A SYSTEMS APPROACH

TO BUILDING DESIGN

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The methodology of building design is discussed together with existing and potential uses of the digital computer for building design processing. It is concluded that interdisciplinary information sharing through computer aided design procedures is severely restricted by methodology problems and by the relatively small quantity of common information. Appraisal of design alternatives will remain the major role of the digital computer in the building design process.

Appraisal of building thermal environments is discussed and a differential cost approach is developed to measure both resource and performance differences between design alternatives. A computer model for simulation of the heat flows that occur in commercial buildings with intermittently operated hydronic heating is developed. The model and the differential cost approach are used to establish the significance of dynamic influences on fuel consumption and thermal environmental quality. Differences in capital costs, energy costs, and users' costs of reduced performance due to thermal deficiency are evaluated for a range of equipment sizes for two typical rooms. It is concluded that the extra resources associated with dynamic simulation modelling are not warranted for the design of building heating equipment. Further investigation is required into the empirical adjustments for intermittency and the appropriate level of detail of models for thermal appraisal of architectural decisions.
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LIST OF SYMBOLS

A = area or a constant

$A_f$ = area of floor

$A_i$ = area of surface i

$A_g$ = area of glazing

$A_s$ = area of surface

$A_w$ = area of wall

$[A]$ = tridiagonal matrix

a = absorbance of a wall or a constant

$a_{ij}$ = element in the ith row and jth column of matrix $[A]$

$a_n$ = a constant

$a_o$ = steady state term

B = a constant

$[B]$ = tridiagonal matrix

b = a constant

$b_{ij}$ = element in the ith row and jth column of matrix $[B]$

$b_n$ = a constant

C = heating value of fuel or a constant

$C_b$ = differential cost of boiler above cost of smallest boiler considered

$C_e$ = differential cost of emitter size

$C_f$ = energy cost for a given period

$C_g$ = specific heat of glass

$C_p$ = humid specific heat of air

$C_{pa}$ = specific heat of dry air

$C_{plant}$ = lumped thermal capacity of hydronic heating system
\( C_{ps} \) = specific heat of steam
\( C_u \) = user cost of thermal deficiency over failure period
\( C_{ud} \) = daily user cost due to initial occupancy thermal deficiency
\( C_v \) = volume specific heat of moist air
\( C(i) \) = thermal capacity at node i
\( c \) = specific heat capacity of air
\( c_u(i) \) = thermal capacity per unit volume of material

\( D \) = number of degree days for the estimate period or a constant
\( D_f \) = extraneous short-wave radiant flux incident on the floor
\( D_i \) = extraneous short wave radiant flux incident on surface i
\( D_s \) = direct solar radiative flux transmitted through the glazing
\( DX(i) \) = thickness of element i
\( d_g \) = thickness of glass

\( E \) = efficiency of fuel utilization or illuminance on a horizontal plane
\( E_g \) = diffuse solar radiative flux transmitted through the glazing
\( E_i \) = short wave flux emitted from surface i
\( E[C_{fa}] \) = expected value of annual energy consumption cost for heat from boiler to the room
\( E[C_{ua}] \) = expected annual user cost
\( E[C_{ud}] \) = expected value of daily user cost
\( E[H_a] \) = expected annual energy input to the room from the boiler
\( E[H_b] \) = expected value of daily energy input to the room from the boiler
\( E[H_d] \) = expected value of daily energy input to the room
\( F \) = quantity of fuel consumed or a constant

\( F_{ij} \) = radiation configuration factor for radiation from surface \( i \) to surface \( j \)

\( F_{ji} \) = radiation configuration factor for radiation from surface \( j \) to surface \( i \)

\( f \) = a function of any variables

\( f_{ci} \) = the ratio of the surface area of the clothed body to the surface area of the nude body

\( f_D \) = cloudiness scale factor for direct solar radiation

\( f(x) \) = a periodic function of \( x \)

\( f(\theta_e) \) = heat gain or loss for a bin as a function of the average external air temperature

\( f(\phi_d) \) = function of \( \phi_d \)

\( f(\phi_b) \) = function of \( \phi_b \)

\( \phi_{ij} \) = script \( F \) factor

\( \text{Gr} \) = Grashof Number

\( g \) = constant gradient

\( H \) = equivalent full-load hours

or boiler output capacity minus output capacity of smallest boiler considered

\( H_a \) = quantity of heat input to air

\( H_b \) = daily energy input to room from boiler

\( H_d \) = daily energy input to room from boiler and internal heat sources

\( H_i \) = short wave flux incident on the surface \( i \)

or daily energy input to the room from internal heat sources

\( H_o \) = extraterrestrial daily insolation on a horizontal surface
\( h \) = number of working days in annual heating season

\( h_{al} \) = specific enthalpy of dry air at temperature \( \theta_1 \)

\( h_{a2} \) = specific enthalpy of dry air at temperature \( \theta_2 \)

\( h_{cl} \) = convective heat transfer coefficient for the clothed body [kcal/m\(^2\) hr °C]

\( h_e \) = external surface convective coefficient

\( h_s \) = surface conductance

\( h_{sl} \) = specific enthalpy of steam at temperature \( \theta_1 \)

\( h_{s2} \) = specific enthalpy of steam at temperature \( \theta_2 \)

\( h_1 \) = specific enthalpy of moist air in initial state

\( h_2 \) = specific enthalpy of moist air in final state

\( I \) = volume infiltration rate

\( I_{cl} \) = thermal resistance of clothing [clo]

\( I_{sc} \) = solar constant which is defined as the solar radiation intensity at normal incidence outside the earth's atmosphere when the earth is at its mean distance from the Sun = 442 Btu/hr/ft = 1.395 Kw/m\(^2\) = 2.00 langley/min

\( i-1,i,i+1 \) = indices for distance increments

\( J \) = index for preceding time interval

\( K \) = index for present time interval

\( K_{emit} \) = emitter coefficient

\( K_{emitb} \) = basic emitter coefficient for room

\( K_p \) = heating plant thermal capacity proportionality constant

\( K_e \) = emitter ratio

\( K(I) \) = thermal conductance between node \( i \) and node \( i+1 \)
\( K(I) \) = thermal conductance between surface node \( i \) and internal node \( i+1 \)

\( k \) = thermal conductivity

\( k(i) \) = thermal conductivity of element \( i \)

\( L \) = latitude of location

or characteristic length = mean of rectangular dimensions

\( L_e \) = variation length of emitter element

\( m \) = human metabolic rate per unit nude body surface area

or mass of air

or an exponent

\( m_a \) = mass of dry air

\( N \) = number of hours

\( N_{\text{emit}} \) = exponent dependent upon heat emitter

\( N_b \) = number of building load units

\( N_{CD} \) = normalised daily direct radiation on a horizontal surface for a clear day

\( N_{cd} \) = normalised daily diffuse radiation on a horizontal surface for a clear day

\( N_D \) = normalised daily direct radiation on a horizontal surface appropriate to the constant level of cloudiness

\( N_d \) = normalised daily diffuse radiation on a horizontal surface appropriate to the constant level of cloudiness

\( Nu \) = Nusselt Number

\( n \) = number of surfaces or a constant
\( \text{Pr} \)  = Prandtl Number

\( \text{P}_w \)  = water vapour pressure at ground level

\( \rho \)  = proportion of the occupants' employment worth lost due to reduced performance

\( \text{P}_a \)  = partial pressure of water vapour in room air

\( \text{P}_a \) or partial pressure of dry air in moist air

\( \text{P}_b \)  = standard atmospheric pressure

\( \text{P}_r \)  = combined long wave radiation and convection to room air

\( \text{P}_i \)  = proportion of the total heat output from the artificial lighting

\( \text{P}_s \)  = partial pressure of steam in moist air

\( \mathcal{Q} \)  = maximum heat flow required from plant

\( \mathcal{Q} \) or calculated maximum heat load based on \( \theta_d \) and \( \theta_e \)

\( \mathcal{Q}_a \)  = net rate of heat input to the air

\( \mathcal{Q}_{\text{back}} \)  = heat input to back casing of emitter

\( \mathcal{Q}_{\text{boil}} \)  = rate of heat output from boiler

\( \mathcal{Q}_{\text{boiln}} \)  = maximum boiler output for room

\( \mathcal{Q}_c \)  = rate of convective heat flow from surface to external air

\( \mathcal{Q}_{\text{ec}} \)  = convective heat output from room's emitters

\( \mathcal{Q}_{\text{emit}} \)  = total heat output from room's emitters

\( \mathcal{Q}_{\text{emitb}} \)  = basic emitter output

\( \mathcal{Q}_{\text{ij}} \)  = convective heat flow from room air to windows

\( \mathcal{Q}_{\text{h}} \)  = total sensible heat loss from humans

\( \mathcal{Q}_{\text{hc}} \)  = convective heat loss from humans

\( \mathcal{Q}_{\text{hr}} \)  = long wave radiative heat loss from humans

\( \mathcal{Q}_{\text{ij}} \)  = radiant power leaving surface i that is incident at surface j

\( \mathcal{Q}_{\text{inf}} \)  = rate of heat flow to warm the infiltrating air
\( Q_1 \) = total energy output from artificial lighting
\( Q_{lc} \) = convective heat output from the artificial lighting to the room air
\( Q_r \) = net rate of long wave radiant heat flow from the external surface to the external environment
\( Q_{rad} \) = net long wave radiation emission from the front casing to the other room surfaces
\( Q_{rl} \) = long wave radiant emission to room from artificial lighting
\( Q_s \) = rate of solar heat flow into external surface
\( Q_{sn} \) = net heat flow rate into the surface during the time period .JK
\( Q_{w} \) = total heat flow rate into wall
\( \{Q_s\} \) = vector of net surface input heat flows
\( q_c \) = convective heat flow into glass from room air
\( q_g \) = net heat flow into the glass
\( q_{ij} \) = net long wave radiation flux exchanged from surface i to surface j
\( q_{1,n} \) = conduction heat flow per unit area at surface 1 at time n at
\( q_p \) = rate of heat flow from plenum surface to plenum air
\( q_r \) = net long wave radiant heat flow into glazing from room's internal surfaces
\( q_{ri} \) = net long wave radiant input to surface i
\( q_s \) = net short wave radiant heat flow into glazing from room's internal sources
\( q_{si} \) = net short wave radiant input to surface i

\( R \) = irradiance on a horizontal plane
\( R_d \) = scaling ratio for diffuse solar radiation appropriate to the constant level of cloudiness
\( R_i \) = short wave radiosity from surface \( i \)

\( \{R\} \) = vector of radiosities, \( R_i \)

\( r \) = ratio of density of dry air in dry air and steam mixture to density of dry air at the same total pressure

or ratio of solar radiation intensity at normal incidence outside the earth's atmosphere to the solar constant

or order of the derivative

\( S_D \) = direct solar radiation intensity on a surface for a constant level of cloudiness

\( S_d \) = diffuse solar radiation intensity on a surface for a constant level of cloudiness

\( \{S\} \) = vector of source fluxes, \( \frac{E_i}{\rho_{si}} + D_i \)

\( T \) = absolute temperature of the surface

\( T_D \) = tabulated direct solar radiation intensities on the surface for a clear day

\( T_d \) = tabulated diffuse solar radiation intensities on the surface for a clear day

\( T_e \) = absolute temperature of the external air

\( T_i, T_j \) = absolute temperatures of surfaces \( i \) and \( j \) respectively

\( T_s \) = absolute temperature of external surface

\( t \) = time

\( t' \) = time from start of occupancy

\( t_d \) = duration of initial occupancy thermal deficiency period

\( t_f \) = duration of failure period, i.e. time from start of occupancy until comfort condition is achieved
$U$ = unit fuel consumption, i.e. quantity of fuel used per degree day per building load unit

$U_w$ = thermal transmittance of wall construction

$u_b$ = cost increase per unit output increase

$u_e$ = cost of emitter element per unit length minus cost of distribution pipe per unit length

$U_f$ = unit cost of fuel

$V$ = volume containing air

$V_r$ = volume infiltration rate

$v$ = wind speed in miles per hour

or a variable

$\text{Var}[\phi_d] =$ statistical variance of mean daily external air temperature over the heating season

$\text{Var}[\phi_a] =$ statistical variance of 8 a.m. external air temperature over the heating season

$W =$ humidity ratio = weight of water vapour per unit weight of dry air

or radiant flux emitted from surface

$W_i =$ total radiant flux emitted from surface $i$

$w =$ employment worth of occupants

$x =$ independent variable

$x_p =$ response factor for heat flow per unit area at surface $i$ due to a unit temperature excitation at surface $i$ at time $(n-p)\Delta t$

$[Y] =$ matrix of radiosity coefficients

$[Y]^{-1} =$ inversion of matrix $[Y]$
\( Y_{ij} \) = element in the \( i \)th row and \( j \)th column of inverse matrix \( \{Y\}^{-1} \), named the radiosity factor

\( y_{p} \) = response factor for heat flow per unit area at surface 1 due to a unit temperature excitation at surface 2 at time 

\((n-p)\Delta t\)

\( \alpha \) = thermal diffusivity

or absorptance = fraction of incident radiation absorbed

or a constant

\( \alpha_s \) = short-wave absorptance of surface

\( \alpha_{si} \) = short-wave absorptance of surface 1

\( \beta \) = interpolation parameter

or an exponent

\( \delta \) = solar declination

\( \Delta t \) = duration of the small time interval

\( \Delta x \) = size of small distance increment

\( \Delta \theta \) = temperature difference over small time interval

or magnitude of cool deviation of ambient temperature from optimal thermal comfort ambient temperature

\( \epsilon \) = surface emissivity

\( \epsilon_{e} \) = effective emittance of the external environment

\( \epsilon_s \) = emittance of external surface

\( \epsilon_{sky} \) = effective emittance of the sky on a clear day

\( r \) = density

\( r_g \) = density of glass

\( \eta \) = efficiency
\( T \) = temperature

and, in particular, equivalent ambient temperature

\( \theta_a \) = dry bulb air temperature

\( \theta_{cl} \) = effective mean surface temperature of a clothed body [°C]

\( \theta_d \) = internal temperature for maximum load design

\( \theta_e \) = external air dry bulb temperature

\( \theta_{ei} \) = environmental temperature

\( \theta_f \) = final internal air temperature i.e. when comfort conditions are reached

\( \theta_g \) = temperature of glass

\( \theta_i \) = initial internal air temperature i.e. before heating starts

\( \theta_{l,(n-p)} \) = temperature at surface 1 at time \((n-p)\Delta t\)

\( \theta_m \) = mean external temperature during energy estimation period

\( \theta_p \) = temperature of air in plenum

\( \theta_r \) = mean radiant temperature

\( \theta_s \) = temperature of a surface

\( \theta_w \) = mean water temperature

\( \theta_{x,t} \) = temperature at time, \( t \) and distance, \( x \) through the wall

\{\( \theta_J \}\} = vector of element temperatures at time \( .J \)

\{\( \theta_K \}\} = vector of element temperatures at time \( .K \)

\( \theta_{1,1}, \theta_{1,2} \) = dry bulb temperatures

\( \theta_{2,(n-p)} \) = temperature at surface 2 at time \((n-p)\Delta t\)

\( \rho \) = reflectance = fraction of incident radiation reflected

\( \rho_a \) = density of dry air at pressure \( p_a \)

\( \rho_{a1} \) = density of dry air at dry bulb temperature \( \theta_{1} \)

\( \rho_{a2} \) = density of dry air at dry bulb temperature \( \theta_{2} \)
\( \rho_b \) = density of dry air at pressure \( p_b \)

\( \rho_{si} \) = short wave reflectance of the surface \( i \)

\( \sigma \) = specific heat

or Stefan-Boltzmann constant

\( \tau \) = transmittance = fraction of incident radiation transmitted

\( \tau_D \) = transmittance of glass to direct solar radiation

\( \tau_d \) = transmittance of glass to diffuse solar radiation

\( \phi \) = stimulus magnitude

\( \phi_d \) = mean daily external air temperature

\( \bar{\phi}_d \) = expected value of mean daily external air temperature over the heating season

\( \phi_o \) = approximates the lowest physical value that can be sensed

\( \phi_e \) = 8 a.m. external air temperature

\( \bar{\phi}_e \) = expected value of 8 a.m. external air temperature over the heating season

\( \psi \) = judged sensory magnitude

\( \omega_s \) = sunset hour angle in radians

\( \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} \) = Laplace operator

\( |\nabla| = \text{modulus of variable, } \nabla \)

\( [f(\nabla)]_{\nabla} = \text{function of variable, } \nabla, \text{ evaluated for } \nabla = \bar{\nabla} \)
1.1 CONTEXT

Building design has evolved to become a complex decision-making process. Not only must buildings provide shelter, they must also provide a controlled artificial environment with technological utilities to aid human endeavours. The design of large buildings has become so complex that it is undertaken by a team of highly trained specialists. Their task is to investigate the wide range of performance requirements, detail a satisfactory design, and present a formally documented description of the building so that it may be constructed.

The designers' fundamental tool is the model. A model is a representation and may be verbal, visual, physical, or numerical in form. The most complete model of a building is the building itself. However, this is far more detailed than required for design purposes and is expensive to modify. Hence many incomplete representations are used throughout the building design process to accentuate those characteristics of the building that relate to each component design problem.

Designers' mental processing capabilities have been a natural constraint on the level of detail that can be efficiently included in models. The rapid logical information processing capability of the digital computer has eased this constraint and fostered the development of many new design procedures. With such a marked change in man's
modelling ability there arise the questions of how the capabilities of the
digital computer can be most effectively utilized for building design
and what level of detail to include in building design models. Both
of these questions are addressed in this report.

1.2 RESEARCH SCOPE

The initial objective of the research was to investigate the
building design process for the purpose of establishing both existing
and potential roles of the digital computer. Thus an extensive literature
search was undertaken to investigate:

(1) the building design process
(2) recent developments in design methodology
(3) developments in systems methodology
(4) the characteristics and significance of both existing
uses, and research and development into potential uses, of the digital
computer in the building design process.

Problems associated with inputting building design information into
computer storage, appropriate computer data structures, and uses of common
data were researched with respect to the feasibility of integrated
computer aided design systems for use by all members of the building
design team.

It was concluded that integration of computer procedures was
presently limited to procedures used by single design disciplines and
that, although some automatic computer based interdisciplinary information
sharing was technically feasible, it was unlikely to eventuate because of
design methodology problems and the relatively small set of common
building information used by both existing and potential future computer
aided building design procedures. As it was also concluded that the
major role of the digital computer in building design was to aid appraisal
of design alternatives, the research focused on appraisal of building
thermal environments. The redefined objectives became: to develop a
method for establishing the appropriate level of detail for building
computer aided design appraisal procedures and to apply it to establish
the net benefit of using dynamic simulation models for building heating
design appraisal.

The differential cost approach was developed as an appropriate
method for measuring both resource differences between design alternatives
and differences between their performances. To research its detailed
application building thermal design principles and models were investigated
with particular reference to heating design. As building thermal environ-
mental quality is usually determined from its effects on humans, a user
cost of thermal deficiency model was developed to measure thermal
influences on the performance of building occupants. A differential cost
model, which includes user cost, capital cost of plant, and cost of
consumed fuel, was formulated as an appraisal model for hydronic heating
equipment design proposals.

The significance of the dynamic nature of thermal flows in buildings
was investigated for heating design decisions and the differential cost
model was used to measure it. As commercial buildings are a significant
proportion of professionally designed buildings in New Zealand, the
investigation concentrated on intermittently operated hydronic heating,
which is the most commonly used commercial building heating system.

A digital computer simulation model was developed to study the
dynamic thermal flows in typical rooms of commercial buildings with
intermittently operated hydronic heating. Rooms from two commercial buildings recently constructed in Christchurch were simulated for a range of Christchurch weather profiles, room heat emitter sizes, and centralised boiler capacities.

The results from the simulation study were used for the detailed formulation of the differential cost model for hydronic heating of commercial buildings situated in Christchurch. Evaluation of the differential cost model provided the basis for conclusions about the significance of the dynamic nature of thermal flows in buildings and the value of dynamic simulation models for heating equipment design decisions. Finally, conclusions about both the concept of a user cost as a measure of the quality of environments and the differential cost approach to design decisions were made, together with recommendations for further work on the development of objective design procedures.

1.3 REPORT STRUCTURE

The nature of the building design process and recent developments in building design procedures are reviewed and appraised in Chapter Two with particular reference to the impact of the digital computer on the design of large buildings. The thermal design problem and the mathematical models available to aid its solution are discussed in Chapter Three. Chapter Four develops the concept of a user cost in a thermal environment as part of a differential cost model for thermal environmental appraisal. The next three chapters are concerned with an investigation into the significance of this appraisal measurement model and the use of dynamic simulation models for hydronic heating design decisions. The mathematical
equations that form the basis of the dynamic simulation model used in this investigation are discussed in Chapter Five. Problems encountered with the implementation of the mathematical model to a satisfactory digital computer model are discussed in Chapter Six. The simulation study and the conclusions drawn from it are described in Chapter Seven. Finally, Chapter Eight reviews the research as a whole, presents the conclusions that were drawn from it, and suggests areas for further work.
CHAPTER TWO

THE BUILDING DESIGN PROCESS

2.1 WHAT IS BUILDING DESIGN?

2.1.1 Historical Development

Early humans sought caves for protection from rain, wind, and cold. Their post-nomad descendants rendered man-made abodes from local stone and vegetation. Trial and error construction by their occupants produced the final form of these early buildings. Community activities required larger buildings; larger buildings required construction overseers. Thus the predecessors of today's building designers and constructors designed as they simultaneously constructed and supervised other construction craftsman.

With the development of alternative materials, and alternative forms of construction, came the need for design decisions before construction commenced. Design was partially severed from construction, although the designer remained the chief craftsman. Industrialisation increased the range of materials available for building construction and instigated the advent of the tradesman. More design decisions were required; design and construction became quite distinct: architects designed and oversaw construction; master builders supervised the construction by tradesmen.

Technological development increased the complexity of building design beyond the capabilities of a single designer: specialists emerged to aid the architect. Steel and reinforced concrete released the constraints of compression structures: the structural engineer emerged to design the strength into frames and other tension structures.
The heating engineer became established with the advent of hydronic central heating. With the development of air circulation systems, he became the heating and ventilating engineer; with the use of hot and cold water, and other utility services in buildings, he became the building services engineer. Air conditioning, illumination, and acoustic services also fell within his province. However, with the growth in complexity considered in the design of such building services systems, the latter two have emerged as specialist disciplines in their own right. The need for electrical energy to drive some of these building services, together with the use of telecommunications in buildings, established the electrical engineer as another building design specialist.

Recently the design for colour and texture for the internal decor of a building has been undertaken by interior design consultants. The layout of the land surrounding the building with plants and man-made ornamental structures has also recently become established as a specialist design role for landscape architects. Such specialization of design for buildings is likely to continue as human desires for better services and environments within and around their habitats continue to develop.

The separation of design from construction, and the use of competitive tendering for construction, heralded the need for tender documents. Specifications for the quality of the building, and drawings illustrating the geometry and topology of building components, rightfully remained the design specialists’ responsibility. However, the computation of quantities for tendering purposes became a requirement for design documentation: the quantity surveyor emerged as a non-design member of the building design team. The quantity surveyor's role has grown to include cost estimates for the designed building, progress payments for construction, and cost comparisons for decision-making by the other members of the building design team.
Thus building design has evolved from the trial and error approach of the first occupant builders so that, today, a team of specialists usually produce a detailed design before construction begins. The evolution of the building design process continues with continuing developments in technology and the accompanying changes in society's aspirations for its buildings.

2.1.2 Objective of Building Design

The design of a building is part of a development process. Development is one means of pursuing overall objectives: designing and constructing a building, or building modifications, is one means of development. The building development process can be divided into four functions as outlined in Figure 2.1. The building design team assist the building owner, or developer, with the feasibility study, which is completed before design is commenced. Design and presentation are the full responsibility of the design team, whom also oversee the construction. Although it is necessary for each part of the building to be designed before it is presented, and presented before it is constructed, it is not necessary for each of these three functions to be completed for the whole building, before the next is commenced.
Buildings are developed to house a wide range of objects and activities: large buildings housing people undertaking occupational activities are the concern of this report. Such buildings are developed for organisations.

Markus [1969] presents a conceptual model (Fig. 2.2) of how a building relates to the objectives of an organisation. The main function of a building is to provide an environment in which activities can be performed; the main reason for the activities is to meet the objectives of the organisation. Emphasis is given to the interactive nature of the environment and activity/behaviour systems by a two way arrow. The one way arrows indicate the nature of one system providing for the other. Markus suggests that the diagram can be viewed as the developed surface of a cut cylinder to aid in picturing the two concepts of: the organisation performing activities to meet its objectives, and the organisation using a building to house these activities.
The objectives of organisations suggested by Markus are interesting. In addition to production, he includes adaptability, stability, and morale. They relate to the personal objectives of the people working for the organisation and are necessary for a continuing satisfactory level of production. For a building to be successful it must provide an environment that caters for personal activities in addition to production activities. Markus recognises this point by including informal activity in his activity/behaviour system.

Markus's model only applies completely to buildings that are owned and occupied by the same organisation. Some large buildings are developed primarily as a financial investment to sell or lease. However, to attract buyers or tenants the building must provide a satisfactory environment for their activities.

In addition to the major function of housing activities, the status of the owning organisation and the status of any tenants can be significant. The form of the design can also be influenced by the status of the designers. Financial constraints and responsibilities to the general public as regards safety and the impact of the building on the external environment are further design considerations. Thus the objective of building design is to describe, for construction purposes, a building that, within any financial constraints, will form a satisfactory environment for the organisational activities it is to house, while having regard to the owning organisation's objectives and any requirements for status, safety, and impact on the external environment.

2.1.3 An Information Processing Activity

What activities will occur in the required building? What are the spatial, thermal, acoustic, communication requirements of each activity? How can these be met? What criteria should be used to choose between
Cost of design, construction, labour, material, profit, overheads, etc.

Cost of maintenance: energy, heating, lighting, power, cleaning, repair.

Cost of activity: salaries, materials, non-productive time, absenteeism, morale, advertising, etc.

Value of achieving objectives.

The cost system.

FIGURE 2.2: THE BUILDING-ENVIRONMENT-ACTIVITY-OBJECTIVES-SYSTEM
alternative solutions? Design is a question and answer activity; a problem definition and solution process. The design problem and possible solutions are investigated until a satisfactory final solution is produced. The final solution must then be described so it can be constructed. Information processing is a dominant feature of the design process.

A model of design as an information processing activity is presented in Figure 2.3. The design process is coarsely viewed as involving two activities: solution generation, and appraisal of possible solutions to decide on the final solution. Nine types of information can be identified as components of the information flow. Information that is particular to the design problem flows across the diagram; broad characteristics of the problem are converted to possible solutions, appraisal criteria, and constraints; the possible solutions are appraised to produce a final solution. The possible solutions are influenced by four types of information which have general applicability to all building design problems: experience data, available commodities, design methods, and creative stimuli.

The characteristics of the building design problem are the requirements imposed by the activity/behaviour system the building is to house. The client of the building design team, and the organisation he represents, are sources of this information which the design team must interpret, and expand upon, by drawing upon experience data and design methods. The client's requirements, as perceived early in the design process, are documented as a formal project brief. This explicit statement of the building's performance requirements acts as a checklist for both client/designer communication and later design processing. However, as Luckman [1969] points out, the brief is never a complete statement of the building's requirements. Some characteristics are not apparent at the time of the brief's production; other characteristics are not formally documented.

Criteria are measures of performance for possible solutions.
FIGURE 2.3: DESIGN INFORMATION PROCESSING

KEY:

Information

Activity

SOLUTION GENERATION

Problem Characteristics

Creative Stimuli

Design Methods

Possible Solutions

Criteria

Constraints

Final Solution

Experience Data

Available Commodities
Constraints are ideas that confine the range of solutions which can be considered to be satisfactory. Constraints are often expressed as rules as in codes of practice. These two types of information are detailed from the characteristics of the problem and experience data. They are used to measure the quality and sufficiency of possible solutions.

Experience data is of two types: personal to the particular designer through his association with other projects, and distributed experience through technical publications and codes of practice. Experience data includes information on: what characteristics have been considered as significant for similar design projects, how to express these as performance requirements, and what solutions have been used in the past. Commodities, which are the industrial products available for use as components in any proposed solution, are also publicised in the technical literature.

Design information processing techniques are usually referred to as design methods. Bishop and Alsop [1969] define design methods to be one of four sub-sets of all the procedures used in the building development process. The other sub-sets are management, presentation, which is referred to as design realization by Bishop and Alsop, and construction procedures. Design methods are oriented towards finding a satisfactory solution to the building design problem, whereas the other sub-sets of procedures are concerned with organizing the resources used for the design process, documenting the chosen solution so it can be constructed, and the building construction process.

An important set of information that influences the form of the possible solutions generated is the creative stimuli [Osborn, 1963]. Innovative solutions are the products of creativity; creativity is influenced by the designers' psychological perceptions of themselves and their design problem. Although it is difficult to control creative stimuli, they must be recognised as an information component of the design process.
Figure 2.3 is not meant to imply that the design process occurs as a single sequence. Further problem characteristics become apparent as solutions are generated and appraised; further possible solutions become apparent as other solutions are appraised; design is an iterative process. A hierarchy of subdivision of the design problem accompanies the iteration. The design disciplines of architecture, structural and building services engineering, etc, evolved on the basis of the separable nature of building design problems. Further subdivision is used within each discipline. For example, the building services problems of provision of utilities, such as hot and cold water, and provision of a satisfactory thermal environment are approached as distinct design problems, although a combined solution is often used. The information processing model is valid for both the total design problem and those parts of the total problem that can be separated and approached as distinct design problems.

Although present day building design has been emphasised in describing the information processing model, it is sufficiently flexible to describe both past approaches and future possibilities. It is a valid description of the mental processing that accompanied trial and error construction and is used to classify developments in building design procedures described in Section 2.3 of this report.

2.1.4 Other Models of Design Processing

The wide ranging successful application of the formal scientific method [Ackoff, 1962] has prompted many researchers to produce theoretical models of design processing. The process of design needs to be adequately understood to effectively implement new procedures. However, a review of the published descriptions produces a seemingly conflicting set of models. The apparent contradictions arise, not from any misconceptions about the
process of design, but from the process of classification itself [Pirsig, 1974]. Although a classification system nurtures insight, it also makes it difficult to comprehend features that are incompatible with the classification model. A model is, by definition, an incomplete description. Figure 2.4 is an attempt to remove the apparent contradictions between the various descriptions of design processing by relating them to the simple two step information processing model.

Solution generation can be regarded as consisting of two steps: analysis of the problem and ideation. Appraisal can also be regarded as consisting of two steps: analysis of the solution and evaluation. The resulting four step model is used in Figure 2.4 to classify the terms used by other researchers. Jones [1963] uses the terms analysis and synthesis to describe the two steps of solution generation. His model has only three steps so his use of the term evaluation is taken to be synonymous with appraisal. Markus [1970] uses these three steps of analysis, synthesis, and appraisal in his model, then subdivides appraisal into three terms: representation, measurement, and evaluation. At this level of subdivision a further step, presentation, between measurement and evaluation, would appear appropriate.

Wilson and Wilson [1970] suggest that design consists of the sequence: preparation, synthesis, analysis, selection, and decision. The idea that synthesis precedes analysis appears to conflict with the previously described models. Recognition of two types of analysis: analysis of the problem and analysis of the solution, or possible solutions, resolves the apparent contradiction. Reinschmidt [1972] describes the engineering design process as consisting of four steps: synthesis, analysis, evaluation, and sensitivity analysis. His use of the term synthesis is taken to be synonymous with solution generation. The inclusion of sensitivity analysis demonstrates the necessary evolution
FIGURE 2.4: RELATIONSHIP BETWEEN MODELS OF DESIGN PROCESSING
of descriptions of the design process to reveal the changes of emphasis that occurs with development of design procedures. Computer-aided design procedures have enabled more effort to be expended on the important sensitivity analysis step.

The variation of emphasis for particular design activities is a reflection of the viewpoint of the model builder. It is interesting to note that architectural design requires more effort to be expended on analysis of the problem than engineering design, which requires greater effort for analysis of solutions. This point is reflected in the differences between the models.

A further model is included in Figure 2.4. The four design steps, or energy states, of Koberg's and Bagnall's [1973] model of the problem solving process are included. Analysis of the problem is subdivided into analysis and explicit definition of the problem. Ideation completes solution generation and appraisal is described as selection. Koberg and Bagnall point out that their seven energy states, which involve design, implementation, and feedback, may be traversed in many different ways: linear sequence, circular configuration, constant feedback system, or branching system. Design is an exploratory journey.

The complex nature of large buildings and the team approach have resulted in the use of formal strategies to control the exploratory nature of the building design process. The Royal Institute of British Architects [R.I.B.A., 1965] suggest there are twelve stages to the building development process. Elms [1972] describes the three design stages of present practice as: outline scheme, preliminary design, and detailed design. Markus [1970] suggests that his design processing model is a second dimension of the multi-stage building design process. The stages are progressions from the general and abstract concept for a
building to the particular and concrete final solution. Each design stage involves use of the solution generation, appraisal process at the appropriate level of detail.

2.2 DEVELOPMENT AND THE BUILDING DESIGN PROCESS

2.2.1 Development Stimulus

Society has just come through a period of extensive technological development. It started with the industrial revolution and reached its zenith in the last decade. During this period technology has featured as the means for development; development has featured as the means for success. Ackoff [1962] suggests we have supplemented the ideals of truth, peace, and beauty with the ideal of plenty. The impossibility of achieving this ideal on a world wide scale has only recently become apparent [Meadows et al, 1972].

The ideal of plenty, or materialism as it is often called, is portrayed in society's expectations for buildings and the environments they produce. Pursuit of these expectations stimulates technological development of both products, and design and construction processes for buildings. Their satisfaction leads not to contentment, but to greater expectations. The resultant effect on design processing is increased complexity, arising from greater expectations for buildings and a wider range of products and processes for cognizance.

Our present period of transition from unlimited growth to a more stable state, or even to de-development [Ehrlich & Ehrlich, 1972], is unlikely to reduce the complexity for building design. The relative importance of design requirements may change, but the number of problem
characteristics is unlikely to reduce. The recent energy consciousness has increased the complexity of design as more extensive controls are being considered to reduce energy wastage. Wind concentration, shading, and noise control have evolved as additional considerations for building designers with increased awareness of the importance of the quality of the environment. These developments suggest that environmental consciousness is resulting in even greater complexity in building design.

2.2.2 Development Response

Separation of design from the construction process was a past response to increased complexity, as was the specialisation of designers and the team approach. The effectiveness of these responses is evidenced by their continued use. However, as Elms [1972] points out, communication between all the participants of the building development process is a problem area. Design and build organisations, group design practices, and more emphasis on the management role of the architect are attempts to improve communication difficulties. Although communication problems will tend to constrain further specialisation, increasing complexity will prevent its decline.

The most significant response to recent increased complexity in building design is the extensive development of design methodology. Modification of design procedures has always been a significant mechanism for designers to respond to changes inherent in our industrial development. However, the rate of change of design office procedures has recently increased. Robinson [1969] reports that 90% of design office procedures were superceded in a ten year period. Such rapid change has been aided by a powerful information processing tool: the electronic computer.
Use of computer aided design procedures, like the use of specialist designers, can reduce the extent of effort required by a designer to take account of complexity. A designer must understand the aims, limitations, data requirements, and significance of results of design processing not undertaken by himself. He does not, however, need to know the detailed steps that constitute the processing.

Although the use of computer procedures will reduce the need for further specialist members of the building design team, their use is not a direct substitute for specialists. Computers can rapidly process problems that are modelled objectively. The objective nature of computer processing requires clear communication which is achieved with well defined terms and symbols. A resulting characteristic is the extent of control the designer retains. Associated with this control is the retention of full responsibility for design decisions, which in the end require subjective judgement.

The development and use of new design procedures has been approached from two points of view: efficiency and effectiveness of the building design process. Efficiency is aimed towards minimizing the resources required to produce a building design. Effectiveness is aimed towards maximising the quality of the designed building within the constraints of the resources available. Design procedures that are effective in taking account of increased complexity are appropriate to society's aspirations.
2.3 DEVELOPMENTS IN BUILDING DESIGN PROCEDURES

2.3.1 What Are Building Design Procedures?

Procedures are formalized methods for pursuing an identifiable goal. Building design procedures pursue goals that combine to eventually produce a formal statement of how a building is to be constructed. Bishop and Alsop [1969] classify procedures into two types: determinate and non-determinate. Determinate procedures involve no judgement once selected, so the resulting goals are reached without recourse to subjective decision. Structural calculations for a building frame provide a good example of a determinate procedure. Once the mathematical model has been chosen, a sequence of logical steps is followed which requires the necessary data, but no further subjective decisions. Complex, or time consuming, determinate procedures are obviously well suited to automatic processing, so many have been developed as computerised procedures.

Non-determinate procedures draw upon the experience and knowledge of the person using them throughout their use to assess the non-quantifiable characteristics of the problem. Design of the decor for a building is performed with non-determinate procedures. Most building design procedures are combinations of determinate and non-determinate procedures. Use of the computer with these procedures is restricted to the determinate component with provision for the person using the procedure to perform the non-determinate steps. As the computer is used in a complementary role, the term, 'computer aided procedure', is used.

There has been extensive development of building design procedures in recent years as a means of taking account of the increasing complexities of modern buildings. Although many developments have been possible because of the availability of the digital computer, not all developments require
its use. The developments have been classified on the basis of the information processing model of design illustrated in Figure 2.3.

2.3.2 Analysis of Problem Characteristics

Designers' experience and abilities are obviously significant in the definition and structuring of the characteristics of the design problem. However, Kraus et al [1969] suggest designers can have difficulty finding proper entry points to their problems. Systematic analysis procedures have been developed to help with this difficulty. Osborn's [1963] check lists are well established, both as formal briefing guides, and as an easily applied technique. Jones's [1963] classification of random factors and interaction matrices are useful techniques for unfamiliar problems.

Performance specification, as opposed to direct specification of a partial solution, has been widely promoted [Mainstone et al, 1969] as a technique to prevent unnecessary constrained thinking. As the consideration of possible solutions always reveals previously unrecognised problem characteristics, it is not possible to produce complete performance specifications at an early stage. However, the expression of possible solutions, as they evolve during the design process, in terms of their performance specifications is a worthwhile procedure for problems that offer scope for innovation.

Alexander [1964] was the first to use the sortation capabilities of the digital computer to aid designers with systematic analysis. Hierarchical decomposition is a technique that groups problem characteristics according to their extent of interaction, then suggests a sequence of synthesis to take account of these interactions. Hanson [1969] makes the point that the requirement imposed by the procedure to explicitly
list all problem characteristics and their interactions makes at least a single use of this method worthwhile for any designer. Elms and Clark [1976] combine the ideas behind performance concepts and symbolic pattern languages [Montgomery, 1970] with the hierarchical decomposition procedure to produce a very flexible procedure. They emphasise that these techniques are only aids to a very indeterminate process in which the designer controls the extent of both their use and the guidance he gains from them. It is of interest to note that although the hierarchical decomposition algorithm is a well defined sequence of steps, the problem characteristics that are represented in it may be quite vague concepts.

Luckman [1969], when describing a similar technique to hierarchical decomposition, makes the point that systematic analysis is of value when the problem involves a large quantity of information, the consequences of a poor solution are significant, and the chances of producing a poor solution are high. A better design product, and perhaps less effort at later stages of design, make the additional effort at the analysis stage worthwhile. Simpler problems may warrant less rigorous analysis, but increasing complexity is likely to promote the need for systematic analysis procedures.

2.3.3 Enhancing Creative Stimulus

Innovative solutions are required when design problems are significantly different from past problems as existing solutions are not appropriate. Changes in society's habits and aspirations are producing greater complexity and different priorities for building design problems. New solutions are called for: creativity is required.

Creativity is a subconscious act. Our incomplete understanding of the subconscious mind has enabled the creative act to retain some of
its mystery. Matchett [1968] suggests controlled introspection of one's thought processes can aid understanding and improve communication between the conscious and subconscious realms of thought. Greater control may be achieved, but creativity may also be inhibited. Emotion and irrationality make significant contributions to creativity [Gordon, 1961].

Recognition of this fact has lead to the development of procedures that promote creativity by producing favourable psychological stimuli. Suppression of critical analysis and association of subconscious ideas underlie Osborn's [1963] well known brainstorming procedure. Creative insights can be gained by viewing the problem from different points of view. Gordon's [1961] synectics uses metaphoric mechanisms; Jones's [1970] checklists use a series of questions such as: Put to other use? Adapt? Modify? etc; De Bono's [1967] lateral thinking is a package of procedures for developing new ways of viewing problems. In fact, as creativity is so important for development, there is a great deal of literature on the subject. Koberg and Bagnall [1973] present a good survey of creative techniques in the design context.

Creative stimulus procedures are valuable if used correctly. They are aids to subconscious thought processes; as such, they should only be used if they do, in fact, aid.

2.3.4 Gaining and Disseminating Experience Data

Experience data, which is often termed feedback, or feedforward data, is generally gained and disseminated by informal procedures. Designers supplement their personal appraisal of their products with appraisals from the building constructors, owners, and users. The construction process enforces communication between designers and constructors: construction feedback is effective. Although building owners communicate with designers before, and during, early occupancy, many strengths and weaknesses of buildings do not become apparent until the building has been in use for some time. Opportunities for informal
communication between designers and building users are rare: only major user problems are effectively communicated. As building users display remarkable adaptability in their use of buildings, the feedback of the extent of failure, or success, of the designers' product-in-use is ineffectual.

When formal appraisals are undertaken, it is obviously of value to other designers to learn of solutions used and their effectiveness. However, publication of a building's imperfections is not complimentary to the designers responsible. If building users have their assessment of an inadequacy confirmed, it is likely they will expect the building owner to provide improvements. These pressures act against formal criticism. Most appraisals published in technical journals are quite superficial.

An exceptionally detailed appraisal of a Scottish school was undertaken by the multi-disciplinary Building Performance Research Unit from the University of Strathclyde [B.P.R.U., 1970]. This appraisal demonstrated the excessive effort required to perform a thorough appraisal and produce results that are meaningful for future designs. Appraisal of actual buildings in use appears likely to remain unsystematic.

The Strathclyde Unit put emphasis on developing a range of performance indices; some of which can be used for solution appraisal during the design process. Easily obtained performance indices are useful developments for comparison both between possible design solutions and with the norm for existing buildings. However, there is a danger in their use: as an easily expressed performance measure, they can be given excessive consideration compared with less explicitly measured features. Better perspective is gained by using either a range of indices that measure interacting building features, or by using a common unit of measure for all pertinent performance characteristics.
An example of the unreliability of intuitive rules based on simple indices is reported by Thomsen [1965]. A procedure was developed to estimate the initial cost, operating costs, and revenue for a range of shapes of commercial buildings from all the component costs such as land acquisition, foundations, mechanical services, and other building sub-systems. These estimates were used to compute the percentage return on investment. It was found that the optimum solution did not maximize the ratio of net rentable area to gross area, did not minimize the cost per square foot, and did not cover the entire site.

The significance of this procedure, and a similar one developed by Harper [1968], is that they use performance measures from actual buildings as a basis for formulating the computer simulation procedures. Actual experience is extended by simulation. Although simulation enables interpolation between experience data values and can produce results for a wide range of combinations of the variables, any constraints, such as particular types of buildings, are carried over from the actual experience data to the simulation model. Simulated experience retains the constraints of the actual experience it extends.

Increasing the range for cost simulation models by including many types of buildings increases the problem of paucity of data compared with the number of significant variables. Recognition of the uncertainty of cost estimates is recognised in the model developed by Robertson et al [1974]. Using recorded cost experience data for a wide range of buildings, they established log-normal probability distributions to describe the likely cost of a complete range of building components. These distributions were combined into a cumulative distribution function for total building cost based on size of building. The model provides measures of reliability in addition to an expected value. Although the procedure used to develop this uncertainty model remains valid, the model suffers from a problem
common to all models based on cost experience data. The recorded cost experience data used to produce the model becomes outdated due to rapid and unevenly distributed inflation. The benefits of using such models must be weighed against the effort required to update them.

Real buildings are only one source of experience data. Our technological development is characterised by its objective knowledge of the world [Steadman, 1975]. This collective experience has undergone extensive development with the aid of symbolic documentation and ease of dissemination. Use of such worldly experience data has aided extensive development of research into theoretical models of the nature and behavior of products and processes used for buildings. Designers receive this information as formal theories and codes of practice.

A significant recent development, made possible by the information processing capabilities of the digital computer, is the combination of component theories to produce theory based simulation models of building system behaviour. As with cost simulation models, a wide range of combinations of variables can be researched to establish their interactive sensitivity. Theory based simulation models tend to be less constrained than those based on experience data values.

Hawkes and Stibbs [1969] have developed and used a simulation model to research the effect of a wide range of preliminary architectural design decisions on the environmental performance of buildings. Two types of experience data result from such studies: greater understanding of the building's performance as a system, which can be disseminated as generalisations, and recognition of the variables that require sensitivity analysis for particular design solutions because of the extent of their interaction. In this way the level of detail required in appraisal models for use during the design process can be established.
Thus the present pattern of development of building experience data is away from direct appraisal of real buildings towards research into more detailed definition of building requirements. Controlled conditions are achieved through both physical and theoretical simulation exercises. More detailed specification of performance constraints and criteria, and improved models for appraising design solutions, are resulting from this effort.

2.3.5 Selection of Commodities

The increasing range of commodities available for inclusion in buildings has produced increased complexity for building designers. It is more difficult to remain aware of all available products; it is also more difficult to select the most appropriate product. Advertisements in a wide range of publications aim to reduce this difficulty. However, the number of advertisements is a problem for both designers and advertisers. Designers do not have time to read them all; advertisers believe that half of their advertisements are quite ineffective. As the difficulty lies in determining which half, the problem is likely to remain.

The problem of awareness of the range of commodities has received considerable attention. Classification systems have been developed as aids for retrieving documentation specifically related to the building industry [Gilchrist & Gaster, 1969]. Product classification is a valuable subset of these systems. Pre-classification of documents before dissemination has ensured the successful use of this development.

The problem of selection between competing products has been aided by the development of standards of performance such as fire-ratings. As actual in-use conditions can differ significantly from standard test conditions, standards are not a perfect measure. However, their proven usefulness will ensure their continued development.
In an early innovation, West Sussex County Council [Ray-Jones, 1968] developed computer aided selection of commodities by performance attributes, including construction cost. Selection was from a computer based commodity file containing a range of commodities relating to a method of system building. To give the architect more information for selection, contractors provided unit prices for construction using each of the commodities. More detailed and rapid appraisal of possible solutions was achieved, but at the expense of constraints on the range of possible solutions.

There are other examples of procedures developed for computer-aided selection by performance attributes for particular types of products. A comprehensive computer aided structural design aid [Logcher et al, 1968] includes automatic selection of minimum weight steel sections from a standard range to meet analysed section modulus and area requirements together with any other specified constraints. A heating and air conditioning analysis computer aid [MWD Systems Lab, 1977] includes a procedure for automatic selection of standard wall and roof constructions to meet a required thermal conductance. Automatic selection by computer is only possible for determinate procedures for which selection criteria and constraints can be modelled logically.

The West Sussex County Council developments [Ray-Jones, 1968] included computer aided construction documentation. The documentation procedures were based on the same commodity file as the design procedures, so were also constrained to the one type of systems building components. Unconstrained computer aided building documentation procedures have since been developed. SPEC-2 [MWD System Lab, 1974] is one example of a number of specification computer aids. Specifications can be rapidly output by selecting from a computer file of standard clauses and modifying or extending these as required. Burgess [1970] describes a computer aided
building design presentation system that produces related drawings and technical specification and bill of quantity clauses. The automatic selection is by identifier based on an extension to the CI/SfB building data classification system [Gilchrist & Gaster, 1969]. Computer aided documentation procedures provide more rapid and easily modified construction documentation. They appear to have had some impact on the efficiency of the process; their impact on its effectiveness has not been established.

All these developments, together with the development of appraisal computer aids that use commodities data, led to suggestions for less constrained commodity files. Detailed information on all available commodities would be stored on computer files for direct access by computer procedures. As Reinschmidt [1972] points out, information costs money to provide; large commodity files would be expensive. Bishop and Alsop [1969] suggest industry wide use to distribute the cost. Experimental computer based commodity files have been developed by Whitton [1969] and Degelman [1969] for easily formulated commodities. However, the simulation exercise using specialists on floor finishes to role play a computer based commodity file demonstrated many practical problems [Cross et al, 1970]. The size of a comprehensive commodity file, which Jackson [1970] states would be well in excess of a hundred million characters (bytes) of data, suggests access problems, particularly if many designers are to access the file. It can only be concluded that large scale computer based commodity files will remain infeasible for some time.

Future developments of commodities procedures will include more standards for performance measures and increased use of the existing classification systems. Non-computerised commodity files are likely to become better organised and more widely available. Paucity of data will
be less of a problem for them than the more formally structured computer based commodity files. Computer based indexing procedures may prove useful for aiding retrieval of documents from these manual commodity files. Small computer based commodity files with constrained ranges of products are likely to be developed as appraisal procedures warrant their existence. These may evolve, as Reinschmidt [1972] suggests, into more comprehensive computer based commodity files.

2.3.6 Solution Procedures

As designers learn about their design problems subconscious associations produce ideas for partial solutions. There is usually little difficulty in finding a solution to a design problem; the difficulty is in producing a good solution. Kraus and Myer [1968] point out that psychology and economy are against designers generating full sets of valid alternatives as they are inclined to fall in love with their first proposal. Most of the previously described developments in building design procedures are oriented towards producing better solutions.

Operational research techniques have been used as a basis for formulating automatic solution generation procedures. Aguilar and Hand [1968] used linear programming for an architectural planning procedure. Campion [1968] used queuing theory to aid his design for circulation layout requirements in a restaurant. Willoughby et al [1970] produced a heuristic modelling procedure for generating layouts of University buildings on a site. Gupta and Anson [1972] describe a sequential type search procedure for choosing optimum building envelopes. All these procedures require a single criterion that can be expressed in mathematical form. All other criteria can only be considered by expressing them as constraints. They are examples of constrained optimisation applied to separable parts of the total building design problem.
Brotchie and Linzey [1971] attempt to overcome the sub-optimal approach by formulating the total building design problem as a quadratic programming problem and suggesting a multi-stage optimisation procedure. This procedure attempts to optimise the highly interactive decisions made early in the building design process. However, the difficulty of obtaining building performance data in the form required for this procedure, together with the over-emphasis on automatic synthesis, has impeded its use.

Optimisation procedures must be regarded with some scepticism. As buildings tend to have a variety of uses throughout their life, design for flexibility may be more important than design for an immediate best solution. As all criteria cannot be expressed in a single index, the optimisation is only approximate. Excessive computational resources required for optimisation can also be a problem [Willougby, 1975]. However, operational research techniques should be employed to generate a range of possible solutions for more detailed appraisal.

2.3.7 Appraisal Procedures

Possible solutions to design problems begin as ideas; each idea must be appraised to ensure it can satisfy all performance requirements. When more than one satisfactory solution has been produced, appraisal of relative performance is necessary so that a choice can be made. Ideas are usually initially appraised in a coarse manner involving mainly subjective judgement. Satisfactory ideas continue to be appraised in more detail as they are given more form or modified to meet detailed performance requirements. Detailed appraisal procedures based on objective performance measures have recently undergone extensive development.

Detailed appraisal can be regarded as involving two steps: analysis of the possible solution's performance, and evaluation of this
performance with respect to constraints and criteria. The computer has proved to be a valuable aid for detailed analysis when the solution's behaviour can be modelled mathematically. Formulation of the model, and evaluation of the analysed results, require the designer's subjective judgement, but the mathematical analysis is a determinate procedure. The computer can be used, not only to undertake the calculations, but also to sort and present the results for ease of comprehension and evaluation.

Analysis computer aids have developed to such an extent in the engineering design disciplines that suites of procedures have been produced [Logcher et al, 1968; MWD Systems Lab, 1977]. A distinctive feature of these suites of procedures is the effort expended to improve the man-machine interface: command languages using words that are meaningful to the designers are provided for ease of problem formulation and control of the analysis process; a selection of problem and result data presentation formats, including graphical presentation, are provided for ease of comprehension and selection of pertinent detail; permanent storage and modification capabilities are provided for ease of formulation correction and sensitivity analysis. Generalised procedures have been produced to reduce education overheads and to provide scope for flexibility of design approach. Successful integration of computer capabilities with a single designer has been achieved with these developments where development and maintenance overheads can be shared among many designers.

Analysis models often include constraints as part of the formulation of the problem. Fenves and Goel [1969] demonstrated that checking against complex constraints can be formulated into computer procedures based on logical decision tables. Such automatic checking
procedures are best included as part of a larger suite of procedures where the same logic can be used as part of automatic selection of components from a standard range [Logcher et al, 1971].

Constraints are often used as a more expedient technique than detailed analysis for taking account of performance criteria. As the availability of the computer makes more detailed analysis easier, there has been some development towards explicit measurement of safety or convenience criteria. Mavet [1971] promotes the use of probability theory for measuring building services performance requirements. Simulation of occupants' internal travel times and costs are included as part of the optimisation approach of Brotchie and Linzey [1971].

Further development of technical performance measurement is likely for those functional systems of buildings where consequences of exceeding usual constraint levels can be adequately determined and the cost, or quality, of the building is significantly influenced.

The relative importance assigned to performance criteria reflects society's current values. In the past the capital cost of a building was an extremely dominant criterion. However, the increased quantity of mechanical and electrical equipment in buildings, together with the recent relative increase in energy costs, has increased the significance of operation and maintenance costs. Cost-in-use criteria [Stone, 1970] have been developed using time value of money theory [Lu, 1969] to combine initial and annual costs. Cost-benefit theory has been applied to building design decisions [Brotchie & Linzey, 1971] to measure building occupants' convenience. As Stern [1976] points out, it is easy to abuse this technique by making biased assumptions or ignoring significant factors. Although a total cost value can be a useful evaluation index, it is more easily kept in perspective if component costs,
significant assumptions, and intangibles are explicitly presented for simultaneous evaluation.

Presentation using tabular techniques has been suggested for analysing building design problem characteristics [Jones, 1963], but it has not been promoted as a solution evaluation aid as used by ecologists in their environmental impact assessment matrix [Harrison et al, 1973]. It is obviously easier to achieve balanced judgement when all criteria are well presented in a compact manner.

Development of appraisal procedures will continue as a means of meeting changing requirements for buildings. Although both analysis and evaluation procedures will develop without the need for automatic information processing, the computer will remain a valuable means of implementing many appraisal procedures.

2.4 THE ROLES OF THE DIGITAL COMPUTER

Many of the developments in design procedures have been made possible by the rapid information processing capabilities of the digital computer. Detailed analysis of the interacting nature of problem characteristics [Alexander, 1964; Luckman, 1969] uses the computer's logical sortation capability. Sortation is also used for automatic selection of commodities by both classification identifiers [Burgess, 1970] and performance attributes [Logcher et al, 1968]. The computer's arithmetic capability enables the extension of experience data [Harper, 1968; Hawkes & Stibbs, 1969], automatic solution generation [Brotchie & Linzey, 1971], and analysis of performance as part of appraisal of possible solutions [MWD Systems Lab, 1977]. Automatic storage, retrieval and presentation have been mainly used for building
design as supplementary capabilities. The computer's power is not only in rapid processing of distinct capabilities, but also in its ability to automatically combine its separate capabilities to produce flexible integrated information processing procedures [Logcher et al, 1968].

Increasing use of computer aided design procedures has been accompanied by greater integration. Early developments were oriented entirely towards the computer's arithmetic capability [Gero, 1970]. Problems had to be formulated in terms most meaningful to the computer [Harper, 1968b]. Later, sortation was used for automatic translation from terms oriented towards the designer's conceptual model to a model for arithmetic processing [Harper, 1968b]. Permanent storage of results and problem descriptions was introduced to provide flexible presentation of results and ease of combination of processes into suites of procedures [Roos, 1966]. Integrated use of computer aids by single designers has resulted.

More extensive integration of procedures for use by all members of the design team has been proposed. Degerman [1969] suggests a comprehensive system based on large computer data banks and a dynamic computer based model of the building design which begins as coarse ideas and gains detail as the design progresses. Jackson [1970] and Logcher [1971] discuss the technical requirements for automated interdisciplinary information sharing. The extent of computer storage required for comprehensive computer files [Jackson, 1970] will both technically and economically limit the degree of integration. Recognising that early design decisions have the greatest influence on the cost and quality of a building, Elms [1972] discusses the requirements of a comprehensive computer aided building design system for preliminary design only. He suggests that the computer can aid interdisciplinary communication,
complexity, time, and lack of flexibility problems inherent in present
design methodology. Although the latter problems have been aided by
computer procedures developed for use by individual designers, there
are significant difficulties with information sharing between designers:

1) Subjectivity

Computers cannot make subjective decisions. Although objective
modelling has proved to be a powerful tool, it has limitations. Vague
concepts, such as aesthetics, cannot be objectively modelled. Automatic
selection of components based on performance attributes [Ray-Jones, 1968]
approaches complete decision making by objective means. However, most
objective models only reduce the extent of subjectivity required: they
do not replace it altogether [Stern, 1976]. Further development of
objective models is likely to accompany greater understanding of the
mental processes of decision making [Teague, 1968], but gaining such
understanding will require much time and effort.

The necessity for subjective judgement to be used with objective
models limits the scope of formulated computer models. Even when
formulated, part of the model remains in the designer's mind. If data
that is stored by one designer is to be used by another [Teague, 1968],
the underlying assumptions and uncertainty associated with the data need
to be communicated to avoid its misuse. Manual communication procedures
enable selective communication based on how definite design decisions
are at the time of communication. Accompanying informal discussion
can indicate relative significance and uncertainties associated with the
information.

Uncertainty can be objectively modelled [Benjamin & Cornell, 1970].
Its use in comprehensive computer aided design systems has been suggested
by Reinschmidt [1972], but additional effort will be required for both
data input and computation. However, automatic interdisciplinary
information sharing would remove control of the information from the
designer that produced it. Problems associated with proprietary
considerations and appreciation of context to avoid misuse, or abortive
use, of the information remain unresolved. Computer aided inter-
disciplinary information sharing has only been used successfully for
presentation of construction information when all design decisions have
been made [Burgess, 1970].

(2) Data Input

Inputting information into computer storage and ensuring its
correctness requires significant effort. Shappell [1976] reports that 40%
of structural analysis effort, as measured by elapsed time, involves
input data preparation compared with 5% for computation. Many problems
in architecture and architectural science involve larger quantities of
data with less computation [Gero, 1970], so input becomes even more
significant. Communication aids, such as designer oriented languages,
automatic digitisers, light pens, and sketch facilities [Carter, 1973]
reduce the human effort required, but have development and usage overheads
associated with them. Although much has been written about extensive
graphical communication aids [Degelman, 1969; Negroponte, 1970],
economic feasibility has limited their practical application in building
design [Carter, 1973]. In addition to the effort required to input data,
it can sometimes be difficult to obtain the information in the required
form for computer procedures [Bishop & Alsop, 1969]. Hence data
requirements for models continue to constrain the feasibility of
computer aided design procedures.
(3) Common Data

The idea of information sharing is attractive from the point of view of making the best use of the resources used to store the information [Teague, 1968]. However, the incomplete nature of models often results in different levels of detail required by different procedures. The architectural, structural, and building services preliminary design models all refer to the same building geometry and topology [Jackson, 1970], but the quantity of common information is quite small as different dimensions are relevant to each discipline. A common model would need to be more detailed than any of the individual procedures required. Hence sharing information between procedures may not significantly reduce the quantity of information that must be input to computer storage.

(4) Methodology

Methodology is important; the total procedure for usage of a computer model must be included when considering feasibility. Human factors can render a technically efficient system ineffective [Exley & Harding, 1977]. Reinschmidt [1971] points out that in spite of considerable effort expended on investigation of interdisciplinary information sharing, no satisfactory solution has been produced. This effort [Jackson, 1970] investigated the technical requirements, but the more difficult problems relating to human control of information usage and amendment, human independence, and the influences of personality [Mitroff, 1972] remain unresolved. As Carter [1973] reports, computer aided design has proved to be more difficult than people thought.

As many manual design activities are multi-purpose, new methods must allow for secondary objectives [Exley & Harding, 1977]. For example while undertaking manual calculations a designer can continuously check that parameter values are similar to his expectations. With automatic
computation, checking is still required, so must become a distinct, and sometimes tedious, step in the new procedure. Pilot schemes and simulation exercises, like the role playing to investigate computer based commodity files [Cross et al, 1970], are useful techniques for investigating detailed methodological requirements.

Thus the feasibility of interdisciplinary integration of computer aids must remain uncertain. As significant methodological problems remain unresolved, the extent of integration achieved will depend upon the need for integrated procedures as conceived by the designers whom must use them. Research results influence designers' viewpoints, but many of the computer aids for effective integration require more development effort than research effort. Kane [1971] concurred with this approach when he concluded that further work on components of a computer based building model should be undertaken in a development environment where usage can be studied and feedback from the building designers easily obtained.

Existing developments have established the present roles of the electronic computer. Where problems differ from past problems, and are of significant size and complexity, the computer has a role to aid systematic analysis [Luckman, 1969]. There appears to be little scope for integrating such strategy aids with other computer aids. When designers want to widen the range of possible solutions for subsequent appraisal, and when sufficient information is available in the appropriate form, the computer can aid with automatic solution generation procedures. The integrated use of automatic synthesis and objective appraisal procedures is feasible for use within a single discipline. Further automatic selection procedures, with associated commodity files, are likely to emerge as detailing adjuncts to appraisal procedures. The
major role of the computer will remain as an appraisal aid; not only during the design process, but also for research to produce pseudo experience data applicable to a number of buildings.

2.5 CHAPTER TWO SUMMARY

The building design process has evolved, and continues to evolve, to meet the changing requirements associated with technological development and society's aspirations. Its objective is the production of descriptions for the construction of buildings that will provide acceptable environments to house activities within any financial constraints and taking account of the owner's objectives and any status, safety, and external environmental impact requirements. The detailed description of a large building is developed from a vague concept by information processing activity undertaken by a team of specialists.

Design for large buildings is characterised by increasing complexity that has resulted from society's materialistic expectations and continues to result from emerging environmental consciousness. The most significant response to increasing complexity is the development of design procedures. Design procedures are formalised methods for undertaking the necessary information processing. Much of their development has been made possible by the availability of the efficient objective modelling capability of the digital computer.

Problem characteristics, creative stimuli, experience data, and available commodities are used with design procedures to generate solutions to design problems and to appraise them. Criteria and constraints, which are used for appraisal, are also detailed from this
information. Solutions must take account of all significant problem characteristics and the interactions between them. Systematic analysis procedures have been developed to aid the definition and structuring task. Creative solutions are products of the subconscious mind, and as such, are nurtured by favourable psychological stimuli and informal methodology.

Experience data results from designers' own experience and objective knowledge, which is disseminated informally and as formal theories and codes of practice. The digital computer has proved to be a valuable aid for extending experience information by simulation. The dissemination of information describing available commodities has gained formality with the use of building classification systems and performance standards. Although suggested comprehensive schemes are presently infeasible, restricted computer based commodity files are being used successfully as adjuncts to computer aided appraisal procedures.

Most systematic procedures aim to produce better buildings, particularly through promoting the generation of a number of possible solutions. Automatic solution generation procedures have been developed using optimisation theories. However, their usefulness is limited to generation of a number of possible solutions for appraisal in terms of the criteria that cannot be adequately included in optimisation models. The most extensive use of computer aided design has been for detailed objective appraisal of the performance of possible solutions.

The power of the digital computer lies in its rapid logical sorting and arithmetic processing and their combination with its automatic storage, retrieval, and presentation capabilities. Integrated use of these functions has enhanced its use by single designers. Suggestions for integrated use by the building design team introduces significant
methodological problems which remain unresolved. Computer aided appraisal by a single designer, and as a research exercise, will remain the major role of the digital computer in building design.
3.1 WHAT IS BUILDING THERMAL DESIGN?

3.1.1 Introduction

The major function of a building is to provide an environment in which activities can be performed. As illustrated in Figure 3.1, the thermal environment is one component of a built environment. So thermal design is one component of total building design. The range of control that can be influenced over the thermal environment using heating, ventilating, and air conditioning equipment has meant that status can be an important consideration. However, within the financial resources allocated, the purpose of building thermal design is to ensure the building description includes provisions for a thermal environment that meets the needs of the occupying organisation's activities.

As many design decisions relating to other components of the built environment influence the thermal environment, building thermal design begins as an architectural activity. However, the design of special purpose equipment is a well defined sub-problem well suited to specialisation. Heating, ventilating, and air conditioning specialists usually design thermal equipment as part of total building services design.

Design for thermal equipment has evolved to include many procedures that are based on mathematical models. The availability of the digital computer has aided the development of more detailed mathematical models
FIGURE 3.1: ENVIRONMENTAL COMPONENTS OF A BUILDING
for analysis of the complex heat flows that occur in buildings. Better sizing of equipment and appraisal of both energy usage and the quality of the resulting thermal environment are now possible.

3.1.2 The Thermal Environment in a Building

A building's thermal environment is produced by a complex system of dynamic heat flows. Thermal interchange can occur between the room and four heat sources, or sinks, as illustrated in Figure 3.2. Interchange with the external weather and adjacent rooms is modified by the room's boundaries. Except for the ventilation heatflow path, the thermal interchanges are a combination of conduction, convection, radiation, and latent heat absorption or release.

The sun is the fundamental source of heat for the external weather. Solar radiation is transmitted directly into a room through transparent boundaries and indirectly through absorption by all external boundaries. In combination with wind, clouds, and their moisture release, the sun influences convection, radiation exchange with the ground, sky, and other surrounding surfaces, and latent heat of phase change with the external surfaces of a building's boundaries. Temperature differences across the room's external boundaries produce heat flow by conduction.

Conduction also results from temperature differences across a room's internal boundaries with adjacent rooms. Temperature differences are induced by surface thermal exchanges: convection due to air movement, radiation due to relative temperature and absorption and emittance characteristics of a room's surfaces, radiation due to absorption of solar rays, and latent heat due to condensation and evaporation. Radiation transmission is also possible through any transparent internal boundaries.
**Figure 3.2: Thermal Interchanges of a Building**

- **Ventilation**
- **Internal Heat Sources**
- **External Boundaries**
- **Internal Boundaries**
- **External Weather**
- **Adjacent Rooms**

**Key:**
- Heatflow path
- Heat source or sink
- Heatflow modifier
Similar thermal exchanges can occur at the surfaces of internal heat sources. Humans, animals, artificial lighting, and mechanical and electrical equipment are common sources of heat in rooms. Food is converted to heat by human and animal metabolism, then transferred to the room by surface thermal exchanges and by breathing. Artificial lighting converts electrical energy into thermal radiation, part of which is in the visible range of wavelengths. Heat results from mechanical and electrical processes because of friction and electrical resistance.

Only convective and latent heat interchange occurs with ventilation, which results from either natural infiltration and exfiltration of air through a building, or artificially induced air flow. Any temperature difference between the air in a room and the external air used for ventilation causes a net convective heat exchange to occur. Differences in water content of the air and subsequent evaporation, or condensation, cause the latent heat interchange. Ventilation is necessary in most rooms to maintain a satisfactory chemical composition of the room's air. The strong thermal influence of ventilation and the use of air flow equipment to ventilate and to heat, or cool, a room have resulted in ventilation and thermal design being undertaken simultaneously.

All the heat flows in a room are dynamic. External weather varies on both a diurnal and a seasonal basis. The time-varying nature of other heat flows is echoed in the interchanges between rooms, but on a weaker basis. Heat flows from internal sources reflect the dynamic usage and activity rates associated with them. Variations of wind speed, wind direction, and the temperature of external air influence both natural and artificial ventilation. The significance of each type of thermal interchange with a room and of their dynamic nature depends upon the type of activities occurring and whether humans, animals, or thermally sensitive equipment are involved.
3.1.3 The Thermal Design Process

Thermal design occurs at two levels of the building design process: part of multi-purpose architectural design, and design of special purpose thermal equipment. Architectural decisions on layout and boundary components must take account of the thermal interchanges that occur in buildings together with spatial, physical, visual, and acoustic requirements. Special purpose thermal equipment is designed to improve the resulting enclosed thermal environments and to provide greater control over them.

The general design decision process of solution generation and appraisal is used at both levels of thermal design. Architectural outline schemes are generated from a general knowledge of all environmental influences. Coarse appraisal of the resulting thermal environment is included in the architect's appraisal of all environmental components. However, as detailed analysis of the heat flows is required for analysis of the problem for thermal equipment design, the same analysis is used for detailed thermal appraisal of the architectural solution. The building services engineer usually performs the detailed analysis and may recommend modifications to the architectural solution such as increased insulation for some boundaries or special shading components.

Estimates of the net heat flows required to maintain satisfactory thermal environments are used to generate special purpose thermal equipment solutions. Following choice of room equipment, the required heat flows are converted to energy, or fluid flow, design requirements for distribution system and centralised plant detailed design. A wide range of equipment based on water, steam, air, combustible gases, or electrical distribution systems and coal, oil, gas, solar, or electrical energy sources, together with extensive choice of controls, offers scope for many alternatives. Appraisal has, in the past, been based mainly on capital cost of equipment.
and crude estimates of energy costs. However, the increasing energy consumption in buildings, the greater awareness of the exhaustible nature of some energy resources, and their recent price rises have promoted the need for more detailed appraisal of energy use.

Better procedures for appraising the efficiency of equipment systems, the value of more expensive controls, and the resulting quality of the thermal environments are required. Objective thermal models, on which design procedures are based, are developing to meet this need. Thermal models are reviewed in Section 3.2 under three categories: static plant capacity models, fuel estimation models, and dynamic simulation models. Static plant capacity models either ignore diurnal variation of the external climate or treat the variation as an empirical adjustment to steady state heat flows. The fuel estimation models also treat the dynamic nature of the heat flows very simply. Dynamic simulation models are more detailed than the previous two categories and require the use of a computer. They offer the ability to estimate plant capacity, fuel usage, and the quality of the thermal environment produced by alternative designs.

3.2 MODELS OF THERMAL BEHAVIOUR

3.2.1 Static Plant Capacity Models

3.2.1.1 Steady State Model

The simplest model used for analysing plant capacity is the steady state model [HVE, 1970c; ASHRAE, 1972]. This model is based on the assumption that all parameters are constant with respect to time for the
"design hour". As this assumption is more appropriate to heating design than cooling and air conditioning design, the steady state model will be described in terms of heating. The "heating design hour" is defined by an external air temperature that has a high probability, usually 97\% or 99\%, of being equalled, or exceeded, at a particular location. For heating plant analyses the heat gains from solar radiation are usually ignored because of their low magnitude when maximum heating is required. Internal heat sources were originally ignored as their heat outputs were of much lower magnitude than the high outputs from present day artificial lighting and commercial equipment. The internal air temperature is taken as the measure of the quality of the thermal environment.

Thus the maximum plant output is given by:

\[ Q = \sum_{\text{all walls}} A_w U_w (\theta_a - \theta_e) + \zeta c V_r (\theta_a - \theta_e) \]  

...3.1

where:

- \( Q \) = maximum heat flow required from plant
- \( A_w \) = area of wall
- \( U_w \) = thermal transmittance of wall construction
- \( \theta_a \) = internal air temperature
- \( \theta_e \) = external air temperature for external walls and infiltration, or adjacent room's internal air temperature for internal walls.
- \( \zeta \) = density of air
- \( c \) = specific heat capacity of air
- \( V_r \) = volume infiltration rate.

The thermal transmittance of a wall is the reciprocal of the wall's resistance to steady state heat flow and includes the surface convection...
effects. It can be calculated from the thickness and thermal resistivity, or conductivity, values for each layer of a multi-layered wall together with estimates of the surface convection contributions. Excluding these surface contributions, values of thermal transmittance for a range of standard wall constructions have been measured using laboratory equipment and are published for designers' use [ASHRAE, 1972].

This model was a reasonable approximation when it was first developed as most buildings for which central heating was installed were then designed with continuous heating, low internal heat sources, and heavy external walls. Some cities, such as London, have negligible diurnal variation and cold spells which last for a number of days. Non-dynamic external climate was a reasonable assumption for such cities. Also the large thermal mass of the external walls heavily dampened any effect of diurnal variation in external climatic conditions. Modern buildings have intermittent heating if intermittently occupied, can have external walls of low thermal mass, and suffer high internal heat gains from more intensive artificial lighting. Thus the applicability of the underlying assumptions of the steady state model for modern buildings is questionable.

3.2.1.2 Modifications to the Steady State Model

(a) Intermittent Heating

The first major development away from the conditions that approximated steady state conditions came with the advent of intermittent heating. Pre-heating prior to occupancy, to compensate for overnight cooling, required an increase in the output capacity of the heating plant. The fundamental steady state method was found to be inadequate. The inadequacy was overcome by using empirical factors, based loosely on the building's thermal mass, to increase the basic steady state capacity to allow for the additional heating capacity required to preheat the building prior to occupancy [IHVE, 1970c].
Although lower empirical factors were recommended for buildings of overall lower thermal mass, no recognition was given to the increased significance of diurnal variation of external climate in buildings with light external walls.

An alternative modification to the basic steady state model for intermittent heating problems was proposed by Smith [1941]. From theoretical solutions for the heat flows through internal and external walls during the heat-up period he derived an equation for calculating a parameter called the "absorbance" of a wall. Absorbance is similar to the thermal transmittance of a wall in that it can be calculated from the thermal properties of the wall layers and surface thermal resistance and is used in a similar manner. The absorbance is also a function of the heat-up time. Thus the total heat flow rate into a wall during the heat-up period is the sum of an initial steady state heat flow and a superimposed absorbance heat flow, i.e.:

\[ Q_w = A_w U_w (\theta_e - \theta_i) + A_w a (\theta_f - \theta_i) \]  \hspace{1cm} (3.2)

where:

- \( Q_w \) = total heat flow rate into wall
- \( A_w \) = area of wall
- \( U_w \) = thermal transmittance of wall
- \( a \) = absorbance of wall
- \( \theta_e \) = external air temperature for external walls or adjacent room's internal air temperature for internal walls.
- \( \theta_i \) = initial internal air temperature i.e. before heating starts
- \( \theta_f \) = final internal air temperature i.e. when comfort conditions are reached.
This model has the advantage of simplicity of use for a wide range of buildings if tables of absorbances are available for all types of walls over a range of heat-up times. Smith also included reduction factors dependent upon the length of the heat-up time to be applied to the infiltration and fenestration nil time lag heat flows. However, internal heat loads were ignored as they were still quite small when this model was developed.

The theoretical derivation for the absorbance of a wall is based on the following assumptions:

1. Before heating starts the room air and the internal surfaces of the walls are at the same temperature.

2. The room air temperature suddenly increases and remains a constant amount above the wall surface temperatures.

3. The external air temperature remains constant.

These assumptions are questionable, particularly for short heat-up times which occur in modern buildings of low thermal mass and in locations that have much diurnal variation of climatic conditions.

(b) Internal Heat Sources

The demand for higher luminous intensities resulted in a significant increase in the heat output from artificial lighting. This is crudely accounted for with the steady state model by use of a reduced internal air design temperature [IHVE, 1970c]. A correlation between the room's total heat loss per unit internal/external air temperature difference and the total emission from the room's internal heat sources is implied.

Infiltration heat losses, external wall heat losses, and internal wall heat flows make up the room's total heat loss. Humans, artificial lighting, and equipment are the usual internal heat sources which are
approximately proportional to floor area. As infiltration heat losses are proportional to the volume of the room and the internal/external air temperature difference, they have reasonable correlation with the internal heat output. External wall heat flows depend upon the internal/external air temperature difference in the steady state situation, but the same floor area can have a variable ratio of internal and external walls. Internal wall heat flows do not depend upon the internal/external air temperature difference. Hence the modified steady state method of accounting for internal heat sources must be considered to be crude.

(c) Environmental Temperature

The importance of the thermal radiation exchange between surfaces in an enclosure is well recognised, but this was taken into account in a subjective manner for thermal environmental design. However, its effect was often included in measurements made in existing buildings to assess the quality of the thermal environment. Recently the inclusion of radiation exchange has been advocated for thermal design models [IHVE, 1970c]. The net effect of the radiation exchange between an object at a point in an enclosure and all the surfaces of that enclosure can be measured by the parameter: mean radiant temperature. The mean radiant temperature for an object located at a point is the temperature of a uniform black enclosure that would produce the same net radiant heat exchange with the object at that point. A black enclosure is one with surfaces that are perfect emitters and absorbers of radiant heat.

Modification of the steady state model to include thermal radiation exchange has been suggested [IHVE, 1970c]. Environmental temperature, which is a combination of air temperature and mean radiant temperature in the enclosure, is used instead of internal air temperature to compute the
heat flows through the enclosure boundaries. Environmental temperature is defined by:

\[ \theta_{ei} = \frac{2}{3} \theta_r + \frac{1}{3} \theta_a \] ...

where:

- \( \theta_{ei} \) = environmental temperature
- \( \theta_r \) = mean radiant temperature
- \( \theta_a \) = internal air temperature

In view of the inaccuracies due to the static nature of the steady state model, the benefit of this refinement is questionable.

3.2.1.3 Empirical Models

A number of empirical models were developed for intermittently heated buildings as the steady state model was found wanting [Harrison et al, 1952; Harrison, 1956]. These were based on regression analyses of the performance of a number of typical buildings of the period in which they were developed. Developments in both building technology and thermal models have outdated these models now, but they represent an interesting approach to the problem of producing objective thermal design models. The method used by Cadiergues et al [Harrison, 1956] is most interesting. They analysed fifty typical rooms in buildings of widely differing construction, size, and climatic exposure using influence curves that were derived from a response factor model, which is one of the dynamic simulation models described in Section 3.2.3. The results of these "more accurate" dynamic analyses, which were performed manually, were used to produce the final model by statistical regression analysis of the effect of a range of parameters.
3.2.2 Fuel Estimation Models

A number of fuel estimation models have been developed for estimating the energy consumption of thermal plant in buildings. The dynamic nature of the external climate with its variable winds, solar radiation, air temperature, and water vapour content make this a difficult problem. Dynamic simulation models have been developed to handle the complexity, but before their development some cruder models were used manually. This section reviews these manual models which, because of their crude nature, were only of use in estimating the fuel consumption for storage and handling facility design and for building owners' management purposes.

3.2.2.1 Degree-Day Model

This commonly used model is based on an analysis of the total fuel consumption of a number of residences over a long period as measured by the supply authorities. It was noted that the total fuel consumption had a strong correlation with the difference between a base temperature and the daily mean external air temperature; thus the following equation was derived [ASHRAE, 1973; Kell & Martin, 1958]:

\[ F = U N_b D \] ...

where:

\( F \) = fuel consumption for the estimate period
\( U \) = unit fuel consumption, i.e. quantity of fuel used per degree day per building load unit.
\( N_b \) = number of building load units
\( D \) = number of degree days for the estimate period, i.e. sum of the difference between the daily mean external air temperature and the base temperature for all days in the estimate period.
The building load unit may be based on the calculated heat loss for the building for plant capacity or the installed capacity. In either case, this figure must be divided by the overall efficiency of plant and fuel and the difference between the base temperature and the external design temperature. If the building is intermittently heated, the building load unit must be reduced in proportion to the fraction of the total period that the building is heated.

The American Society of Heating, Refrigerating, and Air Conditioning Engineers list degree days for a base temperature of 65°F (18.3°C) [ASHRAE, 1973]. They also include correction factors for variation of location, i.e. climate, in terms of the external design temperature. In the United Kingdom, the base temperature is 60°F (15.4°C) for a building with an internal temperature of 65°F (18.3°C) [IHVE, 1970b]. One way of viewing the basis of degree-days is to consider that the building requires no heating when the external temperature is greater than, or equal to, the base temperature, and any heating is proportional to the difference between the base temperature and the external air temperature, when this is positive. However, the relationship is not as simple as this, as is shown by the need for correction factors for variation in climate.

The degree-day model is a reasonable model for continuously heated buildings with little diurnal variation in climate or internal heat loads. When this is not the case, the degree day model can be used for a crude estimate of fuel consumption, but is unsatisfactory for energy consumption comparisons between design alternatives.

3.2.2.2 Calculated Heat Loss Model

The calculated heat loss model is based upon adjustment of the calculated maximum heat load for the average climatic conditions [ASHRAE, 1973].
It is a very crude method which usually overestimates the energy consumed owing to the neglect of other energy sources such as solar radiation, artificial lighting, people, and equipment. The energy used is given by:

\[ F = \frac{Q N (\theta_m - \theta_e)}{E C (\theta_d - \theta_e)} \] ... 3.5

where:

\( F \) = quantity of fuel, or energy, used

\( Q \) = calculated maximum heat load based on \( \theta_d \) and \( \theta_e \)

\( N \) = number of heating hours in estimate period

\( E \) = efficiency of fuel utilization over the period

\( C \) = heating value of the fuel

\( \theta \) = mean internal temperature during heating period

\( \theta_m \) = mean external temperature during estimation period

\( \theta_d \) = internal temperature for maximum load design

\( \theta_e \) = external temperature for maximum load design.

3.2.2.3 Equivalent Full-Load Hours Model

This model depends upon the measurement of the energy consumption of existing buildings of similar type and at locations with similar climate to the one under consideration. The measured energy consumption is expressed as the equivalent number of hours of operation at full load. Thus the energy consumption for a similar building is given by [ASHRAE, 1973]:

\[ F = QH \] ... 3.6

where:

\( F \) = fuel consumption for the estimation period

\( Q \) = plant capacity

\( H \) = equivalent full-load hours.
As the model depends upon similar buildings at similar locations, it is of limited use. It was used for crude estimates of cooling energy requirements before the advent of dynamic simulation models.

3.2.2.4 Bin Model

The bin, or temperature frequency occurrence, model uses the monthly frequency of the average external air temperature for each hour of the day [ASHRAE, 1973]. The temperature range is divided into increments or bins, usually of 5 or 10 degrees, and further categorized by periods of the day to allow for the difference between occupied and unoccupied times of the day. The heat gain, or loss, of the building is calculated for each of the bin categories using the steady state model, or a modified version of it. Then the total load for each month is found by summing, over all bins, the product of the heat load and the frequency of occurrence for each bin. Thus the fuel consumption is given by:

\[
F = \frac{1}{EC} \sum_{\text{all bins}} f(e) \cdot N
\]  

...3.7

where:

- \( F \) = fuel consumption for the estimation period
- \( E \) = efficiency of fuel utilization over the period
- \( C \) = heating value of the fuel
- \( f(e) \) = heat gain or loss for a bin as a function of the average external air temperature
- \( N \) = number of hours in that bin.

The bin model is useful for cooling energy estimates as it can take account of solar heat gains, which vary with the time of day, as well as diurnal variation of the external air temperature. The time delay associated
with solar radiation absorption by the room surfaces, then gradual convective transfer to the internal air, is modelled in a crude empirical manner. Also the time delay associated with conduction through thermally heavy walls is neglected. The bin model requires more computational effort than the previously mentioned models, but remains capable of manual computation.

3.2.3 Dynamic Simulation Models

Dynamic simulation models are characterised by their ability to simulate the time-varying nature of thermal behavior. They can simulate the diurnal variation of external air temperature, wind velocity, and solar radiation and take account of the time lags associated with non steady state building heat flows. The basis of the dynamic simulation models is the representation of the dynamic heat flow processes in a building by a system of mathematical equations with time as a variable parameter. Such a system of mathematical equations is detailed in Chapter Five. Where these equations describe explicit relationships that can be solved by simple substitution of values for the parameters, their use in simulation models is quite straightforward. However, the thermal processes of conduction and radiation interchange are described by implicit mathematical relationships which require more complex solution techniques to solve for the heat flow, or temperature values, required for thermal simulation modelling.

As the mathematical representation of conduction posed the most difficult dynamic simulation problem, the models are best categorised by the techniques used to model conduction heat flows through the building's walls. The four types: analogue, numerical discretisation, harmonic, and response factor models are described in the next four sections of this report. The extent of simulation requires that a computer be used for
these models: an analogue computer for analogue models, and a digital computer for numerical discretisation, harmonic, and response factor models.

3.2.3.1 Analogue Models

As the mathematical description of thermal conduction is the same as that describing electrical conduction, thermal processes can be modelled by electrical analogy. The representation requires that the electrical conductance and electrical capacitance of the analogue components correspond to the thermal conductance and thermal capacitance, respectively, of the prototype. This condition is usually achieved by discretisation in the electrical analogue. Thus adjacent electrical resistors and capacitors make up the electrical analogue circuits.

Analogue models have been used mainly for research purposes as they require the reassembly of the analogue for each heat flow problem. The advent of digital computer simulation models, which offer a more efficient modelling process, which can easily be made available to designers, has outdated the use of analogue techniques for building thermal design problems.

3.2.3.2 Numerical Discretisation Models

Physical experiments show that the rate of conduction heat flow in a solid is proportional to the gradient of the temperature difference and is in the opposite direction. This result, together with the principle of conservation of energy, can be used to derive the differential equation that describes thermal conduction [Kreyszig, 1968]:

\[ \nabla^2 \theta = \frac{1}{\alpha} \frac{\partial \theta}{\partial t} \]

...3.8
where:

\[ \nabla^2 = \frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} = \text{Laplace operator} \]

\( \theta \) = temperature at any point at any instant

\( t \) = time

\( \alpha \) = thermal diffusivity

The thermal diffusivity is the ratio of the thermal conductivity (a measure of the ability of the solid to conduct heat through a temperature difference) to the specific thermal capacity of the solid (a measure of the heat required to raise its temperature), i.e.:

\[ \alpha = \frac{k}{\zeta \sigma} \]

where:

\( \alpha \) = thermal diffusivity

\( k \) = thermal conductivity

\( \zeta \) = density

\( \sigma \) = specific heat

These material properties are constant in ordinary physical circumstances, although some variation occurs with variation in moisture content. Numerical models ignore such variation for a specific material. When conduction occurs through materials adjacent to each other, the variation of properties between materials must be provided for in a model.

Numerical discretisation models divide a solid in which thermal conduction occurs into discrete elements. The subdivision occurs at boundaries between adjacent materials and within specific materials. The finer the subdivision, the more accurate the modelling. Fineness of subdivision tends to be limited by the extent of computation required and the accuracy of other modelling characteristics, such as the input data.
One numerical discretisation technique is to represent the partial derivatives in an equation, such as Equation 3.8, by finite differences. For example:

\[ \frac{\Delta \theta}{\Delta t} = \frac{\Delta \theta}{\Delta t} = \frac{\theta_{K} - \theta_{J}}{\Delta t} \quad \ldots 3.10 \]

where:

- \( \Delta \theta \) = temperature difference over small time interval
- \( \Delta t \) = duration of the small time interval
- \( K \) = index for present time interval
- \( J \) = index for preceding time interval

Using the technique for one dimensional heat conduction, which is assumed for building conduction models, Equation 3.8 can be represented by the following finite difference equation:

\[ \frac{a \Delta t}{\Delta x^2} \left[ \beta \theta(i+1)K - 2\theta(i)K + \theta(i-1)K \right] \\
+ (1-\beta)\left[ \theta(i+1)J - 2\theta(i)J + \theta(i-1)J \right] \]

\[ = \theta(i)K - \theta(i)J \quad \ldots 3.11 \]

where:

- \( \Delta t \) = duration of small time interval
- \( \Delta x \) = size of small distance increment
- \( \beta \) = interpolation parameter
- \( i-1, i, i+1 \) = indices for distance increments

This representation assumes constant thermal diffusivity over the distance increments.
Computational instability is one problem that occurs with finite difference representation. It is caused by the errors inherent in the approximations, as indicated in Equation 3.10, increasing at each increment. The relative magnitudes of the time and distance increments need to be chosen to avoid computational instability. Gupta et al [1971] report the conditions for stability for solution of Equation 3.11:

\[ \frac{\alpha \Delta t}{\Delta x^2} > 0 \quad \text{if} \quad 0.5 < \beta < 1 \]

...3.12

\[ 0 < \frac{\alpha \Delta t}{\Delta x^2} < 0.5 \quad \text{if} \quad \beta = 0 \]

The system of simultaneous equations that results from finite difference representation can be easily solved by well established methods [Carslaw & Jaeger, 1959] from any defined initial conditions. An advantage of finite difference representation is the ability to model any sequence of climatic and internal heat source stimuli together with non-linear wall boundary heat flows. Thus radiative and convective boundary heat transfers can be modelled separately.

The assumption of one dimensional conduction heat flow through a boundary of a building is an acceptable approximation because of the small magnitudes of the heat flows that occur within the plane of a boundary compared with the heat flow through the boundary. If a multi-dimensional conduction model was required, matrix inversion techniques would be required to solve the necessary finite element representation equation system.

The finite element technique is an alternative numerical discretisation method which has proved to be useful for multi-dimensional heat flow problems [Desai & Abel, 1972]. It has evolved since the
advent of the digital computer as it requires extensive computation. Rather than use approximations to the partial derivatives, this technique divides the body into small elements and approximates the distribution of the field variable, which for conduction is temperature, within these elements by a simple function; usually a polynomial. Finite element equations are derived for the boundary conditions between the elements using the simple distribution function. This technique will not be detailed as the finite difference method is a better approach for one dimensional heat conduction that is assumed for buildings.

3.2.3.3 Harmonic Models

If the climatic data and other thermal stimulus descriptions can be considered as periodic cycles, they can be represented in sinusoidal form:

\[ f(x) = a_0 + \sum_{n=1}^{\infty} (a_n \cos nx + b_n \sin nx) \]

where:

- \( f(x) = \) a periodic function of \( x \)
- \( x = \) independent variable
- \( a_0 = \) steady state term
- \( a_n, b_n = \) constants

Equation 3.13 represents a steady state term plus harmonic terms. Most data can be adequately represented by a restricted number of harmonic terms. Direct solution to the one dimensional heat flow equation describing conduction through a wall can be obtained for each of these Fourier components and for linear boundary conditions; then the component solutions can be superimposed [Heading, 1963].
Harmonic models have been developed to represent a wide range of boundary conditions including radiative and convective surface heat flows. Multi-layered walls and parallel heat paths can also be modelled [Gupta et al, 1971]. In addition to the constraints of periodic stimuli and linearity, the parameters of the building system must be modelled as independent of time.

3.2.3.4 Response Factor Models

The principle of superposition also provides the basis for response factor models, so linearisation approximations are required for the two stage process. The first stage involves the determination of the system response to a unit excitation under identical boundary conditions to the actual inputs. The second stage approximates any actual excitations by a series of scalar multiples of the unit excitations and computes the system response by summing the products of these scalar factors and the unit response factors for all excitations. Thus the conduction heat flow at the surface of a wall due to temperature excitations at each wall is given by:

\[
q_{1,n} = \sum_{p=0}^{\infty} \theta_{1,(n-p)} x_{p} - \sum_{p=0}^{\infty} \theta_{2,(n-p)} y_{p}
\]

where:

\[q_{1,n} = \text{conduction heat flow per unit area at surface 1 at time } n\Delta t\]

\[\theta_{1,(n-p)} = \text{temperature at surface 1 at time } (n-p)\Delta t\]

\[\theta_{2,(n-p)} = \text{temperature at surface 2 at time } (n-p)\Delta t\]

\[x_{p} = \text{response factor for heat flow per unit area at surface 1 due to a unit temperature excitation at surface 1 at time } (n-p)\Delta t\]
\[ y_p = \text{response factor for heat flow per unit area at surface 1 due to a unit temperature excitation at surface 2 at time (n-p)\Delta t}. \]

As the thermal response factors become a decreasing series after a few terms, it is only necessary to include a small number of terms in the summation. A common time step for the sequences of response factors is one hour and less than ten terms are usually required with a one hour time step for wall conduction response factors [Mitalas & Stevenson, 1967].

The response factors for conduction heat flows in a wall can be determined by any available means. However, an analytical solution to the one-dimensional conduction equation in terms of a sine series can be determined for the boundary conditions implicit in Equation 3.14 [Mitalas & Stevenson, 1967]. The analytical solution is for a homogeneous wall, but the response factors for multi-layered walls can be determined from the response factors computed for each homogeneous layer.

The concept of thermal response factors has been extended from a single wall to the room of a building. The cooling or heating load in a room is regarded as the response to a number of excitations. Temperature changes at the wall surfaces and of the internal air, internal energy source heat emissions, and solar radiation are modelled as excitations. Response of the room load to unit excitations for each of the stimuli can be separately computed and combined for any set of actual excitations [Mitalas & Stevenson, 1967]. This approach is an advantage if the extent of computation required to determine the response factors is greater than the summation computation required when using the response factors.

Kusuda [1971] pursued an interesting approach to the determination of room cooling load response factors. He attempted to obtain room
response factors by regression analysis on a set of data, that had been produced from a detailed computer simulation. However, he was unsuccessful because his use of only three stimuli: external/internal temperature difference, solar radiation, and internal heat generation, proved to be too simplistic.

Muncey et al [1971] extended the response factor concept further in their determination of total building cooling load response factors. Rather than expressing response factors as a list of numeric values corresponding to equi-duration time steps, they represented the response factors to a unit excitation as the summation of a number of periodic time series derived from the use of a harmonic model. The actual stimuli were represented as a regular time sequence of values, but only selected values over a wide time range were used in the response summation similar to Equation 3.14. The technique enabled responses with large time lags to be included without the need for an excessive number of response factors; hence its application to total building response.

As response factors are a property of the responding system and not of the stimuli, any set of stimuli acting with identical boundary conditions to the unit excitation can be modelled with them. However, they require linear boundary conditions for the stimuli and response and invariability of the characteristics of the responding system with time. These conditions are unimportant for individual wall conduction response factors, but can be significant for room and building response factors. Hence the best approach for building analysis and appraisal models appears to be to use individual wall conduction response factors and determine the room and building response by direct computation for each set of stimuli.
3.3 APPRAISAL OF THE THERMAL ENVIRONMENT

3.3.1 The Scope of Thermal Appraisal

Numerical models of thermal behaviour have been developed for analysis of plant requirements and fuel estimation. However, the more accurate simulation of actual thermal environments attainable by using dynamic simulation models offers potential for more extensive appraisal at both levels of thermal design.

The first level, multi-purpose architectural design, has the greater influence on both the quality of the final thermal environment and the cost of producing it. More detailed appraisal of the thermal consequences of orientation, layout, and building form decisions is desirable if more effective energy use is to be gained. The multi-purpose nature of preliminary design decisions means many later design decisions are based upon them. Lack of flexibility results. Any appraisal must be rapid if it is to provide scope for modifications that do not cause consequential abortive design efforts.

The rapid processing capability of the digital computer has potential for aiding detailed thermal appraisal of preliminary design decisions. Traditionally thermal design specialists have undertaken any detailed analysis as a first step towards special purpose plant design. However, the time and effort required to communicate both preliminary design decisions and appraisal results between designers reduces the desired flexibility and thus the effectiveness of the appraisal. Computer aided thermal appraisal procedures for use directly by architects, possibly as part of comprehensive architectural appraisal procedures, may prove more useful. An architect may retain control while avoiding
the detailed processing and can easily investigate the significance of modifications.

Hawkes and Stibbs [1969] have used dynamic simulation to produce some general principles to aid preliminary design solution generation. However, direct appraisal of design alternatives during the design process appears to be warranted because of the complex interactive nature of thermal variables. Gupta [1970] has formulated a systematic approach to the thermal characteristics of architectural design and suggests optimisation theory can be usefully employed. Gupta and Anson [1972] demonstrate the use of this approach incorporating both thermal and lighting characteristics. Interesting features of their model are the availability of options for optimisation criteria and the automatic output of sensitivity information. These features, together with the use of optimisation to synthesise details, appear to provide a rapid and flexible appraisal procedure. Further investigation into the level of detail required in such appraisal models, the availability of their information requirements, and the value of their use is warranted.

Appraisal of architectural decisions can be based on either the heat flows required to maintain a satisfactory thermal environment, or the quality of the thermal environment without special purpose thermal equipment. The sensitivity of the thermal effects of architectural decisions to details of thermal equipment needs to be established for adequate appraisal of heat flows. It appears that the mode of heat transfer, i.e. convection, radiation, or air circulation, may be the only significant detail.

Dynamic simulation models were originally developed for use at the second level of thermal design: special purpose equipment design.
Not only can they be used for analysis of plant size and energy requirements, but also to appraise the quality of the resulting thermal environment produced by equipment alternatives. It is likely that the level of detail appropriate to special purpose equipment appraisal will differ from that required for architectural thermal appraisal. It, too, requires investigation.

Thus the scope of thermal appraisal includes measurement of both heat or energy flows and the quality of the thermal environment for both architectural design and thermal equipment design. Models appropriate to each type of appraisal are required.

3.3.2 The Roles of the Building Thermal Behaviour Models

Static plant capacity models are presently used for heating equipment design. The extent of approximation to actual thermal behaviour limits their use to analysis of maximum plant thermal output requirements. Crude fuel estimation models must now be regarded as limited to the determination of fuel storage facilities if static plant capacity models have been used. Although appraisal of alternative equipment systems was attempted with these models, they proved to be too simplistic for this purpose [GARD, 1969]. The bin model offers scope for refinement and formulation as a computer aided procedure, but dynamic simulation models have been developed specifically to make effective use of the capabilities offered by automatic computation.

Dynamic simulation models are definitely superior to static plant capacity models and crude fuel estimation models for cooling and air conditioning equipment analysis of requirements and energy consumption appraisal. Their use enables better equipment sizing and control design producing savings in both capital costs and fuel costs. However,
the benefit of their use, compared with the additional cost of design processing they incur, for heating equipment design is not so apparent as the capital cost of heating plant is relatively insensitive to minor variation in heat load requirements. The sensitivity of energy costs, and the quality of the thermal environment, to equipment design decisions may warrant dynamic modelling, however. This question is pursued in the simulation study described in Chapter Seven.

Of the dynamic simulation models presently available, the numerical discretisation technique with its non-linearity capability is attractive for research purposes. However for design modelling, and in particular architectural design appraisal, the harmonic and response factor techniques are attractive because of their ability for use over a wide range of detail of representation of the real world situation. The room, and total building, response factor approaches may prove most suitable for architectural design appraisal. Detailed representation can be used to establish the response factors. Then coarse representation of the climate and other thermal stimuli can be used for appraising the effects of extensive climatic variation. The restriction of linearity means that non-linear boundary conditions with any stimulus must be approximated by a set of linear conditions. However, the use of more than one linear relationship for a single stimulus can improve the adequacy of the approximation.

Determination of all the roles of dynamic simulation models requires investigation into the levels of detail required for appraisal for both architectural and thermal equipment design. Energy flow appraisal is based on maintaining a satisfactory thermal environment. Chapter Four investigates the human requirements for a satisfactory thermal environment, together with measures for appraising the quality of human thermal environments.
The purpose of building thermal design is to make provision for the thermal requirements of the housed activities. A room of a building undergoes thermal interchange with the external weather and adjacent rooms through its boundaries, with internal heat sources, and by ventilation. These four heatflow paths consist of sequences of conduction, convection, radiation, and latent heat exchanges that combine in a complex and dynamic manner to form the basic thermal environment of a room. The nature of the activities in a room influences the significance of each type of thermal interchange.

Thermal building design begins as part of multi-purpose architectural design. It continues at a second level with the well defined sub-problem: the provision of special purpose thermal equipment. Thermal equipment design is based on numerical models for both analysis of the heat flow requirements and appraisal of energy consumption. Many simplifying assumptions have been used to reduce the complexity of thermal models for use in efficient design procedures. A range of static plant capacity models and crude fuel estimation models have been used in the past.

The availability of the digital computer has fostered the development of more detailed dynamic simulation models for both analysis of plant requirements and appraisal of their energy consumption. These models offer potential for more extensive appraisal of both architectural design and thermal equipment design through measurement of energy requirements or the quality of the thermal environment resulting from design alternatives.

Dynamic simulation models have outdated static plant capacity and crude fuel estimation models for cooling and air conditioning equipment.
design. The net benefit of their use for heating design requires investigation, as do the modelling requirements for architectural appraisal.
4.1 INTRODUCTION

Use of numerical models for thermal appraisal requires both an adequate definition of a satisfactory thermal environment and a measure of thermal quality when it is unsatisfactory. Thermal requirements depend upon the activities performed in the environment. Buildings are designed to enclose a range of activities involving people, animals, equipment, and commodities. Although the building itself produces some constraints, such as avoidance of condensation, the housed objects usually require more consideration when determining thermal requirements.

The thermal condition of the objects may be influenced by any of the thermal processes of convection, conduction, radiation, and latent heat of phase change. Convection occurs at any surface open to the fluid in the environment if there is a temperature difference at the surface/fluid interface. Temperature gradients in solids cause conduction heat flows. Radiation interchange occurs between all surfaces in an enclosure and results in relative heat exchange depending upon the wavelength of the radiation and the absorption characteristics of the environmental fluid and surfaces. Melting and evaporation require heat input; condensation gives out heat to the condensing surface. The dynamic natures of the external climate and the activities occurring in buildings ensure continual thermal interchange between the housed objects and the building environment.
The thermal interchange can influence the health, comfort, and performance of people and animals and aid deterioration processes of equipment and commodities. Some commodities, such as frozen foods, require special thermal environments; some equipment, such as electronic computers, are very sensitive to thermal conditions; but humans, and sometimes animals, usually produce the most stringent thermal requirements for the majority of buildings.

The influence of thermal conditions on the health, comfort, and performance of humans is discussed in Section 4.2. A model for measuring the quality of thermal environments in terms of monetary cost of thermal deficiency is investigated in Section 4.3. The resulting model for user cost in cool environments is used in Section 4.4 to formulate a total cost difference appraisal model for hydronic heating design alternatives.

4.2 THE EFFECT OF THERMAL ENVIRONMENTS ON HUMANS

4.2.1 Human Thermal Responses

Human physical reactions to the thermal environment are well understood. To survive, the human body must maintain long term thermal balance between heat loss to the environment and bodily heat production. This metabolic heat is produced from the combustion of carbohydrates and fats within the body tissues. Heat loss occurs by convective and radiative interchange between the human surface and the environment and by evaporation of body fluids. Respiration, water vapour diffusion through the skin, and sweat evaporation on the skin surface cause evaporative heat loss.
The human body regulates its heat loss using the physiological mechanisms of vasoconstriction and vasodilation, i.e. the closing and opening respectively, of the capillaries carrying warm blood to the skin surface, and the mechanisms of shivering and sweat secretion. Clothing acts as a thermal insulant by reducing all heat loss processes occurring at the skin surface. The thermal balance of the human body is also influenced by the extent of physical activity undertaken, as metabolic heat production increases with increased physical activity. Further regulation of human heat loss may be achieved by controlling the thermal environment.

Within the wide range of thermal environments in which human survival is possible, thermal influence on human performance depends upon the human's psychological perception of the thermal environment. Usage of the physiological mechanisms required to regulate the thermal balance with the environment is part of the perception process. In fact, it has been found that thermal comfort occurs in environments that produce physiological thermal neutrality, i.e. when there is no physiological thermal regulatory effort required [Gagge et al., 1967].

A human's sense of any deviation from the desired thermal comfort condition is related to the regulatory effort required to maintain thermal balance and any resulting change of average body temperature from the desired homiothermic value. In anisotropic thermal environments extreme temperature differences between different parts of the human body also influence the sensation of thermal discomfort.

Based on the assumption that thermal comfort is the desirable criterion for any human thermal environment, much research has been directed towards determining the thermal conditions that define human thermal comfort and the sensitivity of thermal sensations to variation
of these conditions. Most of the research has used verbal rating scales to measure the sensations of deviation from the thermally neutral condition. Tables 4.1 and 4.2 present two such verbal scales used by experimental subjects to express the size of the thermal, or discomfort, sensations they experience from the range of controlled thermal environments used for the research [Gagge et al, 1969; Fanger, 1970]. As subjects vary in their choice of a comfortable thermal environment due to physiological differences, the mean value of the ratings expressed by a group of subjects is always derived as the significant index.

<table>
<thead>
<tr>
<th>Rating Index</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>-3</td>
<td>Cold</td>
</tr>
<tr>
<td>-2</td>
<td>Cool</td>
</tr>
<tr>
<td>-1</td>
<td>Slightly Cool</td>
</tr>
<tr>
<td>0</td>
<td>Neutral</td>
</tr>
<tr>
<td>1</td>
<td>Slightly Warm</td>
</tr>
<tr>
<td>2</td>
<td>Warm</td>
</tr>
<tr>
<td>3</td>
<td>Hot</td>
</tr>
</tbody>
</table>

**TABLE 4.1: THERMAL SENSATION RATING SCALE**

<table>
<thead>
<tr>
<th>Rating Index</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>Comfortable</td>
</tr>
<tr>
<td>2</td>
<td>Slightly Uncomfortable</td>
</tr>
<tr>
<td>3</td>
<td>Uncomfortable</td>
</tr>
<tr>
<td>4</td>
<td>Very Uncomfortable</td>
</tr>
</tbody>
</table>

**TABLE 4.2: COMFORT SENSATION RATING SCALE**
Research results indicate that discomfort increases more rapidly with cooler, than with warmer, environments, while thermal sensations have a similar rate of variation for both heat and cold [Gagge et al, 1967]. Discomfort sensations have been found to follow the power law, given in Equation 4.1, that governs many dimensions in the domain of sensory psychophysics [Stevens, 1968].

\[ \psi = k(\phi - \phi_o)^\beta \]  

where:

- \( \psi \) = judged sensory magnitude
- \( \phi \) = stimulus magnitude
- \( \phi_o \) = approximates the lowest physical value that can be sensed
- \( \beta \) = exponent dependent upon sensory dimension involved
- \( k \) = proportionality constant dependent upon choice of units.

By using both the thermal sensation rating scale (a magnitude estimation technique) and adjustment of the loudness of a white noise to correspond to the intensity of thermal sensation (a cross-modality matching technique), exponents of 1.7 and 0.7 were found for cold and hot discomfort respectively [Stevens et al, 1969].

4.2.2 Thermal Influences on Human Performance

Recognising that human performance may be influenced in a different manner than human comfort, some researchers have investigated the effect of thermal environments on human activities. Because of the greater difficulty, and thus expense, of controlling warm environments, most of the studies have been on the effect of excess heat on user performance.

Pepler and Warner [1968] include both cool and warm environments in their study of the effect of the thermal environment on learning. By
using a programmed text, they produced records of speed and accuracy of learning for six groups of twelve students, each studying at six different room temperatures. Their results showed that the differences in percentage errors and rate of working (the two significant measures of effectiveness of learning) were not statistically significant over the six temperature conditions tested. However, the sensations of effort experienced in studying the programmed text did vary significantly for large temperature differences. These effort sensations were assessed by the students using the rating scale given in Table 4.3. Pepler and Warner state that the purpose of this scale was "to assess the student's efficiency of learning behaviours on the assumption that sensations of increased effort are indicative of an increased 'cost' of learning." The minimum sensation of effort occurred at the room ambient temperature nearest that producing a neutral thermal environment. Effort sensation ratings increased with increase in temperature difference both sides of the minimum.

<table>
<thead>
<tr>
<th>Rating Index</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>None</td>
</tr>
<tr>
<td>2</td>
<td>Very Slight</td>
</tr>
<tr>
<td>3</td>
<td>Slight</td>
</tr>
<tr>
<td>4</td>
<td>Moderate</td>
</tr>
<tr>
<td>5</td>
<td>Much</td>
</tr>
<tr>
<td>6</td>
<td>Very Much</td>
</tr>
<tr>
<td>7</td>
<td>Extreme</td>
</tr>
</tbody>
</table>

**TABLE 4.3: EFFORT SENSATION RATING SCALE**
All the experimental evidence on the effect of the thermal environment on human performance, which at first appears to be contradictory, can be explained using the concept of arousal, or degree of activation. The level of arousal has been proposed as a measure of intensity of all human behaviour, i.e. a psychological cost. It is affected by the person's attitude towards his activity as well as his sensation of the physical environment. It is hypothesised that all stimulating factors have an additive effect on a person's level of arousal and that human performance has a relationship with arousal of the form shown in Figure 4.1. Evidence suggests that any task has an optimum level of arousal and that performance deteriorates for levels of arousal both less than and greater than the optimum level [Provins, 1966].

Wyon [1973] hypothesises the relationship between level of arousal and ambient temperature to be of the form shown in Figure 4.2. Both low and high ambient temperatures near the thermal comfort condition tend to raise the level of arousal.

The applicability of these relationships is well illustrated by the experimental results of Wilkinson et al [1964]. The effect of raised body temperature upon performance by twelve subjects for a vigilance task and an arithmetic task was studied. Performance of the vigilance task deteriorated with mild body temperature elevation, but improved with higher body temperature as illustrated in Figure 4.3. Variation of body temperature produced the opposite effects on arithmetic performance, i.e. increased performance with mild body temperature elevation and decreased performance with high body temperature as illustrated in Figure 4.4.

As an increase in ambient temperature produces an increase in human body temperature, these results can be explained using Figures 4.1
and 4.2. Firstly consider the effect of temperature changes on the level of arousal. If the control condition in the above experiment is considered to be represented by point C in Figure 4.2 and the mildly elevated and high body temperatures represented by points M and H respectively, it can be seen that the effect of mild increase in temperature is a reduction in level of arousal, but a higher increase in temperature results in an increased level of arousal over the control condition. This is true for both types of task.

Now consider the effect these changes in level of arousal have on performance of the two types of task. If the vigilance task control condition is considered to be represented by point V in Figure 4.1 with a lower than optimal level of arousal due to its monotonous nature, then the positive gradient of performance with respect to level of arousal explains the results obtained. A decrease in level of arousal due to mild temperature increase produces a reduced performance. An increase in level of arousal due to a high temperature produces an increased performance. The opposite results for the arithmetic task can be explained by the negative gradient of performance with respect to level of arousal if the control condition for this task is considered to be represented by point A in Figure 4.1 with a higher level of arousal due to the more challenging nature of the task.

The hypothesised relationship between ambient temperature and level of arousal illustrated in Figure 4.2 appears reasonable for relatively slow changes in ambient temperature and for static conditions. However, some experimental evidence suggests that for mild temperature increases, the initial sensation of warmth produces an initial increase in level of arousal, followed by a decrease as the body adapts to the new thermal conditions [Poulton & Kerslake, 1965].
FIGURE 4.1: HUMAN PERFORMANCE/AROUSAL RELATIONSHIP

FIGURE 4.2: VARIATION OF LEVEL OF AROUSAL WITH AMBIENT TEMPERATURE
FIGURE 4.3: PERFORMANCE VARIATION WITH BODY TEMPERATURE FOR VIGILANCE TASK

FIGURE 4.4: PERFORMANCE VARIATION WITH BODY TEMPERATURE FOR ARITHMETIC TASK
The present state of knowledge on the effect of the thermal environment on human performance contains many voids. The arousal concept provides a framework for future research in this area, but its real deficiency is the lack of an adequate measure of arousal. A person's sensation of effort may provide such a measure, but the influence of the type of task and the personal motivation on this sensation needs to be investigated.

4.2.3 Definition of a Satisfactory Human Thermal Environment

As Wyon [1973] points out, there are three kinds of criteria that are pertinent to satisfactory human environments: survival, comfort, and performance. The physiological adaptability of the human body results in a wide range of thermal environments in which survival is possible. So comfort and performance become the significant criteria for satisfactory human thermal environments.

If one accepts the arousal concept, the required thermal environment for human performance should be the one that produces the optimal level of arousal in the user, i.e. the level of arousal that enables a person to give their best performance. Present evidence suggests that such an environment varies with the type of activity the user is undertaking and with the personal motivation of the user himself [Provins, 1966; Wilkinson et al, 1964]. However, a designer must aim for some optimal condition that will give the best results for the range of activities and motivations that occur in the designed environment.

On present evidence an environment that produces thermal comfort remains the best choice from a human performance point of view. The vigilance and arithmetic tasks discussed in Section 4.2.2 had optimal levels of arousal either side of the thermal comfort condition. In
Pepler's and Warner's [1968] learning study, the lowest effort rating was at the temperature nearest the thermal comfort condition. Simultaneous attainment of both thermal comfort and optimal human performance is convenient from two points of view. It eliminates the need to decide the relative merits of the two criteria and means that research results relating to both criteria can be easily collated.

Thermal comfort has been defined as that condition of mind which expresses satisfaction with the thermal environment [Fanger, 1970]. However, as it results from a lack of consciousness of thermal stimulation, i.e. a lack of thermal discomfort, it could also be defined as that condition of mind which does not express any dissatisfaction with the thermal environment. Due to the biological variance of people it is not possible to produce a set of thermal conditions that will be regarded as thermally comfortable by all individuals. As buildings are rarely designed for individuals, optimal thermal comfort has been proposed as the appropriate criterion for a group of people. It is the condition in which the highest possible percentage of the group is thermally comfortable [Fanger, 1970].

A great deal of research has been directed towards establishing the conditions that will provide optimal thermal comfort for a normal sample group of people. An important variable is the human activity level, which is the amount of physical exertion required for human activities, and influences the rate of metabolic heat production. Experimental investigations have shown that the sensation of thermal comfort at a given activity level is related to mean skin temperature and rate of sweat secretion. Fanger [1970] used the results of many researchers to establish mathematical expressions for the relationship of these two parameters with human activity level. Using these relationships
and a heat balance equation for a clothed human body and its environment, he derived his human thermal comfort equation:

\[
m(l-\eta) - 0.35[43 - 0.061m(l-\eta) - p_a] - 0.42m(l-\eta) - 50
\]

\[
- 0.0023m(44-p_a) - 0.0014m(34-\theta_a) = 3.4 \times 10^{-8} f_c l \left( \left( \theta_{cl} + 273 \right)^4 - \left( \theta_m + 273 \right)^4 \right) + f_c h_{cl}(\theta_{cl} - \theta_a) \quad \ldots 4.2
\]

where:

- \( m \) = human metabolic rate per unit nude body surface area [kcal/hr/m²]
- \( \eta \) = human external mechanical efficiency
- \( p_a \) = partial pressure of water vapour in room air [mm Hg]
- \( \theta_a \) = room air temperature [°C]
- \( f_{cl} \) = the ratio of the surface area of the clothed body to the surface area of the nude body
- \( \theta_{cl} \) = effective mean surface temperature of a clothed body [°C]
- \( \theta_m \) = mean radiant temperature [°C]
- \( h_{cl} \) = convective heat transfer coefficient for the clothed body [kcal/m² hr °C]

Fanger [1970] defines mean radiant temperature, "in relation to a given person placed at a given point with a given body position and a given clothing, as that uniform temperature of a black enclosure which would result in the same heat loss by radiation from the person as the actual enclosure under study." His analysis also produced an expression for the effective mean surface temperature of a clothed body:

\[
\theta_{cl} = 35.7 - 0.032m(l-\eta) - 0.18f_{cl}(m(l-\eta) - 0.35[43 - 0.061m(l-\eta) - p_a] - 0.42m(l-\eta) - 50 - 0.0023m(44-p_a) - 0.0014m(34-\theta_a)) \quad \ldots 4.3
\]
where:

\[ I_{cl} = \text{thermal resistance of clothing [clo]} \]

\( (1 \text{ clo} = 0.88 \text{ ft}^2\text{hr}^\circ\text{F}/\text{Btu} = 0.155 \text{ m}^2\text{C}/\text{W} = 0.18 \text{ m}^2\text{hr}^\circ\text{C}/\text{kcal}) \)

The human thermal comfort equation has been derived to cover quite an extensive range of combinations of the pertinent variables. However, extreme values of particular variables can produce thermal discomfort even though Equation 4.2 is satisfied. Using his comfort equation, Fanger [1970] identified six variables that are pertinent to the thermal environmental design of buildings when humans determine the quality requirements. These human thermal environmental variables are listed in Table 4.4.

<table>
<thead>
<tr>
<th>Human activity level</th>
<th>Thermal resistance of clothing</th>
<th>Dry bulb air temperature</th>
<th>Mean radiant temperature</th>
<th>Relative air velocity</th>
<th>Water vapour pressure</th>
</tr>
</thead>
</table>

**TABLE 4.4: THERMAL ENVIRONMENTAL VARIABLES PERTINENT TO HUMAN COMFORT**

The human activity level is determined by the activities the humans are engaged in. Activities are part of the building function so, although they vary for different types of human activity, they cannot be varied by the building designers. Thus the pertinent activity level, or levels, for each environmental area must be established and designed for. Clothing habits are a product of society conventions and anticipated insulation requirements for the thermal environment. Hence thermal resistance of clothing is a design variable within the range established by society conventions.
The other four variables represent the thermal environmental stimuli, so can be influenced directly by design decisions. These four environmental design variables are also relevant to thermal environments in which the quality requirements are determined by objects other than humans.

Prior to the development of Fanger's human thermal comfort equation (Equation 4.2), a range of simple numerical relationships between two or more of the thermal environmental design variables had been proposed to produce single indices to express human thermal comfort requirements. These indices are restricted to normal clothing levels worn by people in sedentary activities. They were derived from some of the subjective response comfort study results that Fanger also used, or as the basis of a measuring instrument. The Institution of Heating and Ventilating Engineers [IHVE, 1970a] define five such indices: equivalent temperature, effective temperature, globe temperature, resultant temperature, and environmental temperature. They must now be regarded as special cases of Fanger's more general human thermal comfort equation (Equation 4.2).

4.2.4 Measurement of the Quality of Human Thermal Environments

Definition of a satisfactory thermal environment is sufficient for appraising the heat, or energy, flows that are required to produce it. Appraisal of unsatisfactory thermal environments requires a measure of their quality.

As the achievement of perfect thermal conditions for all possible climatic conditions is a luxurious usage of resources, buildings are designed to be thermally satisfactory most of the time. Design alternatives differ in their use of resources and the resulting quality
of the thermal environment. Design decisions must take account of the benefits that result from committing different quantities of resources, such as thermal equipment, compared with the costs associated with the resources. Although prestige is a possible benefit that can result from thermal design solutions, the quality of the resulting environment always requires consideration.

Human thermal comfort has been assumed to be the requirement for satisfactory thermal environments. As comfort is achieved by a lack of discomfort sensations, thermal environmental quality is highest when there is no discomfort. Thus quality of human thermal environments is more easily expressed in terms of its antithesis, deficiency. The extent of deviation from conditions that produce thermal comfort is appropriate to thermal deficiency.

Although it is technically possible to provide individual people with sufficient controls to individualize thermal environments, as is attempted in aircraft, buildings are designed on the basis of enclosed spaces which may house any number of people. Optimal thermal comfort conditions then become the minimum deficiency thermal environment.

Measurement of the deviation from either actual thermal comfort conditions for an individual, or optimal thermal comfort conditions for a normal sample group of people, is a complex problem. Fanger [1970] has identified the six variables listed in Table 4.4 as pertinent to human sensation of comfort, or discomfort, near the comfort condition. Of these six parameters, the four environmental design variables: dry bulb air temperature, mean radiant temperature, relative air velocity, and water vapour pressure, can be directly influenced by building design decisions. Measures of each of these variables can be individually estimated for any design proposal, but their combination into a single
measure that is meaningful in terms of environmental quality and that can be easily related to different resource costs poses a problem.

The equivalent temperature type indices, mentioned in Section 4.2.3 as special cases of the human thermal comfort equation, were developed for this purpose [THVE, 1970a]. They offer the ability to measure any deviation from a desired thermal condition with a single index, but it is difficult to relate the effect of the deviation to its significance on the human occupants and to resource costs.

Gupta [1970] suggests a degree of discomfort measure, which is the ratio of two temperature-time curve areas. He defines a range of temperature conditions as satisfactory and uses the area of the temperature-time curve, which describes the time-varying nature of room temperatures, within this range as the denominator. The numerator is the area of the temperature-time curve outside the satisfactory range. Gupta uses environmental temperature as his temperature index, but any single thermal index could be used to produce a time integrated ratio. The significance of dynamic variation of thermal conditions on human occupants is reasonably well approximated by the degree of discomfort model, but its relationship with resource costs is not clear.

Fanger [1970] developed possible measures for thermal environmental quality based on his human thermal comfort equation. The first measure, called predicted mean vote, is an estimate of the expected thermal sensations that would be expressed by a normal group of people in any thermal environment. Using the research results based on the thermal sensation rating scale and the associated numerical rating scale presented in Table 4.1, he produced a numerical model to predict the mean rating index for any combination of the six thermal environmental variables listed in Table 4.4.
Recognising that it is difficult to interpret the significance of a value of his predicted mean vote, Fanger [1970] developed a second measure called predicted percentage of dissatisfied. From any combination of the six thermal environmental variables his model predicts the percentage of people in a normal sample that would express dissatisfaction with the thermal environment by voting -3, -2, +2, and +3 on the thermal sensation rating scale of Table 4.1. He demonstrates that the minimum possible predicted percentage of dissatisfied is 5% and occurs for optimal thermal comfort conditions.

Fanger suggests a third index for thermally non-uniform enclosures called the lowest possible percentage of dissatisfied. It is the minimum value of the predicted percentage of dissatisfied for all points in the enclosure. He suggests that the difference between the lowest possible percentage of dissatisfied and the ideal minimum of 5% can be used as a measure of the quality of the thermal environment. All Fanger's measures are related to human comfort, but are only indirectly related to human performance of activities. They are also difficult to compare with alternative design solution resource costs.

Although present knowledge provides an understanding of how human performance is affected by the thermal environment, no adequate measure of this performance for a range of activities is available. Pepler's and Warner's [1968] effort rating scale, presented in Table 4.3, may prove to be an adequate measure for the level of arousal, but suffers from the difficulty of evaluating its significance with respect to human performance. A measure that is common to both resources and effects associated with design proposals offers the advantages of greater objectivity. Such a measure is developed in Section 4.3.
4.3 USER COST OF THERMAL DEFICIENCY

4.3.1 The Concept of a User Cost

The purpose of measuring, or estimating, thermal environmental quality is to determine the significance of any deviation from desired conditions. What is the significance of thermal deficiency in a building? Extreme thermal deficiency can cause rapid deterioration of the building structure and its contents. However, such extremes are usually prevented because of more stringent thermal requirements for the activities housed in buildings. This chapter is primarily concerned with environments in which humans determine the thermal requirements as this is most common for buildings. Thus the significance of thermal deficiency on the building's human occupants is of interest.

Human survival, or health, can be affected by extreme thermal conditions, which must be regarded as absolute constraints from a design point of view. Within such constraints, thermal deficiency is significant for both human comfort and human performance. Both of these characteristics are important to the human occupants themselves and to the organisation for whom they work if they are engaged in occupational activities. Human comfort and human performance are factors that influence the success of a building's activity/behaviour system in achieving the organisational objectives as discussed in Section 2.1.2. Comfort is important for morale; performance is important for production as well as morale.

The significance of a small degree of discomfort is difficult to assess. Fanger's dissatisfaction indices, that are discussed in Section 4.2.4, were developed to ease this difficulty. However, even with their use, it is difficult to relate discomfort measures to costs of thermal improvement alternatives.
The significance of reduced performance is easier to relate to resource costs. The direct relationship between performance and production means performance is more easily expressed in the monetary worth measure that is used for resources. Thus monetary worth is proposed as an appropriate measure for reduced human performance owing to thermal deficiency.

On the assumption that maximum human comfort and maximum human performance occur simultaneously, there is obviously a strong correlation between any discomfort and reduction in performance resulting from thermal deficiency. It is possible that a performance measure implicitly measures discomfort. More extensive research is obviously required to determine the complete relationship between performance and discomfort. However, design decisions based on human performance effects can be considered to be compatible with human comfort requirements.

User cost of thermal deficiency is proposed as a measure of reduced human performance in thermally deficient environments. The value of using such a measure is in the objective processing it allows when appraising the quality of thermal environments in relation to the cost of resources used to achieve them. Use of dissimilar measures requires implicit comparison during subjective judgement. Use of similar measures separates the comparison from the necessary subjective judgement of underlying assumptions and significance of the final result. A common measure aids decision making by dividing the appraisal process into less complex steps.

Common measures are open to abuse, however. They can be given excessive consideration compared with intangibles that are not included in the measure. Their underlying assumptions can be overlooked or glossed over. With this type of abuse they can be used to justify preconceived ideas [Stern, 1976]. On the other hand, if intangibles are not the predominant feature and underlying assumptions are made explicit, use of
a common measure can aid understanding of the relative significance of factors, particularly when used with sensitivity studies.

User cost is the monetary value placed on any detrimental effect the performance of a system has on the human user. It has been used in cost models for optimisation of the design of road transport systems [Nicholson, 1974] and building lift systems [Brotchie & Linzey, 1971]. For both of these transport systems the user cost is based on the time the user spends travelling on the system. If the user is in employment during this travelling time, his time can be costed according to his employment worth, which to include overhead costs should be taken as, say, twice his payrate. Then the user cost function is the product of the user's cost per unit time and the expected travel time.

The concept of a user cost of thermal deficiency has not been previously proposed. It is implied, however, by the investment of money in thermal equipment to prevent thermal deficiency. No doubt the main reason for no explicit user cost of thermal deficiency has been the lack of knowledge on how any given thermal environment affects man's intellectual, manual, and perceptual performance. However, the research results described in Section 4.2 provide a basis for the development of such a model.

4.3.2 A User Cost Model for Cool Environments

Maximum human performance is assumed to occur in environments that produce optimal thermal comfort conditions. Such environments are defined to have zero user cost of thermal deficiency. As discussed in Sections 4.2.3 and 4.2.4, the six thermal environmental variables listed in Table 4.4 define thermal environments with respect to their effect on human occupants. To develop a user cost model, appropriate values for,
or relationships with, the variations of these parameters need to be established.

As commercial buildings represent a significant proportion of professionally designed buildings and provide a reasonably well defined environment, this section formulates a user cost model for cool environments that occur in commercial buildings. Applications of the formulated model are discussed in Section 4.3.3.

The human activity level of office inhabitants depends upon the tasks in which they are involved. A range of commercial tasks and their associated activity levels are presented in Table 4.5. Based on this data, a value of 1.2 mets was chosen as representative for commercial activities.

<table>
<thead>
<tr>
<th>Activity</th>
<th>Activity Level (mets)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Seated, quiet</td>
<td>1.0</td>
</tr>
<tr>
<td>Standing, relaxed</td>
<td>1.2</td>
</tr>
<tr>
<td>Walking on the level at:</td>
<td></td>
</tr>
<tr>
<td>2.0 mph (3.2 km/hr)</td>
<td>2.0</td>
</tr>
<tr>
<td>2.5 mph (4.0 km/hr)</td>
<td>2.4</td>
</tr>
<tr>
<td>Typing</td>
<td>0.9-1.2</td>
</tr>
<tr>
<td>Adding machine operation</td>
<td>1.2</td>
</tr>
<tr>
<td>Miscellaneous office work</td>
<td></td>
</tr>
<tr>
<td>(e.g. filing, checking ledgers)</td>
<td>1.0-1.2</td>
</tr>
<tr>
<td>Draughtsman</td>
<td>1.2</td>
</tr>
<tr>
<td>General laboratory work</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Note: met-units measure the rate of energy release per unit body surface area

\(1 \text{ met} = 18.5 \text{ Btu/hr ft}^2 = 58.2 \text{ W/m}^2 = 50 \text{ kcal/hr m}^2\)

Table 4.5: Human Activity Levels [Fanger, 1970]
The clothing worn by office inhabitants varies from person to person depending upon personal views on convention and fashion, as well as insulation requirements. In some developed countries the average insulation value of clothing has decreased over a few decades as both home and work environments have been warmed more effectively. This is shown by the increase in desired ambient temperature in the U.S.A. from 65-70 °F (18.3-21.1 °C) in 1900 to 77-79.5 °F (25.0-26.4 °C) in 1966. This increase has resulted partly from the use of lighter weight clothing by both men and women and partly from changing living and eating patterns [Gagge, 1969].

Wyon [1973] notes that a consequence of the trend to higher temperatures is a reduction in the width of the subjectively tolerated comfort zone, which means fewer people can be comfortable, even in a thermally optimal environment. Any deviation from the optimal thermal environment increases the number of people that feel uncomfortable. The increase is greater for warmer comfort environments than for cooler comfort environments for any unit change to the thermal environment. This fact can be shown by plotting Fanger's predicted mean vote, a measure of thermal sensation [Gagge et al, 1967] for variation in the basic environmental parameters. Figure 4.5 is such a plot for variation in ambient temperature (dry bulb air temperature = mean radiant temperature) for a range of clothing levels. The greater sensitivity for warmer comfort environments is shown by the increase in gradient of predicted mean vote with respect to ambient temperature.

Wyon [1973] suggests a number of contributing factors for the trend to higher preferred temperatures, all of which depend upon the designers' desire to provide temperatures that produce the minimum number of complaints. While this remains a worthwhile goal, the long term
All curves are for: 1.2 met activity level
20 ft/min (0.10 m/s) relative air velocity
50% relative humidity

Predicted Mean Vote

(see table 4-6 for definition and significance of clo-unit of thermal resistance of clothing)

FIGURE 4.5: VARIATION OF PREDICTED MEAN VOTE OF THERMAL SENSATION FOR A RANGE OF CLOTHING LEVELS
consequences indicate that to increase temperatures, is not always the best solution. Obliging people to wear more clothing is often a better solution. In fact, Wyon reports that the solution recommended to alleviate the heat discomfort of surgeons in operating theatres was to lower the temperature and encourage anaesthetists, whose activity level is lower, to wear more clothing.

Recognising these facts, and the fact that slightly warm environments reduce levels of arousal, which is most likely to reduce performance, particularly if motivation to work is low, a reasonably high level of clothing is desirable for determining the required ambient temperature in commercial buildings in winter. Thus a value of 1.25 clo is chosen. Its significance can be gauged by reference to Table 4.6, which gives clothing levels for a number of ensembles. Note that office inhabitants are normally seated, thus the chairs will increase the thermal resistance of clothing above that of the ensembles they wear.

The four environmental variables: dry bulb air temperature, mean radiant temperature, relative air velocity, and water vapour pressure, vary with climatic variation. Thus a range of combinations will produce optimal thermal comfort. There are some constraints on this multi-dimensional comfort space as thermal discomfort can occur due to extreme conditions for a single stimulus, even though Fanger's comfort equation (Equation 4.2) is satisfied by the combination of the thermal environmental variables.

At low human activity levels, the relative air velocity must be small as excessive values will produce the sensation of an unpleasant draught. Ventilation requirements set a minimum value for fresh air supply, which results in air movement within a room. The air movement is so complex it cannot be adequately modelled mathematically. Thus a
<table>
<thead>
<tr>
<th>Clothing Ensemble</th>
<th>Clothing Insulative Level (clo)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Nude</td>
<td>0</td>
</tr>
<tr>
<td>Shorts</td>
<td>0.1</td>
</tr>
<tr>
<td>Light summer clothing:</td>
<td></td>
</tr>
<tr>
<td>Long light-weight trousers, open-neck shirt with long sleeves</td>
<td>0.5</td>
</tr>
<tr>
<td>Light Working Ensemble:</td>
<td></td>
</tr>
<tr>
<td>Athletic shorts, woollen socks, cotton work shirt (open-neck), and work trousers, shirt tail out</td>
<td>0.6</td>
</tr>
<tr>
<td>Light Outdoor Sportswear:</td>
<td></td>
</tr>
<tr>
<td>Cotton shirt, trousers, T-shirt, shorts, socks, shoes and single ply poplin (cotton and dacron) jacket</td>
<td>0.9</td>
</tr>
<tr>
<td>Typical Light Business Suit</td>
<td>1.0</td>
</tr>
<tr>
<td>Typical Light Business Suit and Cotton Coat</td>
<td>1.5</td>
</tr>
<tr>
<td>Heavy Traditional European Business Suit:</td>
<td></td>
</tr>
<tr>
<td>Cotton underwear with long legs and sleeves, shirt, woollen socks, shoes, suit including trousers, jacket and vest</td>
<td>1.5</td>
</tr>
<tr>
<td>Heavy Wool Pile Ensemble:</td>
<td></td>
</tr>
<tr>
<td>(Polar weather suit)</td>
<td>3.4</td>
</tr>
</tbody>
</table>

Note: clo-units measure the thermal resistance of clothing

\[ 1 \text{clo} = 0.88 \text{ft}^2\text{hr}^\circ\text{F}/\text{Btu} = 0.155 \text{m}^2\circ\text{C}/\text{W} = 0.18 \text{m}^2\text{hr}^\circ\text{C}/\text{kcal} \]

**TABLE 4.6: CLOTHING INSULATIVE LEVEL FOR VARIOUS CLOTHING ENSEMBLES [Fanger, 1970]**
typical value of 20 ft per min (0.10 m/s) is assumed for the relative air velocity in a commercial building. The assumption of a constant relative air velocity is reasonable as the range between a minimum value that meets ventilation requirements, and a maximum value for thermal comfort requirements, is small in cool environments when the human activity is low. The IHVE [1970a] suggest the range is 20-30 ft per min (0.10-0.15 m/s) for an ambient temperature of 68°F (20°C), which is proposed below. Some control over air movement is usually available to inhabitants of commercial buildings, either by window opening, or air vent controls, thus the assumed relatively low air velocity can be achieved if the environment is perceived as cool.

Fanger's [1970] investigations show that thermal comfort is quite insensitive to variations in water vapour pressure in thermal environments that occur in New Zealand offices in winter. A constant relative humidity of 50% is therefore assumed.

These assumptions leave the dry bulb air temperature and the mean radiant temperature as the pertinent variables for measuring thermal environmental quality or deficiency. It is convenient to combine these two variables for the user cost model formulation. Many indices have been proposed as combined measures of some, or all, of the thermal environmental design variables [IHVE, 1970a]. Fanger's human thermal comfort equation (Equation 4.2) indicates the complexity of the interrelationship between the variables. Fanger's [1970] research demonstrates that the relative sensitivity of thermal comfort to dry bulb air temperature and mean radiant temperature varies with variation in the values of the other thermal environmental variables. Any simple combination index is limited to a particular range of thermal conditions.

Ambient temperature is proposed as an appropriate combination measure. Fanger [1970] uses ambient temperature to express the condition
when dry bulb air temperature equals mean radiant temperature. As this condition is uncommon in practice, its meaning is extended for present purposes. Ambient temperature is defined as the dry bulb air temperature in an equivalent human thermal environment in which the dry bulb air temperature equals the mean radiant temperature. The relationship between ambient temperature and its two component variables: dry bulb air temperature and mean radiant temperature, depends upon the values of the other thermal environmental variables. Thus a simple relationship is derived for the particular values presently assumed for cool environments in commercial buildings.

By interpolation of values derived from charts produced by Fanger [1970], the gradient of variation of dry bulb air temperature with respect to variation of mean radiant temperature required to maintain thermal comfort when the other variables are at the assumed values was found to be -0.64. This value applies to a range of 9°F (5°C) about the comfort condition when dry bulb air temperature equals mean radiant temperature. Using the gradient, an equivalent ambient temperature relationship for the assumed conditions can be derived as follows:

Assume

\[ \theta = \alpha \theta_a + (1-\alpha) \theta_m \] \( \ldots 4.4 \)

where:

\( \theta \) = equivalent ambient temperature

\( \theta_a \) = dry bulb air temperature

\( \theta_m \) = mean radiant temperature

\( \alpha \) = constant, to be determined.

Differentiating Equation 4.4 with respect to \( \theta_m \):

\[ \frac{\partial \theta}{\partial \theta_m} = \alpha \frac{\partial \theta_a}{\partial \theta_m} + (1-\alpha) \]
For small increments:

\[
\frac{\Delta \theta}{\Delta \theta_m} = \frac{\Delta \theta}{\Delta \theta_m} \quad \text{and} \quad \frac{\Delta \theta_a}{\Delta \theta_m} = \frac{\Delta \theta_a}{\Delta \theta_m}
\]

where:

\[\Delta \theta_i = \text{small increment of } \theta_i.\]

Substituting gives:

\[
\frac{\Delta \theta}{\Delta \theta_m} = a \frac{\Delta \theta_a}{\Delta \theta_m} + (1-a)
\]

For constant comfort, the equivalent ambient temperature remains constant, i.e. \(\Delta \theta = 0\). Thus:

\[
0 = a \frac{\Delta \theta_a}{\Delta \theta_m} + (1-a)
\]

\[\therefore a = \left(1 - \frac{\Delta \theta_a}{\Delta \theta_m}\right)^{-1}\]

\(\frac{\Delta \theta_a}{\Delta \theta_m}\) is the derived gradient which was found to equal -0.64.

Substituting gives \(a = 0.61\), and:

\[
0 = 0.61 \theta_a + 0.39 \theta_m \quad \ldots 4.5
\]

The relationship expressed in Equation 4.5 is used to measure ambient temperature when the thermal environment in commercial buildings deviates on the cool side of the optimal thermal comfort condition. As the thermal effect on the occupants' levels of arousal, and thus on their performance, depends upon their sensory perception of the thermal environment, the psychophysical power law expressed by Equation 4.1
seems most appropriate for a user cost relationship. Thus the proposed form of the user cost in a cool environment is:

\[ p = k \Delta \theta^2 \]  \ldots 4.6

where:

- \( p \) = proportion of the occupants' employment worth lost due to reduced performance
- \( k \) = proportionality constant
- \( \Delta \theta \) = magnitude of cool deviation of ambient temperature from optimal thermal comfort ambient temperature.

The choice of an exponent for Equation 4.6 posed some difficulty. Stevens et al [1969] found that the exponent for cold discomfort sensation was 1.7. As the thermal effect on effort sensation seems more appropriate than the discomfort sensation with respect to user cost, the exponent for Pepler and Warner's increase in effort sensation results was derived. For ambient temperatures below the comfort temperature the exponent was computed to be 3.3. Their effort sensation rating scale has an imposed standard for magnitude estimation so the derived exponent may not be a true value for the power law.

Weighing these results together with the necessary assumption that user cost varies linearly with the sensation of effort or cold discomfort, and the desire for a simple user cost relationship, an integer exponent of 2 is a reasonable choice for the present state of knowledge. The sensitivity of the user cost relationship to the value of the exponent is illustrated later after the proportionality constant has been derived.

The determination of an appropriate proportionality constant for Equation 4.6 requires knowledge of the extent of reduced performance for
a particular set of thermal conditions. One thermal environment that can be related to user cost is that in which no useful work can be achieved even if motivation is high. For such an environment the user cost of thermal deficiency is the total employment worth of the occupants. As cold discomfort correlates well with mean skin temperature [Gagge et al, 1967], a constraint on mean skin temperature is appropriate for defining such an environment. It is unlikely that useful work can be performed if mean skin temperature is below the critical subjective tolerance limit without numbing, which has been reported to be approximately 77°F (25°C) [ASHRAE, 1972a]. Thus a value of 77°F will be used as the constraint on mean skin temperature to define the no work environment.

Use of the mean skin temperature constraint requires that it be converted to a constraint on ambient temperature. As with thermal comfort conditions, the mean skin temperature is influenced by all the thermal environmental variables. Two sets of thermal conditions that produce the constraining mean skin temperature can be established from published research. However, the constraining ambient temperature required is the value appropriate to the thermal conditions assumed for commercial buildings in winter. It can be derived from the published research results by assuming that ambient temperatures which produce the constraining mean skin temperature vary linearly with the variation in thermal comfort ambient temperatures appropriate to the same fixed values of the other thermal environmental variables. The comfort ambient temperatures can be evaluated from Fanger's human thermal comfort equation (Equation 4.2).

The comfort ambient temperature required for the assumed conditions for commercial buildings in winter is 68°F (20°C). This is the winter design temperature currently used for commercial activities in New Zealand [NOW, 1972] and recommended by the Institution of Heating and
Ventilating Engineers [IHVE, 1970a]. The set of values, that are assumed to be appropriate for winter conditions in commercial offices, together with the ambient temperatures that produce both optimal thermal comfort and the no work thermal environment as defined by the constraint on mean skin temperature, are listed as the cool environment case in Table 4.7.

The two sets of parameter values defining constraining ambient temperatures, together with their comfort ambient temperatures, are also listed in Table 4.7. The nude case is derived from Hardy and Stolwijk's [1966] studies on nude sedentary subjects. The light clothing case results were predicted by Gagge et al [1971] using their computer based model of human physiological regulatory response to model conditions representing a sedentary person wearing light clothing.

By linear extrapolation of the variation in the constraining ambient temperatures with respect to the comfort ambient temperatures for the above two sets of data, the constraining ambient temperature for the cool environment case was estimated. This ambient temperature defines the no work environment for the values of the thermal environmental variables that are assumed constant in a commercial building environment that requires heating. In such a thermal environment the user cost due to the cool environment equals the occupants' total employment worth. Using this information and the assumed exponent of 2, the proportionality constant for Equation 4.6 for cool commercial environments is:

\[ k = 2.14 \times 10^{-3} \text{ per } ^\circ\text{F}^2 \]

\[ (= 6.94 \times 10^{-3} \text{ per } ^\circ\text{C}^2) \]

The resulting relationship between user cost in a cool environment and ambient temperature is presented in Figure 4.6 together with a range of relationships with differing exponents and constraining ambient
<table>
<thead>
<tr>
<th>Variable</th>
<th>Units</th>
<th>Nude Case</th>
<th>Light Clothing Case</th>
<th>Cool Environment Case</th>
</tr>
</thead>
<tbody>
<tr>
<td>Activity level</td>
<td>met</td>
<td>1.0</td>
<td>1.0</td>
<td>1.2</td>
</tr>
<tr>
<td>Clothing level</td>
<td>clo</td>
<td>0</td>
<td>0.6</td>
<td>1.25</td>
</tr>
<tr>
<td>Air Velocity</td>
<td>ft/min (m/sec)</td>
<td>20 (0.10)</td>
<td>30 (0.15)</td>
<td>20 (0.10)</td>
</tr>
<tr>
<td>Constraining Ambient Temperature</td>
<td>F (°C)</td>
<td>51.8 (11)</td>
<td>50 (10)</td>
<td>46.4 (8)</td>
</tr>
<tr>
<td>Comfort Ambient Temperature</td>
<td>F (°C)</td>
<td>83.7 (28.7)</td>
<td>78.8 (26.0)</td>
<td>68 (20)</td>
</tr>
</tbody>
</table>

Note: Relative humidity is irrelevant in such cold environments.

**TABLE 4.7: SOME THERMAL ENVIRONMENTS THAT PRODUCE A MEAN SKIN TEMPERATURE OF 77°F (25°C)
FIGURE 4.6: SENSITIVITY OF USER COST FUNCTION TO EXPONENT AND CONSTRAINING AMBIENT TEMPERATURE
temperatures to illustrate the sensitivity of the chosen user cost relationship to these parameters. The user cost model expressed by Equations 4.6 and 4.7 applies to steady exposure to a single ambient temperature and the transient situation from a thermally comfortable condition.

4.3.3 Application of the User Cost Model

As a preview to a discussion of the applicability of the user cost model to building thermal design decisions, it is worthwhile reviewing the assumptions made in the derivation of the model:

(1) The thermal environment that promotes the best performance from building occupants is one that provides thermal comfort. This is assumed to be a static thermal environment, although there is some evidence [Wyon, 1973] that suggests a controlled dynamic thermal environment may prove to be more desirable.

(2) The determination of the desired thermal comfort condition is an average for a normal group of people and is derived from experimental evidence of the physiological comfort conditions for skin temperature and sweat secretion. These are the most significant physiological measures, but are certainly not the only measures; for extreme conditions the effects of each factor as a separate stimulus need to be considered [Wyon, 1973].

(3) The user cost in a cool environment is assumed to be proportional to the square of the deviation of ambient temperature from the thermal comfort ambient temperature.

(4) The user cost is based on the occupants' employment worth and it is assumed that the extreme, when no work can be achieved, occurs when mean skin temperature is at the critical subjective tolerance limit without numbing.
(5) Only variation of the ambient temperature is considered, so variations of the four thermal environmental variables: human activity level, thermal resistance of clothing, relative air velocity, and water vapour pressure, are neglected.

These assumptions should be kept in mind for any application of the user cost model. One possible application is the determination of what thermal environmental condition building designers should design towards. A single value design temperature is commonly used, which implicitly assumes a static thermal environment as the basis of design. An optimum design temperature would be one that minimised the combined resource and user costs. Figure 4.7 illustrates such an optimum cost approach [Markus, 1970], which shows the optimum condition as including some user cost. The optimum ambient temperature would appear to be the logical choice for a design temperature unless assessment of factors not included in the model suggest otherwise.

![Figure 4.7: Optimum Cost Approach](image-url)
Considering the assumptions inherent in the user cost model, it may be likely that some of the other thermal environmental variables will vary from their assumed values, or the design temperature may be for the air temperature and it is likely that the mean radiant temperature will be a few degrees lower than the air temperature. Thus a higher design temperature than the apparent optimum may be chosen. The difference between the chosen temperature and the apparent optimum then becomes a factor of safety, or perhaps more appropriately, a factor of ignorance.

Thermal environments in buildings tend to be time-varying because of the dynamic natures of the external weather and internal heat sources. The thermal storage effect of the building structure and contents dampens both the frequency and magnitude of the dynamic thermal stimuli. As a result, building thermal environments display a sluggish dynamic response. Although the user cost model has been developed from research predominantly based on static thermal environments, it remains applicable to such sluggish dynamic conditions. It therefore provides a measure for the time-varying thermal quality resulting from both architectural design proposals and special purpose thermal equipment design alternatives. Section 4.4 uses the user cost model for cool environments together with total cost theory to develop a procedure for hydronic heating design appraisal.
4.4 A DIFFERENTIAL COST APPROACH TO HYDRONIC HEATING

4.4.1 The Differential Cost Approach

A differential cost model is a numerical model for comparison of alternative solutions to a problem. Regarding the problem area as a system, each solution can be viewed as the allocation of a quantity of resources which modifies the performance of the system. The modification may have both beneficial and non-beneficial effects on the system. For comparison purposes the significant differences between the quantity of resources used and both the beneficial and non-beneficial effects need to be evaluated. Comparison is made easier if common measuring scales are used. Thus resources may involve the allocation of manpower, equipment, and materials, but the quantities of each of these can be combined using the common measure of monetary cost.

Monetary cost, or worth, is the basic measure for trading in our social system. Its use has replaced the previous system, bartering, because it enables trading to function in a more complex and efficient manner. A range of commodities and services can have their values easily compared, or totalled, using the common measure. The resources required to construct a building, which include both building commodities and services in the form of construction skills, are combined in terms of their monetary worth to determine the construction cost of the building.

Time value of money theory [Lu, 1969] has extended the use of the common measure, monetary worth, to include resources used at different times. Resources expended throughout the life of a system, such as a building, can be compared, or totalled, on a common scale
with the initial cost of the system. Commonly used scales are: present worth, which equivalences all costs to their worth at a particular time, and annual equivalent worth, which equivalences them to regularly re-occurring sums [Stone, 1970].

Brill [1970] used time value of money theory to compare the total costs associated with both a building and the activities that occur in it. By describing a government office building as "a piece of a government information processing system, along with men, machines, and methods," he summed the costs of the information processing system over 40 years and produced the comparison given in Table 4.8.

<table>
<thead>
<tr>
<th>System Component</th>
<th>Total Cost</th>
</tr>
</thead>
<tbody>
<tr>
<td>Design and Construction of the Building</td>
<td>2%</td>
</tr>
<tr>
<td>Operation and Maintenance of the Building</td>
<td>6%</td>
</tr>
<tr>
<td>Users' Salaries</td>
<td>92%</td>
</tr>
</tbody>
</table>

**Table 4.8: Ratio of Costs for a Government Information Processing System [Brill, 1970]**

Although building users' productivity can only be influenced in relatively minor ways through building design, any influence is magnified by the high ratio of users' salaries in the total organisational objectives system. Knowledge of cost sensitivity is really required, however.

Total costs are relevant to cost control in building design [MPBW, 1968] for both capital costs and operation and maintenance costs. However for appraisal of design alternatives, the differences between
total costs are pertinent. The differential cost approach is proposed as a direct measure of cost sensitivity. The differential cost of a system component is its total cost measured from any convenient datum.

Two important characteristics are exhibited by differential cost models. All parameters that influence cost variability between the design alternatives must be included in the system of interest. And non-variable cost components are only included for convenience of measurement. Thus most costs that are common to all alternatives will be excluded from the differential cost model.

To distinguish the cost sensitivity of design decisions on building occupants, their activity costs are not included as resource costs for a total organisational system as described by Brill (Table 4.8). Extension of the common measure, monetary worth, to include effects of building design alternatives enables only the variable part of the building users' activity costs to be included. User cost of thermal deficiency measures the detrimental effect of thermal performance on the building activities.

It is pertinent to note that although monetary worth enables objective processing for combining costs of resources and effects, the evaluation of the resulting monetary values requires subjective judgement. Context influences the worth of money itself [Markus, 1970]. In a tight monetary budgetary situation, resource costs gain greater significance than when resources are less constrained. Cost differences require a knowledge of the order of magnitude of total cost for their significance to be appreciated. A differential cost model is an objective aid to subjective decision-making.
A differential cost model consisting of resource and effects cost differences for a hydronic heating system is developed in Sections 4.4.2 and 4.4.3. Resource costs include the capital cost differences between room emitter alternatives and central boiler size alternatives plus the running cost differences due to variation in energy consumption. The user cost model for cool environments is used to measure the effect of design alternatives on the quality of the resulting thermal environments.

4.4.2 Hydronic Heating Resource Costs

A hydronic heating system is one that uses water as the thermal transportation medium. In a commercial building it generally consists of a centralised boiler with furnace, flue and associated controls, distribution pipework, pumps, and controls, and finned tube emitters with optional mechanical dampers on the enclosing panels in each room. Such a system has the installation and running costs outlined in Table 4.9.

As operator costs only vary with very large differences in equipment sizing and maintenance costs depend upon the quality of the equipment rather than the quantity of heat output, these costs are not included in the differential cost model for equipment sizing.

<table>
<thead>
<tr>
<th>Resource</th>
<th>Major Cost Factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>Capital Investment</td>
<td>Equipment type and size</td>
</tr>
<tr>
<td>Energy</td>
<td>Equipment operation</td>
</tr>
<tr>
<td>Operator</td>
<td>Macro size of equipment</td>
</tr>
<tr>
<td>Maintenance</td>
<td>Quality of construction of equipment</td>
</tr>
</tbody>
</table>

**TABLE 4.9: RESOURCE COSTS ASSOCIATED WITH THERMAL EQUIPMENT**
Not all capital costs need to be included; only differences are relevant to the differential cost. The two parameters that vary between alternative hydronic systems are the emitter size and boiler size. As any normally expected variation in water flowrate between alternative emitter and boiler systems is unlikely to cause any change to distribution pipe or pump sizes, distribution cost can be excluded from the differential cost model. The variation in the length of the distribution pipes due to variation in emitter element length is included as part of the emitter differential cost. The furnace, flue, and associated controls are independent of the boiler size variation within the range considered. Thus the capital costs that need to be included are emitter cost and boiler cost. The only pertinent running cost is the cost due to variation in energy consumption for the alternatives.

4.4.2.1 Room Emitter Differential Cost

For any chosen finned tube emitter system, the emitter size, i.e. the heat output capability, is varied by the length of the finned tube element and the mechanical damper, if provided. As the heating system is designed on the basis of its maximum heat output, the mechanical damper must be considered to be set for maximum output for design purposes. Enclosing panels are sized according to the length of the wall the heater will be put on, so are independent of the variation in element length. Support brackets and the number of valves are also independent of the variation in element length. Thus the length of the finned tube element is the only differential cost model variable between alternative designs. Its differential cost must measure the difference in cost between the elements plus a length of distribution pipe, so that element plus distribution pipe span the same length of wall for each alternative. Suppliers cost both element and pipe as a linear function of length for
any given diameters, so the cost of variation in emitter size is:

\[
C_e = u_e L_e
\]

where:

- \( C_e \) = differential cost of emitter size
- \( u_e \) = cost of emitter element per unit length minus cost of distribution pipe per unit length
- \( L_e \) = variation length of emitter element.

Any length of emitter element can be used for the arbitrary zero of the variation length of emitter element as it is the difference between the costs of alternatives that is significant. Although a continuous range of total emitter element lengths is technically possible, suppliers stock, and thus charge for, a range of sizes that combine to give length variations of 1 foot (30.5 cm). These are the practical design alternatives.

4.4.2.2 Boiler Differential Cost

The other equipment variable is the total boiler output capacity for the building. As boiler systems vary in heating efficiency, reliability, and ease of maintenance, the alternatives are considered to be of the same standard and to vary only in total heat output capacity. Output capacity varies discretely for both single boiler systems and multiple boiler installations. Although the total cost of boiler systems does not vary linearly with total output capacity over the large range of boiler sizes available, an analysis of boiler prices showed that a linear function for the increase in cost with respect to the increase in output capacity is a good representation for the range of capacities applicable to any given building. Therefore the differential cost of alternative boiler systems for any building is given by:
\[ C_b = u_b H \] ...4.9

where:

\[ C_b = \text{differential cost of boiler above cost of smallest boiler considered} \]
\[ u_b = \text{cost increase per unit output increase} \]
\[ H = \text{boiler output capacity minus output capacity of smallest boiler considered.} \]

4.4.2.3 Energy Consumption Cost

As fuel is priced using a linear model, the energy consumption cost is a linear function of the quantity of fuel used. In a hydronic heating system the fuel is consumed in the furnace which heats the circulating water in the boiler. It is therefore convenient to model the energy cost as a function of the heat output from the boiler:

\[ C_f = \frac{u_f H}{\eta} \] ...4.10

where:

\[ C_f = \text{energy cost for a given period} \]
\[ u_f = \text{unit cost of fuel} \]
\[ H = \text{boiler energy output for the period} \]
\[ \eta = \text{mean boiler efficiency} \]

The boiler energy output is dependent upon the thermal characteristics of the heating plant and the building, the thermal variability of weather, and the desired thermal environment in the building. These relationships are investigated in the simulation study described in Chapter Seven.
4.4.3 A User Cost Relationship for Intermittent Heating

With the heating of a building, user cost occurs when the heating system fails to produce the required conditions when the building is occupied. For a building intermittently heated by a hydronic system, it was hypothesised that, if any such failure was to occur, it would occur during the initial occupancy period on a cold day. The hypothesis was based on the premise that if the heating system is capable of producing the required conditions at any time during the daily occupancy period, then it is capable of maintaining them for the remainder of the occupancy period. This seemed a reasonable hypothesis as the lowest external air temperatures occur at the start of the heating cycle and the largest heat flow into the building’s fabric occurs during the preheating period prior to occupancy, and during early occupancy. Also heat emission from human occupants, office equipment, and artificial lighting is usually absent and solar radiation is low during the preheating period.

The computer simulation study, described in Chapter Seven proved the hypothesis to be correct with the following minor exception. Additional short failure periods occurred with sudden large increases in the heat load as the heating system took some time to respond. Such conditions occur with sudden increases in the infiltration load when windows are suddenly opened wide during cold periods. It is most likely, however, that occupants would open windows in stages to avoid a large influx of cold air. In fact, the natural control on excessive ventilation is beneficial. Thus the exception can be neglected and the user cost model can be derived for the hypothesised situation that failure only occurs during the initial occupancy period.

Transient thermal conditions occur when people enter a cool office at the start of the working day. It is assumed that the people are
thermally comfortable when they arrive in the office as they will wear overclothes to achieve thermal comfort. On entering their office, the people remove their overclothes and reduce their activity level. If the office is thermally cool for the reduced clothing and activity levels, the people will sense the thermal environment as cool and vasoconstriction will occur. The transience occurs from a thermally neutral condition.

In Chapter Seven it will be shown that the rate of increase in room ambient temperature from a cool condition at the start of occupancy is relatively slow. Thus a quasi-steady state situation occurs once the person has achieved thermal balance with the cool environment. As the environment warms slowly the person's thermal sensation and body temperature will slowly change. This is not the case when a cool environment is suddenly warmed as a person's psychological sensations lead the body temperature changes and the environment is immediately perceived as warm [Stolwijk & Hardy, 1966].

The user cost model derived in Section 4.3 is applicable to transient conditions from the thermally neutral condition and the quasi steady-state thermal environment that occurs during early occupancy. When the heating system fails to produce the required ambient temperature of 68°F (20°C), the user cost of thermal deficiency on that day is:

$$C_u = \int_0^{t_f} w p dt = w k \int_0^{t_f} \Delta \theta^2 \ dt$$

...4.11

where:

- $C_u$ = user cost of thermal deficiency over failure period
- $t_f$ = duration of failure period, i.e. time from start of occupancy until comfort condition is achieved
- $w$ = employment worth of occupants
- $p$ = proportion of the occupants' employment worth lost due to reduced performance
t = time variable
k = proportionality constant

\( (\approx 2.14 \times 10^{-3} \text{ per } ^\circ F^2 = 6.94 \times 10^{-3} \text{ per } ^\circ C^2) \)

\( \Delta \theta \) = ambient temperature decrement,

i.e. magnitude of cool deviation of ambient temperature from optimal thermal comfort ambient temperature.

The application of Equation 4.11 for appraisal of hydronic heating system design alternatives requires knowledge of two relationships with respect to the design variables of emitter size and boiler size. The first relationship is the variation of the duration of the failure period over the heating season. Variation of the ambient temperature with time during the failure period is the second. A computer based dynamic simulation model was developed to research these relationships. Its mathematical basis is described in Chapter Five and problems associated with its development are discussed in Chapter Six. The two required relationships for the user cost model, together with the boiler energy output relationship, are established in the simulation study described in Chapter Seven, which also includes evaluation of the differential cost model for a range of hydronic heating design alternatives.

4.5 CHAPTER FOUR SUMMARY

Humans usually produce the most stringent requirements for building thermal environments. Perception of thermal conditions by people is related to the extent of physiological regulation required to maintain thermal balance between their metabolic heat production and their heat
loss to the surroundings. Human discomfort results from awareness of thermal sensations. Peoples' performance of both mental and manual tasks is influenced by thermal conditions and the suggested concept of level of arousal adequately explains the indirect relationship.

From existing evidence it is assumed that optimal thermal comfort and optimal thermal influence on performance occurs for humans under similar conditions. The human thermal comfort equation (Equation 4.2) defines these conditions in terms of six variables: human activity level, thermal resistance of clothing, dry bulb air temperature, mean radiant temperature, relative air velocity, and water vapour pressure.

Deviation from optimal thermal comfort conditions is indicative of deficiency of human thermal environments. Many measures have been proposed for thermal deficiency, but none of them adequately measure the influence on human performance and the significance of thermal environmental quality with respect to alternative design proposal resource costs. User cost of thermal deficiency, which measures reduced human performance in terms of monetary worth is proposed as an appropriate appraisal measure.

The user cost of thermal deficiency in a cool environment is assumed to be proportional to the square of the ambient temperature decrement from optimal thermal comfort conditions. Ambient temperature is defined as the dry bulb air temperature in an equivalent human thermal environment in which the dry bulb air temperature equals the mean radiant temperature. The proportionality constant for the user cost model is derived by assuming the critical subjective tolerance limit without numbing defines thermal conditions when no useful work can be performed. Constant values appropriate to winter design conditions for commercial buildings are assumed for the other four thermal environmental variables:
human activity level = 1.2 mets,
thermal resistance of clothing = 1.25 clo,
relative air velocity = 20 ft per min (0.10 m/s),
relative humidity = 50%.

An ambient temperature of 68°F (20°C) then produces optimal thermal comfort.

The assumptions used to derive the user cost of thermal deficiency model must be given consideration when applying it. The model has possible application for determining static thermal design conditions. It can also be applied to dynamic thermal appraisal for both architectural design proposals and special purpose thermal equipment design alternatives.

As appraisal of design alternatives is based on differences between total resource costs and resulting quality, direct measurement of total cost differences, which includes both resources and effects associated with the alternatives, is proposed. A differential cost model for hydronic heating design alternatives consists of numerical expressions in monetary worth terms for: room emitter differential cost (Equation 4.8), boiler differential cost (Equation 4.9), energy consumption cost (Equation 4.10), and user cost of thermal deficiency (Equation 4.11). As both energy and user cost models contain unknown relationships, a simulation study is required to establish them.
5.1 INTRODUCTION

5.1.1 Model Purpose

Chapter Four identified three relationships that are required for evaluation of the differential cost model for hydronic heating. Expressions that take account of variation of the design variables: room emitter size and central boiler size, are required for:

(1) Boiler energy output and its variation over the heating season.
(2) Duration of the thermal deficiency period during initial daily occupancy and its variation over the heating season.
(3) Variation of ambient temperature during the thermal deficiency period.

As this information is not available, a computer based model was developed to simulate the dynamic thermal response of typical rooms of commercial buildings heated by low pressure hot water convectors.

The structure of the model and the basis of the mathematical equations that combine to represent the dynamic heat flows in a building are described in this chapter. Representation of the time variation and problems associated with implementing the set of mathematical equations into a computer simulation model are discussed in Chapter Six. Use of the simulation model to establish the required relationships is
described in Chapter Seven. Tabulation of the data requirements of the model, together with the units and some of the simulation study values, is presented in Appendix B. The remaining simulation study values are tabulated in Chapter Seven.

5.1.2 Model Structure

The continuous time-varying thermal response in a building is represented by solution of the set of equations for a series of small, equal sized, discrete time intervals. From a known initial thermal state, heat flows for the small time interval are computed, then used to determine the new thermal state at the end of the time interval. Heat flows for the next time interval are then computed using the new thermal state or temperatures. This process is repeated many times to simulate the dynamic thermal response of a room in a building.

Cumulation of the heat flows from the boiler over all the time intervals that represent a day gives the daily energy requirements. Identification of the simulated time when thermal comfort conditions are achieved after initial occupancy begins gives the duration of the thermal deficiency period for a particular day. Recording of the ambient temperatures at intervals during the thermal deficiency period defines its variation for that particular day. Repeated simulations with various climatic sequences enabled variations over the heating season to be studied.

The model is based on a single room which undergoes thermal interchanges between its components and with heat sources and sinks that are represented as external influences on the room system components. The room convection heat flows, together with the source and sink heat flows, are illustrated in Figure 5.1. Figure 5.2 illustrates the room
radiation heat flows, which occur simultaneously with the room convection heat flows, and repeats the source and sink heat flows for clarity. Pertinent portions of these two figures are repeated throughout this chapter where the associated mathematical expressions are described.

The room heat flow system receives heat from the fuel fired in the boiler and from the casual sources: metabolic heat from the human occupants, and converted electrical energy from the artificial lighting. Adjacent rooms and the external environment act as both sources and sinks for the system. The room's thermal response is modelled as four types of heat flow: conduction, convection, long wave radiation, and short wave radiation. These four types of heat flow occur both within the room system boundary, and through it as source and sink heat flows.

Conduction response occurs for the following room components: wall behind emitter, other external walls, external windows, and internal structures. These components undergo heat transfer at their surfaces by the other heat flow mechanisms and respond to their surface stimuli by conduction, which both transfers heat through them and modifies the quantity of heat stored in them. The room's physical boundary with the external environment is illustrated as three types of conduction components. The windows are considered as a separate type of component as short wave radiation is transmitted through them, in addition to the heat conduction. The wall behind the room's heat emitters is also modelled as a separate component as this boundary condition requires special modelling. The remainder of the room's external boundary is modelled by the basic conduction model. Different constructions are treated as separate walls with separate values describing their thermal properties. They are illustrated in Figures
5.1 and 5.2 as one type of component as the mathematical equations that describe their thermal behaviour are identical. Similarly the room's internal boundary with adjacent rooms is modelled by the basic conduction model using a specific set of parameters for each wall, floor, and ceiling construction.

The room air is the centre of the room convection heat flows, which are represented by lines between the room components in Figure 5.1. The room air gains heat directly by convection, from the heat emitters, and from the casual sources occupying the room. Convective heat transfer also occurs between the room air and the room internal surfaces, the direction depending upon the relative temperatures at the transfer surface. The plenum air undergoes a convective response with the adjacent surfaces of the ceiling and floor. It also receives heat from the artificial lighting fittings if these protrude through the ceiling. The room convection heat flows are directly influenced by infiltrating air which, if at a different temperature than the mean temperature of the room air, causes an effective heat transfer between the room air and the external air. Convective heat transfer also occurs at the room's physical boundaries with the external environment. The adjacent rooms are assumed to undergo similar heat transfer to the room under study, but this is not modelled directly as temperature boundary conditions are used.

The radiation heat flows are modelled as two types: long wave radiation which depends upon the surface temperatures of the room components, and short wave radiation which is independent of these surface temperatures and is emitted from the sun and the room's artificial lighting fixtures. Both types of radiation heat flow for the room components are represented by the lines containing double arrow heads joining the room components in Figure 5.2. All the internal
Figure 5.1: Room Convection Plus Source and Sink Heat Flows
Casual Sources

Energy Input to Casual Sources

Heat Input from Boiler Fuel

Internal Structures

Heat Interchange with Adjacent Rooms:
  - Convection
  - Long Wave Radiation
  - Short Wave Radiation

Room Heat Emitter

Casual Sources

Plenum Air

Internal Structures

Room Radiation Interaction

Wall Behind Emitter

External Walls

External Windows

Heat Interchange with External Environment:
  - Convection
  - Long Wave Radiation
  - Solar Radiation

External Environment

Room System Boundary

Heat Sources

Heat Interchanges with Sources or Sinks

Room Thermal Component

Radiation Interchange

KEY:

FIGURE 5.2: ROOM RADIATION PLUS SOURCE AND SINK HEAT FLOWS
surfaces in the room, including the heat emitter casing, exchange radiation with each other through the diathermic air. Part of the casual heat gain from humans, artificial lighting, and equipment is emitted as long wave radiation to the room's surfaces. The artificial lighting also emits short wave radiation. These numerous radiation paths are represented in Figure 5.2 simply by single paths to a common node in the room.

The external surfaces of the room's boundary components undergo long wave radiation exchange with their surroundings. The surroundings are the adjacent room for the internal structure boundaries and the ground, sky, and adjacent building structures in the external environment for the external boundaries.

Short wave solar radiation, both direct and diffuse, shines on the external wall surfaces. This is partially absorbed, partially reflected, and with glass, partially transmitted.

The room heat flow model integrates all these component heat flows over small time intervals to determine the room response to input system environment data.

5.2 HEATING PLANT

```
  Room System Boundary

Room Heat Emitter
```
A hydronic heating system consists of boiler, room emitters, associated pipework and circulating water. The heat of combustion of fuel in the furnace is input into the hydronic system in the boiler. This heat raises the temperature of the water, which upon circulation raises the temperature of the pipework and room emitter elements. The warmed pipework and emitter elements emit heat by convection to the adjacent air and by long wave radiation to any surrounding surfaces. A heat balance for this system gives [Billington, 1960]:

\[ Q_{\text{boil}} = C_{\text{plant}} \frac{d\theta_w}{dt} + Q_{\text{emit}} \] ...5.1

where:

- \( Q_{\text{boil}} \) = rate of heat output from boiler.
- \( C_{\text{plant}} \) = lumped thermal capacity of hydronic heating system.
- \( \frac{d\theta_w}{dt} \) = rate of rise of mean water temperature.
- \( Q_{\text{emit}} \) = heat emitted from emitter elements and pipework.

The computer model uses Equation 5.1 to compute the mean water temperature at each time step. \( Q_{\text{boil}} \) equals the maximum boiler output, \( Q_{\text{boiln}} \) when no control is in force. If the room ambient temperature is up to comfort level or the mean water temperature is up to its maximum permitted value, \( Q_{\text{boil}} \) is reduced to the value required to maintain the controlling temperature. When the heating system is turned off, \( Q_{\text{boil}} \) equals zero.

The model uses Equation 4.8, repeated below to define the room ambient temperature.

\[ \theta = 0.61 \theta_a + 0.39 \theta_m \] ...4.8
where:

\[ \theta = \text{room ambient temperature} \]
\[ \theta_a = \text{room air temperature} \]
\[ \theta_m = \text{mean radiant temperature} \]

When Equation 5.1 is applied to a single room in a building the parameters \( Q_{\text{boiln}} \) and \( C_{\text{plant}} \) must be the room’s proportion of their values for the whole building’s hydronic heating system. \( Q_{\text{boiln}} \) can be proportioned in the ratio of the room’s emitter capacity to the total emitter capacity for the building. \( C_{\text{plant}} \) could be proportioned in the same manner, but as the thermal capacity of the heating system is not computed for any other purpose, a relationship with known parameters is desirable. Thus the linear relationship of Equation 5.2 was postulated:

\[ C_{\text{plant}} = K_p Q_{\text{emitb}} \quad \ldots 5.2 \]

where:

\[ K_p = \text{heating plant thermal capacity proportionality constant} \]
\[ = 0.008257 \text{ hr}/\theta^0 (0.01486 \text{ hr}/\text{C}^0) \]

for the Christchurch Central Police Station.

\( Q_{\text{emitb}} \) = basic emitter output as defined in Equation 5.4.

The proportionality constant computed for Christchurch Central Police Station is used in the computer model as this is a typical value for a multistorey office building.
5.3 ROOM HEAT EMITTER

Heat is output from the room's heat emitters by convection to the air and by long wave radiation exchange with surrounding surfaces. As the long wave radiation exchange occurs from the panel surface of the emitter, and this panel receives its heat from the emitter element by a combination of convection and long wave radiation, the total output from the emitter is assumed to be described by an equation similar in form to the turbulent natural convection equation. This equation form was found to be a good representation of emitter manufacturer's data. Thus:

\[ Q_{\text{emit}} = K_{\text{emit}} (\theta_w - \theta_a)^{N_{\text{emit}}} \]  

...5.3

where:

- \( Q_{\text{emit}} \) = total heat output from room's emitters
- \( K_{\text{emit}} \) = emitter coefficient
- \( N_{\text{emit}} \) = exponent dependent upon heat emitter
- \( \theta_w \) = mean water temperature
- \( \theta_a \) = room air temperature
is an input parameter specifying the size of the room's heat emitters. \( N_{\text{emit}} \) was derived from manufacturer's data for a number of emitters and a typical value of 1.33 was used in the model.

As present practice is to size a room's heat emitters in terms of an output for a standard water-air temperature difference, a basic emitter output is defined:

\[
Q_{\text{emitb}} = K_{\text{emit}} (100)^{N_{\text{emit}}} \quad \ldots 5.4
\]

where:

- \( Q_{\text{emitb}} \) = basic emitter output [Btu/hr]
- \( K_{\text{emit}} \) = emitter coefficient [Btu/hr/(F\(^0\))\(^N_{\text{emit}}\)]
- \( N_{\text{emit}} \) = exponent

The standard water-air temperature difference of 100 F\(^0\) (55.6C\(^0\)) corresponds to a maximum mean water temperature of 170\(^0\)F as used in the study and a room air temperature of 70\(^0\)F.

Heat flows between the emitter element and the emitter casing by both long wave radiation exchange and by convection using the air between the element and the casing as the transfer medium.

Neglecting the small thermal capacity of the emitter front casing the net radiation component from the heat emitter equals the net long wave radiation emission from the front casing to the other room surfaces. This net long wave radiation emission from the front casing depends upon the surface radiation characteristics, temperature and geometry of all the room surfaces. The mathematical model of long wave radiation is discussed in Section 5.8.2 of this chapter.

The model considers the back casing of the heat emitter to be the surface element of the backing wall. Thus the surface heat
transfer for this wall is an additional component of the emitter heat output. The net convective output is:

\[ Q_{ec} = Q_{emit} - Q_{rad} - Q_{back} \]  \hspace{1cm} \ldots 5.5

where:

- \( Q_{ec} \) = convective heat output from room's emitters
- \( Q_{emit} \) = total heat output from room's emitters as given by Equation 5.3.
- \( Q_{rad} \) = net long wave radiation emission from the front casing to the other room surfaces.
- \( Q_{back} \) = heat input to back casing of emitter.

The model considers the front and back casings to have equal temperatures. This temperature is defined by the following relationship which was derived from manufacturer's data:

\[ \ln(\theta_c - \theta_a) = C_1 \ln(\theta_w - \theta_a) - C_2 \]  \hspace{1cm} \ldots 5.6

where:

- \( \theta_c \) = mean emitter casing temperature
- \( \theta_a \) = room air temperature
- \( \theta_w \) = mean water temperature
- \( C_1 = 0.943 \)
- \( C_2 = 0.570 \) when temperatures are in \( ^\circ F \)
  \hspace{1cm} = 0.603 \) when temperatures are in \( ^\circ C \).

Thus the heat flow into the wall behind the emitter is assumed to be the heat required to maintain the temperature of its surface element equal to the mean emitter casing temperature as defined by Equation 5.6.
5.4 CASUAL SOURCES

The casual heat gains from humans and artificial lighting are modelled for the occupancy hours of the day at constant values of total heat output and as zero output during non-occupancy hours. Office equipment is not included in the model as the heat output from most office equipment is small, it may only be operated intermittently, and some offices house no extraneous heat producing equipment.

5.4.1 Humans

The metabolic processes in the human body produce energy which may be expended as external mechanical work, stored in the body as heat, or transferred to the environment as heat. This transferred heat involves a number of processes: evaporation of secreted sweat, latent and dry heat loss due to breathing, heat conduction through the clothing, convective heat loss and radiative heat loss from the outer surface of the clothed body.

The heat transferred from human occupants when they are present in the room is classified as an environmental heat source. All the
heat transfer processes mentioned above can be summed to give total latent, radiative and convective heat inputs to the room. The latent heat is removed from the human body by evaporation of moisture in or on the surface of the body. It is not, however, given to the room heat flow system, as the water remains in vapour form. This water vapour changes the composition of the room air. Its effect on the thermal properties of the air will be discussed in Section 5.5 of this chapter.

The radiative heat loss is due to long wave radiation exchange between the surface of human bodies in the room and the room's internal surfaces. This exchange depends upon the location of the human occupant within the room, whether he is seated or standing, the temperature of his outer surface and the geometry and surface temperatures of the room's internal surfaces. In practice the location of human occupants within a room varies from room to room. The model assumes a single fixed location and all human occupants are assumed to be seated at this location.

The surface temperature of a human body obviously varies over the surface which is a combination of various garments and nude skin. By considering heat flows from the total body surface, Fanger [1970] defines an effective mean surface temperature of a clothed body, which is the parameter required for use in radiation exchange equations developed later. From a heat balance for the human body, he produced an expression in metric units for his effective mean surface temperature. It was presented as Equation 4.3 in Chapter Four, but is also presented as Equation 5.7 for convenience:
\[ \theta_{cl} = 35.7 - 0.032 \, m(l-\eta) - 0.18 \, I_{cl} \left( m(l-\eta) \right) \\
- 0.35 \left[ 43 - 0.061 \, m(l-\eta) - p_a \right] - 0.42 \left[ m(l-\eta) - 50 \right] \\
- 0.0023 \, m(44-p_a) - 0.0014 \, m(34-\theta_a) \] 

...5.7

where:

- \( \theta_{cl} \) = effective mean surface temperature of a clothed body [°C].
- \( m \) = human metabolic rate per unit nude body surface area [kcal/hr/m\(^2\)].
- \( \eta \) = human external mechanical efficiency.
- \( I_{cl} \) = thermal resistance of clothing [clo].
- \( p_a \) = partial pressure of water vapour in room air [mm Hg].
- \( \theta_a \) = room air temperature [°C].

Table 5.1 lists the values of the effective mean surface temperature of a clothed body computed from Equation 5.7 for a range of values of the other variables. The human external mechanical efficiency is taken as zero as this is the case for all office activities [Fanger, 1970]. It can be seen from Table 5.1 that the effective mean surface temperature of a clothed body is insensitive to the variation of air temperature found in offices. It is sensitive to the other parameters however. In Chapter Four values of 1.2 mets (60 kcal/hr/m\(^2\)) and 1.25 clo were chosen as appropriate values for metabolic rate, or activity level, and thermal resistance of clothing respectively. These values were used to determine the clothing temperature required for radiation emission from the human body. A water vapour pressure of 0.261 in Hg (6.63 mm Hg), which is equivalent to a relative humidity of 50% at a dry bulb temperature
<table>
<thead>
<tr>
<th>Metabolic Rate m kcal/hr m (mets)</th>
<th>Clothing Resistance I_cl</th>
<th>Water Vapour Press. P_a (mm Hg in Hg)</th>
<th>Air Temp. θ_a (°C °F)</th>
<th>Clothed Body Temp. θ_cl (°C °F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>60 (1.2)</td>
<td>1.25</td>
<td>6.63 (0.261)</td>
<td>20.0 (68.0)</td>
<td>25.2 (77.4)</td>
</tr>
<tr>
<td>50 (1.0)</td>
<td>1.25</td>
<td>6.63 (0.261)</td>
<td>20.0 (68.0)</td>
<td>26.7 (80.0)</td>
</tr>
<tr>
<td>70 (1.4)</td>
<td>1.25</td>
<td>6.63 (0.261)</td>
<td>20.0 (68.0)</td>
<td>23.8 (74.8)</td>
</tr>
<tr>
<td>60 (1.2)</td>
<td>1.00</td>
<td>6.63 (0.261)</td>
<td>20.0 (68.0)</td>
<td>26.9 (80.5)</td>
</tr>
<tr>
<td>60 (1.2)</td>
<td>1.50</td>
<td>6.63 (0.261)</td>
<td>20.0 (68.0)</td>
<td>23.5 (74.3)</td>
</tr>
<tr>
<td>60 (1.2)</td>
<td>1.25</td>
<td>2.29 (0.090)</td>
<td>20.0 (68.0)</td>
<td>25.7 (78.3)</td>
</tr>
<tr>
<td>60 (1.2)</td>
<td>1.25</td>
<td>9.39 (0.370)</td>
<td>20.0 (68.0)</td>
<td>24.9 (76.9)</td>
</tr>
<tr>
<td>60 (1.2)</td>
<td>1.25</td>
<td>6.63 (0.261)</td>
<td>10.0 (50.0)</td>
<td>25.4 (77.7)</td>
</tr>
</tbody>
</table>

**TABLE 5.1: SENSITIVITY OF EFFECTIVE MEAN SURFACE TEMPERATURE OF A CLOTHED BODY**

**Note 1:** Zero human external mechanical efficiency is assumed.

**Note 2:** Water vapour pressures of 6.63, 2.29, 9.39 mm Hg are equivalent to relative humidities of 50% at dry bulb temperatures of 60, 32, 70°F (15.6, 0.0, 21.1°C) respectively.
of 60°F (15.6°C), is chosen as representative of office conditions in winter, thus a constant value of 77.4°F (25.2°C) will be used as the effective mean surface temperature of a clothed body. This effective mean surface temperature of a clothed body is used in the long wave radiation exchange described in Section 5.8.2 to determine the net radiative heat loss from the human occupants.

The total convective heat loss from a human occupant includes the dry heat loss due to breathing and the convection from the clothed body surface. As heat gains to the environment from human occupants are usually given as total sensible heat (convective plus radiative) and total latent heat it is convenient to use the total sensible heat in the model. This has been shown to be insensitive to the variation of air temperature found in offices, thus a constant value per human occupant of 250 Btu/hr/person (73.3 W/person) is used in the model [IHVE, 1970a]. The convective heat loss is this sensible heat loss minus the radiative heat loss.

\[
Q_{hc} = Q_h - Q_{hr}
\] ...

where:

- \( Q_{hc} \) = convective heat loss from humans.
- \( Q_h \) = total sensible heat loss from humans.
- \( Q_{hr} \) = long wave radiative heat loss from humans.

The total sensible heat loss from all human occupants is an input parameter for the model.
5.4.2 Artificial Lighting

The artificial lighting converts the electrical energy supplied to it into heat by radiation and convection interchange with the room. The radiant energy emission contains a range of wavelengths, which for the purposes of heat flow can be considered as two types: short wave radiation, part of which is in the visible range, and long wave radiation. The distribution of the energy emitted from lights into these components depends upon the type of lighting, type of installation fittings, and the surroundings above and below the lighting installation.

As only limited research [McIntyre, 1973; Bedocs & Hewit, 1970; Roberts, 1964] has been conducted into the heat gains from artificial lighting, the data is not available for a very refined model of the thermal processes. Thus a model based on constant fractions of the supplied energy has been developed from the available data. As the artificial lighting is operating only during occupancy when the variation in room air temperature is small, and the artificial lighting system has negligible thermal capacity, the omission of dynamic distribution of the supplied energy is quite reasonable.

The model is based on fluorescent lighting as this is the most common type of office lighting, but applies to any diffusely emitted lighting. Bedocs and Hewit [1970] have measured the total heat transferred upwards from fluorescent lighting for a range of installation types. Their value of 53% for a prismatic or opal diffuser is used in the model as an input to the plenum air. All the upward flowing heat from the artificial lights is considered to be convected to the plenum air as division into radiative and convective components would only cause redistribution of this same quantity of heat within the plenum.
structure with small effect on the room response. McIntyre [1973] has developed the following relationship between the short wave and long wave irradiance and illuminance from artificial lighting:

\[ R = k E \]  \quad \text{(5.9)}

where:

- \( R \) = irradiance on a horizontal plane
- \( k \) = constant dependent upon type of lighting
- \( E \) = illuminance on the horizontal plane.

For fluorescent lighting the constant takes the values 0.0027 W/m²/lux and 0.0053 W/m²/lux for short wave and long wave radiation respectively. The model assumes an illuminance of 30 lumens/ft² (322 lux) at the working plane as recommended for offices in the New Zealand Code of Practice [SANZ, 1962]. For this illuminance with fluorescent lighting, Equation 5.9 gives irradiances at the working plane of 0.28 Btu/hr/ft² (0.87 W/m²) and 0.54 Btu/hr/ft² (1.71 W/m²) for short wave and long wave radiation respectively. The equivalent uniform radiant emission at the ceiling level can be computed by dividing by the radiation configuration factor between the ceiling and the working plane. This factor is defined later in this chapter and has a value of 0.38 for the 11 ft x 16 ft x 10 ft office size used in the simulation study. Thus the equivalent uniform radiant emissions at the ceiling level for an illuminance of 30 lumens/ft² (322 lux) at the working plane are 0.73 Btu/hr/ft² (2.3 W/m²) and 1.43 Btu/hr/ft² (4.5 W/m²). The electrical input to achieve this illuminance from recessed fluorescent lighting with an opal diffuser, which is assumed in the model, is 5.93 Btu/hr/ft² (18.7 W/m²). This value is derived
by scaling the values quoted by Bedoc and Hewit [1970] and is in agreement with the general values given in the I.H.V.E. Guide [1970b]. Thus the component heat flows from the artificial lighting that are used in the model are listed in Table 5.2. The net long wave radiant emission and convected heat to the room air are only approximate values as it is convenient to give the lighting surface exposed to the room an equivalent uniform temperature and define its long wave radiant emission in terms of this temperature. As the net emission, which is computed in the long wave radiation exchange model developed in Section 5.8.2, varies slightly with the temperatures of the room's other surfaces, the artificial lighting's convective output to the room air is varied accordingly to keep the total output constant. Thus:

$$Q_{lc} = p_r Q_1 - Q_{rl} \ldots 5.10$$

where:

$$Q_{lc} = \text{convective heat output from the artificial lighting to the room air}$$
\[ p_r = \text{combined long wave radiation and convection to room air} \]

proportion of the total heat output from the artificial lighting = 0.35

\[ Q_l = \text{total energy output from artificial lighting} \]

\[ Q_{rl} = \text{long wave radiant emission to room from artificial lighting} \]

The equivalent uniform temperature of the lighting surface is derived from the approximate proportional emission given in Table 4.4 using the long wave radiation model with temperatures of 68°F (20°C) for all the other room surfaces. This is the desired equivalent ambient temperature for the room and the mean radiant temperature of the room with respect to the surface of the artificial lighting only varies slightly from this value during occupancy when the artificial lighting is on. The lighting surface temperature is an input parameter to the room heat flow model together with the component proportions and the total heat output from the artificial lighting as given in Table 5.2. During the vacancy period the artificial lighting is not operating so its surface temperature is assumed equal to the ceiling surface temperature.

5.5 ROOM AIR

![PLENUM AIR](response) ![ROOM AIR](response)

The room air can be considered to be a mixture of dry air and water vapour. As humidifiers are not usually installed in New
Zealand offices any moisture added to the air comes from environmental heat sources such as human occupants which provide the latent heat themselves. Thus no latent heat considerations need to be included in the model. The most general equation for the thermal response of moist air is [ASHRAE, 1972a]:

\[
H_a = m (h_2 - h_1) \tag{5.11}
\]

where:

- \(H_a\) = quantity of heat input to air
- \(m\) = mass of air
- \(h_2\) = specific enthalpy of moist air in final state
- \(h_1\) = specific enthalpy of moist air in initial state.

As no latent heat is involved, the enthalpy of the moist air can be written in terms of specific heats, thus [ASHRAE, 1972a]:

\[
H_a = m_a (C_{pa} + C_{ps}W)(\theta_2 - \theta_1) \tag{5.12}
\]

where:

- \(C_{pa}\) = specific heat of dry air
- \(C_{ps}\) = specific heat of steam
- \(W\) = humidity ratio = weight of water vapour per unit weight of dry air
- \(\theta_2 - \theta_1\) = dry bulb temperature change
- \(m_a\) = mass of dry air

As the room model has a constant volume, the air's heat capacity per unit volume and per unit temperature rise is required. Thus the volume specific heat of moist air is defined as:
\[ C_v = \rho_a \frac{C_p}{\rho_a} = \rho_a \left( C_{pa} + C_{ps} \text{ W} \right) \]  \( \ldots 5.13 \)

where:

- \( C_v \) = volume specific heat of moist air
- \( \rho_a \) = density of dry air
- \( C_p \) = humid specific heat of air

All the terms in Equation 5.13 vary with dry bulb temperature, although the density of dry air has the most significant variation. It is necessary to include the influence of variable dry bulb air temperatures in the model. As no mathematical expression is available from the literature, the following analysis derives a suitable relationship between volume specific heat of moist air and dry bulb air temperature.

As a tabulation of the volume specific heat of moist air with respect to dry bulb temperature and humidity ratio was not readily available, it was computed from tabulated values of the specific volume and specific enthalpy of dry air at standard atmospheric pressure and the saturation pressure and specific enthalpy of steam [ASHRAE, 1972a].

From Equation 5.11, the specific heat of dry air is given by:

\[ C_{pa} = \frac{dh_a}{d\theta} = \frac{h_{a2} - h_{a1}}{\theta_2 - \theta_1} \]  \( \ldots 5.14 \)

where:

- \( h_{a1} \) = specific enthalpy of dry air at temperature \( \theta_1 \)
- \( h_{a2} \) = specific enthalpy of dry air at temperature \( \theta_2 \)

Similarly, the specific heat of steam is given by:

\[ C_{ps} = \frac{h_{s2} - h_{s1}}{\theta_2 - \theta_1} \]  \( \ldots 5.15 \)
where:

\[ h_{s1} = \text{specific enthalpy of steam at temperature } \theta_1 \]
\[ h_{s2} = \text{specific enthalpy of steam at temperature } \theta_2 \]

Combining Equations 5.14 and 5.15 for a constant humidity ratio, the humid specific heat of air is given by:

\[ C_p = \frac{(h_{a2} + h_{s2} W) - (h_{a1} + h_{s2} W)}{\theta_2 - \theta_1} \]  \hspace{1cm} ...5.16

and the volume specific heat of moist air is given by:

\[ C_v = r \left( \frac{\rho_{a2} (h_{a2} + h_{s2} W) - \rho_{a1} (h_{a1} + h_{s1} W)}{\theta_2 - \theta_1} \right) \]  \hspace{1cm} ...5.17

where:

\[ r = \text{ratio of density of dry air in dry air and steam mixture to density of dry air at the same total pressure.} \]
\[ \rho_{a1} = \text{density of dry air at dry bulb temperature } \theta_1 \]
\[ \rho_{a2} = \text{density of dry air at dry bulb temperature } \theta_2 \]

As air obeys the perfect gas relation [Jennings, 1970], the ratio of the densities of dry air at the same temperature, but at different pressures is given by:

\[ r = \frac{\rho_a}{\rho_b} = \frac{p_a}{p_b} \]  \hspace{1cm} ...5.18

where:

\[ \rho_a = \text{density of dry air at pressure } p_a \]
\[ \rho_b = \text{density of dry air at pressure } p_b \]
\[ p_a = \text{partial pressure of dry air in moist air} \]
\[ p_b = \text{standard atmospheric pressure} \]
As the Gibbs-Dalton law of partial pressures is closely obeyed by atmospheric air-steam mixtures, the partial pressure of dry air in moist air is given by [Jennings, 1970]:

\[ P_a = P_b - P_s \] ...5.19

where:

\( P_s = \) partial pressure of steam in moist air.

As the pressure of steam is essentially independent of whether air is, or is not present [ASHRAE, 1972a], the partial pressure of steam can be found from steam tables for any given humidity ratio.

The variation of the volume specific heat of moist air with dry bulb temperature as computed from Equation 5.17, is plotted for three values of humidity ratio in Figure 5.3. In the modelled system the human occupants dissipate some water vapour into the room through breathing and sweat evaporation. Exfiltrating air removes some of this humidity from the room when it is replaced by infiltrating external air which has a lower humidity ratio. This situation is modelled by assuming a linear variation of the volume specific heat of moist air with dry bulb temperature:

\[ C_v = a \theta_a + b \] ...5.20

where:

\( a, b = \) constants

\( \theta_a = \) dry bulb air temperature

Values of \(-7.65 \times 10^{-5}\) Btu/ft\(^3\) and 0.01997 Btu/ft\(^3\)/\(^\circ\)F are used in the model for \( a \) and \( b \) respectively. This line, which is shown in Figure 5.3 is defined by the points \( A \) and \( B \), where \( A \) is the point with relative humidity of 80\% at 40\(^\circ\)F (4.4\(^\circ\)C) and \( B \) is the point with relative humidity of 50\% at 60\(^\circ\)F (15.6\(^\circ\)C).
FIGURE 5.3: VARIATION OF VOLUME SPECIFIC HEAT OF MOIST AIR WITH DRY BULB TEMPERATURE AND HUMIDITY RATIO
Thus from Equations 5.12, 5.13 and 5.20, the thermal response of the room air is given by:

\[ Q_a = (a \theta_a + b) V \frac{d\theta_a}{dt} \]  

...5.21

where:

- \( Q_a \) = net rate of heat input to the air
- \( \theta_a \) = mean dry bulb air temperature
- \( V \) = volume containing air
- \( \frac{d\theta_a}{dt} \) = rate of rise of mean air temperature

Equation 5.21 is also used to model the thermal response of the air in the plenum between the floor and the ceiling of vertically adjacent rooms. The heat input to the plenum air from these boundaries and the artificial lighting is described in Sections 5.4.2 and 5.8.4.

5.6 INFILTRATING AIR

Air infiltrates through the cracks around windows due to a pressure difference across the cracks. This pressure difference is due to wind and the temperature difference between the internal and external air. Although mathematical relations exist to describe this process
[Jackman, 1970], the specific data required for the wind velocity, its pressure distribution around the building and for the nature of the leakage cracks is not readily available. It is more convenient to make the infiltration rate an input variable of the model. Infiltration rate can be varied to take account of wind direction, the room's location in the building, which affects the external wind pressure, the type of window, and the position of variable opening windows. The model assumes natural ventilation without humidification, which is common in New Zealand office buildings. As condensation is a minor thermal behaviour characteristic, it is neglected.

External air infiltrating into the room gradually mixes with the warmer air and gains heat from it. The heat required to raise the temperature of this infiltrating air is given by Equation 5.12. As no humidification takes place and condensation is neglected, no latent heat is involved, so the derivations that applied to thermal response of room air also apply to the infiltrating air. Thus from Equations 5.12, 5.13 and 5.20, the heat loss due to infiltration is given by:

\[ Q_{\text{inf}} = (a \theta_a + b) \ I \ (\theta_a - \theta_e) \]  
\[ \text{...5.22} \]

where:

- \( Q_{\text{inf}} \) = rate of heat flow to warm the infiltrating air
- \( I \) = volume infiltration rate
- \( \theta_e \) = external air dry bulb temperature

Equation 5.22 is based on air infiltrating directly from the external air, which is true for rooms with windward exposures. This is the condition that produces the greater heating requirements and, as the wind varies in direction, all rooms have windward exposures at some time. Rooms with leeward exposures receive infiltrating air from other rooms in the building. This air is at a similar
temperature to the air already present in the room. Thus leeward exposure can be modelled by a zero infiltration rate.

Exfiltration is actually a part of the same process as infiltration. The warm air that exfiltrates from the room need not be considered in the room heat flow system as the infiltration heat flow is actually the net heat flow of the total process involving both infiltration and exfiltration.

The model provides for three infiltration rates to apply to a room. When the building is unoccupied windows and doors will generally be closed, thus a vacant infiltration rate applies during this period. During occupancy when the room is at comfort conditions some windows will generally be open to create the desired ventilation rate. This is called the comfort infiltration rate. The third infiltration rate, named the heating infiltration rate, has a value between the other two. It applies during occupancy when the room is below comfort conditions as, even with the room's windows closed, the opening of external and internal doors in the building will increase the infiltration rate above that occurring during vacancy.

All three infiltration rates are modelled as constants. They must be considered to be the mean infiltration rates for their situation as in practice the variation of wind strength and position of doors with time causes fluctuations in the infiltration rate.
5.7 CONDUCTION HEAT FLOWS

Heat flows through solid materials by conduction in the opposite direction to any temperature gradient. As the heat flow through the wall is much larger than any heat flow in planes parallel to the wall's surfaces, the latter is neglected. Thus the thermal response of each wall is represented by the one dimensional heat flow equation [Gupta et al, 1971]:

\[
\frac{\partial^2 \theta_{x,t}}{\partial x^2} = \frac{1}{\alpha} \frac{\partial \theta_{x,t}}{\partial t} \tag{5.23}
\]

where:

- \( \theta_{x,t} \) = temperature at time, \( t \) and distance, \( x \) through the wall
- \( \frac{\partial \theta_{x,t}}{\partial t} \) = partial derivative of \( \theta_{x,t} \) with respect to time
- \( \frac{\partial^2 \theta_{x,t}}{\partial x^2} \) = second partial derivative of \( \theta_{x,t} \) with respect to distance
- \( \alpha \) = material's thermal diffusivity

Each wall of the room is modelled by Equation 5.23 which is solved using the Crank-Nicholson finite difference method [Crank, 1956]. The finite difference representation of Equation 5.23 is developed in
Chapter Six along with a discussion of stability and accuracy characteristics of the derived model.

A "wall" in the room heat flow model is a part of the room's structure that can be considered to be a single construction form for modelling the conduction heat flow through it. It does not need to be contiguous but the combination of similar constructions at separate locations on the room boundary is constrained by the nature of the modelled surface heat flows. Similar heat flows on the surface external to the room is one constraint. Convection heat flows on the room's internal surfaces differ for horizontal and vertical surfaces as described in Section 5.8.1. Both long wave and short wave radiation, as described in Sections 5.8.3 and 5.8.4, are based on the geometric radiation configuration factors between all the internal surfaces of the room. Each wall is modelled with one surface internal to the room, thus the values for the configuration factors for this surface must be applicable for all portions of a modelled wall.

No specific furniture component is included in the model because of the low thermal capacity and quick response of most office furniture [Kelles, 1972]. Bookcases which would have a significant thermal capacity could be included as part of a wall structure.

The boundary conditions for each wall are defined by the net heat flow at the surface, which is the sum of convective and long wave and short wave radiative components. The wall behind the heat emitters is a special case as its internal boundary condition is given by the temperature of the emitter casing.
5.8 ROOM'S INTERNAL SURFACES

Heat transmission occurs at a room's internal surfaces by convection with the surrounding air, by long wave radiation exchange with the other surfaces in the room, and by absorption of short wave radiation emitted from the artificial lighting and transmitted through the external glazing.

5.8.1 Convection

Convection between a solid surface and a fluid is a complex process that includes conduction through the fluid layer closest to the surface. Heat is convected by the movement of the surface layer of fluid. In forced convection this movement is caused by some external force, but in free convection the movement is due to the change in buoyancy of the fluid with change in temperature.

Although infiltration and ventilation cause some movement of air within a room, the velocities involved are kept below 30 ft/min (0.15 m/s) to avoid draughts [IHVE, 1970a]. With such low velocities, the criterion for the dominance of free convection over forced convection is easily met [McAdams, 1954],

\[ \frac{Grashof\ Number}{(Reynolds\ Number)^2} > 1.0 \]
Free convection, like many fluid processes, must rely on experimental results to establish the relationships between the pertinent variables as analytical derivations are not possible owing to the complexities involved. General relationships have been predicted from correlations of experimental results using dimensionless ratios. The most widely applicable correlation for free convection is given by Equation 5.24 [McAdams, 1954]:

$$\text{Nu} = C \text{ (Gr.Pr)}^m$$  ...5.24

where:

- $\text{Nu}$ = Nusselt Number
- $\text{Gr}$ = Grashof Number
- $\text{Pr}$ = Prandtl Number
- $C$ = constant
- $m$ = exponent

The values of the constant and exponent depend upon whether laminar or turbulent flow exists as determined by the size of the (Gr.Pr) product, and upon the shape and orientation of the surface.

By assuming the fluid properties of air to be constant for the temperature range involved, simpler relationships have been proposed. These relationships are based on Newton's convection equation [McAdams, 1954], which expresses the variation of convective heat flow with temperature difference as a linear relationship:

$$q_c = h_c A \frac{(\theta - \theta_s)}{s - a}$$  ...5.25

where:

- $q_c$ = rate of convective heat flow from surface to adjacent air
- $h_c$ = surface convective coefficient
$A_s = \text{area of the surface}$

$\theta_s = \text{temperature of the surface}$

$\theta_a = \text{temperature of the air}$

As the variation with temperature difference is not linear, the non-linearity is generally included in expressions for the surface convective coefficient. These expressions depend upon the nature of the flow and the orientation of the surface. Table 5.3 lists some relationships proposed by McAdams [1954] and recommended by others [Hutchison, 1952; Buchberg, 1971; Holman, 1972]. For the range of internal air temperatures used in the simulation study, i.e. 40-70°F (4.4-21.1°C), the maximum error in using the relationships given in Table 5.3, rather than the more general relationships proposed by McAdams [1954] is 3% [Buchberg, 1971]. This error is quite acceptable in view of the 15% accuracy of the correlations for the general relationships. Hence the expressions presented in Table 5.3 together with Equation 5.25 are used in the model for the convective heat flows at the surfaces internal to the room.

<table>
<thead>
<tr>
<th>Surface Orientation</th>
<th>Equation</th>
<th>Flow</th>
</tr>
</thead>
<tbody>
<tr>
<td>Vertical</td>
<td>$h_c = 0.19(\theta_a - \theta_s)^{1/3}$</td>
<td>Turbulent</td>
</tr>
<tr>
<td>Horizontal, heat flow up</td>
<td>$h_c = 0.22(\theta_a - \theta_s)^{1/3}$</td>
<td>Turbulent</td>
</tr>
<tr>
<td>Horizontal, heat flow down</td>
<td>$h_c = 0.12\left(\frac{\theta_a - \theta_s}{L}\right)^{1/4}$</td>
<td>Laminar</td>
</tr>
</tbody>
</table>

**TABLE 5.3: CONVECTIVE SURFACE COEFFICIENT RELATIONSHIPS**

Note 1: $L = \text{characteristic length} = \text{mean of rectangular dimensions}$

Note 2: Units are: $h_c$ in Btu/hr-ft$^2-\circ F$. 
The radiant energy emitted from an isothermal surface of a body is given by the Stefan-Boltzmann law [Weibelt, 1965]:

\[ W = \varepsilon \sigma T^4 \]  \( \ldots 5.26 \)

where:

- \( W \) = radiant flux emitted from surface
- \( \varepsilon \) = surface emissivity
- \( \sigma \) = Stefan-Boltzmann constant
- \( T \) = absolute temperature of the surface
All surfaces emit radiation with a range of wavelengths and the emissivity generally varies with wavelength. If the emissivity is independent of the wavelength of the radiation the surface is called gray. The radiation emitted from surfaces at normal temperatures has a narrow spectrum of wavelengths that are relatively long compared with the short wavelengths of high temperature radiation. Thus this radiation is called long wave radiation. Grayness is a reasonable assumption for long wave radiant emission.

Radiation incident upon a surface is generally absorbed, reflected, and transmitted through the material. Thus by the first law of thermodynamics [ASHRAE, 1972a]:

\[ \alpha + \rho + \tau = 1.0 \]

where:
- \( \alpha \) = absorptance = fraction of incident radiation absorbed
- \( \rho \) = reflectance = fraction of incident radiation reflected
- \( \tau \) = transmittance = fraction of incident radiation transmitted.

Application of the first law of thermodynamics also gives Kirchoff's law which states that for an opaque surface, i.e. one with zero transmittance, the emissivity at a given wavelength equals the absorptivity at that wavelength. For a gray surface the wavelength is irrelevant, thus [ASHRAE, 1972a]:

\[ \varepsilon = \alpha \]

where:
- \( \varepsilon \) = surface emissivity
- \( \alpha \) = surface absorptance
A black body is a body whose surface has an emissivity of unity. The space-wise distribution of the radiation emitted from the surface of a black body is given by Lambert's law. This law states that the intensity of radiant energy over a hemispherical surface above the emitting surface varies as the cosine of the angle between the normal to the radiating surface and the line joining the radiating surface to the point of the hemispherical surface. Radiation that obeys Lambert's law is called diffuse radiation [ASHRAE, 1972a].

The following mathematical model of radiation exchange between the surfaces of an enclosure requires a number of assumptions:

(1) All surfaces are isothermal. This can be achieved by dividing a non-isothermal surface into sections that can be considered to be isothermal. The room is divided into "walls" according to the orientation and the construction form of the structure. Each of these walls and the artificial lighting, glazing and human occupants is considered to have a single isothermal room surface.

(2) Each room surface is assumed to be gray with uniform emittance over the surface. This is a reasonable assumption for long wave radiation exchange between surfaces with temperatures of similar magnitudes [Fanger, 1970].

(3) Each room surface is assumed to be opaque to long wave radiation. All materials used for surfacing walls have negligible transmittance to long wave radiation. This includes glass, which has high transmittance to short-wave radiation only [Hottel, 1954].

(4) All radiation is emitted and reflected diffusely. This is only true for black surfaces but is a close approximation for most non-metallic surfaces [Fanger, 1970].
(5) The room air is considered to be diathermous. Water vapour and carbon dioxide absorb and scatter some radiation but the small concentrations and space sizes concerned make this negligible [Jennings, 1970].

Analysis of the radiation exchange between a room's surfaces requires knowledge of the fraction of radiation emitted from one surface that is incident at each of the other surfaces, and at the emitting surface if it sees itself. This fraction is called the radiation configuration factor and for diffuse surfaces depends only on the geometrical relationship between the emitting and receiving surfaces. It is defined by the following equation [Holman, 1972]:

\[ Q_{ij} = W_i A_i F_{ij} \]  \hspace{1cm} \ldots 5.29

where:

- \( Q_{ij} \) = radiant power leaving surface \( i \) that is incident at surface \( j \)
- \( W_i \) = total radiant flux emitted from surface \( i \)
- \( A_i \) = area of surface \( i \)
- \( F_{ij} \) = radiation configuration factor for radiation from surface \( i \) to surface \( j \).

Application of the first law of thermodynamics gives the reciprocity theorem [Holman, 1972]:

\[ A_i F_{ij} = A_j F_{ji} \]  \hspace{1cm} \ldots 5.30

The radiation configuration factors for all the surfaces in the modelled rooms were computed from equations based on Lambert's law [Weibelt, 1965]. When two surfaces are distributed over a common plane as is the case for the ceiling plus lighting and for the studded walls,
the radiation shape factor for the total plane is divided between the surfaces in the ratio of their areas. The radiation shape factors for the human occupants were taken from figures produced by Fanger [1970] for a seated person.

If all the room's surfaces absorbed all the radiation incident upon them, a relatively simple model of radiation exchange using radiation configuration factors can be used. For surfaces with non-zero reflectivity a set of infinite series of partial reflections is involved. Hottel [1954] presents a method of analysis of this complex process based on a script F factor which is defined by the following equation:

\[ q_{ij} = A_i \Phi_{ij} \sigma (T_i^h - T_j^h) \geq A_j \Phi_{ji} \sigma (T_j^h - T_i^h) \] ...

where:

- \( q_{ij} \) = net long wave radiation flux exchanged from surface i to surface j
- \( A_i \) = area of surface i
- \( \Phi_{ij} \) = script F factor
- \( T_i, T_j \) = absolute temperatures of surfaces i and j respectively.

The net long wave radiation flux includes the direct interchange between the two surfaces plus contributions due to multiple reflection at all room surfaces. The script F factors depend upon the radiation configuration factors, the emissivities, and the reflectivities of all surfaces in the enclosure. Thus the solution of a system of linear simultaneous equations of rank equal to the number of surfaces is required to evaluate the script F factors for an enclosure. Ishimoto and Bevans [1969] have developed an efficient method of solution using a
single matrix inversion. This method was computerised and the script F factors produced were used as input parameters for the room heat flow model.

The net long wave radiant input to each surface is computed by summing the component irradiated from all the surfaces:

\[ q_{ri} = \sum_{j=1}^{n} q_{ji} \]

...5.32

where:

- \( q_{ri} \) = net long wave radiant input to surface \( i \)
- \( n \) = number of surfaces in the room

Thus the room heat flow model uses Equations 5.31 and 5.32 to compute the net long wave radiant input to each surface at any instant in time.

5.8.3 Short Wave Radiation

Short wave radiation is emitted by the artificial lighting and through the external glazing as both direct and diffuse solar radiation. Most room surface materials have relatively high reflectances for short wave radiation, thus a set of infinite series of partial
reflections also occurs for short wave radiation. Hence the short wave radiosity of a surface is defined as the total short wave flux irradiating from the surface. It is the sum of emitted and reflected short wave radiant flux components:

\[ R_i = E_i + \rho_{si} H_i \]  

...5.33

where:

- \( R_i \) = short wave radiosity from surface \( i \)
- \( E_i \) = short wave flux emitted from surface \( i \)
- \( \rho_{si} \) = short wave reflectance of the surface \( i \)
- \( H_i \) = short wave flux incident on the surface \( i \)

The only surfaces that emit short wave radiation are the artificial lighting and the external glazing. The diffuse component of solar radiation that is transmitted through the glazing is considered to be diffusely emitted by the glazing. The direct solar radiation that is transmitted through the glazing is considered to irradiate uniformly to the floor only and the portion that is reflected from the floor is considered to be diffusely reflected. Thus the short wave radiation incident on a surface is the sum of the contributions from the radiosities from all the surfaces in the room plus an extraneous source for the floor only:

\[ H_i A_i = \sum_{j=1}^{n} R_j A_j F_{ji} + D_i A_i \]  

...5.34

where:

- \( A_i \) = area of surface \( i \)
- \( F_{ji} \) = radiation configuration factor for radiation from surface \( j \) to surface \( i \)
- \( D_i \) = extraneous short wave radiant flux incident on surface \( i \)
- \( n \) = number of surfaces
Application of the reciprocity theorem gives:

\[ H_i = \sum_{j=1}^{n} R_{ij} F_{ij} + D_i \] ...5.35

Substituting Equation 5.35 in Equation 5.33 gives:

\[ R_i = E_i + \rho_{si} \sum_{j=1}^{n} R_{ij} F_{ij} + \rho_{si} D_i \] ...5.36

Rearranging gives:

\[ \frac{R_i}{\rho_{si}} - \sum_{j=1}^{n} R_{ij} F_{ij} = \frac{E_i}{\rho_{si}} + D_i \] ...5.37

This is equivalent to the matrix equation:

\[ [Y] \{R\} = \{S\} \] ...5.38

where:

\[ [Y] = \text{matrix of radiosity coefficients} \]
\[ \{R\} = \text{vector of radiosities, } R_i \]
\[ \{S\} = \text{vector of source fluxes, } \frac{E_i}{\rho_{si}} + D_i \]

Inversion gives:

\[ \{R\} = [Y]^{-1} \{S\} \] ...5.39

where:

\[ [Y]^{-1} = \text{inversion of matrix } [Y]. \]

Writing in summation form:

\[ R_i = \sum_{\text{source}} y_{ij} \frac{E_j}{\rho_{sj}} + D_j \] ...5.40
where:

\[ y_{ij} = \text{element in the } i\text{th row and } j\text{th column of inverse matrix } [Y]^{-1}, \text{ named the radiosity factor}. \]

As the source fluxes are zero for all surfaces except the artificial lighting, external glazing, and the floor, the summation reduces to a summation over these latter surfaces, named source surfaces.

The net short wave radiant input to each surface is the absorbed radiation minus the emitted radiation:

\[ q_{si} = (\alpha_{si} H_i - E_i) A_i \] ...5.41

where:

\[ q_{si} = \text{net short wave radiant input to surface } i \]
\[ \alpha_{si} = \text{short wave absorptance of surface } i \]

Substitution from Equation 5.33 gives:

\[ q_{si} = \left\{ \frac{\alpha_{si}}{\rho_{si}} R_i - \left(1 + \frac{\alpha_{si}}{\rho_{si}} \right) E_i \right\} A_i \] ...5.42

Substitution from Equation 5.40 gives:

\[ q_{si} = \left\{ \frac{\alpha_{si}}{\rho_{si}} \sum_{\text{source surfaces}} y_{ij} \left( \frac{E_j}{\rho_{sj}} + D_j \right) - \left(1 + \frac{\alpha_{si}}{\rho_{si}} \right) E_i \right\} A_i \] ...5.43

The matrix of radiosity coefficients is inverted using a matrix inversion computer program and the radiosity factors for each surface with respect to the source surfaces only, together with the short wave absorptance to reflectance ratio for each surface are input to the room heat flow system model. The model uses Equation 5.43 to compute the net short wave radiant input to each surface at any instant in time.
5.8.4 Surfaces in Adjacent Rooms and Plenums

The surfaces of the internal walls in adjacent rooms are assumed to undergo identical net heat flows to the surfaces in a similar position in the room under study. Thus walls that are symmetric in the direction of the heat flow are modelled with half their thickness and a zero heat flow boundary condition.

The assumption of similar heat flows in adjacent rooms means the heat flows for the floor and ceiling of the same room can be used for the plenum surface heat flows. The heat input to the plenum from the artificial lighting is described in Section 5.4.2. The floor and ceiling plenum surface heat flows are modelled using a simplified linear convection model [Jennings, 1970]:

\[ q_p = h_s A_s (\theta_s - \theta_p) \]

where:

- \( q_p \) = rate of heat flow from plenum surface to plenum air
- \( h_s \) = surface conductance
- \( A_s \) = area of surface
- \( \theta_s \) = temperature of the surface
- \( \theta_p \) = temperature of air in plenum

This simple model, which neglects radiation exchange between the plenum surfaces, is used in the room heat flow model as it is assumed that any lack of precision would have negligible effect on the room's thermal response.
5.9 SURFACES EXTERNAL TO THE BUILDING

Heat transmission occurs at surfaces external to the building by convection with the external air, by long wave radiation exchange with the sky, the ground, and any other surrounding surfaces and by absorption of both direct and diffuse solar radiation. Latent heat processes are neglected.

5.9.1 Convection

The wind blowing over the outside surface of a building causes forced convective heat transfer between the surface and the external air. This is described by Newton's convection equation [McAdams, 1954]:

\[ Q_c = h_c A_s (\theta_s - \theta_e) \]  ...5.45

where:

- \( Q_c \) = rate of convective heat flow from surface to external air
- \( h_c \) = external surface convective coefficient
- \( A_s \) = area of external surface
- \( \theta_s \) = temperature of the external surface
- \( \theta_e \) = temperature of the external air
The external surface convective coefficient increases with increasing roughness of the surface, with increasing wind velocity over the surface, and with increasing temperature difference [Jennings, 1970]. As Buchberg [1969] has shown that the thermal response of simple dwellings is relatively insensitive to variation in the value of the external surface convective coefficient, it was assumed that the thermal response of the room with less external exposure would also be insensitive. Thus a constant value for the external surface convective coefficient is used in the model. The values used were computed from the following equations [Jennings, 1970]: \( h_e \) in Btu/hr/ft\(^2\)/°F

\[
\begin{align*}
  h_e &= 1.4 + 0.28 \, v \quad \text{for very smooth surfaces} & \cdots 5.46 \\
  h_e &= 1.6 + 0.3 \, v \quad \text{for smooth wood and plaster} & \cdots 5.47 \\
  h_e &= 2.0 + 0.4 \, v \quad \text{for cast concrete and smooth brick} & \cdots 5.48 \\
  h_e &= 2.1 + 0.5 \, v \quad \text{for rough stucco surfaces} & \cdots 5.49 \\
\end{align*}
\]

where:

\( v = \) wind speed in miles per hour.

Equation 5.46 was used for glass. As overall surface coefficients of heat transfer are commonly based on a 15 mph (67 m/s) wind, this wind velocity was used in the above equations to obtain the appropriate external surface convective coefficient for each external wall to be input to the room heat flow model.
5.9.2 Long Wave Radiation

The long wave radiation exchange between the external surface of the building and its surroundings is modelled by the following equation:

\[
\dot{Q}_r = \sigma (\varepsilon_s T_s^4 - \varepsilon_e T_e^4)
\]  \[\text{...5.50}\]

where:

- \(\dot{Q}_r\) = net rate of long wave radiant heat flow from the external surface to the external environment
- \(\varepsilon_s\) = emittance of external surface
- \(T_s\) = absolute temperature of external surface
- \(\varepsilon_e\) = effective emittance of the external environment
- \(T_e\) = absolute temperature of the external air.

The external environment consists of the sky, the ground, and any adjacent buildings seen by the external surface. The effective emittance of the sky on a clear day has been derived by Brunt [Holden, 1961]:

\[
\varepsilon_{\text{sky}} = a + b \sqrt{p_w}
\]  \[\text{...5.51}\]
where:

\[ \epsilon_{\text{sky}} = \text{effective emittance of the sky on a clear day} \]
\[ P_w = \text{water vapour pressure at ground level} \]
\[ a, b = \text{constants depending upon the surface orientation.} \]

Holden [1961] has produced formulae for the values of these constants for all orientations. Parmelee and Aubele [1951] note that when temperature inversion occurs the emission of the sky would be expected to be greater. Clouds increase the effective emittance of the sky above the values for clear skies. Field data for completely overcast days averaged an effective emittance of 0.96 [Parmelee & Aubele, 1951].

The ground temperature and the temperature of surrounding buildings will vary to a certain extent from the temperature of the external air and these surfaces will have a range of emittances. The choice of an effective emittance for the external environment must take account of these factors. A constant effective emittance neglects any dynamic variation of the temperature difference between these surfaces and the external air, but as the external walls temper this variation, its effect on the room response is assumed to be negligible. A value of 0.9 is used for the effective emittance in the model.

5.9.3 Solar Radiation
In passing through the earth's atmosphere the solar radiation, which has a spectral distribution in the short wavelength range, is partially scattered and absorbed by dust, gas molecules, ozone, and water vapour. Some of the scattered radiation reaches the earth as diffuse solar radiation. The portion of the solar radiation that is neither absorbed nor scattered is transmitted to the earth as direct solar radiation. The intensity of both direct and diffuse radiation on a surface on the earth depends upon the orientation of the surface, the length of the atmospheric path traversed by the sun's rays, and the composition of the atmosphere. Both the length of the atmospheric path and the composition of the atmosphere vary with time. The variation of the former is well understood, thus is described by a precise geometrical model [Spencer, 1965a]. The variation of the composition of the atmosphere cannot be precisely modelled because of the extreme variability in cloudiness [Lui & Jordan, 1960]. Some approximate generalised models have been developed however, from recorded solar radiation intensities at a number of locations [Parmelee, 1954; Lui & Jordan, 1960; Spencer, 1965b].

The room heat flow model requires time profiles of direct and diffuse solar radiation intensities on the vertical surfaces of the room's external walls. The New Zealand Meteorological Service record total solar radiation intensities, i.e. the sum of the direct and diffuse components, on horizontal surfaces only. Although this is sufficient data for determining the total radiation on a vertical surface using the geometrical model of solar position [Spencer, 1965a] and making due allowance for reflection to the vertical surface, further information is required to determine the direct and diffuse components. Tables for a number of Australasian cities, including Christchurch, New Zealand, the city used for this study, have been produced giving the direct and diffuse
solar radiation intensities for clear days for a range of surface orientations [CSIRO, 1966]. These tables are based on data recorded for Melbourne and the model developed from it by Spencer [1965a]. As the distribution of both daily direct and diffuse solar radiation on a surface with respect to the hour of the day for a given day and constant cloudiness is independent of the magnitude of the radiation i.e. the magnitude of the cloudiness [Parmelee, 1954], the distributions given in these tables can be used for other values of constant cloudiness if appropriate scaling factors can be established.

Lui and Jordan [1960] in their analysis of recorded data for a number of localities established a relationship between the daily diffuse radiation on a horizontal surface and the daily total radiation on a horizontal surface. Both these variables are normalised by dividing by the extraterrestrial daily insolation on a horizontal surface which is given by:

\[
H_0 = \frac{24}{\pi} \tau \frac{I_{sc}}{I_s} (\cos L \cos \delta \sin \omega_s + \omega_s \sin L \sin \delta)
\]

...5.52

where:

- \(H_0\) = extraterrestrial daily insolation on a horizontal surface
- \(\tau\) = ratio of solar radiation intensity at normal incidence outside the earth's atmosphere to the solar constant
- \(I_{sc}\) = solar constant which is defined as the solar radiation intensity at normal incidence outside the earth's atmosphere when the earth is at its mean distance from the Sun = 442 Btu/hr/ft\(^2\) = 1.395 Kw/m\(^2\) = 2.00 langley/min
- \(L\) = latitude of location
\[ \delta = \text{solar declination} \]
\[ \omega_s = \text{sunset hour angle in radians.} \]

The sunset hour angle is given by [Parmelee, 1954]:

\[ \cos \omega_s = -\tan L \tan \delta \quad \ldots 5.53 \]

This information is used in the present study to produce a method for determining hourly values of direct and diffuse solar radiation intensities for constant cloudiness on the surface orientations included in Spencer's Tables [CSIRO, 1966]. Equations 5.54 and 5.55 describe the application of this method:

\[ S_d = R_d T_d \quad \ldots 5.54 \]

where:

- \( S_d \) = diffuse solar radiation intensity on a surface for a constant level of cloudiness
- \( R_d \) = scaling ratio for diffuse solar radiation appropriate to the constant level of cloudiness
- \( T_d \) = tabulated diffuse solar radiation intensities on the surface for a clear day.

\[ S_d = R_d T_d \quad \ldots 5.55 \]

where:

- \( S_D \) = direct solar radiation intensity on a surface for a constant level of cloudiness
- \( R_D \) = scaling ratio for direct solar radiation appropriate to the constant level of cloudiness
- \( T_D \) = tabulated direct solar radiation intensities on the surface for a clear day.
The scaling ratios are computed from Equations 5.56 and 5.57.

\[ R_d = \frac{N_d}{N_{cd}} \]  \hspace{1cm} \ldots 5.56

where:

- \( N_d \) = normalised daily diffuse radiation on a horizontal surface appropriate to the constant level of cloudiness.
- \( N_{cd} \) = normalised daily diffuse radiation on a horizontal surface for a clear day.

\[ R_D = \frac{N_D}{N_{cD}} \]  \hspace{1cm} \ldots 5.57

where:

- \( N_D \) = normalised daily direct radiation on a horizontal surface appropriate to the constant level of cloudiness.
- \( N_{cD} \) = normalised daily direct radiation on a horizontal surface for a clear day.

The normalised daily direct and diffuse radiation values for a clear day are obtained by dividing the tabulated daily radiation values by the extraterrestrial daily insolation on a horizontal surface which can be computed from Equation 5.52 for the location and date of interest. The normalised values for the constant level of cloudiness are read from Figures 5 and 8 of Lui and Jordan's [1960] paper after normalising the daily total radiation on a horizontal surface for that level of cloudiness. Monthly maximum, mean, and minimum values for the latter parameter were obtained from New Zealand Meteorological Service records. This method assumes that the scaling ratio is independent of the surface...
orientation. Spencer [1965c] found this to be so when applying Parmelee's [1954] results for Melbourne.

The heat transfer to the external surfaces of the room from the direct and diffuse solar radiation is described by:

\[
Q_s = \alpha_s A_s (S_d + S_d)
\]

where:

- \(Q_s\) = rate of solar heat flow into external surface
- \(\alpha_s\) = short-wave absorptance of surface
- \(A_s\) = area of surface
- \(S_d\) = direct solar radiation intensity on the vertical surface
- \(S_d\) = diffuse solar radiation intensity on the vertical surface.

5.10 WINDOWS

As the thermal capacity of the glass is small, it is modelled as a single lumped system at a uniform temperature for which the
thermal response is given by:

\[ q_g = c_g \cdot \zeta_g \cdot d_g \cdot A_g \cdot \frac{d\theta}{dt} \] ...

where:

- \( c_g \) = specific heat of glass
- \( \zeta_g \) = density of glass
- \( d_g \) = thickness of glass
- \( A_g \) = area of glazing
- \( \frac{d\theta}{dt} \) = rate of rise of temperature of glass.

The net heat flow into this system is the sum of all the surface heat transfers:

\[ q_g = q_c + q_r + q_s - Q_c - Q_r + Q_s + E \cdot A_g \] ...

where:

- \( q_g \) = net heat flow into the glass
- \( q_c \) = convective heat flow into the glazing internal surface as given by Equation 5.24
- \( q_r \) = net long-wave radiant input to the glazing internal surface as given by Equation 5.32
- \( q_s \) = net short-wave radiant input to the glazing internal surface as given by Equation 5.43
- \( Q_c \) = convective heat flow from the glazing external surface as given by Equation 5.45
- \( Q_r \) = net long-wave radiant output from the glazing external surface as given by Equation 5.50
\[ Q_s = \text{solar radiant input into the glazing external surface as given by Equation 5.58.} \]
\[ E_g = \text{diffuse solar radiative flux transmitted through the glazing} \]
\[ A_g = \text{area of glazing.} \]

The transmitted diffuse solar radiation must be included as an input to the glass as Equation 5.44 includes it as an output. This transmitted diffuse solar radiation is given by:

\[ E_g = \tau_d S_d \quad ...5.61 \]

where:

\[ \tau_d = \text{transmittance of glass to diffuse solar radiation.} \]

For \( \frac{1}{4} \) inch clear plate glass, which is assumed in the model, the transmittance to diffuse solar radiation is 0.7 [Spencer, 1965c]. This constant value can be used as the diffuse solar radiation is assumed to obey Lambert's Law thus neglecting the higher intensities irradiating from around the position of the sun [ASHRAE, 1972a].

For direct solar radiation, the transmitted portion is given by a similar equation:

\[ D_s = \tau_D S_D \quad ...5.62 \]

where:

\[ D_s = \text{direct solar radiative flux transmitted through the glazing} \]
\[ \tau_D = \text{transmittance of glass to direct solar radiation.} \]
The transmittance of glass to direct solar radiation varies with the incident angle of the direct radiation [ASHRAE, 1972a]. The direct solar radiative flux that is transmitted through vertical ¼ inch thick clear plate glass is included in the solar radiation tables for Christchurch [CSIRO, 1966]. The cloudiness scaling ratio for direct solar radiation given by Equation 5.57 is also appropriate for these transmitted direct solar radiation values. This radiation is incident on portions of the floor, the portions varying throughout the day. As the model is based upon isothermal layered walls, this transmitted direct solar radiation is considered to be uniformly incident upon the floor as an extraneous short-wave radiant flux. It is computed from the tabulated values [CSIRO, 1966] by:

\[ D_f A_f = f_D D_s A_g \]  \( \ldots 5.63 \)

where:

- \( D_f \) = extraneous short-wave radiant flux incident on the floor
- \( A_f \) = area of floor
- \( f_D \) = cloudiness scale factor for direct solar radiation
- \( D_s \) = tabulated direct solar radiation intensities transmitted through vertical glass
- \( A_g \) = area of glazing.

The absorptance of glass for short-wave radiation also varies with incident angle. This variation is small for most incident angles, thus a constant absorptance of 0.16 is assumed for both direct and diffuse solar radiation [ASHRAE, 1972a]. Thus from Equation 5.28, the reflectance for diffuse solar radiation is 0.14. For glass other than the ¼ inch thick clear plate glass all the solar-optical properties can be
multiplied by the ratio of the shading coefficients [ASHRAE, 1972a]. No external or internal shading devices are allowed for in the model, but these could also be handled using shading coefficient ratios [ASHRAE, 1972a].

5.11 ROOM RESPONSE

All the previously described convective heat flows are combined in the room heat flow system by computing the net rate of heat input to the room air:

\[ Q_a = Q_{ec} + Q_{hc} + Q_{1c} - Q_{gc} - Q_{inf} - \sum_{\text{all walls}} q_c \]  

...5.64

where:

\( Q_a \) = net rate of heat input to the room air to be used in Equation 5.21
\[ Q_{ec} = \text{convective heat output from the room's heat emitters as given by Equation 5.5} \]

\[ Q_{hc} = \text{convective heat loss from humans as given by Equation 5.8} \]

\[ Q_{lc} = \text{convective heat output from the artificial lighting as given by Equation 5.10} \]

\[ Q_{gc} = \text{convective heat flow from room air to fenestration as given by Equation 5.24} \]

\[ Q_{inf} = \text{rate of heat flow to warm the infiltrating air as given by Equation 5.22} \]

\[ q_c = \text{convective heat flow into an internal surface in the room as given by Equation 5.24.} \]

The radiative heat flows influence the room response by influencing the room's internal surface temperatures, and thus the convective flows and the mean radiant temperature of the room.

5.12 CHAPTER FIVE SUMMARY

Heat flows within a building are of four types: conduction, convection, long wave radiation, and short wave radiation. The interaction of these heat flows at a room's surfaces has been used to synthesise a room thermal response model from mathematical descriptions of all the component heat flows. The dynamic thermal response of the room to its thermal stimuli is simulated by solution of this set of mathematical equations for a sequence of time intervals.
6.1 OVERVIEW

The development of the computer simulation model from the equations of Chapter Five posed some interesting problems. A finite difference representation was chosen as this numerical technique offered the least constraints on the modelling process, which is important for a research model. Finite difference representation of the one dimensional conduction equation can cause instability problems. Specific conditions for stability [Gupta et al, 1971] were used, but the particular boundary conditions arising with the room heat flow model were found to cause computational instability. Thus the details of the finite difference representation of the equations of Chapter Five and the computational stability criterion that was established are outlined in this chapter.

As the modelling of the thermal controls causes an abrupt change in the heat flow rate into the wall behind the emitter, it produces a further instability problem. This difficulty and its solution are also outlined in Chapter Six. The problem of attaining a satisfactory level of assurance for the simulation model is also discussed.
6.2 FINITE DIFFERENCE REPRESENTATION

The equations developed in Chapter Five were mapped to a Fortran computer program based on a finite difference representation with respect to time and also with respect to distance through the wall for conduction through the walls. The operational sequence of this computer program is illustrated in Figure 6.1. The initialisation includes reading of input data, echo output of significant run parameters, and computation of run constants and initial values of temperature and energy levels.

The significant steps in the sequence, which is repeated for each time interval, are: computation of heat flow rates, and computation of new levels of temperature and cumulative energy usage. Temperature levels are required for the heat flow rate computations and heat flow rates are required for the new level computations. This conflict is resolved by the discretisation of time. At each time step the heat flow rates are computed from the current values of temperature levels, which, for the first time step, are the initialised values. Then the time is incremented to the next step and these heat flow rates are used to compute the new levels appropriate to the new time value. The sequence is well represented by the notation developed by Forrester [1971] and illustrated in Figure 6.2. Levels are regarded as applying to a particular time value and rates are regarded as applying between time values. Present levels are denoted by a .K suffix. They are computed from the rates for the past to present period denoted by a .JK suffix. The rates were computed from the values of the levels applicable to the past time value and denoted by a .J suffix. When the time is incremented the present levels become past levels. This notation will be used in the derivation of the finite
FIGURE 6.1: OPERATIONAL SEQUENCE OF SIMULATION MODEL
Past

<table>
<thead>
<tr>
<th>Time Interval</th>
</tr>
</thead>
<tbody>
<tr>
<td>Past-to-Present</td>
</tr>
<tr>
<td>Level J</td>
</tr>
<tr>
<td>Rate J/K</td>
</tr>
<tr>
<td>Present</td>
</tr>
<tr>
<td>Level K</td>
</tr>
</tbody>
</table>

FIGURE 6.2: TIME DISCRETISATION

difference equations in the next section.

The computational sequence within the heat flow rate step is illustrated in Figure 6.3. The model stimuli, which include the external environment and the energy inputs to casual sources, vary with time; thus the values appropriate to the present time step need to be determined. The room's internal surface heat flows are computed and summed to give the net heat flow rate for each surface. Net heat flow rates for the surfaces facing the external environment and the plenum are also computed, then all the surface heat flow rates are used in the conduction finite difference model. Plant heat flow rates are computed assuming no controls and the room air response, which is the net convection response, is computed. Trial new levels for the maximum water temperature for the plant and the ambient temperature are computed. If these trial values are above their respective maximums, adjustments are made to the plant heat flow rates. Finally, the glass thermal response, which is the result of the net heat flow into the glass, is computed.

The finite difference representations of the equations of Chapter Five that have first order differentials with respect to time are derived as follows:
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Model Stimuli Heat Flows

Long Wave Radiation Exchange

Short Wave Radiation Exchange

Convection Heat Flows

Net Heat Flow into Room's Surfaces

External Surface Heat Flows

Plenum Surface Heat Flows

Conduction through Walls

Plant and Room Air Response

Adjustment for Boiler Control

Adjustment for Emitter Control

Glass Thermal Response

FIGURE 6.3: HEAT FLOW RATES COMPUTATION SEQUENCE
Consider the equation:

\[ \frac{dv}{dt} = f \]

where:

\[ v = \text{any variable} \]
\[ t = \text{time} \]
\[ f = \text{a function of any number of variables excluding } v \text{ and } t. \]

Then for small time increments, \( \Delta t \):

\[ \frac{dv}{dt} \approx \frac{\Delta v}{\Delta t} = \frac{v.K - v.J}{\Delta t} \]

Thus

\[ v.K = v.J + f \Delta t \]

Equations 5.1 and 5.21 are modelled on this basis. All non-differential equations are unchanged by the mapping from the mathematical model to the fortran computer model.

6.2.1 Conduction Equation Representation

As the one dimensional conduction equation is the most complex of the equations of Chapter Five, it was the major consideration in the mapping from the mathematical model to the fortran computer model. Numerical discretisation, response factor, and harmonic methods have all been used for this purpose, but only the numerical discretisation methods allow for non-linearity of the boundary conditions. The problems of computational instability and inaccuracies due to lumping errors require special consideration, but otherwise, the finite difference solution to the one dimensional conduction equation posed the least constraints on the modelling process. Thus it was chosen for the research model.
The finite difference model of the one dimensional conduction equation is developed below using Forrester's [1971] time discretisation notation, together with a distance discretisation notation as illustrated in Figure 6.4. The distance discretisation divides the wall into layers of isothermal planar elements. The links between the nodes in Figure 6.4 represent the significance of the nodes on the time and distance derivatives of the temperature of element \(i\). As each planar element may have different material properties from its adjacent elements, combined thermal conductances between the element nodes are required for the model. The notation and the relationship between the internodal properties and the element properties, which are considered to act at the nodes, are illustrated in Figure 6.5. All these properties are independent of time.

The thermal capacity at node \(i\) is:

\[
C(i) = cu(i) \cdot DX(i) \cdot A_w \quad \ldots 6.2
\]

where:

- \(C(i)\) = thermal capacity at node \(i\)
- \(cu(i)\) = thermal capacity per unit volume of material of element \(i\)
- \(DX(i)\) = thickness of element \(i\)
- \(A_w\) = area of wall.

The thermal conductance between internal nodes \(i\) and \(i+1\) is:

\[
K(i) = \frac{2 \cdot A_w}{DX(i) + DX(i+1) \cdot k(i) + k(i+1)} \quad \ldots 6.3
\]
**FIGURE 6.4: DISTANCE-TIME NODAL NOTATION**

**Element Properties:**

<table>
<thead>
<tr>
<th>Distance Representation</th>
<th>i-1</th>
<th>i</th>
<th>i+1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness</td>
<td>DX(i-1)</td>
<td>DX(i)</td>
<td>DX(i+1)</td>
</tr>
<tr>
<td>Conductivity</td>
<td>k(i-1)</td>
<td>k(i)</td>
<td>k(i+1)</td>
</tr>
<tr>
<td>Thermal Capacity per unit volume</td>
<td>cu(i-1)</td>
<td>Cu(i)</td>
<td>Cu(i+1)</td>
</tr>
<tr>
<td>Thermal Capacity</td>
<td>C(i-1)</td>
<td>C(i)</td>
<td>C(i+1)</td>
</tr>
</tbody>
</table>

**Internodal Properties:**

<table>
<thead>
<tr>
<th>Distance Representation</th>
<th>I-1</th>
<th>I</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conductance</td>
<td>K(I-1)</td>
<td>K(I)</td>
</tr>
</tbody>
</table>

**FIGURE 6.5: NOTATION FOR ELEMENT AND INTERNODAL PROPERTIES**
where:

\[ K(I) = \text{thermal conductance between node } i \text{ and node } i+1 \]
\[ k(i) = \text{thermal conductivity of element } i \]

If one of the elements is a surface element, its node is considered to be on the surface rather than at the centre of the element. Thus the total thickness of the surface element must be used for the conductance:

\[ K_s(I) = \frac{\lambda_w}{DX(i) + DX(i+1)} \frac{k(i)}{2 k(i+1)} \]

where:

\[ K_s(I) = \text{thermal conductance between surface node } i \text{ and internal node } i+1 \]

Consider the heat flows with respect to element \( i \) over the time period \( .JK \) as illustrated by the lines linking the nodes of Figure 6.4. The net heat flow into element \( i \) raises its temperature from \( \theta(i).J \) to \( \theta(i).K \). This net heat flow is the result of conducted heat flow between element \( i \) and its two adjacent elements. The temperatures of the nodes are defined at the beginning and the end of the time period \( .JK \), thus proportions of the instantaneous heat flows at these times are used to represent the heat flow during the time period \( .JK \). The resulting heat balance for an internal element is:

\[ \text{Heat stored in element } i = \text{Heat conducted from element } i-1 + \text{Heat conducted from element } i+1 \]

i.e.

\[ C(i)[\theta(i).K-\theta(i).J] = \Delta t K(I-1)\{\beta[\theta(i-1).K-\theta(i).K]+[1-\beta][\theta(i-1).J-\theta(i).J]\} \]
\[ + \Delta t K(I)\{\beta[\theta(i+1).K-\theta(i).K]+[1-\beta][\theta(i+1).J-\theta(i).J]\} \]

\[ ...6.5 \]
where:

\[ \beta = \text{interpolation coefficient} \]

\[ \Delta t = \text{size of time period} \]

The heat balance for a surface element is:

\[
\begin{align*}
\text{Heat stored in element } i &= \text{Heat conducted from element } i+1 + \text{Net heat input to the surface} \\
&= C(i)[\theta(i),.K-\theta(i),.J] = \Delta t K(I)\{\beta[\theta(i+1),.K-\theta(i),.K] + [1-\beta]\theta(i+1),.J-\theta(i),.J]\} \\
&+ \Delta t Q_s,.JK.
\end{align*}
\]

where:

\[ Q_s,.JK = \text{net heat flow rate into the surface during the time period} \]

Rearranging Equation 6.6 gives:

\[
\begin{align*}
\{-\beta K(I) - \frac{C(i)}{\Delta t}\theta(i),.K + \beta K(I) \theta(i+1),.K \} &= \left\{[1-\beta] K(I) - \frac{C(i)}{\Delta t}\theta(i),.J - [1-\beta] K(I) \theta(i+1),.J \right\} \\
&- Q_s,.JK.
\end{align*}
\]

Rearranging Equation 6.5 gives:

\[
\begin{align*}
\beta K(I-1) \theta(i-1),.K - \{\beta [K(I-1)+K(I)] + \frac{C(i)}{\Delta t}\} \theta(i),.K + \beta K(I+1) \theta(i+1),.K \\
&= -[1-\beta] K(I-1) \theta(i-1),.J + \left\{[1-\beta] [K(I-1)+K(I)] - \frac{C(i)}{\Delta t}\right\} \theta(i),.J \\
&\quad - [1-\beta] K(I+1) \theta(i+1),.J.
\end{align*}
\]
Thus in matrix form:

\[ [A] \{ \theta \}_{K} = [B] \{ \theta \}_{J} + \{ Q_{s} \} \] ...

where:

\[ [A] = \text{tridiagonal matrix} \]
\[ [B] = \text{tridiagonal matrix} \]
\[ \{ \theta \}_{K} = \text{vector of element temperatures at time } .K \]
\[ \{ \theta \}_{J} = \text{vector of element temperatures at time } .J \]
\[ \{ Q_{s} \} = \text{vector of net surface input heat flows.} \]

The first row of this matrix equation is:

\[ a_{11} \theta(1).K + a_{12} \theta(2).K = b_{11} \theta(1).J + b_{12} \theta(2).J - Q_{s1} \]

where:

\[ a_{ij} = \text{element in the ith row and jth column of matrix } [A] \]
\[ b_{ij} = \text{element in the ith row and jth column of matrix } [B] \]

Rearranging Equation 6.10 gives:

\[ \theta(1).K = m_{12} \theta(2).K + D(1).JK \]

where:

\[ m_{12} = \frac{-a_{12}}{a_{11}} \]
\[ D(1).JK = \frac{b_{11} \theta(1).J + b_{12} \theta(2).J - Q_{s1}}{a_{11}} \]

and where:

\[ Q_{s1}.JK = \text{net heat flow rate into surface 1 during the time period } .JK. \]

The second row of the matrix equation is:

\[ a_{21} \theta(1).K + a_{22} \theta(2).K + a_{23} \theta(3).K = b_{21} \theta(1).J + b_{22} \theta(2).J + b_{23} \theta(3).J \]

...6.14
Substituting Equation 6.11 in Equation 6.14 and rearranging gives:

\[ \Theta(2)K = m_{23}\Theta(3)K + D(2)JK \quad \ldots 6.15 \]

where:

\[ m_{23} = \frac{a_{23}}{a_{22} + a_{21}m_{12}} \quad \ldots 6.16 \]

\[ D(2)JK = \frac{b_{21}\Theta(1)J + b_{22}\Theta(2)J + b_{23}\Theta(3)J - a_{21}D(1)JK}{a_{22} + a_{21}m_{12}} \quad \ldots 6.17 \]

Thus the general recursion formulae are:

\[ \Theta(i)K = m_{i,i+1}\Theta(i+1)K + D(i)JK \quad \ldots 6.18 \]

where:

\[ m_{i,i+1} = \frac{a_{i,i+1}}{a_{i,i} + a_{i-1,i-1}m_{i-1,i}} \quad \ldots 6.19 \]

\[ D(i)JK = \frac{b_{i,i-1}\Theta(i-1)J + b_{i,i}\Theta(i)J + b_{i+1,i}\Theta(i+1)J - a_{i,i-1}D(i-1)JK}{a_{i,i} + a_{i-1,i-1}m_{i-1,i}} \quad \ldots 6.20 \]

The last row of the matrix equation is:

\[ a_{n,n-1}\Theta(n-1)K + a_{n,n}\Theta(n)K = b_{n,n-1}\Theta(n-1)J + b_{n,n}\Theta(n)J + Q_{sn}JK \quad \ldots 6.21 \]

where:

\[ Q_{sn}JK = \text{net heat flow rate into surface } n \text{ during the time period } JK \]

Substituting from Equation 6.18 for \( \Theta(n-1)K \) and rearranging gives:

\[ \Theta(n)K = \frac{b_{n,n-1}\Theta(n-1)J + b_{n,n}\Theta(n)J + Q_{sn}JK - a_{n,n-1}D(n-1)JK}{a_{n,n} + a_{n,n-1}m_{n-1,n}} \quad \ldots 6.22 \]
If the net heat flows at the wall's surfaces are known, the present temperature of surface element \( n \) can be computed from Equation 6.22 using the past temperatures and the recursion formulae (Equations 6.19 and 6.20). Then back substitution in the temperature recursion formula (Equation 6.18) gives the present temperatures of all the other wall elements. Only those variables with a time suffix need to be computed each time step, the others are computed as program constants from the input wall element properties. This method is used for all the walls for which the net heat flows at the two surfaces are known from other equations in the model.

The wall behind the room's heat emitter is an exception as its internal surface boundary condition is a defined surface temperature which equals the emitter casing temperature. The same method is used with its boundary condition, but starting with the second row of the matrix equation (Equation 6.9). Rearranging Equation 6.14 gives:

\[
\theta(2) \cdot K = m_{23}^* \theta(3) \cdot K + D^*(2) \cdot JK
\]

where:

\[
m_{23}^* = -\frac{a_{23}}{a_{22}}
\]

\[
D^*(2) \cdot JK = \frac{b_{21} \theta(1) \cdot J + b_{22} \theta(2) \cdot J + b_{23} \theta(3) \cdot J - a_{21} \theta(1) \cdot K}{a_{22}}
\]

Thus Equations 6.23, 6.24, and 6.25 replace Equations 6.18, 6.19, and 6.20 respectively for the emitter wall.

The heat flow rate into the emitter wall is computed from Equation 6.26 which is a rearrangement of Equation 6.10:

\[
Q_{\text{back}} = b_{11} \theta(1) \cdot J + b_{12} \theta(2) \cdot J - a_{11} \theta(1) \cdot K - a_{12} \theta(2) \cdot K
\]

where:

\( Q_{\text{back}} \) = heat input to back casing of emitter.
6.2.2 Computational Stability and Accuracy

The size of the discretisation steps influences the computational accuracy and, with discretisation of more than one parameter, incorrect relative step sizes can cause computational instability. This can be recognised as it manifests itself as a pronounced oscillation or divergent values for successive steps [Crank, 1956]. The conduction equation is the only equation in the room heat flow model that can exhibit computational instability. Its discretisation parameters are:

\[ DX(i) = \text{thickness of element } i \]

and \[ \Delta t = \text{size of time step.} \]

The equations outlined in the previous section for an internal element pose no constraints on the sizes of these parameters for their stability if the element properties listed in Figure 6.5 are all positive and the interpolation coefficient defined in Equation 6.5 is equal to 0.5 [Gupta et al., 1971]. These characteristics, which map the method to correspond to Crank-Nicholson's method, were used in the modelling process.

The combination of the equations describing the surface heat flows and the surface element conduction poses stability constraints on the discretisation parameters. The net heat flows into the surfaces of the walls are a combination of the convection, long wave radiation, and short wave radiation as described in Chapter Five. All of these heat flows depend upon the temperature of the surface element in addition to other parameters. Their complex relationships are simplified in the following analysis which provides an approximate stability criterion. The approximate criterion proved to be sufficient for the range of values used in the
simulation study as stable behaviour resulted.

Consider the net surface heat flow rate to be defined by the
combined surface resistance equation:

\[ Q_{sJK} = R A_w (\theta_a J - \theta(1) . J) \] ...6.27

where:

- \( Q_{sJK} \) = net heat flow rate into surface during the time period .JK
- \( R \) = combined surface resistance
- \( A_w \) = area of wall
- \( \theta_a J \) = air temperature at past time step
- \( \theta(1) . J \) = surface element temperature at past time step

Substituting Equation 6.27 in Equation 6.6 and rearranging gives:

\[
\left\{ \frac{C(1)}{\Delta t} + \beta K(1) \right\} \theta(1) . K = \beta K(1) \theta(2) . K \\
+ \left\{ \frac{C(1)}{\Delta t} - [1 - \beta] K(I) - R A_w \right\} \theta(1) . J \\
+ [1 - \beta] K(1) \theta(2) . J + R A_w \theta_a J 
\] ...6.28

If all the time invariant parameters have positive values and
\( \beta = 0.5 \), then the only term that can cause instability is the \( \theta(1) . J \) term:

\[
\frac{C(1)}{\Delta t} - [1 - \beta] K(1) - R A_w \theta(1) . J \\
\frac{C(1)}{\Delta t} + \beta K(1) \theta(1) . J 
\]

A sufficient criterion for stability would be that the coefficient of \( \theta(1) . J \) must not be negative, but this condition is not necessary as the other terms in Equation 6.28 could dampen any oscillatory effect from this term. Although no theoretical basis could be established, experimentation proved the following less restrictive criterion to be sufficient for a wide range of parameter values:
\[ \frac{C(1)}{\Delta t} - [1 - \beta] K(1) - R A_w \geq -1 \]

i.e.

\[ \frac{2 C(1)}{\Delta t} + [2\beta - 1] K(1) - R A_w \geq 0 \]

Substituting from Equations 5.2 and 5.4:

\[ \frac{2 \text{cu}(1) \, DX(1)}{\Delta t} + \frac{[2\beta - 1]}{k(1)} - R \geq 0 \]

If \(\beta = 0.5\), then criterion becomes:

\[ \frac{2 \text{cu}(1) \, DX(1)}{\Delta t} \geq R \]

\[ \therefore \frac{DX(1)}{\Delta t} \geq \frac{R}{2 \text{cu}(1)} \]

This criterion is neither necessary nor sufficient for all values of the parameters, but it proved to be a satisfactory criterion for the values of the parameters used in the simulation study.

The computational accuracy of a discretisation model can be determined by successive runs with variation of the discretisation parameters. Smaller discretisation steps reduce lumping errors, which improves computational accuracy. From experimentation with the room heat flow model it was found that a time step of 0.01 hours met accuracy requirements and was a practical value to work with for program output control. The thickness of wall elements varies with each wall as described in Chapter Seven. The program allows for a maximum of ten elements through a wall, which was found to provide adequate scope for the range of walls modelled.
6.3 MODELLING THE THERMAL CONTROLS

The two controls on the rate of heat flow from the boiler are described in Section 5.2. The restriction of a maximum water temperature was infrequently encountered in the simulation study and when encountered caused no modelling problems as the change to the emitter heat flows were of a gradual nature.

The restriction of a maximum room ambient temperature was modelled as a control on the emitter's convective output to the room air. Any reduction in the room emitter's heat output requires a reduced water temperature, and thus a reduced boiler output. The emitter control process is outlined in Figure 6.6. It is necessary to check that the boiler output does not become negative as this is not possible in the real world prototype. The resulting feedback control model allows the room ambient temperature to temporarily rise above the required temperature. In the simulation study very small rises in the room ambient temperature above the required temperature occurred for small time durations only.

The control model caused some unexpected feedback effects on the heat flow rate into the wall behind the room emitter. Equation 6.26 models the heat flow rate into the emitter wall with the emitter casing temperature used as the surface element temperature. The emitter casing temperature is modelled as a function of water temperature and the room air temperature as defined by Equation 5.6. It was found that with this relationship, a sudden large variation in value of the water temperature produced a disproportionate variation in the heat flow rate into the wall behind the room emitter for the next time step. The model responded to this by varying the water temperature along with other parameters, thus reinforcing an unstable behaviour mechanism, which would not occur in
Will the Ambient Temperature at the next time step be greater than the required temperature?

Yes

Reduce convective heat flow to air to produce the required temperature

Thus reduce emitter output, water temperature and boiler output

Has boiler output become negative?

No

Yes

Set boiler output to zero and adjust water temperature, emitter output and convective heat flow to air

FIGURE 6.6: DETAILS OF EMITTER CONTROL ADJUSTMENT
practice. The problem was overcome by restricting the possible variation in the heat flow rate into the wall by preventing it from becoming negative. This restriction dampened the unstable behaviour mechanism in less time steps than a number of more complex trial controls.

Figure 6.7 illustrates the resulting control model. Although it allows some fluctuations to occur in the heat flow through the wall behind the emitter, which would not occur to the same extent in a real world prototype, it is assumed that the resulting average behaviour is a reasonable representation. In the simulation study described in Chapter Seven, the fluctuations only occurred for small time durations and only when very large and sudden boiler outputs occurred.
6.4 TESTING THE MODEL

The development of the computer simulation model was a two stage process as illustrated by Figure 6.8. The real world prototype room heat flow system is mapped to a set of mathematical equations as the first stage, then this set of mathematical equations is mapped to the computer simulation model as the second stage. The second mapping was checked using an electronic calculator to compute all the component heat flows for a number of time steps. Thus the correctness of the second mapping can be assured with a high level of confidence.

The first mapping is more difficult to test. The task of physically measuring the large number of parameters in a room of a commercial building was so large to be ruled out of the study. However a reasonable level of confidence can be achieved by detailed appraisal of the behaviour of the simulation model using one's qualitative experience of the heat flows in commercial buildings. Plots of the component heat flows with respect to time were output from the model and the time variations and relative magnitudes of all component heat flows were analysed with regard to the causes and the feasibility of occurrence in the real world prototype. The process was repeated for numerous iterations as it also provided an initial check on the second mapping. The most significant development of the mathematical model was the separation of the radiative response from the convective response. An early version of the simulation model was developed with linear combined surface resistances but this model exhibited an unlikely dynamic response that suggested an inadequate mapping. The dynamic response exhibited by the final model correlates well with the expected response of real world prototypes as assessed by the author.
FIGURE 6.8: TWO STAGE MAPPING TO A COMPUTER MODEL
6.5 CHAPTER SIX SUMMARY

The mathematical model described in Chapter Five was implemented as a fortran computer simulation model based on the discretisation of time and for the conduction equation, the combined discretisation of time and distance through the wall. The particular boundary conditions arising from the application of finite difference methods caused a numerical instability to occur even though reported stability criteria were met. A further stability criterion is established for the boundary conditions. The numerical instability initiated by the abrupt change in heat flow rate, due to modelling of a maximum ambient temperature for the room, was overcome by restricting the magnitude of the abrupt change.

Assurance of the mapping of the mathematical model to the computer model was adequately resolved by checking computation results by an alternative process. However, the more difficult assurance problem of how well the real world is represented by the mathematical model relies upon the research that established the mathematical model details and an iterative process of evaluating the simulation response using one's expectation gained from experience of the real world situation.
7.1 INTRODUCTION

7.1.1 Study Context

The major role of the digital computer for building design was identified in Chapter Two as computer aided appraisal both by individual designers and for research into building design methodology. Dynamic thermal simulation models were discussed in Chapter Three. There is no doubt that their sensible use for both analysis of cooling and air conditioning equipment requirements and appraisal of energy usage is advantageous. However, it was also noted that they offer scope for appraisal of the quality of thermal environments resulting from both architectural and special purpose thermal equipment proposals and that the net benefit of their use for heating design has not been established.

Appraisal of the quality of thermal environments and energy flows requires an adequate definition of the requirements for satisfactory thermal conditions. Requirements for satisfactory human thermal environments were established in Chapter Four, together with the formulation of a user cost of thermal deficiency model for measuring the quality of thermal environments in terms of monetary worth.

The user cost model was used as part of the formulation for a differential cost model for appraisal of hydronic heating design proposals. Three relationships are needed to evaluate the differential
cost model:

1. Variation of boiler energy output over the heating season.
2. Variation of the duration of the thermal deficiency, or failure, period that may occur during initial daily occupancy.
3. Variation of ambient temperature during the thermal deficiency period. Ambient temperature is defined as the dry bulb air temperature in an equivalent human thermal environment in which the dry bulb air temperature equals the mean radiant temperature.

As room emitter size and central boiler size are the two design variables pertinent to hydronic heating design alternatives, expressions that can be evaluated for a range of values of these two parameters are required.

The formulation of a computer based model, capable of simulating the dynamic thermal response of typical rooms of commercial buildings that are heated by low pressure hot water convectors, is discussed in Chapters Five and Six. The developed model was used in the simulation study to derive some empirical expressions for the three required relationships for a range of hydronic heating design alternatives for two typical commercial building rooms.

Chapter Seven reports on the simulation modelling and the subsequent evaluation of the differential cost model for hydronic heating design proposals.

7.1.2 Study Purpose

The simulation study was undertaken to evaluate the differential cost for a range of hydronic heating alternatives for typical rooms in commercial buildings as a means of:
Appraising the value of dynamic thermal simulation models for building heating design.

Investigating the detailed use of both the differential cost approach and the user cost of thermal deficiency model.

Appraising the effectiveness of the differential cost approach for:

(i) aiding the selection of the level of detail required for a building design procedure.

(ii) appraisal of design alternatives during the building design process.

Gaining further understanding of the dynamic thermal response that occurs in buildings.

7.1.3 Study Scope

Thermal conditions pertinent to human comfort and performance were investigated for two commercial buildings recently constructed in Christchurch, New Zealand. Commercial buildings were chosen as they are a significant proportion of professionally designed buildings and exhibit reasonably well defined thermal environments. They are usually designed to meet the thermal requirements of their human occupants as any other thermal considerations are generally less restrictive. Christchurch weather data was used for the study because of its large diurnal variation of air temperature.

The dynamic response of a single room in each of the buildings to the time-varying heat flows resulting from external weather, internal heat sources, and specialist thermal equipment was simulated using the computer based model detailed in Chapters Five and Six. Ventilating external air was assumed to gain any heat in the room itself. Conduction
convection, long wave radiation, short wave radiation, and thermal storage effects are all included. Rectangular rooms with a single wall with a south exposure were modelled as this orientation produces the greatest demand for heating. Corner rooms were not studied as the results from the rooms with a single wall exposure proved to be sufficient to meet the study aims.

A range of alternative hydronic heating designs were studied by varying the room emitter heat output capacity and the central boiler heat production capacity. Expressions for the three relationships discussed in Section 7.1.1 were derived for each design alternative for a typical room of each of the two buildings: one with relatively small heat requirements, and one with relatively large heat requirements. These expressions were used to evaluate the differential cost model components for each of the hydronic heating design alternatives.

Section 7.2 outlines the formulation for the study. Input data for the simulation model to describe the particular weather, rooms, and range of heating systems is presented. A characteristic structure of the daily intermittent heating cycle became apparent in the study. It is described in Section 7.3. Two dimensionless parameters were derived to aid analysis of the simulation run results. They are defined in Section 7.4. The results in terms of the dimensionless parameters are presented in Section 7.5. Derivation of the expressions for the relationships discussed in Section 7.1.1 and subsequent evaluation of the differential cost model components are presented in Section 7.6. Finally, Section 7.7 restates the major assumptions used for the study, discusses the results, and presents the conclusions drawn from the simulation study.
7.2 FORMULATION

7.2.1 Weather Description

The present state of knowledge of climatic processes is inadequate for precise prediction of future weather. However, extensive records for many locations have been kept. These records have two modes of use for simulation modelling:

(1) The recorded sequences are used directly by the model.

(2) The recorded sequences are analysed and derived statistical parameters that describe the broad characteristics of the data are used as the basis of the simulation modelling.

The first mode of use has become common for building thermal simulation modelling, but it requires simulation of a long time period, such as a year, to provide reasonable results for energy consumption and then the results are only strictly applicable to the past time period simulated. Methods based on the second mode of use are an obvious extension as they will presumably reduce the extent of the simulation duration required.

The first mode of use of the climatic records was inappropriate to the present simulation study because of the excessive computer time that would have been required with the small time increment of the model. Thus a second mode method was developed for the external air temperature based on the monthly mean and standard deviation of hourly recorded values. Both diurnal variation and seasonal variation of external air temperature were pertinent to the study. In particular, the diurnal variation near the coolest seasonal extreme was of interest as the maximum plant capacity is determined from estimates of such extremes.
As monthly means and standard deviations of hourly external air temperatures for Christchurch over a ten year period were available, six diurnal profiles were derived from this data and used for the study. As overnight cooling has a significant influence on the dynamic response of the building, a 36 hour period was chosen for each profile. The first 12 hours of the period are a transition from the standard initial conditions to initial conditions more appropriate to the particular temperature profile. All measurements were taken for the last 24 hours of these profile periods. The profiles, which are illustrated in Figure 7.1, proved to be an adequate set of standardised weather data for meeting the study objectives.

July is the coldest month in the Southern Hemisphere, so Profile JM, the July means of the hourly external air temperature, was used to determine the initial conditions for all other runs. Each room was simulated with this profile to determine a steady periodic state. A six day simulation proved to be necessary to achieve it. Profiles JM1 and JM2, the July means minus the standard deviation for each hour and the July means minus twice the standard deviation respectively, were used as more extreme climatic sequences. Profile SM, the heating season means for each hour, and the two warmer profiles were established to provide a basis for investigating seasonal variations. As each run used considerable computing resources, only the six external air temperature profiles illustrated in Figure 7.1 were used for the study.

Hourly values of solar radiation intensities for Christchurch on a clear day have been tabulated [CSIRO, 1966]. These tabulated values, in combination with recorded values on a horizontal surface, provided the solar radiation data required by the model. Analysis of the recorded solar radiation intensities on a horizontal surface showed that zero
FIGURE 7.1: EXTERNAL AIR TEMPERATURE PROFILES USED IN STUDY
Intensity was a reasonably common occurrence, thus solar radiation was excluded from the runs with the JM2 profile. Its inclusion with the other profiles added another dimension of complexity to the model behaviour. As the significance of the dynamic influence on the fuel and user costs could be assessed at this stage, the remainder of the study was completed excluding the effect of solar radiation.

7.2.2 Room Descriptions

Two room constructions were modelled in the simulation study, each with a range of heating plant sizes. The construction characteristics for Room 1 were derived from the plans of the Christchurch Central Police Station and those for Room 2 from the University of Canterbury Library Arts Building. These buildings were chosen because they are typical of present construction forms for commercial buildings and the contract drawings were available to the author. The geometric layouts of the two rooms are illustrated in Figures 7.2 and 7.3. Data requirements of the model and some of the values are listed in Appendix B. Values for each of the walls and surfaces are listed in Tables 7.1 to 7.8.

A typical size for a multistorey office of 11 feet by 16 feet with a height of 10 feet [Loudon, 1968] was used for both rooms. Different sides were modelled as the external boundaries of the two intermediate storey offices to produce rooms with differing heat requirements from their thermal plant. A number of the sides of the rooms have more than one type of construction as shown in Figures 7.2 and 7.3 and listed in Tables 7.1 and 7.2. The external side for both buildings has a distinct construction type behind the room emitters. This section of the external wall was modelled as the heat emitter wall. Above it adjacent vertical panels of glass and opaque fabric were repeated across the facades of the two buildings.
Ceiling (156ft²) and
Artificial Lighting (20ft²)

External Wall (44ft²)
and Windows (33ft²)

Back with studs
(22ft²) and
Back without studs
(88ft²)

FIGURE 7.2:  GEOMETRIC LAYOUT OF ROOM 1
Ceiling (156 ft²) and
Artificial Lighting (20 ft²)

External Wall (48 ft²)
and Windows (48 ft²)

Back (160 ft²)

Floor (176 ft²)

Combined Sides (220 ft²)

Heat Emitter Wall
(64 ft²)

Plus Humans' Surface (30 ft²)

**Figure 7.3:** Geometric Layout of Room 2
<table>
<thead>
<tr>
<th>Material</th>
<th>Conductivity (Btu-in/ft²/hr/°F)</th>
<th>Thermal Capacity per Unit Volume (Btu/ft²/in/°F)</th>
<th>Element Thicknesses (inches)</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Heat Emitter Wall</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steel</td>
<td>310.</td>
<td>4.87</td>
<td>.02</td>
</tr>
<tr>
<td>Asbestos wallboard</td>
<td>.29</td>
<td>.805</td>
<td>.05, .20, .25</td>
</tr>
<tr>
<td>Sisalation</td>
<td>.90</td>
<td>1.55</td>
<td>.01</td>
</tr>
<tr>
<td>Insulation</td>
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<td>.028</td>
<td>1.25, 1.25</td>
</tr>
<tr>
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<td>.01</td>
</tr>
<tr>
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<td>1.96</td>
<td>.25, .20, .05</td>
</tr>
<tr>
<td><strong>External Wall</strong></td>
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<td></td>
<td></td>
</tr>
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<td>0.77</td>
<td>.05, .20, .25</td>
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<td>1.55</td>
<td>.01</td>
</tr>
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<td>.0015</td>
<td>4.5</td>
</tr>
<tr>
<td>Building paper</td>
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<td>1.55</td>
<td>.01</td>
</tr>
<tr>
<td>Timber (Pine)</td>
<td>.80</td>
<td>1.79</td>
<td>.7, .7, .3</td>
</tr>
<tr>
<td><strong>Side Wall Without Studs &amp; Back Wall Without Studs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plasterboard</td>
<td>1.40</td>
<td>.77</td>
<td>.05, .15, .15, .15</td>
</tr>
<tr>
<td>Air Gap</td>
<td>2.06</td>
<td>.0015</td>
<td>2.0</td>
</tr>
<tr>
<td><strong>Side Wall With Studs &amp; Back Wall With Studs</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plasterboard</td>
<td>1.40</td>
<td>.77</td>
<td>.05, .15, .15, .15</td>
</tr>
<tr>
<td>Timber</td>
<td>.80</td>
<td>1.79</td>
<td>4 @ .5</td>
</tr>
<tr>
<td><strong>Ceiling</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fibreboard</td>
<td>.34</td>
<td>.48</td>
<td>.05, .1, .1, .1, .1, .05</td>
</tr>
<tr>
<td><strong>Floor</strong></td>
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<td></td>
</tr>
<tr>
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<td>3.5</td>
<td>.025, .1</td>
</tr>
<tr>
<td>Concrete</td>
<td>12.0</td>
<td>2.3</td>
<td>6 @ 1.0</td>
</tr>
</tbody>
</table>

**TABLE 7.1: WALL ELEMENT PARAMETERS FOR ROOM 1**
<table>
<thead>
<tr>
<th>Material</th>
<th>Conductivity (Btu-in/ft²/hr/°F)</th>
<th>Thermal Capacity per Unit Volume (Btu/ft²/in/°F)</th>
<th>Element Thicknesses (inches)</th>
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</thead>
<tbody>
<tr>
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</tr>
<tr>
<td>Asbestos wallboard</td>
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<td>.805</td>
<td>.1, .2, .2</td>
</tr>
<tr>
<td>Insulation</td>
<td>.28</td>
<td>.028</td>
<td>3 @ .5</td>
</tr>
<tr>
<td>Concrete</td>
<td>12.0</td>
<td>2.3</td>
<td>4 @ 1.5</td>
</tr>
<tr>
<td><strong>External Wall</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Concrete</td>
<td>12.0</td>
<td>2.3</td>
<td>10 @ .6</td>
</tr>
<tr>
<td><strong>Side Wall &amp; Back Wall</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Plasterboard</td>
<td>1.4</td>
<td>.77</td>
<td>4 @ .125</td>
</tr>
<tr>
<td>Air Gap</td>
<td>4.6</td>
<td>.0015</td>
<td>1.0</td>
</tr>
<tr>
<td>Plasterboard</td>
<td>1.4</td>
<td>.77</td>
<td>4 @ .125</td>
</tr>
<tr>
<td><strong>Ceiling</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fibrous plaster</td>
<td>1.4</td>
<td>.77</td>
<td>.05, .1, .1, .1, .1, .05</td>
</tr>
<tr>
<td><strong>Floor</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Carpet</td>
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<td>1.08</td>
<td>2 @ .125</td>
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<td>Underfelt</td>
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<td>.55</td>
<td>.25</td>
</tr>
<tr>
<td>Concrete</td>
<td>12.0</td>
<td>2.3</td>
<td>5 @ 1.0</td>
</tr>
</tbody>
</table>

**TABLE 7.2: WALL ELEMENT PARAMETERS FOR ROOM 2**
The windows and the opaque panels were modelled as separate walls, but the computation of their radiation configuration factors, as defined in Equation 5.29 in Section 5.8.2, required special attention. As these two walls are actually a group of smaller walls distributed over the same combined boundary, their configuration factors were computed as a proportion, based on their area, of the configuration factor for the combined boundary. The ceiling and the artificial lighting had their configuration factors similarly determined. In Room 1 the internal sides of the room were modelled as two construction types: wall panelling separated by studs, and wall panelling separated by an air gap. The configuration factors for these two constructions were also derived using area proportions. The surface parameters for all the walls are listed in Tables 7.3 and 7.4.

As both the rooms are symmetrical about a central vertical plane perpendicular to their external side, the radiation configuration factors for two internal sides are identical. Thus the internal sides can be combined for modelling purposes. The side of the room opposite the external wall is called the back of the room for ease of reference. Two human occupants are modelled with their combined surface area at the centre of the room. Two sets of radiation configuration, as listed in Tables 7.5 and 7.6 for Room 1 and Tables 7.7 and 7.8 for Room 2, are required to simulate long wave radiation exchange as the presence, or absence, of human occupants in a room changes the number of surfaces in the room.

7.2.3 Range of Heating Systems

Six combinations of room emitter coefficient and boiler capacity for each of the rooms were chosen to cover a range of typical installation combinations. They are listed in Table 7.9 together with the general
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Heat Emitter Wall External Surface Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conduction coefficient</td>
<td>Btu/hr/ft²/°F</td>
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<td>Emittance for long wave radiation</td>
<td>-</td>
<td>.96</td>
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<tr>
<td>Absorptance to solar radiation</td>
<td>-</td>
<td>.70</td>
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<tr>
<td><strong>External Wall External Surface Parameters</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Conduction coefficient</td>
<td>Btu/hr/ft²/°F</td>
<td>6.1</td>
</tr>
<tr>
<td>Emittance for long wave radiation</td>
<td>-</td>
<td>.90</td>
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<tr>
<td>Absorptance to solar radiation</td>
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<td>.50</td>
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<tr>
<td><strong>Plenum Surface Parameter</strong></td>
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</tr>
<tr>
<td>Surface conductance</td>
<td>Btu/hr/ft²/°F</td>
<td>1.63</td>
</tr>
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</table>

**Table 7.3**: Non-Internal Surface Parameters for Rooms 1 & 2
<table>
<thead>
<tr>
<th>Surface</th>
<th>Area (ft²)</th>
<th>Short Wave Radiation</th>
<th>Long Wave Radiation Emittance</th>
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</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Absorptance</td>
<td>Reflectance</td>
</tr>
<tr>
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<td>33</td>
<td>.667</td>
<td>.6</td>
</tr>
<tr>
<td>External wall</td>
<td>44</td>
<td>.667</td>
<td>.6</td>
</tr>
<tr>
<td>Side wall without studs</td>
<td>256</td>
<td>.667</td>
<td>.6</td>
</tr>
<tr>
<td>Side wall with studs</td>
<td>64</td>
<td>.667</td>
<td>.6</td>
</tr>
<tr>
<td>Back wall without studs</td>
<td>88</td>
<td>.667</td>
<td>.6</td>
</tr>
<tr>
<td>Back wall with studs</td>
<td>22</td>
<td>.667</td>
<td>.6</td>
</tr>
<tr>
<td>Ceiling</td>
<td>156</td>
<td>.667</td>
<td>.6</td>
</tr>
<tr>
<td>Floor</td>
<td>176</td>
<td>.667</td>
<td>.6</td>
</tr>
<tr>
<td>Artificial lighting</td>
<td>20</td>
<td>0.</td>
<td>1.0</td>
</tr>
<tr>
<td>Windows</td>
<td>33</td>
<td>1.143</td>
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<td>.2</td>
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**Table 7.4: Internal Surface Parameters for Rooms 1 & 2**
<table>
<thead>
<tr>
<th>Surface Description</th>
<th>Surface Index</th>
<th>HEW</th>
<th>Ext</th>
<th>SW</th>
<th>SS</th>
<th>BW</th>
<th>BS</th>
<th>Ceil</th>
<th>Fl</th>
<th>Lit</th>
<th>Win</th>
<th>Hum</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat emitter wall</td>
<td>HEW</td>
<td>0.</td>
<td>0.</td>
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<td>.077</td>
<td>.080</td>
<td>.020</td>
<td>.108</td>
<td>.365</td>
<td>.013</td>
<td>0.</td>
<td>.029</td>
</tr>
<tr>
<td>External Wall</td>
<td>Ext</td>
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<td>0.</td>
<td>.343</td>
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<td>.084</td>
<td>.021</td>
<td>.244</td>
<td>.170</td>
<td>.031</td>
<td>0.</td>
<td>.022</td>
</tr>
<tr>
<td>Side wall without studs</td>
<td>SW</td>
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<td>.058</td>
<td>.177</td>
<td>.045</td>
<td>.114</td>
<td>.028</td>
<td>.204</td>
<td>.230</td>
<td>.026</td>
<td>.044</td>
<td>.034</td>
</tr>
<tr>
<td>Side wall with studs</td>
<td>SS</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Back wall without studs</td>
<td>BW</td>
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<td>.042</td>
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<td>.083</td>
<td>0.</td>
<td>0.</td>
<td>.202</td>
<td>.228</td>
<td>.025</td>
<td>.032</td>
<td>.024</td>
</tr>
<tr>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ceiling</td>
<td>Ceil</td>
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<td>.069</td>
<td>.339</td>
<td>.084</td>
<td>.114</td>
<td>.028</td>
<td>0.</td>
<td>.271</td>
<td>0.</td>
<td>.051</td>
<td>.021</td>
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<tr>
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<td>Fl</td>
<td>.067</td>
<td>.039</td>
<td>.326</td>
<td>.081</td>
<td>.110</td>
<td>.027</td>
<td>.233</td>
<td>0.</td>
<td>.029</td>
<td>.030</td>
<td>.058</td>
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<tr>
<td>Artificial Lighting</td>
<td>Lit</td>
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<td>.069</td>
<td>.339</td>
<td>.084</td>
<td>.114</td>
<td>.028</td>
<td>0.</td>
<td>.271</td>
<td>0.</td>
<td>.051</td>
<td>.021</td>
</tr>
<tr>
<td>Windows</td>
<td>Win</td>
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<td>0.</td>
<td>.343</td>
<td>.085</td>
<td>.084</td>
<td>.021</td>
<td>.244</td>
<td>.170</td>
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<td>0.</td>
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<td>.033</td>
<td>.290</td>
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<td>.071</td>
<td>.018</td>
<td>.106</td>
<td>.339</td>
<td>.014</td>
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**TABLE 7.5: RADIATION CONFIGURATION FACTORS FOR ROOM 1 WITH HUMAN OCCUPANTS**
<table>
<thead>
<tr>
<th>Surface Description</th>
<th>Surface Index</th>
<th>HEW</th>
<th>Ext</th>
<th>SW</th>
<th>SS</th>
<th>BW</th>
<th>BS</th>
<th>Ceil</th>
<th>Fl</th>
<th>Lit</th>
<th>Win</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat emitter wall</td>
<td>HEW</td>
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<td>0.</td>
<td>0.317</td>
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<td>0.084</td>
<td>0.021</td>
<td>0.113</td>
<td></td>
<td>0.14</td>
<td>0.</td>
</tr>
<tr>
<td>External wall</td>
<td>Ext</td>
<td>0.</td>
<td>0.</td>
<td>0.349</td>
<td>0.087</td>
<td>0.087</td>
<td>0.022</td>
<td>0.248</td>
<td>0.175</td>
<td>0.032</td>
<td>0.</td>
</tr>
<tr>
<td>Side wall without studs</td>
<td>SW (SS)</td>
<td>0.041</td>
<td>0.050</td>
<td>0.187</td>
<td>0.047</td>
<td>0.117</td>
<td>0.029</td>
<td>0.210</td>
<td>0.237</td>
<td>0.027</td>
<td>0.045</td>
</tr>
<tr>
<td>Side wall with studs</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Back wall without studs</td>
<td>BW (BS)</td>
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<td>0.207</td>
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<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Ceiling</td>
<td>Ceil</td>
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<td>0.070</td>
<td>0.346</td>
<td>0.086</td>
<td>0.117</td>
<td>0.029</td>
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<td>0.276</td>
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<tr>
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<td>Fl</td>
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<td>0.346</td>
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<td>0.245</td>
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<tr>
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<td>0.070</td>
<td>0.346</td>
<td>0.086</td>
<td>0.117</td>
<td>0.029</td>
<td>0.</td>
<td>0.276</td>
<td>0.</td>
<td>0.052</td>
</tr>
<tr>
<td>Windows</td>
<td>Win</td>
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<td>0.</td>
<td>0.349</td>
<td>0.087</td>
<td>0.087</td>
<td>0.022</td>
<td>0.248</td>
<td>0.175</td>
<td>0.032</td>
<td>0.</td>
</tr>
</tbody>
</table>

**TABLE 7.6: RADIATION CONFIGURATION FACTORS FOR ROOM 1 WITHOUT HUMAN OCCUPANTS**
<table>
<thead>
<tr>
<th>Surface Description</th>
<th>Surface Index</th>
<th>HEW</th>
<th>Ext</th>
<th>Side</th>
<th>Back</th>
<th>Ceil</th>
<th>Fl</th>
<th>Lit</th>
<th>Win</th>
<th>Hum</th>
</tr>
</thead>
<tbody>
<tr>
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<td>.343</td>
<td>.014</td>
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<td>.042</td>
</tr>
<tr>
<td>External wall</td>
<td>Ext</td>
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<td>.028</td>
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<td>.105</td>
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</tr>
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<td>Back</td>
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<td>.083</td>
<td>.020</td>
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<td>Fl</td>
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<td>.274</td>
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<td>Win</td>
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<td>0.</td>
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<td>.228</td>
<td>.267</td>
<td>.157</td>
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<td>0.</td>
<td>.028</td>
</tr>
<tr>
<td>Human Occupants</td>
<td>Hum</td>
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<td>.045</td>
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<td>.335</td>
<td>.014</td>
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**TABLE 7.7: RADIATION CONFIGURATION FACTORS FOR ROOM 2 WITH HUMAN OCCUPANTS**
<table>
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<th>Surface Description</th>
<th>Surface Index</th>
<th>HEW</th>
<th>Ext</th>
<th>Side</th>
<th>Back</th>
<th>Ceil</th>
<th>Fl</th>
<th>Lit</th>
<th>Win</th>
</tr>
</thead>
<tbody>
<tr>
<td>Heat emitter wall</td>
<td>HEW</td>
<td>0.</td>
<td>0.</td>
<td>.281</td>
<td>.227</td>
<td>.119</td>
<td>.358</td>
<td>.015</td>
<td>0.</td>
</tr>
<tr>
<td>External wall</td>
<td>Ext</td>
<td>0.</td>
<td>0.</td>
<td>.294</td>
<td>.235</td>
<td>.275</td>
<td>.161</td>
<td>.035</td>
<td>0.</td>
</tr>
<tr>
<td>Side wall</td>
<td>Side</td>
<td>.083</td>
<td>.064</td>
<td>.108</td>
<td>.213</td>
<td>.208</td>
<td>.233</td>
<td>.027</td>
<td>.064</td>
</tr>
<tr>
<td>Back wall</td>
<td>Back</td>
<td>.091</td>
<td>.071</td>
<td>.292</td>
<td>.210</td>
<td>.238</td>
<td>.027</td>
<td>.071</td>
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<tr>
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<td>.085</td>
<td>.292</td>
<td>.215</td>
<td>.274</td>
<td></td>
<td></td>
<td>.085</td>
</tr>
<tr>
<td>Floor</td>
<td>Fl</td>
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<td>.044</td>
<td>.291</td>
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<td>.244</td>
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<td>.044</td>
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</tr>
<tr>
<td>Artificial Lighting</td>
<td>Lit</td>
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<td>.085</td>
<td>.295</td>
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<td>.274</td>
<td>.031</td>
<td>.044</td>
<td></td>
</tr>
<tr>
<td>Windows</td>
<td>Win</td>
<td>0.</td>
<td>0.</td>
<td>.294</td>
<td>.235</td>
<td>.275</td>
<td>.161</td>
<td>.035</td>
<td>0.</td>
</tr>
</tbody>
</table>

**TABLE 7.8:** RADIATION CONFIGURATION FACTORS FOR ROOM 2 WITHOUT HUMAN OCCUPANTS
<table>
<thead>
<tr>
<th>Room</th>
<th>Emitter Coefficient</th>
<th>Boiler Capacity</th>
<th>Emitter Ratio</th>
<th>Boiler Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room 1:</td>
<td>8.68</td>
<td>3968.</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>8.68</td>
<td>5158.</td>
<td>1.00</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>8.00</td>
<td>3657.</td>
<td>0.92</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>8.00</td>
<td>3885.</td>
<td>0.92</td>
<td>1.062</td>
</tr>
<tr>
<td></td>
<td>9.00</td>
<td>4114.</td>
<td>1.037</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>9.00</td>
<td>4388.</td>
<td>1.037</td>
<td>1.067</td>
</tr>
<tr>
<td>Room 2:</td>
<td>13.16</td>
<td>6015.</td>
<td>1.00</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>13.16</td>
<td>7820.</td>
<td>1.00</td>
<td>1.30</td>
</tr>
<tr>
<td></td>
<td>12.00</td>
<td>5485.</td>
<td>0.91</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>12.00</td>
<td>6399.</td>
<td>0.91</td>
<td>1.167</td>
</tr>
<tr>
<td></td>
<td>14.00</td>
<td>6399.</td>
<td>1.064</td>
<td>1.00</td>
</tr>
<tr>
<td></td>
<td>14.00</td>
<td>5485.</td>
<td>1.064</td>
<td>0.86</td>
</tr>
</tbody>
</table>

TABLE 7.9: RANGE OF HEATING SYSTEMS STUDIED
7.3 THE INTERMITTENT HEATING CYCLE

Intermittent operation of thermal plant attempts to produce a satisfactory thermal environment when the building is occupied and save fuel costs by shutting down the thermal plant when the building becomes vacant. The plant is started up prior to occupancy to produce satisfactory initial occupancy conditions. Sometimes plant shut down is timed to occur just prior to the end of the occupancy period to save additional fuel at the expense of a slight reduction in environmental quality for this short period.

The simulation study was based on a daily intermittent heating cycle. As a building's thermal stimuli are variable from day to day, a standardised daily cycle was established for the study. Human occupancy was assumed to begin at 8 a.m. and end at 5 p.m. The heat output from both human occupants and artificial lighting was assumed to be constant during occupancy and nil during vacancy as illustrated in Figure 7.4. Constant ventilation rates were also assumed during both occupancy and vacancy, but a lower rate is assumed during vacancy to take account of closed windows and external doors. As discussed in Section 7.2.1, a steady period state was established for the initial conditions of the rooms.

From experimentation with the model, five distinct periods were distinguished in the daily intermittent heating cycle as illustrated in Figure 7.5. Note that the thermal plant is assumed to shut down at the
FIGURE 7.4: ROOM DAILY OPERATIONAL CYCLE ASSUMPTIONS
Figure 7.5: The Intermittent Heating Cycle
end of occupancy, so no occupancy terminal cooling is modelled. The remainder of Section 7.3 discusses each of the five periods of the intermittent heating cycle in turn.

7.3.1 Preheat Period

The preheat period begins when the heating plant is started up and finishes when the required ambient temperature is attained, or human occupancy begins, whichever occurs first. It is preceded by the over-night cooling period and followed by the pre-occupancy maintenance period if the required ambient temperature is attained before the start of human occupancy. Otherwise it is followed by the initial occupancy deficiency period. For the purposes of the study, the start of the preheat period was iteratively chosen for each room, thermal plant, and weather combination so that the required ambient temperature was just attained at the start of occupancy. Thus an ideal thermal plant start up control system was assumed.

Before the thermal plant is started up, all components of the building have attained a state of slow cooling. The wall behind the heat emitter, the external wall, and the windows are all transferring heat from the room to the external environment. The other surfaces in the room are all transferring heat to the room air by convection and to the cooler surfaces of the external boundary walls by long wave radiation. The air temperature in the room is lower than the mean radiant temperature because of relatively high ventilation heat losses.

After the thermal plant is started up, heat from the boiler raises the temperature of the circulating water, thus increasing the total heat output from the room's emitters. Radiant heat emission from the emitter surface increases, as does the heat flow into the wall behind the
emitter. The emitter convective output to the room air should also increase, but because of the unstable behaviour mechanism of the model, described in Section 6.3, the simulated room air temperature suffers a small decrease before increasing as expected. This small air temperature decrease occurs over a very small time duration, thus is considered an acceptable simulation error. The subsequent increase in the room's air temperature is relatively rapid due to its low thermal capacity.

The increasing radiative and convective emission from the room's heat emitters reduces the heat flow from the room surfaces of the internal walls, floor, and ceiling until the direction of net heat flow at these surfaces is reversed. The elapsed time to this net heat flow reversal varies for each wall and is particularly dependent upon the thermal capacity of the wall construction. Thus thermally light walls undergo a rapid heat reversal, but thermally heavy walls may continue to transfer heat to the room air and other room surfaces beyond the preheat period. The heat flow through the external boundary walls and windows is increased during the preheat period due to the increasing room air temperature and surface temperatures of the internal walls, floor, and ceiling.

7.3.2 Pre-occupancy Thermal Maintenance Period

The pre-occupancy thermal maintenance period only occurs if the required ambient temperature is attained in the room before the start of occupancy. During this period the heat from the room's heat emitters is transferred to the room's internal surfaces by a combination of radiation exchange and convection. Heat input to the room's internal surfaces increases the surface temperatures, thus causing heat to be
conducted into the fabric of the walls to increase the quantity of heat stored by them. For the external boundaries, the additional heat is eventually transferred to the external environment, the time lag involved depending upon the thermal response characteristics of the wall and the external environmental stimuli on the boundaries' external surfaces.

The rise in temperature of the room's internal surfaces increases the mean radiant temperature in the room. As the ambient temperature is assumed to be the control on the heat output from the room's emitters, and thus on the boiler output, when the required ambient temperature is attained, an increase in mean radiant temperature results in a decrease in the room's air temperature so that the ambient temperature remains constant and equal to the required temperature.

The thermal response of the room during the initial occupancy deficiency period particularly, but also during the subsequent periods in the intermittent heating cycle, is influenced by the duration of the pre-occupancy thermal maintenance period. Its influence is due to the heat storage in the fabric of the room's walls that occurs during it. Hence it was necessary to establish a consistent basis for determining the time of thermal plant start up and the duration of the thermal maintenance period. A zero duration thermal maintenance period was chosen as it minimises the preheat period and eliminated the minor fluctuations caused by the modelling of the constant ambient temperature control as described in Section 6.3. The minimum preheat period may not be the best solution as a longer preheat period reduces the initial occupancy deficiency period. It also increases the fuel consumption, so the optimum would balance these two factors. However, to achieve an optimum, the measure of the cost of thermal deficiency is required,
which is pursued in the present study. In addition to lack of information, there is the fact that practical control systems only approximate ideal controls. Thus the choice of modelling a minimum preheat period that could produce the required ambient temperature at the start of occupancy is a good first approximation to the optimum and provides a consistent basis for the simulation study. To achieve this condition, it was necessary to iterate over a number of simulation runs for each room, thermal plant, and weather combination.

7.3.3 Initial Occupancy Deficiency Period

At the start of occupancy the ventilation rate changes from the vacant ventilation rate to the heating ventilation rate as illustrated in Figure 7.4. This change, which models the effect of opening doors and windows by the human occupants, is assumed to occur instantaneously. A sudden reduction in the room air temperature, which reduces the ambient temperature, results from the increased ventilation rate. The thermal plant control responds to the lower ambient temperature by increasing the rate of heat output from the boiler to the circulating water. The resulting increased circulating water temperature, together with the decreased room air temperature, produces a larger convective output from the room's emitters. Increased convective output, together with changes to the convective heat flows at the room's internal surfaces due to the reduced room air temperature, offsets the increased ventilation heat load to eventually increase the room air temperature and return the ambient temperature to the required temperature. A return to the required temperature defines the end of the initial occupancy deficiency period.
As the circulating water has thermal capacity it undergoes an increase in temperature as a gradual process. Thus both the convective and radiative output from the room's heat emitters gradually increase. A relatively sudden rise in mean radiant temperature of the room occurs at the start of occupancy due to higher temperature surfaces radiating more heat. Both artificial lighting and human occupants are modelled as instantaneous additional heat sources. The room emitter surface increases rapidly at first but reduces its rate of increase as both radiative and convective output increase.

Thermal lag in the heating plant response meant all simulation runs exhibited an initial occupancy deficiency. However, the duration of the deficiency period varies with each room, thermal plant, and weather combination. For the SMI weather profile, illustrated in Figure 7.1, the duration of the deficiency period was so small, it was regarded as approximating an instantaneous thermal plant response. For Room 1 simulation runs with the SMJ weather profile a similar zero deficiency period can be assumed. All other simulation runs exhibited an initial occupancy deficiency period of significant duration.

Two characteristics of the initial occupancy deficiency period were of primary interest in this study:

(1) Variation of the duration of the thermal deficiency period with the study variables and, in particular, with the weather profiles.

(2) Variation of the ambient temperature during the thermal deficiency period.

The investigations into these characteristics are described in Sections 7.5 and 7.6.
7.3.4 Thermal Adequacy Period

After the ambient temperature has returned to the required temperature, following the increased ventilation rate, the modelled room thermal controls maintain an average ambient temperature equal to the required temperature. As the control operates the rate of heat output from the boiler, the thermal lag in the system, together with the dampened unstable behaviour mechanism described in Section 6.3, causes a small oscillatory variation in ambient temperature through the thermal adequacy period.

During this period the rate of heat flow to the room surfaces reduces as the surfaces gradually increase their temperatures. As the external air temperature is in the higher section of its daily profile, the heat loss through the glass and the ventilation heat loss both decrease as they are nil time lag heat flows. These decreased heat flows from the room reduce the heat output required from the room heat emitters, and thus from the boiler. The thermal adequacy period finishes when human occupancy is assumed to end, and the thermal plant is assumed to be shut down at this time.

7.3.5 Overnight Cooling Period

When the heating plant is shut down, no further heat is input to the circulating water, so it gradually cools towards the room air temperature. The room air temperature itself, cools due to heat flow through the external boundary of the room and due to infiltration heat loss. Infiltration heat loss is reduced from the occupancy ventilation heat loss as windows and doors are assumed closed. The change in ventilation rate is assumed to be sudden as illustrated in Figure 7.4.

No heat is input to the room from the artificial lighting or human occupants during the overnight cooling period. Thus the room's internal
surfaces gradually reduce their temperatures with heat being convected to the cooler room air and radiated to the cooler external boundary surfaces. The extent of the cooling of the fabric of the room influences the heat required to produce the required ambient temperature the following day. Two heat loss mechanisms are significant during this period: heat loss through the glazing, and infiltration heat loss. Thus designs that minimise these overnight cooling heat losses reduce the energy consumption required the next day.

7.4 DERIVATION OF DIMENSIONLESS PARAMETERS

The hydronic heating system variables of room emitter size and boiler capacity were converted to somewhat arbitrarily defined dimensionless parameters, which are more meaningful numbers for analysis of the results of the simulation runs. These dimensionless parameters are called the emitter ratio and the boiler ratio.

7.4.1 Boiler Ratio

The boiler ratio was established as a dimensionless measure for the boiler capacity. Boiler capacity is significant only when the room's thermal response places a demand on the heating plant that requires maximum boiler heat output. Such conditions occur during both the preheat period and the initial occupancy deficiency period in the intermittent heating cycle. As the latter is of primary importance in the present study, the basic emitter output defined by Equation 5.4 is based on a standard water-air temperature difference of $100^\circ F$ ($55.6^\circ C$), which corresponds to the water-air temperature difference achieved in the
simulations towards the end of the initial occupancy deficiency period for the cooler weather profiles.

The boiler ratio is defined as the ratio of the actual boiler capacity to this basic emitter output for the modelled emitter size:

\[ Q_r = \frac{Q_{boiln}}{Q_{emitb}} \] ... 7.1

where:

- \( Q_r \) = boiler ratio
- \( Q_{boiln} \) = maximum boiler output for room
- \( Q_{emitb} \) = basic emitter output as defined in Equation 5.4.

7.4.2 Emitter Ratio

At the start of the study there was a need to choose an emitter size for each room that could be used as a basis for comparing the simulation study results and choosing the emitter size and boiler capacities to use in the study. As the sizing of hydronic heating room emitters is usually undertaken by using a steady state analysis model, Room 1 was analysed using the usual steady state assumptions as presented in Table 7.10. A typical value for the size of the room's heat emitters, as measured by the emitter coefficient, was determined by substituting the analysed steady state heat requirement into Equation 5.4. A modified form of Equation 5.4 is used as the basis of manufacturers' tables for designer's to choose the length of emitter elements required.

The derived value of the emitter coefficient was used in a simulation run with weather profile JM2 and a boiler ratio of 1.0, i.e. a boiler capacity equal to the basic emitter output as defined by Equation 5.4. In this case the boiler capacity equalled the calculated
<table>
<thead>
<tr>
<th>Component Heat Flow Rate</th>
<th>Equation</th>
<th>Value (Btu/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Windows</td>
<td>Area × U-value × Temp. Difference 33 × 1.44 × 30 = 1426.</td>
<td></td>
</tr>
<tr>
<td>Infiltration</td>
<td>Specific heat × No. of air × Room × Temp. per volume changes/hr × Volume × Diff. .018 × 2 × 1760 × 30 = 1901.</td>
<td></td>
</tr>
<tr>
<td>TOTAL</td>
<td>= 3968.</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE 7.10: STEADY STATE ANALYSIS FOR ROOM 1**
steady state heat requirement. A duration of the initial occupancy deficiency period equal to two hours resulted. A basic emitter coefficient was arbitrarily defined for each room from this result as a means of establishing a basis of comparison between the two rooms investigated.

The basic emitter coefficient for a particular room construction and usage is defined as the emitter coefficient used with a boiler ratio of 1.0 that produces an initial occupancy deficiency period equal to two hours for weather profile JM2. Emitter ratio is defined as the ratio of the actual emitter coefficient to the basic emitter coefficient for the room:

\[ K_r = \frac{K_{\text{emit}}}{K_{\text{emitb}}} \]  \( \ldots 7.2 \)

where:

- \( K_r \) = emitter ratio
- \( K_{\text{emit}} \) = emitter coefficient
- \( K_{\text{emitb}} \) = basic emitter coefficient for room

7.5 RESULTS FROM THE SIMULATION RUNS

Typical temperature responses for Room 1 with the thermal maintenance period controlled by choice of the boiler start-up time to have zero duration and for weather profile JM2 are illustrated in Figure 7.6. The influence of the zero duration thermal maintenance period control, which was used for all simulation runs reported in Section 7.5, can be appreciated by comparing Figures 7.5 and 7.6 which illustrate Room 1 responses, on different time scales, that differ only due to their boiler start-up times. Excluding the pre-occupancy thermal maintenance period in Figure 7.5 and the period prior to the boiler...
FIGURE 7.6: ROOM 1 TEMPERATURE PROFILES FOR WEATHER PROFILE JM2
start-up time in Figure 7.6, the shape of the two sets of responses remains almost identical. Their only significant difference is the durations of the initial occupancy deficiency periods. As one would expect, an earlier boiler start-up time causes optimal thermal comfort conditions to be attained earlier during initial occupancy. Zero duration thermal maintenance period responses are similar to the intermittent heating cycle described in Section 7.3 in all other characteristics.

Figure 7.7 illustrates temperature responses for Room 2 for similar conditions to the Room 1 response illustrated in Figure 7.6. The larger heat loss through the external walls of Room 2, due to both larger areas and larger overall conductances than for Room 1, produces lower room temperatures at the end of the overnight cooling period, i.e. before boiler start-up. Thus larger heat input from the heating plant is required for Room 2 than for Room 1. Also longer preheat periods than for Room 1 are required to produce similar durations of initial occupancy deficiency periods. However, the basic shapes of the temperature responses of the two rooms remain very similar. The temperature fluctuations during the thermal adequacy period are of larger magnitude for Room 2, but these fluctuations were of little significance in the present study.

The measured durations of the initial occupancy thermal deficiency period for the range of thermal plant configurations and weather profiles used in the study are listed in Tables 7.11 and 7.12 for Rooms 1 and 2 respectively. Values listed are the time in hours from the start of occupancy to the instant when the required thermal condition of ambient temperature equal to 68°F (20°C) is next attained. Very small durations are listed as zero as these values result from the assumed sudden change in ventilation rate which would be more gradual in practice. The durations are plotted against the 8 a.m. external air temperature in Figures 7.8 and 7.9.
FIGURE 7.7: ROOM 2 TEMPERATURE PROFILES FOR WEATHER PROFILE JM2
<table>
<thead>
<tr>
<th>Emitter Ratio</th>
<th>Boiler Ratio</th>
<th>Duration of Thermal Deficiency Period (Hours) for Weather Profile:</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>JM2</td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>2.00</td>
</tr>
<tr>
<td>1.00</td>
<td>1.30</td>
<td>1.59</td>
</tr>
<tr>
<td>0.92</td>
<td>1.00</td>
<td>2.68</td>
</tr>
<tr>
<td>0.92</td>
<td>1.062</td>
<td>2.26</td>
</tr>
<tr>
<td>1.037</td>
<td>1.00</td>
<td>1.77</td>
</tr>
<tr>
<td>1.037</td>
<td>1.067</td>
<td>1.44</td>
</tr>
</tbody>
</table>

**TABLE 7.11: DURATION OF THERMAL DEFICIENCY PERIOD FOR ROOM 1**

Note: A dash (-) means no value was determined.
### TABLE 7.12: DURATION OF THERMAL DEFICIENCY PERIOD FOR ROOM 2

<table>
<thead>
<tr>
<th>Emitter Ratio</th>
<th>Boiler Ratio</th>
<th>JM2</th>
<th>JM1</th>
<th>JM</th>
<th>SM</th>
<th>SMJ</th>
<th>SML</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>2.00</td>
<td>1.07</td>
<td>0.45</td>
<td>0.33</td>
<td>0.23</td>
<td>0</td>
</tr>
<tr>
<td>1.00</td>
<td>1.30</td>
<td>1.81</td>
<td>0.68</td>
<td>0.21</td>
<td>0</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>0.91</td>
<td>1.00</td>
<td>2.91</td>
<td>1.63</td>
<td>0.65</td>
<td>0.43</td>
<td>0.32</td>
<td>0</td>
</tr>
<tr>
<td>0.91</td>
<td>1.167</td>
<td>2.57</td>
<td>1.37</td>
<td>0.40</td>
<td>0.31</td>
<td>-</td>
<td>0</td>
</tr>
<tr>
<td>1.064</td>
<td>1.00</td>
<td>1.54</td>
<td>0.79</td>
<td>0.38</td>
<td>0.27</td>
<td>0.18</td>
<td>0</td>
</tr>
<tr>
<td>1.064</td>
<td>0.86</td>
<td>2.65</td>
<td>1.47</td>
<td>0.60</td>
<td>0.43</td>
<td>-</td>
<td>0</td>
</tr>
</tbody>
</table>

Note: A dash (-) means no value was determined.
FIGURE 7.8: ROOM 1 THERMAL DEFICIENCY DURATION VERSUS 8 A.M. AIR TEMPERATURE
Figure 7.9: Room 2 Thermal Deficiency Duration Versus 8 A.M. Air Temperature

Graph showing the relationship between hours and degrees Fahrenheit, with various symbols indicating different values of $K_r$ and $Q_r$. The symbols are:

- $\times$ $K_r = 1.07$, $Q_r = 0.86$
- $\times$ $K_r = 1.07$, $Q_r = 1.00$
- $\circ$ $K_r = 0.91$, $Q_r = 1.17$
- $\square$ $K_r = 0.91$, $Q_r = 1.00$
- $+$ $K_r = 1.00$, $Q_r = 1.30$
- $\times$ $K_r = 1.00$, $Q_r = 1.00$
The durations of the preheat period that produced the required thermal condition of an ambient temperature equal to 68°F (20°C) exactly at the start of occupancy, i.e. produced a zero duration pre-occupancy thermal maintenance period, are listed in Tables 7.13 and 7.14 for Rooms 1 and 2 respectively.

The daily energy supplied to Room 1 from the boiler, artificial lighting, and human occupants is listed in Table 7.15 for the range of plant configurations and weather profiles used in the study. Table 7.16 lists the daily energy inflow from these sources for Room 2. The values tabulated in Tables 7.15 and 7.16 are plotted against the mean daily temperature for each weather profile in Figures 7.10 and 7.11.

7.6 ANALYSIS OF THE SIMULATION RESULTS

As discussed in Section 7.1.2, the simulation runs were undertaken to provide data for evaluation of the differential cost model formulated in Chapter Four. The extent of digital computing resources required to iteratively determine the boiler start-up time to achieve zero duration pre-occupancy thermal maintenance periods, as discussed in Section 7.3.2, and to perform the daily intermittent heating cycle simulations limited the quantity of simulation results data for subsequent analysis.

As only relative sensitivity of the differential cost model components is required to meet the purpose of the study, as described in Section 7.1.2, a rather crude analysis of the simulation results is presented in Section 7.6. To obtain expressions for the required relationships described in Section 7.1.1, regression analyses [Daniel & Wood, 1971] were undertaken on the limited quantity of simulation
<table>
<thead>
<tr>
<th>Emitter Ratio</th>
<th>Boiler Ratio</th>
<th>JM2</th>
<th>JM1</th>
<th>JM</th>
<th>SM</th>
<th>SMJ</th>
<th>SMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>1.70</td>
<td>1.51</td>
<td>1.33</td>
<td>1.26</td>
<td>1.19</td>
<td>1.01</td>
</tr>
<tr>
<td>1.00</td>
<td>1.30</td>
<td>1.32</td>
<td>1.19</td>
<td>1.06</td>
<td>1.01</td>
<td>-</td>
<td>0.83</td>
</tr>
<tr>
<td>0.92</td>
<td>1.00</td>
<td>1.88</td>
<td>1.66</td>
<td>1.46</td>
<td>1.37</td>
<td>1.29</td>
<td>1.10</td>
</tr>
<tr>
<td>0.92</td>
<td>1.062</td>
<td>1.76</td>
<td>1.56</td>
<td>1.39</td>
<td>1.30</td>
<td>1.22</td>
<td>1.05</td>
</tr>
<tr>
<td>1.037</td>
<td>1.00</td>
<td>1.63</td>
<td>1.45</td>
<td>1.29</td>
<td>1.21</td>
<td>1.14</td>
<td>0.97</td>
</tr>
<tr>
<td>1.037</td>
<td>1.067</td>
<td>1.53</td>
<td>1.37</td>
<td>1.22</td>
<td>1.15</td>
<td>1.08</td>
<td>0.93</td>
</tr>
</tbody>
</table>

**TABLE 7.13: DURATION OF PRE-HEAT PERIOD FOR ROOM 1**

Note: A dash (-) means no value was determined.
### TABLE 7.14: DURATION OF PRE-HEAT PERIOD FOR ROOM 2

<table>
<thead>
<tr>
<th>Emitter Ratio</th>
<th>Boiler Ratio</th>
<th>JM2</th>
<th>JM1</th>
<th>JM</th>
<th>SM</th>
<th>SMJ</th>
<th>SMI</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>2.33</td>
<td>2.01</td>
<td>1.71</td>
<td>1.60</td>
<td>1.48</td>
<td>1.23</td>
</tr>
<tr>
<td>1.00</td>
<td>1.30</td>
<td>1.68</td>
<td>1.49</td>
<td>1.32</td>
<td>1.23</td>
<td>-</td>
<td>0.98</td>
</tr>
<tr>
<td>0.91</td>
<td>1.00</td>
<td>2.75</td>
<td>2.32</td>
<td>1.94</td>
<td>1.80</td>
<td>1.67</td>
<td>1.36</td>
</tr>
<tr>
<td>0.91</td>
<td>1.167</td>
<td>2.20</td>
<td>1.90</td>
<td>1.63</td>
<td>1.52</td>
<td>-</td>
<td>1.18</td>
</tr>
<tr>
<td>1.064</td>
<td>1.00</td>
<td>2.12</td>
<td>1.84</td>
<td>1.59</td>
<td>1.48</td>
<td>1.38</td>
<td>1.15</td>
</tr>
<tr>
<td>1.064</td>
<td>0.86</td>
<td>2.62</td>
<td>2.23</td>
<td>1.88</td>
<td>1.75</td>
<td>-</td>
<td>1.32</td>
</tr>
</tbody>
</table>

Note: A dash (-) means no value was determined.
### TABLE 7.15: DAILY ENERGY SUPPLIED FROM BOILER, ARTIFICIAL LIGHTING, AND HUMAN OCCUPANTS FOR ROOM 1

Note: A dash (-) means no value was determined.
<table>
<thead>
<tr>
<th>Emitter Ratio</th>
<th>Boiler Ratio</th>
<th>JM2</th>
<th>JM1</th>
<th>JM</th>
<th>SM</th>
<th>SMJ</th>
<th>SML</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>73658.</td>
<td>67426.</td>
<td>59823.</td>
<td>55068.</td>
<td>49995.</td>
<td>40277.</td>
</tr>
<tr>
<td>1.00</td>
<td>1.30</td>
<td>77875.</td>
<td>71343.</td>
<td>63951.</td>
<td>58968.</td>
<td>-</td>
<td>42377.</td>
</tr>
<tr>
<td>0.91</td>
<td>1.00</td>
<td>72883.</td>
<td>66426.</td>
<td>58982.</td>
<td>54351.</td>
<td>49861.</td>
<td>40280.</td>
</tr>
<tr>
<td>0.91</td>
<td>1.167</td>
<td>74311.</td>
<td>68432.</td>
<td>61145.</td>
<td>56475.</td>
<td>-</td>
<td>41196.</td>
</tr>
<tr>
<td>1.064</td>
<td>1.00</td>
<td>74368.</td>
<td>68303.</td>
<td>60727.</td>
<td>56004.</td>
<td>50634.</td>
<td>39541.</td>
</tr>
<tr>
<td>1.064</td>
<td>0.86</td>
<td>71660.</td>
<td>65631.</td>
<td>58540.</td>
<td>54160.</td>
<td>-</td>
<td>37909.</td>
</tr>
</tbody>
</table>

**TABLE 7.16:** DAILY ENERGY SUPPLIED FROM BOILER, ARTIFICIAL LIGHTING, AND HUMAN OCCUPANTS FOR ROOM 2

Note: A dash (−) means no value was determined.
Figure 7.10: Daily energy supplied to Room 1

- $K_r = 1.04$, $Q_r = 1.07$
- $Y K_r = 1.04$, $Q_r = 1.00$
- $G K_r = 0.92$, $Q_r = 1.06$
- $D K_r = 0.92$, $Q_r = 1.00$
- $+$ $K_r = 1.00$, $Q_r = 1.30$
- $X K_r = 1.00$, $Q_r = 1.00$
FIGURE 7.11: DAILY ENERGY SUPPLIED TO ROOM 2
results data presented in Section 7.5. The form that the regression expressions may take is restricted by their subsequent use. Evaluation of both the user cost of thermal deficiency and the energy cost components of the differential cost model to account for their variation over the heating season is achieved using probability moments [Benjamin & Cornell, 1970]. The Taylor Series expansions of the regression expressions derived in Sections 7.6.1 and 7.6.2 must form decreasing series [Hahn & Shapiro, 1967] to enable their easy evaluation. This restriction on the derived regression expressions limited their "goodness-of-fit" [Benjamin & Cornell, 1970] to the analysed data that could be achieved.

The simulation results applicable to the user cost of thermal deficiency are analysed in Section 7.6.1 to produce evaluations of the user cost model for the range of hydronic heating alternatives used in the simulation runs. Section 7.6.2 presents a similar analysis for the energy cost evaluations. Then all components of the differential cost model are presented in Section 7.6.3.

7.6.1 User Cost of Thermal Deficiency

The user cost model for intermittent heating derived in Chapter Four and expressed by Equation 4.10 requires two relationships to be established:

(1) The duration of the thermal deficiency period during initial occupancy and its variation over the heating season.

(2) The variation of ambient temperature in the room during the initial occupancy deficiency period.

Tables 7.12 and 7.13 and Figures 7.8 and 7.9 present the durations of the thermal deficiency period measured in the simulation study. However, a numerical expression in terms of the external air temperature variation
during the heating season is required for the user cost model.

As the thermal deficiency period always starts at 8 a.m., i.e. the occupancy start time, the external air temperature at 8 a.m. was chosen as representative of the influence of the external air temperature on the duration of thermal deficiency. As the annual user cost model, which is derived later in this section, is stochastically related to the variation in external air temperature, the desired numerical expression must have the property of possessing a decreasing Taylor Series expansion with respect to the 8 a.m. external air temperature. An expression that meets this requirement, as demonstrated later in this section, and approximates the curve described by the points in Figures 7.8 and 7.9, is one of the form:

\[ t_d = F e^{D\phi_8} \]  

...7.3

where:

\[ t_d = \text{duration of thermal deficiency period} \]
\[ \phi_8 = 8 \text{ a.m. external air temperature} \]
\[ F, D = \text{constants dependent upon the particular room and heating plant.} \]

Equation 7.3 was chosen, not only for the decreasing Taylor Series expansion property of its probability moment, but also as it can be transformed to the following linear expression so that linear regression analysis could be used to determine values of the two constants:

\[ \log_e t_d = \log_e F + D\phi_8 \]  

...7.4

Table 7.17 lists values of the two constants in Equation 7.3 that were derived using Equation 7.4 and the simulation results listed in Tables 7.11 and 7.12. The resulting curves described by Equation 7.3 with the listed values of its two constants are illustrated in Figures
<table>
<thead>
<tr>
<th>Room</th>
<th>Emitter Ratio</th>
<th>Boiler Ratio</th>
<th>F (hours)</th>
<th>D (°F⁻¹)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room 1:</td>
<td>1.00</td>
<td>1.00</td>
<td>24.0</td>
<td>-0.1012</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.30</td>
<td>31.5</td>
<td>-0.1266</td>
</tr>
<tr>
<td></td>
<td>0.92</td>
<td>1.00</td>
<td>33.2</td>
<td>-0.1021</td>
</tr>
<tr>
<td></td>
<td>0.92</td>
<td>1.062</td>
<td>29.3</td>
<td>-0.1042</td>
</tr>
<tr>
<td></td>
<td>1.037</td>
<td>1.00</td>
<td>20.2</td>
<td>-0.0997</td>
</tr>
<tr>
<td></td>
<td>1.037</td>
<td>1.067</td>
<td>14.5</td>
<td>-0.0952</td>
</tr>
<tr>
<td>Room 2:</td>
<td>1.00</td>
<td>1.00</td>
<td>30.0</td>
<td>-0.1104</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.30</td>
<td>75.1</td>
<td>-0.1539</td>
</tr>
<tr>
<td></td>
<td>0.91</td>
<td>1.00</td>
<td>50.2</td>
<td>-0.1150</td>
</tr>
<tr>
<td></td>
<td>0.91</td>
<td>1.167</td>
<td>66.5</td>
<td>-0.1316</td>
</tr>
<tr>
<td></td>
<td>1.064</td>
<td>1.00</td>
<td>21.0</td>
<td>-0.1070</td>
</tr>
<tr>
<td></td>
<td>1.064</td>
<td>0.86</td>
<td>39.0</td>
<td>-0.1094</td>
</tr>
</tbody>
</table>

**TABLE 7.17:**  CONSTANTS FOR EQUATION 7.3
7.12 through 7.17. The simulation results used to derive the regression curves are also plotted in the figures.

The second relationship required for the user cost model is for the variation of ambient temperature in the rooms during the initial occupancy thermal deficiency period. The complex nature of this variation is illustrated in Figure 7.18. It begins with a large negative gradient due to the increased infiltration rate at the start of occupancy, then curves sharply into a gently increasing gradient. Having regard to the approximations used in the simulation runs and the approximate nature of the present analysis, a simple linear relationship is assumed, as illustrated in Figure 7.18, for the purpose of the user cost model. The straight line is described by the equation:

$$\theta = \theta_r - g(t_d - t')$$

...7.5

where:

- $\theta$ = ambient temperature during initial occupancy thermal deficiency period.
- $\theta_r$ = required ambient temperature of room during occupancy.
- $t_d$ = duration of initial occupancy thermal deficiency period.
- $t'$ = time from start of occupancy.
- $g$ = constant gradient.

Equation 4.10 requires an expression for the ambient temperature deviation from the required ambient temperature during thermal occupancy in terms of the variation in time through the thermal deficiency period.

Figure 7.19 illustrates the derived relationship and its derivation from Equation 7.5 is presented below:
FIGURE 7.12: ROOM 1 THERMAL DEFICIENCY DURATION DERIVED CURVES

+ $K_r = 1.00$, $Q_r = 1.30$

$X$ $K_r = 1.00$, $Q_r = 1.00$
FIGURE 7.13: ROOM 1 THERMAL DEFICIENCY DURATION DERIVED CURVES

○ $K_r = 0.92$, $Q_r = 1.05$

□ $K_r = 0.92$, $Q_r = 1.00$
FIGURE 7.14: ROOM 1 THERMAL DEFICIENCY DURATION DERIVED CURVES

- For curve X, $K_r = 1.04$, $Q_r = 1.07$
- For curve Y, $K_r = 1.04$, $Q_r = 1.00$
FIGURE 7.15: ROOM 2 THERMAL DEFICIENCY DURATION DERIVED CURVES

- ○ $K_r = 1.00, \ Q_r = 1.30$
- × $K_r = 1.00, \ Q_r = 1.00$
FIGURE 7.16: ROOM 2 THERMAL DEFICIENCY DURATION DERIVED CURVES
FIGURE 7.17: ROOM 2 THERMAL DEFICIENCY DURATION DERIVED CURVES

$\times K_r = 1.07, \quad Q_r = 0.86$

$\times K_r = 1.07, \quad Q_r = 1.00$
FIGURE 7.18: VARIATION OF AMBIENT TEMPERATURE DURING INITIAL OCCUPANCY THERMAL DEFICIENCY PERIOD.

FIGURE 7.19: VARIATION OF AMBIENT TEMPERATURE DEVIATION FROM REQUIRED TEMPERATURE DURING THERMAL DEFICIENCY.
\[
\Delta \theta = \theta_r - \theta = g(t_d - t') \quad \therefore \quad \Delta \theta = g t \quad \ldots \text{7.6}
\]

where:
- \( \Delta \theta \) = ambient temperature deviation from required ambient temperature during occupancy
- \( t \) = time through thermal deficiency period measured from the end of this period.

Table 7.18 lists values for the gradient, \( g \), derived by linear regression from a number of simulation run results. The gradient varies with variation of both the heating system and the external air temperature profile.

Having regard to the previous approximations, the average gradient from the simulation run results is adopted for the user cost model.

Substituting Equation 7.6 in Equation 4.10 gives:

\[
C_{ud wk J} = \int_{0}^{t_d} g^2 t^2 dt \quad \text{for } t_d > 0 \quad \ldots \text{7.7}
\]

\[\therefore \quad C_{ud} = \frac{1}{3} \text{wkg}^2 t_d^3 \quad \ldots \text{7.8}\]

Substituting Equation 7.3 into Equation 7.8 gives:

\[
C_{ud} = \frac{1}{3} \text{wkg}^2 F^3 e^{3\Delta \theta} \quad \ldots \text{7.9}
\]

where:
- \( C_{ud} \) = daily user cost due to initial occupancy thermal deficiency
- \( w \) = employment worth of occupants
- \( k \) = proportionality constant as defined by Equation 4.5 and of magnitude as given by Equation 4.6
- \( g \) = constant gradient as defined by Equations 7.5 and 7.6
<table>
<thead>
<tr>
<th>Room</th>
<th>Emitter Ratio</th>
<th>Boiler Ratio</th>
<th>Weather Profile</th>
<th>Gradient, g (°F/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room 1:</td>
<td>1.00</td>
<td>1.00</td>
<td>JM2</td>
<td>2.30</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.00</td>
<td>JM1</td>
<td>2.40</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.00</td>
<td>JM</td>
<td>2.67</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.00</td>
<td>SM</td>
<td>2.30</td>
</tr>
<tr>
<td>Room 2:</td>
<td>1.00</td>
<td>1.00</td>
<td>JM2</td>
<td>2.27</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.00</td>
<td>JM1</td>
<td>2.55</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.00</td>
<td>JM</td>
<td>2.69</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.00</td>
<td>SM</td>
<td>2.84</td>
</tr>
</tbody>
</table>

Mean = 2.50

**TABLE 7.18:** GRADIENT OF AMBIENT TEMPERATURE VARIATION DURING THERMAL DEFICIENCY PERIOD
\[ F, D = \text{constants as defined by Equation 7.3} \]
\[ \phi_8 = \text{8 a.m. external air temperature for the day.} \]

An annual user cost can be estimated from the expected value of the daily user cost over the heating season for the building. As the 8 a.m. external air temperature is the only variable with respect to days through the heating season, the following analysis is applicable [Hahn & Shapiro, 1967]:

\[ E[C_u] = E[f(\phi_8)] \]

\[ E[C_u] \approx f(\bar{\phi}_8) + \frac{1}{2} \frac{d^2f(\phi_8)}{d \phi_8^2} \operatorname{Var}[\phi_8] \quad \text{...7.10} \]

where:

\[ E[C_u] = \text{expected value of daily user cost} \]
\[ f(\phi_8) = \text{function of } \phi_8 \]
\[ \bar{\phi}_8 = \text{expected value of 8 a.m. external air temperature} \]
\[ \text{over the heating season} \]
\[ \operatorname{Var}[\phi_8] = \text{statistical variance of 8 a.m. external air temperature} \]
\[ \text{over the heating season.} \]

Equation 7.10 is only valid if the Taylor Series expansion of \( f(\phi_8) \) with respect to \( \phi_8 \) forms a decreasing series as higher order terms can then be neglected. This is the case for the user cost function as expressed by Equation 7.9 as the ratio of successive derivatives is:

\[ \frac{d^{r+1}f(\phi_8)}{d \phi_8^{r+1}} : \frac{d^r f(\phi_8)}{d \phi_8^r} = 3D \quad \text{...7.11} \]

The Taylor series expansion forms a decreasing series if this ratio meets the constraint:
\[ |3D| < r + 1 \quad \ldots 7.12 \]

where:

\[ r = \text{order of the derivative.} \]

All the values of D listed in Table 7.17 satisfy Equation 7.12.

Substituting Equation 7.9 into Equation 7.10 gives:

\[
E[C_{ud}] = \frac{1}{3} wkg^2 F^3 e^{3D\phi_0} + \frac{3}{2} wkg^2 F^3 d^2 e^{3D\phi_0} \varphi[\phi_0]
\]

\[
= \frac{1}{6} wkg^2 F^3 e^{3D\phi_0} \{2 + 9d^2 \varphi[\phi_0]\} \quad \ldots 7.13
\]

The expected annual user cost is given by:

\[
E[C_{ua}] = h E[C_{ud}]
\]

\[
\therefore E[C_{ua}] = \frac{1}{6} hwkg^2 F^3 e^{3D\phi_0} \{2 + 9d^2 \varphi[\phi_0]\} \quad \ldots 7.14
\]

where:

\[ E[C_{ua}] = \text{expected annual user cost} \]

\[ h = \text{number of working days in annual heating season.} \]

The parameters of Equation 7.14 that are constant for all room and heating system combinations are listed together with their values in Table 7.19. The external air temperature statistical parameters were derived from records for Christchurch, New Zealand for the ten years from 1960 to 1969. The constants that depend upon the room and heating system, together with the corresponding values of the expected annual user cost are listed in Table 7.20.
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of working days in heating season (May 1 - September 30)</td>
<td>h</td>
<td>109</td>
<td>days</td>
</tr>
<tr>
<td>Employment worth of occupants</td>
<td>w</td>
<td>10.</td>
<td>$ per hour</td>
</tr>
<tr>
<td>Constant as defined by Equation 4.5</td>
<td>k</td>
<td>2.14x10^{-3}</td>
<td>per °F^2</td>
</tr>
<tr>
<td>Constant as defined by Equation 7.5</td>
<td>g</td>
<td>2.50</td>
<td>°F/hr</td>
</tr>
<tr>
<td>Mean 8 a.m. external air temperature for Christchurch heating season</td>
<td>Φ_b</td>
<td>40.9</td>
<td>°F</td>
</tr>
<tr>
<td>Variance of 8 a.m. external air temperature for Christchurch heating season</td>
<td>Var[Φ_b]</td>
<td>45.5</td>
<td>°F^2</td>
</tr>
</tbody>
</table>

**TABLE 7.19: CONSTANT PARAMETERS FOR USER COST MODEL**
<table>
<thead>
<tr>
<th>Room</th>
<th>Emitter Ratio</th>
<th>Boiler Ratio</th>
<th>$F$ (hours)</th>
<th>$D$ ($^{\circ} \text{F}^{-1}$)</th>
<th>$E[\text{Cua}]$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room 1:</td>
<td>1.00</td>
<td>1.00</td>
<td>24.0</td>
<td>-.1012</td>
<td>0.84</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.30</td>
<td>31.5</td>
<td>-.1266</td>
<td>0.12</td>
</tr>
<tr>
<td></td>
<td>0.92</td>
<td>1.00</td>
<td>33.2</td>
<td>-.1021</td>
<td>2.02</td>
</tr>
<tr>
<td></td>
<td>0.92</td>
<td>1.062</td>
<td>29.3</td>
<td>-.1042</td>
<td>1.10</td>
</tr>
<tr>
<td></td>
<td>1.037</td>
<td>1.00</td>
<td>20.2</td>
<td>-.0997</td>
<td>0.59</td>
</tr>
<tr>
<td></td>
<td>1.037</td>
<td>1.067</td>
<td>14.5</td>
<td>-.0952</td>
<td>0.36</td>
</tr>
<tr>
<td>Room 2:</td>
<td>1.00</td>
<td>1.00</td>
<td>30.0</td>
<td>-.1104</td>
<td>0.60</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.30</td>
<td>75.1</td>
<td>-.1539</td>
<td>0.08</td>
</tr>
<tr>
<td></td>
<td>0.91</td>
<td>1.00</td>
<td>50.2</td>
<td>-.1150</td>
<td>1.70</td>
</tr>
<tr>
<td></td>
<td>0.91</td>
<td>1.167</td>
<td>66.5</td>
<td>-.1316</td>
<td>0.63</td>
</tr>
<tr>
<td></td>
<td>1.064</td>
<td>1.00</td>
<td>21.0</td>
<td>-.1070</td>
<td>0.30</td>
</tr>
<tr>
<td></td>
<td>1.064</td>
<td>0.86</td>
<td>39.0</td>
<td>-.1094</td>
<td>1.47</td>
</tr>
</tbody>
</table>

**Table 7.20: Expected Annual User Costs**
7.6.2 Energy Cost

Energy consumption cost is expressed in Equation 4.9 in terms of the quantity of energy output by the boiler. Thus the daily energy output by the boiler, for each weather profile, was measured in the simulation study. To obtain an estimate of the annual energy output for each room and heating plant combination, a numerical expression relating the daily energy output to the boiler size and the external air temperature variation during the heating season was required. The mean daily external air temperature was chosen as representative of the diurnal influence of the external air temperature. As for the annual user cost model, a stochastic relationship with the external air temperature variation is to be used, so the desired numerical expression must have the property of possessing a decreasing Taylor Series expansion with respect to the daily mean external air temperature. A suitable expression is one of the form:

\[ H_d = A(B - \phi_d)^n \]  \[ ...7.15 \]

where:

- \( H_d \) = daily energy input to room
- \( \phi_d \) = mean daily external air temperature
- \( A, B, n \) = constants requiring determination for the particular rooms and heating plants.

Combinations of \( A, B, \) and \( n \) were evaluated for the daily energy input by the boiler alone, then for the daily energy input by the boiler plus internal heat sources. The best fit to the data listed in Tables 7.15 and 7.16 and plotted in Figures 7.10 and 7.11 was obtained by including energy input from the internal heat sources and by setting the constant, \( B \), equal to the required ambient temperature for the room. With this value of the constant, \( B \), Equation 7.15 expresses the expected result that no energy input is required for the room when the mean daily external air
temperature equals the room's required ambient temperature. Obviously no heat is required from the boiler unless the mean daily external air temperature is somewhat lower than the required room ambient temperature because of the heat supplied to the room from the internal heat sources of artificial lighting, human occupants, and equipment. Although constant values for the internal heat sources were used in all simulation runs, the value of the mean daily external air temperature at which no heat from the boiler was required varied for each combination of room, heating system, and weather profile simulated. Without inclusion of the heat from internal heat sources, the constant, B, would vary for each study combination.

Using the total daily energy input from boiler and internal heat sources, and setting the constant, B, equal to the room's required ambient temperature, Equation 7.15 becomes:

\[ H_d = H_b + H_i = A(68 - \phi_d)^n \]  \[ 7.16 \]

where:

- \( H_d \) = daily energy input to room from boiler and internal heat sources
- \( H_b \) = daily energy input to room from boiler
- \( H_i \) = daily energy input to room from internal heat sources
- \( \phi_d \) = mean daily external air temperature
- \( A, n \) = constants dependent upon particular room and heating plant.

Values of the two constants, A and n, of Equation 7.16 were derived from the simulation results presented in Tables 7.15 and 7.16. Linear regression of the following transformation of Equation 7.16 was used to derive the values, which are listed in Table 7.21:

\[ \log_e H_d = \log_e A + n \log_e (68 - \phi_d) \]  \[ 7.17 \]
Figures 7.20 through 7.25 present the curves described by Equation 7.16 with these values of its two constants, together with the appropriate simulation results. The figures illustrate that the curves do not fit the data particularly well. As evaluation of the relative sensitivity of the differential cost model components will meet the study objectives and there is insufficient data to warrant a very rigorous analysis, the expressions defined by Equation 7.16 and the constants listed in Table 7.21 were used to evaluate crude estimates of the annual energy costs.

An annual energy usage by the room can be estimated from the expected value of the daily energy input to the room using the same method as for user cost. In the energy usage model the mean daily external air temperature is the only variable with respect to the days through the heating season. Thus the following analysis can be used [Hahn & Shapiro, 1967]:

\[
E[H_d] = E[f(\phi_d)]
\]

\[
E[H_d] = f(\bar{\phi}_d) + \frac{1}{2} \left[ \frac{d^2 f(\phi_d)}{d \phi_d^2} \right] \text{Var}[\phi_d]
\]

where:

- \(E[H_d]\) = expected value of daily energy input to the room
- \(f(\phi_d)\) = function of \(\phi_d\)
- \(\bar{\phi}_d\) = expected value of mean daily external air temperature over the heating season
- \(\text{Var}[\phi_d]\) = statistical variance of mean daily external air temperature over the heating season

Equation 7.18 is valid only if higher terms of the Taylor Series expansion can be neglected. The ratio of successive derivatives is:
Figure 7.20: Room 1 Daily Energy Derived Curves

+ $K_r = 1.00$, $Q_r = 1.30$

$X$ $K_r = 1.00$, $Q_r = 1.00$
Figure 7.21: Room 1 Daily Energy Derived Curves

- Circle: $K_r = 0.92, Q_r = 1.06$
- Square: $K_r = 0.92, Q_r = 1.00$
FIGURE 7.22: ROOM 1 DAILY ENERGY DERIVED CURVES

$\times K_r = 1.04, \quad Q_r = 1.07$

$Y K_r = 1.04, \quad Q_r = 1.00$
FIGURE 7.23: ROOM 2 DAILY ENERGY DERIVED CURVES
**FIGURE 7.24: ROOM 2 DAILY ENERGY DERIVED CURVES**

- $K_r = 0.91$, $Q_r = 1.17$
- $K_r = 0.91$, $Q_r = 1.00$
FIGURE 7.25: ROOM 2 DAILY ENERGY DERIVED CURVES

\[ \times K_r = 1.07, \quad Q_r = 0.66 \]

\[ \triangle K_r = 1.07, \quad Q_r = 1.00 \]
<table>
<thead>
<tr>
<th>Room</th>
<th>Emitter Ratio</th>
<th>Boiler Ratio</th>
<th>$A_{(Btu/^oF^{-n})}$</th>
<th>$n$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room 1:</td>
<td>1.00</td>
<td>1.00</td>
<td>3711.</td>
<td>.733</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.30</td>
<td>3689.</td>
<td>.753</td>
</tr>
<tr>
<td></td>
<td>0.92</td>
<td>1.00</td>
<td>3824.</td>
<td>.718</td>
</tr>
<tr>
<td></td>
<td>0.92</td>
<td>1.062</td>
<td>3992.</td>
<td>.711</td>
</tr>
<tr>
<td></td>
<td>1.037</td>
<td>1.00</td>
<td>3862.</td>
<td>.723</td>
</tr>
<tr>
<td></td>
<td>1.037</td>
<td>1.067</td>
<td>3782.</td>
<td>.734</td>
</tr>
<tr>
<td>Room 2:</td>
<td>1.00</td>
<td>1.00</td>
<td>8209.</td>
<td>.607</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.30</td>
<td>8699.</td>
<td>.608</td>
</tr>
<tr>
<td></td>
<td>0.91</td>
<td>1.00</td>
<td>8501.</td>
<td>.594</td>
</tr>
<tr>
<td></td>
<td>0.91</td>
<td>1.167</td>
<td>8822.</td>
<td>.591</td>
</tr>
<tr>
<td></td>
<td>1.064</td>
<td>1.00</td>
<td>7646.</td>
<td>.632</td>
</tr>
<tr>
<td></td>
<td>1.064</td>
<td>0.86</td>
<td>7259.</td>
<td>.637</td>
</tr>
</tbody>
</table>

**TABLE 7.21:** CONSTANTS FOR EQUATION 7.16
\[
\frac{d^{r+1}f(\phi_d)}{d \phi_d^{r+1}} : \frac{d^r f(\phi_d)}{d \phi_d^r} = \frac{x-n}{(68-\bar{\phi}_d)}
\]

Thus Equation 7.18 is valid if:

\[
\frac{x-n}{(68-\bar{\phi}_d)} < r + 1
\]

where:

\[r = \text{order of the derivative.}\]

As all values of the constant, \(n\), are less than unity and the factor, \((68-\bar{\phi}_d)\), is greater than unity for Christchurch weather, this constraint is met.

Substituting from Equation 7.16 into Equation 7.18 gives:

\[
E[H_d] \approx A(68-\bar{\phi}_d)^n + \frac{A}{2} n(n-1) (68-\bar{\phi}_d)^{n-2} \text{Var}[\phi_d]
\]

\[
\therefore E[H_d] \approx A(68-\bar{\phi}_d)^n \left\{ 1 + \frac{n(n-1) \text{Var}[\phi_d]}{2(68-\bar{\phi}_d)^2} \right\}
\]

As the daily energy input to the room from internal heat sources is treated as constant:

\[
E[H_d] = E[H_d] - H_i
\]

where:

\[E[H_d] = \text{expected value of daily energy input to the room from the boiler}\]

\[H_i = \text{daily energy input to the room from internal heat sources}\]

\[= 13396 \text{ Btu for this study.}\]
Thus:
\[ E[H_b] = A(68-\bar{\phi}_d)^n \left\{ 1 + \frac{n(n-1)\text{Var}[\phi_d]}{2(68-\bar{\phi}_d)^2} \right\} - H_i \] ...7.23

The expected annual energy input to the room from the boiler is given by:
\[ E[H_a] = h \cdot E[H_b] \] ...7.24
\[ \therefore E[H_a] = hA(68-\bar{\phi}_d)^n \left\{ 1 + \frac{n(n-1)\text{Var}[\phi_d]}{2(68-\bar{\phi}_d)^2} \right\} - hH_i \] ...7.25

where:
\[ E[H_a] = \text{expected annual energy input to the room from the boiler} \]
\[ h = \text{number of heating days in annual heating season}. \]

The energy consumption cost was derived in Chapter 4 and is expressed by Equation 4.9. Substituting Equation 7.25 into Equation 4.9 gives:
\[ E[C_{fa}] = \frac{u_f h}{\eta} \left( A(68-\bar{\phi}_d)^n \left\{ 1 + \frac{n(n-1)\text{Var}[\phi_d]}{2(68-\bar{\phi}_d)^2} \right\} - H_i \right) \] ...7.26

where:
\[ E[C_{fa}] = \text{expected value of annual energy consumption cost for heat from boiler to the room} \]
\[ u_f = \text{unit cost of fuel} \]
\[ \eta = \text{mean boiler efficiency}. \]

The parameters of Equation 7.26 that have constant values for all room and heating system combinations are listed together with their values in Table 7.22. Records of the external air temperature for Christchurch, New Zealand, for the ten years from 1960 to 1969 were used
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit cost of fuel</td>
<td>$u_f$</td>
<td>$4.0 \times 10^{-6}$</td>
<td>$$/\text{Btu}$$</td>
</tr>
<tr>
<td>Number of heating days in heating season (May 1 - September 30)</td>
<td>$h$</td>
<td>109</td>
<td>days</td>
</tr>
<tr>
<td>Mean boiler efficiency</td>
<td>$\eta$</td>
<td>0.80</td>
<td>-</td>
</tr>
<tr>
<td>Daily energy input to room from internal heat sources</td>
<td>$H_i$</td>
<td>13396.</td>
<td>Btu/day</td>
</tr>
<tr>
<td>Mean daily external air temperature for Christchurch heating season</td>
<td>$\phi_d$</td>
<td>45.3</td>
<td>$^\circ \text{F}$</td>
</tr>
<tr>
<td>Variance of daily mean external air temperature for Christchurch heating season</td>
<td>$\text{Var}[\phi_d]$</td>
<td>21.9</td>
<td>$^\circ \text{F}^2$</td>
</tr>
</tbody>
</table>

**TABLE 7.22:** CONSTANT PARAMETERS FOR BOILER ENERGY CONSUMPTION

**COST MODEL**
to determine the values of the statistical parameters. Table 7.23 lists the constants that depend upon the room and heating system combination together with the corresponding evaluations of Equations 7.25 and 7.26.

7.6.3 Differential Cost

The results presented in Sections 7.6.1 and 7.6.2 provide the necessary information for evaluating the differential cost model for hydronic heating system sizing. The model was developed in Chapter Four and consists of:

1. Room emitter differential cost as described by Equation 4.7.
2. Boiler differential cost as described by Equation 4.8.
4. User cost of thermal deficiency as described by Equation 7.14.

Tables 7.24 and 7.25 list the values of the parameters used to evaluate these equations for the room and heating system combinations used in the simulation study. The resulting crude estimates for the component and total differential costs are listed in Table 7.26. Note that the annual energy consumption costs and user costs have been converted to equivalent initial value costs using the uniform series present worth factor for 30 years at 8% interest [Lu, 1969]. Also note that arbitrary zeros have been used for the cost scales as it is the differences in costs between the alternatives that are of interest. The differential cost estimates presented in Table 7.26 are for single rooms, so cost differences would be proportionately higher for multi-room buildings.
<table>
<thead>
<tr>
<th>Room</th>
<th>Emitter Ratio</th>
<th>Boiler Ratio</th>
<th>$A$ (Btu/°F$^{-n}$)</th>
<th>n</th>
<th>$E[H_a]$ (Btu x 10$^3$)</th>
<th>$E[C_{fa}]$ ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room 1:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.00</td>
<td>3711.</td>
<td>.733</td>
<td>2521.</td>
<td>12.56</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.30</td>
<td>3689.</td>
<td>.753</td>
<td>2744.</td>
<td>13.72</td>
</tr>
<tr>
<td></td>
<td>0.92</td>
<td>1.00</td>
<td>3824.</td>
<td>.718</td>
<td>2446.</td>
<td>12.23</td>
</tr>
<tr>
<td></td>
<td>0.92</td>
<td>1.062</td>
<td>3992.</td>
<td>.711</td>
<td>2529.</td>
<td>12.64</td>
</tr>
<tr>
<td></td>
<td>1.037</td>
<td>1.00</td>
<td>3862.</td>
<td>.723</td>
<td>2547.</td>
<td>12.73</td>
</tr>
<tr>
<td></td>
<td>1.037</td>
<td>1.067</td>
<td>3762.</td>
<td>.734</td>
<td>2601.</td>
<td>13.01</td>
</tr>
<tr>
<td>Room 2:</td>
<td></td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.00</td>
<td>8209.</td>
<td>.607</td>
<td>4464.</td>
<td>22.32</td>
</tr>
<tr>
<td></td>
<td>1.00</td>
<td>1.30</td>
<td>8699.</td>
<td>.608</td>
<td>4837.</td>
<td>24.19</td>
</tr>
<tr>
<td></td>
<td>0.91</td>
<td>1.00</td>
<td>8501.</td>
<td>.594</td>
<td>4430.</td>
<td>22.15</td>
</tr>
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<td>1.167</td>
<td>8822.</td>
<td>.591</td>
<td>4596.</td>
<td>22.98</td>
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<td>1.064</td>
<td>1.00</td>
<td>7646.</td>
<td>.632</td>
<td>4506.</td>
<td>22.53</td>
</tr>
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<td>0.86</td>
<td>7259.</td>
<td>.637</td>
<td>4294.</td>
<td>21.47</td>
</tr>
</tbody>
</table>

**Table 7.23:** Expected Annual Boiler Energy Costs
### Table 7.24: Constant Parameters for Differential Cost Model

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Equation</th>
<th>Symbol</th>
<th>Value</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Unit incremental capital cost of room emitter</td>
<td>4.7</td>
<td>( u_e )</td>
<td>3.0</td>
<td>$/foot</td>
</tr>
<tr>
<td>Ratio of emitter length to emitter constant</td>
<td>-</td>
<td>-</td>
<td>.703</td>
<td>ft/hr°F^{1.33}/Btu</td>
</tr>
<tr>
<td>Unit incremental capital cost of boiler capacity</td>
<td>4.8</td>
<td>( u_b )</td>
<td>.003</td>
<td>$/Btu/hr</td>
</tr>
<tr>
<td>Uniform series present worth factor over 30 years at 8%</td>
<td>Applied to results from 7.14 &amp; 7.26</td>
<td>-</td>
<td>11.26</td>
<td>-</td>
</tr>
</tbody>
</table>

Note: A dash (-) means not applicable.
<table>
<thead>
<tr>
<th>Emitter Ratio</th>
<th>Boiler Ratio</th>
<th>Emitter Constant</th>
<th>Emitter Length (ft)</th>
<th>Boiler Capacity (Btu/hr)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room 1:</td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>8.68</td>
<td>6.1</td>
<td>3968</td>
</tr>
<tr>
<td>1.00</td>
<td>1.30</td>
<td>8.68</td>
<td>6.1</td>
<td>5158</td>
</tr>
<tr>
<td>0.92</td>
<td>1.00</td>
<td>8.00</td>
<td>5.6</td>
<td>3657</td>
</tr>
<tr>
<td>0.92</td>
<td>1.062</td>
<td>8.00</td>
<td>5.6</td>
<td>3885</td>
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<tr>
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<td>1.00</td>
<td>9.00</td>
<td>6.3</td>
<td>4114</td>
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<tr>
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<td>1.067</td>
<td>9.00</td>
<td>6.3</td>
<td>4388</td>
</tr>
<tr>
<td>Room 2:</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>13.16</td>
<td>9.3</td>
<td>6015</td>
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<tr>
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<td>9.3</td>
<td>7820</td>
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<tr>
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<td>1.00</td>
<td>12.00</td>
<td>8.4</td>
<td>5485</td>
</tr>
<tr>
<td>0.91</td>
<td>1.167</td>
<td>12.00</td>
<td>8.4</td>
<td>6399</td>
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<tr>
<td>1.064</td>
<td>1.00</td>
<td>14.00</td>
<td>9.8</td>
<td>6399</td>
</tr>
<tr>
<td>1.064</td>
<td>0.86</td>
<td>14.00</td>
<td>9.8</td>
<td>5485</td>
</tr>
</tbody>
</table>

**TABLE 7.25:** VARIABLE PARAMETERS FOR DIFFERENTIAL COST MODEL
<table>
<thead>
<tr>
<th>Emitter Ratio</th>
<th>Boiler Ratio</th>
<th>Emitter Diff. Cost ($)</th>
<th>Boiler Diff. Cost ($)</th>
<th>Energy Diff. Cost ($)</th>
<th>User Diff. Cost ($)</th>
<th>Total Diff. Cost ($)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room 1:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>18.23</td>
<td>11.90</td>
<td>141.45</td>
<td>9.49</td>
<td>181.07</td>
</tr>
<tr>
<td>1.00</td>
<td>1.30</td>
<td>18.23</td>
<td>15.47</td>
<td>154.50</td>
<td>1.31</td>
<td>189.52</td>
</tr>
<tr>
<td>0.92</td>
<td>1.00</td>
<td>16.80</td>
<td>10.97</td>
<td>137.68</td>
<td>22.76</td>
<td>188.21</td>
</tr>
<tr>
<td>0.92</td>
<td>1.062</td>
<td>16.80</td>
<td>11.65</td>
<td>142.37</td>
<td>12.43</td>
<td>183.25</td>
</tr>
<tr>
<td>1.037</td>
<td>1.00</td>
<td>18.90</td>
<td>12.34</td>
<td>143.37</td>
<td>6.66</td>
<td>181.28</td>
</tr>
<tr>
<td>1.037</td>
<td>1.067</td>
<td>18.90</td>
<td>13.16</td>
<td>146.44</td>
<td>4.03</td>
<td>182.54</td>
</tr>
<tr>
<td>Range for Room 1:</td>
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<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.12</td>
<td>0.30</td>
<td>2.10</td>
<td>4.50</td>
<td>16.82</td>
<td>21.45</td>
<td>8.45</td>
</tr>
<tr>
<td>Room 2:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>1.00</td>
<td>1.00</td>
<td>27.64</td>
<td>18.04</td>
<td>251.32</td>
<td>6.76</td>
<td>303.76</td>
</tr>
<tr>
<td>1.00</td>
<td>1.30</td>
<td>27.64</td>
<td>23.46</td>
<td>272.33</td>
<td>0.85</td>
<td>324.28</td>
</tr>
<tr>
<td>0.91</td>
<td>1.00</td>
<td>25.20</td>
<td>16.45</td>
<td>249.42</td>
<td>19.11</td>
<td>310.19</td>
</tr>
<tr>
<td>0.91</td>
<td>1.167</td>
<td>25.20</td>
<td>19.20</td>
<td>258.73</td>
<td>7.11</td>
<td>310.23</td>
</tr>
<tr>
<td>1.064</td>
<td>1.00</td>
<td>29.40</td>
<td>19.20</td>
<td>253.70</td>
<td>3.37</td>
<td>305.67</td>
</tr>
<tr>
<td>1.064</td>
<td>0.86</td>
<td>29.40</td>
<td>16.45</td>
<td>241.73</td>
<td>16.58</td>
<td>304.17</td>
</tr>
<tr>
<td>Range for Room 2:</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>0.16</td>
<td>0.44</td>
<td>4.20</td>
<td>7.01</td>
<td>30.60</td>
<td>18.26</td>
<td>20.52</td>
</tr>
</tbody>
</table>

**TABLE 7.26: ROOM HEATING SYSTEM DIFFERENTIAL COSTS**
7.7 SIMULATION STUDY DISCUSSION AND CONCLUSIONS

The extreme complexity inherent in the dynamic thermal behaviour of buildings produced the need for many assumptions in the study. Assumptions used to formulate the differential cost model components were stated in Chapter Four. Discussion of the assumptions underlying the simulation model was presented in Chapters Five and Six. Most of the room heat flow processes were simulated in sufficient detail to ensure their influence was adequately modelled. However, the information was not available for very detailed simulation of hydronic heating plant performance. Thus the assumptions underlying its modelling were the most significant for the model; they are restated below.

Sections 7.2, 7.3, and 7.6 enunciated further assumptions used to produce the results presented in Table 7.26. Typical characteristics of New Zealand commercial buildings were assumed, but the coldest and most dynamic typical conditions were studied to meet the study objectives as stated in Section 7.1.2. The most significant assumptions and limitations are:

(1) Only one room from each of two typical construction forms was studied.

(2) Each room has a single external wall with a southern exposure.

(3) External weather was modelled using probabilistically derived hourly values of air temperature from records for Christchurch, New Zealand.

(4) Nil solar radiation intensities were used for the reported results.

(5) Each simulation run commenced from standardised initial conditions.
(6) Intermittently operated hydronic heating systems were studied.

(7) Heating plant was modelled for a single room as a simple single capacitance system (Equation 5.1) that neglected any effects of multiple thermal loads.

(8) Total heat output from a room's heat emitters was described by the turbulent natural convection equation (Equation 5.3).

(9) Radiant emission for the room's heat emitters was described by an empirically derived equation (Equation 5.6) for the mean emitter casing temperature.

(10) Heat flow into the wall behind the emitter was crudely based on maintaining its surface equal to the mean emitter casing temperature (Equation 6.26).

(11) Controls on heat output from the boilers were modelled as a maximum distribution water temperature of 170°F (72.2°C) and a maximum room ambient temperature of 68°F (20°C).

(12) Energy differential cost was based on a constant efficiency of heat production from fuel by the thermal plant.

(13) User cost was assumed to be proportional to the square of the deviation of ambient temperature from thermal comfort conditions.

(14) Constant values appropriate to winter design conditions in commercial buildings were assumed for the other four thermal environmental variables.

(15) User cost was based on the room's occupants' employment worth by assuming no work can be achieved when mean skin temperature is at the critical subjective tolerance limit without numbing.

(16) Constant rates of external air ventilation of 2.0 room changes per hour during occupancy and 0.3 room changes per hour during vacancy were assumed.
(17) Empirical expressions derived from the simulation run results are adequate to estimate the energy and user differential cost components of the hydronic heating design alternatives.

The estimates presented in Table 7.26 indicate that all the differential cost components are relatively insensitive to the hydronic heating equipment design variables: room emitter size and boiler capacity. Both energy consumption and user performance differential cost components exhibit variations that are an order of magnitude greater than the differences in capital costs, though they are still small. However, variations in energy consumption costs are compensated by opposing variations in user performance costs. Hence total cost differences are quite insensitive to hydronic heating sizing within the normal range of design choice.

Energy costs could prove to be more sensitive to design decisions if additional complexities are considered. Multiple thermal loads may modify the performance of the central plant from the assumed simple model used in the present study. Variable boiler efficiencies with heat output from a building's central plant could also prove pertinent. Negation of the effectiveness of equipment controls, due to such mechanisms as building occupants opening windows to produce excessive ventilation rates, can influence energy costs significantly. Large increases in the price of fuel could also increase their significance. However, within the assumptions of the present study, very large increases in fuel costs would be necessary to make energy costs significant with respect to equipment size variation options for individual rooms.

The results from the simulation runs and user cost estimates indicate that the quality of commercial building thermal environments designed for human requirements do not vary to any significant extent during the daily occupancy period in winter. As intermittently operated
hydronic heating is one of the most sensitive heating systems with respect to diurnal variability and the Christchurch weather data, that was used in the study, exhibits the largest diurnal variation for any New Zealand city, the study results represent relatively extreme conditions. It is therefore concluded that diurnal dynamic effects are of minor significance in building heating design.

As the dynamic thermal behaviour of buildings is not very sensitive to heating equipment design decisions it is concluded that the additional design resources, that the use of dynamic simulation models requires, are not justified for appraisal of special purpose heating equipment design alternatives. In Chapter Three it was noted that dynamic simulation was of value for cooling and air conditioning equipment design. Its potential for aiding appraisal of the thermal performance of architectural proposals was also noted. Simulation of both summer and winter conditions would be required.

The simulation study results give some indication of the significance of architectural decisions on winter thermal performance. Comparison of the estimated differential costs for the two rooms is possible because all the scales have similar arbitrary zeros for both rooms. For similar thermal environmental quality, which is measured by user cost, it can be seen that the energy costs for Room 2 are significantly in excess of the energy costs for Room 1. Room 2 requires additional heat input from the heating plant to compensate for high heat loss through its external walls due to both larger external wall area and lower quality thermal response. Detailed examination of the simulation run heat flows showed that glass area, external air infiltration rates, and external wall thermal response contributed most to room winter heating requirements. The study results are an insufficient basis for establishing any recommendations,
but further investigation of the value of dynamic simulation models for thermal appraisal of architectural proposals appears worthwhile.

The minimum of the total differential costs presented in Table 7.26 occurs for both rooms with their dimensionless parameters equal to unity. Judicious choice of both emitter ratio and boiler ratio produced this result. However, the occurrence of larger total differential costs when the boiler ratio equals 1.30 suggests that the common empirical adjustment [Mow, 1972] to the steady state plant capacity estimates to allow for intermittent heating may not be necessary. Although the 30% increase in boiler capacity decreases the user cost, increases in the boiler capital cost and, more significantly, in the energy consumption cost produce a greater total cost than without the 30% adjustment. More detailed simulation of the hydronic heating plant to take account of heat distribution to a number of rooms is required to confirm that such empirical increases in boiler capacity are unnecessary.

A number of conclusions can be drawn from the methodology used for the simulation study. The different behaviour exhibited by the rooms' dry bulb air temperature and mean radiant temperature demonstrated that measurement of both of these temperatures is necessary for detailed appraisal of thermal environmental quality. Their combination into a single variable, ambient temperature, proved to be satisfactory to meet the study objectives. Combination of heating plant energy input with other energy input to the rooms produced more meaningful relationships for energy consumption variation with external climate than use of heating plant energy consumption alone.

Although it proved to be difficult to establish relationships for the variation of thermal environmental quality and energy consumption with variation of external climate through the heating season, the
differential cost approach proved a feasible and valuable means of measuring the significance of dynamic simulation modelling for heating equipment design appraisal. Thus it is concluded that when all pertinent variations can be expressed in terms of monetary worth the differential cost approach is an effective aid for selection of the level of detail appropriate for a building design procedure. Its use for appraisal of design alternatives during the building design process is limited to problems that have all cost data readily available.

The user cost of thermal deficiency proved to be a useful measure of thermal design effects for use as part of the differential cost model. Difficulties associated with its evaluation were due to establishing the dynamic variation of thermal environmental quality, rather than the conversion of measures of such variation into user cost terms. As discussed in Chapter Four the formulated model for user cost of thermal deficiency can be applied to static heating design conditions. The user cost concept could also be applied to warm thermal environments and to visual and acoustic environmental performance measurement.

The problem of paucity of data resulting from the simulation runs was adequately overcome by choosing appropriate numerical expressions for variation with external air temperature so that both linear regression analysis and probabilistic moments could be used to evaluate the energy and user cost models. The present state of knowledge of climatic processes enables the prediction of the macro pattern of variations in weather, but occurrence of particular climatic conditions can only be described by probabilistic models based on past records. Thus simple modelling of functions of climatic variables is difficult. The use of probabilistic moments proved to be both an efficient and an effective modelling procedure.
Dynamic thermal behaviour of building environments is very complex. The finite difference model proved a feasible means of investigating such complex behaviour in detail, but extensive computing resources were required. The need for such extensive resources indicates that the more efficient response factor and harmonic models are better for design simulations. Although static thermal conditions can be simply modelled, no adequate method other than simulation is available for measuring cumulative dynamic effects occurring in building thermal environments.

A brief restatement of the main conclusions drawn from the simulation study follows.

(1) Total cost differences and all differential cost components are insensitive to hydronic heating design sizing.

(2) Diurnal dynamic effects are of minor significance in building heating design.

(3) Dynamic simulation models are not required for special purpose heating equipment design appraisal.

(4) Glazed area of external walls, external air infiltration rates, and external wall thermal response are the major factors that contribute to building winter heating requirements.

(5) The value of the commonly used 30% increase in hydronic heating boiler capacity to allow for intermittency is suspect and requires further investigation.

(6) The benefit of using dynamic simulation models for thermal appraisal of architectural design proposals requires investigation.

(7) Mean radiant temperature can exhibit quite a different dynamic response to dry bulb air temperature in a room of a building.
(8) Total heat input to a room proved easier to relate to external climatic variation than special purpose thermal equipment heat input alone.

(9) The differential cost approach is an effective aid for selection of the level of detail appropriate for building design procedures when all pertinent variations can be expressed in monetary worth terms.

(10) Use of differential cost models during the building design process is limited to problems that have all cost data readily available.

(11) The user cost of thermal deficiency proved to be a useful measure of the quality of thermal environments.

(12) The user cost concept has possible application to performance measurement for static heating design conditions, warm thermal environments, visual environments, and acoustic environments.

(13) Probabilistic moment models offer scope for efficient and effective modelling of any type of variation with external climate.

(14) Finite difference modelling proved a feasible means of simulating detailed thermal dynamic behaviour, but required extensive computing resources.

7.8 CHAPTER SEVEN SUMMARY

A simulation study was undertaken using the model described in Chapters Five and Six to derive empirical expressions for the three relationships required for evaluation of the differential cost model for hydronic heating sizing that was formulated in Chapter Four. The differential cost components for six hydronic heating design alternatives
for each of two typical commercial building rooms were subsequently evaluated. The results were used to appraise the value of dynamic thermal simulation models for building heating design, to investigate the detailed use and effectiveness of the differential cost approach and the user cost of thermal deficiency model, and to gain further understanding of the dynamic thermal response of buildings.

Six standardised diurnal profiles of external air temperature derived from Christchurch weather records were used for the simulation runs. Five distinct periods in the daily intermittent heating cycle were recognised. Durations of two of these periods: the initial occupancy deficiency period and the pre-heat period were recorded, together with values of the simulated daily energy supplied from the boiler and internal heat sources.

Two dimensionless parameters: emitter ratio and boiler ratio, were formulated and derived for each design alternative simulated, to aid analysis of the simulation run results. Regression analysis of the limited number of results was used to produce approximate numerical expressions for the required relationships. Use of probability moments enabled evaluation of both the user cost of thermal deficiency and the energy consumption cost models. Thus all differential cost components were evaluated for the six design alternatives for each of the two rooms and a number of conclusions, which are presented in Section 7.7, were drawn from the results and study methodology.
8.1 BUILDING DESIGN PROCESSING

The objective of building design is to describe, for construction purposes, a building that will provide a satisfactory environment for the activities it is to house. In pursuing this objective due regard must be given to any financial constraints, to the owner's objectives, and to any status, safety, and external environmental impact requirements. Evolution of the building design process continues with continuing technological development, emerging environmental consciousness, and the accompanying changes in society's expectations for its buildings. The greater number of requirements for buildings is increasing the complexity of design problems. Extensive development of design procedures is the most significant response to this increasing complexity.

Design processing involves two major activities: solution generation and appraisal of possible solutions. Each activity includes an analysis step: solution generation begins with analysis of the problem; appraisal begins with analysis of the generated solutions. The recognition of the two types of analysis explains an apparent incompatibility between other descriptions of design processing.

Solution generation uses five types of information: problem characteristics, creative stimuli, experience data, available commodities, and design methods. Criteria and constraints are detailed during solution generation and used for appraisal of the generated solutions.
Systematic analysis procedures have been developed to aid the definition and structuring of problem characteristics and the interactions between them. Simple problems do not usually warrant the additional effort such procedures require. However, as Luckman [1969] points out, systematic analysis is of value when the problem involves a large quantity of information, the consequences of a poor solution are significant, and the changes of producing a poor solution are high.

As creativity is a subconscious act, psychological stimuli are of more significance than formal procedures. The informal techniques that have been developed to aid creative solution generation are valuable if used correctly and with discretion.

Experience data that describes the characteristics of past building designs and their extent of success or failure is generally gained and disseminated by informal procedures. Feedback from building constructors is effective; but feedback about the designers' product-in-use is ineffectual. The inherent pressures that act against both the formal criticism of buildings and the extent of effort required to perform thorough appraisals have meant that most building appraisals published in technical journals are quite superficial.

The dissemination of information describing the increasing range of commodities available for inclusion in buildings has gained formality with the use of building information classification systems. Their successful use has been ensured by the pre-classification of documents before their dissemination. Development of performance standards, such as fire-ratings, for commodities will continue as the standards have proved to be useful aids for selection from competing products.

Finding a solution to a design problem is not the major difficulty in building design; it is the production of an effective solution that
takes account of the wide variety of requirements for present day buildings. Ideas for partial solutions emerge through subconscious associations, but adequate understanding of both the design problems and the performance of possible solutions are important prerequisites for effective solutions. Although some systematic design procedures have been developed to increase the efficiency of the design process, many have been developed primarily as means for aiding the design of better quality buildings. Further development of appraisal procedures appears to offer the greatest scope for taking account of the continuing research into more detailed understanding of building behaviour and the variation in priorities assigned to the various building performance criteria.

8.2 COMPUTER AIDS FOR BUILDING DESIGN

Much of the recent development of building design procedures has been made possible by the rapid information processing capabilities of the digital computer. Its logical sorting capability has been effectively used to aid detailed analysis of the interactive nature of problem characteristics and for automatic selection of commodities. Both performance attributes, when selection criteria and constraints could be modelled logically, and classification identifiers have been used as the basis of selection.

A number of building design procedures have been formulated to take advantage of the computer's arithmetic capability. Automatic solution generation procedures have been developed using optimisation theories; but as all criteria cannot be adequately represented numerically, the use of optimisation procedures is limited to the generation of possible
solutions for further appraisal with respect to the remaining criteria. The major role of the digital computer has been, and will remain, as an aid for the appraisal of design solutions when their behaviour can be modelled numerically and is of sufficient importance to warrant detailed appraisal.

The digital computer's power lies not only in its rapid processing of distinct capabilities, but also in its ability to automatically combine its logical sorting, arithmetic, storage, retrieval, and presentation capabilities in order to produce flexible integrated information processing. Extensive integration of computer capabilities for use by individual designers has been achieved where development and maintenance overheads could be shared amongst many designers. However, the feasibility of any computer aided design procedure is limited by:

(1) the necessity for subjective judgement to be used with objective design models.

(2) the availability of data in the required form for the procedure.

(3) the need for sufficient benefit to accrue from its use to warrant the additional effort required for problem formulation and computation.

(4) its compatibility with human methodological requirements, which are a significant part of any human activity.

The feasibility of automatic interdisciplinary information sharing is influenced by two further factors:

(1) the need for designers to both be responsible for and retain control of the information they produce, which is always to some extent incomplete.
(2) the low proportion of building information common to computer aided design procedures.

Computer aided interdisciplinary information sharing has only been successfully used for presentation of construction information after all design decisions have been made. Any successful integration of design decision-making aids will depend mainly upon the need as perceived by the designers who must use them.

8.3 OBJECTIVE MODELLING

Appraisal of design alternatives is concerned with the significance of differences. Hence the differential cost model that measures both resource and effects differences with the common measure, monetary worth, is proposed as an appraisal aid. It proved to be an effective aid for selection of the level of detail appropriate for a building design procedure when all pertinent differences could be expressed in monetary worth terms. Its use during the building design process is feasible if all cost data is readily available.

Use of a common measure, such as monetary worth, aids decision-making by dividing the appraisal process into less complex steps. Misuse can stem both from excessive consideration given to a common measure compared with that given to intangibles and from insufficient attention to underlying assumptions. However, if intangibles are not the predominant feature and underlying assumptions are made explicit, use of a common measure can aid understanding of the relative significance of factors, particularly when used with sensitivity studies.
8.4 THERMAL DESIGN OF BUILDINGS

A building's thermal environment results from the combination of many time-varying conduction, convection, radiation, and latent heat exchanges. The nature of the activities in a room influences the significance of each type of thermal interchange. Design for satisfactory thermal environments begins as part of multi-purpose architectural design. Thermal specialists continue it at a second level with the well defined sub-problem: the provision of special purpose thermal equipment.

The recently developed dynamic simulation models are of value for design of cooling and air conditioning equipment as they enable better equipment sizing and control design with consequential capital and energy cost savings. Although intermittent heating, external walls with low thermal mass, and high internal heat gains have increased the size of diurnal dynamic effects, these effects are of minor significance for design sizing of building heating equipment. Hence the extra design resources associated with use of dynamic simulation models is not warranted for special purpose heating equipment design.

Empirically derived models have proved of value for thermal design in the past, but have become outdated with changes of building styles. Hence the simplistic steady state model, with empirical modifications to allow for dynamic effects, remains the most appropriate heating load analysis model. However, the commonly used 30% increase in hydronic heating boiler capacity to allow for intermittency is suspect and requires further investigation.

As manual fuel estimation methods are only of value for estimating fuel storage requirements, dynamic simulation models are necessary for
comparison of fuel usage between design alternatives. The simple model of hydronic heating simulated energy consumptions that were insensitive to equipment size variation for individual rooms. However, more detailed modelling of thermal plant and the thermal influence of architectural decisions during both winter and summer may warrant detailed energy cost comparisons. The benefit of using dynamic simulation models for thermal appraisal of architectural design proposals requires investigation.

Four types of dynamic simulation models have been developed: analogue, numerical discretisation, harmonic, and response factor. Digital computer simulation models have outdated the use of analogue techniques for building thermal design problems. The lack of linearity constraints with numerical discretisation models makes them attractive for research purposes, but they require extensive computational resources. For design modelling, the harmonic and response factor models offer efficient, but more approximate, simulation. Response factor models offer scope for a wide range of detail from individual wall conduction response to both room and total building response.

8.5 HUMAN THERMAL ENVIRONMENTS

Thermal design of buildings is usually governed by human thermal requirements for comfort and performance. A person's thermal comfort results from a lack of consciousness of thermal stimulation. Any thermal discomfort is perceived through awareness of the extent of physiological regulation people automatically undertake to maintain thermal balance between their metabolic heat production and their heat loss to the surroundings. It is not possible to produce a set of thermal
conditions that would be regarded as thermally comfortable by all individuals because of the biological variance of people. Thus optimal thermal comfort, which is the condition in which the highest possible percentage of a normal group of people would be thermally comfortable, is the most appropriate human thermal comfort criterion for buildings that are not designed for individual people.

Peoples' performance of both mental and manual tasks is influenced by thermal conditions. The concept of arousal satisfactorily explains the influence of thermal sensations in addition to other influences on performance, such as personal motivation and type of activity. On present evidence, optimal thermal comfort remains the most appropriate criterion for building thermal design from both performance and comfort points of view.

Human thermal comfort is influenced by the six variables: human activity level, thermal resistance of clothing, dry bulb air temperature, mean radiant temperature, relative air velocity, and water vapour pressure. Both the sets of values that produce optimal thermal comfort and the relative influence of each variable on discomfort can be easily established from Fanger's [1970] human thermal comfort equation. Constant values appropriate to winter design conditions for commercial buildings are:

- human activity level = 1.2 mets,
- thermal resistance of clothing = 1.25 clo,
- relative air velocity = 20 ft per min (0.10 m/s),
- relative humidity = 50%,
- ambient temperature = 68°F (20°C).

Ambient temperature is the dry bulb air temperature in an equivalent human thermal environment in which the dry bulb air temperature equals the mean radiant temperature.
Designers' desire to provide thermal environments that produce the minimum number of complaints has resulted in heating plant designs that produce increased temperatures in buildings in winter with consequential reduced widths of the subjectively tolerated comfort zone, i.e. the significance of any deviation from optimal thermal comfort conditions is greater with higher room temperatures. Obliging people to wear clothing with greater thermal resistance is often a better solution to cool thermal sensations than the production of higher room temperatures.

Thermal quality is more easily expressed in terms of its antithesis: deficiency. Thermal deficiency of human environments is related to the extent of deviation of the thermal environmental variables from conditions that produce optimal thermal comfort. Measurement of such deviations in terms of building users' costs of reduced performance owing to thermal influences proved feasible and useful for cool environments.

The value of usage of such a monetary worth measure is in the objective processing it allows for appraisal of the quality of alternative thermal environments in relation to the cost of resources used to achieve them. The user cost model for thermal deficiency, as outlined in this report, has possible application to the determination of optimal static heating design conditions and to measurement of the quality of thermal environments resulting both from architectural alternatives and from cooling and air conditioning equipment options. The user cost concept could also be applied to measurement of the quality of visual and acoustic environments.
REFERENCES


MATCHETT, E., "Control of Thought in Creative Work", The Chartered Mechanical Engineer, April 1968.


APPENDIX B

DATA REQUIREMENTS OF THE MODEL

The data required by the simulation model is listed in Tables B.1 to B.7 together with the units and some of the values used for the simulation study. The external environment data is a time sequence of values for which hourly values over a 48 hour cycle were used in the study. Linear interpolation is included between the hourly values. Where two values are listed for a parameter, as in Tables B.3 and B.4, the first value was used for room A in the study and the second for room B. The values used for the wall and surface data are outlined for each wall and each room in Chapter Seven. A range of output indices and output time sequences were included as optional input for the model for testing and simulation study output. All this data can be optionally input to the model but default values appropriate to the simulation study are included in the model. A different set of units could easily be adopted for the model, but a number of constants would need to be modified.

The script F and radiosity factors listed in Table B.5 were computed from the data listed in Table B.8 using a matrix inversion fortran computer program. The configuration factors were determined from the geometric relationships between pairs of the rooms surfaces using a fortran computer program developed from the relationships presented by Hottel [Weibelt, 1965].
<table>
<thead>
<tr>
<th>Time Sequence Parameters</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>External air temperature</td>
<td>( F )</td>
</tr>
<tr>
<td>Total solar radiation intensity on room's external vertical surfaces</td>
<td>( \text{Btu/hr/ft}^2 )</td>
</tr>
<tr>
<td>Diffuse solar radiation intensity on room's external vertical surfaces</td>
<td>( \text{Btu/hr/ft}^2 )</td>
</tr>
<tr>
<td>Direct solar radiation intensity transmitted through vertical glass</td>
<td>( \text{Btu/hr/ft}^2 )</td>
</tr>
</tbody>
</table>

**TABLE B.1:** EXTERNAL ENVIRONMENT DATA REQUIRED BY THE MODEL
<table>
<thead>
<tr>
<th>Artificial Lighting Parameters</th>
<th>Units</th>
<th>Study Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total heat output during occupancy</td>
<td>Btu/hr</td>
<td>1044.</td>
</tr>
<tr>
<td>Total heat output during vacancy</td>
<td>Btu/hr</td>
<td>0.</td>
</tr>
<tr>
<td>Proportion of heat to plenum</td>
<td>-</td>
<td>.53</td>
</tr>
<tr>
<td>Room short wave proportion</td>
<td>-</td>
<td>.12</td>
</tr>
<tr>
<td>Remainder of room's proportion</td>
<td>-</td>
<td>.35</td>
</tr>
<tr>
<td>Lighting surface area</td>
<td>ft²</td>
<td>20.</td>
</tr>
<tr>
<td>Lighting surface temperature</td>
<td>°F</td>
<td>78.</td>
</tr>
<tr>
<td>Lighting surface short wave reflectance</td>
<td>-</td>
<td>1.0</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Human Parameters</th>
<th>Units</th>
<th>Study Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total heat output during occupancy</td>
<td>Btu/hr</td>
<td>500.</td>
</tr>
<tr>
<td>Total heat output during vacancy</td>
<td>Btu/hr</td>
<td>0.</td>
</tr>
<tr>
<td>Human surface area</td>
<td>ft²</td>
<td>30</td>
</tr>
<tr>
<td>Temperature of clothed surface</td>
<td>°F</td>
<td>77.4</td>
</tr>
<tr>
<td>Emittance to radiation</td>
<td>-</td>
<td>0.97</td>
</tr>
</tbody>
</table>

**TABLE B.2:** CASUAL SOURCE DATA REQUIRED BY THE MODEL
<table>
<thead>
<tr>
<th>Plant Parameters</th>
<th>Units</th>
<th>Study Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Maximum boiler output for the room</td>
<td>Btu/hr</td>
<td>study variable</td>
</tr>
<tr>
<td>Maximum average water temperature</td>
<td>°F</td>
<td>170.</td>
</tr>
<tr>
<td>Room emitter thermal coefficient</td>
<td>Btu/hr/F^N^emit</td>
<td>study variable</td>
</tr>
<tr>
<td>Room emitter exponent (N_{emit})</td>
<td>-</td>
<td>1.53</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Room Parameters</th>
<th>Units</th>
<th>Study Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Room volume</td>
<td>ft^3</td>
<td>1760.</td>
</tr>
<tr>
<td>Plenum volume</td>
<td>ft^3</td>
<td>352.</td>
</tr>
<tr>
<td>Night ventilation rate</td>
<td>ft^3/hr</td>
<td>528.</td>
</tr>
<tr>
<td>Initial occupancy ventilation rate</td>
<td>ft^3/hr</td>
<td>3520.</td>
</tr>
<tr>
<td>Day ventilation rate</td>
<td>ft^3/hr</td>
<td>3520.</td>
</tr>
<tr>
<td>Required ambient temperature</td>
<td>°F</td>
<td>68.</td>
</tr>
<tr>
<td>Effective emittance to long wave radiation of the external environment</td>
<td>-</td>
<td>0.9</td>
</tr>
<tr>
<td>Total number of walls</td>
<td>-</td>
<td>8.6</td>
</tr>
<tr>
<td>Number of external walls</td>
<td>-</td>
<td>2.2</td>
</tr>
<tr>
<td>Number of symmetric internal walls</td>
<td>-</td>
<td>4.2</td>
</tr>
<tr>
<td>Conduction Interpolation Coefficient</td>
<td>-</td>
<td>0.5</td>
</tr>
</tbody>
</table>

**TABLE B.3: PLANT AND GENERAL ROOM DATA REQUIRED BY THE MODEL**
### Glazing Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Study Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Area of glazing</td>
<td>ft²</td>
<td>33.48</td>
</tr>
<tr>
<td>Glass thermal capacity</td>
<td>Btu/ft²/°F</td>
<td>0.682</td>
</tr>
</tbody>
</table>

### Glazing External Surface Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Study Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Convection Coefficient</td>
<td>Btu/hr/ft²/°F</td>
<td>5.6</td>
</tr>
<tr>
<td>Emittance to long wave radiation</td>
<td>-</td>
<td>0.94</td>
</tr>
<tr>
<td>Absorptance to total solar radiation</td>
<td>-</td>
<td>0.16</td>
</tr>
</tbody>
</table>

### Glazing Internal Surface Parameters

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Units</th>
<th>Study Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Reflectance to short wave radiation</td>
<td>-</td>
<td>0.14</td>
</tr>
<tr>
<td>Transmittance to diffuse solar radiation</td>
<td>-</td>
<td>0.7</td>
</tr>
</tbody>
</table>

**TABLE B.4:** GLAZING DATA REQUIRED BY THE MODEL
<table>
<thead>
<tr>
<th>Wall Parameters</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Wall surface area</td>
<td>ft²</td>
</tr>
<tr>
<td>Number of elements</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Element Parameters</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness</td>
<td>in</td>
</tr>
<tr>
<td>Conductivity</td>
<td>Btu-in/ft²/hr/F⁰</td>
</tr>
<tr>
<td>Thermal Capacity per unit volume</td>
<td>Btu/ft²/in/F⁰</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>External Surface Parameters</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Conduction coefficient</td>
<td>Btu/hr/ft²/F⁰</td>
</tr>
<tr>
<td>Emittance to long wave radiation</td>
<td></td>
</tr>
<tr>
<td>Absorptance to solar radiation</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Plenum Surface Parameter</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Surface conductance</td>
<td>Btu/hr/ft²/F⁰</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Internal Surface Parameters</th>
<th>Units</th>
</tr>
</thead>
<tbody>
<tr>
<td>Script F factors for long wave radiation exchange excluding humans</td>
<td></td>
</tr>
<tr>
<td>Script F factors for long wave radiation exchange including humans</td>
<td></td>
</tr>
<tr>
<td>Ratio of absorptance to reflectance for short wave radiation</td>
<td></td>
</tr>
<tr>
<td>Radiosity factor with respect to floor</td>
<td></td>
</tr>
<tr>
<td>Radiosity factor with respect to artificial lighting</td>
<td></td>
</tr>
<tr>
<td>Radiosity factor with respect to glazing</td>
<td></td>
</tr>
</tbody>
</table>

**TABLE B.5: WALL AND SURFACE DATA REQUIRED BY THE MODEL**
<table>
<thead>
<tr>
<th>Internal Surface Convection Parameters</th>
<th>Units</th>
<th>Study Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Coefficient for vertical surfaces</td>
<td>Btu/hr/°F^N</td>
<td>0.19</td>
</tr>
<tr>
<td>Exponent for vertical surfaces</td>
<td>-</td>
<td>0.3333</td>
</tr>
<tr>
<td>Coefficient for horizontal surfaces with upwards heat flow</td>
<td>Btu/hr/°F^N</td>
<td>0.22</td>
</tr>
<tr>
<td>Exponent for horizontal surfaces with upwards heat flow</td>
<td>-</td>
<td>0.3333</td>
</tr>
<tr>
<td>Coefficient for horizontal surfaces with downwards heat flow</td>
<td>Btu/hr/°F^N</td>
<td>0.0626</td>
</tr>
<tr>
<td>Exponent for horizontal surfaces with downwards heat flow</td>
<td>-</td>
<td>0.25</td>
</tr>
</tbody>
</table>

**TABLE B.6: INTERNAL SURFACE CONVECTION PARAMETERS REUOERED BY THE MODEL**

<table>
<thead>
<tr>
<th>Run Parameters</th>
<th>Units</th>
<th>Study Values</th>
</tr>
</thead>
<tbody>
<tr>
<td>Duration of time step</td>
<td>hr</td>
<td>0.01</td>
</tr>
<tr>
<td>Start time</td>
<td>hr</td>
<td>0.</td>
</tr>
<tr>
<td>Finish time</td>
<td>hr</td>
<td>variable</td>
</tr>
<tr>
<td>Output indices</td>
<td>-</td>
<td>variable</td>
</tr>
</tbody>
</table>

**TABLE B.7: RUN DATA REQUIRED BY THE MODEL**