MATHEMATICAL COMPONENT STRENGTHS AND WEAKNESSES OF YEAR 4 AND YEAR 5 PRIMARY SCHOOL STUDENTS

A dissertation

submitted in partial fulfilment

of the requirements for the Degree

of

Master of Education

in the University of Canterbury

by

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March, 2010

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ACKNOWLEDGEMENTS

First, thank you to my supervisors, John Church and Jane McChesney. Thank you for the countless hours you spent thinking this project over, putting up with questions, and reading draft upon draft.

Thank you also to Ruth Humphries. Ruth, I think you know how much of a blessing it has been having a colleague who understands exactly what this project has been about and how much work it has been.

To the staff at each of the schools we visited thanks! Thanks for making us feel welcome at your schools, for putting up with disruptions to your classes, and for letting us take up room in the hall, office, and various other places. A huge thank you also goes to all of the students who completed the Canterbury Speedy Maths Test! We had a bit of fun I think. You have all done your bit to make maths easier for other kids.

Last, but not least, thanks to my husband, John, and our parents. All of you have borne the stress and strain of the last two years. Thank you so very much for this. Without you all I would not be at this point.

ABSTRACT

A lack of skill in particular component skills has been hypothesised as a cause of learning delays in children and this has been found to be the case in previous studies of reading delays (Smith, 2007; Williams, 2002). The present study explored this hypothesis with regard to the development of mathematical skills. The aim of the present study was to investigate whether the delays of children who are delayed in mathematical development are in part due to a lack of skill, particularly a lack of fluency, in particular component skills. Performance on several component skills was investigated: The ability to read and write numbers, to recognise quantities and equality, and to perform simple and more complex operations. Performance of each of these skills was compared in two groups of Year 4 and 5 (8-9 year old) children: a group of typically developing children and a group of children showing delayed development in mathematics. Children whose mathematical development was delayed were likely to be less fluent at performing each of the component skills tested than children whose development was typical. Additionally, children whose development was delayed were more likely to have low levels of fluency in several of the component skills. The results of the present study highlight the importance of building component mathematical skills to fluency.

Many adults lack confidence in mathematics and struggle with the numeracy skills required in everyday life. Being numerate can have a profound effect on an individual’s life: “to be numerate is to have the ability and inclination to use mathematics effectively in our lives – at home, at work, and in the community” (Ministry of Education, 2001, p. 1).

Often, the mathematics problems of adults can be traced back to the development of mathematics skills when they were children. Research has demonstrated that if mathematical delays are not identified early, they are likely to persist throughout schooling. Young children who have poor mathematics ability are likely to continue to have poor mathematics ability as older children (Lyon, 1996). Other negative outcomes are likely if mathematical delays are not remedied. For example, individuals with mathematics delays are likely to leave school early and are unlikely to continue to higher education (Lyon, 1996).

Once a child begins to fall behind, the gap between that student’s achievement and the achievement of successful students tends to increase in size. Stanovich (1986) refers to this as the Matthew effect in reference to the gospel of Matthew, “For to everyone who has will more be given, and he will have an abundance. But from the one who has not, even what he has will be taken away” (Matthew 25:29, English Standard Version). Stanovich discussed the Matthew Effect in relation to the process of learning to read. Good readers experience more success at reading, read more, and as a result their reading skills develop more quickly than the skills of poor readers. Matthew effects have also been observed in the development of mathematic skills (e.g. Cawley & Miller, 1989). Observations of the Matthew effect in mathematical development suggest that delays in mathematics are easier to remedy the earlier they are identified.

Historical conceptualisations of learning delays

During the second half of the twentieth century children with learning delays tended to be referred to as children with learning disabilities. Use of this term prompted researchers to search for a cause of the disabilities and a way to identify those who had them. To the present day, there is no standard way of identifying or diagnosing individuals with learning disabilities either within or between countries (Kavale & Forness, 2000).

A standard that has been commonly used to identify those with learning disabilities is the discrepancy standard. When there is a significant discrepancy between an individual's IQ, and thus their expected level of achievement, and their actual achievement in an academic setting they may be diagnosed with a learning disability (Lyon, 1996). The Diagnostic and Statistical Manual (American Psychiatric Association, 2000) requires the discrepancy between actual and expected achievement to be one which significantly interferes with an individual's daily life or further development in order for a learning disorder to be formally diagnosed. Although discrepancy standards appear to facilitate the identification of individuals with learning disabilities, they have been criticised as being difficult to define operationally (Kavale & Forness, 2000). Discrepancy models have also been labelled as 'wait to fail' models (Lyon, 1996) because they make early identification of individuals with learning disabilities impossible. In order to be identified as having a learning disability an individual must already be failing because of it.

In New Zealand, the learning disabilities concept has had a somewhat troublesome history. Historically the New Zealand Ministry of Education has not formally recognised learning disabilities (Chapman, 1992). More recently the Ministry has recognised one of the disabilities, that is, Dyslexia (Ministry of Education, 2007a). At present, children with learning disabilities are able to access assistance through the Ministry of Education but only if their disability adversely affects their access to the curriculum (Ministry of Education, 2010). For many children assessment, assistance, and support for their learning disabilities has come from private organisations; for example, the Dyslexia Foundation of New Zealand and, in Christchurch, the Seabrook-McKenzie Centre.

Mathematics achievement in New Zealand

Regardless of whether or not they have been identified as having a learning disability, a significant proportion of children in New Zealand achieve at a low level in the area of mathematics. These low levels of achievement have been revealed in several studies. In 1996 a large scale international study, the Third International Mathematics and Science Study, revealed that, on average, New Zealand 9 year olds were achieving at a lower level than similarly aged children in countries comparable to New Zealand such as Australia, England, Canada, and the United States (Chamberlain, 1997). Low levels of achievement by New Zealand children were seen to be linked to the socio-economic level of the school they attended, the language they spoke at home, and the ethnic group they identified with.

The mathematics achievement of Year 5 students (approximately 9 year olds) was assessed again in 2006 by the Trends in International Mathematics and Science Study (TIMSS). This study revealed that, on average, New Zealand students' performance was still significantly lower than students in Australia, the United States, and England. However, the mean score of New Zealand students was higher than it had been in the 1996 study and this improvement seemed to be the result of improved performance by the lowest achieving children (Ministry of Education, 2009a). At the same time, the Programme for International Student Assessment (PISA) examined the mathematical performance of 15 year old New Zealand children. PISA showed that the average mathematical performance of 15 year olds was comparable to that of students in countries such as Canada and Australia (Caygill, Marshall & May, 2008).

Several studies have examined the mathematical performance of New Zealand children at a national level. In particular, the Numeracy Development Project examined the mathematical performance of students in Years 5 to 9 (age 9-13 years), focusing particularly on the effect of a professional development programme for teachers. When Jenny Young-

Loveridge (2007) examined the achievement of more than 16,000 students in Years 5 to 9 she found that 22% of the Year 5 students were achieving at a lower level than they were expected to have been according to the national curriculum. These children were described as a “cause for concern” or “at risk” of failing in mathematics (p. 23). Young-Loveridge also found evidence for the Matthew effect. The proportion of students in “cause for concern” or “at risk” categories was higher for the older age groups. Fifty-three percent of Year 6 students, 44% of Year 7 students, and 58% of Year 8 students were classified as either “cause for concern” or “at risk”.

In their examination of a small longitudinal sample (n=83), Thomas and Tagg (2008) found much lower proportions of children to be achieving poorly at mathematics than Young-Loveridge. When the children in their sample were in Year 5, 12% had been categorised as a “cause for concern” or “at risk”. When the children were in Year 6 this figure increased to 15%, and in Year 7 it was 11%.

Regardless of the true numbers of children experiencing mathematics delays, it is clear that these children exist in sufficient numbers to warrant further study.

Changing New Zealand policies with respect to mathematics teaching

The poor mathematical performance of New Zealand children, as evidenced in the various studies outlined above, prompted the Ministry of Education to place greater emphasis on the development of number skills and concepts. Since 1996, several groups have been commissioned by the Ministry to examine the achievement of New Zealand children in the area of numeracy. These include the 1997 Mathematics and Science Taskforce, the 1999 Junior Mathematics Review Group, the 1999 Literacy Taskforce, the 2000 Numeracy Think Tank, and the 2001 Numeracy Development Project Working Group (Ministry of Education, 2001). These groups suggested several methods to improve achievement in the area of mathematics including professional development for teachers, a focus on teaching number especially early in schooling, development of a number framework and diagnostic tools to measure progress.

Following on from these recommendations, a major professional development programme, the Numeracy Development Project (NDP) was launched in 2001 (Ministry of Education, 2001). A key component of the NDP was the Learning Framework for Number (Ministry of Education, 2005). The Number Framework was conceptualised as a tool to help teachers understand how children learn and the things they need to learn about number. Based on research into mathematical development, it outlined the development of number skills as a series of stages which build on one another. Guidelines were given about the stages in which children will ideally be working at different points during their time at school (Ministry of Education, 2005). Progress through the stages of the Number Framework was to be measured by teachers using a variety of informal assessments together with the specially developed Numeracy Project Assessment (NumPA).

Late in 2009 the Ministry of Education launched National Standards for mathematics, to be implemented in 2010 (Ministry of Education, 2009b). These standards are intended to ensure that all children reach levels of achievement that will be sufficient for them to participate in the New Zealand economy as adults (New Zealand Educational Institute, 2009). All New Zealand schools are required to assess their students' achievement against the national standards regularly throughout the year. Information gained through this assessment must be used to identify children who are at risk of not achieving the required level of performance, and their subsequent learning experiences are to be modified. If children do not meet minimum standards, they are to be taught at a particular level until they do meet them.

Possible causes of mathematics delays

During the past 60 years, many factors have been hypothesised to explain delays in the learning of specific skills such as those involved in mathematics. Some of these have been located within the individual whose learning is delayed and others outside of the individual. For example, a number processing deficit is an internal factor which has been hypothesised to be the cause of delays in learning mathematical skills (e.g. Geary, 2003). In contrast, instruction that is inadequate in various ways has been hypothesised as an external cause of delays in learning mathematical skills (Church, 2008a).

Church (2008b) has outlined a number of the factors which have been hypothesised to explain delays in learning. These include (a) too few learning opportunities, (b) inappropriate teaching aims, (c) inappropriate teaching procedures, (c) failure to build skills to an appropriate level of fluency, and (d) inadequate monitoring of progress. A factor which is particularly important in the learning of mathematics skills is building basic skills to an appropriate level of fluency.

Delay due to lack of fluency building. Being fluent at a particular skill is being able to perform it accurately, and also at a useful speed (Binder, Haughton, Bateman, 2002). Several authors (e.g. Binder et al., 2002; Chiesa & Robertson, 2000; Church, 2008b; Johnson & Layng, 1992; Mercer & Miller, 1992) have highlighted the importance of individuals being able to perform skills fluently: If two individuals can perform the same skill, but one is much faster, it can be considered that the faster individual is more skilled (Chiesa & Robertson, 2000).

The performance of experts in many fields may be considered to be fluent. Bloom (1986) refers to this as automaticity. When skills are able to be performed fluently, or automatically, they require very little conscious attention and so this attentional capacity is freed for other tasks. In addition to freeing attention for other tasks, skills which can be performed fluently can be applied to complex and novel situations. They can also be used for extended periods of time, and be retained over long periods of time without practice (Binder et al., 2002).

Traditionally, in academic areas such as mathematics, most emphasis has been placed on developing a high level of understanding and accuracy rather than fluency. By considering only level of accuracy it is not possible to discriminate between a child who can solve a complex equation instantaneously and a child who takes 10 minutes to do so. A measure of fluency can do so by considering both the accuracy and the speed of a skill. Fluency is thus often measured by counting the number of correct responses per minute.

Delay due to missing component skills. Dowker (1998) argues that mathematical skill is not one entity but is comprised of many sub-skills or processes. She cites studies of typical adults and children and also those with arithmetical disabilities as evidence of the hypothesis and its history. The hypothesis that Dowker (1998) outlines might reasonably be extended to the sub-skills that make up mathematics as a whole, supposing that they are also combinations of several skills. Based on this argument, some children may be delayed in mathematics because they are not sufficiently skilled in these sub-skills or processes.

In the research literature, these sub-skills are known variously as component skills or tool skills. In this report they will be referred to as component skills. Component skills are "the most basic elements of more complex skills" (Johnson & Layng, 1992, p. 1479). In order for an individual to learn a new skill they must have acquired the appropriate component skills and knowledge, and so instruction must be developmentally appropriate, as Church (2008b) has outlined.

An example may be useful in illustrating the concept of component skills. Story writing may be considered a complex skill involving many component skills. In order for a child to write a story competently they need to be able to organise words into coherent sentences, have sufficient vocabulary and be able to spell these words, be able to form letters, and be able to grip a pen or pencil to write. If a child can perform each of these component skills then they should be able to learn to perform the complex skill of story writing.

The importance of fluency in component skills

Complex skills are likely to be acquired more easily and more quickly when their component skills are fluent. Church (2008b) summarises some reasons why this is the case:

a) The individual is then able to focus their attention on the complex skill rather than the component skills.
b) When a skill is fluent the individual is able to practise it more than a non-fluent skill in the same amount of time because they can work faster.
c) The individual can maintain a higher level of motivation throughout practice because they experience more success, and practice is experienced as faster and easier than if the component skill is not fluent.
d) Fluent component skills are more easily generalised and combined with other skills (Mercer and Miller, 1992).

Empirical research has demonstrated that fluency in component skills is important in the development of reading skills. For example, Williams (2002) investigated development of the component skills of phonemic awareness and decoding fluency in a sample of 63 children in Year 4 or 5 (8.5 to 9.5 years), whose development in the area of reading was either ‘normal’ or ‘delayed’. She found no relationship between phonemic awareness and reading skills, although this relationship was expected based on previous literature. Williams did, however, find a strong relationship between reading skills and level of decoding fluency: The children whose development was ‘normal’ had a high level of decoding fluency while that of the ‘delayed’ children was much lower.

In an extension of Williams’ (2002) work, Smith (2007) further investigated the relationships between reading skills, phonemic awareness, and decoding fluency. In her analysis of the reading skills of a sample of 103 Year 7 (11 years old) children who were described as either ‘good or ‘poor readers’ Smith was able to more precisely describe the relationship between the above mentioned component skills.

Smith (2007) found that the ‘poor readers’ all had poor reading comprehension and reading fluency. Additionally, the poorest readers also had poor phonemic awareness and poor decoding fluency. In contrast, the ‘good readers’ all had adequate abilities in phonemic awareness, decoding fluency, and comprehension. The best readers also had adequate levels of reading fluency. Based on this data, it appears that phonemic awareness is an early reading skill to develop. This is followed by development of decoding fluency, comprehension skills, and lastly reading fluency. Smith (2007) argues that the specific development of each of these skills, visible in her analyses, is not detected in routine testing by schools. Therefore, she argues, children’s specific areas of delay in reading are often not identified and appropriate assistance cannot be given when and where it is needed.

Clark (2001) investigated the utility of learning component skills with regard to mathematical development. In particular, he investigated the importance of fluency in single digit multiplication when learning to factorise quadratic equations. Clark found that when children who were fluent in basic multiplication practised and were tested on factorising, their performance increased and they met the fluency criterion after only five practice sessions. In contrast, when children who were not fluent at basic multiplication were tested they were unable to meet the criterion. All of these students reached a plateau in performance below the criterion. Clark gave the children who had low levels of fluency in basic multiplication an opportunity to practise this skill, and built their fluency until it reached the criterion level. When they had completed this practice, Clark once again taught these children to factorise. Their performance increased and they quickly met the criterion level of fluency in factorising.

In the course of his work in remedial teaching, Haughton (1972) observed the importance of practising component skills to fluency. Haughton's colleagues had applied this idea to the teaching of reading, writing, and mathematics. It was this approach which most often increased children's performance in the targeted area. Haughton and his colleagues found that, across all academic areas, a fluency level of between 100 and 200 movements (responses) per minute indicated proficient performance.

Importance of component skill fluency in mathematical development. It seems likely, given the empirical studies of reading development and literature concerning mathematical development, that fluency in component skills will also be important in mathematical development. Since Haughton's initial report, several studies have been undertaken to investigate this idea. A literature search on the subject of fluency building of component skills in children with mathematics delays produced seven reports. These are summarised in Table 1. In general, in these studies it was found that complex skills tended to be acquired more rapidly when component skills had been practised to a high level of fluency.

The development of number skills and understandings

Understandings of mathematical skills and concepts begin to develop during early childhood and research concerning this development has been outlined by several authors (e.g. Geary, 2006; Church, 2008a). Around the time they begin school, many children have begun to understand number concepts, they know some number names, and can count. They recognise that numbers can be represented in different ways. For example, "three" can be represented as a word, a symbol (3), and by three of various objects. They may even be able to solve simple mathematical problems when they are presented in context. However, not all children have this level of number understanding when they begin school. Young-Loveridge (1987, cited in Church, 2008a) found that a third of new entrants could not recognise a

Table 1.

The effects of teaching component skills on the subsequent performance of composite skills.

Participants	Method	Dependent variable and results
Denvir & Brown (1986)		
Participants were part of a larger study. All were described as "low achievers" by authors. Control students, n =2 (7:9 & 8:3) Experimental students, n=5 (7:9-9:7)	All participants were individually taught various new skills (e.g. counting on from larger numbers as a strategy for adding). Participants in the experimental group were taught a skill which required an already learned component skill (based on a "hierarchy" determined by authors in previous study). Participants in the control group were taught a skill which required component skills which had not been learned.	The number of skills acquired: Participants who had acquired important component skills acquired more new skills than did the participants who had not learned the component skills.
Johnson & Layng (1992)		
Single case study (Laurie, age unknown)	Laurie's teacher tried to build fluency in complex multiplication when Laurie's rate of fluency in multiplication facts was low. This was not successful and Laurie's fluency plateaued at a very low rate. Laurie's teacher helped her to build fluency in basic multiplication facts and then reintroduced more complex multiplication. Laurie's performance on the complex task improved steadily.	Number of correctly completed complex multiplication problems per minute: Fluency could be built in the complex skill only when fluency in the component skill was sufficient.
Chiesa & Robertson (2000)		
Participants were identified by their class teacher and a learning support teacher as unable to keep up with their classmates in mathematics, n=5 The other children in the classroom served as a control group, n=20 All participants were 9-	The week prior to the intervention, the entire class was taught the composite skill: Division of two digit numbers by a one digit number, up to and including five. The children in the experimental group participated in a programme of building component skills (multiplication fluency and number writing) using precision teaching methods. While the children in the experimental group participated in	Fluency in the target skill (rate per minute): Level of fluency in the composite skill rapidly increased after fluency in component skills was built. When fluency in component skills was not built, very little improvement in fluency of the composite skill was

Table 1 continued.

10 years	this programme, the other children in the class continued with their normal mathematics programme.	observed.
Clark (2001) Participants were classified according to their level of fluency on a test of basic multiplication facts. The High Fluency group were “able mathematicians” and achieved the fluency criterion, n=4 (Year 10-11) The Low Fluency group were “experiencing a range of difficulties with mathematics” and did not achieve the criterion, n = 7 (Year 10)	All students were instructed on the procedure for solving quadratic equations (composite task) and given opportunity to practise in pairs. This continued until students reached the fluency criterion or their performance plateaued. Students in the Low Fluency group built fluency in basic multiplication facts (component task) using written practise sheets until they achieved the fluency criterion. Following this, these students again practised factorising quadratic equations. Sessions continued until they reached the criterion.	Mean number of correct responses per minute on the factorising task: Students who initially had a high level of fluency in the component task were quickly able to master the composite task. Students who initially had a low level of fluency in the component task were unable to master the composite task until they were more fluent in the component task
Singer-Dudek & Greer (2005) Participants were 8 adolescents “with developmental disabilities and behaviour disorders” attending the same private day school. Participants were selected because they had the prerequisite skills for learning the component skills	The participants were taught the component skills (single digit addition and multiplication facts). Four of the participants were taught these skills to a fluency criterion of 100 written digits per minute. They were reinforced for faster rates of correct responding. The other four participants were taught to an accuracy criterion, they were reinforced for correct responses. Participants in this group continued to practise after they reached a high level of accuracy and until they had been given the same number of learning opportunities as participants who were taught to fluency. Both groups were given feedback regarding errors at the end of each session. After each of the participants met the relevant criteria they were taught the composite skill (multiplication of two digit numbers	(a) The number of lessons required to meet composite skill mastery criteria: On average, the students who had practised the component skills to mastery took fewer lessons to meet mastery criteria on the composite skill than the students who had practised the component skills to fluency. (b) The percentage of correct responses on one and two month maintenance trials: The students who had learned component skills to fluency were more accurate at the composite task after a period of no

Table 1 continued.

	by two digit numbers). They were reinforced for correct responses. This phase continued until the participant met an accuracy criterion. Composite skill teaching took four months to complete.	practise than were the students who had learned the skills to mastery.
Ezbicki (2008) Participants were 22 fourth grade students Participants were assigned to an experimental group and a matched control group	An intervention targeting addition and multiplication fact fluency took place over an 8 week period. The intervention consisted of instruction on various strategies to solve facts, untimed practise, goal setting, and short, timed drills of the targeted skills. After each timed drill, participants were given immediate feedback about their accuracy and speed in graphical form.	Fluency in both addition and multiplication facts increased after the intervention. There was evidence of skill transfer to non-targeted skills (fluency of subtraction, division facts) but this was not statistically significant. There was no evidence of skill transfer to measures of grade-level complex computation problems and applied mathematics problems
VanDerHeyden & Burns (2009) Participants were all second through fifth grade students attending one elementary school, n = 432 26 received special education services.	Children were all involved in a computational fluency-building intervention for four days per week for the school year. The intervention involved teaching of skills in a predetermined sequence (based on the order they were normally taught in classrooms). During each session, children were involved in guided peer practise and short group lessons where necessary. At the end of each session a probe test assessed children's performance of the skill. Within each class, when the median score on the probe test reached mastery level the entire class moved onto next skill in the sequence.	Average digits correct per minute on the probe test: Those children who had mastered skills that were earlier in the teaching sequence were more likely to learn later skills

specific number as representing a group of that many objects, 40% could not recognise specific numerals, and most could not add small numbers of objects.

Mathematics curricula tend to vary between countries. However there are a group of core concepts which are common to the curricula of most western countries. These skills include number, relational, arithmetical, and measurement concepts as well as specific arithmetical and measurement operations (Church, 2008a).

In New Zealand, the Number Framework outlines the mathematical concepts and operations which are taught in several different areas: Number, measurement, geometry, algebra, and statistics. By the end of Year 4, students are expected to be performing at Stage 5, and at the end of Year 5 students are expected to be performing at Stage 5 or 6 of the framework, as can be seen in Tables 2 and 3. If students are not meeting these expectations then their mathematical development is considered “at risk” or “cause for concern” (Ministry of Education, n.d.).

Table 2

Counting operations specified by the New Zealand Number Framework during the first five years at school (Adapted from Ministry of Education, 2005).

Stage	Addition and subtraction	Multiplication and Division	Proportions and ratios
1: One-to-one counting	Counts a set of objects but cannot form sets		Unable to divide set into equal parts
2: Counting from one on materials	Counts all using materials		Can divide a set into equal parts using materials
3: Counting from one by imaging	Images and counts all objects		Can divide a set into equal parts using materials or imaging
4: Advanced counting	Counts on or back	Uses skip counting May still use materials	
5: Early additive part-whole	Begins to use mental strategies to estimate and solve problems based on known basic facts	Uses known multiplication facts and repeated addition	Finds a fraction of a number, Derives from known addition facts

Table 3

Number concepts specified by the New Zealand Number Framework during the first five years at school (Adapted from Ministry of Education, 2005).

Stage	Number identification	Number sequence and order	Grouping/ place value	Basic facts	Written recording
<------1, 2, and 3: Counting from 1------>	Identifies numbers 0-10	Says number word sequences, forwards and backwards, the number before and after, for numbers 0-10	Instantly recognises patterns to five		
	Identifies numbers 0-20	Says number word sequences etc. for numbers 0-20 Orders numbers 0-20	Instantly recognises patterns to 10 Knows groupings within and with 5, and within 10	Recalls addition and subtraction facts to five Doubles to 10	Records the results of counting and operations using symbols, pictures, and diagrams
<------------4: Advanced counting------------> <-------------5: Early Additive------------->	Identifies numbers 0-100 Symbols for simple fractions	Says number word sequences etc. for numbers 0-100 Orders numbers 0-100	Knows groupings with 10 and within 20 Knows the number of tens in decades	Recalls addition and subtraction facts to 10 Recalls doubles to 20 and corresponding halves Recalls “ten and” facts Recalls multiples of 10	Records the results of mental addition and subtraction using equations
	Identifies numbers 0-1000 Symbols for most common fractions and improper fractions	Says number word sequences etc. for numbers 0-1000 Orders numbers 0-1000 and fractions with like denominators	Knows groupings within 100 Rounds three-digit whole numbers to nearest 10 or 100	Recalls addition facts to 20 and subtraction facts to 10 Recalls multiplication facts for the 2, 5, and 10 times tables and corresponding division facts Recalls multiples of 100	Records results of addition, subtraction, and multiplication calculations using equations and diagrams

Number reading and writing. Since many mathematical tasks involve written representations of numbers, it is important that children can read and write digits at a suitable speed. Several authors have pinpointed particular fluency rates for number reading and writing which are predictive of success in learning later skills. Children who are delayed in mathematics often have low levels of fluency in reading and writing numbers. For example, Johnson and Layng (1992) recount the story of Carter, a hypothetical student at one of their schools. When Carter completed initial testing at the school he could write the digits 0 to 9 at around 100 digits per minute students, whereas the fluency goal for this task at their school is 160 to 180 digits per minute.

Number reading can be measured by asking children to read number words aloud or to write them as numbers. Number writing fluency can be measured by asking children to write (or copy) numbers.

Number sense, number concepts, and quantity recognition. At a young age children seem to develop a number sense. Number sense has been defined in different ways by different authors (Berch, 2005). Ell (2001) summarises the idea of number sense as an understanding of what numbers are, how they relate to one another, how they can be used, and being able to work flexibly with numbers. Children who have mathematical disabilities have been shown to have poor number sense (e.g. Mazzocco & Devlin, 2008). Once children have number sense with respect to whole numbers, understandings of other number concepts, such as place value and equivalence, begin to emerge.

Central to number sense is the ability to recognise a given quantity (such as five) represented in different ways. Young-Loveridge refers to this as quantity recognition and argues that fast or instant recognition of small quantities is related to the development of mathematical competence (Young-Loveridge, 1991).

Arithmetical skills. Understanding of the basic number operations of addition, subtraction, multiplication, and division is thought to be essential for children to advance in mathematical understanding. To learn addition and subtraction, children develop additive thinking, that is , the ability to think of numbers as parts and wholes (Young-Loveridge, 2008).

In order to perform basic number operations, children use many strategies. When they first learn addition, children count all of a set of concrete objects. As their understanding develops they will often use more efficient strategies such as counting on from the larger of the sets, or retrieving an answer to a basic fact from their long-term memory (Geary, 2004).

In her study of the mathematical abilities of Year 4 students, Young-Loveridge (1999) details how the level of ability seemed to vary widely within the group. One-third of the children still counted by ones to add 10 to a number while nearly 30% of the students could complete a two-digit subtraction problem. Crooks and Flockton (2001, cited in Church, 2008a) also studied the mathematical abilities of Year 4 students. Most of the children in their sample could complete single-digit subtraction problems with high levels of accuracy.

The performance of children who are delayed in mathematical development

The performance of children who are delayed in their mathematical development tends to be similar to that of younger children (Geary, 1994; Torbeyns, Verschaffel, & Ghesquière , 2004). Torbeyns et al. examined the mathematical performance of students in Grades 4 to 6 who were delayed in mathematics. They found that their levels of performance tended to be similar to students who were in Grade 2. Kameenui & Simmons (1990) have explained that children who are delayed in their development of mathematics tend to make ‘predictable errors’ compared to children whose development is not delayed. For example, these children tend to have difficulty working with quantities which contain zeros.

Typically developing children tend to use more efficient strategies for solving problems than children whose mathematical development is delayed (Geary, 2006). For example, typically developing children are more likely to recall known facts than to count. In contrast, Geary (2004) outlines how children with delayed mathematical development seem to be ‘stuck’ on using counting to solve problems. Because these children continue to count, they continue to make counting errors, especially as the operations become more complex.

Children who are delayed in mathematical development also tend to work more slowly and with lower levels of fluency than typically developing children (Church, 2008a). Children who are delayed in mathematical development tend to have a lack of fluency in basic number facts (Kameenui & Simmons, 1990). Other predictable errors of these children include having poorly developed quantity concepts, being inaccurate at recognising numerals and counting objects in disordered arrays, and having difficulties with operations which contain zeros (Kameenui & Simmons, 1990).

Aims of the present study

The aim of the present study was to investigate whether children who are experiencing delays in the development of mathematical skills are lacking in either accuracy or fluency with respect to component skills. Several component skills that may not be developed in children who are delayed in mathematics were investigated: The ability to read and write numbers, to recognise quantities and equality, and to perform simple and more complex operations. The aim was to compare the performance of each of these skills of students with delayed development in mathematics against the performance of children who were known to be achieving at an average level in mathematics.

METHOD

Participants

Ninety students participated in the present study: Forty males and 50 females. The students were all Year 4 or 5 students during the 2009 school year. The mean age of the students was 9 years and 9 months at the time of testing.

The students attended three different schools. Demographic information describing each of these schools is shown in Table 4. More than half (52%) of the participants in the present study attended School A. School A was a full primary school, enrolling students in Years 1 to 8. It was located in a small city (approximate population 27,000) in New Zealand. School A had a slightly higher proportion of New Zealand European and a lower proportion of Maori students than the national population.

Table 4.

Demographic information for Schools A, B, and C.

	School			
	A	B	C	New Zealand population ^
School decile	7	5	10	
School roll	437	483	410	
Gender (%)				
Male	50	49	51	51
Female	50	51	49	49
Ethnicity (%)				
New Zealand European	81	84	77	79
Maori	13	10	6	15
Other	6	6	17	28

^ Total may be equal to more than 100% because individuals were able to nominate more than one ethnicity; adapted from Statistics New Zealand (2006).

Twenty-nine (32%) of the participants in the present study were from School B. Like School A, School B was a full primary school. The two schools were located in the same city. They had a similar number of students and their gender and ethnic compositions were similar.

The smallest group of participants in the present study (16%) attended School C. School C was in a small settlement near to a large city. A larger proportion of the students at School C came from households with a high socio-economic status than did the students at the other two schools. At School C a smaller proportion of students identified themselves as New Zealand European or Maori and a larger proportion identified themselves as another ethnicity than at the other two schools, possibly because the school was located near to a University. The proportion of New Zealand European students at School C was closer to being representative of the New Zealand population than School A and B.

The aim of the present study required the selection of two groups of students; a group who were making average progress in mathematics and a group of students whose development in mathematics was clearly delayed, that is, below the average. To select the participants, the author obtained the scores of all Year 4 and 5 students on the most recent administration of the Progressive Achievement Test: Mathematics (PAT: Mathematics) test at each school. Selection was based on the stanine score of each student. Stanine scores indicate which of nine equal intervals a particular score falls into. A stanine level of 1 represents the lowest 4% of scores and a level of 9 represents the highest 4%. Students who had scored in stanines 1, 2, 3, or 5 were eligible to participate in the present study. Those who scored in stanine 5 (i.e. the middle 20% of students) were considered to be achieving at an average level in mathematics for their age and were assigned to the “Typically Developing” group. Students with a stanine score of 1, 2, or 3 (i.e. the lowest scoring 22%) were assigned to the “Delayed Development” group. Forty-three of the participants in the present study were classified as being in the “Typically Developing” group and 47 of the participants were classified as being in the "Delayed Development" group. The two groups did not have equal numbers of participants because different numbers of children met the membership criteria for each group and all children who met the criteria were eligible to participate.

After a Principal expressed interest in students from their school participating in the present study, informed consent to participate was sought from the Principal, the school's Board of Trustees, and teachers of Year 4-7 students. The author supplied information sheets to the school and was also available for consultation if requested.

The parents/caregivers of each student selected to participate in the present study received information regarding their child's possible involvement. The task of sending the information sheets and consent forms to parents, receiving the consent forms which were returned, and selecting the students who should attend the testing sessions was undertaken by each of the three schools.

Measures

Progressive Achievement Test: Mathematics. The PAT: Mathematics test is a multiple choice test which was developed and standardised in New Zealand (Darr, Neill, & Stephanou, 2006). The PAT: Mathematics was designed to assess the mathematical skills and level of mathematical understanding of Year 4 to 10 students. It is based on the New Zealand school curriculum. At present, the PAT: Mathematics test is widely used in New Zealand schools and different subtests are provided for each year group. Raw scores from the test are transformed to scaled scores and stanines.

Canterbury Speedy Maths Test. The Canterbury Speedy Maths Test (CSMT) was designed by the author and colleagues with reference to the current New Zealand school curriculum and available research literature. It was designed to identify specific skill areas which may contribute to delayed progress in mathematical development.

The CSMT consists of a booklet of six pen-and-paper tests. The tests are described below. In order to maintain the children's motivation, the tests were arranged so as to alternate tasks which would be familiar to the children (e.g. writing numbers, single digit addition) and those which would be less familiar, or likely to be perceived as more difficult (e.g. solving complex equations). As can be seen in Appendix 1, each test begins with a practice page. Brief instructions for the test were printed on this page followed by four practice questions, similar to questions in the test. The answer to the first practice question was provided.

Number Writing. Test 1 was a test of children's number writing fluency, measured with a copying task. Children were presented with a 5 by 10 grid of digits, each paired with an empty box in which to copy the digit. They were told "I want to see how fast you can write numbers... I'm going to give you half a minute to see how many numbers you can copy". The test yielded two scores: (a) number copying fluency (the number of digits written per minute) and (b) number legibility (the percentage of digits copied legibly). As can be seen in Appendix 1, this test appeared in the test booklet as Task 3: Copy Numbers Fast.

Number Word Reading. Test 2 was a measure of children's ability to read number words. For each question the children were presented with a written instruction such as "write the number three" or "write the number that comes before one hundred." They were instructed to write the answer, for example the number "3" or the number "99" in the empty box alongside the instruction. The children were allowed one minute to complete the 20 questions. This test produced a single score: Number word reading fluency (the number of questions answered correctly per minute). This test appeared in the test booklet as Task 1: Writing numbers.

Quantity Recognition. Test 3 was designed to assess children's quantity recognition. It may also be considered to provide an indication of number sense and understanding of quantity concepts. Quantities were represented in seven different ways: A hen with different numbers of eggs, a crab with different numbers of legs, a building with many windows, some of which were broken, a dial with divisions, a die, a ruler, and a number line with divisions. Children were instructed to "...answer the question 'How many?' and write the number in the circle" as shown in Figure 1. They had 2 minutes to answer as many of the 40 items as they could. Two scores were calculated for Test 3: (a) percent correct (items answered correctly as a percentage of all items attempted) and (b) number correct per minute. As can be seen in Appendix 1, Test 3 appeared in the test booklet as Task 5: How many?

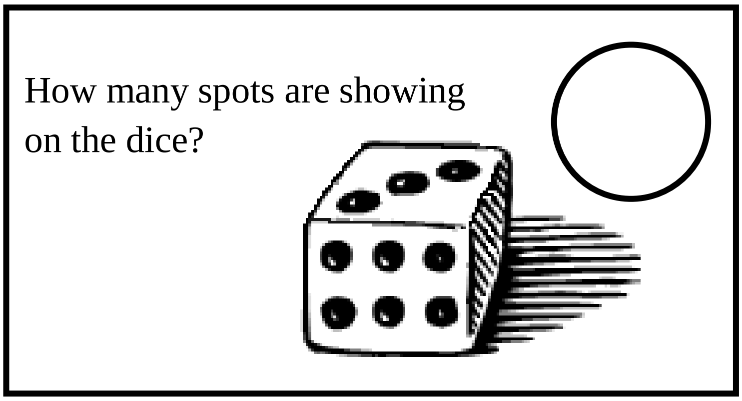

Figure 1. A sample question from Test 3 of the Canterbury Speedy Maths Test.

Single Digit Addition. Test 4 measured the ability of children to add single digit numbers. Children were told that "this task is to see how fast you can add numbers". They were presented with 80 horizontal equations (e.g. 6 + 8 = ☐) and were given one minute to write the answers to as many as they could. Two scores were calculated for Test 4: (a) number correct per minute and (b) percent correct (questions answered correctly as a percentage of questions attempted). Test 4 appeared in the test booklet as Task 4: Adding numbers fast.

Equality Recognition. Test 5 was designed to assess children's ability to recognise equality between two parts of a mathematical equation; for example, being able to recognise that 2 x 3 is equal to 2 + 2 + 2. For each question the child was presented with an equation, which contained either addition, multiplication, or both. Some equations consisted of dollar amounts. Alongside each equation the words “Right” and “Wrong” were printed. Children were instructed to “look at these number stories or equations. Some are correct and some are incorrect. Circle ‘right’ to show which ones are right and circle ‘wrong’ to show which ones are wrong”. They had two minutes to respond to as many of the 20 equations as they could. A score of number correct per minute was calculated for Test 5. This test appeared in the test booklet as Task 2: Which equations are correct?

Complex equations. Test 6 asked children to complete addition and subtraction equations with two and three digit numbers, with and without decomposition or regrouping. Each of the equations required a two digit number to be either added to or subtracted from a three digit number. The equations were selected so that half of the addition problems and half of the subtraction problems were ‘difficult’ (required decomposition or regrouping) while the others were ‘easy’ (did not require decomposition or regrouping). The equations were quasi-randomly arranged to be in the order ‘easy’ addition, ‘easy’ subtraction, ‘difficult’ addition, and ‘difficult’ subtraction. Children were instructed to solve as many of the equations as they could in 4 minutes. This test yielded five scores: number of easy addition equations correct, number of easy subtraction equations correct, number of difficult addition equations correct, number of difficult subtraction equations correct, and total number of equations correct. Test 6 appeared in test booklet as Task 6: Some hard ones.

Pilot testing. The CSMT was piloted with a group of 8 children aged from 7 to 12 years of age in August, 2009. The test was administered according to the instructions in the Administrator version of the CSMT (Appendix) and the procedure was similar to that described below. The pilot testing session lasted approximately 40 minutes.

At the end of the pilot session, the children were asked for feedback regarding the test. For example, they were asked if there were any parts of the test that were confusing or exceptionally difficult. Several minor changes were made to the CSMT following the pilot session: (a) The instruction that children should work as fast as they can was added; (b) The answer was added to the first question in each set of practice questions; (c) In Test 1, terminology was changed from “nought” to “zero”; (d) In Test 6, the font size of the symbols + and – was increased and they were made bold; (e) The time allowed for Test 3 was increased from 1 to 2 minutes and the time allowed for Test 6 was increased from 2 to 4 minutes.

Testing Procedure

Testing of the participants took place during November and December of 2009. The author, her colleague, or both supervised each test session. In all cases testing took place during the morning, in a room which had been designated as convenient by the school principal: a classroom, a meeting room, or the school assembly hall. The atmosphere in each room was similar to that of a typical classroom; the rooms were generally quiet and of a comfortable temperature. Each child sat on a chair with a desk or table at the appropriate height. Prior to children entering the testing room, a test paper was placed on each desk, with information and consent forms on top of it.

Groups of 10 to 12 students were tested at a time. Generally the students in each group were of similar year level. They were withdrawn from their classroom for testing at a time which was convenient for their teacher. Prior to leaving their classroom the students were informed that they were going to do some maths activities and were asked to bring a pen or pencil with them.

Children were directed to choose a desk to sit at and were greeted briefly before the information and consent procedures were explained to them. As part of the consent procedure, the children were informed about what they were being asked to do, why, and what would happen to the information they supplied. They were given an opportunity to ask relevant questions about the present study and were allowed to leave if they decided not to participate. Two children elected to leave at this point. Following this procedure, each testing session was completed according to the instructions outlined in the Administrator Version of the CSMT. Each testing session lasted approximately 30 minutes.

Children were given general information about the test before they began. This information related to, for example, the short duration of the test as a whole and the importance of following any instructions given. Children were also instructed to not worry about writing neatly but to try to complete each task as quickly as possible. They were also told that if they finished a test before the time limit then they were to sit quietly until the time was up.

Eighteen children answered all questions on one of Test 1, Test 2 or Test 3 - eight children in the Delayed Development group and 10 in the Typically Developing group. Of these children, one in the Delayed Development group and two in the Typically Developing group answered all questions on two of Tests 1, 2, and 3. One child in the Typically Developing group answered all the questions on all three of Tests 1, 2, and 3, and these were all scored as correct. Because these 18 children answered all of the questions on one or more tests, their fluency levels may be slightly underestimated. However, their scores are sufficient to establish whether they had mastered certain skills or not; in these cases, clearly they had. Whether any other children completed all of the questions, but not all correctly, on any tests was not recorded.

After general information and instructions were given, the administrator read the instructions and each practice question to the group and they attempted to answer these questions individually. At this time the children could ask questions. The administrator ensured all children had completed the practice questions and that they understood what was required of them to complete the test. The administrator detailed the amount of time which was allowed for the task and instructed the group to begin. Each test was timed. When the time limit was reached the administrator instructed the children to stop and praised them for their efforts. This procedure was repeated for each of the six tests.

RESULTS

Sample Characteristics

The Delayed Development and Typically Developing groups were largely similar with respect to mean age and year level, as can be seen in Table 5. In both groups, more of the children were in Year 5 than Year 4 at school. One child who was in Year 6 was accidentally included in the Typically Developing group. Of the children drawn from each of the three schools, approximately half were assigned into each of the Delayed Development and Typically Developing groups. There were more girls than boys in the sample because more consent forms were received for girls than boys.

Two children in the Delayed Development group appeared to have been misclassified (Participants 79 and 145). Although their PAT: Mathematics scores placed them within the Delayed Development group, their scores across all of the component tests were above the

Table 5.

Sample characteristics

		Delayed Development (N = 47)	Typically Developing (N = 43)	Total
Age (in months; at time of testing)		117.72	116.47	
Year level	4	16	15	31
	5	31	27	58
	6	0	1	1
School	A	23	24	47
	B	16	13	29
	C	8	6	14
Gender	Males	22	18	40
	Females	25	25	50

average of the children in the Typical Development group. This suggested that they were not actually delayed in their mathematical development. These two children were retained in the Delayed Development group, however, because they had met the operational definition of "delayed in mathematics" that is they had received a stanine score of 1, 2, or 3 on their most recent PAT: Mathematics test.

Scores of the Delayed Development and Typically Developing groups on the criterion and the tests of component skills

The mean scores of the Delayed Development and the Typically Developing groups on the PAT: Mathematics (criterion measure) and each of the six component skills tests are presented in Table 6.

Number Copying. Both groups achieved high levels of legibility on this test, and legibility did not differ significantly between the two groups. Mean levels of fluency on the Number Copying test were also similar and a little lower than expected for this age group.

Number Word Reading. On the Number Word Reading test, the children in the Delayed Development group responded significantly more slowly than the children in the Typically Developing group. Given that each question required four or five responses, the mean fluency level of the Typically Developing group may be considered to be adequate.

Quantity Recognition. On the Quantity Recognition task, the Delayed Development group was significantly less accurate than the Typically Developing group. The Typically Developing group was not all that accurate (85%) either.

In addition to not responding very accurately, children in both groups tended to complete the task with low levels of fluency. The children in the Delayed Development group responded significantly more slowly than children in the Typically Developing group.

Single Digit Addition. On the Single Digit Addition task, children in both the Delayed Development and Typically Developing groups responded with high levels of accuracy. Both

Table 6.

Mean scores, standard deviations, t-scores, and p-values for the Delayed Development and Typically Developing groups on the criterion measure and five component skills tests.

	Mean score (Standard deviation)			
	Delayed Development (N = 47)	Typically Developing (N = 43)	t	p (2-tailed)
PAT: Mathematics Stanine	2.38 (0.80)	5.00 (0.00)	-21.5	< 0.001
Test 1: Number Copying				
(a) Percent legible	98.9 (3.31)	98.7 (5.23)	0.27	0.79
(b) Number legible per minute	60.9 (13.9)	64.4 (14.3)	-1.18	0.240
Test 2: Number Word Reading				
Number correct per minute	10.2 (3.4)	13.0 (3.49)	-3.85	< 0.001
Test 3: Quantity Recognition				
(a) Accuracy	76.2 (12.3)	84.3 (10.2)	-3.39	0.001
(b) Fluency	6.77 (2.30)	8.79 (2.61)	-3.91	< 0.001
Test 4: Single Digit Addition				
(a) Accuracy	95.2 (8.00)	97.7 (4.42)	-1.77	0.081
(b) Fluency	17.0 (6.47)	18.9 (6.11)	-1.45	0.150
Test 5: Equality Recognition				
Fluency	4.65 (2.10)	6.43 (2.16)	-3.97	< 0.001
Test 6: Complex Equations				
Fluency	1.23 (0.99)	2.02 (1.14)	-3.49	0.001

groups averaged more than 95% correct. However, accuracy of responding in the Delayed Development group was twice as variable as the accuracy of responding in the Typically Developing group.

Although children in both groups responded accurately on the Single Digit Addition task, children in the Delayed Development group tended to respond with lower levels of fluency than children in the Typically Developing group. Considering that each question in this task required one or two responses, the mean fluency levels of both groups may be considered to be very slow.

Equality Recognition. On the Equality Recognition task, children in the Delayed Development group responded significantly less fluently than children in the Typically Developing group.

Complex Equations. On the Complex Equations task, children in the Delayed Development responded with significantly lower levels of fluency than children in the Typically Developing group. However, children in the Typically Developing group also responded with low levels of fluency (2 correct answers per minute). The results of the Complex Equations task were not considered further because children in both groups tended to respond with such low levels of fluency.

Correlations with the criterion.

Pearson Biserial Correlations between the criterion (group membership) and scores on the first five component skills tests are summarized in Table 7.

The correlations between Number Copying legibility and the criterion and between Number Copying fluency and the criterion were small. Similarly, the correlations between the criterion and (a) Single Digit Addition accuracy and (b) Single Digit Addition fluency were also fairly small.

Table 7.

Pearson Biserial Correlations between the criterion measure and scores on five component skills tests.

	Maths group	Test 1(a)	1(b)	2	3(a)	3(b)	4(a)	4(b)
Test 1: Number Copying								
(a) Legibility	0.115							
(b) Fluency	0.125	0.983						
Test 2: Number Word Reading								
Fluency	0.379	0.471	0.461					
Test 3: Quantity Recognition								
(a) Accuracy	0.339	0.175	0.132	0.261				
(b) Fluency	0.385	0.523	0.515	0.604	0.549			
Test 4: Single Digit Addition								
(a) Accuracy	0.185	-0.037	-0.073	0.301	0.216	0.174		
(b) Fluency	0.153	0.484	0.475	0.629	0.120	0.576	0.365	
Test 5: Equality Recognition								
Fluency	0.390	0.295	0.295	0.627	0.189	0.487	0.177	0.692

There were moderate correlations between the criterion and Number Word Reading fluency, Quantity Recognition accuracy and fluency, and Equality Recognition fluency. These correlations are consistent with the between group differences reported in Table 6 and suggest that the three fluency measures (Number Word Reading fluency, Quantity Recognition fluency, and Equality Recognition fluency) are the three variables which most strongly predict maths delays at this age level.

Analysis of patterns common to children in the Delayed Development group

Fluency scores on Number Word Reading, Quantity Recognition, and Equality Recognition were examined as part of this analysis. Number Copying and Single Digit

Addition scores were excluded from the analysis because they were not highly correlated with the criterion. Additionally, fluency on Quantity Recognition was included instead of accuracy because fluency on this test was more strongly related to the criterion.

For the tests considered in this analysis, the children in the Typically Developing group demonstrated fluency levels much lower than expected on the basis of published fluency criteria. The scores of the children in the Delayed Development group were lower still. Because of these factors, it was decided that the fluency criterion for this analysis would be the mean fluency level of the children in the Typically Developing group.

The percentage of children in the Delayed Development and Typically Developing groups who scored below the above defined fluency criteria are displayed in Figure 2. Around 50% of the children in the Typically Developing group scored below the criterion level for each test. This was expected as their group's mean fluency level on each test was used as the criterion. Of the children in the Delayed Development group, more than 70% scored below the fluency criterion on each of the tests examined here. The percentage of children in the Delayed Development group with fluency below the criterion was approximately the same for each test.

Because considerable numbers of children in the Delayed Development group scored below the criterion on each of the three tests, a further analysis was undertaken to see how many children in this group scored below the fluency criterion on more than one of the component skills tests. As can be seen in Figure 3, 76% of the children in the Delayed Development group scored below the fluency criterion for Equality Recognition. Seventy-two percent of this group scored below the fluency criterion in that test and also the Quantity Recognition test and 66% scored below the fluency criterion on all three of the tests being examined here. A similar trend was found to have occured with children in the Typically

Developing group, however far fewer of these children scored below both, or all three, of the fluency criteria.

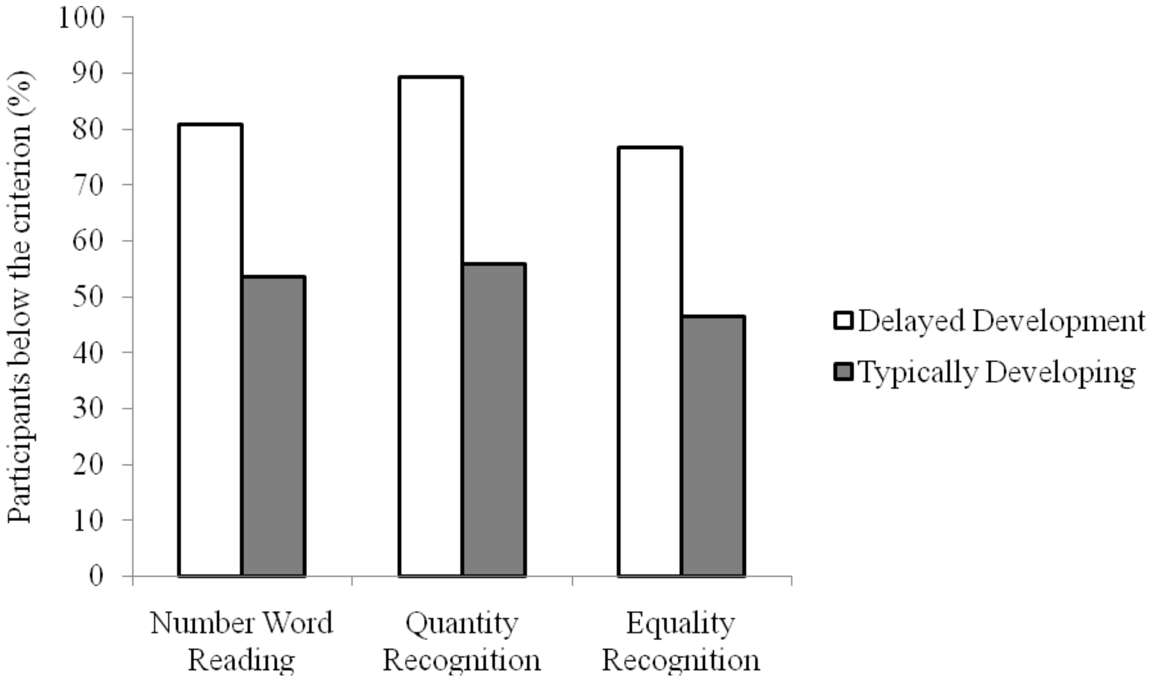

Figure 2. Percentages of children whose fluency was below the mean fluency of the Typically Developing group for three tests of component skills.

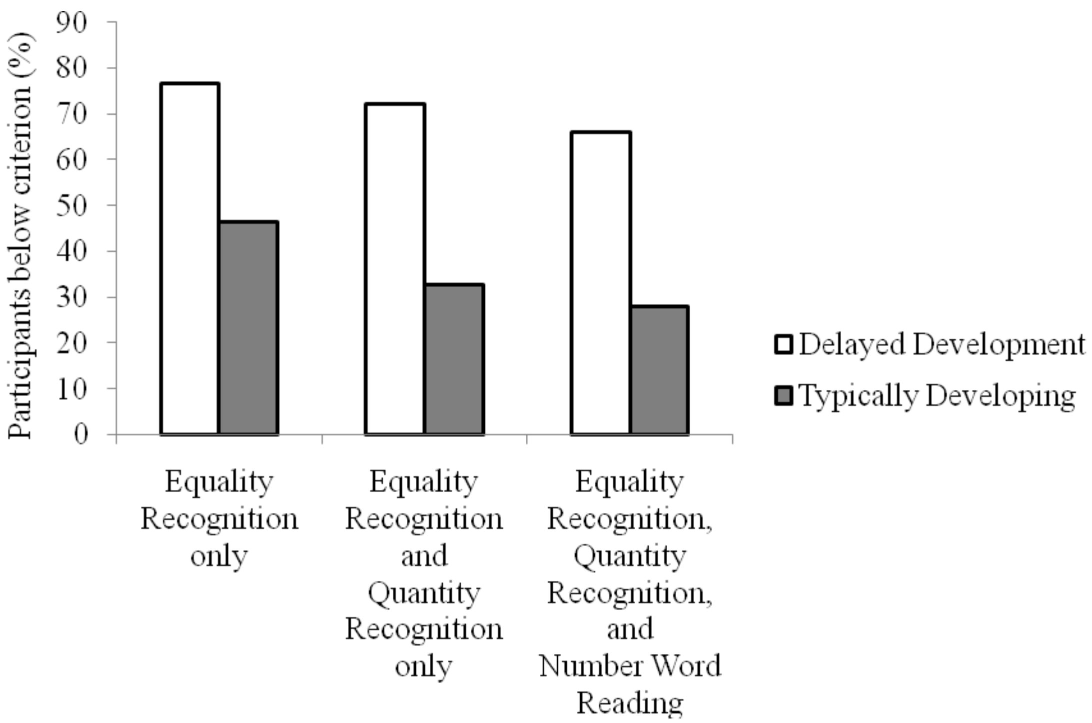

Figure 3. Percentages of children whose fluency was below the mean fluency of the Typically Developing group for combinations of three tests of component skills.

DISCUSSION AND CONCLUSIONS

For the purposes of the present study, children were classified as having "Delayed" or "Typical" development according to their performance on a standardised mathematics test which was based on the New Zealand curriculum. Six tests were used to measure the early number skills and understandings of children in both groups. The skills measured were number writing, number word reading, recognising quantity and equality, completing addition problems with single digits, and completing more complex equations.

Patterns of performance among children whose mathematical development is delayed or typical of children their age.

For each of the skills examined in the present study, children whose mathematical development was considered to be delayed were likely to perform at a lower level than the children whose development was considered typical. However, many children whose development was considered typical also performed at a low level on several of the tests. In the Delayed Development group there were few children who performed at a low level on only one test. It was much more common for these children to perform at a low level on several tests. These results suggest that delays in mathematics are unlikely to be a function of a failure to master just a single component skill.

Accuracy and fluency. Measures of accuracy did not predict delayed development in mathematics in the present study. Both groups of children tended to perform at similar levels of accuracy on the tests of Number Copying, Quantity Recognition, and Single Digit Addition. These results indicate that most children understand the skills required for these tests, and that these skills are well practised. These results may have been expected considering the emphasis placed on early acquisition of these skills in the Number Framework (Table 3).

When compared with measures of accuracy, children's rate of performance, or fluency, tended to vary with level of mathematical development, as indicated by performance on the standardised test. These results suggest that it is children's level of fluency, not accuracy, on component skills that is most likely to predict delayed development in number concepts and skills.

The importance of fluency in mathematical component skills

Number writing. The average levels of fluency of the two groups, around 60 answers correct per minute, were roughly consistent with the published fluency criteria presented in Table 8, but were a little lower than would be expected for children who have been at school for between four and five years. However, there were only 7 children in the Delayed group and 3 children in the Typical group who copied numbers at a rate of less than 48 numbers per minute. Based on these figures, a lack of fluency in this component skill cannot be a major cause of delayed mathematical skill for the children in this sample. The skill of number writing was involved in most of the other tasks. Because the majority of children did not seem to be seriously delayed on this task, it is unlikely that a lack of fluency in number writing is a cause of delayed development in mathematics at this age level.

Number reading. In the present study, the children whose mathematical development was considered delayed were significantly less fluent at reading numbers than were the typically developing children. This suggests that lack of fluency in this area may be hindering the development of more complex skills in these children.

It should be noted that the average fluency of both groups of children on this task seems to be somewhat lower than published fluency criteria suggest. It is difficult to estimate a criterion rate for this task because each question involves an average of six reading, comparing and writing responses. Children needed to read the number word and write the

Table 8.

Some component skills involved in mathematics and the fluency rates demonstrated to predict success in learning later skills.

Skill	Fluency rate
Writing numerals	> 30 digits/minute [1]
	160-180 digits/minute [2]
Copying numerals	80-120 digits/minute [3]
Reading numbers aloud	200-250 digits/minute [2]
	120-150 names/minute [3]
Writing answers to maths facts	70-100 digits/minute [3]
	80-100 facts/minute [2]
Completing basic computation	80 correct digits/minute [1]
(addition, subtraction, multiplication, division)	25-50 problems/minute [1]

[1]Haughton (1972)
[2]Johnson and Layng (1992)
[3]Binder, Haughton, and Bateman (2002)

digits that it requested. At 60 responses a minute, it might be expected that this task would be performed at a rate of around 10 answers per minute. This is approximately equivalent to the mean score of the students in this study. There were 7 children in the Delayed group and two in the Typical group who answered at a rate of less than 8 correct answers (48 responses) per minute and a lack of fluency in this skill may have been hindering the progress of these students.

Recognising quantities. Many of the children in the Delayed group were very slow in recognising quantities represented in various ways. This was also true of the children whose development was considered typical. It was clear during testing that, instead of instantly recognising quantities, many children were using the less efficient strategy of counting out the quantities in each example. In addition, the error rate on this test, especially on the questions which involved dials and number lines, was higher than expected.

Addition with single digits. On average, the children in both groups completed the single digit addition problems to a high level of accuracy and there was no significant difference between the mean scores of the two groups. However, the children in both groups were very slow at this task. The mean of the typical group, at 19 answers (29 digits) per minute, was only about half as fast as might be expected after between four and five years at school. Clearly the children in the sample had practised Single Digit Addition to a high level of accuracy at school and this is consistent with the emphasis in the New Zealand Number Framework. However, the children appeared to have received little practise in committing these number combinations to memory.

Recognising equality. Among the children in the present study, those whose mathematical development was described as typical were able to recognise equality within an equation more rapidly than the children in the Delayed group. Here too the children in both groups were very slow. Given three to four unitary responses per question, a mean of 6.5 answers per two minutes (3.25 per minute) is equivalent to about 11 to 12 responses per minute. While the test involved a task which was novel to most students this is still very slow and suggests that many children must have been counting the answers on this task too.

Complex equations. The children in the present study tended to score at a very low level when completing the test of complex equations. However, scores on this test were more variable for children who were considered to be typically developing. These results suggest that most children simply do not know how to complete addition or subtraction problems when they are presented as vertical algorithms. The higher variability of scores for typically developing children suggests that some children in this group may have received some instruction about completing this type of equation, or that the instructions and practise examples prior to the test were sufficient for them to understand what was required of them.

These findings are consistent with teaching to the New Zealand curriculum, as outlined in the Number Framework. In the Number Framework, it is not expected that children will be able to complete vertical algorithms such as those used in the test of complex equations until they are at Stage 6 (Ministry of Education, 2007b). Children who are achieving at curriculum expectations are not expected to be learning at Stage 6 until the end of Year 6 at school (Ministry of Education, n.d.). Teachers at several of the participating schools indicated to the author that they had not taught most of the participating children about vertical algorithms and that they did not think most of the children would be able to complete them.

Specific characteristics of children who are delayed in mathematics

Based on previous literature, it had been hypothesised that a specific pattern of delays would be associated with children who were delayed in mathematics. However, this was not found to be the case in the present sample. Children who had been assigned to the Delayed group were likely to perform at a low level of fluency, compared to levels which had been expected and compared to children in the Typically developing group, on the Number Word Reading, Quantity Recognition, and Equality Recognition tests. However, many of the typically developing children also performed at a low level of fluency on these tests. There was not one single skill, or a group of skills, in which children whose mathematical development was delayed performed substantially worse than the children whose development was typical.

Children with delayed development in mathematics were likely to perform at a low level of fluency, compared to the typically developing group, on more than one skill. Two-thirds of children in the Delayed Development group demonstrated a low level of fluency on all three of the tests mentioned above, compared to less than one-third of typically developing children. When mastery was defined as the average fluency level of the typically developing children, 20 (43%) of the children in the delayed group had yet to achieve mastery of any of the skills tested, compared to less than 20% of typically developing children.

These findings suggest that if children who are delayed in mathematics are to receive appropriate remedial instruction diagnostic tests need to assess children's levels of fluency, as well as their understanding, of basic number skills. Additionally, a range of skills need to be assessed because different children may be lacking in mastery with respect to different combinations of basic number skills. Some children may demonstrate adequate levels of fluency in most skills, but lack fluency in performing particular number skills. However, those children whose mathematical development is most delayed may not be able to perform any number skills with adequate levels of fluency.

Matthew effects in mathematics

It seems that the ability of the component skills tests used in the present study to predict mathematical achievement is higher with older children than with younger children. In a study parallel to the present one, Humphries (2010) used the same six tests of component skills to examine the performance of Year 6 and 7 students. Over all of the tests, children in Humphries' study tended to perform at a higher level than those in the present study, as is to be expected. Additionally, differences in performance between the children in the Delayed and Typical groups tended to be larger for the 10 and 11 year olds in Humphries' study than they were for the 8 and 9 year olds in the present study. On the Single Digit Addition test, for example, Humphries found that children who were considered to be typically developing tended to perform the task significantly more fluently than those considered to be delayed in mathematical development. Such a difference was not found in the present study. The observation of larger differences between the two groups in the Humphries' study of older children is consistent with previous studies which have found Matthew Effects in mathematical development (e.g. Cawley & Miller, 1989).

Limitations of standardised tests of mathematics

In the present study, performance on a standardised mathematical test (PAT: Mathematics), was used as an operational definition of mathematical ability. Children who performed at a low level on the PAT: Mathematics were classified as “delayed” in their mathematical development and children who performed at an average level on the test were considered to be “typically developing.” However, there were at least two children who were misclassified, and another three who were probably misclassified, in that they obtained low scores on the PAT: Mathematics but above average scores on all or most of the component skills tests. The cause of this discrepancy is unknown. Possibly the PAT: Mathematics tests of these children were incorrectly scored. To guard against this kind of problem in the future, investigators should check the validity of the standardised scores against another source of information. For example, investigators could discuss children’s mathematical development with their class teacher and examine other mathematical tests they have completed, to validate the initial classification of “delayed.”

Limitations of the component skills tests

Appropriateness of the tasks. The complex operations test, while possible having diagnostic value for older children, was clearly inappropriate to use with Year 4 and 5 children. Teachers observed that Year 4 and 5 children have little contact with vertical format addition and subtraction operations and the very low scores on this test confirmed that this is the case.

Multiple-choice format. In the present study, the Equality Recognition test was presented in a multiple-choice format. Children were to select either “right” or “wrong” in response to each equation. The test was presented in this way to limit the amount of writing required to complete the test. However, many children seem to have selected from the two answers semi-randomly, perhaps without even examining the relevant equation. Although only two children in the present study answered all 20 questions of this test correctly, 16 children selected an answer to all of the questions. Examination of these children's test papers indicated that several seemed to have followed some type of pattern, such as alternating right and wrong, when answering. Since the children in the Delayed and Typical groups tended to write numbers with similar levels of fluency, this test might be better presented in a different way in future studies. For example, children could be asked to complete equations by adding either numbers or symbols, so that both sides of the equations are equivalent.

Mathematics and reading. Deficits in mathematics are often comorbid with difficulties in language, whether written or oral (Lyon, Fletcher, Fuchs, & Chhabra, 2006). Therefore, if any of the children in the present study had difficulties with reading, these may have negatively affected their performance on the criterion measure and several of the component skills tests. While none of the children had difficulties that were obvious during testing, some may have more subtle difficulties. In an attempt to limit the effect that any reading difficulties might have on the component skills tests, instructions were always read aloud to participants and their understanding was checked during the completion of the practice questions. If any children were unable to complete the examples, or did not seem to understand what was required, one of the administrators spent a small amount of time with them explaining the task. Considering these factors, the effects of any reading difficulties on the present results should be small.

In future studies it could be useful to consider the reading abilities of participants. This could be done using a formal reading assessment, whether administered specifically as part of the study or prior.

Mathematics and behaviour problems. Learning delays can co-occur with attention disorders and also other social adjustment problems (Lyon, 1996). There seemed to be few children who were reluctant to participate in the present study. The children who were selected to participate seemed happy and willing to do so. When children were withdrawn for testing, it was explained to them that they were going complete “a new kind of maths test” that was made up of several short tests and that they were being done to find out how children learn mathematics, and to try and make it easier for children to learn. Children were given several opportunities to leave if they did not want to participate. Only two children did so.

When the author returned to the schools to continue to test children, those who had already participated seemed to have told their classmates what was involved generally, and many children who had not been selected wanted to participate. On one occasion when testing was not complete before a morning-tea break, one child said that they were happy to stay in during their break “as long as we’re helping other kids.”

Implications for the teaching of mathematics

In the present study quite low levels of fluency were observed on a number of basic mathematics tasks. If these skills are not practised to an adequate level of fluency, the acquisition of more advanced skills is likely to become increasingly difficult. This is already beginning to become visible in the performance of many of the children in the Delayed Development group. Some authors have attributed this to the so called “spiral curriculum” in which many skills are introduced and then revisited from year to year. Binder and his colleagues (2002) have outlined how moving students on to learning different skills before they have adequate fluency levels can undermine their self confidence and diminish the chances they will have learned the skill well enough to remember it later.

In order to ensure that remedial instruction is provided to those students that are in need of it, assessment needs to be used to check what students need to know, monitor how well they have learned it, and to inform future teaching (Gurganus, 2007). In New Zealand, mathematics achievement is assessed using various tools such as PAT: Mathematics, NumPA, Assessment Resource Banks, Assessment Tools for Teaching and Learning, Knowledge Assessment for Numeracy, Global Strategy Stage Assessment, and less formal assessments. Few of these tools provide estimates of fluency regarding particular skills.

After children with less fluent skills have been identified, programs to build fluency in particular skill areas will need to be developed for individual children. Binder and colleagues (2002) outline some important components in a fluency building programme. The programme must provide children with adequate opportunity to practise their target skill. Practice should occur every day, at least initially. It should be for short periods of time (i.e. 2 to 3 minutes of practice), and it should always have a clear goal. Progress toward this goal should be graphically recorded. It is important that progress toward fluency goals be carefully monitored and that fluency continues to improve. Intervention must target the simplest component skill and build this to fluency initially before moving on to more complex skills. This type of practise is embodied in instructional models such as Morningside (Johnson & Layng, 1992) and Direct Instruction (Brown, 1985).

Implications for future research

The present study aimed to investigate whether some children are delayed in mathematical development in part because they are lacking either accuracy or fluency of certain component skills. The study found that Year 4 and 5 children who were classified as delayed in mathematics were likely to have similar levels of accuracy but lower levels of fluency than children who were developing typically in performing the component skills of number reading and writing, recognising quantities and recognising equality. Children who were delayed in mathematics were also more likely to perform several of these component skills with a low level of fluency than were the typically developing children.

Given these findings, future research might profitably focus on further developing assessments of component skills, particularly focusing on the assessment of fluency.

Consideration should be given to which skills are important to assess. It is possible that other important early number skills may be lacking in children in Years 4 and 5, and these too should be assessed. Additionally, it will be important to assess appropriate number skills when considering children at different stages of schooling, independent of their chronological age.

As well as assessment, future research should attempt to continue to review ways to increase component skill fluency in children whose mathematical development is delayed. Given that these children frequently seem to have low levels of fluency in many skills, it will be important to determine the order in which component skills should be targeted for fluency building. At this point a clear hierarchy of mathematical skills is not clear, although some authors have attempted to establish one (e.g. Denvir & Brown, 1986).

Conclusions

Many of the children in the present study who were considered to have delayed development in mathematics were found to have low levels of fluency in the early number skills examined. Given that these students were in their fourth and fifth years of schooling, these results are concerning. It is likely that if the skills deficits of these children had been identified at an earlier stage of their schooling that they may not have been classified as Delayed at Year 4 or 5. If this were the case they may not be at risk for many of the negative outcomes associated with early mathematical delays.

In order that children such as these are identified earlier in their school career, it is important that early mathematics teaching in New Zealand schools be further developed. Continuing emphasis must be placed on the acquisition, and fluent performance, of basic mathematical skills by all children. Additionally, sufficient instruction, opportunities for practice, and assistance needs to be provided for those children who struggle to acquire fluency. This assistance must be provided early on, while the gap between these children and those who are making typical progress is still small, and before a pattern of mathematical failure becomes established.

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APPENDIX

My name ______________________

My age ______________________

My birthday ______________________

My class ______________________

NUMBER:

Canterbury Speedy Maths Test

Administrator Edition
Version 1

Prepared by
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The instructions in *italics* throughout the test are provided for you to read as you administer the Canterbury Speedy Maths Test. Additional advice and guidance is also provided.

The test is to be administered to groups of no more than 10 children at a time ideally with two administrators, one to read the instructions and another to supervise and help with administration. You will also need a timer.

On a table set out a test paper (face down) and a pencil for each child. When all the children are seated begin reading the instructions below:

Today we are going to do a new kind of maths test.
Actually it's six tests, but they are really short.
I'll show you how to do each one and then you'll have a bit of time, like one or two minutes, to complete the test. And then we'll stop and get ready for the next one.
There are lots of questions so you may not finish them but try your best.
On some of the pages there are little "Stop" signs. When you see one of them it means that you don't turn over the page until I tell you to.
I am also going to use this timer [hold up the timer]*, and when it rings it's time for the test to stop and for you to put your pencils down* [Demonstrate the timer ringing]
Please begin by putting your details on the test [Wait for all children to do this]
Now turn to the next page, called Task One.

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Task 1. Writing numbers

Start Here.

Can you read these number words?

Read the number word and write the number in the box.

1. Write the number three	**3**
2. Write the number that comes **before** three	
3. Write the number twenty-three	
4. Write the number that comes **after** twenty-three	

[Set timer to 1 minute]
This is a test to see if you can read number words.
Let's have a practice. The first question says "write the number three", *and the numeral three is written in the box. Lets see if you can do the next one ,*"write the number that comes before three"
[Check answer and do the other two practice examples as a group, give feedback]

Now I want you to do the same thing but for real this time.
When I say go I want you to turn the page and start.
Remember, read the number word and write the number in the box.
Work as fast as you can and see how many you can do.
Don't worry about being neat, just do them as quickly as you can [This is a very important instruction!]
Don't start until I tell you.
Ready? Go [start the timer]
[When the timer rings make sure all children stop writing straight away. Praise them for effort on the task]
Turn to the next page called Task Two

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Task 1

Read the number word and write the number in the box. See how many you can write.
Do all the number words down one side of the page first.
Don't start until the teacher says

1. Write the number nine
2. Write the number that comes **before** one
3. Write the number eighteen
4. Write the number forty-three
5. Write the number that comes **after** sixty-six
6. Write the number that comes **before** thirteen
7. Write the number zero
8. Write the number that comes **after** zero
9. Write the number one hundred and six
10. Write the number that comes **before** fifty-eight
11. Write the number that comes **after** nineteen
12. Write the number seventy-five
13. Write the number that comes **after** ninety-nine
14. Write the number six
15. Write the number three hundred and twelve
16. Write the number that comes **before** one hundred
17. Write the number eleven
18. Write the number five hundred and sixty seven
19. Write the number that comes **after** seven hundred
20. Write the number that comes **before** one hundred and fifty

Task 1

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Task 2. Which equations are correct?

Start here. Look at these number stories. Some are right and some are wrong.

Circle Right or Wrong to show which ones are right and which ones are wrong.

1	3 + 1 = 1 + 3	Right	Wrong
2	3 x 2 = 2 + 2	Right	Wrong
3	$5 + $3 = $2 + $2 + $2	Right	Wrong
4	4 x 2 = 5 + 3	Right	Wrong

[Set the timer to 2 minutes]
Look at these number stories or equations. Some are correct and some are incorrect. Circle "right" to show which ones are right and circle "wrong" to show which ones are wrong.
The first practice one says "3 + 1 = 1 + 3" *Is that correct or incorrect?*
[Wait for a response]
This equation is correct so we have circled right.
The second one says "3 x 2 = 2 + 2" *Is that right or wrong? So circle* wrong.
The third one says "$5 + $3 = $2 + $2 + $2" *Is that right or wrong? So circle* wrong.
The fourth one says "4 x 2 = 5 + 3" Is that right or wrong? So circle right.

So now you are ready to do some more.
When I tell you, you can turn the page and start.
Remember to circle right or wrong to show if the equation is right or wrong.
Work as fast as you can and see how many you can do.
Don't start until I tell you.
Ready? Go. [Start the timer]
[When the timer goes off make sure all children stop writing and praise them for the effort]
Turn over the page to Task Three

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Task 2
Circle Right or Wrong to show which ones are right and which ones are wrong.
Don't start until the teacher tells you to.

1	3 + 5 = 5 + 3	Right	Wrong
2	6 x 1 = 4 x 2	Right	Wrong
3	\$2 + \$7 = \$8 + \$1	Right	Wrong
4	4 x 4 = 8 x 2	Right	Wrong
5	12 + 2 = 6 + 6	Right	Wrong
6	2 x 10 = 4 x 5	Right	Wrong
7	\$5 x 3 = \$4 + \$4 + \$4	Right	Wrong
8	4 + 4 = 4 x 2	Right	Wrong
9	14+ 2 = 3 + 15	Right	Wrong
10	5 + 5 = 6 + 4	Right	Wrong
11	2 x 3 = 3 + 3 + 3	Right	Wrong
12	\$4 + \$7 = \$1 x 10	Right	Wrong
13	6 + 4 = 1 + 10	Right	Wrong
14	8 x 2 = 2 x 8	Right	Wrong
15	\$1 + \$1 + \$1 = \$3 x 1	Right	Wrong
16	3 + 9 = 10 + 1	Right	Wrong
17	7 + 5 = 3 x 4	Right	Wrong
18	16 + 1 = 10 + 7	Right	Wrong
19	12 + 3 = 2 x 7	Right	Wrong
20	5 x 5 = 10 x 2	Right	Wrong

Task 2

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Task 3. Copy numbers fast

How fast can you copy numbers? Copy each number in the box as fast as you can.

1 $\boxed{1}$	5 $\boxed{}$	7 $\boxed{}$

[Set timer to 30 seconds]
This task should be really easy.
I want to see how fast you can write numbers.
Let's have a practice. We have done the first one for you.
Copy the numbers 5, and 7 *into the boxes next to them as fast as you can. Go.*
[Wait for the children to do this]
How did you go?
[Check that the children have understood what to do and done it. Explain if they haven't]

Now you are ready to do some more. I have a whole page for you and I'm going to give you half a minute to see how many numbers you can copy.
When I tell you, turn the page and start.
Remember to work as fast as you can and don't worry about being neat. Don't start until I tell you.
Ready? Go. [Start the timer]
[When the timer goes off make sure that all the children stop writing and then praise them for their effort]
Now turn over the page to Task Four

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Task 3
Copy these numbers as quickly as you can and keep going until you are told to stop.
Don't start until the teacher tells you to.

4	2	5	1	7
5	0	8	9	5
2	3	4	3	5
1	2	5	1	9
5	7	1	3	0
2	8	4	1	4
9	2	3	8	6
4	2	5	8	3
0	3	8	7	0
9	5	9	8	5

Task 3 a)

b)

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Task 4. Adding numbers fast

Start here.
How fast can you add?
Write the answer to each of these additions as fast you can.

$2 + 2 = \boxed{4}$ $4 + 1 = \boxed{}$ $6 + 0 = \boxed{}$ $2 + 8 = \boxed{}$

[Set the timer to 1 minute]
This task is to see how fast you can add numbers.
Let's have a practice. We have done the first one for you.
How quickly can you write the answers to the next three equations? Go
[Allow roughly 10 seconds for the children to finish]
Stop!
[Go through the answers]

Now you are ready to do some more.
On this test I want you to work down the page.
When I tell you, turn the page and start. You need to stop when I say stop.
Remember to work as quickly as you can and don't worry about being neat.
Ready? Go. [Start the timer]
[When the timer goes off make sure that all the children stop writing and praise them for their effort]

We are going to have a short break before we do the rest of the tasks.
So stand up and have a quick stretch…
[After they have had a few minutes break instruct the children to sit down again]

Ok, now turn over the page to Task Five.

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Task 4

Work **down** the page.

Don't start until the teacher says "Go". Stop writing as soon as the teacher says "Stop".

Write the answer to as many of these additions as you can. Go as fast as you can.

6 + 8 =		7 + 4 =		2 + 6 =		0 + 5 =	
2 + 9 =		1 + 3 =		5 + 5 =		6 + 6 =	
1 + 3 =		8 + 5 =		8 + 8 =		2 + 7 =	
0 + 4 =		0 + 3 =		3 + 9 =		9 + 2 =	
7 + 0 =		3 + 9 =		4 + 1 =		4 + 3 =	
3 + 4 =		7 + 5 =		9 + 6 =		7 + 8 =	
1 + 6 =		2 + 0 =		4 + 8 =		5 + 4 =	
5 + 9 =		6 + 8 =		2 + 1 =		2 + 2 =	
2 + 3 =		9 + 9 =		5 + 7 =		3 + 0 =	
6 + 4 =		6 + 7 =		9 + 0 =		1 + 3 =	

6 + 3 =		8 + 0 =		7 + 7 =		4 + 1 =	
4 + 4 =		4 + 5 =		3 + 8 =		5 + 5 =	
7 + 4 =		6 + 1 =		5 + 1 =		6 + 7 =	
9 + 0 =		0 + 7 =		6 + 9 =		4 + 9 =	
5 + 8 =		5 + 9 =		0 + 7 =		7 + 3 =	
7 + 1 =		0 + 4 =		4 + 6 =		2 + 2 =	
3 + 6 =		6 + 2 =		4 + 0 =		9 + 3 =	
1 + 1 =		9 + 8 =		1 + 1 =		1 + 8 =	
8 + 2 =		5 + 6 =		9 + 2 =		0 + 5 =	
3 + 9 =		1 + 2 =		8 + 9 =		9 + 1 =	

Test 4 a)

b)

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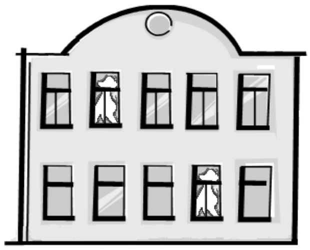
Task 5. How many?

Start Here.
On the next four pages there are a lot of pictures. You have to answer the question “How many?” and write the number in the circle. Here are some practice ones.

How many fingers are being held up?		**3**
How many windows are broken?		
How many umbrellas are there?		
How many legs does the crab have?		

[Set timer to 2 minutes]

Now for something completely different.
On the next four pages there are lots of pictures.
You have to answer the question “how many”? and write the number in the circle.
Here are some practice ones:
The first one says “How many fingers are being held up?” There are three fingers up so the number three has been written in the circle.
How many windows are broken? [Take an answer] *Yes, two. So, you write* two *in the circle.*
Now do the next two for yourself [Give feedback]
When everyone is ready we will start. There are four pages. Work across the page and answer the two on each line before you go to the next line. Remember you have to answer the question “how many”? and write the number in the circle. You need to stop writing when I say stop.
When I tell you to you can turn over the page and start.
Remember to go as fast as you can and don’t worry about being neat
Ready? Go. [Start timer]

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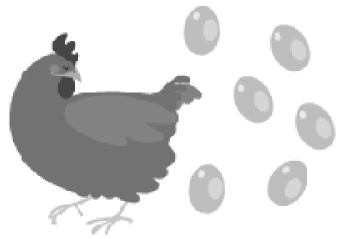
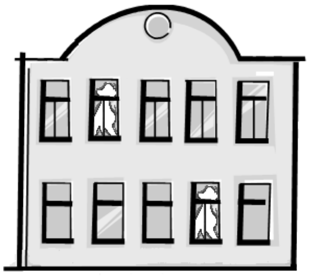
How many eggs does the hen have?



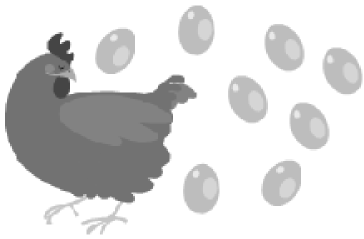
How many legs does the crab have?

How many windows are broken?

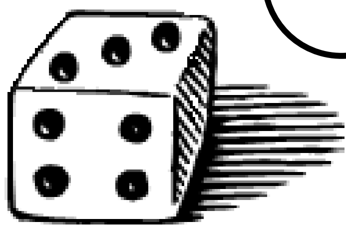


What number is the arrow pointing to?

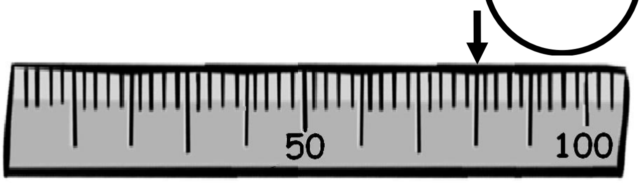
How many eggs does the hen have?



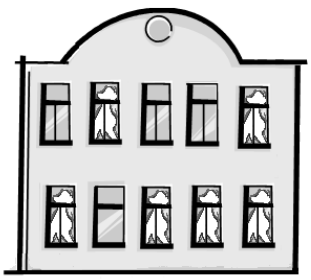
How many spots are showing on the dice?



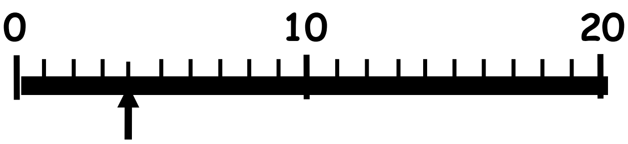
What number is the arrow pointing to?



How many windows are broken?



What number is the arrow pointing to?



What number is the arrow pointing to?

More

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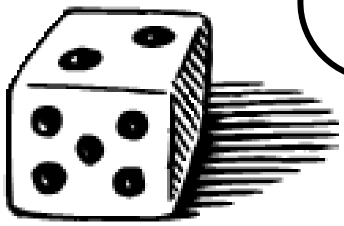
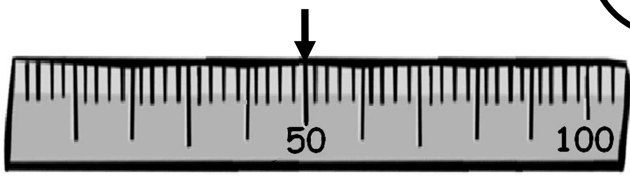
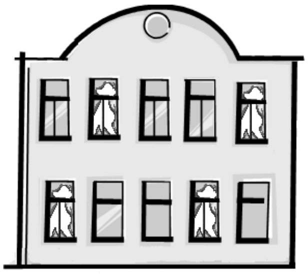
How many spots are showing on the dice?



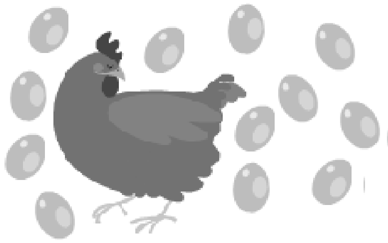
What number is the arrow pointing to?



How many windows are broken?

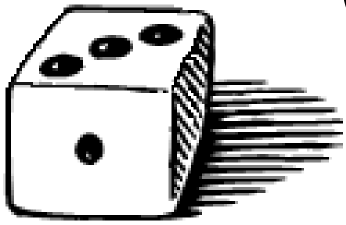


How many eggs does the hen have?



How many legs does the crab have?

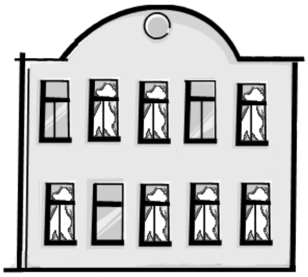
How many spots are showing on the dice?



How many eggs does the hen have?

How many legs does the crab have?

How many windows are broken?



What number is the arrow pointing to?

More

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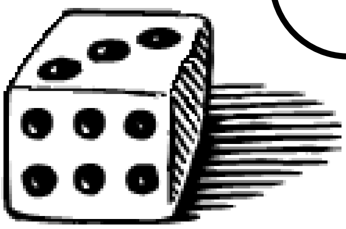
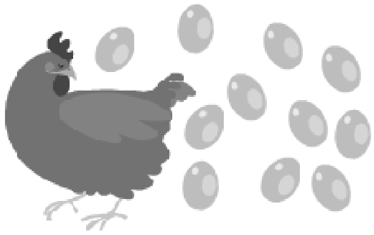
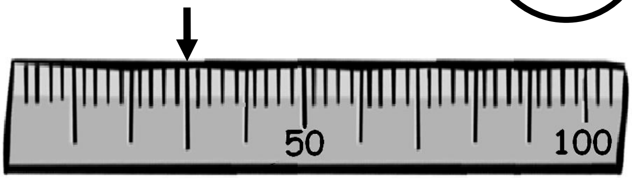
How many spots are showing on the dice?



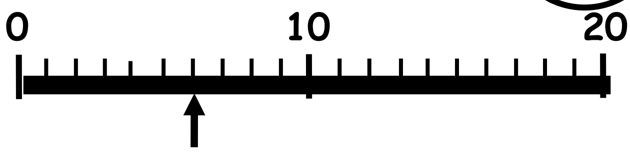
How many eggs does the hen have?



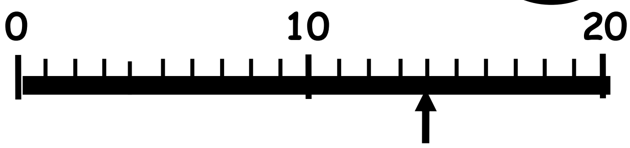
What number is the arrow pointing to?



What number is the arrow pointing to?



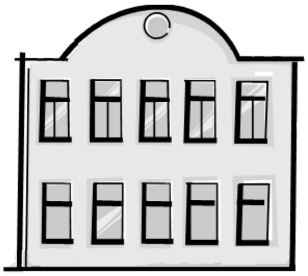
What number is the arrow pointing to?



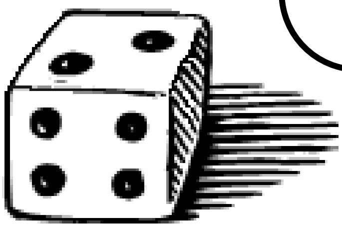
How many legs does the crab have?

What number is the arrow pointing to?

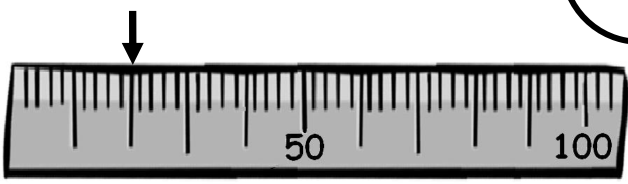
How many windows are broken?



How many spots are showing on the dice?



What number is the arrow pointing to?



More

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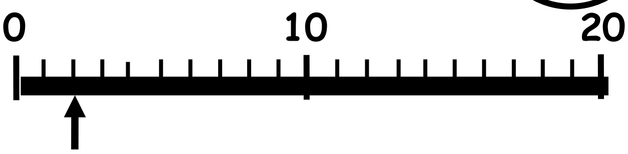
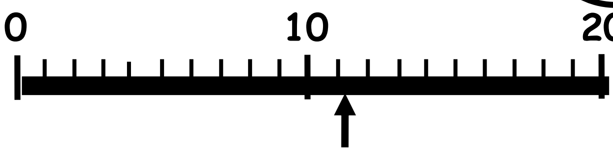
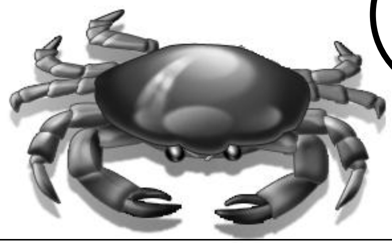
What number is the arrow pointing to?



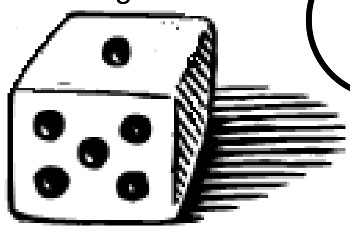
What number is the arrow pointing to?



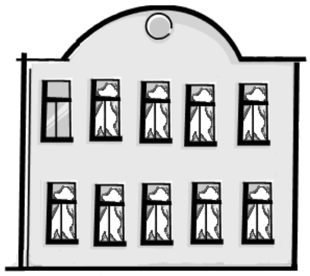
How many legs does the crab have?



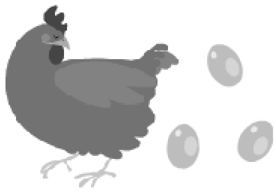
How many dots are showing on the dice?



How many windows are broken?



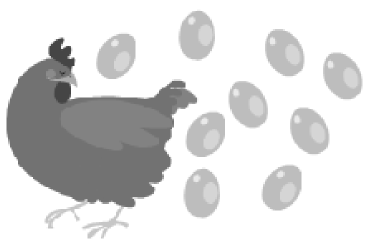
How many eggs does the hen have?



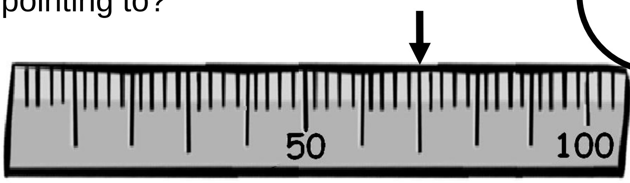
What number is the arrow pointing to?

What number is the arrow pointing to?

How many eggs does the hen have?



What number is the arrow pointing to?



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Task 5 a)	
b)	
c)	

Task 6. Some hard ones

Start Here.
Here are some hard ones. Can you work out the answers to any of these? Some are **addition** questions and some are **subtraction** questions. Here are some practice ones.

4 0 2	1 6 8	7 3 9	2 7 5
+ 1 5	– 3	+ 5 0	– 2 4
4 1 7			

[Set timer to 4 minutes]
This task might be a bit more difficult but try your hardest.
Some of the questions are adding questions and some are subtracting questions.
First there are some practice questions.
The first one is an adding one and we have written the answer for you.
[Go through the question explaining how to solve it]
The second one is a subtraction one [Go through the question]
Now you do the next two [Give feedback]

When everyone is ready we will start.
There are two pages and the questions are just like the ones we practiced.
Go across the page and answer the four on each line before you go to the next line.
You need to stop writing when I say stop.
When I tell you to you can turn the page and start.
Remember to go as fast as you can and don't worry about being neat.
Ready? Go. [Start the timer]
[When the timer goes off make sure all the children stop writing and praise them for their efforts]

Well done. That was the last task.
Thank you very much for your help with our study.
We will come and collect your papers and then you will be able to go.
[At this point you could ask for any feedback – such as which tasks did you enjoy, which ones were the hardest, which was the easiest etc?]

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Task 6

When every one is ready we will start. There are two pages.
Go **across** the page and answer the five on each line before you go to the next line.
Remember, some are **addition** and some are **subtraction**!
Stop writing as soon as the teacher says "Stop".
Write the answer to as many questions as you can. Go as fast as you can.
Ready?

514	877	684	194	326
+ 71	− 6	+ 70	− 8	+ 11
245	261	155	432	569
− 22	+ 69	− 47	+ 237	− 14
759	704	689	197	284
+ 23	− 12	+ 10	− 5	+ 37
194	720	384	516	836
− 76	+ 46	− 70	+ 274	− 40
603	458	236	362	145
+ 22	− 3	+ 58	− 48	+ 652
563	240	762	806	967
− 11	+ 599	− 5	+ 43	− 16
372	356	971	375	418
+ 61	− 78	+ 17	− 42	+ 85

More

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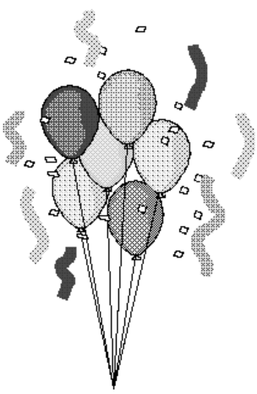
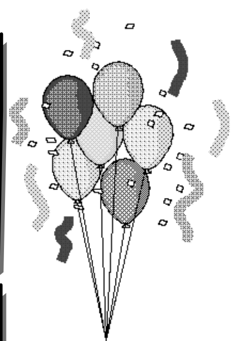
$\begin{array}{r} 541 \\ -\ \ 29 \\ \hline \end{array}$	$\begin{array}{r} 482 \\ +\ \ 16 \\ \hline \end{array}$	$\begin{array}{r} 189 \\ -\ \ 27 \\ \hline \end{array}$	$\begin{array}{r} 415 \\ +\ \ 67 \\ \hline \end{array}$	$\begin{array}{r} 683 \\ -\ \ \ 7 \\ \hline \end{array}$
$\begin{array}{r} 325 \\ +\ \ 34 \\ \hline \end{array}$	$\begin{array}{r} 299 \\ -\ \ \ 7 \\ \hline \end{array}$	$\begin{array}{r} 789 \\ +\ 111 \\ \hline \end{array}$	$\begin{array}{r} 930 \\ -\ \ 61 \\ \hline \end{array}$	$\begin{array}{r} 606 \\ +\ \ 72 \\ \hline \end{array}$
$\begin{array}{r} 619 \\ -\ \ 10 \\ \hline \end{array}$	$\begin{array}{r} 424 \\ +\ \ 69 \\ \hline \end{array}$	$\begin{array}{r} 807 \\ -\ \ 23 \\ \hline \end{array}$	$\begin{array}{r} 313 \\ +\ \ 81 \\ \hline \end{array}$	$\begin{array}{r} 378 \\ -\ \ 45 \\ \hline \end{array}$
$\begin{array}{r} 350 \\ +\ \ 71 \\ \hline \end{array}$	$\begin{array}{r} 245 \\ -\ \ \ 6 \\ \hline \end{array}$	$\begin{array}{r} 284 \\ +\ \ 14 \\ \hline \end{array}$	$\begin{array}{r} 495 \\ -\ \ 73 \\ \hline \end{array}$	$\begin{array}{r} 534 \\ +\ \ 98 \\ \hline \end{array}$
$\begin{array}{r} 510 \\ -\ \ \ 3 \\ \hline \end{array}$	$\begin{array}{r} 740 \\ +\ 109 \\ \hline \end{array}$	$\begin{array}{r} 728 \\ -\ \ \ 2 \\ \hline \end{array}$	$\begin{array}{r} 770 \\ +\ \ 92 \\ \hline \end{array}$	$\begin{array}{r} 612 \\ -\ \ 57 \\ \hline \end{array}$



THE END!



Task 6	
a)	
b)	
c)	
d)	
e)	
f)	

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