Frequency–Dependent A.C. System Equivalents for Harmonic Studies and Transient Convertor Simulation

A thesis presented for the degree of Doctor of Philosophy in Electrical Engineering in the University of Canterbury, New Zealand

by N.R. Watson, B.E.(Hons)

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# Table of Contents

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>List of Figures</td>
<td>v</td>
</tr>
<tr>
<td>List of Tables</td>
<td>x</td>
</tr>
<tr>
<td>List of Principal Symbols</td>
<td>xi</td>
</tr>
<tr>
<td>Abbreviations</td>
<td>xii</td>
</tr>
<tr>
<td>Abstract</td>
<td>xiv</td>
</tr>
<tr>
<td>Acknowledgements</td>
<td>xv</td>
</tr>
<tr>
<td><strong>CHAPTER 1</strong> INTRODUCTION</td>
<td></td>
</tr>
<tr>
<td>1.1 Importance of HVDC Transmission</td>
<td>1</td>
</tr>
<tr>
<td>1.2 The Need for Computer Modelling</td>
<td>2</td>
</tr>
<tr>
<td>1.3 Types of Studies</td>
<td>2</td>
</tr>
<tr>
<td>1.4 Incorporating HVDC Convertors</td>
<td>3</td>
</tr>
<tr>
<td>1.5 The need for Equivalent Circuits</td>
<td>4</td>
</tr>
<tr>
<td>1.6 Thesis Outline</td>
<td>5</td>
</tr>
<tr>
<td><strong>CHAPTER 2</strong> OVERVIEW</td>
<td></td>
</tr>
<tr>
<td>2.1 Computational Aids for Dynamic Simulation</td>
<td>7</td>
</tr>
<tr>
<td>2.2 Digital Computer Simulation</td>
<td>9</td>
</tr>
<tr>
<td>2.3 Development of HVDC Convertor Models</td>
<td>10</td>
</tr>
<tr>
<td><strong>CHAPTER 3</strong> MATHEMATICAL MODEL AND COMPUTER IMPLEMENTATION</td>
<td></td>
</tr>
<tr>
<td>3.1 Introduction</td>
<td>13</td>
</tr>
<tr>
<td>3.2 Representation of Components</td>
<td>13</td>
</tr>
<tr>
<td>3.2.1 Static Convertor</td>
<td>13</td>
</tr>
<tr>
<td>3.2.1.1 Modes of operation</td>
<td>14</td>
</tr>
<tr>
<td>3.2.1.2 Convertor control systems</td>
<td>14</td>
</tr>
<tr>
<td>3.2.1.3 Types of control schemes</td>
<td>15</td>
</tr>
<tr>
<td>3.2.2 Synchronous Machines</td>
<td>16</td>
</tr>
<tr>
<td>3.2.3 Transmission Lines</td>
<td>17</td>
</tr>
<tr>
<td>3.2.4 Transformers</td>
<td>17</td>
</tr>
<tr>
<td>3.2.5 Static Shunt Elements</td>
<td>18</td>
</tr>
<tr>
<td>3.2.6 Static Series Elements</td>
<td>18</td>
</tr>
<tr>
<td>3.2.7 A.C. System</td>
<td>19</td>
</tr>
</tbody>
</table>
3.3 Method of Analysis
  3.3.1 Trapeziodal Integration
  3.3.2 Topology Changes
  3.3.3 Choice of State Variables
  3.3.4 Per Unit System
  3.3.5 Initial Conditions
  3.3.6 Network Equations

3.4 Extensions
  3.4.1 C-type Filter
  3.4.2 Harmonic Current Source
  3.4.3 RLC Networks connected between phases

CHAPTER 4 ACREP: A FLEXIBLE PROGRAM FOR PROCESSING AND PLOTTING POWER SYSTEM DATA

4.1 Capability of the ACREP program
4.2 Objectives, Structure and Operation of ACREP
4.3 Modular Design
4.4 Special Features
4.5 Illustrative Examples
  4.5.1 Spectral Analysis
  4.5.2 Harmonics Graphics package
  4.5.3 Frequency Response of an RLC Network
  4.5.4 General Plot Facility

CHAPTER 5 A.C. SYSTEM MODEL

5.1 Historical Review
5.2 Obtaining the frequency response of an a.c. system
5.3 Synthesis of Frequency-Dependent Equivalent
  5.3.1 Direct Method
    Feature Extraction
    The Scaling Process
    Correction Filters
    Derivation of correction branch parameters
    Matching of correction branches
    Illustrative Examples
5.3.2 Optimization Method

5.3.2.1 Choice of Objective Function

5.3.2.2 Optimization Methods

5.4 The Philosophy of the Frequency-Dependent a.c. System Models

5.5 Implementation of Frequency-Dependent a.c. System Models

5.6 Initialization of Frequency-dependent equivalents

5.7 Validation of the Computer Model

5.8 Extension to Multi-converter systems

5.9 Frequency-dependent d.c. system model

5.10 Negative Resistance Region in Impedance Loci

5.11 Discussion and Conclusions

5.11.1 Explicit versus Implicit Mutual Representation

5.11.2 Comparison between direct and Optimization techniques for forming frequency-dependent equivalent circuits.

5.11.3 Impedance Magnitude and Phase Angle Match

5.11.4 Negative Resistive Component of Mutual Coupling.

5.11.5 Dynamic Instabilities.

5.11.6 Future Work

CHAPTER 6 HARMONIC ASSESSMENT

6.1 Introduction

6.2 Harmonic Penetration

6.3 Characteristic Harmonics

6.4 Incorporation of HVDC Convertors

6.5 Illustrative Examples

6.6 Computational Efficiency

6.7 Harmonic Penetration Example

6.8 Conclusions

6.8 Future Work

CHAPTER 7 DYNAMIC SIMULATIONS

7.1 Introduction

7.2 D.C. Line Fault Simulation

7.3 Rectifier Side A.C. System Disturbance
7.4 Invertor Side A.C. System Disturbance
7.5 Discussion and Conclusions

CHAPTER 8 CONCLUSIONS

REFERENCES

APPENDIX A1 Detailed Synchronous Machine Coefficient Matrix
A2 Transmission Line Representation
A3 Parameters for RLC circuit analysis example
A4 Single Variable Optimization Methods
A5 Published Papers
# LIST OF FIGURES

<table>
<thead>
<tr>
<th>Figure</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>2.1</td>
<td>12</td>
</tr>
<tr>
<td>3.1</td>
<td>15</td>
</tr>
<tr>
<td>3.2</td>
<td>17</td>
</tr>
<tr>
<td>3.3</td>
<td>18</td>
</tr>
<tr>
<td>3.4</td>
<td>19</td>
</tr>
<tr>
<td>3.5</td>
<td>26</td>
</tr>
<tr>
<td>3.6</td>
<td>30</td>
</tr>
<tr>
<td>3.7</td>
<td>32</td>
</tr>
<tr>
<td>3.8</td>
<td>33</td>
</tr>
<tr>
<td>3.9</td>
<td>34</td>
</tr>
<tr>
<td>4.1</td>
<td>39</td>
</tr>
<tr>
<td>4.2</td>
<td>42</td>
</tr>
<tr>
<td>4.3</td>
<td>42</td>
</tr>
<tr>
<td>4.4</td>
<td>43</td>
</tr>
<tr>
<td>4.5</td>
<td>43</td>
</tr>
<tr>
<td>4.6</td>
<td>44</td>
</tr>
<tr>
<td>4.7</td>
<td>44</td>
</tr>
<tr>
<td>4.8</td>
<td>45</td>
</tr>
<tr>
<td>4.9</td>
<td>46</td>
</tr>
<tr>
<td>4.10</td>
<td>46</td>
</tr>
<tr>
<td>4.11</td>
<td>47</td>
</tr>
<tr>
<td>4.12</td>
<td>47</td>
</tr>
<tr>
<td>4.13</td>
<td>48</td>
</tr>
<tr>
<td>4.14</td>
<td>48</td>
</tr>
<tr>
<td>4.15</td>
<td>49</td>
</tr>
<tr>
<td>4.16</td>
<td>49</td>
</tr>
<tr>
<td>4.17</td>
<td>50</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>4.18</td>
<td>General branches</td>
</tr>
<tr>
<td>(a) Series branch</td>
<td>50</td>
</tr>
<tr>
<td>(b) Filter branch</td>
<td>50</td>
</tr>
<tr>
<td>4.19</td>
<td>Frequently used models</td>
</tr>
<tr>
<td>4.20</td>
<td>RLC circuit</td>
</tr>
<tr>
<td>4.21</td>
<td>Impedance loci of RLC circuit</td>
</tr>
<tr>
<td>4.22</td>
<td>Frequency Response of RLC circuit</td>
</tr>
<tr>
<td>(a) Impedance versus frequency plot</td>
<td>53</td>
</tr>
<tr>
<td>(c) Resistance versus frequency plot</td>
<td>53</td>
</tr>
<tr>
<td>(d) Reactance versus frequency plot</td>
<td>53</td>
</tr>
<tr>
<td>4.23</td>
<td>General Plot facility display of six waveforms simultaneously</td>
</tr>
<tr>
<td>5.1</td>
<td>A.C. System Equivalent proposed by Hingorani and Burbery</td>
</tr>
<tr>
<td>5.2</td>
<td>Equivalent proposed by Bowles</td>
</tr>
<tr>
<td>5.3</td>
<td>Data flow and sequence for a Dynamic Study using a frequency-matched a.c. system equivalent</td>
</tr>
<tr>
<td>5.4</td>
<td>The Lower South Island of New Zealand test system</td>
</tr>
<tr>
<td>5.5</td>
<td>Impedance Loci Matrix of the test system</td>
</tr>
<tr>
<td>5.6</td>
<td>Diagonalized Matrix Impedance Loci</td>
</tr>
<tr>
<td>5.7</td>
<td>Impedance versus Frequency Matrix derived from figure 5.5</td>
</tr>
<tr>
<td>5.8</td>
<td>Structure of a frequency-matched a.c. system equivalent</td>
</tr>
<tr>
<td>5.9</td>
<td>Flow Diagram for the Direct and Optimization Algorithms</td>
</tr>
<tr>
<td>5.10</td>
<td>Impedance Loci of the equivalent based on intersection with X axis feature extraction</td>
</tr>
<tr>
<td>5.11</td>
<td>Impedance Loci of the equivalent based on new feature extraction</td>
</tr>
<tr>
<td>5.12</td>
<td>Equivalent cct based on new feature extraction</td>
</tr>
<tr>
<td>5.13</td>
<td>Response of modified equivalent cct</td>
</tr>
<tr>
<td>5.14</td>
<td>Impedance response when the scale factor is optimized</td>
</tr>
<tr>
<td>(a) Impedance Loci</td>
<td>75</td>
</tr>
<tr>
<td>(b) Impedance versus Frequency</td>
<td>75</td>
</tr>
<tr>
<td>5.15</td>
<td>Objective function in conjunction with intersection with X axis feature extraction</td>
</tr>
<tr>
<td>5.16</td>
<td>Objective function in conjunction with new feature extraction</td>
</tr>
<tr>
<td>5.17</td>
<td>Impedance Response</td>
</tr>
<tr>
<td>(a) Equivalent based on intersection with x axis feature extraction</td>
<td>77</td>
</tr>
<tr>
<td>Figure Description</td>
<td>Page</td>
</tr>
<tr>
<td>-----------------------------------------------------------------------------------</td>
<td>------</td>
</tr>
<tr>
<td>(b) Equivalent based on new feature extraction</td>
<td>77</td>
</tr>
<tr>
<td>5.18 Impedance Response</td>
<td></td>
</tr>
<tr>
<td>(a) Equivalent based on intersection with x axis feature extraction and scale factor optimized</td>
<td>78</td>
</tr>
<tr>
<td>(b) Equivalent based on new feature extraction and scale factor optimized</td>
<td>78</td>
</tr>
<tr>
<td>5.19 Frequency response of equivalent when the scale factor is optimized from 50 to 1250 Hz</td>
<td>79</td>
</tr>
<tr>
<td>5.20 Impedance with the scale factor optimized</td>
<td>79</td>
</tr>
<tr>
<td>5.21 Impedance versus frequency</td>
<td>80</td>
</tr>
<tr>
<td>5.22 Phase Angle versus frequency</td>
<td>80</td>
</tr>
<tr>
<td>5.23 Resistance versus frequency</td>
<td>81</td>
</tr>
<tr>
<td>5.24 Reactance versus frequency</td>
<td>81</td>
</tr>
<tr>
<td>5.25 Impedance Match of 3x3 Impedance Matrix</td>
<td>82</td>
</tr>
<tr>
<td>(a) Impedance match of (1,2) mutual element</td>
<td>86</td>
</tr>
<tr>
<td>(b) Impedance match of (1,2) mutual element when one correction branch has been added</td>
<td>86</td>
</tr>
<tr>
<td>(c) Impedance match of (1,2) mutual element when two correction branches are used</td>
<td>86</td>
</tr>
<tr>
<td>5.27 Matching to the Diagonalized Impedance Matrix</td>
<td></td>
</tr>
<tr>
<td>(a) Phase A</td>
<td>88</td>
</tr>
<tr>
<td>(b) Phase B</td>
<td>88</td>
</tr>
<tr>
<td>(c) Phase C</td>
<td>88</td>
</tr>
<tr>
<td>5.28 Impedance Match with a corrective branch for the 50 Hz parameters</td>
<td></td>
</tr>
<tr>
<td>(a) Phase A</td>
<td>89</td>
</tr>
<tr>
<td>(b) Phase B</td>
<td>89</td>
</tr>
<tr>
<td>(c) Phase C</td>
<td>89</td>
</tr>
<tr>
<td>5.29 Phase Angle Match with a corrective branch for the 50 Hz parameters</td>
<td></td>
</tr>
<tr>
<td>(a) Phase A</td>
<td>90</td>
</tr>
<tr>
<td>(b) Phase B</td>
<td>90</td>
</tr>
<tr>
<td>(c) Phase C</td>
<td>90</td>
</tr>
<tr>
<td>5.30 Impedance match obtained using various error criteria</td>
<td></td>
</tr>
<tr>
<td>(a) Least Squares</td>
<td>94</td>
</tr>
<tr>
<td>(b) Mini-max</td>
<td>94</td>
</tr>
<tr>
<td>(c) Mini-average</td>
<td>94</td>
</tr>
<tr>
<td>(d) Mini-area</td>
<td>95</td>
</tr>
<tr>
<td>Figure</td>
<td>Page</td>
</tr>
<tr>
<td>--------</td>
<td>------</td>
</tr>
<tr>
<td>5.31 Pictorial View of the effect of Numerical Noise on Gradient estimate</td>
<td>96</td>
</tr>
<tr>
<td>5.32 Interpretation of Self and Mutual terms</td>
<td>97</td>
</tr>
<tr>
<td>5.33 Extension of Hingorani and Burberry's equivalent by incorporating the mutual voltage as part of the fundamental frequency source</td>
<td>99</td>
</tr>
<tr>
<td>5.34 Full frequency-matched a.c. system equivalent</td>
<td>99</td>
</tr>
<tr>
<td>5.35 The self circuit for the frequency-matched a.c. system as implemented in TCS</td>
<td>100</td>
</tr>
<tr>
<td>5.36 Various a.c. system equivalent circuits</td>
<td>102</td>
</tr>
<tr>
<td>5.37 Self &lt;-&gt; Mutual Circuit Interface</td>
<td>103</td>
</tr>
<tr>
<td>5.38 Two types of interface</td>
<td>104</td>
</tr>
<tr>
<td>(a) Standard TCS circuit &lt;-&gt; torn branches</td>
<td>104</td>
</tr>
<tr>
<td>(b) Standard (Self) circuit &lt;-&gt; Mutual circuits</td>
<td>104</td>
</tr>
<tr>
<td>5.39 Multi-convertor System</td>
<td>111</td>
</tr>
<tr>
<td>5.40 Data Flow in forming a d.c. equivalent circuit</td>
<td>113</td>
</tr>
<tr>
<td>5.41 Structure diagram of DCLINK</td>
<td>114</td>
</tr>
<tr>
<td>5.42 Impedance match of d.c. equivalent</td>
<td>116</td>
</tr>
<tr>
<td>5.43 Partitioned equivalent circuit</td>
<td>117</td>
</tr>
<tr>
<td>5.44 Busbar voltage, displaying a harmonic instability</td>
<td>122</td>
</tr>
<tr>
<td>6.1 Structure diagram of the harmonic penetration program</td>
<td>125</td>
</tr>
<tr>
<td>6.2 Structure diagram of the iterative Harmonic Algorithm</td>
<td>130</td>
</tr>
<tr>
<td>6.3 Data flow with TCS assessment of convertor currents</td>
<td>132</td>
</tr>
<tr>
<td>6.4 Data flow with IHA assessment of convertor currents</td>
<td>133</td>
</tr>
<tr>
<td>6.5 Simple test system</td>
<td>134</td>
</tr>
<tr>
<td>6.6 Modified test system</td>
<td>136</td>
</tr>
<tr>
<td>6.7 Harmonic voltage throughout the Lower South Island of New Zealand test system</td>
<td>142</td>
</tr>
<tr>
<td>6.8 Harmonic Branch currents at sending end for Lower South Island test system</td>
<td>145</td>
</tr>
<tr>
<td>7.1 HVDC Test System</td>
<td>147</td>
</tr>
<tr>
<td>7.2 D.C. Line Fault Simulations</td>
<td>148</td>
</tr>
<tr>
<td>(a) Rectifier Current</td>
<td>148</td>
</tr>
<tr>
<td>(b) D.C. Voltage</td>
<td>148</td>
</tr>
<tr>
<td>7.3 System after Fault application</td>
<td>149</td>
</tr>
<tr>
<td>7.4 Rectifier side a.c. system fault</td>
<td>150</td>
</tr>
</tbody>
</table>
Figure

7.5 Dynamic response due to Rectifier side Fault with Thevenin Representation
   (a) A.C. System Current 151
   (b) Busbar Voltage 151
7.6 Dynamic response due to Rectifier side Fault with Frequency-dependent Representation
   (a) A.C. System Current 152
   (b) Busbar Voltage 152
7.7 Dynamic response due to Rectifier side Fault with Thevenin Representation
   (a) A.C. System Current 153
   (b) Busbar Voltage 153
7.8 Dynamic response due to Rectifier side Fault with Frequency-dependent Representation
   (a) A.C. System Current 154
   (b) Busbar Voltage 154
7.9 Invertor side a.c. system fault 155
7.10 Faulted Busbar voltage
   (a) Thevenin Representation 156
   (b) Frequency-dependent representation without mutual coupling 156
   (c) Frequency-dependent representation 156
7.11 A.C. System Current
   (a) Thevenin Representation 157
   (b) Frequency-dependent representation without mutual coupling 157
   (c) Frequency-dependent representation 157
7.12 Fault current 158

A2.1 Transmission Line \( \pi \)-segment Model 182
A4.1 Dichotomous Search Technique 184
A4.2 Search Technique used in the Fibonacci and Golden Section Searches 185
A4.3 Success/Failure step Algorithm 187
## LIST OF TABLES

<table>
<thead>
<tr>
<th>Table</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.1</td>
<td>CPU requirements for various error criteria</td>
</tr>
<tr>
<td>5.2</td>
<td>Assessed impedances with phase &quot;A&quot; current injection</td>
</tr>
<tr>
<td>5.3</td>
<td>Assessed impedances with phase &quot;B&quot; current injection</td>
</tr>
<tr>
<td>5.4</td>
<td>Assessed impedances with phase &quot;C&quot; current injection</td>
</tr>
<tr>
<td>6.1</td>
<td>Phase &quot;A&quot; Current Harmonics for simple test system</td>
</tr>
<tr>
<td>6.2</td>
<td>Phase &quot;A&quot; Voltage Harmonics for simple test system</td>
</tr>
<tr>
<td>6.3</td>
<td>Phase &quot;A&quot; Current Harmonics for modified test system</td>
</tr>
<tr>
<td>6.4</td>
<td>Phase &quot;A&quot; Voltage Harmonics for modified test system</td>
</tr>
<tr>
<td>6.5</td>
<td>IHA/TCS comparison with Thevenin a.c. system representation used in TCS</td>
</tr>
<tr>
<td>(a) Phase &quot;A&quot; harmonic currents</td>
<td>138</td>
</tr>
<tr>
<td>(b) Harmonic Voltages</td>
<td>138</td>
</tr>
<tr>
<td>(c) Comparison of d.c. Current Harmonics</td>
<td>138</td>
</tr>
<tr>
<td>6.6</td>
<td>IHA/TCS comparison with frequency-matched a.c. system equivalent used in TCS. (Implicit mutual coupling representation)</td>
</tr>
<tr>
<td>(a) Phase &quot;A&quot; harmonic currents</td>
<td>139</td>
</tr>
<tr>
<td>(b) Harmonic Voltages</td>
<td>139</td>
</tr>
<tr>
<td>(c) Comparison of d.c. Current Harmonics</td>
<td>139</td>
</tr>
<tr>
<td>6.7</td>
<td>Comparison of Computational Efficiency</td>
</tr>
<tr>
<td>6.8</td>
<td>Harmonic Voltages throughout the Lower South Island of New Zealand's primary transmission network</td>
</tr>
<tr>
<td>6.9</td>
<td>Harmonic branch currents at the sending end throughout the Lower South Island (N.Z.) test system</td>
</tr>
<tr>
<td>A3.1</td>
<td>Parameters for the circuit of figure 4.20</td>
</tr>
</tbody>
</table>
List of Principal Symbols

SYMBOLS

\( \xi \) - State variable convergence tolerance.
\( \xi_d \) - State variable derivative convergence tolerance.
\( I_d \) - D.C. current.
\( V_d \) - D.C. voltage.
\( I_{dm} \) - D.C. current margin.
\( s \) - Laplace Operator.
\( \alpha \) - Convertor firing angle.
\( \phi \) - Phase angle.
\( \mu \) - Convertor commutation angle.
\( \gamma \) - Convertor extinction angle.
\( f \) - frequency (Hz)
\( h \) - step length.
\( j \) - imaginary unit \((j=\sqrt{-1})\)
\( \Psi \) - Inductor flux.
\( Q \) - Capacitor charge.
\( \Theta \) - Objective function.
\( \Theta \) - Electrical angle \((\Theta=\omega t)\)
\( \sigma \) - Penalty function.
\( \lambda \) - Lagrange multiplier
\( p \) - derivative w.r.t. \( t \) (or \( \omega t \))
\( \omega \) - Angular frequency \((=2\pi f)\)
\( f_{pr} \) - frequency of parallel resonance
\( I_h \) - Current at harmonic \( h \)
\( V_h \) - Voltage at harmonic \( h \)
\( V_t \) - Convertor terminal voltage
\( Y_h \) - Admittance at harmonic \( h \)
\( Y_{rr} \) - Admittance of the actual a.c. system (required response)
\( Y_{ec} \) - Equivalent circuits admittance
\( Y_{cb} \) - Admittance required by the correction branch
Subscripts:

\( \alpha \) - alpha node
\( \beta \) - beta node
\( \gamma \) - gamma node
\( i \) - \( i^{th} \) element of vector
\( ij \) - element in \( i^{th} \) row and \( j^{th} \) column of matrix
\( \ell \) - inductive branch
\( r \) - resistive branch
\( c \) - capacitive branch
\( ec \) - Equivalent circuit
\( cb \) - correction branch
\( rr \) - Required response
\( \cdot \) - denotes a vector

Superscripts:

\( . \) - derivative w.r.t. time
\( t \) - transpose of matrix
\( j \) - Result of \( j^{th} \) iteration
Abbreviations:

a.c. - Alternating current
ABS - Absolute value
ACREP - Program for processing and plotting Power System Data
BFGS - Broyden, Fletcher, Goldfarb and Shanno
C.C.C. - Constant Current Control
CPU - Central Processor Unit of the computer
d.c. - Direct current
DCLINK - Admittance generating program for d.c. side
DEC - Digital Equipment Corporation
E.A.C. - Extinction Angle Control
EFC - Equi-distant Firing Control
EMTP - Electro-Magnetic Transient Program
F - Farads
H - Henries
HARMAC - Harmonic Penetration Program
HVDC - High voltage direct current
IHA - Iterative Harmonic Algorithm
INTER - Interactive data base preparation program
IPC - Individual Phase Control
KCL - Kirchhoff's Current Law
KVL - Kirchhoff's Voltage Law
km - Kilometres
kV - Kilovolts
LINE - Line constants program for an arbitrary number of conductors
ln - Natural logarithm
m - Metres
min. - Minutes
MW - Megawatts
p.u. - Per unit
S.C.R. - Short Circuit Ratio
sec. - seconds
TCS - Transient Convertor Simulator
t - Time
TL - Line constants program for three phase a.c. lines
TNA - Transient Network analyser
w.r.t. - with respect to
ABSTRACT

This thesis describes the algorithms developed for synthesizing frequency matched a.c. system equivalents for use with a Transient Convertor Simulator program. Two synthesis methods are outlined, with the merits of each being illustrated by applying them to the lower South Island portion of New Zealand's primary transmission system.

Validation of the diakoptical technique used to model the frequency dependent mutual coupling, as well as data preparation, is achieved by using the current source model to inject harmonic currents into the frequency-matched a.c. system equivalent.

The need for frequency matched a.c. system equivalents is demonstrated by using the Transient Convertor Simulator program for harmonic penetration studies.

Finally the effect of the frequency-dependence of an a.c. system model on the transient behaviour of a convertor is illustrated by simulating both a.c. and d.c. disturbances. The difference in transient behaviour with asymmetric a.c. faults as well as the harmonic assessment for harmonic penetration studies both demonstrate the need for frequency matched a.c. system equivalents that more accurately model the actual system.
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1.1 Importance of HVDC Transmission.

The technical advantages of HVDC transmission are well documented in the literature and were understood from the early days of electricity. However HVDC transmission had to wait for the development of the Mercury-arc valves before finding its first commercial application in 1954. This was the Sweden-Gotland link which transmitted 20 MW at 100 kV d.c. and was justified because the transmission distance (96 km) was too long for a.c. technology of the time and cheaper than building a new fossil fuel generation plant on the island. Since then there has been a rapid increase in the number and power rating of HVDC schemes.

Mercury-arc valves were exclusively used in the a.c./d.c. conversion process until 1972 when the Eel River scheme was commissioned. This was the first use of the thyristor valve which eventually superseded mercury-arc valves for HVDC transmission schemes. The Kingsnorth HVDC scheme which was commissioned in 1974 was the last scheme using mercury-arc and all subsequent schemes used thyristor valves. By the end of 1979 50 percent of the total installed HVDC capacity of 12,000 MW were thyristor valve systems.

Performances of the HVDC systems throughout the world have been regularly surveyed (Last and Middleton 1970, 1972, Jarrett and Middleton 1974, Rumpf and Jarrett 1976, Rumpf 1980, Knudsen and Albrecht 1982, Knudsen et al 1984). These surveys show that HVDC systems are technically and economically viable due to their high reliability and use of the HVDC links attributes to meet the particular systems requirements.

Advances in convertor stations, such as the use of air or water cooling, and developments in thyristor valves have lead to a reduction in cost relative to a.c. transmission and greater power ratings. The increased competitiveness of d.c. transmission is reflected in the number of links presently under consideration. New Zealand is currently considering a second d.c. link and some third world countries are also implementing d.c. links. In the early schemes the a.c. systems were strong compared to the transmitted power. However as the convertor
ratings increase the relative a.c. system strength diminishes and the problems of dynamic response and harmonics increase greatly.

1.2 Need for Computer Modelling.

The planning, design and operation of a power system requires several different types of studies to ensure that satisfactory performance is maintained. If such studies are carried out at the design stage, solutions to possible problems can be found before they eventuate, thus preventing costly damage or maloperation of power system components.

However the complexity of power systems makes such studies difficult and time consuming. A truly analytical solution is virtually impossible for all except the simplest of problems. The need for computational aids led to the development of special purpose analog computers and the use of scaled down models. Due to the inherent difficulties of such methods and to the rapid development of powerful digital computers, computer simulation has become the preferred method. The appeal of the digital computer is its ability to process a vast amount of data in a systematic way with great accuracy, and to do so in an extremely short time. This has been enhanced by the development of efficient numerical techniques for modelling large electrical power systems. The effectiveness of computer modelling is summarized by the following quotation from IEEE std. 399-1980 "The modern digital computer offers power systems engineers a powerful tool to perform more effective studies to any industrial power system."

1.3 Types of Studies.

The types of studies performed at the planning stage are:
(a) Load flow (or Power flow)
(b) Fault and Short-Circuit Studies
(c) Harmonic Studies
(d) Dynamic Studies

Load flow studies determine the voltage, current, power and power-factor or reactive power at various points in a power system. This is a
fundamental frequency steady state analysis, with all waveforms assumed to be perfectly sinusoidal. Load flow studies find applications in almost all other analysis as they supply the initial conditions for the other studies.

Short-Circuit studies are required to ensure that protection equipment can isolate faults quickly and minimize damage and personnel hazard.

Harmonic studies are needed to assess the effect of discrete harmonic current injections and the resulting voltage distortion at the various power system busbars.

Many diverse types of studies can be labelled as Transient or Dynamic Studies. However, they are distinguished by the objective of the study, which will determine the time scale of interest. The time scale may range from micro-seconds for fast transients such as lightning strikes, to many minutes for long-term dynamic studies. The time scale of interest dictates the type of model required for the various power system components (Concordia and Schulz 1975).

1.4 Incorporating HVDC Convertors.

The influence of HVDC convertors on system behaviour necessitates detailed convertor representation for these various types of studies. Although still relatively few in number, as a result of their large power ratings and diverse control strategies means they greatly influence the power systems they are connected to. This has motivated many people worldwide in the development of computer models to analyse the steady state and dynamic behaviour of HVDC convertors. An early attempt was made at UMIST with the development of a Transient Convertor Simulation (TCS) program which models HVDC convertors to a high degree of accuracy. Further work on TCS has continued at the University of Canterbury by progressively improving the accuracy of the results by developing better models for the various power system components. The work of Joosten (1987) in developing a realistic transformer model that represents both saturation and hysteresis effects is particularly note-worthy.
The work described in this thesis concentrates on the derivation of a.c. system models suitable for TCS, with a time scale of interest from 0.2 to 400 milli-seconds. A three phase a.c./d.c. load flow developed by B. Harker (1980) is used for the derivation of initial conditions. The use of the improved TCS program in harmonic studies is also demonstrated in the thesis.

1.5 The need for equivalent circuits.

The complexity of modern power systems makes it imperative to use equivalent circuits to ease the computational burden. Normally the areas of the power system where detailed information is required must be modelled by explicit representation of each of the power system components, while the remaining parts of the power system are reduced to an equivalent circuit. The credibility of the simulation is then dependent on how well the equivalent circuit performance represents that part of the actual power system. The use of equivalent circuits has become well established since Thevenin first published his theorem in 1883. However, since then considerable effort has been put into refining the equivalents to try and improve their accuracy.

Under fault conditions the a.c. system parameters vary due to the finite time required for the flux to change in components such as synchronous machines. A means of obtaining accurate time varying network equivalents was developed by Heffernan (1980). This involved interfacing the dynamic simulation program with a multi-machine a.c./d.c. transient stability program. The network equivalents obtained from the latter program provided a more accurate indication of the time response of the a.c. system by virtue of the synchronous machine models used in the transient stability program.

It has been known for a long time that the frequency response of the power system components affects the transient response due to the multitude of frequencies present in transient waveforms. However, incorporating such frequency-dependence in dynamic simulation programs has proved difficult. The present work develops methods of obtaining a practical and computationally efficient equivalent circuit that accurately represents the frequency-dependence of the actual system being represented.
1.6 Thesis Outline.

The work in this thesis relates to the development of equivalent circuits that accurately model the frequency-dependence of the systems they represent and are computationally efficient. A further constraint is that the equivalent circuits should be suitable for incorporation into TCS programs.

The material covered is divided into three separate topics as follows:

(i) the development of a dynamic program with accurate representation of HVDC convertors,
(ii) incorporating the frequency-dependence of power system components into dynamic simulation programs,
(iii) harmonic analysis and dynamic simulations.

Chapter 2 sets the scene by giving a brief overview of various methods of performing dynamic studies with particular reference to the representation of HVDC convertors.

Chapter 3 is dedicated to the basic mathematical models by which each type of system component is represented in the digital computer. However, discussion of the frequency dependent a.c. system model is left until chapter 5. The method of network analysis is outlined and the TCS program improvements that have been implemented are detailed.

Chapter 4 describes the flexible program for processing and plotting power system data (ACREP) that has been written. The objectives, structure and operation of the program are discussed as well as its capability. Illustrative examples of its capability are also given.

Chapter 5 begins by reviewing previous work in the development of frequency-dependent power system components for use in time domain studies. Possible methods of obtaining the frequency-response for the equivalent circuit are discussed. The mathematical basis for synthesizing an equivalent circuit with a very similar frequency response is presented. The algorithms are then applied to the lower South Island
Chapter 6 deals with the specialized use of the developed dynamic program for steady-state solutions, namely harmonic studies. The existing techniques for harmonic penetration studies are surveyed and discussed. A brief portrayal of how the programs developed fit in and interact with the existing software at the University of Canterbury is given. The results of a case study are presented and compared with those obtained from an iterative steady state technique.

In Chapter 7 the results of simulating a.c. and d.c. disturbances are presented and the effect of a frequency-dependant a.c. system model discussed. Finally the concluding remarks and suggestions for further work are presented in chapter 8.
2.1 Computational Aids for Dynamic Simulation.

The need to predict the dynamic behaviour of electrical power systems has lead to the development of various computational aids. These are; (i) Analog computer, (ii) Transient Network Analyser (TNA), (iii) Digital computer. Each tool has its strengths and weaknesses and they should be looked on as complementary rather than mutually exclusive. There is also further subdivision possible depending on the different implementation of each computational aid. Combined use of these computational aids has also been proposed to utilize the best features of each approach.

Mathematical modelling techniques assume that transient events can be expressed and solved in terms of mathematical equations, whereas the experimental approaches attempt to simulate the physical behaviour of components, circuits and systems such that the application of a physical input to the model results in an output similar to that of the real system. Transient network analysers (TNA) fall into the latter category, while digital computers and electronic analog computers model the system mathematically. TNA is used here in a broad sense and encompasses all the analysers using the electromagnetic model approach to transient analysis.

The analog computers solve the differential equations that mathematically represent the transient phenomena. They are well suited for preliminary sensitivity studies due to the speed and flexibility of system parameters and control implementation. Their major limitation is the lack of representation of nonlinear circuit elements such as saturable transformers and HVDC convertors.

The TNA has been the traditional method of performing transient power system studies. Peterson (1939) described the TNA developed at GEC for transient studies. Over the years very elaborate TNA's have been constructed and the representation of power system components improved. In the conventional TNA simulators, reactors, capacitors and resistors are connected to duplicate the behaviour of their actual counterpart; tapped inductive reactors are used for transformer simulation. Another
8.

The type of TNA is the scaled down physical simulator, where miniature transformers are used to represent the actual ones. An accurate simulation of transformers is difficult as it requires representation of magnetization characteristics (saturation curve and losses) as well as leakage inductance and capacitance. Requirements in different areas often conflict and compromises have to be made. In particular the transformer resistance tends to be higher in the model than in practice. While it is possible to add resistance, inductance or capacitance, to reduce them is more of a challenge. Transmission lines are represented by lumped networks made up of π or T sections. The TNA is limited in the size and complexity of the system it can simulate due to time and economic considerations. The system parameters are inflexible making the study of different configurations time consuming. Once a particular design has been decided upon, a TNA model can be set up, allowing such parameters as HVDC control strategies to be optimized for a small extra cost. Considerable spread in transient voltages develop due to the instant in the cycle at which a change occurs and the variations between the closing or opening of the different phases. It is relatively easy to perform the switch operation on the TNA repeatedly and collect statistical data. The enormous development in TNA's is evident in the 80 references cited by Concordia (1956).

The digital computer has been applied to many different mathematical formulations of the transient phenomena and the most important ones are outlined in the next section. The digital computer is well suited to studying different system configurations due to the flexible manner in which the power system topology can be set up by software. Greater accuracy is achievable compared to the analog computer approach, but the optimization of system parameters requires many runs of digital simulation, which maybe expensive in terms of computer time. The accurate modelling of nonlinear elements is also easier to accomplish in the digital computer solution compared to both analog computer or TNA approaches. The major factor limiting the complexity and accuracy of digital computer simulations is the computational cost. The type of study required determines the mathematical model necessary and hence the cost.

The use of equivalent circuits tries to alleviate the computational burden without significant loss of accuracy, hence the importance of the present work on a.c. system equivalents.
2.2 Digital Computer Simulation.

Much work has been done to develop ingenious techniques for digital simulation of transients, as well as methods that allow certain difficulties to be circumvented or assist in reducing the amount of computation. Considering the enormous amount of published literature on the topic only a superficial outline can be given here. For a more detailed treatment and list of references the reader is directed to the reviews by Pender (1969), Humpage and Wong (1982), Bickford (1986) and the books by Greenwood (1971) and Bickford et al (1976).

Travelling wave methods are based on the solution of the transmission line equations, which maybe expressed as a combination of forward and backward travelling waves on the line. Two different approaches exist based on the Lattice diagram of Bewley, and graphical method initiated by Allievei and developed by Schnyder and Bergeron (Bickford et al 1976). These techniques were later adapted for solution by digital computers due to the labour involved in applying travelling wave methods. The mathematical basis for this method relies inherently on lossless propagation, but correction terms can be incorporated to account for the losses in real systems. Although travelling wave methods are suited towards distributed parameter elements such as lines and cables, lumped parameter elements like generators, transformers etc., can be approximated by short line stubs. The travelling wave methods are attractive for small transmission line problems but they prove rather cumbersome for realistic power systems studies due to their complexity. Moreover the representation of HVDC convertors in travelling wave methods has not been attempted.

The importance of correctly modelling the frequency-dependence of power elements to include its effect on the transient system response has been recognized for many years. This has led many people to investigate frequency domain techniques based on Fourier series or Fourier transforms, due to their easier representation of frequency dependence. In these methods the frequency-dependence is modelled to an accuracy limited only by the accuracy to which the system parameters are known. The Fourier frequency-domain method involves formulating the problem in terms of a repetitive function, transforming, solving and performing the inverse transform to obtain the time response. However HVDC convertor
representation is a problem that has not been solved with this method.

The z-transform techniques are a class of frequency domain analysis which seek to avoid going from the frequency domain to time domain via the inverse Fourier transform, with its inherent difficulties. As with the other frequency domain analysis techniques, accurate convertor representation is a problem.

Time domain analysis based on step-by-step numerical integration of the system's differential equations is the most suitable method for dynamic studies involving HVDC convertors. The most widely acclaimed transient program is undoubtedly EMTP (Electro Magnetic Transient Program). EMTP uses the trapezoidal integration equation to linearize the differential equations to a set of simultaneous equations, which are then solved. EMTP logic, as coded by H.W. Dommel (Dommel 1969, Dommel and Meyer 1974), was designed when switching operations were few and far between. EMTP features fixed step-length which ensures economic simulation, as many of the matrices are then constant.

In the TCS program the problem is formulated using state variables as a set of first order differential equations, and the trapezoidal integration is then used to solve for the new state variables at each time step. Variable step length is used in TCS to enable time step boundaries to fall exactly on switching operations.

A problem associated with EMTP is numerical noise. Campos-Barros (1985) has demonstrated the superiority of the TCS formulation when simulating convertor valve switchings, due to the oscillations in EMTP simulations caused by numerical noise.

2.3 Development of HVDC Convertor Models.

An early method of digital HVDC dynamic simulation used "The Central Process" technique. In this method HVDC convertor operation is broken into similar consecutive processes. It requires many subroutines, with each subroutine solving a set of differential equations arising from the particular network topology. Only those conduction patterns coded can be simulated, so prior knowledge of all possible network topologies is required. The control and processing of the subroutines were included in the main program. This method is limited in versatility and was applied
to studies where the convertor was connected to an infinite a.c. busbar. Many different sets of equations became necessary to describe the network when the a.c. system impedance became significant making the approach cumbersome and wasteful of computing time (Hay et al 1970a, 1970b, 1971a, 1971b, Hingorani et al 1966a, 1966b, 1967, 1968).

Williams and Smith (1973) applied the tensor-analysis techniques proposed by Kron (1939, 1959) to the modelling of convertor bridges. This led to an elegant and superior method of dealing with the periodically varying topology of the bridge. The main advantages were the logical procedure for automatic assembly and solution of the network equations and superior generality. The programmer no longer needed to be aware of all the reduced sets of equations that described each particular conduction situation. Mesh analysis formed the basis of the work performed by Milias-Argitis (1976, 1977a, 1977b, 1978) which was an extension of Williams and Smith's work.

Kron's techniques are equally applicable to nodal formulation and this formed the basis of TCS (Arrillaga et al 1977a, 1977b). Early dynamic simulation programs used idealised three phase e.m.f. sources in series with an impedance to represent the a.c. system, they included explicit representation of the harmonic filters; but the convertor transformers were represented by their leakage reactance.

The general dynamic simulation program that resulted from the developments at UMIST used a nodal formulation and state space theory. The diakoptical techniques (Kron 1963, Brameller et al 1969) implemented by Al-Khashali (1976) resulted in an extensive reduction in computational burden. A detailed synchronous machine model was developed for the UMIST program by Campos Barros (1976). A more realistic coupled coil representation for transformers was implemented in conjunction with existing models for transmission lines, harmonic filters, a.c. system and convertor. The convertor controller was based on a direct digital control scheme developed by Arrillaga and Galanos (1969, 1970a, 1970b, 1970c), this in principle being similar to the phase locked oscillator of Ainsworth (1968), giving equidistant firing control. Further improvements were achieved at the University of Canterbury by incorporating the a.c. system's time response into the model (Heffernan 1980, Turner 1980). By
this stage four different a.c. system models were catered for. A more realistic d.c. fault model was introduced based on experiments performed by Kohler (1967). At this stage the two main deficiencies were apparent; the transformer model, and the representation of the frequency-dependence of an a.c. system. The development of a transformer model that represents both saturation and hysteresis effects was accomplished by Joosten (1987), while this thesis reports on the later problem.

Figure 2.1  Power System Analysis Overview
3.1 Introduction.

A power system can be represented by a network of inductive, capacitive and resistive elements, where each component has a particular circuit model defined in terms of these three elements. The most suitable approach is a state space formulation which can be used to treat both linear and non-linear circuits. This yields a set of non-linear differential equations. The state variables are related to the energy stored in the electric fields of capacitors, and in magnetic fields of inductors, while resistors are non-state elements. This chapter gives a brief outline of the component models and method of solution with more detail being given to modifications that have been made to the existing TCS algorithm (Heffernan 1980).

3.2 Representation of Components.

In a dynamic simulation, consideration of the major area of interest permits considerable simplification in component models by neglecting effects outside the area of interest. For example, source e.m.f.'s are assumed to be of constant frequency as generator-rotor swings are too slow to have an effect on the convertor or its controller. At the other end of the scale, the effect of frequencies higher than 2.5 kHz are also neglected.

3.2.1 Static Convertor.

The Graetz bridge is well accepted as the basic building block for HVDC schemes and its operation well documented (Adamson and Hingorani 1960, Cory 1965, Kimbark 1971, Uhlmann 1975, Arrillaga 1983). Both 6 and 12 pulse convertor models are available, with the 12 pulse consisting of two 6 pulse bridges connected in series on the d.c. side and via a star-star and star-delta convertor transformers respectively. This configuration of convertor transformers gives the 30 degree phase shift required for 12 pulse operation.
3.2.1.1 Modes of Operation.

In principle any valve conduction is possible as valve switching is determined purely by the conditions it experiences. If a valve is forward biased and a firing pulse is present it will attempt to turn "ON". However the conducting state of one valve influences the conditions and hence behaviour of the other valves and this limits the conduction patterns normally experienced. The normal and abnormal conduction states that can be expected may be categorised by the following modes:

A Non-commutating mode
Two valves on different sides and arms of the bridge are ON.

B Normal commutation
Three valves in ON state; one on each arm and at least one conducting on each side of bridge.

C Non-commutating arm short-circuit
Both valves on one of the arms in ON state with no other valves ON.

D Commutating arm short-circuit
Both valves on one of the arms in ON state with one or both valves on another arm being ON. There is a single commutation process if one valve on the second arm is conducting, otherwise there are commutations taking place on both sides of the bridge simultaneously.

E a.c. short-circuit
Four or more valves conducting, involving all three arms of the bridge. This has the effect of short-circuiting the convertor transformer's three secondary terminals.

Although bypass valves are present in existing schemes, their effect can be modelled implicitly by modes C, D and E.

3.2.1.2 Convertor Control Systems.

The operational details of the control system differ for each particular application, however their basic principles are the same and are well documented (Adamson and Hingorani 1960, Cory 1965, Kimbark 1971, Uhlmann 1975, Jotten 1978, Arrillaga 1983).

The steady state operating characteristics for a basic HVDC link controller are displayed in figure 3.1. The rectifier operates in
constant current control (C.C.C.) mode by varying its delay angle so as to maintain a link current while the inverter determines the link voltage by operating with minimum extinction angle (referred to as extinction angle control E.A.C.). The inverter also has a C.C.C. feature which is set to operate during transient or abnormal conditions.

High link voltage is required to minimize losses, and to minimize reactive power demands the control angles ($\alpha_r$ and $\gamma_i$) need to be as small as practicable. However $\gamma_i$ must be large enough to avoid commutation failures on the occurrence of minor disturbances. Convertor transformer tap changing is used to alter the level of the effective a.c. supply voltage, but this is a relatively slow type of control and therefore not modelled in TCS.

3.2.1.3 Types of Control Schemes.

Individual Phase-Control (IPC) was the norm for early HVDC schemes. With this scheme valve firing instances are determined independently for each valve based on the relevant a.c. voltage zero crossing. IPC reflect any unbalance or distortion in the a.c. supply voltages by producing variations in valve conduction periods. These in turn produce further distortion and unbalance, with the result of uncharacteristic harmonic production in the a.c. system, and harmonic instability if it reinforces the original distortion (Ainsworth 1968). These phenomena are common with weak a.c. systems. Filtering the a.c. system waveform entering the
controller can reduce the problem but not remove it.

Equidistant Firing Control (EFC) produces a train of equidistant firing pulses which are advanced or retarded by the controller. Equal conduction periods are assured regardless of a.c. voltage waveforms, and hence uncharacteristic harmonics and harmonic instability problems are alleviated. Most EFC controllers are variations of the phase locked oscillator technique developed by Ainsworth (1967, 1968). Digital implementation of this firing control has been proposed by Arrillaga and Galanos (1969, 1970a, 1970b, 1970c) and Arrillaga and Baldwin (1974), and is used in TCS.

3.2.2 Synchronous Machines.

When a convertor is present phase-variable representation is required due to the synchronous machines terminal voltages being non-sinusoidal and the need to accurately represent the convertor. Since most manufacturers' data is in the two-axes d-q-o form it must be transformed to direct phase quantities (a,b,c) (Concordia 1951, Kimbark 1968, Fitzgerald et al 1983). A phase-variable model allows easy handling of asymmetries, non-linearities and distortion effects and the extra computation caused by the time varying inductances is partially offset by averting the need of transformations at each time step. A d-q axis representation is retained for the rotor circuit since it fits the actual geometry and winding arrangement. In writing the differential equations the synchronous machine is treated as a motor, with the positive current flow direction being into the positive terminal. In matrix form the terminal voltage may be expressed as:

\[
V_g = p(L_g \cdot I_g) + R_g \cdot I_g
\]

\[
= L_g \frac{d}{dt} I_g + \omega \cdot \frac{d}{dt} L_g \cdot I_g + R_g \cdot I_g
\]

where \( \omega = \frac{d\theta}{dt} \), the angular velocity.

\( L_g \) is a 6x6 matrix and its elements are given in appendix A1. The stator values can accommodate fourth harmonic terms if available, as these can be of significance in convertor-generator units (Campos-Barros 1976).
3.2.3 Transmission Lines.

Wave propagation techniques are classically used to analyse the transient response of a transmission line. However, the implementation is difficult where there are multiple a.c. and d.c. lines of differing lengths, and this has lead to a network approach being adopted. The transmission line model consists of a number of cascaded π-segments where the number of π-segments is determined from the highest frequency of interest (Smith and Bell 1984). Appendix A2 describes the selection of component values for this model.

3.2.4 Transformers.

The single phase coupled coil transformer model, as shown in figure 3.2 is the basic building block by which all other three phase transformers are represented, due to the ease in which winding configuration and associated phase shifts are accounted for.

Neglecting the iron losses, the matrix equation for the coupled coil model is:

\[
\begin{bmatrix}
V_1 \\
V_2
\end{bmatrix} =
\begin{bmatrix}
L_{11} & L_{12} \\
L_{21} & L_{22}
\end{bmatrix}
\frac{d}{dt}\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix} + \begin{bmatrix}
R_1 & 0 \\
0 & R_2
\end{bmatrix}\begin{bmatrix}
I_1 \\
I_2
\end{bmatrix}
\]

(3.2)

where the relationship between the leakage reactance and inductance in the model is (in p.u.)

\[
L_{12} = L_{21} = \sqrt{L_{22}(L_{11} - X_{la})}
\]

(3.3)

where \(X_{la}\) is the total leakage reactance of the transformer.
Accurate representation of non-linearities in core magnetization (saturation and hysteresis) has been incorporated into the transformer model by Joosten (1987).

3.2.5 Static Shunt Elements.

Both a.c. and d.c. harmonic filters and surge capacitors greatly influence the convertor waveforms following a disturbance and even in the steady state, and hence need to be accurately represented. Fig. 3.3 displays the various shunt elements that can be used to represent plant components and faults. Three phase a.c. components are assumed to be star connected with the neutral point earthed.

![Figure 3.3 The various static shunt element models.](image)

3.2.6 Static Series Elements.

These models are essentially the same as for the static shunt elements except that the components are connected between phases. This allows circuit breakers, delta loads and other such components to be modelled. Care must be taken to ensure that the topology constraints, as discussed in section 3.3.6, are not contravened when using these elements.
3.2.7 A.C. System.

The size and complexity of a.c. power systems necessitates the use of less complex, yet sufficiently accurate a.c. system models. Explicit busbar information will generally not be available at all busbars and hence system components of particular interest must be represented in detail. A number of a.c. system equivalents have already been developed and are displayed in figure 3.4. The major feature of the a.c. system models developed by Heffernan (1980) was their time varying nature. The present work involves the incorporation of the frequency response of the system, as expressed by impedance loci, into an a.c. system model. The details of obtaining the impedance loci and the synthesis of frequency matched a.c. system models is presented in chapter 5.

Figure 3.4  A.C. System Equivalents
3.3 Method of Analysis.

Any lumped network obeys three laws; Kirchhoff's voltage law (KVL), Kirchhoff's current law (KCL) and the element laws (branch characteristics). From these a set of first-order differential equations can be derived that characterize the network and this constitutes the state variable approach. In the state variable approach the network is characterized by the following matrix (or state) equations:

\[ \dot{x} = [A]x + [B]y + [E]z \]  
\[ y = [C]x + [D]y \]

where

- \( y \) represents the input voltages and currents.
- \( x \) is the vector of state variables.
- \( y \) represents the output voltages and currents.
- \( z \) represents a set of dependent variables.

The dependent variables are evaluated by the equation:

\[ z = [F]x + [G]y \]

The matrices \([A],[B],[C],[D],[E],[F],[G]\) are the appropriate coefficient matrices which may be non-linear functions of \( x \) or \( y \) and/or time varying.

The attractions of the state variable approach are its amenability to numerical methods of analysis and the ease in which non-linearities are incorporated. Non-linearities which are functions of time, voltage or current magnitude are easily handled, which allows most types of power system non-linearities. The non-linearity not easily simulated is the frequency-dependence.

3.3.1 Trapezoidal Integration.

The basic trapezoidal method is very well known, having been established before the advent of the digital computer. More recently an implicit trapezoidal integration method has gained wide acceptance due to its good stability, accuracy and simplicity (Gear 1971, Arrillaga et al 1983a).

For each time step the change in a state variable is equal to the integral of the area under its derivative and the trapezoidal integration
approximates this area by the trapezoid given by:

$$\Delta x = (h/2)(\dot{x}_t + \dot{x}_{t+h})$$  \hspace{1cm} (3.7)

where $\dot{x}_{t+h}$ is determined iteratively.

The iterative procedure is as follows:

(i) For an initial estimate it is assumed that $\dot{x}_{t+h} = \dot{x}_t$

(ii) An estimate of $x_{t+h}$ based on $\dot{x}_{t+h}$ estimate is obtained.

(iii) $\dot{x}_{t+h}$ is estimated from the current $x_{t+h}$ value using equation (3.9)

$$x_{t+h} = x_t + (h/2)(\dot{x}_t + \dot{x}_{t+h})$$  \hspace{1cm} (3.8)

$$\dot{x}_{t+h} = f(t+h, x_{t+h})$$  \hspace{1cm} (3.9)

(iv) Steps (ii) and (iii) are performed iteratively until convergence is reached. Convergence is deemed to have occurred when all the state variables satisfy:

$$\xi \geq \text{ABS}(x_{t+h}^{j+1} - x_{t+h}^j)$$  \hspace{1cm} (3.10)

where $\xi$ is the convergence tolerance. It is sometimes necessary to specify an additional convergence constraint to ensure the state variable derivatives have converged sufficiently, i.e.

$$\xi_d \geq \text{ABS}(\dot{x}_{t+h}^{j+1} - \dot{x}_{t+h}^j)$$  \hspace{1cm} (3.11)

where $\xi_d$ is the state variable derivative convergence tolerance.

Normally three to four iterations are required with a suitable step length. If convergence fails the step length is halved and the iterative procedure is restarted. The integration step length is also automatically increased or decreased during the simulation based on the past history of the number of iterations needed to reach convergence. This greatly improves efficiency of the simulation.

3.3.2 Topology Changes.

The instances of network connection or parameter value changes need to be determined accurately to achieve an accurate simulation. Two types of changes can be distinguished; those easily predicted before the event and those which are detected only after the event. The time for switching "ON" a convertor valve and fault application are easily determined before the event. In such cases the simulation step size is
lowered so that the simulation step falls exactly on the required time the change is to take place. As the state space coefficient matrices are not functions of step size, changing it does not impose a dramatic computational burden.

The times of zero current in convertor valves and fault branches are not easily predictable before they occur. Hence, these conditions are detected after they occur and linear interpolation (backwards in time) is used to reach the time of the switching. The state variables are also obtained by linear interpolation rather than trapezoidal integration. This is because the linear interpolation gives sufficient accuracy, due to the small simulation step size, while the trapezoidal integration would increase dramatically the storage and computation overheads.

3.3.3 Choice of State Variables.

Although inductor current and capacitor charge are the state variables chosen by most textbooks it is better to use the flux linkage of the inductor ($\Psi$) and the capacitor charge ($Q$) as the state variables. Regardless of the numerical integration algorithm used in solving the differential equations, this will result in a better solution due to less error propagation owing to the presence of local truncation error (Chua and Lin 1975). Also large step lengths are achievable while maintaining convergence within a few iterations (Heffernan 1980).

State space analysis requires the number of state variables to be equal to the number of independent energy storage elements (i.e. independent inductors and capacitors). Therefore it is important to recognize when inductors and capacitors in a network are dependent or independent. For example a dependent inductor is one whose current is a linear combination of the current in k other inductors in the system. This is not always obvious due to the presence of intervening networks.

The use of capacitor charge or voltage as a state variable creates a problem when a set of capacitors form a closed loop. In this case the standard state-variable formulation falls down as one of the chosen state variables is a linear combination of the others, therefore, should not be a state variable. This would be a serious limitation as many power system elements exhibit this situation (e.g. transmission line model). There are several ways of overcoming this problem; TCS uses the
charge at a node rather than capacitor voltage as a state variable to circumvent the problem.

A similar problem arises when only inductive branches are connected to a radial node. If the initialization of state variables was such that the sum of the current at this radial node was non-zero, then this error will remain throughout the simulation. The development of a "Phantom Current source" is one method that has been developed to overcome the problem (Joosten 1985). There are several other possible methods for overcoming this current error. One approach is to choose an inductor at each node with only inductors connected to it, and make its flux a dependent rather than state variable. Another possibility is to transform the inductor to its controlled source equivalent (Chua and Lin 1975).

Each approach has some disadvantages. The "Phantom Current Source" approach can cause very large voltage spikes when trying to overcome inaccurate initial conditions. Partitioning the inductor fluxes into state and dependent variables is hard as it is often difficult to identify if the inductor flux is dependent or independent. An inductor can still be dependent even if it is not connected directly to a radial node consisting of inductive branches, but it has an intervening resistor/capacitor network.

3.3.4 Per Unit System.

In the analysis of power systems Per Unit quantities rather than actual values are normally used. This scales voltages, currents and impedances to the same relative order thus treating each to the same degree of accuracy. In dynamic analysis instantaneous phase quantities and their derivatives are evaluated. The variables may be changing relatively fast causing a large difference between the order of a variable and its derivative. For example consider the sinusoid:

\[ x = A \sin(\omega t + \phi) \]
\[ \dot{x} = \omega A \cos(\omega t + \phi) \]

The relative difference in magnitude between \( x \) and \( \dot{x} \) is \( \omega \), which may be high. Therefore a base frequency \( \omega_0 \) is defined. All the state variables are changed by a factor \( \omega_0 \) and this then necessitates the use of reactance and susceptance matrices rather than inductance.
and capacitance matrices. The integration is now with respect
to electrical angle rather than time.

\[ \psi_k = \omega_0 \Psi_k = (\omega_0 L_k) I_k = L_k I_k \]  
\[ Q_k = \omega_0 q_k = (\omega_0 C_k) V_k = C_k V_k \]

Where:
- \( L_k \) is the inductance
- \( L_k \) the inductive reactance
- \( C_k \) is the capacitance
- \( C_k \) the capacitive susceptance
- \( \omega_0 \) the base angular frequency

3.3.5 Initial Conditions.

The initial conditions required for a simulation to be performed
are the state variables at the start of the simulation. An obvious case
is to start from a de-energized state, however, the extensive start-up
interval would be computationally prohibitive. The results from a steady
state analysis such as provided by a load flow program are better,
although it cannot provide exact initial conditions where convertors
and/or detailed synchronous machines are being modelled, by virtue of the
fact that it is a fundamental frequency analysis only.

Therefore a preliminary dynamic simulation, using the appropriate
load flow values as the starting point, is used in order to obtain the
desired steady state operating point. This information is then stored and
used as the initial conditions for subsequent dynamic studies of the
system. This preliminary study will provide the "dynamic initial
conditions", as they are obtained via a dynamic simulation.

The quality of initial conditions is observable in the severity of
the initial transient before the steady state operating point is reached.
The accuracy of the initial conditions is assessed by performing an FFT
on the TCS waveforms once the steady state operating point has been
achieved. The FFT algorithm allows inspection of current and voltages of
both the fundamental and harmonics. The dynamic initial conditions will
always be better than the user entered load-flow data as the former
inherently contains harmonic information while the latter allows fundamental magnitudes and angles only.

3.3.6 Network Equations.

The nodes are partitioned into three possible groups depending on what types of branches are connected to them. The classification of the three node types are:

- **α nodes**: Nodes that have at least one capacitive branch connection.
- **β nodes**: Nodes that have at least one resistive branch connection but no capacitive branches connected.
- **γ nodes**: Nodes that have only inductive branches connected.

The resulting branch-node incidence (or connection) matrices for the \( r, l \) and \( c \) branches are \( K_{rn}^t, K_{ln}^t \) and \( K_{cn}^t \) respectively. The elements in the branch-node incidence matrices are determined by:

\[
K_{bi}^t = \begin{cases} 
1 & \text{if node } i \text{ is the sending end of branch } b \\
-1 & \text{if node } i \text{ is the receiving end of branch } b \\
0 & \text{if branch } b \text{ is not connected to node } i.
\end{cases}
\]

Partitioning these branch-node incidence matrices on the basis of the above node types yields:

\[
K_{\alpha\alpha}^t, K_{\alpha\beta}^t, K_{\alpha\gamma}^t
\]

\[
K_{rn}^t = \begin{bmatrix} K_{\alpha\alpha}^t & K_{\alpha\beta}^t & 0 \\
K_{\beta\alpha}^t & K_{\beta\beta}^t & 0 \\
0 & 0 & K_{\gamma\gamma}^t
\end{bmatrix}
\]

\[
K_{cn}^t = \begin{bmatrix} K_{\alpha\alpha}^t & 0 & 0 \\
0 & 0 & 0
\end{bmatrix}
\]

The efficiency of solution can be improved significantly by restricting the number of possible network configurations to those normally encountered in practice. The restrictions are:

1. Capacitive branches have no series voltage sources.
2. Resistive branches have no series voltage sources.
3. Capacitive branches are constant valued (i.e., \( pC_c = 0 \)).
4. Every capacitive branch or subnetwork has at least one end as the system reference (or ground node).
5. Resistive branch subnetworks have one end connected to the system reference or an alpha node.
6. Inductive branch subnetworks have at least one non-gamma node.
Figure 3.5 shows the fundamental branches and the following branch equations can be derived. Although the equations that follow are correct as they stand, with $L$ and $C$ being inductance and capacitance matrices respectively and the $p$ operator denoting a derivative w.r.t. time, the TCS implementation necessitates that the $L$ and $C$ be interpreted as inductive reactance and capacitive susceptance matrices respectively. Also the $p$ operator must now represent a derivative w.r.t. electrical angle rather than time.

Resistive branches.

$$I_r = R_r^{-1}(K_t^{r\alpha} V_{\alpha} + K_t^{r\beta} V_{\beta})$$  \hspace{1cm} (3.18)

Inductive branches.

$$E_{L} - p(L I) = R_{L} I + K_{L}^{t\alpha} V_{\alpha} + K_{L}^{t\beta} V_{\beta} + K_{L}^{t\gamma} V_{\gamma}$$  \hspace{1cm} (3.19)

$$p\psi_{L} = E_{L} - pL I - R_{L} I + K_{L}^{t\alpha} V_{\alpha} + K_{L}^{t\beta} V_{\beta} + K_{L}^{t\gamma} V_{\gamma}$$  \hspace{1cm} (3.20)

where $\psi_{L} = L I$

Capacitive branches.

$$C_{C} p(K_{C}^{t\alpha} V_{\alpha}) = I_{C}$$  \hspace{1cm} (3.21)
In deriving the nodal analysis technique Kirchhoff's current law is applied, the resulting nodal equation being:

\[ K_{nc} I_c + K_{nr} I_r + K_{ns} I_s = 0 \]  

where \( I_c, I_r, I_s \) are the branch current vectors. Applying the node type definitions gives the following three equations:

\[ K_{\alpha} I_{\alpha} + K_{\alpha} I_{\alpha} = 0 \] 
\[ K_{\beta} I_{\beta} + K_{\beta} I_{\beta} = 0 \] 
\[ K_{\gamma} I_{\gamma} + K_{\gamma} I_{\gamma} = 0 \]

Premultiplying equation (3.21) by \( K_{ac}^t \) and substituting into equation (3.25) yields:

\[ pQ = -K_{\alpha} I_{\alpha} + K_{\alpha} I_{\alpha} + K_{\alpha} I_{\alpha} \]

where \( Q = C \cdot V_{\alpha} \) and \( C^{-1} = (K_{ac}^t C C \cdot K_{ac})^{-1} \)

The dependent variables \( V_{\beta}, V_{\gamma} \) and \( I_r \) can be entirely eliminated from the solution so only \( I_{\alpha}, V_{\alpha} \) and the input variables are explicit in the equations to be integrated. This is however undesirable due to the resulting loss in computational efficiency even though it reduces the overall number of equations. The reasons for the increased computational burden are:

- Loss of matrix sparsity,
- Incidence matrices no longer have values of -1,0 or 1. This therefore requires actual multiplications rather than simple additions or subtractions when calculating a matrix product.
- Some quantities are not directly available, making it time consuming to recalculate if it is needed at each time step.

Therefore \( V_{\beta}, V_{\gamma} \) and \( I_r \) were retained and extra equations derived to evaluate these dependent variables. To evaluate \( V_{\beta} \) equation (3.18) is premultiplied by \( K_{br}^t \) and rearranged to give:

\[ V_{\beta} = -R_{\beta} (K_{\beta} I_{\beta} + K_{\beta} I_{\beta} + K_{br} R_{br}^{-1} t V) \]  

where \( R_{\beta} = (K_{br} R_{br}^{-1} t)^{-1} \)
Premultiplying equation (3.19) by $K_{Y\ell}$ and applying equation (3.23) gives the following expression for $V_Y$:

$$V_Y = -L_Y K_{Y\ell}^{-1} p I_S - L_Y K_{Y\ell}^{-1} \left( E_\ell - p L_\ell K_\ell^{-1} + R_\ell I_\ell + K_\ell^{-1} V + K_\ell V_\ell \right)$$  \hspace{1cm} (3.28)

where $L_Y = (K_{Y\ell}^{-1} K_{Y\ell}^{-1} )$

$I_r$ is evaluated by use of equation (3.18). The sequence of solution once the trapezoidal integration has converged, for a time step, is as follows; the state-related variables are calculated followed by the dependent variables and lastly the state variable derivatives are obtained from the state equations.

State-Related Variables.

$$I_\ell = L_\ell^{-1} \psi_\ell$$  \hspace{1cm} (3.29)

$$V_\alpha = C^{-1}_\alpha q_\alpha$$  \hspace{1cm} (3.30)

Dependent Variables.

$$V_\beta = -R_\beta \left( K_{\beta\ell} I_\ell + K_{\beta\ell} I_\ell + K_{\beta\ell}^{-1} I_\ell \right)$$  \hspace{1cm} (3.31)

$$I_r = R_r^{-1} (K_r^{-1} V_r + K_r V_r)$$  \hspace{1cm} (3.32)

$$V_Y = -L_Y K_{Y\ell}^{-1} p I_S - L_Y K_{Y\ell}^{-1} \left( E_\ell - p L_\ell K_\ell^{-1} + R_\ell I_\ell + K_\ell^{-1} V + K_\ell V_\ell \right)$$  \hspace{1cm} (3.33)

State Equations.

$$p I_\ell = E_\ell - p L_\ell I_\ell + R_\ell I_\ell + K_\ell^{-1} V + K_\ell V_\ell + K_\ell^{-1} V_\ell$$  \hspace{1cm} (3.34)

$$p Q_\alpha = - K_{\alpha\ell} I_\ell - K_{\alpha\ell} I_\ell - K_{\alpha\ell} I_\ell$$  \hspace{1cm} (3.35)

where

$$C^{-1}_\alpha = (K_{\alpha\ell} C_{\alpha\ell}^{-1} )^{-1}$$

$$L_Y = (K_{Y\ell}^{-1} K_{Y\ell}^{-1} )^{-1}$$

$$R_\beta = (K_{\beta\ell} R_{\beta\ell}^{-1} K_{\beta\ell}^{-1} )^{-1}$$
3.4 Extensions.

3.4.1 C-type Filter.

The use of conventional damped filters for low order harmonics involves large fundamental power loss in the damping resistor, therefore the C-type filter has been designed to reduce the power loss (Arrillaga 1983). The shunt filter branch routines in TCS have been changed to accommodate the C-type filters. The dimension of the capacitive submatrix representing a C-type filter differs from that of the other components and some restrictions must be placed on the order in which the various components are entered.

3.4.2 Harmonic Current Source.

In any work dealing with frequency-dependence, the ability to inject a controlled amount of harmonic current is important in order to derive the impedance at the appropriate harmonic frequency. A current source model has been added to the network equations given in section 3.3.6. For simplicity each current source is designed to inject one frequency component; hence several current sources are required to inject a multitude of harmonics. Although this model has been referred to as a "Harmonic Current Source" the current source can be of any frequency not necessarily a harmonic frequency.

3.4.3 RLC Networks connected between Phases.

The mutual coupling between phases must be modelled in harmonic assessment studies as it greatly influences the harmonic levels, particularly the higher order harmonics. It also needs to be modelled for accurate dynamic studies. However, to complicate matters, the mutual coupling is strongly frequency-dependent and exhibits both resistive and reactive coupling components.

An early attempt to model frequency dependent mutual coupling between the phases was the connection of frequency dependent networks between the phases as illustrated in figure 3.6. However the explicit location of RLC branches between phases would violate a TCS constraint as the capacitor would not be connected to an alpha node or earth. The
removal of this constraint from the TCS formulation would require major modifications to the program. A possible solution is the addition of a very small capacitance between earth and each phase in order to convert the coupling nodes into alpha nodes; however, this would result in small integration step lengths and thus deterioration of computational efficiency.

The approach implemented, based on diakoptical techniques, involves the formation of a set of state equations for the mutual branches which are solved at each time step to obtain the current in each branch. This current is then treated as a current source connected to the appropriate nodes when the main state equations are solved. So as far as the existing TCS algorithm is concerned the mutual branches appear as dependent current sources connected between the phases. Therefore these mutual branches are referred to as "torn" branches as they form an extra network, separate in identity from the standard TCS circuit, with a voltage-current source providing the interface between the two. An example of the state equation for a mutual RLC branch is:
\[
\frac{d}{d\theta} \begin{bmatrix}
  \text{V}_{\text{p.u.}} \\
  \text{Q}_{\text{p.u.}}
\end{bmatrix}
= \begin{bmatrix}
  -\frac{R}{\omega L} & \frac{-1}{\sqrt{3}} \frac{Z_b}{\omega L} \\
  \frac{1}{\sqrt{3}} \frac{Z_b}{\omega L} & 0
\end{bmatrix}
\begin{bmatrix}
  \text{V}_{\text{p.u.}} \\
  \text{Q}_{\text{p.u.}}
\end{bmatrix}
+ \begin{bmatrix}
  1 \\
  0
\end{bmatrix} \text{V}_{\text{p.u.}}
\] (3.36)

where \( \text{V}_{\text{p.u.}} \) is the per unit voltage across the RLC branch.

Figures 3.7 and 3.8 provide some insight into possible iterative algorithms. The internal iterative loop estimates the extra network in figure 3.7 and the standard TCS circuit in figure 3.8. An alternative approach, and the method adopted, consists of embedding the extra state variables as part of the standard variable iterative loop as shown in figure 3.9. The estimates for the new state variables and the state variable derivatives are evaluated for the standard and extra variables at the same stage and the convergence criteria reflects the convergence of both sets of variables. Compared to the use of separate iterative loops for the standard and extra variables this solution permits considerable computational saving. However the separate iterative loop method requires far less modification to the original TCS algorithm.

However the selection of the appropriate parameters for a frequency matched equivalent circuit including the mutual coupling became virtually impossible. With reference to figure 3.6, as the six frequency dependent circuits contribute to each of the self and mutual terms they must be considered simultaneously. This precludes the use of direct approaches such as Hingorani and Burberry's as they match only one circuit to a required frequency response in isolation. Only a multi-variable optimization technique is applicable and since a scalar objective function must represent the match of the self and mutual terms simultaneously the solution will contain significant error due to the compromises. The computational requirements are also prohibitive. Thus this approach had to be abandoned, in spite of its obvious advantages in the modelling of circuit topologies without the restriction of the TCS formulation. Much further work had to be carried out in this area and is reported in chapter 5.
Figure 3.7 Separate Iterative Loop algorithm; Extra Network inner loop
Figure 3.8 Separate Iterative Loop algorithm; Standard TCS circuit inner loop
Estimate new state variables for both standard and extra networks

Obtain state variable derivatives for both standard and extra networks

Has convergence been reached all variables (both standard & extra)?

No

Yes

Figure 3.9 Embedded Iterative Loop approach
CHAPTER 4. ACREP: A FLEXIBLE PROGRAM FOR PROCESSING AND PLOTTING POWER SYSTEM DATA.

4.1 Capability of the ACREP program.

Although the program derives its name from an acronym of "AC system REPresentation", this is somewhat misleading as the program is capable of performing many more functions other than producing AC system equivalents. The main functions performed by ACREP are:

(i) Synthesis of frequency-matched equivalents for use in dynamic simulation.
(ii) Calculation and display of the frequency response (Impedance, phase angle, admittance, etc) of RLC networks.
(iii) Spectral analysis. This option allows forward and reverse Discrete Fourier Transforms to be performed by use of the FFT algorithm. Two FFT algorithms have been implemented; the first is the standard, requiring the number of data points to be some power of two (i.e. 64, 128, 256, 512, 1024, 2048, etc); the second is a mixed radix FFT allowing less restriction. Windowing of the data before performing a transformation has been allowed for. There is also the option to reconstruct the time waveform from the spectral data by summation of the Fourier Series.
(iv) A Complex matrix manipulation facility. This allows the series of complex matrices that represent the frequency response of a system to be read in and arithmetic calculations to be performed on them.
(v) Vector arithmetic facility. This allows almost any arithmetic and/or trigonometric operation to be performed on a set of real valued numbers stored as vectors on a position by position basis, where each position represents the response at a particular frequency. For example this is useful in derivation of the difference between two waveforms and the inversion of an admittance vector to obtain the impedance response.
(vi) General Graphics for displaying data in data files of any format.

(vii) A Harmonics Graphics Facility which interfaces with the existing power system programs (i.e. TL, INTER, HARMAC, IHA) to provide graphical outputs of the content in their data base.

These features of ACREP have already enabled it to be used to perform many diverse jobs. The flexible graphics facility is probably the most widely used feature and has been used for displaying a wide variety of power system data as well as data from electronic experiments such as noise measurements on a telecommunication line. The ability to display several waveforms simultaneously has also proved worthwhile.

The spectral analysis facility has been used for the analysis of earthquake records as well as the time waveforms derived from TCS. The results may be displayed either in tabular form, as a histogram or as a two-dimensional plot.

The Complex matrix facility is used to apply the appropriate transformation to the matrices describing the systems frequency response.

The vector computation facility will be more apparent after reading chapter 5, where it is used to calculate the required impedance of a correction filter in the direct method of frequency-dependent equivalent synthesis. It is also used to provide a direct comparison of Iterative Harmonic Analysis (IHA) and Transient Convertor Simulation (TCS) in harmonic assessment, by allowing for differences in their time reference and p.u. systems.

The calculation of the frequency response of RLC circuits is a useful feature and has been used (Ireland 1986) in the design of a C-type filter for a HVDC physical simulator. When constructed, its frequency response compared very well with the computed estimates.

Examples of graphic displays are two and three dimensional plots of harmonic voltages and currents, cross-sections of the three dimensional plots and impedance loci. This is an extension of the work done by Gellen (Arrillaga et al 1985).
4.2 Objectives, Structure and Operation of ACREP.

At this stage a brief outline of the structure and operation of ACREP is in order. For a more detailed description refer to the ACREP MANUAL. The structure of ACREP was designed to provide fast and accurate operation and minimize the utilization of disc space while being user-friendly. Although these objectives are not entirely compatible there is a clear optimum between these constraints. If a user gets frustrated using the program, it is a clear indication that the happy medium has not been reached. It is then likely that it suffers from either not being sufficiently user-friendly, or being too user-friendly and therefore slow in its operation.

The usual practice in the past has been to have separate programs to perform each distinct major step in the analysis. This therefore required read and write operations from disc at each step. The speed of operation can be greatly increased by minimizing the number of read and write operations. This is achievable by lumping several of the required steps into one program, as has been done in ACREP. The results of one step are stored in the program variables (virtual memory), ready to be used by the next step, rather than writing out a data file and then reading it in. Virtual memory is the area set aside by the computer for storing the program variables while the program is executing. When the program stops execution the contents are lost and this memory is allocated to store the variables of any another image that starts execution.

This use of program vectors rather than data files also leads to increased accuracy and reduced build-up of error. The writing out of a formatted file windows the data in the program variables by the format of the write statement, hence causing a loss of accuracy. Where there are several programs in a chain, each introduces a loss of significance by writing a data file and the total error builds up. By keeping data in program vectors maximum accuracy for the given data type is obtained.

However, the danger then exists that by retaining the data in program vectors allows a large amount of computational work to be lost if the program stops execution prematurely, due to encountering an error. Therefore a very comprehensive error-handling capability has been
incorporated into ACREP to make it virtually impossible for it to stop prematurely, before the results have been stored on disc.

If the error-handler is forced to operate then the program vectors currently being used are left in an indeterminate state. To circumvent the problem this causes, there are two sets of program vectors, a data base and a working set. If the working set is left in an indeterminate state then it is quickly and easily reloaded from the data base.

4.3 Modular Design.

To produce reliable code which is easily understood and modifiable the program has been written in a modular manner. Many subroutine calls are required for each analysis, with each subroutine performing a specific function. The price of this modular structure is a slower speed of program execution, however the benefits of tidy and flexible code out-weight this draw-back. There are 353 subroutines which can be classified into the following classes, depending on the primary function they perform:

(i) DCL (Digital Command Language) mimicking.
(ii) Computational.
(iii) Disc I/O.
(iv) User I/O.
(v) Error-handling
(vi) Debugging and Patching.

The DCL mimicking subroutines allow functions normally given in the computers own command language to be performed from inside the program. For example editing, printing and deleting files can be done from within the program, without terminating execution. Other examples are the changing of the process's priority, the creation of another process by spawning a process and the listing of files in a directory.

Computational subroutines are those that primarily perform the required calculations.
The disc I/O subroutines are devoted to the reading and writing of data files. They also perform the additional function of transferring data between the working set and data base regions of memory.

The User I/O subroutines display information on the terminal or plotter and allow the user to specify commands and parameters. There are two ways in which the program communicates with the user, the distinction being made on the basis of whether "text" or "graphics" modes of the terminal are used for the communication.

An over-view of the ACREP program is given in Figure 4.1.
4.4 Special Features.

A versatile program must have the ability to change many attributes. However to prompt for each characteristic, even with inbuilt default values present, leads to a program that is slow and laborious to use. This also increases the probability of entering erroneous numbers. Therefore the program suppresses the prompts for the characteristics that are varied infrequently, unless specifically asked for. A vector controls what attributes the program user is prompted for and hence can vary. The program user alters the appropriate vector position to enable and disable the prompting for the various characteristics. Forty-five different plot characteristics can be varied at present.

Often the coordinates of important points on the plot need to be determined. Measurement by means of a rule on the plot or screen is less than satisfactory. This is in part due to the time required to translate from a distance measurement to the units of the axes, and also to the very poor accuracy obtained. Therefore the ability to accurately determine the coordinates of a point by means of a cross-hair cursor has been incorporated into the program. To display the coordinates of particular points the cross-hair cursor is first activated, then positioned on the required point and the space-bar key depressed. The use of the cross-hair cursor in conjunction with the zoom ability allows very accurate values to be obtained.

The zoom feature present on many graphic terminals, while allowing the plot to be enlarged provides no improvement in resolution, as the enlarged plot is based on the data points originally evaluated. In this program the zoom recalculates the plot by placing all the data points in the zoom region, and therefore achieves a far better resolution.

The program has a complete set of default attributes inbuilt, such that the program user only needs to specify those required to be different from the default values. These defaults are partitioned into three sets; those that are data dependent, those that are terminal or plot device dependent, and those that are independent of both data and terminal (or plotting device). When using ACREP it is likely that many attributes need to be different from the default settings and yet only a few need to be changed at each stage of analysis. The ability to
customize the default settings without editing, recompiling and linking is an important feature as it would take approximately 33 minutes of CPU to reform the program. The default settings are customized during program execution by the use of NAMELIST-directed I/O statements (DEC 1984). The Namelist write produces a namelist file which contains the current variable values. Next time the program is used the inbuilt defaults can be superseded by the values used in the last session by reading in the namelist file.

In a program as large as ACREP, it is not reasonable to expect every user to know what is going on inside the program, and neither is it reasonable to expect the program to be entirely free of bugs. For this reason a PATCH facility has been written to aid debugging. The PATCH facility performs two functions, the first is to indicate what has gone wrong, the second is to allow the user to by-pass the problem and complete the required work. This allows the present user to obtain the results without requiring a detailed knowledge of the workings of ACREP. The error should be noted when it appears and given to a person familiar with ACREP, for correction at a later date. The Patch facility works by allowing the program variables to be inspected and altered at almost any stage of the program execution. Besides this, there are very extensive error detection and notification facilities in almost all facets of the program's operation.

4.5 Illustrative Examples

4.5.1 Spectral Analysis.

The results can be displayed graphically as a histogram with either the magnitude, phase angle, real part or imaginary part being the quantity displayed. The program has the ability to plot the actual values or the proportion of the maximum value as well as thresholding the data.
By summing the Fourier series the time reconstruction of a spectrum is obtained. This is important for two reasons; first, it allows the time waveform to be inspected when the harmonic data comes from a table. This is the case with the harmonic assessment programs (HARMAC<->IHA). Secondly when assessing the harmonics in a time waveform, the presence of a transient component in the waveform can be tested for by comparing the time waveform with a time reconstruction of the harmonics. Figure 4.2 displays a histogram of the current waveform harmonics while figure 4.3 illustrates its time reconstruction. The information can also be conveyed as a continuous plot rather than a
histogram as illustrated in figures 4.4 and 4.5, the latter being the continuous version of the former. In addition to the graphical output the information can also be written to a data file or terminal screen, in the form of a table.

4.5.2 Harmonics Graphics Package.

There are normally four files produced by the harmonic penetration program (HARMAC, section 6.1). Three of them are used for plotting harmonic voltage, current and impedance respectively. The fourth is a harmonic admittance file to be used by the IHA program.
Three dimensional diagrams of voltage or current magnitudes, phase angle, real component and imaginary part can be displayed for either phase or sequence components. The three possible types of three-dimensional plots are: isometric, two-dimensional perspective and three-dimensional perspective. By specifying one of the perspective views allows the observation point to be varied, which enables the details previously hidden to be seen.

An example of the isometric view, which is the default display, is illustrated in figure 4.6 while figure 4.7 shows the same graph from a different view point using a two-dimensional perspective.
Figure 4.8 Busbar voltage harmonics throughout the Lower South Island of New Zealand when one per unit of each harmonic is injected.

Alternatively a cross-section can be displayed in isolation. With reference to figure 4.8 the third harmonic cross-section is shown in figure 4.9 or in a more conventional form in figure 4.10. Many cross-sections can be overlaid and hence displayed simultaneously as shown in figure 4.11. Cross-sections of the three-dimensional plot in the xz plane can also be displayed as illustrated in figure 4.12.

All the phase or sequence components can be displayed simultaneously or individually. Another option is a point plot which is illustrated in figure 4.13 (a)&(b) for the three-dimensional harmonic current plot of figure 4.14.

The harmonics graphics facility allows the plotting of the impedance loci. In order to produce a smooth loci plot from the data points in the file the impedance is split into its real and imaginary components and cubic spline interpolation is used between the data points. Figure 4.15 shows the impedance loci obtained by this method. Finally, a magnitude against frequency plot can be plotted as illustrated in figure 4.16.
The magnitude of the following quantities can be displayed against frequency, impedance, phase angle, resistance, reactance, admittance, conductance or susceptance. Cubic spline interpolation on each component is also used for these plots and both phase or sequence components can be inspected.
Figure 4.11 Multiple cross-sections

Figure 4.12 Cross-section of the harmonics at a busbar
Figure 4.13 Point Plot of Current Harmonics
HARMONIC CURRENT MAGNITUDES

Figure 4.14 Branch Current Harmonics

HARMONIC VARIATION OF IMPEDANCE

Figure 4.15 Impedance Loci
4.5.3 Frequency-Response of an RLC Network.

The algorithm developed for calculating the frequency response of an RLC network is based on the solution of a ladder network. The network must therefore be representable as a ladder network, as shown in figure 4.17. General filter and series branches are displayed in figure 4.18 (a) & (b). Other simpler filter branches are modelled by eliminating the...
appropriate components from the general filter branch. This is achieved by specifying a zero component value for these components.

The series branches are handled in the same manner. The more frequently used filter branches (and subnetworks) have their own separate model and hence data requirements, thereby allowing a simpler and more economic data input. Figure 4.19 displays these more frequently used models.

Both impedance loci or "quantity" versus frequency plot can be displayed, where the quantity to be plotted is either the impedance, resistance, reactance, phase angle, admittance, conductance or susceptance. In contrast to when the actual system's response is displayed, no interpolation is used as the data points to be plotted can be generated at any required frequency spacing.

The program can also plot the response of an RLC network together with the required response, stored in a file, thereby allowing easy comparison. This shows the accuracy of a frequency-matched a.c. system equivalent.

Another feature is the ability to obtain the over-all frequency response of the parallel combination of the a.c. system response and an RLC network. This is useful in observing the effect of adding harmonic filters to the a.c. system's frequency response.
As an illustration the impedance loci of the circuit displayed in figure 4.20 (circuit parameters given in appendix A3), is shown in figure 4.21 and the impedance, resistance and reactance response displayed in figures 4.22 (a),(b)&(c) respectively.
Figure 4.22 Frequency Response of RLC circuit.
4.5.4 General Plot Facility.

A maximum of 25 waveforms can be displayed simultaneously, of which some can be the sums or differences between waveforms, with freedom to alter the dimensions of the plot and the drawing and labelling of axes. The default is a 30x20 cm plot on an A3 page, however there are five other standard options. Alternatively the program user can enter his own non-standard plot dimensions. To distinguish between the various waveforms different pen colours are used in conjunction with differing line type. There are seven standard line types while the plotter limits the number of available colours (typically 4 or 6). The user can also choose between the two core graphics packages available on the computer system, to obtain the plot. As an example of the capabilities of the general plot facility, figure 4.23 shows six waveforms plotted simultaneously.

Figure 4.23 General Plot facility display of six waveforms simultaneously
5.1 Historical Review.

It is well established that time-domain dynamic simulation is required for accurate representation of non-linearities such as HVDC convertor operation, corona losses, transformer hysteresis and saturation. This makes frequency-dependence modelling difficult because in a step-by-step numerical integration algorithm all frequencies are effectively being simulated at each time step.

The transmission line is the predominant frequency-dependent power system component, and many have concentrated on modelling its characteristics. Hingorani (1970) showed the importance of the frequency-dependence of line parameters on the transient by considering a pole-to-ground fault on a bipolar HVDC scheme.

Budner (1970) developed one of the earliest frequency-dependent line models by convolving weighting functions with the past history of node voltages. The weighting functions were derived by applying an FFT algorithm to obtain the inverse Fourier transform of the frequency dependent admittance matrices. Snelson (1972), Meyer and Dommel (1974) and Carroll and Nozari (1975) made further developments in this area. To increase computational speed the use of recursive convolution was proposed by Semlyen (1975), Dabuleanu and Semlyen (1975) and Ametani (1976).

Another approach to modelling frequency-dependence has emerged based on approximating the transmission line transfer function by rational functions. Once a rational function approximation has been found it may be made amenable to direct incorporation into state equations or realized by a lumped R,L and C network. Semlyen (1982) proposed direct incorporation as state equations while lumped parameter realization was suggested by Martí (1982,1983), Yen et al (1982), Semlyen and Deri (1985), and Naidu and de Lima (1985).
The problem of modelling the frequency-dependence of transformers in transient studies has been tackled by Avila-Rosales and Alvarado (1982). They reviewed seven existing transformer models, none of which modelled the frequency dependence of the transformer.

Most previous approaches concentrate on modelling the frequency dependence of individual transmission lines and transformers. The approaches are based on differential equations, such as the telegrapher's equations for a transmission line, and are therefore not applicable to an arbitrary frequency response. The complexity of power systems makes individual line and transformer representation unreasonable and frequency-dependent a.c. system models necessary.

Hauer (1981) reported a more general approach to modelling transmission lines by curve fitting in the frequency domain via non-linear optimization with a least squares objective function. However, the details of the fitting process were not elaborated on.

Hingorani and Burbery (1970) were the first to propose the use of an RLC network to represent the frequency-dependence of an a.c. system. Their paper described the derivation of the network parameters so that the modelled frequency response matched the required response and discussed the numerical problems that were encountered. Figure 5.1 shows the equivalent circuit proposed by Hingorani and Burbery (1970). The

Figure 5.1  A.C. System Equivalent proposed by Hingorani and Burbery
stages of the circuit synthesis were; first the required L and C values are determined based on a lossless model (i.e., neglecting the resistance in the model of Figure 5.1); resistances are then added which correspond with the real part of the impedance at the appropriate resonant frequencies; finally L and C values are scaled so as to give the correct impedance level at the peaks while maintaining the correct resonant frequencies.

Figure 5.2 Equivalent proposed by Bowles

Bowles (1970) proposed the simpler equivalent displayed in Figure 5.2 using a T-network which has the same impedance as the short circuit impedance at fundamental frequency. The values of R and L were selected to give the required impedance angle. Bowles claimed that Hingorani and Burberry's equivalent was too complicated for HVDC simulation as only the impedance at low frequencies (up to the 5th harmonic) are of importance for most purposes because the harmonic filters swamp the a.c. impedance at higher frequencies. Other authors have also echoed the sentiment that Hingorani and Burberry's equivalent is unduly complicated (Reeve and Subba Rao 1974). Kruempel and Reitan (1970) and Bowles and Clarke (1970) criticized Hingorani and Burberry's equivalent due to the need for the impedance-frequency locus, which was difficult to obtain at the time. However, this is no longer a problem and section 5.2 will discuss its derivation. Reeve (1970), Bowles and Clarke (1970) commented that the impedance-frequency characteristic changes significantly due to switchings in the system on a routine basis. However, any representation is subject to this drawback. Representation for typical conditions is an
improvement on hypothetically and arbitrarily selected conditions. Morched and Brandwajn (1983) proposed an equivalent very similar to Hingorani and Burbery's equivalent and almost identical LC parameter determination based on a lossless model. The resistances are determined iteratively with the real part of the impedance in the appropriate resonant frequency used as the initial estimate, although no information about the iterative loop was supplied by the authors. Do and Gavrilovic (1984) extended this work by using least-squares fitting in an iterative algorithm to obtain an accurate equivalent when system losses were high, and applied it to zero and positive sequence equivalents. Again no details of the fitting process were given. Watson et al (1985) presented Hingorani and Burbery's approach with a few refinements and applied it to three phase dynamic studies with implicit representation of mutual coupling using TCS. More recently Watson and Arrillaga (1987) have applied optimization techniques for synthesizing a.c. equivalents with explicit representation of the mutual coupling.

5.2 Obtaining the frequency response of an a.c. system.

Before any frequency-dependent model can be produced the system's frequency-response must be determined. This may be accomplished by harmonic impedance measurements at a number of frequencies on the system itself or by calculation on a digital computer over a range of frequencies (Breuer et al 1982, Densem 1983). The disadvantages of direct measurement are:

(i) the system must be left in an operating condition and hence the numerous outage conditions which are liable to give rise to resonances at harmonic frequencies cannot be studied,
(ii) the need to make measurements while the system is energized demands the use of a high power source of harmonics,
(iii) the measurements are restricted to a few harmonic frequencies and do not give a satisfactory picture of the resonances at intermediate frequencies,
(iv) the accuracy of measurements made on the actual system are prone to significant experimental error.
The frequency response of a power system at the point of harmonic injection is normally presented in the form of a single impedance locus plot. However, to accurately represent network asymmetries and mutual coupling requires a $3 \times 3$ impedance matrix for each harmonic. The Harmonic Penetration Program HARMAC developed by Densem (1983) has the ability to generate the required series of $3 \times 3$ impedance matrices that describe the frequency-dependence of the a.c. system. To display the enormous amount of information that results, a generalization of the impedance loci is used.

The interaction of the various programs required to generate and use the impedance loci is illustrated in figure 5.3. Referring to figure 5.3, the first program in the chain is the TL (Transmission Line) program. This program reads in the line geometry and conductor data and calculates the transmission line parameters for each frequency over the required range using an Equivalent $\pi$ model. This incorporates Carson's correction (1926) for earth return and the correction for skin effect in the conductor. The data base is completed by the INTER (INTERactive) program. This is achieved by reading the line data produced by TL and adding it to the other component data. HARMAC (abbreviation of AC HARMonics) is then used to calculate the three phase system impedances at the required frequencies. This calculation only uses the first stage of HARMAC, the second stage is used specifically for harmonic assessment and is discussed in Chapter 6. The primary function of the ACREP (AC system REPresentation) program is to input the systems frequency response and derive an equivalent circuit with a matched frequency response. This equivalent is then used in Transient Convertor simulation (TCS). The JAUGPS (Generate Plot Store) program then plots the TCS output.

As an illustration the frequency response of the self and mutual terms of the New Zealand lower South Island test system in figure 5.4, observed from the Tiwai bus, are shown in figure 5.5. Because power systems are generally bilateral, the $(n,m)$ loci are seen to be identical to loci $(m,n)$. Moreover for the particular line geometry of the test system, two phases, "A" and "C", are seen to be similar while phase "B" differs substantially.
Conductor data and line geometry

1. Transmission Line Parameter Program (TL)
2. Interactive Data Program (INTER)
3. Harmonic Penetration Program (HARMAC)
4. A.C. System Representation Program (ACREP)
5. Power-Flow Program (PF)
6. Transient Convertor Simulator (TCS)
7. Graphics and Data manipulation Program (JAUGPS)

1. Parameters of each transmission line over the required frequency range
2. Complete a.c. system data at fundamental and harmonic frequencies of each individual component
3. 3x3 impedance loci matrix
4. Frequency-matched a.c. system equivalent
5. a.c. system data at fundamental frequency
6. Remainder of hvdc system data
7. Initial currents and voltages throughout the hvdc system
8. Voltage and current waveform data at unequally spaced intervals

Figure 5.3 Data Flow and sequence for a dynamic study using a frequency-matched a.c. system equivalent.
Some dynamic models lack mutual coupling capability and in such cases it is possible to reduce the full matrix information to three diagonal terms which contain the mutual impedances implicitly. This is preferable to using the self terms as at some harmonic frequencies the mutual terms have a larger effect than the self term. An assumption regarding the phase sequence must be made in order to perform the reduction. By way of example, assuming a positive sequence current then the reduction is achieved by post multiplying each of the 3x3 matrices by the matrix

\[
\begin{pmatrix}
1 & a & a \\
a & 1 & a \\
a & a & 1
\end{pmatrix}
\]

where \( a = -1/2 + j \sqrt{3}/2 \)
Figure 5.5 Impedance Loci Matrix of the test system

Figure 5.6 Diagonalized Matrix Impedance Loci
and extracting the diagonal terms. This method is valid for systems with little phase current asymmetry, as is normally the case with converter plant. Applying this method to the above test system results in the three loci displayed in figure 5.6.

When comparing the frequency response of an equivalent circuit with the measured or computed response, impedance/frequency plots are more easily comprehended and used rather than the impedance loci. These are illustrated in figure 5.7 for the system of figure 5.4.

5.3 Synthesis of Frequency-Dependent Equivalent.

The development of a multi-phase frequency-matched a.c. system model requires an algorithm that can produce an equivalent circuit matched to an arbitrary frequency response. Two different techniques have been developed to perform this; the first is a direct method based on Hingorani's method, the second on non-linear optimization. Thus the complex impedance-frequency information contained in the loci matrices are converted into equivalent circuits. Each element of the 3x3 matrix must be synthesized into an equivalent circuit, which are then combined to form the overall frequency dependent equivalent model for the a.c. system, the structure of which is shown in figure 5.8. The equivalents matching the self terms are represented explicitly as shunt branches while the mutual circuits are processed separately by using diakoptical techniques. The phase currents are first impressed upon the mutual circuits and the resulting voltages are then incorporated in the overall circuit as additional voltage sources. The computer implementation of this frequency-dependent equivalent model is reviewed in section 5.4.

The equivalent circuit topology is selected to suit the requirements of the time-domain algorithms, such as the use of node-type partitioning and tensor matrix analysis (Arrillaga et al 1983a), and is discussed in chapter 3. Although these considerations restrict the possible topologies, they dramatically increase the solution efficiency in the case of a.c./d.c. dynamic simulation, which require regular topological changes, by avoiding involving the whole network during the localized
Figure 5.7 Impedance versus Frequency Matrix derived from figure 5.5
convertor valve switchings. Since the topology used by Hingorani and Burbery (1970) is amenable to efficient Transient Convertor Simulation this form has been retained as the starting point in the derivation of the equivalent circuit.

The basic steps of the equivalent circuit synthesis for both alternative methods are illustrated in figure 5.9 and discussed in the next two sections. The acceptability of an equivalent is a subjective decision based on the following factors:
(i) The type of study. For steady state harmonic penetration studies minimal error at the harmonic frequencies is required while the errors at intermediate frequencies are unimportant. This is clearly unacceptable for transient convertor simulations as some intermediate frequencies could be excited.

(ii) The feasibility of altering automatically the equivalent circuits to reduce the error to a prescribed accuracy.

(iii) The availability of an economically viable alternative to model the actual system explicitly. This relates to the amount of CPU time required to provide the simulation and accuracy required.
The CPU time used in a.c./d.c. simulation is related to the number of state variables and the time steps of the numerical solution, the latter being related to the time constants of the equivalent circuit. In each case the acceptability of a particular equivalent circuit and the required CPU time are assessed by means of a short-duration dynamic simulation study.

5.3.1 Direct Method.

The basic matching philosophy consists of selecting the values of R, L and C that give the peaks and troughs at the correct frequencies and the Q (quality factor) of each branch such as to cause the equivalent circuit's response between these frequencies to approximate the actual system's response. The mathematical formulation used in initial assessment of the RLC values to give the peaks and troughs at the correct frequencies is as follows.

The first step is to obtain values for the inductors and capacitors of the equivalent circuit based on a lossless model. The resistance of the branches are ignored as they do not affect the resonant frequencies. The admittance of an n branch network is

$$Y(s) = \sum_{k=1}^{n} \left\{ \frac{s}{L_k(s^2 + \omega_k^2)} \right\} = \frac{s}{\sum_{k=1}^{n} (1/L_k) \prod_{l=1}^{k} (s^2 + \omega_l^2)}$$

(5.1)

where \(\omega_l^2 = 1/(L_lC_l)\) and \(s = j\omega\)

For an LC network the impedance is zero at a minima and infinite at a maxima. Impedance minima occur when:

$$\prod_{i=1}^{n} (s^2 + \omega_i^2) = 0$$

(5.2)

Impedance maxima will occur when the numerator of equation (5.1) is zero. i.e.

$$s \sum_{k=1}^{n} ((1/L_k) \prod_{l=1}^{k} (s^2 + \omega_l^2)) = 0$$

(5.3)
Let \( F_1, F_2, \ldots, F_n \) be the frequencies at which the admittance is zero. Then equation (5.3) can be written as:

\[
\sum_{k=1}^{n-1} (s^2 + M_k^2) = 0
\]

(5.4)

where \( M_k = 2\pi F_k \)

By equating the coefficients of \( s \) in equation (5.3) with equation (5.4) the following set of equations result:

Coefficient for \( S^{2n-1} \)

\[
\sum_{k=1}^{n} \frac{1}{L_k} = 1
\]

(5.5)

Coefficient for \( S^{2n-3} \)

\[
\sum_{k=1}^{n} \left( \frac{1}{L_k} \sum_{i=1}^{n} \omega_i^2 \right) = \sum_{k=1}^{n-1} \sum_{i=1}^{n} M_k^2
\]

(5.6)

Coefficient for \( S^{2n-5} \)

\[
\sum_{k=1}^{n} \left( \frac{1}{L_k} \sum_{i=1}^{n} \sum_{j=1}^{n} \omega_i^2 \omega_j^2 \right) = \sum_{k=1}^{n-1} \sum_{i=1}^{n} \sum_{j=1}^{n} M_k^2 M_j^2
\]

(5.7)

Coefficient for \( S^{1} \)

\[
\sum_{k=1}^{n} \frac{1}{L_k} \sum_{j=1}^{n} \omega_j^2 = \prod_{k=1}^{n-1} M_k^2
\]

(5.8)

Thus \( n \) equations are formed with \( 1/L_1, 1/L_2, \ldots, 1/L_n \) as the unknowns to be solved for. i.e.

\[
[A] \ x = b
\]

(5.9)

where

\[
x^t = \left[ \frac{1}{L_1}, \frac{1}{L_2}, \ldots, \frac{1}{L_n} \right]
\]

\[
b^t = \left[ 1, \sum_{k=1}^{n-1} M_k, \sum_{i=1}^{n-1} \sum_{k=1}^{n} M_k M_i, \ldots, \prod_{k=1}^{n} M_k \right]
\]
This is solved using Gaussian elimination with partial pivoting and the result reciprocated to give the inductor values. To obtain the appropriate capacitor values equation (5.2) is used. The capacitor values are given by:

\[ C_k = \frac{1}{(L_k \omega_k)^2} \] (5.10)

where \( k = 1, 2, \ldots n \)

The admittance of the tuned network at any particular frequency is given by:

\[ Y = \sum_{k=1}^{n} \left\{ \frac{R_k - j(L_k/\omega)(\omega^2 - 1/(L_k C_k))}{R_k^2 + [(L_k/\omega)(\omega^2 - 1/(L_k C_k))^2]} \right\} \] (5.11)

Branch \( k \) is in series resonance when \( \omega = \sqrt{1/(L_k C_k)} \). The corresponding admittance term for the \( k \) branch is \( 1/R_k \). This term is significantly larger than the contribution from the other branches and hence dominates the admittance. The resistance that dominates at each impedance minima is inserted in the corresponding branch. This yields impedance maxima that are higher than those of the actual system. This is corrected by scaling all inductor and capacitor values. A common scaling factor is used for inductors and capacitors so that the resonant frequencies remain unchanged. The appropriate scale factor is obtained by dividing the calculated impedance of the RLC network at a particular frequency by the actual systems impedance at this frequency. Some discretion needs to be exercised in determining what frequency is to be used to determine the scale factor. The branch resistance dominates the RLC network's impedance at a minima and maxima, therefore choosing a match frequency in their close vicinity will produce poor results.
If resistance is ignored Equation (5.11) reduces to:

\[ Y = -j \sum_{k=1}^{n} \frac{\omega}{L_k (\omega^2 - \omega_k^2)} \]  

This shows that, as \( L_k \) appears in the denominator, to increase the admittance contribution of each branch, so as to reduce the impedance, the inductors are divided by the scale factor. In order to maintain the same resonant frequencies the capacitor values are multiplied by the scale factor.

A number of numerical errors arise from the direct application of the above equations. Floating point overflow occurs when trying to calculate the coefficients of \( s \) due to the finite range of numbers a digital computer can represent. Angular frequency (\( \omega \)) is in the range 0 to 15000 rad/sec and the coefficient of \( s^k \) is of the order \( \omega^{2(k-1)} \), \( k \geq 1 \), where there are \( n \) parallel branches whose parameters are to be determined. To overcome this difficulty some of the coefficients (elements of matrix \( A \); equation 5.9) are automatically scaled down by a fixed scale factor before their calculation overflows the number representation. This ensures that the resulting coefficient remains within the required range of values for the computer to represent; knowledge of the scale factors allows the actual coefficient to be determined.

This crude scaling using an arbitrary scale factor allows the coefficients (elements of matrix \( A \)) to be formed, however, the values of the coefficients in the matrix equation 5.9 still differ widely between equations. Solving the system of equations directly as they stand results in attempts to divide by a number approaching zero and failing to give a satisfactory solution. This is overcome by scaling each equation, so that the maximum coefficient in the equation has a magnitude of 10. (This means scaling each row of matrix \( A \) to have a maximum value of 10). Gaussian elimination using partial pivoting is then used to solve for \( 1/L_k \); \( k=1,2,...,n \)
Feature Extraction.

The feature extraction algorithm used for determining the parallel and series resonant frequencies differs from that used by Hingorani and Burbery (1970). Hingorani and Burbery used the frequencies at which the impedance loci crossed the Real axis (or X axis) as the resonant frequencies and starting point for the synthesis algorithm. This does not give satisfactory results when some of the loops in the loci are not centred about a point on the real axis and completely misses loops that fail to cross the real axis. Instead the minimum and maximum impedance points of each loop in the impedance loci are taken as the resonant frequencies.

Figure 5.10 displays the phase "B" loci based on equal current for an equivalent circuit formed using the intersection of real axis feature extraction (c.f. figure 5.6). The first loop between 260 to 290 Hz is
completely missing while the loop consisting of the loci from 500 to 860 Hz does not correspond well with the required loop (displayed in Figure 5.6), as the latter only clips the real axis. Using the new feature extraction routine the loci of figure 5.11 is obtained. Although the first loop is too large it can easily be improved by adjusting the resistance values.

Although the impedance loci display both magnitude and phase information it is probably more intuitive to display the frequency response as impedance versus frequency plots. The troughs in these plots correspond with series resonances and the peaks with parallel resonances. They provide a better idea of which parameter, and in what direction, they need to be changed to improve the response. However, two plots (magnitude and phase plots) are required to display the information contained in the impedance loci. The impedance versus frequency plot for the new feature extraction method is illustrated in Figure 5.12. By inspection the second peak can be seen to be too large while the previous
trough too low. By increasing the resistance on the first RLC the trough's minimum value increases and the parallel resonance peaks on either side diminish, hence the resistance of the RL branch needs to be decreased to maintain the level of the first peak. Figure 5.13 illustrates the response after these two resistances have been altered. This gives a better match across the frequency range, but may cause slight errors in the fundamental frequency (50 Hz) parameters. This discrepancy, due to the compromises made to get the troughs and peaks of the correct size, can easily be corrected by the inclusion of a correction filter, as will be discussed later.
The Scaling Process.

The original scaling process relied on the user specifying a match frequency and the impedance at this frequency; the algorithm then scaled the L's and C's to achieve the specified impedance level at this frequency. Although this worked well it required expert judgement of what frequency point to match to. Usually this involved several attempts at scaling as once one choice had been made and the response compared to the actual frequency response, a better choice of match frequency became apparent. Near the troughs the impedance level is relatively insensitive to scaling as the resistance dominates in these areas, hence the match frequency should not be in this region. The peaks are also very dependent on the resistance, so the match frequency should not be specified to coincide with or be in close proximity to the peak. Single variable optimization techniques have been developed and applied to obtain the best scale factor and this is now the preferred method of obtaining the scale factor. The three different optimization techniques that have been applied are:

(i) Dichotomous search,
(ii) Golden section search,
and (iii) Success/failure step search.

All three performed equally well, although unlike the first two, the Success/Failure algorithm has the advantage that it does not need two initial guesses. A discussion of these three optimization techniques is given in appendix A4. Although the optimization technique determines the efficiency of solution, it is the objective function's definition that determines the optimum point and hence ultimately the extent of success of the optimization technique. Of the objective functions tried the least squares function has been found to be the most satisfactory. Applying the least square technique to the total frequency range is undesirable. A better fit may be achieved by accepting a large initial error in a particular region and using a correction branch at a later stage. To stop regions that are going to be corrected by subsequent use of correction filters detrimentally affecting the scale factor, the frequency range can be partitioned into up to five regions, with different weighting factors for each region. Different weightings can also be given depending on the sign of the error.
HARMONIC VARIATION OF IMPEDANCE

(a) Impedance Loci

(b) Impedance versus Frequency

Figure 5.14 Impedance response when the scale factor is optimized
Figures 5.14 (a) & (b) illustrate the response of a frequency-dependent equivalent matched to phase "B" of the actual system where implicit accounting of mutual coupling is present (c.f. loci displayed in figure 5.6) and when the scale factor is optimized. Care must be taken to ensure that the global minimum and not just a local minimum is found, as several minima may exist. This is illustrated in Figure 5.15 which shows the variation of the least square objective function for different scale factors for the (1,3) locus of the full 3x3 loci matrix, (in conjunction with Hingorani and Burbery's original feature extraction technique). The
feature extraction has a great effect on the objective function as shown by Figure 5.16, which corresponds with the (1,3) loci in conjunction with the new feature extraction algorithm. Figure 5.17 (a) & (b) illustrate the comparison between the frequency responses of the actual system and the equivalent circuit using the original and new feature extraction algorithms respectively. The corresponding figures 5.18 (a) & (b) show the same comparison of frequency response when the scale factor has been optimized rather than directly assessed.
With regard to loci (2, 1) of the test system (figure 5.4), figure 5.19 shows the effect of optimizing the scale factor over the whole frequency range from 50 to 1250 Hz with a constant weighting factor. There is a large peak at 884 Hz which requires the use of correction filters, and this peak has adversely affected the optimal solution. This results in a large discrepancy for the first peak. Choosing a weighting factor of zero for the region around the peak stops this peak influencing the optimum scale factor, giving a more favourable
In matching the frequency-response of an RLC circuit to the power systems response, not only is the impedance magnitude important, but also the phase angle match. This information can be inspected by comparing the match in phase angle or the match of both the resistance and reactance. A typical frequency-response match of the impedance
magnitude, phase, resistance and reactance are displayed in figures 5.21, 5.22, 5.23 and 5.24 respectively. The phase "A" equivalent circuit for implicit mutual coupling representation was used for this illustration. If the phase angle match is bad in a region, it may require compromising the impedance magnitude to improve it or the subsequent use of a correction filter branch.
Figure 5.23 Resistance versus frequency

Figure 5.24 Reactance versus frequency

Figure 5.25 displays the comparison in frequency-response for the full 3x3 system with the straight application of the direct method. The (1,2) term obviously requires the use of correction filters.
Figure 5.25 Impedance Match of 3x3 Impedance Matrix
Correction Filters.

When scrutinizing an equivalent circuit it must be kept in mind that it is easier to lower its impedance level in a region than to increase it. This is achieved by means of adding correcting RLC filter branches. Hence it is sometimes desirable to select smaller than optimum scaling factors as it results in larger impedance levels which can be compensated for later.

Derivation of Correction Branch Parameters.

From inspection the region of largest deviation from the required impedance response is found and three matching points (frequencies \(\omega_1, \omega_2, \omega_3\)) are selected in this region. The impedance of the required response and equivalent circuits response are inverted and the difference obtained. This is then inverted to give the required impedance response of the correction filter. The impedance at the required match frequencies is then obtained and an RLC branch is synthesized so that its frequency response passes through these three match points. The admittance of the required correction branch is given by:

\[
Y_{rb} = Y_{rr} - Y_{ec}
\]

(5.13)

where

- \(Y_{rr}\): Required admittance response.
- \(Y_{ec}\): Equivalent circuits admittance.
- \(Y_{rb}\): Admittance required by correction branch.

\(Y_{rb}, Y_{rr}\) and \(Y_{ec}\) are complex vectors containing the frequency response of the admittance of the correction branch, required response and equivalent circuits response respectively. Let \(Z_1, Z_2\) and \(Z_3\) denote the impedance of the \(Z_{rb}\) at the match frequencies \(\omega_1, \omega_2\) and \(\omega_3\), respectively. Then this implies the RLC branch must satisfy the following three equations:

\[
|Z_1|^2 = R^2 + (\omega_1 L - 1/(\omega_1 C))^2 \quad (5.14)
\]

\[
|Z_2|^2 = R^2 + (\omega_2 L - 1/(\omega_2 C))^2 \quad (5.15)
\]

\[
|Z_3|^2 = R^2 + (\omega_3 L - 1/(\omega_3 C))^2 \quad (5.16)
\]
Equation (5.14) minus equation (5.15) gives:

\[ |Z_1^2| - |Z_3^2| = (\omega_1 L - 1/(\omega_1 C))^2 - (\omega_2 L - 1/(\omega_2 C))^2 \]
\[ = (\omega_1 - \omega_2)^2 L^2 + (1/\omega_1 - 1/\omega_2)/C^2 \]  
(5.17)

Similarly using equations (5.14) and (5.16) yields:

\[ |Z_2^2| - |Z_3^2| = (\omega_2^2 - \omega_3^2) L^2 + (1/\omega_2 - 1/\omega_3)/C^2 \]  
(5.18)

Rearranging equation (5.18) gives:

\[ (1/C^2)(1/\omega_2^2 - 1/\omega_3^2) = |Z_2^2| - |Z_3^2| - (\omega_2 - \omega_3)^2 L \]  
(5.19)

Therefore C can be expressed as

\[ C^2 = \frac{(1/\omega_2^2 - 1/\omega_3^2)}{|Z_2^2| - |Z_3^2| - (\omega_2 - \omega_3)^2 L} \]  
(5.20)

Substituting equation (5.20) into equation (5.17) gives:

\[ |Z_1^2| - |Z_2^2| = (\omega_1^2 - \omega_2^2) L^2 - \left( \frac{(\omega_2 - \omega_3)^2 L^2 - (|Z_2^2| - |Z_3^2|)}{(1/\omega_2^2 - 1/\omega_3^2)} \right)(1/\omega_1^2 - 1/\omega_2^2) \]
\[ = L^2 \left[ (\omega_1^2 - \omega_2^2) - \frac{(\omega_2 - \omega_3)^2}{(1/\omega_2^2 - 1/\omega_3^2)} (1/\omega_2^2 - 1/\omega_3^2) \right] + \frac{|Z_2^2| - |Z_3^2|}{(1/\omega_2^2 - 1/\omega_3^2)}(1/\omega_1^2 - 1/\omega_2^2) \]  
(5.21)

Hence

\[ \frac{|Z_1^2| - |Z_2^2| - |Z_3^2|}{(1/\omega_1^2 - 1/\omega_2^2)} = \frac{|Z_2^2| - |Z_3^2|}{(1/\omega_2^2 - 1/\omega_3^2)}(1/\omega_1^2 - 1/\omega_2^2) \]  
(5.22)

Given three points, equation (5.22) is solved to obtain the inductance of the correction branch. Equation (5.20) is solved next to obtain the capacitance value. Lastly equation (5.14) is solved for the required resistance; this completes the derivation of the correction filter branch parameters.
Although the above formulation synthesizes a correction branch to correct for impedance mismatch, other quantities such as resistance, reactance, conductance, susceptance and phase angle can be matched in a similar manner.

**Matching of correction branches.**

The newly derived correction filter is added to the equivalent circuit to form a new equivalent and the process repeated if necessary by identifying a new region of discrepancy and another correction filter. This process continues until the required accuracy is obtained, though the accuracy may be limited by the increasing computing requirements. The CPU time used in a.c./d.c. dynamic simulation is related to the size of the time steps of the numerical solution, which in turn depends on the time constants of the equivalent circuit. The acceptability of a particular equivalent circuit and the required CPU time are assessed in each case by means of a short-duration dynamic simulation study.

**Illustrative Example.**

Figure 5.26(a) displays the frequency response of the (1,2) mutual element of the actual system together with that of the equivalent obtained by the direct method. Although the results are in good agreement at low frequencies the second and third peaks (at 845 and 876 Hz) show substantial disparity. The use of a correction filter in the region of largest discrepancy (862 to 906 Hz) modifies the harmonic response to that shown in Figure 5.26(b). The new branch reduces considerably the error in the regions where the impedances of the equivalent circuit were too high, but no marked improvement is noticeable where the equivalent circuit impedances are too low. As explained earlier these are best corrected by a combination of scaling factor and subsequent use of correction filters.

The region of greatest discrepancy of the new equivalent circuit is now between 750 and 810 Hz. The addition of a second correction filter in this region produces a far better match, as is shown in Figure 5.26(c).
Figure 5.26(a) Impedance match of (1,2) mutual element

Figure 5.26(b) Impedance match of (1,2) mutual element when one correction branch has been added

Figure 5.26(c) Impedance match of (1,2) mutual element when two correction branches are used
The matched equivalent circuit with implicit accounting of the mutual coupling for each of the three phases is displayed in figures 5.27 (a),(b)&(c). The direct synthesis produces an equivalent which compares well with the required response but the 50 Hz impedance of the equivalent is too high. Therefore one correction filter needs to be used in each phase to produce the correct impedance (magnitude and angle) at 50 Hz. The correction filter is given a high Q to avoid any detrimental effect on the response at higher frequencies. Figures 5.28 (a),(b)&(c) are an amplified view of the impedance match in the 50 Hz region showing the effect of incorporating these correction filters. Figures 5.29 (a),(b)&(c) shows an amplified view of the phase angle match.
Figure 5.27 Matching to the Diagonalized Impedance Matrix
Figure 5.28 Impedance Match with a corrective branch for the 50 Hz parameters
Figure 5.29 Phase Angle Match with a corrective branch for the 50 Hz parameters
5.3.2 Optimization Method.

5.3.2.1 Choice of Objective Function.

The mathematical problem of optimization is to minimize a scalar function (known as an objective function), which is dependent on many parameters and to determine the values of the dependent parameters at this minima. In the present work the difference between the required response and equivalent circuits response is to be minimized by optimizing the R, L and C values of the equivalent circuit. As in a large number of optimization problems, data is only available at a number of sample points, regardless of whether the required response is obtained by measurement or frequency domain analysis. Hence only at these data points can comparisons between the required and equivalent circuit's response be made. Let \( x_1, x_2, \ldots, x_n \) denote the \( n \) dependent parameters and \( A_1, A_2, \ldots, A_m \) the values of the independent variables (namely the values of the required response) at the \( m \) available sample points. Then the form of the objective function is:

\[
\theta = h\{g(x,A_1), g(x,A_2), \ldots, g(x,A_m)\} \quad (5.23)
\]

However, the values of the independent variables may be incorporated into the function \( g(x) \) to yield equation (5.24)

\[
\theta = h\{g_1(x), g_2(x), \ldots, g_m(x)\} \quad (5.24)
\]

The form of the \( h \) function is termed the error criterion. The choice of objective function is an important issue in optimization problems as it greatly influences the optimum point and the ease by which it is found.

Four of the more commonly used error criteria are:

(a) Least Squares.
(b) Mini-max.
(c) Mini-average.
(d) Mini-area.

Least Squares is the most widely used error criterion. The general form is:

\[
\text{Minimize } \theta = \sum_{i=1}^{m} \{w_i g_i(x)\}^2 \quad (5.25)
\]
where $w_1, w_2, \ldots, w_m$ are termed weights or penalties and have the effect of emphasizing $G_i(x)$ in regions of importance.

Mini-max error criterion minimizes the maximum element in $G(x)$. Its form is given in equation 5.26. One of the disadvantages with this error criterion is that the derivatives of $\theta$ with respect to the parameters are not defined, as $\theta$ jumps discontinuously as one value of $i$ changes to another during the course of optimization.

\[
\text{Minimize } \theta = \max \{ |w_i G_i(x)| \} \quad (5.26)
\]

Mini-average minimizes the average, however the problem with this is that it can allow large deviations from the required response if it is compensated by a similarly large deviation in the opposite direction.

\[
\text{Minimize } \theta = \sum_{i=1}^{m} \{ w_i G_i(x) \} \quad (5.27)
\]

Mini-area minimizes the area formed by $G(x)$ (area of discrepancy). To maintain versatility, numerical integration techniques such as Simpson's Rule cannot be used as the requirement of an even number of data points is too stringent. Once a particular frequency range for the optimization has been chosen, the number of sample points within this range may be odd or even, and interpolation cannot alter this fact (while still maintaining equally spaced sample points).

\[
\text{Minimize } \theta = \int w_i G_i(x) \quad (5.28)
\]

In the current problem the function $G(x)$ is:

\[
G_i(x) = Z_i(x) - A_i \quad (5.29)
\]

where $Z_i(x)$ is the impedance of the equivalent circuit at the frequency of the $i$th sample point and $A_i$ is the required impedance at this point. $x$ corresponds to the $R, L$ and $C$ values of the equivalent circuit. It should be noted that although this chapter deals almost exclusively with the matching of the impedance response the ACREP program's algorithm can optimize the following parameters: Admittance, Conductance, Susceptance, Resistance, Reactance or Phase angle.
5.3.2.2 Optimization Methods.

Once the objective function has been defined it is necessary to decide on a method of performing the optimization. Although not completely separate, optimization methods fall into two classes, Search and Gradient methods. Search methods use objective function evaluations only while gradient techniques require additional gradient information (Adby and Dempster 1974).

In general Gradient methods are superior to Search methods if the functions involved have continuous derivatives which can be evaluated analytically. Functions for which derivative information is not readily available normally require the application of Search methods. Recently, however, a Gradient method that only requires objective function evaluations (Harwell 1986) has been formulated. It is not necessary to supply derivative information as an estimate of the Gradient is obtained numerically from either the Finite or Central Difference formula. This Gradient method uses a special implementation of the BFGS method to minimize a Lagrange multiplier penalty function (equation 5.30). The Lagrange multiplier penalty function with only equality constraints is:

$$P(x_1,\ldots,g) = \Theta(x) - \frac{1}{2} \sum_{i=1}^{k} (\lambda_i C_i(x))^2 + \frac{1}{2} \sum_{i=1}^{k} (g_i C_i(x))^2$$  \hspace{1cm} (5.30)

where the objective function is

$$\Theta(x) = h\{G_1(x), G_2(x), \ldots, G_m(x)\}$$

and the constraints are

$$C(x) = 0 , \hspace{0.2cm} i=1,2,\ldots,k$$

$$g_i , \hspace{0.2cm} i=1,\ldots,k$$ are the penalty parameters

$$\lambda_i , \hspace{0.2cm} i=1,\ldots,k$$ are the multiplier parameters

Two multi-variable optimization techniques have been implemented in ACREP. The first is the gradient method described above while the second is the pattern search technique.

Figure 5.30 (a),(b),(c)&(d) show the optimum as found by the gradient optimization for the four main error criteria mentioned previously. The optimum using Mini-average and Mini-area are similar as expected as the sample points are equally spaced. The best is the least
Figure 1. Impedance (Ohms) vs. Frequency (Hz)

(a) Least Squares

(b) Mini-max

(c) Mini-average
squares error criterion, which accounts for its popularity. The various CPU requirements to reach these solutions from a nominal starting point are displayed in Table 5.1. In general the Gradient method takes in the order of 1 to 4 minutes of CPU time to optimize a typical equivalent circuit consisting of 11 variables. This compares well with the 20 minutes to 3 hours required for the same optimization problem using the pattern search.

<table>
<thead>
<tr>
<th>Error Criteria</th>
<th>CPU Time</th>
<th>Objective Fcn.</th>
</tr>
</thead>
<tbody>
<tr>
<td>(a) Least Squares</td>
<td>1 Min</td>
<td>223679.26</td>
</tr>
<tr>
<td>(b) Mini-max</td>
<td>31.17 sec</td>
<td>279.92</td>
</tr>
<tr>
<td>(c) Mini-average</td>
<td>38.70 sec</td>
<td>-5329.57</td>
</tr>
<tr>
<td>(d) Mini-area</td>
<td>1 Min</td>
<td>-285796.61</td>
</tr>
</tbody>
</table>

Table 5.1 CPU requirements for various Error Criteria.
One question that comes to mind is that if the Gradient Method is so much better why implement the pattern search method? The reason being that in many cases the Gradient method fails to find the minima. The Gradient search assumes that the objective function can be calculated to the full precision of the variables, and hence ignores any numerical noise present. When well away from the minima the step length is large and hence the required directions are easily determined, regardless of the numerical error. However, when in the proximity of the minima the step length is small, hence the numerical error greatly influences the gradient estimate making it extremely hard to determine in which direction the minima lie. The nett effect is for the algorithm to stagnate and then stop before reaching the minima. Figure 5.31 depicts the effect of numerical noise. When this occurs a search technique such as the Pattern Search can be applied, starting from where the gradient search stopped. Search Methods are more robust than the gradient methods as gradient estimation is always prone to significant numerical errors.
5.4 The Philosophy of the Frequency-Dependent A.C. System Models.

The determination of the self and mutual impedance terms of an a.c. system can be visualized as the ratio of voltage to current obtained by shorting every independent voltage source and injecting a current of frequency \( f \) in one phase, as depicted in figure 5.32.

![A.C. System Diagram](image)

Figure 5.32 Interpretation of Self and Mutual Terms.

Hence the matrix equation describing this situation is:

\[
\begin{bmatrix}
V_1 \\
V_2 \\
V_3
\end{bmatrix} =
\begin{bmatrix}
Z_{11} & Z_{12} & Z_{13} \\
Z_{21} & Z_{22} & Z_{23} \\
Z_{31} & Z_{32} & Z_{33}
\end{bmatrix}
\begin{bmatrix}
I_1 \\
I_2 \\
I_3
\end{bmatrix}
\]

(5.34)

Then to obtain the frequency response this process is performed for a range of frequencies. As the impedance is determined with the independent voltage sources shorted the model must have the fundamental-frequency source incorporated. Hingorani and Burberry's equivalent (figure 5.1) put the voltage source in the RL branch only. A natural extension of this is to also include the mutual voltage in this branch. This has the
advantage of requiring no extra circuit elements to be modelled as the mutual voltage can be incorporated into the existing fundamental-frequency voltage source. However, it is questionable whether this is strictly the correct place for the voltage sources to be incorporated. When the equivalent of figure 5.1 is open circuited the terminal voltage is less than the source voltage as the RLC branches of the a.c. system equivalent loads the voltage source causing a voltage drop across the RL branch. Although this error may be small for a fundamental-frequency source, by virtue of the high impedance of the RLC branches at 50 Hz relative to the RL branch impedance, this may not hold where the mutual voltage source is concerned, due to the higher frequencies present in the mutual voltage waveform.

Consider the situation depicted in figure 5.33, where the mutual voltage source is incorporated into only the RL branch. By applying the Superposition Theorem, consider the effect of the mutual voltage source in isolation (i.e. shorting the fundamental-frequency source). Let phases "A" and "C" be open circuited while phase "B" is loaded. From equation (5.34), under these conditions the terminal voltage of phase "A" should be \( Z_{ab}I_b \), however, the circuit of figure 5.33 has the RLC branches loading the mutual voltage source, therefore the terminal voltage appearing will be:

\[
V_a = Z_{ab}I_b - (R_{RL}I_r + L_{RL} \frac{dI_r}{dt})
\]

where \( I_{RL} \) is the current in the RL branch and \( I_b \) the phase "B" current.

The Substitution Theorem states that "two subnetworks are equivalent if the relationships between the terminal voltages and terminal currents are the same for both and such equivalent subnetworks can be exchanged without affecting conditions in any part of the network external to them" (Skilling 1974). Obviously the circuit of figure 5.33 violates this and hence is not a true equivalent of the actual systems performance as strictly laid down by the Substitution Theorem.

The equivalent displayed in figure 5.34 does however approach this ideal of being a true equivalent as the fundamental-frequency voltage sources are now the open circuit voltages.
In figure 5.34 there are three parts: a frequency dependent impedance ($Z_{aa}', Z_{bb}', Z_{cc}'$) which is implemented by a matched RLC equivalent circuit; a fundamental-frequency voltage source, which is not apparent when impedances are obtained; lastly a controlled voltage source which mimics the mutually induced voltage terms, $Z_{ab}I_b + Z_{ac}I_c$, $Z_{ba}I_a + Z_{bc}I_c$ and $Z_{ca}I_a + Z_{cb}I_b$ terms.
Figure 5.35 The self circuit for the frequency-matched a.c. system as implemented in TCS

The modelling of a voltage source in isolation is not possible in TCS as each voltage source must be associated with an inductive branch (Joosten 1985). This difficulty is overcome by moving the fundamental-frequency and mutual voltage sources into the inductive branches of the RLC network, as depicted in figure 5.35. This change does not alter the performance of the circuit. However, this equivalent is computationally expensive due to the number of voltage sources that need to be updated at each time step. Since the RLC branches are tuned to exhibit series resonance above the fundamental frequency their impedance is large at 50 Hz and the RL branch dominates. Under these conditions the fundamental-frequency voltage sources in the RLC branches can be eliminated. It could also be argued that the mutual voltage source can be eliminated from the RLC branches as its value is several orders of magnitude less than the voltage across the resistor, inductor and capacitor making up the branch; however this is not strictly valid as the harmonic levels are usually several orders of magnitude less than the fundamental and therefore approximations made at harmonic frequencies may result in high relative errors.
Figure 5.36 displays the various a.c. system equivalent circuits which may be used for dynamic studies with the corresponding CPU requirement for a typical 5 cycle simulation. These models represent the actual system performance with varying degrees of accuracy and in each case the purpose of the simulation will dictate the most appropriate model to be used.
<table>
<thead>
<tr>
<th>THEVENIN EQUIVALENT</th>
<th>FREQUENCY-MATCHED EQUIVALENT No. 1</th>
<th>FREQUENCY-MATCHED EQUIVALENT No. 2</th>
<th>FREQUENCY-MATCHED EQUIVALENT No. 3</th>
<th>FREQUENCY-MATCHED EQUIVALENT No. 4</th>
</tr>
</thead>
<tbody>
<tr>
<td>Implicit Representation of Mutual Coupling.</td>
<td>![Diagram of equivalent circuit 1]</td>
<td>![Diagram of equivalent circuit 2]</td>
<td>![Diagram of equivalent circuit 3]</td>
<td>![Diagram of equivalent circuit 4]</td>
</tr>
<tr>
<td>8 min. 50 sec.</td>
<td>15 min. 15 sec.</td>
<td>26 min. 54 sec.</td>
<td>46 min. 54 sec.</td>
<td></td>
</tr>
</tbody>
</table>

Figure 5.36 Various a.c. system equivalent circuits
5.5 Implementation of Frequency-Dependent A.C. System Models.

The solution process for the Self <-> Mutual circuit interface is depicted in figure 5.37.

![Flowchart](image)

Figure 5.37 Self <-> Mutual Circuit Interface.

Direct application of this algorithm would result in this iterative loop being required for each iteration of the integration algorithm (separate iterative loop solution). This is highly undesirable due to the computational burden it imposes. Instead, the above loop is embedded in the existing integration iterative algorithm, as discussed in section 3.4.3 for the modelling branches between phases, thus achieving only one level of looping. Compared to the modelling of "torn" branches (section 3.4.3), the final method for the implementation of frequency-dependent a.c. system equivalent reverses the roles of the voltage and current source interface between the standard and extra circuits. The philosophies of the two approaches however differ greatly. In figure 5.38(a) the extra circuit represents torn branches that are actually part of the network while in figure 5.38(b) the extra circuits
(mutual circuits) are used to calculate the mutual voltage at each time step and do not represent actual branches in the network. The early attempt used tearing to remove branches and the interface consisted of a voltage source input for the torn branches and an equivalent current source input for the main part of the network, as depicted in figure 5.38(a). In the frequency-dependent a.c. system equivalent as implemented, the mutual voltage is the input to the main part of the network while the current source the input to the mutual networks, as shown in figure 5.38(b). In this case the voltage source models the voltage induced due to a current in the other two phases, while the mutual network calculates the voltage to be applied, based on the phase currents and frequency-dependent mutual impedance.

The iterative procedure of the integration routine is as follows: estimate the state variables, hence obtain the phase currents; evaluate the dependent variables, therefore obtain the mutual voltage; test for convergence of all the variables and if required perform another iteration. This approach however results in a larger number of iterations prior to convergence and therefore requires the re-tuning of the parameters governing the step length optimization based on past convergence history, for satisfactory operation.
5.6 Initialization of Frequency-dependent Equivalents.

The method of initialization is based on what experience has shown to be the best for starting convertor operation. Disabling the convertor controller at start-up and enabling it several cycles into the simulation allows a steady state to be reached in a shorter duration than if the controller is active from the start of the simulation.

To avoid prolonged initialization transients the mutual circuits are not excited with the phase currents until sufficient time has been allowed for the main transient in the self circuit to die away (approximately two cycles). A further delay is allowed for the main transient in the mutual circuits to die away before applying the mutual voltage back into the self network.

At the time of energizing the mutual circuits it is imperative to initialize the current in the inductors in the best way possible. Firstly the sum of the currents at a node must be initialized to be zero. This is very important as the phantom current source (Section 3.3.3, Joosten 1985) developed to overcome the gamma node problem does not operate on the mutual circuits. Secondly the sum of the initial inductor currents in the mutual circuit must equal the appropriate self circuit's phase current. This makes the initialization slightly more difficult as we have no idea of what this current will be until the program has reached this point in the simulation. The result of not achieving the correct initial currents in the branches forming the mutual circuit is a constant offset between the self circuits phase current and the current applied to the mutual circuit. This is due to the equation for gamma nodes using rate of change of current (equation 3.23) rather than current magnitude. Once the fluxes in the inductors have been initialized, all the changes occurring during the simulation are governed by the rate of change of current.
5.7 Validation of the Computer Model.

A comparison between the frequency response of the synthesized RLC circuits and their required responses provides a clear indication of how well each of the self or mutual terms response is modelled. But in developing the TCS program to cope with the frequency-dependent a.c. systems, each stage of the development must be validated to ensure the correct implementation. Although difficult and time consuming, the validation exercise is essential for the credibility of the simulated results.

A preliminary test in the process of building up the model involved applying a voltage rather than current excitation to the mutual circuits. By modelling identical RLC networks for both the mutual and self circuits and applying the same fundamental voltage excitation, while the mutual voltage interaction with the self circuit removed, allows the transient responses to be compared directly, hence verifying the mutual circuit. This tests the software associated with setting up the extra networks forming the mutual circuits as well as the algorithm for obtaining the mutual circuit variables at each time step.

When the interaction between the mutual and self circuits is enabled, if the mutual voltage derived from the phase currents is applied with the wrong polarity to the self network, an instability may occur making the error very noticeable. This instability is due to a positive feedback effect; the mutual voltage is of a polarity that aids the fundamental-frequency voltage source thus tending to increase the current magnitude in each phase. The increase in a phase's current due to the mutual voltage results in an increase in the mutual voltage applied to the other two phases, hence a larger phase current results. This increase of phase current in the other two phases produces a bigger mutual voltage to be applied to the original phase and thus increases its current even more.

As mentioned in chapter 3 the current source model was first implemented to allow testing of the complete a.c. system model. Each current source is capable of modelling a single frequency source, therefore a separate source is required for each frequency to be injected. For convenience and to maintain clarity of results, normally
only one frequency at a time was injected. Due to the time considerations the testing of the complete model by this method, at best, can be performed only at a few selected frequencies.

By injecting a current in phase "A" and assessing the voltages of the three phases, the first column of the 3x3 impedance matrix at the current source frequency is obtained. The other two columns are assessed by injecting current in the other two phase in a similar manner. These results can then be compared to the expected values from the designed frequency-response match, to validate both the diakoptical technique used to implement the frequency dependent mutual coupling and the correctness of the input data. Tables 5.2, 5.3 and 5.4 summarize the results of these three current injection tests. The impedances designated by the "A" are the theoretical result from the designed match while the values labelled by the "B" are obtained by using an FFT on the TCS derived waveforms. The results show very good agreement at all the six frequencies the test was performed at. This clearly demonstrates that the diakoptical technique has been correctly coded and the input data has been correctly entered.

A more comprehensive test is the comparison of the harmonic levels with those derived from frequency domain programs, as outlined in chapter 6. This effectively tests the frequency-dependent a.c. system equivalent at all the required harmonic frequencies simultaneously. A quantitative validation of dynamic simulation results is not possible due to the lack of an adequate yard-stick. However, credibility for the dynamic simulations can be provided by the harmonic assessment studies. If the frequency response of the model is accurate so will be its dynamic response. The dynamic response can be considered as the convolution of the input with the impulse response, which corresponds to the multiplication of the input and system responses in the frequency domain. Verification of the system response in the frequency domain ensures the accuracy of the corresponding convolution in the time domain.
<table>
<thead>
<tr>
<th>Freq. (Hz)</th>
<th>Matrix Position</th>
<th>Impedance Magnitude (Ohms)</th>
<th>Phase Angle (Degrees)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>(1,1) A</td>
<td>47.031</td>
<td>46.85</td>
</tr>
<tr>
<td></td>
<td></td>
<td>47.093</td>
<td>46.81</td>
</tr>
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A : Theoretical value obtained from Frequency domain Software.
B : TCS assessed values.

Table 5.2  Current injected into phase "A"
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A : Theoretical value obtained from Frequency domain Software.
B : TCS assessed values.

Table 5.3 Current injected into phase "B"
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A: Theoretical value obtained from Frequency domain Software.
B: TCS assessed values.

Table 5.4 Current injected into phase "C"
5.8 Extension to Multi-convertor Systems.

Consider now the case when two convertors are connected to the same a.c. system as shown in figure 5.39. This system requires two points of observation, i.e. busbars A and B. The convertor connected to busbar A will affect the impedances of the a.c. system when observed from busbar B and vice versa. This inter-dependence increases the number of mutual terms as each phase has a self and five mutual terms.

An extension of the diakoptical technique to this case requires six self and thirty mutual circuits for a complete frequency-dependent model. Although extension of the theory to more than two convertors is straightforward, the number of frequency dependent circuits required increases with the square of the number of convertors. The complexity of each frequency dependent circuit is dependent on the number of loops in the frequency loci. Therefore it becomes impractical to model accurately a large number of convertors connected to the a.c. system.

Figure 5.39 Multi-convertor System
5.9 Frequency-dependent d.c. System model.

The same procedure used in the development of frequency-dependent a.c. system equivalents can be applied to produce frequency-dependent d.c. system equivalents. However, because the TL program (figure 5.3) was designed specifically for three phase transmission lines it cannot easily be used for modelling d.c. lines. Instead the more general line constants program (LINE) developed by Prof Dommel (Dommel 1980) is used to produce the transmission line parameters at all the required frequencies. Figure 5.40 depicts the new chain of data flow and the associated programs required to produce a d.c. equivalent circuit which is frequency-matched.

The LINE program is capable of modelling skin effect and earth return effects for an arbitrary number of conductors. Carson's equations are used to implement earth return effects.

The program DCLINK (figure 5.40) forms the overall admittance matrix by cascading the chain parameter matrices for each different section forming the d.c. system. The structure of DCLINK is shown in figure 5.41. The output from DCLINK is a set of 2x2 admittance matrices, one matrix for each required frequency. The equation describing the totality of the d.c. system with its different sections is:

\[
\begin{bmatrix}
  I_1 \\
  I_2
\end{bmatrix} =
\begin{bmatrix}
  Y_{11} & Y_{12} \\
  Y_{21} & Y_{22}
\end{bmatrix}
\begin{bmatrix}
  V_1 \\
  V_2
\end{bmatrix}
\] (5.31)

where the subscript 1 denotes quantities associated with the positive pole and 2 the negative pole.

If a bipolar link is to be represented in the transient program the set of 2x2 admittance matrices produced by DCLINK result in two frequency-dependent self and two frequency-dependent mutual circuits being required. From knowledge of the operating conditions of the link it is possible to reduce the 2x2 matrices so that only one frequency-dependent equivalent circuit is required in the transient program, with the mutual coupling accounted for implicitly. However the use of a monopolar equivalent of the bipolar link imposes limitations on the d.c. faults that can be simulated.
Conductor data and line geometry of d.c. line

Transmission Line Parameter program (LINE) → Obtain overall admittance matrices (DCLINK) → Synthesis of frequency-matched equivalent (ACREP) → Transient Convertor Simulator (TCS)

Figure 5.40 Data Flow in forming a d.c. equivalent circuit.
Figure 5.41 Structure diagram of DCLINK
If the current in each conductor is identical in magnitude and opposite in sign then the monopolar equivalent is obtained by: inverting the admittance matrices obtained from DCLINK, then post multiplying by the matrix (as $I_1 = -I_2$):

$$
\begin{bmatrix}
1 & -1 \\
-1 & 1
\end{bmatrix}
$$

This gives:

$$
V_1 = (Z_{11} - Z_{12}) \cdot I_1 \tag{5.32}
$$

$$
V_2 = (Z_{22} - Z_{21}) \cdot I_2 \tag{5.33}
$$

The last equation is disregarded and the impedances in equation (5.32) are halved. This is done as $V_1$ must equal $V_2$ in magnitude for the bipolar link to be represented as a monopolar equivalent. This results in twice the current that would appear in one pole of a bipolar link occurring in the monopolar equivalent. As the impedances are halved from that of a single pole and the losses are $I^2R$, the power loss is twice that of one pole, or that of the total bipolar link it represents. However, applying any fault that would cause $I_1$ not to be equal to $I_2$ in the bipolar link it represents cannot be modelled as it would render this monopolar equivalent invalid. In general the full bipole operation must be modelled.
Figure 5.42 compares the frequency response of a d.c. system with one pole de-energized, and hence using earth return, with that of the matching equivalent circuit.

The frequency-dependent d.c. system model is probably best used in harmonic studies and a.c. fault dynamic studies. The matching equivalent is correct for the terminal conditions only, with no information being available on busbars in the system it represents. Therefore when d.c. faults are being modelled the location must be accessible for TCS to use its d.c. arc modelling capability. That is, the line must be represented by \( \pi \)-equivalent models on each side of the fault.
5.10 Negative Resistance Region in the Impedance Loci.

In the case of power transformers the mutual coupling between phases is considered to be purely inductive, however, the mutual coupling between phases in power transmission systems exhibits both resistive and inductive components. When the mutual coupling is inductive the induced voltage in one phase lags or leads the current in the inducing phase by 90 degrees. The resistive component of the mutual coupling produces an induced voltage component which is either in phase or 180 degrees out-of-phase with the current that caused it.

The mutual coupling between phases often exhibits a frequency range in which the resistive component is negative. This is predominantly due to the effect of the transmission lines. In the frequency-dependent equivalents circuits this is modelled by having a negative resistance in the appropriate RLC branch. Although this works well for modelling minor excursions of the impedance loci into the left hand half plane, difficulties arise where complete loops occur in the left hand half plane of the actual systems response.

Figure 5.43 Partitioned equivalent circuit
Consider the frequency-dependent equivalent as depicted in figure 5.43. The equivalent has been partitioned into three regions; the first consists of the RL branch and all RLC branches having a series resonance below \( f_{pr} \); the second portion is the branch to exhibit parallel resonance with the former section; the last section consists of the remaining RLC branches that do not influence the impedance until a higher frequency is reached. Let \( R_1 + jX_1 \) denote the impedance at a frequency \( f_{pr} \) of the first portion and let \( R_2 - jX_2 \) represent the impedance of the \( k \)th RLC branch at \( f_{pr} \). Then the \( k \)th parallel resonance occurs when the susceptance is zero. (i.e. when the capacitive susceptance of the RLC branch equals, in magnitude, the inductive susceptance of the first region). This can be shown to occur when:

\[
\frac{X_1}{X_2} = \frac{(R_1^2 + X_1^2)}{(R_2^2 + X_2^2)} \tag{5.34}
\]

As the resistive components are considerably smaller than the reactances involved, this implies the ratio of \( X_1 \) to \( X_2 \) (equation 5.34) is close to unity at the frequency of parallel resonance. Now the total impedance is given by:

\[
Z_t = \frac{Z_1 Z_2}{Z_1 + Z_2} = \frac{(R_1 + jX_1)(R_2 - jX_2)}{(R_1 + R_2) + j(X_1 - X_2)}
\]

\[
\quad = \frac{(R_1R_2 + X_1X_2) + j(X_1R_2 - X_2R_1)}{(R_1 + R_2) + j(X_1 - X_2)} \tag{5.35}
\]

From equation (5.35) it can be seen that at a parallel resonance, if \( R_2 \) is negative but less than \( R_1 \) in magnitude the impedance peak will occur in the right half on the impedance plane. However if \( (R_1 + R_2) \) is negative the resistance will be negative at the parallel resonance and therefore the maximum impedance point occurs in the left hand half plane. This is also accompanied by change to an anti-clockwise progression of the impedance loci in the region of this resonance. However the frequency dependence of actual power systems always appear to show a clockwise progression of the loci. Therefore, although the impedance magnitude match can be very good, the corresponding phase angle match is bad due to incorrect direction of the loci progression.
5.11 Discussion and Conclusions.

5.11.1 Explicit versus Implicit Mutual Representation.

The frequency-response of the implicit representation is normally far better behaved than that of the full $3 \times 3$ response, allowing a far better match between the required and equivalent circuit to be achieved. However, there are inaccuracies difficult to assess due to the assumptions used to diagonalize the impedance matrix. With the full representation, although the frequency-response match is more complicated, there are no simplifying assumptions and inspection of the frequency-response match can identify the expected error. The computation efficiency is another factor in favour of the implicit mutual coupling representation.

5.11.2 Comparison between Direct and Optimization Techniques for forming frequency-dependent equivalent circuits.

The CPU time required by the direct method of synthesis is several orders of magnitude less than that of the optimization technique. Typically the direct method requires 100 to 200 milli-seconds of CPU compared to 10 seconds to 2 hours for the optimization technique.

The topology of the equivalent circuit is fixed with the direct method whereas the only limitation for the optimization technique is the ability to evaluate the circuits frequency response. However, the topology must be decided upon before starting the optimization.

The ability to optimize the equivalent circuit is dependent on how perverse the objective function is. The application of weighting functions and/or changing the optimization method are often required to overcome stagnation problems in the gradient algorithm.
The optimization technique generally finds the best fit for a given number of components in the equivalent circuit. Therefore if many simulations are to be performed with an equivalent it is probably worth the effort of trying to optimize. On the other hand where several models are to be tried with relatively few simulations of each, the direct method is the best option.

5.11.3 Impedance Magnitude and Phase Angle Match.

The impedance magnitude and phase angle are both very important in harmonic assessment studies. This is due to the time waveforms being dependent on the magnitude as well as phase angle of the harmonics. Therefore after the circuit has been synthesized the match in both needs to be inspected. Both the direct and optimization methods can correct for regions of inadequate matching of phase angle. The direct method can use correction filter branches while the optimization method can incorporate both impedance and phase angle as part of the objective function. The effect of impedance phase angle on the dynamic behaviour depends on the cause of the transient and the system and is difficult to predict in advance.

5.11.4 Negative Resistive component of Mutual Coupling.

Conservation of power is one of the fundamental laws of science. The existence of a negative resistance allows the mutual coupling to supply power to the load resulting in the load's power dissipation exceeding the power supplied by the external sources. At first sight it may appear that energy is coming from no-where, however, this is not the case. Although negative resistive coupling is not physically realizable as it amounts to generation, it can occur as a result of the reduction process required to derive the 3x3 impedance loci for one busbar from the frequency response of all the system components and their inter-connection. Therefore it is a mathematically convenient solution to a non-physical problem.
For instance a transmission line with three phase conductors and two earthwires has a primitive impedance matrix which is 5x5 in dimension and there is energy associated with the mutual coupling between the phases and the earthwires. This primitive impedance matrix is reduced to a 3x3 by absorbing the terms associated with the earthwire into the remaining nine terms. Therefore these nine terms embody the mutual coupling with the earthwires.

\[
[Z]_{\text{Prim.}} = \begin{bmatrix}
[A] & [B] \\
[C] & [D]
\end{bmatrix}
\]

where dimensions of the sub-matrices are:

- \([A]\) 3x3, \([B]\) 3x2, \([C]\) 2x3, \([D]\) 2x2

The reduced impedance matrix is obtained from the following equation:

\[
[Z] = [A] - [B][D]^{-1}[C]
\]

Although the elimination of the earthwires from the formulation may result in a negative real part to the mutual coupling, the corresponding changes in the self terms ensures that the overall power is dissipated and power conserved.

Although negative resistive regions require more care in modelling and are more awkward to handle this is not a major limitation for HVDC dynamic simulations. The negative resistance regions only result due to the presence of long transmission lines together with very light loading conditions. HVDC applications do not include long a.c. transmission lines as the d.c. line is used to cover the larger distances; therefore the a.c. system is unlikely to exhibit major negative resistive regions in its loci. Also the presence of loading would help to reduce any negative resistance regions.
Dynamic instability can occur with some synthesized equivalent circuits. This is primarily due to the impedance loci being in the left hand half plane. Consider for example the case of a small $7^{\text{th}}$ harmonic current in phase "A" that causes a mutual voltage in the other two phases to increase their $7^{\text{th}}$ harmonic current. There is then the possibility of a dynamic instability if the increased current in these two phases tends to increase the phase "A" $7^{\text{th}}$ harmonic current substantially. The equivalent with the mutual voltage applied only to the RL branch is moderately immune to such instabilities as the RLC branches load the mutual voltage source and hence dampens its effect on the external network. The equivalents with the mutual voltage applied to all the parallel branches in the equivalent are susceptible to this form of instability. An example of a dynamic instability occurring is displayed in figure 5.44.

When the resistive component of the mutual coupling is positive the voltages and currents in the system are self regulating due to the negative feedback formed. With a negative resistive mutual coupling the feedback is positive and a stable solution depends on the size of the mutual coupling relative to the self terms. A frequency-matched equivalent may be unstable if the matching compromises causes the equivalent impedance matrix, at a particular frequency, not to be diagonally dominant. If a substantial negative resistive region exists,
it must be made smaller, the reduction depending on the values of the components of the same frequency in other elements of the impedance matrix. If the negative resistive region is small it can be approximated by an equivalent with an impedance loci completely contained in the right half of the impedance plane.

5.11.6 Future Work.

There is still a need for a better understanding of the frequency-response behaviour of power systems. Although a great deal of work has been done on the accurate calculation of the transmission line parameters at any frequency, the combined effect of the interconnection of transmission lines on the frequency-response is not well understood. More specifically, an adequate explanation is required of why the impedance loci always appears to exhibit a clockwise progression with frequency. Also it needs to be determined if there are any exceptions to this apparent power system rule.

The optimization techniques allow a great variety of topologies in the equivalent circuits to be synthesized. There is a need for further research into the different possible topologies with the aim of finding a topology that can emulate the power systems frequency-response better.

Rational functions, previously used to model the frequency response of individual transmission lines, can be extended for an arbitrary a.c. system frequency response and their performance compared with the methods presented here. Once a rational function approximation has been made of the frequency response two possible options are ; (i) RLC network implementation, (ii) a straight incorporation into state equations that can be implemented directly into the existing state variable solution method. This requires a system identification process that obtains the appropriate state equation coefficient matrices from the rational function.
CHAPTER 6  HARMONIC ASSESSMENT.

6.1 Introduction.

The problems associated with the propagation of harmonic currents in power systems are many and diverse in nature. They include extra losses in electrical machines, errors in metering, malfunction of equipment (e.g. Ripple Control), failure of power system components such as power factor capacitors, and interference in communication systems. This has led to an increasing interest in the calculation of harmonic effects on supply systems which has been exemplified by a conference specifically about power system harmonics being held in UMIST (1981), the introduction of legislation limiting the allowable harmonic levels in New Zealand and Australia, an IEEE overview (IEEE 1983) and bibliography (IEEE 1984). The main harmonic source is the static power converter, other sources such as transformer magnetizing currents, fluorescent lights etc. being relatively unimportant. As the use of converter installations rapidly grows in size and number the need for A.C. and D.C. side voltage and current harmonic calculations are becoming an important part of power system planning.

6.2 Harmonic Penetration.

Harmonic penetration is the propagation of harmonic currents in an a.c. system from a harmonic injecting source, thus resulting in voltage distortion at the various busbars. The harmonic Penetration program (HARMAC), already mentioned (Chapter 3) in connection with the derivation of a frequency-dependent model of a power system, is now considered in the context of steady-state harmonic assessment. Therefore a brief outline of the complete operation of HARMAC needs to be given.

Realistic analysis of harmonic levels requires a three phase representation of the power system so that important effects such as system unbalance can be modelled (Densem 1983). Figure 6.1 displays a structured diagram of the program HARMAC developed by Densem. The basis
Input frequency range and busbars at which harmonic impedances are required.

Read shunt capacitors, transformers, filters and unbalanced loads at fundamental frequency and form admittance matrix.

Read line data for all frequencies and include in harmonic admittance matrices.

Calculate system harmonic impedance for a reduced system.

Input current injection busbar and three phase injection data.

Solve \([I_h] = [Y_h][V_h]\) for all frequencies to obtain the three phase voltages.

Output the busbar voltages.

Calculate and output the line current flows for all frequencies.

**Figure 6.1** Structure diagram of the harmonic penetration program
of the harmonic penetration algorithm is the system nodal admittance matrix \([Y_h]\), calculated at each harmonic \(h\). Each element for a three phase study is represented by a \(3 \times 3\) block consisting of self and mutual admittances between the phases and ground. The details of the formulation of the \([Y_h]\) are given by Arrillaga et al (1985). Once \([Y_h]\) have been evaluated the harmonic current injections must be specified and with them equation 6.1 is solved for the system voltage \([V_h]\) at each harmonic of interest. If the individual system element admittances are preserved, the currents in each element can then be calculated, i.e.

\[
[I_h] = [Y_h] . [V_h]
\]  \hspace{1cm} (6.1)

When HVDC convertors are the sources of the harmonic currents then determining the injected harmonic currents is a complex problem as they cannot be considered fixed due to the strong interaction which exists between the system and a large power convertor. HARMAC requires the harmonic current injections to be specified in order to perform a harmonic penetration study. Therefore a program external to HARMAC is required for the simulation of this interaction and thus determination of the injected current harmonics. Section 6.4 deals with two methods while section 6.5 compares their results.

6.3 Characteristic Harmonics.

Idealized conditions such as balanced sinusoidal a.c. voltages at convertor terminals, perfectly constant d.c. current, equal commutation inductances and each valve conducting at equal time intervals, have been in the past commonly used. Under these conditions the harmonics that appear are known as the characteristic harmonics. For a convertor of pulse number \(p\) the characteristic harmonics on the a.c. and d.c. sides are given respectively by equations 6.2 and 6.3.

\[
h = pq \pm 1
\]  \hspace{1cm} (6.2)

\[
h = pq
\]  \hspace{1cm} (6.3)

where \(q\) is any positive integer.
Since the six pulse Graetz bridge is the basic building block of an HVDC convertor the pulse number will normally be 6 or 12 or perhaps some higher multiple of 6. The characteristic harmonics produced by the three phase bridge are well known and were reported as early as 1945 by Reid (1945). With the increase in the number and power of convertors the assumptions used are becoming invalid. The production of non-characteristic harmonics due to non-ideal conditions have become a problem. The reasons for non-ideal conditions are:

(a) firing errors.
(b) a.c. voltage unbalance and/or distortion.
(c) Ripple or modulation of d.c. current.
(d) Unbalance commutation inductance.

As the number and power rating of HVDC convertors increase the operating conditions are becoming less ideal and the problems with non-characteristic harmonics greater. This has necessitated the development of more complex methods of analysis, and invalidates some of the assumptions inherent in the earlier work.

6.4 Incorporation of HVDC Convertors.

The harmonic currents produced by convertors cannot be considered fixed as convertor operation is influenced by the voltage distortion at the terminal busbar. There is an inter-dependence between injected harmonic current and voltage distortion with the a.c. system and filters providing the link between the two. If the level of distortion is reinforced by the action of more harmonic current injection due to the convertor controllers response to the distortion, the convertors operation is unstable. This phenomena is termed "harmonic instability" and has been reported by several authors (Ainsworth 1967, Kauferle et al 1970b).

The use of physical scaled-down models (Simulators) seems an obvious method and was indeed used in early studies (Laurent 1962), however they lacked credibility for quantitative analysis of real systems. One of the main problems with physical simulators is the difficulty of modelling the strong frequency dependence of the various power system components by means of discrete components. With the
limitations of physical models harmonic studies are predominantly being performed by computer modelling.

Time-domain dynamic programs of the types used to assess the dynamic response to disturbances can also be used for steady-state harmonic studies. Without fault application the analysis is used to generate waveforms converging on the steady-state. The spectrum of the derived waveforms are obtained via an FFT algorithm once the waveforms have reached steady-state within sufficient accuracy.

Reeve et al (1974) used this approach to analyse the effect of frequency deviations on the harmonic levels. The main shortcoming of this study was the lack of accuracy of the a.c. system model at harmonic frequencies. The authors acknowledged this problem and settled for a T representation proposed by Bowles (1971).

Kitchin (1981) proposed the use of state-variable analysis for evaluating convertor harmonics and mentioned the need for such a program in conjunction with a Harmonic impedance and penetration program. The representation of the convertor was via "tearing" and replacing by equivalent sources rather than by the use of tensor analysis, and the convertor was assumed to be the only non-linear device. Since a frequency matched equivalent was not used a correct harmonic assessment is not achieved.

The dynamic simulation programs use automatic step changing algorithms to optimize program performance and to force the time intervals to fall close to valve switching instances. Therefore linear interpolation is normally used to generate the equally spaced data points required for the FFT algorithm.

Using dynamic simulation programs for steady-state solutions is expensive in computer time. This is because the steady-state solutions are arrived at only after sufficient time has elapsed to ensure that the initialization transient has decayed. The largest time constant of the transient can be a few orders of magnitude greater than the integration step-size. For reasons of numerical stability the step-size has to be kept smaller than the smallest system time constant (Williams and Smith 1973) and smaller than a quarter of the period of the highest frequency (Hay and Hingorani 1970).
Methods of reducing the computational burden such as exploitation of periodicity have been developed (Lipo 1971, Liou 1972, Ooi et al 1980), however, they greatly restrict the generality of such programs. In these approaches each cycle of the supply frequency is divided into a number of subintervals and the convertor is treated as a piece-wise linear problem. Between two successive switchings, the network is modelled by linear circuit elements, and its behaviour predicted by solving the standard state space equations. The proponents of this method acknowledge (Berube 1983) its impracticability for unbalanced conditions due to the increase in complexity and number of equations. The other shortcomings are the need for prior knowledge of the sequence of network topologies, lack of ability to represent large a.c. systems and the tedious equation formulation.

An alternative to dynamic modelling is to employ an iterative process. Several authors (Reeve and Baron 1971, Yacamini and de Oliveira 1980a,1980b,1986, Harker 1980) have adopted this approach. A structured diagram of an iterative harmonic algorithm (IHA) is given in figure 6.2. Sinusoidal voltage at the convertors a.c. busbar are assumed for the first iteration and the resulting a.c. harmonic current injections calculated. The harmonic current injections are then used to assess the voltage distortion at the convertor terminals. The controller strategy is used to update the firing instants from the distorted terminal voltage waveforms at each iteration. The distorted voltage waveform is then used to reassess the injected harmonic currents. This process continues iteratively until convergence is reached. Although this iterative algorithm is based in the frequency domain it is not entirely in this domain. The calculation of the harmonic current injections requires a time domain representation of the commutation process as well as an FFT to convert this to the frequency domain. Early studies assumed a constant d.c. system, but more recent studies include d.c. system components and their associated d.c. ripple (Yacamini and de Oliveira 1980b,1986, Yacamini and Smith 1983).

The iterative harmonic algorithm accurately represents the frequency-response of the power system by virtue of the independent individual impedances used at each frequency. The second advantage of the iterative algorithm is the extremely small computational effort required compared to a time-domain solution. However, experience with the
Read the harmonic information for the a.c. and d.c. systems.

Read the initial power flow conditions for the convertor.

Iterative Harmonic Algorithm (IHA)

Repeat Until Converged

For each convertor, solve for the a.c. current injections and the d.c. voltage.

The a.c. system voltages at the convertor terminal are found for each harmonic.

The convertor voltage and d.c. system model are used to calculate the d.c. current waveform.

The zero crossings and firing instants are updated because of the a.c. voltage distortion.

Figure 6.2 Structure diagram of the iterative Harmonic Algorithm
iterative algorithm developed at the University of Canterbury has shown that non-convergence can occur and in such cases there is little choice except to use a time-domain program which contains a frequency-matched a.c. system model. Recent studies into the non-convergence of the iterative algorithm and comparisons with time-domain simulation have been reported by Eggleston (1985). A recent publication (Yacamini and de Oliveira 1986) also indicates that multiple solutions may exist, the solutions obtained being dependent on the initial conditions. In view of these concerns a simple test system was analysed using both the iterative algorithm and time-domain algorithm in order to verify that they give results consistent with each other and with theory. The lower South Island (N.Z.) primary system is used as the test system to demonstrate the importance of using a frequency matched a.c. system model.

Figure 6.3 shows the data flow and programs used for a complete harmonic penetration study based on TCS derived assessment of the convertor interaction. ACREP has been split to show the main functions it performs and to aid clarity. A corresponding diagram showing data flow and programs used when the iterative harmonic algorithm is used is given in figure 6.4.

6.5 Illustrative Examples.

A simple system was devised to verify that the iterative algorithm and dynamic results are indeed consistent with each other and theory. The initial test system was chosen to be analytically solvable and is depicted in figure 6.5. This was necessary to enable easy identification of incorrect results when comparing the two approaches of harmonic assessment. The smoothing reactor was chosen to be artificially large (15.4 Henries or 10 p.u.) so that the d.c. current is virtually constant. The 6 pulse convertor is fed via a convertor transformer represented by a leakage reactance $X_2$ and the a.c. system, which is purely inductive, contains no harmonic filters.

Traditionally the commutating voltage is defined as the sinusoidal voltage $E_c$ which drives the commutation current (Arrillaga 1983), while the commutating reactance is the intervening reactance between the commutating voltage and convertor. This is not practical for a complex
Parameters of each transmission line over the required frequency range
2 Complete a.c. system data at fundamental and harmonic frequencies of each individual component
3 3x3 impedance loci matrix
4 Frequency-matched a.c. system equivalent
5 a.c. system data at fundamental frequency
6 Remainder of hvdc system data
7 Initial currents and voltages throughout the hvdc system
8 Voltage and current waveform data at unequally spaced intervals
9 Equally spaced waveform data
10 Harmonic levels
11 Harmonic Penetration results. The voltages and currents throughout the a.c. system

Figure 6.3 Data flow with TCS assessment of convertor interaction
Conductor data and line geometry

Transmission Line Parameter Program (TL)

Interactive Data Program (INTER)

Harmonic Penetration Program (HARMAC)

Graphical Display of the Harmonics (ACREP)

1. Parameters of each transmission line over the required frequency range
2. Complete a.c. system data at fundamental and harmonic frequencies of each individual component
3. a.c. system data at fundamental frequency
4. Remainder of hvdc system data
5. Initial currents and voltages throughout the hvdc system
6. 3x3 impedance loci matrix
7. Harmonic levels
8. Harmonic Penetration results. The voltages and currents throughout the a.c. system

Figure 6.4 Data flow with IHA assessment of convertor interaction
Figure 6.5 Simple test system

power system and the iterative algorithm uses the distorted voltage $V_t$ as the commutating voltage. The delay angle is measured from the zero crossings of the $E_c$ phase-to-phase voltages and 20 degrees was selected for this study. The other parameters were; $V_c = \sqrt{3}$ p.u., $X_1 = X_2 = 0.1$ p.u. and $I_d = 1.0$ p.u. The solution of equation 6.4 yields 19.07 degrees as the commutation angle. This was indeed verified by measurement of the TCS waveforms.

\[
\frac{\sqrt{3} X_c I_d}{V_c} = \cos(\alpha) - \cos(\alpha + \nu)
\]

where $X_c = X_1 + X_2$

A certain amount of data manipulation was required in order to compare the TCS and iterative harmonic algorithm phase angle results. The TCS algorithm uses a cosine wave on phase "A" as its reference while the iterative algorithm uses a sine wave as its reference. The iterative algorithm also treats a.c. current harmonics as injections while TCS treats them as a load current. Therefore to compare results the TCS waveforms were advanced by 1/4 of a cycle and an FFT performed. The iterative algorithm expresses the harmonics as the coefficients of a sine series while the TCS results obtained via ACREP are the coefficients for an exponential or sine series. As, $\sin(\theta + 90) = \cos(\theta)$, 90 degrees is added to the phase angle of the TCS results to obtain the sine series form.

The results for the harmonic current injected into the a.c. system for phase "A" are tabulated in table 6.1. The analytic solution assuming ideal conditions shows a vast disparity with the other results,
especially for the higher order harmonics. When the waveshape of the commutation current is considered in the analytic solution the harmonic levels obtained are far closer to the simulation results of TCS and the iterative harmonic algorithm.

Table 6.2 displays the corresponding harmonic voltages at \( V_t \) for phase "A". The results of Table 6.1 and Table 6.2 show a close agreement between TCS, the iterative algorithm and theory, once a more detailed analysis of the commutation period is carried out. However, assessment
assuming the classical ideal conditions leads to considerable errors (e.g. 245% for the 13th harmonic). This error is small for the low order harmonics and increases with the harmonic order.

Harmonic instabilities generally occur when the a.c. system is resonant near a low order harmonic and the short circuit ratio is small. To investigate problems with convergence of the iterative algorithm a set of typical filters was added to the test system and the a.c. system reactance was chosen to cause a parallel resonance with the harmonic filters near the third harmonic. TCS was used to study the same situation and assess whether a prospective non-convergence was indicative of a harmonic instability or resulted from the numerical solution. The modified test system is displayed in Figure 6.6. As the power rating of the convertor was increased the iterative algorithm took longer to converge and eventually failed to converge. Tables 6.3 & 6.4 show the results of the iterative algorithm and TCS with a convertor power rating low enough to allow convergence. Under these arduous conditions when the iterative algorithm did converge the results compared well with the TCS simulation results. However the non-convergence was found to be due to a numerical instability and thresholding the uncharacteristic harmonics at each iteration overcame this problem in many cases.

The two previous test systems were balanced and therefore no uncharacteristic harmonics were expected except those due to the finite precision of the computer. Also, the potential for modelling the frequency-dependence of an a.c. system accurately, inherent in the iterative algorithm's formulation, was not used.
Table 6.3 Comparison of Phase "A" Harmonic Currents

Table 6.4 Comparison of Phase "A" Harmonic Voltages.

Next the Lower South Island (N.Z.) system was used as the frequency-dependent a.c. test system. Two TCS simulations were performed, the first using a Thevenin equivalent, the second the frequency-matched equivalent circuit with implicit incorporation of mutual effects. The iterative algorithm was then used to assess the convertors interaction with the a.c. system and compared with the TCS results. The results are summarized in Tables 6.5 (a),(b)&(c) and tables 6.6 (a),(b)&(c) for a Thevenin and frequency-matched a.c. equivalent respectively.
<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>Phase A</th>
<th>Phase B</th>
<th>Phase C</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TCS</td>
<td>IHA</td>
<td>TCS</td>
</tr>
<tr>
<td>1</td>
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Table 6.5(a) Phase "A" Harmonic Currents

<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>Phase A</th>
<th>Phase B</th>
<th>Phase C</th>
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<td>TCS</td>
</tr>
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Table 6.5(b) Harmonic Voltages.

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Table 6.5(c) Comparison of d.c. Current Harmonics
### Table 6.6(a) Phase "A" Harmonic Currents

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<th>Phase A (TCS)</th>
<th>Phase A (IHA)</th>
<th>Phase B (TCS)</th>
<th>Phase B (IHA)</th>
<th>Phase C (TCS)</th>
<th>Phase C (IHA)</th>
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</thead>
<tbody>
<tr>
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<td>IHA</td>
<td>TCS</td>
<td>IHA</td>
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### Table 6.6(b) Harmonic Voltages

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<th>DC Current (IHA)</th>
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<td>0</td>
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<td>1.8991</td>
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<td>0.0044</td>
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<td>0.0005</td>
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<td>0.0002</td>
<td>0.0004</td>
</tr>
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<td>0.0025</td>
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<td>0.0001</td>
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<td>0.0016</td>
<td>0.0019</td>
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<td>0.0001</td>
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<td>0.0014</td>
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<tr>
<td>20</td>
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<td>0.0001</td>
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<tr>
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</tr>
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</table>

### Table 6.6(c) Comparison of d.c. Current Harmonics

Table 6.6 TCS/IHA comparison with frequency matched a.c. system equivalent
The TCS/IHA comparison with Thevenin a.c. system equivalent shows reasonable agreement between the harmonic current injections, but considerable disparity between the harmonic voltages at the converter terminals. This indicates that the harmonic current is reasonably insensitive to the frequency response of the a.c. system while the voltage distortion is greatly dependent on it. It can also be noted that the TCS evaluated voltage harmonics are larger than those of the IHA, e.g. 10 times larger for the 25th harmonic, 2 times for 23rd harmonic and 1.5 times for the 19th harmonic. The impedance of the Thevenin equivalent increases with increasing frequency while that of the actual system shows numerous troughs and peaks as frequency increases. The nett result is that the Thevenin impedance tends to be larger than that of the actual system at higher frequencies. Since, as mentioned previously, the harmonic current level is approximately the same the assessed voltage distortion is considerably higher and thus the disparity increases with increasing frequency.

The TCS results using the Frequency-matched a.c. system show good correspondence with the Iterative algorithm (IHA). In particular the higher order voltage harmonics show very good agreement.

Thresholding of the harmonics at each iteration was not used as this made the numerical process less stable and caused non-convergence in some simulations of unbalanced a.c. systems.

6.6 Computational Efficiency.

The CPU requirements of the TCS algorithm with Thevenin or frequency-dependent model and the corresponding IHA simulation have a complex dependence on many factors, such as the relative strength of the a.c. system and convertor rating, convergence tolerances, initial conditions etc. Therefore the numerical comparison made in table 6.7 is only valid for the test cases used to derive them and provides only an approximate indication of their relative CPU requirements. The frequency response of the lower part of the New Zealand South Island system was used for the two cases listed where frequency-dependence was modelled. The results were obtained on a VAX 11/750 computer.
As mentioned previously TCS requires a number of cycles of simulation for the initialisation transient to die down before the harmonic assessment can be made. The number of cycles required depends largely on the time constants and damping of the circuit as well as the specified initial conditions. Generally eight to ten cycles are sufficient.

It should also be noted that TCS has a greatly increased CPU requirement for the last cycle of simulation. This is caused by the larger number of data points being written out into a data file, in order to increase the precision of the FFT process. Moreover, as the TCS program incorporates an adaptive step-size selection algorithm, the program JAUGPS must derive a set of equally spaced data points from TCS's unequally spaced information.

By way of example the TCS solution takes about 32 minutes to assess the harmonic levels with a frequency-dependent a.c. system equivalent. In contrast the IHA program only takes 4 minutes to produce the same information.

<table>
<thead>
<tr>
<th>A.C. System model used in both algorithms.</th>
<th>TCS per cycle</th>
<th>TCS total</th>
<th>IHA total</th>
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<tr>
<td>Thevenin</td>
<td>106</td>
<td>283 (last cycle)</td>
<td>1131</td>
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<tr>
<td>Frequency-dependent</td>
<td>183</td>
<td>439 (last cycle)</td>
<td>1903</td>
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Table 6.7 Comparison of Computational Efficiency (in sec.)
6.7 Harmonic Penetration Example.

To complete the harmonic penetration study the assessed convertor currents, obtained from either TCS or IHA, are supplied to HARMAC, which then calculates the branch harmonic currents and busbar harmonic voltages. When TCS is used to assess the steady state convertor currents the harmonic penetration study is self checking; when the frequency response of the a.c. system has been correctly modelled, the harmonic voltages at the terminal busbar derived from HARMAC will agree with the harmonic voltages obtained from the TCS simulation for this busbar.

To illustrate the use of TCS in conjunction with HARMAC for harmonic penetration studies the assessed current using a frequency-matched a.c. system equivalent (Table 6.6(a)) were used in HARMAC to obtain the harmonic penetration throughout the a.c. system. The busbar voltages and branch currents are summarized in Tables 6.8 and 6.9 while figures 6.7 and 6.8 graphically display these results.

![Harmonic Voltage Magnitudes](image)

**Figure 6.7** Harmonic voltage throughout the Lower South Island of New Zealand test system.
<table>
<thead>
<tr>
<th>Harmonic Order</th>
<th>Busbar</th>
<th>INVERCARGILL</th>
<th>MANAPOURI</th>
<th>ROXBURGH</th>
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<tbody>
<tr>
<td>5</td>
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<td>220 kV</td>
<td>14 kV a</td>
<td>14 kV b</td>
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Table 6.8 Harmonic Voltages Throughout the Lower South Island of New Zealand test system. Based on TCS assessed harmonic currents using implicit accounting for frequency-dependent mutual coupling.
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Branch Designation

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<td>MANAPOUR2014 TO MANAPOUR1220</td>
<td>B</td>
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<td>C</td>
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<td>MANAPOUR1220 TO INVERCARG220</td>
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<td>E</td>
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<tr>
<td>INVERCARG220 TO ROXBURGII-220b</td>
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<tr>
<td>ROXBURGII-220 TO ROXBURGII-011</td>
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Table 6.9 Harmonic Branch Currents at sending end, throughout the Lower South Island of New Zealand test system. Based on TCS assessed harmonic currents using implicit accounting for frequency-dependant mutual coupling.
6.8 Conclusions.

The computational cost clearly makes it desirable to use the iterative algorithm (IHA) wherever possible. However, the possibility of false solutions and numerical instability exist with the iterative algorithm and in such cases a TCS simulation program must be used to obtain reliable results.

In general, increasing the number of sample points used for the FFT improves the accuracy. However, beyond a certain point the accuracy deteriorates as the round-off errors and errors associated with obtaining equally spaced data points, build up. Although the results of the previous sections were all obtained using 2048 sample points, subsequent tests have shown that the use of 1024 sample points provides better results.
6.9 Future Work.

Further work is required into the numerical instabilities of the iterative algorithm. Two factors can contribute to the IHA failing to converge, they are: (i) a very weak a.c. system and/or (ii) a resonance between the a.c. system and harmonic filters near a low order harmonic. It appears that thresholding the harmonics at each iteration helps convergence in the case of balanced systems but hinders convergence if the a.c. system is unbalanced. With reference to the recent work by Yacamini and de Oliveira (1986) the question must be asked of whether multiple solutions can exist depending on the initial conditions. This possibility must be investigated before any faith can be placed on the IHA results.

The elusive goal of studying the effect and interaction of multiple harmonic sources, such as two convertors, needs to be addressed. This is particularly relevant to New Zealand's South Island system where there exist two large convertors, the HVDC link with convertor station at Benmore and an aluminium smelter at Tiwai. At present the TCS approach looks the best for achieving this as it can already model multiple convertor systems and has proved reliable. However, the accuracy of the frequency-dependent model may be the limiting factor. Although the extension of the frequency-dependent equivalent for multiple harmonic sources was detailed in chapter 4 the alterations required to implement this in TCS have yet to be made.
7.1 Introduction.

To demonstrate the effect of the a.c. system representation on the dynamic performance of HVDC convertors various faults are applied to the a.c./d.c. system depicted in figure 7.1.

The lower South Island of New Zealand is again used as the test system and comparisons are made between a Thevenin equivalent of this system and a frequency-matched equivalent. The filter data used is the monopolar equivalent of the New Zealand hvdc link filters (Robinson 1966). A simple Thevenin representation is used for the a.c. system at the receiving end of the d.c. link (system 2) in sections 7.2 and 7.3. Finally, in section 7.4, the detailed a.c. representation and the a.c. fault location are placed at the receiving end (system 2) in order to assess the effect of the disturbance at the invertor end.

![Figure 7.1 HVDC Test System](image-url)
7.2 D.C. Line Fault Simulation.

Figures 7.2 (a)&(b) show the rectifier current and d.c. voltage on the convertor side of the rectifier's smoothing reactor, when a d.c. line fault is applied midway between the two convertors. The control strategy on the rectifier is such that on detection of the fault the convertor is
Figure 7.3 System after Fault application

ordered to operate in the inversion mode. This speeds up the de­
energizing of the line and hence arc extinction. Once the arc has been
extinguished the convertor is in the blocked (non-conducting) state.
During this period the d.c. voltage oscillates with the natural frequency
of the line. After a delay larger than the deionization time the line can
be re-energized and normal operation resumed.

Figure 7.2 shows that the convertor currents estimated by the
different models differ only slightly. This can be explained with
reference to figure 7.3 which displays the circuit looking from the
rectifier end immediately after fault occurrence. Due to the position of
the fault, the transformer, smoothing and d.c. line reactances limit the
rate of change of current during the fault.

The d.c. voltage waveforms show (figure 7.2 (b)) greater variation
between the various alternative models. This is due to its sensitivity to
the trapped energy in the d.c. line once blocking of the convertors has
occurred. The d.c. voltage is primarily determined by the inductor
voltage and hence dependent on the rate of change of current.
7.3 Rectifier Side a.c. System Disturbance.

The fault is a single phase short-circuit applied to phase "A" and the single line diagram of figure 7.4 depicts the fault location. Figures 7.5 (a)&(b) display the a.c. system current and busbar voltage waveforms respectively, when a Thevenin equivalent is used. Figures 7.6 (a)&(b) show the corresponding currents and voltages when a frequency-matched a.c. system equivalent is used. There is considerable difference between the two sets of results.

The interaction of the power system and convertor controller, after a disturbance, complicates the transient behaviour. Thus to observe exclusively the effect of the different a.c. system representations the convertor's C.C.C. and E.A.C. controllers are disabled. Figures 7.7 (a)&(b) and figures 7.8 (a)&(b) display the a.c. system currents and busbar voltage waveforms with the Thevenin and the frequency-matched equivalents respectively. There are considerable differences between the two sets of results; for instance the peak voltage in phase "B" for the frequency-matched equivalent (figure 7.8(b)) is 28% greater than the corresponding peak for the Thevenin alternative (figure 7.7(b)). With the Thevenin model the a.c. system current (figure 7.7(a)) on the faulted phase becomes sinusoidal as the fault forms a simple RL series circuit driven from the fundamental-frequency voltage source, with no possibility of resonances occurring. However the corresponding current with the frequency-matched a.c. system equivalent (figure 7.8(a)) does show some distortion due to the resonances present. Moreover the frequency-matched

![Diagram of Rectifier Side a.c. System Fault](image_url)

Figure 7.4 Rectifier side a.c. system fault
Figure 7.5 Dynamic response due to Rectifier side Fault with Thevenin Representation

The a.c. system equivalent also models the coupling between phases and thus any distortion present in the two unfaulted phases is reflected into the faulted phase. The fault effectively shorts out the harmonic filters on the faulted phase and thus the dynamic response is that of the a.c. system following the sudden application of a short circuit.

It is also interesting to compare figures 7.8 (a) & (b) with figures 7.6 (a) & (b), as this clearly demonstrates how the convertor's controller action has considerable influence on the dynamic performance. The busbar voltage peak in phases "B" & "C" are substantially reduced due to the effect of the controller.
Figure 7.6 Dynamic response due to Rectifier side fault with Frequency-dependent Representation
Figure 7.7  Dynamic response due to Rectifier side Fault with Thevenin Representation
Figure 7.8 Dynamic response due to Rectifier side Fault with Frequency-dependent Representation

In this series of simulations it is the representation of a.c. system 2 that is varied and faulted, as depicted in figure 7.9. An HVDC system is more sensitive to disturbances in the invertor side of the link than the rectifier due to the possibility of commutation failures. Hence any differences in the dynamic response should be more observable. Three alternative a.c. system representations were used i.e. a Thevenin equivalent and a frequency-matched equivalent (No. 3, figure 5.36) with and without mutual coupling. Figures 7.10, 7.11 & 7.12 display the busbar voltage, a.c. system current and fault branch current respectively, for each of the representations.

![Diagram of invertor side a.c. system fault](image)

Figure 7.9 Invertor side a.c. system fault

Figures 7.10 (b)&(c) show that the resonances following fault application cause numerous multiple zero crossings, which always pose a problem for convertor controllers.

Following the voltage collapse in the faulted phase, the corresponding convertor valves cannot conduct, hence with the Thevenin equivalent, phase "A" is a simple RL circuit uncoupled to the other two phases. Therefore the phase "A" a.c. system current (figure 7.11(a)) is sinusoidal for the Thevenin equivalent. Figure 7.11(b) shows that with the frequency-dependent model (without mutual coupling) the phase "A" a.c. system current is distorted immediately after fault application but the distortion dies away with time. This distortion is due to the dynamic
(a) Thevenin Representation

(b) Frequency-dependent representation without mutual coupling

(c) Frequency-dependent representation

Figure 7.10 Faulted Busbar voltage
(a) Thevenin Representation

(b) Frequency-dependent representation without mutual coupling

(c) Frequency-dependent representation

Figure 7.11 A.C. System Current
Figure 7.12 Fault current

response of the circuit that models the frequency-response and hence the resonances of the a.c. system. The remaining two phases experience enormous distortion due to the upset in the valve conduction pattern. When a frequency-matched equivalent with mutual coupling is used (figure 7.11(c)) this distortion is reflected into phase "A" causing the faulted phase current to be grossly distorted.

It is also interesting to note the permanent current offset that occurs in the fault branch current, which is dependent on the point of fault application, when either the Thevenin model or frequency-matched model without mutual coupling are used. Although the offset also appears when the frequency-matched and coupled equivalent is used, in this case it appears to decay with time (figure 7.12).

7.5 Discussion and Conclusions.

The choice of gains for the convertor controller requires considerable experience as dynamic instability can occur due to the interaction between the a.c. system and convertor. The strength of the a.c. system connected to the convertor is one factor that influences this choice. The Thevenin equivalent can suffer from this type of instability, although it is less prone than the frequency-matched equivalent. Such instabilities are also experienced in practice (Gunn 1966) and have been
the subject of much investigation. Although it is not the purpose of the thesis to deal with this problem, it has been demonstrated that the controller does significantly influence the dynamic response. Moreover with a valve in a bridge being fired nominally every 60 degrees the convertor has the ability to respond quickly to changes by advancing or delaying the firing instant.

The rectifier current is relatively insensitive to the a.c. system representation when d.c. line faults are simulated. The reasons for this are; the presence of transformer, smoothing and line reactances, the symmetry of the fault and the presence of the harmonic filters. The d.c. voltage cannot be influenced by the a.c. system representation when the convertor is blocked; however, in the interval between fault application and convertor blocking the a.c. system equivalent will have considerable effect on the dynamic behaviour and hence on the line oscillations that occur after blocking.

The a.c. system representation has a greater influence on the dynamic behaviour following an a.c. system disturbance. The effect of the frequency-dependent self impedances is to distort the faulted phase current due to the dynamic response caused by an abrupt change, the amount of distortion, and its duration, being determined by the characteristics of the a.c. systems frequency-response. Finally the addition of mutual coupling also shows the current distortion induced by the unfaulted phases, which is more realistic.
CHAPTER 8 CONCLUSIONS

With the rapid growth in both size and number of HVDC schemes there is a greater need for the accurate simulation of transient phenomena, particularly in cases where the d.c. convertor station feeds into a relatively weak a.c. system. Although explicit modelling of essential a.c. components (e.g. convertor transformer, harmonic filters) has been retained, due to the size and complexity of modern power systems some form of equivalent circuit must be used for the remainder of the a.c. system. Towards this end the main aim of this thesis has been to present, and show the effect of, a three phase a.c. system model that accurately represents the a.c. system at harmonic frequencies. Both the frequency-dependence of the self and mutual terms associated with each phase have been accurately modelled. Several frequency-matched a.c. system models have been presented with any inherent assumptions outlined.

The accuracy of the proposed frequency-matched a.c. system models has been verified by the use of a Harmonic current source model to inject a specified harmonic component and applying an FFT to the resulting time domain waveforms. These results compared very well with the expected values derived from a harmonic penetration study of the complete system in the frequency domain. Further verification of the proposed a.c. system equivalent was made by comparison between an Iterative Harmonic Algorithm and Transient Convertor Simulation for assessing convertor harmonics.

Both a.c. and d.c. system disturbances have been presented and the response of the proposed models show considerable difference with the Thevenin representation, particularly in the case of an asymmetrical fault.

The use of a Thevenin model based on the S.C.R. of the a.c. system only represents the system accurately at fundamental frequency and therefore gives inaccurate results at harmonic frequencies. The resonances between the a.c. system and harmonic filters as well as resonances within the a.c. system must be modelled for accurate dynamic simulation. It is the combined impedance response of the a.c. system in parallel with harmonic filters that needs to be accurately modelled. The resonances in the parallel combination may be excited by disturbances,
resulting in waveform distortion and possible overvoltages, which will affect the converter controllers and their response. If the harmonic filters dominate the combined response at the high order harmonics then the a.c. system model may only need to be accurate at the lower order harmonics, below the high pass filter's resonant frequency. Bowles (1970) and Giesner (1971) both proposed equivalents that more realistically represented the a.c. system at low order harmonics on this basis. However, a general model should be capable of representing a.c. system resonances at relatively high frequencies (such as the case of the New Zealand Lower South Island system), when these are not swamped out by the presence of filters.

The most significant difference with respect to the models proposed by Hingorani and Burbery (1970), Bowles (1970) and Giesner and Arrillaga (1971) is the modelling of the mutual effect between phases.

With reference to the New Zealand HVDC link the initially installed filters were tuned to the 5th, 7th, 11th & 13th. The N.Z. Electricity Department did this as some HVDC authorities recommended a "wait and see" policy about harmonic problems, for there is no justification in providing expensive filter branches to guard against trouble that might not occur. However, early operating experience showed a need for a high-pass filter and 9th harmonic filters to reduce the widespread telephone interference that occurred. The 9th harmonic problem was a result of a resonance between the filters and a.c. system (Robinson 1966). The development of the ACREP program described in this thesis allows the a.c. system, filter and combined response to be inspected and any resonance problems forecasted. This allows remedies to be found before the troubles occur. Also the combined response of "a.c. system equivalent" and filters can be compared to the combined response of "actual" a.c. system and filters in order to assess the sophistication needed in the a.c. system equivalent for accurate steady state and dynamic studies.

In steady state harmonic penetration studies, frequency-matched a.c. system equivalents are needed as the small levels of higher order harmonics are greatly affected by small errors in the combined response. A table of harmonics can easily display relatively large discrepancies at high order harmonics without any noticeable difference in the waveform.
The mutual coupling between phases has a considerable influence on the harmonic level, particularly at the higher order harmonics. Most power systems try to balance the loading on the phases as best as practicable, and therefore even if mutual coupling is not to be explicitly represented it is still better to diagonalize the 3x3 impedance matrices than to simply ignore the mutual coupling and use the self terms.

The differences between the alternative a.c. system models have been clearly illustrated with reference to an asymmetrical fault applied to the a.c. side of an a.c./d.c. system.

**Future Work.**

The response of a d.c. link to sudden changes in system conditions or requirements is dependent on the convertor controller's response. At present a simple algebraic relationship is used to simulate what is in practice a complex circuit. The TCS convertor controller needs to be enhanced so that its dynamic behaviour more realistically models the controller it represents. As the type and operation of convertor controllers differ for each application, a modular approach is required which allows the controller's transfer function to be built up of standard transfer blocks. Once this has been achieved extension to the fault detection and protection mathematical models can be made. For example the advantages of altering the normal firing sequence once a fault has been detected (Graham 1984) can be demonstrated.

Although the modelling of d.c. line faults has been carried out in detail, further work is needed for the a.c. fault simulation, in particular the arc characteristics of the a.c. fault and more flexible circuit breaker models are required.

The proposed extension of the frequency-matched a.c. system equivalent for multi-convertor systems needs to be implemented and tested. This would permit investigation of the interaction of two harmonic sources connected to the same a.c. system. This is relevant to
the Lower South Island of New Zealand where Tiwai and Benmore busbars both have large a.c./d.c. convertors connected.

With the recent advances in transformer and a.c. system representation, when the improvements in the convertor controller are completed, the Transient Convertor Simulation program will be a very powerful tool for both steady-state and dynamic HVDC system studies.
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APPENDIX A1

DETAILED SYNCHRONOUS MACHINE COEFFICIENT MATRIX.

This matrix, \( L_g \), is a 6x6 matrix which can be partitioned into stator, stator-rotor, and rotor terms as follows:

\[
L_g = \begin{bmatrix}
L_{ss} & L_{sr} \\
L_{rs} & L_{rr}
\end{bmatrix}
\]

where

\[
L_{ss} = \begin{bmatrix}
-L_{aa} - L_{aa2} \cos(2\theta) & -L_{ab} - L_{ab2} \cos(2(\theta + 30)) & -L_{ac} - L_{ac2} \cos(2(\theta + 150)) \\
-L_{ab} - L_{ab2} \cos(2(\theta + 30)) & -L_{bb} - L_{bb2} \cos(2(\theta - 120)) & -L_{bc} - L_{bc2} \cos(2(\theta - 90)) \\
-L_{ac} - L_{ac2} \cos(2(\theta + 150)) & -L_{bc} - L_{bc2} \cos(2(\theta + 90)) & -L_{cc} - L_{cc2} \cos(2(\theta + 120))
\end{bmatrix}
\]

\[
L_{sr} = \begin{bmatrix}
L_{afd} \cos(\theta) & L_{akd} \cos(\theta) & -L_{akq} \sin(\theta) \\
L_{bfd} \cos(\theta - 120) & L_{bkd} \cos(\theta - 120) & -L_{bkq} \sin(\theta - 120) \\
L_{cfd} \cos(\theta + 120) & L_{ckd} \cos(\theta + 120) & -L_{ckq} \sin(\theta + 120)
\end{bmatrix}
\]

\[
L_{rs} = L_{sr}^{t}
\]

\[
L_{rr} = \begin{bmatrix}
L_{fd} & L_{fkd} & 0 \\
L_{fkd} & L_{kd} & 0 \\
0 & 0 & L_{kq}
\end{bmatrix}
\]

The torque matrix is a 6x6 matrix derived from \( L_g \):

\[
\frac{d}{d\theta} L_g = \begin{bmatrix}
\frac{\partial L_{ss}}{\partial \theta} & \frac{\partial L_{sr}}{\partial \theta} \\
\frac{\partial L_{rs}}{\partial \theta} & \frac{\partial L_{rr}}{\partial \theta}
\end{bmatrix}
\]

where \( \frac{\partial L_{rr}}{\partial \theta} = 0 \) and \( \frac{\partial L_{rs}}{\partial \theta} = (\frac{\partial L_{sr}}{\partial \theta})^{t} \)
Overhead transmission lines or cables are defined by their series impedance \((R + X_c)\) and shunt susceptance \((B_c)\) matrices per unit length. The elements of the series impedance and shunt susceptance matrices are derived from the geometry of the tower configuration, the characteristics of the conductors and from the earth resistivity. A method for calculating these matrices is described by Arrillaga et al (1983) and summarized here:

To obtain the capacitance matrix, Maxwell's potential coefficient matrix \([P]\) is first obtained from the geometry of the tower configuration and from the conductors' radii. The diagonal element of \([P]\) is

\[
P_{ii} = \frac{1}{2\pi\varepsilon_0} \ln \frac{2h_i}{r_i} \quad \text{(km/F)}
\]

and the off-diagonal element is

\[
P_{ik} = \frac{1}{2\pi\varepsilon_0} \ln \frac{D_{ik}}{d_{ik}} \quad \text{(km/F)}
\]

where

- \(r_i\) - radius of conductor \(i\)
- \(D_{ik}\) - distance between conductor \(i\) and image of conductor \(k\) (in m)
- \(d_{ik}\) - distance between conductors \(i\) and \(k\) (in m)
- \(h_i\) - average height above ground of conductor \(i\) (in m)
- \(\varepsilon_0\) - Permittivity constant

Note that the potential coefficient matrix is real and symmetric. The capacitance matrix is obtained by inverting matrix \(P\).

\[
[C'] = [P]^{-1}
\]

When only the phase quantities are of interest the capacitance matrix \([C']\) can be reduced to a matrix \([C]\) called the capacitance matrix for the "equivalent phase conductors". Details of the elimination of ground wires and bundling of conductors are given in the following two references; Dommel (1978) and Dommel (1980). The reduction process for the series impedances and Maxwell's potential coefficients are identical.
The diagonal elements for the series impedance is given by:

\[ Z_{ii} = (R_{ii} + \Delta R_{ii}) + j(2\omega 10^{-4} \ln \frac{2h_i}{GMR_i} + \Delta X_{ii}) \]  \( \Omega/km \)

The mutual impedance terms are given by

\[ Z_{ik} = (R_{ik} + \Delta R_{ik}) + j(2\omega 10^{-4} \ln \frac{D_{ik}}{d_{ik}} + \Delta X_{ik}) \]  \( \Omega/km \)

where
- \( R_{ii} \) - a.c. resistance of conductor \( i \)  \( \Omega/km \)
- \( GMR_i \) - geometric mean radius of conductor \( i \)  \( \Omega/km \)
- \( f \) - frequency in Hz,
- \( \omega = 2\pi f \)
- \( \Delta R_{ii}, \Delta X_{ii} \) - are Carson's correction terms for earth return effects

Note for the self impedance terms that the geometric mean radius (GMR) instead of the actual radius \( r \) is used to account for the contribution which the internal inductance makes to the total inductance. The series impedance matrix is also symmetric as \( Z_{ik} = Z_{ki} \).

The equivalent circuit of a typical transmission line segment is illustrated in figure A2.1. As the chosen state variable is the charge at a node rather than capacitor charge, the capacitance matrix \([C''\)] is required. The elements of \([C'\)] are related to the capacitor matrix for the equivalent circuit \([C]\) by:

\[ C_{aa} = C_1 + C_4 + C_5 \]
\[ C_{ab} = -C_4 \]

\[
\begin{array}{ccc}
  a' & b' & c' \\
  a & C_{aa} & C_{ab} & C_{ac} \\
  b & C_{ba} & C_{bb} & C_{bc} \\
  c & C_{ca} & C_{cb} & C_{cc} \\
\end{array}
\]

\[
\begin{array}{cccc}
  L_{aa} & L_{ab} & L_{ac} & L_{ae} \\
  L_{ba} & L_{bb} & L_{bc} & L_{be} \\
  L_{ca} & L_{cb} & L_{cc} & L_{ce} \\
  L_{ea} & L_{eb} & L_{ec} & L_{ee} \\
\end{array}
\]
Figure A2.1 Transmission Line π-segment Model
Appendix A3

Parameters for RLC circuit analysis example.

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Table A3.1 Parameters for the circuit of figure 4.20
Appendix A4

Single Variable Optimization Methods.

Let \( x_1 \) and \( x_2 \) be two \( x \) values that bracket the actual minima. Then the method that successively halves the interval known to contain the minima by placing two test points close together at the centre of the interval, is known as a Dichotomous Search. The dichotomous search is illustrated in figure A4.1. A pair of test points are required at each step in order to be able to distinguish which half of the interval being divided contains the minima. The small difference \( \Delta x \) between the evaluations of each test point is limited by the accuracy in which the objective function can be evaluated and sets a limit on the resolution of the optimization.

![Figure A4.1 Dichotomous Search Technique](image)

The dichotomous search is inefficient in terms of objective function evaluations as at each interval reduction two search points that are very close together are evaluated. By using one of the two search points in the next interval to be searched enhances efficiency as only one additional search point is needed for each successive interval reduction. Figure A4.2 illustrates this process. At the first step the two test points \( x_{a,k} \) and \( x_{b,k} \) are required. This discriminates whether the minimum is in the left or right hand region \( (I_{k+1}^L \text{ or } I_{k+1}^R) \). If say the minima lies in the latter interval then only one new test point \( x_{b,k+1} \) is required for the next interval reduction, where \( I_{k+2}^L \) and \( I_{k+2}^R \) are the two possible regions.
The Fibonacci Search requires the placement of the first two evaluations so that the search ends when the last two evaluations coincide and halve the final interval. If the final interval is $I_n$ then the length of all previous intervals are given by:

$$I_k = I_{k+1} + I_{k+2}$$

(A4.1)

Note that at the final step the interval $I_{n+1}$ is created equal in length to $I_n$ but is not used. However, it is needed in equation (A4.1) to determine $I_{n-1}$. The ratio of the $k^{th}$ interval length to the final interval length then forms the well known Fibonacci sequence.

$$I_{n+1} = I_n$$

$$I_{n-1} = 2I_n$$

$$I_{n-2} = 3I_n$$

$$I_{n-3} = 5I_n$$

$$I_{n-4} = 8I_n$$ etc...

The Fibonacci search achieves the greatest interval reduction of methods requiring a fixed number of function evaluations.
The main drawback of the Fibonacci Search is the need to specify the number of objective function evaluations in advance. The Golden Section Search overcomes this by placement of the test points so that a constant ratio between successive interval lengths exists. The ratio is 1.618034, which is obtained by solving the appropriate quadratic equation which results from imposing this condition.

The problem with the previous methods is the need to specify two points that bracket the true minima. The Step Success/Failure Search, like the dichotomous search, uses two closely positioned test points. However, only one initial starting point is required. The algorithm then steps in the direction of the minima and evaluates a new pair of test points. Based on these results, the algorithm either: steps again in the same direction; halves the step length and steps in the new direction indicated; or remains at the same point and reduce the distance between the two test points. A flow diagram for the Step Success/Failure search is displayed in Figure A4.3.
Evaluate the objective function at two test points that are closely spaced and straddle the current point under consideration.

- Do these two test points indicate the minima lies at higher value?
  - Yes
  - No → Halve the step length

- Do these two test points indicate the minima lies at lower value?
  - Yes → Halve the distance between the two test points
  - No

- Was the last step taken in this direction?
  - Yes → Step in direction of Minima
  - No

- Is the step length smaller than test point spacing?
  - Yes → Halve test point spacing
  - No → Have too many halvings of test point spacing occurred?
    - Yes → Output warning message
    - No

- Is the step length smaller than convergence tolerance?
  - Yes
  - No → Has the maximum number of iterations been reached?
    - Yes → Output Error Message
    - No → Output final solution

- Was the last step taken in this direction?
  - Yes → Halve the step length
  - No → Step in direction of Minima

Figure AN.3 Success/Failure Step Algorithm
Appendix A5

Published Papers.
A.C. SYSTEM EQUIVALENTS FOR THE DYNAMIC SIMULATION OF HVDC CONVERTORS

N.R. Watson, J. Arrillaga, A.P.B. Joosten

University of Canterbury, Christchurch, New Zealand.

INTRODUCTION

The behaviour of hvdc convertors following small or large disturbances can be assessed by means of scaled down physical models or computer analysis in the time domain. In both cases, models with various degrees of complexity are already available to represent the convertor configuration, control, filters and convertor transformers; in contrast the degree of representation of the a.c. network leaves much to be desired.

Given the size and complexity of a power system it is not practical to model each of its components individually. Equivalent circuit models are usually derived to suit the fundamental frequency, the simplest being the short circuit impedances used in fault studies. Such models, however, are inadequate to represent the system behaviour at harmonic frequencies.

AC networks are generally inductive for the lower harmonics. Since the convertor filters are often capacitive for such frequencies, a parallel resonant condition may exist, typically at around the third or fourth harmonic, or even at higher frequencies depending on the a.c. system strength. Moreover a complex system/filter combination will produce several oscillatory harmonic frequencies of relatively low orders. Disturbances may excite these natural frequencies and produce considerable waveform distortion and overvoltages which will in turn interact with the convertor control. Symmetrical impedance loci are currently measured or calculated for the complete harmonic spectrum at various operating conditions. These can be used to derive accurate harmonic equivalents (1)(2) for the a.c. system. However it has been shown (3) that there is considerable diversity between the resonant frequencies seen by different phases and thus in general the use of three-phase harmonic equivalent circuits will be more appropriate.

This paper describes the derivation of three-phase harmonic equivalent circuits to represent a.c. networks and illustrates their effect on the dynamic behaviour of hvdc convertors.

HARMONIC MODELLING OF THE A.C. NETWORK

System Harmonic Impedances

Information of the system harmonic impedances is often presented in the form of impedance loci. These can be derived either experimentally or analytically. The latter provides a more flexible alternative, which can assess any specified critical operating condition and can derive the loci at any number of interharmonic frequencies without the need for interpolation.

Analytic impedance loci can be obtained from a computer algorithm recently developed at the University of Canterbury makes use of the admissibility matrix and linear transformations to interconnect the various system components represented by appropriate equivalent circuits. Transmission lines generally have the greatest influence on the impedance loci and need to be accurately represented i.e. by means of coupled multiconductor Equivalent circuits with frequency dependent distributed parameters. (3)(4)

As an illustration, Fig. 1 shows the New Zealand South Island transmission system in the vicinity of an aluminium smelter (at Tiwai) and the three phases of the corresponding impedance locus is illustrated in Figure 2.

Derivation of Harmonic Equivalents

In an earlier model (1) the selected minima and maxima frequencies used for the derivation of the equivalent circuit are those where the impedance locus crosses the real axis (refer to Figure 2). However the impedance versus frequency plots derived from the harmonic impedance locus provide more explicit information of the maxima and minima harmonic impedances.

The equivalent circuit will consist of a number of single-tuned parallel branches as shown in figure 3 and the first step is to obtain values for the inductors and capacitors of the equivalent circuit that will produce the same minima and maxima resonance frequencies. The resistances of the branches are ignored as they have very little influence on the magnitude of the admittance at frequencies that are sufficiently higher or lower than the series resonant frequency of the branch.

The admittance of an n branch network is:

\[ Y(s) = \sum_{k=1}^{n} \frac{s}{\frac{1}{L_k (s^2 + w_k^2)} - \frac{1}{s}} \]

(1)

where:

\[ w_k^2 = \frac{1}{LC_k} \quad \text{for } k=1,2,\ldots, \text{etc} \]

and \( s = jw \).

For an LC network the impedance is zero at minima and infinite at maxima frequencies.

Impedance minima occur when:

\[ \frac{n}{1} \sum_{i=1}^{n} (s^2 + w_i^2) = 0 \]

(2)

Impedance maxima will occur when the numerator of equation (1) is zero, i.e.
\[
\frac{n}{\sum_{k=1}^{n} \frac{1}{R_k J_k}} \left( s^2 + M_k^2 \right) = 0
\]  
(3)

Let \( F_1, F_2, \ldots, F_n \) be the frequencies at which the admittance is zero, then equation (3) can be written as:

\[
s \sum_{k=1}^{n} \left( s^2 + M_k^2 \right) = 0
\]  
(4)

By equating the coefficients of \( s \) in (3) and (4) the following equations result:

For \( s^{2n-1} \)

\[
\frac{n}{\sum_{k=1}^{n} \frac{1}{R_k J_k}} = 1
\]  
(5)

For \( s^{2n-3} \)

\[
\frac{n}{\sum_{k=1}^{n} \frac{1}{R_k J_k}} = \frac{n-1}{M_k^2}
\]  
(6)

For \( s^{2n-5} \)

\[
\frac{n}{\sum_{k=1}^{n} \frac{1}{R_k J_k}} = \frac{n-1}{M_k^2}
\]  
(7)

For \( s^1 \)

\[
\frac{n}{\sum_{k=1}^{n} \frac{1}{R_k J_k}} = \frac{n-1}{M_k^2}
\]  
(8)

Thus \( n \) equations are formed which are then solved for \( n \) unknown inductances \((L_1, L_2, \ldots, L_n)\). The appropriate capacitor values are found using:

\[
w_k^2 = \frac{1}{M_k^2} \quad k=1,2,\ldots,n
\]  
(9)

where \( w_k \) are the required minima frequencies.

The admittance of the tuned network at any particular frequency is given by:

\[
Y = \sum_{k=1}^{n} Y_k = \sum_{k=1}^{n} \frac{R_k - \frac{1}{J_k}}{R_k^{*} + \frac{1}{L_k^{*}} \left( w^2 - \frac{1}{J_k} \right)}
\]  
(10)

Branch \( k \) is in series resonance when \( w_k^2 = 1/L_k C_k \). The corresponding admittance term for the \( k \)th branch is \( 1/R_k \). This term is significantly larger than the contribution from the branches not in series resonance and hence dominates the admittance. The resistance that dominates at each minima is inserted in the corresponding branch. This yields maxima point impedances for the equivalent circuit which are much higher than those of the actual system. The problem can be corrected by the scaling of all inductor and capacitor values. A common scaling factor is used for all inductors and capacitors so that the resonance frequencies remain unchanged. The appropriate scaling factor is obtained by dividing the calculated impedance of the network at a particular frequency by the actual systems impedance. Equation (10) with \( R \) ignored yields:

\[
Y = \sum_{k=1}^{n} \frac{1}{\left( \frac{1}{R_k} + \frac{1}{L_k^{*}} \right) \left( w^2 - \frac{1}{J_k} \right)}
\]  
(11)

As \( L_k \) appears in the denominator of equation (11), to increase the admittance contribution of each branch, so as to reduce the impedance, the inductors are divided by the scaling factor. In order to keep the same resonance frequencies the capacitor values are multiplied by the scaling factor.

These equivalent circuits, developed in the frequency domain, can also be used in the time domain according to standard circuit laws. When using transfer functions in the frequency domain, multiplication by the Laplace transform of the input and application of the inverse Laplace transform obtains the time solution.

Moreover, in order to prove the general applicability of the equivalent model it is necessary to match the system harmonic impedances in magnitude and phase. Figures 4 and 5 compare the real and imaginary (or resistance and reactance) magnitudes of the actual and equivalent circuits of the system in Figure 1; they show very good agreement. It can thus be expected that the time response of the proposed equivalent circuit will follow that of the actual system, regardless of the input, both on steady and transients states.

**TRANSIENT CONVERTOR SIMULATION (TCS)**

An existing TCS programme (5) solves the following set of state space equations:

\[
\begin{align*}
\dot{V}_a &= E - R \dot{I}_a + K^T \dot{C}_a V_y + K^T \dot{V}_y + K^T \dot{V}_T \\
\dot{Q}_a &= -K^T \dot{C}_a V_y
\end{align*}
\]

where

\( \psi_f \) (magnetic flux in inductive branches) and \( Q_a \) (electric charge at capacitive nodes) are used as state variables and the network nodes are divided into \( a \) (with capacitive branches), \( b \) (with resistive, but not capacitive branches) and \( y \) (with only inductive branches) nodes to provide a diakoptical solution for efficient computation.

The dependent variables are interrelated by the following equations:

\[
\begin{align*}
I_y &= L^{-1} \psi_f \\
V_y &= C^{-1} \dot{Q}_a \\
V_y &= -R_0 (K_{x0} I_y + K_{x0}^{-1} K_{x0} \dot{V}_0) \\
V_y &= -I_y K_{x0} L^{-1} (E_D - R_0 I_y + K_{x0} \dot{V}_y + K_{x0} \dot{V}_y) \\
I_T &= K_T^{T} V_y + K_T \dot{V}_y
\end{align*}
\]

In the above equations the \( R, L, C \) coefficients are assumed constant at each step of an implicit integration procedure based on a trapezoidal approximation. Any attempt to make these parameters frequency-dependent would make the solution extremely complicated.

With the use of the harmonic matched a.c. system equivalents described in the last section it is possible to derive realistic
waveform information from the TCS programme without altering the basic algorithm.

Illustrative Example

Figure 6 illustrates the test system used to demonstrate the effect of alternative a.c. system representations during a d.c. short-circuit.

The a.c. system on the rectifier side is first represented as a Thevenin equivalent to give the specified Short Circuit Ratio. The dynamic response prior to, during, and after fault clearance is illustrated in Figures 7 and 8 (continuous line).

The dynamic study is then repeated with the a.c. system represented by the tuned equivalent circuit developed above with all other parameters, controls and specifications remaining unchanged. The results are plotted in the same figures (dash and dot).

It is not the purpose of this paper to discuss the fault behaviour in any detail, but rather to show that the predicted behaviour can be inaccurate in the absence of a frequency-dependent a.c. system model.

Figure 7 shows that the fault current peak calculated with the improved a.c. model is only slightly lower than that of the Thevenin equivalent. However the voltages across the converter d.c. terminals, illustrated in Figure 8, show considerable differences. In particular the difference in the first voltage peaks following rectifier current extinction is of the order of 50%.

CONCLUSIONS

A general method has been described to derive three-phase a.c. system harmonic equivalents. It has been shown that the equivalent circuits derived match very accurately the actual system impedances, both in phase and magnitude.

The use of this equivalent circuit for the a.c. system provides adequate representation of system resonances, the effect of their associated overvoltages and waveform distortion on the convertor controller is then catered for.

The proposed circuit does not give explicit information at the individual a.c. system busbars, but it provides an extremely efficient simulation in comparison with a full a.c. system representation.

The harmonic equivalent circuit has been used as part of a Transient Convertor Simulation programme to assess the effect of frequency dependence on the development of hvdc disturbances. Results obtained under typical d.c. line fault conditions have indicated substantial differences with respect to earlier simulations based on a simplified Thevenin equivalent a.c. representation.

REFERENCES


Figure 1 Test system

Figure 2 Three-phase impedance loci of the a.c. system
Figure 4 Resistive component of the test system impedance versus frequency
(i) Equivalent model
(ii) Actual system

Figure 5 Reactive component of the test system impedance versus frequency
(i) Equivalent model
(ii) Actual system
Figure 7 Variation of the d.c. current following a line short-circuit
(i) Improved equivalent model
(ii) Thevenin equivalent

Figure 8 Variation of the voltage at the convertor d.c. terminals following a line short-circuit
(i) Improved equivalent model
(ii) Thevenin equivalent
Abstract - A generalization of the impedance loci is first described capable of displaying the complete frequency response of any given power system including multi-phase related effects such as phase asymmetries and mutual couplings. This information is useful to derive frequency dependent equivalent circuits suitable for integration in the time-domain solutions of power system waveforms. Direct and optimization techniques are used in the derivation of the equivalent circuits. Their effect on impedance frequency matching and computational efficiency are compared.

INTRODUCTION

Harmonic studies are beginning to play an important part in power system analysis. The harmonic currents produced by non-linear components are normally derived on the assumption of a strong, i.e., perfectly sinusoidal, voltage supply. These non-linear harmonic currents are then injected into the a.c. network (assumed linear) to determine the levels of voltage distortion. However, when the non-linear load is a large power convertor and the a.c. system harmonic impedances are large, the supply voltage waveforms are not perfectly sinusoidal and the derivation of the injected harmonic currents requires an iterative algorithm. Moreover, if the injected harmonic frequency is close to a parallel resonant the algorithm often diverges. In such cases there is no alternative to the use of a transient convertor simulation (TCS) for the derivation of harmonic impedances. However, TCS harmonic studies depend on the existence of a.c. system equivalents responding accurately to power and harmonic frequencies.

These frequency-dependent equivalents are also required to analyze the behaviour of a.c./d.c. interconnections following convertor or system disturbances. The scaled-down simulators and TCS programs currently used to investigate the dynamic behaviour of h.v.d.c. convertors are by necessity very limited in their frequency dependence representation.

An equivalent circuit consisting of tuned RLC branches has been advocated by R.G. Hingorani [2] as a possible solution for HVDC studies and a three-phase extension of the method has been proposed by the authors at a recent conference.[3] The accuracy of these equivalent circuits is restricted by the lack of mutual coupling representation and their 'matching' capability is limited to one quantity, the harmonic impedance, and to a few selected harmonic frequencies.

This paper presents a generalization of the above techniques capable of matching the impedances of mutually coupled and asymmetrical systems for any required continuous range of frequencies.

SYSTEM FREQUENCY RESPONSE AT A CONVERTOR TERMINAL

The frequency response of a power system at the point of harmonic injection can be derived either from measurements or from frequency-domain studies, and the results are normally presented in the form of a single impedance-locus plot. However the exact frequency response must include network asymmetries and mutual couplings, thus requiring a 3 x 3 matrix impedance for each harmonic. A generalization of the impedance loci is used in this paper to display the harmonic impedances of the system of Figure 1 (the lower part of the New Zealand network) observed from the TIMAI Bus are shown in Figure 2.

Because power systems are generally bilateral, the (n,m) loci are seen to be identical to loci (m,n). Moreover for the particular line symmetry of the test system, two phases, A and C, are seen to be similar while phase A differs substantially. Some dynamic models lack mutual coupling capability and in such cases it is possible to reduce the full matrix information to three diagonal terms which contain the mutual impedances implicitly. The method is valid for systems with little phase current asymmetry, as is normally the case with convertor plant. This is achieved by post multiplying each of the 3 x 3 matrices by the matrix

\[
\begin{pmatrix}
a & a^2 & a \\
1 & a & a^2 \\
a^2 & a & 1 \\
\end{pmatrix}
\]

and extracting the diagonal terms. This is important because for some harmonics the mutual terms have as much effect as the self terms. The resulting loci for the system of Figure 1 are displayed in Figure 4. The frequency response is probably better assessed with reference to impedance-frequency plots rather than impedance loci. These are illustrated in Figure 4 for the system of Figure 1.

Implementation in Time Domain Studies

Generally time domain solutions will be required to derive the voltage waveforms resulting from the interaction between a static convertor and the supply system. Thus the complex impedance-frequency information contained in the loci matrices (such as
Fig. 2. Impedance Loci Matrix of Test System

Fig. 3. Diagonalized Matrix Impedance Loci
Fig. 4. Impedance/Frequency Matrix Derived from Figure 2.

that of Figure 2 must be converted into equivalent circuits. Every element of the $3 \times 3$ matrix must be synthesised into an equivalent circuit.

The equivalents matching the self terms are represented explicitly as shunt branches. The mutual terms are processed separately by using diakoptical techniques; the phase currents are first impressed upon the mutual circuits and the resulting voltages are then incorporated in the overall circuit as additional voltage sources per phase. Thus the overall frequency dependent model of the a.c. system to be used in the time domain solution is as shown in Figure 5.

Extension to Multiconverter Systems

Let us now consider the case of a two-converter system as shown in Figure 6. This system needs two points of observation, i.e. busbars A and B. The converter connected to busbar B will affect the impedances of the a.c. system when observed from busbar A and vice versa. This interdependence increases the number of mutual terms, i.e. each phase has a self and five mutuals.

An extension of the diakoptical technique to this case requires six self and thirty mutual circuits for a complete frequency-dependent model.

Although extension of the theory to more than two converters is straightforward, the number of frequency dependent circuits required increases with the square of the number of converters and the complexity of each frequency dependent circuit is dependent on the number loops of the frequency loci.
functions of frequency). Let $h$ be the Error Criterion Function.

Among the various types of Error Criterion, the Least Squares method has been found to be the most suitable in our problem. Its general form is:

$$\min \phi = \sum (w_i g_i)^2$$

where $w_1, \ldots, w_m$ are weighting factors to emphasize $g_i$ in regions of importance.

The optimization method is more flexible in the selection of equivalent circuit topology; the only limitation in this respect is the possibility of evaluating its frequency response. However, the circuit topology must be decided before the optimization process.

The CPU time requirements of the multi-variable optimization techniques is several orders of magnitude greater than those of the direct approach but they can be substantially reduced if the circuit derived by the direct method is used as a first approximation in the optimization approach.

**Effectiveness of the Algorithm**

Figure 8(a) displays the frequency response of one of the mutual elements of the 'actual' test system of Figure 1, together with that of Hingorani's [21] harmonic equivalent. Although the results are in good agreement at low frequencies the second and third peaks (at 845 and 876 Hz) show substantial disparity. The use of a correction filter in the region of largest discrepancy (862 to 906 Hz) modifies the harmonic response as shown in Figure 8(b). The new branch reduces considerably the error in the regions where the impedances of the equivalent circuit were too high, but no marked improvement is noticeable where the equivalent circuit impedances are too low. As explained earlier these are best corrected by a combination of scaling factor and subsequent use of correction filters.

The region of greatest discrepancy of the new equivalent circuit is now between 750-810 Hz. The addition of a second correction filter in this region produces a far better match as shown in Figure 8(c).

With reference to multi-variable optimization techniques, the effectiveness of the synthesized equivalent circuit is very dependent on the choice of error criteria. If the optimization criterion is the minimisation of the maximum deviation, the response, illustrated in Figure 9(a), is unsatisfactory, i.e. a large discrepancy is introduced in the first peak. In this case it is necessary to use more elaborate weighting functions which are a function of the discrepancy as well as the frequency. The on other hand the use of the least squares as the error criterion, illustrated in Figure 9(b), shows a marked improvement. Although the sum of the errors squared is minimized in this case, there are larger discrepancies at some individual frequencies when compared with the direct solution.

Thus an interactive approach based on the direct method enhanced by the use of correction filters generally has less error over most of the frequency range as compared to an optimization derived equivalent. However, each correction filter increases the complexity of the equivalent circuit and hence the computational burden when carrying out time domain studies. On the other hand the CPU time required for a multi-variable optimization study is several orders of magnitude greater than the direct approach. Hence if a relatively small number of time domain simulations are required the direct approach is preferable while for many, the optimization technique is probably best as a comparable accuracy equivalent can be obtained with fewer circuit elements.
Therefore it becomes impractical to model accurately a large number of convertors connected to the a.c. system.

**Equivalent Circuit Synthesis**

An initial equivalent circuit topology is selected to suit the requirements of the time-domain algorithm, such as the use of node-type partitioning and tensor matrix analysis.[4] These restrictions increase dramatically the solution efficiency in the case of a.c.-d.c. simulation, which require regular topological changes, by avoiding involving the whole network during localised converter valve switchings.

Since the topology used by Hingorani,[2] i.e. RLC shunt branches, is amenable to Efficient Transient Converter Simulation,[4] this form has been retained as the starting point in the derivation of the equivalent circuits. The basic matching philosophy consists of selecting the values of R, L and C that give the peaks and troughs at the correct frequencies and the Q (quality factor) of each branch such as to cause the equivalent circuit response between those frequencies to approximate the actual system response. The analytical part of the algorithm is described in Appendix A.

An acceptable error between the actual system and the synthesised circuit depends mainly on the following factors:

(i) The type of study. For steady state harmonic penetration studies minimal error at the harmonic frequencies is required while the error at intermediate frequencies are unimportant. This is clearly unacceptable for transient converter simulations as some intermediate frequencies could be excited.

(ii) The feasibility of altering automatically the equivalent circuits to get the error down to a prescribed accuracy.

(iii) The availability of an economically viable alternative to model the actual system explicitly. This relates to the amount of CPU time required to provide the simulation and accuracy required.

The basic steps of the equivalent circuit synthesis are illustrated in the flow diagram of Figure 7. This diagram includes two alternative paths, i.e. the direct and optimization methods, which are described in the next two sections.

**Direct Method**

The mathematical formulation used to make an initial assessment of the RLC values to give the peaks and troughs at the correct frequencies is described in Appendix A. Scaling of circuit parameters is then carried out, without altering the position of the peaks and troughs, to obtain the best fit between the equivalent and real system responses.

**Optimization Path**

- Use measurement or analysis to obtain the system frequency response (voltage and current).
- Use Hingorani's topology and direct synthesis to derive a multi-phase equivalent.
- Decide equivalent circuit topology and optimize the circuit parameters.
- Is the equivalent circuit acceptable?
  - Yes
  - Optimize
- Design a correction branch
- Add another branch to the equivalent circuit
- Matching Completed

**Fig. 7 Flow Diagram for the Direct and Optimization Algorithms**

Optimization methods [5][6] use either the Search or Gradient approaches. The latter being more difficult to implement due to the need for derivative information.

A gradient optimization method based on a special implementation of the Broyden-Fletcher-Goldfarb-Shanno (BFGS) technique has been used to minimize the difference (scalar quantity 'g') between the actual system and equivalent circuit responses. Its form is:

\[ g = h(g_{1}, \ldots, g_{2}) \to g_{1, \ldots, 2} \]

where 'g' is termed the objective function and \( g_{1, \ldots, 2} \) are the values of the independent variables at the m sample points (which are complex.
Optimised Mutual Element of the Impedance/Frequency Matrix

Fig. 9. Optimized Mutual Element of the Impedance/Frequency Matrix
Actual system response (continuous line) Using maximum deviation criterion (dotted line (a))
Using least squares criterion (dotted line (b))

Fig. 10. Dynamic Simulation of the d.c. Voltage at the Convertor Terminals following a Line Short-Circuit
(i) Frequency-dependent equivalent
(ii) Thevenin equivalent

CONCLUSIONS

Direct synthesis and multivariable optimization techniques have been described for the derivation of three-phase frequency-dependent a.c. system equivalents. These equivalents model the frequency dependence of the self and mutual impedance terms and are amenable for implementation into time-domain programs. Both methods have proved capable of achieving high levels of matching accuracy, but the optimization techniques require much greater computer requirements.

It should be noted that the equivalent circuit does not give explicit harmonic information at the individual a.c. system buses. It does, however, provide accurate assessment of the voltage and current waveforms at the convertor terminals, which is essential in Transient Convertor Simulation. This information can also be used in harmonic penetration studies and filter design for the accurate derivation of harmonic levels throughout the system. For this purpose the steady state harmonic currents finally obtained from the Time Domain Simulation must be injected into the actual network (represented in the frequency domain) to derive the voltage distortion levels at remote buses. We have already tried this technique with promising preliminary results.

REFERENCES


APPENDICES

A. Derivation of Equivalent Circuit Parameters

The basic equivalent circuit contains a number of single-tuned parallel branches as shown in Figure 5 and the first step is to obtain values for the inductors and capacitors of the equivalent circuit that will produce the same minima and maxima resonance frequencies. The resistances of the branches are ignored as they have very little influence on the magnitude of the admittance at frequencies that are sufficiently higher or lower than the series resonant frequency of the branch.

The admittance of an n branch network is:

\[ Y(s) = \sum_{k=1}^{n} \frac{1}{L_k C_k (s^2 + \omega_k^2)} \]

where:

\[ \omega_k = \frac{1}{\sqrt{L_k C_k}} \]

For an LC network the impedance is zero at minima and infinite at maxima frequencies.

Impedance minima occur when:

\[ \prod_{j=1}^{n} (s^2 + \omega_j^2) = 0 \]

Impedance maxima will occur when the numerator

\[ \prod_{j=1}^{n} (s^2 + \omega_j^2) 

of equation (1) is zero, i.e.
\[ n \prod_{k=1}^{n} \left( 5^2 w_k^2 \right) = 0 \]  
(3)

Let \( F_1, F_2, \ldots, F_n \) be the frequencies at which the admittance is zero, then equation (3) can be written as:
\[ \prod_{k=1}^{n-1} \left( 5^2 w_k^2 \right) \text{ where } w_k = 2T w_k \]  
(4)

By equating the coefficients of \( s \) in (3) and (4) the following equations result:

For \( s = 1 \)
\[ \sum_{k=1}^{n} \frac{1}{k} = 1 \]  
(5)

For \( s = 2 \)
\[ \sum_{k=1}^{n} \frac{1}{k} \sum_{j=k}^{n} w_j^2 = \sum_{k=1}^{n-1} \frac{1}{k} M_k^2 \]  
(6)

For \( s = 3 \)
\[ \sum_{k=1}^{n} \frac{1}{k} \sum_{j=k}^{n} w_j^2 = \sum_{k=1}^{n-1} \frac{1}{k} M_k^2 \]  
(7)

Thus \( n \) equations are formed which are then solved for the \( n \) unknown inductances \( L_1, L_2, \ldots, L_n \).

The appropriate capacitor values are found using:
\[ \frac{1}{w_k^2} = \frac{1}{L_k} k = 1, 2, \ldots, n \]  
(8)

where \( w_k \) are the required minima frequencies.

The admittance of the tuned network at any particular frequency is given by:
\[ Y = \sum \left( \frac{R_k}{L_k} \right) \]  
(9)

Branch \( k \) is in series resonance when \( w_k^2 = 1/C_k L_k \). The corresponding admittance term for the \( k \)th branch is \( 1/R_k \). This term is significantly larger than the contribution from the branches not in series resonance and hence dominates the admittance. The resistance that dominates at each minima is inserted in the corresponding branch. This yields maxima point impedances for the equivalent circuit which are much higher than those of the actual system. The problem can be corrected by the scaling of all inductor and capacitor values. A common scaling factor is used for all inductors and capacitors so that the resonance frequencies remain unchanged. The appropriate scaling factor is obtained by dividing the calculated impedance of the network at a particular frequency by the actual systems impedance. Equation (10) with \( R \) ignored yields:
\[ Y = \sum_{k=1}^{n} \left( \frac{R_k}{L_k} \right) \]  
(10)

As \( L_k \) appears in the denominator of equation (11), to increase the admittance contribution of each branch, so as to reduce the impedance, the inductors are divided by the scaling factor. In order to keep the same resonance frequencies the capacitor values are multiplied by the scaling factor.

B. Derivation of Correction Circuit Parameters

\[ |z_1|^2 = R^2 + \left( w_1 L - \frac{1}{w_1 C}\right)^2 \]  
(12)

\[ |z_2|^2 = R^2 + \left( w_2 L - \frac{1}{w_2 C}\right)^2 \]  
(13)

\[ |z_3|^2 = R^2 + \left( w_3 L - \frac{1}{w_3 C}\right)^2 \]  
(14)

Equation (12) - equation (13) gives:
\[ |z_1|^2 - |z_2|^2 = \frac{1}{w_1 C} - \frac{1}{w_2 C} \]  
(15)

Similarly using equation (13) and (14) yields:
\[ |z_2|^2 - |z_3|^2 = \frac{1}{w_2 C} - \frac{1}{w_3 C} \]  
(16)

Rearranging equation (16) gives:
\[ |z_2|^2 - |z_3|^2 = \frac{1}{w_2 C} L^2 \]  
(17)

Substituting equation (17) into equation (15) gives:
\[ |z_1|^2 - |z_2|^2 = \frac{1}{w_1 C} - \frac{1}{w_2 C} \]  
(18)

Hence
\[ |z_1|^2 - |z_2|^2 = \frac{1}{w_1 C} - \frac{1}{w_2 C} \]  
(19)

Given three points, equation (19) is solved to obtain the admittance of the required correction branch. Equation (12) is solved next to obtain the capacitance value. Lastly equation (12) is solved for the required resistance, this completing the determination of the correction filter parameters.
Comparison of steady-state and dynamic models for the calculation of AC/DC system harmonics

Prof. J. Arrillaga, DSc, FIEE, FRSNZ
N.R. Watson, BE
J.F. Eggleston, BE, PhD
C.D. Callaghan, ME

Abstract: The steady-state and dynamic simulation models currently proposed for the derivation of AC/DC system waveforms are described. Their application to the prediction of harmonic distortion is compared with reference to accuracy, general applicability, and computational efficiency. It is concluded that generally the use of an iterative steady-state algorithm provides the most accurate and efficient solution. However, such an algorithm often presents convergence problems and in such cases dynamic simulation provides the necessary back up to obtain a solution.

1 Introduction

The highly nonlinear and waveform-dependent behaviour of AC/DC converters cannot be modelled purely in the frequency domain. In approximate harmonic studies the converter harmonic currents are calculated from idealised waveforms derived from AC/DC load flows [1], and the harmonic voltages are then obtained by injecting each of the harmonic currents independently into the AC system, which is assumed perfectly linear.

Two basically different techniques have been proposed recently to improve the harmonic calculations. The first is an extension of the approximate steady-state model, but uses time and frequency domain studies in an iterative manner and claims to provide exact solutions upon convergence of the algorithm [2, 3]. The second technique uses only dynamic simulation and thus claims to provide a more realistic representation of the physical behaviour [4].

However, hitherto, no attempt has been made to validate the results and compare the computational efficiency of the proposed algorithms. This paper presents the basic characteristics of the steady-state and dynamic simulation models and compares their performance in terms of general applicability, accuracy and efficiency.

2 Analysis of AC/DC systems in periodic steady-state

The steady-state behaviour of purely AC systems is normally analysed using single-frequency phasors, i.e. in the frequency domain.

However, the AC/DC conversion process involves multiple point-on-wave switching operations within each cycle and these cause a periodic sequence of transients, which needs modelling in the time domain.

Thus in general the modelling of AC/DC power systems in periodic steady-state requires three basic steps and an iterative process. The steps are:

(i) time domain derivation of the alternating-current and direct voltage waveforms
(ii) time-to-frequency conversion of the current waveforms
(iii) harmonic power flow.

2.1 Single-frequency AC/DC power flow

The conventional AC power-flow algorithm assumes undistorted voltage and current waveforms, whereas the AC/DC conversion process produces a spectrum of harmonic frequency components. Therefore, when carrying out AC/DC power flows, three reasonable assumptions are normally made to simplify the convertor voltage and current relationships and to comply with the single-frequency power-flow requirements. These are:

(a) the presence of perfect filtering at the convertor terminals (behind the convertor transformer)
(b) the presence of infinite smoothing inductance on the DC side (thus providing perfect DC)
(c) the convertor transformer leakage (commutation) impedance being purely inductive (thus ensuring that the commutating currents are sinusoidal).

With reference to step (i) of the iterative process described in Section 2, once the firing angle is known, the commutation current waveform and its limits are clearly defined, the rest of the AC system in a unified solution. Finally, the AC/DC power flow (step (iii)) will require either a single-phase or a three-phase algorithm [1] depending on the degree of symmetry of the AC and/or DC system configuration and operation.

2.2 Conventional harmonic power-flow analysis

The Fourier analysis carried out to derive the current phasor in single-frequency power flows can also provide
the frequency components of the phase-current waveforms. In conventional harmonic analysis these are the harmonic currents used for the derivation of harmonic voltages throughout the (otherwise linear) power system.

Each of the harmonic currents is injected in turn into the AC system to derive the individual levels of harmonic voltage distortion.

3 Iterative harmonic analysis (IHA)

The direct solution described in the previous section assumes that the convertor-current waveform derived from the single-frequency power-flow solution is not affected by voltage distortion. However, the approximations used in the power-flow solution can introduce considerable error in the derivation of the harmonic currents.

On completion of the three-phase AC/DC power-flow solution, which provides reasonable initial information for the harmonic analysis, the approximations used can then be removed and an iterative solution carried out involving the following steps:

(i) Input information is read consisting of AC and DC system harmonic impedances as well as (initial) convertor operating conditions derived from a three-phase AC/DC load flow.

(ii) The alternating (harmonic) current injections and the direct (harmonic) voltage are calculated.

(iii) For each harmonic, the AC system voltages at the convertor terminals are found.

(iv) The convertor direct voltage and DC system harmonic impedances are used to calculate the direct-current waveform.

(v) The voltage zero crossings and firing instants are updated with the latest information of AC convertor-voltage waveforms.

Steps (ii) to (v) are repeated until convergence is achieved.

In the calculation of the alternating-current injections from the convertor (step (ii)) the effects of waveform asymmetry and distortion are included, both during and outside the commutation intervals. The basic formulation is given in Appendix 12.1.

The AC and DC system models used in the derivation of the alternating-voltage and direct-current waveforms (steps (iii) and (iv)) are frequency dependent, i.e. the transmission lines are represented by their equivalent-π circuits.

4 Transient convertor simulation (TCS)

State-space and nodal analysis can be used to derive a set of dynamic equations which allows a systematic solution of the network in the time domain.

At each step of the solution the instantaneous values of the network voltages and currents are checked against specified logical constraints, to decide whether a topological change is required.

As a result of initial mismatches between the real and assumed voltages and currents throughout the system, the dynamic simulation must be run for a number of cycles until the solution reaches a steady state.

The structure of the TCS algorithm is described in Appendix 12.2 and a more detailed formulation and the numerical solution can be found in Reference 1.

Finally the steady-state time-domain current waveforms are subject to a fast Fourier transform to obtain their harmonic components.

5 Verification of the algorithms

The IHA and TCS programmes have been developed independently using different conventions, per-unit systems etc. It is thus essential to validate their performance before attempting to compare their relative merits. For a reliable verification, the results of IHA and TCS must be compared with an exact solution.

The AC/DC power-flow algorithm provides an exact solution when the system under consideration satisfies the approximations set out in Section 2.1. The idealised circuit diagram of Fig. 1 represents such a system which includes a six-pulse bridge rectifier.

Fig. 1 Basic equivalent AC/DC system

The AC system voltage $V_g$ (the commutating voltage) behind inductance $X_f$ is assumed perfectly sinusoidal. On the DC side the smoothing inductance is very large so that the current is perfect DC. The actual values of the test-system components are indicated (in p.u.) in the Figure. Details of the per-unit system are given in Appendix 12.3.

For a specified DC current of say 1 p.u., the power-flow solution provides all the information required to derive the phase-current waveform (i.e. $\alpha = 20^\circ$, $\mu = 19^\circ$) and with it the current harmonics. Under these conditions the conventional (direct) solution can be used to calculate the voltage harmonic distortion and the results are shown in Table 1.

5.1 Verification of IHA

Instead of the voltage source, the intermediate node $V_T$, shown in Fig. 1, is used as the commutating voltage in the iterative solution.

The fundamental component of the phase current, derived in the previous section, is used to calculate the corresponding voltage component at $V_T$ and this value, together with those of the direct current $I_d$ and firing

<table>
<thead>
<tr>
<th>Harmonic order</th>
<th>TCS magnitude</th>
<th>TCS phase</th>
<th>Exact magnitude</th>
<th>Exact phase</th>
<th>IHA magnitude</th>
<th>IHA phase</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>0.7668</td>
<td>149.3</td>
<td>0.7762</td>
<td>148.5</td>
<td>0.7658</td>
<td>148.6</td>
</tr>
<tr>
<td>6</td>
<td>0.1384</td>
<td>-152.9</td>
<td>0.1382</td>
<td>-152.6</td>
<td>0.1374</td>
<td>-152.3</td>
</tr>
<tr>
<td>7</td>
<td>0.0874</td>
<td>145.8</td>
<td>0.0887</td>
<td>164.1</td>
<td>0.0876</td>
<td>146.4</td>
</tr>
<tr>
<td>11</td>
<td>0.0381</td>
<td>-159.3</td>
<td>0.0386</td>
<td>-158.3</td>
<td>0.0383</td>
<td>-157.8</td>
</tr>
<tr>
<td>13</td>
<td>0.0246</td>
<td>137.9</td>
<td>0.0244</td>
<td>137.7</td>
<td>0.0243</td>
<td>138.2</td>
</tr>
</tbody>
</table>
angle $\alpha$ constitute the initial conditions for the iterative process described in Section 3. At each iteration the latest value of $V_T$ is used as the commutating voltage and the process is repeated until convergence.

Preliminary results obtained differed markedly from those of Section 5. The deviations were traced to different interpretations of the specified conditions, and in particular the implementation of the zero crossing points used as a reference for the delay angle. Once the above anomalies were corrected, the results, illustrated in Table 1, show very good correspondence with the exact solution.

5.2 TCS verification

The initial conditions for the dynamic simulation are also derived from the results of the AC/DC load flow. With this information the test system required a 15-cycle run to reach a steady-state condition. Voltage and current waveforms for the last cycle of the TCS solution are shown in Figs. 2 and 3 respectively.

![Waveforms derived with the TCS model](image1)

![Phase-current waveform derived with the TCS model](image2)

Application of an FFT to the current waveform of Fig. 3 produced the harmonic currents listed in the last two columns of Table 1.

It is apparent that the harmonic levels derived from the TCS model are practically the same as those obtained with the IHA and exact solutions.

6 Limitations of the iterative solution

The previous section has shown that upon convergence the iterative harmonic analysis provides the right solution. The next question is to determine the range of applicability of the IHA algorithm.

Two main factors affect the level of voltage distortion at the converter terminals, i.e.

(i) the presence of a harmonic frequency close to the parallel resonance between the filters' capacitance and the system inductance

(ii) the relative strength of the AC system, normally represented by the SCR (short circuit ratio).

The sensitivity of the IHA algorithm to these two factors is investigated in Sections 6.1 and 6.2.

6.1 Symmetrical system

Fig. 4 represents a six-pulse rectifier connected to a symmetrical system (represented by its Thevenin equivalent) and characteristic harmonic filters (i.e. the 5th, 7th, 11th and 13th orders). The filters' ratings and characteristics are those of the single-tuned branches in the New Zealand HVDC link.

The system inductance has been chosen to resonate with the filters at a frequency close to the third harmonic and the short circuit ratio can be controlled by altering the convertor power rating.

Fig. 5 shows the effect of altering the short circuit ratio on the number of iterations to reach convergence. The continuous line illustrates that when the SCR was reduced to about 1.8 the algorithm failed to converge and the results showed considerable third-harmonic content. This was an unexpected result because in a perfectly symmetrical system (such as that of Fig. 4) no third-harmonic current can be generated by the convertor. However a closer look at the numerical result of the first iteration showed that small levels of third and all harmonics were generated by the computer as a result of random numerical error and these were amplified by the parallel resonant system. It could be argued that this numerical problem has a replica in a real system, caused by small random variations between the individual firings.

However, to isolate this problem from the investigation, the number of significant figures of the harmonic results was reduced by introducing 'thresholding'. The
new results, illustrated by the dotted line in Fig. 5, show
that the algorithm converges in two or three iterations
irrespective of the SCR.

6.2 Asymmetrical system conditions
The system of Fig. 7 represents the New Zealand South
Island network in the vicinity of a large power conver-
tor (at TIWAI) which, for simplicity, is assumed to be a six-
pulse rectifier. The converter is fed from a long-distance
untransposed transmission line which introduces some
asymmetry into the test system, and therefore produces
tripple harmonic currents.

Fig. 8 illustrates the three-phase impedance loci of the
test system, seen from the converter busbar; these loci
have been derived by injecting 1 p.u. harmonic currents
at that busbar (with the converter disconnected).

With this test system the IHA algorithm failed to
converge when the short circuit ratio was reduced to 5.3.
Earlier studies carried out with iterative algorithms had
concluded that computational divergence reflects a har-
nonic instability in the real system or at least a weakness
which may result in unreliable converter operation
[5–7]. So far, however, no definite proof has been offered
to substantiate such claims. Should they be correct, the
use of dynamic simulation should also fail to settle into a
steady state.

Therefore the system of Fig. 7, with a six-pulse conver-
tor at TIWAI, was analysed in the time domain using the
frequency-dependent model described in Section 7.1.

The TCS results show that it is possible to operate the
system with SCR levels in the region where the IHA solu-
tion does not converge. This conclusion is reached by
observing that the converter alternating-voltage wave-
forms settle down to a steady state. For instance, Fig. 6
shows two cycles of the waveforms predicted by TCS
(with an SCR of 5.3) after ten cycles of simulation.

7 Limitation of TCS in a practical system
The complexity of a power system cannot be accurately
represented by means of a single Thévenin equivalent and
therefore the algorithmic verification of TCS carried out
in Section 5 gives no indication about its accuracy in a
real system. Even with a more comprehensive equivalent
circuit the state-space model makes no provision for the
frequency dependence of the individual parameters
involved.

To assess the error incurred by ignoring such fre-
cquency dependence, the test system of Fig. 7 was used
instead.

In this case the results of an accurate IHA model, with
the transmission lines represented as equivalent-
matrices were used as a reference. These results are
compared in Table 2 with a dynamic simulation using a
Thévenin equivalent of the AC network and they

| Table 2: TCS/IHA comparison for the system of Fig. 7 with
Thévenin AC system equivalent |
<table>
<thead>
<tr>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Harmonic current injections into AC system (phase a)</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>TCS</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>13</td>
</tr>
<tr>
<td>17</td>
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<tr>
<td>19</td>
</tr>
<tr>
<td>23</td>
</tr>
<tr>
<td>25</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Harmonic voltages</th>
</tr>
</thead>
<tbody>
<tr>
<td>Phase a</td>
</tr>
<tr>
<td>-----------</td>
</tr>
<tr>
<td>TCS</td>
</tr>
<tr>
<td>1</td>
</tr>
<tr>
<td>5</td>
</tr>
<tr>
<td>7</td>
</tr>
<tr>
<td>11</td>
</tr>
<tr>
<td>13</td>
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<tr>
<td>17</td>
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<tr>
<td>19</td>
</tr>
<tr>
<td>23</td>
</tr>
<tr>
<td>25</td>
</tr>
</tbody>
</table>
clearly show that, in the absence of detailed three-phase and frequency-dependence representation, the TCS solution cannot provide accurate information.

7.1 Modified TCS to include the effect of frequency dependence

It is not practical to represent the frequency dependence of each individual system component in the dynamic simulation model. A more realistic solution is to derive the TCS simulation model. A more realistic solution is to derive a three-phase dynamic equivalent with reasonably improved frequency match, as shown in Fig. 7.1 Modified system equivalent of Fig. 9 which provides a greatly improved frequency match, as shown in Fig. 10.

Fig. 9 Frequency-dependent equivalent circuit

When the circuit of Fig. 9 is used (instead of the Thévenin equivalent) the harmonic information derived from TCS is very close to that of the IHA solution. The results of the IHA and TCS alternatives are illustrated in Table 3.

8 Computational efficiency

The CPU requirements of the two models described in previous sections have a complex dependence on many factors, such as the relative strength of the AC system and convertor rating, convergence tolerances, initial conditions etc. Therefore the computational requirements in Table 4 provide an approximate indication of the relative computational efficiencies. The test system under consideration relates to the lower part of the New Zealand South Island System and the two cases listed refer to (i) a simplified Thévenin equivalent of the AC system and (ii) an accurate frequency-dependent AC system equivalent. The results were obtained on a VAX11/750 computer.

It has already been explained that the TCS results require a number of cycles of simulation for the initial-condition transient to die down before the harmonic assessment can be made. The number of cycles required to reach a steady state depends largely on the time constants and damping of the circuit as well as the specified initial conditions. In general, however, eight to ten cycles are usually sufficient.

By way of illustration the TCS solution takes about 32 minutes to assess the harmonic levels with the frequency-dependent AC system equivalent. In contrast the IHA programme only takes 4 minutes to produce the same information.

It should also be noted that TCS requires a greatly increased CPU time in the last cycle of simulation. This is caused by the larger number of data points being written out into a data file, in order to increase the precision of the FFT process. Moreover, as the TCS programme has automatic step-size selection, the FFT process has to derive a set of equally-spaced data points from the programme's unequally-spaced information.

9 Conclusions

The relative merits and limitations of harmonic analyses in the time and frequency domains have been compared and the positive aspects of each model have been used to crossverify and improve the accuracy of the two alternative methods.

In general the use of the steady-state iterative harmonic analysis is to be recommended both in terms of computational efficiency and accuracy of system representation at harmonic frequencies. It has been shown, however, that when the relative strength of the

---

**Table 3:** TCS/IHA comparison for the system of Fig. 7 with frequency matched AC system equivalent

<table>
<thead>
<tr>
<th>Phase</th>
<th>Harmonic current injection into the AC system (phase A)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>TCS</td>
</tr>
<tr>
<td>1</td>
<td>1.4794</td>
</tr>
<tr>
<td>5</td>
<td>0.2850</td>
</tr>
<tr>
<td>7</td>
<td>0.1729</td>
</tr>
<tr>
<td>11</td>
<td>0.0916</td>
</tr>
<tr>
<td>13</td>
<td>0.0850</td>
</tr>
<tr>
<td>17</td>
<td>0.0311</td>
</tr>
<tr>
<td>19</td>
<td>0.0208</td>
</tr>
<tr>
<td>23</td>
<td>0.0068</td>
</tr>
<tr>
<td>25</td>
<td>0.0004</td>
</tr>
</tbody>
</table>

**Table 4:** CPU comparison (in seconds) of TCS and IHA derived results

<table>
<thead>
<tr>
<th></th>
<th>TCS</th>
<th>IHA (total)</th>
</tr>
</thead>
<tbody>
<tr>
<td>With Thévenin</td>
<td>106</td>
<td>1131</td>
</tr>
<tr>
<td>equivalent</td>
<td>283</td>
<td>241</td>
</tr>
<tr>
<td>With frequency-</td>
<td></td>
<td></td>
</tr>
<tr>
<td>dependence</td>
<td>183</td>
<td>241</td>
</tr>
</tbody>
</table>

---

**Fig. 10** Impedance match of frequency-dependent equivalent circuit
AC system (determined by the short circuit ratio) is low, the iterative algorithm presents convergence problems. In such cases there is no alternative to the use of transient convertor simulation to derive harmonic information.

The results derived from transient convertor simulation are unrealistic if the AC system equivalent is not frequency dependent. However, a preliminary study of the AC system in the frequency domain provides the necessary information to derive an accurate AC system harmonic equivalent, and thus improve the accuracy of the TCS model.

10 Acknowledgments

The authors wish to acknowledge the financial help received from the UGC and the NZERDC to carry out the investigation reported in this paper.

11 References


12 Appendix

12.1 Derivation of the alternating currents at the convertor terminals

When only two valves conduct, say valves 1 and 2, the instantaneous phase-current injections are derived from the expression:

\[ I = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} I_s \\ 0 \\ -I_s \end{bmatrix} \]

During the commutation from valve 1 to 3 the current injections are

\[ I = \begin{bmatrix} I_1 \\ I_2 \\ I_3 \end{bmatrix} = \begin{bmatrix} I_s - I_e \\ I_e \\ -I_e \end{bmatrix} \]

where \( I_e \), the commutation current, has the following expression [7]:

\[ I_e(t) = \sum_{h=1}^{\infty} (X_h e^{-\omega_T m} + S_h \sin (\text{host} + \phi_h)) + Y(1 - e^{-\omega_T m}) + C_{mn} \]

where

\[ X_h = \frac{B_h}{L_{mn}} \hat{I}_h - 1/T_m \]

\[ Y = \frac{R_e I_t}{R_{mn}} \]

and \( n \) is the incoming phase and \( m \) is the outgoing phase;

\[ R_{mn} = R_m + R_n \]

\[ L_{mn} = L_m + L_n \]

\[ T_{mn} = L_{mn}/R_{mn} \]

\[ A_h = \frac{P_h \cos (\theta_h + \phi_h)}{S_h} - \frac{V_{mn} \cos (\theta_h + \phi_m)}{S_h} \]

\[ B_h = \frac{P_h \sin (\theta_h + \phi_h) - V_{mn} \sin (\theta_h + \phi_m)}{S_h} \]

\[ H_h = \frac{h_o A_h}{B_h} \]

\[ \phi_h = \tan^{-1} \left( \frac{1}{\text{host}} - \frac{1}{\text{host}} \right) \]

where \( P_h \) and \( \phi_h \) are the magnitude and phase of the \( h \)th harmonic of the phase to neutral voltage of the incoming phase, \( P_m \) and \( \phi_m \) are the same, but for the outgoing phase, and \( C_{mn} \) is the integration constant such that \( I_h(0) = 0 \).

However, for a more general transformer model, capable of representing the star-delta connection, the foregoing equation is rewritten in terms of the phase-to-phase voltages, i.e.

\[ V_{mn} = \sum_{h=1}^{12} (A_h \sin (\text{host} + B_h \cos \text{host})) \]

using the first firing angle \( \theta_1 \) as a time reference.

For a star/star transformer

\[ A_h = -\sqrt{3} V_1 \cos (\theta_h + \phi_A) \]

\[ B_h = -\sqrt{3} V_1 \sin (\theta_h + \phi_A) \]

For a star-g/delta transformer (DY11) and a commutation from valve 1 to valve 3

\[ A_h = -\sqrt{3} V_1 \cos (\theta_h + \phi_A) \]

\[ B_h = -\sqrt{3} V_1 \sin (\theta_h + \phi_A) \]

12.2 TCS formulation

The following set of state equations has been found to provide a very efficient computer solution [1]:

\[ \dot{p}Q_h = -(K_{ej} I_e + K_{u} I_s) \]

\[ \dot{p} \psi_h = E_h - f I_t + K_{eq} V_e + K_{eg} \psi_h + K_{e} V_e \]

where \( Q \) and \( \psi \) are state variables representing the node charge and inductive branch flux respectively.

The rest of the symbols and suffixes in the above equations represent the following:

\( l \) inductive branches

\( r \) resistive branches

\( c \) capacitive branches

\( e \) node with at least one capacitive branch connected

\( b \) node with at least one resistive but no capacitive or resistive branches connected

\( y \) node with inductive but no capacitive or resistive branches connected

The above definitions give the topological or branch-node incidence matrices, their general elements being:

\[ IEE PROCEEDINGS, Vol. 134, Pt. C, No. 1, JANUARY 1987 \]
$K_p = \begin{cases} 
1 & \text{if node } i \text{ is the sending end of branch } p \\
-1 & \text{if node } i \text{ is the receiving end of branch } p \\
0 & \text{if node } i \text{ is not connected to branch } p 
\end{cases}$

Details of the formulation and numerical solution can be found in Reference 1.

### 12.3 p.u. system

<table>
<thead>
<tr>
<th>Base quantities</th>
<th>AC side</th>
<th>DC side</th>
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<tbody>
<tr>
<td>Power, MVA</td>
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<td>100</td>
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<tr>
<td>Voltage, kV</td>
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<td>105</td>
</tr>
<tr>
<td>Current, kA</td>
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<tr>
<td>Impedance, $\Omega$</td>
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<td>191</td>
</tr>
<tr>
<td>Frequency, Hz</td>
<td>50</td>
<td>50</td>
</tr>
</tbody>
</table>