

Estimation of site-specific and spatially distributed ground motion in the Christchurch earthquakes: application to liquefaction evaluation and ground motion selection for post-event investigation

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ABSTRACT

This paper presents a methodology by which both site-specific and spatially distributed ground motion intensity can be obtained immediately following an earthquake event. The methodology makes use of both prediction models for ground motion intensity and its correlation over spatial distances. A key benefit of the methodology is that the ground motion intensity at a given location is not a single value but a distribution of values. The distribution is comprised of both a mean and also standard deviation, with the standard deviation being a function of the distance to nearby strong motion stations.

The methodology is illustrated for two applications. Firstly, maps of conditional peak ground acceleration (PGA) have been developed for the major events in the Canterbury earthquake sequence. It is illustrated how these conditional maps can be used for post-event evaluation of liquefaction triggering criteria which have been adopted by the Department of Building and Housing (DBH). Secondly, the conditional distribution of response spectral ordinates is obtained at a specific location for the purposes of determining appropriate ground motion records for use in seismic response analyses of important structures at locations where direct recordings are absent.

1 CONDITIONAL GROUND MOTION DISTRIBUTIONS OVER A SPATIAL REGION

1.1 Theory

Because of the complexity of a ground motion time series, the engineering representation of ground motion severity typically comprises one or more ground motion intensity measures, IM . Below the general IM variable will be used, while subsequent applications will make use of a specific IM (e.g. peak ground acceleration, PGA, or pseudo-acceleration response spectral ordinates, SA(T)).

The representation of IM at a single location i , for the purposes of ground motion prediction, is generally given by:

$$\ln IM_i = \overline{\ln IM}_i(\text{Site}, \text{Rup}) + \eta + \epsilon_i \quad (1)$$

where $\ln IM_i$ is the (natural) logarithm of the observed IM ; $\overline{\ln IM}_i(\text{Site}, \text{Rup})$ is the median of the predicted logarithm of IM as given by an empirical ground motion prediction equation (GMPE), which is a function of the site and earthquake rupture considered; η is the inter-event residual; and ϵ_i is the intra-event residual. Based on equation (1), empirical ground motion prediction equations can provide the (unconditional) distribution of ground motion shaking as:

$$\ln IM_i \sim N(\overline{\ln IM}_l, \sigma_\eta^2 + \sigma_\epsilon^2) \quad (2)$$

where $X \sim N(\mu_X, \sigma_X^2)$ is short-hand notation for X having a normal distribution with mean μ_X and variance σ_X^2 .

By definition, for a given ground motion intensity measure, (e.g. peak ground acceleration, *PGA*) all observations from a single earthquake event have the same inter-event residual, η . In this regard, the inter-event residual represents the correlation between all observations from a single event, which may occur as a result of a unique effect occurring during the earthquake rupture, which subsequently affects the ground motion at all locations in a systematic manner. On the other hand, the intra-event residual, ϵ_i varies from site to site. In this regard the intra-event residual represents all other randomness which leads to a difference between the observed ground motion intensity, the predicted median ground motion intensity, and the systematic inter-event residual. While the intra-event residual varies from site to site, it is correlated spatially as a result of similarities of path and site effects between various locations.

Based on the aforementioned properties of η and ϵ_i , use can be made of recorded *IM* values at strong motion stations to compute a conditional distribution of *IM* at an arbitrary site of interest as discussed further below.

Firstly, an empirical ground motion prediction equation (GMPE) is used to compute the unconditional distribution of ground motion intensity at the strong motion stations where ground motions were recorded. A mixed-effects regression (Abrahamson and Youngs 1992, Pinheiro et al. 2008) can then be used to determine the inter-event residual, η , and the intra-event residuals, ϵ_i 's, for each strong motion station.

Secondly, the covariance matrix of intra-event residuals is computed by accounting for the spatial correlation between all of the strong motion stations and the site of interest. The joint distribution of intra-event residuals at the site of interest and the considered strong motion stations can be represented by:

$$\begin{bmatrix} \epsilon^{site} \\ \epsilon^{SMstation} \end{bmatrix} = N \left(\begin{bmatrix} 0 \\ \mathbf{0} \end{bmatrix}, \begin{bmatrix} \sigma_{\epsilon^{site}}^2 & \Sigma_{12} \\ \Sigma_{21} & \Sigma_{22} \end{bmatrix} \right) \quad (3)$$

where $\mathbf{X} \sim N(\boldsymbol{\mu}_X, \boldsymbol{\Sigma})$ is short-hand notation for \mathbf{X} having a multivariate normal distribution with mean $\boldsymbol{\mu}_X$ and covariance matrix $\boldsymbol{\Sigma}$ (i.e. as before, but in vector form); and $\sigma_{\epsilon^{SMstation}}^2$ is the variance in the intra-event residual. In Equation (3) the covariance matrix has been expressed in a partitioned fashion to elucidate the subsequent computation of the conditional distribution of ϵ^{site} . The individual elements of the covariance matrix can be computed from:

$$\Sigma(i, j) = \rho_{i,j} \sigma_{\epsilon_i} \sigma_{\epsilon_j} \quad (4)$$

where $\rho_{i,j}$ is the spatial correlation of intra-event residuals between the two locations i and j ; and σ_{ϵ_i} and σ_{ϵ_j} are the standard deviations of the intra-event residual at locations i and j . Based on the joint distribution of intra-event residuals given by Equation (3), the conditional distribution of ϵ^{site} can be computed from (Johnson and Wichern 2007):

$$\begin{aligned} [\epsilon^{site} | \epsilon^{SMstation}] &= N(\Sigma_{12} \cdot \Sigma_{22}^{-1} \cdot \epsilon^{SMstation}, \sigma_{\epsilon^{site}}^2 - \Sigma_{12} \cdot \Sigma_{22}^{-1} \cdot \Sigma_{21}) \\ &= N(\mu_{\epsilon^{site} | \epsilon^{SMstation}}, \sigma_{\epsilon^{site} | \epsilon^{SMstation}}^2) \end{aligned} \quad (5)$$

Thirdly, using the conditional distribution of the intra-event residual at the site of interest given by Equation (5), and substituting into Equation (2), the conditional distribution of *IM* at the site of interest, IM_{site} , can be computed from:

$$[\ln IM_{site} | \ln IM_{SMstation}] = N \left(\overline{\ln IM_{site}} + \eta + \mu_{\epsilon_{site} | \epsilon_{SMstation}}, \sigma_{\epsilon_{site} | \epsilon_{SMstation}}^2 \right) \quad (6)$$

That is, the conditional distribution of IM at a specific site is a lognormal random variable (i.e. the log of IM is a normal random variable) which is completely defined via the conditional median and conditional standard deviation.

It should be noted that in cases where the site of interest is located far from any strong motion stations the conditional distribution will be similar to the unconditional distribution, and for sites of interest located very close to a strong motion station the conditional distribution will approach the value observed at the strong motion station.

2 SPATIAL PGA MAPS IN THE CANTERBURY EARTHQUAKES

This section discusses the earthquake sources considered, as well as the ground motion prediction and spatial correlation equations which were adopted to compute the spatial PGA maps.

2.1 Earthquake sources

The spatial distribution of ground motion intensity was considered for major events in the Canterbury earthquake sequence. The finite fault models for the 4 September 2010, 22 February 2011, 13 June 2011 (2:20pm) and 23 December 2011 (2:18pm) events from Beavan et al. (2012) were adopted, while those for the 16 April 2011, 13 June 2011 (1:01pm), and 23 December 2011 (2:18pm) events were obtained in a first-order manner by using the CMT solutions from the GeoNet catalogue (Ristau 2008), and then fault dimensions based on magnitude scaling relationships (Stirling et al. 2007).

2.2 Ground motion prediction and spatial correlation equations

As illustrated from the theory in the previous section, ground motion prediction equations (GMPE's) and spatial correlation equations are required to compute the conditional ground motion at each location. The Bradley (2010, 2013) GMPE is adopted to provide the unconditional PGA distribution. Figure 1a provides a comparative example of the PGA amplitudes observed in the 4 September 2010 Darfield earthquake compared with the model prediction. Further scrutiny of this model against observations in the Canterbury earthquakes can be found in elsewhere (Bradley 2012b, Bradley 2013, Bradley and Hughes 2013). Figure 1b illustrates the adopted spatial correlation model of Goda and Hong (2008) for PGA as a function of the separation distance between two locations. As expected on physical grounds, the correlation is 1.0 for a separation distance of zero (i.e. two points at the same location), and tends toward zero as the separation distance increases. Thus, on the basis of Figure 1b and the previous theoretical discussions, it can be understood that if the recorded ground motion PGA is above the median prediction at a given strong motion station then it is more likely that the PGAs near this station will also be above average. The strength of this statement will decrease as the separation distance from the station and the site of interest increases.

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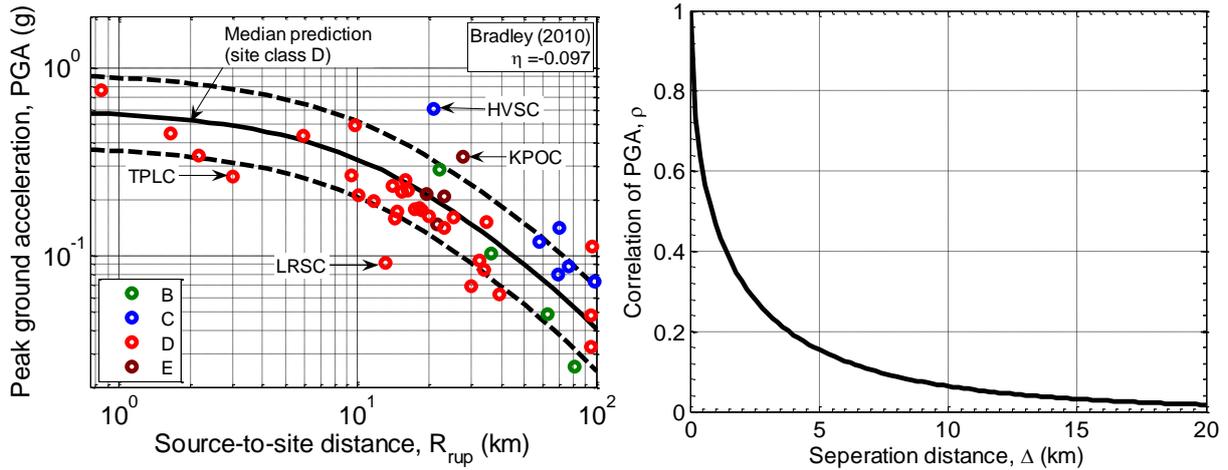


Figure 1: (a) Example comparison of observed PGA values with the empirical prediction of Bradley (2010) for the 4 September 2010 earthquake (M_w 7.1); and (b) correlation of intra-event residuals for PGA as a function of separation distance (Goda and Hong 2008).

2.3 Computed spatial PGA maps for the Canterbury earthquakes

Figure 2 and Figure 3 provide examples of the spatial PGA maps produced for the 4 September 2010 and 22 February 2011 earthquakes, respectively. As illustrated in section 1.1, the distribution of PGA at any given location is lognormal, and therefore defined by a median and standard deviation. In Figure 2 and Figure 3 the median values are portrayed by contours, while the standard deviation is given according to the color map in the figure legend.

Several features are worthy of note in Figure 2 and Figure 3:

- Firstly, the median PGA amplitudes display a typical attenuation in amplitude as the distance from the earthquake source increases.
- In the proximity of strong ground motion stations, the contours can be observed to vary markedly as a result of differences between some observed PGA. This is consistently the case in Heathcote Valley for all events, due to strong basin-edge effects (Bradley 2012d); and also apparent at Kaiapoi High School during the 4 September 2010 earthquake as a result of wave-guide effects (Bradley 2012a). However, as shown by the median PGA contours, these effects are expected to be localised.
- The conditional standard deviations shown at the bottom panel of each of the figures provide an indication of the level of uncertainty in the conditional median PGA prediction. Near strong motion stations the conditional standard deviations decrease toward zero. This implies that the prediction of PGA is more accurate close to strong motion stations, and less accurate as the distance from strong motion stations increases.

3 CONDITIONAL PGA VALUES FOR LIQUEFACTION ASSESSMENT

There are several applications of the conditional ground motion distribution theory, and in particular, the maps of conditional PGA. Here, their application for liquefaction assessment is discussed.

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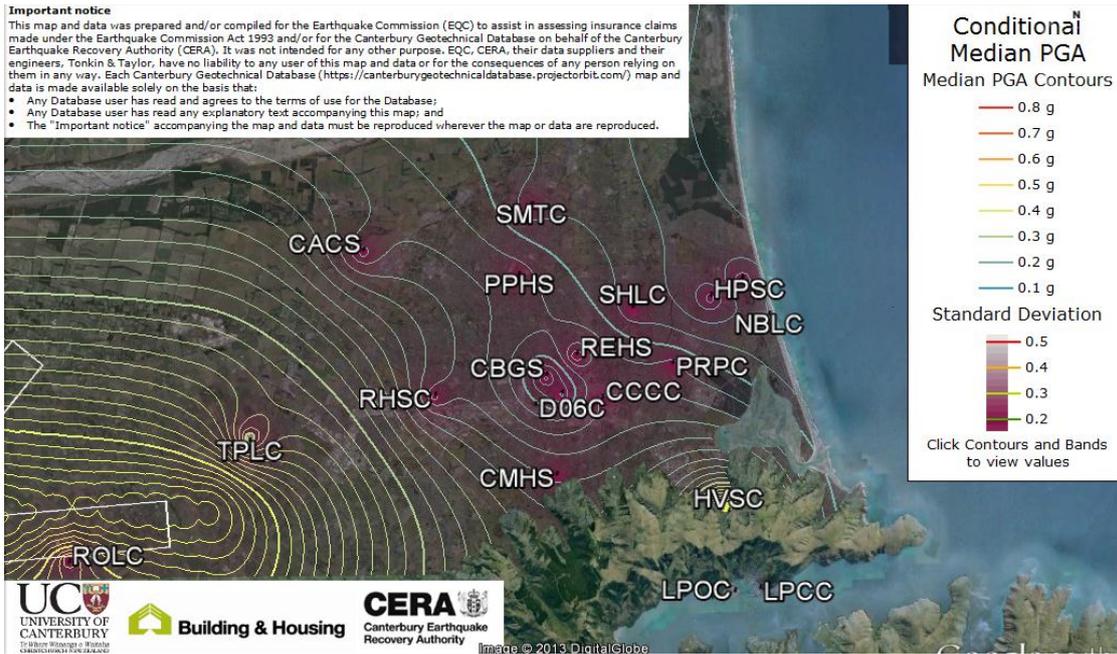


Figure 2: Conditional median (top) and conditional standard deviation (bottom) of PGA predicted in Canterbury from the 4 September 2010 earthquake.

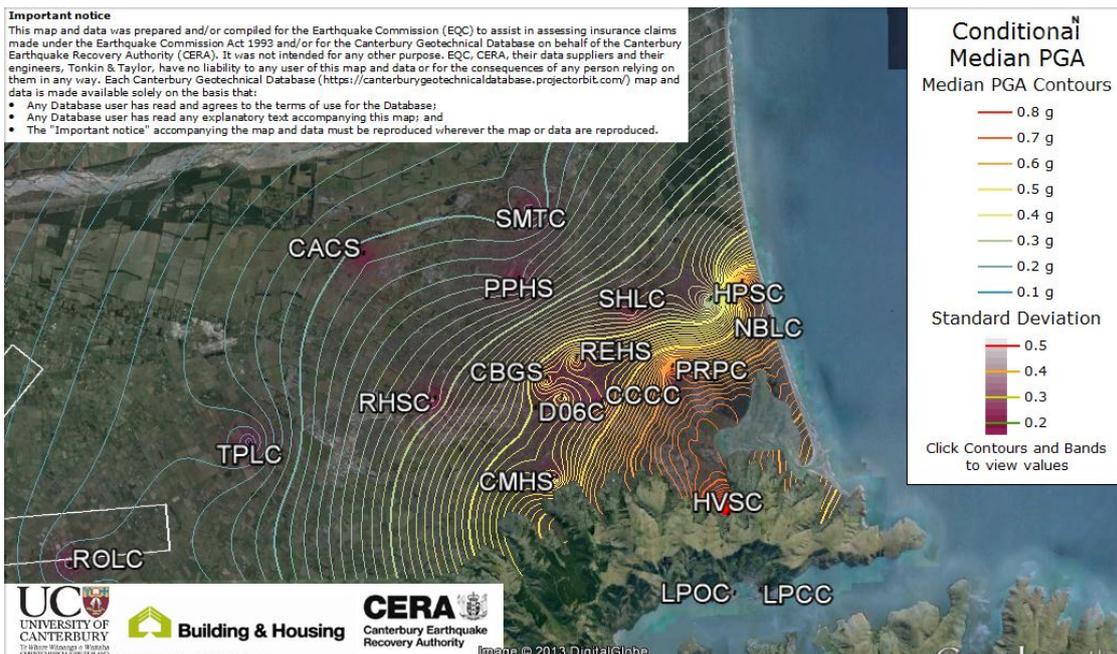


Figure 3: Conditional median (top) and conditional standard deviation (bottom) of PGA predicted in Canterbury from the 22 February 2011 earthquake.

3.1 Simplified method for liquefaction evaluation

Liquefaction assessments conventionally utilise a stress-based approach in which the factor of safety (FS) against liquefaction is obtained from the cyclic stress ratio (CSR) and the cyclic resistance ratio (CRR) (New Zealand Geotechnical Society 2010). Specifically,

$$FS = \frac{CRR_{7.5}}{CSR_{7.5}} \tag{7}$$

where the subscripts in both the denominator and numerator indicate that the ratios are representative of a $M_w 7.5$ earthquake. The CRR can be obtained via various insitu testing methods (e.g. CPT, SPT, Vs) or laboratory data, but importantly is a property of the geotechnical conditions at the site of concern (New Zealand Geotechnical Society 2010). The CSR , which represents the ratio between the cyclic shear stress and vertical effective stress in the soil, can be estimated using the general equation:

$$CSR_{7.5} = 0.65 \frac{a_{max} \sigma_{vo}}{g \sigma'_{vo}} r_d \frac{1}{MSF} \quad (8)$$

where a_{max} is the average horizontal (geometric mean) peak ground acceleration (PGA) at the ground surface; g is the acceleration of gravity; σ_{vo} is the vertical total stress at the depth of interest; σ'_{vo} is the vertical effective stress at the depth of interest; r_d is a reduction factor to account for the soil flexibility; and MSF is a magnitude scaling factor to account for the number of cycles of significant ground motion as a function of moment magnitude, M_w . The value of MSF can be obtained from several published equations, for example (Idriss and Boulanger 2008):

$$MSF = 6.9 * \exp\left(-\frac{M_w}{4}\right) - 0.058 \leq 1.8 \quad (9)$$

It should be noted that the a_{max} in Equation (8) is that provided by the previously discussed spatial PGA maps. Hence, such maps enable a site-specific evaluation of the FS against liquefaction for all major events in the Canterbury earthquake sequence. Comparing field observations of ground deformation following each of these events, with the respective liquefaction FS computed, enables a first-hand assessment of the accuracy and precision of the simplified liquefaction evaluation procedure on a site-by-site basis. Such first-hand validation is critical, because it must be recalled that this methodology is highly simplified, and region-specific soil characteristics and in situ conditions may lead to systematic differences between prediction and reality.

3.2 Implementation by the Engineering Advisory Group (MBIE)

The previously discussed methodology and spatial PGA maps have been adopted by the Engineering Advisory Group to the Ministry of Business, Innovation and Employment (MBIE 2012) for liquefaction evaluation as outlined in the previous section. In order to make use of the spatial PGA contour plots (e.g. Figure 2 and Figure 3) for a site-specific liquefaction assessment, Google Earth files have been created and are publically available on the Canterbury Geotechnical Database (<https://canterburygeotechnicaldatabase.projectorbit.com>).

4 CONDITIONAL RESPONSE SPECTRA VALUES FOR SITE-SPECIFIC GEOTECHNICAL AND STRUCTURAL ASSESSMENT

The theory in section 1.1 can also be extended to examine the prediction of response spectral ordinates of different vibration periods, $SA(T)$. Such assessments can therefore provide a site-specific estimate of the response spectrum of the unknown ground motion which occurred during these past events.

This methodology has been applied to estimate the site-specific response spectrum at the location of the CTV building from the 22 February 2011 earthquake (Bradley 2012c). Figure 4 illustrates the median, 16th and 84th percentiles of the conditional response spectrum predicted at the site as well as the geometric mean response spectra of four ground motions recorded in the general vicinity of the CTV site. In this particular application the observed ground motions can

be seen to provide a good representation of the conditional response spectrum, and therefore these ground motions could be utilized as input in response history analyses of the CTV structure, in lieu of a direct ground motion record at that site.

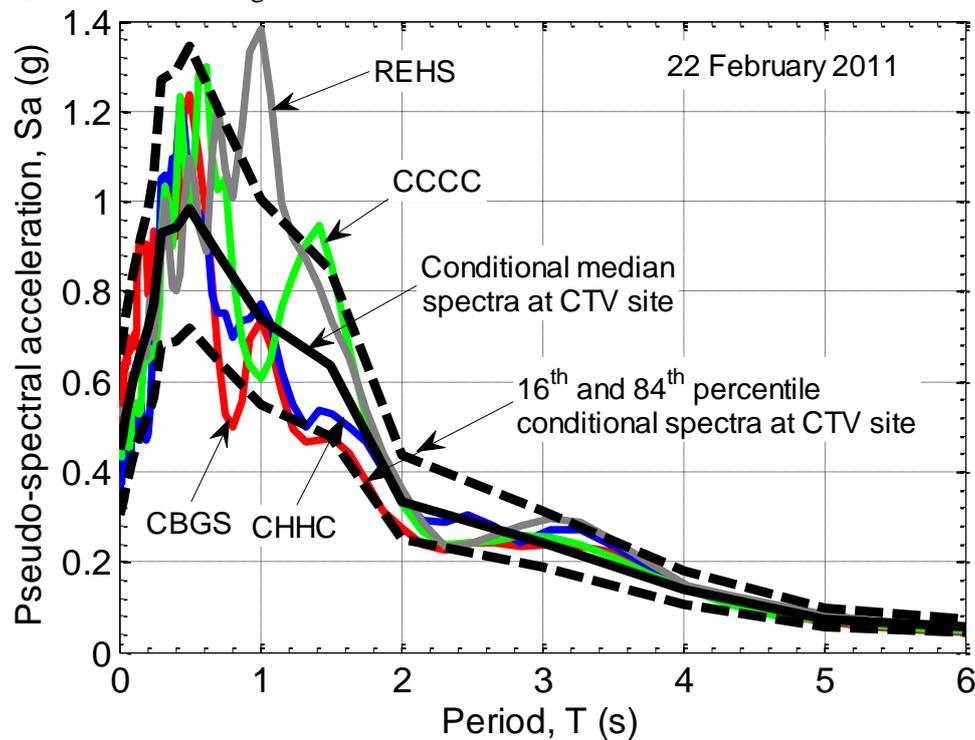


Figure 4: Comparison of the ground motions observed at the four 'CBD' strong motion stations (i.e. CCCC, CHHC, REHS, CBGS) with the conditional response spectrum distribution at the CTV site for the 22 February 2011 Christchurch earthquake.

In addition to using conditional response spectral ordinates for detailed post-event analyses of structures, it can also be used to obtain a rapid estimation of the seismic demand imposed on structures in an urban region for post-event triaging of buildings. While such information is certainly not a substitute for a rigorous field inspection, an understanding of the likely seismic demand (and consequent dynamic response) of the system will assist in identifying what forms of seismic distress may be apparent.

5 CONCLUSIONS

This paper presents a methodology by which a conditional distribution of ground motion intensity can be determined following an earthquake event. The predicted values are dependent on both the general manner in which ground motion intensities are observed to vary over a region from a given causative fault (as predicted by empirical ground motion models), combined with the actual recorded PGA values at various strong motion stations in the region. As such, the predicted intensity values are termed 'conditional', that is, the prediction is conditional on the observations at distinct locations.

The methodology was illustrated by first developing maps of the spatial distribution of PGA for the major events in the Canterbury earthquake sequence for the assessment liquefaction triggering. The methodology was also applied to the site-specific estimation of response spectra for use in determining appropriate ground motions for response history analyses.

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