A COMPARISON OF CONVENTIONAL AND REVERSE TRANSFORMER DESIGN

P.S. Bodger, M.C. Liew and P.T. Johnstone

Department of Electrical and Electronic Engineering
University of Canterbury

Abstract

A reverse approach to transformer design is presented in this paper. The physical characteristics and dimensions of the windings and core are the specifications. By manipulating the amount and type of material actually to be used in the construction of the transformer, its performance can be determined. Such an approach is essentially the opposite of the conventional transformer design method. Both design methods are applied to two sample high voltage transformers. The measured performances of the as-built transformers highlight the limitations in using the conventional method, which are overcome using the reverse approach.

1. INTRODUCTION

From a manufacturer's perspective it is convenient to design and produce a set range of transformer sizes. Usually, the terminal voltages, VA rating and frequency are specified. These specifications decide the materials to be used and their dimensions. This approach to transformer design has been utilised and presented in detail in textbooks [1,2]. It has been used as a design tool for teaching undergraduate power system courses at universities [3-5]. In addition, it has also been used extensively in designing switched mode power supplies [6,7]. Finite Element Analysis has also been applied, concurrent with the above approach, to aid the overall design process [8,9].

However, by designing to rated specifications, consideration is not explicitly given to what materials and sizes are actually available. Core and winding material suppliers offer catalogues of preferred sizes. This reflects the supplier's manufacturing capabilities in extrusion, rolling and forming tools and equipment. It is not economic to offer customers any size and shape they require. It is possible that an engineer, having designed a transformer, may then find the material sizes do not exist. The engineer may then be forced to use available materials. Consequently the performance of the actual transformer built is likely to be different from that of the design calculations.

A reverse design approach is presented in this paper, whereby the physical characteristics and dimensions of the windings and core are the specifications. By manipulating the amount and type of material actually to be used in the transformer construction, its performance can be determined. Such an approach lends itself to designing transformers using what is available from suppliers. This is essentially the opposite of the conventional transformer design method. It allows for customised design, as there is considerable flexibility in meeting the performance required for a particular application.

2. CONVENTIONAL DESIGN

Consideration is given to the layout of the core and windings of a two winding. The laminated core occupies the central space. The windings are wrapped around the core, with the low voltage (LV) winding inside the high voltage (HV) winding. Insulation is allowed for between the core and windings, between windings, around winding wire and in between each layer of winding if required.

The yokes and limbs of the core are additional to this. They depend on whether the transformer is a “core” or “shell” type [10] and have dimensions determined by the boundaries of the windings and cross-section of the core. Usually, for smaller transformers, the core laminations come in discrete sizes. For shell type cores they may be fabricated to eliminate waste from stamping from rolled strip. Such “scrapless” EI cores [11] have specific ratios for their window dimensions and magnetic path sizes.

In the conventional approach to designing transformers, the terminal voltages, V_1, V_2, VA rating, S, and frequency, f, are specified. Material characteristics then lead to calculation of core and winding dimensions. Based on the designer's experience, core steel with known relative permeability, μ_r, and knee point flux density, B, is chosen. A stacking factor, SF_c, is assigned to account for the lamination's metal and insulation composition. A window width factor, WWF, (the ratio of winding space height to width) is also selected, again on experience.

For the primary and secondary windings, acceptable current densities, J_1, J_2, volts per turn, VT_1, VT_2, and space factors, SF_1, SF_2 (amount of copper to winding cross-sectional area) are chosen. The current density estimates are made based on generally accepted thermal capacities of transformer winding material. Typically this is 1-2 A/mm^2 for copper or aluminium.

The volts per turn reflect a designer's experience and may differ from one manufacturer to another. In practice the values vary from under unity to more than 50, with inside this range being most typical. An empirical formula cited in the literature [10] is

$$VT = \frac{\sqrt{S}}{VTF} \tag{1}$$

where VTF = voltage per turn factor

All this achieves is to move the problem of estimating the volts per turn to the factor. No calculation is presented for the latter.

The space factors depend on voltage ratings and the insulation systems used. It is difficult to find any general rules for specifying this. Again experience determines the values.

Having specified the ratings and made estimates of the other factors listed above, the design procedure for the transformer then follows a more calculated path. The current ratings are

$$I_1 = \frac{S}{V_1}, \quad I_2 = \frac{S}{V_2} \tag{2}$$

Hence the areas of the winding wires are

$$A_1 = \frac{I_1}{J_1}, \quad A_2 = \frac{I_2}{J_2} \tag{3}$$

Thicknesses, t_1 and t_2, can be calculated for circular cross-section wires from (in general)

$$t = \sqrt{\frac{4A}{\pi}} \tag{4}$$

From the chosen volts per turn, the number of turns per winding is

$$N_1 = \frac{V_1}{VT_1}, \quad N_2 = \frac{V_2}{VT_2} \tag{5}$$

from which the winding ratio is

$$a = \frac{N_1}{N_2} \tag{6}$$

This is only the same as the voltage ratio if the volts per turn are the same for both windings.

From the ‘Transformer Equation’ [12]

$$V_1 = 4.44 N_1 f \phi \qquad (\phi = BA_c) \tag{7}$$

The area of the core can be calculated from

$$A_c = \frac{V_1}{4.44 N_1 f B} \tag{8}$$

The actual core dimensions include the stacking factor.

$$A'_c = \frac{A_c}{SF_c} \tag{9}$$

The winding window areas are calculated from

$$A_{W1} = \frac{S}{\pi f B SF_c A'_c J_1} \times \frac{1}{SF_1} \tag{10}$$

$$A_{W2} = \frac{S}{\pi f B SF_c A'_c J_2} \times \frac{1}{SF_2} \tag{11}$$

The winding window width and height are

$$WW = \sqrt{\frac{A_{W1} + A_{W2}}{WWF}}, \quad WH = \frac{WWF}{WW} \tag{12}$$

from which the number of winding layers can be calculated.

Having obtained the dimensions of the transformer windings and core, and the material characteristics being known, the components of a transformer equivalent circuit can be calculated. Next, the performance of the transformer under open circuit, short circuit and loaded conditions can be estimated by putting an impedance $Z_L = R_L + jX_L$ across the output and varying its value.

3. REVERSE TRANSFORMER DESIGN

A transformer profile showing known material characteristics and dimensions is depicted in Fig. 1.

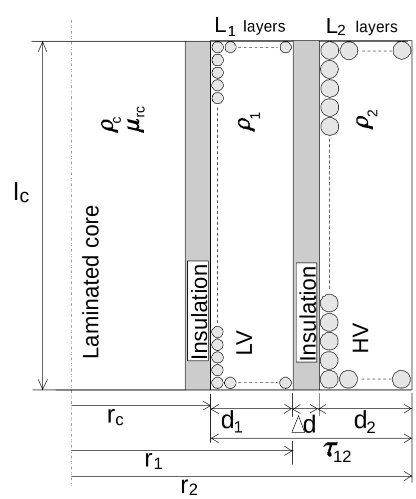

Fig. 1. Axial view of the transformer showing component dimensions and material properties

Instead of 'guessing' the values of SF_1, SF_2 and *WWF*, as required in the conventional approach, these can be accounted for by knowledge of the actual dimensions of materials used. Also winding current densities and volts per turn become a consequence of the design, rather than a design specification.

With respect to Fig. 1, the winding and core material resistivities and permeabilities become specifications. The core cross-section dimensions (diameter for a circular core and side lengths for a rectangular core) are selected from catalogues of available materials. A core length is chosen. Laminations that are available can be specified in thickness and a stacking factor estimated from the ratio of iron to total volume.

For each winding, the wire size can be selected, again from catalogues. They also specify insulation thickness. The designer can then specify how many layers of each winding are wound. Insulation space between layers allows for high voltage applications.

The only rating requirements are the primary voltage and frequency. The secondary voltage and transformer VA rating are a consequence of the construction of the transformer.

The number of turns on the windings is estimated to be:

$$N_1 = \frac{L_1 l_c}{t_1}, \quad N_2 = \frac{L_2 l_c}{t_2} \tag{13}$$

where L_1 = number of primary winding layers
L_2 = number of secondary winding layers
l_c = length of the core

The equivalent circuit shown in Fig. 2 is often used for supply frequencies [12].

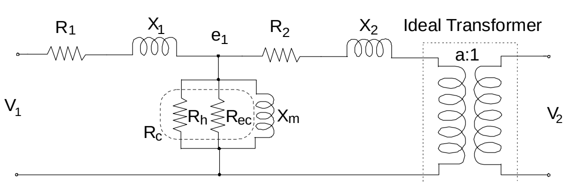

Fig. 2. Transformer equivalent circuit, referred to the primary winding

The analysis of reverse design modelling can be divided into the following parts:

3.1 Resistance Models

3.1.1 Core loss Resistance

The losses in the core consist of two major components; the hysteresis loss and the eddy current loss. The hysteresis loss can be calculated using [13]

$$P_h = k_h f B^x \tag{14}$$

where k_h = a constant (material dependant)
x = *Steinmetz* factor[1]

The eddy current loss is expressed as [13]

$$P_{ec} = \frac{c_l^2}{12\rho_c} \frac{l_c}{N_1^2 A_c} e_1^2 \tag{15}$$

where c_l = lamination thickness
ρ_c = operating resistivity of the core
A_c = cross-sectional area of the core
e_1^2 = induced primary winding voltage

The variation of resistivity with temperature of all materials should be accounted for, since the transformer will be heated up under operation. The operating resistivity at temperature T°C is

$$\rho = \frac{(1 + \Delta\rho T)\rho_{20^\circ C}}{(1 + 20\Delta\rho)} \tag{16}$$

where $\Delta\rho$ = thermal resistivity coefficient
$\rho_{20^\circ C}$ = material resistivity at 20°C

The hysteresis and eddy current losses can be expressed in terms of the induced voltage e_1 as

$$P_h = \frac{e_1^2}{R_h}, \quad P_{ec} = \frac{e_1^2}{R_{ec}} \tag{17}$$

where R_h = hysteresis loss equivalent resistance
R_{ec} = eddy current loss equivalent resistance

Thus, as shown in Fig. 2, both R_h and R_{ec} can be included in the model as the core loss resistance R_c. R_c is expressed as

$$R_c = \frac{R_h R_{ec}}{R_h + R_{ec}} \tag{18}$$

3.1.2 Primary Winding Resistance

The primary winding resistance is

$$R_1 = \frac{\rho_1 l_1}{A_1} \tag{19}$$

where ρ_1 = resistivity of the primary winding wire
l_1 = effective length of the wire
A_1 = cross-sectional area of the wire

---

[1] S*teinmetz* factor has a value between 1.8 and 2.5.

The effective length of the primary winding wire is estimated by calculating the length of wire on each layer of the winding, and then summing over all layers, taking into account the increasing diameter of each layer wound around the previous one.

3.1.3 Secondary Winding Resistance

The secondary winding resistance is

$$R_2 = \frac{\rho_2 l_2}{A_2} \tag{20}$$

where ρ_2 = resistivity of the secondary winding wire
l_2 = effective length of the wire
A_2 = cross-sectional area of the wire

As for the primary winding, the effective length of the secondary winding wire is calculated by approximating the length of the wire on each layer of the secondary winding, and then summing over all layers.

3.2 Inductive Reactance Models

3.2.1 Magnetising Reactance

The magnetising reactance is [14]

$$X_m = \frac{\omega N_1^2 \mu_o \mu_{rc} A_c}{l_c} \tag{21}$$

where μ_o = permeability of free space($4\pi\times10^{-7}$Hm^{-1})
μ_{rc} = relative permeability of core
$\omega = 2\pi f$

3.2.2 Primary and Secondary Leakage Reactances

The primary and secondary leakage reactances are assumed to be the same and are each half of the total transformer leakage reactance [14].

$$X_1 = X_2 = \frac{1}{2}\frac{\omega \mu_o N_1^2 4\tau_{12}\delta'}{l_c} \tag{22}$$

where τ_{12} = winding thickness factor (see Fig. 1)
$\delta' = \frac{d_1 + d_2}{3} + \Delta d$

Having obtained the component values, the equivalent circuit can be solved to calculate the performance of the device. Open circuit, short circuit and loaded circuit performances can be obtained. From this information the secondary voltage and VA rating of the transformer are directly derived. Further, performance measures of voltage regulation and power transfer efficiency for any load condition can be readily calculated. Finally, the current flows and densities in the windings can be calculated and compared to desired levels. These are a result of the design rather than being initial design specifications.

4. DESIGN DATA FOR SAMPLE TRANSFORMERS

As examples of the two approaches to transformer design, two single-phase, 50 Hz, high voltage transformers have been designed, built and tested. Their nominal ratings are listed in Table I. Using the reverse design approach, only the frequency and nominal primary voltage are entered as rated data.

TABLE I
Nominal ratings of high voltage transformers

Transformer	**#1**	**#2**
Primary voltage (V)	240	14
Secondary voltage (kV)	6.24	4.56
VA rating (VA)	200	617

Transformer #1 was designed for the power supply of an electric, water purification device [15]. Transformer #2 was a model, designed to evaluate the harmonic performance of capacitive voltage transformers. Both transformers were built as shell types with rectangular cores. Both transformers were for special applications and not procurable directly from a manufacturer.

Using the conventional design approach, in addition to the rating data above, estimates of the core maximum flux density, stacking factors, current densities, volts per turn factors, and the winding width factor were specified, as listed in Table II.

TABLE II
Data for conventional transformer design

Transformer	**#1**	**#2**
Primary voltage (V)	240	14
Core: Peak flux density (T)	1.5	1.65
Window width factor	3	5
LV winding: Current density (A/mm^2)	2	3
Voltage per turn factor	24	49
Space factor	0.35	0.5
HV winding: Current density (A/mm^2)	2	3
Voltage per turn factor	24	49
Space factor	0.35	0.5

Standard physical values of material permeabilities, resistivities and thermal resistivity coefficients were also entered as data, for the two different steel cores used, and the copper windings, as shown in Table III.

A core stacking factor of 0.95 was estimated for both transformers.

TABLE III
Material constants

	Core	LV Winding	HV Winding
Rel. permeability: Transformer #1 Transformer #2	 3000 9000	 1 1	 1 1
Resistivity at 20°C (Ωm)	1.8×10^{-7}	1.76×10^{-8}	1.76×10^{-8}
Thermal resistivity coeff. (Ωm/°C)	0.006	0.0039	0.0039
Operating temperature (°C)	50	50	50
Density (kg/mm^3)	7870	8960	8960

The dimensions of the various components entered as data for the reverse transformer design method are shown in Table IV.

TABLE IV
Data for reverse transformer design

Transformer	**#1**	**#2**
Core: Length (mm)	 66	 114
Width 1 (mm)	51	152
Width 2 (mm)	44	44
Core/LV insulation thickness (mm)	2	3.25
LV winding: Number of layers	 5	 1
Wire diameter (mm)	0.8	3.55
Interlayer insulation thickness (mm)	0	0
LV/HV insulation thickness (mm)	0.7	6.5
HV winding: Number of layers	 20	 20
Wire diameter (mm)	0.125	0.212
Interlayer insulation thickness (mm)	0	0.09

With the conventional design approach there is a lack of precision to include the interlayer, interwinding and intercore/winding insulation. They are not input parameters. This can have the effect of squashing up the winding space, which affects the calculation of leakage reactances. They are underestimated in value, which affects the calculated performance of the transformer.

5. CALCULATED AND MEASURED PERFORMANCES

The equivalent circuit parameters, referred to the primary, calculated for the transformers using both conventional and reverse design methods, are presented in Table V.

TABLE V
Calculated and measured equivalent circuit parameters for sample transformers

Equi. Circuit Params.	#1			#2		
	Conv.	Revr.	Meas.	Conv.	Revr.	Meas.
R_c(Ω)	3076	3419	3388	6.6	24	18
X_m(Ω)	1755	1982	1987	11	49	41
R_{wind}(Ω)	11.5	8.6	10.0	0.013	0.044	0.043
X_{leak}(Ω)	0.2	1.8	2.8	0.001	0.005	0.012

While there is generally close agreement in the calculation of resistances and magnetising reactance for Transformer #1, the leakage reactance calculation is quite different. However, the equivalent circuit parameters for Transformer #2, calculated by the conventional and reverse design methods, show a significant difference.

The measured values of these equivalent circuit parameters, as determined by open circuit and short circuit tests are also shown in Table V. These show that the reverse design method, with its particular accounting of actual dimensions, most accurately models the equivalent circuit parameters. However, the notable exception is that of the calculations of leakage reactance of both transformers. This is due to the spacial arrangement mismatches between the calculated and the measured results.

A resistance was placed across the secondary of Transformer #1 to the give rated load conditions at unity power factor. On the other hand, since Transformer #2 was designed for capacitive loads, an open circuit condition was used to compare calculated and measured values. All the results are given in Table VI.

TABLE VI
Calculated and measured rated load performance

	#1			#2		
	Conv.	Revr.	Meas.	Conv.	Revr.	Meas.
V_1(V)	240	240	240	14.05	14.05	14.05
I_1(A)	0.87	0.84	0.76	2.5	0.66	1
V_2(kV)	6.1	6.02	6.2	4.56	4.57	4.57
I_2(mA)	30	30	27	0	0	0
P_1(W)	205	200	181	30	8	8
Effy.(%)	89	89	92	0	0	0
Reg.(%)	2.2	2.9	0.7	0.1	0.1	0.1

The values listed in Table VI show that despite the variation in equivalent circuit parameter estimation, both the conventional and the reverse design methods give performance results which are useful in predicting the actual performance of as-built transformers. The difference in regulation in

Transformer #1 reflects the difference in the calculated and measured values of the leakage reactance. However, in the case of Transformer #2, it can be seen that the conventional method doesn't predict the actual performance well. The primary current and hence the total input power are significantly different to the actual values. For Transformer #2, the conventional method specifies a flux density of 1.65T. On the other hand, the reverse design method calculates a flux density of only 0.35T, which approximates that of the actual transformer under operating conditions. In this case the conventional method is not accurate, and highlights a limitation in estimating correct flux densities and hence performance. Usually, the load on a transformer in operation varies so the design is most about size and ultimate ratings. Either of the approaches can be taken depending on the limitations present.

6. CONCLUSIONS

Conventional transformer design starts from a consideration of required frequency, voltage and VA ratings. It estimates a number of factors for the core and winding arrangement, using values that are generally only known to experienced design engineers. The resultant design may not match what is actually available in materials and hence the predicted performance can be in error.

The alternative presented in this paper is to reverse the design procedure. The dimensions of core and winding materials are entered based on what is available. The overall size, ratings and performance of the transformer can then be predicted.

Sample high voltage transformers have been designed, built and tested. The results highlight the problems associated with conventional design and show the usefulness of the reverse design approach. Such a design philosophy allows for the exploration in the design of transformers with alternative construction options, where flexibility in shape and size is required.

7. REFERENCES

[1] Lowdon, E., *Practical Transformer Design Handbook*, McGraw-Hill, Inc., 2nd edition, 1989.

[2] McLyman, W.T., *Transformer and Inductor Design Handbook*, Dekker, New York, USA, 2nd edition, 1988.

[3] Rubaai, A., “Computer aided instruction of power transformer design in the undergraduate power engineering class”, *IEEE Trans. on Power Systems*, Aug 94, v. 9, No. 3, pp. 1174-1181.

[4] Jewell, W.T., “Transformer design in the undergraduate power engineering laboratory”, *IEEE Trans. on Power Systems*, May 90, v. 5, No. 2, pp. 499-505.

[5] Shahzad, F., and Shwehdi, M.H., “Human-computer interaction of single/three phase transformer design and performance”, *Industrial and Commercial Power Systems Technical Conference*, May 97, pp. 193-196.

[6] Hurley, W.G., Wölfle, W.H. and Breslin, J.G., “Optimized transformer design: Inclusive of high-frequency effects”, *IEEE Trans. on Power Electronics*, July 98, v. 13, No. 4, pp. 651-659.

[7] Petkov, R., “Design issues of high-power high-frequency transformer”, *Proc. 1995 International Conference on Power Electronics and Drive Systems*, Feb 95, v. 1, pp. 401-410.

[8] Asensi, R., Cobos, J.A., Garcia, O., Prieto, R., and Uceda, J., “A full procedure to model high frequency transformer windings”, *IEEE Power Electronics Specialist Conference PESC’94*, June 94, v. 2, pp. 856-863.

[9] Allcock, R., McClelland, R., Holland, S.A., and Roué, A., “Transformer design analysis using finite element methods”, *IEE Colloquium on Computation in Electrostatics*, 1995, pp. 8/1-8/3.

[10] Slemon, G.R., and Straughen, A., *Electric Machines*, Addison-Wesley, Inc., USA, 1980.

[11] Flanagan, W.M., *Handbook of Transformer Applications*, McGraw-Hill, 1986.

[12] Paul, C.R., Nasar, S.A., and Unnewehr, L.E., *Introduction to Electrical Engineering*, McGraw-Hill, Inc., Singapore, 1986.

[13] Slemon, G.R., *Magnetoelectric Devices: Transducers, Transformers, and Machines*, John Wiley and Sons, Inc., USA, 1966.

[14] Say, M.G., *Alternating Current Machines*, Longman Scientific & Technical, England, 1983.

[15] Johnstone, P.T., and Bodger, P.S., “High voltage disinfection of liquids”, *IPENZ Trans.*, Nov 97, v. 24, No. 1, EMCh, pp. 30-35.