The Multicriteria Aircraft Landing Problem

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MCDM 2011
Outline

1. Introduction to the Aircraft Landing Problem (ALP)
2. The Multicriteria Problem
3. Sample Results
4. Conclusion
‘Runway congestion is causing delays’

Soubhik Mitra, Hindustan Times
Mumbai, April 19, 2011

The extra time taken by airplanes to vacate the runway is one of the reasons behind the delays at Mumbai airport, stated a report by civil aviation ministry presented in a review meeting on delays held earlier this month. According to the minutes of the meeting (a copy of which is with HT), flights have

Jetstar to sue Sydney Airport for massive flight delays

April 20, 2011 6:06 PM EST
Airports and Delay

“The efficient operation of airports, and runways in particular, is critical to the throughput of the air transportation system” (Balakrishnan and Chandran, 2010)

Even if there is adequate capacity, peaked schedules and/or randomness in aircraft arrivals causes delay.
Wake Vortex Constraints

Aircraft movement creates turbulence especially during take-off and landing notably *wingtip vortices*

Turbulence is especially hazardous during take-off and landing
Wake Vortex Constraints

There are required temporal separations between operations on a common runway

These wake vortex separations depend upon lead and trail aircraft type operation type

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Table 2.1: The Minimum Separation Matrix
The Aircraft Landing Problem

The system is easily modeled via mathematical programming:

- Sequence and schedule aircraft landings on a common runway to maximize capacity
- Sequence-dependent job shop scheduling (see also irrigation scheduling)
Aircraft Landing Problem: decision variables

\[ x_i \equiv \text{the time operation } i \text{ is performed} \]

\[ \delta_{i,j} \equiv \begin{cases} 1 & \text{if operation } i \text{ is performed before operation } j \\ 0 & \text{otherwise} \end{cases} \]
Aircraft Landing Problem: base formulation

\[
\begin{align*}
\text{min} & \quad \sum_{i \in I} x_i \\
\text{s.t.} & \quad E_i \leq x_i \leq L_i & \forall i \in I \\
& \quad \delta_{i,j} + \delta_{j,i} \leq 1 & \forall i, j \in I \\
& \quad x_i - x_j + \delta_{i,j}(S_{j,i} + L_j - E_i) \geq S_{j,i} & \forall i, j \in I \\
& \quad \delta_{i,j} \in \{0, 1\} & \forall i, j \in I
\end{align*}
\]
Aircraft Landing Problem: early and late costs

\[
\begin{align*}
\text{min} & \quad \sum_{i \in I} \alpha x_i + \beta y_i \\
\text{s.t.} & \quad E_i \leq (T_i + x_i - y_i) \leq L_i \quad \forall i \in I \\
& \quad \delta_{i,j} + \delta_{j,i} \leq 1 \quad \forall i, j \in I \\
& \quad (T_i + x_i - y_i) - (T_j + x_j - y_j) + \delta_{i,j}(S_{j,i} + L_j - E_i) \leq S_{j,i} \quad \forall i, j \in I \\
& \quad \delta_{i,j} \in \{0, 1\} \quad \forall i, j \in I \\
& \quad x_i, y_i \geq 0 \quad \forall i \in I
\end{align*}
\]
Aircraft Landing Problem: constrained position shifting

Note. Nodes shaded in black do not belong to any source-sink path and hence can be pruned from the network.
Aircraft Landing Problem: implications

Consider a toy problem: 1 S and 1 H plane waiting to land.

We have two possible sequences: SH and HS.

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Table 2.1: The Minimum Separation Matrix
Aircraft Landing Problem: implications

SH: Small plane lands at time 0, Heavy plane lands at time 60.
HS: Heavy plane lands at time 0, Small plane lands at time 195.

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Table 2.1: The Minimum Separation Matrix

Sequence optimization involves looking for opportunities to move smaller aircraft ahead of larger aircraft in the queue especially if minimizing makespan, if delay costs are equal for all aircraft, if considering long activity sequences.
Aircraft Landing Problem: implications

Favor smaller aircraft ???

Can be bad for the traveling public

Can be bad for environmental impacts

Can be bad for long-term system performance

NB: Benefits of schedule optimization more important when sequencing large numbers of runway operations
Aircraft Landing Problem: alternative objectives

Minimize the costs of fuel burn, noise pollution, and emissions
[ Solveling et al., 2011]

Minimize inequity
[ Soomer and Koole, 2008]
Aircraft Landing Problem: shortcomings

All previous approaches have been single-objective

Require translation to common units
(costs of emissions, inequity)

Results are sensitive to translation models

Only find supported efficient solutions
Outline

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Minimizing Environmental Impacts

\[
\begin{align*}
\text{min} & \quad \sum_{i \in I} x_i & (1) \\
\text{min} & \quad \sum_{i \in I} P_i \ x_i & (2) \\
\text{s.t.} & \quad E_i \leq x_i \leq L_i \quad \forall i \in I & (3) \\
\delta_{i,j} + \delta_{j,i} & \leq 1 \quad \forall i, j \in I \quad (4) \\
x_i - x_j + \delta_{i,j}(S_{j,i} + L_j - E_i) & \geq S_{j,i} \quad \forall i, j \in I \quad (5) \\
\delta_{i,j} & \in \{0, 1\} \quad \forall i, j \in I \quad (6)
\end{align*}
\]
The $\epsilon$-Constraint Method

\[
\begin{align*}
\min & \quad \sum_{i \in I} x_i \\
\text{s.t.} & \quad \sum_{i \in I} P_i x_i \leq \epsilon \\
& \quad E_i \leq x_i \leq L_i \quad \forall i \in I \\
& \quad \delta_{i,j} + \delta_{j,i} \leq 1 \quad \forall i, j \in I \\
& \quad x_i - x_j + \delta_{i,j}(S_{j,i} + L_j - E_i) \geq S_{j,i} \quad \forall i, j \in I \\
& \quad \delta_{i,j} \in \{0, 1\} \quad \forall i, j \in I
\end{align*}
\]
The $\epsilon$-Constraint Method
Minimizing Reversals

\[ \min \sum_{i \in I} x_i \]  
\[ \min \sum_{i,j \in Z} \delta_{i,j} \]  
\[ \text{s.t. } E_i \leq x_i \leq L_i \quad \forall i \in I \]  
\[ \delta_{i,j} + \delta_{j,i} \leq 1 \quad \forall i, j \in I \]  
\[ x_i - x_j + \delta_{i,j}(S_{j,i} + L_j - E_i) \geq S_{j,i} \quad \forall i, j \in I \]  
\[ \delta_{i,j} \in \{0, 1\} \quad \forall i, j \in I \]
The \( \varepsilon \)-Constraint Method

\[
\begin{align*}
\text{min} & \quad \sum_{i \in I} x_i \\
\text{s.t.} & \quad \sum_{i,j \in Z} \delta_{i,j} \leq \varepsilon \\
& \quad E_i \leq x_i \leq L_i \quad \forall i \in I \\
& \quad \delta_{i,j} + \delta_{j,i} \leq 1 \quad \forall i, j \in I \\
& \quad x_i - x_j + \delta_{i,j}(S_{j,i} + L_j - E_i) \geq S_{j,i} \quad \forall i, j \in I \\
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\end{align*}
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Sample Problem

20 aircraft landing on a common runway

Earliest times of arrival are randomly generated (U[0,30])
Latest times of arrival are set to 50

Environmental impact data from [Solveling et al., 2011]
Assume delay can be absorbed in cruise phase of flight
Sample Results

- NO_x Emissions (lbs)
- Delay (flight min)

![Graph showing NO_x emissions vs. delay in flight minutes]
Introduction to the Aircraft Landing Problem (ALP)

The Multicriteria Problem

Sample Results

Conclusion

- Delay (flight min)
- CO Emissions (lbs)

\[\begin{array}{c|c|c|c|c|c|c|c|c}
\hline
\text{Delay} & 400 & 420 & 440 & 460 & 480 \\
\hline
\text{CO Emissions} & 33.8 & 34.0 & 34.2 & 34.4 & \\
\hline
\end{array}\]
Sample Results

The graph shows the relationship between delay (flight minutes) and SO2 emissions (lbs). The data points are scattered across the range of delays from 390 to 420 minutes and emissions from 21.15 to 21.45 lbs.
Passenger Cost (USD) vs. Delay (flight min)
Outline

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Conclusions

We have formulated and solved Bicriteria Aircraft Landing Problems

There are trade-offs between environmental impacts and delay

There are trade-offs between different delay objectives
Future Work

We need to test the approach on larger-scale problems
  Multicriteria problems
  Multiple runway problems

Further research is needed to define appropriate objective functions
  Equity particularly poorly understood