 (a+d)^2] \tan \phi
\]

ie.,

\[
b' = b''
\]
APPENDIX III

The Single Line Method of Wavelength Determination.

We assume that the diffracting crystal is bent to be part of a true circle of Radius $R_0$, and that the spectrometer thus constituted has no errors of focus, radiation of wavelength $\lambda$ being reflected at angle $\phi$ according to the corrected Bragg equation $n\lambda = 2D\sin\phi$ to a point focus on the focal circle of radius $R_0/2$.

![Diagram](image)

Fig. a

With reference to fig. a, suppose that a wavelength $\lambda'$ is in focus at $P$ where the plate cuts the focal circle. Then, in the first order,

$$\lambda' = 2D\sin\phi'.$$

Further, the distance $L'$ of the focal point $P$ from the crystal pole 0 is given by

$$L' = R_0\sin\phi'.$$

Let radiation of wavelength $\lambda_x$ intersect the plate at a point distant $x'$ from $P$ so that

$$\lambda_x = 2D\sin\phi'_x.$$

The angular separation of the two rays is thus ($\phi'_x - \phi'$) and their linear separation on the plate $x'$ is given by

$$x' = L'\tan(\phi'_x - \phi').$$

This definition of $x'$ is perfectly general since it involves no special assumptions as to the magnitude of $\phi'_x$. 
We have that

\[ x' = L' \tan (\phi_x' - \phi') , \]

\[ \therefore (\phi_x' - \phi') = \tan^{-1} \left( \frac{x'}{L'} \right) \]

\[ \therefore \phi_x' = \tan^{-1} \left( \frac{x'}{L'} \right) + \phi'. \]

Further

\[ \lambda_x' = 2D \sin \phi_x', \]

\[ \therefore \lambda_x' = 2D \sin \left\{ \tan^{-1} \left( \frac{x'}{L'} \right) + \phi' \right\}. \]

The last expression gives the value of the unknown wavelength \( \lambda_x' \) which is recorded at a distance \( x' \) from the focal point \( F \).

For some purposes it is desirable to express this wavelength function as a power series. If we write \( \lambda_x' = f(x') \) it may be shown that \( f(x') \) can be expanded as an infinite series in powers of \( x' \), and further that it possesses successive differential coefficients. Maclaurin's theorem is therefore applicable and we may write, finally, that

\[ \lambda_x' = \lambda' + \left( \frac{x'}{L'} \right) \left( 2D \cos \phi' \right) - \left( \frac{x'^2}{L'^2} \right) \left\{ (2D/2!) \sin \phi' \right\} 

+ \left( \frac{x'^3}{L'^3} \right) \left\{ (6D/3!) \cos \phi' \right\} + \ldots \]

The only quantities in the wavelength function which are determined experimentally are the distances \( x' \) and \( L' \). But they do not appear independently, for in fact, it is the ratio \( x'/L' \) whose magnitude must be known. To record \( \lambda' \) exactly in focus the plate is set at a focal distance \( L' \) given by

\[ L' = R_0 \sin \phi' \]

\[ = R_0 \left( \frac{\lambda'}{2D} \right). \]

If there is an error in this setting, either in the positive or negative sense, there will be a broadening of the line as depicted in fig.b.
It is assumed that this broadening is symmetrical, i.e., that there is no relative shift of the intensity centre of the line. Further, it is assumed that all the wavelengths measured are sufficiently close to the focal point when recorded, to be considered in focus on the plate in comparison to the broadening they all suffer due to an error in the plate setting.

Now, \( L' = R_0 \left( \frac{\lambda'}{2D} \right) \), but the true radius \( R_0 \) is not known. We must therefore use a measured value \( R \) in order to set the plate at a distance \( R \left( \frac{\lambda'}{2D} \right) \) in a region where the relevant assumptions hold. Let this distance for the setting under consideration be \( L \). Our knowledge of its value is now simply that which the arm scale can provide, and is independent, under this aspect, of a knowledge of \( R_0 \) (or \( R \)).

If, as shown in fig. c, the experimentally measured distance between the unknown \( \lambda' \) and the known \( \lambda' \) is \( x' \), it follows that \( x/L = x'/L' \).

We may thus substitute the measurable \( x/L \) for the exact ratio \( x'/L' \) in the wavelength function and write, finally, that

\[
\lambda_x = 2D \sin \left\{ \tan^{-1} \left( \frac{x}{L} \right) + \theta' \right\}.
\]
**APPENDIX IV**

**The Multiple Line Method of Wavelength Determination**

This method, which is a modification of that given by Haglund (1942), involves the use of two known lines or wavelengths. The same general considerations apply as in the single line method, i.e., the plate is set at a distance \( L = R \sin \phi \) with accuracy sufficient to ensure that the assumptions made in Appendix III concerning the broadening of the lines are valid. For here, also, the measured distances appear as ratios rather than independently.

Four different cases have been considered, but, since the several derivations are similar, only one is given here, the other results being quoted.

With reference to the figure, the following notation is employed:

- \( \lambda_x \) = the unknown wavelength being measured
- \( \lambda_1, \lambda_2 \) = the known wavelengths, one of which is the reference wavelength
- \( \phi' \) = reflection angle of the reference wavelength
- \( \alpha \) = the angular separation of the known wavelengths
- \( \Delta \phi \) = the angular separation of the unknown wavelength and the reference wavelength
- \( d \) = the linear separation on the plate of the known wavelengths
- \( c \) = the linear separation on the plate of the unknown wavelength and the reference wavelength
- \( y \) = a parameter

**CASE I**: \( \lambda_x \) measured with reference to \( \lambda_2 \); \( \lambda_1 \) being in focus.

By geometry we have that

\[
y = \frac{d}{\sin \alpha},
\]

and

\[
\frac{c}{\sin \Delta \phi} = \frac{y}{\sin \lambda_1 \lambda_0} = \frac{y}{\cos (\alpha + \Delta \phi)}.
\]
whence
\[ \frac{c}{\sin \Delta \phi} = \frac{d}{\sin \alpha \cos(\alpha + \Delta \phi)} . \]

\[ \therefore \frac{d}{c} = \frac{\sin \alpha (\cos \alpha \cos \Delta \phi - \sin \alpha \sin \Delta \phi)}{\sin \Delta \phi} \]

\[ = \sin \alpha \cos \alpha \cot \Delta \phi - \sin^2 \alpha \]

\[ \therefore \cot \Delta \phi = \frac{d}{c} + \sin^2 \alpha \]

\[ \sin \alpha \cos \alpha \cot \Delta \phi \]

\[ = \frac{d}{c} \cosec \alpha + 1 \]

\[ \cot \alpha \]

\[ \therefore \tan \Delta \phi = \frac{c \tan \alpha}{\tan^2 \alpha \cosec \alpha + \frac{c}{d} \tan^2 \alpha} \]

\[ = \frac{\frac{c}{d} \tan \alpha}{1 + \tan^2 \alpha + \frac{c}{d} \tan^2 \alpha} \]

\[ = \frac{\frac{c}{d} \tan \alpha}{1 + \frac{c + d}{d} \tan^2 \alpha} \]

\[ \therefore \Delta \phi = \tan^{-1} \left[ \frac{\frac{c}{d} \tan \alpha}{1 + \frac{c + d}{d} \tan^2 \alpha} \right] . \]

The unknown wavelength is then found by substituting for \( \Delta \phi \) in the equation \( \lambda_x = 2D \sin (\phi' + \Delta \phi) \) giving, finally, that

\[ \lambda_x = 2D \sin \left\{ \phi' + \tan^{-1} \left[ \frac{\frac{c}{d} \tan \alpha}{1 + \frac{c + d}{d} \tan^2 \alpha} \right] \right\} . \]
CASE II: $\lambda_x$ measured with reference to $\lambda_1$; $\lambda_2$ being in focus.

$$\lambda_x = 2D \sin \left\{ \phi' + \tan^{-1} \left[ \frac{\frac{c}{d} \tan \alpha}{1 + \frac{d-c}{d} \tan^2 \theta} \right] \right\}.$$ 

CASE III: $\lambda_x$ measured with reference to $\lambda_2$; $\lambda_2$ being in focus.

$$\lambda_x = 2D \sin \left\{ \phi' + \tan^{-1} \left[ \frac{c}{d} \tan \alpha \right] \right\}.$$ 

CASE IV: $\lambda_x$ measured with reference to $\lambda_1$; $\lambda_1$ being in focus.

$$\lambda_x = 2D \sin \left\{ \phi' + \tan^{-1} \left[ \frac{c}{d} \tan \alpha \right] \right\}.$$
APPENDIX V

A Comparison of the Accuracies of the Single and Multiple Line Methods of Wavelength Measurement.

Suppose that an unknown wavelength \( \lambda_x \) is recorded by a perfect spectrometer together with two known wavelengths \( \lambda_1 \) and \( \lambda_2 \), the latter being in focus. \( \lambda_x \) may then be determined with reference to \( \lambda_2 (\lambda_1) \) by the multiple line method (Appendix IV), or with reference to \( \lambda_x \) by the single line method (Appendix III). The following identities may accordingly be written:

\[
\lambda_2 = \lambda' \quad (\phi_2 = \phi')
\]

\[x = c\]

As shown in Appendix IV, the multiple line wavelength function takes the two general forms

\[
\lambda_x = 2D \sin \left\{ \phi' + \tan^{-1} \left[ \frac{(c/d) \tan \alpha}{1 + (d + d'/d) \tan \alpha} \right] \right\}
\]

and

\[
\lambda_x = 2D \sin \left\{ \phi' + \tan^{-1} \left[ \frac{(c/d) \tan \alpha}{1 + (d + d'/d) \tan \alpha} \right] \right\}
\]

Since the angle \( \alpha \) is small (about 1° in the present work) the denominator in the \( \tan^{-1} \) term is almost equal to 1. Further, the effect on the value of the \( \tan^{-1} \) term of variations in \( \tan \alpha \) is negligible compared with the effect of variations in \( \tan \alpha \) in the numerator. It follows that the two forms of the wavelength function are equivalent as regards the calculation of errors, and we need only consider the simpler expression

\[
\lambda_x = 2D \sin \left\{ \phi' + \tan^{-1} \left[ \frac{(c/d) \tan \alpha}{1 + (d + d'/d) \tan \alpha} \right] \right\}
\]

We have that

\[
\alpha = (\phi_2 - \phi_1)
\]

where

\[
\phi_2 (\neq \phi') = \sin^{-1} \left( \frac{\lambda_2}{2D} \right)
\]
and
\[ \phi_1 = \sin^{-1}\left(\frac{\lambda_1}{2D}\right) \]

whence
\[ \lambda x = 2D \sin\left\{ \sin^{-1}\left(\frac{\lambda x}{2D}\right) + \tan^{-1}\left[\left(\frac{c}{d}\right) \tan\left(\sin^{-1}\left(\frac{\lambda x}{2D}\right) - \sin^{-1}\left(\lambda x/2D\right)\right)\right]\right\} \]

The complete differential of \( \lambda x \) may be written as
\[ \Delta \lambda x = \Delta \lambda_2 \left( \frac{\partial \lambda x}{\partial \lambda_2} + \Delta \lambda_1 \left( \frac{\partial \lambda x}{\partial \lambda_1} + \Delta d \left( \frac{\partial \lambda x}{\partial d} \right) + \Delta c \left( \frac{\partial \lambda x}{\partial c} \right) + \Delta d \left( \frac{\partial \lambda x}{\partial d} \right) \right) \]

the following being the expressions derived for the component partial derivatives.

\[ \frac{\partial \lambda x}{\partial \lambda_2} = \frac{\cos \left\{ \phi_2 + \tan^{-1}\left[\left(\frac{c}{d}\right) \tan \phi_2\right]\right\}}{\cos \phi_2} \left\{ 1 + \frac{\left(\frac{c}{d}\right) \sec^2 \phi_2}{1 + \left[\left(\frac{c}{d}\right) \tan \phi_2\right]^2} \right\} \]

\[ \frac{\partial \lambda x}{\partial \lambda_1} = \frac{\cos \left\{ \phi_2 + \tan^{-1}\left[\left(\frac{c}{d}\right) \tan \phi_2\right]\right\}}{\cos \phi_1} \left\{ \frac{\left(\frac{c}{d}\right) \sec^2 \phi_1}{1 + \left[\left(\frac{c}{d}\right) \tan \phi_1\right]^2} \right\} \]

\[ \frac{\partial \lambda x}{\partial d} = 2 \cos \left\{ \phi_2 + \tan^{-1}\left[\left(\frac{c}{d}\right) \tan \phi_2\right]\right\} \left\{ \frac{\left(\frac{c}{d}\right) \sec^2 \phi_2}{1 + \left[\left(\frac{c}{d}\right) \tan \phi_2\right]^2} \right\} \left( \tan \phi_1 - \tan \phi_2 - \tan \phi_1 \right) \]

\[ + 2 \sin \left\{ \phi_2 + \tan^{-1}\left[\left(\frac{c}{d}\right) \tan \phi_2\right]\right\} \]

\[ \frac{\partial \lambda x}{\partial c} = 2D \cos \left\{ \phi_2 + \tan^{-1}\left[\left(\frac{c}{d}\right) \tan \phi_2\right]\right\} \left\{ \frac{\tan \phi_2 / d}{1 + \left[\left(\frac{c}{d}\right) \tan \phi_2\right]^2} \right\} \]

\[ \frac{\partial \lambda x}{\partial d} = -2D \cos \left\{ \phi_2 + \tan^{-1}\left[\left(\frac{c}{d}\right) \tan \phi_2\right]\right\} \left\{ \frac{\left(\frac{c}{d^2}\right) \tan \phi_2}{1 + \left[\left(\frac{c}{d}\right) \tan \phi_2\right]^2} \right\} \]

As shown in Appendix III, the single line wavelength function has the form
\[ \lambda x = 2D \sin \left\{ \phi' + \tan^{-1}\left(\frac{x}{L}\right) \right\} . \]
We have that
\[ \phi'(z) = \sin^{-1}(\lambda z/2D), \]
whence
\[ \lambda_x = 2D \sin \left\{ \sin^{-1}(\lambda z/2D) + \tan^{-1}(x/L) \right\}. \]

The complete differential of \( \lambda_x \) may be written as
\[ \Delta \lambda_x = \Delta \lambda(\partial \lambda / \partial z) + \Delta \xi(\partial \lambda / \partial \xi) + \Delta \phi(\partial \lambda / \partial \phi) + \Delta \lambda (\partial \lambda / \partial \lambda), \]
the following being the expressions derived for the component partial derivatives.

\[ \left( \partial \lambda / \partial z \right) = \frac{\cos \left\{ \phi + \tan^{-1}(x/L) \right\}}{\cos \phi} \]
\[ \left( \partial \lambda / \partial \xi \right) = 2 \sin \left\{ \phi + \tan^{-1}(x/L) \right\} - 2 \cos \left\{ \phi + \tan^{-1}(x/L) \right\} \tan \phi \]
\[ \left( \partial \lambda / \partial \phi \right) = \frac{2D \cos \left\{ \phi + \tan^{-1}(x/L) \right\}}{L} \left\{ \frac{1}{1 + (x/L)^2} \right\} \]
\[ \left( \partial \lambda / \partial \lambda \right) = -\frac{2D \cos \left\{ \phi + \tan^{-1}(x/L) \right\}}{L} \left\{ \frac{1}{1 + (x/L)^2} \right\} \]

In the present relative measurements the given quantities \( \lambda_1, \lambda_2 \) and \( D \) are regarded as being known exactly. It follows that the wavelength error \( \Delta \lambda_x \) is contributed to by the errors \( \Delta c(\partial \lambda / \partial c) \) and \( \Delta d(\partial \lambda / \partial d) \) in the multiple line method, and by the errors \( \Delta x(\partial \lambda / \partial x) \) and \( \Delta \lambda (\partial \lambda / \partial \lambda) \) in the single line method. But substitution from the equalities \( x = c \) and \( \tan \alpha = d/L \) shows immediately that
\[ \Delta c(\partial \lambda / \partial c) = \Delta x(\partial \lambda / \partial x), \]
while the remaining two errors become

\[
\Delta d(\partial \lambda x / \partial d) = - \frac{2 D x \cos\{\phi_2 + \tan^{-1}(x/L)\}}{L d \{ 1 + (x/L)^2 \}} \Delta d,
\]

and

\[
\Delta L(\partial \lambda x / \partial L) = - \frac{2 D x \cos\{\phi_2 + \tan^{-1}(x/L)\}}{L^2 \{ 1 + (x/L)^2 \}} \Delta L.
\]

From the last expressions we obtain the relationship

\[
\Delta L(\partial \lambda x / \partial L) = \frac{\Delta L/L}{\Delta d/d} \Delta d(\partial \lambda x / \partial d),
\]

which shows that the relative accuracy of the single and multiple line methods, in the case of a perfect spectrometer, is effectively determined by the relative errors in the measurements of the quantities \( L \) and \( d \) respectively.
APPENDIX VI

An Error of Eccentricity in the Spectrometer Axis.

With reference to the figure, suppose that due to an eccentric error in the rotation mechanism of the spectrometer, the crystal pole is not identical with the rotation axis 0 but moves on a small circle of radius e.

If the plate is set at a distance L from axis 0, and two lines are recorded a distance d apart, this distance d will be ascribed to a difference in reflection angle \( \Delta \phi' \), at 0.

Thus,

\[
\Delta \phi' = \frac{d}{L}
\]

But, in fact, the reflection occurs at the point C, say, where the true difference in reflection angle is \( \Delta \phi \) given by

\[
\Delta \phi = \frac{d}{L+e}
\]

The magnitude of the error thus introduced into the estimated reflection angle of a line is given by

\[
(\Delta \phi' - \Delta \phi) = d \left[ \left( \frac{1}{L+e} \right) - \frac{1}{L} \right] = \frac{d}{L} \left\{ L \left[ 1 - e/L + (e/L)^2 + \ldots \right] \right\} = dL' - dL' + de/L^2 - de^2/L^3 \\
\approx de/L^2 \text{ radians.}
\]

From an examination of the manufacturer's data on the roller bearings used in the rotation mechanism it is estimated that \( e \) has an upper limit of 0.01 cm, while \( d \) and \( L^2 \) had magnitudes of about 1 cm and 1600 cm\(^2\) respectively. On substituting these values, the error in the reflection angle is found to have a negligible value of 0.0004°.
APPENDIX VII

An Error in the Angular Setting of the Spectrometer.

The experimental determination of angle in terms of the spectrometer scales is not involved in either the single or multiple line method of wavelength determination. However, the accuracy with which the angular setting of the rotating arm may be accomplished with respect to the crystal could effect the wavelength determination, by its introduction of an error into distances measured on the plate.

With reference to the figure, any two wavelengths $\lambda'$ and $\lambda_x$ are reflected by the crystal $C$ in fixed directions and with angular separation $6\theta$.

In the correct setting, the centre line of the arm is identical with the direction of reflection of $\lambda'$. Let there be an error $6\phi'$ in the angle setting, in the worst sense as shown.

The plate then shifts from its correct position $P$ where the measured line separation would be $x$, to position $P'$ where the actual measurement $x'$ is made. It is required to find by what amount $x'$ differs from $x$.

We have that

$$x' = y + z = x \sec 5\phi' + z.$$

Now,

$$\frac{z}{\sin 6\theta} = \frac{d}{\sin(3\theta - 6\theta - 6\phi')} = \frac{a + x \tan 6\phi'}{\cos (6\theta + 6\phi')}$$

$$z = \frac{a + x \tan 6\phi'}{\cos (6\theta + 6\phi')} \sin 6\theta.$$

And,

$$a = b \tan 6\phi'$$

$$= L \tan \left(\frac{5\phi'}{2}\right) \tan 5\phi'.$$
It follows that

\[ x' = x \sec \theta' + \frac{L \tan (\theta'/2) \tan \theta' + x \tan \theta'}{\cos (\theta' + \theta')} \sin \theta. \]

Expanding and neglecting third and higher order terms not containing \( L \) we have

\[
x' = x \left\{ 1 + (\theta'/2)^2 \right\} + \theta \frac{L \left\{ (\theta'/2)^2 + x \theta \theta' \right\}}{1 - (\theta' + \theta)^2} \]

\[
= x + \frac{(x/2)(\theta')^2 + \left\{ L \theta (\theta')^2/2 + x \theta \theta' \right\}}{1 + (\theta' + \theta)^2} \]

\[
= x + x/2 (\theta')^2 + L \theta (\theta')^2/2 + x \theta \theta' \]

\[
= x + x \left\{ (\theta')^2/2 + \theta \theta' \right\} + L \theta (\theta')^2/2. \]

The spectrometer scales can be set to \( \theta' = \pm 0.001 \) radians, and in the present work \( \theta \) and \( L \) had magnitudes of about 0.02 radians and 400 mm respectively. Substitution of these values shows that the difference between \( x \) and \( x' \) is negligible. Thus, when \( x \) has its upper value of about 10 mm, \( (x' - x) \) has a magnitude of 0.0002 mm.
APPENDIX VIII

The Error in the Wavelength of Neon Kα₁,₂

With reference to Section W-9 and Appendix IV, the wavelength of Ne Kα₁,₂ (λₖ) is given by

\[ \lambdaₖ = 2D \sin \left\{ \phi₂ + \tan^{-1} \left( \frac{p/q}{\tan \alpha} \right) \right\} \]

Here,

\[ \phi₂ = \sin^{-1} \left( \frac{\lambda₂}{2D} \right) \]
\[ \alpha = \left[ \sin^{-1} \left( \frac{\lambda₂}{2D} \right) - \sin^{-1} \left( \frac{\lambda₁}{2D} \right) \right] \]

where \( \lambda₂ \) is the wavelength of Ni Kα₁,₂ and \( \lambda₁ \) is the wavelength of Ni L₂.

Since \( \lambda₁, \lambda₂ \) and \( D \) are taken to be free from error, they need not be expressed explicitly in the wavelength function.

Let \( \Delta \lambda_k \) be the error in \( \lambda_k \) due to the errors \( \Delta p \) and \( \Delta q \) in \( p \) and \( q \), respectively. Then the following differential expression may be written.

\[ \Delta \lambda_k = \Delta p \left( \frac{\partial \lambda_k}{\partial p} \right) + \Delta q \left( \frac{\partial \lambda_k}{\partial q} \right) \]

We have that

\[ \left( \frac{\partial \lambda_k}{\partial p} \right) = 2D \cos \left\{ \phi₂ + \tan^{-1} \left[ \frac{(p/q) \tan \alpha}{1 + [(p/q) \tan \alpha]^2} \right] \right\} \left( \tan \alpha / q \right) \]

which, for the present purpose, may be rewritten with sufficient accuracy as

\[ \left( \frac{\partial \lambda_k}{\partial p} \right) = 2D \cos \left\{ \phi₂ + \tan^{-1} \left[ \frac{(p/q) \tan \alpha}{1 + [(p/q) \tan \alpha]^2} \right] \right\} \left( \tan \alpha / q \right) \cdot \frac{(p/q)}{(\tan \alpha / q)} \]

Also,

\[ \left( \frac{\partial \lambda_k}{\partial q} \right) = 2D \cos \left\{ \phi₂ + \tan^{-1} \left[ \frac{(p/q) \tan \alpha}{1 + [(p/q) \tan \alpha]^2} \right] \right\} \left\{ \frac{-((p/q)(\tan \alpha / q))}{1 + [(p/q) \tan \alpha]^2} \right\} \]

which may be rewritten as

\[ -\left( \frac{\partial \lambda_k}{\partial q} \right) = 2D \cos \left\{ \phi₂ + \tan^{-1} \left[ \frac{(p/q) \tan \alpha}{1 + [(p/q) \tan \alpha]^2} \right] \right\} \left( \frac{p/q}{\tan \alpha / q} \right) \]
The following values are known.

\[
\lambda_2 = 14.536926 \ldots \text{RX} \\
\lambda_1 = 14.250499 \ldots \text{RX} \\
D = 9.927580 \ldots \text{RX} \\
p = 1.29 \text{ mm} \\
q = 7.58 \text{ mm}
\]

\[
\Delta p = \Delta q = \pm 0.01 \text{ mm (Probable)} \\
\Delta p = \Delta q = \pm 0.03 \text{ mm (Possible)}
\]

By substituting these values in the previous expressions we obtain that

\[
(\theta \lambda_k / \theta p) = 35.6 \text{ X/mm}
\]

\[
(\theta \lambda_k / \theta q) = 6.05 \text{ X/mm}
\]

\[
\Delta p (\theta \lambda_k / \theta p) = \pm 0.35 \text{ X (Probable)} \\
\Delta p (\theta \lambda_k / \theta p) = \pm 1.03 \text{ X (Possible)} \\
\Delta q (\theta \lambda_k / \theta q) = \pm 0.06 \text{ X (Probable)} \\
\Delta q (\theta \lambda_k / \theta q) = \pm 0.18 \text{ X (Possible)}
\]

and, finally,

\[
\Delta \lambda_k = \pm 0.4 \text{ X (Probable)} \\
\Delta \lambda_k = \pm 1 \text{ X (Possible)}
\]
APPENDIX IX

The Errors in the Wavelengths of the Neon Satellites (Plate 62/20).

With reference to Section IV-9 and Appendix IV, the wavelengths of the neon satellite lines (\(\lambda_s\)) are given by

\[
\lambda_s = 2D \sin \left\{ \phi_K - \tan^{-1}(v/u) \tan \alpha \right\}
\]

Here,

\[
\phi_K = \sin^{-1}(\lambda_K/2D)
\]

\[
\alpha = \left[ \sin^{-1}(\lambda_K/2D) - \sin^{-1}(\lambda_L/2D) \right]
\]

where \(\lambda_K\) is the wavelength of Ne K\(\alpha\) and \(\lambda_L\) is the wavelength of Ni K\(\alpha\).

By substitution we get that

\[
\lambda_s = 2D \sin \left\{ \sin^{-1}(\lambda_K/2D) + \tan^{-1}(v/u) \tan \left[ \sin^{-1}(\lambda_K/2D) - \sin^{-1}(\lambda_L/2D) \right] \right\}
\]

The error \(\Delta \lambda_s\) in \(\lambda_s\) is contributed to by the errors \(\Delta \lambda_K\), \(\Delta v\), and \(\Delta u\) in \(\lambda_K\), \(v\), and \(u\) respectively.

Thus,

\[
\Delta \lambda_s = \Delta \lambda_K \left( \frac{\partial \lambda_s}{\partial \lambda_K} \right) + \Delta v \left( \frac{\partial \lambda_s}{\partial v} \right) + \Delta u \left( \frac{\partial \lambda_s}{\partial u} \right)
\]

But \(\lambda_K\) is itself a function of \(u\), since \(u\) now represents the quantity denoted by \(p\) in the consideration of \(\Delta \lambda_K\) in Appendix VIII. From the latter we have that

\[
\Delta \lambda_K = \Delta p \left( \frac{\partial \lambda_K}{\partial p} \right) + \Delta q \left( \frac{\partial \lambda_K}{\partial q} \right),
\]

which we now write as

\[
\Delta \lambda_K = \Delta u \left( \frac{\partial \lambda_K}{\partial u} \right) + \Delta q \left( \frac{\partial \lambda_K}{\partial q} \right).
\]
It follows that

\[ \Delta \lambda_s = (\delta\lambda_s/\partial \lambda_k)\left[(\delta \lambda_k/\partial u)\Delta u + (\delta \lambda_k/\partial \phi)\Delta \phi\right] + (\delta \lambda_s/\partial u)\Delta u + (\delta \lambda_s/\partial \phi)\Delta \phi \]

\[ = \Delta u \left[(\delta \lambda_s/\partial \lambda_k)(\delta \lambda_k/\partial u) + (\delta \lambda_s/\partial \lambda_k)(\delta \lambda_k/\partial \phi)\right] + \Delta \phi (\delta \lambda_s/\partial \phi). \]

The various partial derivatives may be expressed, with sufficient accuracy, in the following forms.

\[ (\delta \lambda_s/\partial \lambda_k) = [1 - (v/u)] \]

\[ (\delta \lambda_s/\partial u) = 2D \cos\left\{ \phi_k - \tan^{-1}\left[(v/u)\tan \theta \right]\right\} (v/u^2) \tan \theta \]

\[ - (\delta \lambda_s/\partial \phi) = 2D \cos\left\{ \phi_k - \tan^{-1}\left[(v/u)\tan \theta \right]\right\} \tan \theta /u \]

\[ (\delta \lambda_k/\partial u) = (\delta \lambda_k/\partial \phi) \text{ which is given in Appendix VIII as} \]

\[ = 2D \cos \phi_k \tan \left( \phi_{\lambda_k} - \phi_{\lambda_k} \right) / \phi \]

\[ = 2D \cos \phi_k \tan \left( 1^\circ 12' \right) / 7.58 \]

\[ - (\delta \lambda_k/\partial \phi) \text{ is given in Appendix VIII as} \]

\[ = 2D \cos \phi_k \tan \left( \phi_{\lambda_k} - \phi_{\lambda_k} \right) \phi / \phi^2 \]

\[ = 2D \cos \phi_k \tan \left( 1^\circ 12' \right) 1.29 / (7.58)^2 \]

The following values are known.

\[ \phi = u = 1.29 \text{ mm} \quad \Delta \phi = \Delta u \pm 0.03 \text{ mm} \]

\[ V \text{ (Lines I to IV)} \quad \Delta V \pm 0.04 \text{ mm} \]

\[ D = 9.227580... \text{ mm} \]

\[ \phi_k = 47^\circ 16.264' \]

\[ \alpha = 0^\circ 12.255' \]

\[ q = 7.58 \text{ mm} \quad \Delta q \pm 0.03 \text{ mm}. \]
On substituting for these quantities and evaluating we obtain the various error contributions collected in the table.

<table>
<thead>
<tr>
<th>LINE</th>
<th>$\left[ \frac{\partial^2 \phi}{\partial \lambda_1^2} \right] \Delta \lambda_1 + \left[ \frac{\partial^2 \phi}{\partial \lambda_2^2} \right] \Delta \lambda_2$</th>
<th>$\left[ \frac{\partial^2 \phi}{\partial \lambda_1^2} \frac{\partial^2 \phi}{\partial \lambda_2^2} \right] \Delta \lambda_1 \Delta \lambda_2$</th>
<th>$\left[ \frac{\partial^2 \phi}{\partial \lambda_2^2} \right] \Delta \lambda_2$</th>
<th>$\Delta \lambda_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>$\pm 1.1 \times$</td>
<td>$\pm 0.004 \times$</td>
<td>$\pm 1.5 \times$</td>
<td>$\pm 2.6 \times$</td>
</tr>
<tr>
<td>II</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
<tr>
<td>III</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
<tr>
<td>IV</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
<tr>
<td>V</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
<td>$\times$</td>
</tr>
</tbody>
</table>

This leads to the final value ascribed to all of the lines of

$$\Delta \lambda_3 \text{ (Lines I to V)} \pm 3 \times.$$
APPENDIX X

The Errors in the Wavelengths of the Neon Satellites (Plate Q2/56).

With reference to Section IV.9 and Appendix IV, the wavelengths of the neon satellite lines ($\lambda_s$) are given by

$$\lambda_s = 2D \sin \left\{ \phi_k - \tan^{-1} \left[ \frac{(n/m) \tan \alpha}{1 + (m-n) \tan \alpha} \right] \right\}.$$ 

For the purposes of calculating errors this may be written with sufficient accuracy as

$$\lambda_s = 2D \sin \left\{ \phi_k - \tan^{-1} [(n/m) \tan \alpha] \right\}.$$ 

Here,

$$\phi_k = \sin^{-1} (\lambda_k/2D)$$

$$\alpha = \left[ \sin^{-1} (\lambda_k/2D) - \sin^{-1} (\lambda_F/2D) \right]$$

where $\lambda_k$ is the wavelength of NeKd and $\lambda_F$ is the wavelength calculated to be in focus. By substitution we get that

$$\lambda_s = 2D \sin \left\{ \sin^{-1} (\lambda_k/2D) - \tan^{-1} [(n/m) \tan (\sin^{-1} (\lambda_k/2D) - \sin^{-1} (\lambda_F/2D))] \right\}.$$ 

The error $\Delta \lambda_s$ in $\lambda_s$ is contributed to by the errors $\Delta \lambda_k$, $\Delta n$ and $\Delta m$ in $\lambda_k$, $n$ and $m$ respectively. Thus,

$$\Delta \lambda_s = \Delta \lambda_k (\partial \lambda_s/\partial \lambda_k) + \Delta n (\partial \lambda_s/\partial n) + \Delta m (\partial \lambda_s/\partial m).$$

But, in fact, $m$ is not independent of $\lambda_k$ for we have that

$$m = L \tan \alpha$$

$$= R \sin \phi_F \tan [\sin^{-1} (\lambda_k/2D) - \sin^{-1} (\lambda_F/2D)].$$
It follows that
\[ \Delta m = \Delta R \left( \frac{\vartheta m}{\vartheta R} \right) + \Delta \lambda_k \left( \frac{\vartheta m}{\vartheta \lambda_k} \right), \]
and
\[ \Delta \lambda_s = \Delta \lambda_k \left( \frac{\vartheta \lambda_s}{\vartheta \lambda_k} \right) + \Delta \vartheta \left( \frac{\vartheta \lambda_s}{\vartheta \vartheta} \right) \left[ \left( \frac{\vartheta m}{\vartheta R} \right) \Delta R + \left( \frac{\vartheta m}{\vartheta \lambda_k} \right) \Delta \lambda_k \right] \]
\[ = \Delta \lambda_k \left[ \left( \frac{\vartheta \lambda_s}{\vartheta \lambda_k} \right) + \left( \frac{\vartheta \lambda_s}{\vartheta \vartheta} \right) \left( \frac{\vartheta m}{\vartheta \lambda_k} \right) \right] + \Delta R \left[ \left( \frac{\vartheta \lambda_s}{\vartheta \vartheta} \right) \left( \frac{\vartheta m}{\vartheta R} \right) \right] \]
\[ + \Delta \vartheta \left( \frac{\vartheta \lambda_s}{\vartheta \vartheta} \right). \]

The various partial derivatives may be expressed, with sufficient accuracy, in the following forms:

\[ \left( \frac{\vartheta \lambda_s}{\vartheta \lambda_k} \right) = \left[ 1 - \left( \frac{n}{m} \right) \right] \]
\[ \left( \frac{\vartheta \lambda_s}{\vartheta \vartheta} \right) = 2D \cos \left\{ \phi_k - \tan^{-1} \left[ \left( \frac{n}{m} \right) \tan \alpha \right] \right\} \left( \frac{\vartheta \lambda_s}{\vartheta \vartheta} \right) \]
\[ \left( \frac{\vartheta \lambda_s}{\vartheta \lambda_k} \right) = \frac{R \sin \left[ \sin^{-1} \left( \frac{\vartheta R}{2D} \right) \right]}{2D \cos \phi_k} \]
\[ \left( \frac{\vartheta \lambda_s}{\vartheta \vartheta} \right) = \sin \phi_k \tan \alpha \]
\[ \left( \frac{\vartheta \lambda_s}{\vartheta \vartheta} \right) = -2D \cos \left\{ \phi_k - \tan^{-1} \left[ \left( \frac{n}{m} \right) \tan \alpha \right] \right\} \left( \frac{\vartheta \lambda_s}{\vartheta \vartheta} \right) \]

The following values are known

\[ m = 6.130 \ldots \text{ mm} \]
\[ n = \text{Lines I to XXI} \]
\[ R = 500.3 \text{ mm} \]
\[ D = 9.927580 \ldots \text{ kX} \]
\[ \alpha = 0^\circ 58.264' \]
\[ \phi_k = 47^\circ 16.264' \]
\[ \phi_f = 46^\circ 18.006' \]
On substituting for these quantities and evaluating we obtain the various error contributions collected in the table.

<table>
<thead>
<tr>
<th>LINE</th>
<th>( \left[ \frac{\partial \lambda_5}{\partial \lambda_k} + \frac{\partial \lambda_5}{\partial m} \right] \Delta \lambda_k )</th>
<th>( \left[ \frac{\partial \lambda_5}{\partial m} \right] \Delta \lambda_k )</th>
<th>( \left( \frac{\partial \lambda_5}{\partial \lambda_5} \right) \Delta \lambda_5 )</th>
<th>( \Delta \lambda_5 )</th>
</tr>
</thead>
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<td>( \pm \pm 2.5 ) ( \times )</td>
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<td>( \pm \pm 2.6 ) ( \times )</td>
<td></td>
</tr>
</tbody>
</table>

This leads to the final value ascribed to all of the lines of

\[ \Delta \lambda_5 \text{ (Lines I to XXI)} \pm \pm 2.5 \times \]
The Microdensitometer Slit Effect.

Consider a line spectrum to be recorded on a photographic plate in the plane \((x,y)\) so that the density of blackening is a function \(B(x)\) of \(x\) only, the lines being parallel to the \(y\)-axis.

With reference to the diagram, let a rectangular slit of width \(2s\) and length \(2l\), whose axis makes an angle \(\theta\) with the \(y\)-axis, move so that the locus of its centroid is in the \(x\)-axis.

Let the slit position be determined by the distance \(x_s\) which its centroid is from an arbitrary origin and, further, let any point within the slit be measured with respect to the centroid as the origin \(x'=0\).

Let \(x'\) be the distance from this origin of an element of slit parallel to the \(y\)-axis, of length \(y'\) and width \(dx'\). Then the contribution of this slit element to the microdensitometer reading \(M(x_s)\) will be proportional to \(B(x_s+x')y'dx'\). Putting the proportionality constant equal to unity it follows that the microdensitometer reading
is given by the summation
\[ M(x_s) = \sum_{x'} [B(x_s+x') \, y' \, dx'] . \]

Now, a simple geometrical construction shows that over the range of angle \( 0 \leq \theta \leq \pi/2 \), \( y' \) is a composite function of \( x' \) having a slope \( dy'/dx' \) which is discontinuous. The required summation must therefore be carried out in several stages. By considering the three cases corresponding to the angle values \( 0 < \theta < \tan^{-1} s/l \); \( \theta = \tan^{-1} s/l \); and \( \tan^{-1} s/l < \theta < \pi/2 \), it is possible, by standard calculations which will not be given here, to deduce a general integral expression for \( M(x_s) \). It may be written in the form

\[ M(x_s) = \int_{-a}^{+a} B(x_s+x') \, p(q+x') \, dx' + \int_{-a}^{+a} B(x_s+x') \, r \, dx' + \int_{-b}^{+b} B(x_s+x') \, p(q-x') \, dx' , \]

where

- \( b = q = s \cos \theta + l \sin \theta \) for all values of \( \theta \)
- \( p = \cosec \theta \ \sec \theta \)
- \( a = s \cos \theta - l \sin \theta \) \( 0 \leq \theta \leq \tan^{-1}(s/l) \)
- \( r = 2l \sec \theta \)
- \( a = l \sin \theta - s \cos \theta \) \( \tan^{-1}s/l \leq \theta \leq \pi/2 \).

Referring to Section 11-6, it may be taken that the intensity in a natural line has a symmetrical distribution with wavelength of the form

\[ I(\lambda) = \frac{w_\lambda}{2\pi} \frac{I_0}{(\lambda-\lambda_0)^2+w_\lambda^2/4} , \]

or, on the linear plate scale, of the form

\[ I(x) = \frac{w_x}{2\pi} \frac{I_0}{(x-x_0)^2+w_x^2/4} , \]

where \( w_x \) is full line width at half maximum intensity,
and $I_0$ is the total line intensity. This line shape may be rewritten as

$$I(x) = \frac{\omega_x^2}{4} \frac{I(x_0)}{(x-x_0)^2 + \frac{\omega_x^2}{4}}$$

where $I(x_0)$ is the intensity maximum at the line centre $x_0$.

For the present purpose, the natural line shape may be approximated with sufficient accuracy by a Gaussian error curve adjusted to fit exactly at the points of maximum and half maximum intensity. Such a curve has the form

$$I(x) = I(x_0) \exp \left\{ \frac{-4(x-x_0)^2 \log 2}{\omega_x^2} \right\}$$

It is now assumed that the density of photographic blackening may be taken as sensibly proportional to the incident intensity (cf. Section IV-5). It follows that the variation of blackening in the photographic plate is given by

$$B(x_s) = B(x_0) \exp \left\{ \frac{-4(x_s-x_0)^2 \log 2}{\omega_x^2} \right\}$$

where $x_0$ and $x_s$ are the distances from an arbitrary origin of the line maximum and the point where the blackening is $B(x_s)$ respectively.

We will consider the usual case where the slit is tilted so that $0 \leq \theta \leq \tan^{-1}(\theta/\gamma)$ and substitute the last expression for $B(x_s)$ in the previously given expression for $M(x_s)$. This gives that

$$M(x_s) = \begin{cases} 
\int_{-a}^{-b} B(x_0) \exp \left\{ \frac{-4(x_s+x'-x_0)^2 \log 2}{\omega_x^2} \right\} p(q+x') \, dx' & \text{I} \\
+ \int_{-a}^{-b} B(x_0) \exp \left\{ \frac{-4(x_s+x'-x_0)^2 \log 2}{\omega_x^2} \right\} r(x') \, dx' & \text{II} \\
+ \int_{a}^{b} B(x_0) \exp \left\{ \frac{-4(x_s+x'-x_0)^2 \log 2}{\omega_x^2} \right\} p(q-x') \, dx' & \text{III} 
\end{cases}$$
The component integrals which have been labelled I, II, and III are now treated separately.

Integral I: Take the arbitrary origin at the line centre so that $x_0 = 0$. Then we have that

$$I = \int_{-a}^{a} B(x_0) \exp\left\{ -\frac{4 \log 2}{\omega_z^2} (x_s + x')^2 \right\} (q + x') \, dx'. $$

Substitute

$$W = \frac{4 \log 2}{\omega_z^2},$$

$$\therefore I = B(x_0) \int_{-a}^{a} \exp\left\{ -W(x_s + x')^2 \right\} (q + x') \, dx'. $$

Change of variable,

Let $y = (x_s + x')$,

$$\therefore dy = dx' \quad x_s - a$$

and 

$$I = B(x_0) \int_{x_s - a}^{x_s} \exp(-Wy^2) \{(q - x_s) + y\} \, dy.$$

$$\therefore I = B(x_0) \int_{x_s - b}^{x_s} \exp(-Wy^2) \left\{ (q - x_s) + y \right\} \, dy + \int_{x_s - b}^{x_s} \exp(-Wy^2) y \, dy$$

$$= B(x_0) \int_{x_s - b}^{x_s} \exp(-Wy^2) \, dy + \frac{1}{2} \left. \exp(-Wy^2) y^2 \right|_{x_s - b}^{x_s}$$

since we may write $dy^2 = 2y \, dy$ in the latter part of the expression.
Now, \[
\int_{x_{S-b}}^{x_{S-a}} \exp(-W y^2) \, dy = \int_{x_{S-b}}^{x_{S-a}} \exp(-W y^2) \, dy + \int_{x_{S-b}}^{x_{S-a}} \exp(-W y^2) \, dy
\]
\[
= \int_{x_{S-b}}^{x_{S-a}} \exp(-W y^2) \, dy - \int_{x_{S-b}}^{x_{S-a}} \exp(-W y^2) \, dy.
\]

It follows that
\[
I = \mathcal{B}(x_0) \left\{ (q \cdot x_s) \left[ \int_{x_{S-b}}^{x_{S-a}} \exp(-W y^2) \, dy - \int_{x_{S-b}}^{x_{S-a}} \exp(-W y^2) \, dy \right] + \frac{1}{2} \int \exp(-W y^2) \, d(y^2) \right\}_{(x_{S-b})^2}^{(x_{S-a})^2}.
\]

By similarly treating the original forms of integrals II and III they become
\[
II = \mathcal{B}(x_0) \left\{ \right\}_{(x_{S-b})^2}^{(x_{S-a})^2} \frac{1}{2} \int \exp(-W y^2) \, d(y^2)
\]
\[
III = \mathcal{B}(x_0) \left\{ (q+x_s) \left[ \int_{x_{S-b}}^{x_{S-a}} \exp(-W y^2) \, dy - \int_{x_{S-b}}^{x_{S-a}} \exp(-W y^2) \, dy \right] - \frac{1}{2} \int \exp(-W y^2) \, d(y^2) \right\}_{(x_{S-a})^2}^{(x_{S+b})^2}.
\]

Consequently, \( M(x_s) \) is now given by the expression
\[
M(x_s) = I + II + III \text{ which may be rearranged to give}
\]
\[
M(x_s) = \int_{x_{S-b}}^{x_{S-a}} \exp(-W y^2) \, dy \left\{ \mathcal{B}(x_s) \left[ \rho(q-x_s) - \rho(q+x_s) \right] + \exp(-W y^2) \, d(y^2) \right\}_{(x_{S-a})^2}^{(x_{S-b})^2} + \int_{x_{S-b}}^{x_{S+a}} \exp(-W y^2) \, d(y^2) \left\{ \mathcal{B}(x_s) \rho(q+x_s) + \frac{1}{2} \int \exp(-W y^2) \, d(y^2) \right\}_{(x_{S-b})^2}^{(x_{S+a})^2}.
\]
Consider the integral \[ \int_0^{x_s-a} \exp(-Wy^2) \, dy. \]

It is required in a form amenable to evaluation from tables of the probability integral.

Let \( t = \sqrt{Wy} \),

\[ t^2 = Wy^2 \quad \text{and} \quad dy = \frac{1}{\sqrt{W}} \, dt. \]

\[ \therefore \int_0^{x_s-a} \exp(-Wy^2) \, dy = \frac{1}{\sqrt{W}} \int_0^{\sqrt{W}(x_s-a)} \exp(-t^2) \, dt, \quad \text{as required}. \]

Similarly,

\[ \int_0^{x_{s+b}} \exp(-Wy^2) \, dy = \frac{1}{\sqrt{W}} \int_0^{\sqrt{W}(x_{s+b})} \exp(-t^2) \, dt, \]

\[ \int_0^{x_{s-b}} \exp(-Wy^2) \, dy = \frac{1}{\sqrt{W}} \int_0^{\sqrt{W}(x_{s-b})} \exp(-t^2) \, dt. \]

On substitution we get

\[ M(x_s) = \frac{\sqrt{W}(x_s-a)}{\sqrt{W}} \int_0^{\sqrt{W}(x_s-a)} \exp(-t^2) \, dt \left\{ B(x_s) \left[ p(q-x_s) - r \right] \right\} + \frac{\sqrt{W}(x_{s+a})}{\sqrt{W}} \int_0^{\sqrt{W}(x_{s+a})} \exp(-t^2) \, dt \left\{ B(x_s) [r - p(q+x_s)] \right\} \]

\[ + \frac{\sqrt{W}(x_{s+b})}{\sqrt{W}} \int_0^{\sqrt{W}(x_{s+b})} \exp(-t^2) \, dt \left\{ B(x_s) p(q+x_s) \right\} - \frac{\sqrt{W}(x_{s-b})}{\sqrt{W}} \int_0^{\sqrt{W}(x_{s-b})} \exp(-t^2) \, dt \left\{ B(x_s) p(q-x_s) \right\} \]

\[ + \frac{1}{2W} B(x_0) p \left\{ \exp[-W(x_{s-b})^2] - \exp[-W(x_s-a)^2] + \exp[-W(x_{s+b})^2] - \exp[-W(x_{s+a})^2] \right\} \]
This expression may be rearranged to give

\[
M(x_s) = B(x_0) \left\{ \left[ \frac{p + q - r}{\sqrt{w}} \right] \frac{\sqrt{w(x_s-a)}}{\sqrt{w}} \int_0^{\sqrt{w}(x_s+a)} e^{-(t^2)} dt + \left[ r - p(q+x_s) \right] \frac{1}{\sqrt{w}} \int_0^{\sqrt{w}(x_s-b)} e^{-(t^2)} dt \right. \\
\left. + \frac{1}{2w} \left[ \exp(-w(x_s-a)^2) + \exp(-w(x_s+b)^2) - \exp(-w(x_s-a)^2) - \exp(-w(x_s+b)^2) \right] \right\}
\]

where

\[
\begin{align*}
\theta &= \cos\theta \sec\theta \\
b &= q + s \cos\theta \pm \sin\theta \\
a &= q - s \cos\theta - \sin\theta \\
r &= 2s \sin\theta \\
w &= \frac{\log 2}{\omega_x^2} \\
\end{align*}
\]

Also

\[
\int_0^{T} e^{-(t^2)} dt = \frac{\sqrt{\pi}}{2} (\text{Probability Integral})
\]

where

\[
t = \sqrt{w} y \\
y = (x_s + x')
\]

\[
T = \begin{cases} 
\sqrt{w} (x_s+a) \\
\sqrt{w} (x_s-a) \\
\sqrt{w} (x_s+b) \\
\sqrt{w} (x_s-b) 
\end{cases}
\]
The final expression for $M(x_s)$ has been evaluated in terms of the values used in the present work.

In Sections 11-6,7 the measured halfwidth of the line Ne Ka is given as $0.40\,\text{mm}$, and this is also the order of breadth of the other neon lines considered.

\[ W_x = 0.40\,\text{mm} \]
\[ = 400\,\mu \]

The m.densitometer slit is of length $1.0\,\text{mm}$ while the greatest slit width used was $0.05\,\text{mm}$.

\[ \therefore l = 0.5\,\text{mm} \]
\[ = 500\,\mu \]

and $s = 0.025\,\text{mm}$
\[ = 25\,\mu \]

From a consideration of the variation in the settings of the m.densitometer slit when repeatedly adjusted to be parallel with a recorded line it is estimated that the lack of parallelism in any setting is probably less than $\theta = 1^\circ$.

It is consequently found that
\[ p = 57.307436. \]
\[ b = 9 = 33.722393\,\mu. \]
\[ a = 16.268993\,\mu. \]
\[ r = 1000.1523\,\mu. \]
\[ W = 17.328679 \times 10^{-6} \,\mu^2. \]

In the following tables are given the calculated values for the experimental line shape $M(x_s)$, and also for the Weisskopf-Wigner natural line shape and the assumed Gaussian line shape. The three expressions have been evaluated for $50\,\mu$ increments of the slit displacement $x_s$ and have been normalised to unity value when $x_s (=x) = 0$. 
The tabulated values are plotted as the line shapes of Fig. 55. It is clear that the densitometer slit has no effect on the shape of the Gaussian line. But the Gaussian line follows closely the shape of the Weisskopf-Wigner line; the deviation near the base being unimportant since here the Gaussian line is narrower than the other. It is concluded that the densitometer slit used in the present work has not contributed to the measured line halfwidths.

Footnote: With reference to Section IV-9, it follows that if the slit does not alter the shape of the line, the position of the line peak is known with an accuracy determined by the magnitude of the slit displacement. Evidently the positional error cannot be greater than

\[ \pm \frac{\ell}{2} \text{(slit displacement)}. \]
Deviations due to Ring Electrode.
Fig. 4

[Graph showing a linear relationship between peak kilovolts and gap spacing (cm)].

- Vertical axis: Peak Kilovolts
- Horizontal axis: Gap Spacing (cm)
Fig. 6
Spark Gap No. 6
Fig. 7

$L = 22 \text{ cm}$

Tube Current (mA)

<table>
<thead>
<tr>
<th>Pressure (gµ)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5</td>
</tr>
<tr>
<td>10</td>
</tr>
<tr>
<td>15</td>
</tr>
<tr>
<td>20</td>
</tr>
<tr>
<td>25</td>
</tr>
<tr>
<td>30</td>
</tr>
<tr>
<td>35</td>
</tr>
</tbody>
</table>

Spark Gap No. 6

No. 4

No. 2
Fig. 8

Spark Gap No. 8

Tube Current (mA)

Pressure (gμ)

$L (cm)$

- $22 - O$
- $17 - X$
- $12 - Δ$
IONISATION CHAMBER

Fig. 9
Fig. 10

Ion Current ($\mu$A) vs. Chamber Voltage
Fig. 11

$L = 22$ cm

![Graph showing ion current vs. pressure for Spark Gap No. 6, No. 4, and No. 2.](image-url)
Fig. 12

Spark Gap No. 6

Ion Current (μA)

Pressure (gμ)

$L (cm) \{22 - o, 17 - x, 12 - \triangle\}$
Fig. 13

Spark Gap No. 6
L = 22 cm
Fig. 15

External Electrode Tube

Vertical Section

Plan
Fig. 24  Oil Cooling System (Vertical Section)
Fig. 26
Mounting Tube

A, B, C, D, E, F, G, H, I
Fig. 28
Mounting of Formvar Film

Water Surface

A

B

C

D

E

3 mm

16 mm

25 mm
Fig. 30

Vacuum System

[Diagram of a vacuum system with labeled components such as P, S, A, B, C, O, D, U, T, W, N, M, and J.]
Fig. 31
Fig. 34
Fig. 38
Fig. 39

Microdensitometer Records

Showing Effect of Altering Focus

N.B. The curves have been arbitrarily adjusted on the densitometer scale.
Fig. 4-0
Microdensitometer Records

(a) Neon Exposure: Plate Q2/36
Ne Kα1,2

(b) Air Exposure: Plate Q2/37
Closed Tube End

N.B. The curves have been arbitrarily adjusted on the densitometer scale
Fig. 41

Blackening Curve: Plate Q1/1

Intensity

Microdensitometer Reading
Fig. 42

Blackening Curve: Plate Q1/9
Fig. 43

Blackening Curve: Plate A2/20
Fig. 44

Blackening Curve: Plate 02/36
Fig. 45
SHAPE OF NEON Kα1,2

Plate G/7  ○
Plate G/9  +
Fig. 46
Satellite Analysis Plate 02/20

Experiment

Theoretical Contour

Kale Axis

Intensity

Linear Plate Scale
Fig. 47
Satellite Analysis Plate 02/36

[Diagram with axes labeled 'Intensity' and 'Linear Plate Scale']

- Experimental Contour
- Theoretical Contour
- Background

Satellite Analysis Plate 02/36
Fig. 48
Satellite Analysis Plate 02/36
Fig. 4.9
Microdensitometer Record
Plate 02/36

Linear Plate Scale
Fig. 50

Microdensitometer Record

Plate Q2/22

Ni Lβ₁, Ni Lα₁,₂, Ne Kα₁,₂
Fig. 51
Moseley Diagram

The circles indicate diagram lines. The vertical lines cover the ranges of the satellite lines.
Fig. 52
Moseley Diagram

The circles indicate diagram lines. The vertical lines cover the ranges of the satellite lines.
Fig. 54
Semi-Moseley Diagram
Fig. 55
Microdensitometer Slit Effect

O = Microdensitometer Reading