RADIO-METEOR INVESTIGATIONS OF ATMOSPHERIC MOTION

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by

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A coherent pulse radar has been addressed to the problem of tracking drifting meteor trains to deduce neutral atmosphere velocities in the height range 80 to 110 km. The present work describes the principles of operation, the implementation of these principles and the calibration of such a meteor radar. The doppler frequency shift or equivalent phase changes of the reflected signal are considered to be due to a radial movement of the ionized column. The theory of radio wave reflections from meteor trains is considered in detail, with particular regard to the interpretation of the received signal phase changes as being due to radial movements. It is shown that spurious radial velocities can arise from echoes from meteor trains with electron line densities in excess of \(10^{14} \text{ m}^{-1}\). Amplitude characteristics including echo decays and polarization phenomena are also considered. Results are presented from a nine day observing period in July 1977. The wind velocity results reveal the dominance of a regular semi-diurnal tidal component, the irregularity of the diurnal component and the existence of longer period (70 to 80 hours) oscillations at meteor heights.
An experimental project of this nature includes contributions from many people. I would especially like to thank:

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Dr R.G.T. Bennett for numerous helpful discussions and assistance with data gathering;
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My wife Anne for her continued support despite the disruptions an experimental project entails;
and finally Professor A.G. McLellan for providing the opportunity to pursue the project and financial support in the form of a teaching fellowship.
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INTRODUCTION

Most of the information concerning the observational state of motions in the lower thermosphere has to date been provided by the "meteor wind" experiment. The majority of this data refers to mid-latitudes, but is almost exclusively confined to the northern hemisphere. A meteor wind experiment was first established at Christchurch (13°S) by Wilkinson (1973) however preliminary data revealed several shortcomings of this system.

In the experiment of Wilkinson, the radial velocity component of the radiowave reflection point was deduced from the doppler shift of the reflected wave by observing changes in the relative echo phase angle. These phase changes must be observed over a suitable time interval and so echoes whose durations are less than this value fail to yield velocity information. This effect, first considered by Manning, Peterson and Villard (1954), introduces a bias against low radial velocities. Some idea of the low velocity loss may be gained by considering the echo duration to be that of an underdense meteor echo. The echo amplitude from such a trail decays with a time constant

$$T = \frac{\lambda^2}{16\pi^2 D_a}$$

due to radial ambipolar diffusion of the trail plasma, where $\lambda$ is the radio wavelength and $D_a$ the ambipolar diffusion coefficient. If one half period of the doppler beat frequency (corresponding to a phase change of $\pi$ radians) is required to deduce the radial velocity, the minimum distance the trail must move is $\lambda/4$. The minimum radial velocity that can be resolved in two decay time constants is
This has been evaluated for typical meteor altitudes using values of \( D_a \) from Barnes and Paznickas (1972) and a representative wavelength of \( 10 \text{m} \) (Table 1).

Table 1. Minimum radial velocity

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<th>Altitude (km)</th>
<th>Velocity ( V_r ) (ms(^{-1}))</th>
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<td>80</td>
<td>0.9</td>
</tr>
<tr>
<td>85</td>
<td>2.4</td>
</tr>
<tr>
<td>90</td>
<td>5.9</td>
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<td>95</td>
<td>16.7</td>
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<tr>
<td>100</td>
<td>43.9</td>
</tr>
<tr>
<td>105</td>
<td>115.3</td>
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<td>110</td>
<td>274.8</td>
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The consequences of these low velocity losses will be considered in section 8.5.

The echo location technique used by Wilkinson relied on the use of narrow azimuth antennas and echo altitude derived from the decay rate of underdense echoes. The ambipolar diffusion coefficient is assumed to be inversely proportional to the neutral atmospheric density, so the decay time constant is altitude dependent. Although this relationship appears to be satisfied for accumulated data, the scatter of ambipolar diffusion coefficient values at a given altitude casts doubt on the validity of such a scheme applied to individual echoes.

The third limitation of the previous system relates to the data recording and analysis. The receiver outputs were displayed on an oscilloscope and recorded photographically. The subsequent analysis of the film records proved time consuming and prevented operation of the system over extended periods.
The primary aim of the present work was to produce a meteor wind system that circumvents these limitations. The first section (Chapters 1 and 2) details the developments of methods in use at other meteor wind stations and the methods and parameters chosen for the present equipment. In the second section (Chapters 3 and 4) the implementation of these methods is presented, along with a description of the calibration and the consequences of the various measurement techniques. The theoretical aspects of radiowave reflections from meteoric ionization are considered in the third section (Chapters 5 and 6), especially those aspects related to the interpretation of meteor wind results. Finally, in the fourth section (Chapters 7 to 9) preliminary results obtained during a nine day observing period in July 1977 are presented. As well as the observed neutral wind velocities, these results provide information on meteor echo parameters and the operation of the system.
CHAPTER 1

MEASUREMENTS OF ATMOSPHERIC MOTIONS IN THE LOWER THERMOSPHERE

INTRODUCTION

The types of experiments performed in the lower thermosphere to deduce neutral atmospheric velocities are primarily determined by the properties of the region. Most experiments track the motion of some form of tracer, which is assumed to share the motion of the neutral atmosphere and is usually a minority constituent. Many sub-divisions of the types of experiments based on the property to be measured and the measurement technique are possible. These include categories such as those that measure neutral atmosphere velocities and those that obtain velocities from the drift of plasma irregularities. Further sub-division is possible into direct and indirect methods, ground based or vehicle borne methods and those using natural or artificial tracers. The most commonly used experiments will be considered following the categories of Kent (1970), where a more complete list of techniques may be found.

1.1 MEASUREMENT OF NEUTRAL ATMOSPHERE VELOCITIES

The most direct evidence for lower thermosphere motions is provided by the visual observation of phenomena such as noctilucent clouds and luminous trails. Noctilucent clouds reveal information on the horizontal structure of small scale motions but have limited height and latitude coverage (Fogle and Haurwitz, 1966). Irregularity of occurrence prevents observation of all but short period motions.

Luminous trails are short lived, and in the case of enduring visual meteor trains, a rare occurrence. However, small scale structure may be deduced from train distortions using photographic records obtained at
several sites (Liller and Whipple, 1954). Since luminous trails offer such
direct evidence of atmospheric movements, considerable effort has been
directed to injecting artificial trails into the atmosphere. Early
experiments used rockets carrying payloads of an alkali metal such as
sodium which was released along the rocket path above 80 km altitude.
Resonant scattering of sunlight produces a visible trail, restricting the
observations to short periods near sunrise and sunset. As the trail
diffuses it also distorts under the action of local winds. Photography at
spaced ground sites and subsequent triangulation determines the position of
the trail as it evolves (Blamont and de Jager, 1961, Kochanski, 1964).

The time limitation for sodium observations has led to the develop­
ment of chemiluminescent materials such as trimethylaluminiunum (T.M.A.),
which when released reacts with atomic oxygen to produce visual trails
during the hours of darkness (Rees et al., 1972). The lack of continuous
coverage prevents the observation of long period phenomena, although some
tidal features have been inferred from such data (Hines, 1966). Optical
tracers are usually restricted to atomic dimensions and may be expected to
share the motion of the ambient atmosphere since they are electrically
neutral and undergo sufficient collisions (Kent, 1970). The cost of
particle injections prevents their use as a routine observation method.

1.2 MEASUREMENTS OF THE DRIFT OF IONIZATION IRREGULARITIES

Several methods exist for inferring velocity values from radiowave
reflections from the D and E regions of the ionosphere. These include
phase path variations, doppler shifts, angle of arrival measurements and
the ionospheric drifts experiment using close spaced receivers. In this
experiment, radio waves reflected from ionization irregularities produce a
diffraction pattern at ground level. Changes in the diffraction pattern,
in particular its drift velocity across the antennas are interpreted in
terms of the atmospheric motion of the irregularities responsible for the
diffraction pattern. Variations of this method include the use of partially
or totally reflected radio waves and either normal or oblique incidence.
The use of partial reflections enables the determination of drifts over a
range of altitudes (Fraser, 1965).

The moving diffraction pattern produces fading signals at the spaced
antennas. The time delay between similar features in the fading record may
be associated with the horizontal displacement of the irregularity, provided
its form remains unchanged. If however the line of diffraction pattern
maxima is inclined to the drift direction, peak amplitudes cross the sampling
antennas at times not representative of the drift. The correlation
analysis method of Briggs et al. (1950) takes into account irregular
variations in the diffraction pattern and the extension of Phillips and
Spencer (1955) allows for its anisometry. Assuming a statistically
stationary amplitude time series from each antenna, cross and auto-
correlation functions are computed which yield information on the time
delays and the nature of the irregularities. The use of three such time
delays from three antenna pairs serves as a check of experimental consisten-
cy. The ionospheric drifts experiment also has a low velocity cut-off,
dependent on the antenna separations and the maximum length of the
amplitude sampling period.

The indirect feature of the experiment is the interpretation of
fading time delays as being representative of neutral gas motion. Since
the irregularities are illuminated by spherical radio waves, the size of the
diffraction pattern at ground level is twice that of the features aloft and
hence the derived velocities are usually halved. This is the so-called
point source effect. The experimental results of Wright (1968) questioned
this scaling factor, but later experiments (Golley and Rossiter, 1970) have
shown the drift results to depend on equipment parameters, small antenna
spacing being responsible for Wright's results. The theoretical properties
of diffracting screens have been treated by Ratcliffe (1956) but the influence on drifts of the formation of irregularities and the reflection processes are not well determined. Hines (1968) conjectured the irregularities may be formed by neutral atmosphere wave motion or by the dissipation of internal gravity waves. The measured drift would then be the combination of the horizontal phase velocity of the wave and the background motion and not representative of the background flow. Noctilucent cloud observations have shown that bulk movement and wave motion may move in different directions (Haurwitz, 1961). This has prompted several comparisons of ionospheric drifts with other velocity measuring experiments. These include comparisons with meteor winds (Stubbs, 1973, Felgate et al., 1975) and with rocket trails (Lloyd et al., 1972). The comparisons reveal similar motions for these methods when statistically reliable mean velocities are determined by averaging over the same volume of atmosphere over appreciable time intervals. Using partial reflections, the ionospheric drift experiment is capable of covering the altitude region from 75 to 100 km, with a time scale down to the order of minutes, although suitable ionospheric reflections are often not available at all heights during night hours.

1.3 THE RADIO METEOR METHOD

A meteoroid entering the earth's atmosphere suffers increasingly frequent collisions with individual atmospheric molecules as it descends. These have sufficient kinetic energy of impact to produce evaporation of the meteoroid atoms. The ablated atoms undergo collisions with neutral atmosphere molecules and since the ionization potentials of constituent meteoroid atoms are lower than that for any atmospheric molecule, most of the ionization and excitation produced is attributed to them. The
nature of the ionized column left in the meteoroid wake depends on the
parent meteoroid properties such as mass, shape, density and velocity, its
trajectory and neutral atmosphere parameters. As the meteoroid encounters
increasing atmospheric density in its descent, the rate of ionization
production increases and subsequently decays at lower levels as the
meteoroid material is exhausted. Meteor trains are typically 10 km in
length and occur mostly in the altitude range 80 to 110 km, the so-called
meteor region. A summary of the theory and observational results of train
formation is contained in McKinley (1961). The presence rather than the
formation process of the ionized column is of importance for the meteor
wind experiment.

The rate of occurrence and spatial distribution of meteor trains is
important however since these will ultimately determine the spatial and
temporal scales of motion that can be deduced by their use. Meteors may be
divided into shower meteors, which are those that travel in well defined
heliocentric orbits and appear to an earth observer to emanate from a
single point, the radiant, and sporadic meteors. Due to their continual
occurrence, chiefly sporadic meteor will be used in a typical meteor wind
experiment, the radiants of which may be considered to be randomly
distributed over the celestial sphere. Because of the earth's annual
motion there will be a concentration of radiants and an increase in
meteoroid geocentric velocity in the direction of the apex of the earth's
way. Combined with the earth's daily rotation, this results in a diurnal
variation in both the observed visual and radio echo rates and mean
meteoroid velocity. Meteors in the morning sector are swept up by the
passage of the earth whereas those on the evening side must overtake it.
The diurnal radio echo rate variation will influence the statistical
reliability of the determination of wind velocity averages, while the
diurnal mean meteoroid velocity variation would be expected to result in an
inverse variation of the mean height of maximum ionization production. Although extending over the altitude range 80 to 110 km, the height distribution of ionized trains for sporadic meteors is approximately Gaussian with a peak near 95 km. The exact form of this distribution and the radio echo rate depend on the parameters of the meteoroid population and the radar used to observe the trains; detailed considerations may be found in Kaiser (1953).

Once created the ionized column expands under radial ambipolar diffusion and is assumed to move bodily with the motion of the neutral atmosphere. The ion collision frequency with neutrals is much greater than the ion gyrofrequency in the meteor region, at least by a factor of 10 at 110 km (Kent, 1970). Any drift of the meteoric ionization is expected to be controlled by the more massive positive ions. The formation of space charge fields due to the separation of electrons and positive ions results in the electrons (which are responsible for radiowave scattering) sharing the motion of the ambient neutral gases. This may not be the case when the column is closely aligned with the geomagnetic field at altitudes greater than 95 km (Kaiser et al., 1969).

The electron column is capable of reflecting V.H.F. radio waves under specular conditions (Chapter 5). The echo range variations may be tracked by radar to yield the radial component of the train drift. This is usually carried out by observing relative echo phase changes with time, since the range drift is small during the echo lifetime. Such phase changes may be interpreted as being due to radial drift provided the nature of the radar target is unchanged during the observing period. Equivalently the drift may be viewed as a doppler frequency shift due to the motion of the target. The meteor wind experiment is usually considered to be a direct technique measuring neutral air motions. However, the drift is that of a charged particle ensemble which may be subject to interactions with other
charged particles or the geomagnetic field, but is generally believed to share the motion of the surrounding atmosphere.

1.3.1 Early Meteor Wind Experiments

The first reported use of radiowave reflections from meteor trains to deduce information about upper atmosphere winds was that of Manning et al. (1950). (It is interesting to note that these authors (Manning et al., 1949)) had earlier attributed the low frequency body doppler shift to a radial contraction of the fully formed ionized column. Continuous wave emissions were used in conjunction with a separate pulse transmitter for range determinations. The doppler shift was determined by examining the beat between the reflected and transmitted continuous waves, a radial movement of \( \lambda/2 \) resulting in a relative phase change of \( 2\pi \) for the round trip signal. The determination of the sign of the doppler shift and hence the radial velocity was achieved by the use of an additional beat of the echo with the transmitted wave advanced by \( \pi/2 \). This resulted in two rotating phasors corresponding to the two doppler beats separated by \( \pi/2 \). The sense of rotation could then be determined by observing which of the components had the leading phase.

Manning et al. (1950) analyzed results on the basis of a uniform horizontal wind. By observing over all possible azimuthal sectors, a sinusoidal variation of the radial drift with azimuth should be apparent. A weighted Fourier analysis was used to determine the magnitude and direction of such a uniform wind. The interpretation of the body doppler as being due to a radial drift was also investigated. Using two carrier frequencies simultaneously, the doppler beat for a given reflection was found to be proportional to the carrier frequency, indicating the body doppler was indeed due to a radial drift of the ionized column.
A pulsed radar was first addressed to the problem of drift determinations by Greenhaw (1952a). A high range resolution radar was used, but observations were necessarily restricted to echoes with durations in excess of 0.5 seconds to provide a uniform resolvable displacement. Results of a similar magnitude to those of Manning et al. were obtained for a restricted observation period. The mechanism responsible for the amplitude fluctuations in long duration echoes was also considered. The creation of multiple specular reflection points by non-uniform winds would introduce interference and hence fading. Several velocities would then be apparent, an effect that would manifest itself as a broadening of the returned pulse in time, in conjunction with the amplitude fading, as observed by Greenhaw. The high resolution pulse technique however was quickly superseded by the coherent pulse method of Greenhaw (1954).

Many variations on the basic techniques for determining radio meteor drifts exist and are conveniently divided into the possible operating modes; the continuous wave methods and the coherent pulse methods.

1.3.2 The Continuous Wave Systems

(a) The Adelaide system

Manning et al. recognized the need to measure the height or position of the reflection point to investigate possible atmospheric stratification. A system capable of determining the echo position has been described by Robertson et al. (1953). Although the recording techniques have since been modified, the principles for determining the drift and the location of the reflection point are still in use and have been adapted for other systems.

The continuous wave mode necessitates the separation of transmitting and receiving sites. In the method in use at Adelaide the transmitted continuous wave reference is
present at the receiving site, so during the reception of an echo the vector sum of both direct and reflected waves is presented to the receivers. The subsequent detected output contains the doppler beat as in the method of Manning et al. The sense of the radial drift is obtained by the use of a phase modulated system in which the phase of the transmitted wave is periodically retarded by \( \pi/2 \). This wave reaches the receiving antenna before the phase retarded reflected wave and so briefly, the ground wave is retarded by \( \pi/2 \) relative to its original value. This sudden change of phase is applied at 50 Hz, resulting in a doppler beat output with superimposed spikes representing the phase quadrature beat, determining the sense of rotation of the doppler beat phasor.

The echo position is determined by measurement of the angles of arrival, and echo range, for which an additional pulse transmitter is employed. Consideration of the resultant signal vectors shows that any phase difference between adjacent antennas due to the arrival angle of the reflected wave is preserved in the doppler beat. The phase differences are present as time displacements of the doppler beat outputs, two orthogonal pairs of antennas being required to determine the two angles of arrival. Additional antennas are used to resolve any phase ambiguities that may arise. By having the phase reference established at the antenna, the sign of the phase difference between adjacent antennas is dependent upon the sign of the doppler shift, which must be determined before the echo position can be found. Receiver outputs were displayed on cathode ray tubes and the data reduced from photographic records by measuring beat periods and delays between the various beat outputs (Elford, 1966, Roper, 1972). Some consequences of this reduction method will be discussed in part (c).

(b) The Garchy System

The continuous wave system located at Garchy (Spizzichino et al., 1965) represents an alternative approach to the Adelaide system. Rather than allowing the ground wave to be present at the receiving antennas, it
is suppressed by separating the transmitting and receiving sites by high
ground and by the use of an auxiliary anti-phase transmitting antenna to
suppress the rear lobe of the main transmitting antenna. Since all measure-
ments are carried out as phase measurements, a phase reference is required
at the receiving site. This is achieved by the use of a vertically
polarized direct link, while all echo antennas are horizontally polarized.
By comparing phase differences in adjacent antennas not with respect to the
ground wave reference, but with respect to one of the receiving antennas
considered as a reference, phase differences and hence arrival angles may
be determined independently of the doppler beat. Three antennas situated
at the vertices of a right angled triangle are coupled to receivers whose
synchronous quadrature outputs represent the sine and cosine of the phase
angle difference.

The doppler shift is obtained as a continuously varying phase record
by beating the reflected signal with the transmitter reference. This method
does not require a specific fraction of a doppler cycle to be present for a
drift determination and so reduces the loss of low velocities.

The companion pulses of the Adelaide system are avoided by the use
of a phase modulated transmission to facilitate range measurements. Two
waves with frequency difference $\Delta f$ (Hz) will be in phase every $\frac{1}{\Delta f}$ (seconds).
A measurement of the phase difference between the two frequencies is
equivalent to a measurement of the propagation delay time. By a suitable
choice of the two radio frequencies, a phase change of $2\pi$ corresponds to the
maximum expected echo range. The range resolution is increased by the use
of an additional transmitted frequency serving as an ambiguous Vernier range
measurement. The continuous wave transmitter required to produce these
three stable signals simultaneously is necessarily complicated.
Data from the Garchy experiment were initially recorded by applying the in phase and quadrature phase detector outputs to the horizontal and vertical deflection plates of cathode ray tubes, resulting in a direct polar phase angle display which was photographically recorded. Subsequent development of the system now has these signals recorded in analogue form on magnetic tape for automated reduction (Spizzichino, 1972). This system established new standards in precision for meteor wind experiments, the claimed resolutions for a signal to noise ratio of 20 dB being 0.3 km in range, 1 ms\(^{-1}\) in radial velocity and 0.2° in arrival angles.

(c) The Atlanta system

Features from both the Adelaide and Garchy systems have been incorporated into the Atlanta facility (Roper, 1975) and also the similar system at College, Alaska (Hook, 1970). The aim of the Atlanta system is continuous operation and so emphasis has been placed on reliability and the economy of recorded data. The use of narrow band receivers (±50 Hz) results in good sensitivity for relatively low transmitter output power.

The determination of echo range is accomplished in a similar manner to that at Garchy, by using a double sideband suppressed carrier transmitter. The necessary phase differences however are recorded as time differences or delays rather than a phase angle or representative voltages. The arrival angles are determined in a like manner by measuring the time differences between zero crossings of the doppler beat frequency outputs from several spaced receivers with similar layout to the Adelaide system. Receiver outputs are connected to zero crossing detectors, an output from one of which (channel 1) enables a counter on either a positive or negative transition. The next like transition from channel 1 disables the counter hence determining a single doppler beat period. All other channels begin counting with the initiating transition from channel 1, but their counts are terminated at the first like transition from their associated
receivers. This count is then the time delay corresponding to the phase difference between the appropriate channel and channel 1, expressed as a fraction of the beat period. For consistency an additional counter measures the channel 1 doppler period for transitions of the opposite polarity.

Since this is a digitized form of the Adelaide system, the sense of the radial drift must be known before phase differences and arrival angles can be unambiguously determined. By recording mostly single zero crossing positions of the beat outputs, considerable economy of data recording is achieved.

Since the doppler beat period is measured twice, one full cycle for each zero crossing polarity, a duration of three half cycles of the beat is required for a drift determination. The available time to obtain a single record is 0.75 seconds so the minimum doppler frequency is 2 Hz, corresponding to a minimum radial velocity of $9.2 \text{ ms}^{-1}$. More serious however is the loss of low radial velocities associated with the duration of underdense type echoes. The results of table 1 are for a doppler beat duration of one half cycle so to be applied to the above format these minimum velocities must be multiplied by a factor of 3.

Because phase differences are not determined at the same instant but as time delays during the beat duration, there is an implicit assumption that the doppler beat is uniform over this time interval. For this reason Roper employs the additional doppler beat detector and accepts records whose opposite polarity doppler periods agree within 20%. The acceptance of non-uniform dopplers may introduce systematic uncertainties into the phases and hence angles of arrival and echo position. Radial velocities determined this way are an average over a doppler cycle, during which time the velocity may change appreciably due to shearing (or rotation) of the ionized column (Muller, 1968). Phase changes due to radiowave scattering processes are also expected for large electron density columns (section 6.3.1), the type of train responsible for enduring echoes which may have
sufficiently large durations to produce the necessary length of doppler beat.

1.3.3 The Coherent Pulse Systems

(a) The Jodrell Bank system

Although no longer in operation, the coherent pulse equipment of Greenhaw (1954) forms the basis from which all subsequent pulse apparatus has developed. In this method a continuous oscillator is gated in the lower power stages of the transmitter, resulting in good frequency stability between the transmitted and reference signals, necessary for the relatively small doppler frequency to be obtained. The magnitude and sign of the doppler frequency were determined from echo phase changes by using a pair of phase detectors with references separated by $\pi/2$. The output obtained is a pulsed sampling of the doppler beat. For echo location Greenhow relied on the use of highly directional antennas to restrict the observed azimuths. In addition, the decay rate of underdense echoes was used to provide echo height information. Two orthogonal wind components were obtained by observing on two perpendicular directed antennas, relying on a high echo rate to smooth velocity variations across the narrow azimuthal sector. The previous system in use at Canterbury (Wilkinson, 1973) was based on that of Greenhow.

(b) The A.F.C.R.L. system

This system began with a consideration of the Greenhow method, with the objectives of reducing the data in real time and of an independent technique for echo height measurement so that atmospheric densities could be deduced from echo decay rates (Ramsay and Myers, 1968). The height finding method used was first applied to meteor echoes by Clegg and Davidson (1950). Antenna radiation patterns in the vertical are a function of the antenna height above ground, so the elevation angle can be deduced from the ratio of echo amplitudes in a pair of antennas at different
heights. The doppler frequency was obtained using the method of Greenhow and Neufeld, however the output of both phase detectors was in binary form, compatible with the digital recording format. The data gathering was controlled by a small computer and data obtained during the period 1964 to 1967 with this equipment are presented in Barnes and Pazniokas (1972).

At the end of its development at A.F.C.R.L. this system suffered from two shortcomings. Firstly, the loss of low velocities associated with short duration echoes, and secondly the lack of azimuthal information within the broad antenna beams used. Without azimuthal information a good estimate of the true drift component along the antenna axis can only be obtained with narrow beam antennas.

(c) The Durham system

In 1967 the A.F.C.R.L. system was moved to Durham (N.H.) where azimuth determining equipment was added (Rudman et al., 1970). This utilized two antennas in an interferometric mode. The effective interferometer amplitude pattern may be considered as a combination of the antenna pattern and the interferometer array function

$$\cos \left( \frac{2\pi d}{\lambda} \cos \alpha + \phi \right)$$

where $d$ is the antenna separation, $\alpha$ the azimuthal angle and $\phi$ is any additional phase change between signals introduced into the antennas. The conventional radar practice of having the antenna follow the target is not possible for typical short duration meteor echoes, so rather than the antenna, the interferometer pattern is rotated to place the target in a null. This is achieved by placing known delays (phase changes) in the line to one of the receiving antennas, and stepping the delays for each received pulse. The position of the null output or zero crossing of a phase detector occurs at a position in the sequence, whose delay corresponds to the phase
difference of the plane wave arriving at the two receiving antennas. This introduces a phase quantization depending on the number of delays introduced and is also limited to one sequence per echo. The claimed accuracy of $\pm 1^\circ$ in azimuth was in general confirmed by calibration using a satellite beacon by Clarke et al. (1970). This system has been updated to include a similar phase sequenced interferometer for determination of the elevation angle as well, replacing the previous amplitude ratio method (Clark, 1975).

(d) The Stanford system

The meteor radar system developed at Stanford was also supported by A.F.C.R.L. but differed from the earlier Stanford measurements of Manning et al. in that the pulse mode was adopted. Major design criteria were for unattended operation and a data format that was suitable for digital recording and processing (Nowak, 1967). Consideration of various height finding techniques (Nowak, 1966) led to the conclusion that for a portable system the method of Clegg and Davidson (1950) was most appropriate. The height resolution at all but the largest elevation angles was considered unsatisfactory however, so the height finding was split into two parts. At large elevation angles the apparent radial velocity is small and the echo rate low, so the amplitude ratio method was used at high elevation angles as a calibration for decay heights which were then used at low elevations in subsequent wind analysis.

The problem of low velocity loss has also been considered by Nowak (1964). The particular solution involved beating the return signal with the transmitter reference offset by 40 Hz. A zero velocity then produces a doppler beat frequency of 40 Hz and the sign of the frequency shift is specified without recourse to a quadrature phase detector. This method has also been incorporated into the Durham equipment (Clark, 1975).

Nowak considered a very short pulse necessary for range accuracy and hence for echo height resolution. The long (280 $\mu$sec) pulses employed led
to the use of a pulse compression technique in the form of a phase modulation to decrease the effective pulse length. The transmitted pulse was phase encoded with a pseudo-random binary \(0^\circ\) and \(180^\circ\) code whose autocorrelation function contained only a single significant peak. A running cross-correlation was performed between the received pulse and the transmitted code, a peak output corresponding to the return of the entire pulse. The range accuracy is then 1 bit of the code which in Nowak's case was 10 \(\mu\)secs. The use of a 28 bit code results in a transmitter power equivalent to a 70 kW, 10 \(\mu\)sec pulse for ranging purposes from the 2.5 kW output. The implementation of these techniques is described in Nowak et al. (1970). This particular system (the mark II) was recommended at the 1971 I.U.G.G. meeting in Moscow and copies are in existence (Barnes 1973).

(e) The Sheffield system

The original Sheffield equipment (Muller, 1966) was similar to that of Greenhow (1954). In order to measure echo height directly, a pulsed system using phase comparisons as at Garchy has been implemented (Muller, 1970, 1974). The outputs from two sets of phase detectors operating in quadrature treating one antenna as the phase reference are applied to cathode ray tubes in a similar manner to the Garchy system. The Greenhow and Neufeld method is retained for doppler beat determinations. The claimed height resolution of 1 km seems optimistic in view of the quoted range resolution of 0.4 km and the nature of the phase angle display reproduced in Muller (1974).

Muller (1970) remarks that he could find no previous reference to the application of spaced antennas to determine arrival angles in a pulsed radar. In 1957 however; a single station coherent pulse radar was installed at Mawson in Antarctica (Elford, 1966). This system was essentially identical to the Adelaide system, using the phase differences from spaced antennas to determine arrival angles. By passing the receiver pulsed outputs through narrowband filters, an output format similar to the Adelaide system was obtained.
(f) The Illinois system

The most recent and advanced coherent pulse radar is the Illinois facility (Lee and Geller, 1973, Geller et al., 1977). The use of high peak transmitter power (up to 4 MW) enables large numbers of echoes to be recorded and all facets of data gathering are handled by a computer. The arrival angles are determined from the phase differences in adjacent antennas, with an additional antenna over the usual configuration of three at a large distance to serve as a Vernier measurement for the elevation direction cosine. Instead of having a central antenna provide the phase reference, all phases are measured with respect to the local transmitter oscillator. This is introduced as a reference into the receivers, rather than as a ground wave at the antennas. The receiver outputs are then a pulsed sampling of the sinusoidal doppler beat including the arrival angle phase differences. Zero crossings are not used to deduce the doppler frequency and phase differences; the individual pulse amplitudes are recorded. Each receiver contains two synchronous detectors operating in phase quadrature. A single output is digitized at 10 \mu sec intervals and loaded into a buffer during an inter-pulse period. Subsequent sweeps sample all six (or eight) receiver outputs in a cyclic mode. Since they are not sampled simultaneously, amplitude changes due to echo decays are included in the software reduction to prevent their interpretation as phase changes (Hess and Geller, 1976). The cyclic sampling also means the doppler frequency must be determined prior to finding the arrival angles. The echo range is determined to within a delay of 10 \mu sec by the location of an output sample in a single scan and further refined by fitting a pulse envelope (approximated by a parabola) to adjacent amplitude samples. The meteor echo parameters are determined in real time and recorded on magnetic tape. This system has the ability to distinguish and record simultaneous echoes whose ranges are separated by more than a pulse width.
Numerous other meteor wind stations exist and are still in operation (Barnes, 1973), however those discussed above are considered to have introduced significant variations to the measurement techniques. Most stations favour the pulsed mode, including a large contingent of Russian sets (Teptin, 1972) and many are involved as well with the gathering of astronomical data on meteors. Other variations include the use of two radio-frequencies to obtain consistent drift results (Müller, 1970) and the use of multiple receiving sites (Elford, 1966, Grossi et al., 1972). With widely spaced (of the order of kilometres) receiving sites, multiple reflection points on the same train may be used to determine train orientation (Elford, 1966), to study turbulence from spaced velocity determinations (Greenhow and Neufeld, 1959, Roper, 1966), or simply to increase the rate of velocity determinations (Kingsley et al., 1976). The overriding principle common to all however is the determination of the radial drift of a meteor train by observing changes with time of the relative echo phase angle, the essence of the meteor wind experiment.
CHAPTER 2

METHODS OF OPERATION

INTRODUCTION

To describe the wind field in the meteor region requires the measurement of the train radial drift and if spatial variations are not to be smoothed, the echo position. Several approaches to the measurement of these parameters have been outlined in Chapter 1. These will now be considered in detail and the way in which these principles have been applied to a coherent pulse meteor wind system will be discussed.

2.1 MODE OF OPERATION

It can be shown that both pulsed and continuous wave (meteor) radars can achieve the same echo sensitivity for identical transmitted average powers. The use of continuous wave (C.W.) emissions enables the use of narrowband (down to the maximum doppler beat frequency) receivers, resulting in very low noise levels and hence usable signal to noise ratios for low transmitted powers. Since the absolute input signal and noise levels are lower in a C.W. system, discrimination against broadband interference may be reduced. The use of C.W. necessitates the use of separated transmitting and receiving sites and reflection from a meteor train is via a forward scatter path. This results in increased durations for underdense echoes by the factor \( \sec^2 \phi \), where \( 2\phi \) is the forward scatter angle. In general \( 2\phi < 45^\circ \) so there is little increase and the size of phase changes for a purely horizontal drift are smaller than for the equivalent backscatter case.

In order to deduce echo range either frequency or phase modulation must be impressed on the C.W. transmission if a companion pulse transmitter is to be avoided. A C.W. transmission also suffers from reflections from
nearby targets, in particular aircraft, giving rise to a strong doppler beat due to their close proximity. With pulsed techniques ground clutter may be eliminated by the use of receiver suppression. Although aircraft echoes are easily distinguished in any data record, their presence may prevent the recording of meteor echoes. Roper (1975) quotes a figure of typically 50% of all records as being contaminated by aircraft reflections. Although identical sensitivities can be achieved, the complexity of a C.W. operation, physics department experience in pulse techniques and the availability of a coherent pulse transmitter dictated the use of the pulsed mode for the present investigation.

2.2 OPERATING FREQUENCY

The previous meteor wind system operated at 27.12 MHz, a frequency with very few restrictions on its use. Regular operation was often hindered by local interference and also by long range transmissions during certain periods of the day. For the continuous recording of data a shift to an adjacent frequency was necessary.

The most important experimental features are the ability to measure the radial drift, its resolution and the ability to detect sufficient numbers of suitable echoes. With a transmitted power of tens of kilowatts, most echoes will be of the underdense type for which the duration is proportional to $\lambda^2$. The measurement of radial drift involves observing the rate of relative echo phase variation, favouring a low radar operating frequency. Related to the echo duration is the echo ceiling phenomenon imposed on underdense echoes by the finite meteoroid velocity, the electron volume density being reduced by ambipolar diffusion before the principal Fresnel zone of length $\sqrt{2R\lambda}$ is traversed (R is the echo range). The underdense duration must exceed the traverse time for the echo to be observed,
\[
\frac{\lambda^2}{16\pi^2 D_a} \gg \frac{\sqrt{2} R \lambda}{V}
\]

where \( V \) is the meteoroid velocity in the earth's atmosphere. Hence

\[
\lambda^3 \gg \frac{512 \pi^4 D_a^2}{V^2}
\]

and for \( D_a = 8.48 \text{m s}^{-1} \) (appropriate to 95 km), \( R = 250 \text{ km} \) and \( V = 40 \text{ km s}^{-1} \), (approximately the average sporadic meteor velocity), \( \lambda > 8 \text{ m} \) (36 MHz).

From the radar equation the power received from an underdense echo is expected to vary as

\[
\frac{P_R}{P_T} \propto \lambda^3 \alpha^2
\]

where \( P_R \) and \( P_T \) are the received and transmitted powers respectively and \( \alpha \) the electron line density. Rather than absolute power received, it is the signal to noise ratio that determines signal detectability and from McKinley (1961), the cosmic noise power varies as \( P_N \propto \lambda^{2.3} \) for typical meteor radar frequencies. The signal to noise power is then

\[
\frac{P_R}{P_N} \propto \lambda^{0.7} \alpha^2
\]

which is constant for a given sensitivity. The limiting electron line density then varies as \( \lambda^{-0.35} \). The number of trains having line densities exceeding \( \alpha_m \) is (Kaiser, 1953)

\[
N(>\alpha_m) \propto \alpha_m^{(1-S)}
\]

and \( S \), the mass exponent (section 7.1) may be taken as 2 for sporadic meteors. Hence the echo rate for a given system is expected to vary as \( \lambda^{0.35} \), a relatively weak wavelength dependence.
Most of the contribution to the echo power comes from the first Fresnel zone centred on the perpendicular from the station to the train, and the velocity is a non-linear average over this region. Although the reflection point can be located more precisely than this, there is little point for the meteor wind experiment since this represents the smallest scale of motion to be determined. For a representative range of 250 km and a height resolution of 2.5 km this represents a wavelength of 12.5 m (24 MHz) as an upper limit.

The duration related aspects are of most consequence for the meteor wind experiment, supporting the use of low radio frequencies. However, at longer wavelengths interference is a problem and disturbed ionospheric conditions may lead to oblique signals being received from sporadic E and also auroral ionization. Interference levels were recorded at the Rolleston field station site during August to October 1973, usually during daylight hours when interference was expected to be worst. The audio output from an Eddystone 680X receiver was integrated and applied to a chart recorder while the frequency was continually swept between 25 and 30 MHz. The twin Yagi antenna described in Wilkinson (1973) was used and the dependence of gain on signal frequency ignored. Visual inspection of the records revealed considerably daily interference from 0900 to 1400 hours local time in the frequency range 26.75 to 27.25 MHz, coincident with the expected opening of the propagation circuit from Christchurch to the west coast of North America (Wilkinson, private communication). An approach to the local transmitter licensing authority with several alternative frequencies resulted in the allocation of 26.36 MHz (11.38 m). Further recording of interference levels on this frequency showed it to be acceptable.
2.3 THE MEASUREMENT OF ECHO POSITION OR HEIGHT

(a) Decay Heights

The use of amplitude time envelopes of decay type echoes to deduce echo height relies on the properties of the ionization and surrounding atmosphere and hinges on two assumptions. Firstly, that the decay does obey the underdense decay expression and secondly, that the ambipolar diffusion coefficient is appropriate and is inversely proportional to the neutral atmospheric density and hence easily related to altitude. The restrictions of the first assumption will be discussed in section 6.2 where it is shown that only for \( n < 10^{-13} \) \( \text{m}^{-1} \) and for a radio wave polarized along the train axis is this assumption appropriate. Although there is a statistical relationship between reflection height atmospheric density and echo decay time constants, the exact form of the relationship is unknown and the reliability of individual values uncertain (Barnes, 1968). The effects of the formation processes of the train, its length, ionization irregularities and wind shear rotating the train on individual echoes are unknown, but must be expected to contribute to variation from the theoretical models. Until more stringent criteria for acceptance of decay data are employed, the reliability of a single decay height determination remains in doubt.

(b) Distance Measurements

At different ground locations echoes will be received from different parts of the initially thin straight column of ionization. A single transmitting site and three receiving sites each measuring the echo range determine the path of the meteoroid which is tangent to the three ellipses formed with the transmitter and each receiver in turn as foci. The meteoroid velocity, found from the Fresnel diffraction fringes as the echo is formed, and the time of occurrence at each receiving site is used to determine the
position and separations of the various reflection points (Elford, 1966).
The need for multiple receiving sites makes this method unsuitable for the
present investigation.

(c) Angle Measurements

The alternative echo location method to triangulation is the specifi-
cation of two arrival angles and echo range, or range and the elevation
angle if only echo height is required. The arrival angles may be determined
by phase comparison methods, or the elevation angle obtained using the
amplitude comparison method used by Clegg and Davidson (1950). When an
antenna is placed above a perfectly conducting ground, the resultant
radiation pattern in the vertical plane is a combination of the direct and
reflected waves and so depends on the height of the antenna above ground.
If two antennas with identical azimuthal radiation patterns are vertically
positioned so the ratio of their signal strengths varies uniformly over an
elevation sector of interest, this ratio may be used to determine the
elevation angle independently of the azimuthal direction.

The theoretical accuracies for several antenna combinations have
been considered by Nowak (1966), who concluded that the pattern comparison
method was only suitable for elevation angles in excess of $45^\circ$. A thorough
knowledge of the vertical antenna patterns can only be provided by calibration
measurements, which are difficult to accomplish in the vertical plane.
Stability of the antenna characteristics is also essential for an amplitude
comparison system. A calibration was made of the amplitude ratio system at
Durham by Frost and Clark (1971) using passes of a satellite beacon. This
revealed large discrepancies between the measured and true elevation angles,
an asymmetrical distortion with respect to the azimuthal axis and the
occurrence of double valued amplitude ratios for a given azimuth. Frost and
Clark concluded that pattern ratios alone cannot be used to determine
source elevation, but must be used in conjunction with azimuth related information. Since any rapid variation in the horizontal antenna pattern is also likely to result in a variation from the ideal elevation ratio, the use of highly directional antennas is precluded. Pattern comparison measurements then also require azimuthal information to determine reliable velocity components. For example, two identical horizontal velocities of opposite sign directed perpendicular to the horizontal antenna beam axis occurring at azimuths of $45^\circ$ and $-45^\circ$ respectively yield identical radial velocity components.

2.4 PHASE COMPARISON METHODS

The alternative method of angle determination considers the phase difference due to the additional path length in two or more adjacent antennas as a result of the antenna separations and source angles of arrival. In Fig. 2.1 the phase difference is

$$\phi = \frac{2\pi}{\lambda} d \cos \theta$$

(2.1)

for an incident plane wavefront and is independent of the antenna radiation pattern within a specific lobe for a free space antenna.

Fig. 2.1. Difference in ray path length to two spaced antennas.
The direction cosine of the arriving wave is then specified by a measurement of the radio frequency (r.f.) phase difference. In practice the phase difference is not measured at the antennas but at appropriate receiver outputs, allowing several possible measurement schemes.

The phase difference, originally occurring at radio frequency, remains unchanged in any frequency shifts that maintain phase coherence by sharing a common local oscillator. The phase difference can be obtained at any intermediate frequency down to the doppler beat itself. Since a phase measurement is necessarily relative, the phase detection must be with respect to a reference, the introduction of which can be accomplished in three ways. The reference may consist of the transmitter oscillator which can be introduced at the antennas by flooding the region with a low level continuous wave signal, or it may be introduced into the receivers. Alternatively a receiving antenna can be chosen as a phase reference supplied by the radio frequency echo. For instantaneous phase difference determinations it is necessary to produce a receiving scheme whose instantaneous output signals are simply related, usually in the form of trigonometric functions to the phase difference.

In all situations the signals arriving in two adjacent antennas can be considered as

\[ \cos[(\omega + \omega_d) t + \phi_1] \] \[ \text{and} \] \[ \cos[(\omega + \omega_d) t + \phi_2] \]

where \( \omega \) and \( \omega_d \) are the radio and doppler angular frequencies and \( \phi_2 - \phi_1 \) is the phase difference defined in equation (2.1). In the case of the C.W. reference ground wave being present at the antennas, serving as an additive interferometer, the process of beating or phase detection has taken place prior to the signals entering the receiver. In vector form, the resultant signals in two antennas are shown in fig. 2.2, where
geometrical considerations show that the relative phase $\phi$ is preserved provided $G$ is constant and $|S_1| = |S_2|$. 

![Diagram (Fig. 2.2)](image)

Fig. 2.2. Signals in the additive interferometer. $G$ is the ground wave reference, $S$ the reflected sky wave and $R$ the resultant signal introduced to the receiver.

This restriction indicates only low directivity antennas are suitable for such a phase comparison. The receiver output after envelope detection usually has a cusped form (the resultants in fig. 2.2) and frequency $\omega_d$, but cannot in general be related to a simple trigonometric expression. Since the doppler frequency is included in the output its magnitude and sign must be determined before the phases are found as equivalent time delays. The consequences of this type of measurement have been considered in chapter 1. Using this phase detection method, the relative phases are established in the antennas and the radio frequency component is filtered out at detection, hence the output is independent of any variations in the radio frequency phase paths through the antennas and receivers.

In the other two phase measurement schemes the phase difference is determined at the detection stage and any differential radio frequency phase propagation through the equipment must be subtracted. If the transmitter is used as a phase reference, the synchronous phase detection may be considered as a multiplication of the received and local signals followed by a low pass filtering to remove the r.f. component, equivalent to a
cross-correlation process. In this case we have

$$A(t) \cos[(\omega_d t + \phi_1^1) + \phi_1] \cos(\omega t)$$

where now \( \phi_1 \) is the phase of the sky wave at antenna 1 with respect to the local reference. After low pass filtering this becomes proportional to

$$A(t) \cos(\omega_d t + \phi_1^1)$$

Similarly for antenna 2 the output is proportional to

$$A(t) \cos(\omega_d t + \phi_2)$$

If the reference is also introduced in phase quadrature (i.e. \( \sin \omega t \)) to an additional pair of phase detectors, the outputs are

$$A(t) \sin(\omega_d t + \phi_1) \quad \text{and} \quad A(t) \sin(\omega_d t + \phi_2).$$

Taking ratios of these quantities for each antenna gives the instantaneous phases from the inverse tangent

$$\psi_1 = \frac{\omega_d t + \phi_1}{\omega_d}$$

$$\psi_2 = \frac{\omega_d t + \phi_2}{\omega_d}$$

independently of the amplitude variations. A single determination yields \( \phi_1 - \phi_2 \). The doppler frequency is obtained from two sets of measurements performed at different times. For example if

$$\psi_3 = \frac{\omega_d (t + \Delta t) + \phi_1}{\omega_d}$$

then

$$\omega_d \Delta t = \psi_3 - \psi_1 \quad \text{determining} \quad \omega_d.$$

In principle, in a pulsed system a single received pulse is capable of determining the arrival angles, while two adjacent pulses are required to furnish the doppler frequency, provided all phases are determined con-
currently. If the phases are instead obtained on successive pulses (for example, the cosine then sine components) the doppler frequency must be determined before the phase differences can be found. This also introduces the possibility of influence by effects such as wind shear changing the apparent doppler frequency. The apparent additional phase change $\Delta \phi$ due to wind shear is $\Delta \omega_d \Delta t$ and from (2.5)

$$\frac{\Delta \nu_r}{\Delta t} = \lambda \frac{\Delta \omega_d}{4\pi \Delta t}$$

$$\sim 100 \text{ ms}^{-2} \quad \text{(Muller, 1968)}$$

For typical signal to noise ratios (section 2.8.1) $\Delta \phi \lesssim 0.1 \text{ rad.}$ and if $\Delta \omega_d \Delta t \lesssim 0.05 \text{ rad.}$ then $\Delta t \lesssim 0.02 \text{ sec.}$ for a similar phase tolerance, setting the upper limit between phase determinations from successive pulses. This method of phase determination also necessitates the recording of large quantities of data since for a single arrival angle determination four voltage outputs are required per received pulse.

If the signal at one antenna is treated as the phase reference, then applying the principles of phase detection as above results in a signal of the form

$$A^2(t) \cos(\omega_d t) \cos[(\omega + \omega_d)t + \phi]$$

giving, after low pass filtering,

$$A^2(t) \cos \phi \text{ and } A^2(t) \sin \phi$$

for a quadrature detector. All phase variations other than the arrival angle phase difference such as those due to the doppler beat, Fresnel diffraction effects and radiowave reflection effects (chapter 5) have been eliminated. The ratio of these outputs enables the direction cosines to be determined from a single received
pulse but necessitates a separate method for determination of the radial drift. A differential phase calibration is also required for this method and the multiplication of two signals with similar amplitude variations results in a doubling of the logarithmic dynamic range of the outputs (in the absence of practical circuit limitations). The phase detection process is here equivalent to an autocorrelation, since the signal is multiplied by itself shifted in phase (or delayed in time) and integrated. This results in an increase of the signal to noise ratio over an asynchronous detector since in general the noise is uncorrelated. The signal to noise ratio is however, inferior to the previous method since now two noisy signals serve as inputs, rather than a noisy signal and a reference for a single phase determination. However, to determine the angle of arrival the previous method requires two such phase measurements to obtain the phase difference. Comparisons of the three methods of providing a local phase reference are summarized in table 2.1.

2.5 THE ADOPTED ECHO LOCATION TECHNIQUE

For economy of data the present system has adopted the method of treating one antenna as a phase reference and measuring relative phase differences directly. In general the echo does not lie along the line of sight of the antennas and two arrival angles are necessary to specify its position. An additional antenna and its phase difference yields information on the second direction cosine provided it is not co-linear with the first pair. Neglecting earth curvature the situation is shown in fig. 2.3.

The required angles for a reflection point located at P are the elevation \( \theta \) and azimuth \( \alpha \). The phase differences at the receiving antennas are
Table 2.1. Comparison of the methods of phase reference introduction.

<table>
<thead>
<tr>
<th>Reference phase</th>
<th>Local (Tx) oscillator</th>
<th>Local (Tx) oscillator</th>
<th>Receiving antenna</th>
</tr>
</thead>
<tbody>
<tr>
<td>Injection point</td>
<td>Antenna</td>
<td>Receiver</td>
<td>Receiver</td>
</tr>
<tr>
<td>Doppler beat present in output</td>
<td>Yes</td>
<td>Yes</td>
<td>No</td>
</tr>
<tr>
<td>δ required for δ determination</td>
<td>Yes</td>
<td>No&lt;sup&gt;1&lt;/sup&gt;</td>
<td>No</td>
</tr>
<tr>
<td>Minimum time for reliable δ determination</td>
<td>1 doppler cycle</td>
<td>1 pulse</td>
<td>1 pulse</td>
</tr>
<tr>
<td>Parameter measured for δ</td>
<td>Doppler period and time delays</td>
<td>Voltages</td>
<td>Voltages</td>
</tr>
<tr>
<td>Detection method</td>
<td>Asynchronous envelope</td>
<td>Synchronous phase</td>
<td>Synchronous phase</td>
</tr>
<tr>
<td>Antenna similarity required</td>
<td>Yes</td>
<td>No</td>
<td>No&lt;sup&gt;2&lt;/sup&gt;</td>
</tr>
<tr>
<td>Identical receiver gains required</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
<tr>
<td>Differential phase calibration and r.f. phase stability necessary</td>
<td>No</td>
<td>Yes</td>
<td>Yes</td>
</tr>
</tbody>
</table>

1. Provided all outputs are measured simultaneously.
2. See section 3.3.4.
Fig. 2.3. Echo location geometry.

\[ \phi_{12} = \frac{2\pi d_{12}}{\lambda} \cos \beta \]

so in general \( \alpha \) must be found before \( \theta \) and the echo height \( z \) can be determined. For the perpendicular 1,3 antenna pair

\[ \phi_{13} = \frac{2\pi d_{13}}{\lambda} \sin \gamma \]

from which both \( \theta \) and \( \alpha \) can be obtained since

\[ \tan \alpha = \frac{\phi_{13}}{\phi_{12}} \cdot \frac{d_{12}}{d_{13}} \]

and

\[ \cos \theta = \left[ \left( \frac{\phi_{12}}{2\pi d_{12}/\lambda} \right)^2 + \left( \frac{\phi_{13}}{2\pi d_{13}/\lambda} \right)^2 \right]^{\frac{1}{2}} \]
and with the echo range $R$ known

$$z = R \sin \theta.$$ 

The phase detection process involves the multiplication and low pass filtering of the signals from the two orthogonal pairs of antennas, in phase and in quadrature to provide outputs proportional to the cosine and sine respectively of the phase differences. A multiphase (not necessarily $\pi/2$) reference is required to extend the phase angle detection range from $\pi/2$ to the possible $2\pi$. A phase shift of $\pi/2$ results in outputs that may be simply reduced and are amenable to direct polar displays. By observing both sine and cosine outputs during the same received pulse all time dependent amplitude variations are eliminated. This is important for high altitude underdense echoes since the adjacent received pulses from a pair of detectors in quadrature would be

$$\lambda^2(t) \cos \phi \text{ and } \lambda^2(t+\Delta t) \sin \phi.$$ 

Including the decay expression, the output ratio is then

$$\exp\left[\frac{-32\pi^2 D \Delta t}{a^2} \right] \tan \phi.$$ 

For an interpulse period of 3.3 msec, wavelength 11.38 m and altitude 105 km ($D_a = 58.4 \text{ m}^2 \text{s}^{-1}$, Barnes and Pazniokas, 1972), this results in a systematic phase variation of 0.23 rad. for a phase angle of $\pi/4$. The present phase detection process may be regarded as two interferometer patterns with angular separation $\pi/2$ viewing the same source.
2.6 THE STACKED INTERFEROMETER

A simplification of the preceding system is capable of yielding meteor echo heights independently of the azimuthal angle. A phase rather than amplitude comparison of two vertically separated antennas gives the elevation angle directly since

$$\phi = \frac{2\pi d}{\lambda} \sin \theta$$

in this case. This neglects the reflected wave from the ground beneath the antennas however, so three cases arise: a perfectly conducting ground; an imperfect ground; the absence of a ground as above.

In the case of the perfectly conducting ground a construction of the sums of direct and reflected vectors in each antenna shows that there is no phase difference regardless of elevation angle. This is equivalent to considering the phase centres of both antennas to be at ground level since the total reflection may be represented by an image antenna the same height below ground, hence there is no phase difference. The practical situation is one of an imperfect ground and phase differences are now apparent but depend on the complex reflection coefficient of the ground. The measured phase differences are not in general simply related to $\phi$ defined above so such a system must be calibrated in all possible angles. A stacked interferometer has been employed by Hess and Geller (1976) but this configuration has no azimuthal information, which is desirable for a meteor wind experiment.

2.7 RECEIVING ANTENNA SEPARATIONS

Regardless of the phase comparison method used the direction cosine is ultimately derived from an expression like (2.1). The resolution of the wave arrival angle then depends on the resolution of the phase angle
differences and the antenna separation. The phase uncertainty is
\[ d\phi = \frac{\partial \phi}{\partial \theta} d\theta \]

and so from (2.1)
\[ d\theta = \frac{\lambda}{2\pi} \frac{1}{d \sin \theta} d\phi \]

and is inversely proportional to the antenna separation. However, the variation in \( \phi \) becomes ambiguous (>2\pi) for a separation in excess of one wavelength, unless the arrival angles are restricted. The choice of antenna separations must consider the accuracy, possible phase ambiguities and the possibility of coupling between antennas for small separations. Although not strictly for azimuthal determination alone, the 1,3 antenna pair (fig. 2.3) will be referred to as the azimuthal antenna pair and likewise for the 1,2 or elevation antenna pair.

2.7.1 Separation of the Azimuthal Antennas \( d_{13} \).

To avoid phase ambiguities the total phase variation in the sector of interest must not exceed 2\pi, that is
\[ \left( \frac{2\pi d_{13}}{\lambda} \cos \theta \sin \alpha \right)_{\text{max}} - \left( \frac{2\pi d_{13}}{\lambda} \cos \theta \sin \alpha \right)_{\text{min}} \leq 2\pi. \]

Since there is symmetry about the line of sight
\[ \gamma_{\text{max}} = -\gamma_{\text{min}} \]

so
\[ d_{13} (\text{in } \lambda) \leq \frac{1}{2 \sin \gamma_{\text{max}}} \]

In the main antenna lobe, the azimuthal extreme for an echo will occur at the elevation angle corresponding to the radiation pattern maximum; however,
the worst case, corresponding to the minimum elevation angle will be con-
sidered. The minimum elevation angle is

\[ \theta_{\text{min}} \sim \sin^{-1} \frac{Z_{\text{min}}}{R_{\text{max}}} \]

for a flat earth, corresponding to $10^\circ$ ($Z = 80, R = 480$ km) for the present
equipment.

Since a two antenna array is to be used as the transmitting antenna
(section 3.6.3) the width of the system major lobe will be decided by the
position of the first zero in the array factor $|\cos \left( \frac{\pi d_t}{\lambda} \sin \alpha \right)|$ where $d_t$
is the transmitting antenna separation, given by

\[ \sin \alpha_{\text{max}} = \frac{1}{2d_t} \]

So now

\[ d_{13} \leq \frac{d_t}{\cos \theta_{\text{min}}} \leq d_t \]

(This result is expected since both sets of antennas are operating in an
interferometric mode). In section 3.6.3, the optimum separation for the
transmitting antenna pair is shown to be $1.4\lambda$, giving first nulls in the
radiation pattern (and hence a maximum unambiguous azimuthal phase
determination) at $\pm 21^\circ$. Any echoes arriving at an azimuthal angle $|\alpha|
greater than $21^\circ$ will have a corresponding phase difference that will be
translated back into this region producing a spurious result. In general

\[ \gamma = \sin^{-1} \left( \frac{\phi_{13} + 2n\pi}{2\pi d_{13}/\lambda} \right) \]

where $n = 0$ for the main lobe and only such echoes will be assumed present.
2.7.2 Separation of the Elevation Antennas $d_{12}$

The phase ambiguity considerations again result in

$$\frac{2\pi d_{12}}{\lambda} \left\{ \left( \cos \theta \cos \alpha \right)_{\max} - \left( \cos \theta \cos \alpha \right)_{\min} \right\} \leq 2\pi$$  \hspace{1cm} (2.2)

Only the major lobe will be considered and since all antennas are placed 0.59$\lambda$ above ground (section 3.6.2) the first null in the radiation pattern is at an elevation of $58^\circ$ for the case of a perfect ground. Substitution of the worst case values for $\theta$ and $\alpha$ in (2.2) results in $d \leq 2\lambda$. The degree of coupling between co-linear antennas may be large and such a small separation is unacceptable for resolution purposes. Since the meteor echo heights have a finite distribution we may use this property to eliminate some phase ambiguity on the basis of echo range. Most echoes lie in the height region 80 to 110 km. From fig. 2.4 the additional constraint is

$$\frac{R_{\max}}{\sin \theta_{\max}} \leq \frac{R_{\min}}{\sin \theta_{\min}}$$  \hspace{1cm} (2.3)

Fig. 2.4. Extremes of range and elevation angle for the finite meteor echo height distribution.
Substitution of extreme values of $\theta$ and $\alpha$ now reveals

$$d_{12}(\cos \theta \cos 21^\circ - \cos 58^\circ \cos 0^\circ) \leq 1$$

where

$$\sin \theta = \frac{z_{\min}}{z_{\max}} \cdot \sin 58^\circ$$

hence

$$d_{12} \leq 4.9\lambda.$$

The separation can be further increased only at the expense of the loss of some echoes, since two or more heights within the meteor region may be assigned to a given echo range and phase angles. Ambiguous results will appear initially at the extremes of the echo height distribution resulting in a small decrease in the echo rate. The values of ambiguous direction cosines for $d_{12} > 4.9\lambda$ requires the simultaneous solution of (2.2) and (2.3) over all possible angles and heights. A computer program was written that scanned all possible angles for given antenna separations and as output gave angles that produce ambiguous phases for given heights. A summary of these results is shown in fig. (2.5).

Fig. 2.5. Ambiguous meteor echo regions for various antenna separations.

$$d_{12} = 6.5\lambda(1), 7\lambda(2), 7.5\lambda(3), 8\lambda(4).$$
The results are of a three-dimensional nature so for ease of presentation the worst elevation case for each azimuth has been considered. Shaded regions indicate the regions of ambiguity for the chosen separation of \(7\lambda\), the ambiguities becoming worse for larger elevation angles (where the phase variation is more rapid) and for increasing separations. For the case of \(d_{12} = 7\lambda\) no ambiguities are observed at elevation angles less than \(30^\circ\) and consideration of the vertical radiation pattern maximum (\(25^\circ\)) and the echo rate variation with altitude, (proportional to \(\cos \theta \sin^{-\frac{3}{2}}\theta\) for constant gain, Kaiser, 1953) suggest the system sensitivity is rapidly decreasing above this value.

2.8 ANGLE OF ARRIVAL UNCERTAINTIES AND HEIGHT RESOLUTION

Assuming that phase uncertainties \(d\phi_{12}\) and \(d\phi_{13}\) are the result of Gaussian noise, the resolutions for arrival angles and echo height are derived in Appendix A. For the arrival angles

\[
|d\theta| = \frac{\cos \alpha \, d\phi_{12}}{2\pi d_{12} \sin \theta} + \frac{|\sin \alpha| \, d\phi_{13}}{2\pi d_{13} \sin \theta}
\]

and

\[
|d\alpha| = \frac{\cos \alpha \, d\phi_{13}}{2\pi d_{13} \cos \theta} + \frac{|\sin \alpha| \, d\phi_{12}}{2\pi d_{12} \cos \theta}
\]

where \(d_{12}\) and \(d_{13}\) are in terms of \(\lambda\), resulting in a height resolution

\[
|dz| = dR \sin \theta + \frac{R \cot \theta \cos \alpha}{2\pi d_{12}} \, d\phi_{12} + \frac{R \cot \theta \sin \alpha}{2\pi d_{13}} \, d\phi_{13}
\]

(2.4)

Most of the uncertainty in height is contributed by the second or elevation term in this expression. The requirement for a large antenna separation for the elevation pair is seen to be much more important than for the azimuthal pair. For similar uncertainties the ratio of the contributions
to $dz$ from the two phase terms is $\frac{\cos \alpha}{d_{12}} : \frac{\sin \alpha}{d_{13}}$. The use of an additional elevation antenna at a large spacing (for example $20\lambda$) producing ambiguous but more accurate determinations of $\theta$ is contemplated by Hess and Geller (1976). The contributions from the phase terms in (2.4) both decrease with increasing elevation. This is a direct consequence of the increasing rate of variation of $\phi_{12}$ at higher elevation angles and also leads to the increased phase ambiguities. Although the expressions for the horizontal coordinate resolutions are included in Appendix A, for the scales of motions usually considered in a meteor wind experiment the most important spatial resolution is that of altitude.

2.8.1 Contributions to the Arrival Angle Uncertainties

(a) Noise

The major contribution to the phase uncertainties will be from noise introduced into the system. For radio frequencies near 30 MHz most of the noise contribution is a result of cosmic noise (McKinley, 1961). A phase measurement can be considered as the measurement of a zero crossing position of a signal, as in fig. 2.6a.

---

Fig. 2.6a. Zero crossing of signal plus noise.

b. Signal phasor plus noise.
At the zero crossing the slope of the signal is

\[ \frac{dS}{dt} = \omega S = N/\Delta t \]

The phase uncertainty is \( \Delta \phi = \omega \Delta t = \left( \frac{S}{N} \right)^{-1} \).

Alternatively the noise contribution may be regarded as an additive phasor to the signal (fig. 2.6b) where typically

\[ \Delta \phi \sim \tan^{-1} \frac{N}{S} \]

\[ \sim \left( \frac{S}{N} \right)^{-1} \text{ for large } \left( \frac{S}{N} \right) \]

In the phase detection method used, the phase difference is extracted from two noisy signals. Assuming these are independent the variances may be added to give the phase standard deviation

\[ \sigma_\phi = \sqrt{2} \left( \frac{S}{N} \right)^{-1} \sim d\phi_{12} \]

\[ = 0.14 \text{ rad.} \]

for a typical signal to noise ratio \( \left( \frac{S}{N} \right) \) of 20 db.

(b) **Mutual coupling**

Two or more antennas in close proximity may experience appreciable mutual coupling and the current in each is then the resultant of induced and coupled components. The resultant phase differences may differ from those of the free space waves incident upon them. The equivalent circuit for two coupled antennas in free space is shown in fig. 2.7 (Fedor and Plywaski, 1972).
Fig. 2.7. Equivalent circuit for two coupled antennas. $E_1$, $E_2$ are the signals applied to the antennas, $Z_{11}$, $Z_{22}$ the antenna self impedances, $Z_{L1}$, $Z_{L2}$ the load impedances and $Z_{12}$ the mutual impedance.

Considering these to be the 12 antenna pair,

$$E_1 = I_1(Z_{11} + Z_{L1}) + I_2 Z_{12}$$

$$= E_0(t) \exp(j(\omega t - \phi_{12}/2)) \text{ the free space signal.}$$

($\phi_{12}$ is the angle of arrival phase difference and is not to be confused with the phase angles $\alpha_{11}$, $\alpha_{12}$ associated with the complex impedances $Z_{11}$, $Z_{12}$). Similarly

$$E_2 = I_1 Z_{12} + I_2(Z_{22} + Z_{L2})$$

$$= E_0(t) \exp(j(\omega t + \phi_{12}/2))$$

Neglecting the time variation and setting $Z_{11} = Z_{22}$, $Z_{L1} = Z_{L2}$ and solving for $I_1$, $I_2$
\[ I_1 = \frac{E_0}{(Z_{11} + Z_{L1})^2} \left[ |Z_{11}| \exp j(\alpha_{11} + \frac{\phi_{12}}{2}) + |Z_{12}| \exp j(\alpha_{12} \pm \frac{\phi_{12}}{2}) \right] \]

\[ I_2 = \frac{E_0}{(Z_{11} + Z_{L1})^2} \left[ |Z_{11}| \exp j(\alpha_{11} - \frac{\phi_{12}}{2}) + |Z_{12}| \exp j(\alpha_{12} \pm \frac{\phi_{12}}{2}) \right] \]

provided \( |Z_{12}|^2 < |Z_{11}|^2 \).

Clearly the relative phase difference between the two currents is preserved only when \( Z_{12} = 0 \). Consider the resultant current phasor for each antenna as shown for antenna 1 in fig. 2.8.

![Fig. 2.8. Resultant current phasor in antenna 1.](image)

We may rotate \( |Z_{11}| \) by an arbitrary amount to account for \( \alpha_{11} \). Since in general \( Z_{12} \) is unknown, consider the worst case resultant phasor when \( Z_{12} \) is perpendicular to \( Z_{11} \). Then

\[ \epsilon < \tan^{-1} \frac{|Z_{12}|}{|Z_{11}|} \]

for each resultant in the case of identical antennas. Provided \( |Z_{12}| \lesssim 0.2|Z_{11}| \), the extreme phase difference uncertainty between two antennas as a result of mutual coupling will be
\[ \Delta \phi \leq 2 \frac{|z_{12}|}{|z_{11}|} \quad \text{or} \quad \Delta \phi_{\text{rms}} \leq \sqrt{2} \frac{|z_{12}|}{|z_{11}|} \]

Antenna measurements (section 3.6.5) show that for the close spaced 1,3 pair

\[ |z_{12}| \leq 0.08 |z_{11}| \]

and for the 1,2 pair

\[ |z_{12}| \leq 0.02 |z_{11}| \]

both of which are less than the noise contribution.

(c) The reflecting ground

The receiving antennas may also contribute additional uncertainty to the phase resolution depending on the nature of the reflecting ground beneath them. In the absence of a ground (and any coupling effects) the phase differences in the resultant currents are precisely those of the arriving sky waves. In the general case, the ground may be represented by a complex reflection coefficient which may be a function of the spatial coordinates,

\[ g(x,y) = |R(x,y)| \exp j\theta_R(x,y) \]

where \( \theta_R \) includes the \( \pi \) phase change necessary for the continuity of the electric field vector. Neglecting the time variation, the signals in antennas 1 and 2 whose amplitude gains are \( A_1 \) and \( A_2 \) will be

\[ E_1 = A_1 [\exp j\phi_{12} + R(x_1, y_1) \exp j(\theta_R(x_1, y_1) + 3\phi_{12})] \]

\[ E_2 = A_2 [1 + R(x_2, y_2) \exp j(\theta_R(x_2, y_2) + 2\phi_{12})] \]
and relative phases are preserved provided

\[ g(x_1, y_1) = g(x_2, y_2) \]

These signals may be represented vectorially as in fig. 2.9.

![Diagram](https://via.placeholder.com/150)

Fig. 2.9. Resultant signals for direct wave and ground reflection in two antennas.

The antenna and ground are seen to function as an additive interferometer so the situation is similar to the phase detection method employed at Adelaide. Relative phase is preserved only if the magnitudes and phases of the ground reflection coefficients are equal for all antennas. Note that the relative phase is preserved regardless of antenna pattern amplitude variations since these result in alterations to both the direct and reflected signal vector lengths. In general the exact nature of the electrical ground and its variations are unknown, but any differences must be expected to contribute further uncertainty to the determination of arrival angles.
(d) **Refractive effects**

Refraction of the outgoing and reflected waves during their passage through the troposphere and lower ionosphere may introduce systematic deviations into the derived elevation angles and echo heights. The variation from unity of the refractive index is generally less than 1 part in $10^3$ in the troposphere and 2 parts in $10^2$ in the ionospheric E region, for typical meteor radar frequencies. Both cases have been considered by Revah (1968) who concludes the elevation angle uncertainty in both cases is less than $10^{-3}$ rad. for arrival angles near $\frac{\pi}{4}$ and much less than noise contributions.

2.8.2 **Echo range measurement and uncertainty**

The simplest method of obtaining echo range measures the propagation time of the received pulse, accomplished digitally by running a counter during this time interval. Counting to the leading edge of the received pulse necessitates the use of a time correction to secure the true start of the pulse, whose shape is altered as a result of the finite receiver bandwidth. Since the rise time of the pulse is constant (for constant receiver bandwidth), disabling the counter at a preset pulse amplitude level implies the correction must be dependent on the pulse amplitude and trigger level. An amplitude dependent correction can be avoided by locating a more permanent feature of the pulse such as its position of maximum amplitude or the pulse centre. The first could be obtained by differentiating the pulse and locating the zero crossing. The adopted method locates the pulse centre by counting to the leading edge of the pulse, then halving the counting rate and continuing the count until the trailing edge is reached. The time delay correction is then simply one half the transmitted pulse width.

The major uncertainty in a measurement of echo range is introduced by noise and the associated uncertainty in the determination of the pulse
envelope position. Any propagation delay associated with either transmitter triggering or signal passage through the receiver is a constant delay that can be removed by calibration (section 4.5). If the pulse rise time is $T_R$, then from fig. 2.10

![Fig. 2.10. Received pulse envelope plus noise.](image)

$$\frac{T_R}{\Delta t} \sim \frac{S}{N}$$

and $$T_R = \frac{1}{2B} \text{ (Hz)}$$

where $B$ is the receiver bandwidth. The range uncertainty is

$$\Delta R = \frac{c}{2} \Delta t \quad \text{where } c \text{ is the velocity of light}$$

$$= \frac{c}{4B} \left(\frac{S}{N}\right)^{-1}$$

The present technique measures the position of two such delays and so the range uncertainty is approximately double the above value. For the present equipment the relevant bandwidth is 40 KHz and the range uncertainty is typically 375 m for a signal to noise ratio of 20 db.
2.8.3 Numerical values

From section 2.8.1 the phase uncertainty due to quantifiable effects may be expressed as

\[ d\phi_{12} = \sqrt{2} \left( \frac{S}{N} \right)^{-1} + \sqrt{2} \left| \frac{Z_{12}}{Z_{11}} \right| \]

\[ d\phi_{13} = \sqrt{2} \left( \frac{S}{N} \right)^{-1} + \sqrt{2} \left| \frac{Z_{13}}{Z_{11}} \right| \quad \text{(in radians)} \]

and hence the height uncertainty is

\[ dz = \frac{c}{2B} \left( \frac{S}{N} \right)^{-1} \sin \theta + \frac{R \cot \theta \cot \alpha}{2\pi d_{12}} \left( \sqrt{2} \left( \frac{S}{N} \right)^{-1} + \sqrt{2} \left| \frac{Z_{12}}{Z_{11}} \right| \right) \]

\[ + \frac{R \cot \theta \sin \alpha}{2\pi d_{13}} \left( \sqrt{2} \left( \frac{S}{N} \right)^{-1} + \sqrt{2} \left| \frac{Z_{13}}{Z_{11}} \right| \right) \]

Using the following values; \( d_{12} = 7\lambda, d_{13} = 1.4\lambda, B = 40 \text{ kHz}, \left| \frac{Z_{12}}{Z_{11}} \right| = 0.02, \left| \frac{Z_{13}}{Z_{11}} \right| = 0.08 \) and an echo height of 95 km, the anticipated height resolution has been calculated for a variety of situations. Not included however are contributions to the variance from quantization effects such as the period of the range counter or the quantization levels of any analogue to digital processing of the phase detector outputs. The results (fig. 2.11) apply to an azimuthal angle of 11°, the 3 db beamwidth of the antenna system used.

The variance due to random components may be reduced by repeated determinations of the echo range and phase angles. In the present system typical echoes have a signal to noise ratio of approximately 20 db, for which the resolution is 2 to 5 km. The length of the principle Fresnel zone varies from 2 to 2.5 km in these situations.
Fig. 2.11. Echo height resolution.

2.9 RADIAL VELOCITY MEASUREMENTS

Since the range drift of the meteor echo reflection point is less than 100 m for typical echo durations (1 sec.), the radial velocity measurement is considered as a frequency shift or equivalently as changes in the relative echo phase angle. The frequency of the received signal is

\[ f_r = \frac{1 + v_x/c}{1 - v_x/c} f_t \]

for a positive radial velocity toward the observer, where \( f_t \) is the transmitted frequency. Since \( v_x \ll c \) the doppler frequency is
\[ f_d = f_r - f_t \]
\[ = \frac{2v_r}{\lambda} \]

so
\[ v_r = \frac{\lambda}{4\pi} \omega d = \frac{\lambda}{4\pi} \frac{d\phi}{dt} \]  
(2.5)

where \( \phi \) is the relative echo phase angle. A movement of the reflection point by \( \frac{\lambda}{2} \) results in a \( 2\pi \) phase change since the signal travels out to the target and back. A phase detection of the return signal with respect to the transmitter reference \( f_t \) results in an output that is a pulsed sampling of the doppler beat frequency \( f_d \). Phase changes not associated with a uniform radial drift may also result from effects such as Fresnel diffraction fringes during echo formation, radiowave scattering processes or the presence of non-uniform horizontal winds resulting in wind shear. In addition, non-uniform winds may also create multiple specular reflection points some time after train formation which result in echo amplitude fading and non-usable velocity data. Generally, reliable velocities may be obtained from approximately 0.05 to 0.2 seconds after train formation from low electron density columns.

Since a change in phase with time is required to determine \( v_r \), any system will suffer some velocity losses due to short duration echoes. When doppler phase determinations are included with angle of arrival measurements, a minimum of two pulses is required to determine the radial velocity (section 2.4). The rate of information gathering for such a system is large, although a specific fraction of a doppler beat cycle is not needed. The amount of data may be reduced by considering only zero crossings of the doppler waveform. This may be achieved by reducing the pulsed phase detector outputs to binary form and recording the individual pulse levels (Ramsay and Myers, 1968). This process requires the presence of at least a half doppler cycle since the phase information at all points but the zero
crossing is lost, although no economy of data gathering is achieved.

The adopted technique of using a zero crossing scheme that minimizes data gathering and low velocity losses is a variation of that of Nowak (1964). Instead of beating the return frequency with the transmitter frequency, it is compared with the transmitter frequency offset by a small amount $f_0$. A stationary target then has a doppler output frequency $f_0$; the difference of any beat from $f_0$ specifying both the magnitude of the radial velocity and its direction. Meteor wind results with no expected velocity bias (Hess and Geller, 1976) show that the radial velocity distribution and hence the distribution of $f_d$ is approximately Gaussian with a mean near zero. The use of an offset reference frequency translates such a distribution to a mean value near $f_0$, so that most echoes will furnish radial velocity information since the offset doppler half cycles have correspondingly smaller durations. There is still a bias however, since large radial velocities in one direction will now have long doppler periods and suffer from losses associated with short duration echoes. This bias is reduced by considering an additional offset frequency $-f_0$, which also provides a check of the internal consistency of the zero crossing determinations.

The size of the frequency offset must be at least as large as the highest expected doppler frequency to avoid offset doppler frequencies of ambiguous sign. The maximum expected radial velocity is approximately 100 ms$^{-1}$ (Rosenburg, 1968, Hess and Geller, 1976) so $f_0 > 17.6$ Hz. The offset must be as large as possible to minimize the doppler durations, however the upper limit to $f_0$ is set by the sampling theorem

$$f_0 + f_{d \text{ max}} < \frac{\text{P.R.F.}}{2}$$

(where P.R.F. is the pulse sampling rate)

A more practical upper limit is set by the need to retrieve the continuous doppler beat from the pulsed sampling of it by the use of a realizable low
pass filter. The amplitude spectrum of the pulsed sinusoidal doppler beat is given in Appendix B and shown in fig. 2.12.

![Diagram](attachment:image.png)

**Fig. 2.12. Pulsed doppler amplitude spectrum.**

The frequency components are $nw \pm \omega_d$ in general so to filter the offset doppler frequency

$$(f_0 + f_d \text{ max.}) \ll \text{P.R.F.} - (f_0 + f_d \text{ max.}).$$

The effect of the size of $f_0$ on the radial velocity resolution must also be considered. From fig. 2.6a the uncertainty in a single zero crossing position is

$$\Delta t = \frac{1}{2\pi} \left( \frac{S}{N} \right)^{-1}$$

so

$$\frac{df}{f} = \frac{1}{2\pi} \left( \frac{S}{N} \right)^{-1}$$

The output frequency after low pass filtering is

$$f = |f_0 + f_d|$$

and in practice two independent transitions are required to specify a half period, so
\[ df_d = \frac{f}{\pi \left( \frac{S}{N} \right)^{-1}} \sqrt{2} \]

and
\[ dv_r = \frac{\lambda}{2\pi} \left| \frac{2v_r}{\lambda} + f_0 \right| \left( \frac{S}{N} \right)^{-1} \sqrt{2} \]

indicating a low value of \( f_0 \) to minimize the velocity uncertainty since the major contribution is from the second term in this expression.

The offset frequencies chosen are \( \pm 30 \) Hz, resulting in a maximum unambiguous velocity of 170 ms\(^{-1}\) on each offset. After low pass filtering, the available power in the offset doppler beat is \( \frac{T}{T} \) of that in the sampled waveform, resulting in a considerable reduction in the doppler signal strength (Appendix B). The noise power contribution is also greatly reduced however, since the effective noise bandwidth will now be that of the low pass filter used to extract the continuous beat. For the present system, \( \frac{T}{T} = 50 \) and the 3 db bandwidth is reduced by a factor of approximately 250 resulting in a 7 db increase in signal to noise ratio in the ideal case. The expected radial velocity uncertainties using a signal to noise ratio of 27 db are given in table 2.2.

<table>
<thead>
<tr>
<th>( v_r )</th>
<th>( dv_r (+f_0) )</th>
<th>( dv_r (-f_0) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>3.4</td>
<td>3.4</td>
</tr>
<tr>
<td>20</td>
<td>3.8</td>
<td>3.0</td>
</tr>
<tr>
<td>40</td>
<td>4.2</td>
<td>2.6</td>
</tr>
<tr>
<td>60</td>
<td>4.6</td>
<td>2.2</td>
</tr>
<tr>
<td>80</td>
<td>5.0</td>
<td>1.8</td>
</tr>
<tr>
<td>100</td>
<td>5.4</td>
<td>1.4</td>
</tr>
</tbody>
</table>

For negative radial velocities the symmetry of the offsets simply reverses the positive and negative offset values in table 2.2.
2.10 TRANSMITTER PARAMETERS

Having first specified the radio frequency and the pulsed mode of operation, the pulse period or repetition rate and pulse length can be considered. The ratio of these two parameters determines the transmitter duty cycle and hence the available peak power for a given average power dissipation.

(a) Pulse repetition frequency (P.R.F.)

It was shown in section 2.9 that the P.R.F. should be considerably greater than the largest expected offset doppler output of approximately 50 Hz. The P.R.F. however determines the maximum unambiguous echo range that can be determined. Echoes were observed by Wilkinson (1973) with ranges up to 600 km. The P.R.F. chosen for the present equipment is 300 Hz for which the maximum unambiguous range is 500 km. Echoes observed at larger ranges are most likely of the overdense type and unsuitable for radial velocity determinations. The largest expected offset doppler frequency is now separated from the next component in the frequency spectrum by approximately 200 Hz. A high P.R.F. also improves the sampling of the received pulse amplitude variations.

(b) Pulse length

The usual radar criterion of short pulses for range resolution is not applicable here since the transmitted pulse and receiver response are unmatched and the meteor echo is a discrete target, so range may be determined to less than a pulse length. The use of very short pulses also requires wide band receivers, increasing the noise power. The upper limit to the pulse length is set by the transmitter power output capability and the requirement that no significant amplitude or relative echo phase changes occur in this time interval. The largest expected offset doppler frequency
output is 50 Hz and so
\[
\frac{d\phi}{t} \sim 100 \text{ rad sec}^{-1}
\]
For \( d\phi = 0.05 \text{ rad} \), \( t \leq 160 \mu\text{sec} \). The present pulse length is 66 \( \mu\text{sec} \) (equivalent to 9.9 km) resulting in a transmitter duty cycle of 0.02.

2.11 SUMMARY

The aim of the present experiment is to measure the apparent radial velocity component and position of radio meteor echoes, while economizing on the quantity of data recorded per echo. The radial velocity is obtained from the duration of half cycles of the doppler beat frequency, offset by \( \pm 30 \text{ Hz} \) to reduce the losses of low velocity values caused by short duration echoes. Echo position is obtained from the echo range and angles of arrival, the latter determined from the phase differences in three antennas situated at the vertices of a right angled triangle. One antenna serves as the phase reference eliminating all but the angle of arrival phase differences. The echo range is obtained by timing the propagation delay to the centre of the received pulse, eliminating the need for an amplitude dependent correction factor.
CHAPTER 3

SYSTEM DESCRIPTION

INTRODUCTION

The implementation of the principles outlined in chapter 2 for a coherent pulse meteor wind experiment will be discussed. This particular radar system has been constructed at the Rolleston field station (geographic 43°S, 172°E) of the physics department. The design and performance of individual elements, including receivers, their associated control and recording circuits and the transmitting and receiving antennas will be considered.

3.1 THE OVERALL SYSTEM

A schematic block diagram of the meteor radar system is shown in fig. 3.1 with the objectives of measuring echo amplitude, range, doppler beat frequency and relative phase angle differences between spaced antennas. Each receiving antenna has an associated r.f. amplifier and mixer, using a common local oscillator to maintain phase coherence. Subsequent phase detection is performed at the receiver intermediate frequency (i.f.) of 1.62 MHz. This simplifies the phase detectors and reduces r.f. leakage problems associated with direct conversion receivers. For each of the antenna pairs, 1;2 and 1;3, two phase detectors operating in phase and in quadrature produce the sine and cosine outputs necessary to determine $\phi_{12}$ and $\phi_{13}$, with the signal from antenna 1 serving as the phase reference. The i.f. signal is also fed into an amplitude detector whose output is used for echo range measurements and for the initiation of a recording sequence by the meteor echo discriminator.
RECEIVING ANTENNAS.

Fig. 3.1. Meteor wind radar schematic block diagram.

The amplitude and four phase output voltages are selected by track and hold circuits. The pulse amplitudes are multiplexed into a single analogue to digital converter and stored in solid state memories. A count representing the propagation delay for echo range is also stored in its own memory. The 1.62 MHz i.f. signal is introduced into two doppler beat phase detectors, one for each offset reference frequency, which are synthesized from the transmitter 1.62 MHz oscillator by a single sideband technique. After low pass filtering, the durations of consecutive offset beat half cycles obtained from zero crossings are stored in the doppler memory. On completion of the recording format an echo record is punched out on paper tape.
3.2 THE TRANSMITTER

The transmitter has undergone minor modifications since the description of Wilkinson (1973). The low power stages consist of two crystal controlled oscillators whose frequencies are 24.74 MHz and 1.62 MHz, providing reference signals for the receiving system. These are subsequently mixed, gated and amplified. The present doppler measurements require a P.R.F. of 300 sec\textsuperscript{-1} compared with the 150 sec\textsuperscript{-1} used previously, doubling the average power if the 66 \mu sec pulse length were to be retained. The final power amplifier stages are not capable of sustaining this level and so the P.R.F is doubled only during the recording of an echo. The system dead-time due to the punching out of an echo results in a small increase (approximately 20%) in average power if recording and punching out occurs continually. The extra current drawn during echo recording results in a H.T. voltage drop as a result of power supply internal resistance, decreasing the peak power from approximately 40 to 35 kW. Modifications to the transmitter power supplies and overload protection circuits have enabled reliable operation in this mode.

3.3 RECEIVER AMPLITUDE AND PHASE CIRCUITS

3.3.1 R.F. Amplifier, Mixer and Phase Shifters

The r.f. amplifier (fig. 3.2) consists of two identical cascoded pair tuned amplifiers adopted for low noise. The input impedance is approximately 50\Omega and the use of a F.E.T. as the first stage matches the high impedance of the parallel tuned input. The gain is typically 40 db overall and to achieve satisfactory isolation the individual stages are mounted in copper boxes.
Fig. 3.2 R.F. amplifier.

Fig. 3.3 Mixer

Fig. 3.4 Phase shifters

(All capacitors in μf unless otherwise stated.)
The r.f. output is mixed with the 24.74 MHz transmitter oscillator signal in a double balanced modulator (LM 1496) with a conversion gain of 3. (fig. 3.3) An adjustable gate generated in the meteor echo discriminator (section 3.5.2) is used to gate the mixers, providing variable receiver blanking. This signal suppresses the transmitter ground pulse preventing saturation of the filtered doppler outputs and affords some immunity against unwanted echoes such as sporadic E ground reflections and radio aurora.

The phase shifters (fig. 3.4) provide the \( \pi/2 \) phase shift required to produce the quadrature outputs. These are applied to the 1.62 MHz signal input lines of the phase detectors (fig. 3.12) and comprise \( \pm \pi/4 \) RC networks with additional components for impedance matching. The phase shifted outputs have equal gain, required for the phase detectors, when the phase shifts are precisely \( \pm \pi/4 \). It must be remembered the outputs in this case are proportional to \( \cos(\phi+\pi/4) \) and \( \sin(\phi+\pi/4) \).

3.3.2 I.F. amplifier and limiting amplifier

The i.f. amplifier (fig. 3.5) provides additional gain and serves as a buffer to drive the doppler phase detectors, the amplitude detector and the phase detector reference line.

Using one receiving antenna as a phase reference results in a doubling of the logarithmic dynamic range unless limiting action occurs. This is provided by the limiting amplifiers (fig. 3.6) whose maximum output of 300 mV is suitable as a reference signal for the double balanced modulators employed as phase detectors. Two limiting amplifiers are required, one for each pair of phase detectors to ensure both cosine and sine output amplitudes maintain a constant ratio for inputs below the limiting condition. The wideband limiters introduce differential phase changes of the output with respect to the input as limiting action occurs. Extensive
Fig. 3.5 I.F. amplifier

Fig. 3.6 Limiting amplifier.
measurements indicate the range of phase changes is less than 0.1 rad. over a dynamic range of 40 db. Limiting action begins for an r.f. input of approximately 10 µV r.m.s.

3.3.3 Phase Detector, Filter and Video Amplifier

The phase detector, low pass filter and video amplifier are shown in fig. 3.7. The double balanced modulator (LM 1496) essentially performs a multiplication of the signal and reference inputs. The upper sideband (3.24 MHz) is removed by the balanced π section low pass filter. This filter determines the video bandwidth and hence the overall synchronous receiver bandwidth. Since the noise power is proportional to bandwidth and the power spectrum of a rectangular r.f. pulse train has a \( \left| \frac{\sin x}{x} \right|^2 \) form (Appendix B), the resultant signal to noise ratio varies with receiver bandwidth. Lawson and Uhlenbeck (1950) show that the signal to noise ratio is a maximum when \( BT \sim 1.2 \), that is when the bandwidth is 18 kHz for a pulse length of 66 µsec. However, this does not take account of the pulse shape and a bandwidth as narrow as this has no portion of the received pulse with slowly varying amplitude (compared with the pulse length). The angle of arrival phase angles are obtained from the ratio of the sine and cosine phase detector outputs. For the amplitude contributions to cancel, the received pulses must be sampled at identical points, preferably near the pulse centre. Since track and hold circuits must simultaneously obtain all phase detector output signals, a bandwidth greater than the optimum is required, in order to reduce errors due to differing propagation delays through the phase detectors or timing differences in the track and hold circuits. Therefore, the bandwidth used is 25 kHz (at 3 db down) for the phase detectors, for which the decrease of the signal to noise ratio from optimum is negligible.
Fig. 3.7 Phase detector, filter and video amplifier.

Fig. 3.8 Phase detector response.
To be reliable, the phase detector output signals must also have identical amplitude factors. Inspection of fig. 3.12 however reveals that identical gains need only be preserved from the phase shifters and phase detectors onward, hence these are provided with a gain control.

The video amplifier (fig. 3.7) has a gain of 1.5 and acts as a buffer, the LM308 being chosen for its low d.c. drift characteristic. The bi-polar output has a d.c. offset control since any offset will produce spurious phase results. The stability of the amplifier is such that during a twelve day observation period, the drift was less than 20 mv on all four phase detectors.

The response of a pair of phase detectors may be checked under c.w. conditions by applying the outputs to an XY recorder creating a direct polar output. Using this method both sets of detectors were found to produce non-linear outputs, the locus of the cosine and sine components exhibiting considerable curvature for low signal inputs. This is equivalent to a varying phase angle with signal amplitude and probably results from carrier breakthrough on the phase detector signal inputs. The limiting action of the reference channel is delayed by the inclusion of a series resistor in the i.f. amplifier, alleviating this problem, resulting in the phase behaviour shown in fig. 3.8, and a slightly increased dynamic range. A large difference between either input to the phase detector pairs will produce this non-linearity, emphasising the need for similar receiving antennas in this particular configuration. The uniformity of the phase angle output in fig. 3.8 also shows the limiting amplifier differential phase changes to be negligible.

3.3.4 Amplitude Detector and Video Amplifier

The amplitude response of the phase detector outputs is a function of the limiting amplifier behaviour, so a separate asynchronous detector (fig. 3.9) is required. Inclusion of the diodes in the feedback loop
Fig. 3.9 Amplitude detector.

Fig. 3.10 Video amplifier.
reduces the linearity threshold to approximately 5 mv, giving the detector a dynamic range in excess of 60 db.

The associated video amplifier (fig. 3.10) produces an output offset by -5v to ensure similarity with the phase outputs presented to the common analogue to digital (A/D) convertor. Drift of the video amplifier can be up to 120 mv over long periods but this is generally less than noise.

In addition to optimum signal to noise ratio and pulse shape considerations, the range uncertainty due to noise is

$$\Delta R = \frac{C}{S} \left\langle \frac{S}{N} \right\rangle^{-1}$$

and the noise power is

$$P_N = kTB$$ where \(k\) is Boltzmann's constant and \(T\) the noise temperature, hence

$$\Delta R \propto B^{-\frac{k}{2}}$$

supporting a larger than optimum bandwidth. The amplitude channel bandwidth is 40 kHz \((BT \sim 2)\) producing an output whose rise time is approximately 12.5 \(\mu\)sec and so the pulse shape has a broad maximum amplitude region.

For a noise temperature of \(10^5 \text{°K}\) (McKinley, 1961) this bandwidth results in a noise input voltage of 1.7 \(\mu\)V in a 50Ω circuit. The maximum signal expected can be estimated using equation (5.4). Including a typical echo range of 220 km and representative transmission parameters

$$\nu_{\text{max}} \sim 95|q_{\text{max}}|$$

where \(q_{\text{max}}\) is the maximum reflection coefficient and is approximately 2, although the majority of echoes observed will be from trains with electron
line densities less than $10^{16} \text{m}^{-1}$. This suggests a dynamic range of approximately 40 db is sufficient for the majority of meteor echoes. Although linearity in excess of 60 db is available from the detector, the useful linear range is determined by the receiver as a whole, the upper limit resulting from saturation of the i.f. amplifier. The receiver response is shown in fig. 3.11 where the useful dynamic range is approximately 37 db. In practice this range encompasses most observed echoes.

![Fig. 3.11. Receiver amplitude response.](image)

### 3.3.5 Phase Signals in the Receivers

The working layout of the amplitude and phase sections of the receivers is shown in fig. (3.12). Consider the 1;2 antenna pair presenting
Fig. 3.12 Receiver functional layout.
\[ A_1(t)\cos(\omega t + \phi_1) \text{ and } A_2(t)\cos(\omega t + \phi_2 + \Delta\phi_{12}) \]

to the receivers, where \( \omega \) includes any doppler shift and \( \Delta\phi_{12} \) is any differential phase shift. If the pre-phase shift and phase detector gains are \( g_1 \) and \( g_2 \) respectively and the local oscillator \((\omega_0, \phi_0)\), the signals multiplied in the phase detectors are

\[ g_1 A_1(t)\cos[(\omega - \omega_0)t + (\phi_1 - \phi_0)] \times g_2 A_2(t)\cos[(\omega - \omega_0)t + (\phi_2 - \phi_0) + \frac{\pi}{4} + \Delta\phi_{12}] \]

and a similar term with \(-\pi/4\) phase shift for the quadrature phase detector. Any differential phase introduced in the receivers is now included in \( \Delta\phi_{12} \).

After low pass filtering, the outputs from the two phase detectors whose gains are \( g_3 \) and \( g_4 \) are

\[ g_1 g_2 g_3 A_1(t) A_2(t)\cos(\phi_1 - \phi_2 + \frac{\pi}{4} \Delta\phi_{12}) + \Delta A_1 \]
\[ g_1 g_2 g_4 A_1(t) A_2(t)\sin(\phi_1 - \phi_2 + \frac{\pi}{4} \Delta\phi_{12}) + \Delta A_2 \]

In order to find the true angle of arrival phase difference, any d.c. offsets \( \Delta A_1 \) and \( \Delta A_2 \) must first be subtracted. The only gains that need to be kept constant are those associated with phase detection processes since all other amplitude variations are common. The inverse tangent of the ratio of these outputs includes the differential phases through the various stages, which cannot be distinguished and must be removed in a calibration including the entire signal path within the system before the true phase difference is available.
3.4  DOPPLER CIRCUITS

3.4.1  Offset Frequency Generator

Offsetting the local reference 1.62 MHz by ±30 Hz requires the use of single sideband techniques to achieve cancellation of unwanted frequency components. A block diagram of the method is shown in fig. 3.13a and the corresponding circuit in fig. 3.13b. The sine and cosine identities are achieved by ±π/4 phase shifts before being multiplied in the double balanced modulators. By applying the appropriate polarity differential outputs to the two tuned loads, both upper and lower sidebands are recovered separately. In practice complete cancellation of the unwanted sideband does not occur because of circuit asymmetries and the resultant signal contains some 60 Hz amplitude modulation. This is eliminated by hard limiting the signal and subsequently filtering the fundamental component (fig. 3.14). After buffering, this signal is suitable as a reference for the doppler phase detectors.

3.4.2  Low Frequency Oscillator

The frequency offset for the single sideband generator is produced in a 30 Hz Wien bridge oscillator circuit (fig. 3.15). Extra precautions are necessary with the frequency stability of this oscillator since it determines the lower limit of the instrumental accuracy of the radial velocity, the 1.62 MHz being common to both transmitted and detected signals. Output amplitude stability is maintained by negative temperature coefficient resistors in the feedback loop.

3.4.3  Doppler Phase Detector and Low Pass Filter

To retrieve the offset doppler beat, the offset reference and i.f. signals are applied to a double balanced modulator serving as a phase detector (fig. 3.16).
Fig. 3.13a Offset frequency generator block diagram.
Fig. 3.13b Offset frequency generator.
Fig. 3.14 Sideband limiter

Fig. 3.15 Low Frequency oscillator

Fig. 3.16 Doppler phase detector
Fig. 3.18. Doppler filter response.

This output (a pulsed sampling of the offset doppler beat) is applied to an active low pass filter (fig. 3.17) taken from Nowak (1967). In practice two identical filters are cascaded. The response of a single filter and two filters cascaded is shown in fig. 3.18. Also included are the frequencies of typical radial velocities and the frequency components for zero radial velocity. At 270 Hz (PRF-30Hz) the response is reduced by 46 dB for the case of two filters. The low frequency roll-off is a consequence of the input coupling capacitor to the filter. The digital recording technique (section 3.5.7) precludes the recording of frequencies below 9.8 Hz so the response below this value is unimportant.
Fig. 3.17 Low pass filter.

Fig. 3.19 Gated i.f. amplifier.
3.4.4 Gated i.f. Amplifier

Initial results from both doppler channels with the above system showed the doppler signals contained an unacceptable amount of noise and resulted in few echoes producing reliable velocities. Simulations using generated echoes produced reliable results indicating the problem was due to noise rather than instrumental causes. Although the signal to noise ratio for individual pulses is of order 20 dB, since the low pass filter performs as an integrator, the doppler output contains the noise power from the entire period the receiver is gated on.

The noise power can be reduced by a factor of approximately 50 by gating the i.f. input to the phase detector on only during the received pulse, resulting in a signal to noise ratio equivalent to the continuous wave doppler case. This has been implemented (fig. 3.19) by using three monostable multivibrators triggered from the received echo pulse, setting up a gate one interpulse period later and three pulse lengths in duration. The gating signal is applied to a F.E.T. connected to a buffer amplifier and the resultant gated i.f. input is applied to the doppler phase detector, giving reliable velocities for both offset channels.

3.5 DIGITAL CIRCUITS

The digital circuits provide control for all facets of the data handling and storage. The unit is self contained and all clock frequencies are synthesized from a 1.2 MHz crystal oscillator. On reception of a meteor echo, pre-determined quantities of data are accumulated in solid state memories before being punched out on paper tape. The logical elements used are mainly 74 series T.T.L. apart from the memories which are P.M.O.S. static shift registers. Component identification numbers, for example F1, refer to positions on the relevant circuit board. The construction uses a wire-wrap technique, allowing some design flexibility.
Fig. 3.20 Frequency generator.
3.5.1 Frequency Generator

The square wave output from the 1.2 MHz oscillator (fig. 3.20) is divided to maintain synchronization and provides signals at 1.2 MHz and 600 kHz for the range counter, 5 kHz for the doppler counter, 300 Hz and 150 Hz for the transmitter P.R.F. and 1 Hz for the date/time clock.

3.5.2 Meteor Echo Discriminator

In order to restrict the data recorded, the system remains inactive and in a reset condition until triggered by the recognition of a meteor echo by the discriminator (fig. 3.21). Return pulses must have

1) sufficient amplitude above the noise level
2) sufficiently large pulse width
3) approximately the same range (delay) as the previous received pulse which satisfied both 1) and 2).

The comparator verifies pulse amplitude, while the presence of an output exceeding the 50 μsec delay (F1) results in the identification of a pulse of sufficient duration. In addition, the discriminator is inhibited by the transmitter suppression (H1) and range gate (H2), the latter to avoid echoes with ranges greater than the desired maximum of 500 km (≥ 300 Hz P.R.F.). These signals are both generated from the P.R.F. signal and are combined (E34) to produce the range window applied as receiver blanking to the mixers (fig. 3.23).

The delay criterion is checked by three monostables (A4, F2, H3) producing a gate one interpulse period later and so two received pulses (at 150 Hz) are required to verify an echo. The discriminator is also inhibited at this point (E22) during the punching out of a previous echo. Once the echo is recognized, the single pulse output enables a record sequence and doubles the P.R.F. for a period of approximately 0.8 sec. (F3), the length of the longest reliable doppler count (section 3.5.7). The
Fig. 3.21 Meteor echo discriminator.
Fig. 3.22 Echo amplitude comparator

Fig. 3.23 Receiver suppression

Fig. 3.24 Transmitter line driver
P.R.F. signal is taken to the transmitter via the line driver (fig. 3.24).

The pulse amplitude comparator (fig. 3.22) has a variable trigger level to cope with the variability of ambient noise levels and is compatible with the -5v video amplifier offset. A similar comparator is used for ranging and also used to trigger the gated doppler i.f. amplifier and the track and hold circuits. The use of two comparators enables the pulse sampling level (that is, the track and hold level) to be set independently of the echo recognition or discriminator level. In this way an echo need not be recorded unless it is well above the noise but the recording of the pulse amplitudes may continue down to a lower level above the noise.

3.5.3 Range Measurement Circuit

Since the expected range uncertainty due to noise is approximately 375 m (section 2.8.2) the timing intervals for any count must be less than 2.5 μsec. The clock rate is to be halved during the received pulse interval, so the quantization for each transition and its appropriate clock rate must be considered. For the two transitions, at the leading and trailing edges of the received pulse, 2.5 μsec corresponds to a frequency of 1.2 MHz (125m) halving to 500 kHz (250 m) during the pulse. For a maximum range of 500 km the usual 8 bit word (for punched papertape) would be inadequate, since the corresponding range increment is approximately 2 km. The 1.2 MHz clock rate gives a maximum count of 4000 and so the range values are 12 bit words.

In the range measuring circuit (fig. 3.25), the recognition of an echo resets B6 and enables range values to be read in. Counting at 1.2 MHz is initiated from the P.R.F. signal (A11) and continues until the leading edge of the pulse (falling within the range window, C7) exceeds the comparator threshold (C10). The rate is then halved (A10) and terminated at the pulse trailing edge, resetting A11. After a 20 μsec delay (A7,A8)
Fig. 3.25 Range measurement circuit.
Fig. 3.26 Range memory
the 12 bit count is clocked into the range shift registers by A7, C74. The 12 bit counter (A9,B9,C9) is then reset (B7) and the counter C8 is incremented, determining the number of range values obtained. On acquiring 16 range values All2 inhibits further reading and changes the state of C73 and C74 so that future clocking of the range memory will come from the punch-out routine.

3.5.4 Range Memory

The range counts are stored as 12 bit words in 16 word P.M.O.S. shift registers (MM 5040) and are clocked from the range circuit. On output however they each consist of two six bit segments (fig. 3.26) compatible with the paper tape format.

3.5.5 Track and Hold Circuit

The collection of individual pulse amplitudes is controlled from the received pulse resulting in one set of results per inter-pulse period. This technique reduces the A/D speed requirement but only a single echo at a time may be recorded. Track and hold amplifiers are necessary, enabling the simultaneous sampling of the phase and amplitude channels by a single A/D convertor.

The track and hold circuit (fig. 3.27) is based on RCA application note ICAN-6668. The CA 3080A maximum differential input is ±5 v. When the voltage is held for a period greater than the duration of the applied pulse the differential input, due to feedback, may exceed 5 v and the output falls to this value. The inclusion of a diode on the input sustains the input and the differential voltage drop introduced by the diode is eliminated by placing a similar diode in the feedback loop. The output voltage decay (on the hold mode) is approximately 20 mv per 100 μsec, which is satisfactory in terms of the A/D conversion time of 20 μsec and the voltage levels for each increment (bit size) of 39.2 mv for amplitude and 19.6 mv for phases.
Fig. 3.27 Track and hold circuit.
3.5.6 A/D Control

A single echo pulse produces five output pulse levels to be recorded; four associated with the two phase angles, and the echo amplitude. For the first fifteen pulses all five levels are recorded, followed by a further 125 values of the echo amplitude only. The A/D control circuit is shown in fig. 3.28. A record sequence is initiated by the discriminator and further recording inhibited when the 200 word memory is filled. D5₂ gates a pulse formed from the range comparator and delayed (A1) to the following two monostables (B6,C6) acting as pulse formers. The gate and delay (3 msec) on the input of A1 prevent multiple triggering, so only the echo at the shortest range is recorded. The output pulse from B6 causes D3 to set the track and hold circuits to the hold mode and a conversion is then initiated by C6. The A/D status level indicates when a conversion is taking place and the trailing edge of this signal is converted to a pulse by D6. This pulse is used to increment the 200 word counter (B2,B3). For the first fifteen received pulses, B5₄ is enabled by C3 and C4₂ and the output from D6 ('end of conversion') increments the analogue multiplexer address (D4) and initiates the next conversion via D5₁, until five conversions have been performed for each received pulse. The five word counter output (B5₃) then causes the track and hold amplifiers to be released (D2₁, D3). After fifteen such cycles B5₄ is inhibited, the multiplexer address is fixed at the amplitude input and only a single conversion per received pulse via D5₂ is allowed. Amplitude values are recorded up to a maximum count of 200 (B2,B3) when any further conversion is inhibited until the reception of the next satisfactory echo. The amplitude memories are not shown, but are 200 word static shift registers (MM 5053) and are similar to the doppler memories (fig. 3.30).
Fig. 3.28 A/D control
Analogue to Digital Convertor

The Analog Devices ADC 10Z is a 10 bit successive approximation device with 20 \( \mu \)sec conversion time. The phase detector and amplitude pulses are presented to the common A/D convertor whose input voltage range is determined by external components. Only the lowest 8 bits are used, the maximum inputs corresponding to \( \pm 5 \) v for the amplitude and \( \pm 2.5 \) v for the phase inputs. The choice of the input signal range does not affect the resolution of the device but must be compatible with preceding amplifier slew rates, video amplifier drift and the voltage decay of the track and hold amplifiers. The A/D outputs also drive 8 l.e.d.s to monitor performance.

3.5.7 Doppler Digitizing Circuit

The doppler beat is recorded in the form of half cycle periods of the filtered continuous offset frequency. The half cycle periods are determined in the zero crossing detector (fig. 3.29, Lenk, 1973). Choosing identical reference voltages (specifying the level at which a zero crossing is considered to have occurred), results in no hysteresis, determining the true half cycle periods. The output pulse duration is

\[
\Delta t = \frac{V_{\text{ref}}}{2\pi f \times \text{amplitude}}
\]

\( \approx 100 \mu \text{sec} \) for typical values and is much smaller than typical half cycle periods.

The zero crossing pulses are shaped in F4 (fig. 3.31) and initiate counting by the 8 bit counter C2,D2. The largest expected radial velocity is approximately 100 m s\(^{-1}\), for which the two offset beat frequencies are 47.6 Hz and 12.4 Hz. The maximum permissible count is 255 bits and the clock rate corresponding to this value is 6.3 kHz for a half period of 12.4 Hz. Although a larger count decreases the quantization error, the
Fig. 3.29 Zero-crossing detector

Fig. 3.30 Doppler memory
Fig. 3.31 Doppler digitizing circuit
the clock frequency used is 5 kHz giving a maximum radial velocity on both offsets of 115 ms\(^{-1}\).

Three delays (C5, D5 and E5) control the reading and resetting of the counter, while the reading and punch control of the memories is achieved in a similar manner to the range circuit (B2, B3, B6, C3, D3). Two identical doppler digitizing circuits are required, one for each offset. Two bits of the doppler memory are shown in fig. 3.30. The maximum duration of 8 cycles at the lowest unambiguous frequency of 9.8 Hz, is 0.82 sec, thus setting the maximum period for which the P.R.F. need be doubled.

### 3.5.8 Date/Time Registers and Multiplexing

For the investigation of meteor winds, which are a time dependent phenomenon, local time is available, obtained by successive division of the 1 Hz clock (fig. 3.32). Time may be reset manually, or is reset automatically by turning the equipment on. The various time element outputs from the dividers are multiplexed to give four 8 bit words with the following coding.

<table>
<thead>
<tr>
<th>word</th>
<th>bit 7</th>
<th>bit 6</th>
<th>bit 5</th>
<th>bit 4</th>
<th>bit 3</th>
<th>bit 2</th>
<th>bit 1</th>
<th>bit 0</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>30s</td>
<td>20s</td>
<td>10s</td>
<td>8s</td>
<td>4s</td>
<td>2s</td>
<td>1s</td>
<td>N.C.</td>
</tr>
<tr>
<td>2</td>
<td>1 hour</td>
<td>30 min</td>
<td>20</td>
<td>10</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1 min</td>
</tr>
<tr>
<td>3</td>
<td>8</td>
<td>4</td>
<td>2</td>
<td>1 day</td>
<td>12 hour</td>
<td>6</td>
<td>4</td>
<td>2</td>
</tr>
<tr>
<td>4</td>
<td>800 day</td>
<td>400</td>
<td>200</td>
<td>100</td>
<td>80</td>
<td>40</td>
<td>20</td>
<td>10</td>
</tr>
</tbody>
</table>

Bit 0 of word 1 is retained as a user option and has been used to indicate whether the antenna direction is south or west, or to indicate the injection of a calibration echo.
Fig. 3.32 Local time clock.
3.5.9 End of Echo Detector

If an echo does not persist for the full duration of the data collection interval, the memories will only be filled up to the corresponding location and when punched out, the echoes will have random starts. A re-triggerable monostable with duration greater than the interpulse period (fig. 3.33) serves as an 'end of echo' detector once the recording sequence is initiated. The premature end of an echo (below the range comparator threshold) causes all remaining memory locations to be filled with known physically unreal values. For example, the doppler memories contain a count of 33, equivalent to a doppler frequency of 150 Hz. This enables the end of an echo to be readily found and the beginning of each echo is then at the front of the record.

3.5.10 Output Multiplexing

As each memory is filled (the doppler clock-in rate depends on the doppler frequency and so is variable) a signal indicating completion of the echo recording is produced. The punch out sequence is controlled by the multiplexer (fig. 3.35) and punch driver logic (fig. 3.34). A delay of approximately 2 s allows the punch to obtain a speed sufficient for 100 characters per second. The punch timing is synchronized with the punch relay drivers. The memories are then clocked out via the appropriate register control, for example C7 for the range circuit. The counters relevant to the number of values in each set of memories are incremented on receipt of a character punched command from the punch, while the data are multiplexed through H2 to H5 and F2 to F5, being addressed by H1. To complete an echo a portion of blank tape is punched and the entire system reset, the process taking approximately 4 to 5 seconds. This dead-time reduces the re-triggering of the system by some persistent echoes whose velocity data is unreliable due to echo fading.
Fig. 3.33 End of echo detector.

Fig. 3.34 Punch driver logic.
Fig. 3.35 Output multiplexing
Fig. 3.36a Amplitude and phase circuits

Fig. 3.36b Doppler circuits
Fig. 3.37 Single receiving Yagi antenna

Fig. 3.38 Meteor echo receiving apparatus
The above system is self-contained and includes its own power supplies. Additional test equipment has also been constructed to enable calibrations of both the linear and logic sections. Many test points are included and manual controls allow several of the data gathering sequences to be overridden. Views of receiving equipment are shown in fig. 3.36a and b, showing the construction of the individual elements. In fig. 3.38 the complete system is shown in typical operation. The four racks contain (from top to bottom) the linear receiving circuits (figs 3.36a and b), the logic circuits, the punch drivers and the power supplies.

3.6 **ANTENNAS**

The characteristics of both transmitting and receiving antennas affect many aspects of a meteor wind experiment that utilizes spaced antennas to measure echo arrival angles. The size of the effective pattern main lobe dictates the separation of the azimuthal receiving antenna pair. Further, since only the main lobe is considered for analysis and reduction purposes minor lobes must be sufficiently suppressed. Tailoring of the transmitting array may relieve some of these criteria, however the transmitting and receiving arrays must have a similar main lobe structure if the effective gain is to be kept large. Although the number of underdense echoes observed is largely independent of antenna gain (for $S \sim 2$, Kaiser, 1953), higher gain implies a narrow main lobe and reduces the detectable electron density threshold, increasing the rate of echoes suitable for reliable wind determinations.

The receiving antennas must be of a physical construction that allows close spacing of the phase centres of the azimuthal pair without introducing appreciable mutual coupling (precluding the use of large non-resonant antennas). The use of three similar receiving antennas eliminates the need for precise location of the phase centres. The number of echoes
observed with a system with constant gain in the vertical plane varies as \( \cos \theta \sin^{-1} \theta \) for \( \theta \geq 10^\circ \) (Kaiser, 1953), since a larger volume of the meteor region is covered for lower wave angles \( \theta \). The vertical radiation pattern is determined by the antenna and its height above a reflecting ground, large wave angles having the following implications in the present context: increased elevation angle accuracy, increased elevation phase ambiguities, decreased echo rate and decreased doppler shift for horizontal drifts. For ease of matching the similar receiving antennas to their respective receivers, a simple feeding arrangement is desirable. The above criteria are most easily met by a horizontally polarized Yagi-Uda or Yagi array.

3.6.1 The Yagi Antenna

If a conductor is placed in close proximity to a radiating element a current will be induced and give rise to subsequent radiation from the conductor. The magnitude and phase of this radiated field and hence its effect on the total radiated field depends on the conductor's length, diameter and position. If the length of the parasitic element is \( \leq 0.45 \lambda \) the phase is such that the radiation is increased in the forward direction by this director, while longer elements act as reflectors. A Yagi array usually consists of a single driven element, one reflector and several directors.

Theoretical predictions of the behaviour of Yagi arrays require a knowledge of the current distributions on all elements and hence expressions for the mutual impedances. Walkinshaw (1946) treated Yagis with up to four directors assuming a sinusoidal current distribution on each element. A more general trigonometric expression for the current distribution has been used by King et al. (1968) who considered arrays with up to 10 directors, with special attention directed to conditions that yield a maximum forward gain or maximum front to back ratio. For more specific cases a numerical
technique has been presented by Thiele (1969). In this method the current
distribution on each element is represented by a Fourier series expansion
of even terms and each element is divided into a large number of components.
The integral equation for the electric field vector is then required to be satisfied at all points along the element axis resulting in a set of algebraic equations amenable to numerical solution. The variation in the resultant radiated field may be considered by constructing the vector sum of the individual element currents, allowing for the phase delays between adjacent elements.

Experimental approaches such as those of Fishenden and Wiblin (1948)
have also contributed to the general properties of Yagi antennas. These may be summarized; 1) although overall gain increases with the number of directors, the gain per element decreases, 2) little improvement results from the use of more than one reflector, 3) for director lengths $\geq 0.43\lambda$ there is a sharp decrease in both gain and front to back ratio, 4) a reduction in side lobe levels can be achieved with director lengths tapered towards the front of the array.

3.6.2 The Receiving Antenna

The optimum eight element Yagi considered by Thiele had the following parameters.

- director length $0.4275\lambda$
- director spacing $0.34\lambda$
- reflector length $0.5\lambda$
- reflector spacing $0.125\lambda$
- excitor length $0.47\lambda$
- element diameter $0.003\lambda$

Initially a single antenna with these dimensions was constructed, the elements consisting of one inch diameter ($0.002\lambda$) dural tubing (fig. 3.37).

The most important parameters for a meteor wind experiment, main lobe width and minor lobe levels relate to the radiation pattern. The method of horizontal pattern measurement is described in appendix C. To be representative of the far field pattern, the distance $d$ to the test dipole
must be (Blake, 1966)

\[ d > \frac{(A+a)^2}{\lambda} > 80 \text{m}. \]

where \( A \) is the largest dimension of the array \((2.17\lambda)\) and \( a \) is the dimension of the test antenna. The radiation pattern was initially measured from a radial distance of 120 m and found to be in good agreement with the calculations of Thiele (fig. 3.39). The main lobe has a total 3 db width of approximately 32° while the first nulls occur at ±42°. Subsequent measurement of all receiving antennas from a distance of 1.5 km yielded similar results.

The performance of individual Yagis may be checked by altering the position of the front director, since the current in this element is at the tip of the field vector sum. Positional changes and hence shortening and/or rotation of the vector will alter the resultant field and cause variations in the front to back ratio, indicating the optimum position. Results in table 3.1 show that the optimum front to back ratio of -18 db occurs at the predicted spacing of 0.34\( \lambda \) for the front director.

Table 3.1. Optimum front director position

<table>
<thead>
<tr>
<th>Front director spacing (( \lambda ))</th>
<th>Front to back ratio (db)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.29</td>
<td>-15</td>
</tr>
<tr>
<td>0.34</td>
<td>-18</td>
</tr>
<tr>
<td>0.39</td>
<td>-13</td>
</tr>
<tr>
<td>0.44</td>
<td>-10</td>
</tr>
<tr>
<td>0.49</td>
<td>-9</td>
</tr>
</tbody>
</table>

The height above ground of the antennas was dictated by the availability of materials for the array support members, 6 cm outside
diameter water pipe of length 22 feet (0.59λ). For a perfectly conducting ground, interference between direct and reflected waves produces a beam in the vertical plane with maximum at 25° and first null at 58°. No measurement of the vertical polar diagram has been attempted although some properties may be inferred from the meteor echo statistics presented in chapter 7.

3.6.3 The Transmitting Antenna

For a given number of elements and total length, the main lobe width of a single Yagi can only be decreased at the expense of increased minor lobe levels. The main lobe width of the receiving antennas (84°) is too wide in terms of the azimuthal interferometer spacing, so the transmitting antenna must be used to decrease the echo gathering range of azimuthal angles for the overall system. For two Yagis placed side by side, the mutual coupling is sufficiently small for separations \( d_t \geq 1.2\lambda \) for the polar diagram to be simply that of a single unit multiplied by the array factor

\[
\cos \left( \frac{\pi d_t}{\lambda} \sin \alpha \right)
\]

allowing for the propagation phase delays due to the separation.

The effective total pattern for meteor echo collection is a combination of both transmitting and receiving arrays. If the radiation pattern for a single Yagi is \( E_R(\alpha) \) then that of the transmitting pair is

\[
E_T(\alpha) = E_R(\alpha) \left| \cos \left( \frac{\pi d_t}{\lambda} \sin \alpha \right) \right| \sqrt{2}
\]

since the power gain is doubled. The effective total radiation pattern is then
Fig. 3.39 Single Yagi horizontal radiation pattern.

Fig. 3.40 Combined and transmitting horizontal antenna patterns.
\[ E(\alpha) = E_R^2(\alpha) \left[ \sqrt{2} \cos \left( \frac{\pi d}{\lambda} \sin \alpha \right) \right] \]

which is a function of the array factor and single Yagi patterns. For separations greater than \( d_t = 1\lambda \), the width of the major lobe will be determined by the array factor (table 3.2).

Table 3.2 Major lobe width.

<table>
<thead>
<tr>
<th>( \frac{d_t}{\lambda} )</th>
<th>1.0</th>
<th>1.2</th>
<th>1.4</th>
<th>1.6</th>
<th>1.8</th>
</tr>
</thead>
<tbody>
<tr>
<td>array factor</td>
<td>30°</td>
<td>25°</td>
<td>21°</td>
<td>18°</td>
<td>16°</td>
</tr>
<tr>
<td>first zero ( \alpha )</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

As well as decreasing the main lobe width, the array factor can be used to reduce effective sidelobe levels by arranging array factor nulls to coincide with pattern maxima. Using experimental radiation patterns for a single Yagi, the expected transmitter and overall radiation patterns were computed for various separations. These show the expected reduction in main lobe width and a subsequent growth of the first sidelobe for \( d > 1.5 \lambda \). A computed radiation pattern for the adopted separation of \( 1.4 \lambda \) is shown in fig. 3.40, with a comparison with the measured pattern for the twin Yagi array. The construction of the transmitting array is identical to the receiving antennas, being also placed 0.59\( \lambda \) above ground.

3.6.4 Antenna Impedance Matching and Feed Arrangements

In order to increase the antenna input impedances the driven elements are folded dipoles of length 0.47\( \lambda \). The typical feed arrangement for a receiving antenna is shown in fig. 3.41a. Open wire lines of 16 SWG copper with three inch polystyrene spacers (impedance ~ 550\( \Omega \)) are used on most sections. The antennas are resonated by the addition of balanced capacitive stubs added to the driven elements, the input impedances of all six receiving antennas being measured as in the range 188 to 205\( \Omega \). The
Fig. 3.4la Receiving antenna feed arrangement.

Fig. 3.4lb Transmitting array feed arrangement.
folded dipole is connected to the open wire line, supported by a shorted
\(\lambda/4\) section, by a \(\lambda/2\) balanced co-axial section. The impedance of the
antenna is presented an integral number of wavelengths along the line, again
supported by a shorted \(\lambda/4\) section and introduced into a \(\lambda/2\) co-axial balun,
resulting in an unbalanced 50\(\Omega\) output, matched to the receivers.

The matching arrangement for the transmitting antennas is shown in
fig. 3.4lb. Again the driven elements are resonated by capacitive stubs
and introduced to the line through balanced \(\lambda/2\) co-axial sections. In
order to match the parallel antennas to the transmitter output impedance of
approximately 600\(\Omega\), a \(\lambda/4\) matching section of twin 1 inch outside diameter
tubes with variable spacing is included, whose impedance is adjusted near
260\(\Omega\), transforming the array impedance of approximately 110\(\Omega\) up to 620\(\Omega\). A
radio frequency ammeter enables the transmitter output power to be monitored,
attached by a balanced \(\lambda/4\) section.

3.6.5 **Mutual Impedance and Coupling**

The mutual impedances between the close spaced antennas have been
estimated using the open circuit-short circuit method. Referring to fig.
2.7, with antenna 2 open circuit, \(I_2 = 0\)

\[
\text{and } Z_{o.c.} = \frac{E_1}{I_1} = Z_{11}, \text{ the self impedance.}
\]

Now with antenna 2 in a short circuit condition

\[
Z_{s.c.} = \frac{E_1}{I_1} = Z_{11} - \frac{Z_{12}^2}{Z_{11}}
\]

Solving for \(Z_{12}\)

\[
|Z_{12}| = \left|Z_{o.c.}(Z_{o.c.}-Z_{s.c.})\right|^{1/2}
\]
An additional or alternative method is required if the sign of \( Z_{12} \) is to be determined (Altshuler, 1960), however only the magnitude is required to estimate the phase uncertainty. Mutual impedances obtained this way are

\[
\frac{|Z_{13}|}{|Z_{11}|} = 0.08 \quad \text{(West directed antennas)}
\]

\[
= 0.07 \quad \text{(South)}
\]

This method is only reliable for \( \frac{|Z_{13}|}{|Z_{11}|} \) large otherwise the term \( Z_{o.c.} - Z_{s.c.} \) is small. The resolution in the above case was approximately 0.02 and a result of this magnitude was obtained for both \( 1;2 \) antenna pairs.

The mutual coupling may also be investigated by driving one antenna and observing the signal induced in any adjacent antenna. The situation is then (fig. 2.7)

\[
Z_{L2} = Z_{RX}, \quad Z_{L1} = Z_{\text{generator}} = Z_g, \quad E_2 = 0.
\]

and the observed signal is \( V = I_2^* Z_{RX} \).

So

\[
E_1 = I_1(Z_{11} + Z_g) + I_2 Z_{12}
\]

\[
0 = I_2(Z_{22} + Z_{RX}) + I_1 Z_{12}
\]

If \( Z_{12} \) is small then \( I_2 << I_1 \) and so

\[
I_1 \sim \frac{E_1}{Z_{11} + Z_g}
\]

and

\[
I_2 \sim \frac{-I_1 Z_{12}}{Z_{22} + Z_{RX}}
\]

Now \( Z_{11} = Z_{22} \) and \( Z_{RX} = Z_g = 50\Omega \),

so

\[
\frac{|V|}{E_1} \sim \frac{Z_{12} Z_{RX}}{(Z_{11} + Z_{RX})^2} \sim 0.6 \frac{|Z_{12}|}{|Z_{11}|}
\]
Using this method the results of table 3.3 were obtained for the south (s) and west (w) antennas.

Table 3.3. Antenna isolation and mutual coupling

<table>
<thead>
<tr>
<th></th>
<th>( S_{12} )</th>
<th>( S_{13} )</th>
<th>( W_{12} )</th>
<th>( W_{13} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \frac{V}{E_1} )</td>
<td>-43 db</td>
<td>-33 db</td>
<td>-42 db</td>
<td>-35 db</td>
</tr>
<tr>
<td>( \frac{Z_{12}}{Z_{11}} )</td>
<td>0.01</td>
<td>0.04</td>
<td>0.01</td>
<td>0.03</td>
</tr>
</tbody>
</table>

Neither method is satisfactory for small mutual impedances and the second method may include any coupling effects between adjacent transmission lines. An outward travelling wave may be induced into the passive line and combine with the signal coupled via the antennas and produce a spurious result. Both methods however yielded results of a similar magnitude and indicate the degree of coupling is small and can be neglected.
CHAPTER 4

SYSTEM OPERATION AND CALIBRATION

INTRODUCTION

A major problem for meteor wind experiments is the calibration of echo location techniques, including the differential phase paths through the equipment. Verification of the operation of the echo location technique will be considered along with the calibration of the receiving equipment. The deduction of meteor echo characteristics from the individual records and the consequences of the particular measurement techniques used in the present equipment are also examined.

4.1 METEOR ECHO FORMAT

Since in this experiment the recording medium is punched paper tape, considerable economy in the gathering of data is necessary. For a medium sensitivity meteor radar most echoes are of the decay type, those whose amplitude rises rapidly to the maximum value and subsequently decays under the action of ambipolar diffusion. Maximum signal to noise ratio usually occurs early in the echo lifetime. The relative phase angles and echo range are not expected to change appreciably throughout the echo lifetime and so are measured during the first 15 and 16 pulses respectively. Although the Fresnel diffraction produces amplitude and phase fluctuations in the initial stages of the echo, these do not affect the arrival angle phases since it is only the phase differences between adjacent antennas that are recorded. If the range and phase variances are a result of Gaussian noise, they are reduced by repeated measurements of these quantities.
The amplitude and phase memory consists of 200 words in total, 60 being allocated to the four phase detector outputs, leaving 140 amplitude words. The durations of decay type echoes depend primarily on the echo altitude. The distribution of echo durations, defined as duration above noise, obtained by Wilkinson (1973) with similar equipment, had a most probable value of 0.2s and few values greater than 0.5s. The 140 amplitude words gives a maximum record length of 0.47s, sufficient for the majority of echoes. In addition, overdense echoes (those that have large electron densities and reflect in a manner analogous to metal cylinders) often exhibit amplitude fading after approximately 0.2s duration (Philips, 1969) and are then unsuitable for radial velocity measurements.

The required number of doppler half cycles is also determined by the expected echo duration. The lowest unambiguous frequency is 9.8 Hz, corresponding to a half cycle period of 0.05s. The modal duration of 0.2s implies at least four values are required. For each offset channel, sixteen doppler half periods are recorded, corresponding to a completed record duration of 0.27s for each offset for a zero radial velocity.

The resulting echo output format on punched paper tape is:

- time/date : 4 words
- phase and amplitude : 75 words
- amplitude : 125 words
- range : 32 words (16 x 12 bit words)
- dopplers : 32 words (two offset channels).

At the end of each record 16 words of blank tape are also provided to separate consecutive echo records and minimize tape reading errors.
4.2 DATA REDUCTION

The paper tape records are reduced in three steps using the University's B6700 computer, the first simply transferring the paper tape images directly onto magnetic tape for ease of handling. This continuous record is then sorted into records of individual meteor echoes, decoded where necessary and again stored on magnetic tape. The computer program (appendix H) searches for a non-zero word considered as the echo start and then completes the record by including the following 267 words. The date/time words and the 12 bit range words are decoded from the 8 bit tape format. After a skip of 8 words from the record end, allowing up to ±8 read errors per record, the process is repeated. Records with read errors are usually distinguished by unreal date/time values.

The compressed (253 word) echo records are then processed to yield the meteor echo parameters. The differential phase paths through the equipment were initially a separate measurement (section 4.4) and the data from these were initially read from cards. Later developments enabled this calibration to be included in the echo reduction computer program (appendix I) which determines echo amplitude, range, doppler and position information.

4.2.1 Amplitude

The parameters determined are; 1) pulse amplitude duration above approximately twice the noise amplitude, 2) maximum amplitude and its time of occurrence, 3) echo rise time, 4) mean signal to noise ratio for the first fifteen pulses, 5) echo decay characteristics. Some care is required in determining the echo decay constant since it is shown in chapter 6 that the exponential slope does not necessarily begin at maximum amplitude, especially if a perpendicularly polarized reflection component is present. To reduce the effect of such systematic decay slope increases the decay is not considered to begin until 25% of the duration from maximum amplitude.
to echo termination has elapsed. The instrumental effect of the transmitter power drop (during P.R.F. doubling) is eliminated by ensuring at least fifteen return pulses precede the start of the decay. The natural logarithms of the successive pulse amplitudes are fitted to a straight line using linear regression, yielding values of the gradient, intercept and the correlation coefficient. The fitting process is carried out only if at least ten points are available, resulting in a minimum amplitude decay length of twenty five pulses. This imposes a ceiling on the heights of echoes whose decay rate may be deduced, since the decay time constant is

\[ T = \frac{\lambda^2}{16\pi^2 D_a} \]

and if two such time constants constitute the echo lifetime of 0.083s (25 pulses), \( D_a \leq 19.7 \text{ m s}^{-1} \). This corresponds to a height of approximately 100 km (Barnes and Pazniokas, 1972), below that due to the finite meteor velocity effect. Although corrections for the transmitter power drop could be made, the time interval where this is taking place is generally accounted for by the echo rise time to maximum amplitude.

Two examples of echo amplitude output both characteristic of decay echoes are shown in natural logarithmic form in fig. 4.1.

![Graph](image)

**Fig. 4.1.** Echo amplitude output
Both echoes rise rapidly to maximum amplitude and subsequently decay. Diffraction fringes are evident on both records early in the echo lifetime. The decay rate for the first example is uniform throughout most of the echo lifetime whereas the second echo initially decays at a faster rate. This indicates the possible presence of plasma resonance (Chapter 5) and the need to begin the decay slope determination some time after maximum amplitude has been reached.

4.2.2 Range

Range values corresponding to end of echo values, or those from low amplitude pulses (less than twice noise) are discarded and the mean and standard deviation of the 1.2 MHz counts is found. If the standard deviation exceeds 1 km then individual values greater than one standard deviation from the mean are eliminated. These values may be due to noise spikes triggering the range comparator resulting in spurious values. Provided the spread in range values is not large the modified mean is found, giving the true radial range once the time delay through the system and half the transmitted pulse width have been subtracted.

4.2.3 Radial Velocity

If the echo does not persist for eight full doppler cycles on both offsets, the memory locations will be completed with end of echo values. The number of reliable doppler counts for each offset is considered to extend up to the final zero crossing prior to the echo amplitude duration. Since the doppler waveform may have an arbitrary initial phase, the initial count is ignored for both channels. This may help to eliminate Fresnel diffraction phase variations whose rate of change (given by the Cornu spiral) is largest in the initial stages. These phase changes are only appreciable before the meteoroid passes the perpendicular point on the train from the common transmitting and receiving site, at which time
the amplitude has risen to half its maximum value. Typical decay echo amplitude rise times are 0.01 to 0.04s (4 to 12 pulses) while a single doppler half cycle duration is 0.017s for zero radial velocity.

If there are less than eight doppler values remaining the echo is of short duration and probably a decay type echo and the radial velocity is obtained from the mean value. If more than eight values remain, the first four are discarded. The long duration above noise level indicates the echo may be from a column with a large electron line density. In this case the deduced radial velocity may be systematically increased (Chapter 6) for up to 0.5s depending on the echo altitude, although the zero crossing method will smooth this variation. Discarding the four values (a duration of 0.07s at 30 Hz) while not eliminating this effect, reduces the size of the error. The doppler filter output is in general asymmetric, so for reliable velocity determinations at least two half cycle durations are required on each offset channel.

4.2.4 Reflection Point Position

For the first fifteen pulses phase angle differences are found for the two arrival angles from the ratios of the sine and cosine output phase detectors. Values from low amplitude pulses are rejected and the arrival angles determined from the mean phase angles. Since the elevation antenna pair separation exceeds 4.9λ there are several possible phase angles separated by 2π. However, usually only a single value will correspond to an elevation angle whose associated height lies within the 80 to 110 km altitude range. The standard deviations of the two phase angles also provide an additional rejection criterion.
In addition to the parameters and associated standard deviations, several error indicators are included in the output from the computer program for future data selection. Long duration overdense echoes ($\geq 5$ s) may re-trigger the system and in general these produce no worthwhile information since severe amplitude fading caused by the formation of multiple reflection points may accompany the echo. These effects may be eliminated by considering the occurrence of an echo within a time interval of one minute at approximately the same range (within 15 km) to be the result of this type of situation. In general the amplitude fading is accompanied by non-representative doppler values. Although the amplitude may only be recorded up to the first fade due to the action of the end of echo detector, the reflection points will have begun interfering before this time and the doppler information will be unreliable. Shearing doppler values may also be contributed by an echo from a trail that has become specular through rotation in the background wind field. A slow echo rise-time ($\geq 0.3$ s) may be due to this effect or attributed to a large electron density column, neither of which are expected to produce true background wind velocities. Simulated data with additive Gaussian noise has been applied to the reduction program yielding results with uncertainties of a similar magnitude to those obtained in the expressions of chapter 2.

4.3 RECEIVER CALIBRATION.

By disabling all transmitter power supplies above the driver stage, sufficient power leaks through to produce approximately 30 µv at the receiver inputs and at this level the transmitter may be gated on for continuous operation. If a signal generator is then introduced as the phase reference (antenna 1), the phase detector outputs are the beats between the two sources. This allows monitoring of the phase detector gains on the
correct frequency and also a check of the bandwidth as the signal generator frequency is swept. For low beat frequencies (≤ 20 Hz) the beat may be sampled and punched out providing a check of the π/2 phase shift, phase detector gains, d.c. offsets and verification of the A/D operation (fig. 4.2).

$$\frac{\text{Gain 1}}{\text{Gain 2}} = 0.977$$

\[\text{d.c. offset 1} = +9 \text{ bits} \]

\[2 = +2 \text{ bits}\]

Fig. 4.2. Sampled beat frequency output from a pair of quadrature phase detectors.

Alternatively the π/2 phase shift may be verified by observing the beat outputs on an XY oscilloscope resulting in a circular locus for the correct phase shift and gains. Phase detector linearity may be checked by
introducing a pulsed r.f. generator into the appropriate two receiver inputs, resulting in a pulsed output dependent on the input amplitudes and relative phases. The oscilloscope amplifier gain controls may then be used to reduce the cosine and sine pulse amplitudes to identical levels and hence check the phase uniformity over the desired input range by varying the input amplitude. The pulse amplitudes may be punched out or alternatively an XY display may be used which, however, is more successful on c.w. operation.

Correct operation of the two doppler channels is verified by again allowing transmitter leakage into the reference receiver at low levels and observing the filter and punched doppler outputs, both of which should be 30 Hz.

4.4 DIFFERENTIAL PHASE CALIBRATION

Before correct phase angle differences relating to arrival angles can be obtained, differential phase paths through the antennas, feeders and receivers must be subtracted. This is achieved by placing a r.f. source on the interferometer baseline axis and measuring the resultant phase detector outputs yielding \( \Delta \phi_{12} \) and \( \Delta \phi_{13} \). At ground level \(( \theta = 0 )\) the phase differences are

\[
\phi_{12} = \frac{2\pi d_{12}}{\lambda} \cos \alpha + \Delta \phi_{12}
\]

and

\[
\phi_{13} = \frac{2\pi d_{13}}{\lambda} \sin \alpha + \Delta \phi_{13}.
\]

Correct operation of the system can be verified, at least at ground level, by checking these dependences.

The phases were measured by using the portable c.w. oscillator (appendix C) and test dipole in the far field of the interferometer,
Fig. 4.3: Relative phase angle output, $\theta_\alpha$. (Numbers refer to calibration transmitter locations.)
given by the condition

\[ d \geq \frac{(A+a)^2}{\lambda} \]

where now \( A \sim 9.5\lambda \) and so \( d \geq 1 \) km. The measurements were performed at distances exceeding 4 km over terrain flat to within \( \pm 0.5\degree \). The phase detector outputs were connected to an XY recorder and produced a d.c. output for the c.w. inputs. Any frequency difference between the oscillator and transmitter reference does not appear as a doppler shift in the present phase detection method, simplifying the test oscillator requirements. Since there is an additional tuned circuit in the i.f. amplifier in the phase reference receiver (compared with receivers 2 and 3 in fig. 3.12), the resultant phase difference is frequency dependent so the test oscillator must maintain frequency stability. The phase dependence on frequency is

\[ \frac{\Delta \phi}{\Delta f} = 0.029 \text{ rad.kHz}^{-1}. \]

and the oscillator frequency dependence on temperature (Appendix C) is such that a phase error of only \( 6 \times 10^{-3} \) rad. occurs for a \( 10\degree \) change in oscillator temperature. By smoothly decreasing the test oscillator power output using a synchronous motor, the phase angle is drawn directly onto the chart, also serving to check the phase detector linearity (fig. 4.3). This method forms the basis of the differential phase calibration used during the data collection.

The results for \( \phi_{12} \) for the west antennas (fig. 4.4a) are shown with the expected azimuthal dependence including the differential phase of 200°. Linear regression of \( \phi_{12} \) on \( \cos \alpha \) results in a slope of 6.96\( \lambda \) compared with the physical spacing of 7\( \lambda \). Regression of \( \phi_{13} \) (fig. 4.4b) on \( \sin \alpha \) places the azimuthal pair 1.37\( \lambda \) apart rather than 1.4\( \lambda \). The agreement between experimental and predicted values indicates that the degree of coupling between the antennas is small enough to be neglected and that the phase centres of the antennas are at similar positions to the physical
Fig. 4.4a \( \phi_{12} \) dependence on azimuth \( \alpha \).

Fig. 4.4b \( \phi_{13} \) dependence on azimuth \( \alpha \).
spacing. Due to the symmetry of the Yagi arrays the phase centre would be expected to lie along the array axis. The effect of differential phase centres on the array axes would result in an increased apparent antenna separation, however this is not observed. Treating positive and negative azimuths separately yields similar results for both phases whereas if coupling was appreciable, the effects may be dependent on the direction of arrival of the incident wave. If the results of fig. 4.4b are translated into true azimuth and measured azimuth after removing the 16° differential phase, the r.m.s. angle error over the main lobe is ±0.7° for the azimuthal angle. These results have been repeated with the phase detector pairs interchanged, and for the south receiving antennas, all yielding similar results.

This type of differential phase calibration requires considerable time and could only be performed twice per day during data collection runs. During a calibration the system is not operational and the resulting chart must be analyzed separately before being applied to the meteor echo data. The calibration process was streamlined by the construction of an echo transponder, which consists of a simple low current receiver, delayed pulse oscillator and C.M.O.S. control logic. A calibration command results in the transmitter P.R.F. being increased briefly to 200 Hz. The transponder discriminator recognizes this P.R.F. and gates on the oscillator for approximately one second and produces a signal which when received produces a simulated echo output. The user option bit (bit 0 of word 1) of the echo record is set to indicate a calibration echo from which the differential phases are obtained in the same manner as the phases from meteor echoes. The time to record a calibration is that for a normal echo and so frequent calibrations are possible.
4.5 DIRECTION FINDER VERIFICATION

Despite the number of meteor wind systems using some form of reflection point location, little information is available on calibration methods used. The most detailed accounts are those of Spizzichino (1972), Barnes (1972) and Robertson et al. (1953). Spizzichino considers the calibration in two sections. Firstly, the differential phases introduced by receiving components are found in a similar method to the above with a source at ground level. Secondly, corrections to the actual arrival angles are determined in a similar manner by suspending the source from a helicopter and traversing the antenna beam. Barnes (1972) outlines several calibration attempts performed at A.F.C.R.L., including r.f. sources carried aloft by balloons, aircraft and a satellite. The situation of the Rolleston field station on the southern approach to Christchurch airport precludes the use of balloons in the interferometer far field. Since the c.w. measurements of Robertson et al. (1953) rely on the beating of the direct and reflected waves, aircraft echoes are suitable for the calibration of their particular system. Aircraft reflections have been observed with the present equipment while c.w. differential phase calibrations were being carried out, producing beats at the receiver outputs. Active source measurements such as these however do not require the system to operate in its intended pulse mode unless the transponder is used.

The use of passive targets requires a suitable scattering cross section, a target beyond the receiver suppression minimum range of 30 km and also an independent position determination. Although it is intended to use a passive satellite as a reflector at Illinois (Hess and Geller, 1976), this was not feasible with the present transmitted power of 40 kW. The most suitable target proved to be a Boeing 737 aircraft, which made a single over-flight per day through a region of the south interferometer at an altitude of approximately 11 km, giving a maximum elevation angle $\theta$ of
21.5°. The aircraft location was established by the Meteorological Service Cossor 353 radar, whose operating parameters are; frequency 2800 MHz, pulse length 1 μs, P.R.F. 466 sec⁻¹, range accuracy ΔR ~ 25m, arrival angle accuracy Δθ, Δα ~ 0.07° for θ ≥ 4°. This unit is situated at Christchurch airport 18 km north-east of Rolleston and records the target position at 30s intervals. Timing was synchronized between both stations and the target coordinates translated to an origin at Rolleston using flat earth geometry.

The first successful run enabled the range delay through the system to be measured as shown in fig. 4.5 (example 1). This provides good agreement with range data for subsequent runs, the r.m.s. error in range being approximately 0.5 km for this data. For the initial run, no differential phase calibration was available, however the expected phase differences derived from the target arrival angles are compared with the recorded phase values in fig. 4.6. The variation with time is similar, the displacement indicating the differential phases are approximately uniform across the area of the beam considered. There is however a slight decrease in Δϕ₁₂ and Δϕ₁₃ as the beam is traversed.

Subsequent runs included a differential phase calibration enabling a comparison of arrival angles, the azimuth being shown in fig. 4.7 (examples 2 and 3). The agreement of both sets of data is good, although there are individual departures up to 1.5°. The elevation angles appear to show systematic differences larger than can be expected by the use of flat earth geometry in example 3 of fig. 4.9. Although the elevation angles presented are not representative of those expected for meteor echoes, correct operation of the interferometer is expected over the entire beam. The elevation angles observed by the Meteorological Service radar are not sufficiently small (≤ 4°) for tropospheric refraction to be a problem.
Fig. 4.5 Echo range comparison

Fig. 4.6 Echo phase angle comparison
Fig. 4.7 Azimuthal angle comparison
Fig. 4.8 Radial velocity comparison

$V_r \text{ (ms}^{-1}\text{)}$

Met. service radar

Positive freq. offset

Negative freq. offset
Fig. 4.9 Elevation angle comparison.

Although the Cossor radar does not give instantaneous radial velocity output, average values may be obtained from the rate of change of the translated range coordinate. The comparison with deduced radial velocities is shown in fig. 4.8 for both offset channels, one of which is expected to become ambiguous at 115 ms\(^{-1}\). The other continues recording up to 160 ms\(^{-1}\) when the echo is lost, this verification also proving useful for the correct assignment of the radial velocity sign or direction. Although \(v_r\) is slightly larger than \(\frac{dR}{dt}\) in example 2, possibly due to the 30 Hz offset, they generally agree to within 10 ms\(^{-1}\).
The reflections from the slowly varying target also reveal the amplitude effect of the transmitter power drop during the P.R.F. doubling. Results indicate the echo amplitude decreases by approximately 8% over a period of 15 pulses (0.05s). The amplitude variations due to noise are usually in the range ±3 bits, corresponding to a r.f. input of approximately 2 µV. These amplitudes also serve as a crude check of the interferometer effective antenna pattern, since the target, whose dimensions are typically several wavelengths is not expected to be especially aspect sensitive.

While these results (examples 2 and 3 of figs 4.7 through 4.9) provide reasonable agreement, they were obtained from echoes whose signal to noise ratios were at best only 20 db. Increased return signals have been observed during non-comparison times indicating that course changes by the aircraft were a major factor affecting signal strength. An approach was made to Christchurch air traffic control, gaining approval for an alternative course with a more favourable position within the interferometer beam. Results from this run are included in figs 4.5 through 4.9 (example 4). Although signal strength increased to approximately 50 µV maximum, the results are similar to previous runs but have extended the elevation angles up to those where large numbers of echoes are observed. A complete check of all look angles with differential phases from the transponder is desirable to provide the direction finder calibration, but requires the extended use of an aircraft for this type of measurement.

4.6 DIFFERENTIAL PHASE VARIATIONS

A 24 hour c.w. differential phase calibration was performed on the west directed antennas with results being punched out at intervals of 1 hour or less. In addition the short term variations in $\Phi_{12}$ were measured by applying the d.c. phase detector outputs to an XY recorder. The area
Fig. 4.10 Receiver differential phase and temperature variations.
covered by the pen during any interval then represents the limits of the phase variation within that interval (fig. 4.10). Also included in fig. 4.10 are the phases $\phi_{12}$ and $\phi_{13}$ from the punched data and the temperature in the vicinity of the receivers. This temperature is well correlated with the differential phase variations, indicating a receiver origin for the phase drifts. The temperature variation is basically diurnal but was varied intentionally after 0800 h. The drift is approximately $0.1 \text{ rad} \ C^{-1}$ and probably originates in the phase reference receiver channel since it is common to both phases. It may be largely caused by the additional i.f. tuned circuit since this will accentuate any changes in the receiver tuning. Although the phases are unaffected by input amplitude changes, any differential gain changes of the phase detectors will also result in an apparent phase change. This effect is unlikely to result in similar variations for the two pairs of phase detectors however. Some of the differences between the chart recorder data and punched data may be a result of d.c. offset changes in the phase detector video amplifiers since the chart records were referred to the true d.c. zero for each example.

In any one hour period the differential phase drift is usually less than 0.05 rad, indicating differential phase calibrations need only be carried out at like intervals. Similar results to the above have also been obtained using the transponder for calibrations.

If the receivers suffer a diurnal temperature variation $\Delta T$ the differential phases will vary accordingly and if not continually monitored, will result in a spurious diurnal variation of the arrival angles. The variation in the elevation angle is (for $\alpha = 0$)

$$\Delta \theta = \frac{\Delta \phi_{12}}{2\pi d \sin \theta}$$

and so
\[
\Delta z = \frac{z_0 \cot \theta \Delta T \times 0.1 \text{ rad} \cdot \theta^{-1}}{2\pi d_{12} \sin \theta} \\
= 1.10 \Delta T (^\circ \text{C}) \text{ km for } \theta = 25^\circ, z_0 = 95 \text{ km.}
\]

Such a diurnal modulation will affect the observation of neutral atmosphere velocities, especially atmospheric tides.

Making use of material presented in chapter 8, a linearly polarized tidal component propagating in the absence of dissipation may have its velocity expressed as

\[
u(t, z) = u_0 \cos(2\pi T - \frac{z}{\lambda z}) \exp(z/2H)
\]

where \( T \) is the tidal period, \( \lambda_z \) the vertical wavelength and \( H \) the atmospheric scale height. The height of observation \( z \) now becomes

\[
z = z_0 + \Delta z \cos\left(\frac{2\pi t}{24} + \phi\right)
\]

introducing a non-linear modulation of the apparent tidal component. This effect has been numerically evaluated for an apparatus temperature variation of \( \pm 5^\circ \text{C} \) and representative tidal parameters; a single propagating diurnal and semi-diurnal components and a mean wind. The results are presented in table 4.1.

Table 4.1. Simulated distortions of tidal components

<table>
<thead>
<tr>
<th>Tidal parameters</th>
<th>Apparent tides</th>
</tr>
</thead>
<tbody>
<tr>
<td>Period(h)</td>
<td>( \lambda_z ) (km)</td>
</tr>
<tr>
<td>d.c.</td>
<td>(-)</td>
</tr>
<tr>
<td>24</td>
<td>26</td>
</tr>
<tr>
<td>12</td>
<td>200</td>
</tr>
<tr>
<td>8</td>
<td>(-)</td>
</tr>
</tbody>
</table>
The tidal parameters all had zero initial phase and the phase of the diurnal temperature variation was arbitrarily chosen as 0.7 rad. The diurnal component is expected to be most seriously affected since the applied modulation was diurnal also and an appreciable fraction (0.42) of the vertical wavelength. The long wavelength semi-diurnal component suffers little phase distortion but has its amplitude increased. The appearance of an 8 hour component is also to be expected since the modulation of the 12 hour component by the 24 hour Az variation produces the usual sum and difference frequencies, also contributing to the diurnal distortion. These conclusions are similar to those of Gavrilov et al. (1976) who considered the effect the diurnal variation of the mean echo height on a meteor wind experiment with no height measurement.

4.7 RADIAL VELOCITY SENSITIVITY

The relative sensitivity of the present system in measuring radial velocities is determined by the relative response of the low pass doppler filters, the digitizing method and the numerical data reduction. Since the offset output frequency and the radial velocity are directly related, the velocity response is precisely that of the cascaded low pass filters. While reducing unwanted frequency components, the use of two filters also has the detrimental effect of decreasing the high frequency or large velocity response. Each individual channel then has a response tapering down from the maximum velocity of opposite sign to that particular offset as shown in fig. 4.11.

To achieve reliable results the reduction program requires a velocity value from both offset channels, setting the maximum absolute velocity to the digitized limit for each individual channel, $\pm 115 \text{ ms}^{-1}$. 
Fig. 4.11. Relative radial velocity sensitivity

This also has the effect of reducing the individual channel sensitivities to large velocities giving the composite reduced sensitivity shown in fig. 4.11. Although the sensitivity is not uniform there is no bias in any particular direction as with the single offset case. The overall response is decreased by 18% at ±50 ms$^{-1}$, encompassing the majority of radial velocities.

This velocity sensitivity is purely instrumental and takes no account of the characteristics of meteor echoes. Losses associated with finite echo durations are expected to produce a high velocity cut-off instead of the usual low velocity losses associated with doppler beat determinations. As before we consider only decay type echoes whose velocity producing durations are two decay time constants, corresponding to a full offset doppler cycle from each channel. Due to the symmetry of the frequency offsets, the effect of the echo durations will reduce the maximum observed radial velocities of both signs equally, since values from both offsets
are required for a reliable velocity determination. Using the values of $D_a$ from Barnes and Pazniokas (1972) yields the maximum observable radial velocities of table 4.2.

Table 4.2 Maximum observable radial velocity.

| z (km) | $|v_r|_{\text{max}}$ (ms$^{-1}$) |
|--------|----------------------------------|
| 98     | 115                              |
| 100    | 94                               |
| 102    | 56                               |
| 104    | 2                                |

Fortunately little bias is introduced below approximately 102 km, the region where the majority of echoes are received. Although increased echo durations are associated with electron line densities $\alpha \gtrsim 10^{13}$ m$^{-1}$, the radial velocities from these echoes may contain systematic errors.

The 1.62 MHz transmitter oscillator is common to both the synthesized offset references and the transmitted frequency. This coherence leaves only the drift of the 30 Hz oscillator as an additional source of uncertainty. Over the temperature range 5°C to 30°C the frequency is linear with temperature, with

$$\frac{\Delta f}{\Delta T} = 0.0083 \text{ Hz } ^\circ \text{C}^{-1}$$

Over an operating temperature range of 10°C, the corresponding apparent radial velocity drift is 0.5 ms$^{-1}$, considerably less than the estimated contribution to the uncertainty by noise.
4.8 ATMOSPHERIC VELOCITY DETERMINATIONS

The output of the present system is reduced to be interpreted as the radial velocity component and spatial coordinates of the reflection point. The radial velocity $v_r$ depends on the three components of the velocity vector $v$, shown for a flat earth in fig. 4.12,

$$v_r = v_x \sin \gamma + v_y \cos \gamma \cos \beta + v_z \cos \gamma \sin \beta$$

(4.1)

where the cartesian velocity components are

$$v_x = v \cos \delta \sin \varepsilon$$
$$v_y = v \cos \delta \cos \varepsilon$$
$$v_z = v \sin \delta$$

Fig. 4.12. Radial velocity components.
To reconstruct the velocity vector in three dimensions requires three determinations of the radial components in a small spatial region while \( v \) remains constant, resulting in three simultaneous equations involving the arrival angles. In practice the echo rate is usually insufficient to allow this and the range of azimuthal angles is generally too small to achieve sufficient accuracy. Large azimuthal differences also imply large horizontal separations over which the velocity may not be constant. The velocity component in the antenna direction \( v_y \) may be obtained provided the other two components in (4.1) may be ignored. Two sets of perpendicular antennas are then used to determine two orthogonal components of the horizontal velocity vector. This method was used in the present investigation.

Because of the horizontal stratification of the atmosphere and the types of motion present at meteor echo heights, the vertical component of velocity \( v_z \) is expected to be small. This has been experimentally verified by early workers such as Elford and Robertson (1953) and Greenhow (1954) showing in general

\[
v_z \lesssim 5-10 \text{ ms}^{-1}, \quad \delta \lesssim 10^\circ.
\]

Consider the velocity vector \( v \) observed at elevation \( \theta \) in fig. 4.13.

Fig. 4.13. Non-horizontal velocity vector.

The radial velocity component is
\[ v_x = v \cos(\theta - \delta) \]

and the ratio of derived to true horizontal component is

\[ \frac{v_y'}{v_y} = 1 + \tan \theta \tan \delta \]

The systematic uncertainty in \( v_y \) introduced by the neglect of \( v_z \) is less than 8% of \( v_y \) for \( |\delta| < 10^\circ \), for typical elevations of 25°.

Although round earth geometry is essential for true echo heights, earth curvature may be neglected for the horizontal velocities. At the reflection point, the angle between the true and equivalent (flat earth) horizontal surfaces is equal to that subtended by the two earth radials to the radar and the true horizontal range of the reflection point. This is always less than the flat earth horizontal range \( R \cos \theta \) and so a worst estimate for the inclination \( \delta \) to the true horizontal using flat earth geometry is

\[ \delta \leq \frac{R \cos \theta}{R_E} \quad \text{where } R_E \text{ is the earth radius} \]

\[ \leq 4^\circ \text{ for } R = 500 \text{ km, } \theta = 25^\circ, \]

for which the uncertainty in the true \( v_y \) is 3%.

Although \( v_z \) may be neglected, for typical horizontal velocities \( v_y \sim v_x \). For an individual determination of \( v_x \), the contribution from \( v_x \) may be as large as

\[ v_x \sin \gamma_{max} = 0.35 v_x \]

Such an uncertainty is unacceptably large for an individual determination of the supposed \( v_y \). The horizontal tidal components however are deduced from averages across the antenna beam. The geometry is similar to that
of fig. 4.13 considered now in the xy plane with a velocity vector directed at an angle \( \varepsilon \) (\( \equiv \delta \)) to the y axis and observed at azimuth \( \alpha \) (\( \equiv \theta \)). Again

\[
v'_y = v_y (1 + \tan \varepsilon \tan \alpha)
\]

and the expected values are

\[
E[v'_y] = E[v_y] + E[\tan \varepsilon]E[\tan \alpha]
\]

For the symmetric antenna beam

\[
E[\tan \alpha] = 0
\]

resulting in a measurement of the true average component \( v'_y \), since the contributions from \( v_x \) integrate to zero for a constant velocity. If the azimuthal angle \( \alpha \) is not available and \( v_y \) is approximated by \( v_x \) in the beam direction then

\[
E[v'_y] = E[v_y]E[\cos \alpha]
\]

an underestimate, indicating the need for a narrow antenna beam if no azimuth is available, or a correction to the measured mean values. The mean horizontal velocity component \( v_y \) in the direction of the antenna is derived from individual radial velocity determinations

\[
v_y = v_x / \cos \beta \cos \gamma \]

The integration over the finite beamwidth while not altering the mean component will make a significant contribution to the velocity variance.

4.9 WIND SHEARS

Since the vertical length of a meteor train is typically ten kilometres, regions of differing horizontal velocity will cause the train to shear. The velocity determining principal Fresnel zone then shifts along.
the train to regions of larger radial velocity, if a radial velocity toward the observer is considered as positive. Statistical properties of shears can be obtained from a single station meteor wind experiment (Muller, 1968). These results and rocket trail measurements (Rosenburg, 1968) show typical shear sizes to be 10-20 m s\(^{-1}\) km\(^{-1}\). Following the analysis of Muller, such shears would produce radial accelerations of 25-100 m s\(^{-2}\) for a slant range of 250 km. For a zero radial velocity (30 Hz) the maximum duration of the doppler output is 0.2s (12 half cycles) resulting in a velocity change of 5-20 m s\(^{-1}\). During this time the reflection point moves 0.5-1 km and we regard the changing velocity as an average over this height interval.

Muller found approximately 10% of meteor echoes contained resolvable shears. In the present system the measurement of half cycle durations will have a smoothing effect on any apparent radial velocity determinations and the short duration of the doppler recording will help suppress the effects of vertical wind gradients.

4.10 ECHOES IN MINOR LOBES

Considerations so far have only dealt with meteor echoes within the antenna pattern effective main lobe. The results will be contaminated by the occurrence of echoes in minor lobes producing both spurious arrival angles and velocities. The limiting electron line density \(n_m\) able to be observed by the system is obtained from the radar equation (5.4) expressed as

\[
P_R = \frac{P_T G G \lambda^3}{32\pi R^3} |g|^2
\]

\[
= \frac{V_R Z}{Z_{in}}
\]

Using the following parameters
\[ P_T = \text{transmitted power} = 35 \text{ kW} \]

\[ G_1 = \text{Tx antenna gain} = 12 \text{ db over isotropic} \]

\[ G_2 = \text{Rx antenna gain} = 9 \text{ db over isotropic} \]

\[ R = 250 \text{ km}, \quad Z_{\text{in}} = 50\Omega \text{ and input voltage } v_R = 5\mu\text{V} \]

The limiting reflection coefficient is

\[ g_{\text{lim}} = 1.5 \times 10^{-2} \]

\[ = \alpha |r_e| \text{ for underdense echoes (Chapter 5)}, \]

where \( r_e \) is the classical electron radius. Hence \( \alpha_m = 7 \times 10^{12} \text{ m}^{-1} \) and the majority of echoes observed will be of the underdense type. Since, for \( s = 2 \)

\[ n(>\alpha_m) \propto \frac{1}{\alpha_m} \]

then \( n(>\alpha_m) \propto (P_T G_1 G_2)^{1/2} \) for underdense echoes. (This is simply verified by observing the number of echoes above a detection threshold as the transmitter output power is varied).

From Kaiser (1953) the total echo rate is

\[ N \propto \frac{1}{\alpha_m} \int \int \int S^2(\theta, \alpha) \, d\theta d\alpha \]

where \( S \) is the antenna amplitude pattern.

Although the power gain of individual minor lobes is low (fig. 3.37) the solid angle covered by a particular lobe may be large resulting in an appreciable echo rate contribution. For lobes of identical shape

\[ N \propto \frac{1}{\alpha_m} \propto (G_1 G_2)^{1/2} \]
the effective antenna gain, and the proportion of echoes in minor lobes is
less than 5% of the total.

This analysis also shows antennas such as λ/2 dipoles are
unsuitable for the present echo location system. Although the sidelobe
levels are reduced by the use of λ/2 dipoles, the front to back ratio is
simply that of the transmitting array, resulting in an appreciable proportion
of echoes being observed in the rear lobe.

As well as having a maximum value of unity at θ = 25°, the ground
reflection factor has a secondary maximum of relative amplitude 0.54 at
θ = 90°. The vertical antenna pattern of the system is unknown, but side-
lobes in the vicinity of θ = 90° are expected to be small and the echo
rate for constant gain decreases as cos θ sin⁻¹θ resulting in very few
echoes at large elevations.

4.11 SUMMARY

The aim of the meteor wind system is to deduce horizontal velocity
components in the meteor region. The measured quantities are echo range
(return pulse delay), arrival angles (phase differences) and radial velocity
(doppler frequency shift), from which the echo position and horizontal
velocity component in the antenna direction are derived. Two orthogonal
sets of antennas provide two components of the horizontal velocity. Factors
having a significant effect on the horizontal velocity determination
include;

- Echo position
- Range
- noise
- propagation delay through the system
- quantization
- θ, α
- noise
- ionospheric, tropospheric refraction
- antenna phase response including the reflecting ground
- antenna mutual coupling
- receiver phase response linearity
- receiver gain variations
- receiver d.c. offset variations
- quantization
- differential phase variations
- phase ambiguities.

\[ v_r \] - noise
- filter response and sensitivity
- wind shears
- echo duration associated losses

\[ v_y \] - non-horizontal winds
- echoes in minor lobes
- echo distribution asymmetric to the antenna beam axis.

Consideration of the above affects has shown that the present equipment is capable of producing reliable mean horizontal velocity values from the drifts of meteoric ionization. One major assumption is that of supposing that the derived apparent velocity is indeed due to the horizontal drift of the ionized column. The theory of radiowave reflection from meteor trains will be considered in the next two chapters where it is shown that this assumption has important consequences.
CHAPTER 5

THE REFLECTION OF RADIO WAVES FROM METEORIC IONIZATION

INTRODUCTION

The interpretation of radio meteor echo data and subsequent deduction of meteoric and atmospheric parameters requires a knowledge of radiowave reflection processes. The validity of approximate scattering models will be investigated by comparison with numerical full wave solutions. Results for a range of electron line densities $\alpha = 10^{13} \text{m}^{-1}$ to $\alpha = 10^{16} \text{m}^{-1}$, thought to include the sensitivities of most meteor radars will be presented for the backscatter mode. S.I. units are used throughout.

5.1 DEFINITION OF THE REFLECTION COEFFICIENT $g$

The scattering of radio waves may be described in terms of an effective scattering cross section $\sigma$ in the conventional radar equation

$$P_R = \frac{P_T G^2 \lambda^2 \sigma}{64\pi^3 R^4}$$

(5.1)

where $P_R, P_T$ are the received and transmitted powers respectively, $G$ the gain of a common transmit/receive antenna, $\lambda$ the radio wavelength and $R$ the distance to the target. Equivalently, a complex reflection coefficient $g$ may be employed, which can be related to the echo amplitude and phase, the quantities obtained in most experiments. Neglecting wave polarization, the full wave expression for a wave reflected from a meteor train can be written as (section 5.4)

$$E^{ref} = \sum_m j_m^m H_m^1 (kr) \cos m\phi$$
where $t_m$ is the $m^{th}$ mode reflection coefficient, $m$ is an integer, $(r, \phi, z)$ are cylindrical polar coordinates, $H_{m}^{(1)}$ is the $m^{th}$ order Hankel function of the first kind and $k = \frac{2\pi}{\lambda}$. Applying the asymptotic expansion of $H_{m}^{(1)}$ gives the expression for the field at a large distance $R$ from the train,

$$E_R^{\text{ref}} = \sum_{m} j_m t_m (\frac{2}{\pi kr})^{\frac{1}{2}} \exp[-j(kR - \pi/2)] \cos m\phi$$

which is a plane wave.

The reflection coefficient $g$ is defined as

$$g = \sum_{m} j_m t_m \cos m\phi \quad (5.2)$$

so

$$|E_R^{\text{ref}}| = g (\frac{2}{\pi kr})^{\frac{1}{2}}$$

for a unit amplitude incident wave. In general

$$g = (\frac{\pi kr}{2})^{\frac{1}{2}} \frac{E_R^{\text{ref}}}{E^{\text{inc}}} \quad (5.3)$$

where $E^{\text{inc}}$ is the field incident upon the train.

The reflection coefficient may be related to the scattering cross section. The power flux density incident upon the train is $P_T G/4\pi R^2$, so from (5.3) the received power will be

$$P_R = |g|^2 \frac{2}{\pi kr} \cdot \frac{\lambda G}{4\pi^2} \cdot \frac{P_T G}{4\pi R^2}$$

The contribution to the received power is effectively that from the principle Fresnel zone of length $2\sqrt{kr}$ for the case of plane wave incidence. However, in the backscatter mode, the zone size for a $\pi$ phase change is reduced to $\sqrt{2kr}$ seen from a radius $R$, resulting in a reduction in the reflected power by a factor 2. Hence
Comparison with (5.1) yields

\[ \sigma = \frac{2RL}{\pi} |g|^2 \]  

(5.5)

The reflection coefficient \( g \) is in general complex and represents the amplitude and phase of the reflected wave with respect to the incident wave.

5.2 THE IONIZATION TRAIN MODEL

Although the passage and subsequent ablation of a meteoroid in the earth's atmosphere produces both free electrons and ions, the more massive ions contribute negligibly to the scattering of radiowaves, so the train is considered as an electron ensemble. The train is typically many Fresnel zones in length and here will be considered infinitely long. The finite meteoroid velocity produces diffraction fringes early in the echo lifetime, however these and the general Fresnel diffraction case for the reflected wave are not considered. Typical echo lifetimes are greater than the creation (by the finite velocity meteoroid) time of the train for altitudes \( \lesssim 110 \) km, so the train is assumed to be formed instantaneously.

Once created the train undergoes radial ambipolar diffusion since the more mobile electrons create a space charge electric field, and so are retarded in their motion by the slower diffusion of much heavier positive ions. Solution of the radial diffusion equation (McKinley, 1961) shows that this diffusion produces a train with a Gaussian radial distribution

\[ n(r,t) = \frac{\alpha}{2} \frac{e^{-r^2/a^2}}{\pi a^2} \]  

(5.6)

where

\[ a^2 = r^2_0 + \frac{4D \cdot t}{a} \]  

(5.7)
where $r_0$ is the train initial radius. A Gaussian radial ionization distribution results from the diffusion of a line of electrons and positive ions, or from a Gaussian column with non-zero initial radius. The collisional processes leading to the formation of the train are likely to result in a Gaussian radial profile. The train is considered uniform in cross section.

The electron gyro-frequency in the meteor region is approximately 1 MHz, and since most radio meteor experiments are conducted at frequencies near 30 MHz, the effect of the geomagnetic field is ignored, as are thermal motions of the plasma and the radiation damping of individual electrons. The electron collision frequency is also much less than typical radio frequencies, but collisions will be retained in the subsequent full wave scattering formulation.

5.3 APPROXIMATE MODEL SOLUTIONS

Using the above train model, the full wave expressions for the wave equations contain variable coefficients. Previous workers have chosen restricted electron line density regions to deduce approximate expressions for the reflection coefficient.

The first theoretical work on the scattering of radio wave energy from a column of meteoric ionization appears to be that of Pierce (1938). Pierce suggested the column has a volume electron density greater than the critical density out to a radius of several wavelengths, resulting in total reflection from the region. The measurements of scattered wave energy by early experimenters however were found to be considerably less than those predicted. Blackett and Lovell (1941) formulated a theory of reflection from ionized columns expected to result from cosmic rays incident on the atmosphere. This approach was applied to meteor trains by Lovell and Clegg (1948) using the assumption that each electron scatters independently and
coherently (the Born approximation) to yield the total reflected power. The full wave treatment of Herlofson (1951) discussed the validity of the independent scatterer model and drew attention to the possibility of a resonant plasma oscillation, due to charge separation when the incident wave has polarization perpendicular to the column. Herlofson obtained polarization ratios for the particular cases of a dielectric cylinder and of a cylinder having a linear radial decrease in density. Herlofson also showed that ambipolar diffusion would lead to a loss of coherence of the independently scattering electrons as the column's radial dimension increased, resulting in a decrease in the reflected energy. The Gaussian ionization profile produces an exponential decay of echo amplitude with time for the underdense train, given by

\[ g = \alpha n_0 r_e \exp\left[-(ka)^2\right] \]

where \( r_e \) is the classical electron radius.

The effect of an ionized medium on electromagnetic waves can be represented by a reduced, continuous dielectric constant (appendix D).

\[ \kappa = 1 - \frac{n_0 e^2}{\varepsilon_0 m^2 \nu (1+j\nu/\omega)} \]  

(5.8)

where \( e \) and \( m \) are the electron charge and mass respectively, \( \nu \) the electron collision frequency and \( \varepsilon_0 \) the permittivity of free space. The expression (5.8) is for the case of an incident plane wave whose associated field quantities have a time dependence represented by \( \exp(-j\nu t) \). The independent scatterer model is necessarily restricted to low electron densities for the Born approximation to be justified. The train is termed dilute (or underdense or unsaturated) when

\[ (1-\kappa) \ll 1 \]  

(Kaiser and Closs, 1952)
i.e. \[ \frac{\alpha e^2}{\pi a^2 \varepsilon_0 m_0^2} \, e^{-r^2/a^2} \ll 1 \] neglecting collisions.

The minimum value of \( K \) occurs at \( r = 0, t = 0 \) for the Gaussian column, and so to satisfy the underdense approximation

\[ \alpha << \frac{(kr_0)^2 \pi \varepsilon_0 mc^2}{e^2} \]

where \( c \) is the velocity of light. For a wavelength of 10m and initial train radius of 0.5m (Baggaley, 1970), this condition becomes

\[ \alpha << 8.8 \times 10^{-12} \text{ m}^{-1}. \]

The region of validity of the underdense model has also been investigated by Brysk (1959), who considered the maximum permissible phase change due to refractive effects as the incident wave passes through the ionized column.

For dense trains, Greenhow (1952b) suggested that total reflection takes place at the radius of critical density in a manner analogous to that for a metallic cylinder. Equating the dielectric constant to zero yields the critical radius

\[ r_c^2 = a^2 \log_e \left[ \frac{\alpha e^2}{\pi \varepsilon_0 m c^2 (ka)^2} \right] \tag{5.9} \]

In the geometrical optics limit \( (kr_c \gg 1) \) the reflection coefficient for a totally reflecting cylinder is

\[ g \sim \left[ \frac{\pi}{4} kr_c \right]^{1/2} \]

and is independent of polarization.
For a Gaussian ionization distribution there is considerable ionization outside the critical radius. As the incident wave passes through this region, refraction increases the effective wavelength 'seen' by the critical radius cylinder. Equivalently the effective critical radius and hence the reflection coefficient are reduced. The ray tracing approach of Manning (1953) showed this refractive de-focussing of the incident energy reduced the maximum critical radius by approximately 30%.

For these overdense echoes, total reflection is expected to take place while \( r_c > 0 \), and once the incident wave penetrates the column independent scatter may commence. The lower limit of electron line density for an overdense column may be estimated by considering the column to have diffused to a radial size such that the electron scatter is incoherent once the collapsing critical radius becomes zero. The electron oscillations will destructively interfere when the column is of the order of a wavelength radially

\[
i.e. \ a \geq \frac{\lambda}{2\pi}
\]

and from (5.9) the critical radius becomes zero when

\[
(ka)^2 = \frac{ae^2}{\pi\epsilon_0 mc^2} \geq 1
\]

So

\[
a \geq 10^{14} \text{ m}^{-1}.
\]

This is a minimum value since refraction in the column will increase the wavelength within the column and reduce the loss of coherence.

Kaiser and Closs (1952) reduced the wave equations to a formal electrostatic problem whose solutions predicted both the independent scatterer and metallic cylinder types of behaviour, depending on whether or not the incident wave was able to penetrate the ionized column. Kaiser and
Closs also investigated the plasma resonance phenomenon predicted by Herlofson and found the polarization ratio to depend on the ionization gradient of the column, the maximum ratio of 2 for a Gaussian column being considerably less than that for Herlofson's homogeneous cylinder. Kaiser and Closs also predicted that radiation damping in the column would allow observation of resonance only in underdense trains.

5.4 FULL WAVE SCATTERING THEORY

Fields within the column must satisfy Maxwell's equations which may be combined to give (appendix D)

\[ \nabla^2 E + k^2 k_E = 0 \tag{5.10} \]

\[ \nabla^2 H - (\nabla \times H) \times \frac{\nabla k}{k} + k^2 k_H = 0 \tag{5.11} \]

A plane wave of arbitrary polarization incident normally upon the column can be resolved into two plane waves whose polarizations are parallel to and perpendicular to the column; each component wave may be treated separately. For the case of parallel polarization and using cylindrical polar coordinates \((r, \phi, z)\) (fig. 5.1), with \(E\) along the meteor train or column axis

\[ \frac{\partial^2 E_z}{\partial r^2} + \frac{1}{r} \frac{\partial E_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 E_z}{\partial \phi^2} + k^2 k_E = 0 \]

For transverse polarization \(H\) is along the train axis and

\[ \frac{\partial^2 H_z}{\partial r^2} + \left[ \frac{1}{r} - \frac{1}{k} \frac{\partial k}{\partial r} \right] \frac{\partial H_z}{\partial r} + \frac{1}{r^2} \frac{\partial^2 H_z}{\partial \phi^2} + k^2 k_H = 0 \]

Using separation of variables the fields within the column may be expressed as
Fig. 5.1 Meteor scatter geometry

Fig. 5.2 Radial equation solutions $T_m(r)$. $\alpha = 5 \times 10^{-13} \text{m}^{-1}$, $ka = 0.2$, $m = 1$. 
with \( m \) an integer and \( a_m, b_m \) arbitrary constants. The function \( P_m (r) \) now satisfy

\[
\frac{d^2 P_m}{dr^2} + \frac{1}{r} \frac{dP_m}{dr} + \left( k^2 - \frac{m^2}{r^2} \right) P_m = 0
\]  

(5.14)

for the parallel polarization case, and \( T_m (r) \) must satisfy

\[
\frac{d^2 T_m}{dr^2} + \frac{1}{r} \frac{dT_m}{dr} + \left( \kappa^2 - \frac{m^2}{r^2} \right) T_m = 0
\]  

(5.15)

for the transverse case.

The incident plane wave may be expanded as a series of cylindrical waves. Considering the case of parallel polarization and suppressing the time varying factor

\[
E_{z}^{inc} = \exp(jkx)
\]

\[
= \sum_{m} j^{m} J_{m} (kr) \cos m\phi
\]  

(5.16)

where \( J_{m} \) is the \( m^{th} \) order Bessel function. In addition to satisfying the wave equation, the reflected wave must describe a diverging plane wave at large distances from the column and is singular at the column axis. The reflected wave is represented by

\[
E_{z}^{ref} = \sum_{m} j^{m} J_{m}^{(1)} (kr) \cos m\phi
\]  

(5.17)
where $t_m$ is the $m^{th}$ mode reflection coefficient. The choice of the Hankel function $H_m^{(1)}$ of the first or second kind must be consistent with the choice of the incident wave time variation and represent a wave travelling radially outward.

Outside the column the field is the sum of the incident and reflected waves ((5.16) and (5.17)), while inside the field is given by equation (5.12) for parallel polarization. Matching these fields and their radial derivatives at the ionization radial boundary

$$J_m(kr_b) + t H_m^{(1)}(kr_b) = a P_m(r_b)$$

$$kJ_m'(kr_b) + kt H_m^{(1)'}(kr_b) = a P_m'(r_b)$$

yielding for the $m^{th}$ mode

$$t_m = \frac{kJ_m'(kr_b)P_m(r_b) - J_m(kr_b)P_m'(r_b)}{H_m^{(1)}(kr_b)P_m'(r_b) - kH_m^{(1)'}(kr_b)P_m(r_b)} \quad (5.18)$$

where $r_b$ is the matching radius.

The total reflection coefficient for a wave whose incident polarization is parallel to the column axis is

$$g_\parallel = \sum_m t_m \cos m\phi \quad (5.2)$$

and for backscatter $\phi = 180^\circ$ (fig. 5.1).

The expressions for the perpendicular reflection coefficient $g_\perp$ are similar with $T_m$ replacing $P_m$. However, an additional factor $(-1)$ must be included in (5.18) since the $H$ field component is now along the column axis and $E$ must suffer a $\pi$ phase change for the direction of the Poynting vector to be reversed.
5.5 METHOD OF SOLUTION

To calculate the reflection coefficient the field components $P_m (r_b)$ and $T_m (r_b)$ must be found by numerical integration of equations (5.14) and (5.15). Integration was carried out in the same manner as Lebedinets and Sosnova (1968) and Jones and Collins (1974). The only essential difference between equations (5.14) and (5.15) is the term $\frac{1}{\kappa} \frac{d \kappa}{dr}$, which represents charge separation and hence the appearance of conditions for plasma resonance in the case of transverse polarization. This term also introduces an additional singularity into (5.15). However, by the use of a complex dielectric constant, including electron collisions, this singularity may be shifted off the real axis along which the integration is carried out. Complex solutions are retained for $P_m (r)$ and $T_m (r)$ although these are only necessary in the case of transverse polarization.

The second order complex equations (5.14) and (5.15) may each be reduced to a set of four simultaneous first order equations (Appendix E), suitable for integration using a Runge-Kutta routine. Two boundary conditions are required to commence the integration, namely the field component and its first derivative near the column axis to avoid the singularity for both polarizations when $r = 0$. Near the column axis $\kappa$ may be considered a constant and (5.14) is then Bessel's equation whose solution is

$$P_m (r) = J_m (\kappa \frac{1}{2} r) \text{ for } \kappa > 0 \text{ and } r \text{ small.}$$

(The modified Bessel function $I_m$ is chosen for $\kappa < 0$. ) The starting conditions for (5.14) are not critical and the appropriate Bessel function or its expansion for small argument may be used, with the imaginary part of the starting solution set to zero. The coupling term between the separated real and imaginary equations of (5.14) is proportional to $\kappa_i$ (Appendix E),
the imaginary part of the dielectric constant, and $K_i \sim K_r \frac{\nu}{\omega}$ which is small since typically $\frac{\nu}{\omega} \sim 0.001$, so the imaginary solutions are always negligible.

Although the transverse polarization wave equation (5.15) also approximates Bessel's equation for small arguments, the coupling term between the separated real and imaginary equations is proportional to $\frac{1}{|K|^2}$ and the imaginary solutions grow once the singular point on the axis has been reached as shown in fig. (5.2), so the imaginary starting solutions are more important than in the parallel case. The field components near the axis are approximated by a series solution to (5.15), valid for small arguments (Appendix F).

An automatic fourth order Runge-Kutta routine with an accuracy of $10^{-4}$ at each step was used to integrate along the axis until the ionization radial boundary was reached (Appendix J). Since the Gaussian column has a diffuse boundary, the matching radius is chosen where the dielectric constant is essentially unity. Consideration of the values in table (5.1) led to a choice of $K = 0.999$ for the radial boundary, any smaller increment from unity resulting in a increased matching radius and hence longer integration computing time.

Table 5.1 Reflection coefficients deduced at various boundaries

| $\kappa$ | $|g_\parallel|$ | $\angle g_\parallel$ (rads) |
|---|---|---|
| 0.9 | 0.0763478 | -1.6471575 |
| 0.99 | 0.0769109 | -1.6477592 |
| 0.999 | 0.0769669 | -1.6478206 |
| 0.9999 | 0.0769729 | -1.6478274 |
| 0.99999 | 0.0769747 | -1.6478293 |
The individual mode reflection coefficients \( t_m \) were found for higher \( m \) until the contributions of each successive term altered the resultant reflection coefficient by less than 1 part in \( 10^4 \). The computational time increases with increasing electron density and increasing \( \text{ka} \), since for large \( \text{ka} \) the matching radius is at a large distance from the radial axis. Larger electron densities require a smaller integration step to maintain accuracy and a greater number of modes to ensure convergence of the reflection coefficient.

These calculations may be extended to the forward scattering case for normal incidence through variations in the scatter angle \( \phi \) in equation (5.2). The more difficult treatment in the case of oblique forward scattering involves the solution of equations (5.14) and (5.15) simultaneously since geometrical considerations and the gradient of the dielectric constant leads to coupling of the incident polarizations.

5.6 RESULTS

Calculations were carried out for electron line densities in the range \( 10^{13} \) to \( 10^{16} \text{m}^{-1} \). The lower limit (approximate meteor magnitude +11) corresponds to the detection threshold of most backscatter meteor radars while the upper limit is in the overdense regime. This range encompasses the vast majority of observed radio meteors.

5.6.1 Parallel Polarization Magnitudes

The magnitude of the reflection coefficient is proportional to echo amplitude, and its variation with \( (\text{ka})^2 \), which is approximately proportional to time, is shown in fig. 5.3. For electron line densities \( \alpha \lesssim 10^{13} \text{m}^{-1} \), the expression for underdense echoes of Kaiser and Closs including the exponential decay predicted by Herlofson gives a good description of echo
Fig. 5.3. The reflection coefficient magnitude for the case of parallel polarization. $(ka)^2$ is proportional to time.

- full wave theory
- metallic cylinder
- metallic cylinder with refraction
- underdense expression

behaviour. For progressively larger $\alpha$ the approximation of independent scatterers breaks down as the wave (and hence phase relations between the electrons) is modified by passage through the column. Also shown is the reflection coefficient for a totally reflecting cylinder (Mentzer, 1955)

$$g_{II} = \sum_{m} (-1)^m \frac{J_m(kr_c)}{H_m(2)(kr_c)}$$

$$\rightarrow \left[\frac{kr_c}{4c}\right]^\frac{1}{2} \text{ for } kr_c \gg 1.$$
The form of this result can be inferred from (5.16) and (5.17). The totally reflecting cylinder has no internal fields and so the incident and reflected waves must be matched at the cylinder boundary \( r_c \). This is an over-estimate and the correction applied by Manning (1953) allowing for the refracting sheath outside the critical radius, reduces the effective scattering radius and hence the reflection coefficient providing close agreement with full wave solutions for \( \alpha \gtrsim 10^{16} \text{ m}^{-1} \).

Trains with transition line densities \( 10^{13} < \alpha < 10^{16} \text{ m}^{-1} \) exhibit both overdense and underdense types of behaviour. In the initial stages reflection takes place near the surface of critical density provided that the axial dielectric constant is sufficiently negative. The reflection coefficient shows an increase as the effective reflecting cylinder diameter increases due to radial diffusion. The diffusion however causes the electron volume density to decrease, and once the axial dielectric constant becomes only slightly negative the wave can penetrate the column as the skin depth is large and individual electron scatter commences. The expression for the underdense reflection coefficient assumes the incident wave is unmodified by passage through the column and that all electrons 'see' the same incident field. However, in the sub-critical density region refraction will increase the phase velocity within the column. In consequence the larger effective wavelength reduces the loss of coherence of electron scattering, resulting in a larger reflection coefficient and a modified decay rate as illustrated in fig. 5.4. A slope of \(-1\) for the underdense case is satisfied for \( \alpha \lesssim 10^{13} \text{ m}^{-1} \). Although for higher \( \alpha \) the subsequent decay is exponential once the incident wave has penetrated the column, (i.e. when

\[
(ka)^2 = \frac{\alpha e^2}{\pi e_0 m c^2}
\]

indicated by crosses in fig. 5.4) the decay slope decreases with increasing line density. By summing the scattered field from individual electrons the echo amplitude is expected to vary as
Fig. 5.4 Natural logarithm of the parallel reflection coefficient magnitude

- full wave theory
- underdense expression
- underdense expression including the effective wavelength within the column

\[
\frac{g^0_{\parallel}}{g^0_{\parallel}} = \frac{2\pi \int_0^{\infty} n J_0 \left( \frac{4\pi r}{\lambda} \right) rdr}{2\pi \int_0^{\infty} nrdr} \tag{5.20}
\]

(McKinley 1961), where \( g^0_{\parallel} \) is the initial reflection coefficient and \( J_0 \) the zero order Bessel function. Some account may be taken of the effect of refraction by including a term for the effective wavelength within the column in (5.20). Numerical integration for \( \alpha = 10^{14} \text{ m}^{-1} \) is shown in fig. 5.4, indicating a larger reflection coefficient compared with the usual underdense model expression and the decay slope change due to refraction. The applied correction to the underdense model is too large however since no bending and resulting de-focussing of the incident wave were included, and the correction only applies to the axial line along which the rays travel.
For larger values of $\alpha$ the electron content in the sub-critical region is greater, refractive effects become more important and produce further departure of the decay slope from the underdense value of $-1$. Once the wave has penetrated the column, the subsequent decay is purely exponential, indicating an analytic solution may be possible.

The results for parallel reflection coefficients are in agreement with the numerical solutions of Brysk and Buchanan (1965), who considered the scattering cross-section of a generalized Gaussian potential, a formulation that excludes the possibility of plasma resonance. The present results may be compared with those of Brysk and Buchanan by the use of (5.5).

5.6.2 Transverse Polarization Magnitudes

Herlofson suggested that only the dipole mode ($m=1$) was important for the transverse polarization case and Kaiser and Closs considered only this mode in their electrostatic approach. Polar diagram plots of $q_\perp$ are shown in fig. 5.5 indicating that most of the scattered energy is contributed by the dipole mode for $\alpha \leq 10^{14} \text{ m}^{-1}$. For $\alpha = 10^{15} \text{ m}^{-1}$ other higher order modes are also becoming significant, whereas for the case of $q_\parallel$ the $m=0$ mode is the most significant.

Reflection coefficients for the case of transverse polarization are shown in fig. 5.6. For $\alpha = 10^{13} \text{ m}^{-1}$ the present results for $q_\perp$ are in agreement with those of Kaiser and Closs, and also those of Keitel (1955), who employed the wave matching technique of Herlofson to verify the approximate solutions for $\alpha = 10^{13} \text{ m}^{-1}$ and $\alpha = 10^{17} \text{ m}^{-1}$. Enhanced scattering is also found for transition line densities giving train reflections that exhibit overdense, resonant and underdense types of behaviour in their lifetimes.
Fig. 5.5 Polar diagrams of the transverse reflection coefficient $g_1$. 

$a = 10^{13} \text{ m}^{-1}, ka = 0.2$

$a = 10^{15} \text{ m}^{-1}, ka = 2.0$

$a = 10^{14} \text{ m}^{-1}, ka = 0.8$
Although the totally reflecting cylinder with 

\[ g_L = \sum (-1)^m \frac{J'_m(kr)}{m \frac{H'_m(kr)}{m}} \]  

(Mentzer, 1955) closely approximates the reflection coefficient for 
\( \alpha \geq 10^{16} \text{ m}^{-1} \), the scattering resonances are only apparent early in the echo lifetime indicating that the sharp critical density boundary is short-lived as diffusion decreases the ionization gradient.

The condition for plasma resonance is dependent on the geometry of the plasma (Herlofsson). Kaiser and Closs predicted that for a cylinder
with a Gaussian radial ionization distribution the resonance will occur when the axial dielectric constant is \(-1.4\). The peak values of \(g_\parallel\) from the present full wave solutions are in accord with this behaviour. From the resulting polarization ratios \(\rho\) (defined as \(g_\perp/g_\parallel\)) in fig. 5.7, the radiation damping predicted by Kaiser and Closs for \(\alpha \geq 10^{14} \text{m}^{-1}\) is not present for transition line densities. For \(\alpha = 10^{13} \text{m}^{-1}\) the polarization ratio \(\rho\) is close to the value of 2 given by Kaiser and Closs considering only the dipole mode. The polarization ratios tend to unity near the end of an echo, since as the ionization density gradient decreases (due to diffusion) the value of

\[
\frac{1}{K}\frac{dK}{dr} \ll \frac{1}{r}
\]

in equation (5.15) and for large \(ka\), \(g_\perp \to g_\parallel\). A peak ratio of \(\rho = 2.6\) occurs for \(\alpha = 2 \times 10^{14} \text{m}^{-1}\) and most values are close to 2.0. For increasing \(\alpha\) the condition \(\kappa_{r=0} = -1.4\) occurs later in the echo lifetime.
and for the transition region there is a gradual rise of the polarization ratio to a maximum near the end of the echo. For $\alpha = 10^{16} \text{ m}^{-1}$ $\rho$ is essentially unity in the initial stages.

5.6.3 Parallel Polarization Phases

The phase angle $\psi$ of the reflection coefficient is defined as

$$\psi = \tan^{-1} \frac{q_{\text{imag}}}{q_{\text{real}}}$$

and represents the phase of the reflected wave with respect to that of the incident wave. Fig. 5.8 shows the parallel reflection coefficient phase variations with increasing $ka$. The behaviour is similar to that for parallel reflection coefficient magnitudes. For low density columns $\alpha \lesssim 10^{13} \text{ m}^{-1}$ the incident wave is virtually unchanged in its passage through the column resulting in a constant phase. For $\alpha \gtrsim 10^{16} \text{ m}^{-1}$ the phase is that appropriate to the case of a totally reflecting cylinder, the decreasing phase corresponding to an increasing effective scattering radius. For transition line densities the initial phase decrease corresponds to the scattering cylinder undergoing expansion but as the volume density is reduced by diffusion effects the cylinder collapses, resulting in a phase increase, the phase then showing no significant change when the axial dielectric constant is positive. The total electron content that the central wave travels through is unchanged as the trail expands, so the phase retardation is constant once the wave penetrates the column. The phase however does not return to the underdense value of $-\pi/2$. The train cannot be considered underdense and refractive effects will alter the phase relations within the column and the resultant reflected phase. Increases in the total electron content in the train then causes the final phase to decrease with increasing electron line density.
Fig. 5.8  Phase angle (radians) of the reflected wave for parallel polarization.

--- full wave theory

--- metallic cylinder

5.6.4 Transverse Polarization Phases.

A phase change of $180^\circ$ should accompany the plasma resonance (Kaiser 1955) since the situation may be regarded as a forced damped oscillation with varying resonant frequency. Phase changes for the case of transverse polarization are shown in fig. 5.9.
Fig. 5.9 Phase angle (radians) of the reflected wave for various electron line densities.

- full wave theory transverse polarization
- full wave theory parallel polarization
- metallic cylinder transverse polarization
- Kaiser (1955)

Although resonant phase changes do occur these are generally smaller than predicted by Kaiser for low density trains. The phase changes associated with a totally reflecting cylinder for $\alpha = 10^{16} \text{ m}^{-1}$ are a good approximation to the full wave solutions, the decreasing phase again corresponding to an expansion of the reflecting cylinder. As for amplitude behaviour the phases of transition echoes show metal cylinder, resonant and underdense type behaviour characterized by a constant phase early in the echo lifetime.
followed by a corresponding expansion. The combined effects of a collapsing cylinder and a resonance phase change then cause the phase to increase, the resonance effect being dominant later in the echo lifetime, until the phase finally becomes constant and of similar value to the parallel case at the echo termination. Since the effects on the phase of a cylinder expansion and of resonance are of opposite sign the resulting phases for the case of transverse polarization would be expected to peak earlier than those for the parallel polarization case. This is illustrated in Fig. 5.9 for $\alpha = 5\times10^{15}$ m$^{-1}$ and $10^{15}$ m$^{-1}$.

5.7 COMPARISON WITH OTHER FULL WAVE SOLUTIONS

Full wave solutions may be compared by considering the variation with electron line density of the maximum reflection coefficients for both polarizations as shown in fig. 5.10. Approximate model solutions are also shown to be appropriate for both polarizations for $\alpha \leq 10^{13}$ m$^{-1}$. Although the metal cylinder with refraction shows good agreement for $g_{\parallel}^{\text{max}}$ for $\alpha = 10^{15}$ m$^{-1}$ it is evident from fig. 5.3 this model is not satisfactory for the entire amplitude profile for line densities as low as this. For transverse polarization no approximate model solution is satisfactory.

The conclusions of the present approach for transverse polarization are in marked contrast to those obtained by both Jones and Collins (1974) and Lebedinets and Sosnova (1968) using similar methods. Rather than a maximum $g_{\perp}$ and polarization ratio $\rho$ of $\sim 1.4$ and 2.6 respectively, they find resonances with polarization ratios of order 100, corresponding to the large values of $g_{\perp}^{\text{max}}$ shown in fig. 5.10 (curve E). The two peaks are identified by Jones and Collins as corresponding to the dipole and quadrupole modes.
Fig. 5.10 The maximum value of the reflection coefficient $g_{\text{max}}$ as a function of electron line density.

A $g_{\parallel \text{max}}$ independent scatterer model
B $g_{\perp \text{max}}$ Kaiser and Closs (1952)
C metallic cylinder model
D metallic cylinder with refraction
E $g_{\perp \text{max}}$ Jones and Collins (1974)
F $g_{\parallel \text{max}}$ present work
G $g_{\perp \text{max}}$ present work

Within the full wave method, the governing equation of motion for the column electrons is
\[ m\ddot{x} + m\dot{x} = e^{i\omega t} \]

The source of the inconsistency appears to lie in the representation of the time variation of the field quantities \( E \) and \( H \). In the complex dielectric constant in equation (5.8) the sign of the imaginary part depends on the adopted formulation of the time variation of the incident wave. Despite the small size of the imaginary term, \( (v/\omega = 0.001) \) the sign is important both mathematically and physically. Herlofson (1951) integrated equation (5.15) in the complex plane and justified the path taken around the singularity on the basis of electron collisions. Physically the incorrect sign is equivalent to a negative collision frequency, collisions with neutral particles supplying energy to the electrons motion rather than providing a damping effect. In consequence there results larger resonances than are physically real. Specifically equation (2) of Lebedinets and Sosnova appears to be inconsistent with the adopted form of the incident wave time variation. The polarization ratios which result from their treatment exceed by a factor 40 those of the present work and which are also considerably larger than can be supported by experimental evidence (Chapter 6).

For the transverse polarization case the coupling term between separated real and imaginary wave equations (Appendix E) is proportional to \( \frac{1}{|\kappa|^2} \). As a result the imaginary solutions grow once the singular point on the axis has been reached, in a direction dependent on the sign of the imaginary part of \( \kappa \). The incorrect sign has the effect of reversing the direction of the resonance phase variations. For the parallel polarization case complex solutions are unnecessary since the imaginary solutions are negligible and an error in the choice of sign for the imaginary part of \( \kappa \) will have a negligible effect on the results.
INTRODUCTION

The large discrepancy between the present full wave solutions and those of previous workers has been discussed in chapter 5. In this chapter we will compare the theoretical reflection coefficients with available published experimental data and present applications of the theoretical results. The results for transverse polarization apply mainly to behaviour associated with the plasma resonance phenomenon. Phase variations are of considerable importance for the meteor wind experiment and the deduction of neutral atmosphere velocities.

6.1 TRANSVERSE POLARIZATION AND PLASMA RESONANCE

The experimental approaches to the polarization problem in scattering from meteoric ionization have followed three approaches: the use of linearly polarized antennas to measure two mutually perpendicular reflection coefficients; the use of circularly polarized antennas to obtain quantities related to the polarization ratio; the measurement of phase changes associated with the plasma resonance.

Using corrections to the results of Clegg and Closs (1951) for differing equipment sensitivities for the two polarizations, Closs et al. (1953) obtained a mean resonant polarization ratio, \( \rho \), of 1.7 and observed a decrease in \( \rho \) with increasing line density as predicted by Kaiser and Closs (1952). The method of estimating line densities was based on the approximate theory and applied to transition type echoes. The nett effect would be to concentrate the apparent line densities about values of 2 to 3 \( \times 10^{14} \text{ m}^{-1} \). By carrying out observations during periods of known meteor shower activity
the orientation of the train was known. This experiment was repeated by Billam and Browne (1956) using an improved antenna system and larger wavelength to facilitate the easier observation of polarization effects. Twin crossed Yagi antennas of similar radiation pattern were used and the polarization ratios at the end of an echo were normalized to unity to allow for any sensitivity differences. For short duration echoes (corresponding to low electron line densities $\alpha < 10^{14} \text{m}^{-1}$) $\rho$ was observed to rise to a maximum value early in the echo lifetime and subsequently to decrease to unity, a behaviour in agreement with Kaiser and Closs and with the present calculations. For the case in which two mutually perpendicular linear polarizations illuminate the train the observed polarization ratio will be a maximum only when the train is perfectly aligned with these polarizations. Any misalignment will have the effect of reducing the observed polarization ratio (to unity for an angle of incidence of $\pi/4$) so sporadic meteors would be expected to contaminate the results but only produce a decrease in $\rho$. An increase in the observed value of $\rho$ may result if the transmitting antennas are not in phase so that the resultant transmitted wave has elliptical polarization. These effects are small for small phase errors but can lead for example to a value of $\rho \sim 7.7$ for the case of $\alpha = 5 \times 10^{13} \text{m}^{-1}$ and incidence angle of $45^\circ$ for a circularly polarized wave. An increase in $\rho$ might also occur as a consequence of anisotropic diffusion due to the geomagnetic field (Kaiser and Closs, 1952). The meteor train is not instantaneously created and exhibits Fresnel diffraction fringes during formation. For low density trains the resonance occurs early in the echo lifetime so the effect of the meteor finite velocity may increase $\rho$ (Billam and Browne). Lebedinets and Sosnova have calculated reflected wave amplitudes for several meteor velocities, calculations which, however, are dependent on reflection coefficients obtained using their particular full wave treatment.
For long duration echoes ($\alpha \geq 10^{16} \text{ m}^{-1}$) Billam and Browne (1956) find $\rho$ to be essentially constant throughout the echo lifetime although some variations were found for very long echoes. Radio meteor echoes of very long duration are subject to severe fading as atmospheric winds distort the trains creating multiple reflection centres. Any differential effects could produce changes in the polarization ratio and these effects might be expected for echoes having durations in excess of about 0.2s (Greenhow, 1952b; Philips, 1969). The results for $\alpha = 10^{16} \text{ m}^{-1}$ and $10^{15} \text{ m}^{-1}$ in Figs 5.3 and 5.6 are to be compared with Fig. 6.1. For transition line density echoes Billam and Browne frequently find polarization ratios which increase towards the end of an echo to a maximum value $\rho \sim 2$ in the absence of other effects (the finite velocity effect would have no effect on transitional type echoes unless the meteor is very slow). Billam and Browne found a mean value of $\rho$ near 2 and very few cases of $\rho > 5$. Errors were estimated as 25% but polarization ratios of the order of those predicted by published full wave calculations were not obtained.

Using the twin circularly polarized helical antenna method of Van Valkenberg (1954) it is only possible to deduce the polarization ratio provided the relative phases of both return signals are known. The experiment however, yielded the ratio of the semi-major to semi-minor axis of the polarization ellipse and was not restricted to shower meteors. This ratio is always greater than the value $\rho$ and for the 89 echoes measured 80% gave $\rho < 4$ with a most probable value near 2, showing values consistent with Billam and Browne and the present results.

Greenhow and Neufeld (1956) made use of meteor wind records which indicated very small phase changes due to reflection point motion and attributed residual phase variations to polarization effects provided these were also evident in echo amplitude behaviour. Since the meteor trail orientation was unknown, the phases could be expected to yield values up to
Fig. 6.1 Experimental reflection coefficient magnitudes and polarization ratios (after Billam and Brown, 1956).

Fig. 6.2 Experimental resonance phase changes (after Greenhow and Neufeld, 1956).
the maximum value corresponding to transverse polarization. Horizontally polarized antennas were used so most echoes would have large perpendicular components. From Fig. 5.8 the phase changes that occur after the resonance peak range (for low density trains) from approximately 22° (\( \alpha = 10^{13} \text{ m}^{-1} \)) to 45° (\( \alpha = 10^{14} \text{ m}^{-1} \)). Greenhow and Neufeld observed a mean phase change of 38°, most values being between 20° and 60°. Typical records are shown in fig. 6.2 which may be compared with low density examples in fig. 5.8. For a single incident linearly polarized wave the reflection coefficient
\[ g(\theta) = g(\pi - \theta) \]
\[ g(\theta) = g_{||} \cos^2 \theta + g_{\perp} \sin^2 \theta \]  
(6.1)

where \( \theta \) is the angle between the incident wave E field and the train axis. Hence phase changes are of the same sign and the variation of phase change with \( \theta \) is shown in Fig. 6.3. Although Greenhow and Neufeld measured the phase change after the resonance peak, the maximum phase change observed was 120° compared with a maximum total phase change of 113° (\( \alpha = 5 \times 10^{13} \text{ m}^{-1} \)) in fig. 6.3. The durations of the observed phase shifts (defined by the authors as being measured from the peak echo amplitude to when the phase was sensibly constant) had a most probable value of 20 ms and few values above 100 ms. The theoretical durations depend on the ambipolar diffusion coefficient and hence upon echo height. Assuming an echo height of 92 km with \( D_a = 4.6 \text{ m}^2 \text{s}^{-1} \) (Barnes and Pazniokas, 1972) Table 6.1 gives the theoretical resonant phase shift durations for underdense echoes where the durations are defined as commencing at the peak echo amplitude.
Fig. 6.3 Relative phase angle (radians) of the reflected wave for various polarizations $\theta$ of the incident wave for $a = 5 \times 10^{13} \text{m}^{-1}$.

$\theta = 0^\circ$: parallel polarization

$\theta = 90^\circ$: perpendicular polarization.
Table 6.1

Resonance phase shift duration

<table>
<thead>
<tr>
<th>Line density (m⁻¹)</th>
<th>Phase shift duration (msec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1 x 10^{13}</td>
<td>17.1</td>
</tr>
<tr>
<td>2 x 10^{13}</td>
<td>22.3</td>
</tr>
<tr>
<td>5 x 10^{13}</td>
<td>36.6</td>
</tr>
<tr>
<td>1 x 10^{14}</td>
<td>83.4</td>
</tr>
</tbody>
</table>

The wave inclination to the train will not affect these durations and the results of Greenhow and Neufeld are of the same order. The spread in echo heights will result in a corresponding spread in observed durations. Phase changes due to wind shear and the finite velocity effect (given by the familiar Cornu Spiral) could not be eliminated in the method of Greenhow and Neufeld.

In the experimental arrangement of Sidorov et al. (1965) two linear waves having orthogonal polarizations were transmitted. Measurement of the phase difference between the two reflected waves permitted the elimination of effects due to meteor velocity and any reflection point motion. The theoretical phase differences between parallel and perpendicular polarizations are shown in Fig. 6.4. With the transmitted waves in phase, Fig. 6.4 represents the maximum phase differences to be observed since these will decrease with increasing inclination to the train becoming zero for $\theta = 45^\circ$. Phase changes for several line densities are given by Sidorov et al. in their Fig. 5. For increasing line densities the resonant phase changes occur later in the echo lifetime in accord with the present calculations. The phase reversal at the beginning of the echoes was attributed to diffraction effects by Greenhow and Neufeld but is however still present in the low
Fig. 6.4 The phase angle differences $\phi_\parallel - \phi_\perp$ (radians).

density examples of Sidorov et al. in agreement with Fig. 6.4. Account may be taken of a finite meteor train initial radius by starting the abscissa at a non-zero value ($kr_0$), and for low density trains this will remove some of the initial phase reversals and possibly some of the resonant phase change as in Fig. 5(a,b,c) of Sidorov et al. The values of their observed total phase changes are also in agreement with the present calculations while for low density trains the phase difference returns to zero before the end of the echo as predicted (see Fig. 5(b) of Sidorov et al.). For transition type echoes Sidorov et al. note that the gradient of the phase variation often changed during the echo lifetime, again in accord with the present results. The size of the phase changes are found to be smaller for transition type echoes. The occurrence of resonant effects in transition echoes is consistent with the echo amplitude results of Billam and Browne (1956) but does not
support the suggestion of Kaiser and Closs (1952) that radiation damping would be dominant. Changes of the phase gradient are evident for the case of transition echoes in Fig. 6.4 (for $\alpha = 5 \times 10^{14}$ and $10^{15}$ m$^{-1}$). For large electron densities the examples of Sidorov et al. show slow phase variations which are a result of the expansion of the boundary of the totally reflecting cylinder for both polarizations. One example (Sidorov et al. Fig. 5(1)), shows a rapid initial decrease in the phase difference. The present calculations indicate that such a decrease will occur (for $\alpha \geq 10^{16}$ m$^{-1}$) until $kr_c - 1$, which is very early in the echo lifetime and very likely to be excluded by the effects of finite initial radius. For two linear polarizations the size of the phase difference changes decreases with increasing angle of incidence to the train (being zero for $\theta = 45^\circ$) but retain the same form. For circular polarization, however, the phase changes are of a similar magnitude regardless of the value of $\theta$ but have varying directions depending on which quadrant $\theta$ lies in. Sidorov et al. employed circular polarization for their observations but the distribution of total phase difference changes (their fig. 4) show phases much less than the $180^\circ$ they expected. The most probable value of near $20^\circ$ is lower than that of Greenhow and Neufeld (1956) whose phase changes were measured after the resonance peak. There is no indication in the former report however of the dependence of phase change upon line density, $\alpha$; the inclusion of a large range of $\alpha$ (underdense, transition and overdense) will have the effect of reducing the average observed phase change. The magnitude of the phase changes observed are not in disagreement with theory, with most experimental values lying between $0^\circ$ and $90^\circ$ with a maximum value of $120^\circ$. Sidorov et al. noted that their phase change durations are a factor of about three longer than those of Greenhow and Neufeld. However the durations of Sidorov et al. were measured from the echo beginning until the end of the phase change whereas
those of Greenhow and Neufeld were measured from the time of the resonance peak.

For a height of 92 km and $r_0 = 0.5$ m (Baggaley 1970) then for $\alpha = 10^{14}$ m$^{-1}$ the theoretical resonance duration is 0.179s and will decrease with increasing $r_0$ or decreasing $\alpha$; a result in accord with experiment. Further, when the transition echoes are identified the durations so defined are longer, again in accord with the present calculations. Thus, for $\alpha = 5 \times 10^{14}$ m$^{-1}$, $D_a = 4.6$ m s$^{-1}$ and $r_0 = 0.5$ m the phase change duration is 0.309s. Again it is to be noticed that a spread in heights and hence values of $D_a$ will result in a spread of observed duration values.

A comparison of the present full wave calculations with a variety of experimental data shows good agreement for both echo amplitudes and phases. Polarization ratios and resonance phase changes are in accord with the theoretical values and the presence of resonance phenomena in trains with line densities in the transition region is also supported by experimental evidence. This suggests the adopted scattering model is appropriate for radio-meteor echoes, and further consequences of these particular calculations will now be considered.

6.2 AMPLITUDE CHARACTERISTICS

6.2.1 Echo Decay Time Constants

The exponential decay of the echo from an underdense train may be described by the time $T$ for the amplitude to decrease by a factor of $\exp(1)$ corresponding to $(ka)^2 = 1$

$$T = \frac{\lambda^2}{16\pi^2 D_a}$$

(6.2)

Experimentally an estimate of $T$ may be found from the time taken for the echo amplitude to decrease by a constant factor from the peak value $T_{1/e}$
(Greenhow and Neufeld, 1955 who employed $T_{1/2}$) or instead by fitting an exponential curve to the echo record thus matching the gradient, $T_g$, (NcWak, 1967). Jones and Collins (1974) have considered the total time for the amplitude to decrease by $\exp(+1)$ as an estimate of $T$; however, this measurement is not usually obtained in experiment. Echo durations based on the above three definitions together with durations for the underdense and overdense models of the theory of Kaiser and Closs (1952) are given in Fig. 6.5. For transition type echoes an exponential decay does not begin at the instant of maximum amplitude so that measures of $T$ based on the amplitude reduction method are a worse estimate than those based on the decay curve slope. The time-constant however is systematically increased for both cases for $\alpha > 10^{13}$ m$^{-1}$. In most experimental situations the inclination of the meteor column to the incident wave is unknown and for an angle of incidence, $\theta$, the reflection coefficient is given by (6.1). Since $g_\bot$ tends to $g_\|$, for large $ka$ the slope of any echo decay will eventually be independent of polarization. If, however, an estimate of $T$ is made from constant factor amplitude reduction, the inclusion of a perpendicularly polarized component will reduce the time-constant since plasma resonance results in enhanced peak amplitudes.

Results for ambipolar diffusion coefficients deduced from measurements of $T$ are generally expressed as

$$\log_e D_\alpha = h/H_D + C$$

(6.3)

where $H_D$ is the diffusion scale height and $C$ a constant. From the full wave solutions a general expression for the echo decay slope may be written $g \sim \exp[-\beta(ka)^2]$ where $\beta \leq 1$ is a function of $\alpha$. Then

$$T = \lambda^2/16\pi^2 D_\alpha \beta$$

(6.4)
Fig. 6.5 Echo duration

a Totally reflecting cylinder duration
b Total time for amplitude to decrease by a factor e (= 2.718)
c Time for maximum amplitude to decrease by e for parallel polarization, measured from the time of the amplitude maximum
d As for c for the case of perpendicular polarization
e Duration from the exponential decay slope
f Underdense echo duration.

resulting in an echo decay diffusion coefficient equal to $SD_a$, so that the echo height $h_d$ given by the echo decay slope will be related to the true echo height, $h$, by

$$h_d = h + H_d \log_e \beta$$  \hspace{1cm} (6.5)$$

resulting in systematically decreased heights. Such errors occur if values
of α are in the transition region ($2 \times 10^{13} \leq \alpha \leq 10^{15} \text{ m}^{-1}$) while for a very sensitive radar ($\alpha \ll 10^{13} \text{ m}^{-1}$) no significant error results. For illustration, an indication of the likely height error may be found in the following way. The height of maximum ionization production in a meteor train decreases both with increasing mass and therefore $\alpha_{\text{max}}$ and also with decreasing meteoroid velocity. Taking the height of maximum ionization production as representative of the reflection point height using the expression of McKinley (1961) and using the value 7.4 km for $H_D$ appropriate to the height range 90-100 km (Jones 1970), the decay heights for different meteor velocity groups are shown in fig. 6.6. Serious under-estimates of echo height may result for high meteor velocities and low altitudes.

6.2.2 Ambipolar Diffusion Coefficients and Scale Heights

Ambipolar diffusion coefficients for a particular height derived from echo decay time constants exhibit considerable spread and as a result diffusion scale heights, $H_D$, are determined experimentally using least squares analyses. Measured values of $H_D$ in the height interval 90-100 km range from 6.5 km (Greenhow and Neufeld 1955) to 26 km (Murray 1959) although selection effects inherent in the continuous wave method are believed responsible for Murray's large value. Greenhow and Hall (1961) used a least squares analysis assuming errors in both height measurement and $T$ to obtain a mean value of $H_D = 7.8 \pm 0.3$ km. A subsequent re-analysis of the data by Jones (1970) gave 7.4 km. The effect of including echoes from trains of unknown orientation and $\alpha$ will produce a spread in the data of Greenhow and Hall since $T$ was a measure for the echo amplitude to decrease by a constant factor. This introduces generally reduced values of $D_a$ for lower altitudes since larger line densities are generally produced at lower heights so that the slope of any $\log D_a$-height plot will be consequently increased. However, the results of Jones (1975) based on laboratory
Fig. 6.6 Echo height versus height obtained from the exponential decay slope.

measurements suggest that the gradients of the plots of Greenhow and Hall are already too small. Greenhow and Hall also considered the variation of $\log D_e$ versus height for two reflection points on the same meteor train and obtained results consistent with those for individual reflections. Trains with transition values of $\alpha$ however would produce similar increases in both values of $T$ and therefore the results would indeed be expected to be consistent with the single echo experiment. More recently Brown (1976) has measured echo decay times $T_{1/e}$ using the relatively low radio frequency of
When compared with time-constants calculated using standard atmospheric models, most experimental results exhibit greater values of $T$ with an increasing spread at lower heights. Although consistent with the inclusion of transition type echoes, these results for low heights are particularly susceptible to train distortion by atmospheric winds. The considerable scatter in decay time-constant data is a feature of both backscatter and forward scatter modes (Rice and Forsyth, 1963). Values of $D_a$ using a very high resolution radar (Southworth, 1968) also exhibit considerable spread suggesting that additional factors have a large effect on train diffusion.

Using multi-frequency radio reflections from the same train neutral atmosphere variations can be eliminated. The dual frequency experiment of Greenhow (1952b) found that although the most probable exponent for the wavelength dependence of $T_{1/e}$ was 2 there was considerable scatter about this value. Since $g_{||} \sim \exp[-\beta (ka)^2]$ a $\lambda^2$ variation would still be expected for decay time-constants for echoes from transition type trains. Using forward scatter observations Rice and Forsyth (1963) selected only those echoes showing exponential decays and occurring simultaneously on three radio frequencies. Results confirmed the statistical validity of the wavelength dependence but still revealed a considerable scatter for individual trains. Rice and Forsyth also considered the effect of a sinusoidally modulated ionization profile and showed that the observed spread in values of $T$ could result from the irregularities along the column. However, Brown and Elford (1971) employing a random modulation concluded that the effect of wind shear rotating the meteor train was more important.

Both diffusion theory and train chemistry have been considered by Jones (1975) who concluded that the interpretation of that ambipolar diffusion determines echo decay rate is correct. The effect of transition
type echoes on derived $D_a$ values cannot be determined without a knowledge of the train orientation, the method of analysis and most importantly, $\alpha$.

Since transition echoes generally occur at lower heights where the values of $D_a$ are smaller, some of the slow decay echoes may be lost because of the limited sampling time (often $\lesssim 1$ s) of many experiments. Inclusion of such transition echoes, however, must be expected to produce a spread in observed decay height values, and the magnitude of the spread in $D_a$ as deduced from decay rates is not at variance with this effect. For purposes of illustration Table 6.2 shows the estimates of $D_a$ that would result from echo decay measurements for three definitions of $T$ and different $\alpha$ for a reflection height of 95 km. $T_g$ is the decay constant measured from the slope of the exponential part of the echo profile, $T_{1/e} (g_{||})$ that measured from amplitude decrease for the case of parallel reflection coefficients, and $T_{1/e} (g_{\perp})$ that measured for the case of transverse coefficients.

Table 6.2 Dependence of measured $D_a$ ($m^2 s^{-1}$) on $\alpha (m^{-1})$ and method of measurement.

| $\alpha$  | $T_g$ | $T_{1/e} (g_{||})$ | $T_{1/e} (g_{\perp})$ |
|-----------|-------|--------------------|-----------------------|
| $1 \times 10^{13}$ | 8.48  | 7.57               | 19.7                  |
| $5 \times 10^{13}$ | 6.78  | 5.65               | 10.6                  |
| $1 \times 10^{14}$ | 6.04  | 4.35               | 8.48                  |
| $5 \times 10^{14}$ | 3.44  | 1.64               | 2.52                  |
| $1 \times 10^{15}$ | 2.37  | 0.43               | 1.00                  |

The calculations are based on an actual ambipolar diffusion coefficient at 95 km of 8.48 $m^2 s^{-1}$ (Barnes and Pazniokas, 1972). Only for measurements of
the slope of the exponential part of the echo decay and for $a < 10^{-13}$ m$^{-1}$ is the estimate of $D_a$ good.

6.2.3 Non-isothermicity

After formation the meteor train undergoes radial expansion controlled by the heavy positive ions. Assuming electrical neutrality and in the absence of the geomagnetic field, the ambipolar diffusion coefficient is

$$D_a = D_i (1 + T_e/T_i)$$

where $D_i$ is the ion diffusion coefficient and $T_e, T_i$ the electron and ion kinetic temperatures respectively. It is usually assumed that the electron and ion temperatures are identical. The collision processes leading to the train formation result in the formation of thermal electrons and ions. The heat exchange between electrons and atmospheric molecules may be less efficient than that for meteoric ions and atoms, resulting in slower electron cooling and hence a time varying ambipolar diffusion coefficient reducing to $D_a = 2D_i$ in the isothermal case. Measurements of changes in $D_a$ with time obtained from decay rates of meteor echoes may be interpreted as being due to electron cooling within the train.

Such an experiment has been carried out by Delov (1975) using a pulsed meteor radar ($\lambda = 8.13$ m). Individual pulse amplitudes were recorded and the decay rate (and hence $D_a$) deduced from short segments of the echo envelope. For an underdense echo

$$g \propto \exp \left( \frac{-16\pi^2 D_a}{\lambda^2} t \right)$$

so sequential pulse amplitudes $A_n, A_{n+1}$ separated by $\Delta t$ may be represented by

$$A_{n+1} = A_n \exp \left( \frac{-16\pi^2 D_a \Delta t}{\lambda^2} \right)$$
Fig. 6.7 Reflection coefficient magnitude
\[ g_\theta \cdot (\alpha = 5 \times 10^{-13} \text{m}^{-1}) \]

Fig. 6.8 Apparent diffusion coefficient variation with time. \( (D_a = 13 \text{m}^2 \text{s}^{-1}, \ r_0 = 0.85 \text{m}, \ \alpha = 5 \times 10^{-13} \text{m}^{-1}) \)
Several values of $D_a$ can then be estimated during an echo duration.

Delov's results may be summarized:

1) In 96% of echoes $D_a$ decreases with time.
2) 78% of $\Delta D_a$ values lie in the range $0-16\,\text{m}\,\text{s}^{-1}$.
3) On average, $D_a$ becomes constant approximately 0.17s after maximum amplitude.

Apart from electron cooling effects in the meteor train, an apparent decrease with time of $D_a$ can be produced by any mechanism that increases the echo amplitude above the expected decay expression early in the echo lifetime. The plasma resonance phenomenon has this effect for low $\alpha$ trains, the type considered by Delov. Antennas of either horizontal or vertical polarization will receive echoes whose reflection coefficients are composed of contributions from both $g_\parallel$ and $g_\perp$ depending on the train orientation. The inclusion of a perpendicularly polarized component results in amplitude enhancement, and the use of sequential amplitude ratios will produce an apparent increase in $D_a$ during the resonant scattering period (section 6.2.2). Only for purely parallel polarization will $\beta D_a$ remain constant. The deduced value of $D_a$ then becomes constant ($=\beta D_a$), when $g_\perp$, or more generally $g_\theta + g_\parallel$, later in the echo lifetime. The nature of the apparent time variation of $D_a$ will be influenced by $\alpha$, train inclination to the incident wave polarization $\theta$, radio wavelength used and echo altitude.

Delov used decay echoes and the reflection coefficient for $\alpha = 5 \times 10^{13}\,\text{m}^{-1}$, thought to be typical, is shown for various inclinations $\theta$ in fig. 6.7. The value of $D_a$ becomes constant when $g_\theta + g_\parallel$ or when the polarization ratio $\rho = 1.0$. From fig. 5.6, the times from maximum amplitude to $\rho = 1.0$ have been calculated for $\lambda = 8.13\,\text{m}$, $D_a = 13\,\text{m}\,\text{s}^{-1}$ (Delov) and $r_0 = 0.85\,\text{m}$ (Baggaley 1970), values appropriate to Delov's experiment, (although for the typical altitude of Delov's echoes, 92 kms, Barnes and
Pazniokas quote $D_a = 4.6 \text{ m s}^{-1}$). These results are presented in table 6.3, and are in agreement with those of Delov, however a spread of echo heights will produce a corresponding variation in these values.

Table 6.3 Time from $\lambda_{\text{max}}$ to $\rho = 1.0$ for various $\alpha$

<table>
<thead>
<tr>
<th>$\alpha$ (m$^{-1}$)</th>
<th>t (secs)</th>
</tr>
</thead>
<tbody>
<tr>
<td>$1 \times 10^{13}$</td>
<td>0.10</td>
</tr>
<tr>
<td>$5 \times 10^{13}$</td>
<td>0.15</td>
</tr>
<tr>
<td>$1 \times 10^{14}$</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Beginning at maximum amplitude, four values of $D_a$ have been deduced from the amplitude ratios at $\Delta t = 0.036s$ apart for $\alpha = 5 \times 10^{13}$ m$^{-1}$. These are shown in fig. 6.8. Early values of deduced $D_a$ are increased due to resonant scattering, the values subsequently reducing to $S_D D_a$, with $\Delta D = 8.2 \text{ m s}^{-1}$, a similar magnitude to Delov's results.

Fig. 6.8 represents the single case when $\alpha = 5 \times 10^{13}$ for transverse polarization, representing the largest expected variation. Any intermediate inclination to the incident wave will have smaller changes $\Delta D$. For increasing $\alpha$, the plasma resonance peak occurs later in the echo lifetime (fig. 5.6), and the polarization ratio takes longer to reduce from its maximum value to unity. Hence $D_a$ measured at constant time intervals after the peak amplitude value may not have reduced to the base value $D_5 = S_D D_a$ after 5 measurements, depending on what portion of the curve $D_5$ is obtained.

When $D_a$ is large (i.e. at high altitudes) the reduction of $g_0 \rightarrow g_1$ is accomplished rapidly so the apparent variation in $D_a$ with time will also be larger. However, the increase with altitude of $r_0$ may reduce the occurrence of resonance peaks for low electron densities. The effect of
using a shorter radio wavelength also increases the observed reduction rate of the polarization ratio, resulting in an inverse relationship of the apparent rate of change of $D_a$ with wavelength.

The possible increase of $\Delta D$ with increasing $D_a$ (or altitude) suggested above may explain the observed dependence of $\Delta D$ on meteoroid velocity observed by Delov. Higher velocity meteoroids tend to produce their ionization at greater altitudes where the ambient $D_a$ is larger, and hence $\Delta D$ may be greater. However, data pertaining to all heights were used in Delov's analysis, and the true dependence of $\Delta D$ on meteor velocity should be considered at separate heights to eliminate this possibility.

The above interpretation of Delov's results suggest that scattering phenomena and not electron cooling may be responsible for the observed apparent time dependence and reduction of $D_a$. Also, calculations by Baggaley and Webb (1977) indicate that electron cooling times are of the order of $10^{-2}$ s rather than the value of 0.17s of Delov. Two experiments are suggested to determine if scattering or cooling processes are responsible for these effects.

The first utilizes two radio frequencies and would apply the above analysis to reflections on both frequencies from a common train. The apparent variations of $D_a$ deduced in this manner would be expected to show a wavelength dependence that could not be attributed to electron cooling, an identical result for both frequencies being anticipated in this case. More conclusively, the plasma resonance is restricted to echoes with a transversely polarized component, whereas only a parallel polarized component shows a uniform exponential decay. An experiment to measure two perpendicularly polarized components, specifically $g_\perp$ and $g_\parallel$, and subjected to Delov's analysis would be expected to yield a time varying and a constant $D_a$ respectively. Electron cooling and hence a true time dependence of the diffusion coefficient should be independent of polarization. In general
meteor train orientation is unknown, however this deficiency may be reduced by observing during periods of known meteor shower activity. Observations for both polarizations need not be confined to the same meteor train, and although amplitude reduction was shown to produce unreliable estimates of $D_a$ in section 6.2.2, only changes in $D_a$ are required so this method may be employed. Any contamination by sporadic meteors of unknown orientation will increase $\Delta D$ for the parallel polarization case and reduce it for the transverse case, and so will not enhance any differential result. A further complication is Faraday rotation of the plane of polarization which may be appreciable since the signal travels both out to the train and back through the lower ionosphere. This effect can be reduced by the choice of short radio wavelengths, or a location and antenna arrangement such that the majority of detected echoes lie in a direction that is perpendicular to the geomagnetic field direction. A true decay-type thermalization experiment must eliminate polarization effects, and also avoid any Fresnel diffraction effects which may produce enhanced amplitudes near maximum amplitude for low electron line density echoes.

6.3 PHASE CHARACTERISTICS

6.3.1 Apparent Radial Velocities of the Reflection Point

A Gaussian radial distribution of ionization may be regarded as a totally reflecting cylinder of critical radius $r_c$ given by the condition that $\kappa = 0$

$$r_c^2 = a^2 \log_e \left[ \frac{\alpha e^2 \pi mc^2 \varepsilon_0 (ka)^2}{2} \right]$$

with $e$ and $m$ electron charge and mass, $\varepsilon_0$ the free space permittivity and $c$ the velocity of light. Because of diffusion, $r_c$ initially increases and subsequently decreases again as the electron volume density decreases in
the region of the column axis. Kashcheyev and Delov (1974) have suggested that such a movement of the reflection point will lead to additional radial velocity components and also to a changing velocity when the wave finally penetrates the column. The corresponding apparent radial velocity is given by

\[
\frac{dr}{dt} = \frac{c}{2D_a} \left[ \log_e \left( \frac{ae^2}{\pi mc \varepsilon_0 (ka)^2} \right) - 1 \right] / \left[ a^2 \log_e \left( \frac{ae^2}{\pi mc \varepsilon_0 (ka)^2} \right) \right]
\]

This apparent velocity, initially large and positive and which continually decreases to become large and negative when \( \kappa(r=0) = 0 \), is shown in Fig. 6.9 for a height of 92 km and \( \lambda = 10 \) m. A velocity towards the observer is taken as positive. Refraction by ionization outside the critical radius (Manning 1953) has the effect of slightly reducing both the critical radius and also the associated apparent velocities.

If measured phase changes, \( \frac{d\phi}{dt} \), are interpreted as radial movements, the radial doppler velocity is

\[
v_r = \frac{\lambda}{4\pi} \frac{d\phi}{dt}
\]

Velocities derived from phase changes of the parallel reflection coefficient are shown in Fig. 6.10 for a height of 92 km and \( \lambda = 10 \) m. The velocity, \( \frac{dr}{dt} \), in Fig. 6.9 could equivalently have been obtained from the phase of the totally reflecting cylinder reflection coefficient. Rather than using the continuous differential \( d\phi/dt \) discrete changes of phase have been employed since this is the method of most meteor wind experiments. It is clear that significant apparent radial velocities are introduced for line densities as low as \( a = 10^{14} \) m\(^{-1} \). For transition line densities these velocities decrease to approximately zero rather than attain the large negative values predicted on the basis of metallic cylinder scattering.
Fig. 6.9 Behaviour of the critical radius and its associated velocity.

\[ \lambda = 10 \text{m}, D_a = 5 \text{m}^2\text{s}^{-1}, r_0 = 0.5 \text{m}. \]

Total reflection requires not only that \( \kappa(r=0) < 0 \) but also that the ionization density gradient be sufficiently large so that at the critical radius

\[ \left. \frac{d\kappa}{dr} \right|_{r_c} \gg 0. \]

Although the condition of a non-zero critical radius may persist for most of the echo lifetime for the case of large \( a \), the diffuse nature of the Gaussian ionization profile implies that the second condition is only satisfied early in the lifetime. Hence the totally reflecting model is only appropriate for small \( (ka) \) as shown by the reflection coefficient behaviour for transition type line densities. Velocities derived on the basis of a totally
reflecting cylinder model are applicable only in the early stages of an echo and such velocities later assume zero values rather than large negative values. The reflection coefficient phase changes for transverse incidence or for an arbitrary angle of incidence are smaller than for the parallel case and also exhibit a more complicated behaviour (Chapter 5). For large $\alpha$ however, an increasing effective scattering cylinder radius again produces a decreasing radial velocity component towards the observer.

Although the general Fresnel diffraction case has not been included in the present discussion, its effect on velocities deduced from meteor echoes has been considered by Kaiser (1955). Once the train is formed a constant drift results in a steady phase shift equivalent to doppler shifted frequency, provided the train has a uniform ionization profile. The case
of non-uniform ionization has been considered by Rice and Forsyth (1963) whose model consisted of a sinusoidal modulation on a realistic ionization profile. With this model the phase fluctuations only become appreciable (~ π/4 rads) later in the echo lifetime. The random modulation of the ionization profile considered by Brown and Elford (1971) also exhibits no large phase fluctuations for the Fresnel diffraction echo.

6.3.2 Comparison With Experiment

The effect on wind data of the inclusion of echoes from overdense trains has received little attention. Baggaley and Wilkinson (1974) have considered the correlation between half hour velocity averages derived from underdense and overdense type echoes. Although representative scattering radius movements (Fig. 6.10) are less than the sample standard deviation of about 30 ms⁻¹, they are expected to be a systematic feature. The regression lines of Baggaley and Wilkinson however, reveal no significant displacement and hence no systematic difference. Although the sample of underdense echoes used showed the form of an exponential decay it was shown above that these may in fact contain significant numbers of transition echoes. In a low sensitivity radar system the decay distinction may be insufficient to reveal any systematic differences between echo types. Greenhow and Neufeld (1956) have analysed meteor wind records to resolve the phase changes associated with plasma resonance. It has been shown (section 6.1) that such phase changes are in accord with theory. Dual frequency operation employed by Muller (1970) has been used to eliminate the effects of plasma resonance to obtain consistent wind results. The results of Manning, Villard and Peterson (1950) from dual frequency observations revealed some spread in the velocities derived from each radio-frequency, but could not be expected to yield systematic differences unless reflection point movement both towards and away from the observer were considered
separately. Kashcheyev and Delov (1974) used meteor wind data in an attempt to identify the effects of train diffusion on overdense echo reflection point motions but considered only the times late in the echoes when the amplitudes decrease. However, velocity changes for overdense echoes occur throughout the echo lifetime and not only when the density becomes subcritical. In most transition echoes the major velocity changes are expected to occur early in the echo lifetime, any change when the wave penetrates the column being small. In addition the creation of multiple reflection centres on wind distorted trains and the associated interference effects restrict the observation of velocity data to times $\lesssim 0.2$ s (Philips, 1969). Diffraction phenomena and associated phase changes during train formation generally obscure radial velocity effects and may therefore restrict the observation of such velocity changes in low density echoes and for slow meteors.

Kashcheyev and Delov presented data to support the observation of a velocity change at times of amplitude decrease in echoes from overdense trains; however such changes cannot be differentiated from those corresponding to shearing movements of the reflection point along the meteor column. The single station technique has been used by Muller (1968) to resolve such shears by observing second order phase changes in the returned signal. Adopting the convention that a velocity towards the observer is positive, then for linear shears all observed accelerations should have a positive sign. Muller's data, however, produced a significant proportion (20-40 percent) of echoes showing shears of negative accelerations. Muller considered that such accelerations could arise from either the creation of two reflection centres having opposite velocities one of which becomes non-specular, or, since the negative accelerations were essentially instantaneous and were not accompanied by echo fading they were a product of non-linear shears along oscillatory velocity profiles having very small scale.
Fig 6.10 shows in fact, that instantaneous negative accelerations are to be expected (quite independent of any train shearing motion) for $\alpha \geq 10^{14} \text{ m}^{-1}$.

The magnitudes of the radial accelerations predicted using Fig. 6.10 (25-100 ms$^{-2}$) are resolvable by Muller's experiment, whose minimum resolution was 5 ms$^{-2}$. For an echo slant range of 250 km the associated apparent train shears are in accord with the shears observed by Muller. For numerical illustration Table 6.4 gives the theoretical radial velocities, $V_r$, (measured at $T = 0.05$ s), accelerations $\Delta V_r/\Delta T$ (for $0.05 < T < 0.15$ s) and associated apparent shears $\Delta V_r/\Delta x$ (after Muller 1968) where $x$ is measured along the meteor train. Parameters used were $\lambda = 10$ m, $\alpha = 10^{15} \text{ m}^{-1}$ with initial radius $r_0$ after Baggaley (1970) and $D_a$ after Barnes and Pazniokas (1972).

Table 6.4 Theoretical motion of echo reflection point.

<table>
<thead>
<tr>
<th>height (km)</th>
<th>$V_r$ (m s$^{-1}$)</th>
<th>accel. ($\text{m} \text{s}^{-2}$)</th>
<th>apparent shear $\Delta V_r/\Delta x$ ($\text{m} \text{s}^{-1} \text{km}^{-1}$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>80</td>
<td>2.7</td>
<td>- 7.3</td>
<td>5.4</td>
</tr>
<tr>
<td>85</td>
<td>4.7</td>
<td>- 16.7</td>
<td>8.2</td>
</tr>
<tr>
<td>90</td>
<td>7.9</td>
<td>- 36.5</td>
<td>12.1</td>
</tr>
<tr>
<td>95</td>
<td>11.3</td>
<td>- 61.5</td>
<td>15.7</td>
</tr>
<tr>
<td>100</td>
<td>14.0</td>
<td>-123</td>
<td>22.2</td>
</tr>
<tr>
<td>105</td>
<td>5.5</td>
<td>-155</td>
<td>24.9</td>
</tr>
</tbody>
</table>

Because of the effects of diffraction at train formation and signal facing later in the echo lifetime the majority of experiments have in fact measured $V_r$ in the interval $0.5 < t < 0.15$. 
CONCLUSIONS

The wave matching technique has been used to obtain complex reflection coefficients for radiowave scattering from a column of meteoric ionization. The behaviour of reflection coefficients has been used to examine parameters associated with radio-meteor characteristics and in particular their relevance to meteor wind observations. Calculations for the backscatter mode indicate that for transition line density trains an exponential echo decay may indeed be observed but the decay rate is not described by the usual underdense decay expression. The decay slope is systematically decreased for increasing $\alpha$ as a result of refractive effects in the subcritical density column. Measured ambipolar diffusion coefficients are consequently in error and may also exhibit considerable spread at a given height depending on the inclination of the column to the incident wave and on the echo analysis technique employed. Such systematic errors will be reflected in incorrect values of diffusion scale height for the meteor region.

A serious observational effect of train diffusion is the appearance of spurious radial velocities and velocity changes in any train where the dielectric constant is negative for a sufficiently long period. For parallel polarization the associated acceleration is always negative and is present for trains $\alpha \geq 10^{-14} \text{ m}^{-1}$. Data from a low sensitivity meteor radar may include significant numbers of transition and overdense type echoes in velocity samples. Systematic errors may be introduced into derived prevailing wind components and, since trains of larger electron density are generally produced at lower heights, into the corresponding velocity gradients. Contamination of observed tidal components may also arise from diurnal variations of parameters involved in train formation. The present calculations indicate that a knowledge of echo amplitude and particularly of train
electron line density is of considerable importance in assessing the reliability of doppler velocity data. The use of high sensitivity radars, i.e. $\alpha << 10^{13} m^{-1}$ is also seen to be important, not only to provide statistical reliability of the derived wind components but to reduce the possible systematic variations of both radial velocity and decay time-constants introduced by the selection of a substantial fraction of echoes from high density trains.

Although the apparent radial velocities are consistent with shear values observed by Muller (1968), the possible shear interpretation of a changing radial velocity indicates a need to consider individual echoes. The parallel reflection coefficient phase changes exhibit a $\lambda^2$ dependence suggesting the employment of a multi-frequency experimental arrangement. An experimental knowledge of both $\alpha$ and train orientation is essential. Some control of orientation can be realized by observing during periods of known meteor shower activity. Such an approach would permit the determination of the contribution of diffusion contaminated radial velocity measurements and hence their effect on the results of radio-meteor wind data.

The difference between the terms decay and underdense as applied to meteor echoes should be emphasized. To obey the underdense scattering expression, a trail should have $\alpha \leq 10^{13} m^{-1}$, whereas echoes from transition line density trains also exhibit decay behaviour.
CHAPTER 7

OBSERVATIONS OF SPORADIC METEOR ECHOES

INTRODUCTION

Results using the equipment described previously are presented, and in particular information pertaining to the performance of the system gathering meteor echoes. Most of the data referred to were obtained during a ten day period beginning July 7th 1977. Where possible the apparatus was operated continuously with differential phase calibrations being carried out twice per day using the c.w. method (section 4.4). During much of this period echoes thought to be associated with auroral activity were observed, mostly on the south directed antennas. Such echoes may be avoided by the use of the range gate, but severe conditions resulted in no data being available for approximately six hours during the run. The sensitivity was also reduced by the effect of moisture due to atmospheric conditions altering the impedance matching sections to the transmitting antennas for short periods. Observations were carried out by observing for successive thirty minute periods on the south and west antenna sets. A subsequent run of five days duration was performed in August in conjunction with the partial reflection ionospheric drift experiment located at Birdling's Flat, however this data has not yet been fully analysed.

7.1 SYSTEM SENSITIVITY

The sensitivity of the system can be estimated from the received echo parameters and compared with the value of $\alpha_m \sim 7 \times 10^{12} m^{-1}$ obtained for the limiting line density in section (4.10). The mass-frequency distribution of incident sporadic meteoroids $\nu_m$ is (Kaiser, 1953)
\[ v_m \, dm = b \, m^{-S} \, dm \]

where \( b \) is a constant, \( m \) the meteoroid mass and \( S \) the mass exponent. Hence the number of meteoroids with masses in excess of \( m \) will be

\[ n(>m) \propto m^{1-S} \]

and since the electron line density is proportional to the parent meteoroid mass, a similar expression results for \( n(\alpha_m^\alpha) \), so

\[ \frac{d \log n(>\alpha_m^\alpha)}{d \log \alpha_m^\alpha} = 1-S \]

The radar equation (5.4) shows the maximum received amplitude \( A_m \) to be proportional to the maximum reflection coefficient which is

\[ g_{\| \text{max}} = \alpha \pi r_e \text{ for underdense,} \]

and

\[ g_{\| \text{max}} = \left[ \frac{\pi}{4} \left( \frac{e^2}{\pi \bar{E}_0 mc^2} \exp(-1) \right)^{1/2} \right]^{1/2} \text{ for overdense echoes.} \]

For underdense echoes \( A_m \propto \alpha \) and hence

\[ \frac{d \log (>A_m^\alpha)}{d \log A_m^\alpha} = 1-S \]

while for overdense echoes \( A_m \propto \alpha^{1/4} \) so

\[ \frac{d \log n(>A_m^\alpha)}{d \log A_m^\alpha} = 4(1-S) \]

Hence a plot of the logarithmic cumulative distribution of maximum amplitudes results in two distinct linear regions corresponding to the two echo regimes. The transition echo region may be included by using the results
of fig. 5.10 for an assumed $S$ value. Jones and Collins (1974a) conclude $S = 2.17$ but recent results from the high sensitivity Illinois meteor radar (Hess and Geller, 1976) suggest a value closer to 2. With $S = 2$ the expected form of the $\log n(A_m)$ (equivalent to $(1-S)\log a$) versus $\log A_m$ (equivalent to $\log g_{\text{max}}$) is shown in fig. 7.1a using the full wave solutions for parallel polarization (section 5.7).

If the experimental curve clearly defines both asymptotic regions, the point of intersection corresponding to $a = 5.8 \times 10^{13} m^{-1}$ may be used to estimate the sensitivity from the corresponding echo amplitude. This only accounts for the case of parallel polarization however and in general the train orientation is unknown and the polarization may be influenced by Faraday rotation. For an inclination of $\pi/4$ to the incident polarization, the echo amplitude is increased by 50%, the intersection then corresponding to $a = 3.4 \times 10^{13} m^{-1}$. For $S$ values greater than 2, the intersection is shifted to larger electron line densities, for example $a = 2.4 \times 10^{14} m^{-1}$ for $S = 2.2$, so the appropriate $S$ value must be determined prior to estimating the sensitivity.

To encompass both the underdense and overdense portions of the curve a range of approximately four orders of magnitude for $a$ is required. The present receiver offers a dynamic range of less than 40 db which is further reduced by the system triggering level to approximately 28 db, reducing the equivalent electron line densities to two orders of magnitude near the point of intersection. Portions of a restricted $\log n(A_m)$ versus $\log A_m$ curve identified as straight lines will shift the point of intersection and produce apparent values $S_a$ identified from the underdense region that exceed the true $S$ value. The intersection will be further smoothed by the effects of finite initial train radius reducing the maximum amplitude for short duration echoes and the inclusion of echoes of unknown polarization from throughout the entire antenna beam.
Fig. 7.1a Cumulative maximum amplitude distribution for S=2.

Fig. 7.1b Experimental cumulative maximum amplitude distribution

\[ \log_{10} N(\%) \]
An experimental curve using four days data during which the transmitter output power was maintained substantially constant is shown in fig. 7.1b. The use of large numbers of echoes (8658 for south, 6479 for west) will help overcome the effect of smoothing over the large antenna gain variations, a trial correction of these results by $R^{3/2}$ introducing little variation to the resulting curve. Most echoes are expected to reach maximum amplitude within the present sampling time of 0.47s. The departure from the straight line at low amplitudes is a result of the trigger level setting while the curve is prevented from extending to lower log $n$ values by receiver saturation. The slope of the linear portion for the south echoes is -1.10, and a second lines whose slope is four times this value is also fitted. The intersection occurs near $A_m = 80$ $\mu$V input and if the polarization correction is ignored to counter the bias introduced by the restricted dynamic range, the line density corresponding to 5 $\mu$V is approximately $3 \times 10^{12}$ $\text{m}^{-1}$ (from fig. 5.10). The west directed antennas are less sensitive during this period, recording 25% fewer echoes, consistent with the departure from the underdense slope for lower amplitudes than the south echoes. An estimate of the point of intersection for the west data shows $A_m = 64$ $\mu$V, an amplitude decrease of 21%. Because of the narrow range of values and the corresponding difficulty in determining the true intersection, these values may be optimistic, but are similar to those arrived at by considering the equipment parameters.

Alternatively, if the system sensitivity can be reliably determined the apparent mass exponent $S_a$ may be corrected. Following Jones and Collins (1974a)

$$
\frac{d \log n}{d \log m} = 1 - S = \frac{d \log n}{d \log A} \frac{d \log A}{d \log m}
$$

$$
= (1 - S) \frac{d \log q}{d \log a} = (1 - S) \beta
$$
where \( \alpha \) may be used for \( g \) since the slope is unchanged by polarization effects and \( \beta \) may be evaluated from fig. 5.10. From section (4.10), the limiting line density was estimated at \( \alpha = 7 \times 10^{12} \text{ m}^{-1} \) and the average observed echo probably exceeds the limiting value by 1 magnitude (Jones and Collins), equivalent to \( 1.8 \times 10^{13} \text{ m}^{-1} \). For this line density \( \beta = 0.85 \) and hence \( S = 1.95 \).

As a third alternative the system sensitivity may be estimated by assuming a value of \( S \) and hence finding an experimental value for

\[
\beta = \frac{1-S}{1-S_a}
\]

The corresponding \( \alpha \) may then be found using fig. 5.10 and subtracting one magnitude. These estimates indicate a typical sensitivity around \( 10^{13} \text{ m}^{-1} \) hence at least 90% of recorded echoes will have \( \alpha < 10^{14} \text{ m}^{-1} \) and are suitable for radial velocity measurements although echo decay slopes may contain some systematic decreases.

### 7.2 THE METEOR ECHO RATE

During the ten day observing period a total of 34,000 echoes were recorded, 19,000 of these with the south directed antennas. Despite the use of the variable range window there is little day to day variation in the total echo rate (approximately \( \pm 100 \) echoes) and the ratio of \( n_{\text{north}} \) to \( n_{\text{west}} \) varies little from the mean value of 1.27. The echo rate may be increased by simply reducing the triggering level to a value closer to the noise level, however since the major objective was to determine atmospheric wind velocities, the system was triggered at higher levels required to produce reliable velocity data. The number of acceptable radial velocity
Fig. 7.2 Usable meteor echo rate

Fig. 7.3 Distribution of echo signal to noise ratios.
estimates is reduced by requiring similar valued complete cycle determin-
ations of the boat from both offset channels. With these restrictions the
recovery rate of velocity data is 59% for the south data and 61% for the
west. Departures from these values on individual days are also small,
indicating a consistency of operation. A further reduction is made when
only echoes with altitudes in the 80 to 110 km range are considered, 46%
and 47% of all recorded echoes being now usable from the south and west
respectively.

Hourly echo rates reveal the characteristic diurnal variation. More
important for statistically reliable velocity averages however is the rate
of obtaining usable velocities in any 30 minute observing period as shown
in fig. 7.2. Also shown is the least squares fit of an average and diurnal
term. The ratio of the mean values is 1.23 indicating a similar recovery
and the phases are separated by 34 minutes, the south rate peaking at
approximately 0400h L.T. The very low rate around 1800h prevents reliable
velocity determinations during this period, the ratio of maximum to
minimum rate being accentuated by the use of narrow beam antennas.

7.3 EXPERIMENTAL UNCERTAINTIES

Assuming 2 \mu V to be representative of the equivalent noise voltage
at the receiver input, the average signal to noise ratio is calculated by
the program during the first 15 pulses, during which time the reflection
point position is determined. The distribution of the average signal to
noise ratios is shown in fig. 7.3 where the lower overall sensitivity of the
west system results in a larger proportion of echoes with low values. The
mean values are 19.7 db and 18.4 db for south and west respectively and
although taken only over the first part of the echo are probably represen-
tative of typical values.
7.3.1 Reflection Point Location

In section (2.8) estimates of the resolutions of \((R, \theta, \phi)\) were obtained assuming a Gaussian noise origin for the variances, for a single determination of these quantities. The phase uncertainty due to quantization may be expressed as

\[
\Delta \phi \sim \sin^{-1} \frac{\Delta A}{2A}
\]

where \(\Delta A\) is the quantization interval (19.6 mv) and \(A\) the signal amplitude. This quantity is generally less than the noise contribution and may simply be added to it, so the sample standard deviation is

\[
S_\phi = \sqrt{\frac{C}{2B} \left( \frac{S_\phi}{N} \right)^{-1} + \frac{\Delta A}{2A} \frac{1}{\sqrt{n}}}
\]

where \(n\) is the number of determinations of \(\phi\). This quantity is shown in fig. 7.4a for \(n = 15\) together with experimental data for \(S_{\phi_{13}}\). Although the experimental values exceed the estimates by a factor of 2, typical echoes with signal to noise ratios near 20 db have \(S_\phi \sim 0.1\) rads, similar to the values used in section (2.8). Contributions to the increased variance may come from non-uniform receiver phase response. These uncertainties however are independent of system effects originating in the antennas such as ground effects and mutual coupling. Results for \(S_{\phi_{12}}\) are similar to fig. 7.4a and the correlation between \(S_{\phi_{12}}\) and \(S_{\phi_{13}}\) indicates a predominantly noise origin for the variance.

Similarly for the echo range,

\[
S_R = \frac{C}{2B} \left( \frac{S_R}{N} \right)^{-1} + \frac{R_Q}{\sqrt{n}}
\]

where \(R_Q\) (= 375 m) is the quantization for both pulse edges, and this is plotted in fig. 7.4b where typical standard deviations are \(\leq 500\) m. The reason that estimates for both echo range and phase angles are optimistic.
may be that the estimate of the noise level is too low. The experimental values obtained are similar to those used in section (2.8), indicating that the earlier resolution estimates are realistic in the absence of systematic effects.

7.3.2 **Radial Velocity Uncertainty**

If the variance of each of the two radial velocity determinations, considered independent, is due to similar Gaussian noise, the distribution of velocity differences will also be Gaussian with zero mean and standard deviation $\sqrt{2}$ times that of the individual determinations. The radial
Fig. 7.5 Radial velocity difference distribution.

velocity difference distribution (fig. 7.5) is approximately symmetrical indicating little bias in the radial velocity sensitivity. The increase near \(-12 \text{ m.s}^{-1}\) however is a systematic feature and is also present when results from individual days are considered, but cannot be related to equipment parameters. The major source of bias will be drift of the 30 Hz oscillator however this is expected to be small (section 4.7). Radial velocity values are accepted for further analysis provided \(|\Delta v_r| < 15 \text{ m.s}^{-1}\) which implies a probable radial velocity uncertainty of less than \(\pm 7.5 \text{ m.s}^{-1}\).

The radial velocity uncertainty may be expressed as
\[ \Delta V_\pm = \left\{ \sqrt{2} \frac{f_\pm}{\pi} \left( \frac{S}{N} \right)^{-1} + \frac{2f_\pm^2}{f_c} \right\} \frac{1}{2} \sqrt{n} \]

where \( f_\pm = \left| \frac{2V_c}{\lambda} \pm 30 \text{ Hz} \right| \)

and \( n \) is the integral part of

\[ \left| \frac{2V_c}{\lambda} \pm 30 \text{ Hz} \right| \times 2 \times 0.2s \]

where 0.2s is the mean duration and \( f_c \) the doppler clock frequency (5 kHz, section 3.8.7). In general two velocity values are obtained from signals with similar signal to noise ratios and

\[ |\Delta V_r| = |\Delta V_+ + \Delta V_-| \]

Due to the symmetry of the ±30 Hz offsets, only velocities in one direction need be considered. Theoretical estimates of \(|\Delta V_r|\) vary little with velocity since for an increasing positive velocity the effects of quantization and filter response increase the uncertainty on the positive offset, while reducing that for the negative offset. In section (2.9) it was shown that the signal to noise ratio in the narrow-band doppler channel exceeded the usual amplitude output value (20 dB) by 7 dB. This value is subsequently reduced by the overall velocity sensitivity (section 4.7) to 22.4 dB for a zero radial velocity. Using this value, the appropriate velocity sensitivity curve and an echo duration of 0.2s the expected value of \(|\Delta V_r|\) is 4.7 m.s\(^{-1}\). This compares well with the mean experimental value of 5.9 m.s\(^{-1}\), although the experimental values exhibit a slight increase with increasing radial velocity, for example, \(|\Delta V_r|\) is 7.4 m.s\(^{-1}\) when \(|V_r|\) is 85 m.s\(^{-1}\).

The variation of \(|\Delta V_r|\) with signal to noise ratio has also been considered, however the observed decrease with increasing signal to noise ratio is less than predicted. The experimental signal to noise ratio
determined during the first fifteen pulses may not be appropriate to the
doppler case. Although the mean duration is 0.2 s, the echo durations
above a level sufficient to produce reliable velocities are probably less
than this value.

These considerations check the ability of each offset to measure the
same radial velocity. Determinations may be influenced by wind shear or
scattering effects which may not be apparent in the above type of analysis,
so these figures reflect instrumental effects only. For typical signal to
noise ratios \(|\frac{\Delta V}{V}\| \leq 6 \text{ m.s}^{-1}\) indicating the uncertainty limit for radial
velocity determinations.

7.4 RADIAL VELOCITY DISTRIBUTIONS

The operation of the offset frequency scheme can be verified by
considering the distribution of radial velocities (fig. 7.6) split into
three 10 km height intervals centred on 85, 95 and 105 kms. These distri-
butions have a similar shape indicating that echo duration associated
features, which are expected to be altitude dependent, are not large. The
calculation in section (4.7) indicated the losses of high velocities would
only be appreciable for underdense echoes above an altitude of 102 km,
since most velocities observed are less than 50 ms\(^{-1}\), even above 100 km
(Wilkinson, 1973). The present distributions will be optimistic since the
echo height distribution (fig. 7.11) is approximately Gaussian with a
peak near 95 km. Radial velocity data in the two extreme height intervals
will be biased towards this peak height, reducing any high velocity losses
in the 100 to 110 km group. Although the echo height distribution is
symmetrical about 95 km, the radial velocities centred on 105 km number
2057, whereas 566 more were observed in the lowest stratum (80 to 90 km),
indicating some loss of high altitude velocities.
Fig. 7.6 Radial velocity distributions.

The distribution of echo durations (above twice the noise value, fig. 7.7) shows that although a higher proportion of echoes have shorter durations as the altitude range is increased, the durations are still sufficiently long to permit determinations of radial velocities in all three height intervals. The distribution of echo durations will also be biased towards the modal values by the effect of the echo height distribution. Radial velocities from a higher altitude range, for example 105 to 110 km, have not been considered due to the low echo rate, however some losses of high velocities must be expected at high altitudes. The radial velocity distribution of Hess and Geller (1976) is similar to the present results, but will also contain most echoes from the 90 to 100 km altitude region. Further contamination of the present results may arise from the possible variations of deduced echo heights due to receiver differential phase.
Fig. 7.7 Distribution of echo durations.

changes (section 4.6) and the relative radial velocity response (section 4.7).

The most significant differences between each of the three radial velocity distributions (fig. 7.6) are the shifts in the mean value. These may be translated to horizontal velocities and interpreted as a positive gradient for the meridional mean wind. In general the mean velocity from such a distribution does not equal the mean flow due to the contamination by the diurnal variation in echo rate. For the present three distributions this will be common, but takes no account of tidal variations with altitude, the diurnal sampling variation of which renders the distribution means unreliable. Comparison of the radial velocity distributions with other
published distributions from both meteor wind and rocket data indicate the success of the offset frequency scheme, at least up to altitudes near 100 km.

7.5 **ECHO RATE AS A FUNCTION OF EQUIPMENT PARAMETERS**

Theoretical expressions for the dependence of the echo rate on antenna parameters for shower and sporadic meteoroids are contained in Kaiser (1960, 1961). Assuming a uniform apparent radiant distribution for sporadic meteors, the expression for echo rate dependence is proportional to

\[
\frac{\cos \theta}{\cos \theta_m} \cdot \frac{\sin \theta}{\sin \theta_m} \cdot e^{(3S-7)/2} \cdot s^{2(S-1)}(\theta)
\]

for \( \theta \geq 10^\circ \) (flat earth), provided the antenna amplitude pattern \( S(\theta, \alpha) \) can be expressed as

\[
S(\theta, \alpha) = S(\theta) \cdot S(\alpha)
\]

and for an antenna pattern symmetric about azimuth \( \alpha = 0 \). \( \theta_m \) is the direction of the maximum observed echo rate, which because of the above expression is usually at a lower elevation than the beam maximum, since the echo rate decreases with increasing elevation angle for constant antenna gain.

Experimentally, the vertical radiation pattern may be found from the relative variation in the number of echoes observed at given elevations \( n(\theta) \). For \( S = 2 \),

\[
S(\theta) \propto \left[ \frac{n(\theta)}{\cos \theta \sin \frac{1}{2} \theta_m} \right]^2
\]

and the resulting normalized values for both antenna directions are shown in fig. 7.8. Also indicated are the minimum expected elevation angle based on flat earth geometry and the position of the first maximum and the
Fig. 7.8 Antenna vertical radiation patterns

Fig. 7.9 Antenna horizontal radiation patterns.

--- Previous measurements.
first null in the ground reflection factor. The radiation pattern in the vertical plane is a combination of the free space antenna pattern, which generally has a broader main lobe than the horizontal pattern, and the ground reflection factor. This combination results in an antenna pattern maximum in the vertical at a lower elevation than the ground reflection factor maximum, as observed for both the south and west data. The west antennas have a larger proportion of echoes at greater elevation angles, indicating possible ground variations which may have consequences for the echo location technique (section 2.8.1c). Both sets of data have irregular variations above 36° which may arise from minor lobe behaviour or as a result of possible ambiguous arrival angles for these elevations. Most echoes however are observed at elevation angles less than 30° as expected.

If the elevation pattern is considered uniform across the azimuthal sector of interest, the relative echo rate for given azimuths may be used to verify the horizontal radiation patterns (fig. 7.9). While the south data are in reasonable agreement, those for the west indicate a wider beam than anticipated and a displacement to positive azimuth values. Towards the end of the data run problems associated with the west transmitting array were observed, manifest as occasional arcing across the feed point of the northern-most yagi. This may have altered both the amplitude and phase of the signal delivered to this antenna resulting in radiation pattern variations. Incorrect phasing of the feeds will result in a displacement of the beam direction and a reduction in maximum amplitude, but will generally be accompanied by a reduction in main lobe width. These problems with the west transmitting array, observed as departures from the expected radiation pattern are probably responsible for the overall lower echo sensitivity in this direction. A slowly varying phasing problem would be expected to yield variations in the ratio of echo rates $n_s/n_w$ associated with changes in the mean echo azimuth, however such data are inconclusive.
The effect of an offset beam maximum will bias the average velocity components observed in this direction. For a displacement of $\alpha = 4^\circ$, and considering the radiation pattern as having constant gain across the main lobe results in an uncertainty of 6%, which is less serious than the decreased echo rate for average velocity determinations.

The theory of the echo range distribution and its dependence on equipment parameters is given in Kaiser (1953, 1961). The number of echoes observed in a range interval $dR$ is

$$n_R dR \propto \frac{1}{\alpha_m} \cos \theta \ S^2(\theta) \quad \text{for underdense echoes}$$

and

$$\alpha_m \propto R^{3/2} \quad (\alpha_m \text{ is the limiting line density})$$

so

$$n_R \propto \frac{S^2(\theta) \cos \theta}{R_E^{3/2} \left[ (\sin^2 \theta + \frac{2h}{R_E^2} \right]^{1/2} - \sin \theta \right]^{3/2}}$$

where $R_E$ is the earth's radius. This expression assumes a uniform apparent radiant distribution, a constant echo height and flat earth geometry, the latter restricting the validity to $\theta << 90^\circ$ and $\theta > 10^\circ$. The echo amplitude distribution (section 7.1) indicates most echoes observed are of the underdense type, so the range distribution may also be used to estimate the system elevation radiation pattern $S(\theta)$. Alternatively the pattern deduced earlier may be used to predict the echo range distribution. This has been calculated assuming an echo height of 95 km and is compared with 48 hours data when the range gate was not extensively varied in fig. 7.10. The derived range distribution for overdense echoes peaks at the same value as the underdense case but reduces to zero more rapidly. The agreement suggests these range data are consistent with the patterns derived earlier and support the conclusion that the majority of echoes received are of the underdense type.
7.6 **METEOR ECHO HEIGHTS**

The total meteor echo height distribution depends on the physical characteristics of the meteoroid, its trajectory, the neutral atmosphere and the observing radar parameters. The theoretical height distribution for sporadic meteors using classical ablation theory has been considered by Kaiser (1954). For $S$ values near 2 the equipment has little effect on the observed distribution, which is approximately Gaussian with a standard deviation equal to the atmospheric scale height $H$. Weiss (1960) has considered effects that may increase the standard deviation, such as meteoroid fragmentation, the spread of limiting electron line density values and variations of $H$ itself. The observed echo height distributions for both antenna directions are shown in fig. 7.11, the mean heights for the south
and west being 94.8 and 95.6 km respectively. The similarity of both sets of data is supported by the constancy of the ratio of relative numbers for each height interval and its r.m.s. deviation

\[
\frac{n_s}{n_w} = 1.31 \pm 0.17
\]

For both directions the standard deviation for all height values is approximately 10 km and the appropriate Gaussian function is also shown for the south data. Instrumental effects may influence the height distribution since the interferometer parameters were chosen for an altitude range of 80 to 110 km, and ambiguous phase results occur initially at the extremes of this region. The reduction program assigns heights outside
this region by considering the closest possible alternative to 95 km with the appropriate phase differences. The continuity of the distributions at 80 and 110 km indicate this is satisfactory. Phase angles become ambiguous above approximately 30° elevation for the range 80 to 110 km. However, few echoes are expected above this angle as the influence of the antenna gain and echo rate decrease lead to rapidly reducing echo sensitivity with elevation. Consideration of the echo height distributions over 5° elevation intervals reveal similar distributions to those of fig. 7.11. The observed height distribution is similar to those observed using similar methods, for example Hess and Geller (1976), Spizzichino (1972) and Elford (1966).

Any receiver differential phase changes will result in a variation of the apparent heights of echoes as the corresponding elevation angles vary. This effect would be expected to broaden the central region of the observed echo height distribution, however this is not apparent in 12 hour sets of data. Such height variations should manifest themselves as changes in the mean echo height for sufficiently large samples and corresponding variations in the mean elevation angle. Since the echo rate is greatest using the south directed antennas the mean echo heights will be more reliable, 48 hours of which are shown in fig. 7.12. The uncertainties for each mean height are one standard error of the mean, equivalent to an r.m.s. deviation. Also included are the mean elevation angle for the corresponding period (mean azimuths are generally within ±1° of zero) and the mean decay height (fitted echo amplitude decays with correlation coefficient $|r| > 0.9$) using 2 hour averages of both south and west data to increase sample sizes. The decay heights exhibit a predominantly diurnal variation with a peak value near the observed peak in the echo rate, expected to coincide with maximum meteoroid velocity. The higher mean velocity implies a greater mean height (McKinley, 1961) and similar decay height behaviour is reported.
by Hess and Geller (1976). The departures of the mean echo height from the decay height variation are also shown in fig. 7.12, but these exhibit little correspondence with the mean elevation angles. The mean echo height variation is expected to be larger than that for decay heights since the two hour averages will introduce some smoothing. During the times of high echo activity the mean elevation angles show little variation from the overall mean value of 21°, indicating there are no large changes due to
differential phase changes.

Apparent decay height variations may be a result of variations of the true mean meteor echo height or changes in the diffusion coefficient used to deduce the decay height as a result of neutral atmosphere density variations of tidal origin. Although individual decay heights are considered unreliable, mean values are thought to represent true height variations. Haurwitz (1964) has deduced expected tidal atmospheric pressure variations from experimental meteor wind velocities using the constitutive tidal equations. Provided atmospheric temperature variations are small

\[
\frac{\delta h}{H} \sim \frac{\delta p}{P} \sim \frac{\delta \rho}{\rho}
\]

where \(\delta P\) and \(\delta \rho\) are pressure and density perturbations respectively.
Assuming the pressure variations have similar latitudinal variations to the ground level pressure data; diurnal proportional to \( \sin \theta \) and semi-diurnal proportional to \( \sin^3 \theta \), and translating Haurwitz's results to latitude 43°S, the respective percentual density and hence \( D_a \) variations are

\[
\frac{\delta D_a}{D_a} \leq 1\% \quad \text{for the diurnal and}
\]

\[
\leq 5.5\% \quad \text{for the semi-diurnal variation.}
\]

To be responsible for a mean decay height variation of ±1.5 km the variation required in \( D_a \) is \( \pm 2.3 \, \text{m}^2\text{s}^{-1} \) so

\[
\frac{\delta D_a}{D_a} \sim 29\%.
\]

Hence the mean height changes are too large to be accounted for by tidal variations, which should result in a dominant semi-diurnal oscillation. The occurrence of peak height near the time of maximum echo rate and meteoroid velocity also supports a meteoroid rather than atmospheric origin for the variation.

The distribution of decay heights (fig. 7.13) shows that no echoes are observed above 105 km due to the finite meteoroid velocity effect and more importantly, the data length required to determine the decay rate. Also shown are the results obtained by Wilkinson (1973) where the duration aspect was less restrictive, thus resulting in larger numbers above 96 km. The peak height will depend on the expression chosen for the vertical variation of \( D_a \). The present results used values from Barnes and Pazniokas (1972), however in the region 90 to 100 km there is little difference between most published experimental values. Values below approximately 87 km may have been eliminated from Wilkinson's data as a result of film reading, by having the slow amplitude variations considered as being due to overdense echoes.
The decay height distribution is deficient at both altitude extremes when compared with the interferometer height distribution. This feature is also seen in the data of Muller (1972) and Revah (1968). A possible reason for the low altitude loss may be the onset of fading due to the creation by wind shears of multiple reflection points. For an altitude of 90 km \( (D_a = 3.16 \text{ m s}^{-1}, \text{Barnes and Pazniokas}) \) the decay time-constant is 0.26s, compared with the typical time to the onset of fading of 0.2s (Philips, 1969). A low altitude decaying echo may begin to fade and hence be rejected from decay echo data, or interpreted as a more rapid decay placing it at a higher altitude. The relationship of echo decays to neutral atmosphere parameters is not clear, especially on an individual echo basis.

7.7 ECHO DURATION CHARACTERISTICS AS CALIBRATION

In section (7.1) the distinctly different amplitude characteristics of underdense and overdense echoes enabled the equipment sensitivity to be estimated. Since the durations of the two types of echoes are dissimilar, might not this also be used as a measure of sensitivity? Care must be exercised however since the definition of durations are different for the two regimes; a decay time-constant for underdense echoes and the duration to zero critical radius for the overdense case. The first of these is normally assumed independent of electron density while the overdense expression is proportional to \( \alpha \) for the simple overdense model. If the variation in echo height, range, antenna gain and initial train radius are ignored, the duration above a given reflection coefficient threshold will depend on the electron line density and may be expected to show different behaviour when the underdense and overdense approximations are satisfied.

If we assume ambipolar diffusion is the only mechanism responsible for the volume electron density decrease and
and the overdense echo terminates when

\[(ka)^2 = \frac{\alpha e^2}{\pi c_0 mc^2}\]

then \[T \propto \alpha \text{ (for } r_0 = 0) \text{ and } \nu_T \propto T^{-S}\]

hence

\[
\frac{d \log(n > T)}{d \log T} = 1 - S
\]

This expression considers a train with uniform cross section. However, Kaiser (1953) considers the maximum duration that may arise when the specular reflection point is centred on the ionization profile maximum and finds

\[T \propto \alpha^{4/3}\]

and \[\nu_T \propto T^{-3S/4}\]

No similar expression is possible for the exponential decay, but the duration to a particular reflection coefficient \(g_{\parallel \text{ min}}\) is given by

\[(ka)^2 = \log_e \left( \frac{\alpha \pi r}{g_{\parallel \text{ min}}} \right) \text{ (for } r_0 = 0)\].

These expressions together with full wave solutions (section 5.6.1) are shown in fig. 7.14a for a detectable level of \(g_{\parallel} = 0.05\) and \(S = 2\). Unfortunately no distinct intersection of the curves is present as occurs in the amplitude case. The determination of \(S\) will also be doubtful for \(\alpha \leq 10^{16} \text{ m}^{-1}\) and echoes with line densities in excess of this value are infrequent and may be subject to fading unless very high radar frequencies are used. The effect of a non-zero initial radius \(r_0\) is to reduce all durations, and the variation of antenna gain, echo altitude and echo range
Fig. 7.14a Cumulative echo duration distribution.

Fig. 7.14b Experimental cumulative echo duration distribution.
would be expected to smooth any experimental results.

The present results include durations based on the above definition, but only up to a maximum value of 0.47s, when the amplitude memory is filled. These are shown in fig. 7.14b for the south data and are consistent with most echoes being of the low density type, but yield no further useful information. The 0.47s time interval includes 77.9% and 74.9% for the south and west echoes respectively, consistent with the higher sensitivity of the south system (due to antenna variations), since the additional echoes would be of the underdense type and have short durations. Although the echo duration above is largely independent of the effects of plasma resonance, it cannot be used to estimate the system sensitivity in a similar manner to maximum amplitudes.
CHAPTER 8

ATMOSPHERIC VELOCITIES

INTRODUCTION

In this chapter the theoretical descriptions of neutral gas motions expected in the meteor region will be discussed. The velocities used in a typical meteor wind experiment are mean horizontal velocities and the method of obtaining mean velocities and their statistical reliability is considered. Contributions to the variance associated with the determination of the mean velocity are also considered. Methods of spectral analysis are presented and applied to the present velocity data, revealing the significant frequency components suitable for study by the present meteor wind experiment.

8.1 NEUTRAL GAS MOTIONS IN THE LOWER THERMOSPHERE

The overall motion of the atmosphere may be separated for theoretical consideration into several superimposed dynamical systems. The motions considered are not peculiar to the lower thermosphere, but we are restricted to the 80 to 110 km altitude range by the meteor wind experiment. The separation is based on the dominant time scales of the systems, since these and the spatial scales determine the appropriate equation of motion for a neutral gas parcel and any boundary conditions. The dominant motion of the atmosphere is one of rotation, introducing a Coriolis force term into the equation of motion.

The scales of motion that will be considered are the prevailing wind, planetary waves, atmospheric tides, short period internal gravity waves and turbulence. The oscillatory modes are different theoretical manifestations of the internal gravity wave type. These are hydrodynamic
waves whose restoring force is due to the earth's gravitational attraction in the horizontally stratified atmosphere. For example, atmospheric tides are examples of internal gravity waves on a rotating sphere, while they may also be thought of as planetary waves whose forcing and periods are explicitly determined. Wave motions are important in the upper atmosphere since they carry most of the kinetic energy, while their sources are not in general within the region. The vertical propagation of such waves depends on the refractive index 'presented' to them by the atmosphere, in which the vertical temperature structure plays a major role. Although the above separation of the dynamical systems is convenient for theoretical purposes, non-linear interactions between these components may also take place.

8.1.1 The Prevailing Wind

The prevailing wind is a gross global motion whose mean pattern is dominated by an annual cycle. Since data obtained with the present equipment is typically several days in length, the prevailing wind will be present as a constant drift, although day to day variations may result from unresolved long period oscillations or trends. The prevailing winds are predominantly zonal and are approximately geostrophic, that is, the horizontal component of the Coriolis force and the pressure gradient force are in equilibrium. The resulting equation of motion for the zonal wind $u$ (positive eastward) is

$$2p u \Omega \cos \theta = -\frac{\partial P}{\partial y}$$

where the $y$ axis is northward, $\theta$ is the co-latitude, $\Omega$ the earth's angular rotation frequency and $p$ the neutral gas density. This equation may be combined with the condition for hydrostatic equilibrium, using the ideal gas law, resulting in the thermal wind equation

$$\frac{\partial T}{\partial y} = -(2\Omega \cos \theta \frac{T^2}{g}) \frac{\partial}{\partial z} \left( \frac{u}{T} \right)$$
relating $u$ to the meridional temperature gradient. This expression gives a reasonable approximation of the prevailing wind in mid-latitudes, and temperatures derived from the mean zonal flow are in agreement with those from other sources (Murgatroyd, 1965). A discussion of the limitations of the thermal wind equation including dissipation and energy source considerations is contained in Wilkinson (1973).

8.1.2 Planetary Waves

Planetary waves are primarily a lower atmosphere phenomenon although this may be a consequence of the observational state at higher altitudes. Considerable experimental data on the global scale oscillations exists, mainly confined to the troposphere and lower stratosphere (Craig, 1965). Recently evidence in support of long period oscillations, thought to be of planetary scale, has been presented by Pokrovskiy (1970), Muller (1972), Glass et al. (1975) and Portnyagin and Svetogorova (1977) for the meteor region, and also by Cavalieri (1976) and Brown and Williams (1971) for the ionospheric E region.

The theoretical base for a description of planetary waves begins with Rossby waves, which are a consequence of the variation with latitude of the Coriolis force (Platzman, 1968). Large scale motion of an air parcel from a high pressure to low pressure area is deflected parallel to the isobars by the Coriolis force. Departures from this geostrophic equilibrium lead to long period oscillations and since the spherical earth is a closed surface, an integral number of wavelengths exist around constant latitude circles. The theoretical formulation approximates restricted latitude regions with flat surfaces ($\beta$ planes) that include the Coriolis force variation with latitude. Rossby waves are the idealization of geostrophic planetary waves on a plane. They may be free, corresponding to the resonant normal modes.
of the atmosphere, which may be excited by weak forcing such as thermal perturbation or departures from geostrophic equilibrium, or forced waves, generated by and locked to topographical features.

The problem of vertical propagation of planetary waves (which contain a considerable part of the dynamic energy of the troposphere) has been considered by Charney and Drazin (1961). These authors considered a mid-latitude $\beta$ plane using the geostrophic approximation and including a mean zonal flow independent of latitude. The effective refractive index was found to depend principally on the vertical distribution of mean zonal wind. In this model, vertical propagation is only possible in the presence of eastward winds smaller than a critical value dependent on the horizontal scale of the oscillation. If the zonal phase velocity of the wave approaches that of the zonal background wind $u$, the angular wave frequency $\omega$ (as observed in a coordinate system moving with the zonal flow) is Doppler shifted to

$$\omega' = \omega - k.u$$

where $k$ is the horizontal wave-number. When the zonal phase velocity is equal to the mean flow the vertical wavelength tends to zero and the wave is dissipated, dumping its energy and momentum at this critical level. During summer the stratospheric mean zonal winds are westward so all modes should be trapped. In winter the flow is eastward and all but the largest scales are trapped. Only during the equinoxes should any appreciable planetary wave energy leak into the mesosphere.

A spherical, rotating earth model was used by Dickinson (1968), using the linear perturbation theory for stationary waves commonly used in tidal theory. Mean zonal winds were included with variations in both latitude and altitude. The model indicated that planetary wave energy may be ducted (by the zonal winds) away from a mid-latitude source towards the
pole or equatorward. The strong mean zonal winds are insufficient to account for the observed trapping of most planetary wave energy in the lower atmosphere in this spherical model. Dickinson (1969) seeks an alternative mechanism for the trapping of energy in Newtonian cooling as a dissipation mechanism. This results in a reduction of the amplitudes of waves reaching the mesosphere by factors of 2 and 10 for winter and equinoctial periods respectively. Support for the ducting of planetary wave energy may be found in McNulty (1976). This author also finds the upward energy flux is a maximum in winter for stationary waves, in contrast to the results of Charney and Drazin.

These theoretical models indicate that at all times of the year very little energy is expected to leak out of the stratosphere in planetary wave modes. If this were not the case, Charney and Drazin suggest that since large quantities of energy are contained in these modes, extreme heating of the upper atmosphere would take place. The degree of permeability of the upper atmosphere to planetary scale motions is not yet well determined. However, experiments with extended latitudinal and altitude coverage such as the Nimbus IV SCR experiments (Harwood, 1975) may improve the observational state in the mesosphere.

8.1.3 Atmospheric Tides

Atmospheric tides are those oscillations resulting from the earth's rotation in gravitational and radiation fields and so have periods that are fractions of a solar or lunar day. Consider the solar heat input: this has a non-uniform distribution over a single hemisphere which rotates once per day. Hence several modes of oscillation which may be described as a series of spatial waves that follow the apparent motion of the sun are expected. Similarly for the gravitational field case. Equivalently, at a given location, the daily solar heat input has a harmonic content giving rise to harmonically related periods dominated by the diurnal term. Historically, tidal theory was developed to explain the predominance of the
semi-diurnal variation in ground level pressure data, good accounts of which are contained in Siebert (1961), Craig (1965) and Chapman and Lindzen (1970), who also detail the following theoretical considerations.

The spatial extent of tidal oscillations necessitates the use of rotating spherical earth geometry rather than the $\beta$ planes appropriate to Rossby waves. The problem is to determine and mathematically express the forcing and the atmospheric response to the particular modes, weighted by the spatial distribution of the forcing terms. The most important modes are the migrating (sun following) tides whose zonal phase velocities are much greater than the mean zonal flow. The numerous approximations and their regions of validity are discussed in Chapman and Lindzen (1970). Tidal fields are expressed as linearizable perturbations on a mean state and terms higher than first order neglected. By assuming periodic solutions in time and single valued longitudinal solutions, the constitutive equations of motion (including forcing terms), continuity and the first law of thermodynamics are reduced to a single equation in altitude and co-latitude. Assuming the variables are separable results in two second order differential equations - Laplace's tidal equation and the vertical structure equation.

Laplace's tidal equation, appropriate to a uniform fluid on a rotating sphere, is independent of any forcing terms and hence is also relevant for planetary waves where the frequency is an additional free parameter. Its solution is in terms of Hough functions (expansions of associated Legendre polynomials) the set of which is assumed complete so that the latitudinal forcing may be expressed in terms of them. These solutions represent the latitudinal free normal modes of oscillation, and the matching of these by the latitudinal distribution of the forcing function will determine the relative excitation of each mode. Tidal modes are specified by their frequency $\sigma$, longitudinal wave number $s$ and the
order of the latitudinal mode \( n \) introduced by the separation of variables. For example

\[
\theta_2^2(\theta) \quad \theta_n^\sigma s(\theta)
\]

is the first symmetric \((n=2)\), semi-diurnal \((\sigma=2)\) migrating \((s=2)\) Hough mode. In general only migrating modes are considered, \( s \) is suppressed and the above notation becomes the \((2,2)\) mode. Craig (1965) refers to \( n \) as a wave type and \( \sigma=s \) as a wave family. The solution of Laplace's tidal equation is an eigenfunction \((\theta_n^{\sigma} s(\theta))\), eigenvalue problem with associated eigenvalue \( h_n \), called the equivalent depth by analogy with the rotating spherical fluid. The eigenvalue \( h_n \) appears in the separation constant for the horizontal and vertical equations.

The forcing term is expressed as a separable function of altitude and latitude and with the tidal period specified, the equivalent depth \( h_n \) and Hough modes can be determined. The vertical propagation is determined by substitution of the appropriate vertical forcing component and \( h_n \) into the vertical structure equation. The effective atmospheric refractive index depends mainly on the vertical temperature structure and \( h_n \), a positive value of \( h_n \) corresponding to a positive refractive index and hence conditions for propagation. The two major sources of tidal excitation are thought to be solar insolation by water vapour in the troposphere (Siebert, 1961), and a larger contribution by absorption by ozone in the stratosphere (Butler and Small, 1963). In general only symmetric Hough modes (equinoctial conditions) are considered.

**The migrating solar semi-diurnal tide**

The latitudinal expansions of the solar insolation reveals that for both sources the \((2,2)\) mode receives the bulk of the semi-diurnal excitation since it closely approximates the latitudinal form of the inputs. For the \((2,2)\) mode, \( h_n = 7.82 \) implying a propagating mode with a large wavelength in the vertical, so the vertical forcing will also be efficient (the vertical forcing distributions span 10 to 20 km). The \((2,2)\) mode not only receives
the bulk of the excitation, but also responds efficiently and propagates in the vertical, except in the mesosphere where it is evanescent. Sufficient energy leaks through however and this mode is expected to dominate in the meteor region.

**The migrating solar diurnal tide**

The expansion coefficients for the latitudinal excitation are much larger than the semidiurnal coefficients, consistent with the dominance of this period. However, unlike the semi-diurnal case, no single mode is well matched to the forcing function. Lindzen (1966) and Kato (1966) suggested it is necessary to include Hough functions with negative eigenvalues to completely describe the latitudinal response. In the expansion, the positive $h_n$ values are all small implying short vertical wavelengths and inefficient forcing of these modes, while the negative $h_n$ values correspond to evanescent modes which decay away from the region of generation. The propagating modes are dominant at low latitudes, and the diurnal tide at meteor heights has a marked latitude dependence. The vertical structure of the diurnal tide may be expected to be complex as a result of the superposition of propagating and evanescent modes. This feature and the inefficiency of generation of the propagating modes accounts for the relative weakness of the diurnal tide observed in ground level pressure data.

The effect of dissipation has been considered by Lindzen (1970, 1971) who developed an equivalent gravity wave model for the tidal oscillations and included molecular viscosity, thermal conductivity and Newtonian cooling. Although the modes considered begin to attenuate, this is generally at altitudes greater than 110 km, except for the (1,2) mode which may become unstable near 85 km.
Lindzen and Hong (1974) have included background mean zonal winds and corresponding temperatures satisfying a geostrophic flow in the full tidal formulation. These authors find that mode coupling takes place; modes not directly forced by the solar input receive excitation. The mid-latitude semi-diurnal tide is then dominated by the (2,4) rather than (2,2) mode, whose amplitude may be reduced by 40% near 100 km. The background velocities considered were the seasonal mean zonal velocities and so departures from this behaviour may be expected on individual days. A seasonal variation in the atmospheric tides is also to be expected since the dominant driving force annually changes its latitudinal distribution.

8.1.4 The Irregular Component

Early meteor wind observations revealed the existence of considerable velocities (up to 30 ms\(^{-1}\)) after the mean wind and tidal periods had been subtracted. Such motions are considered as the irregular component. This irregular motion may be theoretically split into two components - ordered, propagating short period internal gravity waves and disordered local motions considered as turbulence. The present radar system is unable to uniquely specify either, however the presence of such motions will contribute to the observed scatter in velocities obtained over appreciable time intervals or volumes.

Internal Gravity Waves

Although atmospheric tides are examples of internal gravity waves, the short period examples do not in general have such regular identified generation processes. The theoretical aspects of internal gravity waves are covered in Hines (1974) and Jones (1976). Since only short periods are considered, the Coriolis force may be neglected and cartesian geometry adopted. Linearized perturbation theory in applied and the plane wave propagation characteristics described in terms of the dispersion relation.
for the particular model. Internal gravity waves have a low period limit of approximately 5 minutes in the meteor region and the strongest members occur in the 1 to 3 hour period range. Proposed sources of generation include meteorological sources at low altitudes, resulting in a constantly changing interference of superimposed waves. A summary of possible generating and dissipating mechanisms is contained in Justus and Woodrum (1972). Again, the effective refractive index is largely dependent on the vertical temperature structure, resulting in the trapping and ducting of waves below the mesosphere. A feature of all internal gravity waves is their exponential amplitude growth with increasing altitude in the absence of dissipative processes, and the upward energy (group velocity) propagation associated with downward phase propagation. Short period internal gravity waves are linearly polarized whereas the inclusion of the Coriolis force leads to the longer period modes being elliptically polarized.

Turbulence

In contrast to the orderly internal gravity wave motions are those ascribed to turbulence. Turbulent motions are irregular in nature, diffusive, dissipative and described only by their statistical properties. The atmosphere is thought to contain small scale eddies and be well mixed below approximately 110 km, the so-called turbopause. Since it is a dissipative phenomenon, continuing turbulent motion must have its energy replenished. This may be provided by shearing in the background flow, increased as a result of vertical wave amplitude growth, and from wave critical layers.
The theoretical divisions of the neutral atmosphere motions emphasizes the difficulty in dealing with realistic atmospheric models. Each time scale is considered separately and its linear behaviour is studied, although non-linear second order gravity wave interactions have been considered by Spizzichino (1969). A dynamical description of the atmosphere requires a knowledge of each class of motion, its interaction with members of that class and other time scales. For example, the exponential amplitude growth of an internal gravity wave with increasing altitude results in increasing vertical wind shear and hence instability and turbulence, which may supply energy to the mean flow or generate a new regime of gravity waves.

The division in terms of time scales is also convenient observationally. In the present experiment 30 minute averages of velocities are used from which the average or mean flow can be extracted. Harmonic analysis (Appendix G) reveals the strength of tidal contributions while the variance for each time interval may be partially attributed to irregular components.

8.2 THIRTY MINUTE VELOCITY AVERAGES

Since a single velocity determination is the resultant of all contributing scales of motion in the reflecting region (essentially the principal Fresnel zone), multiple determinations are necessary to specify the individual components. Although the equipment spatial resolutions are important, the primary criterion is the usable echo rate. The determinations are in general unequally spaced in time and are distributed throughout the observing volume. To reduce these to uniformly spaced data amenable to further analysis, integration or interpolation of the raw data is required, introducing smoothing appropriate to the chosen intervals. Several schemes exist for reconstituting the data, sharing the common assumption of a uniform horizontal velocity across the observing region.
The method in use at Garchy (Spizzichino, 1972) is an interpolation model, reconstituting data equally spaced in time and height. Within a given interpolation region about \((z_0, t_0)\) a regression plane (the interpolation is two dimensional) can be determined using weighted least squares provided three or more echoes occur in this region. The weighting chosen is unity at \((z_0, t_0)\), falling linearly to zero at the edge of the interpolation region. Once the regression plane coefficients have been determined, the interpolated wind value at \((z_0, t_0)\) can be found by substitution. The size of the interpolation region is determined from the data by considering the two dimensional auto-correlation function \(r(\Delta z, \Delta t)\), and determining the region where

\[
1 - r(\Delta z, \Delta t) \leq 0.2.
\]

Spizzichino (1972) outlines the validity of the assumptions, precautions, associated errors and smoothing which result from the method. A uniform horizontal wind is assumed and orthogonal wind components must be gathered and treated separately.

The analysis method developed by Groves (1959) is in use at Adelaide (Elford, 1966) and Atlanta (Roper, 1977). The motions are assumed to consist of velocity components with specified (tidal) periods whose height variation may be represented by a polynomial, including vertical velocity components and orthogonal horizontal components, provided the echo azimuth is known. The data are then reconstituted using least squares in a generalization of the approach of Manning et al. (1950) for constant horizontal velocities. As the periods are specified this method is unsuitable for studying short period internal gravity waves. The low meteor echo rate at altitude extremes may also introduce instability into the end points of the altitude polynomial fit, although this will be reflected in the concurrent error analysis. Both of the above interpolation
methods yield the same results within the quoted uncertainty magnitude for identical data (Fellous et al., 1974).

In the absence of interpolation routines data are accumulated over time and height intervals chosen to yield a satisfactory echo rate. The velocities are again assumed uniform across the horizontal section of the echo gathering area, which for the present system extends to approximately 300 km by 300 km (although the majority of usable echoes are observed within the 3 db antenna regions whose extent is approximately 150 x 150 km). A horizontal section of velocity values is shown in table 8.1 and although most velocities are positive, there are occasionally large differences across adjacent intervals. These may be a result of the horizontal separation, minor lobe echoes or motions whose periods are of the same order or less than the 30 minutes over which these data were accumulated. The usable echo rate is insufficient to afford determinations of short period motions in anything but a statistical sense. The most appropriate periods for consideration in the present experiment are then atmospheric tides, for which a velocity averaging time interval of 30 minutes introduces little smoothing. During this time sufficient echoes must be recorded in a given height interval to integrate out the effects of shorter period motions and provide a statistically reliable average velocity.

The radial velocity distributions (fig. 7.6) are approximately Gaussian. This is also true when shorter time intervals are considered; the velocity may be considered as a constant value during the 30 minute period (due to tidal, prevailing and other long period components), with superimposed Gaussian 'noise'. In fig. 8.1 the radial velocity distribution for a twelve hour observing period (to emphasize the distortion) is shown. This distribution is translated to a zero mean Gaussian when the appropriate prevailing and tidal components are subtracted. Pokrovskiy et al. (1969) consider shorter observing intervals and suggest departures from a normal
Table 8.1 Horizontal velocity section
(Velocities are in ms\(^{-1}\))

<table>
<thead>
<tr>
<th>Horizontal Range (km)</th>
<th>163</th>
<th>188</th>
<th>213</th>
<th>238</th>
<th>263</th>
<th>288</th>
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<td>5</td>
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<td>13</td>
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<tr>
<td>103</td>
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<td>27*</td>
</tr>
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<td>97</td>
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<td>26*</td>
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<td>59</td>
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<td>25</td>
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<td>-19</td>
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<tr>
<td>85</td>
<td>16</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td>23</td>
</tr>
</tbody>
</table>

* Indicates a multiple determination.
distribution may result from the effect of meteor showers, non-uniform velocity sensitivity or the influence of the antenna pattern. A Gaussian function of the same standard deviation (26.3 ms\(^{-1}\)) is appropriate (fig. 8.1) and so inferential statistics applicable to normally distributed data may be used.

Consider the individual horizontal velocity values to be drawn from a population consisting of the velocities at all points within the sampling space. From this a sample of size \(N\) is drawn whose mean, \(\bar{V}\), is an estimate of the population mean. The distribution of such sample means is normal with a mean equal to the population mean and standard deviation

\[
\sigma_{\bar{V}} = \frac{\sigma}{\sqrt{N}}
\]

the standard error of the mean (S.E.M.), where \(\sigma\) is the population standard deviation and in general is unknown. An individual sample mean has a 95% chance of lying within \(\pm 2\) S.E.M. of the population mean. The S.E.M. can be used to determine the level of confidence for a particular sample mean being an estimate of the true mean value. The S.E.M. can be estimated from the sample standard deviation \(s\) by
Muller (1966) quotes an uncertainty of 6% at the 95% confidence level for a sample containing 5 echoes. This calculation takes into account the effects of velocity determination error, the finite antenna beamwidth and errors due to lack of height information, but neglects the dominant 'geophysical noise'. For a similar mean velocity (47 ms\(^{-1}\)) and taking \(\sigma = 30\) ms\(^{-1}\), the 6% uncertainty for 95% confidence corresponds to \(2\sigma_v = 2.8\) ms\(^{-1}\) and hence \(N = 453\) echoes are required. In general the uncertainty is better expressed as an absolute value since the mean velocity may assume values in the range \(\pm 50\) ms\(^{-1}\).

For a scheme considering average horizontal velocities, the above arguments may be used to estimate the number of velocity determinations required to yield satisfactory altitude coverage. The meteor echo height distribution is approximately Gaussian with a standard deviation of 10 km (section 7.6). The two extreme 5 km intervals in the height range 80 to 110 km levels each receive 9% of the total echo rate. In any 5 km interval, the sample standard deviations are typically 25 ms\(^{-1}\) (section 8.3) and so an uncertainty of 10 ms\(^{-1}\) at the 95% confidence level requires at least 26 echoes in any stratum. Hence for this uncertainty the total usable echo rate must exceed 289 in a 30 minute observing period. In this situation the central height intervals will of course be more accurate. The size of the height interval over which velocity averages are to be determined is seen to be dependent on the usable echo rate as well as the instrumental height resolution.

A common practice in the past has been to average over all heights, relying on the relatively narrow echo height distribution to yield a velocity appropriate to the modal height. The consequences of integrating over appreciable height intervals on tidal components has been investigated
by Glass et al. (1975) for a Gaussian height distribution, and by Hess and Geller (1976) for the same but with a finite ceiling. As expected the short vertical wavelength propagating modes are most seriously affected by the integration. The choice of a height interval must also take into account the possible suppression or distortion of these modes.

A further problem introduced in the present observing scheme is the gathering of orthogonal data from two regions whose horizontal separation is approximately 350 km. This is not a serious limitation when scales of large horizontal extent such as atmospheric tides are considered. The work of Muller and Kingsley (1974a) using two separated meteor wind stations has confirmed the validity of this approximation when average velocities are considered.

The individual meridional horizontal velocities and 30 minute averages for a 24 hour period in the height range 85 to 90 km are shown in fig. 8.2. Although there is considerable scatter in any 30 minute interval there are periods where the motion is almost exclusively in one direction. Also apparent is the diurnal echo rate variation and the continuity of the velocity determinations through the region near zero velocity. The motion appears to be dominated by a semi-diurnal oscillation and indicates the need for average velocities rather than individual values for reliable tidal determinations.

8.3 THE ATMOSPHERIC VARIANCE

The standard deviation (s) of a velocity sample obtained over appreciable height and time intervals comprises contributions from:

1) instrumental uncertainties
2) positional differences within the antenna beam
3) short period internal gravity waves or turbulence
4) vertical wind gradients.
The effects of these contributions have been considered by Baggaley and Wilkinson (1974) who find a mean standard deviation of 31 ms$^{-1}$ (variance 961 m$^2$s$^{-2}$) for all heights, compared with the present results of 27.9 ms$^{-1}$ and 28.7 ms$^{-1}$ for the south and west data respectively. The present instrumental contribution to the variance is generally $< 36$ m$^2$s$^{-2}$ (section 7.3.2) and is negligible. The effect of a finite antenna beamwidth was evaluated by Wilkinson (1973) for the case of a directional antenna similar to the present equipment and a constant horizontal velocity. For an arbitrarily directed mean velocity the variance is 0.26V$^2$. For a typical mean velocity of 30 ms$^{-1}$, this leaves an atmospheric contribution of 630 m$^2$s$^{-2}$, equivalent to an r.m.s. velocity of 25 ms$^{-1}$, similar to the value obtained by Greenhow and Neufeld (1959) and the 20 ms$^{-1}$ of Fellous and Glass (1976). Further calculations by Wilkinson indicate that vertical wind gradients of sufficient size exist in the meteor region (Muller, 1968) to account for all the variance, considering a uniform shear. Hess and Geller (1976) identify a period with large variance with the existence of large vertical wind shears, however Greenhow and Neufeld (1959) report no significant correlation between shears and r.m.s. velocities. In general the contributions from shears, internal
gravity waves and turbulence cannot be distinguished in the present experiment.

A 30 minute period beginning at 0400 \textsuperscript{h} (on 16/7/77) has been analyzed in detail to investigate contributions to the velocity variance over cumulative spatial and temporal scales, centred on 95 km, the results of which are presented in table 8.2.

\begin{table}[h]
\centering
\caption{Velocity standard deviations (ms\textsuperscript{-1})}
\begin{tabular}{|c|c|c|c|c|c|}
\hline
\textit{\Delta t} (min) & 5 & 10 & 15 & 20 & 25 & 30 \\
\hline
\textit{\Delta z} (km) & 1 & 3 & 3 & 9 & 10 & 15 & 14 \\
& 2 & 23 & 23 & 24 & 21 & 23 & 23 \\
& 5 & 22 & 22 & 23 & 22 & 22 & 21 \\
& 10 & 22 & 23 & 22 & 21 & 26 & 25 \\
& \textit{All heights} & 24 & 26 & 25 & 27 & 28 & 27 \\
\hline
\end{tabular}
\end{table}

For the smallest altitude interval there is an increase of the standard deviation with time, however other examples over such small intervals do not always reveal such a trend, which may be fortuitous due to the small sample sizes. There is also a general increase with increasing height interval. Fellous and Glass (1976) report only a slight increase with time for periods in excess of 5 minutes up to 1 hour. These authors also find an increasing standard deviation with increasing altitude interval for \textit{\Delta t} = 5 \text{ min}, and a marked decrease with altitude during summer months. The mean values for all samples exceeding 10 usable echoes for 5 km height intervals are given in table 8.3.

The averages over separate 5 km intervals are mostly less than those including all heights and are uniform apart from the 106 to 110 km interval. The apparent increase may be a consequence of the lower usable echo rate in this interval, or a true atmospheric variation.
Table 8.3

<table>
<thead>
<tr>
<th>z</th>
<th>s(south)</th>
<th>s(west) ms⁻¹</th>
</tr>
</thead>
<tbody>
<tr>
<td>81-85</td>
<td>24.5</td>
<td>24.5</td>
</tr>
<tr>
<td>86-90</td>
<td>24.0</td>
<td>23.5</td>
</tr>
<tr>
<td>91-95</td>
<td>23.1</td>
<td>22.7</td>
</tr>
<tr>
<td>96-100</td>
<td>23.9</td>
<td>24.9</td>
</tr>
<tr>
<td>101-105</td>
<td>25.8</td>
<td>23.7</td>
</tr>
<tr>
<td>106-110</td>
<td>28.1</td>
<td>26.7</td>
</tr>
</tbody>
</table>

In the absence of dissipation an internal gravity wave will increase in amplitude as it propagates vertically, but in doing so produces increasing vertical wind shear and possibly turbulence, any of which may be responsible for the increase. The dependence on altitude interval is also revealed in the distribution of sample standard deviations for the 96-100 km values, and those obtained over all heights (fig. 8.3) for the south data. Also shown are the values of Baggaley and Wilkinson (1974), obtained using similar equipment.

On individual days little regular behaviour is observed in the standard deviation (over all heights) that can be related to the diurnal echo rate. For much of the time however the variations for both south and west are similar, suggesting a common origin (fig. 8.4). Hess and Geller (1976) find little variation over a 24 hour period, with the echo rate variation eliminated by considering groups of 50 echoes. Wilkinson (1973) shows that the contribution to the variance introduced by considering a uniform horizontal velocity over the antenna beam depends on the orientation of the velocity, being largest when it is transverse to the observing direction. Such a uniform velocity will be longitudinal and transverse respectively to the orthogonal observing directions, hence any variation in the standard deviation due to averaging over the beam will be of opposite
sign for the south and west data respectively. The observing regions however are horizontally separated by approximately 350 km although the results of Muller and Kingsley (1974a) suggest that a similar mean value may be expected. However, variations with opposite phases for the south and west data are not apparent. The similarity of the south and west variations is reinforced by considering the daily average standard deviation (fig. 8.5). Similar results are also apparent over a typical 5 km height interval (91-95 km), although as expected the magnitudes are generally smaller. The
Fig. 8.4 Sample standard deviation variations (all heights).

Fig. 8.5 Daily average velocity sample standard deviation variations.
extreme variation corresponds to an equivalent r.m.s. velocity of 22 ms$^{-1}$, which cannot be accounted for by instrumental effects and is independent of the diurnal echo rate variation. To be responsible for these daily variations, increases in the vertical shear, internal gravity wave activity or turbulence, would require an energy flux with a large time scale and a spatial distribution encompassing both separated observing regions. The cause of this variation may lie in variations of tidal origin since the spatial scales are large, or with the conditions of generation or propagation of shorter period internal gravity waves, whose meteor region behaviour may be responsible for shears or turbulence. Since gravity wave propagation is strongly influenced by background winds, radio-sonde data for the appropriate period were investigated. At individual heights, similar variations to those of the standard deviations could be found in the daily average velocities. However, the variations in background winds considered over several heights revealed no such systematic variations.

Identification of internal gravity wave modes has been made in the past using the method of velocity profile differences (Fraser and Kochansky, 1970, Justus and Woodrum, 1972, Manson, Gregory and Stephenson, 1973). Provided the longer period components, mainly tides, remain constant then the sequential velocity profile differences reveal any component whose variation is significant over the difference time interval. Profiles formed from 3 km average velocities are shown in fig. 8.6 for 10 minute intervals, together with the mean 30 minute profile and profile differences (fig. 8.6b). While it is tempting to interpret the resulting oscillatory behaviour as being due to internal gravity waves, it must be remembered that the velocities comprising these averages may come from a considerable horizontal extent in the present experiment. When the standard deviations for each height interval are considered, the velocity differences are not significant at the 95% confidence level. Only a single velocity component (meridional)
Fig. 8.6 Horizontal velocity profiles and profile differences. (Meridional velocities, 0400 h, Day 9).

is here represented and such a monochromatic mode would require considerable atmospheric filtering. The profile variations over such short time scales indicate that unless average velocities are considered, comparison type experiments in this region must be performed concomitantly.
8.4 THE VELOCITY SPECTRUM

Data on atmospheric tides is usually presented in the form of the amplitude and phase of the particular period using harmonic analysis (Appendix G), necessitating a knowledge of the frequency components present. These may be inferred from theoretical considerations or revealed by some form of spectral estimation. Several methods exist for transforming information in the time domain to the frequency domain and although each has limitations peculiar to it, several concepts are common to all.

The time series (of average horizontal velocities) is considered to be characteristic of the physical process under investigation. The sampling interval \( t \) (30 minutes) and data length \( T \) (9 days) introduce limitations since components with frequencies greater than \( \frac{1}{2t} \) will be filtered out. More importantly, short periods may not be resolved by this sampling interval and may be aliased to appear as spurious lower frequencies. This may be eliminated by the appropriate choice of the sampling interval, pre-supposing some knowledge of the phenomenon under study, or the removal of such frequencies before sampling. Greenhow and Neufeld (1959) find that the auto-correlation of velocities falls to zero in approximately 90 minutes, indicating that periods considerably larger than the sampling period are important. Since a finite length of record is available, the data length may be regarded as a window to the continuing process. The longest period that can be completely specified by the time series is one of length \( T \). Unresolved longer periods may be present however as trends in the data.

8.4.1 Fourier Series Methods

A function, periodic over the interval \( T \) may be transformed into the frequency domain using the discrete Fourier transform, appropriate to sampled data. The data is expressed as coefficients of orthogonal sine and cosine terms, determining the amplitude and phase of the particular frequency
components. The frequency components fitted to the time series are harmonically related multiples of $\frac{1}{T}$ up to a maximum of $\frac{1}{2T}$, and the coefficients are readily calculated using the fast Fourier transform algorithm (Bergland, 1969). The expressions for the coefficients are given in Appendix B and it can be shown that these are identical to those obtained from the method of harmonic analysis using least squares methods (Appendix G). The resolution of the frequency components is $\frac{1}{T}$ and only harmonic frequencies are fitted to the time series, which is assumed infinite in length.

A serious limitation of the discrete Fourier series is the effect of the data window. The time series is considered as a continuing phenomenon, multiplied by a rectangular observing window and hence the resulting line spectrum will be the convolution of the time series spectrum and the function $\left(\pi fT\right)^{-1} \sin(\pi fT)$ corresponding to the window. Although the rectangular window has best resolution (minimum main lobe width in the frequency domain), it has large sidelobes which allow the leakage of energy into adjacent frequencies. The effects of leakage can be reduced by applying different windows to the time series, but only at the expense of decreasing the frequency resolution. The characteristics of various windows are discussed in detail in Jenkins and Watts (1968). Windows may be applied to the time series, or their transform may be used to smooth the frequency estimates.

Periods of less than $T$ may be present in the data, and the work of Toman (1965) shows that a sinusoid whose period exceeds $1.75T$ will produce a spectral maximum at zero frequency. In the present context this has the effect of introducing bias into the average or mean wind term. Trends whose time scale is much larger than this value may also be present. The removal of a trend is analogous to applying a high pass filter and may involve the subtraction of a mean value, a linear trend, or a parabolic
trend to approximate a long period component. The type of trend removed should be chosen on the basis of known physical processes associated with the data.

The discrete Fourier transform may be adapted to determine frequencies that are not multiples of the basic frequency by either extending the time series length with zeroes, or by using different sampling intervals and data length if the time series can be represented continuously. The latter method has been used by Muller (1972) to identify long period meteor wind oscillations. However, simulations with this 'jittering' method have shown that line frequencies are considerably broadened, although the peak value is still easily identifiable.

8.4.2 Anharmonic Frequency Analysis

Anharmonic periods can also be revealed using the method proposed by Paul (1972), which can locate such frequency components and determine their amplitude and phase. A frequency dependent transform is defined which when applied to the time series produces a rectangular window in the frequency domain. By changing the transform in a manner so that the window scans the frequency domain, the existence of energy within these frequency intervals can be determined. The resolution is still $\frac{1}{T}$ and in the practical situation a cosine window is used and the transform applied by use of the fast Fourier transform algorithm. Although it has not yet been applied to the present data, this method appears well suited to long period meteor wind observations, where the tidal components and possible long periods are in general well separated in frequency and not necessarily harmonically related.

8.4.3 The Maximum Entropy Method (M.E.M.)

The recently developed maximum entropy method claims increased frequency resolution over Fourier methods, mainly as a result of the easing of restrictions on the cyclic data extensions or window problems associated
with them. The theoretical basis for the method is contained in Lacoss (1971) and Ulrych and Bishop (1975) and a comparison with Fourier methods in Frost (1977). Consider a time series \( x(t) \) whose frequency spectrum is \( X(\omega) \).

If \( H(\omega) \) is the transfer function of a unique filter that whitens \( X(\omega) \) then

\[
|X(\omega)H(\omega)| = k \quad \text{a constant for all } \omega
\]

and

\[
|X(\omega)|^2 = \frac{k^2}{|H(\omega)|^2}
\]

is an estimate of the power spectrum of \( x(t) \). The whitening process can be obtained by a prediction-error filter, whose coefficients contain the spectral estimates. In an information sense, entropy is a measure of the uncertainty and the spectral estimate must be the most random, that is, have maximum entropy and be consistent with the data. The filter coefficients are obtained by maximizing the entropy, using only the available data and requiring no extension to it, hence alleviating some of the window problems associated with Fourier methods.

The maximum entropy method has been applied to simulated time series to demonstrate its superior resolution by Ulrych (1972) and Ulrych et al. (1973). More recent work by Chen and Stegun (1974) suggest some of these earlier results may have been due to fortuitous data choices. The major limitations associated with the method are the choice of the filter length and the lack of a test of the reliability of the results. The resolution is directly related to the filter length, however an excessive length leads to the splitting and shifting of spectral peaks. Chen and Stegun consider the effects of data length, initial phase, filter length and noise on a variety of simulated time series and reveal the limitations of the method. Provided the data lengths cover more than a full cycle of the lowest frequency component, a substantial range of filter lengths up to approximately \( 3\sqrt{N} \) each yield a reasonably good spectrum. (\( N \) is the number of points in the
time series.) For increased noise levels, increases in filter length are tolerable for up to approximately 40% noise. The practical limitations have also been discussed by Courtillot et al. (1977) who applied the method to real data. These authors consider the removal of trends by either linear or parabolic approximation to be important, especially for low frequency stability in the spectrum. A maximum filter length of N/2 is considered satisfactory in agreement with Ulrych and Bishop (1975). As yet no really satisfactory check on the reliability and stability of a M.E.M. spectrum, apart from visual observation, is available.

8.4.4 Velocity Spectrum Results

Since most methods of estimating the spectrum require uniformly spaced data, the time series of horizontal velocity averages taken over all heights has been used. Any gaps still remaining have been filled using linear interpolation, necessary for six points in each of the 220 hour zonal and meridional time series (fig. 8.7). These data have been smoothed using a three hour running mean to reduce high frequency noise. This filter has a frequency response

\[ R(f) = (\pi \tau f)^{-1} \sin(\pi \tau f) \]

where \( \tau \) is the filtering interval and so the attenuation of tidal periods is small, being 20% for an 8 hour component. The smoothed time series reveal the dominance of a 12 hour component, with the zonal component appearing to lead the meridional. The zonal velocities also show a constant term directed eastward and a possible long period (\( \gg 220 \) hours) variation, whereas the meridional velocities show little evidence for prevailing motion.
The unsmoothed time series have been subjected to spectral analysis using the maximum entropy method (fig. 8.8). For both zonal and meridional examples a linear trend has been removed, which consisted mainly of a constant term, the velocity gradient magnitude being less than 0.5 ms\(^{-1}\) day\(^{-1}\) in both cases. After experimentation the filter length used for these examples was chosen at \(3\sqrt{N}\). The extremes of the logarithmic spectra cover a range of approximately 20 db indicating a noise level of at least 10%. For this value, Chen and Stegun (1974) suggest satisfactory spectra are obtained for filter lengths up to \(\frac{15}{24} \sim 3\sqrt{N}\) of the data length, for a single sinusoid. Examples of spectra obtained show little change for a filter length of \(4\sqrt{N}\), while \(5\sqrt{N}\) data exhibit splitting of the spectral peaks which is probably due to the method (fig. 8.8c).
Fig. 8.8 M.E.M. velocity spectra.

Zonal (3√N)

Meridional (3√N)

Meridional (5√N)
The M.E.M. results indicate a dominant 12 hour variation for both directions of observation, and further components near 24 and 8 hours which may also be of tidal origin. Although the M.E.M. shifts the location of spectral peaks it has been reported by Spizzichino (1969) that the apparent period of the diurnal oscillation may lie in the range 17 to 35 hours as a result of phase changes of this component. Spectral peaks located at periods less than 8 hours have been interpreted as internal gravity wave modes in shorter data lengths. However, their appearance in a nine day time series implies a coherence that is difficult to attach to such modes. The M.E.M. however does appear to have the ability to extract short lived oscillations that do not persist for the entire data length. In both sets of data a long period oscillation also exists with a period near 80 hours for the zonal velocities and 70 hours in the meridional case. With increased filter lengths this component splits into periods near 48 and 80 hours, but this is usually accompanied by the splitting of other components as well. The origin of this oscillation will be considered in section 9.3.

Fourier transform methods have also been applied to the time series obtained over all heights. The method of 'jittering' the time series length was used and its effect is shown in fig. 8.9, along with an individual discrete velocity amplitude spectrum for the meridional velocities. The individual spectra have been smoothed by \((\frac{1}{4}, \frac{1}{2}, \frac{1}{4})\) weights, reducing leakage and also the frequency resolution. The dominant peaks are again observed near 12, 24 and approximately 60 to 80 hours, although the results for the zonal data contain considerably more noise.

Although no rigorous analysis of the reliability of these spectra has been performed, some idea may be gained by considering the periods less than 8 hours to be due to white noise. Referring to the M.E.M. spectra, the long period, 24, 12 and possibly 8 hour velocity components are then significant features of the velocity time series.
Fig. 8.9 Velocity spectra using Fourier methods.
8.5 SIMULATED LOW VELOCITY LOSSES

The agreement of the present radial velocity distributions (section 7.4) indicate the reliability of the present method of determining radial velocities. These results can be used to simulate the bias introduced by non-offset zero crossing doppler schemes with their associated low velocity losses. By employing a discrete low velocity (10 ms\(^{-1}\)) cut-off, Hess and Geller (1976) have estimated the bias introduced into the mean velocity component for the case of a Gaussian velocity distribution with a mean and standard deviation of 25 ms\(^{-1}\), resulting in a mean velocity change of 5.7 ms\(^{-1}\). Simulated duration associated losses have been applied to 24 hours data (Day 9) using data considered over all heights in order to have sufficient echoes in the samples. The individual radial velocities were converted to the appropriate non-offset doppler frequency and subsequently discarded if either one or later three half cycle periods did not exceed the echo amplitude duration.

The resulting radial velocity distributions (fig. 8.10) show the duration associated low velocity losses, accounting for almost ±20 ms\(^{-1}\) if three doppler half periods are required. The single half cycle requirement applied to the results of Wilkinson (1973) and the corresponding distribution is similar to the 90 to 100 km example of Baggaley and Wilkinson (1974). These zero crossing schemes also result in a corresponding reduction in the number of suitable echoes for radial velocity determinations. For the 24 hour period considered 3510 echoes were recorded, 48% of these producing reliable velocities in the 80 to 110 km altitude range. These are subsequently reduced to 37% and 19% for the two cases considered, reducing sample sizes and hence the statistical reliability of sample means.

Mean horizontal velocities for 30 minute intervals are shown in fig. 8.11, where the low velocity losses systematically increase the average velocity values. Uncertainties on the 'true' values are the standard error
Fig. 8.10 Radial velocity distributions for various velocity measurement schemes. (1) Present results, (2) one half doppler cycle, (3) three half doppler cycles.

of the mean. Also shown are the best fit tidal components including a prevailing term and 24, 12 and 8 hour periodic components determined by harmonic analysis (Appendix G). The effect of the low velocity losses on derived tidal components is demonstrated in table 8.4. As expected the phase variations are small and so have been excluded.

Table 8.4 Derived tidal component magnitudes (zonal, meridional) (ms⁻¹)

<table>
<thead>
<tr>
<th>Component</th>
<th>'True'</th>
<th>Alternate echoes</th>
<th>&gt; 1/2 doppler</th>
<th>&gt; 3/2 doppler</th>
</tr>
</thead>
<tbody>
<tr>
<td>24ʰ</td>
<td>(27.0,10.8)</td>
<td>(23.3,12.4)</td>
<td>(36.4,13.2)</td>
<td>(49.3,11.9)</td>
</tr>
<tr>
<td>12ʰ</td>
<td>(27.4,24.9)</td>
<td>(23.2,24.1)</td>
<td>(30.1,30.8)</td>
<td>(30.8,39.6)</td>
</tr>
<tr>
<td>8ʰ</td>
<td>(14.3,10.8)</td>
<td>(16.6,11.6)</td>
<td>(18.5,13.3)</td>
<td>(18.8,12.9)</td>
</tr>
<tr>
<td>Prevailing</td>
<td>(8.8,-11.2)</td>
<td>(8.7,-13.0)</td>
<td>(12.3,-12.3)</td>
<td>(19.8,-15.9)</td>
</tr>
<tr>
<td>No. of echoes</td>
<td>1670</td>
<td>835</td>
<td>1288</td>
<td>651</td>
</tr>
</tbody>
</table>
Fig. 8.11 Average horizontal velocities for various measurement schemes (Day 9).
Also considered are the velocity averages and tidal components derived by considering alternate echoes, indicating the increased coefficients are not simply due to the smaller sample sizes associated with the velocity losses. The major effect of non-offset zero crossing schemes is a bias against low velocities, increasing the mean and tidal component determinations and reducing the usable echo rate. Since the losses are related to echo durations, the increase of the cut-off velocity with altitude may have contributed some of the increase in tidal amplitudes with height observed in the past.
CHAPTER 9

THE OBSERVED VELOCITY COMPONENTS

9.1 ATMOSPHERIC TIDES

The velocity spectra for both observing directions confirm the existence of tidal period oscillations in the meteor region. The amplitude of components with periods much greater than the data length have been shown to be small in the trend removal of the section 8.4, so harmonic analysis (Appendix G) can be used to obtain the amplitude and phase of each tidal component from successive 24 hour intervals. The presence of the 60 to 80 hour component will introduce some uncertainty into the derived tidal components and should also appear as changes in the mean wind. Its amplitude however is smaller than that of the major tidal components.

Data from individual days are subject to harmonic analysis, with the individual velocity values weighted by the number of observations N in each 30 minute period, equivalent to considering each velocity determination individually. This will counter some of the bias introduced by the diurnal echo rate variation, but is included principally to deal with gaps in the data. Weighting by the number of points has been used by Muller (1966), while Zadorina et al. (1967) used N/S, where S is the sample standard deviation. A more appropriate weight to take the uncertainty of the 30 minute averages into account would be the inverse of the S.E.M., 1/√N-1/S. All three weighting schemes have been applied to portions of the present data, resulting in no significant variations in the derived harmonic coefficients.

The methods of estimating the uncertainty of the harmonic coefficients are considered in Appendix G. The running harmonic analysis method yields standard deviations that are typically 7 ms⁻¹ at the extreme 5 km height.
intervals, that is 81-85 and 106-110, and approximately 4 ms$^{-1}$ for the central regions. The corresponding phase uncertainties are typically 0.4 rad. for the semi-diurnal component and 0.5 rad. for the diurnal on account of this component's phase variability. For individual determinations, the covariance matrix method (Appendix G) using the S.E.M. as an equivalent r.m.s. velocity deviation yields an r.m.s. deviation of approximately 3 ms$^{-1}$ for data over all heights, 8 ms$^{-1}$ for 10 km height intervals and 14 ms$^{-1}$ for 5 km intervals.

The present data have in general been considered in 5 km height intervals and the daily tidal parameters refer to mean values obtained from the running harmonic analysis. Data were rejected from any 24 hour period where less than 70% of the time series (17 hours) was present, eliminating some data from the altitude extremes. Clearly the extreme data is not as reliable as that from central portions of the meteor region. The velocity data have been fitted with a prevailing or mean wind term, and 24, 12 and 8 hour periodic components.

9.1.1 The Semi-Diurnal Tide

The semi-diurnal component dominates the velocity spectra and its amplitude as revealed by harmonic analysis is usually larger than any other component. The variation of both zonal and meridional components with altitude is shown for the 9 day observing period in fig. 9.1. In the absence of dissipation the velocity should increase exponentially with altitude to maintain a constant energy density. On specific days, for example day 9, amplitude growth is observed, but in general the height structure is complicated and occasionally exhibits severe dissipation (day 5). Some of the apparent amplitude decrease at high altitudes may be due to the high velocity losses described in section 4.7. The behaviour is generally similar for both orthogonal components, although the observing
Fig. 9.1 Semi-diurnal tide, amplitude (Day 1 is July 7th, 1977)
regions are separated by approximately 350 km, indicating real tidal variations of this horizontal scale. The amplitude observed is similar to that observed by northern hemisphere mid-latitude stations, although the day to day amplitude variability implies that only mean seasonal comparisons are meaningful.

The results of Greenhow and Neufeld (1961) and Muller (1966) both refer to latitude 53°N and to the entire vertical extent of the meteor region. The winter velocities of these authors are 20 ms\(^{-1}\) and 20-25 ms\(^{-1}\) respectively, in good agreement with the present mean values of 25 and 23 ms\(^{-1}\) for meridional and zonal components at 95 km. The closest equivalent northern hemisphere station to the present latitude of 43°S with echo height determination equipment is that at Garchy (lat. 47°N). The winter results presented by Glass and Fellous (1975) are compared with the present mean values in table 9.1.

| Table 9.1 Semi-diurnal tide characteristics at 90 km. |
|---------------------------------|----------------|----------------|
|                                 | Garchy (47°N)  | Christchurch (43°S) |
|                                 | Zonal          | Zonal          | Meridional |
| amplitude                       | 21             | 21             | 23 ms\(^{-1}\) |
| amplitude gradient              | 2              | 0.3            | 0.5 ms\(^{-1}\) km\(^{-1}\) |
| phase                           | 09.15          | 10.40          | 01.35 hours (L.T.) |
| phase gradient                  | 11             | 1.8            | 2.3° km\(^{-1}\) |

The amplitude behaviour with height at Garchy in general exhibits larger growth than the present results, but also displays similar amplitude variability, as do the results of Hess and Geller (1976) for the semi-diurnal meridional component. These results cannot be readily compared with southern hemisphere observations since apart from the results of
Fig. 9.2 Semi-diurnal tide, phase. (The phase angles (radians) refer to the position of the zero crossing, whereas the phases expressed as local time refer to the position of the first maxima.)
Wilkinson and Baggaley (1974), the only available data refer to Adelaide at latitude 35°S (Elford, 1959, Vincent and Stubbs (1977)). The latter authors however observed the semi-diurnal tide to be approximately 20 ms\(^{-1}\) in winter. The calculations of Lindzen and Hong (1974) predict an increasing amplitude with height, from approximately 19 ms\(^{-1}\) at 80 km to 35 ms\(^{-1}\) at 100 km and thereafter a decrease in amplitude, for a latitude of 45° in winter. The present mean amplitudes however exhibit a much smaller growth with altitude, although this type of behaviour is observed on individual days. The inclusion of background winds in the tidal formulation does not alter the amplitude of the semi-diurnal tide appreciably, and the present results are of the same magnitude. The calculations of Lindzen and Hong used mean zonal winds from which there will be a day to day variation and hence a variation in the atmospheric transmission characteristics.

The phase variation with altitude for the semi-diurnal tide (fig. 9.2) is regular on most days, showing a phase increase with height, corresponding to a mode or modes with downward phase propagation, expected for propagating tidal modes. In the absence of background winds, the long vertical wavelength (\(\lambda_z \sim 200\) km) propagating (2,2) mode is expected to dominate in the meteor region, while the inclusion of background winds enhances the shorter wavelength (54 km) (2,4) mode by mode coupling. The phase gradients for all examples are less than 5° km\(^{-1}\), implying a single mode vertical wavelength in excess of 75 km. The actual phase gradient may be slightly reduced by the effect of the Gaussian meteor echo height distribution on the adopted 5 km height stratification. Although only six points at most are available from this analysis, the phase gradients have been determined using linear regression, with altitude as the independent variable. These results are shown in table 9.2 for those examples whose correlation coefficient exceeded 0.5.
Table 9.2 Equivalent Vertical Wavelength (km)

<table>
<thead>
<tr>
<th>DAY</th>
<th>ZONAL</th>
<th>MERIDIONAL</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>79</td>
<td>194</td>
</tr>
<tr>
<td>2</td>
<td>108</td>
<td>-</td>
</tr>
<tr>
<td>3</td>
<td>93</td>
<td>121 (below 100 km)</td>
</tr>
<tr>
<td>4</td>
<td>290</td>
<td>-</td>
</tr>
<tr>
<td>5</td>
<td>-</td>
<td>95</td>
</tr>
<tr>
<td>6</td>
<td>250</td>
<td>-</td>
</tr>
<tr>
<td>7</td>
<td>229</td>
<td>-</td>
</tr>
<tr>
<td>8</td>
<td>230</td>
<td>273</td>
</tr>
<tr>
<td>9</td>
<td>134</td>
<td>169</td>
</tr>
</tbody>
</table>

Although the phase uncertainties on individual points are approximately 0.4 rad., the best fit linear phase gradients indicate predominantly long vertical wavelength modes. There is reasonable agreement for the two orthogonal components apart from day 1.

In general several modes will contribute to the semi-diurnal component and either partial or total reflections (from temperature discontinuities or negative temperature gradients of large horizontal extent) of the various modes will complicate the vertical phase structure and may even reverse the apparent direction of propagation. Over a limited altitude observing interval such as the meteor region, the vertical phase gradient may not then be representative of the dominant propagating mode. A combination of the (2,2) and (2,4) modes with equal amplitudes leads to an effective vertical wavelength of approximately 100 km derived from vertical phase gradients. The observed vertical wavelengths in excess of 200 km may be appropriate to the propagating (2,2) mode, while the shorter examples suggest the superposition of this and shorter wavelength modes. Since the relative strengths of these two main propagating modes are dependent on the
mean zonal velocity of the intervening atmosphere, the variation in apparent vertical wavelength may reflect the day to day variability of the zonal flow at lower altitudes.

The average vertical wavelength for day 9 is approximately 150 km. The profiles of 30 minute velocity averages for 12 hours of this day have been constructed for both orthogonal components (fig. 9.3). These averages should represent the actual tidal components, the irregular component being integrated out. Although the altitude coverage is small, many features of the profiles are consistent with a downward propagating wave of large vertical extent. After 12 hours the zonal profile has assumed similar values, while features may be identified that are consistent with the three hour phase shift of the meridional component. Although the semi-diurnal component was dominant on this particular day, other components must be expected to contribute to the profiles. However the general form is not inconsistent with the downward propagation of a mode with long vertical wavelength.

These results contrast with those of Fellows et al. (1975) who find long wavelength modes dominant in summer, while the wavelength in winter is closer to that expected for the (2,4) mode, its dominance being in agreement with the calculations of Lindzen and Hong (1974). Incoherent scatter results (Salah et al., 1975) also support the dominance of the (2,4) mode at altitude 110 km. Slow phase variation with height was observed in winter by Muller and Kingsley (1977), although the phase often decreased with increasing altitude.

The semi-diurnal zonal velocity is usually a maximum three hours before the meridional and so the tidal velocity may be considered as a vector rotating with an anti-clockwise sense when viewed from above. This phase difference (fig. 9.2) is also apparent in the velocity time series (fig. 8.7) and as demonstrated in the average velocity vector diagram
Fig. 9.3 30 minute average velocity profiles.

(fig. 9.4). In this diagram the 30 minute sampling difference for the two observing directions has been ignored, but it is apparent the velocity is dominated by an anti-clockwise rotation whose period is approximately 12 hours. The presence of other periods prevents the form from being a simple rotation.
Such rotations have been used in the past to identify tidal modes since a simple propagating pressure oscillation, periodic in longitude, will be accompanied by a clockwise rotating velocity vector in the northern hemisphere and the opposite sense of rotation in the southern hemisphere. The presence of several superimposed modes will complicate this picture, however if the amplitude of a single anti-clockwise mode is dominant, the rotational sense will be preserved regardless of the polarization of the minor modes. The theoretical tidal formulation (Chapman and Lindzen, 1970) shows that the phase difference between zonal and meridional components depends on the appropriate Hough function and its latitudinal derivative and a \( \pi/2 \) phase shift does not necessarily result. This problem has been considered by Blamont and Teitelbaum (1968) using explicit Hough mode solutions. These authors find the rotational sense of the tidal velocities is latitude.
dependent and may reverse for higher order modes. The dominant modes of
the semi-diurnal tide in mid-latitudes are expected to preserve the anti-
clockwise rotation. The observed rotation accords with southern hemisphere
results observed at Adelaide, in contrast to the clockwise rotation usually
observed in the northern hemisphere.

The phase variation with height is small in the present results and
so meaningful phase comparisons can be made with other data, including
meteor wind stations with no altitude determination. According to Chapman
and Lindzen (1970), there is little variation of the semi-diurnal tidal
phase (~ 1 hour) with latitude, and the winter results for several locations
are presented in table 9.3.

Apart from the results of Wilkinson (1973) there is good agreement
for the southern hemisphere stations, again illustrating the phase differ-
ence of approximately three hours between the orthogonal components. The
calculations in Chapman and Lindzen are for equinoctial conditions and
assume an equatorial vertical temperature profile, independent of latitude
and so can be expected to represent gross tidal features. The northern
hemisphere results are also in general agreement, although phases at Garchy
refer to dominant short vertical wavelength modes and hence rapid variation
with height, and so may not be appropriate for a comparison. The phase
differences with latitude, especially those of Roper (1977) may be due to
the modal structure, however for purely migrating tides similar phases
with respect to local time are expected. Glass et al. (1975) in a comparison
of tidal phases between Garchy and Obninsk observe a scatter in results
between the two sites larger than expected from experimental uncertainties.
These authors suggest features such as a semi-diurnal standing wave or
assymmetrical heat input to the tidal generation may be responsible.
Table 9.3 Semi-diurnal winter phases (hours, L.T.)

<table>
<thead>
<tr>
<th>LATITUDE</th>
<th>ZONAL</th>
<th>MERIDIONAL</th>
<th>SOURCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>50°</td>
<td>0900-1200</td>
<td>1000</td>
<td>Chapman and Lindzen, 1970</td>
</tr>
<tr>
<td>35°S</td>
<td>0100</td>
<td>0500</td>
<td>Elford (in Glass and Spizzichino, 1974)</td>
</tr>
<tr>
<td>43°S</td>
<td>2300-0100</td>
<td>0130-0400</td>
<td>Vincent and Stubbs, 1977 (June)</td>
</tr>
<tr>
<td>34°N</td>
<td>0630</td>
<td>0545</td>
<td>Wilkinson, 1973 (July)</td>
</tr>
<tr>
<td>40°N</td>
<td>0300-0600</td>
<td>-</td>
<td>Hess and Geller, 1976 (95 km)</td>
</tr>
<tr>
<td>47°N</td>
<td>0600-0700</td>
<td>0300-0400</td>
<td>Glass and Fellous, 1975 (95 km)</td>
</tr>
<tr>
<td>53°N</td>
<td>0930</td>
<td>0630</td>
<td>Greenhow and Neufeld, 1961</td>
</tr>
<tr>
<td>54°N</td>
<td>0800</td>
<td>-</td>
<td>Muller, 1966</td>
</tr>
<tr>
<td>56°N</td>
<td>0900</td>
<td>0600</td>
<td>Zadorina et al. 1967</td>
</tr>
</tbody>
</table>

Although the observed phase of the semi-diurnal tide is similar at each altitude for the 9 day interval (fig. 9.2), there are small day to day changes as shown in fig. 9.5. Evidence of this can also be seen in the data averaged over all heights in the zonal and meridional time series (fig. 8.7). For both observing directions and for most altitudes there is a systematic phase decrease, that is, the peak velocity occurs later in time. Note also the $\pi/2$ (3 hours) phase shift in the examples given in fig. 9.5. The average phase decrease is approximately $0.17 \text{ rad.day}^{-1}$. Seasonal phase variations are presented in Greenhow and Neufeld (1961) where a phase increase of $0.13 \text{ rad.day}^{-1}$ is observed, in agreement with the value of Sprenger and Schminder (in Glass and Spizzichino, 1974) for the changeover from summer to winter regimes. During the winter months however only small phase changes are observed by these groups, being less than $0.02 \text{ rad}$.
Fig. 9.5 Semi-diurnal phase, daily variations.

day$^{-1}$ average for the period from December to February. The results of Vincent and Stubbs (1977) also indicate that the semi-diurnal phase at a given altitude may vary by up to four hours (2 rad.) although a systematic variation is not observed. The present rapid (by comparison with Greenhow and Neufeld) phase changes are consistent for both directions of observation and indicate that data from individual isolated days may be insufficient to define a seasonal average. Data from short observing periods however often displays a repeatable pattern from year to year, (Muller, 1970). A continuation of this apparent phase decrease would produce results in disagreement with other southern hemisphere winter data, so presumably it is a short term fluctuation caused by localized variations in the generation or propagation of the semi-diurnal tide, the exact nature of which are as yet unknown.
9.1.2 The Diurnal Tide

The velocity spectra (section 8.4) show the amplitude of the diurnal component to be approximately half that of the semi-diurnal. Harmonic analysis reveals a similar amplitude, indicating that any short wavelength propagating diurnal modes were not suppressed by averaging over all heights. The theory of the diurnal tide requires both short wavelength propagating modes and evanescent modes to describe the atmospheric response, and so will be sensitive to local solar input variations. The propagating modes are confined mainly to low latitudes, while the evanescent modes are larger in high latitude regions, resulting in this component being more latitude dependent than the semi-diurnal. The present results exhibit almost constant amplitude for all heights, being typically 10 to 15 ms\(^{-1}\) (fig. 9.6). Chapman and Lindzen (1970) suggest the diurnal amplitude should show little variation with height for latitudes greater than 45°. Comparisons with other data are only relevant for similar latitudes due to the marked latitude dependence of the diurnal tide. Results obtained at Adelaide (35°S) are dominated by the diurnal tide whose amplitude is typically 20 to 30 ms\(^{-1}\) (Elford, 1959). This dominance is also confirmed by incoherent scatter results at Arecibo (18°N) (Mathews, 1976). The diurnal amplitude at Garchy (47°N) is typically 10 to 20 ms\(^{-1}\) (Glass and Spizzichino, 1974), while Greenhow and Neufeld (1961) find winter values of approximately 4 ms\(^{-1}\) for 53°N. The magnitude of the present results are in reasonable agreement with those of Glass and Spizzichino and intermediate between the results at 35°S and 53°N for winter.

Although the amplitude of the diurnal components suggests evanescent modes, the vertical phase behaviour (fig. 9.7) exhibits little regular behaviour and large day to day variations. Complicated vertical phase structure of the diurnal tide is also reported by Spizzichino (1969) who suggests it be called a 'diurnal oscillation' as a result of the severe
phase changes altering the apparent period. In fig. 9.7 there is also no consistent sense of rotation of the tidal vector, and Bobysunov and Karimov (1971) have observed diurnal components with both clockwise and anticlockwise senses of rotation in meteor wind data. Since the diurnal tide is expected,
Fig. 9.7 Diurnal tide, phase.
to be composed of both short wavelength propagating and evanescent modes, a two dimensional spectral analysis in both frequency and wave number spaces is required to identify the modes. Application of this method at Garchy has identified the propagating (1,2) mode in winter, while the phase behaviour in summer is consistent with the predominance of evanescent modes (Spizzichino 1969, 1972, Fellous et al., 1974, 1975). Two dimensional spectral analysis is impractical with the present results divided into 5 km strata.

In contrast to the semi-diurnal phases, those of the diurnal tide exhibit large variations from day to day (fig. 9.8) as observed by other experimenters. Although there is some tendency for similar variations, the results are in general irregular and are probably responsible for the suggested lack of coherence of this period in spectral analysis (Clark, 1975).

9.1.3 The ter-diurnal tide

An eight hour oscillation exists in the daily ground level pressure oscillations (fig. 9.16), so it is usual to include a ter-diurnal component in the harmonic analysis of upper atmosphere winds. The velocity spectra (fig. 8.7) reveal peaks at periods near eight hours, but these are only of a similar magnitude to the higher frequency components considered as noise. The M.E.M. spectra of single days observations also show periods near eight hours, while spectral analysis by Glass and Fellous (1975), and Pokrovskiy and Teptin (1970) showed significant energy near a period of eight hours. Results at Adelaide show this component has an amplitude slightly in excess of the r.m.s. deviation in the tidal components. Greenhow and Neufeld (1961) consider it only significant in yearly averages, with amplitudes of 2 to 4 ms\(^{-1}\). Muller (1966) finds similar amplitudes and presents evidence for a clockwise vector rotation.
In the present results the eight hour component has been deduced from 10 km height intervals (centred on 85, 95 and 105 km) in an attempt to increase the sample sizes and hence the reliability of the velocity determinations. The average horizontal velocities over the central height interval are shown in fig. 9.9 for a 24 hour period (DAY 9) with the prevailing 24 and 12 hour oscillations subtracted. The residual velocities are reasonably well approximated by the fitted eight hour components, the amplitudes being 10 and 12 ms\(^{-1}\) for the meridional and zonal components respectively. The zonal component's phase leads that of the meridional, although the two orthogonal components have almost opposite phases. The variation of the daily average eight hour components is shown in fig. 9.10.
for the meridional velocities. The average value is near 8 ms\(^{-1}\) for all heights, and although larger than the values of Muller (1966) and Greenhow and Neufeld (1961), it is of a similar magnitude to the results of Glass and Fellows (1975), who also find insignificant growth of this component with altitude. The eight hour component shows no significant variation from day to day. It is however the tidal component most susceptible to gaps in any time series, since it has the lowest effective sampling rate. The corresponding phases show considerable day to day variability (fig. 9.11) and although only determinations are available at only three altitudes, both positive and negative phase gradients exist. It is not considered worthwhile to assign apparent vertical wavelengths to these data, but to note the variability of the phases. The phases for both zonal and meridional components in the central height interval (fig. 9.12) show a tendency for the zonal component to have the leading phase however this is not a regular anticlockwise rotating vector.
Fig. 9.10  Ter-diurnal tide, daily amplitude variations (meridional)

Fig. 9.11  Ter-diurnal tide, daily phase variations (meridional)

Fig. 9.12  Orthogonal phase variations (95 km)
The eight hour component is of interest since it has several possible origins. It may be the third tidal harmonic, an internal gravity wave or the result of non-linear tidal interactions. Glass and Fellous (1975) have calculated the tidal variation expected to result from solar insolation by ozone in an isothermal atmosphere and conclude such a mode has a vertical wavelength in excess of 100 km. The observed wavelengths however are often approximately 35 km (Glass and Fellous). The observed regularity of occurrence and hence regularity of generation suggests it is unlikely that this mode can be ascribed to internal gravity wave activity. The theory of non-linear wave interactions of Spizzichino (1969) shows the process to be similar to the mixing or multiplying of two waves $\omega_1, k_1$, $\omega_2, k_2$ producing 'beat' oscillations $\omega_1 \pm \omega_2, k_1 \pm k_2$. If the dominant 12 and 24 hour propagating modes are the (2,2) and (1,2) modes respectively, the resulting 8 hour oscillation will have a vertical wavelength slightly less than 25 km. Spizzichino suggests such an interaction requires a region where both diurnal and semi-diurnal components are present simultaneously with amplitudes in excess of 20 ms$^{-1}$, restricting it therefore to altitudes greater than 95 km confined to within 40° of the equator.

To determine whether a non-linear interaction takes place requires the parameters of all tidal oscillations to be well determined, preferably with and without the interaction present. If the interaction results in short wavelength modes this may explain the smaller amplitudes observed by Greenhow and Neufeld (1961) and Muller (1966), since they both used experiments with little or no height discrimination. However, an oscillation of 5 to 10 ms$^{-1}$ is still present when the present data are considered over all heights. In these circumstances, contamination by the diurnal mean meteor echo height variation is also possible (section 4.6). Although the non-linear theory provides a possible origin for the eight hour component at meteor heights, extensive downward propagation would be required for it to account for the observation of an eight hour component at ground level.
Evidence for the dominance of the zonal prevailing wind is provided by the velocity time series of fig. 8.7. The average prevailing components from these examples, including all altitudes are compared with other meteor wind data in table 9.4.

<table>
<thead>
<tr>
<th>LOCATION</th>
<th>LAT.</th>
<th>ZONAL</th>
<th>MERIDIONAL</th>
<th>TIME</th>
<th>SOURCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Christchurch</td>
<td>43°S</td>
<td>15</td>
<td>0</td>
<td>July</td>
<td>Present results</td>
</tr>
<tr>
<td>Adelaide</td>
<td>35°S</td>
<td>15</td>
<td>-10</td>
<td></td>
<td>Elford (1959)</td>
</tr>
<tr>
<td>Garchy</td>
<td>47°N</td>
<td>14</td>
<td>-</td>
<td>Jan.</td>
<td>Spizzichino (1972)</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>LOCATION</th>
<th>PREVAILING GRADIENT (ms⁻¹·km⁻¹)</th>
<th>TIME</th>
<th>SOURCE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Christchurch</td>
<td>-0.8</td>
<td>July</td>
<td>Present</td>
</tr>
<tr>
<td>Christchurch</td>
<td>-1.6</td>
<td>Winter</td>
<td>Wilkinson (1973)</td>
</tr>
<tr>
<td>Adelaide</td>
<td>-3.3</td>
<td>June</td>
<td></td>
</tr>
<tr>
<td>Jodrell Bank</td>
<td>0.8</td>
<td>Jan.</td>
<td></td>
</tr>
</tbody>
</table>

Since the mean prevailing component represents the general circulation, comparisons are only meaningful for restricted latitude regions. The present results are dominated by an eastward flow of 15 ms⁻¹ consistent with the mean winter flow for mid-latitudes at meteor heights. In general the meridional component is smaller than the zonal, again in keeping with the present results.

The average prevailing component variation with altitude has been compared with the model of Groves (1969), interpolated for a latitude of 45° and northern hemisphere January (fig. 9.13). The direction of the zonal flow is in agreement although the magnitudes are smaller. At lower altitudes the flow is strongly eastward but the magnitude decreases with
increasing height and accords with the results for Adelaide (winter) of Vincent and Stubbs (1977). The meridional component is again much smaller than the zonal and is poleward at heights below 100 km, but briefly reverses above this height. This result is also in agreement with the Adelaide winter data but conflicts with Groves' model, having a mean velocity gradient of opposite signs for a latitude of 45°. The velocity gradients of mean prevailing components are also included in table 9.4 where a small positive
The meridional gradient in winter is also found in the results of Wilkinson (1973) and Greenhow and Neufeld (1961). The zonal gradient while agreeing with Groves' model does not agree with the winter results of Greenhow and Neufeld.

A possible reason for the discrepancy is the use of short term averages to provide the seasonal mean flow. Day to day fluctuations about the mean value have been observed by Greenhow and Neufeld, however the usual practice is for meteor wind experiments to operate for 1 to 10 days per month and the derived prevailing terms refer to these periods. The velocity spectra indicate periodicities in excess of 24 hours are present and these will be expected to contribute to the variations in the prevailing components. The daily prevailing components for 5 km height intervals are shown in fig. 9.14, where the general trend for the mean values of fig. 9.13 is also apparent, for example the negative gradient of the zonal component.

The meridional components reveal the presence of variations whose periods are similar to those of the meridional spectrum, that is, 70 to 80 hours. These variations exhibit little phase or amplitude variation with altitude. Similar regular variations have been observed by Greenhaw and Neufeld (1961) and Vincent and Stubbs (1977). Although the zonal velocity spectrum contains a similar long period component to that in the meridional, there is no evidence for similar prevailing component variations (fig. 9.14).

There is however confirmation of the long term variation apparent in the zonal time series not resolvable by spectral analysis methods. Some evidence for a longer term trend is also provided by the meridional variations, for example the 92 and 97 km velocities (fig. 9.14).

The apparent absence of the expected 70 to 80 hour variations in the zonal mean winds is probably due to the method used to obtain the mean values. Individual prevailing wind values span 24 hours and are not sampled until another like period has elapsed, hence the Nyquist period is
Fig. 9.14 The prevailing wind, daily variations.
48 hours. Long period variations are better revealed by low pass filtering of the average velocities. A suitable filter consists of a 24 hour running mean, effectively removing the tidal components within any such interval. Alternatively it may be viewed as a filter with \( \sin X/X \) type response whose zeroes fall exactly on the tidal frequencies. This method has been used by Glass et al. (1975) and applied to the present results for all altitudes (fig. 9.15). Variations with periods appropriate to those suggested by the velocity spectra are present, superimposed on the much slower variation in the zonal case.

9.3 **LONG PERIOD COMPONENTS**

The existence of periods in excess of the longest tidal period, that is the earth's rotation period, have been demonstrated in the velocity spectra and the filtered velocity time series. The presence of planetary scale oscillations is well established in the troposphere and stratosphere and has been extended into the mesosphere by satellite radiance measurements (Harwood, 1975). Other results detailing the existence of long period oscillations in the lower thermosphere are present in meteor wind results and variations of ionospheric parameters. In order to determine the origin of these variations, attempts have been made to correlate them with similar variations at lower levels, or to examine their horizontal extent and behaviour.

The ionospheric E region is thought to be well approximated by a Chapman layer and so the height of constant electron density isopleths should be a surface of constant pressure. Brown and Williams (1971) find the electron density isopleths have similar variations to the height of the 10 mb pressure surface in the lower atmosphere, the variations being in phase. This data is also used by Deland and Cavalieri (1973) and the isopleths are also found to be correlated with the ionospheric parameter \( f_{\text{min}} \).
Fig. 9.15 Filtered velocity time series (all heights)

representative of D region absorption. From the size of the height variation of each isobaric surface (E region and 10 mb) these authors conclude the increased $\Delta p$ with height corresponds to a wind satisfying the geostrophic approximation of $\pm 100 \text{ ms}^{-1}$ in the E region, larger than is usually observed.
A similar approach in the southern hemisphere by Fraser and Thorpe (1976) showed a peak in the partial coherence spectra between $f_{\text{min}}$ and 30 mb temperatures at a period near five days. Geisler and Dickinson (1976) have shown this period to correspond to the gravest natural mode of the atmosphere, for which significant velocities extend up to 100 kms. Using an approximate latitude circle of ionosondes, Cavalieri (1976) showed the minimum virtual height parameter $h'E$ exhibits periods in the range 10 to 15 days. Lag correlations for the various stations show a slow westward drift of the periodic features consistent with wavenumber 1. There is also some correspondence with concurrent satellite radiance data, indicative of stratospheric and mesospheric temperatures. Other ionosphere/stratosphere correspondences are also observed (see for example the review of Murata (1974)) however the ionospheric indicators are usually indirectly related to the parameters of the neutral atmosphere.

The first attempt to observe prevailing wind variations and relate these to surface pressure variations was that of Greenhow and Neufeld (1960). In the examples presented, the variations also appear to be in phase. This approach has been continued by Muller (1972) and Muller and Kingsley (1974b) who find examples with periods near 51 hours (and occasionally longer) and corresponding periods in the spectra of corresponding surface pressure data. Teptin (1972) presents evidence for the existence of a four day period. Portnyagin and Svetogorova (1977) have analysed a 128 day time series of meteor wind data, and radiosonde velocities at 5, 10, 15, 20 and 25 km altitudes. The meteor wind data exhibit peaks near periods of 4 to 5 days (this may be the mode theoretically described by Geisler and Dickinson) and 18 days and corresponding peaks are observed in the lower atmosphere velocities. The spectra of Barnes (1972) however for two orthogonal wind components extending over 173 days reveal no significant peaks other than the diurnal and semi-diurnal tides.
A considerable body of evidence is now accumulating for the presence of a 48 hour oscillation in the meteor region, as revealed by the spectra of Muller (1972) (51 hour), Clark (1975), Glass et al. (1975) and the results of Roper (1977) and Vincent and Stubbs. (1977). Roper includes a 48 hour period in the periodic velocity analysis and finds amplitudes as large as 50 m s$^{-1}$ in summer and winter for the zonal component, and smaller values for the meridional. Clark (1975) reports a 48 hour component with amplitudes of 5 to 15 m s$^{-1}$ from the Fourier transform of 256 hour time series. Glass et al. (1975) used a 24 hour running mean on meteor wind data obtained at Garchy and Obninsk. Cross-correlations of the data show a regular period of 48±6 hours in one example while spectra include this component and additional peaks in the 5 to 6 day period range. At Garchy there is very little height variation in either the amplitude or phase of the 48 hour oscillation, but the time delay between Garchy and Obninsk is consistent with a horizontally propagating planetary scale wavenumber 3 oscillation.

Periodic regression (Appendix G) has been applied to the present velocities separated into 10 km height intervals, weighting each 30 minute average by the number of values comprising the average (table 9.5). The largest component is the semi-diurnal tide, showing an amplitude decrease with increase of height, slowly varying phase and a lead of approximately $\pi/2$ of the zonal component. The diurnal amplitudes are smaller than recorded on individual days probably due to the lack of coherence of this period. The prevailing components are consistent with the magnitudes and gradients detailed in section 9.2. Although not present in the spectra (section 8.4), a 48 hour component has been included and the meridional value shows a decreasing amplitude with height and approximately constant phase. The zonal values exhibit much greater variability. The mean period of the zonal and meridional long periods, 75 hours, has also been fitted.
Table 9.5 Periodic Components.

<table>
<thead>
<tr>
<th>Period: 75h</th>
<th>48h</th>
<th>24h</th>
<th>12h</th>
<th>Prevailing</th>
<th>N</th>
</tr>
</thead>
<tbody>
<tr>
<td>Meridional</td>
<td>A</td>
<td>φ</td>
<td>A</td>
<td>φ</td>
<td>A</td>
</tr>
<tr>
<td>105 km</td>
<td>3.2</td>
<td>2.9</td>
<td>4.9</td>
<td>-2.8</td>
<td>0.6</td>
</tr>
<tr>
<td>95</td>
<td>5.0</td>
<td>-0.3</td>
<td>5.0</td>
<td>-2.4</td>
<td>2.3</td>
</tr>
<tr>
<td>85</td>
<td>5.5</td>
<td>-3.1</td>
<td>8.0</td>
<td>-2.7</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Zonal</td>
<td>A</td>
<td>φ</td>
<td>A</td>
<td>φ</td>
<td>A</td>
</tr>
<tr>
<td>105</td>
<td>1.9</td>
<td>2.6</td>
<td>5.7</td>
<td>-0.7</td>
<td>10.0</td>
</tr>
<tr>
<td>95</td>
<td>2.0</td>
<td>0.6</td>
<td>1.7</td>
<td>-1.6</td>
<td>3.5</td>
</tr>
<tr>
<td>85</td>
<td>4.2</td>
<td>0.4</td>
<td>2.0</td>
<td>2.0</td>
<td>2.9</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

N is the number of values present in the 220 hour time series.

to both sets of data. Both directions share a decreasing amplitude with height and a rapid phase increase with height, although there is little difference in phase between the two components for the upper two height intervals. Least squares fitting in this manner may be subject to contamination of the longer periods by the phase variability of shorter periods, especially the diurnal components, during the observing interval. The amplitudes are in general less than those observed by other authors, some of which may be attributable to this feature. Since the periods are no longer orthogonal, the choice of their number and magnitudes will also have an effect on the coefficients so determined. It is considered that the above choice has included all the major contributors.
The origin of the long period components may lie with planetary scale motions at lower levels, sufficient energy of which leaks through the intervening atmosphere to the meteor region. The M.E.M. spectrum of the surface pressure recorded at Christchurch Airport is shown in fig. 9.16. The pressure values are hourly readings and extend over a period of 24 days spanning the wind observation period. In contrast to the upper atmosphere winds, which are dominated by the semi-diurnal tide, most energy is contained in the long period part of the spectrum. The M.E.M. spectrum does resolve the regular semi-diurnal tide, whose amplitude is approximately 0.5 mb and an 8 hour oscillation. A peak in the spectrum near three days is observed in reasonable agreement with those observed in the meteor wind data. The spectrum of the pressure data obtained using the discrete Fourier transform attributes an amplitude of 4 mb to this period during the 24 day interval.

The time series of pressure data (fig. 9.17) shows very large variations near July 1st whose time scale is three days and amplitude range 30 mb. A similar time scale can also be associated with a smaller variation (approximately 10 mb) near July 10th. Either of these may be associated with the velocity variation aloft, but if some form of propagation is implied, a study of the intervening regions should also provide similar variations. Unfortunately little such data are available, especially above 30 km, while below this level a periodic disturbance may be masked by the large scale synoptic variations. If the large scale 1st of July variation is responsible then there is a considerable phase delay, equivalent to a vertical phase velocity of 0.1 ms⁻¹. If this phase velocity is uniform the disturbance should reach the 20 mb (27 km) level in the stratosphere approximately three days later. Also shown in fig. 9.17 are the radiosonde temperatures above Christchurch at the 100, 30 and 20 mb levels for the relevant interval. However, no such correspondence is observed in the
early sections of any of the stratospheric temperature soundings. The second three day variation does appear to have a counterpart in the 100 mb temperature data, but superior levels again fail to reveal any evidence of vertical propagation, despite the similarity of the spectral periods observed. The resolution of the temperature data is 1°C and the leakage of variations smaller than this amount may be responsible for significant velocities in the meteor region.

Several previous authors, for example Greenhow and Neufeld (1960), Brown and Williams (1971) and Muller (1972), find the variations near 100 km are in phase with those at ground level. Since integral wavelength phase differences are unlikely (the variations are not always regular), this may suggest evanescent modes and hence no energy transport. Deland
Fig. 9.17 Surface pressure, stratospheric temperature and meteor wind variations.
and Johnson (1968) find the amplitude of planetary scale travelling waves increases by 50 to 100% from ground level up to the 300 mb level, and then decreases up to the 100 mb level. Observational results of the vertical energy flux of planetary waves up to the 10 mb level (30 km) has been presented by McNulty (1976). For stationary waves the energy flux is a maximum in winter, in contrast to the predictions of Charney and Drazin (1961). All modes show an upward decay of energy in mid-latitudes in agreement with the calculations of Dickinson (1969). Approximating the upward energy flux by a function of the form $A \exp(bz)$ shows the winter average vertical flux to have decreased to 1% at an altitude of 38.8 km. For travelling waves, the results of McNulty are more complicated but again a decreasing vertical energy flux in the winter stratosphere is observed. The size of the oscillations observed at ground level are typically 4 to 12 mb (Muller, 1972) and if evanescent, these may have decayed to small amplitudes at 100 km. Deland and Cavalieri (1973) however find the changes in the isobaric surfaces have increased with altitude, by a factor of 10 from 30 to 110 km. Even for irregular variations, no common phase variation is observed in the present data. Meridional and zonal velocities derived from radiosonde data are available up to the 100 mb level, and an inspection of these also fails to provide supporting evidence for the upward propagation of these periods from levels below 30 km.

Another possibility for the appearance of planetary scale features in the lower thermosphere is generation within the region. Leovy and Ackerman (1973) present data showing the presence of transient variations with periods of 1½ to 4 days in the winter mesosphere near 45 km. The velocity and temperature fluctuations are approximately 20 ms$^{-1}$ and 7°C respectively, however these oscillations decay rapidly above 45 km. The mechanism proposed by Simmons (1974) considers the conditions for baroclinic instability at the stratopause and finds the growth rates for disturbances
of the observed periods to be appreciable. A similar situation may be
considered at the mesopause, however when Newtonian cooling with the
appropriate time scale is considered the disturbance growth rates are
negligible. It therefore seems theoretically unlikely that the observed
long period variations are the result of locally generated instabilities.

The observation of periods near 48 hours suggests rotational or
tidal origins. Hines has proposed that the 51 hour period observed by
Muller (1972) is caused by the diurnal tide propagating to meteor heights,
causing turbulence and restricting the propagation of the next diurnal tide
and so on. This scheme should be accompanied by a varying day to day energy
loss of the diurnal tide and would require the process to be regularly
repeated, since the 48 hour oscillation of Glass et al. (1975) extends
over a period of 10 days. Turbulence as severe as this would also be
expected to restrict the propagation of other oscillatory modes. Alternate
propagation and absence of the diurnal tide will reveal itself in a running
harmonic analysis of the velocity time series. The results are shown in
fig. 9.18 for the meridional component averaged over all heights, since it
is this type of data to which the present spectra and those of Muller refer.
Large excursions of the diurnal amplitude and phase are observed, however
it does not seem appropriate to assign periods to these seemingly irregular
variations. The diurnal phase however does appear to contain a long term
trend. These variations are not matched by those of the semi-diurnal
component whose phase is much more regular although there are day to day
amplitude variations. The absence of appreciable changes in the diurnal
dissipation rate (the amplitudes have little variation with height, fig.
9.6) suggests that if varying amounts of turbulence impedes the diurnal
propagation then it must be generated at lower altitudes. This reduces
the likelihood of the diurnal tide producing turbulence since at lower
altitudes the amplitude and hence vertical wind shear are smaller, although
Lindzen (1971) suggests the (1,2) mode may become unstable near 85 km. It may be more appropriate to test this mechanism at lower latitudes, where the diurnal amplitude is larger. It is interesting to note in this context that the largest amplitude observed for a 48 hour component has been the

Fig. 9.18 Diurnal tide, amplitude and phase variations.
(relatively) low latitude results of Roper (1977).

Muller (1972) states that the 48 hour period is too large to be associated with tidal phenomena. Sub-harmonic periods may in general be generated when the temporal wave equation in canonical form contains a periodic coefficient with the same frequency (that is, it is a Mathieu equation). The tidal formulation usually assumes explicit time dependences for the perturbations in order to eliminate the time derivatives. The constitutive relations include the atmospheric scale height $H$, assumed time independent. In general $H$ has a diurnal variation which may give rise to the conditions for sub-harmonic generation. Although this idea is not continued in the present work, there is a natural extension to longer period sub-harmonic frequencies, however the mathematical complexity is not amenable to simple solutions.

As an alternative to direct vertical propagation of energy, Muller (1972) suggests a modulation, de-modulation process involving internal gravity waves propagating obliquely, whose transmission is governed by the planetary scale properties of the lower atmosphere. On reaching the meteor region this energy and momentum is deposited as turbulence and modifies the background flow, impressing the lower atmosphere time scales upon it. The propagation of internal gravity waves is influenced mainly by the vertical distribution of temperature and horizontal winds and so the dominant scale in the meteor region may reflect the conditions in any intermediate region and not just those at ground level. This mechanism cannot explain the observation of in phase variations due to the propagation delay. The varying internal gravity wave flux should be able to be observed in any intermediate region, for example by the partial reflection ionospheric drifts experiment, whose altitude range extends lower than that of the meteor wind experiment.
The variation of the semi-diurnal tide with background winds suggests a further indirect path for the transfer of planetary scale periods from lower levels. In the theory of Lindzen and Hong (1974), seasonal mean zonal winds and associated temperature gradients are used. Variations from the seasonal mean will produce variable atmospheric transmission characteristics. These variations may be the result of lower atmosphere planetary scale motions, resulting in a possible amplitude modulation of the semi-diurnal or any other tide. This process could also reflect the variations in any atmospheric level but the effects are likely to be more severe at lower levels where the tidal amplitudes are small. A non-linear process is required however to extract the modulation in the meteor region and the results of Lindzen and Hong indicate the phase variation introduced into the semi-diurnal tide by the inclusion of background winds is much larger than the amplitude variation.

The variation of the semi-diurnal tidal amplitudes for the present data is shown in fig. 9.19 where similar variations occur for both orthogonal components and with time scales similar to those of the prevailing component. The r.m.s. velocity change associated with these variations is approximately 25 ms\(^{-1}\) and is considerably larger than that associated with the long period oscillations. The variation of the apparent semi-diurnal vertical wavelength reveals no correspondence with the tidal amplitudes. Modulation of the semi-diurnal tide is reported by Portnyagin and Svetogorova (1977) who find significant peaks in the range 2, 4 and 20 days and similar peaks in the prevailing wind spectra. These authors suggest the variation may result from the influence of the global circulation of the stratosphere on the total ozone distribution, and hence the principal agency for tidal generation. If the generation of the tides is responsible then a similar effect may be expected for the diurnal tide, although it may prove difficult to extract due to this component's apparently irregular
behaviour. The present results reveal no similarity between the amplitude variations of the diurnal and semi-diurnal tides.

Also shown in fig. 9.19 are the daily average standard deviations, shown inverted to reveal the similarity to the semi-diurnal tidal amplitudes. The comparison suggests the additional energy for the variations of the mean...
SD may be contributed by the semi-diurnal tide and hence alters the prevailing component. Since the observations of the standard deviations are similar for both observing directions, if this is a real variation then a source of large horizontal extent is required. If the standard deviation is associated with a r.m.s. velocity, the range of variations corresponds to $22 \text{ ms}^{-1}$ and $19 \text{ ms}^{-1}$ for the zonal and meridional examples. The tidal amplitudes refer to peak values and the equivalent r.m.s. variation is $25 \text{ ms}^{-1}$. There is no significant agreement between the dissipation rate of the semi-diurnal tide with altitude and the standard deviation variations, although it is interesting to note that on day 9 the maximum amplitude growth is observed, coincident with the smallest standard deviation for both directions. Observation of these parameters over a longer time scale with a reliable determination of the nature of the irregular component contribution is required before its effect on the prevailing wind can be assessed. A possible mechanism for the appearance of planetary time scales in the meteor region may consist of the modulation of tidal components by background winds in the lower atmosphere which dissipate some of this energy in the meteor region leading to variations in the irregular component and the deduced prevailing winds.

**Summary of wind velocity results**

1. **The irregular component**

   The major contribution to the irregular component, which is typically $30 \text{ ms}^{-1}$, is of atmospheric origin. It shows little variation over height intervals exceeding 5 km and time scales in excess of 30 minutes. This component may be thought of as Gaussian geophysical noise and hence may be used to determine the statistical reliability of any time averaged velocity sample. Apparent variations have been observed which are similar over a large horizontal extent (350 km) and cannot be attributed to instrumental effects.
(2) **The prevailing wind**

The prevailing wind is predominantly zonal with larger velocities at lower altitudes. Both the zonal and smaller meridional components have day to day variations of 5 to 10 ms\(^{-1}\) which may be periodic in nature.

(3) **The semi-diurnal tide**

This is the largest periodic component and dominates the velocity spectra and harmonic coefficients. The amplitudes are typically 20 ms\(^{-1}\) and there is a slow phase drift during the observing period. The phase differences between the orthogonal velocity components is consistent with an anticlockwise vector rotation. The phase increase with altitude for both zonal and meridional components corresponds to a large vertical wavelength mode or modes, with downward phase propagation. Although there is some day to day variability in the apparent wavelength, this may not be significant when the uncertainties of the individual phases is considered. The downward phase propagation for a tidal mode corresponds to the upward propagation of energy, however the vertical amplitude structure in general suggests an energy loss with altitude.

(4) **The diurnal tide**

This component has approximately half the amplitude of the semi-diurnal tide and the amplitude shows little consistent variation with height. The vertical phase behaviour is very irregular and cannot be associated with a dominant propagating mode. The daily phase variation is also irregular, although there is a tendency for both meridional and zonal components to have similar variations.

(5) **The ter-diurnal tide**

Although the amplitude is usually less than 10 ms\(^{-1}\), the ter-diurnal tide appears to be a real feature of the upper atmosphere winds. Amplitude variations are not significant and vertical phase gradients of either sign occur. There is a tendency for an anticlockwise rotation of the velocity
vector, but not on all days. The possible origins for an oscillation of this period cannot be distinguished in the present data, but are of considerable interest in assessing the importance of non-linear wave-wave interactions.

(6) **Long period oscillations**

Although long period oscillations are observed in the lower thermosphere data, their origins are not yet undetermined. Several possible mechanisms have been discussed including:

1) planetary wave propagation from lower levels
2) local generation of transient long period oscillations
3) variations in the diurnal tide
4) sub-harmonic generation
5) modulation of internal gravity wave fluxes
6) modulation of tides by background winds
7) modulation of tidal generation by the total ozone content.

To identify any of these possible mechanisms as a real contributor requires more than a single experiment. The difference between long period oscillations observed at a single site and planetary waves, which require spatial identification, should be emphasized. Long time series are also required in order to identify true periods associated with a well-established wave regime rather than short-lived transient fluctuations. The present data highlight the difficulties of separating motions into various time scales when these may vary across the observing interval. The presence of long periods contaminates tidal estimates using harmonic analysis, while the variations of the tides, especially the diurnal phases, will contaminate the estimates of the long period coefficients. Severe phase changes of the diurnal tide result in an apparent change of period and the use of a 24 hour running mean as a filter will not then cancel this component. This effect will also contaminate the coefficients determined using a least
squares periodic regression type of analysis. A phase variation of the 24 hour component will be expected to produce sidebands in the spectrum. The amplitude variations of the semi-diurnal tide are commensurate with those of the irregular component and the prevailing wind. This suggests the semi-diurnal tide may be giving up energy to the irregular component which in turn alters the prevailing wind. The lack of a marked change in dissipation rates of the tide indicates the energy is lost outside of the meteor region, however observation of these phenomena on a longer time scale is warranted.
CONCLUSIONS

The meteor wind experiment implemented at Rolleston, described in chapters 2 and 3, has largely met the aims outlined in the introduction. The use of a frequency offset scheme for deducing the train radial velocity produces results that are similar to those obtained by other experimenters (section 7.4) and circumvents the low velocity losses described by Wilkinson (1973). Although the present scheme incurs high velocity losses, experimental results imply that these losses are not significant, and verify the successful operation of the system, at least up to an altitude of 100 km. The success of the interferometric echo location technique has in general been verified by the use of an aircraft as a target, while the use of punched paper tape as a recording medium alleviates the problems associated with the reduction of filmed echo records. The present system is compact and self contained; however, several improvements are possible. These mainly concern the type of data to be recorded and the recording medium.

Although data from paper tape is simply reduced and visual checks of the output are possible while the system is running, large quantities are accumulated over extended observing periods and constant attendance is required. A change to the higher density recording medium of magnetic tape would reduce these problems and also allow conceptual changes further forward in the system. The maximum information to be obtained from individual return pulses is the amplitude and relative phase at each receiving antenna. The use of more than three antennas and associated receivers is unnecessary for a meteor wind experiment, since an accuracy of 2 to 3 km is ideally possible with the present configuration of three antennas, and this represents the smallest scale necessary for radial velocity studies.
If the transmitter is introduced into the three receivers as a local phase reference, a single received pulse can determine the arrival angles and a subsequent sweep will provide the radial velocity. The resulting cosine and sine phase output data are best obtained using six fast analogue to digital convertors operating in parallel providing all the necessary information. This is conditional on the controlling logic being capable of handling input information at six times the appropriate A/D sampling rate. Some pre-processing is also required to detect the presence of an echo and retain the appropriate echo information before beginning the subsequent scan of the six parallel receiver outputs. Such a system would be capable of recording multiple echoes, but the method of phase reference introduction requires a stable oscillator for differential phase calibrations, since any frequency shift from the transmitter reference will be present in the output as a doppler beat. The task of controlling the data gathering functions of such an experiment is suited to a micro-processor, allowing more flexibility in experiment design.

In the present receiver configuration, the undesirable dependence of the differential phase on receiver temperature, while not being serious when frequent calibrations are possible, may be reduced by attention to the additional i.f. tuned circuit in the phase reference channels. To this end, new receivers are at present being developed.

Although the direction finder verification using an aircraft as a target indicates consistent operation, it cannot be described as a calibration. Of other airborne methods, the transponder offers the most promise, since this may be carried aloft by a helicopter. The short pulses may reduce the effect of fading due to the rotation of the helicopter blades, while the target position can be determined visually. Such a calibration is essential since all antenna directions can be covered and hence reveal any irregularities due to features such as a non-uniform reflecting ground.
Most of the random uncertainties in target location can be attributed to noise, while the majority of unknown, possibly systematic, errors have their origin in the receiving antennas. In general, ground effects are unknown and the construction of reflecting grounds presents some difficulty at radio frequencies near 30 MHz. The effects of antenna mutual coupling are difficult to determine, but consistent results from an all sky calibration could confirm the magnitude of these effects. Also of importance is the presence of echoes received outside the major antenna lobe. These problems indicate that considerable attention should be directed towards antenna characteristics for a similar echo location system.

In the absence of systematic effects the altitude resolution of the system is 2 to 3 km, however equally important for atmospheric motions is the usable echo rate. The velocity variance may be associated with Gaussian 'geophysical noise' and hence determines the statistical reliability of any mean velocity value. The diurnal echo rate variation has a large effect on the ability of a meteor wind system to determine reliable velocity estimates over long periods. The meteor echo rate can be increased by raising the transmitter output power $P_T$, but (section 4.10) the echo rate $N$ is proportional only to $P_T^{1/2}$. The statistical reliability of a mean value is best expressed in terms of the standard error of the mean

$$\text{S.E.M.} = \frac{S}{\sqrt{N-1}}$$

and so $\text{S.E.M.} \sim P_T^{-1/4}$, a relatively weak dependence. Even with a high echo rate, some form of velocity interpolation is necessary to provide data equally spaced in time and height for further analysis. Such a scheme is used by Hess and Geller (1976) with an available peak transmitter power of 4 MW. Medium sensitivity meteor radars such as the present system are best suited to deriving tidal and other long period components of atmospheric motion.
Results of atmospheric movements in the lower thermosphere for southern hemisphere locations are not common and the successful operation of the present system has contributed mid-latitude winter data. Winds at these heights are dominated by the semi-diurnal tide as in the northern hemisphere. The phase behaviour of this component is consistent with a long vertical wavelength propagating mode which can be represented by a velocity vector rotating in an anticlockwise sense. The semi-diurnal tidal amplitude of approximately 20 ms$^{-1}$ is typically twice that of the diurnal component, whose phase behaviour is irregular. Prevailing winds are predominantly zonal in agreement with other observations. Long period components exist in the present data but in general their origins are not clear. Several mechanisms have been discussed but each of these requires further surveillance in conjunction with other techniques. In this respect the present location is well served since available data include: ground level pressure, stratospheric velocities and temperatures from radiosonde data, ionospheric parameters relevant to the D and E regions ($f_{\text{min}}$ and $h'E$), S.C.R. radiance data, partial reflection ionospheric drifts and meteor winds. It may be appropriate to verify the long period variations in the ionospheric indicators by considering the neutral gas velocities provided by meteor winds and a comparison with ionospheric drifts. Unfortunately, the respective location prevents coverage around a constant

The meteor train reflection theory (chapters 5 and 6) has important consequences for the meteor wind experiment. The method used to deduce echo amplitude decay data is seen to influence the derived decay parameters, and for transition echoes, the exponential part of the decay slope is not independent of electron density. This parameter can be estimated from the received echo amplitude, so it may be worthwhile to consider echo decay time constants at a given height to see if any dependence on electron density is
from the initial expansion of the effective scattering cylinder introduce spurious radial velocities and shears into meteor wind data. It may be difficult to distinguish these apparent motions from train shears, although the sign of the apparent acceleration is always negative. For large electron line densities ($> 10^{15} \text{ m}^{-1}$) the column reflects in a manner analogous to a metal cylinder, at least in the early stages of the echo lifetime. For a metal cylinder

$$\frac{\Delta \phi}{\Delta (kr_c)} \sim 2$$

and $r_c \sim a$ early in the echo lifetime so

$$\phi \sim ka \sim \frac{t^2}{\lambda},$$

whereas for a uniform radial velocity

$$\phi \sim \frac{t}{\lambda}$$

and for a uniform acceleration

$$\phi \sim \frac{t^2}{\lambda}$$

The observation of phase characteristics of overdense echoes and the use of multiple radio frequencies may help reveal the importance of such scattering radius movements.
APPENDIX A

ECHO ANGLES OF ARRIVAL AND POSITION ACCURACY

The uncertainties in the echo angles of arrival and position can be expressed in terms of the phase angle and echo range uncertainties. For definitions of the quantities used refer to section 2.5 and fig. 2.3. The two measured phase angles are \( \phi_{12}, \phi_{13} \) in terms of \( \lambda \)

\[
\phi_{12} = 2\pi d_{12} \cos \theta \cos \alpha \quad \text{(A1)}
\]

\[
\phi_{13} = 2\pi d_{13} \cos \theta \sin \alpha \quad \text{(A2)}
\]

From fig. 2.3, neglecting earth curvature

\[
x = R \cos \theta \cos \alpha
\]

\[
y = R \cos \theta \sin \alpha
\]

\[
z = R \sin \theta = R(\cos^2 \gamma - \cos^2 \beta)^{\frac{1}{2}}
\]

The height uncertainty is

\[
dz = \frac{\partial z}{\partial R} dR + \frac{\partial z}{\partial \theta} d\theta
\]

\[
= dR \sin \theta + R \cos \theta d\theta \quad \text{(A3)}
\]

and

\[
d\theta = \frac{\partial \theta}{\partial \phi_{12}} d\phi_{12} + \frac{\partial \theta}{\partial \alpha} d\alpha \quad \text{(A4)}
\]

From (A1)

\[
\frac{\partial}{\partial \phi_{12}} \cos \phi_{12} = \frac{1}{2\pi d_{12} \cos \alpha}
\]

\[
= -\sin \theta \frac{\partial \theta}{\partial \phi_{12}}
\]

so

\[
\frac{\partial \theta}{\partial \phi_{12}} = \frac{-1}{2\pi d_{12} \sin \theta \cos \alpha} \quad \text{(A5)}
\]
and \[ \frac{\partial}{\partial \alpha} \cos \theta = \frac{\phi_{12}}{2\pi d_{12}} \frac{\partial}{\partial \alpha} \] (sec \( \alpha \))

so \[ \frac{\partial \theta}{\partial \alpha} = -\frac{\phi_{12}}{2\pi d_{12} \sin \theta} \tag{A6} \]

From (A1) and (A2)

\[ \tan \alpha = \frac{d_{12}}{d_{13}} \frac{\phi_{12}}{\phi_{13}} \]

\[ d\alpha = \frac{\partial \alpha}{\partial \phi_{12}} d\phi_{12} + \frac{\partial \alpha}{\partial \phi_{13}} d\phi_{13} \tag{A7} \]

so \[ \frac{\partial}{\partial \phi_{12}} \tan \alpha = \frac{d_{12}}{d_{13}} \frac{1}{\phi_{12}} \]

and \[ \frac{\partial}{\partial \phi_{13}} \tan \alpha = \frac{d_{12}}{d_{13}} \frac{\cos^2 \alpha}{\phi_{12}} \tag{A8} \]

Similarly \[ \frac{\partial}{\partial \phi_{12}} \tan \alpha = \frac{-d_{12}}{d_{13}} \frac{\cos^2 \alpha}{\phi_{12}} \tag{A9} \]

Substituting (A8), (A9) into (A7)

\[ d\alpha = \frac{\cos \alpha}{2\pi d_{13} \cos \theta} \frac{d\phi_{13}}{2\pi d_{12} \cos \theta} \tag{A10} \]

Substituting (A10), (A6), (A5) into (A4)

\[ d\theta = \frac{-\cos \alpha}{2\pi d_{12} \sin \theta} \frac{d\phi_{12}}{2\pi d_{13} \sin \theta} \tag{A11} \]

((A10) and (A11) could equivalently have been obtained by applying a similar process to (A2) instead of (A1)). Substituting (A11) into (A3)

\[ |dz| = dR \sin \theta + \frac{R \cot \theta \cos \alpha}{2\pi d_{12}} d\phi_{12} + \frac{R \cot \theta \sin \alpha}{2\pi d_{13}} d\phi_{13} \]
Similarly

\[ |dx| = dR \cos \theta \cos \alpha + \frac{R \sin \alpha (\sin \alpha + \cos \alpha)}{2\pi d_{12}} d\phi_{12} + \frac{2R \cos \alpha \sin \alpha}{2\pi d_{13}} d\phi_{13} \]

\[ |dy| = dR \cos \theta \sin \alpha + \frac{2R \cos \alpha \sin \alpha}{2\pi d_{12}} d\phi_{12} + \frac{R \phi_{13}}{2\pi d_{13}} \]

These uncertainties may be assumed to result from zero mean Gaussian noise. Since the phase angle and range determinations are independent the associated variances may be added, however in general the phase contribution is much larger so contributions are simply added. The above result for \( dz \) could also have been obtained using the angles \( \beta \) and \( \gamma \) (fig. 2.3) although \( d\theta \) and \( d\alpha \) are not then obtained directly.
APPENDIX B

THE DOPPLER OUTPUT AMPLITUDE SPECTRUM

The doppler beat output signal is a pulsed sampling of the generally sinusoidal beat, which may include any local frequency offset. A periodic function may be represented by the appropriate Fourier series

\[ S(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos \frac{2\pi}{T} nt + b_n \sin \frac{2\pi}{T} nt \quad \text{(B1)} \]

where \( T \) is the fundamental period of the function and the expansion coefficients are

\[ a_n = \frac{2}{T} \int_{-T/2}^{T/2} S(t) \cos \frac{2\pi}{T} nt \, dt \]
\[ b_n = \frac{2}{T} \int_{-T/2}^{T/2} S(t) \sin \frac{2\pi}{T} nt \, dt \]
\[ a_0 = \frac{2}{T} \int_{-T/2}^{T/2} S(t) \, dt \quad \text{the average or d.c. term.} \]

Consider the infinite pulse train of fig. B1

Fig. B1. Rectangular pulse

\[ S(t) = \begin{cases} 1 & -\tau/2 < t < \tau/2 \\ 0 & \text{elsewhere} \end{cases} \]

= \( S(t+T) \)
Since $S(t)$ is an even function $b_n = 0$.

$$a_0 = \frac{2\pi}{T}$$

and

$$a_n = \frac{2\pi}{T} \frac{\sin n\frac{\pi}{T}}{n\frac{\pi}{T}}$$

So

$$S(t) = \frac{\pi}{T} (1 + 2 \sum_{n=1}^{\infty} \frac{\sin n\frac{\pi}{T}}{n\frac{\pi}{T}} \cos \frac{2\pi}{T} nt)$$

Assume the doppler sinusoid has constant unit amplitude

$$D(t) = \cos \omega_d t$$

The sampled output will be

$$f(t) = S(t).D(t)$$

$$= \frac{\pi}{T} \left( \cos \omega_d t + \sum_{n=1}^{\infty} \frac{\sin n\frac{\pi}{T}}{n\frac{\pi}{T}} \left[ \cos \left( n\omega - \omega_d \right) t + \cos \left( n\omega + \omega_d \right) t \right] \right)$$

where $\omega = \frac{2\pi}{T}$

This is the convolution of the individual pulse train and doppler beat spectra. No restriction has been placed on $\omega_d$ and this is also the spectrum for a pulsed radio frequency oscillation defined by

$$A(t) = \cos \omega_d t \quad -\frac{T}{2} < t < \frac{T}{2}$$

$$= 0 \quad \text{elsewhere}$$

except now $\frac{2\pi}{\omega_d} \ll T$ and the spectrum has a sin X/X type envelope centered about the frequency $\omega_d$.

From (B2) the pulsed doppler beat spectrum has terms at discrete angular frequencies

$$nw \pm \omega_d \quad n = 0,1,2,\ldots$$
Provided $\omega_d$ is sufficiently small compared with $\omega - \omega_d$, the doppler beat may be filtered out from this spectrum and we wish to find the proportion of the total power contained in this component. The Fourier series (Bl) may equivalently be expressed as a complex series

$$f(t) = \sum_{n=-\infty}^{\infty} C_n \exp\left(\frac{2\pi}{T} nt\right)$$

where in this representation both positive and negative frequencies (representing two phasors rotating with opposite senses) are included. Comparison with (Bl) reveals

$$|C_n|^2 = \frac{a_n^2 + b_n^2}{4}$$

The power in a complex waveform is

$$\frac{1}{T} \int_{-T/2}^{T/2} f^2(t) dt = \sum_{n=-\infty}^{\infty} |C_n|^2 \quad \text{(Parseval's theorem)}$$

$$= |C_0|^2 + 2 \sum_{n=1}^{\infty} |C_n|^2$$

the sum of the powers of the complex Fourier components. The average power in a pulsed sinusoid is

$$P_{av} = \frac{1}{T} \int_{-T/2}^{T/2} f^2(t) \, dt$$

$$= \frac{1}{T} \int_{-T/2}^{T/2} \cos^2 \omega_d t \, dt$$

$$= k_2(T) \cdot \frac{T}{2}.$$
\[ P_1 = 2 |c_1|^2 \]
\[ = \frac{a_1^2}{2} \]
\[ = \frac{T}{T} \]

The fraction of the power contained in the doppler sinusoid component is

\[ \frac{P_1}{P_{av}} = \frac{T}{T} \]

which is just the ratio of the pulse length to the inter-pulse period.
Although the antenna pattern may be determined under transmitting or receiving conditions, the latter necessitates a stable portable oscillator and test antenna rather than mobile receiving and recording apparatus. The portable crystal controlled c.w. oscillator (fig. C.1) consists of an oscillator, driver and class B output stage, valves being used to obtain high power levels. The output is fed via a toroidal balun into a $\lambda/2$ dipole mounted on a pneumatic ram, capable of being elevated to a height of 7 metres. Power output can be continuously adjusted and this feature was used for differential phase calibrations (section 4.4). The oscillator frequency dependence on temperature is approximately $\frac{\Delta f}{\Delta T} \sim 21 \text{ Hz } ^{\circ} \text{C}^{-1}$.

Since the mobile source is unmodulated a d.c. coupled receiver output is required. Alternatively the a.g.c. signal of a communications receiver (Eddystone 680X) may be used. This produces a logarithmic characteristic over a range of 30 db for a.g.c. voltages down to $-6$v (fig. C.2). The maximum signal may be set to this level by r.f. gain adjustment. The high impedance a.g.c. line requires the use of a high impedance voltmeter. Alternatively if the antenna under test can be uniformly rotated, the output applied to a chart recorder produces an immediate direct logarithmic radiation pattern. The method has been verified by rotation of the test dipole and by comparison with the theoretical $\lambda/2$ dipole pattern (fig. C.3). The expected resolution is approximately 1 db and the agreement with the theoretical dipole pattern in general confirms this.
Fig. C.1 Portable oscillator

A.G.C.(V)

Fig. C.2 A.G.C. characteristic
APPENDIX D

THE DIELECTRIC CONSTANT AND WAVE EQUATIONS FOR AN IONIZED MEDIUM

The constituent electrons are acted upon by the electric field vector $\mathbf{E}$ of an applied wave. In a cold plasma with no magnetic field present, the resulting equation of motion is

$$m\ddot{x} + m\dot{v} = eE$$

$$= eE_0 \exp(-j\omega t)$$

where $m, e$ are the electron mass and charge respectively and $\nu$ the electron collision frequency. Consider a solution

$$x = A \exp(-j\omega t)$$

then

$$A = \frac{-eE_0}{m\omega^2(1+j\nu/\omega)}$$

so the electron velocity is

$$\dot{x} = \frac{jeE}{m\omega(1+j\nu/\omega)}$$

as a result of the applied field. The resulting current density is

$$J = ne \frac{jeE}{m\omega(1+j\nu/\omega)}$$

where $n$ is the electron volume density. The wave within the ionized medium must satisfy Maxwell's equations.
\[ \nabla \times H = J + \frac{\partial D}{\partial t} \]

\[ = \frac{jne^2}{\varepsilon_0 (1 + j \nu/\omega)} + \varepsilon_0 (-j\omega)E \]

\[ = j\omega \varepsilon_0 E \left( \frac{ne^2}{\varepsilon_0 \omega^2 (1 + j \nu/\omega)} - 1 \right) \]

\[ = j\omega \varepsilon_0 E \left( \frac{ne^2}{\varepsilon_0 \omega^2 (1 + j \nu/\omega)} - 1 \right) \]

So \[ \nabla \times H = -j\omega \varepsilon_0 E \]

where \( \kappa = 1 - \frac{ne^2}{\varepsilon_0 \omega^2 (1 + j \nu/\omega)} \)

the effective dielectric constant. Note that the sign of the imaginary part of the dielectric constant introduced by including electron collisions depends on the form of the time variation chosen for the incident wave.

**The Wave Equations**

\[ \nabla \times E = -\frac{\partial B}{\partial t} \]

\[ = j\omega \mu_0 H \]

\[ \nabla \times (\nabla \times E) = j\omega \mu_0 \nabla \times H \]

\[ = \omega^2 \mu_0 \varepsilon_0 \kappa E \]

\[ = \nabla (\nabla \cdot E) - \nabla^2 E \]

so \[ \nabla^2 E + k^2 \kappa E = 0 \]

Similarly \[ \nabla \times (\nabla \times H) = \nabla \times (-j\omega \varepsilon_0 \kappa E) \]

\[ = -j\omega \varepsilon_0 (k \nabla \times E + \nabla \kappa \times E) \]

\[ = \omega^2 \varepsilon_0 \mu_0 \kappa H - j\omega \varepsilon_0 \frac{\nabla \kappa}{K} \times \kappa E \]

\[ = k^2 \kappa H + \frac{\nabla \kappa}{K} \times (\nabla \times H) \]
So \( \nabla^2 H + \frac{\nabla K}{K} \times (\nabla \times H) + k^2 K H = 0 \).
1. **Parallel polarization**

\[
\frac{d^2 P_m}{dr^2} + \frac{1}{r} \frac{dP_m}{dr} + (k^2 \kappa - \frac{m^2}{r^2}) P_m = 0
\]

let \( P_m(r) = y_1 + jy_2 \)

and \( \kappa = \kappa_r + j\kappa_i \) for complex solutions.

The separated real and imaginary equations are

\[
\begin{align*}
\frac{d}{dr} \left[ y_1'' + \frac{1}{r} \left( y_1' + k^2 \kappa_r y_1 - \frac{m^2}{r^2} y_1 - k^2 \kappa_i y_2 \right) \right] &= 0 \\
j \left[ y_2'' + \frac{1}{r} \left( y_2' + k^2 \kappa_r y_2 - \frac{m^2}{r^2} y_2 + k^2 \kappa_i y_1 \right) \right] &= 0
\end{align*}
\]

which are coupled by the small term in \( \kappa_i \). These may be reduced to four first order simultaneous equations

\[
\begin{align*}
y_1' &= y_3 \\
y_3' &= -\frac{1}{r} y_3 - \left( k^2 \kappa_r - \frac{m^2}{r^2} \right) y_1 + k^2 \kappa_i y_2 \\
y_2' &= y_4 \\
y_4' &= -\frac{1}{r} y_4 - \left( k^2 \kappa_r - \frac{m^2}{r^2} \right) y_2 - k^2 \kappa_i y_1
\end{align*}
\]

2. **Transverse polarization**

\[
\frac{d^2 T_m}{dr^2} + \frac{1}{r} \frac{dT_m}{dr} + \frac{1}{r} \frac{d(k^2 \kappa - \frac{m^2}{r^2}) T_m}{dr} + (k^2 \kappa - \frac{m^2}{r^2}) T_m = 0
\]
let \( r_m(r) = y_1 + jy_2 \)

then \( y_1'' + \frac{1}{r} y_1' - \frac{1}{|\kappa|^2} [\eta y_1' - \xi y_2'] + (k^2 \kappa_r - \frac{m^2}{r^2}) y_1 - k^2 \kappa_i y_2 = 0 \)

and \( j[y_2'' + \frac{1}{r} y_2' - \frac{1}{|\kappa|^2} [\eta y_2' - \xi y_1'] + (k^2 \kappa_r - \frac{m^2}{r^2}) y_2 + k^2 \kappa_i y_1] = 0 \)

where \( \eta = \kappa_r \kappa_r' + \kappa_i \kappa_i' \)

\( \xi = \kappa_r \kappa_i' - \kappa_i \kappa_r' \)

The coupling term is now dependent on \( \frac{1}{|\kappa|^2} \) which may be large as \( \kappa_r \) passes through zero. The corresponding four first order equations are:

\[
\begin{align*}
y_1' &= y_3 \\
y_3' &= -\frac{1}{r} y_3 + \frac{1}{|\kappa|^2} [\eta y_3' - \xi y_4'] - (k^2 \kappa_r - \frac{m^2}{r^2}) y_1 + k^2 \kappa_i y_2 \\
y_2' &= y_4 \\
y_4' &= -\frac{1}{r} y_4 + \frac{1}{|\kappa|^2} [\eta y_4' - \xi y_3'] - (k^2 \kappa_r - \frac{m^2}{r^2}) y_2 - k^2 \kappa_i y_2
\end{align*}
\]
APPENDIX F

THE TRANSVERSE WAVE EQUATION STARTING SOLUTIONS

Use the method of Frobenius to solve

\[ \frac{d^2 T_m}{dr^2} + \left( \frac{1}{r} - \frac{\kappa}{d} \right) \frac{dT_m}{dr} + \left( \kappa^2 \kappa - \frac{m^2}{r^2} \right) T_m = 0 \]  \hspace{1cm} (C1)

let \[ T_m = r^c \sum_{n=0}^{\infty} a_n r^n. \]  \hspace{1cm} (C2)

then \[ T_m' = a_0 c r^{c-1} + a_1 (c+1) r^c + a_2 (c+2) r^{c+1} + \ldots \]  \hspace{1cm} (C3)

\[ T_m'' = a_0 c (c-1) r^{c-2} + a_1 (c+1) r^{c-1} + a_2 (c+2) (c+1) r^c + \ldots \]  \hspace{1cm} (C4)

substitute (C3) and (C4) into (C1) and collect like terms up to \( r^{c+2} \), since starting solution derivatives are also required. Expanding \( \kappa \)

\[ \kappa = 1 - \frac{ne^2}{\varepsilon_0 \mu_0^2} \frac{r^2}{a^2} \text{ e } r^2/a^2 \text{ neglecting collisions.} \]

\[ = 1 - p(1 - \frac{r^2}{a^2} + \frac{r^4}{4a^4} + \ldots) \]

and since we require solutions near the axis, \( r^2 << a^2 \).

\[ r^2 T_m'' = a_0 c (c-1) r^c + a_1 (c+1) r^{c+1} + a_2 (c+2) (c+1) r^{c+2} \]

\[ r T_m' = a_0 c r^c + a_1 (c+1) r^{c+1} + a_2 (c+2) r^{c+2} \]

\[ \frac{r^2}{\kappa} \kappa' T_m = \frac{2pa_0 c r^{c+2}}{a^2 (1-p)} \]

\[ \kappa^2 r^2 T_m = k^2 a_0 r^{c+2} (1-p) \]
Gathering like terms, the coefficients of $r^c$ are

$$a_0 r^c (c(c-1)+c-m^2) = 0 \quad a_0 \neq 0$$

so $$c = \pm m$$

for $r^{c+1}$

$$a_1 r^{c+1} (c(c+1)+c+1-m^2) = 0$$

$$a_1 (2m+1) = 0 \quad \text{since} \quad c^2 = m^2$$

and so $$a_1 = 0$$

for $r^{c+2}$

$$a_2 [(c+2)(c+1)+(c+2) - \frac{2pc_0 c}{a^2 (1-p)} + k^2 a_0 (1-p)-m^2] = 0$$

hence $$a_2 = \frac{2pm}{a_0^2 (1-p)} - \frac{(1-p)k^2}{4(m+1)}$$

So for $r^2 \ll a^2$, the solution is

$$T_m = r^m \left[ 1 + \left( \frac{2mp}{a_0^2 (1-p)} - \frac{(1-p)k^2}{4(m+1)} \right) r^2 \right]$$

When $p = 0, k = 1$

$$T_m = r^m \left[ 1 - \frac{r^2}{4(m+1)} + \ldots \right],$$

the appropriate solution for Bessel's equation.

In general $\kappa$ and hence $p$ defined above is complex and may be written

$$p = \frac{\alpha e^2}{\pi a^2 \varepsilon_0 \omega^2 (1+j \nu/\omega)}$$

$$= s + j\alpha.$$
Substitution into C5 yields

\[ T_m = r^m \left( 1 + \frac{2ms(1-s) - a^2(1-s)j^2}{4(m+1)a^2((1-s)^2 + q^2)} j^2 \right) \]

\[ \sim 2 \left( \frac{2mq + a^2(1-s)j^2}{4(m+1)a^2((1-s)^2 + q^2)} \right) j^2 \]
APPENDIX G

PERIODIC REGRESSION AND HARMONIC ANALYSIS

Harmonic analysis is a particular case of fitting a time series of data \( v(t_i), i = 1, \ldots, N \) by a set of functions \( f_2, \ldots, f_m \), in general periodic, such that

\[
v(t_i) = a_1 + a_2 f_2(t_i) + a_3 f_3(t_i) \ldots + a_m f_m(t_i) + \epsilon(t_i) \quad (Gl)
\]

for all \( i \), where \( \epsilon \) is the residual. The coefficients \( a_1 \ldots a_m \) are to be determined subject to the condition that the sum of the squares of differences \( \Sigma \epsilon_i^2(t_i) \) is minimized. If in addition the data are each weighted by \( \omega_i \), the function to be minimized is

\[
s = \sum_{i=1}^{N} \omega_i (v(t_i) - a_1 - a_2 f_2(t_i) \ldots)^2
\]

So

\[
\frac{\partial s}{\partial a_1} = -2 \sum_{i=1}^{N} \omega_i (v(t_i) - a_1 - a_2 f_2(t_i) \ldots) \cdot 1 = 0
\]

\[
\frac{\partial s}{\partial a_2} = -2 \sum_{i=1}^{N} \omega_i (v(t_i) - a_1 - a_2 f_2(t_i) \ldots) \cdot f_2(t_i) = 0
\]

\[
\vdots
\]

\[
\frac{\partial s}{\partial a_m} = -2 \sum_{i=1}^{N} \omega_i (v(t_i) - a_1 - a_2 f_2(t_i) \ldots) \cdot f_m(t_i) = 0
\]

must be satisfied simultaneously. Reducing the \( m \) equations to matrix form with the summations and \( t_i \) understood as above.
that is \( Y = FF \times A \) and \( A = FF^{-1} \times Y \) determining the coefficients. (A similar formulation applies to the case of a weighted linear least squares fit for \( v_i = a_1 + a_2 t_i \) with \( FF \) a 2x2 matrix). For the case of periodic regression the functions are

\[
f_2(t_i) = \cos \frac{2\pi t_i}{T_2}
\]

and

\[
f_3(t_i) = \sin \frac{2\pi t_i}{T_2} \text{ etc.}
\]

determining the coefficients \( a_2 \) and \( a_3 \) and hence the amplitude and phase of the component whose period is \( T_2 \). In general \( m = 2J+1 \) where \( J \) is the number of periods to be fitted and \( m \) includes a constant term \( a_1 \).

Harmonic analysis considers equally weighted data, harmonically related periods and the data length an integral multiple of the longest period. The off-diagonal elements of \( FF \) are then zero due to the orthogonality of terms like

\[
\sum_{i=1}^{N} f_2(t_i) \equiv \int_0^{2\pi} \cos \theta \, d\theta = 0
\]

and

\[
\sum_{i=1}^{N} f_2(t_i) f_4(t_i) \equiv \int_0^{2\pi} \cos \theta \cos 2\theta \, d\theta = 0.
\]

The matrix \( FF \) is then diagonal and the coefficients may be evaluated independently, for example
identical to the result contained in Appendix I of Craig (1965). In the harmonic case the \((1,1)\) element of FF must equal \(N\) to determine the mean value. In the general case the constant term is not equal to the mean value.

The more generalized formulation deals with cases where the weights are not uniform and hence with gaps in the data, or equivalently, with unequally spaced data. In this case the off-diagonal terms are non-zero even for harmonically related data and the coefficients may be determined for any data length. Provided the sum of the weights (element \((1,1)\)) is normalized to unity, the true constant term is evaluated regardless of the data length. A computer program to calculate the coefficients is presented in Appendix K, and as input requires the data series, its length, weights, number of periods to be fitted and their values, all with consistent time intervals.

Although the number of points constituting the time series need only exceed the number of coefficients to be determined, in practice this is insufficient. This is only true for data with negligible noise, and in the case of the present noisy data, large gaps in the series, or the use of a large number of coefficients can lead to a situation where excessively large coefficients are determined. These may however be a satisfactory approximation to the individual points present in the input time series, but the determination for the entire series is poor as a result of the large noise contribution. This situation is aggravated by fitting many periods, especially high frequencies where any gap in the data means such a frequency is not well defined by the points present.
Confidence limits:

Chapman and Bartels (1940) suggest subdividing the time series, determining the coefficients for these limited lengths of data and observing the scatter. Meteor wind data however are usually too short to consider individual results from individual days in this manner and there is evidence the tidal modes are not constant over periods of a few days. A variation of this method is to use a running harmonic analysis, advancing in steps of a few hours and calculating the deviations from a mean value obtained over a 24 hour period. Again this assumes a slow variation of the tidal components and gives no indication of the uncertainty for the coefficients from an individual determination.

Wilkinson (1973) used a Fourier series method and considered all components with periods less than 6 hours as white noise, hence the average value of these components could be applied to the tidal components as an uncertainty. This is equivalent to considering the mean r.m.s. deviation of the data from the fitted harmonic series. In both cases the uncertainty in all coefficients is characterized by a single parameter.

The following uncertainty estimation is adapted from Brandt (1970) and considers the uncertainty of the individual points of the time series and the functions to be fitted. The equations (Gl) are the normal equations for the system and if

\[ V + AB = 0 \]

with A defined as before and

\[ V = \begin{bmatrix} v(t_1) \\ v(t_2) \\ \vdots \\ v(t_N) \end{bmatrix} \]
then \[ B = - \begin{bmatrix} 1 & f_2(t_1) & f_3(t_1) & \cdots & f_m(t_1) \\ 1 & f_2(t_2) & f_3(t_2) & \cdots \\ \vdots & \vdots & \ddots & \vdots \\ 1 & & & f_m(t_N) \end{bmatrix} \]

The variance matrix for the data points is

\[
C = \begin{bmatrix}
\sigma^2_{t_1} & 0 \\
0 & \sigma^2_{t_2} \\
0 & 0 & \cdots & \sigma^2_{t_N}
\end{bmatrix}
\]

and from Brandt the covariance matrix for the coefficients \(a_1\ldots a_m\) is

\[
C_A = (B^T C^{-1} B)^{-1}
\]

The square root of the diagonal elements is interpreted as the r.m.s. deviation of the coefficient corresponding to the particular diagonal element. A computer program to evaluate these elements in conjunction with the determination of the coefficients is also included in Appendix K.
APPENDIX H

86700/B7000 FORTRAN COMPILATION MARK 2,8,060 FRIDAY, 04/20/78 12:28 PM

C PROGRAM TO READ PAPER TAPE RECORDS
C
DIMENSION A(271),B(481),RANGE(16),C(253),THF(12)
DIMENSION THF(4)
END=1000

C INITIALIZE
C K IS THE IDENTIFIER OF THE LAST WORD IN A BLOCK

1 IFK=0
NCOUNT=0
K=0

C READ IN 3 RECORDS IE 240 8 BIT WORDS

1 CALL READOPT(B,IERR)
C SHIFil RECORDS NEWLY READ IN ALONGSIDE ANYTHING
C PREVIOUSLY IN ARRAY A
C
IF(I,L.T.0) K=0
DO 2 I=1,240
2 A(K+I)=B(I)
C TEST FOR FIRST NON BLANK WORD RECORD START
C 3 KK=1
IF(A(KK)) 5,5,6
IF(KK,GT,268) GO TO 11
5 KK=KK+1
GO TO 4
C ADJUST K TO ACCOUNT FOR LEADING BLANKS
C 6 K=K+240-RR+1
IF(K,GT,0) GO TO 8
C SHIFil NON BLANK RECORD TO ARRAY START
DO 7 I=1,K
I=I+KK-1
7 A(I)=A(I)
C IS THE REMAINDER A FULL METEOR RECORD THEN OUTPUT
C 8 IF(K,LT,268) GO TO 1
C RANGE WORDS ARE 12 BIT
C
JR=0
DO 18 I=205,236,2
JR=JR+1
18 RANGE(JR)=CONCAT(A(1),A(1)+1,11,5,6)
C
C COMPLEMENT
C
DO 28 I=1,204
28 A(I)=A(I)
DO 38 I=1,16
38 RANGE(I)=RANGE(I)
DO 48 I=237,260
48 A(I)=A(I)
DIREC=CONCAT(DIREC,A(1),0,0,1)
C
C TIME DECODE
C
THF(1)=CONCAT(THF(1),A(1),11,3,4,4)
THF(2)=CONCAT(THF(2),A(1),11,3,4,4)
THF(3)=CONCAT(THF(3),A(1),0,7,1)
THF(4)=CONCAT(THF(4),A(2),0,7,1)
THF(5)=CONCAT(THF(5),A(3),12,7,1)
THF(6)=CONCAT(THF(6),A(2),0,6,1)
THF(7)=CONCAT(THF(7),A(1),0,7,1)
THF(8)=CONCAT(THF(8),A(1),0,7,1)
THF(9)=CONCAT(THF(9),A(3),0,3,1)
THF(10)=CONCAT(THF(10),A(3),0,3,1)
THF(11)=CONCAT(THF(11),A(3),0,3,1)
THF(12)=CONCAT(THF(12),A(3),0,3,1)

C AL SECS A2 HILLS A3 HOURS A4 DAYS A5 HOURS A6 DAYS
A(I)=THF(I)+10,10,THF(I)+30,10,THF(I)
A(I)=THF(I)+10,10,THF(I)+30,10,THF(I)
A(I)=THF(I)+12,12,THF(I)
A(I)=THF(I)+12,12,THF(I)
A(I)=THF(I)+10,10,THF(I)+10,10,THF(I)
A(I)=THF(I)+10,10,THF(I)+10,10,THF(I)

480 THF(I)=A(I)
DO 488 I=1,12
488 THF(I)=0.0
PACK INTO 1 RECORD FOR WRITE TO TAPE
DO 50 I=1,16
50 A(I+204)=RANGE(I)
DO 60 I=1,32
60 A(I+220)=A(I+236)
DO 70 I=1,16
70 C(I)=A(I)
DO 80 I=1,16
C(I+19)=A(I+19)
C(I+34)=A(I+34)
C(I+69)=A(I+69)
80 C(I+64)=A(I+64)
DO 90 I=80,252
90 C(I+64)=A(I)
C(253)=DIR

CAN NOW WRITE C TAPE FORMAT OR A DATA FORMAT
200 FORMAT(*X,12(*F5,0))
WRITE(*,76) (C(I),I=1,253)
N=COUNT=COUNT+1
IF(MCOUNT.GE.50) WRITE(*,6) THFI
IF(MCOUNT.GE.50) N=COUNT=0
IF(IERR.EQ.2) WRITE(*,100)
100 FORMAT(*X,END OF TAPE!)
IF(IERR.EQ.2) GO TO 1000
C
SHIFT READER TO START OF ARRAY
MOVE ALONG 8 WORDS TO ALLOW FOR +/-8 READ ERRORS
DO 9 J=277,K
J=J-268-8
9 A(J)=A(I)
C
CLEAR REMAINDER OF ARRAY
DO 19 I=K+1,530
19 A(I)=0.0
C
ADJUST K EXTRA 240 IS FOR LINE 6
10 K=268+B-240
C
IF IN FIRST HALF OF GAP RESTART
IF(K,LT.-240) GO TO 11
GO TO 3
1000 STOP
END

002:0007:3 IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 007:0006
SEGMENT 002 IS 00E1 LONG

READ 3 PAPER TAPE RECORDS  DEFAULT
SUBROUTINE READPT(B, IERR)
DIMENSION B(1)
1 DO 2 I=1,3
2 M=80*(I-1)+1
MB=0
READ(8,100,END=3) (B(J),J=1,N)
100 FORMAT(10DI)
2 CONTINUE
GO TO 4
3 I=ERR+2
4 RETURN
END

007:0002:2 IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 007:0008
SEGMENT 007 IS 0023 LONG
APPENDIX I

B6700/B7000 FORTRAN COMPIILATION MARK 2,8,060 FRIDAY, 04/28/78 04:1

C
C REDUCTION OF BASIC DATA A(1-253)
C
DIMENSION H(35),Z(10)
DUM1=0:0
C0RREL/NLAKA(A(253))
NAMELIST/HAME10
REAL NOISE,
RETRAN,PREVZ=0,
COUNT=0
C READ DIFFERENTIAL PHASES
READ(5,100) TCAF,NOISE
C
100 FORMAT(2(FS,2))
READ(5,101) DFS12,DFS13,DFW12,DFW13
READ(5,101) TOF512,TOF513,TOF512,TOF513
101 FORMAT(4(FS,2))
WRITE(6,9) TCAL,NOISE
WRITE(6,9) DFS12,DFS13,DFW12,DFW13
WRITE(6,9) TOF512,TOF513,TOF512,TOF513
C
C READ 1 (ECHO)
1 READ(66,END=1000) (A(I),I=1,1,253)

DO 2 I=1,4
2 B(I)=A(I)
C ACCOUNT FOR TIME RESET
DHOUR=1.0
DOUH=30.0
B(2)=G(2)*DIN
IF (B(2),GT,60.) DOUR=2.
IF (B(2),GT,60.) B(2)=B(2)-60.
B(3)=B(3)+DOUR
IF (B(3),GT,23) B(4)=B(4)+1
IF (B(3),GT,23) B(3)=B(3)-24

C AMPLITUDE AND DECAY DATA
CALL AMPLE(AINAX,TINAX,INSPEC,TADURH,2,SIR,NOISE,IAERR)
IF (I.bmp, B(2)) GO TO 46
B(5)=IAERR
B(6)=AINAX
B(7)=TINAX
B(8)=INSPEC
B(9)=TADURH
B(10)=SIR
B(11)=B(11)+264.07
B(12)=B(12)
B(13)=B(15)
B(16)=B(16)
CALL RANGE1(RANGE,REARR,IRADUR,IREARR)
C CHECK FOR LONG PERSISTENTS DI=13MS DI=1 MIN
I=DABS(RANGE+RANGE).LT.15.AND.PREVZ-B(2).LT.1.) ILP=1
C OUTPUT ARRAY
B(I5)=ILP
B(I6)=RANGE
B(I7)=IREARR
B(I8)=IRADUR
B(I9)=IREARR
CALL DMPLE(AVW1,AVW2,SVIG1,SVIG2,IREARR,IRADUR)
B(20)=AVW1
B(21)=SVIG1
B(22)=AVW2
B(23)=SVIG2
B(24)=IREARR
B(25)=SO0.
B(26)=G(2),E(2),G(2) B(25) BAVW1-AVW2
C HITE CALCULATION
IF (I.bmp,GT,1) GO TO 13
CALL HITEC(ELEV,AZI,HITE,DHTE,IREARR,DFS12,DFS13,DFW12,DFW13,
TOF512,TOF513,TOF512,TOF513,TCAF,SH,RANGE,REARR,
IFS1,FSG,SSG1,SSG2)
B(26)=FSG1
B(27)=FSG1
B(28)=SSG1
B(29)=SSG1
B(30)=ELEV
B(31)=AZIM
B(32)=HITE
IF (B(31),GT,0.0) B(33)=B(33)-111.01DEG(B(31))+84.139
B(34)=INCRE
B(35)=A(253)
13 CONTINUE
WRITE(66) (B(I),I=1,1,253)

END
A111AX=YA(I)

105 J=O

DO 3 I=1,10

IF(YA(I).LE.0), YA(I)=1.0

3 YA(J)=ALOG(YA(J))

GO TO 1

RETURN

IAERR=1

RETURN

7 IAERR=2

RETURN

END
SUBROUTINE REGRES(I, XA, YA, Z)

PERFORMS A REGRESSION ANALYSIS ON THE N ELEMENTS OF THE INPUT
ARRAYS XA AND YA, TO FIND AN OUTPUT ARRAY Z SUCH THAT
Z(1) = MEAN OF YA
Z(2) = STD. ERROR OF ESTIMATE OF YA
YA = Z(3)*XA + Z(4) IS THE REGRESSION OF YA ON XA
Z(5) = MEAN OF XA
Z(6) = STD. ERROR OF ESTIMATE OF XA
XA = Z(7)*YA + Z(8) IS THE REGRESSION OF XA ON YA
Z(9) = CORRELATION COEFF. R

DIMENSION XA(I), YA(I), Z(I), XD(I)
EQUIVALENCE (XO(1), XX, (XO(2), X), (XO(3), XY), (XO(4), YY), (XO(5), Y)

DO 1 I = 1, 10
1 Z(I) = 0.0
Z(1) = 1.
CUT = 0.0
100 M = 0
DO 5 I = 1, 15
5 XD(I) = 0.
C
DO 6 J = 1, N
CUT = CUT + 1
XR = XA(J)
YR = YA(J)
X = XR + XR
Y = YR
XY = XY + XR * YR
XX = XX + XR * XR
YY = YY + YR * YR
6 CONTINUE
XR = XR / M
DO 7 J = 1, 15
7 XD(J) = XD(J) / XR
XY = XY / XR
XX = XX / XR
YY = YY / XR
A = XX * YY
IF (A, LE, 0.) GO TO 8
YR = XY / SGRT(A)
XR = 1. - YR * YR
C
Z(1) = Y
B = YR * XR
Z(2) = SGRT(YR * XR)
M = XY / XX
Z(4) = Y - Z(3)*X
Z(5) = X
B = XX * XR
Z(6) = SGRT(XX * XR)
Z(7) = XY / YY
Z(8) = Z(7) * Y
Z(9) = YR
Z(10) = CUT
8 RETURN
END

START OF SEGMENT OOD
SUBROUTINE REGRES(I, XA, YA, Z)

PERFORMS A REGRESSION ANALYSIS ON THE N ELEMENTS OF THE INPUT
ARRAYS XA AND YA, TO FIND AN OUTPUT ARRAY Z SUCH THAT
Z(1) = MEAN OF YA
Z(2) = STD. ERROR OF ESTIMATE OF YA
YA = Z(3)*XA + Z(4) IS THE REGRESSION OF YA ON XA
Z(5) = MEAN OF XA
Z(6) = STD. ERROR OF ESTIMATE OF XA
XA = Z(7)*YA + Z(8) IS THE REGRESSION OF XA ON YA
Z(9) = CORRELATION COEFF. R

DIMENSION XA(I), YA(I), Z(I), XD(I)
EQUIVALENCE (XO(1), XX, (XO(2), X), (XO(3), XY), (XO(4), YY), (XO(5), Y)

DO 1 I = 1, 10
1 Z(I) = 0.0
Z(1) = 1.
CUT = 0.0
100 M = 0
DO 5 I = 1, 15
5 XD(I) = 0.
C
DO 6 J = 1, N
CUT = CUT + 1
XR = XA(J)
YR = YA(J)
X = XR + XR
Y = YR
XY = XY + XR * YR
XX = XX + XR * XR
YY = YY + YR * YR
6 CONTINUE
XR = XR / M
DO 7 J = 1, 15
7 XD(J) = XD(J) / XR
XY = XY / XR
XX = XX / XR
YY = YY / XR
A = XX * YY
IF (A, LE, 0.) GO TO 8
YR = XY / SGRT(A)
XR = 1. - YR * YR
C
Z(1) = Y
B = YR * XR
Z(2) = SGRT(YR * XR)
M = XY / XX
Z(4) = Y - Z(3)*X
Z(5) = X
B = XX * XR
Z(6) = SGRT(XX * XR)
Z(7) = XY / YY
Z(8) = Z(7) * Y
Z(9) = YR
Z(10) = CUT
8 RETURN
END

SEGMENT OOD IS 6950 LONG
SUBROUTINE TO CALCULATE ECHO RANGE AND ERROR
SUBROUTINE RANGE(RANGE, ERR, IERROR, IRANG, ERR)
ERR=ERR2.  ALL VALUES LT 0.04 OR FROM LOW AMPL
ERR=1 LARGE RANGE SPREAD
A(205-220) ARE 12 BIT RANGE WORDS
HYDROGRAPHE RANGE IF SD GT 1 NM
HYDRONU R(16), DEL(16)
CONV/R/H2A/A2(16) 
IRERR=IRANGE-0
IRANGE=0
ERR=0
RE-ZERO
DO 9 I=1,15
R(I)=0.
9  DEL(1)=0.
DELI=3,10
DELI=3,15
ERR=ERR+16
2  K=IRDUR
K DISCARD UNAIDTED VALUES FROM END OF ECHO OR LOW AMPL
DO 1 I=1,16
K=0
R(I)=A(I+204)
1 IF(R(I),LT,1800) K=1
12 IF(A(I+60),LT,10,0) K=1
12 IF(R(I),EQ,0) GO TO 1
12 ERR=IRANGE
12 IRANGE=IRANGE-1
1 CONTINUE
1 IF(K,GT,0) GO TO 11
1 FIND AVERAGE VALUE
DO 4 I=1,K
4 RANGE=RANGE+R(I)
RANGE=RANGE/K
ERR=ERR+1
ERR=ERR+1
11 CONTINUE
CONTINUE
IF(RANGE,LT,0) GO TO 12
RANGE=RANGE/IRDUR
ERR=ERR+1
ERR=ERR+1
12 CONTINUE
IF(RANGE,LT,0) GO TO 13
RANGE=RANGE+R(I)
RANGE=RANGE/K
ERR=ERR+1
ERR=ERR+1
13 CONTINUE
CONTINUE
IF(RANGE,LT,0) GO TO 14
RANGE=RANGE/IRDUR
ERR=ERR+1
ERR=ERR+1
14 CONTINUE
CONTINUE
IF(RANGE,LT,0) GO TO 15
RANGE=RANGE+R(I)
RANGE=RANGE/K
ERR=ERR+1
ERR=ERR+1
15 CONTINUE
END
SUBSECT 000 IS 0071 LONG
C RADIAL VELOCITY
SUBROUTINE DOPPLER( AVVI, AVV2, SVIG1, SVIG2, IDERR, IADURH)
C VEL TOWARDS IS POSITIVE
DIMENSION D(16),D(16),
C OOM/HK/4K/A(22)
IDERR=0
AVV1=0.0
AVV2=0.0
SVIG1=0.0
SVIG2=0.0
CLOCK=5000
OFFSET=30.0
H=16
N=16
C PUT A INTO DOPPLER ARRAYS
DO 1 I=1,16
K=I+220
NC=I+236
D(I)=A(K)
1 DO (I)=A(K)
C FIND RELIABLE? DOPPLER DURATION
C IF DOPPLER DURATION EXCEEDS 140 PULSES?
IF ((IDERR.GT.10) .OR. (DURH.EQ.3)) GO TO 19
TELEH=10DURH/300.0
DSUN=0.0
DO 111 I=1,16
DSUN=DSUN+D(I)
111 CONTINUE
IF((I.GE.16) .OR. (DURH.EQ.3)) GO TO 115
2 DO 112 I=I+1,16
112 D(I)=0.
C IGNORE FIRST COUNT
115 DSUN=DSUN-D(I)
MEM=1
C OTHER OFFSET
DO 116 D(I)=0.
DO 3 I=1,16
DSUN=DSUN+D(I)
DSUN=DSUN/5000.
IF((DSUN.LT.7.5)) GO TO 1
H=1
DO 117 D(I)=0.
117 CONTINUE
IF((I.GE.16) .OR. (DURH.EQ.3)) GO TO 116
4 DO 113 I=I+1,16
113 D(I)=0.
116 DSUN=DSUN-D(I)
MEM=2
C REJECT SPURIOUS SHORT COUNTS FILTER ASSYM ???
C 240 COUNTS=11H/S
C 35 COUNTS=130H/S
DO 14 H=2,214
IF(DO(I).LT.25.0.AND.DO(I).LT.240.) GO TO 14
DSUN=DSUN-D(I)
MEM=1
14 CONTINUE
DO 24 H=2,214
IF(DO(I).LT.120.AND.DO(I).LT.240.) GO TO 24
DSUN=DSUN-D(I)
MEM=1
C FIND AVG
IF((I.GE.15) .AND. (H.NE.1)) DO TO 224
DSUN=DSUN-D(2)-D(3)-D(4)
H=I-2
I=I+1
D(2)=D(0)
D(3)=0.0
DO(I)=0.
224 SUM=DSUN/H
C FIND SD
SVIG1=0.0
DN 5 H=2,16
IF(DO(I).LT.0.0) GO TO 5
SVIG1=SVIG1+D(I-1)*D(I)
5 CONTINUE
SVIG1=SVIG1/(D+H)
F1C=CLOCK/(2.*SVIG)
AVV1=AVV1+(F1C-OFFSET)
SVIG1=(SVIG1+CLOCK)/(2.*SVIG1)^2
1 DO 34 I=I+1,16
34 DO 34 I=I+1,16
IF((I.GE.16) .OR. (DURH.EQ.3)) GO TO 35
IF((I.GE.16) .OR. (DURH.EQ.3)) GO TO 35
IF((I.GE.16) .OR. (DURH.EQ.3)) GO TO 35
C C ECHM LOCATION
C SUBROUTINE HITEC(ELEV, AZIM, HITE, DTIME, HREAR, I0FS12, DFS12, DFU12, DFV13, I1TDFS12, DFS13, DFSU12, DFSU13, TDFSN, SWH, RANGE, REAR, FSN, FSNH, SWSH, SWSHH)
C DIMENSION FI(15), SI(15), BETA(4), ALFA(4), ETA(4), SDASH(4)
C COMMON/ALS/A(23)
C DO = 7.0
C DO = 1.495
C PI = 3.14159
C RE = 6.31
C TERE = 0.0
C ELEV = 0.
C AZIM = 0.
C C SUBTRACT HALF A/D RANGE 128 BITS
C GO TO 158.
C A(1)=A(1)-128.
C IF(A(1),EN0), A(1)=1.0E-6
C C CORRECT DIFF PHASES
C IF(A(1),GT, TDFSN) GO TO 111
C DF12=DFS12
C DF13=DFS13
C IF(SW, GT, 30.) DF12=DFU12
C IF(SJ, GT, 30.) DF13=DFU13
C GO TO 111
C 111 DF12=DFS12
C DF13=DFS13
C IF(SW, GT, 30.) DF12=DFU12
C IF(SJ, GT, 30.) DF13=DFU13
C GO TO 112
C 112 DF(A(3), GT, 35.) GO TO 101
C TIPH=43.
C DELPH=0.
C ARG2=2.9PI*(TIPH-10.)/24
C DELPHDELPHPSI=ARG2
C DF12=DF12+DELPH
C DF13=DF13+DELPH
C 101 DD 3 = 5.19
C IF INDIVIDUAL PHASE ANGLES FROM RATIOS
C IF(A(1), LE0), A(1)=0.0
C F(I)=ATAN2(A(I+15),A(I))
C SI(I)=ATAN2(A(I+16),A(I)+10)
C IF(SI, LT, 0.0) SI(I)=SI(I)+2.0PI
C IF(SI(1), LT, 0.0) FI(I)=FI(I)+2.0PI
C C REJECT FROM LOW AMPS
C IPMURH=16
C DO 4 = 1, 16
C KOM = 0
C IF(A(1),GT, 30.), KOM=1
C IF(KOM, FI, 0.) GO TO 4
C F(I)=0.
C SI(I)=0.
C IPMURH=IPMURH+1
C 4 CONTINUE
C FIND INDIVIDUAL PHASE ANGLES FROM RATIOS
C IF(IPMURH, EN0), GO TO 9
C IF(KOM, EN0), GO TO 4
C C FIND HEAN PHASE ANGLES
C FSNH=0.
C SSUP=0.
C DD 5 = 1.15
C FSN1=SSUP+FI(1)
C 5 SSUP=SSUP+SI(1)
C FSNH=FSN1/16PHN
C SSUP=FSN1/16PHN
C GO TO 235
C C RETURN FOR THE NEGATIVE OFFSET
C SVG2=0.
C IF(A(1),LT, 0.0) GO TO 15
C SVG2=SVG2+D(B(1)-SSUM)*9.2
C 15 CONTINUE
C SVG2=SVG2
C C NON FIND ACTUAL VELOCITIES FOR EACH OFFSET
C IF(A(1), LE0), GO TO 19
C F2=2+D(2)*TDFMN
C AV2=AV2+D(2)*TDFMN
C SVG2=AV2+D(2)*TDFMN
C DFL=DFB(F1+F2)
C 17 RETURN
C 18 IDERH=IDERH+1
C RETURN
C 19 IDERH=5
C RETURN
C END
C SEGMENT OGF IS 00B2 LONG
FSIG = 0,
SSIG = 0.
DO 6 I = 1, 15
IF (FI(I) .LE. 0.) FI(I) = FSUM
IF (SI(I) .LE. 0.) SI(I) = SSUM
FSIG = FI(I) + (FI(I) - FSIG)**2
SSIG = SI(I) + (SI(I) - SSIG)**2
FSIG = FSIG/10000
SSIG = SSIG/10000
FSU(I) = FSIG - FSIG + 0.40
IF (FSU(I) .LT. 0.) FSU(I) = FSU(I) + 2.00
IF (SSU(I) .LT. 0.) SSU(I) = SSU(I) + 2.00
DO 16 I = 1, 15
IF (FSU(I) .LT. 0.) FSU(I) = FSU(I) + 2.00
IF (SSU(I) .LT. 0.) SSU(I) = SSU(I) + 2.00
DO 16 I = 1, 15
BETA(I) = (FI(I) + FSIG) / 2.00
IF (BETA(I) .GT. I, 0.) BETA(I) = I, 00
CETA(I) = ARCCOS (BETA(I))
HDASH(I) = SQRT (RI**2 + RAIG**2 - 2.00 * RI * RAIG * COS (PI / 2.00 + CETA(I)))
IF (HDASH(I) .LT. 0.) HDASH(I) = 0.00
IF (HDASH(I) .GT. HO/SH(I)) HDASH(I) = HO/SH(I)
ELSE HDASH(I) = HDASH(I)
END
8 HITE = HDASH(I)
ELEV = ETA(I) * 57.296
AZIM = ETA(I) * 57.296
RETURN
9 ERR = 1
RETURN
10 HITE = HDASH(I)
ELEV = ETA(I) * 57.296
AZIM = ETA(I) * 57.296
RETURN
11 DELH(I) = HDASH(I) - 95.0
JK = 1
DELH = DELH(I)
DO 12 I = 2, 4
K = 0
IF (DELH .LT. 0.) K = 1
IF (K .EQ. 0.) Go To 12
DELH = DELH(I)
JK = 2
END
12 HITE = HDASH(JK)
ELEV = ETA(JK) * 57.296
AZIM = ETA(JK) * 57.296
RETURN
END

SEGMENT OOF IS OOF9 LONG
PROGRAM TO EVALUATE METEREON TRAIN REFLECTION COEFFICIENTS

COMMON X,Y,YPASH,HI,HI(),HI()=0,0,0

DIMENSION PRMT(5),Y(4),DERV(4),AMH(20),AMH(20)
COMPLEX YC,YOASH,HI(1),HI(),HI(),TH(15),
4(20),F1,Y,YOASH,SPRI
END=1000
EXTERNAL OYPF,FCT
REAL Adx(0,0)
N=4
P(2)=1,1,0,10
WAVE=10,
CAY=(2**P)/WAVE
C2=0.5
CM1=1.2638-16
COLL=0.001
COLL=-COLL
DEL=0.06
PRMT(3)=0.02
PRMT(4)=1.013
C=1

STARTING SOLUTIONS
P0=CONV(0,0,ALPHA/(CAYAY**2))
F1=F0-PP/(PP-PP(0)**2+(1-PP))
F1=F0-(1-PP)**2
F1=F0-(1-(1/T)**(1))
M=1
H=1

SHOULDER SMALL
FF1=CARS(F1)
X2=ADX/2+1,FF1
X2=ADX/2+1,FF1
X1=2**Y1
PRMT(1)=X1
F1=1**X1**0
Y1=2**Y1**0
IF((2,2,2,1))
')

DERIVATIVES
42 YOASH=0
GO TO 93
41 YOASH=XP1**0-(1+F1**0(N**2))

Y(1)=REAL(Y1)
Y(2)=IMAG(Y1)
Y(3)=REAL(YOASH)
Y(4)=IMAG(YOASH)

FIND RC
E=ALG(E(P0),CAYAY,**2)/(CONV(0,ADX))
PRMT(3)=ADX/2**5
S=0.0,0,0,0,0
DERY IS UNITY
DERY(1)=0.25
DERY(2)=0.25
DERY(3)=0.25
DERY(4)=0.25

INTEGRATE
X=PRMT(1)
CALL BKGS(PRMT(1),DERY,END,CM1,CM2,CMF,FCT,OUTP,OUTP,OUTP,OUTP,OUTP,OUTP,AUX,CAPPA,CON,AA,

X=XX
X=PRMT(2)

END
C FOUR
C DIELECTRIC
C EQUATIONS TO BE INTEGRATED
C BACKSCATTER (-1)*TH
C AHI (TH) = REAL(ATH)
C AVG(TH) = IMAG(ATH)
C WRITE(6,150) AHI (TH)

150 FORMAT(5X,2(F12.6))

C REFLECTION COEFF. G
C IF(WT,GT,1) GO TO 47
C SUM=0
C GO TO 2
C SUM=SUM+2*AHI (TH)
C IF(CABS(SUM)+SG,GT,1,OE,3) GO TO 2
C ADDITIONAL (-1) FOR TH
C SUM=SUM
C AHI=CAVS(SUM)
C AARG=ATAN2(IMAG(SUM),REAL(SUM))
C WRITE(6,100) AHI, AARG

100 FORMAT(5X,3(F11.2,4X))
C IF(CAYAY,LT,1,005) GO TO 1
C 1000 STOP

END

SUBROUTINE FCT(X,Y,DERY,CAPPA,CON,AA,C2,ALPHA,HI)
C EQUATIONS TO BE INTEGRATED
C REAL Y(1),DERY(1)
C COMPLEX OXY,OERY,CY,CAP,CDCAP
CAYAY=ASYNCT(0)
C S=2 GAUSSIAN
C X=1(X/AA)0.5
C BI ELECTRIC CONSTANT COMPLEX
C CAPPA=C1+CAPPA*EXP(-XX)/(CAYAY+2)
C COLLE=0.001
C COLLE=ORLFL
C CAP=CAPPA
C CDCAP=CAP+1.0)*COLLFL
C CDCAP=CDCAP*CAPPA,CD CAP
C BI ELECTRIC CONSTANT, DERIVATIVE
C DCAP=C1+CAPPA*XX/AA+2
C CDCAP=CDCAP/(DCAP,CD CAP)
C FOUR FIRST ORDER EQUATIONS
C GERY
C CY=CUMXY(Y(1),Y(2))
C DERY(Y1) Y4
C OERY=EXPL(DERY(1),DERY(2))
C OXY=CDCAP*CAPPA*(X/Y)*OERY+(H/X)*2-CAPPA*CY
C DERY(3) =REAL(OXY)
C DERY(4) =IMAG(OXY)
C RETURN
C END

SUBROUTINE OUTPUT(X,Y,DERY,HLF,HRH,PRINT,CAPPA,HI)
C REAL Y(1),DERY(1),PRINT(1)
C IF(HLF,JL,11) WRITE(6,12)

17 FORMAT(5X,'TOO MANY BISECTIONS')
C RETURN
C END
APPENDIX K

86700/17700 FORTRAN COMPILATION MARK 2,8,060
MONDAY, 05/01/78

C PERIODIC REGRESSION
C
C REQUIRES QMOPQ, QHTR, NLVHY, L.B.H1, SCI, SUBROUTINES
C
C DIMENSIONS ARE FOR 6 HARMONICS AND 250 PTS
C
C DIMENSION A(170), Y(13), COEFF(13), F(13,250), T(6),
C DATA(250), AN(6), PH(6), DATA(250), DATA(250), TPH(6), M(250),
C SIG(250), DELV(11), OUTPUT(26), DPHT(6)
C INTEGER N sig(250)
C REAL M(13,1,13)
C END=1000
C P1=3,141593
C
C SET UP DATA AND PERIODS
C
C NHARM=3
C F(1,j)=2**j,0
C T(3)=6,0
C NPTS=24
C TSTART=200
C WRITE(6,0/2) TSTART

111 NPTS=240
C THE5=2,0
C
C NO VEL IS 999
C NMIN IS 0
C NO SIG IS 999
C
C READ(5,100,END=1000) (DATA(I),I=1,NHT)
C READ(5,100) (M(I),I=1,NHT)
C 100 FORMAT(12(F5,1))
C 102 FORMAT(12(I4))
C WRITE(6,111) (DATA(I),I=1,NHT)
C
C START OF SEGMENT
C 112 TMIN=3
C TIME=TIME+2,0
C DO 50 I=1,13
C Y(I)=0
C DO 50 J=1,250
C F(I,J)=0
C DO 51 J=1,170
C A(I)=0
C DO 52 J=1,250
C DATA(I,J)=0
C 50 CONTINUE
C 51 CONTINUE
C 52 DATA(I,J)=0
C
C NORMALIZED WEIGHTS
C DO 40 I=1,NHT,2,2
C M(I)=M(I**2)
C 40 CONTINUE
C
C U(1)=0,0,0
C U(2)=0,0,0
C U(3)=0,0,0
C U(4)=0,0,0
C U(5)=0,0,0
C U(6)=0,0,0
C U(7)=0,0,0
C U(8)=0,0,0
C U(9)=0,0,0
C U(10)=0,0,0
C U(11)=0,0,0
C U(12)=0,0,0
C U(13)=0,0,0
C U(14)=0,0,0
C U(15)=0,0,0
C U(16)=0,0,0
C U(17)=0,0,0
C U(18)=0,0,0
C U(19)=0,0,0
C U(20)=0,0,0
C U(21)=0,0,0
C U(22)=0,0,0
C U(23)=0,0,0
C U(24)=0,0,0
C U(25)=0,0,0
C U(26)=0,0,0
C
C SET UP FREQUENCY ARRAYS F2 - F13
C F1=0
C DELT=1,0
C TIME=TIMES
C DO 1 I=1,NHT,2,2
C T(I)=T(I**2+1)
C 1 CONTINUE
C
C Y VECTOR
C DO 2 J=1,NHT,2,2
C Y(J)=Y(J)+DATA(I)**U(I)
C 2 Y(J)=Y(J)+DATA(I)**U(I)
C
C Y(1)
C Y(1)=Y(1)+DATA(I)**U(I)
C FF MATRIX COL 1
C FOR NORMALIZED WEIGHTS A(I)=1.0
C OTHERWISE A(I)=NPTS
A(I)=1.0
GO TO 1
GO TO 2
A(J)=A(J)+F(J,I)*W(I)

C FF MATRIX OTHER COLS
THRU XAR(A)+XAR(I-1)
XAR=1
GO TO 6
XAR=3
GO TO 6
6 A(J)=A(J)+F(K,I)*F(J,I)*W(I)
IF(XAR.LT.WGO) GO TO 7

C FF MATRIX ROW I
GO 7=2,XARH
GO TO 6
7 A(I)=A(I)
CALL XAR(I,A,IHAR,D,L,H)
L=1
CALL DRODO(A,Y,COEFF,HARHAR,HARHAR,LL)
GO 17=1,IHAR
XAR(I)+XAR(I-1)=COEFF(2(I-1)+2)*COEFF(2(I-1)+2)
IF(I+1 .EQ. 2) GO TO 2

C RECONSTRUCT THE SERIES
TIME=TIME+DEL
10 TIME=TIME
GO TO 73
71 DATA(I)=DATA(I)+H(X,J)*COS(T/T(J)+PH(J))
72 DATA(I)=DATA(I)+COEFF(I)

C TIME SERIES AND BEST FIT DIFFERENCES
10 IF(H(I).EQ.O) GO TO 11
IF(H(I).EQ.O) GO TO 10
IF((I+1).EQ.1) GO TO 73
CATA(I)=DATA(I)-DATA(I)

C SQUARED DEVIATION SUM AND MEAN DEVIATION
ASR=0.0
GO 74=1,HARHAR,HPTSHARH
GO TO 74
74 ASR=ASR+DATA(I)
75=2,2
ASR=ABM(ASR/ASR)
CALL DRODO(HARHAR,HPTSHARH,G,DELV,PHI,SIG,HIC)
GO 76=2,2
XAR=2
STEP=STEP
76 DELV(I)=ABS(COEFF(I)*DELV(I))-ABS(COEFF(I+1)*DELV(I+1))
GO 77=1,IHAR
DELV(I)=DELV(I)+HARHAR
IF(DELV(I).EQ.O) GO TO 77
IF(DELV(I).EQ.O) GO TO 77
DELV(I)=ABM(ABS(DELV(I)/HARHAR))
77 CONTINUE
78 CONTINUE
79 CONTINUE
79=2,0
79 XAR=2
STEP=STEP
79=2,0
DELV(1)=COEFF(I)
79 CONTINUE
79 CONTINUE
79 CONTINUE
79=3,0
79=2,0
IF(DELV(I).EQ.O) GO TO 79
IF(DELV(I).EQ.O) GO TO 79
DELV(I)=ABS(DELV(I)/HARHAR)
79 CONTINUE
79 CONTINUE
79=4,0
79=2,0
WRITE(6,200) (OUTPUT(I),I=1,20)
200 FORMAT(10H1,2F5.1,1X,F1:1,1X,(F5.1,F5.1,3X),3X,F4.1,1X,13)
GO TO 999
1000 CONTINUE
STOP
END
002017F12 IS THE LOCATION FOR EXCEPTIONAL ACTION ON THE I/O STATEMENT AT 0020013
SEGMENT 002 IS 0193 LONG
COVARIANCE MATRIX

SUBROUTINE COVAR(NPTS, IHARR, G, DELV, HI, SIG, NINC)
DIMENSION DELV(1), SIG(1), G(1), HI(250), AA(217), AT(217), GY(577), AG(217), COV(169)
REAL L(11), M(11)
INTEGER HI(1)

ZERO ARRAYS
DO 11 I=1,NPTS
AA(I)=0,
AT(I)=0,
11 AG(I)=0.
DO 12 I=1,NPTS
12 GY(I)=0.
DO 14 I=1+HI, NPTS+HIINC
SEJ(I+HI)=100,
PY(I)=0. GO TO 14
SEJ(I+HI)=SIG(I)
14 CONTINUE

AA MATRIX COL 1
1 AA(I)=G(I)

AA MATRIX OTHER COLS
DO 2 J=2,NPTS
2 AA(I+1)=G(I,J)

GY MATRIX
N=1
DO 3 HI=1,NPTS
3 IF(HI.GE.NPTS) GO TO 4
NED=HI*NPTS*HI+1
GY(HED)=SEJ(HI)
GO TO 3
4 CONTINUE

TRANSPOSE AA - AAT
CALL GTR(AA, AT, NPTS, IHARR)

MULTIPLY AAT * GY = AG
CALL GMRD(AA, GY, AG, IHARR, NPTS, NPTS)

MULTIPLY RESULTANT AG*AA=COV
CALL GMRD(AG, AA, COV, IHARR, NPTS, IHARR)

INVERT COV
CALL MINV(COV, IHARR, 0, 1, 1, 1, 11)

SORT NF DIAGONALS
N=1
5 HI=1+1
IF(HI.GE.NHARR) GO TO 6
HEL=HI*(NHRN+1)+1
IF(COV(HEL), LE, 0.) COV(HEL)=1.0E-6
DELV(1)=SORT(COV(HEL))
GO TO 5
6 RETURN
END

START OF SEGMENT OOC
SEGMENT OOC IS 0063 LOC


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