THE CONTROL OF FLEXIBLE-LINK ROBOTS CARRYING LARGE PAYLOADS: FROM THEORY TO EXPERIMENTS

A thesis submitted in partial fulfilment of the requirements for the Degree of Doctor of Philosophy in Mechanical Engineering in the University of Canterbury by Eftychios G. Christoforou

UNIVERSITY OF CANTERBURY
CHRISTCHURCH • NEW ZEALAND

1999
To my parents

George and Erinoulla Christoforou.
Abstract

Robotics is a very large engineering field with a wide and constantly expanding range of applications. Apart from its economic significance, the elegant nature of the problem and its multidisciplinary character make it a most interesting as well as challenging field of study. Flexible-link robots in particular, constitute one of the richest and at the same time most difficult problems ever encountered in the field. Conventional rigid-link robots, both in terms of dynamic analysis and control, can be viewed as a special case of flexible ones which results from suppressing the elastic configuration dependence.

Flexible-link robots have already been employed in space operations and a large potential is seen for a new generation of flexible robots to be used in earth-based applications. Such robots are endowed with certain attractive features but at the same time flexibility affects both their tracking and positioning capabilities and dramatically complicates the control problem. The control of flexible-link robots, which constitutes an engineering challenge, is the key to their success.

The present thesis deals with the control problem for flexible multilink robots manipulating large payloads, which is a case that commonly occurs in a space robotic manipulation scenario. Before the control problem is actually tackled, both the dynamics and control problems are reviewed in detail and the current state of technology in the field is surveyed. Then, building on previous research, solutions to the problem are proposed and thoroughly investigated. In particular, a model-based control technique is considered together with its adaptive counterpart, which is able to deal with the problem of uncertainty in the mass properties. Both schemes belong to a family of controllers called passivity-based, which by nature exhibit good robustness characteristics. Apart from the theoretical results and the extensive simulation
studies that were involved in the research, one further step was taken with the actual hardware implementation of the proposed control techniques. The experimental work involved a specially designed robotics facility which will be described in detail. Case studies will be presented in order to demonstrate the applicability and the value of the controllers, provide insight into their nature and investigate their characteristics. Because of the multidisciplinary nature of the problem, the thesis involves the integration of knowledge from various scientific areas such as dynamics, control, sensor technology, mechanical design, materials science, etc.
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Eftychios Christoforou
October 1999
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<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$B_n$</td>
<td>Rigid or flexible body (n-th in the chain)</td>
</tr>
<tr>
<td>$B$</td>
<td>Control influence matrix</td>
</tr>
<tr>
<td>$C$</td>
<td>Matrix with the nonlinear terms of the flexible dynamics</td>
</tr>
<tr>
<td>$C_\theta$</td>
<td>Matrix with the nonlinear terms of the joint–based rigid dynamics</td>
</tr>
<tr>
<td>$C_\rho$</td>
<td>Matrix with the nonlinear terms of the task–space rigid dynamics</td>
</tr>
<tr>
<td>$\sigma_{klj}$</td>
<td>Christoffel symbols of the first kind</td>
</tr>
<tr>
<td>$c_z$</td>
<td>First moment of inertia of a link</td>
</tr>
<tr>
<td>$D$</td>
<td>Structural damping matrix of the flexible dynamics</td>
</tr>
<tr>
<td>$D_{ee}$</td>
<td>&quot;Elastic–elastic&quot; part of $D$</td>
</tr>
<tr>
<td>$E$</td>
<td>Eigenmatrix</td>
</tr>
<tr>
<td>$E$</td>
<td>Young’s modulus</td>
</tr>
<tr>
<td>$\mathcal{F}_n$</td>
<td>Reference frame attached to $B_n$</td>
</tr>
<tr>
<td>$\mathcal{F}_r(\cdot)$</td>
<td>Rigid forward kinematics map</td>
</tr>
<tr>
<td>$F$</td>
<td>Generalized end–effector force</td>
</tr>
<tr>
<td>$F_{\text{non}}$</td>
<td>Nonlinear terms of the flexible dynamics</td>
</tr>
<tr>
<td>$F_{\text{non},e}$</td>
<td>&quot;Elastic&quot; part of $F_{\text{non}}$</td>
</tr>
<tr>
<td>$F_{\text{non},\theta}$</td>
<td>&quot;Rigid&quot; part of $F_{\text{non}}$</td>
</tr>
<tr>
<td>$F_{\text{nc}}$</td>
<td>Nonconservative forces</td>
</tr>
<tr>
<td>$f_{\text{non},e}$</td>
<td>Nonlinear terms of the payload dominated elastic dynamics</td>
</tr>
<tr>
<td>$G$</td>
<td>The controlled system’s transfer matrix</td>
</tr>
<tr>
<td>$g$</td>
<td>Gravity terms of the flexible dynamics</td>
</tr>
<tr>
<td>Symbol</td>
<td>Description</td>
</tr>
<tr>
<td>--------</td>
<td>-------------</td>
</tr>
<tr>
<td>$H$</td>
<td>The controller’s transfer matrix</td>
</tr>
<tr>
<td>$I_r$</td>
<td>Matrix with joint rotor inertias</td>
</tr>
<tr>
<td>$I$</td>
<td>Second moment of inertia of a link</td>
</tr>
<tr>
<td>$I_r$</td>
<td>Joint rotor inertia</td>
</tr>
<tr>
<td>$J$</td>
<td>Jacobian</td>
</tr>
<tr>
<td>$J_e$</td>
<td>Elastic Jacobian</td>
</tr>
<tr>
<td>$J_\theta$</td>
<td>Rigid Jacobian</td>
</tr>
<tr>
<td>$K$</td>
<td>Stiffness matrix of the flexible dynamics</td>
</tr>
<tr>
<td>$K_{ee}$</td>
<td>“Elastic–elastic” part of $K$</td>
</tr>
<tr>
<td>$K_d$</td>
<td>Derivative feedback gain matrix</td>
</tr>
<tr>
<td>$K_i$</td>
<td>Integral feedback gain matrix</td>
</tr>
<tr>
<td>$K_p$</td>
<td>Proportional feedback gain matrix</td>
</tr>
<tr>
<td>$k_d$</td>
<td>Derivative feedback gain scaling factor</td>
</tr>
<tr>
<td>$k_i$</td>
<td>Integral feedback gain scaling factor</td>
</tr>
<tr>
<td>$k_p$</td>
<td>Proportional feedback gain scaling factor</td>
</tr>
<tr>
<td>$L_2$</td>
<td>The standard norm bounded signal space as used in i/o theory</td>
</tr>
<tr>
<td>$L_{2e}$</td>
<td>The extended version of $L_2$</td>
</tr>
<tr>
<td>$L$</td>
<td>Lagrangian</td>
</tr>
<tr>
<td>$l$</td>
<td>Length of a link</td>
</tr>
<tr>
<td>$M$</td>
<td>Mass matrix of the flexible dynamics</td>
</tr>
<tr>
<td>$M_{ee}$</td>
<td>“Elastic–elastic” part of $M$</td>
</tr>
<tr>
<td>$M_{\theta e}$</td>
<td>“Rigid–elastic” part of $M$</td>
</tr>
<tr>
<td>$M_{\theta \theta}$</td>
<td>Mass matrix of the joint-space rigid dynamics</td>
</tr>
<tr>
<td>$M_{pp}$</td>
<td>Mass matrix of the task-space rigid dynamics</td>
</tr>
<tr>
<td>$m$</td>
<td>Mass of a link</td>
</tr>
<tr>
<td>$\overline{M}_{ee}$</td>
<td>Mass matrix of the payload dominated elastic dynamics</td>
</tr>
<tr>
<td>$N_e$</td>
<td>Number of mode shapes</td>
</tr>
<tr>
<td>$q$</td>
<td>Generalized coordinates</td>
</tr>
<tr>
<td>$q_e$</td>
<td>Elastic coordinates</td>
</tr>
<tr>
<td>$q_{ed}$</td>
<td>Estimate for the elastic coordinates</td>
</tr>
<tr>
<td>$q_{e\alpha}$</td>
<td>“Elastic” part of $q_\alpha$</td>
</tr>
</tbody>
</table>
\( q_{n,c} \) Elastic coordinates of \( B_n \)
\( q_\alpha \) Unconstrained mode shape
\( \tilde{q}_c \) Error between the actual and estimated elastic motion
\( R \) Rayleigh's dissipation function
\( r_n \) Position vector with respect to \( F_n \)
\( s \) Scheduling signal
\( S_\mu \) Suitably defined nonnegative function
\( s_\mu \) Filtered errors
\( T \) Kinetic energy
\( t \) Time
\( t_f \) Final time for the maneuver
\( t_i \) Initial time for the maneuver
\( U_e \) Elastic deflection
\( U_{e,i}(\text{tip}) \) Elastic deflection at the outboard end of the \( i \)-th flexible link
\( U_n \) Shape function for \( B_n \)
\( u_n \) Elastic deformation in \( B_n \)
\( u \) System's input
\( u_c \) Controller's input
\( u_d \) Feedforward or disturbance
\( V \) Potential energy
\( W \) Regressor matrix (in task-space coordinates)
\( x \) State of a linear system
\( x \) Translational task-space DOF
\( Y \) Regressor matrix (in body coordinates)
\( y \) System's output
\( y_c \) Controller's output
\( y_d \) Reference signal or sensor noise
\( y \) Translational task-space DOF
\( \alpha \) Vector of suitably selected manipulator parameters
\( \tilde{\alpha} \) Parameter estimation error
\( \hat{\alpha} \) Parameter updates
\(\Gamma\) Adaptation gain matrix
\(\gamma\) Adaptation gain scaling factor
\(\Delta T\) Sampling period
\(\delta W_{nc}\) Virtual work
\(\varepsilon\) Strain
\(\eta\) Modal coordinates
\(\theta\) Joint rotations vector
\(\theta_d\) "Fictitious" joint angles (nonadaptive case)
\(\theta_f\) Final joint-space configuration for the maneuver
\(\theta_i\) Initial joint-space configuration for the maneuver
\(\theta_r\) "Fictitious" joint angles (adaptive case)
\(\theta_t\) Rigid inverse kinematics joint configuration
\(\theta_\alpha\) "Rigid" part of \(q_\alpha\)
\(\theta\) Joint rotation
\(\ddot{\theta}\) Joint-space tracking error
\(\Lambda\) Weighting matrix
\(\mu\) Real parameter
\(\mu^*\) Critical value for \(\mu\)
\(\rho\) Generalized end-effector position
\(\rho_d\) Desired end-effector trajectory
\(\rho_f\) Final task-space configuration for the maneuver
\(\rho_i\) Initial task-space configuration for the maneuver
\(\rho_r\) Filtered position
\(\rho_\mu\) Modified output of the flexible system (\(\mu\)-tip position)
\(\rho_{\mu d}\) Desired trajectory for \(\rho_\mu\)
\(\ddot{\rho}\) Task-space tracking error
\(\ddot{\rho}_r\) Filtered position tracking error
\(\ddot{\rho}_\mu\) Modified output tracking error
\(\sigma(\cdot)\) Singular value of a matrix
\(\tilde{\sigma}(\cdot)\) Maximum singular value of a matrix
\(\underline{\sigma}(\cdot)\) Minimum singular value of a matrix
\[ \begin{bmatrix} A & B \\ C & 0 \end{bmatrix} \]

Standard state-space realization of a linear system

\[ \begin{bmatrix} A_c & K_c \\ K_c & 0 \end{bmatrix} \]

State-space realization of the dynamic controller
<table>
<thead>
<tr>
<th>Abbrev.</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>ACS</td>
<td>Attitude control system</td>
</tr>
<tr>
<td>CCD</td>
<td>Charge coupled device</td>
</tr>
<tr>
<td>CTM</td>
<td>Computed torque method</td>
</tr>
<tr>
<td>DOF</td>
<td>Degree of freedom</td>
</tr>
<tr>
<td>DSP</td>
<td>Digital signal processor</td>
</tr>
<tr>
<td>EVA</td>
<td>Extravehicular activity</td>
</tr>
<tr>
<td>i/o</td>
<td>Input–output</td>
</tr>
<tr>
<td>KYP</td>
<td>Kalman–Yakubovich–Popov</td>
</tr>
<tr>
<td>LHP</td>
<td>Left-half-plane</td>
</tr>
<tr>
<td>LQR</td>
<td>Linear quadratic regulator</td>
</tr>
<tr>
<td>LSS</td>
<td>Large space structures</td>
</tr>
<tr>
<td>MIMO</td>
<td>Multi-input/multi-output</td>
</tr>
<tr>
<td>MRAC</td>
<td>Model reference adaptive control</td>
</tr>
<tr>
<td>P</td>
<td>Proportional</td>
</tr>
<tr>
<td>PC</td>
<td>Personal computer</td>
</tr>
<tr>
<td>PD</td>
<td>Proportional-derivative</td>
</tr>
<tr>
<td>PE</td>
<td>Persistency of excitation</td>
</tr>
<tr>
<td>PI</td>
<td>Proportional-integral</td>
</tr>
<tr>
<td>PID</td>
<td>Proportional-integral-derivative</td>
</tr>
<tr>
<td>PR</td>
<td>Positive real</td>
</tr>
<tr>
<td>RHP</td>
<td>Right-half-plane</td>
</tr>
<tr>
<td>RP</td>
<td>Robust performance</td>
</tr>
<tr>
<td>RS</td>
<td>Robust stability</td>
</tr>
<tr>
<td>SISO</td>
<td>Single-input/single-output</td>
</tr>
<tr>
<td>SPR</td>
<td>Strictly positive real</td>
</tr>
<tr>
<td>STR</td>
<td>Self tuning regulators</td>
</tr>
<tr>
<td>SV</td>
<td>Singular value</td>
</tr>
<tr>
<td>SVD</td>
<td>Singular value decomposition</td>
</tr>
</tbody>
</table>
Chapter 1

Introduction

In this introductory chapter, a brief overview of the contents of the thesis is presented. Present as well as potential future applications for flexible–link robots are discussed in order to emphasize the practical and economic significance of the issue and motivate the subsequent discussions on the control problem. A good understanding of the operations that flexible robots will be required to perform and the characteristics of the environment in which they will be called to serve is of course a prerequisite for both the design and the control of such systems.

The thesis is structured as follows. In Chapter 2, the dynamic modelling of articulated flexible body chains is considered. Various modelling techniques are reviewed and the derivation of the motion equations as used in the present work is discussed in more detail. Important properties of the model are pointed out and unmodelled dynamic effects are discussed. In Chapter 3, an overview of the control problem is provided, its distinct nature is explained and various control techniques proposed in the literature are briefly reviewed. Then, some background on the important concept of passivity is provided, which plays a key role in robotic control and provides the cornerstone for the controllers to be proposed later in the thesis. The focus is then shifted towards the dynamics of a flexible robot manipulating a large payload. An approximate description of the payload dominated dynamics is summarized and the passivity framework related to a suitably defined modified input and output of the plant is considered. Chapter 4 begins with a consideration of standard joint–based techniques commonly used in rigid robot control. A passivity–based scheme suitable
for the tip-position tracking control of flexible-link robots carrying large payloads is then presented together with the relevant proofs of stability and various implementation issues are discussed. A review of adaptive control is made in Chapter 5 and current achievements in the field are highlighted. An adaptive counterpart of the previously mentioned scheme is proposed thereafter, which is supported by proofs of asymptotic stability and relevant issues are treated in detail. In Chapter 6, a detailed description of the facility that was used for the experimental part of the research is made. Its special features are described and practical experimental issues are exposed. The general framework for the experimental work is established, payloads and trajectories are standardized and suitable tools for analyzing the results are defined. Chapters 7 and 8 present experimental case studies on the nonadaptive and adaptive schemes respectively and the results are discussed. Through the case studies, the applicability and the value of the controllers is demonstrated, their nature and characteristics are exposed and their behaviour is investigated. Apart from the results appearing in the main body of the thesis, further experimental results were collected in the appendices. General concluding remarks and directions for further research are found in the closing Chapter 9.

In the present thesis a number of original contributions to the control problem of structurally flexible robots have been made. The thesis provides an extensive, up to date literature review related to this important technological problem and other relevant fields. With the background material, the problem is defined and the major achievements in modelling and control of flexible robotic systems are summarized. A class of controllers proposed in previous work for the case of flexible-link robots carrying large payloads is revisited and some important contributions to their theoretical foundation are made. In particular, a model-based controller together with its adaptive counterpart are considered and their latest versions are proposed in the thesis. These controllers can result in superior performance when compared with their predecessors. They are suitable for the tracking of a desired task-space trajectory with the simultaneous active damping of the vibrations.

The original form of the nonadaptive scheme to be considered is based on an approximate model of the dynamics, whereas here a more complete model of the
system is used. The controller exploits a suitably defined modified input and output for the system. A systematic method for calculating the reference trajectory for the modified output is proposed and investigated in the thesis. A well-defined method for selecting a key property $\mu$ related to the modified output is also presented. For the adaptive case, the proposed scheme identifies parameters corresponding to the mass properties of the entire robot, not just the payload as the original scheme does. A suitable transformation is also suggested, which uses the joint-based rather than task-space implementation of the scheme and avoids difficulties in formulating the regressor matrix involved.

A most important contribution has also been made in the form of experimental work carried out as part of the thesis. For this purpose, a robotics facility was designed and built and various practical problems were tackled. For example, a method that uses strain-gauge measurements to calculate the position and orientation of the end-effector was successfully used. The proposed schemes, which represent a large class of controllers for flexible robots carrying large payloads, were successfully implemented for the first time. The hardware implementation of the controllers provided a bridge over the gap that usually exists between theory and practical applications. Important insights into this class of controllers, as well as the control problem for structurally flexible robots in general was provided.

Due to their distinct characteristics, it is convenient to divide the applications for flexible robots into space and earth-based ones. Each class will be considered here individually. Relevant references are provided by Book [24], [25], where the motivation for using flexible robots is explained, the distinct nature of the problem is discussed, historical, current and future perspectives are illustrated and modelling and control issues are considered. It is important to note that flexible-link robotics technology is closely related to the field of large space structures (LSS), from which various concepts and techniques on modelling and control were borrowed. A general paper on issues related to the control of LSS was provided by Balas [17] and includes a representative list of references on the topic. Important modelling and control issues on LSS can also be found in Kelkar and Joshi [72].
1.1 Flexible–link robots in space

Robots have been successfully employed in space operations in order to improve their productivity and reduce the cost and risk involved, by enhancing astronaut capabilities and executing tasks that would otherwise have to be performed by humans. A comprehensive overview of the space manipulation problem and the current state of technology can be found in the collection of papers in [138]. Like any other space system, robots are weight-critical structures simply because launch mass increases the transportation costs. Such applications demand that the robots are able to deal with large-scale objects and therefore need to have long-reach capabilities. The simultaneous requirements for reducing the launch mass and a large workspace yield robots with long, lightweight links which are inherently flexible.

Certain operations in the space environment require astronauts working outside the spacecraft, which is commonly referred to as extravehicular activity (EVA). Such activity in the unfriendly space environment involves much risk for humans and the time the astronauts can spend outside the spacecraft is limited by the required life support systems. Some of these operations can alternatively be performed using robots operated by crew members while situated inside the spacecraft. In other words, the use of teleoperated robots can transform certain operations from EVA into intravehicular and also considerably reduce the workload for the astronauts.

Due to the lack of gravity, static deflections as would occur on earth are not experienced. Flexible robots operating in space deflect only under the effect of contact and/or inertial forces due to accelerating motion. Consequently, space applications allow the use of robots with flimsy links. At the same time though, the zero gravity space environment further complicates the manipulation problem. Any object manipulated by the robot should only be grasped or released with extreme care because if it escapes the retrieval can be a serious problem.

Space robots are typically mounted on the Shuttle, with the attachment point being inside the cargo bay. They are capable of performing a variety of tasks such as the deployment and retrieval of objects from the Shuttle's cargo bay, the capture and despinning of communication satellites and the performance of various scientific
experiments. Of great importance is their involvement in the assembly of LSS, which due to their size can only be launched in parts and assembled in space. Robots can facilitate the assembly work by transporting the various parts and securely hold them in position while being connected. The EVA involved might also be facilitated by robots, which can serve as transportation and positioning mechanisms for the astronauts themselves while performing assembly operations. Specially designed foot restraints can be attached to the robot's end-point and provide a base that securely holds and locates the astronaut and the various tools involved in performing a certain task [138].

The potential of having earth-based operators performing tasks with robots in space while in teleoperation mode is an active area of research. Man-hours in the unhospitable space environment are extremely costly. Alternatively, an earth-based control station can provide a safe and comfortable working environment for the human operators and will allow for a variety of tasks to be executed more efficiently. At the same time more computational power can be dedicated to the tasks. The main problem in implementing such approaches is the communication delays that do not allow closing stable control loops with earth. Technological achievements in this area are expected to enhance the existing space robotic applications and also give rise to new ones. A general perspective on the issue of telerobotics can be found in Sheridan [130], which summarizes the related terminology, reviews historical, present as well as future applications and raises relevant man–machine interaction issues. The discussions in Pennington [112] are more focused on the hurdles that need to be overcome before the use of a space telerobotics system becomes more feasible. In Backes et al. [14], the development of a control station for the on-board or Earth-based teleoperation of a robot operating in space is reviewed.
1.2 Flexible-link robots in terrestrial-based applications

A different philosophy prevails in the design of industrial robots, which have traditionally been very heavy and bulky structures so that undesirable flexibility effects are not encountered. Consequently, for a typical manipulator that operates in an industrial environment, the ability to perform rapid maneuvers is limited by its own weight, high capacity actuators are required, the capital investment is high and the operation is energy inefficient. The mass carrying capacity of industrial robots is restricted so that deflections due to static or inertial forces do not affect their tracking and positioning accuracy. In other words, they are characterized by low ratios of load to arm weight, which depend on how flexible the system is. In order to avoid elastic deflections due to inertial forces, their speed of operation is also restricted. In Aleksander and Piers [5], the fact that very often the massive components of the robot itself are being moved about in order to accomplish a relatively trivial effect with the end-effector, is described as one of the most striking impressions one gets while watching robots in operation. That constitutes a tremendous waste of energy which will become even more considerable with the rapid growth of the robot population. Lightweight, flexible models on the other hand, exhibit many desirable features and carry the potential to replace conventional designs in many terrestrial-based applications. In the future, it is expected that commercially available flexible models will appear. In a dynamic technological environment, changing requirements often necessitate that engineering concepts are reconsidered and design concepts evolve accordingly. Sometimes radically different solutions emerge but more commonly they are differentiated versions of the existing ones. In robotics, a new generation of flexible-link robots can be seen as the result of such an evolution process.

Strictly speaking, every physical body possesses some degree of flexibility. Flexibility is a phenomenon exhibited even by the most conservatively-designed rigid robots but is usually small enough to allow ignoring it. When it comes to high precision manipulation that necessitates extreme tracking or positioning accuracy, flexibility can no longer be ignored and ways to deal with it need to be found. Therefore, whether
an arm should be treated as rigid or flexible depends on the task to be performed, the size of the manipulated payload and the speed of operation.

A terrestrial–based application for flexible robots can be found in nuclear sites where the presence of humans is prohibited. They can be used in radioactive material handling, facilitate nuclear waste management and perform relevant maintenance and repair work at the site. The use of robotic technology can prevent personnel radiation exposure. In the past, hazardous waste materials from nuclear energy production or research facilities were commonly stored in underground tanks or aboveground concrete silos. An immediate need to remediate contaminated sites has become a high priority issue in various cases. One of the most feasible approaches considered is the deployment of suitable remediation tools inside the tanks by using long reach robotic arms. A good description of the problem, the application requirements and the retrieval approaches can be found in Jansen et al. [70]. Robotic arms for such applications will be required to penetrate the storage facilities through small access holes and, apart from long reach, they will need to have high positioning accuracy and high lifting capacity as well.

A different potential application for earth–based, long–reach flexible robots can be identified in the assembly of large scale structures such as commercial airplanes or ship building. Among the desirable features of lightweight robots is that they can be transported easily. Therefore, they are very well suited for mounting them on mobile platforms or walking machines. A lot of interest has been expressed from robot manufacturers in developing such models and satisfactory control solutions have been requested. The operation of flexible robots can also be safer, and if an accidental collision occurs, damage of equipment can be prevented due to the low inertia and the compliance of the flexible members. In an environment where robots coexist with humans, flexible robots can also be seen as a step towards “human friendlier” robots because in the event of an accident a serious injury might be avoided. In cases where robots are used for performing tasks that require contact with delicate objects, e.g., the cleaning of a glass surface, mechanical compliance of the links can even be a desirable feature.
Chapter 2

System Modelling

Before any solutions to the control problem are considered, a sound understanding and insight to the flexible-link robot dynamics must be ensured. It is worth pointing out that dynamics and control, which are often described as "twin disciplines", cannot be treated in isolation. Robotic control for example would have never reached its current level of maturity without the simultaneous study of the two disciplines. In this chapter, general modelling issues will be considered, the derivation of the model used will be described, fundamental properties of the model will be pointed out and unmodelled dynamic effects will be discussed.

In general, the dynamic equations of motion serve two purposes. The first is the solution to the "forward dynamics" problem; in other words, find the motion given the applied torques. Alternatively, this is called the simulation problem and is used for predicting the behaviour of the system. In the present research, simulation studies allowed testing and refining the theoretical results and provided a useful tool for controller design as well. Simulations were also used during the design of the facility and assisted sizing the actuators. The second problem related to the dynamics is the "inverse dynamics", i.e., find the inputs that produce a certain motion. As it will be discussed later, this problem has a special feature in the flexible-link case. Moreover, the dynamic equations of motion are useful for control purposes and the controllers that make an efficient use of the model and its properties are called model-based controllers. The trajectory tracking problem commonly requires the employment of such techniques.
One issue that needs to be emphasized is that a mathematical description of a physical system can never be exact and there is always some uncertainty related to it. Strictly speaking a model is an idealization of reality. Increasing the accuracy of the model also builds upon its complexity and including minor dynamic effects sometimes adds much more to the complexity than it does on accuracy. Therefore, in practice we deliberately choose to ignore some effects and unavoidably increase the uncertainty in order to facilitate the practical usefulness of the model. Complicated models require more computational power and take more time when calculated on computers. Computational efficiency is very important for simulation studies and necessary for real-time control or man-in-the-loop simulations. Very simple models on the other hand may become inaccurate. Therefore, a compromise between the two conflicting requirements has to be established. Engineering experience, judgment and intuition play an important role in this critical decision.

To derive the equations of motion, fundamental physical laws can be applied. There are basically two widely used and well established avenues to be followed. The first is the Newton-Euler method, which is a direct application of Newton's second law to each individual body of the system. All the forces and moments acting on each member need to be taken into account. This also includes all the constraint ones, which act at the joints and maintain the kinematical conditions of the mechanism. After formulating the equations for each body, additional mathematical manipulations are required to eliminate all of the constraint forces from the description. On the other hand, the Lagrangian approach is an energy-based method that can be more systematic and more efficient since all of the constraint forces are automatically eliminated from the formulation. It is a very general and unifying method in mechanics that provides a framework for modelling complex systems comprising of a variety of different components like mechanical, electrical, hydraulic, thermodynamic, etc.

Studies on the equivalence of the Lagrangian and the Newton-Euler formulation were made by Silver [134], for the case of rigid robot manipulators. It was shown that the two methods are indeed equivalent and yield the same numerical solutions. The specific needs of a problem in hand dictate which method one should use. For example, when analyzing the mechanism for design purposes, an analytic vectorial
method that also yields the constraint forces is the most suitable one.

2.1 Modelling of flexible robots – An overview

One option is to derive the model of the flexible plant by completely neglecting its compliance, i.e., treat it as rigid. Depending on the degree of flexibility, this can be a very rough approximation. In the thesis, we will often refer to the mechanism that has the same geometric and inertial properties as the flexible arm but is composed of absolutely stiff links, as the “corresponding rigid robot”.

For the flexible single-link case, the behaviour of the system can be accurately described using partial differential equation models. Such models are suitable for describing simple beams but cannot easily be obtained for the multilink case. Moreover, such a form is not particularly useful for control purposes. For the one-link flexible arm, the behaviour can be adequately represented by a linear model, which provides acceptable accuracy. An example of such a model was presented in Hastings and Book [63], where a linear state-space description for a single flexible-link manipulator was built, various modelling issues were considered and experimental verification was performed. In the multilink case though, linearized models provide rough approximations that fail to capture the dynamic behaviour of the plant with acceptable accuracy. The contribution of the nonlinear terms to the overall behaviour of the system becomes even more significant when fast maneuvers are involved. It is also important to point out that linearized models are only valid in the neighbourhood of a given operating point which was used for the linearization.

A few of the most important contributions to the modelling of flexible-link robots will be briefly mentioned here. The modelling techniques proposed in the literature either provide models in a closed analytic form or models implemented recursively. In Book [23], a computationally efficient recursive Lagrangian approach was presented. A Lagrangian approach was also used in Cetinkunt and Book [30], to derive the model of the robot in symbolic form. The closed-form dynamic modelling for planar multilink robots was treated in De Luca and Siciliano [51], using the method of Lagrange.
A finite element method was suggested by Naganathan and Soni [98], [99] and employs Newton–Euler laws of motion and Timoshenko beam theory. Important work on the modelling of elastic multibody chains was presented in a series of papers with contributions from Hughes, Sincarsin and D’Eleuterio [136], [69], [56] and [135]. The matrix notational framework introduced therein, results in an elegant and compact description of the dynamics, which not only is well suited to computer implementation but also facilitates better understanding of the dynamic system.

A different class of modelling techniques is the one that treats each flexible body in the chain as if it consisted of a series of rigid ones connected to each other by discrete elements. In Yoshikawa and Hosoda [165] for example, each flexible member is represented by a series of assumed rigid links connected by passive springs and dampers. The parameters involved are selected in order to match the behaviour of the model with actual static and dynamic measurements from the real plant, such as static deflections and vibration frequencies. Such methods have not gained wide popularity among researchers. As claimed by Hughes [67], they might converge to the correct solution for a very large number of bodies but other methods are preferable.

### 2.2 Discretization of link deflections and the Lagrangian approach

The model used in the present work was built using the Lagrangian approach and the whole procedure will be explained here without going too much into the notational details. For building the model, the description of the kinematics involved standard frame transformation matrices which account for both the rigid and the elastic motion. Attempting a manual derivation is definitely a laborious task, time consuming, prone to mistakes, extremely difficult and almost impossible for complicated geometries. Alternatively, a symbolic mathematics computer package proved to be a powerful tool for carrying out this task. A closed form for the equations was obtained, which is suitable for simulating the dynamic behaviour of the system as well as the design and the implementation of the controllers. The availability of the model in an expanded
form allowed more insight to the physics of the system and aided understanding the interplay of the parameters involved.

Figure 2.1: An open-loop chain of rigid and/or flexible bodies.

The robot is modelled as a chain of $N$ flexible and/or rigid bodies $\{B_0, B_1, \ldots, B_N\}$ and a reference frame $\mathcal{F}_n$, $n = 1, \ldots, N$, is rigidly attached to the inboard end of each one to describe its position and orientation, as shown in Figure 2.1. $B_0$ is taken to be rigid and fixed, whereas body $B_{N+1}$ is a rigid payload attached to the tip of the robot. Its body frame $\mathcal{F}_{N+1}$ locates the end-effector within the operational space. Since our motivation stems from space applications, gravity effects are not taken into account and single degree-of-freedom (DOF) rotational joints between bodies are considered, with $\theta_n$ being the corresponding joint rotations. Multiple DOF rotational joints can be modelled by additional links of zero mass and length. It is assumed that one actuator corresponds to each rotational DOF. Translational DOFs between the links will not be considered since they add more complexity to the problem.

Each flexible link is a continuous system with distributed flexibility, and theoretically, it possesses an infinite number of DOFs. In practice though, a finite-dimensional model is needed in order to be useful for simulation and control purposes, in the context of the problem under study. The most widely used methods for describing the link deflections are the finite elements and the assumed modes method (Ritz method). The latter approach was used here. This method is based on the assumption that the
deflected shape of a flexible member can be adequately described by appropriately selected shape functions with time-varying coefficients. The elastic deformation in \( B_n \) is represented as:

\[
\mathbf{u}_n(\mathbf{r}_n, t) = \mathbf{\Psi}_n(\mathbf{r}_n)\mathbf{q}_{n,e}(t)
\]  

(2.1)

with \( \mathbf{\Psi}_n \) being the row matrix of shape functions, \( \mathbf{q}_{n,e} \) the column of the elastic (deformation) coordinates for \( B_n \) and the vector \( \mathbf{r}_n \) denotes position with respect to \( \mathcal{F}_n \). Using this method, a finite-dimensional model for the system can be obtained which captures a sufficiently large number of vibration modes and truncates the rest. In other words, a small number of states is used to describe a distributed system of theoretically infinite DOFs and the accuracy of the model increases with the number of modes captured. On the other hand, increasing the number of modes represented in the model over a sufficient number does not significantly add to the accuracy of the model but only increase its complexity. In practical control applications, very high frequency content in signals cannot be distinguished from noise. Furthermore, the structural damping inherent in real structures causes more damping to the high frequency oscillations. For the above reasons, it does not make much sense when the model accounts for high frequency system dynamics.

The selection of suitable shape functions is crucial for the success of the method. In principle, the shape functions need only be admissible functions in the sense that they have to satisfy the geometric boundary conditions of the problem. As far as which shape functions (type and order) can best capture the behaviour of the flexible members, there is no universal answer. Engineering intuition and experience need to be employed. Considering the geometry of the arm, the distribution of mass and the size of joint inertias can provide some general guidelines as to which functions constitute a suitable choice. A most important contribution to the issue of selecting shape functions was made by Oakley and Cannon [106], who carried out experimental modal analysis investigations.

A comparative study involving infinite-dimensional transfer functions for a single flexible link without any modal approximation and finite-dimensional ones with various shape functions, were performed in Cetinkunt and Yu [32]. Two of the most popular shape functions among researchers, i.e., pinned-free and clamped-free ones
were considered and the predictions provided under closed-loop control were examined. The role played by the feedback control algorithm in determining the suitable shape functions was emphasized. All their studies were in favor of the clamped-free choice which consistently yielded more accurate results than the pinned-free one. The explanation was that closed-loop bandwidth requirements commonly result in stiff joints, which closely resemble the clamped boundary condition rather than the pinned one. It was also found that the clamped-free model gives very accurate predictions even when only the first two modes are captured by the assumed polynomial. Of relevance is the work of Bellezza et al. [21] who derived the "exact" eigenfunctions of a single flexible slewing link, and provided useful information for the selection of suitable shape functions to be used in conjunction with the assumed modes method.

In the case of the experimental arm used for our investigations, the use of geared actuation results in large moments of inertia at the joints and this fact also favors the use of clamped boundary conditions. Given the shape of the cross section of the flexible links and the fact that the system is restricted to move in the horizontal plane, deflections only occur in the plane normal to their axis of rotation. No stretching or torsional modes were considered on the model. Third order shape functions satisfying cantilever boundary conditions at the inboard end were found to be a suitable choice for the bending modes. The shape functions for the \( n \)-th flexible link were chosen as follows:

\[
U_n(x_n, t) = q_{e,n1}(t)x_n^2 + q_{e,n2}(t)x_n^3
\]  

(2.2)

with \( x_n \) being the distance from the inboard end, measured along the undeflected axis of the link. It was found that increasing the order of the polynomials does not considerably improve the accuracy. In particular, the vibration frequencies of the arm were found to converge with good accuracy to the same numerical values when higher order polynomials were used.

The assumed modes method in its standard form does not account for the foreshortening effect. This effect arises due to the fact that in the deformed state the projected length of the link on its undeflected axis varies. The assumed polynomials are taken to vary from zero to the actual length of the link and not the projected one. A modification to the assumed modes method suitable for including this effect was
proposed in [105]. Foreshortening becomes particularly important when long links and large deflections are involved. In our case, its contribution is only minor and was not considered for simplicity.

The above discretization approach is very well suited to a Lagrangian formulation of the dynamics such as adopted in the present case and allows us to retain the nonlinear terms in the formulation. The Lagrangian approach can be summarized by the following three distinct steps:

1. A suitable set of generalized coordinates, \( \mathbf{q} \), is selected, i.e., a set of parameters that is necessary and sufficient to uniquely describe the configuration of the system [80]. The generalized coordinates may or may not have geometrical interpretation and their number is equal to the number of DOFs possessed by the system. In dynamics, such set of coordinates is not unique but properly selected, can facilitate the formulation of the problem. For the case of a rigid robot, the most suitable and natural choice is the joint rotations. For the flexible case, the joint rotations can be used together with a set of coordinates that capture the elastic motion of the system, i.e., \( \mathbf{q} \triangleq \text{col}\{\mathbf{\theta}, \mathbf{q}_e\} \). The column vector \( \mathbf{\theta} \) contains the joint rotations and \( \mathbf{q}_e \) is the totality of elastic coordinates. Given the above choice of shape functions, two elastic coordinates per flexible link are considered.

2. The relevant energy functions are formulated. The expressions for the kinetic energy, \( T \), and the potential energy (also referred to as strain or deformation energy), \( V \), are formulated which are quadratic functions in the generalized velocities and positions respectively:

\[
T = \frac{1}{2} \mathbf{q}^T \mathbf{M}(\mathbf{q}) \dot{\mathbf{q}},
\]

\[
V = \frac{1}{2} \mathbf{q}^T \mathbf{K} \mathbf{q}.
\]

The strain energy is the energy stored within the body upon deformation, and for the case of a perfectly elastic body, this amount of energy is fully recoverable upon unloading. The matrix \( \mathbf{M} \) is the mass matrix of the system and
describes its inertial characteristics, and $K$ is the stiffness matrix which describes its elastic behaviour. The stiffness matrix was taken to be configuration independent.

The virtual work expression is then considered, $\delta W_{nc}$, which corresponds to a virtual displacement of the system, i.e., an arbitrary infinitesimal change in the vector of generalized coordinates consistent with the kinematic constraints imposed on the system [127], [128]. The term virtual is used to distinguish it from an actual displacement. The fact that the virtual displacements are already in agreement with the kinematical conditions of the problem explains why the constraint forces do not participate in the formulation. To build the virtual work expression it was assumed that the joint torques, $\tau_n$, are the only external nonconservative influences:

$$\delta W_{nc} = \sum_{n=1}^{N} \tau_n(t) \delta \theta_n(t)$$

$$= \tau^T \delta \theta$$

$$= (B \tau)^T \delta q,$$

$$F_{nc}^T \delta q$$

(2.5)

where the joint torques were collected in the column vector $\tau$. Conservative forces, e.g., gravity forces, are the ones that satisfy the law of conservation of energy and all their elements can be calculated from a single scalar energy function. The virtual work of all constraint forces is equal to zero and therefore they are also excluded from the formulation. The vector $F_{nc}$ contains the nonconservative forces, which are readily determined from the expression of the virtual work. The control influence matrix, $B$, consists of a unit block over a zero matrix. The structure of the upper part expresses the fact that there is collocation of actuation torques and joint rotations and the lower part the fact that the system is underactuated, i.e., there are less control inputs than actual DOFs. For a fully actuated and collocated system, the matrix $B$ is a unit matrix such as typically occurs in the rigid–robot case.

Finally, the Lagrangian of the system is formulated which by definition is the
difference between the kinetic and the potential energy of the system:

\[ L = T - V \]  \hspace{1cm} (2.6)

3. Lagrange’s equations are finally applied and all necessary differentiations are performed in order to obtain the equations of motion:

\[ \frac{d}{dt} \left( \frac{\partial L}{\partial \dot{q}} \right) - \left( \frac{\partial L}{\partial q} \right) = F_{nc} = B\tau. \]  \hspace{1cm} (2.7)

### 2.3 Equations of motion and important properties

The multibody flexible dynamics obtained as above are described by a set of second-order simultaneous differential equations, whose number is equal to the number of DOFs of the system. They can be written in a compact form using matrix notation as follows:

\[ M(q)\ddot{q} + K\dot{q} + C(q, \dot{q})\dot{q} = B\tau(t), \quad q = \text{col}\{\theta, q_e\} \]  \hspace{1cm} (2.8)

A term for the nonconservative inherent structural damping forces, \( D\dot{q} \), can also be added to the above equations. This can be defined on the basis of Rayleigh’s dissipation function:

\[ R = \frac{1}{2}q^T Dq, \]  \hspace{1cm} (2.9)

where a viscous damping model was considered and \( D \) is the structural damping matrix. Viscous joint friction can also be directly incorporated within the structure of \( D \) by adding a diagonal matrix with the friction coefficients, to the upper left block. The matrices involved in the description of the dynamics can be partitioned consistent with the definition of \( q \) as:

\[ M = \begin{bmatrix} M_{\theta\theta} & M_{\theta e} \\ M_{e\theta}^T & M_{ee} \end{bmatrix}, \quad K = \begin{bmatrix} O & O \\ O & K_{ee} \end{bmatrix}, \quad D = \begin{bmatrix} O & O \\ O & D_{ee} \end{bmatrix} \]  \hspace{1cm} (2.10)

with \( M = M^T > O \), \( K_{ee} = K_{ee}^T > O \) and \( D_{ee} = D_{ee}^T > O \). The standard notations (\( \geq 0 \)) and (\( > 0 \)) are used to denote positive semidefiniteness and positive definiteness respectively. The vector \( \tau \) is the control input and \( C \) is the matrix with the nonlinear centrifugal and Coriolis terms. In certain occasions it will be convenient to write the
nonlinear terms as \( F_{\text{non}} = -C(q, \dot{q}) \dot{q} \) and partition them in the same manner as the above matrices:

\[
F_{\text{non}} = \begin{bmatrix} F_{\text{non},\theta} \\ F_{\text{non},c} \end{bmatrix}.
\]  

(2.11)

Given that \( M \) is an \( n \times n \) matrix, the \( i \)-th element of the nonlinear terms matrix can be defined using

\[
F_{\text{non},i} = -\sum_{j=1}^{n} \sum_{k=1}^{n} c_{kij} q_j \dot{q}_k,
\]  

(2.12)

where \( c_{kij} \) are called the Christoffel symbols of the first kind [108], [55] and they are defined as:

\[
c_{kij} = \frac{1}{2} \left( \frac{\partial M_{ij}}{\partial q_k} + \frac{\partial M_{ik}}{\partial q_j} - \frac{\partial M_{jk}}{\partial q_i} \right)
\]  

(2.13)

with \( M_{ij} \) the \((i,j)\)-th element of \( M \) and \( q_i \) the \( i \)-th element of \( q \).

The mass matrix \( M \) is always positive definite and this implies that it remains nonsingular and invertible for any configuration of the system. This property can be understood on the basis of the kinetic energy which is a quadratic form of the generalized coordinate rates. Any motion of the system, \((\dot{q} \neq 0)\), corresponds to nonzero kinetic energy. It is only possible for the system to have zero kinetic energy \((T = 0)\) when there is no motion, \((\dot{q} = 0)\).

The off-diagonal blocks in the mass matrix, \( M_{\theta e} \), represent the coupling between the rigid and the flexible motion of the arm. In the present case, for deriving the model that was used for our simulations, dependence of the mass matrix on the elastic configuration was not considered. This is a very common approximation when modelling flexible systems [55]. Consequently, the upper left block, \( M_{\theta \theta} \), becomes identical to the mass matrix of the corresponding rigid robot when formulated in joint space, the dynamics of which are represented by the following equation:

\[
M_{\theta \theta}(\theta) \ddot{\theta} + C_{\theta}(\theta, \dot{\theta}) \dot{\theta} = \tau
\]  

(2.14)

with \( M_{\theta \theta} \) the mass matrix and \( C_{\theta} \) the matrix with the nonlinear terms. Furthermore, by suppressing the elastic configuration dependence of the mass matrix, many of the Christoffel symbols will vanish. Various simulation results based on our model were compared with other results, which were obtained using more complete dynamic models. Very close agreement was found between the two sets of results, and this
fact further justifies the validity of the above assumption. It should of course be mentioned that such an assumption does not affect the structure nor the properties of the dynamics model of the system.

The effect of rotor inertias can easily be included in the flexible robot’s dynamics by simply adding a diagonal matrix $I_r$ to the $M_{\theta\theta}$ part of the mass matrix:

$$I_r = \begin{bmatrix} I_{r1} & 0 & \cdots & 0 \\ 0 & I_{r2} & \cdots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \cdots & I_{rN} \end{bmatrix}$$

(2.15)

with $I_{ri}$, $i = 1, \ldots, N$ being the rotor inertia reflected to the output shaft of the gearbox of the $i$–th joint.

For earth–based manipulators, the effect of gravity needs to be included in the model by adding the relevant terms to the potential energy expression. That will effectively augment the dynamic equations by an extra nonlinear term, $g(q)$, which depends on both the rigid and elastic coordinates of the system.

Cancelling the rows and columns corresponding to the rigid DOFs, the equations capture the vibration dynamics when the joints are locked at the constant configuration $\theta = \bar{\theta}$ at which the elastic equations are evaluated. That is equivalent to the limiting case of infinitely big rotor inertias lumped at each joint which basically allow only for the flexible appendages to vibrate:

$$M_{ee}(\bar{\theta})\ddot{q}_e + K_{ee}q_e = 0.$$  

(2.16)

The importance of the nonlinear terms in the dynamic behaviour of the system was investigated in Naganathan and Soni [99], where the simulated motion predicted by the full nonlinear model was compared with the one predicted by the linear one and showed the inadequacy of the latter. In the same work, a further simplification was attempted by ignoring the inertial as well as the damping terms in order to obtain a quasi–static approximation for the deflections that occur during the motion. Such an approximation was found to be too rough but in certain cases it might provide a quick approximation to the elastic motion. The response predicted by the complete model was shown to exhibit oscillations about such a quasi–static solution.
More detailed studies on the various types of nonlinear terms were made in Damaren and Sharf [49]. The nonlinearities originate from either inertial or geometric considerations and were classified according to that nature. A number of simulation studies were performed therein, each time retaining certain nonlinear terms in the model. It was demonstrated that both the inertial and geometric nonlinearities can have a significant effect on the accuracy of the dynamic model. The inability of the fully linearized model to capture the dynamic behaviour with acceptable accuracy was also clearly shown.

In the study of a physical system, the derivation of a model is not only useful for generating numerical solutions but more importantly to gain insight in the problem by studying the properties of the model. A few of the most important properties of the dynamic model of the flexible arm were listed in [55] and a more complete collection was provided by Arteaga [8], who classified them into three categories. In the first group are properties characterizing the matrices of the model and are closely related to the physics of the system. Most of these properties have already been mentioned above. An important property in that group is related to the passive structure of the system and states that given a suitable factorization of matrix $C$, the matrix $\dot{M}(q) - 2C(q, \dot{q})$ is skew-symmetric. It is important to note that the nonlinear terms can be defined in a number of different ways. The above skew-symmetry property is always valid when the nonlinear terms are defined using the Christoffel symbols and it is frequently used in the stability proofs for robotic control schemes. The second group involves norm bounds on the matrices of the model and can be useful in arguments related to Lyapunov stability analysis. The third group includes two properties related to the whole model. The first one states that with a proper definition of the model parameters, the model can be written as

$$M(q)\ddot{q} + Kq + C(q, \dot{q})\dot{q} = Y(\ddot{q}, \dot{q}, q)\alpha,$$

where $Y$ is called the regressor matrix and $\alpha$ is the vector of suitably selected parameters. Adaptive control has been particularly successful for plants whose dynamics can be written in such a linear form [108]. These plants will be called "linearly parameterizable" and include a large variety of systems found in engineering practice.
including rigid robots. The second property in the same class refers to the passivity property satisfied by collocated inputs and outputs of the system and will be treated later in more detail.

2.4 Modal analysis

A linearized form of the equations of motion in the neighborhood of a constant configuration \( \bar{q} = \text{col} \{ \bar{\theta}, 0 \} \) can be written as

\[
M(\bar{q}) \ddot{\bar{q}} + K \dot{\bar{q}} = B r(t)
\]

(2.18)

with \( \delta \bar{q} = \bar{q} - \bar{q} \). The overbar notation \( \bar{\cdot} \) denotes configuration dependent quantities evaluated at the setpoint undeflected configuration \( \bar{\theta} \). For the linearization, the mass matrix is taken as constant, \( M(\bar{\theta}, 0) \), and the nonlinear terms vanish. Such a linear form of the dynamics allows access to eigenanalysis [47]. For the unforced motion the corresponding eigenproblem can be written as

\[
-\omega^2 \alpha M q_\alpha + K q_\alpha = 0
\]

(2.19)

where \( \omega_\alpha \) are the unconstrained vibration frequencies, i.e., with the joints unlocked so that both the rigid and the elastic DOFs participate to the motion. The eigenvectors \( q_\alpha = \text{col} \{ \theta_\alpha, q_{e\alpha} \}, \alpha = 1 \ldots N_e \), are the corresponding mode shapes. A mode shape is the deformed shape of the structure when vibrating at one of its natural (resonance) frequencies during which all displacements along the member reach their zero and their maximum values simultaneously. The mode shapes can be collected into the eigenmatrix \( E \triangleq \text{row} \{ q_\alpha \} \).

There are as many vibration modes as there are DOFs of the system. The mode shapes related to the elastic DOFs are commonly referred to as elastic or deformation modes. When the system is excited by time-varying inputs the overall response of the system will be the sum of all its vibration modes. Theoretically, flexible-link robots have an infinite number of vibration frequencies. Hughes [68], deals with the question of how many vibration modes exist in a physical system and how many should be incorporated in a dynamic model. It is explained that the vibration mode is just
a mathematical notion which is suitable for describing the behaviour of the actual
dynamic system. While the number of modes considered in the model increases, this
notion becomes inappropriate in the sense that it no longer represents the actual
plant. Having an infinite number of vibration modes for a practical system is just a
misconception that does not correspond to reality.

For the flexible arm, one or more of the eigenvalues are zero. The eigenvectors
associated with the zero eigenvalues correspond to a rigid–body mode of vibration
[128], i.e., the system moves as a rigid body without any deformations occurring in
the elastic elements and consequently no change in the system’s strain energy. This
fact is reflected by the semidefinite nature of the stiffness matrix $K$. Given that the
potential energy is a quadratic form of the generalized coordinates it is possible to
have nonzero values of $\mathbf{q}$ that yield zero strain energy.

A very important property of the system is the orthogonality of the mode shapes.
The mode shapes are assumed to be normalized with respect to the mass matrix and
the orthogonality relations can be expressed as

$$
\begin{align*}
\mathbf{q}^T_{\alpha} \mathbf{M} \mathbf{q}_{\beta} &= \delta_{\alpha\beta}, \\
\mathbf{q}^T_{\alpha} \mathbf{K} \mathbf{q}_{\beta} &= \omega^2_{\alpha} \delta_{\alpha\beta}
\end{align*}
$$

(2.20)

with the function $\delta_{\alpha\beta}$ defined as follows:

$$
\delta_{\alpha\beta} = \begin{cases}
1 & (\alpha = \beta) \\
0 & (\alpha \neq \beta)
\end{cases}
$$

More compactly the orthogonality relations can be written as

$$
\mathbf{E}^T \mathbf{M} \mathbf{E} = \mathbf{1}, \quad \mathbf{E}^T \mathbf{K} \mathbf{E} = \text{diag} \{\omega^2_{\alpha}\}.
$$

(2.21)

These properties allow for the system of linear coupled differential equations that
describe the plant to be transformed to a set of decoupled ones. The resulting equations
are expressed in a set of new coordinates which are called modal coordinates, $\eta(t)$,
and they are defined by the modal transformation, $\delta \mathbf{q}(t) = \mathbf{E} \eta(t)$. Then, a state–
space description for the plant can be obtained and used for control system design
purposes [46].
2.5 Velocity kinematics

For the description of the task-space motion it is convenient to use the generalized position vector of the end-effector, \( \rho \), whose upper part consists of the position coordinates and at the bottom are three Euler angles parameterizing its orientation with respect to the base frame. It will be assumed that the number of controlled task-space DOFs is equal to the number of control inputs (and joint DOFs). The velocity of the end-effector is related to the generalized coordinate rates by

\[
\dot{\rho} = J_\theta(\theta, q_e)\dot{\theta} + J_e(\theta, q_e)q_e,
\]

where \( J_\theta \) is the rigid Jacobian and \( J_e \) the elastic one. The two Jacobians capture the individual contribution that the joint and elastic motion have on the end-point motion. For more details on the formulation of the above matrices refer to [43].

When the elastic dependence of \( J_\theta \) is suppressed, it reduces to the Jacobian matrix of the corresponding rigid robot, which relates the joint-space to the task-space velocities. The significance of this matrix quantity in the kinematics, dynamics and control of a rigid robotic mechanism is very well known. The Jacobian is a configuration dependent quantity and at certain configurations of the mechanism, it loses its full rank and becomes noninvertible. More details on this issue can be found in any standard robotics textbook [10], [39], [147], [125], [164]. Such configurations which are related to the geometry of the mechanism are called singularities. When a singular configuration is encountered the arm loses one of its task-space DOFs, i.e., the tip becomes unable to move in a certain direction. Singularities depend on the geometry of the mechanism and they always appear at the boundary of the manipulator's workspace. At singularities, the inverse velocity kinematics problem collapses and it does not provide a solution. Problems might also be encountered in the vicinity of singularities where the joint velocities corresponding to a small task-space motion become excessively large. It is also a very well known fact that the Jacobian naturally appears in force control applications. Its transpose relates the generalized forces, \( F \), exerted by the end-effector of a rigid robot to the corresponding joint torques:

\[
\tau = J_\theta^T(\theta, 0)F.
\]
At a singularity, the end-point of the rigid robot loses the ability to apply force at a certain direction. Furthermore, in the neighborhood of the singularity small joint torques result to very large end-effector forces in that direction.

In general, a rigid manipulator is required to have six joint-DOFs in order to be able to locate its end-effector with an arbitrary position and orientation in the six-dimensional space. If the arm has more DOFs, an infinite number of joint angles can give the same tip position. In other words, the inverse kinematics problem does not have a uniquely defined solution. Such a manipulator is said to be redundant. It is important to note that whether a rigid robot is redundant or not, depends on the number of end-effector DOFs required for performing a certain task. Assuming that the Jacobian matrix $\mathbf{J}(\theta, 0)$ is a nonsingular $(m \times n)$ matrix with $n > m$, the inverse velocity kinematics solution gives $(n - m)$ arbitrary variables [10]. In the present thesis, we will assume that the corresponding rigid robot is nonredundant for the tasks under consideration.

An important comment that needs to be made is related to the dynamic equations of the corresponding rigid robot. Given the kinematical correspondence that exists between the joint rotations and tip positions, the dynamics of the system can be equivalently formulated either in joint-space coordinates as in Eq. (2.14), or in task-space ones as follows:

$$M_{\rho\rho}(\rho)\ddot{\rho} + C_\rho(\rho, \dot{\rho})\dot{\rho} = J_\theta^T(\theta, 0)\tau,$$

where $M_{\rho\rho}$ is the mass matrix and $C_\rho$ the corresponding matrix with the nonlinear terms.

### 2.6 Model uncertainty and assumptions

As has already been pointed out, the model of a system can never be exact and any departure from the nominal model constitutes a source of uncertainty, which can be attributed to many different sources. In the case of the present research where experimental work is also involved, being aware of the unmodelled dynamics and their effect on the system's behaviour was found necessary for bridging the gap that exists
between model and reality. Various effects commonly excluded from modelling and some assumptions made will be discussed here and their effect will be explained.

Generally speaking uncertainty can be categorized into two groups [38]:

1. **Structured uncertainty** exists when the model’s structure allows for certain phenomena to be captured by the mathematical description but the numerical values of the relevant parameters are unknown. Familiar examples of such parametric uncertainty from the field of robotics are related to the values for the mass and geometric properties of the links, coefficients of the friction model, size of rotor inertias, etc. The adaptive control scheme that will be considered later in this work is aimed at dealing with the structured uncertainty problem.

2. **Unstructured uncertainty** is attributed to effects that were not considered in the model, either intentionally (too difficult to model or omitted for the sake of simplicity) or unintentionally (ignorance). In other words, the structure of the model is not general enough to embrace certain effects that contribute to the overall behaviour of the plant. Examples from the robotics context include neglected link or joint flexibility effects, absent or inaccurate models for friction, ignored structural damping or backlash, etc.

One of the most important sources of uncertainty when modelling robots is the friction at the joints, which is even more significant when geared actuators are used. Friction always affects the tracking performance of a robotic arm, by causing tracking lags and shows up as steady-state error when the arm is regulated at its final target configuration. Because of the importance of the issue, it will be treated in more detail in the following section, where the various ways of dealing with the problem will also be discussed.

Some structured uncertainty in the model might result from inaccurate or incorrect data provided by the manufacturers of various components such as motors and gearboxes. As an example, a small error related to the size of the rotor inertias of the electric motors when projected at the output shaft of the corresponding gearbox are amplified by the square of the gear ratio and become significant. The rotary inertia of
the components within the gearboxes is usually completely unknown. Such information is very difficult to obtain because the reduction is usually done in multiple stages and the amplification of the inertia of each component as seen at the output shaft will be different. In general, we found that data provided by manufacturers were often inaccurate and whenever possible were checked before using them. A different source of uncertainty can be attributed to data related to the geometry of the mechanism and is caused by manufacturing and assembly tolerances or inaccuracies.

The modal truncation related to the use of the assumed modes method constitutes an important source of unstructured uncertainty unless a sufficiently large number of vibration modes is retained in the model. When a controller is designed on the basis of a reduced order model certain problems may arise. The following terms related to this problem are attributed to Balas [17], who originally identified it in the context of LSS. The excitation of the truncated modes by the input is called “control spillover” and their presence in the sensed output is called “observation spillover”. Spillover can have undesirable effects on performance or even destabilize the closed-loop system. Relevant discussions can also be found in [72]. The robustness characteristics of the controller determine how well it deals with this problem.

Two different sources of flexibility are encountered in robotic systems, i.e., flexibility of the links and flexibility of the joints. For typical industrial robot designs link flexibility is not significant and the principal source of compliance is identified at the joints. Joint flexibility can be thought of as a spring (rotational for a rotational joint) connecting the actuator to the link. Such an effect is very common when highly geared reducers are used or when the transmission system involves long/thin shafts or elastic belts. Joint flexibility introduces extra DOFs to the system that affect both the dynamic behaviour of the arm as well as its tracking and positioning accuracy and complicates the control problem. The study of flexible-joint robots is itself a large area of research and more information on relevant modelling and control issues can be found in Readman [116]. For systems with large dimensions like space robots, link flexibility prevails over the elasticity of the transmission elements. Therefore, joint flexibility effects were omitted from our model and constitute a source of unstructured uncertainty. In the present thesis, whenever we refer to flexible robots, link flexibility
will be implied.

It is obvious that any physical structure always possesses some damping. Otherwise, no mechanism exists to extract the vibration energy and once oscillations are excited they would continue for ever. Structural damping causes dissipation of energy due to the relative motion of particles within the material. The damping exhibited by structures is often very small and difficult to measure, thus neglecting it can be a reasonable approximation. Its effect was completely excluded from our model.

The presence of backlash at the actuators is another effect excluded from our model, although it is present on our experimental robot and arises from the mechanical clearance at the joints. Backlash is very common in practice when geared systems are involved, due to gaps between each pair of mating gears. It deteriorates the positioning accuracy, induces vibrations and noise and might also cause wear or damage to the mechanism. Its contribution to the dynamic behaviour of a multibody system can also be significant and in general it lowers the vibration frequencies of the system. Another assumption made about the actuators of the system is that they have no torque saturation limits or more realistically that these limits are high enough so they are not encountered. Otherwise, saturation becomes problematic upon implementation of any control algorithm.

2.7 Friction

The damping effect of friction at the joints plays a significant role in the dynamic behaviour of robots and at the same time is very difficult to model and deal with. Due to its relevance with our experimental work, the issue will be treated here in more detail and relevant work on dealing with the problem will be referenced. A very important survey paper on the topic is by Armstrong-Hélouvry et al. [7]. It brings together contributions from tribology, lubrication and the physics literature and provides an exhaustive list of references.

It is very common in engineering applications to divide friction into two different types which often coexist. First is the Coulomb friction which is sometimes referred to as dry friction, since it usually describes the friction between two unlubricated
surfaces in relative motion. The second type is the viscous friction and is characterized by velocity dependence which might be taken as linear or nonlinear. The Coulomb friction is taken to be proportional to the normal contact force between the two surfaces and a constant coefficient which depends on various factors like the material and the finish of the two surfaces. A simple model for friction is shown in Figure 2.2, which was found sufficient for our case. It is known that in the case of Coulomb friction, the size of the force differs before and after the relative motion begins. When a force is applied, the resistance due to friction before the motion starts is called stiction, $T_s$, and after the relative motion of the two surfaces begins, it is called sliding friction, $T_c$. The stiction force is always higher than the sliding friction. Here, it will be assumed that an equal friction torque acts in either direction. In certain cases though, friction might also exhibit joint position dependence due to eccentricities in gearing or other effects [38]. This fact was investigated in Popović and Goldenberg [114], who experimentally verified the positional dependency of friction using a rigid industrial manipulator and proposed a modelling technique that describes friction both as a function of position and velocity.

In [7], the various ways of dealing with the effect of friction are classified into nonmodel–based and model–based. A short discussion and some examples for each one of the approaches will follow.

**Nonmodel–based strategies:**
1. High proportional and derivative gains commonly referred to as “tight” or “stiff” feedback loops can deal with the problem. Discussion of this issue will be postponed until the experimental case studies where the effect of feedback gain sizes on the steady-state error will be investigated.

2. The use of an integral term as part of a feedback controller has the effect of increasing the restoring force with time and is commonly incorporated in a proportional-derivative (PD) controller for dealing with the positioning error problem. This approach will also be investigated experimentally.

3. The injection of a high-frequency bias signal, also called dither, is commonly used in hydraulic systems for dealing with the effects of friction. In the case of geared actuators this method has often been rejected by robot manufacturers because of the wear that is caused to the mechanism. Its use might also interact with the controller, excite high resonance frequencies and cause instabilities. The implementation of the technique involves augmenting the commanded torques by the dither term [54]:

\[ \tau_{dither} = A_0 \sin(\omega_0 t) \]  (2.25)

with the elements of the diagonal matrix \( A_0 \) being higher than the corresponding stiction torques and the frequency \( \omega_0 \) is taken to be higher than the bandwidth of the closed-loop system.

4. Learning control is an approach proposed for the case of mechanical manipulators that perform repetitive tasks and is aimed at dealing with friction effects. More details on the approach can be found later in Chapter 5.

Model-based strategies:

1. Open-loop fixed compensation based on a certain model is commonly used in order to cancel out the friction torques. Many researchers have suggested different models to be used in friction compensation and vary from very simple to fairly complicated ones that aim to capture some specific friction phenomena. Given that friction always opposes motion, in most models the direction at
which the friction torque acts is decided by the sign of the corresponding joint velocity, \( \text{sgn}(\dot{\theta}) \). The signum function is defined as follows:

\[
\text{sgn}(x) = \begin{cases} 
+1 & (x > 0) \\
0 & (x = 0) \\
-1 & (x < 0) 
\end{cases}
\]

and is a discontinuous function at zero velocities. In practice, this might cause problems in the absence of good quality velocity measurements, especially at near-zero velocities where the sign of the velocities might oscillate about the zero value. It might result in compensation acting in the wrong direction and causing instabilities. In [54], replacing \( \text{sgn}(\dot{\theta}) \) by \( \text{sgn}(\dot{\theta}_d) \) was proposed, i.e., base the compensation on the desired joint angles rather than the actual ones. In the case of flexible-link robots such a strategy is less viable because the desired joint rotations are usually not known \textit{a priori}.

2. An adaptive approach can be very useful in cases where the friction characteristics are unknown or change with time. Varying friction characteristics was a problem encountered in our experimental work and will be discussed later. For the case of task-space adaptive controllers like the one to be considered in the present thesis, including friction effect in the adaptation becomes more complicated than in the case of joint-based schemes. In general, for the adaptive treatment of friction, a model which is linear with respect to its parameters needs to be assumed. In [54], an adaptive scheme for friction compensation was proposed and tested on a rigid industrial robot. Adaptation was based on a model that also captures a distinct phenomenon related to friction, namely, downward bends. According to this phenomenon, after the stiction torque has been surmounted the friction torque decreases exponentially to a much lower value and finally starts increasing proportionally to velocity. Including the downward bends to the model was claimed to help avoid overcompensation problems. Another example of adaptive friction compensation is the scheme proposed by Friedland and Park [61].
2.8 Dynamic interaction of the robot with the base

A very important issue related to space manipulators is the dynamic interaction between the robot and the base to which it is attached. Although we will not deal with this problem, a brief discussion will be useful due to its relevance in space robotics. The system can be viewed as a three-body chain comprising the base, the arm and a payload attached to the tip. The base will normally be the spacecraft or a space structure, which is a free floating body not fixed in inertial space. While the robot moves, it applies inertial forces to the base and affects both its position and orientation, especially when massive payloads are manipulated. Of course, the more massive the base, the less the interaction. In Papadopoulos and Dubowsky [111], the dynamics of a rigid robot attached on a floating base are discussed and is observed that the equations of motion and the Jacobian matrix have exactly the same form as the conventional terrestrial-based robots. In a terrestrial-based rigid manipulator, the Jacobians depend on the joint angles only, whereas in space they depend on the spacecraft orientation as well. In the former case, singularities are functions of the joint configuration (kinematic singularities) but in the latter they depend on the mass distribution as well (dynamic singularities). The paper concludes that because of the similar structure of the equations describing the problem, controllers already used for earth-bound robots can be applicable to the space floating-base case, given that measurements of the spacecraft's orientation are available and dynamic singularities are avoided. A collection of papers on the interaction between the robot and its floating base with contributions from various researchers can be found in [161], although only the rigid robot case was treated.

The spacecraft usually has an active attitude control system (ACS), the aim of which is to maintain its attitude with respect to a reference frame. In [161], the role of the ACS and its different modes of operation are discussed. The actuators used by an ACS can normally be jet thrusters and it was explained that there might be certain reasons for not having the ACS active while the manipulator is in operation. If one of the thrusters suddenly fires, it might cause a jerky motion and result in collision of the arm with a nearby object. Moreover, when the system is turned
off, energy is conserved and possible gas impingement on the manipulated object is avoided. Reaction wheels using photo-voltaic energy can be used as an alternative to jet thrusters and exhibit certain advantages. For the development of our controllers, it will be assumed that the base remains fixed while the robot moves, i.e., the spacecraft attitude control system is active and its operation does not interact with the arm's dynamics nor causes any other anomalies within the manipulator's workspace.
Chapter 3

The Control Problem for Flexible Robots – The Concept of Passivity

In this chapter, the special nature of the control problem for flexible robots will be reviewed and various control schemes proposed in the literature will be referenced. Then, the important concept of passivity will be discussed, some basic definitions will be presented and the relevance to robotic control will be explained. Finally, the case of flexible-link robots carrying large payloads will be reviewed and the passivity framework that will be exploited later for control will be introduced.

3.1 The nature of the control problem

The modelling of multibody flexible dynamics has already been studied extensively in the literature and one might consider it as an already solved problem without much to be added to it. The corresponding control problem though has not yet reached the same level of maturity. The special nature of this problem will be explained here. Relevant discussions on different aspects of the control problem for flexible robots can be found in Book [24], [25].

Generally speaking, space systems need to be extremely reliable since human lives and highly expensive equipment are involved. Thus, decision making is usually governed by conservatism. The common approach to controlling flexible space robots has been to perform very slow motions and then wait for the oscillation to decay
before an action is performed by the end-effector. Regarding the efficiency of this approach, it was pointed out that in a realistic space manipulation case [138], [3], approximately one third of the time spent by the astronaut while operating the arm is just waiting for the oscillations to decay. This fact not only causes delays but also frustration to the operators and suggests that the control technology currently used is not sufficient. It also suggests that the control objective needs to be redefined in order to incorporate the active damping of the vibrations as well.

When dealing with the control problem for flexible manipulators, the engineer is confronted with a nonlinear problem. The problem is highly nonlinear due to the interplay of nonlinear multibody and structural dynamics combined with a difficult nonlinear control problem. It belongs to the class of problems with less control inputs than the number of DOFs, i.e., the system is not fully actuated. When a robot performs an operation with its end-effector, the control objective is naturally defined in task-space. In the rigid case though, it can be equivalently defined as the tracking of the corresponding joint trajectories. Such strategy requires having both the sensors and actuators at the joints, which is also the most convenient location from a practical point of view. The systems for which the locations of the sensors coincides with those of the actuators are said to be collocated. Strictly speaking, collocation not only implies that the physical locations of sensors and actuators coincide but also includes the element of "duality" of the corresponding input and output. In the flexible-link case, the end-point position also depends on the elastic motion and the correspondence between task-space and joint-space trajectories is no longer a simple one. When end-point positions are defined as being the output and the joint torques being the control input, the flexible system is noncollocated. Noncollocation in flexible-link robots significantly complicates the control problem.

For the single-link flexible case, any attempt to control the tip position by using end-point measurements and control torques at the root typically results in stability problems. For a linear model of the system, it is known that the mapping from joint torques to end-point rates is nonminimum phase and this nature is attributed to noncollocation. As explained in Book [24], [25], the behaviour of such systems is
characterized by reverse initial action (undershoot), response delays due to wave propagation and phase in the frequency response that is not "minimum". Nonminimum phase transfer functions have zeros in the right-half-plane (RHP) and inversion-based control schemes always become unstable. By inverting the transfer function, the RHP zeros become unstable poles for the closed-loop system. For the case of a single flexible link, a time delay is typically observed between the application of the torque and the start of the tip-motion. Due to this behaviour, the solution to the inverse dynamics problem leads to noncausal solutions and that basically means that torques start before the motion starts. In a multilink nonlinear context, the equivalent nonminimum phase problem is known as instability of the zero dynamics [52]. Zero dynamics are the dynamics left in the system when the output is forced to be zero or constant.

Flexible-link robots can be seen from the perspective of a more general trend in modern engineering practice: the desire to accomplish demanding performance, economic or other objectives often results in technological products which are inherently more difficult to control. Therefore, control becomes a critical factor for their success. The case of a high performance aircraft identified in Vidyasagar [151], provides a familiar example of this trend. Its stability often has to be achieved by exclusively active means on account of enjoying higher speed and better maneuverability at certain flying conditions. In the same article, given the increasing importance of control in robotics, it is foreseen that in the near future a large part of the total manipulator cost will be attributed to its control processor.

3.2 Controllers for flexible robots – Literature review

Control for rigid robots has traditionally been joint-based. Given a desired task-space trajectory, the control objective translates to having the joint DOFs follow the corresponding joint-space one, which can be obtained through inverse kinematics as a purely geometric solution. In the flexible-link case, it is obvious that such strategies are in general not suitable and something more sophisticated is required which takes
into consideration the elastic nature of the plant. Most of the control approaches proposed in the literature for flexible robots aim at closely following the corresponding rigid inverse kinematics trajectories and rely on structural damping to extract the vibration energy from the elastic modes. Passive damping augmentation techniques are often employed to facilitate the desirable energy dissipation effect. Given the requirements of modern applications, a further step needs to be taken by addressing the tip-tracking problem together with active damping augmentation, which is the philosophy of the present research. Indicative of the limited potential of certain joint feedback schemes is the work of Cetinkunt and Book [31], which investigates the limitations imposed on their performance due to structural flexibility.

A review of various control techniques for flexible arms proposed in the literature will be made here. The review is not exhaustive but it focuses on various techniques that have either received a lot of attention in the control literature and indicate the current state of knowledge in the field, or are somehow related to the present work. Some distinct approaches to the problem will be considered separately and these include active damping schemes that require additional actuators, passive damping techniques and the micro/macro manipulation approach.

Active damping is of great importance and a lot of theoretical and applied research has been conducted in that direction. Indicative of relevant practical research is the work of Demeo et al. [57]. An active damping augmentation scheme is presented for the suppression of the elastic motion, which is sensed by a three-axis accelerometer mounted on the flexible arm. Astronaut operators were used for the evaluation of the system through a real-time, man-in-the-loop simulator.

A large part of the research carried out in the field, concerns the single flexible link case. Although such results have both theoretical and practical value, they do not always extend to the more useful multilink case. In Wang and Vidyasagar [154] for example, the applicability of feedback techniques which succeeded in the single-link case was examined for a class of three-DOF multilink manipulators with the last link being flexible. In particular, linearization techniques were investigated, which commonly use a nonlinear feedback that renders the dynamics linear and allows well established linear system control techniques to be used in stabilizing the plant. The
transition to the multilink case was shown to be problematic.

One example of experimental work carried out on a single flexible-link setup can be found in Yurkovich and Tzes [168], which investigates various identification and control techniques. In Cannon and Schmitz [28], experimental studies on tip-tracking control for a one-link robot revealed some problems related to the noncollocated nature of the problem. The controller used was a linear quadratic regulator (LQR) together with an estimator that uses the available measurements to reconstruct the states. Typical symptoms of nonminimum phase behaviour were clearly observed: with the application of the torque the tip initially remained stationary, then moved in the wrong direction and finally a rapid motion occurred to reach the desired setpoint configuration.

Investigations on input-output inversion control for the single-link case were made in De Luca and Siciliano [50]. Two different outputs were considered, the one being the joint rotations and the other the angular position at a point selected along the length of the link. For the system with the joint rotations as the output, the closed-loop dynamics were found to be stable. For the latter case, it was found that beyond a particular location on the arm the nonlinear inversion-based technique results in instability and end-point trajectories always go unstable. This behaviour is of course attributed to the noncollocated nature of the plant.

In Oakley and Cannon [105] the case of a two-link robot with one flexible member was treated, important modelling issues were considered and tip-position tracking controllers were tested experimentally. A model-based nonlinear inversion control strategy for the multilink case was proposed by De Luca and Siciliano [52], for the accurate joint trajectory tracking. Like most other joint-based approaches, the aim is to follow the corresponding rigid robot's trajectories and structural damping is left to deal with the induced oscillations. The controller consists of a nonlinear state feedback decoupling term and a linear feedback part. In its standard form, the implementation requires full state feedback. A modification was also proposed that avoids the measurement of the elastic states. It involves replacing them with suitable estimates obtained through off-line computation based on the available mathematical model and the desired joint-based motion. The approach demonstrates that stable
joint-based inversion control strategies for flexible-link manipulators are possible.

A distinct approach for dealing with the problem of less control inputs than output DOFs was proposed by Siciliano and Book [132]. A singular perturbation approach was followed and used a decomposition of the system into two reduced order subsystems of different time scales. The slow subsystem was shown to be the joint-based model of the equivalent rigid manipulator and the fast subsystem is related to the elastic motion. A composite control strategy was then applied, which consists of a slow control that deals with the rigid motion and a fast control that stabilizes the resulting fast subsystem's motion. For the separation to be possible, the approach assumes sufficiently stiff flexible links and full state availability is required for the implementation of the technique. In Siciliano et al. [133], the above two time-scale control approach is revisited in order to deal with the lack of full state measurements problem. A conceptually similar approach was followed in Moallem et al. [93], where the tip-position tracking problem is considered. Using the concept of integral manifolds and perturbation theory, two different time-scale subsystems were identified. For the implementation of their composite controller for the two subsystems, only tip position, joint position and joint rate measurements are required.

Trajectory tracking for a controlled system commonly requires the inversion of the input-output map. In the flexible tip-position tracking case, the implications related to the inverse dynamics problem have already been mentioned. Contributions from various researchers aim to providing suitable solutions to the inverse dynamics problem, such as [18], [19], [126] and [97]. The inverse dynamics and kinematics problems, which are coupled in the flexible case, are treated in Bayo et al. [20] who proposed an iterative solution in the frequency domain. Open-loop control experimental results based on that method were presented. For the implementation, the control law was allowed not to be causally dependent on the reference input given that the inverse dynamics solution is noncausal. In other words, the torques were allowed to be applied before the actual tip motion begins in order to avoid the typical initial undershoot behaviour of a nonminimum phase system.

An interesting approach to the inversion problem is the one of Asada and coworkers [9]. A method for solving the inverse dynamics problem was proposed, which can be
used in conjunction with feedforward compensation control techniques. The notion of rigid virtual links was used for describing the dynamics of the flexible system. The virtual links corresponding to each one of the flexible members rigidly connect adjacent joints by straight lines and an angle is used to describe the rotation of each one. Small deformations are assumed so that the distance between adjacent joints can be taken as constant. The approach consists of determining the virtual link motion, calculation of the actual deflections relative to the virtual links and finally calculation of the corresponding torques.

Paden et al. [109], deal with the nonminimum phase problem by using a feedforward based on a causal approximation to the inverse dynamics in conjunction with a passive joint-based feedback control, in order to track a tip trajectory. The feedforward was defined on the basis of a delayed version of the nominal task-space trajectory. Exponential stability was shown for sufficiently stiff and damped links. In Singh and Schy [137], a nonlinear inversion technique is proposed for the case of a multilink robot with a flexible boom and allows the decoupling of the joint motion from the elastic one. Joint torques can then be used for controlling the joint motion and additional force actuators at the tip of the robot are responsible for damping out the induced elastic vibrations. In Boyer and Khalil [26], a computationally efficient recursive Newton–Euler implementation of the above scheme is proposed and the approach was viewed as a generalization of the Luh, Walker and Paul [86] algorithm for rigid robots to the flexible case.

Until today, the major uses of flexible robots have been in space and most of the research was carried out under the assumption of absence of gravity. Such an assumption simplifies the problem and researchers tend to embrace it in their investigations. In the face of new terrestrial-based applications for flexible manipulators further studies on the effect of gravity need to be conducted. In De Luca and Siciliano [53], a very simple joint-based PD controller together with a gravity compensation term was examined for the regulation of a flexible robot operating within the gravity environment. Structural damping was not necessary for demonstrating the stability of the closed-loop system. When joint-feedback controllers are used, positioning of
the tip requires the knowledge of suitable joint rotations that counteract the deflections due to gravity. The problem of finding the joint and deflection variables for a given tip position and constraint force as well, was investigated by Siciliano [131]. An inverse kinematics algorithm was proposed, which is based on a technique earlier suggested for the rigid robot case and is known as CLIK [125]. The method employs a Jacobian matrix that accounts for the static deformations due to gravity and the contact with the environment as well.

An important approach to the control problem for flexible robots is the one of Damaren [45], which deals with multilink manipulators carrying large payloads. The scheme was derived on the basis of a nonlinear approximate form of the dynamics that governs the motion of a structurally flexible manipulator with a large payload. The dynamics were built using a Lagrangian approach under the assumption that the kinetic energy of the system can be approximated only by the portion residing within the large payload. An effective separation of the task-space dynamics from the elastic ones was achieved. The task-space motion equation was shown to be equivalent to a version of the corresponding rigid robot’s task-space dynamics, built upon the mass properties of the large payload alone. These equations were effectively combined with the idea of a suitably defined modified passive input-output (i/o) of the plant in order to derive the control strategy. In [44], a passivity-based adaptive version of the scheme was developed. Simulation studies on both schemes showed that they are suitable for the tracking of tip trajectories and also incorporate active damping of the vibration modes. The above two controllers are the original forms of the nonadaptive and adaptive approaches that will be proposed later in the present thesis.

An area in robotics which has started to attract a lot of attention by researchers is that of cooperating robotic arms, which have the ability to handle payloads of large size or more complicated geometries. This situation resembles the two arms of the human body, the cooperation of which enhances our manipulation capabilities. In the rigid robot case, the problem has received a lot of attention by various researchers who investigated the problem of coordinated motion and internal force control as well [33], [170]. In spite of the achievements related to rigid robots, not many results have
been reported for flexible arms. In [33], Uchiyama briefly describes experimental work carried out using two cooperating flexible arms. In Matsuno and Hatayama [90], a control scheme suitable for this case is presented and tested experimentally but the scheme is not accompanied by proofs of stability. Damaren [48], extends previous results on open-loop flexible arms carrying large payloads to the closed-loop case by using the passivity property of a suitably defined input and output for the system. The output is constructed from the task-space position of the payload with contributions from the joint motion of the two arms as well. The input combines the control torques of the two arms and permits adjusting their relative contribution for load sharing purposes. Experimental results demonstrated the applicability of the approach.

3.2.1 Active vibration suppression using additional actuators

For the case of rigid robots, actuators are typically located at the joints lumped with each rotational DOF of the mechanism. In the flexible-link case, due to compliance there are less control inputs than DOFs. An option for the active damping of vibrations is to consider additional actuators that directly deal with the problem, which of course complicates the structure of the arm. Such vibration suppression techniques can either be discrete, i.e., involve actuators acting at discrete locations on the system or distributed ones, i.e., the actuators are evenly dispersed along the flexible members.

An example of the discrete approach is the case of Singh and Schy [137], already mentioned above. The work of Balas [16], also belongs to the same class of controllers and involves actuators and sensors at discrete locations along the structure which are not necessarily collocated. For practical reasons though, this strategy is not well suited to robotic applications.

The use of additional actuators located at the tip of the arm for improving the tracking performance and suppressing the vibrations was examined in Montgomery et al. [96]. In particular, actuation using torque-wheels was investigated and simulation studies based on the dynamic model of a two-link manipulator with a torque wheel at the free end were performed. Such controllers were shown capable of dealing with
the end-point deflections that occur during motion. Other inertial devices that can be used as actuators attached at the tip might range from control moment gyros to linear reaction–mass actuators or even reaction jets. Such approaches have attracted little attention by researchers due to practical implementation problems.

A distributed control approach consists of using sensors and actuators whose action can be distributed along the length of each flexible member. Well suited for such applications are the piezoelectric materials which convert mechanical stress to electrical charge and generate a mechanical force when an electrical charge is applied to the material. The former is called the direct piezoelectric effect and the latter the converse piezoelectric effect. In other words, piezoelectricity is an electromechanical phenomenon. Layers of such materials can be bonded to the flexible members to serve as distributed sensors and others as distributed actuators for active vibration control. Such an approach was proposed and investigated by Tzou [150] and involves sensing the deflections using the voltage generated by the sensor piezoelectric layer, which is then amplified and injected to the actuator layer. Experimental and simulation case studies were presented in order to demonstrate the effectiveness of the technique.

### 3.2.2 Passive damping control techniques

The approaches dealing with the structural vibrations problem can in general be divided into two groups, active and passive. Active approaches imply the use of actuators that exert forces on the structure, whereas passive ones do not rely on any external source of energy. They rather involve properly selected materials or devices which extract the energy from the vibration modes using dissipation mechanisms. Active damping usually provides more versatility in control, since it only requires writing software programs and does not normally involve any changes to the structure or the materials of the components involved.

The design of flexible robots can take advantage of nonconventional materials like high damping alloys or composites in order to improve the stiffness and the damping characteristics of the structure. Composite materials technology has already been extensively used in many space applications [124]. Very often the benefits are obtained
at the expense of some disadvantages that the mechanical designer has to be aware of [101]. High damping alloys sometimes suffer from low strength, low corrosion resistance, high cost and poor machinability. Composite materials are often costly, exhibit low resistance to impact damage and the repair of damaged structures is difficult.

Common structural materials like aluminium usually possess poor inherent damping characteristics, which can be enhanced by using surface treatments and viscoelastic coatings as detailed in Nashif et al. [101]. When such materials are deformed they dissipate energy in the form of heat due to their internal structure. They are usually rubberlike materials and are widely used in many commercial vibration damping and noise control applications. Surface damping treatments can be divided into two categories. First, is the unconstrained (or extensional) damping treatment where the treatment is bonded on one or both sides of the flexible member. When the member deflects, the viscoelastic material deforms under tension or compression accordingly. The second case is the constrained (or shear) damping treatment. The viscoelastic material is again bonded on the flexible member and its outside surface is also constrained by a metal layer. Upon vibrations the viscoelastic material is forced to deform in shear and the desired effect is produced. In both cases, using multi-layer treatments can enhance the results. Passive damping has been used in a wide spectrum of applications and the relevant technology has reached a high level of maturity. In [123], various applications of passive damping are mentioned ranging from the automotive industry, high performance computer disk-drive systems, sporting goods like snow skis, tennis rackets and baseball bats, medical equipment, noise reduction, etc.

In the context of flexible-link robots, the constrained approach was examined by Alberts and coworkers in [2] and [3]. The approach consists of sandwiching a thin layer of viscoelastic material between the surface of the link and a constraining layer of stiff material as shown in Figure 3.1. Since the viscoelastic material is bonded between two materials of different stiffness, when the flexible member bends the elastomeric material is forced to deform in shear and cause energy dissipation.

When implementing such approaches it needs to ensure that the damping is added
in a safe, reliable, durable, efficient and cost-effective way. A serious limitation to the approach is that viscoelastic materials are commonly sensitive to environmental exposure and high temperatures and also vulnerable to mechanical damage. Another drawback is that the extra material might add considerably to the weight of the structure. Augmenting passive damping to deal with vibrations occurring in more than one direction or the case of both bending and torsional vibrations, becomes more complicated.

Passive techniques can either be used alone or in conjunction with active control. In general, they are more suitable for adding damping to the high frequency content of the vibrations. Active damping techniques on the other hand are more effective in dealing with the low frequency content of the vibrations, which is much easier to sense. High frequency oscillations are very difficult to control by active means because they require wide bandwidth controllers with poor robustness characteristics. Using passive techniques as an augmentation to active control can give the best results, with the active control compensating for the low frequency content and the passive means dealing with the uncompensated high frequencies. Concurrent design of the two techniques can provide more efficient solutions. In the present work, no damping augmentation techniques are considered and the focus will be on active vibration control.

When lightweight arms with long links are designed, given the existing weight limitations it is wise to try and get the maximum possible rigidity for the system. As observed by Book [25], the links' shape can be optimized so that they have increased
rigidity without adding to mass, simply by selecting suitable shapes for their cross-section. Such issues are very important in practice but fell outside the scope of the present research.

3.2.3 The micro/macro manipulation control techniques

Space robotic applications often require that the manipulator performs tasks that need accuracy which cannot be provided by long flexible-link arms. A smaller-scale rigid system known as a micro-manipulator can be attached to the end of the large-scale one. The micro-manipulator will be capable of performing tasks which require fine, accurate and sometimes fast motions. The purpose of the large scale system, which is called the macro-manipulator, will be to locate the micro system in different positions within its workspace. A similar arrangement can also be useful in the nuclear waste removal application already mentioned. The long flexible system will be used to insert and locate the small manipulator inside the waste tank and the micro system will perform the necessary operations therein. A similar geometry can be identified on the architecture of the human body, with the arm (macro-manipulator) used to locate the hand (micro-manipulator) within its workspace in order to perform various smaller scale and accurate operations.

Yoshikawa et al. [166], [167], examined the trajectory following control of such systems and proposed the use of the micro-manipulator motion to suitably compensate for the tracking error due to the deformations at the large scale system, so that the system can track an end-point trajectory. The micro system was assumed to have enough DOFs so that the redundant joint DOFs of the composite system are used in compensating for the elastic deformations. Given the desired tip trajectory, suitable joint-space trajectories were designed and simulation and experimental results were reported. For these strategies, no stability proofs were provided and the issue of active vibration damping was not addressed. Other researchers considered the control problem from a different perspective and raised the issue of suitably using the inertial forces generated by the micro system in order to actively damp the elastic deflections at the links of the large-scale system. Efforts in that direction were presented in Lew
and Trudnowski [81] and involved experimental work as well. In Van Vliet and Sharf [153], a specially designed experimental facility was presented, which can be used as a testbed for the validation of relevant theoretical results. The dynamic modelling of the system was also considered.

### 3.3 The Passivity property

The concept of passivity, which was originally used in network theory, has become a fundamental concept in feedback control. A network is defined as passive if it contains no sources of energy, i.e., it consists of passive R-L-C electric circuit elements which only dissipate or store energy, but no active elements like voltage or current generators. Generally speaking, for passive systems the increase in the stored energy is always lower or equal to the energy supplied to the system by external sources. The difference between the two is the energy dissipated by the system. A notion more general than passivity is dissipativity, a comprehensive treatment of which is provided in the pioneering work of Willems [159], [160]. According to the definitions therein, the storage function is a generalization of the concept of stored energy and satisfies a dissipation inequality involving a function called the supply rate. For the storage function, a lower bound exists which is set by the available storage and an upper one determined by the required supply. The available storage was defined as the amount of internal storage which may be recovered from the system and the required supply as the amount of supply which has to be delivered to the system in order to transfer it from a state of minimum storage to a given state.

![Figure 3.2: Multivariable input–output system.](image-url)
Some basic definitions from passivity theory necessary for the development to follow will be presented here. A complete reference can be found in Desoer and Vidyasagar [58]. A system $G$ with input $u \in L_{2e}$ and output $y = G u \in L_{2e}$, as shown in Figure 3.2, is strictly passive if there exists $\epsilon > 0$ such that:

$$\int_0^T u^T y \, dt \geq \epsilon \int_0^T u^T u \, dt , \forall u \in L_{2e} , \forall T > 0.$$  \hspace{1cm} (3.1)

If the above is satisfied with $\epsilon = 0$, the system is passive.

The importance of passivity in control engineering relies on the close relation with stability as expressed by the passivity theorem, which states that the feedback interconnection of a passive and a strictly passive system with finite gain is $L_2$-stable, meaning that finite energy inputs yield finite energy outputs.

![Figure 3.3: The basic multivariable feedback setup.](image-url)

Figure 3.3 shows the basic multivariable feedback control setup, with $G$ being the controlled plant and the signals $u$ and $y$ the corresponding input and output. The block at the bottom is the controller, $H$, with $u_c$ and $y_c$ the controller's input and output respectively. The external signal $u_d$ is either a feedforward or a disturbance and $y_d$ is either a reference signal or sensor noise. Given the above definitions, if $G$ is passive and $H$ is strictly passive with finite gain then the feedback system is $L_2$-stable. The definitions of the $L_2$ space of norm bounded functions, its extended companion $L_{2e}$ and other relevant definitions as used in the input–output stability theory can be found in [152] or [107]. For the case with $y_d \equiv 0$, the only condition for stability is that the feedback controller $H$ is strictly passive, i.e., it is no longer
required to have finite gain (see Desoer and Vidyasagar [58], Theorem VI.5.1, page 181).

It is interesting to note that controllers originally derived using more conventional techniques based on Lyapunov's method were later interpreted in terms of the passivity theory. A classic example is provided by Ortega and Spong [108], where the term passivity-based control was coined. A complete treatment of this elegant approach can be found in Ortega et al. [107], which examines various control system design applications to Euler–Lagrange systems. This category embraces a wide spectrum of engineering systems which can be modelled using the Lagrangian approach. Therein, it was recognized that such systems define passive maps with the storage function being the total energy of the system. Contrary to the class of linearizing controllers, passivity-based ones do not rely on the exact cancellation of the nonlinear effects and the passivity theorem that underlies their design yields a "strong" form of stability.

In a linear context, it is known that passive systems are characterized by positive real (PR) transfer functions with relative degree not greater than one and exhibit minimum phase characteristics. The linear equivalent of a strictly passive system is a system characterized by a strictly positive real (SPR) transfer function. According to Tao and Ioannou [149], a controller characterized by the transfer function $H(s)$ is SPR if and only if:

1. $H(s)$ is real for real $s$ and $H(s)$ is analytic for $\Re\{s\} \geq 0$;
2. $H(j\omega) + H^H(j\omega) > 0$, $-\infty < \omega < \infty$;
3. $\lim_{\omega \to \infty} \omega^2[H(j\omega) + H^H(j\omega)] > 0$,

where $(\cdot)^H$ denotes the Hermitian of a matrix, i.e., its conjugate transpose.

An equivalent statement for the SPR condition related to the state-space description of the system is provided by the Kalman–Yakubovich–Popov (KYP) Lemma or Positive Real Lemma [149]. Consider a stable linear–time–invariant system with a minimal state–space realization:

$$H = \begin{bmatrix} A_c & K_e \\ K_c & 0 \end{bmatrix}.$$

(3.2)
A necessary and sufficient condition for the transfer function $H(s) = K_c(sI - A_c)^{-1}K_e$ to be SPR, is that there exist matrices $P_o = P_o^T > 0$ and $Q_o = Q_o^T > 0$ such that

\begin{align*}
A_c^T P_o + P_o A_c &= -Q_o, \quad (3.3) \\
P_o K_e &= K_e^T. \quad (3.4)
\end{align*}

It is known that on the grounds of passivity theory, any linear dynamic SPR controller can stabilize a passive plant and provides robust stability independent of the particular plant and control design [22]. By dynamic, it is meant that the controller's output depends not only on the corresponding input but also requires knowledge of the state of the system, which carries all necessary information of its history. Given the current input and the state of the controller, the output can be uniquely determined. On the other hand, for a static controller like a constant gain PD, its input is enough to uniquely determine its output.

As intuition suggests, high feedback gains yield better performance. In practice though this is not absolutely correct. For example, a static constant gain PD controller has infinite bandwidth and provides amplification not only to the useful signals but to the high frequency content as well, which basically consists of noise. Therefore, when high gains are used the closed-loop system can be easily destabilized. Linear dynamic SPR controllers are characterized by Bode plots whose gain "rolls-off" at high frequencies, effectively acting as a filter to the high frequency content of the signals. That further enhances robustness to noisy measurements.

### 3.3.1 Passivity property and robotic control – The payload dominated case

In the study of flexible robots, it is useful to look at the system from the point of view of energy tradeoff and balance. Each term in the equations of motion is related to a certain form of energy. Part of the work provided by the actuation torques is transformed into kinetic energy and involves both the joint rotations and the elastic motion. A different portion is transformed to strain energy and stored in the deflected flexible members. The rest is dissipated to the environment in the form of heat due
to friction and structural damping. While the flexible links oscillate, a part of the externally provided energy alternates between the strain and kinetic form.

In the case of rigid robots, it is well known that the mapping from torques to joint rates is passive. This result relies on the collocation of control inputs and outputs and explains why most of the traditional robotic control schemes actually work. The PD controller of Takegaki and Arimoto [148], provides a most familiar example of this fact. As was pointed out in [107], the passivity property might not directly involve the signal that needs to be controlled. From the rigid robot example, it is the positions and not the velocity trajectories that need to be followed. In other words, the actual objective can sometimes be achieved indirectly by imposing a certain behaviour to the passive output. As it will be seen in the sequel, the approaches examined in the present thesis involve a passive output without direct physical meaning which suits well our control requirements. In the context of flexible-link robots, the passivity of the torques to joints map persists given the collocation. The property is independent of any uncertainty involved and the number of vibration modes retained in the model or their characteristics. In this case, the property is of course less strategic for controller design purposes than it is for the rigid case, the reason being that the actual objective is usually the tracking of a prescribed task-space trajectory and not a joint-based one.

A distinct control approach was proposed by Wang and Vidyasagar [155] for the one-link flexible arm case. The transfer function of the system was examined with the actuation torque being the input and the output being the net displacement at the tip, i.e., the sum of the displacement due to the rigid motion plus the elastic displacement. It was found that while increasing the number of modes included in the model the relative degree of the transfer function becomes ill defined and the control of the plant becomes problematic. The result was shown to be independent of the modelling technique. To overcome these remedies, they suitably defined a modified output for the system called the reflected tip position, which was defined as the displacement at the tip due to the rigid motion minus the elastic displacement. Given the new output, it was found that a transfer function of relative degree two exists independent of the number of modelled modes. In Pota and Vidyasagar [115],
the modified output was further examined and it was shown that the mapping from joint torques to the time derivative of the reflected tip position is passive. Further studies and experimental results related to this control approach were presented in [157]. For the above-mentioned transfer function to be passive, it is required that the hub inertia seen by the flexible link is sufficiently small. A modification to the reflected tip position was proposed in Rossi et al. [117], by introducing a weighting factor that multiplies the elastic contribution part of the output and effectively preserves the passivity property in the presence of large hub inertias. Large hub inertias typically occur when geared actuators are used. Another example of such a case is the class of multilink rigid manipulators with the last link being flexible, which were investigated in [156].

Redefinition of the system's output in order to deal with the distinct nature of the flexible inverse dynamics problem was also proposed by Moallem et al. [95], by considering suitable outputs located near the actual end-point ones. The approach is suitable for multilink flexible arms with the link deflections occurring in a plane normal to the corresponding joint axis. An inverse dynamics control strategy for the asymptotically stable tracking of the new output was proposed, which was built upon an input–output linearization of the plant. The control technique results in an approximate tracking of the desired tip tracking. The latest version of the scheme motivated by the concept of sliding surfaces appeared in [94] and improves the performance and robustness characteristics of the original version.

Given the desirability of the passivity property, Damaren [43] examined the multilink case and showed that a conceptually similar approach to [155] is possible given that a large payload is manipulated by the robot. The importance of this case stems from the fact that large payloads cause larger deformations and the vibration frequencies of the system become lower as the payload increases. Such cases commonly occur in space manipulation scenarios where robots are required to manipulate objects whose mass is much larger than their own. It is sufficient to mention that robots which will assist in building the International Space Station will also be required to perform the docking of the Space Shuttle to the Station. In particular, the following
modified i/o for the system was defined:

\[ \hat{\tau}(t) \triangleq J_\theta^T \tau, \quad \hat{\rho}_\mu \triangleq J_\theta \dot{\theta} + \mu J_\varepsilon \dot{q}_\varepsilon = \dot{\rho} - (1 - \mu) J_\varepsilon(\theta, q_\varepsilon) \dot{q}_\varepsilon. \]  

(3.5)

The modified output \( \hat{\rho}_\mu \) is called the \( \mu \)-tip rate, where \( \mu \) is a real parameter. The true tip rates are captured by \( \mu = 1 \), while \( \mu = 0 \) considers only joint-induced motion. It was shown that the mapping from \( \hat{\tau} \) to \( \hat{\rho}_\mu \) is passive for \( \mu < 1 \) when large payloads are involved. For \( \mu = 1 \) the mapping remains passive but the vibration modes become unobservable from tip rates. It is important to note that \( \hat{\rho}_\mu \) provides an effective way to introduce the elastic motion into the control input of a suitable feedback controller and add active damping to the vibration modes. The definition of the modified input of course requires that the rigid Jacobian matrix remains nonsingular so that its inverse exists. In other words, it means that the joint motion can at all times produce a motion at the tip level. This assumption will be retained for the development of the controllers to be proposed later in the thesis.

In [47], the vibration modes of flexible robots were studied, and necessary and sufficient conditions for all vibration modes to exhibit a node at the manipulator’s end-point were derived. It was found that the vibrations are internalized among the joint rotations and link deformations, i.e.,

\[ J_\theta \theta + J_\varepsilon q_\varepsilon = 0, \quad \alpha = 1, \ldots, N_e, \]

when

\[ M_{\theta\theta} J_\theta^{-1} J_\varepsilon = M_{\theta\varepsilon}. \]  

(3.6)

It was shown that this property can be closely achieved for large tip/link mass ratio and sufficiently small rotor inertias. Large joint inertias as occur in the case of highly geared actuation tend to reduce the dynamic interaction between the links. Given a general trend in robotic actuation towards gear-free, direct-drive motors, this assumption will become even more valid in the near future. Actuators of this kind tend to gain more popularity as they are more suitable for accurate manipulation. The localized nature of the vibration modes can be intuitively understood if the limiting case of an infinitely big payload is considered, which can be effectively interpreted as a clamping boundary condition.

In the same work, a Lagrangian approach was employed to derive the dynamic equations of motion for the large payload case. By large payload it is meant that both
its mass as well the moment of inertia are large enough so that all vibration modes of the arm exhibit a node at the tip. As an example, for the arm that was used for performing the large payload experiments to be presented later in the thesis, the mass of the payload was 30 to 90% bigger than the total mass of the arm. More discussions as to what constitutes a large payload will be made in Chapter 6. For the payload dominated case, it was shown that an effective separation of the task-space dynamics from the elastic ones is possible. The nonlinear torques to end-effector dynamics were shown to become essentially equivalent to the corresponding rigid robot case:

$$M_{pp}\ddot{\rho} + C_{p}\dot{\rho} = J_\theta^{-T}(\theta, q_e)\tau$$  \hspace{1cm} (3.7)

where

$$M_{pp} \triangleq J_\theta^{-T}M_{\theta\theta}J_\theta^{-1}, \hspace{0.5cm} C_{p}(\rho, \dot{\rho})\dot{\rho} \triangleq \dot{M}_{pp}\dot{\rho} - \frac{1}{2}\partial(\rho^T M_{pp}\dot{\rho})/\partial\rho.$$  \hspace{1cm} (3.8)

This is a consequence of the vibration modes localization property which renders oscillations unobservable from the tip. The matrix $M_{pp}$ is evaluated at the configuration $\theta = \mathcal{F}_r^{-1}(\rho)$ and $q_e = 0$, where $\mathcal{F}_r(\cdot)$ is the rigid forward kinematics map. The matrix $C_{p}$ can be constructed so that $(2C_{p} - \dot{M}_{pp})$ is skew-symmetric. Similar to the task-space dynamics of the rigid robot case, the rigid Jacobian naturally appears at the actuation side of the equation. It is important to point out that the above Jacobian is calculated over the actual joint configuration.

The elastic coordinates were shown to obey

$$\ddot{q}_e + D_{ee}\dot{q}_e + K_{ee}q_e = -J_e^T J_\theta^{-T} \tau + f_{\text{non},e}$$  \hspace{1cm} (3.9)

where

$$\ddot{q}_e \triangleq M_{ee}^{\frac{1}{2}} M_{ee}^{\frac{1}{2}} q_e,$$

$$f_{\text{non},e} \triangleq -(\ddot{q}_e - \frac{1}{2}\partial(\rho^T M_{pp}\dot{\rho})/\partial q_e - \frac{1}{2}\partial(\dot{q}_e^T M_{ee}\dot{q}_e)/\partial q_e).$$  \hspace{1cm} (3.11)

Suppressing the elastic dependence in $M_{pp}$ leads to neglecting the second term in $f_{\text{non},e}$, and the localized nature of the vibration modes advocates forming $\ddot{q}_e$ with $\dot{\rho} = 0$. The nonlinear terms can then be written as $f_{\text{non},e} = -C_{e}(q, \dot{q}_e)\dot{q}_e$, and the matrix $C_{e}$ can be constructed so that $(2C_{e} - \ddot{q}_e)$ is skew symmetric.
Regarding the nature of Eqs. (3.7) and (3.9), and the idea of separating the rigid motion from the elastic one, an analogy with the results of the singular perturbation technique of Siciliano and Book [132] can be identified. In [132], the slow subsystem used to be the joint-based dynamics of the corresponding rigid robot but here is the task-space version of the dynamics given by Eq. (3.7) which plays the equivalent role. The elastic motion Eq. (3.9) can be identified as the analogous of the fast subsystem.
Chapter 4

Nonadaptive Control Schemes

This chapter begins with a brief discussion of traditional joint-based nonadaptive techniques commonly used in the control of rigid robots. In particular, the joint-based proportional-integral-derivative controller, the computed torque method (CTM) and a passivity-based modification of the CTM will be reviewed. Building on previous results, a passivity-based approach for the tip tracking problem for flexible multilink robots manipulating large payloads will then be proposed. Global asymptotic stability for the closed-loop system will be demonstrated and various controller design and implementation issues will be treated in detail.

4.1 Joint-based schemes used for rigid robot control

Well-established control techniques used for controlling rigid robots have typically been joint-based. This is a rather natural consequence of the fact that for a rigid robot the tracking of a task-space trajectory can be alternatively defined as the tracking of the corresponding rigid inverse kinematics joint trajectories.

The most extensively used scheme is the individual joint proportional-integral-derivative (PID) controller and its success relies on the inherent collocation of control inputs and outputs. The scheme totally ignores the dynamics of the system, each controlled DOF is viewed as a single-input/single-output (SISO) system and the coupling
effects between them are treated as disturbances:

$$\tau(t) = -K_p (\theta - \theta_d) - K_d (\dot{\theta} - \dot{\theta}_d) - K_i \int_0^t (\theta - \theta_d) \, dt,$$

(4.1)

with $K_p$, $K_d$ and $K_i$ being the proportional, derivative and integral gain matrices respectively. Local PID controllers are in general suitable for positioning applications and global asymptotic stability is guaranteed for $K_i = 0$. The proportional and derivative action can be interpreted as additional active springs and dampers which suitably modify its energy functions. The integral term deals with the steady-state positioning error problem. When the positioning problem is considered, the desired joint values $\theta_d$ are constant and $\dot{\theta}_d = 0$. Otherwise, both the desired positions and velocities are functions of time. The direct implementation of the scheme in the task-space is also possible. Although such controllers are well suited for the regulation problem, they are not adequate for tracking and something more sophisticated that takes into consideration the dynamics of the system and the coupling between the joint DOFs is required. Controllers designed for the tracking problem commonly consist of a model-based feedforward and a feedback part. The feedforward action becomes more important in the case of nonlinear plants and its role is to drive the motion, while feedback stabilizes the resulting tracking error dynamics.

For the rigid robot case, a method that uses the mathematical model to define a suitable feedforward which will counterbalance the actual motion dynamics and effectively decouple and linearize the system is commonly referred to as the "computed torque method". The method was originally proposed by Luh Walker and Paul [87]. The aim of the feedback linearization is to render the system linear in closed loop and allow the design of a feedback controller based on well established linear techniques. The availability of a good model is a critical factor for the stability and performance of the scheme and that is the "Achilles's heel" of the method. In the case of rigid robots, a joint-space and a task-space version of the method can be used equivalently. A general study of the control of rigid robots by exact linearization can be found in [74].

Given the dynamics of the rigid robot are described by Eq. (2.14), the joint-based
version of the CTM will be [55]:

\[ \tau(t) = M_{\theta\theta}(\theta)\tau_o + C_{\theta}(\theta, \dot{\theta})\dot{\theta} \]  
(4.2)

\[ \tau_o(t) = \dot{\theta}_d - K_p(\theta - \theta_d) - K_d(\dot{\theta} - \dot{\theta}_d) \]  
(4.3)

so that a linear system of decoupled equations is obtained:

\[ \ddot{\theta} = \tau_o. \]  
(4.4)

The following second order error equation arises for the closed-loop system and allows selecting the feedback gains in order to assign specific pole locations to the system:

\[ \ddot{\theta} + K_d\dot{\theta} + K_p\dot{\theta} = 0 \]  
(4.5)

where \( \dot{\theta}(t) = \theta - \theta_d \) is the tracking error. An integral term can also be incorporated to the feedback compensator.

For the case of flexible-link arms, the joint-based form of the decoupled PD controller is guaranteed to stabilize the plant when used for the regulation problem. The relevant stability proofs can be found in [55]. Stability can also be demonstrated in the absence of structural damping. For the flexible case, the CTM will treat link compliance as uncertainty due to unmodelled dynamics. Therefore, the scheme will fail to decouple and linearize the closed-loop system, resulting in deterioration of performance or even in instabilities.

A different scheme suitable for rigid robot tracking control can be obtained by modifying the structure of the CTM as follows:

\[ \tau(t) = M_{\theta\theta}(\theta)\ddot{\theta}_d + C_{\theta}(\theta, \dot{\theta})\dot{\theta}_d - K_p(\theta - \theta_d) - K_d(\dot{\theta} - \dot{\theta}_d), \]  
(4.6)

i.e., the mass matrix is now considered outside the position and velocity feedback loops. The controller was originally proposed by Paden and Panja [110] and it was named as “PD+” controller given that its structure consists of a PD feedback controller augmented by a nonlinear feedforward term. Proofs of global asymptotic stability were also presented. An interpretation of the scheme on the basis of passivity theory is possible and will be briefly discussed here given its relevance with the analysis to follow.
For a trajectory tracking controller the control torques are often written as

\[ \tau = \tau_d + \tilde{\tau}, \]  

(4.7)

where \( \tau_d \) is a feedforward and \( \tilde{\tau} \) a feedback part. By selecting the feedforward as

\[ \tau_d(t) = M_{\theta\theta}(\theta)\ddot{\theta}_d + C_\theta(\theta, \dot{\theta})\dot{\theta}_d \]  

(4.8)

and combining with Eqs. (4.7) and (2.14) one obtains:

\[ M_{\theta\theta}(\theta)\dddot{\theta} + C_\theta(\theta, \dot{\theta})\dot{\theta} = \tilde{\tau}. \]  

(4.9)

The above error dynamics can be shown to define a passive mapping from \( \tilde{\tau} \) to \( \dot{\theta} \), which is one of the characteristics of Euler–Lagrange dynamic systems [107]. On the basis of the passivity theorem, a strictly passive feedback controller can stabilize the error dynamics. A PD controller provides a suitable choice resulting in the control law given by Eq. (4.6). Given the passivity foundation of the scheme, better robustness characteristics are expected when compared with the CTM, the success of which relies on the exact linearization of the dynamics. The performance and the characteristics of the scheme when used in conjunction with flexible-link robots will be investigated later as part of our experimental case studies.

4.2 Payload dominated inverse dynamics and passive feedback controller

A model-based nonlinear approach for the control of flexible robots will be proposed here, being the product of a passivity-based approach that effectively exploits the physical properties of the system. The control objective for flexible-link robots has commonly been addressed as the tracking of the corresponding rigid joint trajectories, while trying to eliminate the arm deflections using passive or more rarely active control techniques. Here, the control objective will be redefined as the direct tracking of the desired Cartesian trajectories together with the active suppression of the vibration modes. The scheme effectively exploits the approximate description of the payload dominated dynamics as described by Eqs. (3.7) and (3.9), in conjunction with the
\( \mu \)-tip notion. The proposed controller is a natural extension of the ideas presented in [45], where a feedforward based on the payload alone was employed. Our subsequent experimental and theoretical work has shown that the feedforward action can be improved. For example, the rotor inertias corresponding to highly geared actuators make a systematic contribution to the dynamics which cannot be ignored.

### 4.2.1 Feedforward and the passivity framework

For the problem of tracking a time-varying trajectory, \( \rho_d \), the applied torques will be taken as in Eq. (4.7), i.e., divided into a feedforward, \( \tau_d \), and a feedback part, \( \bar{\tau} \).

Eq. (3.7) suggests the following nonlinear choice for the feedforward:

\[
\tau = \tau_d + \bar{\tau}, \quad \tau_d = J^T_\theta(\theta, q_e)[M_{\rho d}(\rho)\dot{\rho}_d + C_{\rho}(\rho, \dot{\rho})\dot{\rho}_d]. \tag{4.10}
\]

It is interesting to point out that \( \tau_d \) is not strictly feedforward, in the sense that is constructed upon desired as well as measured quantities. Eq. (3.9) can be used to define an estimate for the elastic coordinates, \( q_{ed} \), produced by the application of \( \tau_d \):

\[
\overline{M}_{ee}q_{ed} + D_{ee}q_{ed} + K_{ee}q_{ed} + C_{e}(q, q_e)\dot{q}_{ed} = -J^T_eJ_\theta^T\tau_d. \tag{4.11}
\]

It is clear that the trajectories for \( q_{ed} \) involve desired as well as measured quantities. In other words, these trajectories cannot be precalculated, but can only be generated on-line. For the development of the control schemes in the present thesis, when the arguments of the matrices \( J_\theta, J_e \) and \( M_{ee} \) are omitted, the actual rigid and elastic configuration dependence, \( (\theta, q_e) \), will be implied. Given \( q_{ed} \) and the expression for \( \dot{\rho}_\mu \) in Eq. (3.5), the desired trajectory \( \dot{\rho}_{\mu d} \) is defined as

\[
\dot{\rho}_{\mu d} = \dot{\rho}_d - (1 - \mu)J_e(\theta, q_e)\dot{q}_{ed} \tag{4.12}
\]

and the error trajectory for \( \dot{\rho}_\mu \) becomes

\[
\dot{\rho}_\mu = \dot{\rho}_\mu - \dot{\rho}_{\mu d} = \dot{\rho} - (1 - \mu)J_e\dot{q}_e, \tag{4.13}
\]

where

\[
\dot{\rho} \triangleq \rho - \rho_d, \quad \dot{q}_e \triangleq q_e - q_{ed}. \tag{4.14}
\]
Here it is important to emphasize that designing a controller for the modified i/o plant to give \( \dot{\rho}_\mu \equiv 0 \) and \( \ddot{q}_e \equiv 0 \), will result in \( \dot{\rho} \equiv 0 \), i.e., the true tip errors will converge to zero.

Since our interest is on the trajectory following problem, substituting Eq. (4.10) into (3.7) and subtracting (4.11) from (3.9), the following description of the tracking error dynamics is obtained:

\[
M_{\rho\rho}(\rho)\ddot{\rho} + C_\rho(\rho, \dot{\rho})\dot{\rho} = J_\theta^T \tau, \tag{4.15}
\]
\[
\tilde{M}_{ee}\ddot{q}_e + D_{ee}\dot{q}_e + K_{ee}q_e + C_e(q, q_e)\dot{q}_e = -J_e^T J_\theta^{-T} \tau. \tag{4.16}
\]

**Theorem 1** The mapping \( \dot{\rho}_\mu = G(J_\theta^{-T} \tau) \) is passive for \( \mu < 1 \).

**Proof.** Define the nonnegative function \( S_\mu \):

\[
S_\mu = \frac{1}{2} \dot{\rho}^T M_{\rho\rho} \dot{\rho} + \frac{1}{2} (1 - \mu) [\dot{q}_e^T \tilde{M}_{ee} \ddot{q}_e + \dot{q}_e^T K_{ee} \ddot{q}_e], \quad \mu < 1 \tag{4.17}
\]

Differentiating with respect to time and using Eqs. (4.15), (4.16), and the skew-symmetry property of the matrices \( 2C_\rho - \tilde{M}_{\rho\rho} \) and \( 2C_e - \tilde{M}_{ee} \) gives

\[
\dot{S}_\mu = \dot{\rho}^T [M_{\rho\rho} \ddot{\rho} + \frac{1}{2} \tilde{M}_{\rho\rho} \dot{\rho}] + (1 - \mu) \dot{q}_e^T [\tilde{M}_{ee} \ddot{q}_e + \frac{1}{2} \tilde{M}_{ee} \ddot{q}_e + K_{ee} \ddot{q}_e]

= \dot{\rho}^T [J_\theta^{-T} \tau - C_\rho(\rho, \dot{\rho})\dot{\rho}] + \frac{1}{2} \tilde{M}_{\rho\rho} \dot{\rho} + (1 - \mu) \dot{q}_e^T [-J_e^T J_\theta^{-T} \tau - C_e(q, q_e) \ddot{q}_e + \frac{1}{2} \tilde{M}_{ee} \ddot{q}_e - D_{ee} \dot{q}_e]

= \dot{\rho} - (1 - \mu) J_e q_e \hat{q}_e^T \dot{J}_\theta^{-T} \tau - (1 - \mu) \dot{q}_e^T D_{ee} \dot{q}_e

= \dot{\rho}^T (J_\theta^{-T} \tau) - (1 - \mu) \dot{q}_e^T D_{ee} \dot{q}_e. \tag{4.18}
\]

Integrating the above and setting \( S_\mu(0) = 0 \) establishes the result for \( \mu < 1 \):

\[
\int_0^T \dot{\rho}^T (J_\theta^{-T} \tau) \, dt = S_\mu(T) - S_\mu(0) + (1 - \mu) \dot{q}_e^T D_{ee} \dot{q}_e \geq S_\mu(T) - S_\mu(0). \tag{4.19}
\]

**4.2.2 Feedback design**

Since the modified i/o plant preserves the passivity property in the error dynamics for the suggested feedforward, the passivity theorem guarantees that any strictly passive
feedback compensator $H$ with finite gain, as shown in Figure 4.1, will stabilize the system.

The very simple proportional–integral (PI) controller provides the most obvious choice. The integral operator is passive, the proportional term is strictly passive and their combination will be strictly passive. A PI controller in rates corresponds to a PD in positions:

$$\tau = -J^T_\theta(\theta, q_e)[K_p\dot{\rho}_\mu + K_d\ddot{\rho}_\mu]$$  \hspace{1cm} (4.20)

where the corresponding gains $K_p$, $K_d$ are symmetric positive definite matrices.

Global asymptotic stability of the closed-loop system can be proved by employing the following Lyapunov function:

$$V = S_\mu + \frac{1}{2}\ddot{\rho}_\mu^T K_p \dot{\rho}_\mu \geq 0, \quad \mu < 1.$$  \hspace{1cm} (4.21)

Using the previous theorem and the equation for the feedback law,

$$\dot{V} = \dot{S}_\mu + \ddot{\rho}_\mu^T K_p \dot{\rho}_\mu$$  
$$= \ddot{\rho}_\mu^T [J_\theta^{-T} \tau + K_p \dot{\rho}_\mu] - (1 - \mu)\ddot{q}_e^T D_{ee} \ddot{q}_e$$  
$$\leq -\ddot{\rho}_\mu^T K_d \ddot{\rho}_\mu - (1 - \mu)\ddot{q}_e^T D_{ee} \ddot{q}_e \leq 0.$$  \hspace{1cm} (4.22)

Because of the configuration dependence of the system on the desired trajectory, $\rho_d(t)$ and $\dot{\rho}_d(t)$, the system is nonautonomous and LaSalle's Theorem does not apply. To complete the proof, one can use the same arguments as used in [45], which we will repeat here. As explained therein, the invariance principle can be extended to systems which are asymptotically autonomous. Such is the case with our system, because after
the completion of the useful motion, the problem reduces to a regulation at a setpoint, i.e., \( \rho_d(t) \rightarrow \bar{\rho}_d \) while \( t \rightarrow \infty \). If \( \dot{V} \leq -W(x) \) where \( W(x) \geq 0 \), then all bounded solutions tend to the largest invariant set which satisfies \( W(x) \equiv 0 \). From Eqs. (4.21), (4.22) it follows that all solutions are bounded. Using the invariance principle and setting \( \dot{\rho}_\mu = \dot{q}_e \equiv 0 \), it follows from Eq. (4.13) that \( \dot{\rho} \equiv 0 \). From Eq. (4.15) it follows that \( \dot{\tau} \equiv 0 \), and from Eq. (4.16) that \( \ddot{q}_e \equiv 0 \). From the feedback control law given by Eq. (4.20), we obtain \( \dot{\rho}_\mu \equiv 0 \) and conclude that \( \dot{\rho} \rightarrow 0 \).

The combined feedforward/feedback controller becomes:

\[
\tau(t) = J_\theta^T(\theta, q_e)[M_{\rho\rho}(\rho)\ddot{\rho}_d + C_\rho(\rho, \dot{\rho})\dot{\rho}_d] - J_\theta^T(\theta, q_e)[K_d\dot{\rho}_\mu + K_p\ddot{\rho}_\mu].
\] (4.23)

### 4.2.3 Joint-based implementation of the feedforward

The feedforward part of the scheme requires the calculation of the task-space dynamic equations of the system, which can be a nontrivial task in the multilink case. As an alternative, a set of "fictitious" joint quantities will be introduced which can be used in conjunction with the corresponding rigid joint-based dynamic equations, the construction of which is much easier.

It is known that for a rigid robot the task-space dynamics (Eq. 2.24) are related to the equivalent joint-space ones (Eq. 2.14) as follows [55]:

\[
M_{\rho\rho}(\rho) = J_\theta^{-T}(\theta, \dot{\theta}) M_{\theta\theta}(\theta, \dot{\theta}) J_\theta^{-1}(\theta, \dot{\theta})
\] (4.24)

\[
C_\rho(\rho, \dot{\rho}) = J_\theta^{-T}[C_\theta(\theta, \dot{\theta}) - M_{\theta\theta}(\theta, \dot{\theta}) J_\theta^{-1} \dot{J}_\theta] J_\theta^{-1}
\] (4.25)

All matrix quantities at the right-hand-side of the above equations are evaluated at the joint rigid inverse kinematics configuration \( \theta_t = \mathcal{T}_r^{-1}(\rho) \), \( \dot{\theta}_t = \mathcal{J}_r^{-1}(\theta_t, 0) \dot{\rho} \).

In developing the controller, the assumption that the rigid Jacobian calculated over the actual configuration remains nonsingular was made. Here, it will be further assumed that \( \det J_\theta(\theta_t) \neq 0 \). The fictitious desired joint trajectories, \( \theta_d(t) \), are defined using the relation \( \dot{\rho}_d = J_\theta(\theta_t, 0) \dot{\theta}_d \) so that:

\[
\dot{\theta}_d \triangleq J_\theta^{-1}(\theta_t, 0) \dot{\rho}_d
\] (4.26)

\[
\ddot{\theta}_d \triangleq J_\theta^{-1}(\theta_t, 0)[\ddot{\rho}_d - \dot{J}_\theta(\theta_t, 0) \dot{\theta}_d],
\] (4.27)
and the feedforward part of the controller becomes:

$$\tau_d = J^T_\theta(\theta, q_d)J_\theta^{-T}(\theta, 0)[M_{\theta\theta}(\theta)\ddot{\theta}_d + C_\theta(\theta, \dot{\theta})\dot{\theta}_d].$$

(4.28)

The definition of the fictitious joint quantities can be viewed as a generalized inverse velocity kinematics problem for the present case. For absolutely stiff links, this transformation reduces to the familiar rigid inverse velocity kinematics relationship that maps the task-space velocities to the actual joint rates.

Here, it will be useful to make some brief comments regarding the inverse kinematics problem for rigid robots, which appears in the above approach. The inverse kinematics problem involves finding the joint-space positions and velocities that correspond to a set of task-space ones. For simple geometries, such solution can be a relatively easy exercise but for complicated ones it becomes more difficult. When dealing with the inverse kinematics problem, certain important issues need to be considered [10], [39]. First is the solvability issue, i.e., whether the given task-space configuration can be reached by the end-effector of the mechanism. In [39], the volume that can be reached with any possible orientation is defined as dextrous workspace and the volume that can be reached in at least one orientation as reachable workspace. Here, the dextrous workspace will be referred to simply as the workspace of the arm. The second important issue is the existence of multiple solutions to the problem. For example in the case of the three-DoF planar robot used in our experimental investigations, certain positions and orientations of the end-effector can be reached in two different configurations (elbow-in/elbow-out). When implementing a control scheme, the inverse kinematics routine is required to have the built-in ability to distinguish between such multiple solutions.

A different alternative for a joint-based implementation of the feedforward will also be investigated experimentally and we will refer to it as the "approximate form of the feedforward". The feedforward part of the controller will be approximated by constructing it on the basis of the desired trajectory rather than any measured quantities:

$$\tau_d(t) = J^T_\theta(F_r^{-1}(\rho_d), 0)[M_{\rho\rho}(\rho_d)\ddot{\rho_d} + C_\rho(\rho_d, \dot{\rho}_d)\dot{\rho}_d]$$

(4.29)
Given that the above Jacobian is calculated for the rigid inverse kinematics configuration corresponding to the desired task-space one, the above form for the feedforward reduces to the forward joint-based dynamics of the corresponding rigid robot. The discussion on this approximate form for the feedforward will be concluded later in the experimental case studies.

4.2.4 Selection of controller gains

The simplest choice for the feedback gains is a diagonal matrix with positive entries. Alternatively, a choice similar to the one in [45] can be made as follows:

\[
K_p = k_p \tilde{J}_\theta^T \tilde{M}_{\theta \theta} \tilde{J}_\theta^{-1}, \quad (4.30)
\]

\[
K_d = k_d \tilde{J}_\theta^T \tilde{M}_{\theta \theta} \tilde{J}_\theta^{-1}. \quad (4.31)
\]

The overbar notation, \(\bar{\cdot}\), denotes the matrix quantity evaluated at a constant undeflected configuration, which can be taken to be the "middle" configuration of the maneuver, \(\rho_d(t_f/2)\). A choice based on the above structure provides a natural scaling between the individual entries of the gain matrices and the constants \(k_p\) and \(k_d\) define the overall scaling of the gains. The gain matrices obtained as above are commonly diagonally dominant. When a diagonal gain matrix is to be used, the above structure provides a good indication about the relative size that the diagonal entries should have. In practice, a diagonal gain matrix might be preferable because it is much easier to tune experimentally than a full matrix.

4.2.5 The value of \(\mu\) and the stability of the system

To examine the effect of the value of \(\mu\) on stability, a linearization of the plant at a constant configuration will be considered. Then, a state-space description for the closed-loop system will be constructed with \(\mu\) being a variable. Both the motion dynamics Eq. (2.8), as well as \(\rho_\mu\) which was defined in Eq. (3.5), are linearized close to a target configuration \(\rho_\delta = \bar{\rho}\), that corresponds to \(\bar{q} = \text{col}\{\bar{\theta}, 0\}\). Letting \(\delta q = q - \bar{q}\) and \(\delta \rho_\mu = \rho_\mu - \bar{\rho}\):

\[
\bar{M} \delta q + K \delta q = B \tau, \quad (4.32)
\]
\[ \delta \rho_\mu = \bar{J}_\theta \delta \theta + \mu \bar{J}_e \dot{q}_e = \bar{J} \delta q, \quad \bar{J} = \begin{bmatrix} \bar{J}_\theta & \mu \bar{J}_e \end{bmatrix}. \] (4.33)

The overbar notation, (\(\bar{\cdot}\)), as before it denotes configuration dependent quantities evaluated at the setpoint rigid configuration \(\bar{\theta}\). For the regulation problem the feedback controller can be written as:

\[ \tau = -\bar{J}_\theta^T \left[ K_p \delta \rho_\mu + K_d \delta \dot{\rho}_\mu \right] \]
\[ = -\bar{J}_\theta^T \left[ K_p \bar{J} \delta q + K_d \bar{J} \delta \dot{q} \right]. \] (4.34)

The above torque expression is substituted into the linearized dynamics, the accelerations are extracted from the formulation and finally the system is written in a state–space form as follows:

\[
\begin{bmatrix}
\delta \ddot{q} \\
\delta \dot{q}
\end{bmatrix} = \begin{bmatrix}
-M^{-1} B \bar{J}_\theta^T K_d \bar{J} & -M^{-1} (K + B \bar{J}_\theta^T K_p \bar{J}) \\
1 & 0
\end{bmatrix} \begin{bmatrix}
\delta \ddot{q} \\
\delta \dot{q}
\end{bmatrix}
\] (4.35)

The stability of the linearized closed–loop system for specific feedback gains can be determined from the locations of the poles, i.e., the sign and the size of the eigenvalues of the above system’s matrix. In general, it was found that low values of \(\mu\) yield stable poles. Increasing the value of \(\mu\) shifts the poles towards the RHP and after a critical value, which will be denoted as \(\mu^*\), the system becomes unstable. The relation between the value of \(\mu\) and the location of the poles can provide a systematic procedure for selecting the value of \(\mu\), given a specific set of feedback gains and the desired trajectory \(\rho_d(t)\). Starting from a small value of \(\mu\) and increasing it in small steps, the values of \(\mu^*\) can be determined for every rigid inverse kinematics configuration to be encountered en route and select

\[ \mu \leq \min \mu^*(\rho_d). \] (4.36)

### 4.2.6 Assumptions and implementation issues

When implementing the above class of controllers, direct measurements of the elastic coordinates are required for the construction of the \(\mu\)-tip positions and rates and the Jacobian matrix \(J_\theta(\theta, q_e)\) as well. To avoid these problems, the following simplifications suggested in [45], can be used:

\[ J_\theta(\theta, q_e) \equiv J_\theta(\theta, 0), \quad \dot{\rho}_{\mu d} \equiv \dot{\rho}_d, \quad \rho_{\mu d} \equiv \rho_d \] (4.37)
where \( \vDash \) denotes approximation, so that

\[
\dot{\rho}_\mu(t) \vDash [\mu \dot{\rho}(t) + (1 - \mu) J_\theta(\theta, 0) \dot{\theta}(t)] - \dot{\rho}_d(t),
\]

\[
\ddot{\rho}_\mu(t) \vDash [\mu \dot{\rho}(t) + (1 - \mu) \mathcal{F}_r(\theta)] - \ddot{\rho}_d(t).
\]

As defined before, \( \mathcal{F}_r(\theta) \) is the forward kinematics map that corresponds to the joint configuration \( \theta \). The use of the above assumptions removes the need to calculate the trajectories for the elastic coordinates, \( q_{ed} \) and \( \dot{q}_{ed} \), as were defined earlier by Eq. (4.11). On the basis of our numerical simulation and experimental studies, for a sufficiently large payload an appropriate value for \( \mu \) is very close to 1 and that supports the validity of the above simplifications. We will refer to the above assumption as the "standard implementation assumption".

4.2.7 Desired trajectories for the modified output

When the scheme is implemented with a value of \( \mu \) much lower than 1, the standard implementation assumption becomes less valid. An alternative approximate numerical procedure will be proposed here for precomputing the desired trajectories for the modified output, based on the motion predicted by the model. This procedure will provide more insight into the nature of the scheme and further support the validity of the standard implementation assumption.

An estimate for the applied torques can be obtained using Eq. (3.7):

\[
\tau_d = J^T_{\theta} \mathcal{F}^{-1}_r(\rho_d, 0) [M_{pp}(\rho_d) \dot{\rho}_d + C_p(\rho_d, \dot{\rho}_d, \ddot{\rho}_d)].
\]

(4.40)

The corresponding "desired trajectories" for the joint and elastic coordinates will be denoted here as \( q_d = \text{col}\{\theta_d, q_{ed}\} \). By desired joint trajectories it is meant that these trajectories naturally evolve consistently with the desired task-space ones. The joint trajectories can be calculated on the basis of Eq. (2.22) and the corresponding elastic ones are provided by the approximate Eq. (3.9):

\[
\dot{\theta}_d = J^{-1}_{\theta}(\theta_d, q_{ed}) [\dot{\rho}_d - J_e(\theta_d, q_{ed}) \dot{q}_{ed}],
\]

(4.41)

\[
\ddot{q}_{ed} = -M^{-1}_{ee}(\dot{q}_d) [K_{ee} q_{ed} + C_e(q_d, \dot{q}_d) + J^T_e(\theta_d, q_{ed}) J^T_{\theta}(\theta_d, q_{ed}) \tau_d].
\]

(4.42)
Rather than using the above prediction for the elastic motion, the complete dynamics given by Eq. (2.8) can alternatively be used. Extracting the rigid accelerations from the upper part of the equations and substituting them into the lower part, the expression for the desired elastic coordinates is obtained:

\[ \ddot{q}_{ed} = \left( M_{ee}(q_d) - M_{\theta e}(q_d)M_{\theta\theta}^{-1}(q_d)M_{ee}(q_d) \right)^{-1} \left[ F_{nons}(q_d, \dot{q}_d) - K_{ee}q_{ed} - M_{\theta e}(q_d)M_{\theta\theta}^{-1}(q_d)(\tau_d + F_{nons}(q_d, \dot{q}_d)) \right] \] (4.43)

The simultaneous integration of Eqs. (4.40), (4.41) and (4.43), with initial conditions \( \theta_d(0) = \mathcal{F}_r^{-1}(\rho_d(0)) \) and \( q_{ed}(0) = 0 \), yields the numerical values for the estimated trajectories \( (\theta_d(t), q_{ed}(t)) \) and their rates as well. For simplicity, the elastic dependence of the rigid Jacobian in the above equations will be suppressed. To pre-calculate the desired trajectory for \( \rho_{\mu} \), its definition in Eq. (3.5) can be used, where the configuration dependence of \( J_e \) will be replaced by the above estimates:

\[ \dot{\rho}_{\mu d} = \dot{\rho}_d - (1 - \mu)J_e(\theta_d, q_{ed})\dot{q}_{ed} \]
\[ = \mu \dot{\rho}_d + (1 - \mu)J_{\theta}(\theta_d, q_{ed})\dot{\theta}_d. \] (4.44)

Suppressing the elastic configuration dependence of the above rigid Jacobian, the estimates for the desired modified output trajectories are obtained as follows:

\[ \dot{\rho}_{\mu d} = \mu \dot{\rho}_d + (1 - \mu)J_{\theta}(\theta_d, 0)\dot{\theta}_d; \] (4.45)
\[ \rho_{\mu d} = \mu \rho_d + (1 - \mu)\mathcal{F}_r(\theta_d). \] (4.46)

The proposed procedure begins with an estimate for the applied torques provided by the open-loop dynamics, which of course will be different from the actual closed-loop ones. Therefore, the predicted torques contain extra oscillations that will also be introduced in the calculated desired trajectories of the modified output. In order to extract these oscillations and just retain the contribution of the large deflections that occur during the motion, a high order polynomial fit to the calculated numerical values can be used. That effectively "smoothes" the trajectories by acting as a filter to all the small amplitude vibrations.

Given that the method is approximate, following the precalculated desired trajectories for the modified output will result in a task–space motion slightly different
from the desired one. For the forward integration solution that is involved only the
initial condition for the joint angles can be imposed and not the final one and thus
some steady-state error will result. Studies relevant to this method will be presented
as part of the experimental case studies in Chapter 7 and will demonstrate that for
a value of $\mu$ close to 1, the calculated desired trajectories converge to the ones of the
standard implementation assumption.

4.2.8 SPR feedback compensation and the gain-scheduled
version

One of the characteristics of passivity-based-control is that it provides versatility in
feedback design. The feedback part of our controllers consists of the very simple PD
compensator but a repertoire of different choices exists that is guaranteed to stabilize
the system. It has already been mentioned that any linear dynamic SPR controller
provides robust stabilization of a passive plant. In the case of our controller, the
proportional term in the feedback part can be maintained and the derivative term
can be replaced by a dynamic velocity feedback SPR controller.

The studies of Benhabib et al. [22] within the area of LSS, showed that a transfer
matrix that characterizes a structure is PR given the collocation of actuators and rate
sensors. This property is independent of the specifics of the vibration frequencies,
mode shapes and modal truncation. It was shown that an SPR controller is guaranteed
to stabilize a PR plant and a controller design procedure for the robust stabilization of
the plant was proposed. Their design was based on the KYP Lemma, which provides
the cornerstone for most of the other systematic SPR design procedures proposed in
literature such as [89] and [83]. An experimental study with comparisons between
various SPR controller design methodologies when used for the control of a single-link
flexible arm was made in [41]. The design of Benhabib et al. was used in [46] for
the control of multilink flexible arms. A summary of the design procedure will be
presented here. A minimal state-space description of the passive plant $G$ is considered
as follows:

$$\dot{x} = Ax + Bu, \ y = Cx,$$

(4.47)
the matrices of which satisfy the standard controllability and observability conditions
and a stabilizing SPR controller described by the following transfer matrix needs to
be designed:

\[ H(s) = K_c(s1 - A_c)^{-1}K_c. \]  \hspace{1cm} (4.48)

Given the plant \( G \), a matrix \( K_c \) is selected such that all the eigenvalues of \( A_c = (A - BK_c) \) are in the left-half-plane (LHP). Such a matrix can be obtained from the
solution of the algebraic Riccati equation related to the LQR problem which minimizes
the cost function

\[ J = \frac{1}{2} \int_0^\infty (x^TQx + u^TRu)dt \]  \hspace{1cm} (4.49)

with \( Q = Q^T > 0 \) and \( R = R^T > 0 \). Given another positive definite matrix \( Q_o \), the
Lyapunov Eq. (3.3) is solved for its unique positive definite solution \( P_o \) and then we
take \( K_c = P_o^{-1}K_c^T \) in order to satisfy the KYP Lemma, as in Eq. (3.4). The design
procedure guarantees that \( H \) is SPR.

In some control applications, a constant-gain controller might not be adequate
due to varying operating conditions and a very popular technique to deal with this
problem is the gain scheduling. The technique is comprised of a series of control gains,
each one designed and tuned for a different operating point. Once one of these op­
erating conditions is detected the corresponding gain is automatically selected. This
intuitive idea involves a lookup table with a collection of controller gains, which are
precomputed and stored off-line. A mechanism exists which detects the current oper­
ing condition on the basis of suitable measurements and selects the corresponding
set of gains in an open-loop fashion. The choice of the individual gains usually in­
volves extensive simulations as well as tests on the actual plant. Special attention
is required for the transition between the different operating conditions, where the
gains are commonly calculated using interpolation. The signals used for monitoring
the operating condition are called scheduling variables.

Such a technique was originally motivated and then gained popularity as a stan­
andard control technique in the control of high performance aircraft in order to deal with
the significant variations related to flight conditions. Measurements of the speed and
the dynamic pressure usually provide the necessary scheduling variables. As detailed
in Åström and Wittenmark [13], apart from flight control systems, gain scheduling provides solution to other control applications like ship steering, pH control in chemical processes, combustion control of boilers, fuel-air control in a car engine and much more.

Historically there has been a debate as to whether gain scheduling should be considered as an adaptive control technique or not. Although the controller has the ability to change according to changes in the operating conditions, the adaptation is dynamically decoupled from the control, it does not involve feedback and operates in a preprogrammed fashion [13]. As pointed out in the same source, an advantage of gain scheduling over adaptive control is that the controller parameters can change very quickly in response to changes in the operation and the rate of change is only limited by the speed the auxiliary measurements are made available. In the case of adaptive techniques on the other hand, an estimation process is involved and introduces some inertia to tracing any changes. One of the drawbacks of gain scheduling is that the design of each individual controller can be time consuming and costly. The related design and implementation cost increases with the number of operating points considered. The gain scheduling techniques already proposed are usually not accompanied by theoretical results that guarantee their stability. Their implementation is based upon intuition and in spite of their success there are potential hazards due to the lack of stability proofs, especially when nonlinear applications are considered. Relevant discussions and results on this problem can be found in Shamma and Athans [129].

An important result in the area which is supported by stability proofs is the result of Damaren [46], which is a systematic passivity–based gain scheduling technique. The block-diagram of the scheme is shown in Figure 4.2. A number of $N$ SPR controllers are involved and suitable scheduling signals $s_i$, $i = 1, \ldots, N$ are used to multiply the input and the output of each individual controller (the symbol $\otimes$ denotes multiplication). The stability of the overall system requires that the same scheduling signals are used to form both the overall controller output and the individual controller inputs. (Notice that $N$ represents the total number of scheduled SPR controllers, whereas in the modelling section it was used to denote the number of interconnected bodies.)
At this stage it is important to elaborate further on multivariable feedback design issues. Various theoretical tools have been proposed for the design and analysis of multi-input/multi-output (MIMO) control systems. Certain tools tend to be established as standard ones after they were included in commercial computer aided control toolboxes. Such is the structured singular value (SSV) criterion for examining the stability robustness and performance robustness characteristics of a closed-loop system, which was introduced by Doyle [59] and Safonov [121] independently. Nevertheless, the field has not yet reached the same level of maturity as in the SISO case. The interaction between the inputs and the outputs is the main complicating factor in the MIMO case, i.e., a small change in one of the inputs will affect all the outputs. In the SISO case one deals with scalar quantities but in the MIMO case vectors and matrices are encountered. As explained in [139], the main difference between the SISO and the MIMO case is the existence of directions in the latter. For the linear case, the gain of a transfer function, $G$, depends on the frequency but not the magnitude of the input. The same is true for the MIMO case but the gain also depends on the direction of the inputs. The singular value decomposition (SVD) provides the natural way of quantifying the multivariable directionality.
The physical interpretation of SVD in conjunction with the frequency response of the system is also provided in [139] and the important role played by the singular values in analyzing the gains of a MIMO system is explained. For a given frequency, the \( i \)-th singular value (sv), \( \sigma_i \), is the gain of \( G \) in the associated direction. The largest gain for any input direction of the plant is represented by the maximum sv, \( \sigma(G) \), over the whole range of frequencies. The smallest gain for any input direction is equal to the minimum sv, \( \sigma(G) \). The importance of the SVD in analyzing the SPR designs for the present case will be discussed further as part of the experimental case studies to be presented.

4.2.9 Practical applications

The control scheme presented in this chapter was designed assuming that gravity is absent and is well suited to space robotic applications. In its standard form it of course applies to the case of earth–based planar robots operating on the horizontal plane given that a large payload is manipulated. One example of earth–based application where the results can also be applicable is the case of overhead–mounted gantry robots such as described in Baicu et al. [15]. Such a system is SISO with one task–space translational DOF and its analysis is therefore simpler than the more general case of articulated flexible–link body chains considered in the thesis. Such mechanisms constitute a special case of robotic systems and are commonly used in precision manufacturing and material handling in electronic, nuclear and automotive industries. Like the experimental setup used in [15], such systems typically consist of a link clamped to an overhead–mounted translational gantry. Their operation often involves transporting heavy payloads and the induced vibrations impose serious performance limitations. The use of lightweight flexible links will result in the same advantages already mentioned for the case of the articulated robot.
Chapter 5

Adaptive Control Schemes

Flexible-link robots either used in space or earth-based applications will be called to manipulate large scale, heavy objects with unknown mass properties so that adaptive control capabilities will be desirable. This chapter begins with a general discussion of adaptive techniques and a useful classification. Some practical applications for adaptive control are then considered in order to view the topic from a more general perspective and raise important issues to be encountered later. Then, the focus is shifted to robotics and adaptive control for rigid robots is briefly reviewed. A passivity-based adaptive controller suitable for flexible arms manipulating large payloads is then presented together with the corresponding proofs of asymptotic stability. Various characteristics of the scheme and relevant implementation issues are also considered in detail.

5.1 Adaptive control – General considerations

Control engineers are often confronted with the problem of uncertainty regarding the controlled plant or systems with time-varying parameters. Changes might also occur due to ageing of the system or unexpected component failures. Therefore, a fixed-gain controller might be insufficient and when designed to deal with a range of different modes of operation might have to be very conservative, thus unable to respond to high performance requirements. Robust and adaptive control techniques evolved in order to deal with this problem.
Robust control is characterized by the ability to maintain stability and good performance levels in the face of parameter uncertainty, unmodelled dynamics and disturbances. Adaptive control on the other hand is endowed with learning capabilities which allow it to deal with the parameter uncertainty problem and improve performance with time. Generally speaking, it can be said that robust control can cope with uncertainty while adaptive control nullifies it. As noted by Zames [169], the performance of fixed parameter robust controllers is optimized on the basis of a priori information, whereas in the adaptive case the extra a posteriori information is used towards improving performance. Robustness characteristics can of course be incorporated within the adaptive context and one can speak of robust adaptive control. A good review of the subject and a survey on various robust control approaches proposed for the control of rigid robots can be found in Abdallah et al. [1]. A more recent survey paper is the one by Sage et al. [122].

Adaptive controllers are able to deal both with internal uncertainty, i.e., ignorance about the parameters of the system as well as external uncertainty due to changes in the environment. They were developed upon realizing that the response of the system carries information not only on its state but on the model parameters as well. The aim of adaptation is to extract this information and then use it towards improving the performance of the closed-loop system.

Research on adaptive control started in the early 1950's and it was motivated by the control problem for the high performance aircraft, after realizing that constant gain controllers were inefficient in dealing with the different operating conditions that the aircraft undergoes during flight. The field has grown to reach a mature level so that various techniques supported by global asymptotic stability proofs exist. Most of the work carried out in the area involves linear plants and controllers and less results exist in the nonlinear arena. In spite of the theoretical results that exist, adaptive control has not yet been established as a widely-used technique in engineering practice. This fact can be attributed to the computational demands involved in the implementation which was an important issue until recently, conservatism which typically favors the conventional ways of doing things and the level of education of
practicing engineers as well. Due to the limited teaching time at universities, adaptive control goes beyond the scope of most undergraduate engineering courses and is usually taught at the postgraduate level only.

In literature there has not been a consensus on a unique definition of adaptive control algorithms. Defining adaptive control on the basis of the ability to change behaviour in response to changes in the operating conditions is a very general one. Given such a definition it has even been questioned whether feedback controllers should also be considered as such, since the control effort is adjusted according to changes affecting the operation. In order to avoid any confusion, control algorithms will be referred to as adaptive if parameter estimation is explicitly incorporated in the control law [108].

Adaptive control should of course not be considered as the universal solution to any control problem. A big mistake often made is to view adaptive control as an easy alternative to plant modelling and calculation of the model parameters. Even the variations of the open-loop dynamics do not necessarily advocate using an adaptive compensation technique since a fixed controller might still be able to do equally well as the adaptive one [12] and should be the natural choice. This issue was emphasized in the control literature by referring to "uses and abuses" of adaptive control.

It is interesting to point out that adaptive control behaviour is also found in nature and characterizes both animal and plant species. A familiar example is the case of a walking man whose control effort is adjusted according to the changes on the load he is carrying, the slope of the path he is following, the wind blowing and so on. In that case though, the control capabilities are much more advanced than any man-made engineering system and go far beyond adaptation to very complicated learning by experience and intelligent control behaviour.

5.2 Classification of adaptive controllers

A general classification of adaptive controllers will be presented in order to locate the scheme to be examined in the thesis within the sphere of adaptive control, point out its relation with other schemes and yield better understanding. Many different
classifications have been proposed in the adaptive control literature and terms have often been used interchangeably. A general and clear classification is the one provided by Slotine and Li [145], where adaptive techniques are divided into two broad categories according to the signal that drives the adaptation. Model reference adaptive controllers (MRAC) are the ones that extract parameter information from the tracking errors. It can be understood by intuition that tracking errors carry parameter information given that parameter ignorance leads to tracking errors. The structure of MRAC consists of a control law and a parameter update law.

It has also been recognized that another possible source of parameter information is the relation between the input and output signals, which is the driving force for the adaptation in the case of self tuning regulators (STR). Such schemes use a model of the plant and its parameters are updated so that the difference between the predicted and the actual i/o behaviour is driven to zero. The controller receives the parameter estimates and treats them as if they were the true parameters. That is commonly called the certainty equivalence principle. Obviously, the known parameter case provides an upper bound for the performance of the controller. For STR it might be important for both the stability and the performance that the reference trajectories are such that the exact parameters of the plant can be estimated. Controllers that exploit both of the above sources of parameter information are called composite adaptive controllers. The adaptive scheme considered in the present thesis belongs to the MRAC class.

The class of STR controllers can be further categorized into direct and indirect [145], [77]. When the adaptation mechanism updates the actual controller parameters the scheme is called direct. Otherwise, when the plant parameters are estimated and then the intermediate underlying design problem [12] needs to be solved to calculate the corresponding parameters of the controller it is called indirect.
5.3 Applications of adaptive control

Before the focus is shifted to robotic applications where significant benefits can be obtained, some adaptive control applications from different areas will be briefly reviewed. A list of applications and related references can be found in Åström [12].

Adaptive control can play a significant role in the control of an aircraft whose dynamics vary considerably according to flight conditions like speed, height, load and weather conditions. Adaptive controllers for aeroplane applications have also been described as universal autopilots. A kind of extreme case of parameter variation is the fire-fighting airplane whose mass properties will vary tremendously while loading or dispensing large quantities of water.

The steering of large ships like tankers is a very difficult problem which is affected by varying factors like loading, water depth, sea waves and wind. Such factors cause drastic changes to the dynamic behaviour of the ship and manual control becomes very difficult. An adaptive control technique is very well suited to such an application and is discussed in [11] and [82].

One of the case studies in [71], refers to an adaptive drug delivery control system. In patient treatment it is very common that the infusion rate of a drug has to be regulated according to some measured physiological parameters. Such operation has traditionally been performed manually by attending medical personnel and important benefits are expected by automating the process through the use of a suitable adaptive control technique. In the above reference, the difficulties due to the complexity of the controlled plant are discussed and an adaptive control strategy is proposed. Experimental tests on animals were reported.

Other adaptive control applications involve the control of electrical motors with unknown or changing load characteristics and chemical process applications such as pH control. Another most important application for adaptive control techniques is found in signal processing and basically involves adaptive signal filtering for communication applications.
5.4 Adaptive control in robotics

One very common problem in robotic control is the uncertainty related to the geometric and/or mass properties of the arm itself or more likely of the grasped object. Adaptive control techniques carry the potential of consistent performance in the presence of uncertainty. Various globally stable adaptive control techniques have been proposed for the rigid robot case. A collection of some of the most significant ones is provided by Ortega and Spong [108], in a unified tutorial form. One of the most referenced nonlinear schemes in the area was proposed by Craig et al. [40], but exhibits two major weak points related to implementation. It requires the inversion of the inertia matrix, which is a computationally intensive operation, and the availability of acceleration measurements. Both drawbacks were altered by a most elegant technique proposed by Slotine and Li [142], on which experimental results were presented in [143]. The extension of the scheme to the task-space control was also treated, which can be viewed as the rigid equivalent of the adaptive controller for flexible robots that will be considered in the present thesis. The scheme was further examined by Niemeyer and Slotine [104], and a version suitable for the altitude tracking control of a rigid spacecraft with large uncertain loads was proposed by Slotine and Di Benedetto [141]. In [140], the scheme was interpreted using energy considerations from the physics point of view. In Ortega and Spong [108], the algorithm was examined within a more general framework together with other schemes and interpreted in terms of the passivity theory. A composite adaptation version of the scheme was proposed in [144].

A similar technique which was also examined within the passivity-based adaptive control framework of [108], was introduced by Sadegh and Horowitz [119]. Similar toSlotine and Li's algorithm it does not require any acceleration measurements nor any matrix inversions. A modified version of the scheme appeared later in [120] and was named as the "desired trajectory adaptive control". An exclusive feature of that version is that the regressor matrix was entirely built upon quantities of the desired trajectory rather than the measured ones and this fact enhanced the robustness characteristics of the controller. Another important characteristic of this
modification is that the regressor can be calculated and stored off-line and that reduces the real-time computational burden.

A distinct class of controllers for rigid robots with adaptive characteristics are the so called learning controllers. The approach followed in that case is conceptually different from the scheme to be examined here and it will be useful to clarify the relevant terminology and review the characteristics of learning controllers. Such techniques are suitable for arms which perform repetitive tasks, such as pick-and-place operations which are very common in industrial practice. As explained in Narendra and Venkataraman [100], the term learning in an engineering context refers to the gradual change in behaviour pattern under the exposure to the same situation. In [38], learning control is defined as any control scheme that improves the performance of the controlled device as actions are repeated and does so without the necessity of a parametric model. The performance improvement from trial to trial is achieved by monitoring the tracking error from the previous cycles and adjusting the control input accordingly. One of the pioneering works in the area is the scheme proposed by Craig [37], which adaptively constructs suitable feedforward torque profiles based on the error histories from all previous trials. It was emphasized that such an approach is very well suited for dealing with the friction problem, which is extremely difficult to model and compensate as part of a fixed-parameter control law. Since the adaptation is not model-based, the scheme might compensate for unknown dynamics of any kind. Of course the dynamics of the system must be repeatable and any nonrepeatability is viewed as a disturbance to the system. Other indicative algorithms of this approach are the ones proposed by Arimoto et al. [6], Sadegh and Guglielmo [118], and Kuc et al. [78].

5.5 Adaptive control for flexible-link robots carrying large payloads

In spite of the achievements in rigid robot adaptive control, in the flexible case there has not been the same success and the few schemes that have been proposed typically
address the joint trajectory tracking problem. An interesting approach is the one of Lammerts et al. [79], which exploits the structure of the dynamics in dealing both with the joint and link flexibility problem. Another example of adaptive control reported in the area is the scheme of Yang et al. [162]. For the case of flexible-joint robots, Spong [146] derived a passivity-based adaptive strategy by suitably using the approach of Slotine and Li. Some isolated cases of results on learning approaches for flexible-link arms have also been reported such as the approach of Lucibello et al. [85], which deals with the repositioning control of a two-link robot with one flexible arm that moves between two given equilibrium configurations. In Lucibello and Panzieri [84], a learning algorithm was presented which is suitable for the tip tracking problem for a single flexible-link arm. Experimental results for both the above learning techniques were presented.

Important properties for the development of rigid–robot adaptive controllers are the linear dependence of a model-based feedforward law on a set of suitably defined parameters and the passivity of the torque to joint-rates map. In the context of flexible-link robots, Damaren [44] established such a framework by effectively exploiting a payload dominated version of the dynamic equations in conjunction with the passive modified i/o notion and developed an adaptive control strategy. The scheme permitted only the properties of the payload to be estimated on-line. An important extension of the above scheme constructed upon Eqs. (3.7) and (3.9) will be presented here, which opens the door to the mass properties of the robot itself to participate in the set of adaptively updated parameters. It should also be noted that the mass or geometric parameters of the corresponding rigid robot will be updated on-line but not the elastic properties of the links. The controller architecture consists of a control law coupled with a parameter estimation mechanism known as a parameter update law, which keeps extracting parameter information from the tracking errors so that performance improves with time.

The controller design approach consists of the following distinct steps. First, a suitable feedforward law is selected based on the system dynamics for the known-parameter case, which is then modified after introducing some new quantities. The passivity property involving a suitably defined modified input and output for the
plant is established and the passivity theorem suggests the control and adaptation laws and sets the coupling between them. Finally, suitable Lyapunov arguments are employed to prove global asymptotic stability.

5.5.1 Fixed parameter control law

In adaptive control it is very common that the development of a scheme begins by considering the algorithm for the known-parameter case. Here, Eq. (3.7) suggests the following feedforward law:

\[ \tau_d = J_d^T W(\ddot{\rho}_d, \dot{\rho}_d, \dot{\rho}, \rho) \alpha, \]

\[ W(\ddot{\rho}_d, \dot{\rho}_d, \dot{\rho}, \rho) \alpha = M_{\rho \rho}(\rho) \ddot{\rho}_d + C_{\rho}(\rho, \dot{\rho}) \dot{\rho}_d \]

which can be expressed as a linear function of suitably defined parameters. The matrix \( W \) is the regressor matrix and \( \alpha \) is the vector with the parameters to be adaptively updated. The construction of the regressor matrix can be facilitated by the use of a mathematics computer package with symbolic capabilities, which basically becomes a necessity when dealing with complicated robot geometries.

An estimate of the elastic displacements was given earlier in Eq. (4.11) and the reference trajectory for the modified output, \( \rho_{\text{ref}} \), together with the corresponding tracking errors \( \ddot{\rho}_\mu \) and \( \dot{\rho}_\mu \) were defined by Eqs. (4.12) and (4.13) respectively. The filtered position rates, \( \dot{\rho}_r \), and the filtered errors, \( s_\mu \), which can be thought as a measure of tracking accuracy, are defined as follows:

\[ \ddot{\rho}_r \triangleq \ddot{\rho}_d - \Lambda \ddot{\rho}_\mu, \quad s_\mu \triangleq \dot{\rho}_\mu + \Lambda \dot{\rho}_\mu \]

with \( \Lambda = \Lambda^T > 0 \) being a weighting matrix. The idea of using \( s_\mu \) was borrowed from the rigid adaptive scheme of Slotine and Li [142], [143] and is introduced in order to guarantee the convergence of the steady-state positioning errors to zero. As in [44], it is important to emphasize that \( s_\mu \in L_2 \) implies that \( \ddot{\rho}_\mu(t) \in L_2 \cap L_\infty, \dot{\rho}_\mu(t) \in L_2, \) and \( \ddot{\rho}_\mu(t) \to 0 \) as \( t \to \infty \). Furthermore, if \( s_\mu(t) \to 0 \) as \( t \to \infty \), then \( \dot{\rho}_\mu(t) \to 0 \). The errors for the filtered rates and filtered positions are defined by:

\[ \ddot{\rho}_r \triangleq \rho - \rho_r, \quad \dot{\rho}_r \triangleq \dot{\rho} - \dot{\rho}_r = \ddot{\rho} + \Lambda \dot{\rho}_\mu \]
Replacing \( \rho_d \) with \( \rho_r \) in the fixed parameter feedforward,

\[
\tau_d = J^T_\theta W(\dot{\rho}_r, \dot{\rho}_r, \dot{\rho}) = J^T_\theta [M_{\rho r}(\rho) \dot{\rho}_r + C_\rho(\rho, \dot{\rho}) \dot{\rho}_r],
\]

a description of the tracking error dynamics is obtained by subtracting Eq. (5.5) from (3.7), and (4.11) from (3.9):

\[
M_{\rho r}(\rho) \ddot{\rho}_r + C_\rho(\rho, \dot{\rho}) \dot{\rho}_r = J^T_\theta \ddot{\tau}_d,
\]

\[
\bar{M}_{ee} \ddot{q}_e + D_{ee} \dot{q}_e + K_{ee} \dot{q}_e = -J^T_\theta J^T_\theta \ddot{\tau}_d - C_e(q, \dot{q}_e) \dot{q}_e,
\]

with

\[
\ddot{\tau}_d = \ddot{\tau}_d - \tau_d, \quad \ddot{q}_e = q_e - q_e^{cd}.
\]

At this point, the system will be viewed as the mapping from \( J^T_\theta \ddot{\tau}_d \) to \( s_\mu \). As was stressed in [107], the use of \( s_\mu \) effectively reduces the relative degree of the system's output to one and opens the road to a passivity-based adaptive control approach.

**Theorem 2** The mapping \( s_\mu = G(J^T_\theta \ddot{\tau}_d) \) is passive for \( \mu < 1 \).

**Proof.** Define the nonnegative function

\[
S_\mu = \frac{1}{2} \dot{\rho}_r^T M_{\rho r} \dot{\rho}_r + \frac{1}{2} (1 - \mu) \ddot{q}_e^T \bar{M}_{ee} \ddot{q}_e + \ddot{q}_e^T K_{ee} \dot{q}_e , \quad \mu < 1
\]

Differentiating with respect to time and using Eqs. (5.6) and (5.7), the skew-symmetry property of the matrices \( (2C_r - \bar{M}_{ee}) \) and \( (2C_r - \bar{M}_{ee}) \) and the definitions of \( \dot{\rho}_\mu \) and \( \dot{\rho}_r \) gives:

\[
\dot{S}_\mu = \dot{\rho}_r^T [M_{\rho r} \dot{\rho}_r + \frac{1}{2} \dot{M}_{\rho r} \dot{\rho}_r] + (1 - \mu) \ddot{q}_e^T \bar{M}_{ee} \ddot{q}_e + \ddot{q}_e^T K_{ee} \dot{q}_e
\]

\[
= \dot{\rho}_r^T [J^T_\theta \ddot{\tau}_d - C_r \dot{\rho}_r + \frac{1}{2} \dot{M}_{\rho r} \dot{\rho}_r] +
\]

\[
(1 - \mu) \ddot{q}_e^T [-J^T_\theta J^T_\theta \ddot{\tau}_d - C_e \dot{q}_e + \frac{1}{2} \bar{M}_{ee} \ddot{q}_e - D_{ee} \dot{q}_e]
\]

\[
= [\dot{\rho}_r - (1 - \mu) J \dot{q}_e] J^T_\theta \ddot{\tau}_d - (1 - \mu) \ddot{q}_e^T D_{ee} \dot{q}_e
\]

\[
= [\dot{\rho}_r + \Delta \dot{\rho}_\mu + \dot{\rho}_\mu - \dot{\rho}_r \dot{J}^T_\theta \ddot{\tau}_d - (1 - \mu) \ddot{q}_e^T D_{ee} \dot{q}_e
\]

\[
= s_\mu^T (J^T_\theta \ddot{\tau}_d) - (1 - \mu) \ddot{q}_e^T D_{ee} \dot{q}_e.
\]

Integrating the above and setting \( S_\mu(0) = 0 \) establishes the result upon noting that \( S_\mu \geq 0 \) when \( \mu < 1 \):

\[
\int_0^T s_\mu^T (J^T_\theta \ddot{\tau}_d) dt = S_\mu(T) - S_\mu(0) + (1 - \mu) \ddot{q}_e^T D_{ee} \dot{q}_e \geq S_\mu(T) - S_\mu(0).
\]

□
5.5.2 Adaptive version

It is obvious by now that the system possesses all the necessary properties for extending the ideas of [44] to the present case. Given that the modified i/o of the system preserves the passivity property in the error dynamics, a stabilizing feedback controller will be designed as shown in Figure 5.1.

![Controller design for the modified i/o plant.](image)

Figure 5.1: Controller design for the modified i/o plant.

The feedback part of the controller is selected so that the passivity theorem is satisfied:

\[
\ddot{\tau} = -J_\theta^T(\theta, q_0)Hs_\mu, \quad \int_0^T s_\mu^T H s_\mu \, dt \geq \epsilon \int_0^T s_\mu^T s_\mu \, dt, \quad \forall T > 0
\]  

(5.12)

for some constant \( \epsilon > 0 \), i.e., \( H \) is a strictly passive operator. A controller suitable for trajectory following often consists of a feedforward and a feedback part, \( \ddot{\tau}(t) \). Since the parameters \( \alpha \) are unknown we take the applied torque to be:

\[
\tau = J_\theta^T W(\dot{\rho}_r, \ddot{\rho}_r, \dot{\rho}, \rho) \ddot{\alpha} + \ddot{\tau},
\]

(5.13)

with \( \ddot{\alpha}(t) \) the vector of the parameter estimates.

Subtracting Eq. (5.5) from (5.13) yields:

\[
\ddot{\tau} = \tau - \tau_d = J_\theta^{-1} J_\theta^T \ddot{\tau} = J_\theta^{-1} \ddot{\alpha}(t) = \ddot{\alpha}(t) - \alpha
\]

(5.14)

The above relation is premultiplied by \( J_\theta^{-1} \), transposed and then postmultiplied by \( s_\mu \). Finally, the integral of both parts is taken:

\[
\int_0^T (J_\theta^{-1} \ddot{\tau})^T s_\mu \, dt = \int_0^T \ddot{\alpha}^T W^T s_\mu \, dt + \int_0^T (J_\theta^{-1} \ddot{\tau})^T s_\mu \, dt.
\]

(5.15)
If $\bar{a}$ is chosen to be a passive function of $-W^Ts_\mu(t)$ and the map from $-s_\mu$ to $J_{\theta}^{-T}\bar{\tau}$ is strictly passive, then $H$ in Eq. (5.12) will be a strictly passive operator. These observations suggest the following control and parameter update laws:

$$\tau(t) = J_{\theta}^T(\theta, q_e)W(\dot{\theta}, \dot{q}_r, \dot{r}, \rho)\bar{\alpha}(t) - J_{\theta}^T(\theta, q_e)K_ds_\mu, \quad (5.16)$$

$$\dot{\bar{\alpha}}(t) = \hat{\alpha}(t) = -\Gamma W^Ts_\mu \quad (5.17)$$

with $K_d = K_d^T > 0$ and $\Gamma = \Gamma^T > 0$ being the matrices with the feedback and adaptation gains respectively. For the feedback part of the controller a very simple proportional (P) term was selected.

The adaptive algorithm yields global asymptotic stability for the tracking errors $\hat{\rho}$ and $\dot{\hat{\rho}}$ for $\mu < 1$. To prove the above statement, the following Lyapunov function can be employed:

$$V(t) = S_\mu + \frac{1}{2}\dot{\alpha}^T\Gamma^{-1}\dot{\alpha} \geq 0. \quad (5.18)$$

Differentiating the above,

$$\dot{V}(t) = \dot{S}_\mu + \dot{\alpha}^T\Gamma^{-1}\dot{\alpha}$$

$$= s_\mu^T(J_{\theta}^{-T}\bar{\tau}) - (1 - \mu)\dot{q}_e^TD_{ee}\dot{q}_e - (1 - \mu)\dot{q}_e^TD_{ee}\dot{q}_e$$

$$= -s_\mu^TJ_{\theta}^{-T}\bar{\tau} - (1 - \mu)\dot{q}_e^TD_{ee}\dot{q}_e$$

$$\leq 0 \quad (5.19)$$

where the relations $J_{\theta}^{-T}\bar{\tau} = W\bar{\alpha} + J_{\theta}^{-T}\bar{\tau}$ and $J_{\theta}^{-T}\bar{\tau} = -K_ds_\mu$ were used. To complete the proof, exactly the same arguments as used in [44] can be applied. As mentioned therein, a much simpler proof is also possible by using the extension of LaSalle's Theorem to the case of asymptotically autonomous systems, which was also used in the case of our nonadaptive scheme. The Lyapunov function $V$ is positive definite in the state $\text{col}\{\hat{\rho}, \dot{\hat{q}}_e, \hat{\rho}, \dot{\hat{q}}_e, \bar{\alpha}\}$. The invariant set consistent with $\dot{q}_e \equiv 0$, is given by $s_\mu = \dot{q}_e \equiv 0$. Using Eqs. (5.6), (5.7), (5.14) and (5.17), it can be concluded that $\dot{\hat{\rho}} = \hat{\rho} \equiv 0$.

Due to the gradient nature of the adaptation law, the speed of parameter estimation is determined by the size of the adaptation gain matrix, $\Gamma$, and is limited by various factors. By increasing the adaptation gains the learning time is reduced.
and performance improves faster. Learning time is the time required until the adaptation mechanism takes full control of the plant. It is the transition period during which the parameter estimates settle close to their steady-state values. As indicated by Zames [169], parameter information takes time to acquire and depending on the plant's dynamics there is a minimum time required to shrink uncertainty within a narrow tolerance.

For the ability to respond rapidly to time-varying parameters sufficiently fast adaptation is required, i.e., the adaptation gains need to be sufficiently high. The size of adaptation gains is also related to the robustness characteristics of the scheme. Low adaptation gains translate to inertia in tracing any changes which renders the system less vulnerable to disturbances and measurement noise. On the other hand, very high adaptation gains might lead to oscillatory behaviour in the parameter estimates, more time is required until the estimates settle to their steady-state values and instabilities may also occur.

For the development of the controller the estimated parameters were assumed to be constant. This assumption is very common when analyzing adaptive control schemes and considerably simplifies the analysis. In practice though, adaptive controllers are very useful in dealing with the parameter variation problem. For the theoretical results to be valid [145], parameters can only be slow-varying with a change rate much slower than the adaptation rate. Otherwise, the problem has to be reconsidered and the time-varying nature of the parameters needs to be addressed in the analysis.

Our scheme is a good example of adaptive algorithms which effectively exploit the known structure of the system dynamics. By constructing the regressor matrix we effectively focus upon a family of candidate models that describe the plant. Then, adaptation is responsible for selecting the particular member of the family which will best represent the plant. Another interesting remark to be made is that for a value of $\mu = 0$, the scheme reduces to the task-space version of the rigid robot adaptive algorithm of Slotine and Li [142]. This scheme is not suitable for controlling flexible arms and when examined on our experimental flexible arm it became unstable due to the unmodelled flexible dynamics.

The scheme presented is claimed to exhibit very good robustness characteristics.
Robustness is usually treated at two different levels [139], robust stability (RS) and robust performance (RP). The first implies that the system remains stable for all plants in the uncertainty set. Given RS is satisfied, RP means that the system meets the performance specifications for all plants in the uncertainty set. As has already been pointed out, RS is an inherent characteristic of passivity-based controllers. As far as RP is concerned, the observation made in [107] that any comments are redundant because adaptive control is a performance oriented technique also applies here.

The feedback part of the controller was selected to be a very simple proportional term. Given the passivity nature of the scheme, similar to the nonadaptive scheme any SPR controller is guaranteed to stabilize the plant. The gain-scheduled SPR controllers version can also be used. The use of a gain scheduling technique in conjunction with the adaptive scheme is of course somewhat of a controversy. The design of a number of controllers which are optimal for certain configurations requires a priori knowledge of the dynamic parameters, the lack of which is the motivation for using an adaptive technique.

5.5.3 Joint-based parametrization of the regressor

The construction of the regressor matrix using a parametrization in task-space coordinates can be a very difficult exercise especially for complicated geometries. This problem can be overcome in a similar fashion to the nonadaptive case by using a suitable transformation. The “fictitious” joint quantities, $\theta_r(t)$, are defined by $\dot{\theta}_r = J_\theta(\theta_t, 0) \dot{\theta}_r$ so that:

$$\dot{\theta}_r \triangleq J_\theta^{-1}(\theta_t, 0) \dot{\theta}_r, \quad (5.20)$$

$$\ddot{\theta}_r \triangleq J_\theta^{-1}(\theta_t, 0) [\ddot{\theta}_r - J_\theta(\theta_t, 0) \dot{\theta}_r]. \quad (5.21)$$

Given the above definitions,

$$M_{\rho \rho}(\rho) \ddot{\rho}_r + C_{\rho}(\rho, \dot{\rho}) \dot{\rho}_r = J_\theta^{-T}(\theta_t, 0) [M_{\theta \rho}(\theta_t) \ddot{\theta}_r + C_{\theta}(\theta_t, \dot{\theta}_t) \dot{\theta}_r] \quad (5.22)$$

or

$$W(\ddot{\theta}_r, \dot{\theta}_r, \theta, \rho) \alpha = J_\theta^{-T}(\theta_t, 0) Y(\ddot{\theta}_r, \dot{\theta}_r, \theta, \rho) \alpha \quad (5.23)$$
with $Y$ the corresponding joint–based rigid version of the regressor matrix. Then, the applied torque and the adaptation law become:

$$\tau(t) = J^T_{θ}(θ, q_e) J^{-T}_{θ}(θ_t, 0) Y (\ddot{θ}_r, \dot{θ}_r, \dot{θ}_t, \dot{θ}_t) \ddot{α}(t) - J^T_{θ}(θ, q_e) K_d s_{μ},$$

$$\ddot{α}(t) = -Γ Y^T J^{-1}_{θ}(θ_t, 0) s_{μ}. \tag{5.25}$$

The transformation introduced here allows parametrizing the regressor matrix in joint coordinates and its construction becomes much easier since the Jacobian matrix relating the joint coordinates to the task-space ones is effectively eliminated from the formulation. Lifting the flexibility from the system it reduces to the transformation suggested in [142] for the Cartesian–space implementation of Slotine and Li's algorithm. In such a rigid context it also becomes mathematically similar to the transformation proposed by Fossen [60] for the adaptive control of a spacecraft. In that case, a parametrization of the regressor in terms of body frame rather than inertial frame coordinates was effectively achieved.

### 5.5.4 Parameter estimates and persistency of excitation issues

Based on the previous analysis it is clear that the stability and the convergence of the tracking errors to zero is independent of the convergence of the parameter estimates to their exact values, as would be required in parameter identification applications. The parameter updates can rather take such values that drive the tracking error trajectories to zero and preserve stability as well. As very common in adaptive control, the convergence of the parameters to their exact values depends on the desired trajectory rather than the actual one. Loosely speaking, the trajectory needs to “fluctuate” enough so that the resulting dynamics are sufficiently rich and allow the adaptation mechanism to extract all necessary information about the estimated parameters. Such trajectories are called “persistently exciting”. Alternatively, they have been referred to as “general enough”, “rich enough”, “sufficiently exciting” and “sufficiently spanning”. In the case of the regulation problem for example, intuition suggests that for
a zero or constant input there is not persistency of excitation (PE) and the controller will not be able to yield full parameter information. Although PE is not a necessary condition for the stability of our adaptive scheme it is in general a desirable property which is related to the robustness characteristics of the closed-loop system.

Strictly speaking parametric conversion can never be exact in practice. This requires that the assumed structure for the system on which the adaptation is based is identical to the one of the actual plant and also that perfect measurements are available. The above two conditions can only be realized in a simulation context. Examining the issue from the frequency domain perspective provides more insight to the PE property. Loosely speaking, the desired trajectory is persistently exciting if it is able to excite all the modes of the system. In general, for linear systems the convergence of $m$ parameters to their exact values requires that a least $m/2$ sinusoidal components are present in the reference input [145]. In the nonlinear case this simple condition does not hold in general.

A question that naturally arises is whether the estimated parameters converge or not in the absence of PE. In [76], this issue is examined for the case of a family of setpoint stabilizing adaptive controllers and is shown that the estimates will still converge to some stabilizing values. Therefore, adaptation can be switched off after a sufficient learning time, which will allow one to reach these values without violating stability. It was also shown that these values will be stabilizing even when used in conjunction with the equivalent nonadaptive fixed controller. It was concluded that although the results are quite general for the regulation problem, the extension to the tracking adaptive control is not straightforward and still remains an open issue.

Mathematically, the PE property is satisfied [143], [38], [40], if there exist constants $\delta$, $\alpha_1$, and $\alpha_2$, such that for all $t_1 \geq 0$,

$$
\alpha_1 t_1^+ \delta \leq \int_{t_1}^{t_1 + \delta} W^T_d W_d dt \leq \alpha_2 t_1
$$

(5.26)

where $W_d = W(\hat{p}_d, \check{p}_d, \hat{p}_d, \check{p}_d)$, i.e., the regressor matrix is calculated at the desired configuration.
5.5.5 Selection of controller gains

The feedback gain matrix can be selected in a fashion similar to the nonadaptive case as follows:

\[ K_d = k_d \tilde{J}^{-T} \tilde{M}_{\theta\theta} \tilde{J}^{-1} \]  

(5.27)

with the constant \( k_d \) being an overall scaling factor. An even simpler choice is a diagonal matrix with positive entries which can be more easily tuned experimentally.

Both the weighting matrix, \( \Lambda \), and the adaptation gain matrix, \( \Gamma \), can be selected as diagonal with positive entries. The relative size of the entries in the adaptation gain matrix is directly related to the PE property concerning the desired trajectory, i.e., parameters difficult to identify require that the corresponding entry is large. In Niemeyer and Slotine [104], the adaptation gains were taken proportional to the integral describing the PE property. A similar selection based on the mathematical definition of the property was also made in Damaren [44], in a way that provides a rather natural choice.

5.5.6 Minimal parametrization of the regressor

The linear form of the dynamics which was exploited for the adaptation can be realized using different updated parameter vectors. It is important to mention that when systems of interconnected bodies are considered not all the mass properties of the mechanism are required for a complete description of the dynamics. This fact arises from the kinematical constraints imposed on their relative motion due to the interconnections. From the adaptation point of view any redundant inertial parameters are not accessible by the estimation mechanism.

When constructing the regressor the dynamics should be formulated on the basis of a minimal number of inertial properties which are called the base parameters. An analytical treatment of the size of the base parameters set can be found in Mayeda et al. [91], for the case of rigid manipulators with rotational joints having the adjacent joint axes either parallel or perpendicular to each other. It was found that the number of base parameters is \( 7N - 4B \), when the axis of the first joint is not parallel to the gravity vector. Otherwise, the number becomes \( 7N - 4B - 2 \), where \( N \) as used here
is the number of links and \( B \) is the number of links parallel to the base one.

For a body that is free to move in the three-dimensional space, ten parameters are required in order to determine its dynamic equations: the mass, the three Cartesian position coordinates of the center of mass and six elements of the symmetric moment of inertia matrix. When the body is restricted to move on the horizontal plane and only rotate about an axis normal to the plane, the number of inertial parameters is reduced to three. For the case of our experimental arm the corresponding rigid robot is restricted to move on the horizontal plane and has three rotational DOFs. Therefore, the number of base parameters is seven and not all of the nine relevant mass properties are required to describe the system. The mass as well as the first moment of inertia of the first rigid link in the chain do not contribute to the dynamic equations of the system because of the kinematical constrains at the base.

Even when a minimal parametrization of the dynamics is considered, some of the base parameters might be known with adequate accuracy and not expected to vary during operation. Such parameters can be excluded from the adaptation. It is pointless to waste computational resources towards gaining information on something that is already known. In practice, it is more likely that the parameters of the robot itself are known with good accuracy and that uncertainty is related to the parameters of the grasped payload only. In such cases, the problem can be formulated by writing the dynamics in a linear form in terms of two regressor matrices with the first one, \( W_1 \), related to the known parameters collected in vector \( \alpha_1 \) and the other, \( W_2 \), corresponding to the unknown parameters found in vector \( \alpha_2 \):

\[
M_{\rho\rho}(\rho)\ddot{\rho}_d + C_{\rho}(\rho, \dot{\rho})\dot{\rho}_d = W(\dot{\rho}_d, \dot{\dot{\rho}}_d, \dot{\rho}, \rho)\alpha
= W_1\alpha_1 + W_2\alpha_2, \tag{5.28}
\]

i.e., \( W = [W_1 \ W_2] \) and \( \alpha = [\alpha_1 \ \alpha_2]^T \). The control and adaptation for this case are written as follows:

\[
\tau(t) = J^T_\theta(\theta, q_e)W_1(\dot{\rho}_r, \dot{\rho}_r, \dot{\rho}, \rho)\alpha_1 + J^T_\theta(\theta, q_e)W_2(\dot{\rho}_r, \dot{\rho}_r, \dot{\rho}, \rho)\ddot{\alpha}_2(t) - J^T_\theta(\theta, q_e)K_ds_{1\mu}, \tag{5.29}
\]

\[
\alpha_1 = \text{const.}, \quad \ddot{\alpha}_2(t) = -\Gamma W_2^T s_{1\mu}. \tag{5.30}
\]
Chapter 6

Preparation for Experiments

Theoretical proofs of stability are necessary for completeness when deriving a control scheme and an important step usually taken towards examining the behaviour of the closed-loop system is computer simulation studies. Strictly speaking, any conclusions from simulations will be valid for the mathematical model used but not necessarily for the real plant. The ultimate proof for the applicability and value of a controller remains the actual hardware implementation. Ortega et al. [107], stress the importance of experimental work by ironically referring to the "conventional wisdom that it is scientifically pointless to build an experiment to test a mathematical statement that is self-consistent and provably correct". Experimental work is also a necessary step for bridging the gap that exists between theory and practical applications. The philosophy of the present work is that theory and experiment go hand-in-hand and cannot be treated separately. The present chapter gives a detailed description of a special robotics facility that was used for our experimental investigations. Various related issues are discussed, a general framework for the experimental work is established, payloads and trajectories are standardized and the tools for analyzing the results are decided.
Experimental results involving flexible-link robots are of great value since flimsy-link space robots can only undergo limited testing prior to launch. Various types of facilities have been developed to investigate their behaviour. Manipulation experiments can be performed within the earth's gravitational field as done in [88], but more realistic ones involve facilities which allow performing zero-gravity experiments on Earth. The various approaches to emulate a pseudo-zero-gravity environment were categorized by Yoshida [163] into the following groups and indicative examples from each one of them are provided here:

1. Perform the experiment inside an aeroplane while in a parabolic trajectory or a free-falling chamber. This approach involves very expensive equipment and it is not practical.

2. Sink the apparatus inside a water pool and support every component using neutral buoyancy. The drawback of such an approach is that any movement within a high viscosity fluid involves resistance to motion and introduces new dynamic considerations. Such a facility was used in [158].

3. Suspend the whole system from tethers and implement a method for counter-balancing in a way that it does not interact with the dynamics of the arm, as done in [103].

4. Use planar systems and restrict motion in the horizontal plane so that gravity effects are not encountered. Such systems commonly support the system by air-cushion or air-bearings to ensure low friction between the bearing and the supporting surface. Such facilities were used in [163], [153], [113], [73] and [29].

5. Use "hybrid" simulation techniques which involve the calculation of the motion in zero-gravity environment based on a mathematical model and then imposing this behaviour on the experimental system.

Some of the above approaches can be very expensive to implement or impractical. The approach followed in the present work belongs to the fourth category. Although
the motion is restricted to one plane it still provides a valuable testbed for validating dynamic modelling techniques and testing control algorithms. Unlike most of the experimental work carried out in the field, which was focused on the one-link setup or arms with two links of which only one is flexible, our experiments involve the more realistic three-link configuration with two flexible members. This is an important excursion from single-link experimentation because of the nonlinear couplings due to the joint interconnections. The multiple joint and deformation DOFs interact with each other and greatly complicate the problem.

A significant amount of time and effort involved in the project was devoted to the mechanical design of the facility and its accessories. The facility was completely designed and built within the Department of Mechanical Engineering. Rather than presenting any of our manufacturing drawings, a detailed description of the facility will be attempted and the design objectives as well as the special features of the facility will be emphasized.

The experimental rig is shown in Figure 6.1. It consists of two robotic arms possessing three rotational DOFs and two flexible links each. The basic difference between the two arms is the quality of the gearboxes used, with respect to the amount of backlash they exhibit. We will refer to them as the "low-backlash" and the "high-backlash" arm respectively. The low-backlash one, a close posture of which is shown in Figure 6.2, is the one used for all the case studies presented in the thesis. The arms are constrained to move in the horizontal plane so that gravity effects are not encountered. They possess all three DOFs required for a planar arm in order to locate its end effector within the workspace with an arbitrary position and orientation. The arms have redundant DOFs as far as positioning tasks in the horizontal plane are concerned, which allow locating the end-effector at a specific position using an infinite set of different joint configurations. They are no longer redundant when the positioning task involves prescribing the orientational DOF of the end-effector as well. Each arm is supported on air-pads floating on a glass-topped table in an almost frictionless fashion. Continuous pressurized air is supplied to the air-pad system. The table's glass surface is supported on a grid of jacking bolts, which were carefully adjusted using a spirit level to ensure that the arms do not move against gravity.
Figure 6.1: The experimental facility.

Figure 6.2: The low-backlash flexible arm.
A specially designed payload shown in Figure 6.3, can be rigidly attached to the tip of each arm and both its mass and rotary inertia can be varied. Additional mass in the form of standard-size cylindrical plates can be added inside the payload's canister and change its inertial properties. Given that the focus of our studies is on the large payload case, the payload is required not only to have big mass but also a large moment of inertia. This property is increased by using two heavy blocks made of lead, supported on two cantilever beams extending on two diametrically opposite sides of the payload, so that the payload has a "satellite-like" shape. By sliding the blocks and setting their position along the supporting beam the moment of inertia can be adjusted accordingly. Multiple connections on the payload allow for both arms to be connected to it simultaneously as shown in Figure 6.4 and perform closed-loop geometry experiments.

The actuators used are direct current (DC) motors with gear reduction, the current of which is controlled by a PWM amplifier. The popularity of such actuators in robotics relies on the high torque-to-weight ratio they exhibit. When geared actuators are
used, the price to pay is some undesirable effects that are difficult to model or deal with and have already been discussed in the modelling chapter. These are the backlash at the joints, friction, large rotor inertias projected at the gearbox output shaft and joint flexibility. Each motor is equipped with incremental encoders which provide accurate measurements of the joint rotations. For both arms, the resolution of the encoders is 500, 1000 and 500 counts per turn for the shoulder, elbow and wrist motors respectively.

Figure 6.4: The closed-loop chain geometry.

One of the design philosophies was the modularity of the system which allows different arm geometries to be used and various combinations of rigid and/or flexible links. All the parts were designed so they are interchangeable. Another main design objective was to keep the center of mass of each individual component at the same height as the neutral axis of the flexible links so that torsional oscillations are not excited. Most parts of the arms were produced using accurate CNC machining. The main material used for the construction of the arms was aluminium which was selected due to its machinability, the low weight and the resistance to rust. The supporting
table structure was made of steel and the air-pads were made of plexiglass.

The control of the system is based on a personal computer (PC) and a high-speed digital signal processor (DSP), connected to the PC bus. For controlling the arms' motion two separate programs need to run simultaneously, one on the PC and one on the DSP. The DSP program is responsible for collecting all the sensor information, performing all the calculations required by the control algorithm and returning the related torque commands to the actuators. The PC program is used as a user interface and stores all the experimental data. Communication between the two programs is realized using a shared memory region in the DSP. The use of a DSP allows the execution of complex control algorithms at fast sampling rates as required by real-time applications.

The sampling frequency is an issue of crucial importance and its choice is related to the control algorithm that needs to be implemented as well as the presence of disturbances and measurement noise. The choice is often a compromise between contradicting requirements. For example, the sampling frequency has to be high enough to embrace the entire frequency range targeted by the controller. At the same time, when velocities are computed by differencing the available position measurements, their quality deteriorates at higher sampling frequencies on account of quantization effects. Quantization is a special nonlinearity related to digital control and causes numerical errors during the process of converting an analog signal which has infinite resolution, to a digital number with finite resolution. Quantization errors are unavoidable because analog to digital (A/D) and digital to analog (D/A) conversion are necessary for the communication of the digital control computer with the controlled plant. High sampling rates also require the availability of enough computational power. It was found that 200 Hz provides a suitable choice for implementing our control schemes.

The facility is equipped with a number of optical sensors that do not allow for the arm to exceed the boundaries of the table. Optical sensors on every joint prevent it from overlapping the previous one thus protecting the facility and the user. Torque saturation limits are imposed on the command torques so that the operation of the robot is automatically terminated in the case of unstable algorithms. That also
ensures that the saturation limits are not exceeded, which would violate our modelling assumptions.

6.2 Friction compensation

It has already been mentioned that friction is a very difficult phenomenon to model. Generally speaking, unmodelled or uncompensated friction is not expected to cause stability problems with our passivity-based controllers. Due to its dissipative nature, it has a stabilizing effect but at the same time it deteriorates the tracking performance. For our experimental case studies, open-loop friction compensation based on a simple Coulomb model was used, which only considers sliding friction and does not account for stiction. Some velocity dependence of friction was identified which was weak and very difficult to quantify, thus no viscous term was used. A friction term, $\tau_f$, was added to the controller torque output as follows:

$$
\tau_f = [\tau_{f1} \text{sgn}(\dot{\theta}_1), \tau_{f2} \text{sgn}(\dot{\theta}_2), \tau_{f3} \text{sgn}(\dot{\theta}_3)]^T,
$$

(6.1)

where $\tau_{fi}, i = 1, 2, 3$ are the values for the sliding friction corresponding to each joint. These values were determined experimentally by carefully examining each individual motor and are such that when the link connected to the actuator is given a push its motion continues uninterrupted. It was found that overcompensating for friction might cause instabilities and can be as bad or even worse than undercompensating. In Canudas de Wit et al. [54], the existence of possible limit cycles caused by overcompensation is analyzed and it is demonstrated that both underestimating and overestimating friction can be problematic.

When the motors/gearboxes were initially installed, the corresponding friction characteristics for the compensation were determined. After many hours of operation the friction torques were checked and found to be much lower. This fact suggests that the coefficients of the friction compensation have to be periodically checked and if necessary updated. The friction coefficients are subject to change due to ageing and wear of the mechanism, cleaning, lubrication or replacement of parts.

The direction at which the friction compensation acts is decided by the sign of
the velocity signals. Given that no tachometers were used for the direct measurement of the joint velocities, differencing the corresponding position measurements was used. Such measurements become particularly noisy at near-zero velocities so that the direction of the friction compensation alternates and introduces high frequency oscillations at the commanded torques. The amplitude of these oscillations though, is very small and does not cause any instabilities. This problem is expected to be more critical in the case of adaptive techniques and their success will depend on the quality of the available velocity measurements.

The use of a dither signal as discussed earlier in the modelling section was also considered. In general, it was found able to deal well with the positioning errors problem but in some cases it deteriorated the performance of the closed-loop system and its use was finally abandoned.

6.3 Filtering

In order to improve the quality of the available velocity measurements, filtering using low order Butterworth techniques was successfully employed. The order of the filters was found to be an important decision. High order ones were found to cause problems due to the time-delays introduced into the signals. Time-delays in the feedback measurements translate to making decisions based on old information and finally leads to instability. Moreover, heavy filtering causes distortion of the signals. Second order filters were designed and their continuous-time, state-space description was converted to the corresponding discrete form for their digital implementation in real-time. Both the joint and the end-effector velocity signals were filtered to reject all frequency content higher than 70 Hz.

According to Balas [17], a different filtering approach which can be useful in situations where truncated models are used for control purposes, is the “pre- and post-filtering”. Prefiltering, i.e., filtering of the measured signals prevents the truncated mode content from entering the controller and therefore deals with the observation spillover problem. Postfiltering on the other hand, i.e., filtering the controller output rejects all the residual frequency excitations by the controller and acts against the
control spillover problem.

A different way of obtaining better quality velocity measurements can be through state estimation techniques. Such approaches have not been tested here, but they are definitely worth more attention, since they can be key to improving the achievable performance of the controllers.

6.4 Calibration of motors

Before any experiments were performed the motors were calibrated to determine their characteristic curves, i.e., to find the relation between the torque commands and the output torques. An accurate calibration ensures that the controller outputs are translated to the correct torque commands supplied to the actuators. Every motor was tested in isolation and a rigid link of known mass properties was attached to it. The facility's large payload was connected at the end of the link so that its moment of inertia dominates over that of the actuator's rotating parts, which is poorly known. Constant torque commands were supplied to the actuator, the corresponding acceleration was calculated and the applied torque was determined. The characteristics of the actuators were found to be linear but a different characteristic was obtained for the acceleration and deceleration case. When using the arms, the control program monitors the acceleration mode and the corresponding characteristic is used accordingly.

The accelerations used for determining the characteristics were calculated from the available encoder position measurements and their quality was one of the main difficulties encountered. Other difficulties were due to the limited swap space on the table which did not allow high-acceleration tests. The backlash of the gearboxes was another source of difficulty because it induces vibrations at the beginning of the motion and when the direction of the torque is reversed.
6.5 End-effector position using strain-gauge measurements

Apart from joint rotation information, tip measurements are also required for the implementation of our schemes. In the case of rigid arms, the end-point position can be uniquely determined from the joint angles and the known geometry of the mechanism. For the flexible case though, due to the additional elastic DOFs, a different approach needs to be used. Various position detection camera systems are commercially available but their cost is usually high. Specially designed optical systems can be an alternative for reducing cost. As an example, a system external to the manipulator was proposed by Heeren and Veldpaus [64] and measures the translation coordinates of the end-effector of a flexible-link arm that operates on a planar surface. Such systems can sometimes be fairly complicated and the cost of the components involved can still be quite high.

Our facility is equipped with a two-dimensional charge coupled device (CCD) camera located above the table and provides accurate tip position and orientation measurements by detecting two light sources attached on the payload. Such devices consist of a number of parallel light sensitive CCD charge cells whose basic operation is to convert light into electrical charge. The amount of charge at each cell depends on the light intensity, its spectrum and the integration (collection) time. The position of a point light source can be determined after analyzing the light intensity profile captured by the CCD. The CCD camera is supported on a specially designed frame attached on the ceiling right above the table by using threaded rods and adjusting nuts that allow leveling of the system. This arrangement is shown in Figure 6.5. The supporting frame was designed in order to allow shifting the camera to different locations and focus on certain areas of interest. The camera can be positioned in one of three parallel guides running along the table and then slid along their length. The frame also provides support to the camera’s power supply.

This optical system needs to be carefully calibrated to find how the monitored area is mapped within the CCD array. Some practical issues which need special attention are briefly discussed here. Our controllers require feedback information related to
both the end-point and the joint rotations, so that problems might arise when the two sets of measurements are not compatible. Compatible refers to the agreement of the tip measurements obtained by the camera and the forward kinematics solution based on the measured set of generalized coordinates. Such compatibility requires accurate calibration of the camera, exact knowledge of the geometric characteristics of the mechanism and the ability to accurately position the mechanism at any initial geometric posture. All of the above hinge on the accuracy of machining and assembly of the components, the backlash which permits small rotations not sensed by the encoders, etc.

As an alternative to vision systems, a method based on strain-gauge measurements might be preferable due to its simplicity, low cost and compactness. Such a method does not require components external to the system like vision systems do. Resistive strain-gauges, which operate on the principle of change of electrical resistance due to mechanical deformation, have many desirable characteristics that make them suitable for such an application. Commercially available resistance-type strain-gauges
have gained popularity in strain measurement applications and a summary of their advantages over other strain measurement methods is provided by [42]. They have small size and low weight, provide accurate measurements, have stable calibration constants, exhibit linear behaviour over a wide range of strains, respond rapidly to changes and are easy to install and operate. Their measurements are basically independent of small temperature variations. A strain-gauge measures the average strain in its active length but due to their small size, the strain which is a point measurement is approximated quite accurately. In practical applications, changes in resistance are typically measured using a Wheatstone bridge circuit and strain measurements in the form of electric signals are well suited for computer control applications.

A method based on strain measurements was successfully utilized for implementing our controllers and will be described here. It involves a set of $k$ strain-gauges attached on each flexible link at discrete positions along its length, which were calibrated using static loading in order to provide the corresponding strain measurements. An $n$-th order polynomial was assumed to describe the deflected shape of each flexible link and capture equal number of vibration modes. Given the success of cantilevered shape functions when used in conjunction with the assumed modes method in modelling the system, polynomials satisfying clamped boundary conditions at the base were selected:

$$U_e(x, t) = q_{e,1}x^2 + q_{e,2}x^3 + \ldots + q_{e,n-1}x^n, \quad n \leq k + 1 \quad (6.2)$$

with $U_e$ being the deflection measured from the undeflected axis of the link and $x$ the distance from its inboard end. Mechanics of materials provides the strain-curvature relation for a flexible beam:

$$\varepsilon(x, t) = -\frac{b}{2} \frac{\partial^2 U_e(x, t)}{\partial x^2}, \quad (6.3)$$

where $b$ is its thickness. Evaluating $\varepsilon$ at each of the $k$ measurement sites gives a linear system of simultaneous equations, the solution of which provides a set of elastic DOFs for the arm. When $n < k + 1$ the problem is overdetermined and a pseudoinverse least squares solution for the system can be obtained. Forward kinematics can then be applied to give the position of the end-point, and tip velocity measurements can be obtained by differencing the position signals. Given that the method provides
the vector of elastic coordinates, it is very well suited for implementing controllers requiring full state feedback information.

At each measurement location, two strain-gauges were cemented on either side of the link. This is very common in applications because by suitably connecting them to the bridge arrangement, torsional and thermal effects have opposite influence and therefore cancel each other. Shielded cables were used to prevent electrical noise problems due to the proximity with magnetic fields generated around the actuators. Three sets of strain-gauges were attached evenly distributed along the length of each flexible link \((k=3)\) and third order polynomials were considered \((n=3)\). It is known that cubic polynomials can describe the exact deflected shape of a cantilevered beam with a static bending load at its outer end. Cubic polynomials were tested in conjunction with \(k=2\) and \(k=3\), and fourth order ones with \(k=3\). Sinusoidal inputs were used and it was seen that the calculated deflections were almost identical for each one of the above cases. Static loading experiments were performed for the cantilever case yielding very accurate and repeatable results. For the dynamic case, the maximum deflections at the tip of the flexible links were also measured and found to compare accurately with the estimated ones. These deflections were recorded using a pencil attached to the tip of the link, leaving a mark on a rigid follower arm while the flexible link deforms. The rigid follower link was attached as a cantilever to the base of the joint. This test does not account for the foreshortening effect, whose contribution does not play a significant role in the present case anyway.

The accuracy of the method was further assessed using the available measurements from the CCD camera and the two sets of measurements were found to compare very well. The difference between the two sets is within the motion due to backlash, which can only be seen by the camera. The elastic motion sensed using the strain-gauges was also found to be in good agreement with the simulated one. The method is based on the assumption that the selected form of the polynomial can describe the deflected shape of the link. Although no theoretical justification for the method exists all testing seems to verify its validity. The same assumption basically applies to the case of the widely used assumed modes method, which has been established as a standard technique in dynamic modelling.
Experimental results utilizing such a strain-gauge method were reported in [62]. Investigations of a similar approach to the one described here can be found in Miller and Piedboeuf [92], the findings of which verify some rather intuitive facts. It was found that increasing the values of $k$ and $n$ improves the accuracy of the calculations but after a sufficiently large number the improvements are not significant. The size of those two numbers should provide the best compromise between the desired accuracy and practical considerations. For example, attaching a very large number of strain-gauges on a link is not possible and unnecessarily increases the computational load on the control computer. Regarding the optimal positions for attaching the strain-gauges on the link, they found using simulation studies that the best results are obtained when the strain-gauges are evenly distributed along its length. In spite of those results, one would normally expect that the optimal positions of the strain-gauges will depend on the characteristics of the system under consideration.

A different method using strain-gauge measurements in flexible-link control was presented in Nemir et al. [102], where a pseudolink concept was developed for describing the end-position of a flexible link. Joint encoder measurements together with strain-gauge measurements taken along the flexible link are used to calculate the length and the angle of the pseudolink and then this information was used for on-line control.

![Figure 6.6: Calibration of strain-gauges.](image)

In a practical robotics application like the one we are interested in, when selecting the strain-gauges, it might be important to carefully consult the specifications of the manufacturer related to the allowable elongation of the strain-gauge and its fatigue life. The installation of the strain-gauges was done according to the instructions of the
manufacturer and involved carefully preparing the surface and using special adhesives for the bonding. Standard procedures from experimental mechanics handbooks are available for calibrating strain-gauges attached on a flexible beam. Each method involves a certain way of supporting the beam and applying static loads in order to measure the resulting displacement. Figure 6.6 shows the arrangement that was used in the present case for the calibration. The flexible member has length \( l \), and the strain-gauge to be calibrated is located at distance \( x \) along the length of the arm. A whole range of different static loads, \( F \), were applied at the tip of the link. The resulting deflection at the loading point was recorded together with the corresponding raw measurements from the computer. From mechanics of materials the static force is related to the static displacement at the tip, \( U_e(tip) \), and the strain along the length of the arm, \( \varepsilon(x) \), according to the following relations [42], [27]:

\[
U_e(tip) = \frac{F l^3}{3EI} = \frac{4F l^3}{Eb h^3},
\]

\[
\varepsilon(x) = \frac{6F}{Eb h^2} (l - x).
\] (6.5)

Combining the two equations:

\[
\varepsilon(x) = \frac{3h(l - x)}{2l^3} U_e(tip),
\] (6.6)

where \( E \) is the Young’s modulus and \( I = h^3 b/12 \) is the area moment of inertia of the link’s cross-section, with \( h \) the depth and \( b \) the width of the cantilever beam. Eq. (6.6) was used for translating the tip deflections to the corresponding strains at the measuring point as needed for the calibration. A linear relation between the strain and the corresponding measurement was found. A different arrangement for the calibration of strain-gauges attached on a beam is the calibrator described in [65]. In that case the beam is simply supported at two points which are symmetric on either side of the strain-gauge and a static load is applied on each free end.

It is very interesting to note that special strain-gauges find applications in control applications in nature. The article [171], refers to living creatures like cockroaches, crabs and spiders equipped with biological strain-gauges which detect deformations at certain critical strain regions of their exoskeleton. This information is utilized for controlling their movements and also to memorize stepping patterns. This fact
raises the important issue of studying the example of nature when dealing with certain technological problems. In the field of robotics, benefits can be obtained in many areas such as locomotion, vision and sensor technology, needless to mention that robotic manipulators themselves were originally designed to mimic the human arm. Another example of biologically inspired robots is the case of snake–like robots studied in [66].

6.6 Arm geometry and mass properties

The flexible links on both arms of the facility are 6063 aluminium beams with Young’s modulus $E = 68.3 \text{ GPa}$ and cross section $6 \times 30 \text{ mm}$. A schematic view of the arm is shown in Figure 6.7, with the base motor rigidly attached on the supporting table and the rest of the arm as well as the payload supported on the air–pads which float on the glass surface. Due to the design geometry of the arm, mechanical offsets exist between each flexible link and the actuators. These offsets correspond to additional rigid links that were modelled as part of the arm. Consequently, the arm’s model consists of seven interconnected bodies two of which are flexible and a large payload attached to the outer end of the chain.

The mass properties of each individual component of the two arms were calculated using solid models produced by computer aided design (CAD) software, as shown in Figure 6.8 for the case of the low–backlash arm. For the calculation, it was assumed that the material of each component is homogeneous and small manufacturing details that do not contribute much to the dynamics were not considered. Cables and air tubes were also left out of the model although they definitely contribute to the dynamics, especially to the structural damping. Several assumptions had to be made for the distribution of mass within the actuators assembly given that the stator and the rotor of each motor had to be considered as part of a different body in the chain.

The mass properties for each one of the two arms are shown in Tables 6.1 and 6.2 respectively, where $l$ is the length, $m$ is the mass, $c_x$ is the first moment of inertia measured along its length and $I$ is the second moment of inertia about the vertical axis. All properties refer to a frame $\mathcal{F}_n$ rigidly attached to the inboard end of each body as indicated in Figure 6.7. The rotor inertias as “seen” at the output shaft of
Figure 6.7: Schematic view of the arm.

Figure 6.8: Solid models.
each gearbox are $I_{r1} = 127.8$, $I_{r2} = 306.9$, $I_{r3} = 16.1 \text{g} \cdot \text{m}^2$ for the low-backlash arm and $I_{r1} = 88.8$, $I_{r2} = 150.4$, $I_{r3} = 8.3 \text{g} \cdot \text{m}^2$ for the high-backlash one.

For our tests four standard payload sizes were used, ranked in increasing size order as follows:

- **Payload 1**: Empty payload casing with inertia elements at the inboard end.
- **Payload 2**: Empty payload casing with inertia elements at the outboard end.
- **Payload 3**: Half available extra mass inside the payload casing and inertia elements at the outboard end.
- **Payload 4**: All available extra mass inside the payload casing and inertia elements at the outboard end.

No smaller sizes were used because that would violate our fundamental large payload assumption. Larger ones are restricted by the available torques and by the fact that they would overload the structure and reduce the life of the links. The mass properties for each one of the above payloads are collected in Table 6.3.

Given the above nominal values of the mass properties, the lower four unconstrained vibration frequencies for the low-backlash arm predicted by the model are $\omega_1 = 4.80$, $\omega_2 = 10.26$, $\omega_3 = 27.87$ and $\omega_4 = 69.39 \text{Hz}$ respectively. The above frequencies were calculated for the fully-stretched configuration when Payload 2 is attached at the tip of the arm. For the high-backlash arm, the corresponding vibration frequencies are $\omega_1 = 5.84$, $\omega_2 = 11.93$, $\omega_3 = 33.86$ and $\omega_4 = 78.39 \text{Hz}$ respectively. The natural frequencies of the actual arm can be found by simply injecting sinusoidal inputs to the system and varying their frequency until resonance occurs. The matching of the vibration frequencies and mode shapes with the ones predicted by the model is usually a good measure of the accuracy of the model and it is a common model validation technique. In our case, very close agreement between the theoretical and experimental frequencies was observed especially for the lower ones.

A very reasonable question to be asked is how large a payload should be in order to be considered as such. Of course the answer is that it has to be large enough to satisfy the clamped nature of the vibration modes. To explain this further the effect
<table>
<thead>
<tr>
<th>Body</th>
<th>$l$ (mm)</th>
<th>$m$ (kg)</th>
<th>$I$ (g·m²)</th>
<th>$c_x$ (g·m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37</td>
<td>2.53</td>
<td>0.12</td>
<td>1.61</td>
</tr>
<tr>
<td>2</td>
<td>392</td>
<td>0.19</td>
<td>9.79</td>
<td>37.48</td>
</tr>
<tr>
<td>3</td>
<td>77</td>
<td>2.23</td>
<td>14.93</td>
<td>161.77</td>
</tr>
<tr>
<td>4</td>
<td>112</td>
<td>2.10</td>
<td>4.60</td>
<td>29.52</td>
</tr>
<tr>
<td>5</td>
<td>327</td>
<td>0.16</td>
<td>5.69</td>
<td>26.08</td>
</tr>
<tr>
<td>6</td>
<td>66.7</td>
<td>0.93</td>
<td>4.15</td>
<td>54.61</td>
</tr>
<tr>
<td>7</td>
<td>70</td>
<td>0.97</td>
<td>1.67</td>
<td>9.36</td>
</tr>
</tbody>
</table>

Table 6.1: Mass properties of the low-backlash arm.

<table>
<thead>
<tr>
<th>Body</th>
<th>$l$ (mm)</th>
<th>$m$ (kg)</th>
<th>$I$ (g·m²)</th>
<th>$c_x$ (g·m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>37</td>
<td>2.66</td>
<td>0.23</td>
<td>1.84</td>
</tr>
<tr>
<td>2</td>
<td>401</td>
<td>0.19</td>
<td>10.49</td>
<td>39.22</td>
</tr>
<tr>
<td>3</td>
<td>62</td>
<td>1.92</td>
<td>8.43</td>
<td>111.74</td>
</tr>
<tr>
<td>4</td>
<td>82</td>
<td>1.80</td>
<td>2.19</td>
<td>15.82</td>
</tr>
<tr>
<td>5</td>
<td>362.3</td>
<td>0.18</td>
<td>7.73</td>
<td>32.01</td>
</tr>
<tr>
<td>6</td>
<td>55.7</td>
<td>0.95</td>
<td>2.88</td>
<td>46.08</td>
</tr>
<tr>
<td>7</td>
<td>63</td>
<td>0.95</td>
<td>1.50</td>
<td>7.32</td>
</tr>
</tbody>
</table>

Table 6.2: Mass properties of the high-backlash arm.

<table>
<thead>
<tr>
<th>Payload</th>
<th>$m$ (kg)</th>
<th>$I$ (g·m²)</th>
<th>$c_x$ (g·m)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Payload 1</td>
<td>8.659</td>
<td>216.31</td>
<td>971.03</td>
</tr>
<tr>
<td>Payload 2</td>
<td>8.659</td>
<td>480.36</td>
<td>971.03</td>
</tr>
<tr>
<td>Payload 3</td>
<td>10.545</td>
<td>507.78</td>
<td>1189.54</td>
</tr>
<tr>
<td>Payload 4</td>
<td>12.432</td>
<td>535.20</td>
<td>1408.05</td>
</tr>
</tbody>
</table>

Table 6.3: Mass properties of the standard payloads.
of the payload size on the vibration frequencies was examined. Figure 6.9 shows the
lowest four unconstrained vibration frequencies captured by the model of the low-
backlash arm while the size of the payload varies. The plant was linearized at the
fully-stretched configuration and Payload 2 was used with its mass properties scaled
by a material density factor (%). As expected, while increasing the size of the payload
the vibration frequencies drop. That explains why one of the cases flexible-link robots
need to be treated as such is when they manipulate large payloads. For a large increase
of the payload though, the frequencies tend to converge to some constant values which
are the ones corresponding to a clamped boundary condition at the tip of the robot.
The closeness of the frequencies to those values provides a quantitative measure of
whether the payload is large enough to satisfy the fundamental assumption.

As has already been explained, a very important quantity related to a robotic
mechanism is the Jacobian matrix. For the case of our experimental arm, the Jacobian
of the corresponding rigid robot is given by:

\[ J_\theta(\theta, 0) = \begin{bmatrix} -l_1s_1 - l_2s_{12} - l_3s_{123} & -l_2s_{12} - l_3s_{123} & -l_3s_{123} \\ l_1c_1 + l_2c_{12} + l_3c_{123} & l_2c_{12} + l_3c_{123} & l_3c_{123} \\ 1 & 1 & 1 \end{bmatrix} \] (6.7)

where \( l_i \) the length of the \( i \)-th rigid link, \( s_i = \sin(\theta_i) \) and \( c_i = \cos(\theta_i) \). More subscript indices imply the sum of the corresponding sinusoidal functions, e.g., \( s_{ijk} = \sin(\theta_i) + \sin(\theta_j) + \sin(\theta_k) \) and \( c_{ijk} = \cos(\theta_i) + \cos(\theta_j) + \cos(\theta_k) \). The singularities of the mechanism can be identified by solving the equation \( \det J_\theta(\theta, 0) = 0 \), which in the present case gives \( \theta_2 = 0, \pi \).

### 6.7 Desired trajectories

The motion planning problem in robotics is commonly divided into two categories. First, is the positioning or regulation problem, which involves moving from an initial to a final prescribed configuration and is related to a step command often called the desired setpoint. Second, is the more useful case of trajectory following, i.e., the robot has to move between an initial and a final configuration in a certain length of time and the intermediate positions are also prescribed as functions of time. The computation of suitable functions is called trajectory generation. Trajectory tracking control is of course a more difficult problem than regulation, which can be seen as a tracking problem with constant input. It can be useful to point out an important distinction made in [125] that clarifies the difference between the terms path and trajectory, which often tend to be used interchangeably. A path is the geometric description of the motion and does not include the element of time. On the other hand, trajectory is a time law that the motion has to obey.

Since tasks are defined for the end-effector, a trajectory is naturally defined in task-space. In the case of rigid robots, due to the direct correspondence between the task and the joint-space motion, desired trajectories can alternatively be defined in joint or task space and if necessary the kinematics solution (forward or inverse accordingly) can be applied to provide the other description. In the case of flexible-link robots though, the kinematics problem is not just a geometric solution but also
a dynamics problem, therefore direct planning of the trajectories in task-space is more appropriate. The trajectories need to be collision-free and also ensure that the control action remains within the capabilities of the actuators. In a practical situation, trajectory generators need to satisfy additional requirements which will not be considered here. For example, time-optimal or energy-optimal trajectories can be particularly useful especially for the case of repetitive tasks. The design of suitable trajectories that do not excite large deflections during motion is also an approach relevant to trajectory planning for flexible robots. When a trajectory is designed directly in task-space, special issues that need to be considered are end-effector positions which can be reached in multiple joint configurations and the existence of singularities, in the vicinity of which the corresponding joint rates become too high.

The trajectory generator should not be computationally demanding. Low order polynomials have been extensively used in rigid-robot motion planning. They are described by simple mathematical relations and the user only has to define the initial and final configuration, as well as the duration of the maneuver. Commonly used are the cubic polynomials but quintic ones are better since they can be smooth both in velocities and accelerations and will be used here. Our schemes were tested using a number of different, twice differentiable trajectories. In general, it was found that smooth (without sharp peaks) task-space functions were the most trackable. Due to the nonlinear nature of the kinematics problem, smooth joint trajectories often correspond to task-space ones which exhibit sharp peaks and therefore impose a serious challenge to any task-space controller. Our standard trajectories were selected so that singular configurations are not encountered during motion.

In the case of our planar three-DOF arm there are three controlled task-space DOFs, namely the two Cartesian positions plus the orientation of the end-point, all of which are measured with respect to the inertial frame located at the base:

$$\rho = [x, y, \phi]^T.$$  
(6.8)

Three different trajectories were used for our tests:

- **Trajectory 1.** The desired end-effector trajectory, $\rho_d$, is such that all three end-effector DOFs follow a quintic polynomial between an initial, $\rho_i$, and a final
position, $\rho_f$, in a certain length of time, $t_f$:

$$\rho_d(t) = \left[ 10 \left( \frac{t}{t_f} \right)^3 - 15 \left( \frac{t}{t_f} \right)^4 + 6 \left( \frac{t}{t_f} \right)^5 \right] (\rho_f - \rho_i) + \rho_i. \quad (6.9)$$

The initial position is taken to be the one that corresponds to the undeflected joint posture $\theta_i = \left[ -\frac{\pi}{8}, \frac{\pi}{4}, 0 \right]^T$ rad and the final one to $\theta_f = \left[ \frac{\pi}{8}, \frac{7\pi}{16}, \frac{\pi}{3} \right]^T$ rad. The duration of the motion is taken to be $t_f = 3$ seconds.

- **Trajectory 2.** The task-space trajectory is the trajectory that the corresponding rigid robot would follow if all the joints were tracing a fifth order polynomial between an initial, $\theta_i$, and a final, $\theta_f$, joint configuration:

  $$\rho_d(t) = \mathcal{F}_r(\bar{\theta}_d), \quad (6.10)$$

  $$\bar{\theta}_d(t) = \left[ 10 \left( \frac{t}{t_f} \right)^3 - 15 \left( \frac{t}{t_f} \right)^4 + 6 \left( \frac{t}{t_f} \right)^5 \right] (\theta_f - \theta_i) + \theta_i. \quad (6.11)$$

  The initial undeflected joint configuration is taken to be $\theta_i = \left[ \frac{\pi}{4}, \frac{\pi}{4}, 0 \right]^T$ rad and the final one $\theta_f = \left[ \frac{7\pi}{16}, \frac{7\pi}{16}, \frac{\pi}{3} \right]^T$ rad. The duration of the motion is taken to be $t_f = 3$ seconds.

- **Trajectory 3.** A complete circle of radius 25 cm in task-space, whose axes are parallel to the base frame. Initially the tip rests at the right-hand-side quadrant of the circle and its orientational DOF remains constant during the motion. The angular velocity of the tip follows a sinusoid (from low to high peak) to reach its maximum value and then starts to reduce to zero again, following the same sinusoid. The acceleration and deceleration stages were taken to correspond to half of the circle and each stage lasts $3.5$ sec. The initial and final task-space configuration corresponds to the rigid joint configuration $\theta_i = \left[ \frac{\pi}{8}, \frac{3\pi}{8}, 0 \right]^T$ rad. This trajectory is control demanding in the sense that it involves passing through a whole range of different configurations and also requires the inversion of the direction of rotation for all three joints.

For all the above trajectories, the control action is maintained for 8 seconds after the completion of the useful motion in order to allow examination of the steady-state behaviour of the system and the ability of the controller to actively damp the
oscillations. For the adaptive case, the maneuvers will be used repeatedly in a periodic fashion in order to allow enough time for learning and better demonstrate the merits of adaptation. Between each reversal, a 3 second resting time will be considered.

6.8 Performance criteria

To describe the performance in quantitative terms, it is very common to use the size of certain signals of interest and define suitable norm quantities. Three time-domain quantitative criteria were defined in order to accommodate our needs for evaluating the controllers and comparing them. Both the tracking as well as the transient behaviour of the closed-loop system is considered.

- **Task-space trajectory-following accuracy.** The standard \( L_{2T} \) and \( L_{\infty T} \) norms for the task-space errors will be used to describe the tracking capabilities of the controllers. The former provides an overall measure of the error and the latter its maximum value. These two norms will be calculated for the duration of the useful motion only, i.e., the time required until the final configuration is reached:

\[
\| \bar{\rho}_i \|_{2T} \overset{\triangle} = \sqrt{\int_0^{t_f} [\rho_i(t) - \rho_{id}(t)]^2 dt}, \quad (6.12)
\]

\[
\| \bar{\rho}_i \|_{\infty T} \overset{\triangle} = \max_{t \in [0,t_f]} [\rho_i(t) - \rho_{id}(t)] \quad (6.13)
\]

with \( \rho_i \) \((i = 1, 2, 3)\) the \(i\)-th component of the generalized position vector \( \rho \) and \( \rho_{id} \) the corresponding desired value. Since the experimental data are collected in discrete intervals of time, \( t_1, t_2, \ldots, t_k \), with a constant sampling period \( \Delta T = t_n - t_{n-1} \) \( \forall \ n \), the discrete form of the above norms will be considered. The error measures will be defined as follows:

\[
| \bar{\rho}_i | \overset{\triangle} = \sqrt{\Delta T} \sqrt{\sum_{j=1}^{k} [\rho_i(t_j) - \rho_{id}(t_j)]^2}, \quad (6.14)
\]

\[
\bar{\rho}_{i,\text{max}} \overset{\triangle} = \max_{1 \leq j \leq k} [\rho_i(t) - \rho_{id}(t)]. \quad (6.15)
\]

All three controlled DOFs of \( \rho = [x, y, \theta]^T \) can be considered individually and \( [|\ddot{x}|, |\dot{y}|, |\ddot{\theta}|]^T \) calculated using the \( L_{2T} \) norm and \( [\ddot{x}_{\text{max}}, \dot{y}_{\text{max}}, \ddot{\theta}_{\text{max}}]^T \) calculated
using the $L_{\infty T}$ one. For the translational task-space DOFs, it is meaningful to combine the $x$ and $y$ tracking errors since they are measured in the same physical units but they will rather be considered separately given that they are two independently controlled DOFs. The purpose of defining the norm quantities is to quantify performance and allow comparisons, so that it is the relative size of the norms that is important and not their actual values. For that reason the discrete form for the $L_{2T}$ error defined above will be divided by the constant $\sqrt{\Delta T}$.

- **Ability to actively damp oscillations.** This ability will be judged by the residual vibrations after the completion of the useful motion. Instead of using the raw strain measurements for that purpose we will rather consider a more meaningful quantity, i.e., the elastic displacements at the tip of the two flexible members, $U_{e,i}(\text{tip})$, $i = 1, 2$. All the individual strain measurements should also be consulted to ensure that the residual vibration modes do not exhibit a node at the tip of the flexible links. In such a case, the above measure will be inappropriate because vibrations will not be observable. The standard $L_{2T}$ norm will be used to describe the overall deflections at the tip of each link by defining $|U_{e,1}(\text{tip})|$ and $|U_{e,2}(\text{tip})|$. The maximum deflections for each one of the two flexible links will be described using the relevant $L_{\infty T}$ norms and denoted as $U_{e,1}(\text{tip})_{\text{max}}$ and $U_{e,2}(\text{tip})_{\text{max}}$ respectively. The norms will be defined similar to the ones proposed for quantifying the tip tracking accuracy and will be calculated for the time after the completion of the useful motion, during which the control action is maintained. The corresponding discrete form of the norms will be used and division by the time constant $\sqrt{\Delta T}$ will be applied as before.

- **Steady-state error.** Since not all task-space coordinates are measured in the same physical units, it is natural to consider the total absolute steady-state error at the joints level (in rad) that remains at a certain time after the completion of the useful motion and the decay of the residual oscillations. Although the controller treats each one of the controlled DOFs individually, the
above will provide an overall measure for the steady-state error which will be useful for comparisons. For the case of our experimental arm, this will be equal to the steady-state error at the orientational task-space DOF, $\phi_{ss}$. Before the steady-state error is measured, enough time should be allowed to ensure that the oscillations have completely decayed.
Chapter 7

Experimental Results – Nonadaptive Case

In this chapter, the typical behaviour exhibited by conventional joint-based techniques commonly used in rigid robot control will be investigated and their inadequacy for the flexible-link case will be pointed out through experimental results. Then, detailed experimental case studies on the nonadaptive approach for the payload dominated case will demonstrate the applicability and the value of the scheme and provide insight to its nature and characteristics. The design of the controllers as well as their stability and robustness characteristics will be investigated.

7.1 Joint-based rigid-robot schemes

It has already been mentioned that most of the traditional control techniques used for rigid robot control are joint-based, with the control objective being to follow the inverse kinematics trajectories that correspond to the desired task-space motion. The case studies in this section demonstrate the behaviour exhibited by those schemes and explain why such control strategy is in general not suitable for the flexible-link case.

When the joint-based PD controller described by Eq. (4.1) was implemented on the experimental flexible arm, it was found to be stable but unable to follow the desired task-space trajectory due to the deflections of the flexible members. Oscillations persisted long after the completion of the useful motion and were very difficult
to damp. The joint-based form of the CTM described by Eqs. (4.2) and (4.3), which is suitable for the rigid-robot tracking problem was also tested and found to easily go unstable. More successful was the passivity-based PD+ controller described by Eq. (4.6), which was found to be more robust than the CTM and its performance was much better than the joint-based PD case. Case studies using this controller will be presented, the behaviour of which can be regarded as typical of the one exhibited by joint-based schemes for rigid robots when used for controlling flexible-link ones. Experimental results for the case of Trajectory 1 are presented and discussed here. More results for the cases of Trajectory 2 and Trajectory 3 can be found in Appendix A. Payload 2 was used for all relevant case studies and the gains were selected to be diagonal matrices with \( K_p = \text{diag}\{250, 125, 50\} \text{ N-m/rad} \) and \( K_d = \text{diag}\{10, 5, 2\} \text{ N-m-s/rad} \). The feedback gains for the controller of this example and all the others reported in the thesis, were selected and tuned manually so that the performance of the controlled system was optimized. In other words, a combination of gains was found that results in stable behaviour, good trajectory following and also minimizes overshoot and residual oscillations after the completion of the useful motion. Given that some of these requirements are sometimes in competition with each other, the best compromise had to be found by adjusting the size of the gains accordingly.

Figure 7.1 shows the joint-space tracking using the PD+ controller. The continuous line represents the actual motion and the dashed one is the commanded trajectory, i.e., the rigid inverse kinematics solution corresponding to the desired task-space motion. The tracking of the commanded joint trajectories is possible but it becomes more difficult than it is for the rigid case and requires tight feedback loops. The target configuration is finally reached but it takes a considerable amount of time for the oscillations to decay. The corresponding task-space motion together with the desired one is shown in Figure 7.2. The tracing of the Cartesian tip-position path is shown in Figure 7.3 together with the desired one. It is clear from these graphs that good tracking of the commanded rigid joint trajectories results in large tip-tracking errors because of the induced elastic motion. Figure 7.4 shows the size of the deflections, \( U_{ei}(\text{tip}) \), and rotations, \( \Phi_{ei}(\text{tip}) \), that occur at the outboard end of the \( i \)-th flexible link. Oscillations are shown to persist long after the completion of the useful motion.
The above results clearly demonstrate that the joint-based controllers commonly used in rigid-robot control are inefficient when used for controlling flexible ones.

7.2 Payload dominated inverse dynamics and passive feedback

Simulation studies on the latest versions of both the nonadaptive and the adaptive schemes for large payloads as proposed in the thesis were presented in [34] and experimental results can be found in [35] and [36]. Three case studies will be presented in the thesis in order to demonstrate the nature and the characteristics of the non-adaptive scheme, each one corresponding to one of the standard trajectories. The results on Trajectory 1 are presented and discussed here and the ones for Trajectory 2 and Trajectory 3 can be found in Appendix B. The ability of the scheme to combine task-space tracking with simultaneous active damping of the vibrations will be demonstrated.

For all the examples reported here, Payload 2 was manipulated by the arm and the controllers were designed and tuned for this case. For the case study based on Trajectory 1, the feedback gains were selected to be diagonal matrices, \( K_p = k_p \text{ diag}\{15 \text{ N/m}, \ 20 \text{ N/m}, \ 0.6 \text{ N \cdot m/rad}\} \) and \( K_d = k_d \text{ diag}\{15 \text{ N/m \cdot s}, \ 20 \text{ N/m \cdot s}, \ 0.6 \text{ N \cdot m/rad \cdot s}\} \), with \( k_p = 28 \) and \( k_d = 0.8 \). For the other two case studies the gains were selected according to Eqs. (4.30) and (4.31), with \( k_p = 27, \ k_d = 1.3 \) for Trajectory 2 and \( k_p = 35, \ k_d = 1.3 \) for Trajectory 3. Unless otherwise indicated, the above controllers are the ones used throughout the investigations to be reported.

The selection of a suitable value for \( \mu \) was based on the eigenvalue analysis procedure proposed earlier in Chapter 4. According to the closed-loop system matrix in Eq. (4.35), the value of \( \mu^* \) depends on the mass properties of the arm, its configuration and the size of the feedback gains as well. In general, the size of \( K_p \) was found to have little effect on \( \mu^* \) but the size of \( K_d \) was more relevant. That was somehow expected because the two quantities are both related to the damping behaviour of the closed-loop system. Numerical studies have shown that a high value for \( K_d \) results
Figure 7.1: Joint-space tracking using the PD+ controller (--- rigid, — actual).

Figure 7.2: Task-space tracking using the PD+ controller (---- desired, ---- actual).
Figure 7.3: Cartesian tip-position path following for the PD+ controller case (--- desired, ---- actual).

Figure 7.4: Deflections and rotations at the outboard end of each flexible link for the PD+ case.
in a lower value for $\mu^*$. The value of $\mu^*$ with respect to the payload size is examined in Figure 7.5, for a linearization of the plant at the undeflected middle configuration of Trajectory 1, $\rho_d(t_f/2)$, and constant feedback gains. Payload 2 was used and its size was varied by scaling the density of the material, thus causing a proportional change to all mass properties. It is clear that while the payload gets bigger, the value of $\mu^*$ increases and tends asymptotically to 1. For small payloads, a lower value for $\mu$ needs to be used. From this graph it should not be concluded that the scheme will also work for small payloads given that a very small value of $\mu$ is used, because that constitutes a violation of the fundamental large payload assumption.

Figure 7.6 demonstrates the procedure for selecting a suitable value of $\mu$ for the case of our nonadaptive controller designed for Trajectory 1. The critical values for stability were calculated on the basis of the desired trajectory. The area below the graph is the stability area and the minimum value of $\mu^*$ on the graph provides an upper limit for the choice. On the other hand, for the validity of the standard implementation assumption it is important to maintain the value of $\mu$ as close to 1 as possible. In the present case, a value of $\mu = 0.92$ was selected and was used for all case studies in the thesis involving Trajectory 1, both the nonadaptive and adaptive ones. For the case of Trajectory 2 and Trajectory 3, $\mu = 0.92$ and $\mu = 0.90$ respectively were used. In conclusion, the proposed procedure was found to give a very good indication of how big the value of $\mu$ can be without causing any stability problems.

In Figure 7.7, the three graphs at the top show the tip-position tracking for each one of the three controlled task-space DOFs for the case of the proposed nonadaptive scheme. The continuous line represents the actual task-space trajectory and the dashed line is the desired one. Very good tracking is evident without overshoot or any residual vibrations after the completion of the useful motion. A small positioning error exists at the steady-state due to the friction at the joints and the air-bearing system. The three graphs at the bottom show the corresponding joint-space tracking, with the solid line being the actual joint motion and the dashed one the trajectory that the corresponding rigid robot would have to follow in order to yield the desired
Figure 7.5: The effect of the payload size on $\mu^*$. 

Figure 7.6: The value of $\mu^*$ during the desired motion.
Figure 7.7: Trajectory tracking (--- desired/rigid, —— actual).

Figure 7.8: Rigid forward kinematics map based on measured angles (--- desired trajectory, —— rigid forward kinematics map).
task-space one. It is not surprising that the two trajectories are different as the endpoint position depends not only on the joint rotations but on the link deflections as well. The joint DOFs evolve in such a way that allows compensation for the elastic deflections, so that the tip remains on the desired task-space trajectory and damping is added to the elastic motion. This fact once again explains why traditional joint-based, rigid-robot control approaches are condemned to failure when used in the flexible-link case.

Figure 7.8 shows the rigid forward kinematics map $\mathcal{F}_r(\theta)$ based on the measured joint rotations together with the desired tip motion for each one of the controlled task-space DOFs. This further emphasizes how flexibility affects the tip motion and how well the proposed scheme deals with this problem.

![Figure 7.8: Rigid forward kinematics map $\mathcal{F}_r(\theta)$](image)

The velocity tracking for both the task and the joint-space DOFs is shown in Figure 7.9. These graphs provide us with additional information about the nature as well as the tracking and the vibration damping capabilities of the controller. Good tracking at the rates level implies good tracking at the positions as well. The lack of
residual oscillations after the completion of the useful motion at both the task and the joint level implies good damping of the elastic modes. Furthermore, the velocity signals convey important information about the stability of the system because any instabilities bound to occur will first "show up" at the rates. The high frequency oscillations present in the velocity signals is another reminder of the quality of the measurements used for implementing the schemes.

![Graph](image)

Figure 7.10: Cartesian tip-position path following (— desired, — actual).

Figure 7.10 shows the Cartesian tip-position path following for the maneuver. Rather than looking at the tracking of each DOF individually, this graph can be more easily conceptualized. Of course it does not convey any information about the orientational DOF of the end-effector. The element of time is also absent, i.e., the agreement between the actual and desired paths although a necessary condition, does not imply good trajectory tracking.

In Figure 7.11, the two graphs at the top show the size of the deflections at the outboard end of each one of the two flexible links that occur during the motion. The graphs at the bottom show the corresponding rotations due to the elastic motion.
at the same positions. These graphs clearly demonstrate the good active vibration suppression capabilities of the controller, given that there are almost no residual oscillations after the completion of the useful motion. It is obvious that the strain energy residing within the flexible members is almost completely dissipated by the time the arm reaches its final target position.

The control torque histories are shown in Figure 7.12. Each plot shows the feedforward part of the torques together with the combined action and the closeness of the two graphs is a measure of how good the model-based feedforward is. Even better agreement between the two graphs is expected for larger payload sizes. The feedforward alone is in general not adequate and when tested it was found unable to properly track the trajectory. The noise in the control torque signals after the final configuration is reached is due to the poor quality of the rate measurements at close to zero velocities. Both the feedback action as well as the friction compensation torques contribute to these oscillations, the amplitude of which is very small and the system is not destabilized.
Figure 7.12: Control torques (--- feedforward part, — combined action).

Figure 7.13: Tracking for the feedback alone case (--- desired/rigid, — actual).
For tracking control applications, it is very common to use controllers that combine a feedback together with a feedforward part. Feedback alone is usually not adequate and that also applies to the present case. Figure 7.13 shows the tracking and Figure 7.14 the corresponding vibrations at the tip of each flexible link for the case where the feedback part of the controller was used alone. Although the system remains stable, both the tracking and the vibration damping performance deteriorate. In this test the control torques were higher and more oscillatory than before and the corresponding link deflections were also larger.

![Figure 7.13: Tracking and Figure 7.14: Vibrations](image)

Figure 7.14: Deflections at the outboard end of each flexible link for the feedback alone case.

Figure 7.15 shows the exact together with the approximate form of the feedforward as given by Eq. (4.29) and it is clear that they are very close to each other. Numerous simulations and experiments have shown that the approximate form of the feedforward can be very successful. This conclusion will not be generalized, because for other systems with more complicated geometries and larger deflections this assumption might be inappropriate and the use of the exact implementation of the feedforward is recommended.
Figure 7.15: Feedforward part of torques (--- exact, - - - approximate).

Figure 7.16: Task-space tracking for $\mu = 0$ (--- desired, - - - actual).
The behaviour of the controller when implemented with a value of $\mu = 0$ was also examined and this case can be referred to as the corresponding rigid scheme. In this case, the modified output is constructed upon the joint angles and the controller basically reduces to the task-space version of the passivity-based PD+ scheme. Such a controller does not take into account the elastic motion of the system and treats the deflections as disturbances due to unmodelled dynamics. Typical behaviour of this case is provided by the task-space tracking results shown in Figure 7.16, based on Trajectory 1. The same controller design as before was used here and when we tried to tune the controller it was not possible to achieve better behaviour. Although the closed-loop system maintained its stability for $\mu = 0$, very poor tracking behaviour was evidenced and the oscillations excited by the link deflections were very difficult to damp. The corresponding control torque signals were particularly oscillatory.

Generally speaking, all conclusions drawn from our experimental investigations were also verified in simulation, which turned out to be a valuable tool in predicting the dynamic behaviour of the system and examining its closed-loop behaviour. Simulation was found to predict the resulting joint motion, the elastic deflections of the flexible links and the corresponding applied torques quite well. MATLAB was used for the numerical integration of the equations of motion which were available in symbolic form. It is worth mentioning that although a structural damping term in the model was necessary for the stability proofs, its absence in the simulation model was not problematic. Given that no structural damping or joint friction terms were used in the model, the simulated behaviour of the controlled system was slightly underdamped when compared with the experiment. This was typically shown as residual oscillations after the completion of the useful motion. In cases where the closed-loop system was well behaved in the experiment, the corresponding simulation required a bit higher feedback gains in order to add adequate damping to the system.

It should also be mentioned that although the present results are for the three-dimensional planar case, the derivation and the proofs of stability were general. Therefore, we will not hesitate to generalize any conclusions made from the experiments to the more general six-DOF case. It should also be remembered that this class of controllers has already been examined in a simulation context for the more
general case, in [45] and [44]. For this case though, obtaining the dynamic model of the system to be used for the feedforward part of the controller can be a fairly difficult task. The computational load on the control computer also increases considerably. Other problems to be encountered from an implementation point of view, are related to the tip–position measurements required. With the transition from the planar to the six-DOF case, apart from the bending vibration modes it is more likely to encounter torsional ones as well. In such a case, the strain–gauge method used in implementing our controllers cannot be directly applied. Using a camera system to provide the necessary tip measurements may also be inappropriate because it requires direct sight contact between the camera and the end–effector at all times. Depending on the application, a suitable sensing technology will have to be used.

7.2.1 The effect of the feedback gains size

Investigations related to the effect of the feedback gains on the performance of the system were also conducted. The size of the feedback gains was scaled by varying the values of $k_p$ and $k_d$. Results of the effect of the proportional action while the derivative gains remain unchanged are shown in Table 7.1. The tracking accuracy and the steady–state error are shown for a range of proportional gains. For zero proportional gains, the feedforward alone was found unable to properly drive the motion and the errors were excessively large. When the proportional gains are increased, the tracking performance is improved and the steady–state error is reduced. The consistency of the results related to the steady–state error, in some extent also implies the repeatability of the steady–state error for different trials. Very high gains were found to cause instability because of the high frequency noise which is present in the measurements and is also amplified by the feedback controller. Large amplitude, high frequency noise when combined with large feedback gains results in large control signals. When the saturation limits of the actuators are exceeded, instability of the closed-loop system may occur.

Studies related to the size of the derivative feedback gains are summarized in Table 7.2, for the case of constant proportional gains. To better demonstrate the
Table 7.1: The size of proportional feedback gains and performance.

| Traj. | $k_p$ | $|\dot{x}|$ | $|\dot{y}|$ | $|\phi|$ | $x_{\text{max}}$ (mm) | $y_{\text{max}}$ (mm) | $\phi_{\text{max}}$ (rad) | $\phi_{ss}$ (rad $\cdot 10^{-3}$) |
|-------|-------|-------------|-------------|-----------|-----------------|-----------------|----------------|-----------------|
| 1     | 0     | 2.3664      | 0.9357      | 1.9530    | 158.03          | 63.06           | 0.156          | 378.77          |
| 1     | 5     | 0.6773      | 0.8228      | 1.9963    | 41.67           | 67.21           | 0.186          | 6.42            |
| 1     | 10    | 0.3036      | 0.4065      | 1.0575    | 23.61           | 33.89           | 0.091          | 5.64            |
| 1     | 15    | 0.1754      | 0.2508      | 0.6270    | 13.95           | 21.27           | 0.041          | 1.52            |
| 1     | 20    | 0.1408      | 0.1711      | 0.5197    | 11.98           | 13.14           | 0.044          | 2.12            |
| 1     | 25    | 0.1202      | 0.1459      | 0.5594    | 10.96           | 9.45            | 0.042          | 2.09            |
| 1     | 28    | 0.1085      | 0.1336      | 0.5383    | 10.73           | 8.11            | 0.038          | 1.42            |
| 2     | 0     | 1.3259      | 2.1684      | 2.8617    | 92.03           | 142.03          | 0.218          | 415.78          |
| 2     | 5     | 0.9480      | 0.8197      | 0.7830    | 67.36           | 56.76           | 0.062          | 12.58           |
| 2     | 10    | 0.4656      | 0.4681      | 0.5539    | 32.60           | 37.01           | 0.047          | 4.87            |
| 2     | 15    | 0.2593      | 0.3353      | 0.5628    | 18.76           | 29.07           | 0.060          | 1.21            |
| 2     | 20    | 0.1641      | 0.2618      | 0.4979    | 12.75           | 23.72           | 0.050          | 5.25            |
| 2     | 25    | 0.1097      | 0.2114      | 0.4919    | 8.27            | 19.33           | 0.050          | 2.51            |
| 2     | 30    | 0.0800      | 0.1879      | 0.5344    | 6.22            | 18.07           | 0.055          | 1.10            |
| 2     | 35    | 0.0577      | 0.1562      | 0.4548    | 4.76            | 14.72           | 0.045          | 0.20            |
| 2     | 40    | 0.0535      | 0.1475      | 0.5075    | 4.58            | 14.51           | 0.055          | 0.87            |
| 3     | 0     | 2.4071      | 3.4493      | 19.5194   | 139.52          | 206.84          | 0.925          | 1062.65         |
| 3     | 5     | 0.8859      | 2.3294      | 2.4184    | 50.22           | 119.85          | 0.157          | 52.65           |
| 3     | 10    | 0.5321      | 1.3262      | 1.4550    | 39.52           | 66.28           | 0.096          | 13.66           |
| 3     | 15    | 0.3673      | 0.9339      | 1.0307    | 27.35           | 46.55           | 0.063          | 7.97            |
| 3     | 20    | 0.2765      | 0.7319      | 0.8563    | 20.55           | 38.96           | 0.057          | 6.33            |
| 3     | 25    | 0.2095      | 0.5871      | 0.7372    | 14.33           | 32.13           | 0.048          | 11.50           |
| 3     | 30    | 0.1690      | 0.4820      | 0.6675    | 10.67           | 27.35           | 0.052          | 7.25            |
| 3     | 35    | 0.1429      | 0.4128      | 0.6505    | 8.50            | 23.84           | 0.049          | 4.41            |

The effect of the derivative action, the feedforward part of the controller was frozen and the feedback part was used alone leading to more oscillatory behaviour. It is clear that for low derivative gains the damping action of the controller is very weak, the tracking accuracy deteriorates and the motion becomes oscillatory. For zero derivative gains the system operates very close to instability and induced vibrations with almost constant amplitude were found to decay at a very slow rate. For the case of Trajectory 3, the system was unstable when very low values of $k_d$ were used. For very high gains on the other hand, high frequency oscillations appear in the torque signals and eventually instability occurs. No clear conclusions on the effect of the derivative action on the steady-state error were made although for a heavily damped system
| Traj. | $k_d$ | $|\tilde{x}|$ | $|\tilde{y}|$ | $|\phi|$ | $\tilde{x}_{max}$ (mm) | $\tilde{y}_{max}$ (mm) | $\phi_{max}$ (rad) | $|U_{e,1}(tip)|$ | $|U_{e,2}(tip)|$ | $U_{e,1}(tip)_{max}$ (mm) | $U_{e,2}(tip)_{max}$ (mm) |
|-------|-------|-----------------|-----------------|--------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|-----------------|
| 1     | 0.0   | 0.2656          | 0.3108          | 1.8157       | 23.02           | 22.76           | 0.169           | 0.1491          | 0.0101          | 11.00           | 0.76            |
| 1     | 0.2   | 0.2547          | 0.2767          | 1.6837       | 22.11           | 21.77           | 0.160           | 0.0610          | 0.0049          | 5.95            | 0.45            |
| 1     | 0.4   | 0.2511          | 0.2429          | 1.5154       | 20.36           | 20.32           | 0.149           | 0.0602          | 0.0051          | 5.94            | 0.40            |
| 1     | 0.6   | 0.2520          | 0.2415          | 1.5273       | 20.53           | 21.83           | 0.149           | 0.0523          | 0.0047          | 5.44            | 0.37            |
| 1     | 0.8   | 0.2623          | 0.2137          | 1.4691       | 21.42           | 18.70           | 0.142           | 0.0633          | 0.0059          | 5.77            | 0.40            |
| 2     | 0.0   | 0.4322          | 0.2376          | 1.8394       | 33.316          | 18.98           | 0.166           | 0.2418          | 0.0200          | 10.23           | 1.11            |
| 2     | 0.2   | 0.4193          | 0.2219          | 1.7702       | 32.84           | 18.14           | 0.165           | 0.2060          | 0.0178          | 9.59            | 1.10            |
| 2     | 0.4   | 0.4029          | 0.2153          | 1.7276       | 32.1711         | 18.55           | 0.164           | 0.1699          | 0.0154          | 8.86            | 1.09            |
| 2     | 0.6   | 0.3992          | 0.2128          | 1.7288       | 32.34           | 18.42           | 0.166           | 0.1405          | 0.0131          | 8.21            | 1.02            |
| 2     | 0.8   | 0.3907          | 0.2076          | 1.6921       | 31.40           | 17.53           | 0.161           | 0.1093          | 0.0106          | 6.91            | 0.91            |
| 2     | 1.0   | 0.3875          | 0.1981          | 1.6374       | 30.95           | 17.08           | 0.156           | 0.0896          | 0.0090          | 5.65            | 0.73            |
| 2     | 1.2   | 0.3816          | 0.1955          | 1.6328       | 30.46           | 17.01           | 0.159           | 0.0726          | 0.0075          | 5.18            | 0.67            |
| 2     | 1.4   | 0.3772          | 0.1933          | 1.6494       | 30.17           | 16.46           | 0.163           | 0.0715          | 0.0075          | 5.29            | 0.68            |
| 2     | 1.6   | 0.3764          | 0.1940          | 1.5609       | 29.38           | 16.76           | 0.152           | 0.0594          | 0.0062          | 4.55            | 0.58            |
| 3     | 0.4   | 0.4175          | 0.6122          | 1.6819       | 37.21           | 45.18           | 0.128           | 0.3047          | 0.0320          | 15.16           | 1.63            |
| 3     | 0.6   | 0.4061          | 0.5741          | 1.4387       | 37.01           | 44.47           | 0.115           | 0.1910          | 0.0204          | 11.12           | 1.21            |
| 3     | 0.8   | 0.4024          | 0.5524          | 1.2434       | 35.78           | 43.64           | 0.107           | 0.1280          | 0.0140          | 7.34            | 0.83            |
| 3     | 1.0   | 0.3948          | 0.5344          | 1.0985       | 35.74           | 43.12           | 0.095           | 0.0839          | 0.0093          | 5.18            | 0.62            |
| 3     | 1.2   | 0.3911          | 0.5232          | 1.0253       | 35.34           | 42.72           | 0.086           | 0.0495          | 0.0056          | 3.26            | 0.40            |
| 3     | 1.4   | 0.3852          | 0.5170          | 0.9708       | 34.74           | 42.70           | 0.080           | 0.0317          | 0.0035          | 2.52            | 0.29            |
| 3     | 1.6   | 0.3784          | 0.5035          | 0.9251       | 34.44           | 41.44           | 0.076           | 0.0104          | 0.0012          | 0.94            | 0.13            |

Table 7.2: The size of derivative feedback gains and performance.
one would normally expect sluggish behaviour and increased steady-state error.

In many practical applications the three-term PID feedback controller is commonly used, with the integral term targeting the steady-state positioning error caused by friction. When a positioning offset remains at the end of the useful motion, the integral term increases the restoring force with time in order to exceed the friction levels and drive the steady-state error to zero. In its standard form, the feedback part of our scheme consists of a proportional plus a derivative term and the addition of an integral feedback term was investigated experimentally. It is important to mention that such a modification is not in accordance with the passivity theorem which suggested the structure of the feedback part of the controller. Of course, any violation of the passivity theorem does not imply that the closed-loop system will be unstable, given that the theorem provides a sufficient but not necessary condition for stability. In order to better demonstrate the effect of the integral gain on the steady-state error, the friction compensation was turned off and for the case of Trajectory 1 and Trajectory 3 the proportional gains were reduced to $k_p = 15$ and $k_p = 30$ respectively. The integral gains were taken to be $K_i = k_i \text{ diag}\{0.38 \text{ N/m/s, } 2.18 \text{ N/m/s, } 1.89 \text{ N·m/rad/s}\}$ for Trajectory 1, $K_i = k_i \text{ diag}\{1.24 \text{ N/m/s, } 0.22 \text{ N/m/s, } 1.11 \text{ N·m/rad/s}\}$ for Trajectory 2 and $K_i = k_i \text{ diag}\{0.75 \text{ N/m/s, } 2.67 \text{ N/m/s, } 2.01 \text{ N·m/rad/s}\}$ for Trajectory 3. The parameter $k_i$ was used to scale their overall size and the relevant results are shown in Table 7.3. The integral action was definitely not destabilizing and while increasing the relevant gains the steady-state error was considerably reduced. Its effect on the tracking performance was not significant. High integral gains though, were found to induce oscillations after the end of the useful motion and further increasing them results in instability.

### 7.2.2 The effect of singular configurations

When the scheme was derived it was assumed that the rigid Jacobian matrix remains nonsingular and invertible at all times. In spite of this fact, the inverse of the rigid Jacobian matrix does not explicitly appear in the control law and experimental tests were performed in order to examine the ability of the scheme to deal with singularities.
When a joint-based implementation of the scheme is employed using the fictitious joint quantities, the rigid Jacobian calculated over the actual rigid inverse kinematics configuration needs to remain nonsingular otherwise the scheme collapses. For the joint-based implementation, a way to bypass the above singularity problem is to use the approximate form of the feedforward as given by Eq. (4.29). This form was used for examining the effect of singularities on the closed-loop behaviour of the system and the desired trajectories were designed directly in joint-space. Three different cases were tested with the singularity encountered at the initial, the final and an intermediate configuration respectively.

Generally speaking, although singularities were found to cause problems on performance the closed-loop system maintained its stability. To interpret the observed
behaviour it is important to remember that the controller properly uses the joint motion to compensate for the effect of the link deflections on the tip position. When a singularity is encountered, $\det J_\theta(\theta, 0) = 0$, the controller is deprived of the ability to maintain the tip on the desired track and the desired task-space trajectory becomes unreachable.

The least problematic case among the three cases examined was the one with the singularity at the initial configuration. Based on the above comments, that was expected because the arm is initially undeflected and therefore no joint motion is required to compensate for any deflections. By the time any large deflections occur the arm has already moved away from the singularity and regained its ability to use the joint motion in order to compensate for them. The case with the singularity at the final configuration was the most problematic among the three, with the controller facing difficulties while trying to regulate the tip at the target setpoint.

### 7.2.3 Robustness characteristics

The fact that the controller was successfully implemented with the realistic-quality measurements available was the first robustness test successfully passed. A number of tests were performed towards examining the ability of the scheme to deal with the dynamics uncertainty problem. In particular, the controllers which were originally designed and tuned for Payload 2 were tested for all the other standard payload sizes. Our standard performance criteria were used in order to quantify the effect of uncertainty on performance and the results are collected in Table 7.4. Stability was maintained for the whole range of payloads examined. Although the variations on the payload were found to affect the tracking performance, a satisfactory level was maintained for all tests. If gains of larger size are used, the effect of the payload variations on performance is expected to be marginal.

The scheme was designed under the assumption that a rigid large payload is attached to the tip of the robot. A special robustness test was performed to examine whether this requirement can be compromised. The test involved inserting a water container in the payload’s canister, which represents the realistic case of a flexible
arm manipulating a heavy tank containing liquids. The motion of the liquid inside the tank adds dynamics to the payload which cannot be sensed or modelled easily. In general, the scheme was found to maintain its stability and good performance for this special case. Another example of a payload with internal dynamics arises during the manipulation of a satellite with flexible appendages such as solar panels. Our test is reminiscent of the work of Alder and Rock [4], who developed a special experimental facility to investigate the sensitivity of controllers to payload dynamics. Their facility consists of a one–link flexible planar arm manipulating a pendulum, which is allowed to swing in a direction perpendicular to the axis of the link.

7.2.4 The desired trajectories for the modified output

The success of the standard implementation assumption when used in conjunction with a value of $\mu$ close to 1 has already been demonstrated with all the above tests. Its use significantly simplifies the implementation but it becomes less valid when low values of $\mu$ are used. Figure 7.17 shows the task–space tracking behaviour of the controller for the case of Trajectory 1, when the standard implementation assumption is used with a value of $\mu = 0.4$, i.e., the errors for the modified output were calculated using Eqs. (4.38),(4.39). Large tracking errors were encountered and oscillations were found to persist long after the completion of the useful motion. The same figure shows the task–space tracking when the desired trajectories for the modified output are calculated according to the procedure proposed in Paragraph 4.2.7 and the same value of $\mu$ as before. The results demonstrate the success of the proposed method.

The method for designing the desired trajectories was further investigated in order to point out its relevance with the standard implementation assumption. The desired trajectories were calculated using this method for a whole range of values of $\mu$ and their closeness to the ones provided by the standard implementation assumption was quantified using the standard $L_{2T}$ norm as follows:

$$\text{Traj. distance} = \frac{\|\rho_{\text{nd}} - \rho_d\|_{2T}}{\|\rho_d\|_{2T}}$$  \hspace{1cm} (7.1)

Figure 7.18 shows the above trajectory distance measure for each one of the three DOFs involved, calculated at discrete values of $\mu$ for the case of Trajectory 1. The
| Traj. | Payl. | $|\ddot{x}|$ | $|\ddot{y}|$ | $|\dot{\phi}|$ | $\ddot{x}_{\text{max}}$ (mm) | $\ddot{y}_{\text{max}}$ (mm) | $\dot{\phi}_{\text{max}}$ (rad) | $|U_{e,1}(\text{tip})|$ | $|U_{e,2}(\text{tip})|$ | $U_{e,1}(\text{tip})_{\text{max}}$ (mm) | $U_{e,2}(\text{tip})_{\text{max}}$ (mm) |
|-------|-------|-----------|-----------|-----------|-----------------|-----------------|-----------------|----------------|----------------|----------------|----------------|
| 1     | 1     | 0.0802    | 0.1365    | 0.5878    | 8.11            | 10.78           | 0.044           | 0.0392         | 0.0028         | 4.34            | 0.27            |
| 1     | 2     | 0.0834    | 0.1264    | 0.4967    | 7.51            | 8.11            | 0.038           | 0.0125         | 0.0014         | 1.78            | 0.13            |
| 1     | 3     | 0.0706    | 0.1302    | 0.6187    | 5.83            | 10.13           | 0.056           | 0.0168         | 0.0028         | 1.34            | 0.30            |
| 1     | 4     | 0.0986    | 0.1780    | 0.8269    | 9.73            | 15.11           | 0.083           | 0.0339         | 0.0050         | 2.46            | 0.36            |
| 2     | 1     | 0.1088    | 0.1870    | 0.4565    | 11.61           | 18.34           | 0.044           | 0.0122         | 0.0022         | 0.95            | 0.28            |
| 2     | 2     | 0.0979    | 0.1988    | 0.5459    | 8.62            | 19.28           | 0.055           | 0.0098         | 0.0009         | 0.57            | 0.07            |
| 2     | 3     | 0.0800    | 0.2032    | 0.7196    | 5.92            | 20.18           | 0.080           | 0.0277         | 0.0026         | 2.21            | 0.24            |
| 2     | 4     | 0.1674    | 0.2032    | 0.8879    | 13.11           | 19.73           | 0.092           | 0.0272         | 0.0306         | 2.30            | 0.27            |
| 3     | 1     | 0.1384    | 0.4714    | 0.5645    | 7.62            | 30.12           | 0.039           | 0.0092         | 0.0010         | 1.14            | 0.15            |
| 3     | 2     | 0.1470    | 0.4759    | 0.6604    | 9.04            | 30.28           | 0.052           | 0.0052         | 0.0010         | 0.49            | 0.11            |
| 3     | 3     | 0.1519    | 0.4959    | 0.8048    | 9.05            | 30.92           | 0.063           | 0.0057         | 0.0009         | 0.45            | 0.11            |
| 3     | 4     | 0.1844    | 0.5194    | 0.9820    | 13.71           | 31.21           | 0.067           | 0.0504         | 0.0057         | 3.24            | 0.37            |

Table 7.4: Robustness tests.
distance tends to zero while the value of $\mu$ approaches 1, i.e., the calculated desired trajectories converge to the ones given by the standard implementation assumption. When the same exercise was repeated for other trajectories the same conclusion was made. These results further support the validity of the standard implementation assumption and justify its use in conjunction with a high value of $\mu$.

7.2.5 SPR feedback compensation and the gain-scheduled version

Experimental studies have shown that the feedback part of the scheme can also be realized as a proportional term in conjunction with an SPR rate feedback compensator, or the gain-scheduled version of Damaren [46] as discussed earlier in Chapter 4. The closed-loop system for both the above cases was found to be stable and comparisons with the performance of the standard PD feedback version were made.

All dynamic SPR controllers used were designed in continuous time and their
state-space description was obtained. The bilinear transformation was then used to provide their discrete-time equivalent representation, which was used for the digital implementation. Our experience with dynamic SPR controllers has shown that controller designs which are stable and perform well in a simulation context, were sometimes not as successful when implemented experimentally or even went unstable. Balas [17] pointed out this issue in the context of LSS. The common belief that once a continuous-time version of a feedback controller is determined it can also be successfully implemented in practice was described as a "folk theorem". Problems arising from the digital implementation might be related to sampling frequency considerations, zero order hold circuits which might cause spillover problems, etc. It is also important to note that the passive nature of a system is often violated when digital implementation of the controller is involved because of unmodelled discrete-time effects, sensor and actuator dynamics and lack of quality rate measurements [41].

It is very common that the design of a controller is based on linearized models. Then, simulations based on more complete nonlinear models are used in order to test
the behaviour of the final design. In our case, a state-space description of the modified
i/o plant was constructed after linearization of the actual plant at the configuration
for which the controller is intended and transformed to modal coordinates. The effect
of the proportional feedback action was properly incorporated within the model to
effectively augment the stiffness matrix and allow focus on the velocity feedback
design. The linear model of the modified system is described by an equation of the
standard state-space form:

\[
\dot{x} = Ax + Bu, \quad y = Cx
\]  

(7.2)

For the single SPR case, the controller was designed for the the middle configu-
ration. For the gain scheduling tests two SPR controllers were considered, one de-
signed for the initial and one for the final configuration. The dynamic SPR controllers
for each individual setpoint were designed according to the method of Benhabib et
al. [22], which was summarized earlier in Chapter 4 and the controllers were ob-
tained in the form of Eq. (4.48). The design parameters were selected as: \( Q = 501 \),
\( R = 2 \text{ diag\{10^{-4}, 10^{-4}, 10^{-3}\}, } Q_o = 800001 \). For both trajectories examined here,
the proportional gains for the PD as well as the SPR case were the same as before with
\( k_p = 27 \). For the derivative part of the PD controller that was used for comparisons
\( k_d = 0.8 \) was selected. Very useful for the analysis of the controller design are the
eigenvalues of the closed-loop system matrix:

\[
\begin{bmatrix}
\hat{A} & \hat{B}K_c \\
-K_c\hat{C} & A_c
\end{bmatrix}
\]  

(7.3)

The singular value plots of the SPR controller's transfer matrix were also found
to be important in analyzing the feedback controller design. Figure 7.19 shows the
SV plots for the SPR controller's transfer matrix, which was designed for the middle
configuration of Trajectory 1 and the above design parameters. Such graphs are
typically characterized by almost constant values at low frequencies and "roll-off"
at higher ones. This fact expresses the ability of the controller to effectively reject
rather than amplify the high frequency content of the signals which basically consists
of noise. It also suggests that better robustness characteristics are to be expected by
SPR feedback controllers when compared with the very simple constant gain PD ones.
For the gain–scheduled version, the scheduling signals (see Figure 4.2) were defined on the basis of time as follows:

\[ s_1 = \frac{t_f - t}{t_f - t_i}, \quad s_2 = \frac{t - t_i}{t_f - t_i} \quad (7.4) \]

with \( t_i \) and \( t_f \) the initial and final time for the maneuver respectively.

The tests have demonstrated the stability of the closed–loop system as predicted by the passivity theorem and the ability of the \( \text{SPR} \) controllers to add active damping to the system. For both the single and the gain–scheduled \( \text{SPR} \) cases, the performance was comparable and in some cases even better than the corresponding PD case. Relevant results are shown in Table 7.5. No quantitative comparisons will be attempted on the basis of these results because of the difficulty to be subjective, given that the controllers were not designed using an optimal procedure. In general, the \( \text{SPR} \) controller designs proposed in literature although they guarantee stability do not yield optimal controllers and further research is required in that direction.
| Traj. | Contr.       | $|\hat{x}|$ | $|\hat{y}|$ | $|\phi|$ | $\hat{x}_{\text{max}}$ (mm) | $\hat{y}_{\text{max}}$ (mm) | $\phi_{\text{max}}$ (rad) | $|U_{e,1}(\text{tip})|$ | $|U_{e,2}(\text{tip})|$ | $U_{e,1}(\text{tip})_{\text{max}}$ (mm) | $U_{e,2}(\text{tip})_{\text{max}}$ (mm) |
|-------|--------------|--------|--------|--------|-----------------|-----------------|-----------------|----------------|----------------|---------------------------------|---------------------------------|
| 1     | PD           | 0.0888 | 0.1385 | 0.4984 | 8.66            | 10.55           | 0.041           | 0.0194         | 0.0066           | 1.34                            | 0.43                            |
| 1     | SPR          | 0.0994 | 0.1288 | 0.4775 | 10.19           | 9.78            | 0.037           | 0.0207         | 0.0095           | 1.79                            | 0.53                            |
| 1     | Sch. SPR     | 0.1368 | 0.1654 | 0.7202 | 13.13           | 13.59           | 0.068           | 0.0205         | 0.0060           | 1.62                            | 0.60                            |
| 2     | PD           | 0.0776 | 0.1805 | 0.5558 | 6.44            | 18.21           | 0.046           | 0.0208         | 0.0054           | 1.11                            | 0.38                            |
| 2     | SPR          | 0.0635 | 0.1792 | 0.4778 | 5.32            | 17.94           | 0.033           | 0.0096         | 0.0052           | 0.71                            | 0.55                            |
| 2     | Sch. SPR     | 0.1269 | 0.1638 | 0.6772 | 13.22           | 14.86           | 0.048           | 0.0096         | 0.0048           | 0.78                            | 0.45                            |

Table 7.5: Strictly positive real feedback compensation tests.
Chapter 8

Experimental Results – Adaptive Case

In this chapter, experimental results are presented of the adaptive scheme for large payloads proposed earlier in the thesis. The applicability and the value of the scheme is demonstrated and its nature and characteristics are investigated. Results related to Trajectory 1 will be presented and discussed here and further results based on Trajectory 2 can be found in Appendix C. Trajectory 3 is a case of a non-PE trajectory for which the adaptive scheme was found to yield stable closed-loop behaviour but the performance was not particularly good. Results based on this trajectory will not be presented. By examining the PE condition given by Eq. (5.26), the integral calculated over this trajectory yields a zero value related to the second moment of inertia of the last link of the corresponding rigid arm, i.e., the adaptation mechanism will be unable to extract any information about it. This property also carries the contribution of the large payload itself and thus plays a predominant role in the dynamics of the system.

For the adaptive control case studies, ten parameters of the system with contributions from both the robot itself and the manipulated payload were updated on-line:

\[ \mathbf{\alpha} = [m_2, m_3, I_1, I_2, I_3, c_{x2}, c_{x3}, I_{r1}, I_{r2}, I_{r3}]^T, \]  

where \( m_i \) is the mass, \( c_{xi} \) is the first moment of inertia measured along the length and \( I_i \) is the second moment of inertia about the vertical axis of the \( i \)-th link of the corresponding three-link rigid robot. Parameter \( I_{ri} \) is the inertia of rotor \( i \), which is
lumped with the corresponding joint DOF.

For the results related to Trajectory 1, the feedback gain matrix was selected as $K_d = k_d \text{ diag } \{15 \text{ N/m} \cdot \text{s}, 20 \text{ N/m} \cdot \text{s}, 0.6 \text{ N} \cdot \text{m/rad} \cdot \text{s} \}$, with $k_d = 1.8$. For Trajectory 2 it was selected according to Eq. (5.27), with $k_d = 1.8$. For all tests on the adaptive scheme presented in the thesis the weighting matrix was taken to be $\Lambda = 1 \text{ s}^{-1}$. The adaptation gains were also selected to be diagonal, $\Gamma = \gamma \text{ diag } \{1, 10, 0.4, 0.7, 0.05, 15, 0.3, 0.03, 0.0005, 0.2 \}$ with $\gamma = 1$ for Trajectory 1 and $\Gamma = \gamma \text{ diag } \{14, 10, 8, 0.7, 0.05, 15, 1, 8, 7, 2 \}$ with $\gamma = 1$ for Trajectory 2. The size of each entry was tuned manually. Starting from low values they were increased or decreased accordingly based on the rate the corresponding estimated parameters evolved. Given that the size of the individual entries is directly related to the PE property, the integral that describes the property was found to provide a good indication about their relative size. Unless otherwise indicated, the above gains are the ones used throughout the case studies. A value of $\mu = 0.92$ was used for all the following case studies on adaptive control.

![Figure 8.1: Task-space tracking (--- desired, —— actual).](image)
Figure 8.1 shows the task-space tracking for each one of the three controlled task-space DOFs. The continuous line represents the actual trajectory and the dashed one is the desired one. By comparing each subsequent cycle it becomes evident that performance improves with time due to adaptation. For this example, all updated parameters in $\alpha$ were considered to be completely unknown and their initial estimates were set to zero. The positioning error at the intermediate resting points is attributed to friction and the relatively small size of the feedback gains used, which could not be adequately increased due to the quality of the available measurements. Higher feedback gains are expected to reduce this problem. Figure 8.2 shows the time histories for the updated parameters. All parameter estimates increase during the learning stage and finally settle close to a certain value or fluctuate around that value, which is not necessarily the nominal value of the corresponding parameter. The difference between the two can be attributed both to the contribution of the unmodelled dynamics, to inexact nominal values for certain parameters of our model, as well as to inaccurate calibration constants for the actuators.

Figure 8.3 shows the control torques for the maneuver, which are repeated in an almost periodic fashion from one cycle to the next. The adaptively updated feedforward part of the torques is shown in Figure 8.4. At the beginning of the motion their value is zero given that zero initial estimates were considered. During the learning time adaptation tends to “take over” the control of the plant and the size of these torques increases until they finally reach their steady-state values. Initially, the feedback part of the controller has the most significant contribution to the overall control effort. During learning the relative importance is reversed until finally the feedforward torques represent a large percentage of the overall size of the torques. Given higher feedback gains better performance is expected both at the beginning as well as the later stages of the motion.

Examining the feedforward torques some high frequency oscillations can be observed during the intermediate stops. These are excited by the velocity measurements which are particularly noisy at near-zero velocities. The parameter update mechanism cannot distinguish noise or any other external disturbances from the useful signal that drives the adaptation. The amplitude of these oscillations increases during the
Figure 8.2: Parameter updates.
Figure 8.3: Control torques.

Figure 8.4: Adaptively updated feedforward part of the torques.
Figure 8.5: Deflections and rotations at the outboard end of each flexible member.

Figure 8.6: Task-space tracking for high adaptation gains.
learning time due to the increasing size of the parameter updates. This fact explains why very high adaptation gains finally lead to instabilities, which in most occasions were found to occur during one of the intermediate stops, after a number of initial cycles have elapsed. Small adaptation gains on the other hand, result in enhanced robustness of the system with respect to measurement noise. In [145], a stability problem in the estimates is discussed, namely parameter drift, which is more related to the non-PE trajectories case. The use of a dead-zone was proposed for dealing with this problem, i.e., freezing of adaptation once the tracking errors enter within a specified range.

Figure 8.5 shows the size of the elastic deflections and rotations that occur during the maneuver at the outboard end of the two flexible members. It is clear that after the completion of the useful motion the elastic motion decays to zero very quickly due to the vibration damping action provided by the controller.

While the adaptation gains are increased the tracking performance was found to improve considerably, especially during the beginning of the motion because the length of the learning time shrinks. This can be seen in Figure 8.6 which presents the task-space tracking for the same case examined above, the only difference being the size of the adaptation gains which were increased to $\gamma = 1.5$.

### 8.1 The effect of the feedback and adaptation gains

Investigations related to the size of the feedback and adaptation gains are reported in this paragraph. Their effect on performance and stability of the closed-loop system is considered and limitations on their size are discussed.

The results in Table 8.1 show the effect of the feedback gains size on the tracking performance and the steady-state error as well. For the investigations the gains were scaled using the value of $k_d$. For $k_d = 0$ the adaptation acts alone, the arm is unable to follow the desired trajectory and the parameter estimates evolve in a rather desperate and confused manner. This fact illustrates that adaptation alone cannot drive the motion and that the combined action with the feedback part is necessary. As expected, while the size of the feedback gains is increased the tracking performance
The size of the feedback gains though, is limited by the quality of the measurement signals. Given higher gains much better performance is expected and this fact was verified through simulation studies.

| Traj. | $k_d$ | $|\bar{x}|$ | $|\bar{y}|$ | $|\phi|$ | $\bar{x}_{\text{max}}$ (mm) | $\bar{y}_{\text{max}}$ (mm) | $\phi_{\text{max}}$ (rad) | $\phi_{ss}$ (rad $\cdot 10^{-3}$) |
|-------|-------|-------------|-------------|-----------|----------------------|----------------------|----------------------|----------------------|
| 1     | 0.0   | 22.0995     | 20.6652     | 72.1087   | 509.30               | 456.04               | 1.734                | 1318.75              |
| 1     | 0.3   | 6.5098      | 10.7831     | 33.4857   | 235.69               | 307.63               | 1.228                | 574.61               |
| 1     | 0.6   | 3.9193      | 7.3545      | 24.2467   | 156.05               | 244.66               | 0.940                | 512.76               |
| 1     | 0.9   | 3.0410      | 5.4719      | 18.9563   | 122.91               | 197.10               | 0.754                | 437.73               |
| 1     | 1.2   | 2.5671      | 4.4461      | 15.5333   | 93.79                | 164.21               | 0.636                | 335.82               |
| 1     | 1.5   | 2.3704      | 3.7788      | 13.3279   | 81.89                | 141.73               | 0.541                | 281.57               |
| 1     | 1.8   | 2.1843      | 3.2051      | 11.6813   | 78.26                | 123.36               | 0.471                | 249.21               |
| 1     | 2.1   | 2.0580      | 2.8079      | 10.6664   | 72.95                | 107.84               | 0.418                | 220.46               |
| 2     | 0.0   | 23.3889     | 11.2720     | 55.1913   | 495.23               | 224.99               | 1.059                | 1493.12              |
| 2     | 0.3   | 7.2384      | 2.8937      | 17.5196   | 287.27               | 119.36               | 0.733                | 169.88               |
| 2     | 0.6   | 4.4721      | 1.6795      | 11.5986   | 212.09               | 65.86                | 0.502                | 206.53               |
| 2     | 0.9   | 3.2674      | 1.5498      | 9.3373    | 171.86               | 69.14                | 0.510                | 153.92               |
| 2     | 1.2   | 2.6148      | 1.4982      | 7.8323    | 140.23               | 66.47                | 0.442                | 136.38               |
| 2     | 1.5   | 2.1873      | 1.4701      | 6.7712    | 118.37               | 66.21                | 0.384                | 126.27               |
| 2     | 1.8   | 1.9487      | 1.4266      | 6.0089    | 100.19               | 56.67                | 0.003                | 134.64               |

Table 8.1: The size of feedback gains and performance.

Augmenting the feedback part of the controller with an integral term in order to deal with the steady-state positioning offsets was also examined experimentally. In other words the feedback part of the controller in Eq. (5.13) is taken to be a PI term:

$$\tau = -J_T^T(\theta, q_e)K_ds_\mu - J_T^T(\theta, q_e)K_i \int_0^t s_\mu dt \quad (8.2)$$

This choice of course violates the passivity theorem which suggested the form of the feedback part of the controller, because the theorem not only requires the controller to be strictly passive but also to have finite gain. Table 8.2 shows experimental results related to the effect of the integral gains. For both trajectories the integral gains were selected to be $K_i = k_i \text{ diag}\{0.5\text{ N/m}, 0.5\text{ N/m}, 0.5\text{ N \cdot m/rad}\}$, with $k_i$ scaling their overall size. The results have shown that the stability of the closed-loop system is preserved after the addition of the integral term. Its use was found to have little effect on the tracking performance but considerably improved the steady-state positioning
accuracy. Increasing the integral gains above a certain size though, was shown to cause stability problems.

<table>
<thead>
<tr>
<th>$k_i$</th>
<th>$\psi_{ss} \ (rad \cdot 10^{-3})$ / Traj.1</th>
<th>$\psi_{ss} \ (rad \cdot 10^{-3})$ / Traj.2</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>297.50</td>
<td>152.58</td>
</tr>
<tr>
<td>1</td>
<td>160.12</td>
<td>134.21</td>
</tr>
<tr>
<td>2</td>
<td>144.74</td>
<td>129.10</td>
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<td>125.43</td>
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<td>116.03</td>
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<td>97.90</td>
<td>101.52</td>
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<td>7</td>
<td>81.31</td>
<td>104.71</td>
</tr>
<tr>
<td>8</td>
<td>32.39</td>
<td>94.47</td>
</tr>
</tbody>
</table>

Table 8.2: The effect of an integral term on the steady-state error.

Table 8.3: The size of adaptation gains and performance.

Table 8.3 summarizes studies related to the size of the adaptation gains, which were scaled using the value of $\gamma$. It was found that while the adaptation gains are increased, the tracking performance over the whole maneuver improves. The learning period is shortened and less time is taken by the parameter estimates to converge to their steady-state values. This fact is reflected in the performance which improves much faster. Very high adaptation gains though, weaken the robustness of the system.
which becomes easily destabilized.

8.2 Update and convergence of parameters

Given the initial values, the parameter update Eq. (5.17) is integrated online to provide the estimates at each time interval using a step-by-step method. A very simple Euler-Cauchy scheme can be used for the integration:

$$\dot{\alpha}(t) = \dot{\alpha}(t - \Delta T) + \dot{\alpha}(t - \Delta T) \Delta T,$$

(8.3)

where $\Delta T$ is the sampling period. This is a one-step method, i.e., it uses values from the preceding step only. Better behaviour is possible when a multi-step method is used such as an Adams-Bashforth integration scheme [75]. Second and fourth order ones were both tested:

$$\dot{\alpha}(t) = \dot{\alpha}(t - \Delta T) + \left[\frac{3}{2} \dot{\alpha}(t - \Delta T) - \frac{1}{2} \dot{\alpha}(t - 2\Delta T)\right] \Delta T,$$

(8.4)

$$\dot{\alpha}(t) = \dot{\alpha}(t - \Delta T) + \frac{\Delta T}{24} [55 \dot{\alpha}(t - \Delta T) - 59 \dot{\alpha}(t - 2\Delta T) + 37 \dot{\alpha}(t - 3\Delta T) - 9 \dot{\alpha}(t - 4\Delta T)],$$

(8.5)

The Euler-Cauchy integration scheme was found to perform well but better robustness characteristics are expected by the higher order methods. For the results reported in the thesis the above fourth order one was used.

A very important issue that was investigated and is related to the performance of the scheme is the initialization of the updated parameters. For the relevant studies reported in this paragraph only three parameters were updated, the ones of the third link in the chain, i.e., $\alpha = [m_3, I_3, c_{x3}]^T$. These properties include the contribution of the large payload to the dynamics. The same feedback gains as before were used with $k_d = 1.8$ for Trajectory 1 and $k_d = 1.6$ for Trajectory 2. Unless otherwise indicated, for all the results in the thesis involving the above set of three updated parameters, the adaptation gains were $\Gamma = \gamma \text{ diag}\{30, 0.15, 0.9\}$ with $\gamma = 1$ and $\Gamma = \gamma \text{ diag}\{30, 0.15, 3\}$ with $\gamma = 1$ for Trajectory 1 and Trajectory 2 respectively.
The time histories for the parameter estimates are shown in Figure 8.7 for the case of Trajectory 1. The graphs include the zero initial estimates case as well as the one with the initial estimates being the converged values obtained from a previous experiment. In the former case, while the estimates converge to their steady-state values their trajectories merge with the ones from the latter case. It is clear that in the latter case the estimates remained very close to their initial values. When the estimates were initialized to the nominal values of the model, they departed and finally converged to the same values as the other two cases. Among the above three different cases the best performance at the initial stage of the maneuver was observed in the case with the converged values being the initial estimates. The same conclusions were made when all ten parameters of the system were adaptively updated. Results of the performance for the relevant tests are collected in Table 8.4.

A number of tests were performed to investigate the ability of the scheme to identify and adapt to changes related to the manipulated payload. For the results reported
Table 8.4: Initialization of updated parameters and performance.

| Size of $\alpha$ | Traj. | Initial estimates | $|\bar{X}|$ | $|\bar{Y}|$ | $|\bar{\phi}|$ | $\ddot{x}_{\text{max}}$ (mm) | $\ddot{y}_{\text{max}}$ (mm) | $\dot{\theta}_{\text{max}}$ (rad) |
|-----------------|------|------------------|----------|----------|-------------|-----------------|-----------------|-------------------|
| 3               | 1    | zero             | 2.0896   | 2.2549   | 9.4310      | 66.08           | 81.31           | 0.423             |
|                 |      | converged        | 2.4041   | 1.9557   | 4.4221      | 48.01           | 58.28           | 0.123             |
| 3               | 2    | zero             | 1.6754   | 2.4666   | 8.5942      | 70.11           | 56.21           | 0.408             |
|                 |      | converged        | 1.2538   | 2.4876   | 4.4172      | 26.50           | 61.02           | 0.117             |
| 10              | 1    | zero             | 2.2866   | 3.4888   | 12.8031     | 82.71           | 130.90          | 0.512             |
|                 |      | converged        | 2.1042   | 2.1139   | 6.8535      | 59.74           | 65.46           | 0.165             |
| 10              | 2    | zero             | 2.1481   | 1.5223   | 6.6420      | 110.38          | 62.09           | 0.378             |
|                 |      | converged        | 0.7064   | 1.3014   | 2.8979      | 18.64           | 34.01           | 0.071             |

here Trajectory 1 was used, three parameters were adaptively updated and the feedback gains were scaled using $k_d = 1.3$. Figure 8.8 shows the parameter updates for the case where an additional mass was manually placed inside the payload's canister during one of the intermediate resting intervals. The properties of the added mass with respect to frame $\mathcal{F}_{N+1}$ were $m = 3.77$ kg, $I = 53.58$ g·m$^2$ and $c_x = 436.40$ g·m. The ability of the controller to perceive the change and react accordingly can be seen from the parameter updates. It is clear that just after the extra mass is added the parameters tend to converge to higher values in order to accommodate for the change. Both stability and good performance are maintained after the payload increase. The change in the parameter updates is more noticeable in the case of $m_3$ and $c_{x3}$ but less in $I_3$. The reason is that the compact shape of the added mass does not affect the payload's second moment of inertia as much as it affects the other two parameters.

### 8.3 Robustness characteristics

To examine the robustness characteristics of the scheme with respect to ignorance in the payload's mass properties, the controllers which were originally designed and tuned for the case of Payload 2 were also used for maneuvering the other standard payloads. For the case of Trajectory 1, the controller was implemented with $k_d = 1.6$ and $\gamma = 3$ and for the case of Trajectory 2, $k_d = 1.4$ and $\gamma = 3$ were used.
Figure 8.8: Ability of the adaptation to track changes in the updated parameters.
Only the three mass properties as above were updated on-line and the corresponding performance results are shown in Table 8.5. The parameter estimates were found to adjust their size according to the actual size of the payload. The fact that for larger payloads the performance slightly deteriorates is attributed to the relatively low size of the feedback gains, which is limited by the quality of the available velocity measurements.

| Traj. | Payl. | $|\tilde{x}|$ | $|\tilde{y}|$ | $|\tilde{\phi}|$ | $\tilde{x}_{\text{max}}$ (mm) | $\tilde{y}_{\text{max}}$ (mm) | $\tilde{\phi}_{\text{max}}$ (rad) |
|-------|------|---------|---------|---------|-----------------|-----------------|-----------------|
| 1     | 1    | 2.2089  | 2.6089  | 9.4223  | 66.29           | 94.43           | 0.364           |
| 1     | 2    | 2.2512  | 2.7926  | 11.1642 | 69.20           | 97.50           | 0.499           |
| 1     | 3    | 2.3646  | 3.0020  | 12.6251 | 69.46           | 107.61          | 0.575           |
| 1     | 4    | 2.5126  | 3.3160  | 14.0772 | 73.04           | 116.46          | 0.642           |
| 2     | 1    | 1.6236  | 2.3479  | 6.0763  | 68.18           | 51.19           | 0.314           |
| 2     | 2    | 1.6317  | 2.4195  | 8.4808  | 70.09           | 54.83           | 0.414           |
| 2     | 3    | 2.0943  | 2.6527  | 9.3977  | 87.54           | 61.16           | 0.456           |
| 2     | 4    | 2.4483  | 2.7999  | 10.4260 | 102.43          | 65.30           | 0.512           |

Table 8.5: Robustness tests.
Chapter 9

Conclusions

Flexible-link robots have already been used in space operations and a big potential is seen for using them in industrial or other terrestrial-based applications. The success of such robots will depend on their control which constitutes a real engineering challenge. The problem is complicated by the fact that the system has less control inputs than control outputs, the noncollocation of input and output degrees of freedom, as well as the highly nonlinear nature of the controlled system.

In the case of rigid robots, the control objective is commonly defined as the tracking of the joint trajectories that correspond to the desired task-space ones. Most of the control approaches already proposed for flexible robots adopt the above strategy and rely on passive and less rarely active techniques to deal with the induced oscillations. Given the elastic nature of the system, such techniques have limited potential when used in tip trajectory tracking applications. To respond to high performance requirements, the control objective needs to be redefined as the tracking of the desired task-space trajectories together with simultaneous active vibration suppression.

Such was the philosophy of a control scheme proposed and investigated in the present thesis, which is suitable for the case of flexible-link robots carrying large payloads. The scheme effectively exploits an approximate form of the payload dominated dynamics together with a suitably defined passive input and output of the plant. Based on this approach, problems inherent in flexible robot control can be bypassed when large payloads are involved. The scheme consists of a model-based feedforward together with a feedback part and is supported by the necessary proofs.
of asymptotic stability. The approach directly deals with the task-space trajectory tracking and as such it does not require solving the inverse kinematics or dynamics problem. An asymptotically stable adaptive counterpart of the scheme was also proposed, which is able to deal with the uncertainty problem related to the mass properties of the arm and/or the manipulated payload. This problem is commonly encountered in a realistic manipulation scenario. The scheme consists of a control law coupled with a parameter update mechanism, which keeps extracting parameter information from tracking errors so that performance improves with time.

Both schemes proposed are extensions of earlier results which employed a feedforward part built upon the mass properties of the large payload alone. Here, the mass properties of the arm itself also contribute to the feedforward action and superior performance over their original predecessors becomes possible. The control schemes are model-based and effectively exploit the dynamics of the system and the properties of its model.

Although various theoretical results exist in flexible robotic systems literature, not much experimental work has been carried out. In the present work, after the theoretical results, a step further was taken in order to bridge the gap that usually exists between theory and practice and that is one of the major contributions of the thesis. A special robotics facility was designed and built for that purpose. The experimental work undertaken involved the three-DOF planar geometry with two flexible links. That is an important excursion from much simpler geometries used by other researchers. The most commonly used geometry has been the single flexible link arm or two-link arms with only one member being flexible. Various experimental issues that were encountered together with the corresponding solutions provided were discussed in detail. These can provide useful guidelines to anyone dealing with experimental studies on flexible robots or the development of flexible robotic systems for specific applications.

Experimental results with traditional joint-space schemes for rigid robots have clearly demonstrated their inadequacy when used for flexible-link ones. It was shown that even though the schemes might maintain stability and also manage to track the commanded joint trajectories, large task-space tracking errors occur. The reason is
simply that they do not account for the contribution of the elastic displacements to
the position of the tip. Furthermore, vibrations were found very difficult to damp
and persist long after the completion of the useful motion. The results suggest that
something more sophisticated is required that takes into consideration the flexible
nature of the plant and does not solely rely on the structural damping of the system
to extract the energy from the vibration modes.

Experimental studies demonstrated for the first time the applicability and the
value of a class of controllers for flexible-link robots carrying large payloads, the
latest versions of which are the ones proposed in the present thesis. It was shown
that these controllers are indeed suitable for task-space tracking and combine active
damping of the vibrations as well. It was shown that the \( \mu \)-tip notion is an effective
way to overcome the typical nonminimum phase characteristics of flexible arms, when
large payloads are involved. The experimental studies provided more insight into the
controllers, their nature and characteristics, as well as the control problem for flexible­
link robots in general. For the case of the proposed adaptive scheme, its ability to
update the parameters of the model and improve performance with time was also
demonstrated.

Both the nonadaptive and the adaptive schemes considered, belong to the class of
passivity–based controllers. The passivity property that underlies their design pro­
vides a strong foundation for the controllers which is reflected in their good robustness
behaviour. Their robust nature was verified by numerous tests which showed that
a controller design can maintain stability and a good level of performance in the
presence of measurement noise and uncertainty related to the mass properties of the
system. Given the flexibility that the passivity property provides to the design of
feedback controllers, the use of SPR compensation for the feedback part of the con­
trollers was also successfully implemented experimentally. A gain–scheduled version
was tested as well and found to be a very promising approach. The need for further
research on the development of optimal design procedures for SPR controllers was
identified.

Given a suitable assumption, it was shown that the proposed controllers can be
implemented without the need for direct measurements of the elastic coordinates.
In that case, the joint rotations and tip positions together with their corresponding rates are the only measurements required. Tip measurements can be provided by the available sensor technology. An alternative low-cost method that uses strain-gauge measurements together with joint encoder information can also provide the required tip-position measurements. Such a method was successfully used for implementing our controllers.

One of the main difficulties encountered when implementing our controllers was the lack of good quality rate measurements, which were obtained by differencing the corresponding position signals. Such measurements are typically contaminated with noise, and that imposes limitations on the size of the feedback gains and also the performance of the controllers. Although filtering of the velocity signals using Butterworth techniques was found to considerably improve their quality, even better performance of the controllers will be possible given better measurements. The use of state estimation techniques can be a key to obtaining such measurements, which will enhance the performance of our controllers. Although such techniques have not been investigated as part of the present thesis, their implementation in conjunction with the proposed control schemes is recommended as a direction for further research.

A different direction for future research should be the extension of the capabilities of flexible-link robots beyond the positioning and trajectory tracking applications, by examining the constrained force control problem. One of the main characteristics of flexible robots carrying large payloads as examined in the present thesis, is that all vibration modes exhibit a node at the manipulator's end-point. In the asymptotic case of an infinitely big payload, a clamping boundary condition at the tip of the robot will be encountered. This provides a connection with the constrained force control case and might provide a key to exploring such control applications.

Although all our experimental results involved a planar three-DOF arm, it is expected that they will extent to the more general six-DOF case, given that the theory and the proofs of stability were considered in generality. With the results of the present thesis the class of controllers for flexible-link robots carrying large payloads has reached a mature level, which will allow for practical robotic applications to be considered seriously, either space or earth-based ones.
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Appendix A

Further Results on the PD+ Controller – Trajectory 2 and 3

Figure A.1: Joint-space tracking, Trajectory 2 (— rigid, — actual).
Figure A.2: Task-space tracking, Trajectory 2 (--- desired, ---- actual).

Figure A.3: Cartesian tip-position path following, Trajectory 2 (--- desired, ---- actual).
Figure A.4: Deflections and rotations at the outboard end of each flexible link, Trajectory 2.

Figure A.5: Joint-space tracking, Trajectory 3 (--- rigid, --- actual).
Figure A.6: Task–space tracking, Trajectory 3 (--- desired, —— actual).

Figure A.7: Cartesian tip-position path following, Trajectory 3 (--- desired, —— actual).
Figure A.8: Deflections and rotations at the outboard end of each flexible link, Trajectory 3.
Appendix B

Further Results on the Nonadaptive Scheme – Trajectory 2 and 3

Figure B.1: Trajectory tracking, Trajectory 2 (--- desired/rigid, — actual).
Figure B.2: Rate tracking, Trajectory 2 (- - - desired/rigid, - - - actual).

Figure B.3: Cartesian tip-position path following, Trajectory 2 (- - - desired, - - - actual).
Figure B.4: Deflections and rotations at the outboard end of each flexible link, Trajectory 2.

Figure B.5: Control torques, Trajectory 2 (--- feedforward, ---- combined action).
Figure B.6: Task-space tracking for the feedback-alone case, Trajectory 2 (--- desired/rigid, ------ actual).
Figure B.7: Deflections and rotations at the outboard end of each flexible link for the feedback-alone case, Trajectory 2.

Figure B.8: Feedforward part of torques, Trajectory 2 (— exact, —— approximate).
Figure B.9: Trajectory tracking, Trajectory 3 (— desired/rigid, --- actual).

Figure B.10: Rate tracking, Trajectory 3 (— desired/rigid, --- actual).
Figure B.11: Cartesian tip–position path following, Trajectory 3 (— desired, — actual).

Figure B.12: Deflections and rotations at the outboard end of each flexible link, Trajectory 3.
Figure B.13: Control torques, Trajectory 3 (--- feedforward, -- combined action).

Figure B.14: Task-space tracking for the feedback-alone case, Trajectory 3 (--- desired/rigid, --- actual).
Figure B.15: Deflections and rotations at the outboard end of each flexible link for the feedback-alone case, Trajectory 3.

Figure B.16: Feedforward part of torques, Trajectory 3 (— — — exact, —— approximate).
Appendix C

Further Results on the Adaptive Scheme – Trajectory 2

Figure C.1: Task-space tracking, Trajectory 2 (—— desired, —— actual).
Figure C.2: Parameter updates, Trajectory 2.
Figure C.3: Control torques, Trajectory 2.

Figure C.4: Adaptively updated feedforward part of the torques, Trajectory 2.
Figure C.5: Deflections and rotations at the outboard end of each flexible member, Trajectory 2.