POWER TRANSFORMER DESIGN USING MAGNETIC CIRCUIT THEORY AND FINITE ELEMENT ANALYSIS – A COMPARISON OF TECHNIQUES

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Abstract

This paper summarises a reverse method of transformer design where the construction details of the transformer are directly specified and are used to determine the device performance and ratings. Two magnetic models are presented for the inductive-reactance components of the Steinmetz 'exact' transformer equivalent circuit. The first model, based on magnetic circuit theory, is frequently taught in undergraduate power system courses at universities. The second model is based on magneto-static finite element analysis. The reverse design method is used to design two sample high voltage transformers. The performance of the two magnetic models is compared to the measured performance of the as-built transformers. The magnetic model based on finite element analysis is shown to be more accurate than the model based on magnetic circuit theory, though at the expense of complexity of programming.

1. Introduction

From a manufacturer's perspective it is convenient to design and produce a set range of transformer sizes. Usually, the terminal voltages, VA rating and frequency are specified. In the conventional method of transformer design these specifications decide the materials to be used and their dimensions. This approach to transformer design has been utilised and presented in detail in textbooks [1, 2]. It has been used as a design tool for teaching undergraduate power system courses at universities [3-5]. In addition, it has also been used extensively in designing switched mode power supplies [6, 7]. Finite element analysis has also been applied, concurrent with the above approach, to aid the overall design process [8, 9].

However, by designing to rated specifications, consideration is not explicitly given to what materials and sizes are actually available. It is possible that an engineer, having designed a transformer, may then find the material sizes do not exist. The engineer may then be forced to use available materials. Consequently the performance of the actual transformer built is likely to be different from that of the design calculations.

In the reverse design approach, the physical characteristics and dimensions of the windings and core are the specifications. By manipulating the amount and type of material actually to be used in the transformer construction, its performance can be determined. This is essentially the opposite of the conventional transformer design method. It allows for customised design, as there is considerable flexibility in meeting the performance required for a particular application.

This paper first summarises the reverse method of transformer design. Models for the resistive and inductive-reactance components of the Steinmetz 'exact' transformer equivalent circuit are developed from fundamental theory, as previously presented in [10]. Several anomalies are corrected. Then two- and three-dimensional linear and non-linear magneto-static finite element models are introduced as an alternative model for the inductive-reactance components. The performance of the two magnetic models is compared to the measured performance of two as-built transformers.

2. Reverse Transformer Design

A transformer profile showing known material characteristics and dimensions is depicted in Figure 1.

In the reverse design method, the transformer is built up from the core outwards. The core cross-section dimensions (diameter for a circular core and side lengths for a rectangular core) are selected from catalogues of available materials. A core length is
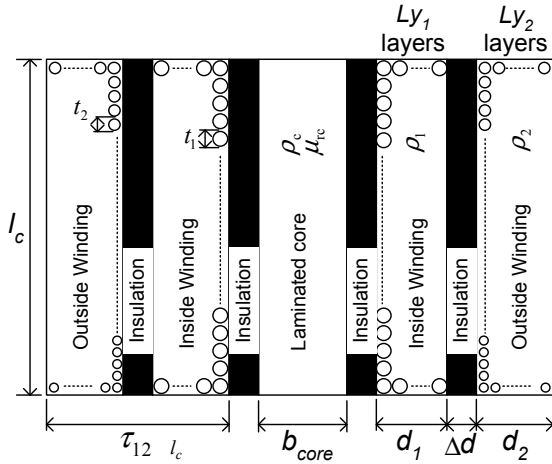
Figure 1 Centre limb of a transformer showing component dimensions and material properties.

chosen. Laminations that are available can be specified in thickness. A core stacking factor can be estimated from the ratio of iron to total volume.

Given the core length, l_c, and diameter, DC (or b_{core} and w_{core} for a rectangular core), the inside winding (usually the low voltage winding) is wound on layer by layer. The wire size can be selected from catalogues. They also specify insulation thickness. The designer can then specify how many layers of each winding are wound.

Insulation is placed between the core and the inside winding (former) and between each layer for high voltage applications. Insulation can also be placed between each winding. The outer winding (usually the HV winding) is wound over the inside winding, with insulation between layers according to the voltage between them.

Winding current densities and volts per turn become a consequence of the design, rather than a design specification. The only rating requirements are the primary voltage and frequency. The secondary voltage and transformer VA rating are a consequence of the construction of the transformer.

The number of turns on the windings are estimated to be:

$$N_1 = \frac{l_c L_1}{t_1}, N_2 = \frac{l_c L_2}{t_2} \qquad (1)$$

where:

l_c = length of the core

L_1, L_2 = number of primary and secondary winding layers

t_1, t_2 = axial thickness of primary and secondary winding wire

This calculation assumes that the winding length is equal to the core length. The actual winding lengths may be used if the primary and secondary winding lengths are different and do not fully occupy the winding window height.

3. Equivalent Circuit Models

The Steinmetz 'exact' transformer equivalent circuit shown in Figure 2 is often used to represent the transformer at supply frequencies [11]. Each component of the equivalent circuit can be calculated from the transformer material characteristics and dimensions.
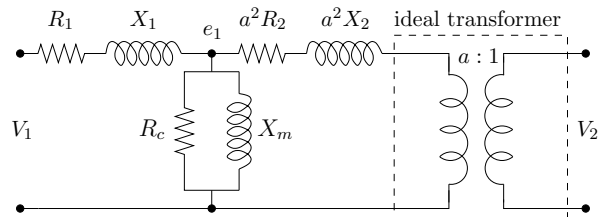

Figure 2 Steinmetz 'exact' transformer equivalent circuit, referred to the primary winding.

3.1. Resistance models

3.1.1. Core loss resistance

The losses in the core consist of two major components; the hysteresis loss and the eddy current loss. The hysteresis loss can be calculated using [11]

$$P_h = k_h f B^x WT \qquad (2)$$

where:

k_h = constant depending on the material, typically 0.11

x = *Steinmetz* factor, typically 1.85

WT = weight of the core

B = peak flux density, calculated from the 'Transformer equation' as [12]

$$V_1 = 4.44 f N_1 \phi \quad (\phi = B A_c) \qquad (3)$$

The eddy current loss is expressed as [13]

$$P_{ec} = \frac{c_l^2}{12 \rho_c} \frac{l_c}{N_1^2 A_c} e_1^2 k_v \qquad (4)$$

where:

c_l = lamination thickness

ρ_c = operating resistivity of the core

A_c = cross-sectional area of the core

e_1^2 = induced primary winding voltage

k_v = total core volume / central limb volume

The variation of resistivity with temperature should be accounted for, since the transformer will be heated up under operation. The operating resistivity for a material at temperature T^oC is

$$\rho = \rho_{20^o C}(1 + \Delta\rho(T - 20)) \qquad (5)$$

where:

$\Delta\rho$ = thermal resistivity coefficient

$\rho_{20^o C}$ = material resistivity at 20°C

The hysteresis and eddy current losses can be expressed in terms of the induced voltage e_1 as

$$P_h = \frac{e_1^2}{R_h}, P_{ec} = \frac{e_1^2}{R_{ec}} \qquad (6)$$

where:

R_h = hysteresis loss equivalent resistance

R_{ec} = eddy current loss equivalent resistance

Thus, both R_h and R_{ec} can be included in the model as the core loss resistance R_c, calculated as

$$R_c = \frac{R_h R_{ec}}{R_h + R_{ec}} \qquad (7)$$

3.1.2. Primary winding resistance

The primary winding resistance is

$$R_1 = \frac{\rho_1 l_1}{A_1} \qquad (8)$$

where:

ρ_1 = resistivity of the primary winding wire

l_1 = effective length of the wire

A_1 = cross-sectional area of the wire

The resistivity is temperature dependent and should be adjusted according to Eq. 5. The effective length of the primary winding wire is estimated by calculating the length of wire on each layer of the winding, and then summing over all layers.

3.1.3. Secondary winding resistance

The secondary winding resistance is

$$R_2 = \frac{\rho_2 l_2}{A_2} \qquad (9)$$

where:

ρ_2 = resistivity of the secondary winding wire

l_2 = effective length of the wire

A_2 = cross-sectional area of the wire

The effective length of the secondary winding wire is calculated in a similar manner to that for the primary winding wire. As for the primary winding, the resistivity is adjusted for the operating temperature.

3.2. Inductive reactive models

3.2.1. Magnetising reactance

The magnetising reactance is [13]

$$X_m = \frac{\omega N_1^2 \mu_0 \mu_{rc} A_c}{l_{eff}} \qquad (10)$$

where:

ω = $2\pi f$

μ_0 = permeability of free space ($4\pi \times 10^{-7}$ H/m)

μ_{rc} = relative permeability of core

l_{eff} = effective path length for mutual flux

3.2.2. Leakage reactances

The primary and secondary leakage reactances are assumed to be the same, when referred to the primary, and are each half of the total transformer leakage reactance. One form of expression is [14]

$$X_1 = a^2 X_2 = \frac{1}{2}\frac{\mu_0 N_1^2}{l_c}\left(\frac{\bar{l}_p d_1 + \bar{l}_s d_2}{3} + \bar{l}_{ps}\Delta d\right) \qquad (11)$$

where:

$\bar{l}_p, \bar{l}_s$ = mean circumferential length of primary and secondary windings

$\bar{l}_{ps}$ = mean circumferential length of interwinding space

d_1, d_2 = thickness of primary and secondary windings

Δd = thickness of interwinding space

Having obtained the component values, the equivalent circuit can be solved. Open circuit, short circuit and loaded circuit performances can be estimated by putting an impedance $Z_L = R_L + jX_L$ across the output and varying its value. Further, performance measures of voltage regulation and power transfer efficiency for any load condition can be readily calculated. Current flows and densities in the windings can be calculated and compared to desired levels.

4. Incorporating Finite Element Analysis into the Reverse Design Method

4.1. Transformer design program

A transformer design program was written in a MS Excel workbook. A module, written in Visual Basic for Applications (VBA) code, was used to couple the workbook to the commercial finite element analysis software package MagNet [15]. By automating the process of finite element modelling, much time is saved and the likelihood of user error is reduced.

4.2. Model detail

Each winding was modelled as a single block of non-magnetic material encompassing all turns over all layers. Uniform current density was assumed. The core was modelled as a single non-conducting isotropic material. A constant relative permeability of 3000 was used for the linear model, and a generic B-H curve for non-oriented core steel was used for the non-linear model. The transformer was enclosed by a rectangular air-space with dimensions twice that of the core, to which a tangential flux boundary condition was applied. The default mesh was automatically refined using the in-built h-adaptation feature and the solution polynomial order was set to 3. Solving time was reduced for the three-dimensional models by making use of transformer symmetry, where only $1/8^{th}$ of the device was modelled. The model geometry for an example transformer, TX1, along with the initial mesh, is shown in Figure 3.
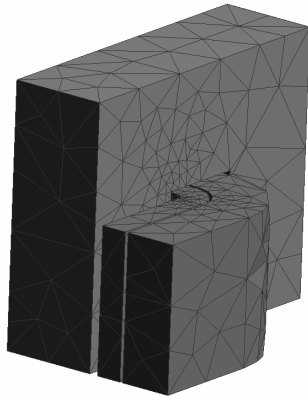

Figure 3 Geometry and initial mesh for transformer TX1 (air-space mesh not shown).

4.3. Reactance calculations

The winding inductances are defined as [16]
$$L_{ij} = N_i N_j P_{ij} \tag{12}$$

where:

N_i, N_j = number of turns on winding i and j

P_{ji} = magnetic permeance, defined as

$$P_{ij} = \frac{\lambda_i}{i_j} \tag{13}$$

λ_i = flux-linkage of winding i due to an excitation current in winding j.

The three magnetic permeances of the two-winding transformer, P_{11}, P_{12} ($=P_{21}$) and P_{22}, are calculated in two simulations. The winding self- and mutual-inductances are converted into components of a T equivalent circuit. Together with the core and winding resistances, this forms the transformer equivalent circuit of Figure 2. The reactance values are given by

$$X_m = a\omega L_{12} \tag{14}$$
$$X_1 = \omega L_{11} - a\omega L_{12} \tag{15}$$
$$a^2 X_2 = a^2 \omega L_{22} - a\omega L_{12} \tag{16}$$

4.4. Alternative calculation of leakage reactances

An alternative method of calculating the leakage reactance is based on energy techniques [17]. This provides a simple calculation check, and is less prone to numerical errors than the self- and mutual inductance method, where the (typically small) value of leakage inductance is given by the difference between two large numbers [18]. However, this method cannot resolve the individual leakage reactance values. For transformers with different primary and secondary winding lengths, or incomplete magnetic cores, the common assumption that the leakage reactances are equal when referred to the primary is no longer valid [19].

The total leakage reactance referred to the primary winding is computed from the calculated total stored energy W_s. The number of primary and secondary turns are both set to N_1, the primary winding is energised with current $+i_s$ and the secondary winding is energised with current $-i_s$. The leakage reactance is given by:

$$X_1 + a^2 X_2 = \frac{2\omega W_s}{i_s^2} \tag{17}$$

5. Two Examples of Transformer Design using the Reverse Design Method

To illustrate the reverse design method, two single-phase, 50 Hz, high voltage transformers have been designed, built and tested. The transformers were designed using the magnetic model based on circuit theory and have been subsequently re-analysed using the finite element magnetic model. Their nominal ratings are listed in Table 1.

Table 1 Transformer nominal ratings

Transformer	TX1	TX2
Primary voltage (V)	240	14
Secondary voltage (kV)	6.24	4.56
VA rating (VA)	200	617

Transformer TX1 was designed for the power supply of an electric water purification device [20]. Transformer TX2 was a model, designed to evaluate the harmonic performance of capacitive voltage transformers. Both transformers were built as shell types with rectangular cores.

Standard physical values of material permeabilities, resistivities and thermal resistivity coefficients were also entered as data, for the core steel and copper windings, as shown in Table 2. The two transformers were constructed using different core steel but the equivalent circuit models do not account for this.

Table 2 Material constants

	Core	LV Winding	HV Winding
Rel. permeability	3000	1	1
Resistivity at 20°C (Ωm)	$1.8{\times}10^{-7}$	$1.76{\times}10^{-8}$	$1.76{\times}10^{-8}$
Thermal resistivity coeff. (/°C)	0.006	0.0039	0.0039
Operating temperature (°C)	50	50	50
Density (kg/m³)	7870	8960	8960

Consideration was given to the wire gauges, insulation material, and core dimensions that were actually available. The dimensions of the various components that were to be used to construct the transformers were entered as data for the reverse design method, shown in Table 3.

5.1 Equivalent Circuit Parameters

The transformer calculated equivalent circuit parameters referred to the primary, along with the measured values as determined by open circuit and short circuit tests are presented in Table 4. The magnetic models are abbreviated as: 'CTM' – circuit theory model, 'l FEM' – linear finite element model, 'nl FEM' – non-linear finite element model. Load tests were also performed but the results have not been included here for space reasons.

The magnetising reactance values of Table 4 for the finite element model were calculated using a two dimensional model of the transformer. A three dimensional model was not required because the mutual flux is mostly constrained to within the plane of the core laminations. Both linear and non- linear models were constructed.

Table 3 Transformer design data

Transformer	TX1	TX2
Core:		
Length (mm)	68	114
Width 1 (mm)	51	44
Width 2 (mm)	44	152
Core/LV insulation thickness (mm)	2	3.25
LV winding:		
Length	66	114
Number of layers	5	1
Wire diameter (mm)	0.8	3.55
Interlayer insulation thickness (mm)	0.5	0
LV/HV insulation thickness (mm)	0.7	6.5
HV winding:		
Length	66	114
Number of layers	20	20
Wire diameter (mm)	0.125	0.212
Interlayer insulation thickness (mm)	0.5	0.09

Table 4 Calculated and measured equivalent circuit parameters for sample transformers

Value	Equivalent circuit parameters			
	R_c, Ω	X_m, Ω	R_{wind}, Ω	L_{leak}, Ω
Transformer TX1				
Meas.	3388	1987	10.0	2.8
CTM	1342	1383	11.5	1.9
l FEM	-	1905	-	1.6
nl FEM	-	1883	-	-
Transformer TX2				
Meas.	18	41	0.043	0.012
CTM	9.9	20	0.055	0.016
l FEM	-	25	-	0.015
nl FEM	-	54	-	-

For the non-linear model the magnetising value was calculated under open-circuit conditions with a static solver using an iterative procedure. The value of excitation current was adjusted until its product with the calculated value of magnetising reactance was equal to the peak value of the primary voltage. This is an approximation to the actual magnetising reactance value, as measured by true RMS meters. A transient solver could have been employed for higher accuracy at the expense of greatly increased computation time.

In practice, the actual value of magnetising reactance is unimportant, but the field distribution, calculated at the instant in time where the field peaks, can be used for loss calculations. More advanced models account for the anisotropic properties of the core and the core construction details. B-H curves and loss data, measured in both the rolling and transverse directions, can be incorporated into the finite element model. Such models are currently used in industry for highly accurate calculation of core losses [21].

A three-dimensional model was used to calculate the leakage reactance values. The two-dimensional model does not accurately calculate the leakage reactance

values because the majority of the leakage flux occurs in the end-winding region. Typically, the leakage flux density is greatest in the duct between the primary and secondary windings, and drops to negligibly low values once inside the core. Thus, only a linear model is required.

The results show that the non-linear finite element model most accurately calculated the magnetising reactance value of the two sample transformers. For transformer TX1, the finite element model was less accurate than the existing model for calculating the leakage reactance value. This may be due to the approximations made in the geometry of the finite element model.

There is a significant difference between the calculated and measured values of core losses. The hysteresis formula (Eq. 2) calculates a loss of 13W/kg for a peak flux density of 1.6T. This is a gross overestimation and should be addressed in a subsequent paper. The intrinsic losses of modern core steel are typically below 1W/kg and most transformer manufacturers obtain a building factor of less than 1.5.

6. Conclusion

A finite element magnetic model has been introduced into the reverse method of transformer design. The model was found to be more accurate than the existing model, which was based on magnetic circuit theory, though at the expense of complexity of programming. This has strengthened the use of the reverse design method as an entry-level design tool, from which more accurate models can be developed.

7. References

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