Identification of level shifts in stationary processes

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Abstract: ART was introduced as a modified use of CART methodology for quick detection of structural breaks in the mean levels. In this paper simulations are presented to test ART against a number of different types of time series, to find a good pruning method, and to compare with alternative approaches.

Keywords: Time Series; Regression Trees; Structural Breaks.

1 Introduction

Atheoretical Regression Trees (ART) (Cappelli and Reale, 2005) is a simple and fast approach to detect structural breaks in the mean in time series. It is an ordinary regression tree procedure where the dependent variable is the time series under consideration and the covariate is a strictly monotonic positive or negative sequence. In this paper a simulation study is presented to further investigate strengths and weaknesses of the method and compare it to the established procedure proposed by Bai and Perron (BPP) (1998, 2003)

2 Simulations

Simulations were run with series of uncorrelated observations drawn from an N(0,1) population with a single break at the mid-point giving two regimes. There where 16 regime sizes, 5^2 to 20^2 observations in length, with break sizes ranging from 0.05 to 2 standard deviations in steps of 0.05 standard deviations and 1,000 replications of each combination of regime length and break size. Our results indicate ART performs well, with $\alpha = 0$, when the regime length is long and the break size is large.
Figure (1) shows the results of one simulation to test ART's ability to correctly locate a single break in a series. These results show the break size is the important criteria.

This set of simulations exposed a problem with ART finding substantial numbers of spurious breaks when the regime length is small and using the default, $\alpha = 0$, cost-complexity pruning. Su *et al.* (2004) found that cost-complexity pruning as developed in standard CART (Breiman *et al.*, 1984) methodology was inferior to several other pruning criteria. They found the BIC (Schwarz, 1978) and RIC (Shi and Tsai, 2002) to give the best results. This issue has also been addressed in the noisy square wave simulations below. For routine tree selection BIC is recommended.

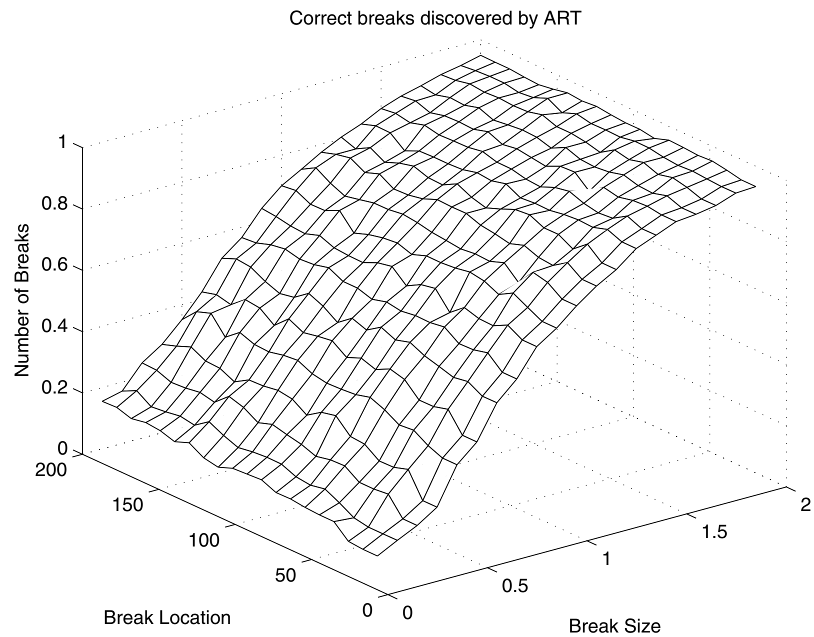

FIGURE 1. Number of breaks correctly found by ART as a function of break location and break size for series length 400. BIC tree selection.

After these first simulations a more complex model was used, i.e.

$$y_t = \mu_{r_i} + \epsilon_t \tag{1}$$

where μ_{r_i} =the mean of regime r_i ;$i = 1, \ldots, 5$ and ϵ_t = noise terms drawn from an N(0,1) distribution.
In all simulations $\mu_{r_i} = 0$ for $i = 1, 3, 5$ and $\mu_{r_4} = -\mu_{r_2}$. When ART was used to test the series the value of μ_{r_2} started at 2 standard deviations and was decremented to 0.05 in steps of 0.05. Because of the large amount of computation required when BPP was used the value of μ_{r_2} was decremented to 0.1 in steps of 0.1.

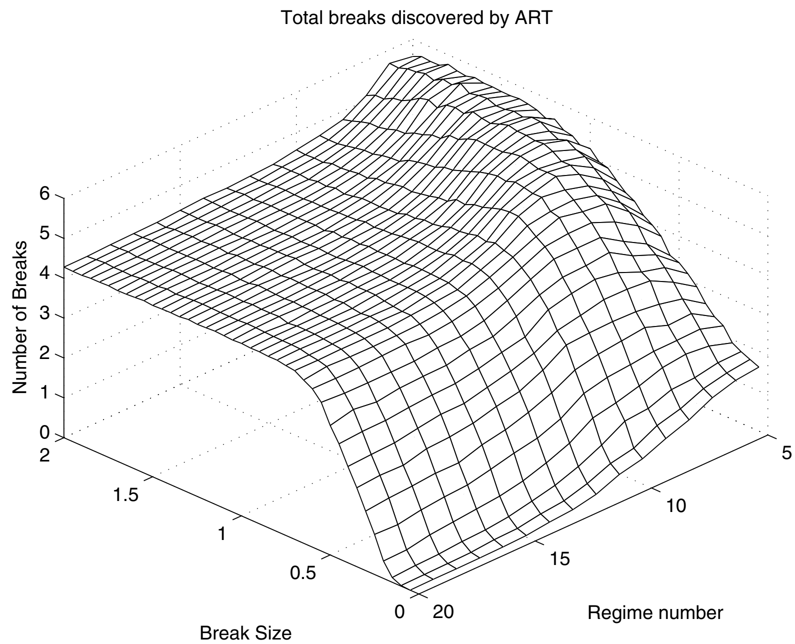

FIGURE 2. Total number of breaks found by ART in the noisy square wave simulations. BIC tree selection.

In essence the resultant series are square waves with an amplitude of break size and gaussian noise of constant variance imposed on them. The noisy square wave simulations were tested by BPP and by ART with $\alpha = 0$, tree selection by BIC, and tree selection by T-fold cross-validation where T is the length of the series. Consistent with Su *et.al.* we found BIC was a good choice. T-fold cross-validation was better for short series but is computationally expensive.

The results from these simulations using ART are presented in Figure (2). BPP results are presented in Figure (3). BPP is computationally expensive. It took 633,918 seconds of CPU time on an 750Mhz UltraSPARC III to generate and analyze 2000 series of length 720 with BPP compared to 1368 seconds on an 1.5Ghz UltraSPARC IIIi processor to generate and analyze 40,000 series of the same length with ART.

When the break-size is large and the regime size is long ART consistently finds the real breaks and generating few spurious candidate breaks without needing to prune T_{max}.

ART's robustness to non-normal noise was tested with series with gamma and geometric noise structures. Our results (not presented here) show only a slight reduction in ART's ability to correctly locate the break with similar

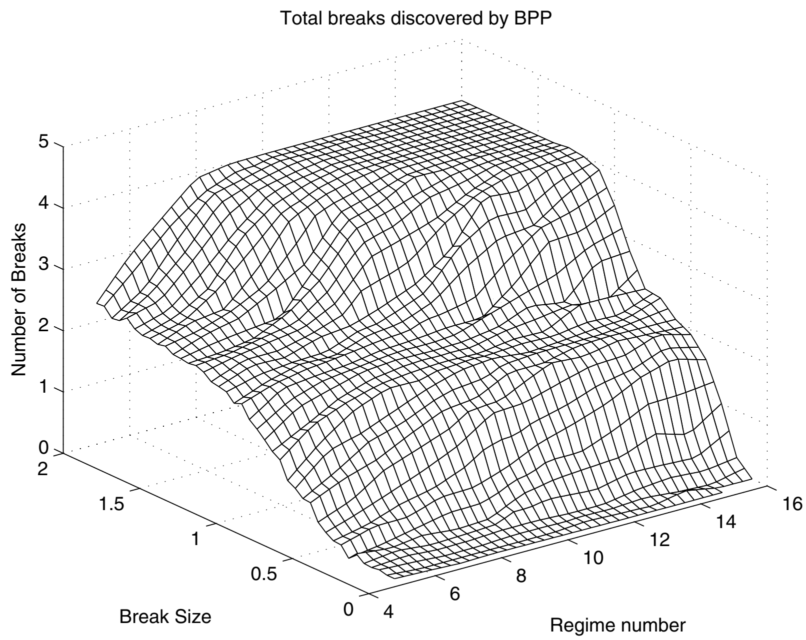

FIGURE 3. Total number of breaks found by BPP in the noisy square wave simulations.

numbers of spurious breaks reported.

References

Breiman, L., Friedman, J.H., Olshen, R.A., and Stone, C.J. (1984). *Classification and Regression Trees*. Boca Raton: Chapman & Hall.

Cappelli, C., and Reale, M. (2004). Detecting multiple structural breaks in the mean via Atheoretical Regression Trees. In *Proceedings of the 20th International Workshop on Statistical Modelling*. 131-134, Sydney, Australia.

Schwarz, G. (1978). Estimating the dimension of a model. *Annals of Statistics*, **6**, 461-464.

Shi, P., and Tsai, C.L. (2002). Regression model selection - a residual likelihood approach. *Journal of the Royal Statistical Society, Series B*, **64**, 237-252.

Su, X.G., Wang, M., and Fan, J.J. (2004) Maximum likelihood regression trees. *Journal of Computational and Graphical Statistics*, **13**, 586-598.