COMPUTER-AIDED DESIGN
OF EXTRUSION DIES FOR THERMOPLASTICS

A thesis
submitted in fulfilment of
the requirements for the Degree
of
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at the
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by
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B.E. (Hons)

University of Canterbury
March 1975 - June 1976
TO MY PARENTS
"NUSIANU MIATENU AWO LA, MI WOE,
GAKE NUSI AME MATENU AWO
HADE O LA, MI LE WOGE."

YULIUS KAISAR.

"ANYTHING WHICH IS POSSIBLE HAS
BEEN DONE, ANYTHING IMPOSSIBLE
WILL BE DONE."

GAIUS JULIUS CAESAR.
ACKNOWLEDGEMENTS

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The author wishes to express gratitude to Helen Oteng for her motherly care and constant encouragement given to me during the course of study. My sincere appreciation goes also to Mrs A.J. Dellow for her expert typing of the manuscript.

June 1976.

J. BADU

B.E. (Hons).
SUMMARY

This report presents a survey of the literature on the design of extrusion dies for thermoplastics. The need for scientific die design and the major problem areas are discussed.

In the literature survey, the major problems isolated were:

1. Material Properties
2. Die Flow Analysis
3. Flow Instability and Melt Fracture

Attention was then confined to Die Flow Analysis. The finite element method was proposed for solving two dimensional flow problems in complex geometrical configurations commonly encountered in polymer extrusions.

The two finite element methods adopted used the variational principle in the formulation of the problem. Also, different variational functionals and nodal variables were used.

The results of the two methods were successfully compared with finite difference, analytical and existing numerical solutions.

The flexibility of the finite element methods makes them very suitable for problems involving complex boundary geometries. It is shown that the finite element method has great potential for use in flow problems, and represents a powerful new tool for the analysis of viscous flows.
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<td>a, b</td>
<td>local nodal point coordinates.</td>
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<tr>
<td>A</td>
<td>area.</td>
</tr>
<tr>
<td>c₁ to c₆</td>
<td>constants in stream function distribution.</td>
</tr>
<tr>
<td>F</td>
<td>viscous dissipation functional.</td>
</tr>
<tr>
<td>H</td>
<td>depth at any section along the axial length.</td>
</tr>
<tr>
<td>H₁</td>
<td>channel depth at inlet.</td>
</tr>
<tr>
<td>I₂</td>
<td>second invariant of rate of deformation.</td>
</tr>
<tr>
<td>i, j, k</td>
<td>subscripts referencing nodal points.</td>
</tr>
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<td>[K]</td>
<td>system viscous stiffness matrix.</td>
</tr>
<tr>
<td>[Ke]</td>
<td>elemental viscous stiffness matrix.</td>
</tr>
<tr>
<td>L</td>
<td>length of converging channel.</td>
</tr>
<tr>
<td>m</td>
<td>subscript referencing element number.</td>
</tr>
<tr>
<td>n</td>
<td>flow behaviour index of a polymer; power-law index.</td>
</tr>
<tr>
<td>N with subscripts</td>
<td>shape functions.</td>
</tr>
<tr>
<td>P</td>
<td>pressure.</td>
</tr>
<tr>
<td>ΔP</td>
<td>overall pressure drop.</td>
</tr>
<tr>
<td>Q</td>
<td>volumetric flowrate.</td>
</tr>
<tr>
<td>u</td>
<td>velocity in the x direction</td>
</tr>
<tr>
<td>u'</td>
<td>dimensionless value of u.</td>
</tr>
<tr>
<td>v</td>
<td>velocity in the y direction; volume of an element.</td>
</tr>
<tr>
<td>v'</td>
<td>dimensionless value of v.</td>
</tr>
<tr>
<td>vₑ</td>
<td>characteristic velocity.</td>
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<tr>
<td>w</td>
<td>width of flow normal to x-y plane.</td>
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\( x, y \) Cartesian coordinates.
\( X', Y' \) dimensionless Cartesian coordinates.

**Greek letters.**

- \( \alpha \): channel taper angle.
- \( \alpha_1, \alpha_2, \alpha_3 \): constants in the velocity distribution.
- \( \gamma_0 \): reference shear rate.
- \( \gamma_c \): characteristic shear rate.
- \( \Delta \): elemental area.
- \( \mu \): viscosity.
- \( \mu_o \): effective viscosity at \( \gamma_0 \).
- \( \mu_c \): characteristic viscosity.
- \( \tau \) with subscripts: viscous stress.
- \( \tau_c \): characteristic viscous stress.
- \( \phi \): sum of velocity components \( u + v \).
- \( \phi' \): dimensionless \( \phi \).
- \( \chi \): integral of functional \( F \).
- \( \psi \): stream function.
- \( \psi' \): dimensionless stream function.
- \( \omega \): overrelaxation factor.
- \( \omega_{opt} \): optimum overrelaxation factor.
- \( \pi_p \): dimensionless pressure drop.
- \( \pi_Q \): dimensionless flowrate.
## ABBREVIATIONS

The following abbreviations were used in this work.

<table>
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<td>FD</td>
<td>Finite Difference</td>
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<tr>
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<td>Finite Element</td>
</tr>
<tr>
<td>FEM</td>
<td>Finite Element Method</td>
</tr>
<tr>
<td>LA</td>
<td>Lubrication Approximation</td>
</tr>
<tr>
<td>LP</td>
<td>Linear Polyethylene</td>
</tr>
<tr>
<td>BP</td>
<td>Branched Polyethylene</td>
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INTRODUCTION

Many flow problems in polymer processing are regarded as difficult to solve without gross over-simplifications or excessive labour. The two main sources of difficulty are the intricate geometrical configurations which are encountered and the complex rheological properties of polymer melts. Consequently one finds that theoretical treatments are often restricted to simple, sometimes unrealistic, geometries and Newtonian fluids.

Industrial processes involving viscous fluids are invariably associated with geometries of a complex nature; for example, mixing vessels, screw extruders and extrusion heads. Herein lies the potential of the finite element method as a design procedure; because it is not limited to geometries associated with the major coordinate systems which, in practical terms, tends to be the case with other methods. Furthermore, because of the way modifications to the geometry can be effected the potential also exists for "Computer-aided Design" in its entirety.

The present study has been restricted to the analysis of two-dimensional, creeping, non-Newtonian flows using the finite element method.

Finite element method approximates the flow by dividing the flow region into small subregions or elements and analysing the flow in terms of, say, velocities at the corners of these elements.
The FE methods are well established in the fields of structural and solid mechanics (Zienckiewicz (60)), but have not been widely used in solving fluid mechanics and heat transfer problems. General examples include Martin (25) and Oden and Somogyi (13). A general FE formulation for fluid flow problems has been developed by Oden (35). Zienkiewicz (60) has also described a FE formulation of two-dimensional flow problems in terms of u, v and p. Various authors (Oden (35), Martin (25) and Card (12)) used the variational principle and demonstrated the FE analysis of two-dimensional fluid flow. Atkinson et al. (1,3) applied FE methods to two-dimensional Newtonian flow problems. Palit and Fenner (37) also applied the FE method to two-dimensional slow non-Newtonian flows.

The object of this work is to compare the type of analysis used by Palit and Fenner (37) with the results using a new type of analysis; a different variational functional and nodal variables.

The present work contains two major sections:

Section 1 contains the literature survey. It critically reviews the published literature on extrusion die design for thermoplastics and lists the major problem areas.

Section 2 presents the analysis of the flow of a Newtonian and non-Newtonian fluid within a two-dimensional channel. Two FE methods were adopted and the differences would be pointed out in the course of this work.
SECTION 1

LITERATURE SURVEY
Fig. 1 Elements of an extruder.
CHAPTER 1

INTRODUCTION

Little has been written about the detailed design of extrusion dies and it is unlikely that this state will change appreciably for some years to come. The die maker is a craftsman who depends on personal experience to an extent which is uncommon in modern industry. The mystique which appears to surround the craft is increased by the reluctance of companies to talk about their success - the solution to the problems of a difficult die may give a distinct advantage over a competitor.

(a) DESCRIPTION OF THE EXTRUSION PROCESS

Fundamentally, the process of extrusion consists of converting a suitable raw material into a product of specific cross-section by forcing the material through an orifice or die under controlled conditions. This definition, as applied to the extrusion of thermoplastic materials, covers two general processes - screw extrusion and ram extrusion.

Thermoplastics are extruded predominantly through screw extruders; more specifically, through single-screw extruders. The elements of a typical single-screw extruder are shown in Fig. 1.
The plastic material is fed from a hopper through the feed throat into the channel of the screw. The screw rotates in a barrel which has a hardened liner. The screw is driven by a motor through a gear reducer, and the rearward thrust of the screw is absorbed by a thrust bearing. Heat is applied to the barrel from external heaters, and the temperature is measured by thermocouples. As the plastic granules are conveyed along the screw channel, they are melted. The melt is forced through a breaker plate which, in some cases, supports a screenpack. The melt then flows through the adapter and through the die, where it is given the required form.

When the thermoplastic material leaves the extrusion die it is usually in the form of a melt or a very soft mass. It has often little decisive form at this stage and even this would rapidly be lost unless it were handled in a suitable manner as it leaves the die. The product from the extrusion die therefore must, in a large number of cases, be looked upon as a semi-finished raw material which may be given its correct form and dimensions by subsequent processing whilst it is still mouldable. The methods of doing this naturally depend on the thermoplastic being used and on the desired product. This subject will not be discussed here. The reader is referred to a suitable work on this topic.

(b) **REGULAR PRACTICE IN DIE DESIGN**

In most extrusion shops, it is still a regular practice to shape the die cross-section as judged
appropriate from previous experience and to proceed from there by trial and error. The die is then installed, and a run is begun. Samples of the chilled extrudate are taken, and the dimensions are checked. Where the thickness is excessive, the corresponding points along the die lips are peened with hammer and punch while the extruder is running, so as to close them slightly and reduce the section thickness. It is hoped that some day plastic flow would be understood well enough so that the crude method of peening extrusion dies would be eschewed by die makers forever.

The work presented here reviews the present state of scientific die design and investigates methods of analysing flow in extrusion dies.
CHAPTER 2

PROBLEMS OF EXTRUSION DIE DESIGN FOR THERMOPLASTICS

INTRODUCTION

The rheologist makes measurements of viscosity and elasticity under carefully defined steady flow conditions; the practical processor commonly works in less ideal circumstances. An example of this contrast is extrusion, where the rheologist measures the flow through a long cylindrical die, but the practical processor operates with relatively short, tapered dies, often of slit or other profile.

A typical tubular extrusion die of 150 mm diameter costs about a thousand dollars to make. At this price the processor has a problem which is expensive to solve by trial and error, and he legitimately turns to the rheologist for assistance. The processor needs to know what will be the swell ratio as the melt leaves the die and what will be the pressure drop through this die. He also needs to know at what throughput rate non-laminar flow will occur. The ideal die will maximise output rate of smooth extrudate and minimise pressure drop and swell ratio, and such optimisation commonly requires an accurate choice of taper for the converging flow regions in the die.

A survey of all available literature on die design
was carried out to ascertain the various problems. The following major problem areas were isolated.

These are:

(1) Material Properties.
(2) Die Flow Analysis.
(3) Flow Instability and Melt Fracture.
(4) Die Swell.

The above areas are further discussed in the next four chapters.
CHAPTER 3

MATERIAL PROPERTIES

INTRODUCTION

Ideally, during extrusion, different screws are required for each material because of the change in extrusion viscosity values from one material to another. The same requirement applies to die design, although the necessity is, perhaps, not so stringent. A change in material viscosity brings about a corresponding change in flow properties, with the result that a die arrangement suitable for a polyamide, for example, would be completely inadequate for an unplasticised vinyl material. In general, the higher the extrusion viscosity of a material the greater the necessity for streamlining the interior of the die.

The dependence of die design on material characteristics need not be overemphasised. This part of the work therefore briefly looks at non-Newtonian fluids, flow behaviour and properties of thermoplastics. The rheological equation describing thermoplastics and its limitations are also discussed.
NON-NEWTONIAN FLUIDS

Classification

The Newtonian viscosity, \( \mu \), depends only on temperature and pressure and is independent of the rate of shear. The diagram relating shear stress and shear rate for Newtonian fluids is the so-called "flow curve", and is therefore a straight line of slope \( \mu \).

Non-Newtonian fluids are those for which the flow curve is not linear. The viscosity is not constant at a given temperature and pressure, but depends on other factors like shear rate, the apparatus in which the fluid is contained, or even on the previous history of the fluid.

These fluids can be classified into three broad types:

1. Fluids for which the shear rate is a function of the shear stress only.
2. Those for which the relation between shear rate and shear stress depends on the time the fluid has been sheared or on its previous history.
3. Systems which have the characteristics of both solids and liquids and show partial elastic recovery after deformation - viscoelastic fluids.

Viscoelasticity results in stresses normal to the direction of shear stress. When viscoelastic jets emerge from capillaries, the normal stresses result in an expansion of the jet as opposed to the usual vena-contracta in Newtonian fluids. This is the Barus effect.
Shear stress

Fig. 2 General flow curve for a pseudoplastic fluid.

\[ \tau = k \dot{\gamma}^n \]

- \( k \) = constant
- \( n \) = flow behaviour index < 1.
Rheological Equation and Limitations

In this work, attention is confined to pseudoplastic fluids which are described by the equation

\[ \tau = k\gamma^n \]

where \( k \) is a constant and \( n \) (less than one) is the flow behaviour index.

A typical flow curve for this fluid is as shown in Fig. 2. It can be seen that \( n \) is not constant over the whole range of the shear rate. This is not a serious drawback because all that is required is the value of \( n \) which describes the flow over the particular range encountered in a particular problem. The chief limitation of this equation is its inability to portray correctly flow behaviour at shear rates in which the fluid is approaching Newtonian behaviour. Thus, extrapolating of data taken over a modest range of shear rates from the non-Newtonian into Newtonian region may result in appreciable errors.

Unfortunately there is no ready solution to this problem and hence one must obtain rheological data at shear rates corresponding to those used in process.

(b) FLOW BEHAVIOUR OF THERMOPLASTICS

Introduction

Flow through dies, tubes and viscometers all represent steady-state processes. Thus steady-state behaviour represents a convenient starting point from which to analyse flow behaviour. Then time-dependent effects may
be superimposed in the form of simple additions.

The non-Newtonian behaviour of polymer melts in steady flow is attributed to two characteristics of the molecular structure of these materials.

(1) The asymmetric shape (great length as compared to the radial dimensions) of the molecules themselves results in an orientation of the particles when a velocity gradient is imposed on the polymer molecules.

(2) The size of the flowing elements - if they are in groups of molecules rather than single particles - is decreased as the shear rate is increased. The restoring tendencies are the intermolecular forces.

**Solid or fluid-like behaviour**

Polymeric materials may exhibit either solid-like or fluid-like behaviour. The kind of response is determined by the Deborah number.

Deborah number is defined as the ratio of natural time to duration time of a process (or the residence time of the material in the process). Natural time is the time required for the relaxation moduli of the material to decay.

Due to the large forces applied during processing of polymers the residence times are very short. Analyses of flow of polymers neglect these short residence times and consider the deforming polymer to be a purely viscous fluid. Obviously this is not a generally valid approach.

It has been shown by Metzner et al. (29, 30, 42) that the behaviour of polymer materials under short residence times is not at all well proximated by fluid-like behaviour and that solid-like behaviour may instead be exhibited under
conditions of sudden acceleration of the fluid.

The importance of memory of a fluid element for its previous deformation history and the possibility of either a solid-like or fluid-like behaviour of a given material may be illustrated clearly through consideration of the behaviour of polymeric melts in short dies.

Deborah no. as we recall is defined as:

\[
\frac{\text{duration of fluid memory}}{\text{duration of deforming process}}
\]

Thus, large resident times and small Deborah nos. imply fluid-like behaviour, while small residence times and large Deborah nos. imply solid-like behaviour.

If the span of the fluid memory exceeds the residence time in the die, i.e. large Deborah no., the emerging extrudate is able to recall its state prior to entry. Otherwise the extrudate is able to recall only the flow conditions in the die. The behaviour of the fluid on emergence from the die should reflect such differences in memory.

Swelling of extrudates is known to decrease approximately exponentially to a constant value as the die length is increased. In a sense therefore, the tendency of the extrudate to return to a state representing its original configuration before entering the die increases as the Deborah no. increases.

The entry to a die is subjected to rapidly changing deformation rates and the fluid should exhibit solid-like behaviour because the Deborah no. is high. Metzner et al. (31) reported that the instability which occurs is due to
the high stresses imposed being relieved.

Any changes which decrease the Deborah no. in
the entry region, such as the use of conical rather than
sharp-edged entry, should therefore result in postponement
of the instability to higher flowrates.

Other characteristics

One of the major problems encountered in extrusion
die design and the study of plast flow is the specification
of the flow properties of commonly extruded thermoplastics.
For generality, simplifications are made in the
representation of the flow properties.

Two chief, readily observable characteristics of
thermoplastics are:

(1) A decrease in apparent viscosity with increasing
shear rate, and

(2) A capacity for elastic recovery when the stress is
suddenly removed.

These two are strongly dependent on the material, the
temperature and in some cases on the previous shear history
of the melt.

Due to the very high coefficients of bulk elasticity
of polymers, incompressibility can also be assumed in the
study and the analyses of their flows.
INTRODUCTION

Published work so far on die flow analysis presents results as either algebraic formulae or numerical solutions obtained from finite difference computations.

The recently developed finite element method, an alternative numerical technique, has been applied to non-Newtonian flow and die design analysis.

This method approximates the flow behaviour by dividing the flow region into small (triangular) subregions or elements and analysing flow in terms of, say, velocities at the corners of these elements.

The principal advantage is the ease of application to complicated geometries. Analysis would enable us to predict pressure drop, distribution of flow in dies, etc. In wire-coating dies, analysis can be done to compute the relationship between pressure drop and flowrate. Useful predictions can be made for the wire tension and shear stresses which are important when the strength of wire and likelihood of melt fracture are considered.

The following discusses briefly, entrance effects, flow channel design, dead spots, flow past obstructions and how die design affects the quality of a product.
(a) **ENTRANCE EFFECTS**

When a polymer enters a capillary, a drop in the available pressure occurs. This drop is referred to as the entrance loss and results in there being less pressure available to drive the polymer through the capillary. The same effect is observed in the flow within a die.

The original available pressure, $p$, is given by:

$$p = \Delta p_e + \Delta p_{cap}.$$ 

where $\Delta p_e$ is the entrance drop and $\Delta p_{cap}$ is the actual pressure loss within the capillary.

The entrance loss is most usually explained in terms of the energy needed to suddenly change the stream from a fat slow stream to a thin fast stream with stretched molecules. Calculations have shown that some of this loss is used up in forcing the melt to flow through the wide chamber upstream of the orifice itself.

Various suggestions have been put forward as to how to determine the entrance drop. These are given in references (45, 46 and 57). Bagley (4) has shown a method of determining the entrance drop which is applicable to all thermoplastics. He proposed that the entrance loss be expressed as an effective length in orifice diameters. This was found to be between 2 and 3 diameters. To correct for entrance loss, this is added to the actual tube length to give a fictitious effective length. Bagley (4, 45) also gave a method based on end correction curves which may be used to apply an approximate correction to flow curves for which no entrance corrections have been made.
Given the dimensions and required output rate of an extruded product, one of the tasks of the die designer is to calculate the dimensions of the die at the exit and define an appropriate flow channel in the body of the die in order to achieve an acceptable pressure drop and minimise the likelihood of flow defects.

The internal shape of the die adaptor (the part of the die body which precedes the land) is designed to allow the material to flow easily towards the smaller cross-sectional area of the die.

Dead space and consequences

When a melt encounters an abrupt change in section during its flow the material conforms to a natural angle of streamline flow in the region in front of the die entry. This pattern of flow results in the presence of dead spots.

Materials which stagnate in the die are subjected to a change in viscosity. This alters the flow properties of that part of the polymer, and polymer degradation is liable to occur if the temperature is high enough. There is therefore a variation in the quality of the extrudate.

This could, however, be avoided by reducing the half angle of entry to the die from $90^0$ to the natural half angle given by:

$$\tan \alpha = \sqrt{2n/\lambda}$$

- quoting Powell (44)

where $n$ = the apparent viscosity corresponding to the shear rate (or shear stress) at the die entry; and $\lambda$ = tensile viscosity corresponding to the tensile rate at the die entry,
Fig. 3 Obstruction introduced by a spider leg.
which in approximate calculations may be taken at the critical tensile rate or tensile stress.

The convergent angle for high viscosity polymer, e.g. UPVC, should not exceed $60^\circ$ and for a low viscosity material like nylon, the angle can be relatively obtuse. High viscosity materials tend to stagnate at the walls while flow takes place through the centre.

Flow past obstructions

With some materials of high melt viscosity it is difficult to achieve proper homogeneity after the melt has been broken by an obstruction like a spider leg. Obstructions by spider legs and breaker plates often result in weld line formation.

Weld lines are liable to occur whenever two flow fronts meet. They are formed when the flow during extrusion becomes divided due to an obstruction and rejoins again. A diagrammatic representation is as shown in Fig. 3.

The flow lines part around the webs and there is a region of high shear adjacent to them. Because polymer which has flowed around the web is sheared to a greater extent than the polymer on either side of it the die is presented with a non-uniform polymer melt. In film or extruded pipe this lack of uniformity shows up as visible lines known as weld lines or memory lines. These lines cause local extrudate weakness.

Weld lines arise from the fact that a molten polymer takes some time to relax to a normal, unoriented state after being sheared. In view of this, rigid materials require greater compaction to erase weld lines.
The following ways have been suggested as to how to eliminate the non-uniformity introduced.

(1) The web should be placed some way from the die so as to give the polymer a long time in which to relax after being parted.

(2) Some degree of mixing of the melt should be encouraged after it had rejoined on the die side of the web.

(3) An adequate hydrostatic pressure should be maintained.

(4) For the tube dies another suggestion is to utilise an off-set die in which the melt flows round the cone from a side entry.

The first suggestion is proven in practice as one way of achieving the second by which mixing is encouraged by designing spiral or overlapping flow channels in the die body. A theoretical solution to the second suggestion is to place a constriction in the flow channel between the web and the die itself so as to create a region of mildly distorted or turbulent flow.

The problem that arises in practice is that of estimating the effect of obstructions placed in the channel of flow, or of asymmetries at the entrances or the exits.

Pearson (39) reported that where the index of power law dependence is \( n \), the effect of the obstruction is \( n \) times as strong as for a Newtonian fluid (\( n = 1 \)). In other words, the entry or exit length required to smooth out the effects of obstructions in a channel of uniform depth is proportional to \( n \).

Using a slender-body approximation, Pearson (39) designed a spider leg. This resulted in a thin obstruction
aligned in the flow direction without producing any disturbance to the flow.

Channel Design

The resistance to flow in the shaping zone is higher than in the feed channels. Nevertheless the feed channels may have considerable influence on the flow. Their lengths may be different for the separate sections of the orifice, and accordingly the cross-sections along the lines of flow have to be wide where the path is shorter, so that the resistance to flow in them shall be identical.

The correct choice of flow passages from extruder to die-lips is the essential prerequisite of a successful die. Clearly the general topological configuration of these passages will be governed by the basic type of die required, but their detailed shape must be determined by reference to the detailed flow behaviour of the materials to be extruded.

Pearson (40) developed an analytical process to predict channel geometry required to give a specified output. The choice of die channel length is determined only by the duration of the relaxation process, which in turn depends on the rate of deformation (the output from the extruder) and the temperature of the melt in the die.

Blymental et al. (11b) reported that the channel length, \( L \), is given by:

\[
\frac{L}{h_o} = \frac{1}{6} K^{1/V} \tau_p^{1-1/V}
\]

where \( K \) and \( V \) are constants dependent on the temperature and \( \tau_p \) is the relaxation time of stress.
The same reference reported "τ_p" as being given by:

\[ \tau_p = K \dot{\gamma}^{n-1} \]

where \( \dot{\gamma} \) is the rate of shear.

(c) THE EFFECT OF DIE DESIGN ON THE QUALITY OF PRODUCTS

It would in many ways be convenient if during extrusion the polymer remained essentially unaltered, but this is not so. It is well known that polymers may be modified through being "worked" (i.e., subjected to high shear stresses in the melt) and that, furthermore, the effect of exposure to high temperatures during fabrication processes may be to modify yet further the structure of the polymer. For a given extruder, the degree of working achieved on the screw is affected by the die design, as this largely controls the back-pressure. Alterations to die design may thus affect the quality of the extrudate in two ways:

1. By providing a different flow path for the polymer and indirectly,
2. By modifying the polymer itself through alterations in the degree of shear experienced on the screw.

Quality of a product may be defined, among others, in terms of uniformity of flow. In order to achieve a uniform rate of melt flow to all points of a die, lip analysis of the complex flow behaviour is required. Pearson (39) showed that it is possible to obtain uniform output by using a narrow-channel approximation. Pearson (40) also developed a numerical technique of designing cross-head dies to achieve uniform output.
CHAPTER 5

FLOW INSTABILITY AND MELT FRACTURE

One of the primary criteria of extrudate quality is the presence or absence of extrudate roughness or melt fracture.

It is widely observed that during extrusion a breakdown from uniform to irregular flow takes place above a certain critical flowrate. Below this rate the extrudate is smooth, but above it the extruded polymer is rough, twisted and distorted. This phenomena sets the upper limit for polymer processing and is known as melt fracture.

No unanimous opinion has yet been reached to account for the onset of melt fracture. Some authors (Tordella et al. (53, 5, 7, 14)) favour a die entry effect; Spencer et al. (50) are in favour of die exit effect, while others support the concept of slip at the walls.

(a) DIE ENTRY EFFECT

It has been acknowledged widely by Tordella et al. (53, 7, 47) that melt fracture is a result of a tearing mechanism occurring at the die entry. However, evidence exists which favours the fact that under certain conditions melt fracture may be induced in the die land. A study by Han et al. (20) gave a clear indication that extrudate
Recoverable shear strain

Fig. 4 A sketch of recoverable shear strain versus shear stress for "Styron".
distortion is attributable to irregularities in the flowrate at the tube wall.

Change of Properties

A sudden change of elastic properties was reported at the critical point by Han et al. (20). A sketch of such a change for 'Styron' is as shown in Fig. 4.

Above the critical point, the storage capacity (recoverable shear strain) suddenly increased with shear stress. This increase in the amount of stored energy is released at the tube exit and may well go to increase the severity of the extrudate distortions.

The "entry region" theory was also supported by Howells and Benbow (21) who reported the cause to be due to the failure of the polymer melt to sustain high tensile stresses arising in the entry region. The breakdown of the network which occurs causes the noticeable backflow or recoil near the entry region.

Schulken and Boy (48) pointed out that shear rate and shear acceleration at the inlet may be the cause of melt fracture. Elastic oscillations, as a result of a sudden change in velocity at the entry region, are no longer damped by the viscosity of the medium when the critical point has been passed.

(b) THE CONCEPT OF WALL SLIPPAGE

Another explanation so far given for flow instability is the concept of slip within the capillary walls.

Benbow and Lamb (8) showed that local slipping causes
bulk flow effects throughout the whole volume of the melt. Slipping at the wall, as reported by Tordella et al. (54, 8) is suggested to be the initiator of flow instability. Lupton (24) and Maxwell and Galt (26) suggested that boundary slip is a relevant and perhaps a determining factor in the breakdown phenomenon.

Several authors (Yun et al. (58, 18, 9)) have indicated that wall slippage occurs when high-density polyethylene melts are extruded. There are some serious grounds to consider local slip of polymer systems relative to the capillary wall as one of the causes of melt fracture at high shear stresses. Nevertheless, melt fracture is sometimes detected for branched polyethylene, propylene and polystyrene in the absence of wall slippage.

Although slip flow is the dominant mode of transport during melt fracture, several authors favour the idea that slip is an effect rather than a cause of melt fracture.

(c) DIE EXIT EFFECTS

There is not much evidence in the literature in favour of flow instability being associated with die exit effects. Spencer (50) explains, however, that due to the high velocity gradient at the walls, there is a very large amount of elastic energy stored and a core of a relatively small amount of elastic energy. The system is unstable and instability takes place at the exit.
Fig. 5a: Shear stress versus apparent strain rate for L.P.

Fig. 5b: Shear stress versus apparent strain rate for B.P.
BEHAVIOUR OF LINEAR POLYETHYLENE (L.P.)
AND BRANCHED POLYETHYLENE (B.P.)

It was observed and reported by Bagley et al. (6) that discontinuity occurred in the flow curve for linear polyethylene (L.P.). With reference to Fig. 5a, a sudden increase in the flowrate without any increase in differential pressure was noted at a critical pressure value. On decreasing the shear stress from an unstable flow condition, discontinuity occurred at a lower shear stress than on increasing it from a stable condition. Consequently a hysteresis loop is formed.

Branched polyethylene (B.P.) exhibited melt fracture without any discontinuity in the flow curve as evident in Fig. 5b.

Flow visualisation studies on L.P. and B.P. have shown that, at the die entry, B.P. shows very large circulatory dead space while for L.P. the dead space is extremely small.

Other studies also show that flow disturbance occurred at the inlet region for B.P., and that for L.P. the breakdown of the flow occurred within the capillary.

General conclusions of other workers can be summarised as below:

(1) Melt elasticity is responsible for flow instability.
(2) Extrudate distortion is primarily due to uneven, elastic strain recovery, and
(3) Instability initiates at the capillary inlet for B.P. and within the capillary at the wall for L.P.
An important result established for L.P. is that melt slip at or near the wall is the cause of flow curve discontinuity occurring at the onset of unstable flow.

(e) WHAT IS TO BE DONE?

In order to understand the phenomenon of slip, it is necessary to resolve whether it occurs at the melt-die wall interface or between layers of melt. Quite a large number of major problems regarding capillary flow instability are still left unresolved. In particular, the rheological differences between linear and branched polyethylene which result in the different instability sites and flow patterns within the reservoir feeding the capillary are unexplained. Additionally, the precise nature of the flow breakdown must be elucidated, although the slip process which occurs within the capillary for L.P. lends credence to the fracture mechanism proposed by Tordella (53).

(f) WHAT CAN BE DONE?

It has been fairly well established that melt fracture occurs only at and above a certain critical shear rate. By changing certain variables a quality product can still be obtained by extruding even above this value. One such variable is viscosity which can be lowered by increasing the temperature or using a resin of higher melt index. Higher temperatures lessen the resistance to flow and result in high shear rate without melt fracture
occurring. Since resistance to flow is lessened, the pressure is lowered and the shear stress is increased. The use of a resin that is less viscous or a higher melt index has the same effect as raising the temperature. Reducing the angle of entry and increasing the die parallel length can also result in extrudates of excellent quality at high output rates.
MELT ELASTICITY

(a) CAUSE OF DIE SWELL

Die swelling is the most obvious measure of the elasticity of a polymer. It is caused by reversible elastic deformations developing in a polymer melt while it is flowing in the die.

An extrudate, on issuing from a die, is noted to have a cross-section much larger than the hole from which it emerges. The main cause of which is the viscoelasticity of the resin.

Stresses developed in the melt by the screw extruder tend to relax on its flow through the die. At the die exit the residual stresses cause the transverse widening of the stream. Also, during flow in the die, the transverse velocity gradient leads to orientation of the molecules along the stream. It is reported by Clegg et al. (14, 49, 59) that at the die exit disorientation of the molecules occurs and leads to the deformation of the extrudate. A certain increase in the cross-section has also been reported to be due to the equalisation of the velocity profile at the exit.

Observations

For one-dimensional flow, the swelling is in the
direction of the velocity gradient, e.g. a round rod swells to a diameter larger than that of the round hole. For two-dimensional flow, however, e.g. in a square-section hole, the shear rate (or shear stress) is not uniform in all directions. It is highest in directions parallel to the sides of square and least along the diagonals. Uneven swelling occurs and the stream swells more at the sides and less at the corners of the square. The corners are all rounded and the final shape is between a square and a circle.

The situation is messier for non-symmetrical sections.

Other effects of melt elasticity

The effects of melt elasticity are not confined to die swell alone. A polymer of high swelling ratio corresponds to one having a long elastic memory (long relaxation time) and hence possesses poor drawdown characteristics. Because of the relatively high melt tension generated during drawdown, such a polymer extruded as a tubular film would have good bubble stability. In the case of wire covering, however, the polymer would yield a covering showing a greater degree of retraction than one made from a polymer with a low swelling ratio.

(b) VARIOUS STUDIES CARRIED OUT ON DIE SWELL

Investigations were carried out by many authors on the problem of die swell. Their findings are briefly presented here.

Metzner and associates (27) measured the swelling of
round strands of polyethylene and polypropylene at various shear rate and L/D (length to diameter) ratios. They observed that for a small L/D ratio there is less time for the stresses set up to relax; therefore, more swelling results.

An experimental investigation by Kaplun et al. (23) revealed an exponential dependency of swelling ratio on the relative length of the channel. An attempt was made at obtaining the swelling ratio theoretically. A comparison of the calculated and the measured values showed a relative error of more than 10% only with a conventional shear rate of more than 300 g/cm³.

Several other authors (Poller et al. (43, 23)) confirmed that die swell decreases with increase in L/D ratio. The maximum value of L/D for which swelling reaches a minimum and remains virtually constant is very important. Kaplun et al. (23) find it to be 15-20 for high pressure polyethylene.

Another important aspect of die swell is the effect of shear rate on it. Several authors have tried to correlate die swell ratio with critical shear rate.

Clegg (13) observed that swelling increases with shear rate up to a maximum (critical point) and decreases thereafter. Kaplun et al. (23) reported the increase in swelling with increase in shear rate; but reaches a limit (probably the critical point) beyond which swelling is constant.

From these investigations, we can say that die swell is constant beyond a particular shear rate. Thus, when the
cross-section of an extrudate is distorted (as a consequence of uneven swelling on sections with non-uniform shear rate, e.g. a square section), by using the above relationship it is possible to select the optimal rate of extrusion.

Kaplun et al. (23) studied "swelling" under conditions which excluded the effects of contraction during cooling and of elongation under the weight of the extrudate itself. The following were established:

(1) The optimal extrusion conditions.
(2) The most suitable length of the moulding channel, and how to predict
(3) The increase in thickness of extrudate resulting from the elastic after effect only at the given extrusion rate.

(c) WHAT CAN BE DONE TO REDUCE DIE SWELL?

It has already been established that the amount of swelling is not uniform for most extrudates. There is more drag at certain points and the resulting shape of the extrudate is different from the die orifice. The "swelling" effect has become very important for "free" extrusions - where the extrudate is not subjected to any mechanical treatment and therefore viscoelastic effect and thermal contraction are at a maximum.

Is it possible therefore to compensate for the effect of swell in the initial die design? A suggestion has been made that this can be done by cutting a "hole" of the appropriate shape in a mass of compound and compressing the mass under pressure, thus finding the shape of the "void" which represents the shape of the required die.
Fig. 6a: The shape of the required die by the compression method.
This is as evident in Fig. 6a.

However, while this method gives some idea of a die for square, oblong and straight sided solid sections, the method gives quite a reverse effect when applied to more complicated sections. The illustrations in Fig. 6b show the type of die required to produce a square rod and also show the suggested die by the "compression" method.

Other suggestions as to how to compensate for die swell are:

(1) To reduce the die opening.

(2) To extrude at the optimal extrusion rate where the swelling ratio is constant, and

(3) For "drawdown" extrusion, the extrudate can be pulled out at a velocity greater than the average velocity in the die.
Profile of die required to produce the square object shown.

Profile of die as suggested by the compression method.

Square object.

Fig. 6b: Die profile.
The previous four chapters discussed the various problems in the design of extrusion dies for thermoplastics. The following conclusions were drawn and suggestions were made under the four main headings.

**MATERIAL PROPERTIES**

A brief survey was made into the flow behaviour and flow properties of polymers. It was established that polymeric materials could exhibit either solid-like or fluid-like behaviour.

It would represent a tremendous advance if the type of behaviour to be exhibited could be predicted under any set of conditions. This would probably require the setting up of a criteria under which the polymer could be described as either behaving like a fluid or a solid. An investigation of such nature will therefore be suggested.

The class of non-Newtonian behaviour which is most important industrially is pseudoplastic. Thus, Newtonian together with pseudoplastic behaviour in steady flow are of primary importance in the processing of thermoplastics.
FLOW ANALYSIS

Obstructions like spider legs, etc. are known to cause weld lines and thus non-uniformity in products. Obviously the extent of non-uniformity introduced by obstructions will differ for various polymers because of the differences in their flow behaviour. Thus the degree of obstructive effect on different sets of polymers could be looked at.

Pressure drop occurs at the entry region of every die. Various suggestions have been put forward (discussed in Chapter 4) as to how to determine it. The type of flow pattern displayed by a polymer at the die entry could be related to the entry pressure drop. Branched and linear polyethylene have been shown (Chapter 5) to exhibit different flow patterns at the die entry. Perhaps, the rheological differences between L.P. and B.P. could give an insight into the mechanism of the pressure loss.

In order to make die design automatic and scientific, it is required that the flow of polymers be really understood. Analysis of the flow of polymers within various channels would enable us to predict their flow behaviour in industrial processes.

The FE method briefly discussed in Chapter 4 could be used to obtain the flow characteristics in terms of dimensionless parameters. From these, the physical parameters may be obtained using the geometric dimensions of the problem region.
FLOW INSTABILITY AND MELT FRACTURE

Flow instability has been thoroughly discussed but the exact nature of flow breakdown is still not clear. Suggestions have already been put forward as to how to minimise the occurrence of melt fracture.

The site of flow instability is still in dispute. In my opinion, evidence seems to be more in favour of either "entry region" or "within the capillary" than at the "exit region".

A study on branched polyethylene supported the "entry region" theory and linear polyethylene went in favour of the "slip theory within the capillary".

It would add tremendous knowledge to melt fracture if the rheological differences between linear and branched polyethylene that led to the different instability sites could be stated.

Linear polyethylene is known to exhibit melt fracture together with a discontinuity in the flow curve while branched polyethylene shows no such discontinuity. The "hysteresis loop" effect displayed by linear polyethylene is still not understood. These two polymers are also known to show different flow patterns at die entries.

Slip has been established to be the cause of flow curve discontinuity and it occurs between polymer to polymer layers very near to the die walls.

Extrudates are known to distort severely beyond the critical point. This is due to the sudden increase in stored energy which is released at the die exit. The reason for the sudden increase in elastic recoverable properties
for polymers might give an explanation for flow instability and could be investigated.

Investigations into various aspects of flow defects suggested in this work, if approved, could be carried out with silicone rubber at room temperature because it exhibits the same flow defects as polymers at high temperatures.

**MELT ELASTICITY**

Die swell was thoroughly discussed but its distribution among various flow layers is still not very clear. Several authors have reported the maximum die swell to occur at the outer layers because shear rate (and shear stress) is maximum there, while others favour the idea of the maximum occurring along the axis of flow. This is because the fastest flow is along the axis and would have had the least time for its stresses to be relaxed.

Most of the work on die swell so far included the effect of contraction during cooling and the weight of the extrudate itself. More realistic results about elastic after-effects of polymers would be obtained if die swell is measured under conditions which exclude the effects of contraction and elongation.

It has been established that die swell is constant beyond a particular shear rate. Non-uniform shear rate (or shear stress) distribution is known to exist for sections like square dies. This results in non-even swelling. If the extrusion can, however, be carried out at the optimum extrusion rate for the polymer in question, "swelling" will be uniform for such a profile.
Internal stresses are known to exist in extrudates after processing, partly due to the redistribution of flow at the die exit. The effects of these residual stresses on the mechanical properties of polymers have so far not been covered. Assessment of the quality of extrudates could also be suggested by the determination of the internal stresses.

The stress distribution determination could lead to birefringence studies and would also be recommended.
SECTION 2

ANALYSIS OF FLUID FLOW
Fig. 7: Converging channel flow.
CHAPTER 8

BASIC THEORY

(a) INTRODUCTION

The equations of continuity, momentum, and energy are mathematical formulations of fundamental physical principles and are independent of the nature of the fluid.

One or more of them, together with the rheological equation of the fluid, are always involved in the mathematical formulation of flow problems.

In general, a flow problem cannot be formulated completely by using only the equations of continuity, momentum and energy. A rheological equation will be required to relate the stress components to the rate of strain of the fluid, and an equation of state will be required for all problems where the density is not constant. In addition, equations giving the temperature and pressure dependence of the other fluid properties may be required.

(b) EQUATIONS GOVERNING FLOW

Fig. 7 shows the geometry of the problem region in Cartesian coordinates.

In this analysis, the flow is assumed to be incompressible, isothermal and creeping (thus the inertia forces are negligible compared with the viscous pressure
According to the principle of conservation of mass, \( u \) and \( v \) must satisfy the equation of continuity. That is:

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{8.1}
\]

The equations of momentum can be written as:

\[
\frac{\partial p}{\partial x} = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \tag{8.2}
\]

\[
\frac{\partial p}{\partial y} = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y} \tag{8.3}
\]

Neglecting the cross-viscosity term in the general constitutive equation the viscous stress components become:

\[
\tau_{xx} = 2\mu \frac{\partial u}{\partial x}
\]

\[
\tau_{yy} = 2\mu \frac{\partial v}{\partial y} \tag{8.4}
\]

and

\[
\tau_{xy} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)
\]

Assuming also that viscosity is a function of the second invariant of the rate of deformation tensor, the viscosity can be written as:

\[
\mu = \mu(I_2) \tag{8.5}
\]

where

\[
4I_2 = 2\left( \frac{\partial u}{\partial x} \right)^2 + 2\left( \frac{\partial v}{\partial y} \right)^2 + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2
\]

In general, the melt viscosity depends on the pressure temperature and the strain rates. The effect of the pressure and temperature is ignored for the present purpose.

Attention is confined to the power-law form variation which represents more closely the stress-strain rate relationship of a wide variety of polymer melts.
Thus the equation of state becomes;

\[ \mu = (4 I_2)^{n-1/2} \]  

(8.6)

The solution of the above equations completes the analysis of the flow of the fluid within the specified geometry.

The results were generalised by obtaining the required variables in dimensionless form. Further details on this are presented in Appendix A1.
Fig. 8: Triangular element showing nodal points.
CHAPTER 9

STREAM FUNCTION ANALYSIS

(a) **INTRODUCTION**

This FE analysis adopts an approach similar to that of Palit and Fenner (37).

The flow region was divided into subregions and the relevant variables required were in terms of the values at the discrete nodal points in the mesh. An appropriate (polynomial) function of position is assumed for each variable within each element. A variational formulation is then used to find the solution for the nodal point variables by minimising the functional.

(b) **FINITE ELEMENT FORMULATION OF THE PROBLEM**

This sub-heading is further divided into four main groups. These are:
1. Elements and choice of mesh.
2. Nodal variables and shape functions.
3. Variational function.
4. Solution procedure.

1. **Elements and choice of mesh**

A mesh of triangular elements with nodal points at the corners (Fig. 8) was selected as being the simplest and
Fig. 9: Mesh used in the analysis.
The principal advantage is the ease of application to complicated flow geometries. For non-Newtonian flow problems, there is the further advantage that the viscosity is constant over each element, simplifying the analysis.

The flow region was divided up into a mesh of triangular elements as shown in Fig. 9.

2. Nodal variables and shape functions

The nodal points chosen are at the vertices of the triangular elements. In this work $\psi$ and $\phi$, the stream function and linear sum of velocity components, were used as the nodal variables.

A quadratic distribution of the stream function was assumed within each element.

Thus;

$$\psi = C_1 + C_2x + C_3y + C_4x^2 + C_5y + C_6xy^2 \quad (9.1)$$

where the $C$'s are constants, expressible in terms of the nodal point variables.

It is not always easy to ensure that the chosen function will satisfy the requirement of continuity between adjacent elements. Thus the compatibility condition on such lines may be violated (though within each element it is obviously satisfied).

3. Variational function

The nodal point variables are required to satisfy the condition that the total viscous dissipation over the flow region should be a minimum. This is equivalent to the
equilibrium condition expressed by equations (8.2) and (8.3).

The functional used was obtained by integrating the local viscous dissipation of \( 4\mu I_2 \) over the region.

That is;

\[
\chi = \int 4\mu I_2 \, dv 
\]  

(9.2)

where \( \mu \) is the viscosity and \( I_2 \) is the second invariant rate of deformation of an element.

The functional \( \chi \) was differentiated with respect to each nodal variable and equated to zero for each element. The condition to be satisfied for every nodal point variable such as \( \psi_i \) is;

\[
\frac{\partial \chi}{\partial \psi_i} = 0
\]  

(9.3)

This leads directly to the elemental stiffness equation which takes the form:

\[
[K_e] [\psi_e, \phi_e] = \begin{bmatrix}
\frac{\partial \chi_e}{\partial \psi_e} & \frac{\partial \chi_e}{\partial \phi_e}
\end{bmatrix}
\]  

(9.4)

where \([K_e]\) is the elemental stiffness matrix, and \([\psi_e, \phi_e]\) is a matrix of the required elemental nodal variables.

For details on the formation of matrices \([K_e]\) and \([\psi_e, \phi_e]\) refer to Appendix A2.

The contribution from all the various elements results in the system matrix equation of similar form, i.e.

\[
[K] [\psi, \phi] = [0]
\]  

(9.5)

where \([K]\) is the system stiffness matrix and \([\psi, \phi]\) is a column matrix containing the nodal values of \( \psi \) and \( \phi \).

4. Solution procedure

Before an attempt is made at solving the equation (9.5), some basic information necessary for the FE
formulation must be obtained. These are: nodal point coordinates, elements and their associated nodal numbers, degree of freedom numbers associated with each node, and the boundary conditions.

Nodal and elemental information

These constitute a considerable amount of data which was very time consuming and required a large amount of manual effort. The nodal variables were numbered in such a way as to minimise the bandwidth of the system stiffness matrix and thereby reduce the amount of storage space required. Refer to Appendix P1. These numbers also refer to positions in the solution vector-matrix where the variables are stored. Within this matrix, the nodal variables were numbered in this fashion: the unknown values first, followed by the known non-zero values and then the zero nodal values.

Boundary conditions

The type of velocity distribution used for the boundary conditions depended on the physical situation of the problem. Following the work of Palit (36), fully developed flow boundary conditions were used, both at the inlet and outlet. The boundary conditions in dimensionless form are as shown in Appendix A3.

Solution of equations

After the introduction of the boundary conditions, the equations were solved for \( \psi \) and \( \phi \) with the aid of a digital computer. The solution subroutine used is SymSol. Refer to Appendix P1.

The viscosity of all elements for the Newtonian flow
was known in advance, but not for non-Newtonian flow. It was therefore necessary after every cycle of iteration to update the elemental viscosities for the non-Newtonian case.

**Convergence and solution accuracy**

In FE analysis, the following types of convergency must be considered:

(a) As the element sizes are reduced, the results must tend to the true solution of the physical problem.

(b) A decrease in the convergent criterion results in an increase in the number of iterations and the nodal point variables should tend to constant values.

Convergence test was applied after every cycle. The relative change of each nodal point variable was computed and the magnitude of these changes summed up. The criterion used was that the above sum should be less than $10^{-5}$.

A check on the FE accuracy can be obtained by comparing the FE and LA solutions. For parallel channel flow, the solutions should be identical. The flowrates at every section of the channel could also be used as a further check on the FE accuracy. Any variation in the flowrates from section to section is a measure of the solution accuracy. This is particularly useful for tapered channels.

(c) **PROBLEMS IN THIS ANALYSIS**

The various problems encountered in using $\psi$ and $\phi$ as the nodal variables can be divided into the following headings:

1. Boundary conditions.
2. Compatibility.
3. Extracting pressure and velocity values.


**Boundary conditions**

The boundary conditions often used for FD formulations are stream function and vorticity. Vorticity is often very difficult to prescribe at the boundaries. In this work, $\psi$ and $\phi$ were used as the boundary conditions. Even though they are relatively straightforward to specify, compared with the FD boundary conditions, they did introduce some problems in the analysis.

First of all, the boundary conditions depended on the flow behaviour index of the polymer whose flow was being analysed. A different set of boundary conditions was therefore required for the Newtonian and non-Newtonian flows. Thus, if the analysis for another polymer (whose flow behaviour is known) is required, a different set of boundary conditions has to be written. For further details on the boundary conditions refer to Appendix A3.

The boundary conditions are also dependent on the inclination of the channel.

The preparation of the boundary conditions is thus laborious and time-consuming.

A solution to the above problem would be to find a set of boundary conditions independent of both the taper of the channel and the type of polymer in use.

**Compatibility**

This analysis leads to continuous velocities over each element. There is compatibility across element
interfaces in \( \psi \) and \( \phi \), the sum of the velocity components, but not in the individual velocities. This is therefore a great drawback to this type of analysis since there is discontinuity in velocities between adjacent elements.

**Extracting pressure and velocity values**

The nature of this analysis requires the pressure and the velocities to be obtained from the nodal variables. The solution of the matrix equation (9.5) yields \( \psi \) and \( \phi \) (at a nodal point).

But

\[
\phi = u + v. \tag{9.6}
\]

Therefore, if either \( u \) or \( v \) is found, the other is obtained directly from the above equation.

The derivation of the velocity \( u \) of an element is as shown in Appendix A4. Once the velocity of an element is found, the nodal point velocities can be found using an averaging process over all the elements involving that point.

The pressure drop across the channel was obtained by integrating equation (8.2) along the bottom boundary of the channel. The stress gradients were obtained in terms of the nodal viscosities and velocities. Finite difference approximations were used to evaluate the derivatives in equation (8.4). Further detail on the estimation of the pressure drop is evident in Appendix A5.

**Geometric problems**

From the look at the coefficients of the shape functions in Appendix A2, it is found that some of the coefficients become infinite when \( E = 0 \).
Fig. 10: Initial mesh.
i.e., when

\[ a_j + b_j = a_k + b_k \]

or when

\[ a_j + b_j = 0 \quad \text{or} \quad a_k + b_k = 0 \]

With regards to the mesh (Fig. 10) isosceles right angled triangles with their hypotenuse making an angle of 135° with the x-axis satisfy this condition. The initial mesh (Fig. 10) used, contained elements as described above and the problem was realised.

Palit (36) also reported a similar problem in his study. Similar difficulties were also experienced by Card (12). A modified area coordinate system as described by Card (12) and Zienkiewicz (60) can be used to get over these difficulties.

Another solution to this problem as adopted by the present author is to divide the flow region into a mesh as shown in Fig. 9.

For the converging channel, the mesh had to be modified to suit the actual boundary. The y coordinates of all the nodal points were changed to \( y' \), given by

\[ y' = y (1 - \frac{\tan \alpha}{L}) \quad (9.7) \]

where \( \alpha \) is the angle of taper of the channel and \( L \) is the length of the channel.
CHAPTER 10

ANALYSIS USING VELOCITIES AND PRESSURE

(a) INTRODUCTION

When the literature on computational methods in fluid mechanics is examined, the overwhelming tendency is to use stream function for the incompressible plane flow class of problems. Generally speaking, the sequence of calculations is to find the stream function, obtain the velocities and then the pressure drop.

This, however, seems to be a lengthy approach. The FE analysis following solves the same flow problem as was discussed in Chapter 9, but obtains the pressures and velocities directly.

In this approach also, a variational principle formulation of the problem was adopted and a different set of nodal variables used. This work also serves as a check on the first analysis of Chapter 9.

Further details of this analysis are presented under the subsequent headings.

(b) FINITE ELEMENT FORMULATION OF THE PROBLEM

The first step towards the FE formulation is to represent the problem region with a finite number of
elements interconnected at a discrete number of points—nodal points. Any arbitrary shape of the element can be chosen. The parameters relevant to the physical problem are then selected. The values of these parameters (e.g., velocity, pressure, temperature, etc.) at the nodal points are called the nodal variables. Nodeless variables; for example, element stress, could also be used. Element functions for the variables are then expressed in the form of polynomials of the coordinates. The choice of the element shape and the form of the element shape function affects the degree of accuracy of the solution, the numerical analysis and the computing time.

1. Elements and choice of mesh

In general, the elements can be of any shape, but triangular elements were chosen in this work to represent the continuum. They are particularly suitable for approximating arbitrary geometries. A typical element is as shown in Fig. 8. The mesh used is similar to that of the first analysis (refer to Fig. 9).

2. Nodal variables and shape functions

The simplest triangular elements (i.e. elements with three nodal points at the vertices) were used in order to make the analysis simple and suitable for a wide range of applications. The velocities (u and v) and pressure (p) were used as the nodal variables.

A linear variation pressure and velocities were assumed over each element. This is a better approximation than the first analysis where the pressure was assumed to
Fig. 11: Typical finite element.
be constant within each finite element. Fig. 11 describes the geometry of a typical triangular element \( m \) with the nodal points \( i,j,k \). The three components of element velocities are:

\[
[u] = \begin{bmatrix}
u_i \\
v_j \\
u_k
\end{bmatrix}
\]

The velocity distribution within the element is thus given by the linear polynomial

\[
u = \alpha_1 + \alpha_2 x + \alpha_3 y \quad (10.1)
\]

where the constants \( \alpha_1, \alpha_2 \) and \( \alpha_3 \) can be evaluated in terms of the element dimensions and the nodal viscosities.

In general, the nodal variables \( u, v \) and \( p \) are given by:

\[
u = N_i u_i + N_j u_j + N_k u_k \quad (10.2)
\]

\[
v = N_i v_i + N_j v_j + N_k v_k \quad (10.3)
\]

and

\[
p = N_i p_i + N_j p_j + N_k p_k \quad (10.4)
\]

where the \( N \)'s are the shape functions given by;

\[
N_r = \frac{1}{2\Delta} [a_r + b_r x + c_r y] \quad (10.5)
\]

For further details see Appendix B1.

3. Variational function

The formulation of the problem requires the satisfaction of equations (8.1) through to (8.6). A formulation technique using the concepts of variational calculus was developed.
The variational method requires in general the minimisation of a functional $\chi$ over the continuum. $\chi$ is an integral quantity obtained mathematically. Although it is apparently treated as a mathematical quantity, it has a physical meaning associated with the energy of the system.

For the equilibrium of the system, the functional chosen must be minimised, with respect to the nodal variables. The functional $\chi$ chosen for the present problem is given by;

$$
\chi = \int_A (4\mu I_2 + 2\mathbf{u} \cdot \mathbf{Vp}) \, \text{d}A .
$$

(10.6)

Since the differential of $\chi$, with respect to the nodal variables, is to be zero, it is only necessary to consider the local viscous dissipation term as being proportional to the equivalent term for the pressure forces.

For a typical element the differential of $\chi$ with respect to the nodal variables is given by;

$$
\begin{bmatrix}
\frac{\partial \chi^e}{\partial u_i} \\
\frac{\partial \chi^e}{\partial v_i} \\
\vdots \\
\frac{\partial \chi^e}{\partial p_k}
\end{bmatrix}
= [K_e]
\begin{bmatrix}
u_i \\
v_i \\
\vdots \\
p_k
\end{bmatrix}
$$

(10.7)

where $[K_e]$ is the elemental stiffness matrix.

An outline of general variational procedures and the evaluation of $[K_e]$ is given in Appendix B1.

The final equations are obtained by adding the
contributions of each element and equating to zero, thus minimising the total integral.

The resulting equation is of the form:

\[
[K] \begin{bmatrix} u \\ v \\ p \end{bmatrix} = [0].
\]

(10.8)

where \([K]\) is the system stiffness matrix and \(\begin{bmatrix} u \\ v \\ p \end{bmatrix}\) is a column matrix containing the nodal variables of \(u, v\) and \(p\).

4. Solution procedure

The basic information required in the solution of equation (10.8) is similar to those listed under Chapter 9. The only differences are in the degree of freedom numbers and the boundary conditions.

There were three degree of freedom numbers per node denoting the three nodal variables. The coordinates and the degree of freedom numbers were generated with the aid of a digital computer (refer to Subroutine Genode, Appendix P2). This considerably reduced the effort in preparing the data manually.

A set of boundary conditions (a great deal simpler than those of the previous analysis) was employed here. A dimensionless pressure of unity and zero was prescribed at the inlet and outlet, respectively, of the channel.

A solution procedure, similar to the previous one, was also adopted here (Reference: Symbol Subroutine Appendix P2).
This analysis was found to have great advantages over the previous one. Most of the problems encountered previously were eliminated here.

No geometric problem was encountered in the analysis. The only geometric problem which may have occurred was already solved in the first analysis.

The boundary conditions used were independent of the flow behaviour index of the polymer. Thus, the same set of boundary conditions could be used to analyse the flow of a Newtonian as well as a non-Newtonian fluid.

The pressure and the velocities being the nodal variables needed not be extracted after the solution was obtained. The required pressure drop encountered during the flow could be obtained without any further numerical integration. The pressure and velocities were found to be continuous over each element and compatible between adjacent elements. Hence the condition of compatibility is satisfied over the whole domain; a great advantage indeed.
CHAPTER 11

RESULTS AND DISCUSSION

Some results are presented for parallel and converging channel flows. The main objective is to compare the results of the two analyses and to demonstrate the power of the FE method.

Part (a) shows the list of results; which are discussed under Part (b).

The FE results are compared with the analytical and FD solutions in terms of dimensionless flowrate, velocities, pressure gradients and viscosities.

The results of the two analyses are presented together for (9 x 9) and (17 x 17) meshes; and at various taper angles of the channel of $\theta = 0^\circ, 10^\circ, 20^\circ, 30^\circ$ and $40^\circ$. 
PART (a): LIST OF RESULTS
NUMBER OF CYCLES OF ITERATION FOR NON-NEWTONIAN \((n = 0.5)\) PARALLEL CHANNEL FLOW USING A \(9 \times 9\) MESH.

Table 1

<table>
<thead>
<tr>
<th>CONVERGENCE CRITERION USED</th>
<th>NO. OF CYCLES OF ITERATION</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.001</td>
<td>6</td>
</tr>
<tr>
<td>0.0001</td>
<td>8</td>
</tr>
<tr>
<td>0.00001</td>
<td>12</td>
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</table>
FLOWRATE RESULTS FOR FIRST ANALYSIS.
DIMENSIONLESS FLOWRATE FOR PARALLEL CHANNEL FLOW

Table 2
NEWTONIAN FLUID

<table>
<thead>
<tr>
<th>POSITION X ALONG CHANNEL</th>
<th>MESH SIZE (9×9)</th>
<th>MESH SIZE (17×17)</th>
<th>EXACT</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.98042</td>
<td>0.99465</td>
<td>1.0</td>
</tr>
<tr>
<td>0.5</td>
<td>0.97761</td>
<td>0.99420</td>
<td>1.0</td>
</tr>
<tr>
<td>1.0</td>
<td>0.98042</td>
<td>0.99465</td>
<td>1.0</td>
</tr>
<tr>
<td>MAXIMUM % ERROR FROM THEORETICAL VALUE</td>
<td>2.239</td>
<td>0.580</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 3
NON-NEWTONIAN FLUID (n = 0.5)

<table>
<thead>
<tr>
<th>POSITION X ALONG CHANNEL</th>
<th>MESH SIZE (9×9)</th>
<th>MESH SIZE (17×17)</th>
<th>EXACT</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.98536</td>
<td>0.99629</td>
<td>1.0</td>
</tr>
<tr>
<td>0.5</td>
<td>0.98409</td>
<td>0.99607</td>
<td>1.0</td>
</tr>
<tr>
<td>1.0</td>
<td>0.98536</td>
<td>0.99629</td>
<td>1.0</td>
</tr>
<tr>
<td>MAXIMUM % ERROR FROM THEORETICAL VALUE</td>
<td>1.591</td>
<td>0.393</td>
<td>0</td>
</tr>
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</table>
DIMENSIONLESS FLOWRATE FOR CONVERGING CHANNEL FLOW AT X = 0.5

THEORETICAL FLOWRATE = 1.0

Table 4
NEWTONIAN FLUID

<table>
<thead>
<tr>
<th>INCLINATION OF CHANNEL</th>
<th>MESH (17×17)</th>
<th>% ERROR FROM THEORETICAL VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0.99420</td>
<td>0.580</td>
</tr>
<tr>
<td>10°</td>
<td>0.99408</td>
<td>0.592</td>
</tr>
<tr>
<td>20°</td>
<td>0.99340</td>
<td>0.660</td>
</tr>
<tr>
<td>30°</td>
<td>0.99196</td>
<td>0.804</td>
</tr>
<tr>
<td>40°</td>
<td>0.98890</td>
<td>1.110</td>
</tr>
</tbody>
</table>

Table 5
NON-NEWTONIAN FLUID (n = 0.5)

<table>
<thead>
<tr>
<th>INCLINATION OF CHANNEL</th>
<th>MESH (17×17)</th>
<th>% ERROR FROM THEORETICAL VALUE</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>0.99607</td>
<td>0.393</td>
</tr>
<tr>
<td>10°</td>
<td>0.99515</td>
<td>0.485</td>
</tr>
<tr>
<td>20°</td>
<td>0.99341</td>
<td>0.659</td>
</tr>
<tr>
<td>30°</td>
<td>0.99056</td>
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<td>40°</td>
<td>0.98591</td>
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VELOCITY RESULTS FOR BOTH ANALYSES
DIMENSIONLESS VELOCITIES FOR PARALLEL CHANNEL FLOW
(FIRST ANALYSIS)

Table 6
NEWTONIAN FLUID

<table>
<thead>
<tr>
<th>POSITION ACROSS THE CHANNEL (y)</th>
<th>THEORETICAL VALUE</th>
<th>POSITION ALONG CHANNEL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>MESH (9x9)</td>
<td>MESH (17x17)</td>
</tr>
<tr>
<td>0.00</td>
<td>0.00000</td>
<td>0.01910</td>
</tr>
<tr>
<td>0.125</td>
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<td>0.64640</td>
</tr>
<tr>
<td>0.250</td>
<td>1.12500</td>
<td>1.10824</td>
</tr>
<tr>
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<td>1.40625</td>
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</tr>
<tr>
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<td>1.50000</td>
<td>1.48496</td>
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<tr>
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<td>1.40625</td>
<td>1.39445</td>
</tr>
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<td>1.12500</td>
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<tr>
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<td>0.00000</td>
<td>0.02384</td>
</tr>
<tr>
<td>MAXIMUM % ERROR</td>
<td>1.501</td>
<td>0.457</td>
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DIMENSIONLESS VELOCITIES FOR PARALLEL CHANNEL FLOW
(SECOND ANALYSIS)

Table 7
NEWTONIAN FLUID

<table>
<thead>
<tr>
<th>POSITION ACROSS THE CHANNEL (y)</th>
<th>THEORETICAL VALUE</th>
<th>POSITION ALONG CHANNEL</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>x = 0.25</td>
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<td></td>
<td>MESH (9×9)</td>
<td>MESH (17×17)</td>
</tr>
<tr>
<td>0.0000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>0.125</td>
<td>0.65625</td>
<td>0.66667</td>
</tr>
<tr>
<td>0.250</td>
<td>1.12500</td>
<td>1.14286</td>
</tr>
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<td>1.40625</td>
<td>1.42857</td>
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</tr>
<tr>
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<td>1.40625</td>
<td>1.42857</td>
</tr>
<tr>
<td>0.750</td>
<td>1.12500</td>
<td>1.14286</td>
</tr>
<tr>
<td>1.875</td>
<td>0.65625</td>
<td>0.66667</td>
</tr>
<tr>
<td>1.000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>MAXIMUM % ERROR</td>
<td>1.588</td>
<td>0.392</td>
</tr>
</tbody>
</table>
DIMENSIONLESS VELOCITIES FOR PARALLEL CHANNEL FLOW
(FIRST ANALYSIS)

Table 8
NON-NEWTONIAN FLUID (n = 0.5)

<table>
<thead>
<tr>
<th>POSITION ACROSS THE CHANNEL (y)</th>
<th>THEORETICAL VALUE</th>
<th>POSITION ALONG CHANNEL</th>
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</thead>
<tbody>
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</tr>
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<td>1.33333</td>
<td>1.33094</td>
</tr>
<tr>
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<td>1.31250</td>
<td>1.31234</td>
</tr>
<tr>
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<tr>
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<td>0.78126</td>
</tr>
<tr>
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<td>0.00000</td>
<td>0.02729</td>
</tr>
<tr>
<td>MAXIMUM % ERROR</td>
<td>1.353</td>
<td>0.310</td>
</tr>
</tbody>
</table>
DIMENSIONLESS VELOCITIES FOR PARALLEL CHANNEL FLOW  
(SECOND ANALYSIS)

Table 9  
NON-NEWTONIAN FLUID (n = 0.5)

<table>
<thead>
<tr>
<th>POSITION ACROSS THE CHANNEL (y)</th>
<th>THEORETICAL VALUE</th>
<th>POSITION ALONG CHANNEL</th>
</tr>
</thead>
<tbody>
<tr>
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<td>MESH (9x9)</td>
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<td>MESH (17x17)</td>
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<td></td>
<td></td>
<td>X = 0.75</td>
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<tr>
<td>0.125</td>
<td>0.77083</td>
<td>0.79027</td>
</tr>
<tr>
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<td></td>
<td>0.77554</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.79027</td>
</tr>
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<td></td>
<td>0.77554</td>
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<td>1.16667</td>
<td>1.19354</td>
</tr>
<tr>
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<td></td>
<td>1.17322</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.19354</td>
</tr>
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<td></td>
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<tr>
<td>0.375</td>
<td>1.31250</td>
<td>1.33875</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.31894</td>
</tr>
<tr>
<td></td>
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<td>1.33875</td>
</tr>
<tr>
<td></td>
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<td>1.33864</td>
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<td>1.35489</td>
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<td>1.31894</td>
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<td>1.17322</td>
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<td></td>
<td>1.19354</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.17322</td>
</tr>
<tr>
<td>0.875</td>
<td>0.77083</td>
<td>0.79027</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.77554</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.79027</td>
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<tr>
<td></td>
<td></td>
<td>0.77554</td>
</tr>
<tr>
<td>1.000</td>
<td>0.00000</td>
<td>0.00000</td>
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</tr>
<tr>
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</tr>
<tr>
<td></td>
<td></td>
<td>0.00000</td>
</tr>
<tr>
<td><strong>MAXIMUM % ERROR</strong></td>
<td>2.522</td>
<td>0.611</td>
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<td></td>
<td>2.522</td>
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<td>0.611</td>
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<td></td>
<td></td>
<td>2.522</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.611</td>
</tr>
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</table>
PRESSURE RESULTS FOR BOTH ANALYSES
DIMENSIONLESS PRESSURE DROPS FOR CONVERGING CHANNEL FLOWS
(FIRST ANALYSIS)

Table 10
NEWTONIAN FLUID

<table>
<thead>
<tr>
<th>ANGLE OF TAPER OF CHANNEL</th>
<th>MESH SIZE (9x9)</th>
<th>MESH SIZE (17x17)</th>
<th>LUBRICATION APPROXIMATION VALUES</th>
<th>PALIT</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>11.64146</td>
<td>11.91259</td>
<td>12.00</td>
<td>12.43</td>
</tr>
<tr>
<td>10°</td>
<td>16.54130</td>
<td></td>
<td>16.13</td>
<td>17.36</td>
</tr>
<tr>
<td>20°</td>
<td>27.36746</td>
<td>24.27</td>
<td>29.05</td>
<td></td>
</tr>
<tr>
<td>30°</td>
<td>63.33640</td>
<td>47.78</td>
<td>69.41</td>
<td></td>
</tr>
<tr>
<td>40°</td>
<td>382.60895</td>
<td>269.05</td>
<td>546.70</td>
<td></td>
</tr>
</tbody>
</table>

Palit did not state the mesh size for his results.

Table 11
NON-NEWTONIAN FLUID (n = 0.5)

<table>
<thead>
<tr>
<th>ANGLE OF TAPER OF CHANNEL</th>
<th>MESH SIZE (9x9)</th>
<th>MESH SIZE (17x17)</th>
<th>L.A. VALUES</th>
</tr>
</thead>
<tbody>
<tr>
<td>0°</td>
<td>6.00431</td>
<td>5.49810</td>
<td>5.65685</td>
</tr>
<tr>
<td>10°</td>
<td>6.63719</td>
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<td>6.86762</td>
</tr>
<tr>
<td>20°</td>
<td>9.01633</td>
<td></td>
<td>8.89443</td>
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<tr>
<td>30°</td>
<td>14.63545</td>
<td></td>
<td>13.38584</td>
</tr>
<tr>
<td>40°</td>
<td>38.62187</td>
<td></td>
<td>35.15758</td>
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</table>
**DIMENSIONLESS PRESSURE VALUES FOR FLOW IN PARALLEL CHANNEL**

(SECOND ANALYSIS)

Table 12

**NEWTONIAN FLUID**

<table>
<thead>
<tr>
<th>POSITION ALONG CHANNEL (X)</th>
<th>THEORETICAL VALUE</th>
<th>POSITION ACROSS THE CHANNEL</th>
<th>Y = 0.0</th>
<th>Y = 0.25</th>
<th>Y = 0.5</th>
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</thead>
<tbody>
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<td></td>
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<td>MESH (9x9)</td>
<td>MESH (17x17)</td>
<td>MESH (9x9)</td>
</tr>
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<td>12.04706</td>
<td>12.19048</td>
<td>12.04706</td>
</tr>
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<td>0.125</td>
<td>10.5000</td>
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<td>10.54118</td>
<td>10.66667</td>
<td>10.54118</td>
</tr>
<tr>
<td>0.250</td>
<td>9.0000</td>
<td>9.14286</td>
<td>9.03529</td>
<td>9.14286</td>
<td>9.03529</td>
</tr>
<tr>
<td>0.375</td>
<td>7.5000</td>
<td>7.61905</td>
<td>7.52941</td>
<td>7.61905</td>
<td>7.52941</td>
</tr>
<tr>
<td>0.500</td>
<td>6.0000</td>
<td>6.09524</td>
<td>6.02353</td>
<td>6.09524</td>
<td>6.02353</td>
</tr>
<tr>
<td>0.625</td>
<td>4.5000</td>
<td>4.57143</td>
<td>4.51765</td>
<td>4.57143</td>
<td>4.51765</td>
</tr>
<tr>
<td>0.750</td>
<td>3.0000</td>
<td>3.04762</td>
<td>3.01176</td>
<td>3.04762</td>
<td>3.01176</td>
</tr>
<tr>
<td>0.875</td>
<td>1.5000</td>
<td>1.52381</td>
<td>1.50588</td>
<td>1.52381</td>
<td>1.50588</td>
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<td>1.000</td>
<td>0.0000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>MAXIMUM % ERROR</td>
<td>1.587</td>
<td>0.392</td>
<td>1.587</td>
<td>0.392</td>
<td>1.587</td>
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</table>
DIMENSIONLESS PRESSURE VALUES FOR FLOW IN PARALLEL CHANNEL
(SECOND ANALYSIS)

Table 13
NON-NEWTONIAN FLUID \( (n = 0.5) \)

<table>
<thead>
<tr>
<th>POSITION ALONG CHANNEL ((X))</th>
<th>THEORETICAL VALUE</th>
<th>POSITION ACROSS THE CHANNEL</th>
<th>(Y = 0.0)</th>
<th>(Y = 0.25)</th>
<th>(Y = 0.5)</th>
</tr>
</thead>
<tbody>
<tr>
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<td>MESH (9x9)</td>
<td>MESH (17x17)</td>
<td>MESH (9x9)</td>
<td>MESH (17x17)</td>
<td>MESH (9x9)</td>
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<td>5.74602</td>
<td>5.67777</td>
<td>5.74602</td>
<td>5.67777</td>
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<td>0.125</td>
<td>4.94975</td>
<td>5.02777</td>
<td>4.96805</td>
<td>5.02777</td>
<td>4.96805</td>
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<td>4.25833</td>
<td>4.30952</td>
<td>4.25833</td>
</tr>
<tr>
<td>0.375</td>
<td>3.53553</td>
<td>3.59126</td>
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<td>3.59126</td>
<td>3.54860</td>
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<td>0.500</td>
<td>2.82843</td>
<td>2.87301</td>
<td>2.83888</td>
<td>2.87301</td>
<td>2.83888</td>
</tr>
<tr>
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<td>2.12131</td>
<td>2.15476</td>
<td>2.12916</td>
<td>2.15476</td>
<td>2.12916</td>
</tr>
<tr>
<td>0.725</td>
<td>1.41421</td>
<td>1.43651</td>
<td>1.41944</td>
<td>1.43651</td>
<td>1.41944</td>
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<td>0.875</td>
<td>0.70711</td>
<td>0.71825</td>
<td>0.70972</td>
<td>0.71825</td>
<td>0.70972</td>
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<td>1.000</td>
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<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
<td>0.00000</td>
</tr>
<tr>
<td>MAXIMUM % ERROR</td>
<td>1.577</td>
<td>0.370</td>
<td>1.577</td>
<td>0.370</td>
<td>1.577</td>
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</table>
VISCOSITY RESULTS OF FIRST ANALYSIS.
### DIMENSIONLESS NODAL VISCOSITIES FOR PARALLEL CHANNEL FLOW

**USING A (9×9) MESH**

Table 14

**NON-NEWTONIAN FLUID (n = 0.5)**

<table>
<thead>
<tr>
<th>POSITION ACROSS THE CHANNEL</th>
<th>THEORETICAL VALUES</th>
<th>AVERAGING PROCEDURE VALUES</th>
<th>% ERRORS</th>
<th>DIRECT EVALUATION VALUES</th>
<th>% ERRORS</th>
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<tr>
<td>0.000</td>
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<td>3.40</td>
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<td>0.48024</td>
<td>1.87</td>
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<td>0.67</td>
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<td>0.70711</td>
<td>0.71983</td>
<td>1.80</td>
<td>0.68850</td>
<td>2.63</td>
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DIMENSIONLESS NODAL VISCOSITIES
FOR PARALLEL CHANNEL FLOW AT X = 0.5
(FIRST ANALYSIS)

Table 15
NON-NEWTONIAN FLUID (n = 0.5)

<table>
<thead>
<tr>
<th>POSITION ACROSS THE CHANNEL</th>
<th>THEORETICAL VALUES</th>
<th>FINITE ELEMENT ESTIMATION</th>
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<td>MESH SIZE (9x9)</td>
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<td>MESH SIZE (17x17)</td>
</tr>
<tr>
<td>0.000</td>
<td>0.35355</td>
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<td>0.35659</td>
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<tr>
<td>0.125</td>
<td>0.47141</td>
<td>0.46827</td>
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<td></td>
<td>0.47118</td>
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<tr>
<td>0.250</td>
<td>0.70711</td>
<td>0.68850</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.70523</td>
</tr>
<tr>
<td>0.375</td>
<td>1.41421</td>
<td>1.25519</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.39177</td>
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<tr>
<td>0.500</td>
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<td>-</td>
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<tr>
<td></td>
<td></td>
<td>-</td>
</tr>
<tr>
<td>0.625</td>
<td>1.41421</td>
<td>1.25519</td>
</tr>
<tr>
<td></td>
<td></td>
<td>1.39177</td>
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<tr>
<td>0.750</td>
<td>0.70711</td>
<td>0.68850</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.70523</td>
</tr>
<tr>
<td>0.875</td>
<td>0.47141</td>
<td>0.46827</td>
</tr>
<tr>
<td></td>
<td></td>
<td>0.47118</td>
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<td>1.000</td>
<td>0.35355</td>
<td>0.36567</td>
</tr>
<tr>
<td></td>
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<td>0.35659</td>
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<tr>
<td>MAXIMUM % ERROR</td>
<td>11.244</td>
<td>1.587</td>
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COST OF THE RESULTING PROGRAMMES.
COST AND STORAGE REQUIREMENTS FOR THE COMPUTER PROGRAMMES
RESULTING FROM THE ANALYSES

Let Programme 1 result from the stream function analysis and Programme 2 from the velocity and pressure analysis.

Table 16

PROGRAMME 1

<table>
<thead>
<tr>
<th>MESH SIZE</th>
<th>RUNNING TIME</th>
<th>COST OF PROGRAMME</th>
<th>STORAGE CAPACITY</th>
<th>STORAGE COST AS A % OF THE TOTAL COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>9x9</td>
<td>53 secs</td>
<td>$2.10</td>
<td>7 K</td>
<td>1</td>
</tr>
<tr>
<td>17x17</td>
<td>12.5 mins</td>
<td>$20.00</td>
<td>20 K</td>
<td>25</td>
</tr>
</tbody>
</table>

PROGRAMME 2

<table>
<thead>
<tr>
<th>MESH SIZE</th>
<th>RUNNING TIME</th>
<th>COST OF PROGRAMME</th>
<th>STORAGE CAPACITY</th>
<th>STORAGE COST AS A % OF THE TOTAL COST</th>
</tr>
</thead>
<tbody>
<tr>
<td>9x9</td>
<td>3 mins</td>
<td>$5.25</td>
<td>10 K</td>
<td>14.5</td>
</tr>
<tr>
<td>17x17</td>
<td>36 mins</td>
<td>$70.00</td>
<td>41 K</td>
<td>47</td>
</tr>
</tbody>
</table>
PART (b): DISCUSSION
PRELIMINARY RESULTS

An initial investigation was performed to determine the most economical convergence criterion to be used in the analysis. This was done for a non-Newtonian fluid \((n = 0.5)\) in a parallel channel. The results are as shown in Table 1.

It is seen that a convergent criterion of \(10^{-3}\) resulted in 6 cycles of iteration, and for a convergent criterion of \(10^{-5}\), 12 cycles of iteration were required.

The latter value was therefore chosen and used for all subsequent analysis.

Flowrate

For two-dimensional plane flows, the accuracy of the FE solution is investigated by analysing the flowrate across a parallel channel.

In Table 2, the dimensionless flowrates at various positions along the channel are compared with the analytical solutions for the Newtonian case.

For steady, isothermal, incompressible flow the dimensionless flowrate at each section along the channel is, by definition, equal to unity.

It is thus seen that the FE results approach the true solution as the element size is decreased.

Table 3 shows the comparison between the flowrates for \(n = 0.5\) using the two meshes. It is found that, similar to the Newtonian case, the results converge to the true values as the mesh is refined.

Using the finer mesh alone, the flowrates at position \(x = 0.5\) were obtained for various angles of taper
Fig. 12: Velocity profile for Newtonian flow in a parallel channel.
Fig. 13: Velocity profile for non-Newtonian ($n = 0.5$) flow in a parallel channel.
of the channel as presented in Tables 4 and 5. The accuracy of the FE method is thus seen to decrease as the inclination of the channel is increased.

**Velocities**

The velocity values across the channel were found for parallel flow at positions \( X = 0.25, 0.5 \) and 1.0. The results for both analyses are as shown in Tables 6 through to 9.

The velocity plots (Figs 12 and 13) of the first analysis are given to demonstrate the nature of the profile. For the Newtonian flow, the profile is parabolic while the parabola flattens for the non-Newtonian case.

All the results illustrate the accuracy of the FE method and also the fact that the finer mesh values tend towards the analytical solution. Along any streamline, the results from the first analysis reveal that the velocities vary from position to position. This is probably due to the compatibility condition which is not satisfied in this analysis. With the second analysis, where compatibility is satisfied over the whole domain, it is noted that velocity is constant for any chosen streamline.

The velocities obtained from the first analysis are, on the average, lower than those from the second analysis. This could be explained by the fact that the dimensionless flowrate was unity for the second analysis while slightly lower flowrate values were obtained with the first analysis.

Both analyses showed almost the same errors. The largest error was obtained with the non-Newtonian flow using the second analysis as evident in Table 9. In this case,
however, there was a considerable improvement in the results, as a finer mesh was used.

**Pressure**

The first analysis compares the FE results of the pressure drops for converging channels of taper ranging from $0^\circ$ to $40^\circ$ with LA solutions and the solution by Palit (36). (Refer to Tables 10 and 11). The results of the second analysis, which only dealt with parallel channel flow, are presented in Tables 12 and 13.

For parallel channel flow, Palit (36) reported an error of the order of 4% in the pressure drop for Newtonian flow with a $(9\times9)$ mesh. The first analysis of this work showed an error of 3% for the same mesh and 0.73% for a $(17\times17)$ mesh. The results of the second analysis are seen to be the best of the three. Errors in the order of 1.59% and 0.39% for the coarse and fine mesh respectively were obtained in this analysis.

For the non-Newtonian case, however, the first analysis yielded errors in the range of 2.8 to 6.2%. One of the major sources of error in the evaluation of the pressure drop is the differentiation process used. Error is also due to the fact that the viscosity is assumed to be constant for each element. This results from the assumption that the stream function varies quadratically over each element. This assumption has the advantage of ensuring correct convergence as the element size is decreased. The price to be paid for these simplifications is a loss of accuracy at the nodal points.
Fig. 14: Dimensionless pressure drop versus taper angle.
Fig. 14 shows the plot of the dimensionless pressure drop against the taper angle. The dotted ones show the LA solution; while the solid ones represent the FE results of the first analysis.

Fig. 14 reveals that the FE and LA solutions diverge as the angle of inclination of the channel is increased. The difference between the two solutions is more noticeable from about the 15° angle onwards. A breakdown of the LA solution beyond 10° was also reported by Benis (10). The FE solution is also seen to be higher than the LA solution because of the two-dimensional nature of the flow.

Table 10 shows differences in the author's results and those of Palit (36). (It was not obvious from Palit's report what mesh size he used.) The author's approach to the evaluation of Δp was different from that of Palit. In both cases, the Δp was evaluated along the bottom boundary of the channel. However, Δp is known to be very much dependent on the viscosity values. Palit used an averaging procedure to obtain the nodal viscosities along the boundary while the approach adopted here evaluates the viscosity directly by the use of equation (8.6). The value of the theoretical viscosity at the boundary of the channel is 0.354. The values obtained from the averaging procedure and the direct evaluation methods are 0.405 and 0.366 respectively. The corresponding errors, as compared with the theoretical value, are 14.4% and 3.4%. Table 14 shows the values and errors for other positions using the two methods.
It can be seen that the errors in both cases are almost the same except at the boundary, where the error in the averaging procedure is very high. It is then obvious why the estimated values of the pressure drops by Palit are unsatisfactory.

Further explanation as to why the viscosity values, using the two methods, are almost the same and only differ significantly at the boundaries is given under Chapter 12. Also refer to the Visco Subroutine of Appendix P1 for further details on the direct evaluation of the viscosities.

Viscosities

Finite element nodal viscosity predictions using the "direct evaluation" method are presented in Table 15. Also included in this table is a set of results for the analytical viscosities. The analysis for the theoretical viscosities is as shown in Appendix A6.

In both cases it is seen that the row corresponding to the $y = 0.5$ position is left blank. This is because the denominator used during the evaluation tended to be extremely small at that position. It resulted in an infinite value for the viscosity which is physically impossible.

Improvements in the results are still observed as the mesh is refined.

Storage

All the computing for this work was done on a Burrough 6718 at the University of Canterbury Computer Terminus.
The results of the cost and the storage requirements of the programmes are presented in Table 16.

It was found that the storage requirement was largely dependent on the FE mesh. The computing time was also found to be largely dependent on the number of nodal variables required, the degree of non-Newtonian behaviour and to a lesser extent on the inclination of the channel.

With reference to the table, we can see that programme 2 is the more expensive of the two and it demanded a lot of storage capacity.

It is obvious from the results that there is inefficiency in the second programme so far as the (17×17) mesh is concerned. It is therefore essential for this programme to be improved if it is to be used for further melt flow analysis in future. In Chapter 12, some suggestions will be put forward as to how to improve upon this programme.
CHAPTER 12

GENERAL DISCUSSION

In the earlier chapters, melt flows in parallel and converging channels were analysed using the FE method. Two approaches were adopted; the first one was similar to the type of analysis used by Palit and Fenner (37) where $\psi$ and $\phi$ were used as the nodal variables. In the second analysis a different variational functional was used and nodal variables $u$, $v$ and $p$ were required.

The results of the two methods were presented and discussed under Chapter 11.

This chapter discusses features associated with the two methods and the general problems encountered with the solutions.

GENERAL DISCUSSION

(a) Constitutive Equations

Polymeric fluids are viscoelastic. Therefore any constitutive equations describing them should include both viscous and elastic effects of the polymer melts. Viscosity may be very large compared with melt elasticity in some flows, whereas in others the elastic effects may dominate.

Not much is known about melt elasticity. Pickup (41), Metzner et al. (28) and Cogswell and Lamb (15) have shown
Fig. 15: Initial layout of the system matrix equation.
that the stresses caused by elongation are greater than the shear stresses in certain flow situations, for example, converging flow of viscoelastic materials. Pearson (39) discussed constitutive equations which include the elastic effects, but they are very complex for practical use and the required material parameters are difficult to measure.

Throughout this work, the melt was treated as a Stokesian fluid. Viscosity relations which are functions of pressure, temperature and strain rates can be used. However, in this investigation, the temperature and pressure dependence were neglected. And also the cross viscosity term was ignored. The flow was also assumed to be steady, incompressible, laminar, viscous and slow moving. For slow non-Newtonian flows it was further assumed that the inertia forces were negligible, compared with the viscous and pressure forces.

In extrusion processes, elastic effects are not predominant, since melts are subjected to large rates of deformation for relatively long periods. For isothermal flow, therefore, a power-law fluid model was good enough to represent the pseudoplastic nature of polymer melts.

(b) Problems encountered in the solution of the "stream function analysis"

The initial problem encountered in this solution was a storage one.

When the system matrix equation;

\[ [K] [\psi, \phi] = [0] \]

was formed, it was put in the form as shown in Fig. 15.
Fig. 16: A portion of the mesh used.
The original idea was to divide the equation into matrices \( K_1, K_2, D_1, D_2, \) etc. The sizes of the matrices are also shown in the figure.

Attention was confined to matrices \( K_1, K_2 \) and part of the column vector \( [\phi, \psi] \) which is \( D_1 \) and \( D_2 \).

The loading vector is then equal to \( [K_2] \times [D_2] \); and the final matrix equation is

\[
[K_1] \times [D_1] = - [K_2] \times [D_2] \quad (12.1)
\]

This procedure is evident from Appendix C.

The above idea worked well for a \((9\times9)\) mesh which had the size of \([K]\) being \((162\times162)\). For a \((17\times17)\) mesh, however, the size of \([K]\) was \((578\times578)\). Storage problems were therefore encountered because the B6718 computer in use had an array limit of 65535 elements; thus approximately a size of \((256\times256)\).

To overcome this problem, advantage was taken of the symmetric nature of the stiffness matrix, and its banded form. The loading vector \(([K_2] \times [D_2])\) was also formed as each finite element was brought in for processing. This procedure is as outlined in the ASSEMB Subroutine of Appendix P1.

It was mentioned in the previous chapter that Palit's estimation of the pressure drop during flow was unsatisfactory. This was because of the incorrect values of the viscosities obtained along the boundaries of the channel. The following explanation might give an insight into why Palit's averaging procedure of viscosity evaluation led to unsatisfactory results.

With reference to Fig. 16 and using an averaging
procedure, nodal point 2 has contribution towards its viscosity from 6 elements (A, B, C, E, F and G). The last three elements are in the high viscosity region and the first three are in the low viscosity region. The effect of the low and high viscosity regions is that a fairly accurate value for \( \nu_2 \) of 0.48024 is obtained. The theoretical viscosity value for nodal point 2 is 0.47141. But since nodal point 1 is along the boundary, it has contribution from only 3 elements (B, C and D), all in the high viscosity region. Because there are no elements below nodal point 1, the value of its viscosity and, for that matter, all points along the boundary is higher than expected. This method gives the viscosity of nodal point 1 as 0.40496 while the analytical value is 0.35355.

A solution to this problem was to obtain the boundary viscosity values by a "direct evaluation method" as already discussed.

(c) Problems encountered in the solution of the "velocities and pressure analysis"

A problem of "division by zero" often occurred in the SYMSOL subroutine of this part of the work. This subroutine uses the same idea as the Gaussian Elimination process. Division by zero could be caused by either of the following:

(1) The stiffness matrix is singular (i.e. no inverse exists) leading to a "no solution" (i.e. not a physical problem).

OR

(2) An unfavourable choice of degree of freedom numbers was employed.
Fig. 17: Unfavourable way of labelling the nodal variables.
Fig. 18: Favourable way of labelling the nodal variables.
This is further explained below.

In the normal Gaussian Elimination procedure, division by zero is avoided by rearranging the equations. With the symmetric banded matrix, however, division by zero can occur even if the matrix is not singular. This occurs when there is no direct connection between the previously labelled degree of freedom numbers and the present ones. It can best be explained by means of a diagram.

A portion of the nodal variables are labelled as shown in Fig. 17. This results in a string of zeros in the eighth column of the stiffness matrix as shown in the figure. A "division by zero" message is therefore obtained from the computer during the reduction process of the matrix.

A way to overcome this problem is to label the nodal variables in a fashion such that they are all directly connected, as evident in Fig. 18.

A more serious problem encountered was that this analysis gave very good results for the pressure drop in parallel channel flow, but failed to yield any satisfactory results for other inclinations of the channel.

An explanation of this can be given with reference to one of the equations of flow, say equation (8.2), i.e.

\[ \frac{\partial p}{\partial x} = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y}. \]

Assuming a constant viscosity, we can write this equation as;

\[ \frac{\partial p}{\partial x} = 2\mu \frac{\partial^2 u}{\partial x^2} + \mu \frac{\partial^2 u}{\partial y^2} + \mu \frac{\partial^2 v}{\partial y \partial x} \quad (12.2) \]

Comparing both sides of the differential equation we can say that the pressure distribution should be a degree
lower than the velocity distribution. In this work, however, linear distribution was assumed for both the velocities and the pressure, and therefore led into problems. While this research was in progress, a similar work by Tanner (51) reported a solution to the problem by assuming a quadratic distribution for the velocities and linear distribution for the pressure over each element.

The problem of running cost of the programme was also faced in this analysis. Due to the large number of nodal variables required and the extremely large arrays encountered, a considerable amount of money was spent as far as the finer mesh is concerned. Typical arrays stored in this work are:

Element (512,5), Anode (289,5), Delta (867), ABC(512,9) and K(735,53). Refer to Appendix P2 for details on these arrays.

For parallel channel flow, all the elements are right-angled triangles. There are also two types of elements present; one with hypotenuse at $45^\circ$ and the other with hypotenuse at $-135^\circ$ with the x-axis.

For parallel channel flow therefore, the area of the elements needs to be evaluated once. This programme evaluates all entries of the array ABC (512,9), i.e. 4608 of them. For parallel channel flow this is not necessary because there are only two types of elements present. Thus only ABC (1,9) and ABC (2,9) are required, i.e. only 18 elements instead of 4608. This would lead to a lot of savings in computer time.

For converging flow, however, where the elements are all different in area, it would be required to evaluate all
the 4608 entries. It becomes necessary to investigate whether:

(1) It would be cheaper to store all the 4608 entries during the processing time, which could run into more than 30 minutes as mentioned before.

OR

(2) Have no storage facility for array ABC and then evaluate every entry of the array as it is required in the programme.

Improving the rate of convergence of the procedure by using an overrelaxation factor, as suggested by Palit (36), could result in a shorter computing time. Section 2.8 of Palit (36) gives an idea of how to determine the optimum overrelaxation factor for a given mesh.

(d) **Comparison of both analyses**

The second analysis is found to be the more general of the two. The same set of boundary conditions is used for the analyses of both the Newtonian and non-Newtonian flows. Thus, for the second programme, the elemental viscosities were assumed to be constant ($\mu = 1$) during the first iteration and the resulting solution was for Newtonian flow. Subsequent iterations required the viscosities to be updated until convergence was finally achieved. Hence, the solution for the non-Newtonian flow problem. This programme can therefore be used to analyse any type of polymer melt flow so long as the flow behaviour index (n) is known.

With the resulting programme from the first analysis, however, different boundary conditions were used for both
the Newtonian and the non-Newtonian flow problems. The programme had to be run separately to obtain the two solutions. If the analysis for another polymer is required, a different set of boundary conditions has to be written in order to run the programme. This is rather a slow and time consuming approach. The programme would, however, be extremely powerful if a set of boundary conditions could be found independent of the flow behaviour index of the fluid.

The second analysis which uses the pressure as one of the nodal variables gives better results for the pressure drop in parallel channel flow. As regards cost, the second programme was found to be more expensive and demanded a lot of storage capacity.

Both analyses used a mesh of triangular elements with nodal points at the corners. This cut down the algebra involved in generating the stiffness matrix. However, higher order polynomial functions are worth looking into. The use of such polynomials means more nodal points per element. The stiffness matrix formation would involve more algebra and numerical integration. The total number of elements required, however, to achieve a certain accuracy will be less than that for three point triangular elements.

Oden and Somogyi (35) reported shorter computing times for higher degree polynomial function and, correspondingly, more nodal points per element.

Further work on this topic should therefore consider parabolic and cubic functions.
CHAPTER 13

CONCLUSION AND RECOMMENDATIONS

(a) CONCLUSION

The main idea behind this work was to develop a FE technique suitable for analysing more general flow situations. Two methods were adopted and discussed in Chapters 9 and 10.

The FE results of the first analysis were successfully compared with theoretical, LA and other numerical solutions where they exist. The results of the second analysis compared favourably with those of the first. The second method gave better estimates of pressure drops in parallel channels, but failed to give any reasonable result for converging channel flows.

Throughout the work, the melt was treated as Stokesian. Elastic effects are noted not to be predominant in extrusion processes and hence, for isothermal flow, a power-law fluid model is a good enough representation of the pseudoplastic nature of polymer melts.

The second analysis was found to be the more general of the two. The boundary conditions were independent of the fluid. Different boundary conditions were required for each fluid flow when the first programme was used. The boundary conditions, in this case, depend very much on the flow
behaviour index of the fluid in question.

The use of three point triangular elements led to considerable cut down of the algebra involved in generating the stiffness matrix. The use of higher order polynomials (more nodal points per element) involves more algebra and integration in obtaining the stiffness matrix. To achieve the same accuracy, however, higher order polynomials require less elements. They are also noted to lead to shorter computing times.

The input data for flow problems like the type discussed consists of geometric dimensions of the flow region, rheological equations, boundary conditions and the FE representation in terms of nodal points and element interconnections. For the problems considered, the number of nodal points and elements were in the order of several hundreds. It is thus desirable to use an internal mesh generating technique. This considerably reduces the effort involved in the preparation of the mesh data manually.

The running time of each programme is very much dependent on the convergent criterion used. $10^{-5}$ was found to be the most economical convergent criterion and was therefore used throughout the analysis.

The velocity plots from the first analysis showed the normal expected trends. The Newtonian velocity profile was parabolic, while for the non-Newtonian flow the parabola flattened. The velocity results from the first analysis were lower, on the average, than the analytical values. The characteristic velocity was chosen so that the flowrate became unity. For the second analysis, the FE predictions
for the velocities were slightly higher than the analytical values.

The results presented in this work demonstrate the type of accuracy achieved by the FE method. Increasing mesh refinement results in convergency to the analytical solution. The accuracy of the FE method was observed to decrease as the taper of the channel was increased.

Pressure drop predictions by the author are better than those of Palit (36) due to a more direct method of evaluating the viscosities. The evaluation of the viscosity by an averaging procedure is known to be unsatisfactory. In the second analysis, linear variation of pressure over each element was assumed. This led to even better results for the pressure drop. FE predictions could be improved further if the elastic nature of polymer melts is taken into account in the analysis.

In general, it is observed that the pressure drop decreased with increase in non-Newtonian effect.

The computer in use, Burrough 6718, placed a restriction on the size of the system stiffness matrix. The array limit of the computer is 65535 elements; thus approximately a size of \((256 \times 256)\). The original method (Appendix C) of obtaining the system stiffness matrix and the loading vector was therefore abandoned. The new method adopted is shown in the ASSEMB Subroutine of Appendices P1 and P.2.

The reduction process of the system stiffness matrix can yield a "division by zero" message, even if the matrix is not singular. This occurs if the degree of freedom numbers are unfavourably arranged as explained in Chapter 12.
The second programme was the more expensive. The cost was mainly in the storage capacity requirements. It is therefore essential to reduce the running time and the storage capacity of the second programme. Several suggestions as to how this can be achieved have been put forward in Chapter 12(c) of this work.

The rate of convergence of the programmes can be increased by the introduction of an overrelaxation factor \( \omega \) as suggested by Palit (36). For a given mesh the value of \( \omega \) at which the total number of cycles of iteration is minimum, is termed the optimum overrelaxation factor \( \omega_{\text{opt}} \). It is desirable to determine \( \omega_{\text{opt}} \) for the meshes in order to economise on the computing times.

The present work shows that the use of the FE method for 2D flow problems is elegant; and results of acceptable accuracy can be produced with reasonable expenditure of computing effort. The accuracy attained in a particular problem is clearly a function of the number of elements used and their distribution, which in turn governs the time for solution.

To predict polymer melt flow behaviour in industrial processes, the first step is to generate a suitable mesh for the flow geometry. With the rheological equation and the boundary conditions known, the two FE methods discussed in Chapters 9 and 10 may be applied to obtain the flow characteristics in terms of dimensionless parameters such as pressure, flowrate, velocity, etc. From these, the physical parameters may be obtained using the geometric dimensions of the problem region. Alternatively, practical problems can be formulated and solved in dimensionless
variables.

We have used the two programmes discussed to solve a complex flow problem. They give us some insight into the type of problems that might be encountered in the study of non-Newtonian flow in extrusion dies.

(b) RECOMMENDATIONS

Early attempts to analyse the extrusion process considered only the assumed Newtonian flow of a polymeric material after melting. With the advent of readily accessible, high-speed digital computers, it was possible to include the non-Newtonian behaviour.

Polymeric fluids are viscoelastic. Thus, more realistic predictions by the FE method would be obtained if the constitutive equation includes both viscous and elastic effects. Other parameters like temperature-sensitivity behaviour of polymer melt viscosity could also be included.

In this work a mesh of triangular elements with nodal points at the corners was selected as being the simplest and easiest to use. For the second analysis, a linear polynomial was also used to define uniquely the variables within each element. The use of higher order polynomial functions and improved elements to investigate the accuracy and computing times of the present solutions could be suggested. An efficient matrix assembly and solution procedures should be developed to deal with these functions and elements. The technique of matrix handling and solving the algebraic equations for the present formulations may
also be improved, thereby economising on computer storage and time. Palit (36) pointed out that three point elements whose shapes are roughly equilateral give well-behaved stiffness matrices for more general flow problems. It may, therefore, be useful to develop a technique of generating a mesh of equilateral triangles. It might also be desirable to generate all the necessary data within the computer to get rid of the manual effort and the time involved.

With both programmes discussed, the first pass computes the constant viscosity solution, and iteration proceeds from this. We then recompute the viscosity at each point and use the new values in the next cycle of computation. The process was noted to be very slow. It is therefore recommended that a modified value of the viscosity is used each time, and also that an overrelaxation factor be introduced in order to increase the rate of convergence of the solution.

The second programme gave very good results for parallel channel flow. It is, however, known to be inefficient and it is essential to improve upon it and extend it to the analysis of converging channel flow.

Attention has so far been confined to isothermal, two-dimensional flows. The FE method, however, can be applied to more general problems. The Cartesian FE formulation can easily be extended to axi-symmetric problems, such as melt flow in converging cylindrical and annular dies, and wire coating dies.

The formulations presented here are applicable to solid stress analysis and lubrication problems. For
example, the flow in the narrow clearance between journal and bearing which is identical to couette flow with pressure gradient. These formulations are also applicable to free surface flows (for example, open channel flows) and there are possibilities of extension to unsteady fluid flow and heat transfer problems.

Different element properties (e.g. viscosity, conductivity, elasticity, etc.) can easily be introduced in the FE formulations. The methods presented, therefore, may be applied to two-phase flows which are very common in the compressing and metering sections of screw extruders and other inhomogeneity problems.

The use of solid elements like tetrahedrons may make the methods extendible to three-dimensional flow problems.
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APPENDIX A1

Dimensional Analysis

Consider the various equations governing the flow of the fluid as below:

\[
\frac{\partial p}{\partial x} = \frac{\partial^2 \tau_{xx}}{\partial x^2} + \frac{\partial \tau_{xy}}{\partial y} \tag{1}
\]

\[
\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{2}
\]

\[
\mu = \mu_0 \left( \frac{\sqrt{\gamma^2 T_2}}{\gamma_o} \right)^{n-1} \tag{3}
\]

Let us define a characteristic velocity and height being \(v_c\) and \(H\) respectively. We then have the following dimensionless quantities (variables with dashes):

\[
u' = \frac{u}{v_c}, \quad x' = \frac{x}{H}, \quad v' = \frac{v}{v_c}, \quad y' = \frac{y}{H}, \quad \gamma_c = \frac{v_c}{H} \quad \text{and} \quad \mu_c = \mu_0 \left( \frac{v_c}{H \gamma_o} \right)^{n-1} \tag{4}
\]

where \(\gamma_o\) is the reference shear rate and \(\mu_0\) is the effective viscosity at a reference shear rate of \(\gamma_o\).

\[
\tau_c = \mu_c \gamma_c \quad \text{i.e.} \quad \mu_0 \left( \frac{v_c}{H \gamma_o} \right)^{n-1} \frac{v_c}{H}
\]
i.e. \[ \mu \left( \frac{v_c}{H} \right)^n \frac{1}{\gamma_n - 1} \] (5)

Also \[ \mu' = \frac{\mu}{\mu_c}, \quad \tau_{xx}' = \frac{\tau_{xx}}{\tau_c} \]

and \[ \rho' = \frac{\rho}{\rho_c} \] (6)

from (1),
\[ \frac{\partial p'}{\partial x'} = \frac{\partial \tau_{xx}'}{\partial x'} + \frac{\partial \tau_{yx}'}{\partial y'} . \]

Substituting dimensionless quantities in the above equation we obtain
\[ \frac{\tau_c}{H} \frac{\partial p'}{\partial x'} = \frac{\tau_c}{H} \frac{\partial \tau_{xx}'}{\partial x'} + \frac{\tau_c}{H} \frac{\partial \tau_{yx}'}{\partial y'} \] (7)

This implies that
\[ \frac{\partial p'}{\partial x'} = \frac{\partial \tau_{xx}'}{\partial x'} + \frac{\partial \tau_{yx}'}{\partial y'} \] (8)

which is similar to equation (1).

Similarly, the continuity equation in dimensionless form becomes
\[ \frac{\partial u'}{\partial x'} + \frac{\partial v'}{\partial y'} = 0 \] (9)

Now for the equation of state:
\[ 4I_2 = 2 \left( \frac{\partial u'}{\partial x'} \right)^2 + 2 \left( \frac{\partial v'}{\partial y'} \right)^2 + \left( \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'} \right)^2 \]
\[ 4I_2 = \frac{v_c^2}{H^2} \left[ 2 \left( \frac{\partial u'}{\partial x'} \right)^2 + 2 \left( \frac{\partial v'}{\partial y'} \right)^2 + \left( \frac{\partial u'}{\partial y'} + \frac{\partial v'}{\partial x'} \right)^2 \right] \]
Now define the dimensionless $4I_2$ thus: $4I_2'$ as the terms in the bracket on the previous page.

\[ 4I_2 = \frac{v_c^2}{H^2} 4I_2' \]  

(10)

From equation (6),

\[ \mu' = \frac{\mu}{\mu_c} \]

i.e.

\[ \mu_o \left( \frac{\sqrt{4I_2}}{\gamma_o} \right)^{n-1} \frac{1}{\mu_o} \left( \frac{H \gamma_o}{v_c} \right)^{n-1} \]

i.e.

\[ \left( \frac{\sqrt{4I_2}}{\gamma_o} \frac{H \gamma_o}{v_c} \right)^{n-1} \]

From equation (10),

\[ \sqrt{4I_2} \frac{v_c}{H} \sqrt{4I_2}' \]

\[ \therefore \quad \mu' \left( \frac{v_c}{H} \frac{\sqrt{4I_2}'}{\gamma_o} \frac{H}{v_c} \right)^{n-1} \]

i.e.

\[ \mu' = \left( \frac{\sqrt{4I_2}'}{\gamma_o} \right)^{n-1} \]

which is also in the same form as the original equation of state. Hence, variables without dashes will be used to represent dimensionless quantities.
APPENDIX A2

Finite element analysis of fluid flow
within a converging channel using $\psi$ and $\phi$.

Consider the geometry of the problem region as shown in Fig. 7.

The equilibrium conditions to be satisfied are:

$$\frac{\partial p}{\partial x} = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{yx}}{\partial y}$$ (1)

$$\frac{\partial p}{\partial y} = \frac{\partial \tau_{xy}}{\partial x} + \frac{\partial \tau_{yy}}{\partial y}$$ (2)

Variational approach

The total potential energy of the system is given by

$$F = 2 \mu I_2$$ (3)

The integral of this energy must be minimised in terms of the nodal values. The above expresses the equilibrium condition as in (1) and (2).

To satisfy the continuity equation:

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

we introduce the concept of a stream function ($\psi$).

i.e. $u = -\frac{\partial \psi}{\partial y}$ and $v = \frac{\partial \psi}{\partial x}$ (4)

Now for $u$ and $v$ to vary linearly over the element, the stream function over an element must be given by
where the c's are constants expressible in terms of nodal point variables. Now

\[ 4I_2 = 2 \frac{\partial u^2}{\partial x} + 2 \frac{\partial v^2}{\partial y} + \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right)^2 \]  

(6)

Thus \( u = -\frac{\partial \psi}{\partial y} \) and \( v = \frac{\partial \psi}{\partial x} \)

Thus \( u = -\frac{\partial \psi}{\partial y} = -c_3 - 2c_5y - c_6x \)

\[ \therefore \frac{\partial u}{\partial y} = -2c_5 \]  

(7)

And \( v = \frac{\partial \psi}{\partial x} = c_2 + 2c_4x + c_6y \)

\[ \therefore \frac{\partial v}{\partial x} = 2c_4 \]  

(8)

Also \( \frac{\partial u}{\partial x} = -c_6 \)  

(9)

and \( \frac{\partial v}{\partial y} = -c_6 \)  

(10)

Substitution in (6) gives

\[ I_2 = c_6^2 + (c_4 - c_5)^2. \]  

(11)

Consider an element (m) as shown in Fig. 11.

The nodal variables are:

\( u_i, v_i, \psi_i, \quad u_j, v_j, \psi_j, \quad u_k, v_k, \psi_k \)

corresponding to the points i, j, k respectively.

But \( \psi = c_1 + c_2x + c_3y + c_4x^2 + c_5y^2 + c_6xy \).

Thus, six constants to be determined with the nine nodal variables.
We therefore define a new variable

\[ \phi = u + v = \frac{\partial \psi}{\partial x} - \frac{\partial \psi}{\partial y} \]  

(12)

The nodal variables are therefore reduced to

\[ \phi_i, \psi_i, \phi_j, \psi_j, \text{ and } \phi_k, \psi_k. \]

Substitution in the equation for \( \psi \) gives

\[ \psi_i = c_1 \]  

(13)

\[ \psi_j = c_1 + c_2 a_j + c_3 b_j + c_4 a_j^2 + c_5 b_j^2 + c_6 a_j b_j \]  

(14)

\[ \psi_k = c_1 + c_2 a_k + c_3 b_k + c_4 a_k^2 + c_5 b_k^2 + c_6 a_k b_k \]  

(15)

From (12),

\[ \phi = c_2 - c_3 + (2 c_4 - c_6) x + (c_6 - 2 c_5) y \]

Thus \( \phi = A + Bx + Dy \)

(16)

where \( A = c_2 - c_3 \), \( B = 2 c_4 - c_6 \) and \( D = c_6 - 2 c_5 \).

Substitution in (16) gives

\[ \phi_i = A \]  

(17)

\[ \phi_j = A + Ba_j + Db_j \]  

(18)

\[ \phi_k = A + Ba_k + Db_k \]  

(19)

(17, 18) and (19) give

\[ \frac{B + D}{2} = c_4 - c_5 \]

i.e. \( c_4 - c_5 = \frac{1}{4A} \left[ (a_k - a_j + b_j - b_k), (b_k - a_k), (a_j - b_j) \right] \)

\[ \begin{bmatrix} \phi_i \\ \phi_j \\ \phi_k \end{bmatrix} \]  

(20)

where \( A \) is the area of the element given by
Further elimination gives

\[ 2\Delta = a_j b_k - a_k b_j. \]  \hfill (21)

\[ a_1 = a_j + b_j; \quad a_2 = a_k + b_k; \quad a_3 = a_j - a_k. \]

\[ c_6 = \frac{1}{E} \begin{bmatrix} 4\Delta (J-K) \\ 4\Delta K \\ -4\Delta J \\ a_1 \\ a_2 \\ a_3 \end{bmatrix}^T \begin{bmatrix} \psi_i \\ \psi_j \\ \psi_k \\ \phi_i \\ \phi_j \\ \phi_k \end{bmatrix} \]  \hfill (22)

where \( E = 2\Delta JK(J-K) \)

\[ J = a_j + b_j; \]

\[ K = a_k + b_k; \]

\[ \alpha_1 = -[4\Delta a_j K - 4\Delta a_k J + (b_j - b_k) (a_j a_k^2 - a_k a_j^2 K) + (a_j - a_k) (b_j a_k^2 - b_k a_j^2 J)]; \]

\[ \alpha_2 = -[b_k a_j^2 K - b_k a_k^2 J + a_k b_j^2 K - a_k b_k^2 J]; \]

and

\[ \alpha_3 = -[-b_j a_j^2 K + b_j a_k^2 J - a_j b_j^2 K + a_j b_k^2 J]. \]

From (11), \( I_2 = c_6^2 + (c_4 - c_5)^2. \)

Thus \( I_2 \) can be found by substituting (20) and (22) into (11).

Now the viscous dissipation functional is

\[ F = 2\mu I_2 \quad \text{from (3)}. \]

The integral required to be minimised in terms of the nodal variables is therefore

\[ \chi = \iiint F \, dv \]

\[ = \int \int 2\mu I_2 \, dA \]

\hfill (23)
where \( W \) is the width of flow normal to the x-y plane.

Substituting for \( I_2 \) gives

\[
\chi = 2\mu W\Delta \left[ c_6^2 + (c_4 - c_5)^2 \right].
\] (24)

The condition to be satisfied for every nodal point variable such as \( \phi_i \) is

\[
\frac{\partial \chi}{\partial \phi_i} = 0.
\]

This leads directly to the fluid element stiffness equation, which takes the form:

\[
[K_e] \begin{bmatrix} \psi_e \end{bmatrix} = \begin{bmatrix} \frac{\partial \chi_e}{\partial \psi_e}, \frac{\partial \chi_e}{\partial \phi_e} \end{bmatrix}
\]

(25)

Differentiation of (24) with respect to the nodal variables gives equation (25) as

\[
\begin{bmatrix}
\gamma_1\gamma_2 & \gamma_1\gamma_3 & \gamma_1\gamma_4 & \gamma_1\gamma_5 & \gamma_1\gamma_6 & \psi_i \\
\gamma_2\gamma_3 & \gamma_2\gamma_4 & \gamma_2\gamma_5 & \gamma_2\gamma_6 & & \phi_1 \\
\gamma_3\gamma_4 & \gamma_3\gamma_5 & \gamma_3\gamma_6 & & & \psi_j \\
\beta_1^2 + \gamma_4^2 & \beta_1^2 + \gamma_4^2 & \beta_1^2 + \gamma_4^2 & \beta_1\beta_2^2 + \gamma_5\gamma_6 & \beta_1\beta_2^2 + \gamma_5\gamma_6 & \phi_j \\
\beta_1^2 + \gamma_4^2 & \beta_2^2 + \gamma_5^2 & \beta_2^2 + \gamma_5^2 & \beta_2\beta_3^2 + \gamma_6^2 & \beta_2\beta_3^2 + \gamma_6^2 & \psi_k \\
\beta_1^2 + \gamma_4^2 & \beta_2^2 + \gamma_5^2 & \beta_2^2 + \gamma_5^2 & \beta_3^2 + \gamma_6^2 & \beta_3^2 + \gamma_6^2 & \phi_k
\end{bmatrix}
\]

\[
4\mu W
\]

where

\[
\gamma_1 = \frac{4\Delta(J-K)}{E};
\]

\[
\gamma_2 = \frac{4\Delta K}{E};
\]

\[
\gamma_3 = \frac{-4\Delta J}{E};
\]

\[
\gamma_4 = \frac{\alpha_1}{E};
\]

\[
\gamma_5 = \frac{\alpha_2}{E};
\]
\[ \gamma_6 = \frac{\alpha_3}{E}; \]

\[ \beta_1 = (a_k - a_j + b_j - b_k)/4\Delta; \]

\[ \beta_2 = \frac{b_k - a_k}{4\Delta}; \]

and

\[ \beta_3 = \frac{a_j - b_j}{4\Delta}. \]
APPENDIX A3

Dimensionless boundary conditions ($\psi$ and $\phi$).

The boundary conditions prescribed in this work are the stream function $\psi$ and the sum of the velocity components $\phi$.

Conditions at the top of the channel are:

$$\psi = 0 \quad \text{and} \quad \phi = 0.$$  

At the bottom we have

$$\psi = -1 \quad \text{and} \quad \phi = 0.$$  

In general,

$$\psi = -\frac{1}{2(1+n)} \left[ (1+2n) \left( \frac{2YH}{H} -1 \right) - n \left( \frac{2YH}{H} - 1 \right) \right] - \frac{1}{2}$$

and

$$\phi = \frac{(1+2n) Y}{H} \left[ 1 - \left( \frac{2YH}{H} - 1 \right) \frac{1}{n} \right]$$

where $H$ is the depth of the channel at the point of interest.

$$Y = \frac{Y}{H_1} \; ; \; \text{is the dimensionless coordinate}$$

and $H_1$ is the height at the inlet of the channel.
APPENDIX A4

Evaluation of u (the x-component velocity) of an element.

It was assumed in the formulation of the problem (Appendix A2) that the stream function distribution over an element is given by
\[ \psi = c_1 + c_2 x + c_3 y + c_4 x^2 + c_5 y^2 + c_6 xy \] (1)
where the c's are constants expressible in terms of nodal point variables. But
\[ u = -\frac{\partial \psi}{\partial y} \] (2)
i.e.
\[ u = -(c_3 + 2c_5 y + c_6 x) \] (3)
All that is required here is to determine \( c_3 \), \( c_5 \) and \( c_6 \) for an element. Now
\[ \phi = u + v \] (4)
i.e.
\[ c_2 - c_3 + (2c_4 - c_6)x + (c_6 - 2c_5)y \] (5)
It is assumed that \( \psi_i, \phi_i, \psi_j, \phi_j, \psi_k \) and \( \phi_k \) are known at the three vertices of the element.
\( c_6 \) has already been determined in Appendix A2. So also is \( (c_4 - c_5) \) in the same appendix.
By the use of equations (1) and (5) and the nodal values, \( c_3 \) and \( c_4 \) can be found.
Thus enabling \( u \) [equation (3)] to be obtained.
Fig. 19: Channel boundary division for the forward difference formula.

Fig. 20: Channel boundary division for the central difference formula.

Fig. 21: Channel boundary division for the backward difference formula.
APPENDIX A5

Pressure drop (Δp) across the channel.

From the basic equations of flow we have

\[ \frac{\partial p}{\partial x} = \frac{\partial \tau_{xx}}{\partial x} + \frac{\partial \tau_{xy}}{\partial y} \]  

The integral of the above gives the required pressure drop.

i.e. \[ \int \frac{\partial p}{\partial x} \, dx = \int_{0}^{L} \frac{\partial}{\partial x} \left( 2\mu \frac{\partial u}{\partial x} \right) \, dx + \frac{\partial u}{\partial y} \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \, dx \]  

where \( \tau_{xx} = 2\mu \frac{\partial u}{\partial x} \) and \( \tau_{xy} = \mu \left( \frac{\partial u}{\partial y} + \frac{\partial v}{\partial x} \right) \).  

The \( \mu \) here refers to the node at which the integral is evaluated.

Now the partial derivatives can be approximated by finite differences.

The development of Taylor's series for \( f(x + \Delta x) \) about \( (x,y) \) gives

\[ f(x+\Delta x, y) = f(x, y) + \Delta x \frac{\partial f}{\partial x} (x, y) + \frac{(\Delta x)^2 \partial^2 f}{2!} \frac{\partial^2 f}{\partial x^2} (x, y) \].  

With this, the finite difference formulae are derived.

**Forward Difference Formulae**

Consider an interval as shown in Fig. 19.

\[ f_{n+1} = f_n + a f'_n + \frac{a^2}{2} f''_n \]  

\[ f_{n+2} = f_n + b f'_n + \frac{b^2}{2} f''_n \]
Fig. 22: Flow region showing the nodal points along the bottom boundary.
Eliminating $f''_n$ from (5) and (6) we obtain

$$f'_n = \frac{b^2 f_{n+1} - a^2 f_{n+2} - (b^2 - a^2) f_n}{ab(b-a)}$$

and hence

$$f''_n = \frac{2[b f_{n+1} - a f_{n+2} - (b-a) f_n]}{ab(a-b)}$$

Central Difference Formula

With reference to Fig. 20, we obtain similar expression for the central difference formula as

$$f'_n = \frac{a^2 f_{n+1} - (b-a)^2 f_{n-1} - (a^2 - (b-a)^2) f_n}{ab(b-a)}$$

and

$$f''_n = \frac{2[a f_{n+1} + (b-a) f_{n-1} - b f_n]}{ab(b-a)}$$

Backward Difference Formula

The backward difference formula with reference to Fig. 21 is

$$f'_n = \frac{a^2 f_{n-2} - b^2 f_{n-1} + (b^2 - a^2) f_n}{ab(b-a)}$$

and

$$f''_n = \frac{2[a f_{n-2} - b f_{n-1} - (a-b) f_n]}{ab(b-a)} .$$

For a channel as shown in Fig. 22, the forward and backward difference formulae were applied at nodes 1 and 9 respectively. For the intermediate points, the central difference formula was used.
APPENDIX A6

Theoretical velocities and viscosities

The second invariant of the rate of deformation tensor is given by:

$$4I_2 = 2\frac{\partial u}{\partial x}^2 + 2\frac{\partial v}{\partial y}^2 + \left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right)^2 \quad (1)$$

For fully developed flow,

$$\frac{\partial u}{\partial x} = 0, \quad \frac{\partial v}{\partial y} = 0, \quad \frac{\partial u}{\partial y} = 0 \quad \text{and} \quad \frac{\partial v}{\partial x} = 0. \quad (2)$$

$$\therefore 4I_2 = \frac{\partial u}{\partial y}^2. \quad (3)$$

Now

$$\mu = (\sqrt{4I_2})^{n-1} \quad (4)$$

i.e.

$$\mu = \left(\frac{\partial u}{\partial y}\right)^{n-1}. \quad (5)$$

Also

$$\frac{\partial p}{\partial x} = \frac{\tau_{xy}}{\partial y}. \quad (6)$$

But

$$\tau_{xy} = \mu \frac{\partial u}{\partial y}. \quad (7)$$

$$\therefore \frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left(\mu \frac{\partial u}{\partial y}\right). \quad (8)$$

i.e.

$$\frac{\partial p}{\partial x} = \frac{\partial}{\partial y} \left(\frac{\partial u}{\partial y}\right)^{n-1} \frac{\partial u}{\partial y} \quad (9)$$

Integrating (9) we obtain
\[ \left( \frac{du}{dy} \right)^n = \frac{dp}{dx} y + A. \]  
(10)

For non-Newtonian flow, \( n = 0.5 \), we have

\[ \frac{du}{dy} = \left( \frac{dp}{dx} \right)^2 y^2 \]  
(11)

Integrating (11) we have

\[ u = \left( \frac{dp}{dx} \right)^2 \left( \frac{y^3}{3} + B \right) \]  
(12)

at \( y = -\frac{1}{2} \), \( u = 0 \)

\[ \therefore B = \frac{1}{24} \left( \frac{dp}{dx} \right)^2. \]

\[ \therefore u = \left( \frac{dp}{dx} \right)^2 \left( \frac{y^3}{3} + \frac{1}{24} \right). \]

Let \( \frac{dp}{dx} = \pi_p \) (the dimensionless pressure drop).

\[ \therefore u = \frac{\pi_p^2}{3} (y^3 + \frac{1}{8}). \]  
(14)

The dimensionless flowrate \( \pi_Q \) is given by

\[ \pi_Q = \int u \, dy \]  
(15)

i.e.

\[ 2 \int_{0}^{-\frac{1}{2}} \frac{\pi_p^2}{3} (y^3 + \frac{1}{8}) \, dy \]  
(16)

i.e.

\[ \pi_Q = -\frac{\pi_p^2}{32}. \]

But \( \pi_Q = -1 \)

\[ \therefore \pi_p^2 = 32 \]

\[ \therefore \pi_p = \sqrt{32} \]

\[ = 4\sqrt{2}. \]

From (14), we have

\[ u = \frac{32}{3} \left( y^3 + \frac{1}{8} \right). \]
As a check, at $y = 0$, $u = 4/3$ and at $y = -\frac{1}{3}$, $u = 0$.

Now the dimensionless viscosity $\mu$ as from (4) is given by

$$\mu = (\sqrt{4I_2})^{n-1}$$

i.e.

$$\left[ \frac{\partial u}{\partial y} \right]^{n-1}$$

Substitution for $u = \frac{32}{3} (y^3 + \frac{1}{8})$ and $n = 0.5$ gives

$$u = \frac{1}{4\sqrt{2}y}$$

at $y = \frac{1}{2}$ : $\mu = 0.354$.

Similar procedure is adopted for Newtonian flow.

With $n = 1$ in equation (10), we obtain

$$u = 6[y^2 - \frac{1}{4}]$$

and $\frac{\pi}{p} = 12$.
APPENDIX B1


Consider an element as shown in Fig. 11. Let the velocity variation within this element be linear, as shown below.

$$u = \alpha_1 + \alpha_2 x + \alpha_3 y$$  \hspace{1cm} (1)

where $\alpha_1$, $\alpha_2$ and $\alpha_3$ are constants to be determined.

Applying the known nodal values we obtain

$$u_i = \alpha_1 + \alpha_2 x_i + \alpha_3 y_i$$  \hspace{1cm} (2)

$$u_j = \alpha_1 + \alpha_2 x_j + \alpha_3 y_j$$  \hspace{1cm} (3)

$$u_k = \alpha_1 + \alpha_2 x_k + \alpha_3 y_k$$  \hspace{1cm} (4)

Elimination gives

$$\alpha_2 = \frac{1}{2\Delta} \begin{bmatrix} y_j - y_k, & y_k - y_i, & y_i - y_j \end{bmatrix} \begin{bmatrix} u_i \\ u_j \\ u_k \end{bmatrix}$$  \hspace{1cm} (5)

where $\Delta$ is the area of an element. $2\Delta$ is given by:

$$2\Delta = \begin{vmatrix} 1 & x_i & y_i \\ 1 & x_j & y_j \\ 1 & x_k & y_k \end{vmatrix}$$  \hspace{1cm} (6)

Further elimination gives $\alpha_1$ and $\alpha_3$ as

$$\alpha_1 = \frac{1}{2\Delta} \begin{bmatrix} (y_j y_k - x_k y_j), & (y_k y_i - x_i y_k), & (y_i y_j - x_j y_i) \end{bmatrix} \begin{bmatrix} u_i \\ u_j \\ u_k \end{bmatrix}$$  \hspace{1cm} (7)
and

\[ \alpha_3 = \frac{1}{2\Delta} \left[ x_k - x_j, x_i - x_k, x_j - x_i \right] \begin{bmatrix} u_i \\ u_j \\ u_k \end{bmatrix} \]  

These can be written as

\[ \alpha_1 = \frac{1}{2\Delta} \left[ a_i, a_j, a_k \right] \begin{bmatrix} u_i \\ u_j \\ u_k \end{bmatrix} \]  

\[ \alpha_2 = \frac{1}{2\Delta} \left[ b_i, b_j, b_k \right] \begin{bmatrix} u_i \\ u_j \\ u_k \end{bmatrix} \]  

and

\[ \alpha_3 = \frac{1}{2\Delta} \left[ c_i, c_j, c_k \right] \begin{bmatrix} u_i \\ u_j \\ u_k \end{bmatrix} \]

where

\[ a_i = x_j y_k - x_k y_i \]
\[ a_j = x_k y_i - x_i y_k \]
\[ a_k = x_i y_j - x_j y_i \]
\[ b_i = y_j - y_k \]
\[ b_j = y_k - y_i \]
\[ b_k = y_i - y_j \]
\[ c_i = x_k - x_j \]
\[ c_j = x_i - x_k \]
\[ c_k = x_j - x_i \]
Substitution of (9), (10) and (11) into (1) yields

\[ u = \frac{1}{2A} \left( a_1 + b_1 x + c_1 y \right), \left( a_j + b_j x + c_j y \right), \left( a_k + b_k x + c_k y \right) \begin{bmatrix} u_i \\ u_j \\ u_k \end{bmatrix} \]

i.e.

\[ u = N_i u_i + N_j u_j + N_k u_k \]

where the \( N \)'s are the shape functions given by

\[ N_r = \frac{1}{2A} \left[ a_r + b_r x + c_r y \right] \]

Similarly,

\[ v = N_i v_i + N_j v_j + N_k v_k \]

and

\[ p = N_i p_i + N_j p_j + N_k p_k \]

Now

\[ 4I_2 = 2 \frac{\partial u}{\partial x}^2 + 2 \frac{\partial v}{\partial y}^2 + \frac{\partial u}{\partial x} \frac{\partial v}{\partial y} + \frac{\partial u}{\partial y} \frac{\partial v}{\partial x} + \frac{\partial u}{\partial y} \frac{\partial v}{\partial y} \]

\[ \frac{\partial u}{\partial x} = \frac{1}{2A} \left[ b_1 u_i + b_j u_j + b_k u_k \right] \]

\[ \frac{\partial v}{\partial y} = \frac{1}{2A} \left[ c_1 v_i + c_j v_j + c_k v_k \right] \]

\[ \frac{\partial u}{\partial y} = \frac{1}{2A} \left[ c_1 u_i + c_j u_j + c_k u_k \right] \]

\[ \frac{\partial v}{\partial x} = \frac{1}{2A} \left[ b_1 v_i + b_j v_j + b_k v_k \right] \]

i.e.

\[ 4I_2 = (u_i, u_j, u_k) \frac{1}{2A^2} [B] \begin{bmatrix} u_i \\ u_j \\ u_k \end{bmatrix} + (v_i, v_j, v_k) \frac{1}{2A^2} [C] \begin{bmatrix} v_i \\ v_j \\ v_k \end{bmatrix} \]

\[ + (u_i, u_j, u_k) \frac{1}{4A^2} [D] \begin{bmatrix} v_i \\ v_j \\ v_k \end{bmatrix} + (u_i, u_j, u_k) \frac{1}{2A^2} [C] \begin{bmatrix} u_i \\ u_j \\ u_k \end{bmatrix} \]
where

\[
[B] = \begin{bmatrix}
    b_i^2 & b_i b_j & b_i b_k \\
    b_j b_i & b_j^2 & b_j b_k \\
    b_k b_i & b_k b_j & b_k^2 
\end{bmatrix}
\]

and

\[
[C] = \begin{bmatrix}
    c_i^2 & c_i c_j & c_i c_k \\
    c_j c_i & c_j^2 & c_j c_k \\
    c_k c_i & c_k c_j & c_k^2 
\end{bmatrix}
\]

i.e.

\[4I_2 = (u_i, u_j, u_k, v_i, v_j, v_k) \times \]

\[\frac{1}{4\Delta^2} \begin{bmatrix}
    2b_i^2 + c_i^2, 2b_i b_j + c_i c_j, 2b_i b_k + c_i c_k, c_i b_i + c_i c_j, c_i b_j + c_i c_k, c_i b_k \\
    2b_j^2 + c_j^2, 2b_j b_i + c_j c_i, 2b_j b_k + c_j c_k, c_j b_i + c_j c_i, c_j b_j + c_j c_k, c_j b_k \\
    2b_k^2 + c_k^2, 2b_k b_i + c_k c_i, 2b_k b_j + c_k c_j, 2b_k b_k + c_k c_k, c_k b_i + c_k c_i, c_k b_j + c_k c_j, c_k b_k \\
    c_i b_i + c_i c_i, c_i b_j + c_i c_j, c_i b_k + c_i c_k, 2c_i^2 + b_i^2, 2c_i b_j, 2c_i b_k, 2c_i c_i, 2c_i c_j, 2c_i c_k \\
    c_j b_i + c_j c_i, c_j b_j + c_j c_j, c_j b_k + c_j c_k, 2c_j^2 + b_j^2, 2c_j b_i, 2c_j b_k, 2c_j c_i, 2c_j c_j, 2c_j c_k \\
    c_k b_i + c_k c_i, c_k b_j + c_k c_j, c_k b_k + c_k c_k, 2c_k^2 + b_k^2, 2c_k b_i, 2c_k b_j, 2c_k b_k, 2c_k c_i, 2c_k c_j, 2c_k c_k 
\end{bmatrix} \begin{bmatrix}
    u_i \\
    u_j \\
    u_k \\
    v_i \\
    v_j \\
    v_k 
\end{bmatrix} \]

(23)

Now

\[\chi = \int_A (4u I_2 + 2u \cdot \nabla p) \, dA \]
Let
\[ x_1 = \int_A 4\mu I_2 \, dA \]  
and
\[ x_2 = \int_A 2u \cdot \nabla p \, dA \]  
\[ x_1 = \int_A 4\mu I_2 \, dA \]
\[ \therefore x_1 = 4I_2 \mu \Delta \]  
(27)

\(4I_2\) has already been obtained as in equation (23).

\[ x_2 = \int_A 2u \cdot \nabla p \, dA \]
\[ u \cdot \nabla p = u_{\partial x} + v_{\partial y} \]  
(28)

Using equations (13), (15) and (16) we obtain

\[ x_2 = (u_i, u_j, u_k) \frac{1}{2\Delta} [AB] \begin{bmatrix} p_i \\ p_j \\ p_k \end{bmatrix} + (v_i, v_j, v_k) \frac{1}{2\Delta} [AC] \begin{bmatrix} p_i \\ p_j \\ p_k \end{bmatrix} \]  
(29)

where

\[ [AB] = \begin{bmatrix} a_i b_i, & a_i b_j, & a_i b_k \\ a_j b_i, & a_j b_j, & a_j b_k \\ a_k b_i, & a_k b_j, & a_k b_k \end{bmatrix} \]

and

\[ [AC] = \begin{bmatrix} a_i c_i, & a_i c_j, & a_i c_k \\ a_j c_i, & a_j c_j, & a_j c_k \\ a_k c_i, & a_k c_j, & a_k c_k \end{bmatrix} \]

For an element, the partial differential of \( \chi \) with respect to the nodal variables \((u_i, v_i, p_i), (u_j, v_j, p_j)\) and \((u_k, v_k, p_k)\) is given by
\[
\begin{bmatrix}
\frac{\partial x^e}{\partial u_i} \\
\frac{\partial x^e}{\partial v_i} \\
\vdots \\
\frac{\partial x^e}{\partial p_k}
\end{bmatrix}
= [K_e]
\begin{bmatrix}
u_i \\
v_i \\
\vdots \\
p_k
\end{bmatrix}
\]

where \([K_e]\) equals.
\[
\begin{bmatrix}
2b_i^2 + c_i^2, & 2b_i b_j + c_i c_j, & 2b_i b_k + c_i c_k, & c_i b_i, & c_i b_j, & c_i b_k, & \frac{2a_i b_j}{\mu}, & \frac{2a_i b_j}{\mu}, & \frac{2a_i b_k}{\mu} \\
2b_j^2 + c_j^2, & 2b_j b_i + c_j b_i, & 2b_j b_k + c_j c_k, & c_j b_i, & c_j b_j, & c_j b_k, & \frac{2a_j b_i}{\mu}, & \frac{2a_j b_i}{\mu}, & \frac{2a_j b_k}{\mu} \\
2b_k^2 + c_k^2, & 2b_k b_i + c_k b_i, & 2b_k b_j + c_k c_j, & c_k b_i, & c_k b_j, & c_k b_k, & \frac{2a_k b_i}{\mu}, & \frac{2a_k b_i}{\mu}, & \frac{2a_k b_k}{\mu} \\
\end{bmatrix}
\]

\[
\frac{\mu}{4\Delta} 	imes
\]

(30)

\[
\begin{bmatrix}
\frac{2a_i b_j}{\mu}, & \frac{2a_i b_j}{\mu}, & \frac{2a_i b_k}{\mu}, & \frac{2a_i c_j}{\mu}, & \frac{2a_i c_j}{\mu}, & \frac{2a_i c_k}{\mu}, & 0, & 0, & 0 \\
\frac{2a_j b_i}{\mu}, & \frac{2a_j b_i}{\mu}, & \frac{2a_j b_k}{\mu}, & \frac{2a_j c_i}{\mu}, & \frac{2a_j c_i}{\mu}, & \frac{2a_j c_k}{\mu}, & 0, & 0, & 0 \\
\frac{2a_k b_i}{\mu}, & \frac{2a_k b_i}{\mu}, & \frac{2a_k b_k}{\mu}, & \frac{2a_k c_i}{\mu}, & \frac{2a_k c_i}{\mu}, & \frac{2a_k c_k}{\mu}, & 0, & 0, & 0 \\
\end{bmatrix}
\]
APPENDIX C

Part of the initial computer programme
NOTATIONS

AJ, BJ, AK, NK = LOCAL COORDINATES OF AN ELEMENT WITH RESPECT TO THE 11 NODE. AN ELEMENT IS LABELED 11, 12, 13 IN AN ANTICLOCKWISE FASHION.

C(NDF, 1) = STORES THE PRODUCT OF K2(NDF, 1) AND DELT2(1).

DELTA(NK) = ARRAY OF KNOWN AND UNKNOWN NODAL VARIABLES.

DELTA1(NDF) = REQUIRED NODAL VARIABLES.

DELTA2(1) = KNOWN NODAL VALUES (PART OF).

K(NK, NK) = SYSTEM STIFFNESS MATRIX.

KE(6, 6) = ELEMENTAL STIFFNESS MATRIX.

K1(NDF, NDF) = A SUB-MATRIX OF THE SYSTEM STIFFNESS MATRIX.

K2(NDF, N) = A SUB-MATRIX OF THE SYSTEM STIFFNESS MATRIX TO BE MULTIPLIED BY DELT2(1).

K3(NDF, N) = THE UPPER TRIANGULAR ELEMENTS OF K1(NDF, N) ARE STORED IN THIS RECTANGULAR MATRIX. THIS FORMS THE RESULTING STIFFNESS MATRIX TO BE USED IN FINDING THE NODAL VARIABLES.

NDelta(6) = ARRAY CONTAINING THE D.O.F. NUMBERS OF A NODE.
SUBROUTINE CALCK(KE, ELEHER, AIODE, I, HE, NK, 
THI, V, H) 

******************************************************************************
CALCULATES THE ELEMENTAL STIFFNESS MATRIX
******************************************************************************

REAL KE(6, 6) 
DIMENSION ELEHER(HE, 6), AIODE(MN, 4)

ELEMENTAL GEOMETRIC CONSTANTS REQUIRED.

CALL GAMMET(G1, G2, G3, G4, G5, G6, B1, B2, 
THI, AI, DJ, AK, BK, JAY, KAY, F, 
TELHER, AIODE, I, FAMII, FAM, HE, NK, THI, V, H) 
F1=FAMII*G1 
F2=FAMII*G2 
F3=FAMII*G3

CALCULATE KE NON.

KE(1, 1)=F1*G1 
KE(1, 2)=F1*G4 
KE(1, 3)=F1*G2 
KE(1, 4)=F1*G5 
KE(1, 5)=F1*G3 
KE(1, 6)=F1*G6 
KE(2, 2)=FAMII*((B1**2)*(G1**2)) 
KE(2, 3)=F2*G4 
KE(2, 4)=FAMII*((B1*B2+G3*G5)) 
KE(2, 5)=F3*G4 
KE(2, 6)=FAMII*((B1*B3+G3*G6)) 
KE(3, 3)=F2*G4 
KE(3, 4)=F2*G5 
KE(3, 5)=F2*G3 
KE(3, 6)=F2*G6 
KE(4, 4)=FAMII*((B2**2)*(G5**2)) 
KE(4, 5)=F3*G5 
KE(4, 6)=FAMII*((B2*B3+G5*G6)) 
KE(5, 5)=F3*G3 
KE(5, 6)=F3*G6 
KE(6, 6)=FAMII*((B3**2)*(G6**2)) 
00 26 12=I, J 
00 26 J=I*12+6 
KE(I, J)=KE(J, I) 
26 CONTINUE 
RETURN
END
SUBROUTINE INSERT(K,ELEMT,ANODE,I,
THE,HK,HM,HI)

******************************************************************************
THIS INSERTS THE ELEMENTAL STIFFNESS
MATRIX INTO THE SYSTEM STIFFNESS MATRIX.
******************************************************************************
REAL K(HK,HK),KE(6,6)
DIMENSION ELEMT(NE,4),ANODE(NI,4),
INDelta(6)

NOGAL NUMBERS.
I1=ELEMT(I,1)
I2=ELEMT(I,2)
I3=ELEMT(I,3)

D.O.F. NUMBERS.

NDELT(1)=ANODE(I1,3)
NDELT(2)=ANODE(I1,4)
NDELT(3)=ANODE(I2,3)
NDELT(4)=ANODE(I2,4)
NDELT(5)=ANODE(I3,3)
NDELT(6)=ANODE(I3,4)

OBTAIN LOCAL COORDINATES WITH
RESPECT TO THE I1 NODE OF EACH ELEMENT (AS ORIGIN)
AJ=ANODE(I2,1)-ANODE(I1,1)
BJ=ANODE(I2,2)-ANODE(I1,2)
AK=ANODE(I3,1)-ANODE(I1,1)
BK=ANODE(I3,2)-ANODE(I1,2)

KE CAN NOW BE CALCULATED.

CALL CALCK(KE,ELEMT,ANODE,I,NE,HK,HM,
TV,J)
DO 25 I1=1,6
DO 25 J=1,6

INSERT KE INTO K NOW.

K(INDELTA(I1),INDELTA(J))=K(INDELTA(I1),INDELTA(J))+KE(I1,J)
25 CONTINUE
RETURN
END
SUBROUTINE SUM(K, ELEM, ANode, HE, HK, THN, V, H)
C
C **************************************************************
C THE SYSTEM STIFFNESS MATRIX IS ASSEMBLED
C **************************************************************
C
REAL K(HK, HK)
DIMENSION ELEM(HE, 4), ANode(HN, 4)
DO 23 I = 1, HK
DO 23 J = 1, HK
23 K(I, J) = 0.0
DO 24 IC = 1, HE
C
C ELEMENTAL STIFFNESS MATRIX IS REQUIRED TO BE INSERTED INTO THE SYSTEM STIFFNESS MATRIX.
C
CALL INSERT(K, ELEM, ANode, IC, HE, HK, HN, V, H)
24 CONTINUE
RETURN
END
SUBROUTINE ISOLAT(K1,K2,DELT1,DELT2,K;
TOELTA,HDF,H,UK)

**********************************************************************
THIS ISOLATES K1,K2,DELT1,DELT2, FROM
SYSTEM MATRIX EQUATION
**********************************************************************

REAL K(HK,HK),K1(HDF,HDF),
TK2(HDF,H)
DIMENSION DELTA(HK),DELT1(HDF),DELT2(H)

K1 ISOLATED.
DO 28 I=1,HDF
DO 28 J=1,HDF
K1(I,J)=K(I,J)
28 CONTINUE

K2 ISOLATED.
DO 29 I=1,HDF
DO 29 J=HDF+1,HDF+H
K2(I,J-HDF)=K(I,J)
29 CONTINUE

DELT1 ISOLATED.
DO 30 I=1,HDF
DELT1(I)=DELT(A(I)
30 CONTINUE

DELT2 ISOLATED.
DO 31 I=1,H
DELT2(I)=DELT(A(I+HDF)
31 CONTINUE
RETURN
SUBROUTINE KAY3(K3, K1, NDF, IBW, K, DELTA, 
TK2, DELT1, DELT2, NE, NK, NN, N)

***********************************************************************
SUBROUTINE TO TAKE THE UPPER TRIANGULAR ELEMENTS OF K1 AND PUT THEM INTO K3.
***********************************************************************

REAL K(NK,NK), K1(NDF,NDF),
TK2(NDF,N), K3(NDF,IBW)
DIMENSION DELTA(NK), DELT1(NDF),
TDELT2(N)
DO 27 I=1,NDF
DO 27 J=1,IBW
K3(I,J)=0.0
27 CONTINUE

K1 REQUIRED.

CALL ISOLAT(K1,K2,DELT1,DELT2,K,DELTA,
TNDF,N,NK)
DO 127 J=1,IBW
DO 127 I=1,NDF+1-J

K1 INTO K3.

K3(I,J)=K1(I,I+J-1)
127 CONTINUE
RETURN
END
SUBROUTINE PRODUCT(C,K2,DELT2,NDF,N,N)

******************************************************************************

THIS MULTIPLIES TWO MATRICES K2 AND DELT2 AND STORES THE RESULT IN C.
******************************************************************************

REAL K2(NDF,N)
DIMENSION C(NDF,N),DELT2(N,N)
DO 32 I=1,NDF
DO 32 J=1,N
Y=0.0

OBTAIN THE PRODUCT OF THE TWO MATRICES.

DC

DO 33 K=1,N
33 Y=Y+K2(I,K)*DELT2(K,J)
32 C(I,J)=Y
RETURN
END
APPENDIX D

Estimation of costs involved in the research work

The various costs encountered during the course of computation are as detailed below.

<table>
<thead>
<tr>
<th>Cost Description</th>
<th>Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>B6718 CPU Time</td>
<td>$210.58</td>
</tr>
<tr>
<td>I/O Processor Time</td>
<td>67.90</td>
</tr>
<tr>
<td>Memory Integral</td>
<td>111.82</td>
</tr>
<tr>
<td>Slow I/O Operations</td>
<td>170.65</td>
</tr>
<tr>
<td>File Open Time</td>
<td>18.28</td>
</tr>
<tr>
<td><strong>Total</strong></td>
<td><strong>$578.23</strong></td>
</tr>
</tbody>
</table>
APPENDIX P1

Resulting computer programme from the first analysis.
*****
* A *
*****

$A_I, A_J, A_K, A_L =$ LOCAL COORDINATES OF AN ELEMENT WITH RESPECT TO THE $I$ NODE, AN ELEMENT IS LABELED $I, J, K, L$ IN AN ANTICLOCKWISE FASHION.

$\alpha =$ ANGLE OF TAPER OF CHANNEL.

$A = (i, j, k, l) =$ ARRAY CONTAINING INFORMATION ABOUT ALL THE NODES, THE FIRST TWO POSITIONS FOR THE $X$ AND $Y$ COORDINATES (GLOBAL) AND THE THIRD AND FOURTH FOR THE D.O.F. NOS.

$\text{AREA} =$ AREA OF AN ELEMENT

*****
* B *
*****

$\beta =$ BETAS (GEOMETRIC CONSTANTS OF EACH ELEMENT)

*****
* C *
*****

$C = (\text{NDF}) =$ ARRAY IN WHICH PREVIOUS SOLUTION FROM DELTA IS STORED

$C = (\text{NDF}) =$ ARRAY IN WHICH SOLUTION FOR THE Unknowns IS STORED, LATER TRANSFERED INTO DELTA

$C_{45} = C_4 - C_5$

*****
* D *
*****

$\delta =$ Unknown Nodal Values of an Element.

$\text{DELTA} = (\text{NDF}) =$ ARRAY OF REQUIRED NODAL VALUES. THESE ARE STREAM FUNCTION($\psi$) AND SUM OF VELOCITIES($\phi$)

$\text{DH} =$ DIVISOR TO DIMENSIONALISE ALL COORDINATES

$\text{DIVO} =$ VALUE OF $I_2$

$\text{DL} =$ DIE LENGTH

$\text{DMUX} =$ (ARRAY) DERIVATIVE OF $\mu$ W.R.T. $X$

$\text{DMUY} =$ (ARRAY) DERIVATIVE OF $\mu$ W.R.T. $Y$

$\text{DP} =$ DIFFERENCE IN PRESSURE BETWEEN INLET AND OUTLET
DUX( )=(ARRAY) DERIVATIVE OF (U) W.R.T.X
DUX( )=(ARRAY) DERIVATIVE OF (U) W.R.T.Y
DVX( )=(ARRAY) DERIVATIVE OF (V) W.R.T.X
DVY( )=(ARRAY) DERIVATIVE OF (V) W.R.T.Y
DXDUX( )=(ARRAY) SECOND DERIVATIVE OF (U) W.R.T.X
DXDVY( )=(ARRAY) DERIVATIVE OF (DVY) W.R.T.X
DYDUX( )=(ARRAY) SECOND DERIVATIVE OF (U) W.R.T.Y

*****
* E *
*****
ELEMT(1E,4)=ARRAY CONTAINING INFORMATION
ABOUT ALL THE ELEMENTS. THE FIRST THREE
POSITIONS FOR THE NODAL NOS. AND THE
FOURTH FOR THE ELEMENT VISCOSITY.

*****
* F *
*****
FBI=FLOW BEHAVIOUR INDEX OF THE POLYMER
FAR=FOUR TIMES AN ELEMENTAL AREA

*****
* G *
*****
G'S=GAHNAS(GEOMETRIC CONSTANTS OF EACH ELEMENT)

*****
* H *
*****
H=HEIGHT OF CHANNEL AT INLET

*****
* I *
*****
I'S=COUNTER; DEGREE OF FREEDOM NUMBERS; NODE NUMBERS
IBW= Bandwidth

*****
* J *
*****
J'S=COUNTER, DEGREE OF FREEDOM NUMBERS; NODE NUMBERS.

*****
* K *
*****
KE(6,6)=ELEMENTAL STIFFNESS MATRIX.
K(NDW,IBW)=SYSTEM STIFFNESS MATRIX IN BANDED AND RECTANGULAR FORM
MU = ELEMENT VISCOSITY.

NUNODE(NNI) = ARRAY CONTAINING THE VISCOSITY OF EACH NODE.

N = NON-ZERO NO OF D.O.F.

NDELT A(6) = ARRAY CONTAINING THE D.O.F. NO. OF AN ELEMENT

NDF = UNKNOWN NO. OF D.O.F.

NE = NO OF ELEMENTS

IIK = INITIAL NO. OF D.O.F (NO. OF NODES * 2)

NNI = NO OF NODES

Q1, Q2, Q3 = FLOW RATE AT INLET, MIDWAY AND OUTLET OF THE DIE.

T = TANGENT OF AN ANGLE

U(NI,3) = X-VELOCITY ARRAY OF ALL THE ELEMENTS. THE FIRST ENTRY REFERS TO THE 11 POSITION OF THE ELEMENT, THE SECOND TO THE 12 POSITION AND THE THIRD TO THE 13 POSITION.

UNODE(NNI) = X-VELOCITY ARRAY OF ALL THE NODES


VNODE(NNI) = Y-VELOCITY ARRAY OF ALL THE NODES

W = WIDTH OF DIE
SUBROUTINE INPUT(NK,NE,NN,NDVF,N,IBV,DL,H,W, 
TELEMET,ANODE,DELTA,DH,ALPHA)

******************************************************************************
THIS INPUTS ALL THE AVAILABLE DATA
******************************************************************************

DIMENSION ELEMET(NE,4),ANODE(NN,4),DELTA(NK)

DATA ABOUT ALL THE ELEMENTS. THE FIRST THREE POSITIONS IS FOR THE
NOdal NUMBERS AND THE FOURTH FOR THE VISCOSITY.

WRITE(6,8)

8 FORMAT(31X,50H ELEMENT INFORMATION:**********ELEMENT INFORMATION/, 
T31X,10H ELEMENT,10H NODE NO.,10H NODE NO.,10H V 
TISCOSITY)
DO 9 I=1,NE
READ(5,/) I!,I2,I3
ELEMET(I,4)=0.0
WRITE(6,10) I!,I1,I2,I3,ELEMET(I,4)
10 FORMAT(31X,15,4X,15,5X,15,5X,15,5X,F10.5)
ELEMET(I,1)=I1+0.1
ELEMET(I,2)=I2+0.1
ELEMET(I,3)=I3+0.1
9 CONTINUE

INFORMATION ABOUT ALL THE NODES IS READ IN

WRITE(6,11)

11 FORMAT(110,30X,' NODAL INFORMATION:**********NODAL INFORMATION',/ 
T31X,' NODE NO.',' X COORD ',' Y COORD ',' DOF NO.',' 
TIX,' DOF NO.','/)
DO 12 I=1,NN
READ(5,/) ANODE(I,1),ANODE(I,2),J1,J2
ANODE(I,1)=ANODE(I,1)/DH
ANODE(I,2)=ANODE(I,2)/DH
WRITE(6,13) I!,ANODE(I,1),ANODE(I,2),J1,J2
13 FORMAT(31X,15,4X,15,5X,2F10.5,15,5X,15)
ANODE(I,3)=J1+0.1  
ANODE(I,4)=J2+0.1
12 CONTINUE

READ IN INITIAL NODAL VALUES

WRITE(6,14)

14 FORMAT(110,30X,' BOUNDARY CONDITIONS',/ 
T31X,' **********BOUNDARY CONDITIONS',/ 
T31X,10H DOF NO.,10H PSII ,/ 
TIX,10H DOF NO.,10H PSII ,/)
DO 15 I=NDVF+1,NK
READ(5,/) DELTA(I)
15 CONTINUE
DO 16 I=NDVF+1,NK,2
WRITE(6,17) I!,DELTA(I),I+1,DELTA(I+1)
17 FORMAT(31X,15,4X,F9.6,2X,15,4X,F8.6)
16 CONTINUE
RETURN
END
SUBROUTINE COORD(I, ANODE, N1, DL, ALPHA)
C
THE GLOBAL COORDINATES GIVEN IN
C THE INPUT SUBROUTINE ARE FOR A
C UNIFORM RECTANGULAR MESH, MODIFIED
C BELOW TO ACCOUNT FOR THE TAPERING
C OF THE DIE. THE X COORDINATES
C REMAIN THE SAME.
C
DIMENSION ANODE(N1,4)
T=TAN(ALPHA)
R=1/DL
ANODE(I,2)=ANODE(I,2)*(1.0-ANODE(I,1)*R)
J1=ANODE(I,3)
J2=ANODE(I,4)
RETURN
END
SUBROUTINE UPDATE(ELEMENT, ANODE, DELTA, I, TIE, NK, NI, ITER)

*********************************
C THIS SUBROUTINE UPDATES THE VALUES
C OF VISCOSITY FOR EACH ELEMENT
*********************************

DIMENSION ELEMENT(NI,4), ANODE(NN,4),
TDELTA(NK)
REAL MU

NODAL NUMBERS

11=ELEMENT(1,1)
12=ELEMENT(1,2)
13=ELEMENT(1,3)

D.O.F. NUMBERS

J1=ANODE(11,3)
J2=ANODE(11,4)
J3=ANODE(12,3)
J4=ANODE(12,4)
J5=ANODE(13,3)
J6=ANODE(13,4)

VALUES OF THE UNKNOWNS

D1=DELTA(J1)
D2=DELTA(J2)
D3=DELTA(J3)
D4=DELTA(J4)
D5=DELTA(J5)
D6=DELTA(J6)

VALUES OF ELEMENTAL CONSTANTS REQUIRED

CALL GAMMEL(G1,G2,G3,G4,G5,G6,B1,B2,B3,
TAREA, AJ, OJ, AK, BK, JAY, KAY, F, ELEMENT, ANODE,
TI, FARNU, FAR, NI, NK, NN, NI, V, H)

CALCULATE 12(ITWO) NOW

C6=G1*D1+G2*D3+G3*D5+G4*D2+G5*D4+G6*D6
C45=B1*D2+B2*D4+B3*D6
DITWO=(C6+C6)+(C45+C45)
R=SQRT(DITWO)

CALCULATE THE VISCOSITY (MU) IF THE FLOW
BEHAVIOUR INDEX(FBI) OF THE POLYMER IS KNOWN

FBI=1.0
MU=(2.**(FBI-1.0))**(R***(FBI-1.0))
ELEMENT(1,4)=MU
RETURN
END
SUBROUTINE ASSEMB(K, KE, ELEM, ANODE, DELTA, C, NE, NK, TNN, W, H, HOF, IBM)

C ***************************************************************
C THIS CALCULATES THE STIFFNESS MATRIX
C OF EACH ELEMENT AND INSERTS IT INTO
C THE SYSTEM STIFFNESS MATRIX. THE MATRIX
C IS BANDED AND THE LOADING VECTOR OBTAINED
C IN THE PROCESS
C ***************************************************************

DIMENSION ELEM(NE,4), ANODE(NH,4), DELTA(NK), TC(HOF,1), DELTA(6)
REAL K(NDF,1,IBM), KE(G,6)
DO 70 J=1, NDF
DO 70 J=1, IBM
K(1,J)=0.0
C(1,1)=0.0
70 CONTINUE
DO 50 I=1, NE
DO 71 IC=1, 6
DO 71 J=1, 6
KE(IC,J)=0.0
71 CONTINUE

ELEMENTAL GEOMETRIC CONSTANTS REQUIRED

CALL GAMESL(G1, G2, G3, G4, G5, G6, B1, B2, TB3, AREA, AJ, BJ, AK, BK, JAY, KAY, F, TELEM, ANODE, I, FARMU, FAR, NE, NK, NH, IV, W)

CALCULATE ELEMENTAL STIFFNESS MATRIX NOW

F1=FARMU*G1
F2=FARMU*G2
F3=FARMU*G3
KE(1,1)=F1*G1
KE(1,2)=F1*G4
KE(1,3)=F1*G2
KE(1,4)=F1*G5
KE(1,5)=F1*G3
KE(1,6)=F1*G6
KE(2,2)=FARMU*(((B1*B1)+(G4*G4))
KE(2,3)=F2*G4
KE(2,4)=FARMU*((B1*B2+G4*G5)
KE(2,5)=F3*G4
KE(2,6)=FARMU*((B1*B3+G4*G6)
KE(3,3)=F2*G2
KE(3,4)=F2*G5
KE(3,5)=F2*G3
KE(3,6)=F2*G6
KE(4,4)=FARMU*(((B2*B2)+(G5*G5))
KE(4,5)=F3*G5
KE(4,6)=FARMU*((B2*B3+G5*G6)
KE(5,5)=F3*G3
KE(5,6)=F3*G6
KE(6,6)=FARMU*(((B3*B3)+(G6*G6))
DO 26 12=1, 5
DO 26 J=12+1, 6
KE(J,12)=KE(12,J)
26 CONTINUE
ELEMENTAL STIFFNESS MATRIX IS AVAILABLE; NOW

OBTAIN THE D.O.F. NOS.

I1=ELEMT(1,1)
I2=ELEMT(1,2)
I3=ELEMT(1,3)
NDelta(1)=ANODE(I1,3)
NDelta(2)=ANODE(I1,4)
NDelta(3)=ANODE(I2,3)
NDelta(4)=ANODE(I2,4)
NDelta(5)=ANODE(I3,3)
NDelta(6)=ANODE(I3,4)

CHECK ALONG THE ROW OF MATRIX.
IF IT IS BEYOND THE RANGE OF (K) NEGLECT IT. IF IT IS WITHIN THE RANGE, GO
ALONG THE COLUMN.

DO 2 K1=1,6
IF BEYOND K'S RANGE, TRY NEXT ROW
IF(NDelta(K1).GT.NDF) GO TO 2
NK1=NDelta(K1)

GO COLUMNWISE

DO 3 KJ=1,6
NKJ=NDelta(KJ)
IF BEYOND K'S RANGE, OBTAIN THE LOADING VECTOR
IF(NKJ.GT.NDF) GO TO 4

WITHIN K'S RANGE, NOW IN BANDED FORM

NKJ1=NKJ-1
IF(NKJ1.LT.1) GO TO 3
K(NKI,NKJ1)=K(NKI,NKJ1)+KE(KI,KJ)
GO TO 3
4 C(NKI,1)=C(NKI,1)-KE(KI,KJ)*DELTA(NKJ)
3 CONTINUE
2 CONTINUE
50 CONTINUE
RETURN
END
SUBROUTINE SYMSOL(NN,M1,NDIM,KKK,A,B)

******************************************************************
D IRE

DIMENSION A(1),B(1)

LOC(I,J)=I+(J-1)*NDIM
GO TO (1000,2000),KKK

REDUCE MATRIX

1000 DO 280 N=1,NN
      NI=LOC(N,1)
      DO 260 L=2,M
      NL=LOC(N,L)
      C=A(NL)/A(NI)
      I=N+L-1
      IF(NN.LT.I) GO TO 260
      J=0
      DO 250 K=L,1,1
      J=J+1
      IJ=LOC(I,J)
      NK=LOC(N,K)
      250 A(IJ)=A(IJ)-C*A(NK)
      260 A(NL)=C
      280 CONTINUE
      GO TO 500

REDUCE VECTOR

2000 DO 290 N=1,NN
      NI=LOC(N,1)
      DO 285 L=2,M
      NL=LOC(N,L)
      I=N+L-1
      IF(NN.LT.I) GO TO 290
      285 B(I)=B(I)-A(NL)*B(N)
      290 B(N)=B(N)/A(NI)

BACK SUBSTITUTION

300 N=NN
      IF(H.EQ.0) GO TO 500
      DO 400 K=2,M
      IK=LOC(H,K)
      L=N+K-1
      IF(NN.LT.L) GO TO 400
      400 CONTINUE
      GO TO 300

500 RETURN
END
SUBROUTINE CONVEG(Delta, C, NDF, CODE, HK)

******************************************************************************
THE TEST FOR CONVERGENCE
******************************************************************************

DIMENSION Delta(HK), C(NDF)
CODE=1
DO 34 I=1, NDF
    E=ABS(C(I))-Delta(I)) - 0.0001*ABS(C(I))
34 IF(E, GT. 0.0) CODE=CODE+1
RETURN
END
SUBROUTINE VEL(U,V,I,ELENET,DELTA,AHODE,NE,NK, 
TIN,W,H)

***************************************************************
CALCULATION OF NODAL POINT VELOCITIES, U FOR
THE X COMPONENT AND V FOR THE Y COMPONENT FOR
EACH ELEMENT.
***************************************************************

DIMENSION U(NE,3),V(NE,3),ELENET(NE,4),DELTA(NK),AHODE(NN,4)

NODE NUMBERS
I1=ELENET(I,1)
I2=ELENET(I,2)
I3=ELENET(I,3)

D.O.F. NUMBERS
J1=AHODE(I,1)
J2=AHODE(I,2)
J3=AHODE(I,3)
J4=AHODE(I,4)
J5=AHODE(I,5)
J6=AHODE(I,6)

VALUES OF THE UNKNOWNS
D1=DELTA(J1)
D2=DELTA(J2)
D3=DELTA(J3)
D4=DELTA(J4)
D5=DELTA(J5)
D6=DELTA(J6)

VALUES OF ELEMENTAL CONSTANTS REQUIRED
CALL GAMBET(G1,G2,G3,G4,G5,G6,B1,B2,
TB3,AREA,AJ,BJ,AK,BK,JAY,KAY,F,
TELENET,AHODE,1,FARHU,FAR,NE,NK,NN,W,H)

C6=G1*D1+G2*D3+G3*D5+G4*D2+
T(G5*D4+G6*D6)
C5=G5*C6+(11*(D2*(AJ-AK)))+(D4*(AK))
T-(D6*AJ))/FAR
C45=B1*D2+B2*D4+B3*D6
C4=C45+C5
C3=((H*(D3-D1))-(D2*AJ)-(C4*(AJ-AJ))
T+C5*(BJ*BJ)+C6*(AJ*BJ))/H)/JAY

OBTAIN THE X & Y VELOCITIES NOW
U1=-C3
U2=-(C3+2.0*C5*BJ+C6*AJ)
U3=-(C3+2.0*C5*BJ+C6*AK)

STORE THE X VELOCITY VALUES IN THE (U) ARRAY
AND THE Y VELOCITY VALUES IN THE (V) ARRAY
U(I,1)=U1
V(I,1)=D2-U1
U(I,2)=U2
V(I,2)=D4-U2
U(I,3)=U3
V(I,3)=D6-U3
RETURN
END
SUBROUTINE VELOC(UNODE, VNODE, U, V, ELEM, DELTA, TANODE, NE, NK, NN, N, N1)

**********************************************
C THIS CALCULATES THE X AND THE Y VELOCITY
COMPONENTS OF EACH NODE. THE CONTRIBUTIONS
OF VARIOUS ELEMENTS TO THAT NODE IS TAKEN INTO
ACCOUNT.
**********************************************

DIMENSION UNODE(NN), VNODE(NN), U(NE, 3), V(NE, 3),
TELEMET(NE, 4), DELTA(NK), ANODE(NN, 4), NNODE(N)

DO 773 NI = 1, NN
UNODE(NI) = 0.0
VNODE(NN) = 0.0
NNODE(NI) = 0

73 CONTINUE

NODAL NUMBERS

DO 772 I = 1, NE
11 = ELEM(I, 1)
12 = ELEM(I, 2)
13 = ELEM(I, 3)

NODAL VELOCITIES

UNODE(11) = UNODE(11) + U(I, 1)
VNODE(11) = VNODE(11) + V(I, 1)
NNODE(11) = NNODE(11) + 1
UNODE(12) = UNODE(12) + U(I, 2)
VNODE(12) = VNODE(12) + V(I, 2)
NNODE(12) = NNODE(12) + 1
UNODE(13) = UNODE(13) + U(I, 3)
VNODE(13) = VNODE(13) + V(I, 3)
NNODE(13) = NNODE(13) + 1

772 CONTINUE

NODAL VELOCITY OBTAINED NOW.

DO 773 I = 1, NN
UNODE(I) = UNODE(I) / NNODE(I)
VNODE(I) = VNODE(I) / NNODE(I)

773 CONTINUE

RETURN
END
SUBROUTINE VISCO(MHNODE, ELEM, DELTA, TANODE, UNODE, VNODE, NE, HK, NN, W, H)

************************************************
VISCOSITY OF EACH NODE
************************************************

DIMENSION ELEM(NE, 4), DELTA(HK), ANODE(NN, 4),
TANODE(NN), VNODE(NN), DUX(81), DVX(81), DUY(81), DVY(81)
REAL MHNODE(NN)

NEWTONIAN FLOW THEORETICALLY VISCOSITY EQUALS 1.0

GO TO 560
DO 49 NI=1, NN
MHNODE(NI)=0.0
49 CONTINUE

VISCOSITY AT EACH NODE IS EVALUATED
BY OBTAINING THE VALUE OF 412 AT
EACH POINT

FINITE DIFFERENCE METHOD IS USED TO
OBTAIN THE DIFFERENTIALS

OBTAIN DERIVATIVES W.R.T.X

DO 50 I=1, 73, 9
A=ANODE(I+1, 1)-ANODE(I, 1)
B=ANODE(I+2, 1)-ANODE(I, 1)
DUX(I)=(B*B*UNODE(I+1)-A*A*UNODE(I+2)-(B*B*A*A))
T*UNODE(I))/(A*A*B*(B-A))
DVX(I)=(B*B*VNODE(I+1)-A*A*VNODE(I+2)-(B*B*A*A))
T*VNODE(I))/(A*A*B*(B-A))
50 CONTINUE

DO 51 I=9, 81, 9
A1=ANODE(I, 1)-ANODE(I-1, 1)
B1=ANODE(I, 1)-ANODE(I-2, 1)
51 CONTINUE

DO 52 J=2, 8
DO 53 I=0, 8
I=J+1L*9
A2=ANODE(I, 1)-ANODE(I-1, 1)
B2=ANODE(I+1, 1)-ANODE(I-1, 1)
DVX(I)=(A2*A2*VNODE(I+1)-(B2-A2)*B2*VNODE(I-1)
53 CONTINUE
52 CONTINUE
NOW OBTAIN DERIVATIVES W.R.T.Y.

DO 54 1=1,9
AY=ANODE(I+9,2)-ANODE(I,2)
BY=ANODE(I+18,2)-ANODE(I,2)
DUY(I)=((BY*BY*UNODE(I+9))-AY*AY*UNODE(I+18))
T=((BY*BY)-(AY*AY))**UNODE(I))/((AY*BY*(BY-AY)))
DVY(I)=((BY*BY*UNODE(I+9))-AY*AY*UNODE(I+18))-((BY*BY)
T=(AY*AY)*VNODE(I))/((AY*BY*(BY-AY)))

54 CONTINUE
DO 55 1=73,81
AY=ANODE(I,2)-ANODE(I-9,2)
BY=ANODE(I,2)-ANODE(I-18,2)
DUY(I)=((AY*AY*UNODE(I-18))-BY*BY*UNODE(I-9))
T=((BY*BY)-(AY*AY))**UNODE(I))/((AY*BY*(BY-AY)))
DVY(I)=(AY*AY*VNODE(I-18))-BY*BY*VNODE(I-9)
T=((BY*BY)-(AY*AY)*VNODE(I))/((AY*BY*(BY-AY)))

55 CONTINUE
DO 56 J=10,64,9
DO 57 I=0,8
I=J+1*1
AY=ANODE(I,2)-ANODE(I-9,2)
BY=ANODE(I+9,2)-ANODE(I-9,2)
DUY(I)=((AY*AY*UNODE(I+9))-BY*BY*UNODE(I))
T=((BY*BY)-(AY*AY))**UNODE(I))/((AY*BY*(BY-AY)))
DVY(I)=(AY*AY*VNODE(I+9))-BY*BY*VNODE(I)
T=((AY*AY)-(BY*BY))**VNODE(I))/((AY*BY*(BY-AY)))

57 CONTINUE
56 CONTINUE

VALUE OF 412

DO 58 I=1,111
FOUR2=2.0*DX(I)*DX(I)+2.0*DVY(I)
T*DVY(I)+(DUY(I)+DVX(I))*(DVY(I)+DVX(I))
R=SQR2(FOUR2)

VISCOITY OF EACH NODE:

FOR LARGE VALUES OF THE VISCOITY, IT IS SET = 1000

MUNODE(I)=1000.0
IF(R.GE.0.000001) MUNODE(I)=1.0/SQR2(R)

58 CONTINUE
560 DO 561 NI=1,111
MUNODE(NI)=1.0
561 CONTINUE
RETURN
END
SUBROUTINE DELTA(P, ELEMET, DELTA, ANODE, U, V, HE,
TH, NE, W, N)

******************************************************************************
THIS CALCULATES THE CHANGE IN PRESSURE BETWEEN INLET AND OUTLET
ALONG THE X-AXIS
******************************************************************************

DIMENSION ELESET(HE, 4), DELTA(NK), ANODE(NH, 4),
THODE(81, U(HE, 3), VNODE(81), V(NE, 3), DUX(9),
TXDXUX(9), DUX(9), DUX(9), FYDUX(9), DVY(9),
TDXDVY(9), DVX(9), DVY(9), REAL VNODE(81))

VALUE OF ALL NODAL VELOCITIES REQUIRED
CALL VELOC(VNODE, VIODE, U, V, ELEMET, DELTA, ANODE,
TH, NE, NK, NN, W, H)

VALUE OF ALL NODAL VISCOSITIES REQUIRED
CALL VISCO(VNODE, ELEMET, DELTA, ANODE, VNODE, HE, NE, NK, NN, W, H)

FINITE DIFFERENCE METHOD IS USED HERE
THE FORWARD AND BACKWARD DIFFERENCE FORMULA
IS USED FOR THE END POINTS
CENTRAL DIFFERENCE FORMULA FOR OTHERS

OBTAIN FIRST AND SECOND DIFFERENTIAL OF
U V R.T X

A=ANODE(2,1)-ANODE(1,1)
B=ANODE(3,1)-ANODE(1,1)
DUX(1)=(B*B*UNODE(2)-A*ANODE(3))-(B*B-A*A)*UNODE(1))
T/(A*B*(B-A))

DXUX(1)=(2.0*BNODE(2)-A*ANODE(3)-(B-A)*ANODE(1))

DUX(9)=(ANODE(9,1)-ANODE(7,1))

DO 50 I=2,8

A2=ANODE(I,1)-ANODE(I-1,1)
B2=ANODE(I+1,1)-ANODE(I-1,1)
DUX(I)=(A2*A2*UNODE(I+1)-(B2-A2)*U(I+1)*UNODE(I-1))

DO 50 CONTINUE
OBTAIN FIRST AND SECOND DIFFERENTIAL OF U AND V W.R.T. X AND Y.

DO 51 I=1,9
   AY=ANODE(I+9,2)-ANODE(I,2)
   BY=ANODE(I+18,2)-ANODE(I,2)
   DXY(I)=(BY*BY*UNODE(I+9)-AY*AY*UNODE(I+18)-((BY*BY)
   T-(AY*AY))/UNODE(1))/(AY*BY*(BY-AY))
   DXDXY(I)=(2,0*BY*UNODE(I+9)-AY*AY*UNODE(I+18)-((BY*BY)
   T*UNODE(1)))/(AY*BY*(AY-BY))
   DXY(1)=(BY*BY*UNODE(I+9)-AY*AY*UNODE(I+18)-((BY*BY)
   T-(AY*AY))/UNODE(1))/(AY*BY*(AY-BY))

51 CONTINUE
   DXDXY(1)=(B*BY*DYVY(2)-A*AY*DYVY(3)-((B*B)-(A*A))*DVY(1))
   T*(A*B*(B-A))
   DO 52 I=2,8
      A2=ANODE(I,1)-ANODE(I-1,1)
      B2=ANODE(I+1,1)-ANODE(I-1,1)
      DXDXY(I)=(A2*A2*DYVY(I+1)-(B2-A2)*(B2-A2))*DYVY(I-1)

52 CONTINUE

OBTAIN THE DIFFERENTIAL OF U AND V

DO 54 I=2,8
      A2=ANODE(I,1)-ANODE(I-1,1)
      B2=ANODE(I+1,1)-ANODE(I-1,1)
      DXVY(I)=(A2*A2*VNODE(I+1)-(B2-A2)*(B2-A2)*VNODE(I-1)
      T-(A2*A2)-(B2-A2)*VNODE(I-1))/(A2*B2*
      T(B2-A2))

54 CONTINUE

OBTAIN THE DIFFERENTIAL OF VISCOSITY.

DO 56 I=1,9
      AY=ANODE(I+9,2)-ANODE(I,2)
      BY=ANODE(I+18,2)-ANODE(I,2)
      DXVY(I)=(BY*BY*UNODE(I+9)-AY*AY*UNODE(I+18)
      T-(BY*BY)-(AY*AY))/UNODE(1))/(AY*BY*(AY-BY))

56 CONTINUE
   DXVY(1)=(B*BY*MUNODE(2)-A*AY*MUNODE(3)-((B*B)-(A*A))*MUNODE(1))
   T/(A*B*(B-A))
   DO 58 I=2,8
      A2=ANODE(I,1)-ANODE(I-1,1)
      B2=ANODE(I+1,1)-ANODE(I-1,1)
      DXVY(I)=(A2*A2*MUNODE(I+1)-(B2-A2)*(B2-A2)*MUNODE(I-1)

58 CONTINUE
THE INTEGRAND OF CHANGE IN PRESSURE
(DELTA-P) IS OBTAINED FOR EACH ONE OF THE
POINTS ALONG THE BOUNDARY.

DO 55 I=1,9
F(I)=2.0*(MUODE(I)*OXU(X(I))+OMUX(I)*OUX(I))
T+MUODE(I)*(DYUY(I)+DXVY(I))+OMUY(I)*(DUY(I))
55 CONTINUE
SUM=0.0
DO 56 I=2,8
SUM=SUM+F(I)
56 CONTINUE

INTEGRATE USING TRAPEZOIDAL RULE TO OBTAIN THE
CHANGE IN PRESSURE
DP=-(F(1)+2.0*SUM+F(9))/16.0
RETURN
END
SUBROUTINE FLORET(Q1,Q2,Q3,ELEMET,
TDELTA,ANODE,U,UNODE,NE,NK,NH,U,H)

*******************************************************************
THIS CALCULATES THE FLOWRATE AT DIFFERENT
SECTIONS OF THE CHANNEL
Q1 FOR THE INLET
Q2 FOR THE MIDDLE SECTION
Q3 FOR THE OUTLET
*******************************************************************

DIMENSION ELEMET(NE,4),DELTA(NK),ANODE(NN,4),
TU(NE,3),UNODE(NH)
SUM1=0.0
SUM2=0.0
SUM3=0.0
DO 7 1=2,8
SUM1=SUM1+UNODE((1*9)-8)
SUM2=SUM2+UNODE((1*9)-4)
SUM3=SUM3+UNODE(1*9)
7 CONTINUE

TO OBTAIN THE FLOWRATE, THE PRODUCT
(U*Dy) WAS INTEGRATE ACROSS THE CHANNEL

Q1=(UNODE(1)+2.0*SUM1+UNODE(73))/16.0
Q2=(UNODE(5)+2.0*SUM2+UNODE(77))/16.0
Q3=(UNODE(9)+2.0*SUM3+UNODE(81))/16.0
RETURN
END
SUBROUTINE GAEBET(G1,G2,G3,G4,G5,G6,B1,B2,B3,
TAREA,AJ,BJ,AK,BK,JAY,KAY,F,ELE,ET,ANODE,
I1,FAR,HU,FAR,NE,NK,NI,V,H)

******************************************************************************
THIS EVALUATES VARIOUS ELEMENTAL CONSTANTS 
THUS THE GAMMAS(G'S) THE BETAS(B'S), AND THE
AREA OF AN ELEMENT
******************************************************************************

DIMENSION ELE,ET(NE,4), ANODE(164,4)
REAL IU,JU,KU

NODAL NUMBERS AND VISCOSITY

I1=ELE,ET(I,1)
I2=ELE,ET(I,2)
I3=ELE,ET(I,3)
I4=ELE,ET(I,4)

OBTAIN THE LOCAL COORDINATES NOW WITH RESPECT
TO THE I NODE.

AJ=ANODE(I2,1)-ANODE(I1,1)
BJ=ANODE(I2,2)-ANODE(I1,2)
AK=ANODE(I3,1)-ANODE(I1,1)
BK=ANODE(I3,2)-ANODE(I1,2)

OTHER GEOMETRIC CONSTANTS.

AREA=ABS(0.5*((AJ*BK)-(AK*BJ)))
JAY=AJ+BK
KAY=AK+BK
FAR=0.5*AREA
F=2.0*AREA*KAY/JAY*(JAY-KAY)
V=1.0
H=1.0
FARMU=FAR*MU*W

GAMMAS AND BETAS

G1=(FAR*(JAY-KAY)*(H*H))/F
G2=((FAR*KAY)*(H*H))/F
G3=(-1.0*(FAR*JAY)*(H*H))/F
T(KAY)=-2.0*(AJ+BK)*AK+AJ*(BK+AK)
T(JAY)=(-2.0*(AK+BK))
ALPHA1=KAY*(JAY-KAY-*-(BJ+AJ)+JAY*AK)-AJ*BJ*(KAY*KAY)
ALPHA2=KAY*(JAY+AJ+AK*-BJ+AK)*KAY*(H*H)/F
ALPHA3=JAY*(KAY*BK-2.0*(AJ+BK)*AJ-2.0*(AK+BJ))
G4=ALPHA1/F
G5=ALPHA2/F
G6=ALPHA3/F
B1=(((AK+BK)-(AJ+BK)*H))/FAR
B2=((BK-AK)*H)/FAR
B3=((AJ-BK)*H)/FAR
RETURN
END
DIMENSION ALL ARRAYS

DIMENSION ANODE (31,4), C(98,1), DELTA(162),
              TELENET(123,4), U(123,3), DELTA(6), UMODE(81),
              TVMODE(31), V(123,3), CELTA(98)
REAL K(98,13), KE(6,6), UNMODE(81)
WRITE(6,998)

998 FORMAT(1HO,30X,'*************'+,999)
  T**,*
  T'*************'+/ 
  T31X,'** ANALYSIS FOR NEWTONIAN FLOW; FLOW BEHAVIOUR
  T [INDEX=1]*'/
  T31X,'** THROUGH A DIE OF AN ANGLE SHANK
  T' BELOW
  T/31X,'*************'+,999)
  T
  T'*************'+//

***** CONTROL INTERGERS ***** AND OTHER INFORMATION
READ 1:

1 FORMAT(31X,4H4 CONTROL INTERGERS **********CONTROL INTERGERS,//,
T31X,28H INITIAL NO. OF D.O.F.  UK=,15,/, 
T31X,28H NO. OF ELEMENTS  NE=,15,/, 
T31X,28H NO. OF NODES  NII=,15,/, 
T31X,28H REDUCED NO. OF D.O.F.  HDF=,15,/, 
T31X,28H NO. OF KNOWN D.O.F.  H=,15,/, 
T31X,28H BANDWIDTH  IBV=,15,/, 
T31X,28H DIE LENGTH  DL=,F10.4,/, 
T31X,28H HEIGHT OF DIE AT INLET  H=,F10.4,/, 
T31X,28H WIDTH OF DIE  V=,F10.4,/,,
T31X,28H NON-DIMENSIONALIZER  DH=,F10.4,/,,,//)
TEST=0,00001
ALO=0.
ALPHA=(3.141592658*AL)/18.0
WRITE(6,999) ALPHA

999 FORMAT(1HO,30X,'*************'+,999)
  T**,*
  T'*************'+/ 
  T31X,'* THIS DIE IS TAPERED AT AN ANGLE=1,E12.4,
  T' RADIANS */ 
  T/31X,'*************'+,999)
  T
  T'*************'+//
OBTAIN ALL THE DATA REQUIRED.

CALL INPUT(NK, NE, HH, NDF, IE, IBV, DL, NW, ELEMT, ANODE, DELTA,  
TH, ALPHA)

LISTING THE SUBROUTINES AND PRINTING ELEMT, ANODE,  
AND THE BOUNDARY CONDITIONS ONLY.

GO TO 103

DO 38 I=1, HH

MODIFY THE COORDINATES OF EACH  
NODE BECAUSE THE DIE IS TAPERED AT AN  
ANGLE ALPHA.

CALL COORD(I, ANODE, HH, DL, ALPHA)

38 CONTINUE

IF(ITER.EQ.0) GO TO 41

39 DO 40 I=1, NE

NEW VALUES OF VISCOSITY REQUIRED.

CALL UPDATE(ELEMT, ANODE, DELTA, I, NE, NK, HH, ITER)

40 CONTINUE

OBTAIN THE STIFFNESS MATRIX AND THE  
LOADING VECTOR.

41 CALL ASSEMB(K, KE, ELEMT, ANODE, DELTA, C, NE,  
THK, NN, M, HH, NDF, IBV)

SOLVE THE MATRIX EQUATION TO OBTAIN THE  
UNKNOWN.

CALL SYRISOL(NDF, IBV, HDF, 1, K, C)

CALL SYRISOL(NDF, IBV, HDF, 2, K, C)

ITER=ITER+1

IF(ITER-1)43, 43, 55

FIRST TIME ROUND

INSERT VALUES OF UNKNOWN INTO DELTA.

43 DO 44, I=1, HDF

DELTA(I)=C(I, 1)

44 CONTINUE

WRITE(6, 45) ITER

45 FORMAT(10H, 30X, 39H, ITERATION NUMBER***ITERATION NUMBER =, I5, /)  
WRITE(6, 46)

46 FORMAT(31X, 10H DOF NO., 10H, PSI1 ,  
TIX, 10H DOF NO., 10H, PSI3 , /)  
DO 47 I=1, HDF

WRITE(6, 48) I, DELTA(I), I+1, DELTA(I+1)

48 FORMAT(31X, I5, 5X, F10.5, I5, 5X, F10.5)

47 CONTINUE
C CALCULATE THE MODAL VELOCITIES OF EACH ELEMENT
DO 40 I=1,N
CALL VEL(U,V,I,ELEMT,DELTA,ANODE,HE,NK,NN
TI,H)
40 CONTINUE
C CALCULATE THE VELOCITY OF EACH NODE
CALL VELOC(UNODE,VNODE,U,V,ELEMT,DELTA,ANODE,
THE,NK,NN,V,H)
C NODAL VISCO
CALL VISCO(UNODE,ELEMT,DELTA,ANODE,UNODE,VNODE,HE,NK,NN,V,H)
WRITE(6,450) ITER
450 FORMAT(1HO,30X,'ITERATION ** NO.=',15,/) WRITE(5,51)
451 FORMAT(31X,'NO. **',5X,'X-VELOCITY',5X,
'Y-VELOCITY',5X,'VISCOILITY') DO 452 I=1,N
WRITE(6,653)I,UNODE(V),VNODE(V),UNODE(V)
452 CONTINUE
C CALCULATE THE CHANGE IN PRESSURE
CALL DELTAP(DP,ELEMT,DELTA,ANODE,U,V,HE,NK,NN,V,H)
WRITE(6,53)ITER,DP
53 FORMAT(1HO,30X,'ITERATION NO. =',15,/, T31X,23H CHANGE IN PRESSURE DP=',F10.5,/) WRITE(6,54) ITER,DP
54 FORMAT(31X,16H ITERATION NO. =',15,/, T31X,22H CHANGE IN PRESSURE DP=',F10.5,/) GO TO 39
C CALCULATE THE FLOW RATE
CALL FLORET(Q1,Q2,Q3,ELEMT,DELTA,
THE,UNODE,VNODE,HE,NK,NN,V,H)
WRITE(6,55)ITER,Q1,Q2,Q3
55 FORMAT(31X,16H ITERATION NO. =',15,/, T31X,22H FLOW RATE Q1=',F10.5,/, T31X,22H FLOW RATE Q2=',F10.5,/, T31X,22H FLOW RATE Q3=',F10.5,/) GO TO 39
C TEST FOR CONVERGENCE.
C CALL CONVEG(Delta,C,HDF,KODE,NK)
WRITE(6,53)ITER,KODE
53 FORMAT(1HO,30X,'ITERATION ** NO.=',15,/, T31X,'NO. OF UNIONS NOT CONVERGED YET=',15)
DO 59 I=1,N
DELTA(I)=DELTA(I)
DELTA(I)=C(I,1)
59 CONTINUE
IF(KODE,HE,1) GO TO 39
WRITE(6,65)TEST
65 FORMAT(1HO,30X,'*************************************************/ T**', T****************/
T31X,'VALUE OF CONVERGENT CRITERION IS',F10.5, T31X,'*',T31X,'*************************************************/
T****************/}
WRITE(6,650) ITER
56 FORMAT(1HO,30X,5NH ITERATION NUMBER,5NH ITERATION NUMBER,5NH ITERATION NUMBER)
57 FORMAT(31X,15,5X,2F10.5)
58 FORMAT(31X,15,5X,2F10.5)
59 CONTINUE

CALCULATE THE HODAL VELOCITIES OF EACH ELEMENT
DO 60 I=1,NE
60 CALL VELOC(U,V,1,ELEM,DELTA,ANODE,NE,NK,II,TW,H)
CONTINUE

CALCULATE THE VELOCITY OF EACH NODE
DO 61 I=1,NE
61 CALL VELOC(UVNODE,VNODE,U,V,ELEM,DELTA,ANODE,
62 THE,NK,NN,W,H)
CONTINUE

NODEL VISCOITY
DO 63 I=1,NE
63 CALL VISCO(MUNODE,ELEM,DELTA,ANODE,UNODE,VNODE,NE,NK,NN,W,H)
WRITE(6,650) ITER
650 FORMAT(1HO,30X,'ITERATION**NO.=',I5,/) 
651 FORMAT(31X,'HODE**NO.',5X,'X-VELOCITY',5X,
66 T,'Y-VELOCITY',5X,'VISCOSITY')
DO 62 I=1,NN
WRITE(6,453) I,UNODE(I),VNODE(I),MUNODE(I)
652 CONTINUE

CALCULATE THE CHANGE IN PRESSURE
DO 65 I=1,NE
65 CALL DELTAP(DP,ELEM,DELTA,ANODE,U,V,NE,NK,NN,W,H)
WRITE(6,53) ITER,DP
63 CALL FLORET(Q1,Q2,Q3,ELEM,DELTA,
64 TANODE,UNODE,NNODE,NE,NK,NN,W,H)
WRITE(6,54) ITER,Q1,Q2,Q3
64 IF(KODE.EQ.1)GO TO 100
65 GO TO 39
100 WRITE(6,45) ITER
WRITE(6,46)
DO 63 I=1,NDF+N2
63 CONTINUE
WRITE(6,101) KODE
101 FORMAT(31X,'KODE =',I1,' THEOREY CONVERGENCE TOOK PLACE')
STOP
END
ANALYSIS FOR NEWTONIAN FLOW; FLOW BEHAVIOUR INDEX=1
THROUGH A DIE OF AN ANGLE SHOWN BELOW

CONTROL INTEGERS

INITIAL NO. OF D.O.F.  NK=  162
NO. OF ELEMENTS   NE=  128
NO. OF NODES      NH=  81
REDUCED NO. OF D.O.F.  NDF=  98
NO. OF KNOWN D.O.F.  N=  46
BANDWIDTH         IBW=  10
DIE LENGTH         DL=  1.0000
HEIGHT OF DIE AT INLET  H=  1.0000
WIDTH OF DIE        W=  1.0000
NON-DIMENSIONALIZER DH=  3.0000

THIS DIE IS TAPERED AT AN ANGLE= 0, RADIANS

ELEMENT INFORMATION

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APPENDIX P2

Resulting computer programme from the second analysis.
***********
*NOTATIONS*
***********

A(3)=CONTAINS THE FIRST THREE GEOMETRIC CONSTANTS OF AN ELEMENT.

ABC(NE,9)=ARRAY OF ELEMENTAL GEOMETRIC CONSTANTS.


ALPHA=ANGLE OF TAPER OF CHANNEL.

B(3)=CONTAINS THE SECOND THREE GEOMETRIC CONSTANTS OF AN ELEMENT.

C(3)=CONTAINS THE THIRD THREE GEOMETRIC CONSTANTS OF AN ELEMENT.

CE(NK,1)=ARRAY IN WHICH THE SOLUTION FOR THE Unknowns IS STORED.

CELTA(NK)=PREVIOUS VALUE OF THE SOLUTION VECTOR(CE) IS STORED IN.

DELTA(NK)=ARRAY OF REQUIRED NODAL VALUES. THESE ARE THE (X) AND (Y) VELOCITIES, AND THE PRESSURE.

DL=DIE LENGTH.

FB1 = FLOW BEHAVIOUR INDEX OF THE POLYMER.

FOURI2 = VALUE OF 412

H = HEIGHT OF CHANNEL AT INLET.

I'S = COUNTER; DEGREE OF FREEDOM NUMBERS; NODE NUMBERS

IBW = BANDWIDTH

ITER = COUNTER OF THE NO. OF ITERATIONS DONE.

J'S = COUNTER, DEGREE OF FREEDOM NUMBERS; NODE NUMBERS.

K(NDF, IBW) = SYSTEM STIFFNESS MATRIX IN RECTANGULAR
AND BANDED FORM.

KE(9, 9) = ELEMENTAL STIFFNESS MATRIX.

KODE = KEEPS COUNT OF THE NO. OF UNKNOWNS NOT CONVERGED.

K1(6, 6) = A SUB-MATRIX OF 412.

K2(6, 3) = A SUB-MATRIX OF KE(9, 9)

L = COUNT = KEEPS COUNT OF THE NUMBER OF ELEMENTS CONTRIBUTING TO
A NODE.

M = VISCOSITY OF AN ELEMENT

MUNODE(N) = ARRAY OF THE VISCOSITY OF EACH NODE.
N=NO. OF NON ZERO D.O.F.
NDELT(6)=ARRAY CONTAINING THE D.O.F. NO. OF EACH NODE.
NDELT(9)=ARRAY CONTAINING THE D.O.F. NO. OF AN ELEMENT.
NDF=UNKNOWN NO. OF D.O.F.
NDOF=COUNTER FOR THE D.O.F. NUMBERS
NE=NO. OF ELEMENTS.
NK=INITIAL NO. OF D.O.F.
NKI,NKJ=D.O.F. NO. OF AN ELEMENT.
NN=NO. OF NODES.
NNODE=NODAL NUMBER.

Q(I)=FLOW RATE AT VARIOUS SECTIONS OF THE CHANNEL.

T=TANGENT OF AN ANGLE.

W=WIDTH OF DIE

X=X-COORDINATE(GLOBAL) OF A NODE.
XH=X-COORDINATE OF THE CENTROID OF AN ELEMENT.

X1,X2,X3=THE X LOCAL COORDINATES OF THE VERTICES OF AN ELEMENT WITH respect TO its CENTROID.

Y=Y-COORDINATE(GLOBAL) OF A NODE.
YH=Y-COORDINATE OF THE CENTROID OF AN ELEMENT.

Y1,Y2,Y3=THE Y LOCAL COORDINATES OF THE VERTICES OF AN ELEMENT WITH respect TO its CENTROID.
SUBROUTINE CONTRO(NK, NE, NN, NDF, N, IBW, DL, H, W, ALPHA)

*********************************
C
THIS READS IN CONTROL INFORMATION
*********************************
C
READ(5,/)NK, NE, NN, NDF, N, IBW, DL, H, W
C
OBTAIN ANGLE OF TAPER OF THE DIE
ALPHA=0.0
ALPHA=(3.141592655*AL)/18.0
WRITE(6,1)NK, NE, NN, NDF, N, IBW, DL, H, W, ALPHA
1 FORMAT(/31X,'CONTROL INFORMATION*****CONTROL
INFORMATION'//
T31X,'INITIAL NO. OF D.O.F.',6X,'NK=',I5/
T31X,'NO. OF ELEMENTS',12X,'NE=',I5/
T31X,'NO. OF NODES',16X,'NN=',I5/
T31X,'UNKNOWN NO. OF D.O.F.',5X,'NDF=',I5/
T31X,'NON ZERO NO. OF D.O.F.',6X,'N=',I5/
T31X,'BANDWIDTH',17X,'IBW=',I5/
T31X,'DIE LENGTH',17X,'DL=',F10.5/
T31X,'HEIGHT OF DIE AT INLET',6X,'H=',F10.5/
T31X,'WIDTH OF DIE',16X,'W=',F10.5/
T31X,'ANGLE OF TAPER OF DIE ALPHA=',F10.5,3X,'RADIAN'///)
RETURN
END
SUBROUTINE SOLVE(ELEMET, ANODE, DELTA, ABC, NK, NE, NN, NDF, N, IBW, DL, H, TW, ALPHA, K1, K2, K, KE, CE)

DIMENSION ELEMET(NE, 5), ANODE(NN, 5), DELTA(NK), TABC(NE, 9), CE(NK, 1), Q(9), CELTA(243)
REAL K1(6, 6), K2(6, 3), K(NDF, IBW), KE(9, 9)

ELEMENTAL AND NODAL INFORMATION REQUIRED

CALL INPUT(ELEMET, ANODE, DELTA, ABC, NK, NE, NN, NDF, N, IBW, DL, H, W, ALPHA)

JUST LISTING ANODE AND DELTA ONLY.

GO TO 650
ITER=0
KODE=0

200 ITER=ITER+1

SYSTEM STIFFNESS MATRIX (K) AND LOADING VECTOR (CE) REQUIRED

CALL ASSEMB(K, KE, ELEMET, ANODE, DELTA, CE, ABC, K1, K2, NK, NE, NN, NDF, N, IBW, DL, H, W, ALPHA, ITER)

MATRIX EQUATION SOLVER REQUIRED.

CALL SYMSOL(NDF, IBW, NDF, 1, K, CE)
CALL SYMSOL(NDF, IBW, NDF, 2, K, CE)
IF(ITER.GT.1) GO TO 400

NEWTONIAN SOLUTION IS NOW AVAILABLE TO BE PRINTED.

CALL OUTPUT(ITER, DELTA, ANODE, ELEMET, Q, CE, NK, TNE, NN, NDF, N)
GO TO 200

TEST FOR CONVERGENCY.

400 CALL CONVEG(DELTA, CE, NDF, KODE, NK)
WRITE(6, 3) ITER, KODE-1
3 FORMAT(1HO, 'ITERATION***NO.=', IS//
T1X, 'NO. OF UNKNOWNS NOT CONVERGED YET=', IS)
DO 450 I=1, NDF

PREVIOUS VALUE OF THE SOLUTION IS STORED IN (CELTA) AND THE PRESENT VALUE IN (DELTA).

CELTA(I) = DELTA(I)
450 DELTA(I) = CE(I, 1)
IF (KODE.NE.1) GO TO 200

NON-NEWTONIAN SOLUTION IS NOW AVAILABLE TO BE PRINTED.

CALL FINALE(CELTA, DELTA, ANODE, ELEMET, NK, NN, NE, NDF, N, Q, ITER)
550 WRITE (6, 600) KODE
600 FORMAT(1X, 'KODE=', I1, 2X, 'THEREFORE CONVERGENCE TOOK PLACE')
650 CONTINUE
RETURN
END
SUBROUTINE INPUT(ELEMENT, ANODE, DELTA, ABC,
TIR, HE, HN, HD, H, IBO, DL, H, V, ALPHA)

**********************************************************************
THIS INPUTS ALL THE AVAILABLE DATA
**********************************************************************

DIMENSION ELEMENT(HE, 5), ANODE(HN, 5), DELTA(HN),
TABC(HE, 9), A(?), B(3), C(3)

NODAL INFORMATION IS OBTAINED
THE ENTRIES OF ANODE ARE GENERATED.
THE FIRST AND SECOND ENTRIES ARE FOR THE
COORDINATES OF THE NODE AND THE OTHERS
ARE THE DEGREE OF FREEDOM NUMBERS
ASSOCIATED TO THE NODE.

CALL GEONODE(ANODE, HN, I, HD)
WRITE(6, 20)
20 FORMAT('3IX, 'NODAL INFORMATION**********,
'NODAL INFORMATION**********NODAL INFORMATION!//
3IX, 'NODE NO., 4X, 'X COORD', 3X, 'Y COORD', 5X, 'DOF',
T' NO. ', 2X, 'DOF NO. ', 2X, 'DOF NO. !)
DO 21 I=1, HN

0, DO. NUMBERS
11 =ANODE(1,3)
12 =ANODE(1,4)
13 =ANODE(1,5)
WRITE(6, 22)1, ANODE(1,1), ANODE(1,2), 11, 12, 13
22 FORMAT(31X, 15, 5X, 2F10.5, 4X, 15, 4X, 15, 5X, 15)
21 CONTINUE

JUST LISTING ANODE AND DELTA ONLY.
GO TO 260

THE GLOBAL COORDINATES GIVEN ABOVE ARE FOR A UNIFORM
RECTANGULAR MESH. MODIFIED BELOW TO ACCOUNT FOR THE
TAPERING OF THE DIE. X COORDINATES REMAIN THE SAME.

WRITE(6, 23)
23 FORMAT(110, 'CORRECTED NODAL INFORMATION********',
'CORRECTED NODAL INFORMATION!//
31X, 'NODE NO., 4X, 'X COORD', 3X, 'Y COORD', 5X, 'DOF NO. ',
T' NO. ', 2X, 'DOF NO. ', 2X, 'DOF NO. !)
T =TAN(ALPHA)
R =T/ DL
DO 24 I=1, HN
ANODE(1,2) =ANODE(1,2) *(1-ANODE(1,1)*R)
11 =ANODE(1,3)
12 =ANODE(1,4)
13 =ANODE(1,5)
WRITE(6, 22)1, ANODE(1,1), ANODE(1,2), 11, 12, 13
24 CONTINUE

DATA ABOUT THE ELEMENTS IS READ IN.
THE FIRST THREE POSITIONS FOR THE NODAL NUMBERS, THE
FOURTH FOR THE VISCOSITY AND THE FIFTH FOR THE AREA.

WRITE(6, 25)
25 FORMAT(110, 'ELEMENT INFORMATION***************',
'ELEMENT INFORMATION!//
T1X, 'VISCOITY', 4X, 'AREA')
ELEMENT VISCOSITY
ELEMET(1,4)=1.0

NODE NUMBERS
ELEMET(1,1)=11+0.1
ELEMET(1,2)=12+0.1
ELEMET(1,3)=13+0.1

CENTROID OF EACH ELEMENT IS NOW OBTAINED
XH=(ANODE(1,1)+ANODE(12,1)+ANODE(13,1))/3.0
YH=(ANODE(1,2)+ANODE(12,2)+ANODE(13,2))/3.0

LOCAL COORDINATES WITH RESPECT TO THE CENTROID
X1=ANODE(11,1)-XH
X2=ANODE(12,1)-XH
X3=ANODE(13,1)-XH
Y1=ANODE(11,2)-YH
Y2=ANODE(12,2)-YH
Y3=ANODE(13,2)-YH

AREA OF AN ELEMENT
AREA=1.5*(X2*Y3-X3*Y2)
ELEMET(1,5)=AREA

GEOMETRIC CONSTANTS OF EACH ELEMENT IS OBTAINED. EACH QUANTITY IS DIVIDED BY TWICE THE AREA FOR CONVENIENCE
A(1)=(X2*Y3-X3*Y2)/(2.0*AREA)
A(2)=(X3*Y1-X1*Y3)/(2.0*AREA)
A(3)=(X1*Y2-X2*Y1)/(2.0*AREA)
B(1)=(Y2-Y3)/(2.0*AREA)
B(2)=(Y3-Y1)/(2.0*AREA)
B(3)=(Y1-Y2)/(2.0*AREA)
C(1)=(X3-X2)/(2.0*AREA)
C(2)=(X1-X3)/(2.0*AREA)
C(3)=(X2-X1)/(2.0*AREA)
DO 27 J=1,3
ABC(1,J)=A(J)
ABC(1,J+3)=B(J)
ABC(1,J+6)=C(J)
27 CONTINUE
WRITE(6,28) 1,11,12,13,ELEMET(1,4)
28 FORMAT(1X,I5,4X,I5,5X,I5,5X,I5,5X,2F10,5)
26 CONTINUE

BOUNDARY CONDITIONS STORED IN DELTA().

260 WRITE(6,32)
32 FORMAT(110,32X,'BOUNDARY CONDITIONS '/
T31X,***************'/
T31X,' DOF DO ',' PRESSURE ')
DO 33 I=10F+1, 10F+1
DELTA(I)=1.0
33 WRITE(6,34) I, DELTA(I)
34 FORMAT(31X,I5,5X,F10,5)
RETURN
END
SUBROUTINE GENODE(ANODE,NN,N,NDF)

C ************************************
C THIS GENERATES THE NODAL COORDINATES
AND THE DEGREE OF FREEDOM NUMBERS
C
DIMENSION ANODE(NN,5)
C
IN THIS PROCEDURE, THE UNKNOWN NODAL VALUES
ARE NUMBERED FIRST, FOLLOWED BY THE KNOWN
NON-ZERO VALUES AND THEN THE ZERO VALUES.
C
NDOF=1
DO 1 I=1,9
DO 1 JJ=1,9
J=10-JJ
I1=0
I2=0
I3=0

OBTAIN NODAL NUMBER

NNODE=1+9*(J-I)
C
IF NODE LIES ALONG THE TOP OR BOTTOM
OF THE CHANNEL WHERE THE X-COMPONENT OF
VELOCITY IS ZERO, THE X-VELOCITY
D.O.F. NUMBER IS LEFT BLANK IN THE MEANTIME
C
IF((J.EQ.1).OR.(J.EQ.9)) GO TO 2
C
NODE IS NOT ALONG THE TOP OR BOTTOM OF
THE CHANNEL. ASSIGN A DEGREE OF FREEDOM NUMBER
TO THE X-VELOCITY.
C
I1=NDOF
NDOF=NDOF+1

IF NODE LIES ALONG THE ENTRANCE OR EXIT
OF THE CHANNEL, WHERE THE Y-COMPONENT OF
VELOCITY IS ZERO, THE Y-VELOCITY D.O.F.
NUMBER IS LEFT BLANK IN THE MEANTIME.
C
IF((I.EQ.1).OR.(I.EQ.9)) GO TO 2
C
NODE IS NOT ALONG THE BOUNDARIES OF THE
CHANNEL. ASSIGN A DEGREE OF FREEDOM NUMBER TO
THE Y-VELOCITY.
C
I2=NDOF
NDOF=NDOF+1

IF NODE LIES ALONG THE ENTRANCE OR EXIT
OF THE CHANNEL, WHERE THE PRESSURE IS KNOWN,
THE PRESSURE D.O.F. NUMBER IS LEFT BLANK IN THE MEANTIME.
C
2 IF((I.EQ.1).OR.(I.EQ.9)) GO TO 3
C
NODE IS NOT ALONG THE ENTRANCE OR EXIT
OF THE CHANNEL. ASSIGN A DEGREE OF FREEDOM
NUMBER TO THE PRESSURE.
C
I3=NDOF
NDOF=NDOF+1
C OBTAIN THE COORDINATES OF A NODE.

3 \( X = (I-1) \times 0.125 \)
\( Y = (J-1) \times 0.125 \)

C STORE THE ABOVE VALUES IN ANODE,

\[ \text{ANODE}(\text{INODE}, 1) = X \]
\[ \text{ANODE}(\text{INODE}, 2) = Y \]
\[ \text{ANODE}(\text{INODE}, 3) = I + 0.1 \]
\[ \text{ANODE}(\text{INODE}, 4) = I + 0.1 \]
\[ \text{ANODE}(\text{INODE}, 5) = I + 0.1 \]

1 CONTINUE

C THE KNOWN NON-ZERO NODAL VALUES ARE NOW ASSIGNED DEGREE OF FREEDOM NUMBERS.

DO 4 \( J=1,9 \)
\[ \text{ANODE}(1 + 9 \times (J-1), 5) = \text{IDOF} + 0.1 \]
\[ \text{IDOF} = \text{IDOF} + 1 \]
4 CONTINUE

C THE ZERO NODAL VALUES ARE NOW ASSIGNED DEGREE OF FREEDOM NUMBERS.

DO 5 \( I=1,9 \)
DO 5 \( J=1,9 \)
\[ \text{INODE} = 1 + 9 \times (J-1) \]
\[ I1 = \text{ANODE}(\text{INODE}, 3) \]
\[ I2 = \text{ANODE}(\text{INODE}, 4) \]
\[ I3 = \text{ANODE}(\text{INODE}, 5) \]
IF(I1, GE, 1) GO TO 6
\[ \text{ANODE}(\text{INODE}, 3) = I + 0.1 \]
6 IF(I2, GE, 1) GO TO 7
\[ \text{ANODE}(\text{INODE}, 4) = I + 0.1 \]
7 IF(I3, GE, 1) GO TO 5
\[ \text{ANODE}(\text{INODE}, 5) = I + 0.1 \]
5 CONTINUE
RETURN
END

******************************************************************************
THIS CALCULATES THE STIFFNESS MATRIX OF EACH ELEMENT AND
INSERTS IT INTO THE SYSTEM STIFFNESS MATRIX. THE
MATRIX IS BANDED AND THE LOADING VECTOR OBTAINED IN
THE PROCESS
******************************************************************************

DIMENSION ELEMET (HE, 5), ANODE(III, 5), DELTA(HK), C(HOF, 1)
1, A(HOF, 10), UDEL(T)
REAL K(HOF, 10), KE(9, 9), K1(6, 6), K2(6, 3)
DO 40 I=1, HM
DO 40 J=1, IV
K(I, J) = 0.0
CK(I, I) = 0.0
40 CONTINUE

EACH ELEMENT IS BROUGHT IN FOR PROCESSING

DO 41 I=1, HE
DO 42 IC=1, 9
DO 42 J=1, 9
KE(1C, J) = 0.0
42 CONTINUE

CALL MATRIC (K1, K2, A, C, I, HE)
IF(ITER.GT.1) CALL UPDATE(ELEMET, ANODE, DELTA, K1, K2, I, HE, HM, HK)

ELEMETAL STIFFNESS MATRIX IS CALCULATED

DO 43 IC=1, 6
DO 43 J=1, 6
KE(1C, J) = K1(1C, J) * ELEMET(1, 4) * 2.0 * ELEMET(1, 5)
43 CONTINUE
DO 44 IC=1, 6
DO 44 J=1, 3
KE(1C, J+6) = K2(1C, J) * 2.0 * ELEMET(1, 5)
44 CONTINUE
DO 45 IC=1, 3
DO 45 J=1, 6
KE(1C+6, J) = KE(J, IC+6)
45 CONTINUE
ELEMENTAL STIFFNESS MATRIX IS AVAILABLE; NOW

OBTAIN THE D.O.F. NOS.

\[ \begin{align*}
11 & = \text{ELEDE}(1,1) \\
12 & = \text{ELEDE}(1,2) \\
13 & = \text{ELEDE}(1,3) \\
14 & = \text{ELEDE}(1,4) \\
15 & = \text{ELEDE}(1,5) \\
16 & = \text{ELEDE}(1,6) \\
17 & = \text{ELEDE}(1,7) \\
18 & = \text{ELEDE}(1,8) \\
21 & = \text{ELEDE}(2,1) \\
22 & = \text{ELEDE}(2,2) \\
23 & = \text{ELEDE}(2,3) \\
24 & = \text{ELEDE}(2,4) \\
25 & = \text{ELEDE}(2,5) \\
26 & = \text{ELEDE}(2,6) \\
27 & = \text{ELEDE}(2,7) \\
28 & = \text{ELEDE}(2,8) \\
31 & = \text{ELEDE}(3,1) \\
32 & = \text{ELEDE}(3,2) \\
33 & = \text{ELEDE}(3,3) \\
34 & = \text{ELEDE}(3,4) \\
35 & = \text{ELEDE}(3,5) \\
36 & = \text{ELEDE}(3,6) \\
37 & = \text{ELEDE}(3,7) \\
38 & = \text{ELEDE}(3,8) \\
41 & = \text{ELEDE}(4,1) \\
42 & = \text{ELEDE}(4,2) \\
43 & = \text{ELEDE}(4,3) \\
44 & = \text{ELEDE}(4,4) \\
45 & = \text{ELEDE}(4,5) \\
46 & = \text{ELEDE}(4,6) \\
47 & = \text{ELEDE}(4,7) \\
48 & = \text{ELEDE}(4,8) \\
51 & = \text{ELEDE}(5,1) \\
52 & = \text{ELEDE}(5,2) \\
53 & = \text{ELEDE}(5,3) \\
54 & = \text{ELEDE}(5,4) \\
55 & = \text{ELEDE}(5,5) \\
56 & = \text{ELEDE}(5,6) \\
57 & = \text{ELEDE}(5,7) \\
58 & = \text{ELEDE}(5,8) \\
61 & = \text{ELEDE}(6,1) \\
62 & = \text{ELEDE}(6,2) \\
63 & = \text{ELEDE}(6,3) \\
64 & = \text{ELEDE}(6,4) \\
65 & = \text{ELEDE}(6,5) \\
66 & = \text{ELEDE}(6,6) \\
67 & = \text{ELEDE}(6,7) \\
68 & = \text{ELEDE}(6,8) \\
71 & = \text{ELEDE}(7,1) \\
72 & = \text{ELEDE}(7,2) \\
73 & = \text{ELEDE}(7,3) \\
74 & = \text{ELEDE}(7,4) \\
75 & = \text{ELEDE}(7,5) \\
76 & = \text{ELEDE}(7,6) \\
77 & = \text{ELEDE}(7,7) \\
78 & = \text{ELEDE}(7,8) \\
81 & = \text{ELEDE}(8,1) \\
82 & = \text{ELEDE}(8,2) \\
83 & = \text{ELEDE}(8,3) \\
84 & = \text{ELEDE}(8,4) \\
85 & = \text{ELEDE}(8,5) \\
86 & = \text{ELEDE}(8,6) \\
87 & = \text{ELEDE}(8,7) \\
88 & = \text{ELEDE}(8,8) \\
91 & = \text{ELEDE}(9,1) \\
92 & = \text{ELEDE}(9,2) \\
93 & = \text{ELEDE}(9,3) \\
94 & = \text{ELEDE}(9,4) \\
95 & = \text{ELEDE}(9,5) \\
96 & = \text{ELEDE}(9,6) \\
97 & = \text{ELEDE}(9,7) \\
98 & = \text{ELEDE}(9,8) \\
\end{align*} \]

CHECK ALONG THE ROW OF MATRIX.

IF IT IS BEYOND THE RANGE OF (K) NEGLECT IT

IF IT IS WITHIN THE RANGE, GO ALONG THE COLUMN

DO 2 KI=1,9

IF BEYOND K'S RANGE, TRY NEXT ROW

IF(1DELT(KI),GT,NDI)GO TO 2

KII=1DELT(KI)

GO COLUMNWISE

DO 3 KJ=1,9

HKJ=1DELT(KJ)

IF BEYOND K'S RANGE, OBTAIN THE LOADING VECTOR

IF (HKJ,GT,NDI)GO TO 4

WITHIN K'S RANGE, NOW IN BANDED FORM

HKJJ=HKJ-HKI+1

IF(HKJJ,LT,1)GO TO 3

K(HKI,HKJJ)=K(HKI,HKJJ)+KE(KI,KJ)

GO TO 3

4 CE(HKI,1)=CE(HKI,1)-KE(KI,KJ)*DELT(A(HKJ)

3 CONTINUE

2 CONTINUE

41 CONTINUE

RETURN

END
SUBROUTINE MATRIX(K1,K2,ABC,I,NE)

*******************************************************************************
THIS OBTAINS THE MATRICES K1 AND K2 FOR EACH ELEMENT (I)
*******************************************************************************

DIMENSION ABC(NE,9),A(3),B(3),C(3)
REAL K1(6,6),K2(6,3)

GEOMETRIC CONSTANTS OF EACH ELEMENT

DO 49 J=1,3
A(J)=ABC(I,J)
B(J)=ABC(I,J+3)
C(J)=ABC(I,J+6)
49 CONTINUE

OBTAIN MATRIX K1

DO 50 I=1,3
DO 50 J=1,3
K1(I,J)=2.0*B(I)*B(J)+C(I)*C(J)
K1(I,J+3)=2.0*C(I)*C(J)+B(I)*B(J)
K1(I,J+6)=C(I)*B(J)
50 CONTINUE

OBTAIN MATRIX K2.

DO 51 I=1,3
DO 51 J=1,3
K2(I,J)=A(I)*C(J)
K2(I,J+3)=A(I)*C(J)
51 CONTINUE
RETURN
END
SUBROUTINE UPDATE(ELEMET, ANODE, DELTA, K1, K2, I, NE, NN, NK)

*************************************************************************
THIS SUBROUTINE UPDATES THE VISCOSITY OF EACH ELEMENT
*************************************************************************

DIMENSION ELEMET(NE,5), ANODE(NN,5), DELTA(NK), NDELTA(6)
REAL K1(6,6), K2(6,3)
REAL MU

OBTAIN THE NODAL NOS. OF EACH ELEMENT

I1 = ELEMET(I,1)
I2 = ELEMET(I,2)
I3 = ELEMET(I,3)

OBTAIN THE REQUIRED D.O.F. NOS. OF EACH NODE

NDELTA(1) = ANODE(I1,3)
NDELTA(2) = ANODE(I2,3)
NDELTA(3) = ANODE(I3,3)
NDELTA(4) = ANODE(I1,4)
NDELTA(5) = ANODE(I2,4)
NDELTA(6) = ANODE(I3,4)

CALCULATE 4I2(FOURI2) NOW.

FOURI2 = 0.0
DO 60 IC = 1, 6
   DO 60 J = 1, 6
      FOURI2 = FOURI2 + K1(IC,J) * DELTA(NDELTA(IC)) * DELTA(NDELTA(J))
60 CONTINUE
R = SQRT(FOURI2)

CALCULATE THE VISCOSITY (MU) IF THE FLOW BEHAVIOUR INDEX (FBI) OF THE POLYMER IS KNOWN

FBI = 0.5
MU = (R**(FBI-1.0))
ELEMET(I,4) = MU
RETURN
END
SUBROUTINE SYMOLS(NN,IM,HDIM,KKK,A,B)

************* BANDED SYMMETRIC MATRIX EQUATION SOLVER, CROUT METHOD *************

DIMENSION A(1),B(1)

LOC(I,J)=I*(J-1)*HDIM
GO TO (1000,2000),KKK

REDUCE MATRIX

1000 DO 280 N=1,NN
71=LOC(N,N)
DO 260 L=2,NM
NL=LOC(N,L)
C=A(NL)/A(N1)
I=I+L-1
IF(NN.LT.I) GO TO 260
J=0
DO 250 K=L,MM
J=J+1
IJ=LOC(I,J)
NK=LOC(N,K)
250 A(IJ)=A(IJ)-C*A(NK)
260 A(NL)=C
280 CONTINUE
GO TO 500

REDUCE VECTOR

2000 DO 290 N=1,NN
71=LOC(N,N)
DO 285 L=2,IM
NL=LOC(N,L)
I=I+L-1
IF(NN.LT.I) GO TO 290
285 B(I)=B(I)-A(NL)*B(N)
290 B(N)=B(N)/A(N1)

BACK SUBSTITUTION

300 N=NN
IF(N.EQ.0) GO TO 500
DO 400 K=2,MM
NK=LOC(N,K)
L=I+K-1
IF(NN.LT.L) GO TO 400
B(I)=B(I)-A(NK)*B(L)
400 CONTINUE
GO TO 300

500 RETURN
END
SUBROUTINE OUTPUT (ITER, DELTA, ANODE, ELEMET, Q, CF, NC, NE, NN, HDF, !)

************************************************************************************
THIS OUTPUT THE SOLUTION FOR NEWTONIAN FLOW.
*************************************************************************************
DIMENSION DELTA(NK), ANODE('HI',5), ELEMET('E',5), Q(9), CE(NK,1),
TOLET(243)
100 WRITE(6,9)
9 FORMAT(1HO, 'THIS IS THE UNSCALED-SOLUTION FOR NEWTONIAN FLOW****
ITERATION NO. = !//IX, 'NODE***NO.', '5X, 'X-VELOCITY', '5X, 'Y-VELOCITY',
25X, 'PRESSURE')

C INSERT VALUES OF UNKNOWNS INTO DELTA
DO 10 I = 1, NN
10 CONTINUE
DO 11 I = 1, NN
11 WRITE(6,12)I, DELTA(I)
12 FORMAT(1X, 'O, 'F6.2)!

C CALL FLORET(Q, DELTA, ANODE, NC, NN)
WRITE(6,13)
13 FORMAT(1HO, 'FLOURATE AT VARIOUS SECTIONS OF THE!//6X, 'SECTION', '8X,
'FLOURATE')
WRITE(6,14)(I, Q(I), I=1,9)
14 FORMAT(8X, 'O, 'I2, '8X, F10.5)
WRITE(6,90)
90 FORMAT(1HO, '27X, 'THIS IS THE SCALED-SOLUTION FOR NEWTONIAN FLOW, SC
TALE SOLUTION FOR NEWTONIAN FLOW. !//16X, 'SOLUTION !//F10.5 FORMAT
2,5OX, 'SOLUTION IN E12.5 FORMAT //IX, 'NODE***NO.', '5X, 'X-VELOCITY', '5X,
'Y-VELOCITY', '5X, 'PRESSURE', '20X, 'X-VELOCITY', '5X, 'Y-VELOCITY', '5X,'
4PRESSURE')
NC=NN
DO 15 I = 1, NN
D.O.F. NUMBERS.

J1 = ANODE(1,3)
J2 = ANODE(1,4)
J3 = ANODE(1,5)

THE NODAL VARIABLES ARE NOW SCALED BY THE FLOW RATE.

MULTIPLICATION BY ZERO IS AVOIDED BY THE IF STATEMENTS

IF(J1.GT.I) GO TO 20
DELT(J1) = DELTA(J1)/Q(1)
20 IF(J2.GT.I) GO TO 30
DELT(J2) = DELTA(J2)/Q(1)
30 IF(J3.GT.I) GO TO 15
DELT(J3) = DELTA(J3)/Q(1)
15 WRITE(6,95), DELT(J1), DELT(J2), DELT(J3),
IDELT(J1), IDELT(J2), IDELT(J3)
95 FORMAT(1X,15,10x,F10.5,5x,F10.5,6x,F10.5,18x,E12.4,4x,E12.4,4x,E12.4,4x,E12
1.4)
999 RETURN
END
SUBROUTINE FLORET(Q, DELTA, ANODE, NK, NH)
C
***********************************************************************
C THIS CALCULATES THE FLOWRATE AT DIFFERENT
C SECTIONS OF THE DIE
C ***********************************************************************
C
DIMENSION Q(N), DELTA(NK), ANODE(NH,5)
C
SET NO. OF SECTIONS(N) OVER THE DIE.
C
1 = 9
DO 2 I = 1, M
2 Q(I) = 0.0
DO 3 I = 1, M
3 J = 1, M - 1
C
NODE NUMBER.
C
NHODE = I + NH(J-1)
C
D.O.F. NO. CORRESPONDING TO THE X VELOCITY.
C
J1 = ANODE(NHODE, 3)
J2 = ANODE(NHODE+1, 3)
D = ANODE(NHODE+M, 2) - ANODE(NHODE, 2)
C
INTEGRATE (U, AOY) ACROSS THE CHANNEL TO OBTAIN THE
C
FLOWRATE.
C
3 Q(I) = Q(I) + (DELTA(J1) + DELTA(J2)) / D
DO 4 I = 1, M
4 Q(I) = Q(I) / 2.0
RETURN
END
SUBROUTINE CONVEG(DELTA, CE, NDF, KODE, NK)

C ********************
C TEST FOR CONVERGENCY
C ********************

DIMENSION DELTA(NK), CE(NDF)
KODE=1
DO 34 I=1, NDF
Y=DELTA(I)

C VERY SMALL NODAL VALUES ARE NOT
C TESTED FOR CONVERGENCY.
C
IF(Y.LT.0.00001) GO TO 34
E=ABS(CE(I)-DELTA(I))-0.001*ABS(CE(I))
IF(E.GT.0.0)KODE=KODE+1
34 CONTINUE
RETURN
END
SUBROUTINE FLORET(Q, DELTA, ANODE, CELTA, N, M, NN, ITER)

*******************************************************************************
THIS SUBROUTINE OBTAINS THE FINAL RESULT
EACH 'DISPLACEMENT' IS OPERATED UNTIL BY THE
APPROPRIATE VALUE OF THE FLOWRATE
*******************************************************************************

DIMENSION DELTA(N), ANODE(I, 5), CELTA(N, 5), Q(I), CELTA(N)
REAL ANODE(I), DELTA(I)
DO 500 I=1, NN

500 DELTA(I)=1.0

WRITE(6,1) ITER
1 FORMAT(1HO, 'ITERATION'***NUMBER'***ITERATION'***NUMBER'***ITERATION'
        'ITX'***NUMBER'***ITERATION'***NUMBER'***ITERATION'
        'ITY'***NUMBER'***ITERATION'***NUMBER', 5X,
        'ITX'***NUMBER'***ITERATION'***NUMBER'***ITERATION'
        'ITY'***NUMBER'***ITERATION'***NUMBER', 5X
    T'PREVIOUS'***NUMBER'***ITERATION'***NUMBER'***ITERATION'
    T'PREVIOUS'***NUMBER'***ITERATION'***NUMBER'***ITERATION'
    DO 2 I=1, NN

2 DO 3 J=1, 4
3 WRITE(6,3) J, DELTA(J), CELTA(J)

4 WRITE(6,4) ITER
4 FORMAT(1HO, 'D.O.F. NUMBERS, OF A NODE
4 J1=ANODE(I, 3)
4 J2=ANODE(I, 4)
4 J3=ANODE(I, 5)

5 WRITE(6,5) J1, J2, J3
5 FORMAT(1HO, 'OUTPUT THE VARIABLES OF THAT NODE.

6 WRITE(6,6) ITER
6 FORMAT(1HO, 'CALCULATE AND PRINT THE FLOWRATE THROUGH THE
6 D.T.
6 CALL FLORET(Q, DELTA, ANODE, NN, N)
6 WRITE(6,7) ITER
7 FORMAT(1HO, 'FLORET AT VARIOUS SECTIONS OF D.T.'
7 T6X, 'SECTION'***NUMBER'***FLOWRATE', 5X
    WRITE(6,14) (I, Q(I), I=1, 9)

99 PRINT()
FINAL SOLUTION IS NOW OBTAINED.

WRITE(6,700)
700 FORMAT(1X,'THIS IS THE SCALED-SOLUTION FOR NON-NEWTONIAN FLOW
    1, SCALED SOLUTION FOR NON-NEWTONIAN FLOW'//16X,'SOLUTION IN F10.5
2FORMAT',50X, 'SOLUTION IN E12.4', FORMAT'//1X,'NODE**HO.1,6X,'X-VELO
3CITY',5X, 'Y-VELOCITY', 5X, 'PRESSURE',20X,'X-VELOCITY', 5X, 'Y-VELOCIT
4Y',5X, 'PRESSURE')
   DO GO TO 17
    J1=ANODE(1,1)
    J2=ANODE(1,4)
    J3=ANODE(1,5)
    FBI=0.5
   THE NODEAL VARIABLES ARE NOW SCALLED BY THE
   FLOWRATE.
   MULTIPLICATION BY ZERO IS AVOIDED BY THE "IF STATEMENTS"
   IF(J1,GT,1.0) GO TO 20
   DELTA(J1)=DELTA(J1)/Q(1)
20 IF(J2,GT,1.0) GO TO 30
   DELTA(J2)=DELTA(J2)/Q(1)
30 IF(J3,GT,1.0) GO TO 60
   DELTA(J3)=DELTA(J3)/(Q(1)**FBI)
60 WRITE(6,722),DELTA(J1),DELTA(J2),DELTA(J3),DELTA(J1),DELTA(J2),
    DELTA(J3)
600 CONTINUE
   DO 725 I=1,HE
725 ELEET(I,1)=ELEET(I,1)/Q(1)**(FBI-1.0)
   VISCOSITY OF EACH NODE CALCULATED AND PRINTED.
   CALL VISCO(NUMODE,ELEET,NN,HE)
   WRITE(6,750)
750 FORMAT(110,30X,' NODE VISCOSITY')
   WRITE(6,800)((1,HEMODE(I)),I=1,NN)
800 FORMAT(1X,15,8X,F10.5)
   RETURN
   END
SUBROUTINE VISCO(MUNODE,ELEMET,HN,HE)

************************************************************************************
THIS CALCULATES THE VISCOSITY OF EACH NODE BY AN AVERAGING PROCEDURE.
************************************************************************************

DIMENSION ELEMET(HE,5),LCOUNT(31)
REAL MUNODE(HN)
DO 33 HN=1,HN
   MUNODE(HN)=0
33 CONTINUE
DO 82 I=1,HE

MODAL NUMBERS.
I1=ELEMET(I,1)
I2=ELEMET(I,2)
I3=ELEMET(I,3)

MUNODE() STORES THE VALUE OF THE MODAL VISCOSITY
AND LCOUNT() KEEPS COUNT OF THE NUMBER OF
ELEMENTS CONTRIBUTING TO THAT NODE.

MUNODE(I1)=MUNODE(I1)+ELEMET(I,4)
LCOUNT(I1)=LCOUNT(I1)+1
MUNODE(I2)=MUNODE(I2)+ELEMET(I,4)
LCOUNT(I2)=LCOUNT(I2)+1
MUNODE(I3)=MUNODE(I3)+ELEMET(I,4)
LCOUNT(I3)=LCOUNT(I3)+1
82 CONTINUE
DO 94 I=1,HN

AVERAGE VALUE OF VISCOSITY.
MUNODE(I)=MUNODE(I)/LCOUNT(I)
94 CONTINUE
RETURN
END
THIS READS IN ALL THE INFORMATION REQUIRED, OBTAINS THE SYSTEM MATRIX EQUATION, THE LOADING VECTOR AND SOLVES THE EQUATIONS.

DIMENSION ALL ARRAYS

DIMENSION DELTA(243), AINDE(81,5), ELEMET(128,5), T0(9), ABC(123,9), A(3), D(3), C(3), IDELTA(6), IDEL(9), TDELTA(243), CE(243), IDELTA(243), CE(243), REAL(K(175,9), KE(9,9), K1(6,6), K2(6,3), UINDE(81)

998 FORMAT(110,30X,'*******************************************/
T**', T                       '/
T**', T                       '///)

CONTROL INFORMATION REQUIRED

CALL CONTROL(1K, IHE, III, HDF, 1H, IHI, DL, H, V, ALPHA)

THE MAIN SUBROUTINE REQUIRED.

CALL SOLVE(ELEMET, AINDE, DELTA, ABC, 1K, IHE, III, HDF, 1H, IHI, DL, H, V, ALPHA, K1, K2, KE, CE)
STOP
END
SECOND PROGRAM FOR THE ANALYSIS OF FLUID FLOW

CONTROL INFORMATION

INITIAL NO. OF D.O.F. \( NK = 243 \)
NO. OF ELEMENTS \( NE = 128 \)
NO OF NODES \( NN = 81 \)
UNKNOWN NO. OF D.O.F. \( NDF = 175 \)
NON ZERO NO. OF D.O.F. \( N = 9 \)
BANDWIDTH \( IBW = 29 \)
DIE LENGTH \( DL = 1.00000 \)
HEIGHT OF DIE AT INLET \( H = 1.00000 \)
WIDTH OF DIE \( W = 1.00000 \)
ANGLE OF TAPER OF DIE ALPHA = 0.00000 RADIAN

NODAL INFORMATION

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