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The Probability Density Functions and the Uncertainty of the EN12354 Prediction Method

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ABSTRACT

The probability density functions (PDF's) of each of the terms described in the prediction method of EN12354-1 and ISO10848-1 were examined in this study using experimental data and Monte Carlo simulations. The results of this study will provide a necessary insight for the calculation of the uncertainty of the apparent sound reduction index. A knowledge of the PDF's is particularly needed if the prediction method is to be applied to lightweight building constructions which may exhibit larger variances in experimental measurements than heavy, monolithic constructions. Based on the results of this study, it is expected that the uncertainty of the logarithmic terms of the prediction method may be calculated using the guidelines of the ISO Guide to the Expression of Uncertainty in Measurement. However, further analysis is needed to clarify the PDF's of measurements made in a reverberant room before definitive conclusions about the use of the GUM method can be made.

1 INTRODUCTION

The standards EN12354-1 [1] and ISO10848-1 [2] currently do not include estimates of the uncertainty of the prediction methods they describe. If the uncertainty of these methods is to be understood, an essential first step is to determine the probability density functions (PDF's) of the terms described in the standards. If the PDF's of the terms can be described by a Gaussian distribution, then the ISO Guide to the Expression of Uncertainty in Measurement (GUM) [3] may be used as a guideline for determining the uncertainty of the prediction methods. However, if the terms are described by PDF's other than the Gaussian distribution, the method of propagating uncertainties according to GUM should not be used [4]. The currently unreleased supplement to GUM [5] proposes the use of Monte Carlo simulations (MCS) in these cases [6]. MCS are able to provide much richer information about the prediction method by propagating the distributions of the inputs through the measurement model to provide the distribution and the uncertainty of the output [7].

As part of this study, the PDF's of the input terms for the EN12354 method were evaluated using laboratory data. For each term, the PDF was assessed in each 1/3 octave band between the 100 Hz and 5000 Hz center frequencies. MCS were then used to propagate the PDF's of the input terms through the mathematical models outlined in EN12354-1 and ISO10848-1 with the goal of determining the PDF's of the predicted terms. The results of the study will indicate if the GUM methods can be used to determine the uncertainty and the expanded uncertainty of the calculations, since the expanded uncertainty is also affected by the PDF's of the terms being described. Knowledge of the PDF of the terms will also provide

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an insight into the prediction methods since all of the terms described in the standards were assumed to have a Gaussian PDF.

2 METHODS

2.1 Monte Carlo simulations

The process of MCS begins with a mathematical model which describes the system. For this study, the models were the equations of EN12354-1 and ISO10840-1. Random observations from the PDF's of the inputs of the model were generated using a pseudo-random number generator. The process of synthesizing the input quantities was done in MATLAB using the Hill-Wichmann algorithm [8, 9] to generate pseudo-random values drawn from a uniform distribution. The synthesized values could then be transformed into a Gaussian distribution using the Box-Muller method [10, 11]. Other distributions could be generated by transforming the uniform and the Gaussian distributions using the transformation described by Hahn [12]. Statistical tests were performed on the synthesized data from the PDF's to ensure that the generated values accurately represented the distributions. The tests included goodness-of-fit tests as well as comparisons of the shape and scale functions of the synthesized and original data sets.

The synthesized values were then propagated through the model to evaluate the performance of the synthesized system. This process is shown in Figure 1 where the input terms x_1, x_2 and x_3 are propagated through the mathematical model to determine the PDF of term y which is a function of x_1, x_2 and x_3.

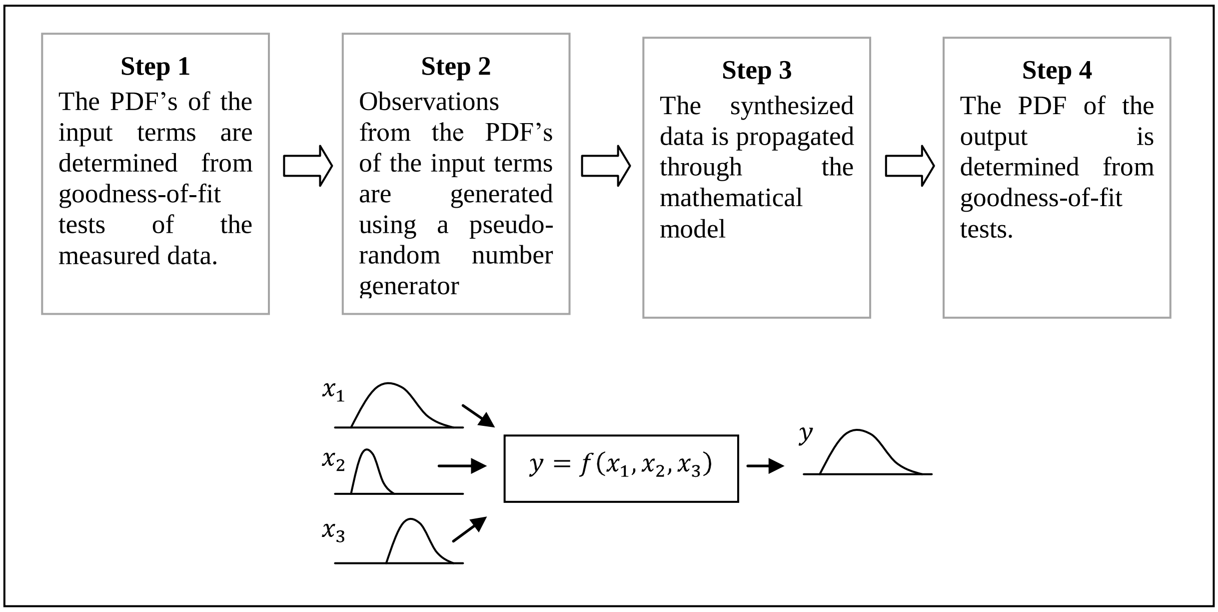

Figure 1: Illustration of the procedure for evaluating the PDF of the output terms.

The last step is to perform goodness-of-fit and other statistical tests were performed on the output data from the simulations to determine the PDF and the uncertainty of the EN12354 predictions.

2.2 Measurements

The PDF's and the scale and shape parameters of the input terms for the EN12354 method were based on an analysis of experimental data. The experimental data included the time and spatially averaged mean square velocity (mean square velocity) measured according to ISO10840-1 on several test panels, the mean square sound pressure level and the reverberation time of a reverberation room, the sound intensity level measured as part of transmission loss measurements according to ISO15186-1:2000 [13] and the structural reverberation time of test panels. In each case, a large number of measurements were made. Histograms of the measured observations were generated and goodness-of-fit tests were used to determine the PDF's of the measured values.

2.3 Assessment of Distributions

The distributions assessed in this study included the Gaussian, exponential, log-normal and the gamma distribution. The choice of the distributions was based on prior studies of the probability density function of several of the input terms [14-17]. The PDF's for each term were assessed in each 1/3 octave band using goodness-of-fit tests including the chi-square test (χ^2), the Kolmogorov-Smirnov test (K-S), and the Anderson-Darling statistic [18, 19]. The PDF that was assigned to the term was the one that fit the data in all of the 1/3 octave bands.

3 RESULTS

The PDF's of the terms of the EN12354 prediction method are described in Table 1. The table includes the standard and equation number where the term is defined, if applicable.

Table 1: Summary of the probability density functions of the terms used in EN12354 and ISO10848. Also noted are input terms from ISO3741:1988 [20], ISO15186-1:2000 and ISO140-3:1995.

Term	Symbol	Unit	PDF	Standard	Equation
Mean square velocity on a panel	v^2	$(m/s)^2$	Log-normal	-	-
Velocity level on a panel	L_v	dB	Gaussian	-	-
Mean square sound pressure in a reverberant room	p^2	Pa^2	Log-normal or Gamma	-	-
Sound pressure level in a reverberant room	L_p	dB	Gaussian or Gamma	-	-
Reverberation time in a reverberant room	T	s	Log-normal or Gamma	-	-
Sound power in a reverberant room	L_w	dB	Gaussian or Gamma	ISO3741:1988	8.2.2
Sound Intensity Level	L_I	dB	Gaussian or Gamma	ISO15186-1:2000	4
Sound power from intensity	L_W	dB	Gaussian or Gamma	-	-
Sound reduction index	R	dB	Gaussian or Gamma	ISO140-3:1995 ISO15186-1:2000	5 7
Transmission loss	τ	-	Log-normal or Gamma	-	-
Radiation efficiency	σ	-	Log-normal	-	-
Vibration transmission factor	d_{ij}		Log-normal	-	-
Velocity level difference	$D_{v,ij}$	dB	Gaussian	ISO10848:2006	8
Direction averaged velocity level difference	$\overline{D_{v,ij}}$	dB	Gaussian	EN12354-1:2000	12
Structural reverberation time	T_s	s	Gamma	EN12354-1:2000	-
Equivalent absorption length	a	m	Log-normal	EN12354-1:2000	11
Vibration reduction index	K_{ij}	dB	Gaussian	EN12354-1:2000	10
Flanking transmission loss	τ_{ij}	-	Log-normal or Gamma	EN12354-1:2000	13
Flanking sound reduction index	R_{ij}		Gaussian	EN12354-1:2000	13
Apparent sound reduction index	R'	dB	Gaussian	EN12354-1:2000	1

4 DISCUSSION

4.1 Mean Square Pressure

The results show that it is possible to describe several of the terms with more than one PDF. For example, the mean square sound pressure in a reverberant room was equally well described within the assessed frequency range as having a log-normal or a gamma PDF. These results agree with prior studies which have assessed the PDF as being either log-normal or gamma distribution [14-17]. Furthermore the gamma distribution can have a wide variety of shapes based on the shape and scale functions and therefore empirical data can often be fitted equally well by either the gamma or the log-normal distribution [12].

Additional measurements of the mean square pressure are currently being analyzed to determine if one of the distributions describes the mean square pressure better, particularly at the lower frequencies where the assumption of diffuse sound field may not hold. The additional data is expected to also yield further insight into the PDF's of other terms that are calculated using the mean square pressure as an input such as the pressure level in the reverberant room and the sound reduction index.

4.2 Mean Square Velocity

In the calculation of the average velocity level, the standard ISO10848 assumes that the mean square velocity has a Gaussian distribution. This assumption is valid as long as the variance of the measured data is no larger than one-third the mean of the data [21]. Above this value, the assumption of a Gaussian distribution no longer holds. For heavy building constructions such as concrete, the assumption of a Gaussian distribution may be acceptable. However, in the case of lightweight building construction which may have a large variance in the mean square velocity, the PDF should be considered as log-normal which affects how the average velocity level is calculated.

4.3 Calculation of Uncertainty

Based on the results of the simulations, it is expected that the method of GUM may be used to calculate the uncertainty of the logarithmic EN12354 predictions. These terms include the apparent sound reduction index, the average velocity level difference and the vibration reduction index, all of which have Gaussian PDF's. However, a definitive conclusion regarding the use of GUM can not be made until the additional analysis work is completed. Future work will also include the calculation of the uncertainty of all of the terms.

5 CONCLUSIONS

Monte Carlo simulations based on experimental data have been used to describe the probability density functions of the terms for the prediction of the apparent sound reduction index according to EN12354. The knowledge of the PDF's is an essential step in the determination of the uncertainty of the predictions as well as the expanded uncertainty. Further analysis is needed to identify the correct PDF of the mean square pressure in a reverberant room and of terms which are calculated from the mean square pressure. Based on the results, it is expected that the method of GUM can be used to determine the uncertainty of the logarithmic terms.

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