Thruster and Vibration Control of Marine Powertrain Using a Class of Feedforward Approximators

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***Abstract*— In this paper, we consider the tracking problem of propeller shaft speed and simultaneously minimizing torsional vibrations in marine shafting system, in the presence of parametric/functional uncertainties and unmodelled dynamics. Torsional vibrations within the shafting system can be induced by the hydrodynamic forces acting on the propeller and the inertia forces of the crank mechanism. Excessive vibrations will lead to severe consequences such as fractured drive shaft and compromised structural integrity. Due to the difficulty in measuring or modelling the hydrodynamic forces as well as the frictional forces, neural networks are used to compensate for the uncertainties. Simulation results illustrate the effectiveness of the proposed controller.**

I. Introduction

The problem of torsional vibrations within the marine shafting system, in the presence of parametric and functional uncertainties, poses a challenge to both marine engineering practitioners and control theorists alike. Torsional vibrations within the propulsive shafting system of a marine vessel can be induced by the hydrodynamic forces acting on the propeller and inertia forces of the crank mechanism. Excessive torsional vibrations within the shafting system will lead to failure of the drive shaft. Furthermore, the propulsion system is connected to the ship's hull which facilitates the transfer of vibrational energy within the shafting system to other sections of the vessel leading to excessive noise and compromising the structural integrity. A common approach to tackle the problem of excessive vibrations is to design the shafting system such that vibration response is within the allowable limits. Several methods have been proposed for the modelling of a marine shafting system [1] [2]. In this paper, we consider the method presented in [3] wherein the marine shafting system is modelled as a chained multiple mass-spring-damper system.

The main focus in torsional vibration suppression within marine shafting systems has been directed towards the design of the shafting system. The design of an effective control strategy to actively minimize torsional vibrations in marine shafting systems during operation have received relatively little attention. In general, the main objective of marine propulsion control focuses on achieving the desired position and velocity for the marine vessel through the control of shaft speed for fixed pitch (FP) propellers and pitch for controllable pitch (CP) propellers. Through the estimation of the propeller axial flow velocity, a nonlinear output feedback controller was developed in [4] for underwater vehicles where varying propeller thrust due to unsteady flow effects were compensated. Recent developments in marine propulsion control focus on the control of the power from the drive system to achieve the desired thrust [5] which achieves improved performance in moderate seas over conventional shaft speed controller.

Hydrodynamic forces acting on the propeller are highly nonlinear and subjected to variations due to the diverse operating conditions such as air suction, cavitation and partial/full emergence of the propellers [4]- [6]. Furthermore, torsional stiffness of the drive shaft will be subjected to changes during operation due to the mechanical wear and tear of the shafting system. Traditional adaptive controllers are generally useful when dealing with systems whose dynamics can be expressed in the linear-in-parameters form, for which the regressor is exactly known and the uncertainty is parametric and time-invariant. Such a restriction is clearly ill-suited for this system. To overcome the limitations of model-based adaptive controllers, we adopt approximation-based control techniques to compensate for functional uncertainties in the dynamic model of the shafting system. According to the Stone-Weierstrass theorem [7], a universal approximator can approximate, to an arbitrary degree of accuracy, any real continuous function on a compact set. Such approximators can utilize a standard regressor function whose structure is independent of the dynamic characteristics of the forces acting on the shafting system.

Motivated by results in control of flexible joint robots [8]-[10], we propose a Lyapunov-based approximation-based controller for the control of the marine shafting system. The control objective is to track a desired trajectory and simultaneously minimize the torsional vibrations within the shafting system. The main contributions of this paper are:

(i) the use of Radial Basis Functions (RBF) Neural Networks (NN) to compensate for the parametric and functional uncertainties which are commonly faced in thruster control. Rigorous stability analysis shows that semiglobal uniform boundedness of the tracking error is guaranteed, and the residual error can be made arbitrarily small by appropriate choice of the design parameters;

(ii) the investigation of torsional vibrations within the shafting system and the design of an active controller to reduce the vibrations.

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The organization of the remainder of this paper is as follows. Section II introduces some technical preliminaries required in the sequel, the dynamic model of the system and the structure of the neural networks (NN) used in the controller design. Thereafter, we present, in Section III, the adaptive NN control design procedure for marine shafting system under unknown hydrodynamic and frictional forces. A simulation study is conducted to investigate the effectiveness of the proposed control design and the results are illustrated in Section IV. Finally, the conclusions are given in Section V.

II. Mathematical Preliminaries

In this section, we present some notions and assumptions that will be used in the subsequent developments.

Lemma 2.1: For bounded initial conditions, if there exists a C^1 continuous and positive definite Lyapunov function $V(x)$ satisfying $\kappa_1(\|x\|) \leq V(x) \leq \kappa_2(\|x\|)$, such that $\dot{V}(x) \leq -\rho V(x) + c$, where κ_1, $\kappa_2 : R^n \to R$ are class k functions and c is a positive constant, then the solution $x = 0$ is uniformly bounded [11].

Assumption 2.1: The desired propeller trajectory, $q_d(t)$, is a bounded C^{2n+2} function.

A. Dynamic Modelling

From the results presented in [3], the marine shafting system can be effectively modelled as a chained multiple mass-spring-damper system. In this paper, the controller is developed utilizing the following chained multiple mass-spring-damper model of the marine shafting system:

$$\begin{aligned}
I\ddot{q} &= k(\theta_1 - q) + f(q, \dot{q}) \\
I_1\ddot{\theta}_1 &= k_1(\theta_2 - \theta_1) - k(\theta_1 - q) + f_1(\theta_1, \dot{\theta}_1) \\
&\cdots \\
I_i\ddot{\theta}_i &= k_i(\theta_{i+1} - \theta_i) - k_{i-1}(\theta_i - \theta_{i-1}) + f_i(\theta_i, \dot{\theta}_i) \\
&\cdots \\
I_n\ddot{\theta}_n &= u - k_{n-1}(\theta_n - \theta_{n-1}) + f_n(\theta_n, \dot{\theta}_n) \qquad (1)
\end{aligned}$$

where k_i is the unknown torsional stiffness of the massless spring connecting the ith mass unit to the $(i+1)$th mass unit, I_i is the unknown moment of inertia of the ith mass unit about the rotating axis, f is a function of unknown hydrodynamic forces acting on the propeller, f_i is a function of unknown frictional forces acting on the ith unit and u is the torque produced by the motor. In this paper, we consider the motor torque as the control input to the system.

B. Feedforward Neural Networks

Functional approximators can be represented as multi-layer feedforward networks which may be nonlinearly- or linearly-parameterized. A class of linearly parameterized feedforward approximators used to approximate the continuous function $f(Z) : R^q \to R$ may be represented as follows:

$$f_{nn}(Z) = W^T S(Z)$$

where the vector $Z = [z_1, \cdots, z_q]^T \in R^q$ are the input variables to the approximator, $W \in R^l$ is a vector of adaptable weights and $S(Z) = [s_1(Z), \cdots, s_l(Z)]^T \in R^l$ is a vector of known continuous (linear or nonlinear) basis functions, with $s_i(Z)$ being chosen as the commonly used Gaussian functions,

$$s_i(Z) = \exp\left[\frac{-(Z - \mu_i)^T (Z - \mu_i)}{\eta_i^2}\right], \quad i = 1, \cdots, l$$

where $\mu_i = [\mu_{i1}, \mu_{i2}, \cdots, \mu_{iq}]^T$ is the center of the receptive field for node i and η_i is the width of the Gaussian function. Universal approximation results in [7] indicate that if l is chosen sufficiently large, $W^T S(Z)$ can approximate any continuous function, $f(Z)$, to any desired accuracy over a compact set $\Omega_Z \subset R^q$ to any arbitrary degree of accuracy.

$$f(Z) = W^{*T} S(Z) + \varepsilon(Z), \quad \forall Z \in \Omega_Z \subset R^q$$

where W^* is the ideal constant weight vector and $\varepsilon(Z)$ is the approximation error for the special case where $W = W^*$ which is bounded over the compact set, i.e., $|\varepsilon(Z)| \leq \bar{\varepsilon}$, $\forall Z \in \Omega_Z$ where $\bar{\varepsilon} > 0$ is an unknown constant. The ideal weight vector W^*, an artificial quantity required for analytical purposes, is defined as the value of W that minimizes $|\varepsilon(Z)|$ for all $Z \in \Omega_Z \subset R^q$, i.e.,

$$W^* := \arg\min_{W \in R^l}\{\sup_{Z \in \Omega_Z} |f(Z) - W^T S(Z)|\}$$

III. Controller Design

In this section, a controller is developed utilizing neural networks and backstepping where that the individual torsional displacement and velocities, q, $\dot{q}$, θ_i and $\dot{\theta}_i$, are measurable.

Step 1: First, we introduce a virtual control, α_r, and a perturbation $z_1 = \theta_1 - \alpha_r$ into (1), we have

$$I\ddot{q} + kq = f + k\alpha_r + kz_1 \qquad (2)$$

A chain of backstepping is required until the physical control input appears in the error equations. The objective is to make z_1 small so that the propeller dynamics can be controlled for tracking purposes. The derivative of z_1 is given by $\dot{z}_1 = \dot{\theta}_1 - \dot{\alpha}_r$. A virtual control input, α_1, is introduced to drive z_1 to a small value as described by

$$\dot{z}_1 = z_2 + \alpha_1 - \dot{\alpha}_r$$

where $z_2 = \dot{\theta}_1 - \alpha_1$ is the second perturbation term to be controlled. The derivative of z_2 is given by

$$I_1\dot{z}_2 = k_1(\theta_2 - \theta_1) + f_1(\theta_1, \dot{\theta}_1) - I_1\dot{\alpha}_1$$

A virtual control input, α_2, is introduced to drive z_2 to a small value as described by

$$I_1\dot{z}_2 = k_1\alpha_2 - k_1\theta_1 + f_1(\theta_1, \dot{\theta}_1) - I_1\dot{\alpha}_1 + k_1 z_3$$

where $z_3 = \theta_2 - \alpha_2$.

Step i: Applying the above methods recursively for each step,

$$z_{2i-1} = \theta_i - \alpha_{2i-2}$$

where α_{2i-2} is the virtual control input to stabilize the $i-1$th subsystem and the control objective for the ith step is to drive z_{2i-1} to a small value. Introducing a virtual control input,

α_{2i-1}, to drive z_{2i-1} to a small value, the derivative of z_{2i-1} is given by

$$\dot{z}_{2i-1} = z_{2i} + \alpha_{2i-1} - \dot{\alpha}_{2i-2} \tag{3}$$

where $z_{2i} = \dot{\theta}_i - \alpha_{2i-1}$ is a perturbation term to be controlled. The derivative of z_{2i} is given by

$$\begin{aligned} I_i\dot{z}_{2i} &= k_i(\theta_{i+1} - \theta_i) - k_{i-1}(\theta_i - \theta_{i-1}) \\ &\quad + f_i(\theta_i, \dot{\theta}_i) - I_i\dot{\alpha}_{2i-1} \end{aligned}$$

A virtual control input, α_{2i}, is introduced to drive z_{2i} to a small value as described by

$$\begin{aligned} I_i\dot{z}_{2i} &= k_i\alpha_{2i} - k_i\theta_i - k_{i-1}(\theta_i - \theta_{i-1}) \\ &\quad + f_i(\theta_i, \dot{\theta}_i) - I_i\dot{\alpha}_{2i-1} + k_i z_{2i+1} \end{aligned} \tag{4}$$

where $z_{2i+1} = \theta_{i+1} - \alpha_{2i}$.

Step n: For the last step, we have

$$z_{2n-1} = \theta_n - \alpha_{2n-2}$$

where α_{2n-2} is the virtual control input to stabilize the $n-1$th subsystem and the control objective for the nth step is to drive z_{2n-1} to a small value. Introducing a virtual control input, α_{2n-1}, to drive z_{2n-1} to a small value, the derivative of z_{2n-1} is given by

$$\dot{z}_{2n-1} = z_{2n} + \alpha_{2n-1} - \dot{\alpha}_{2n-2}$$

where $z_{2n} = \dot{\theta}_n - \alpha_{2n-1}$ is a perturbation term to be controlled. The derivative of z_{2n} is given by

$$I_n\dot{z}_{2n} = u - k_n(\theta_n - \theta_{n-1}) + f_n(\theta_n, \dot{\theta}_n) - I_n\dot{\alpha}_{2n-1} \tag{5}$$

The controller design now becomes the design of α_r in (2), α_{2i-1} in (3), α_{2i} in (4) and u in (5). First, we define the desired trajectory for the propeller as q_d, where $q_d(t)$ is a C^{2n+2} function. Define the tracking error as

$$e(t) = q_d(t) - q(t)$$

where the control objective is the tracking of the desired trajectory $q_d(t)$. A dynamic compensator is defined as

$$r = \dot{e} + \lambda e$$

where λ is a design parameter. Using the above equations and (2), we obtain

$$I\dot{r} = I(\ddot{q}_d + \lambda\dot{e}) + kq - f - k\alpha_r - kz_1$$

Consider the following control laws

$$\begin{aligned} \alpha_r &= \frac{1}{k}(\kappa_r r + I(\ddot{q}_d + \lambda\dot{e}) - f) + q \\ \alpha_{2i-1} &= -\kappa_{2i-1}z_{2i-1} + \dot{\alpha}_{2i-2} - z_{2i-2} \\ \alpha_{2i} &= \frac{1}{k_i}(-\kappa_{2i}z_{2i} + k_{i-1}(\theta_i - \theta_{i-1}) \\ &\quad + I_i\dot{\alpha}_{2i-1} - f_i) - z_{2i-1} + \theta_i \\ u &= -\kappa_{2n}z_{2n} + k_n(\theta_n - \theta_{n-1}) - f_n \\ &\quad - z_{2n-1} + I_n\dot{\alpha}_{2n-1} \end{aligned}$$

where κ_i are design parameters. The above control laws cannot be realized as the torsional stiffness k_i, inertia I_i and the functions of frictional and hydrodynamic forces f_i are unknown. Therefore, the time derivatives of the virtual control

$$\begin{aligned} \dot{\alpha}_r &= \frac{\partial\alpha_r}{\partial q}\dot{q} + \frac{\partial\alpha_r}{\partial\dot{q}}\ddot{q} + \sum_{j=1}^{3}\frac{\partial\alpha_r}{\partial q_d^{(j-1)}}q_d^{(j)} \\ \dot{\alpha}_{2i-1} &= \frac{\partial\alpha_{2i-1}}{\partial q}\dot{q} + \frac{\partial\alpha_{2i-1}}{\partial\dot{q}}\ddot{q} + \sum_{j=1}^{i}\frac{\partial\alpha_{2i-1}}{\partial\theta_j}\dot{\theta}_j \\ &\quad + \sum_{j=1}^{i-1}\frac{\partial\alpha_{2i-1}}{\partial\dot{\theta}_j}\ddot{\theta}_j + \sum_{j=1}^{2i+2}\frac{\partial\alpha_{2i-1}}{\partial q_d^{(j-1)}}q_d^{(j)} \\ \dot{\alpha}_{2i} &= \frac{\partial\alpha_{2i}}{\partial q}\dot{q} + \frac{\partial\alpha_{2i}}{\partial\dot{q}}\ddot{q} + \sum_{j=1}^{i}\left(\frac{\partial\alpha_{2i}}{\partial\theta_j}\dot{\theta}_j + \frac{\partial\alpha_{2i}}{\partial\dot{\theta}_j}\ddot{\theta}_j\right) \\ &\quad + \sum_{j=1}^{2i+3}\frac{\partial\alpha_{2i}}{\partial q_d^{(j-1)}}q_d^{(j)} \end{aligned}$$

contains unknown terms as well. Define h_i as a function of unknown terms and ϕ_i as a function of known terms in $\dot{\alpha}_i$. By employing feedforward approximators to approximate the unknown functions where

$$\begin{aligned} W_r^{*T}S_r &= \frac{1}{k}(f - I(\ddot{q}_d + \lambda\dot{e})) - \varepsilon_r \\ W_{2i-1}^{*T}S_{2i-1} &= \frac{\partial\alpha_{2i-2}}{\partial\dot{q}}\ddot{q} + \sum_{j=1}^{i-1}\frac{\partial\alpha_{2i-2}}{\partial\dot{\theta}_j}\ddot{\theta}_j - \varepsilon_{2i-1} \\ W_{2i}^{*T}S_{2i} &= \frac{1}{k_i}\left(\frac{\partial\alpha_{2i-1}}{\partial\dot{q}}\ddot{q} + f_i - k_{i-1}(\theta_i - \theta_{i-1})\right. \\ &\quad \left. - I_i\phi_{2i-1} + \sum_{j=1}^{i-1}\frac{\partial\alpha_{2i-1}}{\partial\dot{\theta}_j}\ddot{\theta}_j\right) - \varepsilon_{2i} \\ W_{2n}^{*T}S_{2n} &= \frac{\partial\alpha_{2n-1}}{\partial\dot{q}}\ddot{q} + \sum_{j=1}^{n-1}\frac{\partial\alpha_{2n-1}}{\partial\dot{\theta}_j}\ddot{\theta}_j + f_n \\ &\quad - k_{n-1}(\theta_n - \theta_{n-1}) - I_n\phi_{2n-1} - \varepsilon_{2i} \end{aligned}$$

Thus, the practical approximation-based controllers can be realized as follows:

$$\begin{aligned} \alpha_r &= (\kappa_r + \kappa_{Wr})r - \hat{W}_r^T S_r(Z_r) + q \\ \alpha_1 &= -(\kappa_1 + \kappa_{W1})z_1 + r - \hat{W}_1^T S_1(Z_1) + \phi_r \\ \alpha_{2i-1} &= -(\kappa_{2i-1} + \kappa_{W2i-1})z_{2i-1} - z_{2i-2} \\ &\quad - \hat{W}_{2i-1}^T S_{2i-1}(Z_{2i-1}) + \phi_{2i-2} \\ \alpha_{2i} &= -(\kappa_{2i} + \kappa_{W2i})z_{2i} - \hat{W}_{2i}^T S_{2i}(Z_{2i}) - z_{2i-1} + \theta_i \\ u &= -(\kappa_{2n} + \kappa_{W2n})z_{2n} - \hat{W}_{2n}^T S_{2n}(Z_{2n}) \\ &\quad - z_{2n-1} \end{aligned} \tag{6}$$

where $\hat{W}_i$ approximates W_i^*.

Remark 3.1: Although the terms $\frac{\partial\alpha_i}{\partial\dot{q}}$ and $\frac{\partial\alpha_i}{\partial\dot{\theta}_k}$ contain the neural networks weights $\hat{W}_j$ for $j = 1, 2, ..., i$, the large number of neural network weight estimates $\hat{W}_j$ are not recommended to be taken as inputs to the NN because of the curse of dimensionality of RBF NN [12] [13].

By defining intermediate variables $\frac{\partial\alpha_i}{\partial\dot{q}}$ and $\frac{\partial\alpha_i}{\partial\dot{\theta}_k}$ which are available through computation, the NN approximation $\hat{W}_i^T S_i(Z_i)$ of the unknown terms can be computed using the minimal number of NN inputs where

$$Z_r = [\ddot{q}_d + \lambda\dot{e}, q, \dot{q}]^T$$

$$Z_{2i-1} = \left[q, \dot{q}, \bar{\theta}_i, \bar{\dot{\theta}}_{i-1}, \frac{\partial\alpha_{2i-2}}{\partial\dot{q}}, \frac{\partial\alpha_{2i-2}}{\partial\dot{\theta}_1}, ..., \frac{\partial\alpha_{2i-2}}{\partial\dot{\theta}_{i-1}}\right]^T$$

$$Z_{2i} = \left[q, \dot{q}, \bar{\theta}_i, \bar{\dot{\theta}}_i, \frac{\partial\alpha_{2i-1}}{\partial\dot{q}}, \frac{\partial\alpha_{2i-1}}{\partial\dot{\theta}_1}, ..., \frac{\partial\alpha_{2i-1}}{\partial\dot{\theta}_{i-1}}, \phi_{2i-1}\right]^T$$

$$Z_{2n} = \left[q, \dot{q}, \theta, \dot{\theta}, \frac{\partial\alpha_{2n-1}}{\partial\dot{q}}, \frac{\partial\alpha_{2n-1}}{\partial\dot{\theta}_1}, ..., \frac{\partial\alpha_{2n-1}}{\partial\dot{\theta}_{n-1}}, \phi_{2n-1}\right]^T$$

Consider the following Lyapunov function candidate

$$V = \frac{1}{2}z^T M z + \frac{1}{2}\tilde{W}_r^T\Gamma_r^{-1}\tilde{W}_r + \sum_{i=1}^{2n}\frac{1}{2}\tilde{W}_i^T\Gamma_i^{-1}\tilde{W}_i \tag{7}$$

where $z = [r, z_1, z_2, ..., z_{2n}]^T$, Γ_i are constant gain matrices, $\tilde{W}_i = \hat{W}_i - W_i^*$ and $M \in R^{2n+1\times 2n+1}$ is a diagonal matrix with $\frac{I}{k}$ as its first diagonal element, 1 and $\frac{I_i}{k_i}$ as its $(2i)$th and $(2i+1)$th diagonal element respectively. The time derivative of V is given by

$$\dot{V} = z^T M\dot{z} + \tilde{W}_r^T\Gamma_r^{-1}\dot{\hat{W}}_r + \sum_{i=1}^{2n}\tilde{W}_i^T\Gamma_i^{-1}\dot{\hat{W}}_i \tag{8}$$

Substituting the virtual controllers, α_i, and control input, u, into (8), we obtain

$$\dot{V} = -z^T(K + K_w)z + \tilde{W}_r^T\left(\Gamma_r^{-1}\dot{\hat{W}}_r + S_r(Z_r)r\right) + \sum_{i=1}^{2n}\tilde{W}_i^T\left(\Gamma_i^{-1}\dot{\hat{W}}_i - S_i(Z_i)z_i\right) + \varepsilon z$$

where $\varepsilon = [\varepsilon_r, \varepsilon_1, ..., \varepsilon_{2n}]^T \in R^{2n+1}$, $K_w \in R^{2n+1\times 2n+1}$ is a diagonal matrix with κ_{wr} as its first diagonal element and κ_{wi} as its $(i+1)$th diagonal element and $K \in R^{2n+1\times 2n+1}$ is a diagonal matrix with κ_r as its first diagonal element and κ_i as its $(i+1)$th diagonal element. It is clear that by choosing the adaptation laws

$$\dot{\hat{W}}_r = -\Gamma_r\left(S_r(Z_r)r + \sigma_r\hat{W}_r\right)$$
$$\dot{\hat{W}}_i = \Gamma_i\left(S_i(Z_i)z_i - \sigma_i\hat{W}_i\right) \tag{9}$$

where the second term on the RHS imposes a growth condition on the weight vector with $\sigma_i > 0$, the following is obtained

$$\begin{aligned}\dot{V} &< -z^T Kz + \frac{1}{4}\bar{\varepsilon}^T K_w^{-1}\bar{\varepsilon} - \sigma_r\tilde{W}_r^T\left(\tilde{W}_r + W_r^*\right) - \sum_{i=1}^{2n}\sigma_i\tilde{W}_i^T\left(\tilde{W}_i + W_i^*\right)\\ &< -z^T Kz - \frac{\sigma_r}{2}\left\|\tilde{W}_r\right\|^2 - \sum_{i=1}^{2n}\frac{\sigma_i}{2}\left\|\tilde{W}_i\right\|^2 + \frac{1}{4}\bar{\varepsilon}^T K_w^{-1}\bar{\varepsilon} + \frac{\sigma_r}{2}\|W_r^*\|^2 + \sum_{i=1}^{2n}\frac{\sigma_i}{2}\|W_i^*\|^2\\ &< -\rho V + C\end{aligned} \tag{10}$$

where $C := \frac{1}{2}\bar{\varepsilon}^T K_w^{-1}\bar{\varepsilon} + \frac{\sigma_r}{2}\|W_r^*\|^2 + \sum_{i=1}^{2n}\frac{\sigma_i}{2}\|W_i^*\|^2$, and

$$\rho := \min\left(\frac{2\lambda_{\min}(K)}{\lambda_{\max}(M)}, \min_{i=r,1,2,...,2n}\left(\frac{\sigma_i}{\lambda_{\max}(\Gamma_i)}\right)\right)$$

and $\lambda_{\min}(\bullet)$ and $\lambda_{\max}(\bullet)$ denote the minimum and maximum eigenvalues of $\bullet$ respectively.

Theorem 3.1: Consider the powertrain dynamics (1) under Assumption 2.1, with control law (6) and adaptation law (9). Given that the initial conditions are bounded, and that full state information is available, the closed loop system is semiglobally uniformly bounded. The closed-loop error signals $z = [r, z_1, \cdots, z_{2n}]^T$, $\tilde{W}_r$, $\tilde{W}_i$ will remain within the compact sets Σ_z, Σ_{W_r} and Σ_{W_i} respectively, defined by

$$\Sigma_z := \left\{z \in R^{2n+1}\ \middle|\ \|z\| \le \sqrt{\frac{D}{\lambda_{\min}(M)}}\right\}$$

$$\Sigma_{W_r} := \left\{\tilde{W}_r \in R^{l_r}\ \middle|\ \left\|\tilde{W}_r\right\| \le \sqrt{\frac{D}{\lambda_{\min}(\Gamma_r^{-1})}}\right\}$$

$$\Sigma_{W_i} := \left\{\tilde{W}_i \in R^{l_i}\ \middle|\ \left\|\tilde{W}_i\right\| \le \sqrt{\frac{D}{\lambda_{\min}(\Gamma_i^{-1})}}\right\}$$

where $D = 2\left(V(0) + \frac{C}{\rho}\right)$ with C and ρ as defined in (10)

Proof. From (10) and Lemma 2.1, it is straightforward to show that the signals $z, \tilde{W}_r, \tilde{W}_1, ..., \tilde{W}_{2n}$ are semiglobally uniformly bounded. For completeness, the details of the proof are provided here. Multiplying (10) by $e^{\rho t}$, yields

$$\frac{d}{dt}(Ve^{\rho t}) \le Ce^{\rho t}$$

Integrating the above inequality, we obtain

$$V \le \left(V(0) - \frac{C}{\rho}\right)e^{-\rho t} + \frac{C}{\rho} \le V(0) + \frac{C}{\rho} \tag{11}$$

Substituting (7) into (11), we obtain

$$\frac{1}{2}\lambda_{\min}(M)\|z\|^2 \le V(0) + \frac{C}{\rho}$$

The bounds of $\left\|\tilde{W}_r\right\|$ and $\left\|\tilde{W}_i\right\|$ can be similarly shown. This concludes the proof. ∎

IV. Simulation

In this section, simulation studies are carried out to illustrate the proposed state feedback controller.

Consider a system described below

$$\begin{aligned}I\ddot{q} &= k(\theta_1 - q) + f(q, \dot{q})\\ I_1\ddot{\theta}_1 &= k_1(\theta_2 - \theta_1) - k(\theta_1 - q) + f_1\left(\theta_1, \dot{\theta}_1\right)\\ I_2\ddot{\theta}_2 &= u - k_1(\theta_2 - \theta_1) + f_2\left(\theta_2, \dot{\theta}_2\right)\end{aligned}$$

where $f = -0.16\dot{q}|\dot{q}|$ is a function of unknown hydrodynamic forces acting on the propeller, $f_1 = -0.01(\dot{\theta}_1 - \dot{\theta}_1|\dot{\theta}_1|)$

and $f_2 = -0.01(\dot{\theta}_2 - \dot{\theta}_2|\dot{\theta}_2|)$ are functions of unknown frictional forces acting on the first and second unit of the shafting system respectively. The torsional stiffness are given by $k = 1.5 \times 10^6$ and $k_1 = 1.2 \times 10^6$. The moment of inertia of the mass units about the rotating axis are given by $I = I_1 = I_2 = 10$. The desired trajectory is given by

$$q_d(t) = 80t^7 - 233t^9 + 305t^{11} - 215t^{13} + 80t^{15} - 12t^{17}$$

and the NNs inputs are as follows:

$$\begin{aligned} Z_r &= \left[\frac{\ddot{q}_d + \lambda \dot{e}}{\epsilon_1}, q, \dot{q}\right]^T \\ Z_1 &= \left[q, \dot{q}, \theta_1, \frac{1}{\epsilon_2}\frac{\partial \alpha_r}{\partial \dot{q}}\right]^T \\ Z_2 &= \left[q, \dot{q}, \theta_1, \dot{\theta}_1, \frac{1}{\epsilon_3}\frac{\partial \alpha_1}{\partial \dot{q}}, \frac{\phi_1}{\epsilon_4}\right]^T \\ Z_3 &= \left[q, \dot{q}, \theta_1, \dot{\theta}_1, \theta_2, \frac{1}{\epsilon_5}\frac{\partial \alpha_2}{\partial \dot{q}}, \frac{1}{\epsilon_6}\frac{\partial \alpha_2}{\partial \dot{\theta}_1}\right]^T \\ Z_4 &= \left[q, \dot{q}, \theta_1, \dot{\theta}_1, \theta_2, \dot{\theta}_2, \frac{1}{\epsilon_7}\frac{\partial \alpha_3}{\partial \dot{q}}, \frac{1}{\epsilon_8}\frac{\partial \alpha_3}{\partial \dot{\theta}_1}, \frac{\phi_3}{\epsilon_9}\right]^T \end{aligned}$$

where ϵ_i are design parameters chosen such that the inputs of the NNs are normalized to similar orders of magnitude in the input space in order to facilitate the placement of nodes and improve the performance of the NNs.

In the following simulation study, we select the centers and widths of the RBF's as: Neural Network $\hat{W}_r^T S_r$ contains 8 nodes (i.e., $l_r = 8$), with centers $\mu_{rl}(l = 1, ..., l_r)$ evenly spaced in $[-5, 10] \times [-5, 15] \times [-10, 10]$, and widths $\eta_{rl}^2 = 20(l = 1, ..., l_r)$. Neural Network $\hat{W}_1^T S_1$ contains 16 nodes (i.e., $l_1 = 16$), with centers $\mu_{1l}(l = 1, ..., l_1)$ evenly spaced in $[-5, 15] \times [-10, 10] \times [-5, 15] \times [-10, 10]$, and widths $\eta_{1l}^2 = 100(l = 1, ..., l_2)$. Neural Network $\hat{W}_2^T S_2$ contains 64 nodes (i.e., $l_2 = 64$), with centers $\mu_{2l}(l = 1, ..., l_2)$ evenly spaced in $[-5, 15] \times [-10, 10] \times [-5, 15] \times [-10, 10] \times [-10, 10] \times [-10, 10]$, and widths $\eta_{2l}^2 = 300(l = 1, ..., l_3)$. Neural Network $\hat{W}_3^T S_3$ contains 128 nodes (i.e., $l_3 = 128$), with centers $\mu_{3l}(l = 1, ..., l_3)$ evenly spaced in $[-5, 15] \times [-10, 10] \times [-5, 15] \times [-10, 10] \times [-5, 15] \times [-10, 10] \times [-10, 10]$, and widths $\eta_{3l}^2 = 500(l = 1, ..., l_3)$. Neural Network $\hat{W}_4^T S_4$ contains 512 nodes (i.e., $l_4 = 512$), with centers $\mu_{4l}(l = 1, ..., l_4)$ evenly spaced in $[-5, 15] \times [-10, 10] \times [-5, 15] \times [-10, 10] \times [-5, 15] \times [-10, 10] \times [-10, 10] \times [-10, 10] \times [-10, 10]$, and widths $\eta_{4l}^2 = 700(l = 1, ..., l_4)$. The initial weight estimates are set to be 0, i.e., $\hat{W}_r(0) = 0$, $\hat{W}_1(0) = 0$, $\hat{W}_2(0) = 0$, $\hat{W}_3(0) = 0$ and $\hat{W}_4(0) = 0$..

The following initial conditions and controller design parameters are adopted in the simulation: $x(0) = [0, 0, 0.005, 0, 0, 0]^T$, $\Gamma_r = \text{diag}\{65\}$, $\Gamma_1 = \text{diag}\{15\}$, $\Gamma_2 = \text{diag}\{45\}$, $\Gamma_3 = \text{diag}\{15\}$, $\Gamma_4 = \text{diag}\{20\}$, $\sigma_r = \sigma_1 = \sigma_2 = \sigma_3 = 0.1$, $\sigma_4 = 0.01$, $\kappa_r = \kappa_{wr} = 1.5$, $\kappa_1 = \kappa_{w1} = 6$, $\kappa_2 = \kappa_{w2} = 0.00005$, $\kappa_3 = \kappa_{w3} = 2.5$, $\kappa_4 = \kappa_{w4} = 0.00005$, $\lambda = 12$, $\epsilon_1 = 5$, $\epsilon_2 = 0.8$, $\epsilon_3 = 2$, $\epsilon_4 = 80$, $\epsilon_5 = 2$, $\epsilon_6 = 0.5$, $\epsilon_7 = 60$, $\epsilon_8 = 25$ and $\epsilon_9 = 7000$.

Fig. 1 shows that tracking of the desired propeller velocity is achieved. Figs. 2 and 3 shows that torsional vibrations within the shafting system, due to the initial torsional displacements $q(0) - \theta_1(0) = -0.005$ and $\theta_1(0) - \theta_2(0) = 0.005$, is reduced after 1 second. However, it is noted that, under the proposed control law, the torsional vibrations within the shafting system does not converge to zero. This problem may be reduced through a proper selection of the design parameters and further studies are to be conducted in order to improve the performance of the proposed controller. Figs. 4, 5 and 6 show the boundedness of the control input and the NN weights in the control loop.

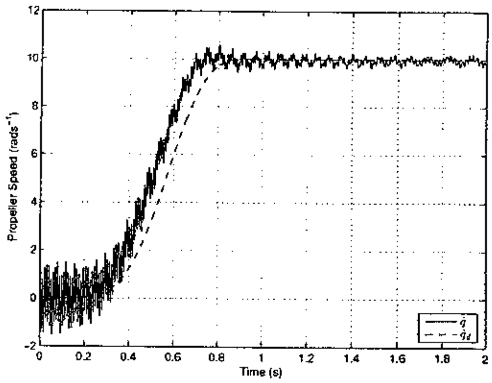

Fig. 1. Propeller velocity, $\dot{q}$, and desired velocity

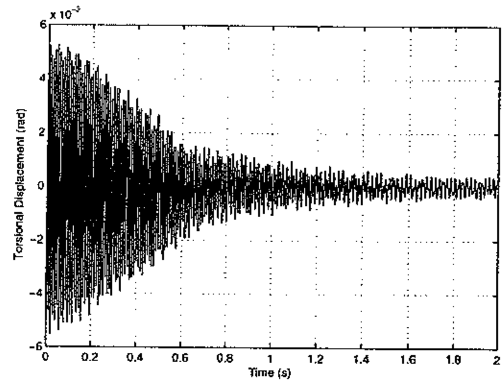

Fig. 2. Torsional Displacement $q - \theta_1$

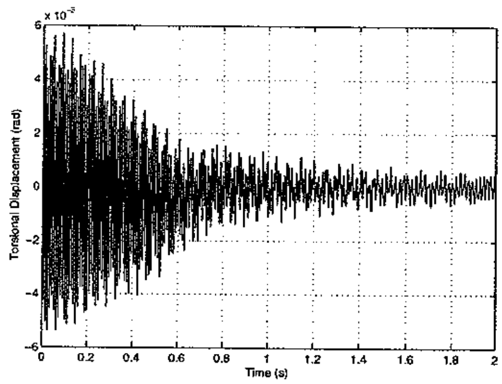

Fig. 3. Torsional Displacement $\theta_1 - \theta_2$

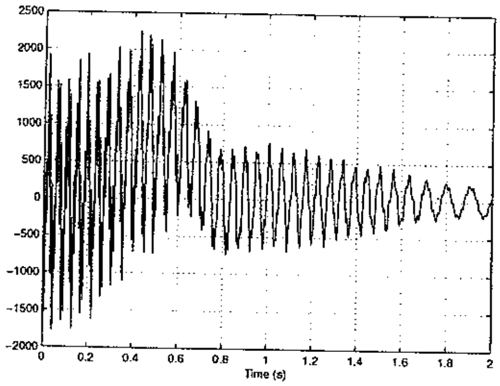

Fig. 4. Control Input $u(t)$

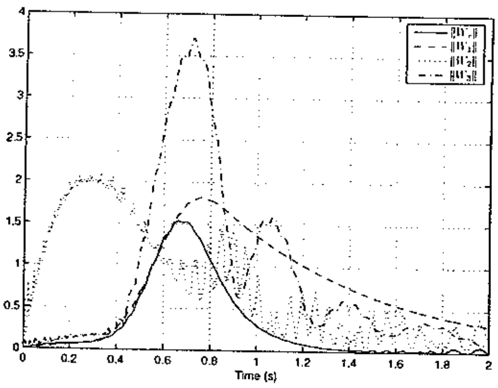

Fig. 5. Norms of NN weights $\|\hat{W}_r\|$, $\|\hat{W}_1\|$, $\|\hat{W}_2\|$ and $\|\hat{W}_3\|$

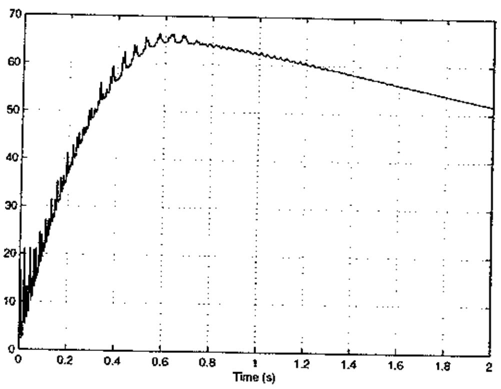

Fig. 6. Norms of NN weight $\|\hat{W}_4\|$

V. Conclusion

In this paper, an approximation-based control design is developed with the objective of solving a marine propulsion control problem while simultaneously suppressing the torsional vibrations within the shafting system, in the presence of unmodelled dynamics, or parametric/functional uncertainties. The controller is mathematically shown to guarantee semiglobally uniformly bounded stability, and the steady state compact set to which the closed loop error signals converge is derived. The size of compact set can be made small through appropriate choice of the control design parameters.

Simulation results demonstrate that the system, under the proposed control and adaptation laws, is able to achieve the desired shaft speed and the torsional vibrations within the shafting system are reduced, with all closed loop signals uniformly bounded. Although the torsional vibrations are noticeably reduced under the proposed control law, it does not converge to zero as illustrated in the simulation results. Further studies are to be conducted in order to improve the performance of the proposed controller.

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