

# Rocket Roll Dynamics and Disturbance – Minimal Modelling and System Identification

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**Abstract**— The roll dynamics of a 5kg, 1.3 m high sounding rocket are analyzed in a vertical wind tunnel. Significant turbulence in the tunnel makes the system identification of the effective inertia, damping and asymmetry with respect to roll challenging. A novel method is developed which decouples the disturbance from the rocket frame’s intrinsic roll dynamics and allows accurate prediction of roll rate and angle. The parameter identification method is integral-based, and treats wind disturbances as equivalent to a movement in the actuator fins. The method is robust, requires minimal computation, and gave a realistic disturbance distribution reflecting the randomness of the turbulent wind flow. The mean absolute roll rate of the rocket frame observed in experiments was 16.4 degree/s and the model predicted the roll rate with a median error of 0.51 degrees/s with a 90th percentile of 1.25 degrees/s. The roll angle (measured by an encoder), was tracked by the model with a median absolute error of 0.25 degrees and a 90th percentile of 0.50 degrees. These results prove the concept of this minimal modeling approach which will be extended to pitch and yaw dynamics in the future.

**Keywords**—rocketry, roll-control, wind-tunnel, minimal modelling, integral-based parameter identification, disturbance

## I. INTRODUCTION AND MOTIVATION

A special topic rocket systems engineering course has been developed at the University of Canterbury in 2009, and has led to a funded summer project with a view for introducing rocketry and aeronautical control into research and teaching for Mechanical Engineering/Mechatronics and Electrical Engineering students. One of the motivations for the course and research is to facilitate a possible entry into the NASA university student launch initiative (USLI) competition [1]. The goal for this competition, is to build and fly a re-usable rocket that lifts a scientific payload to as close to one mile as possible. For details on current progress including rocket propulsion, avionics and control system development see [2].

An important aspect in control system design to stabilize a rocket is the use of mathematical models to describe dynamics in all the axes. The motion of a rocket has been well documented in the literature but is typically very complex involving up to 6-DOF and therefore very complex system identification methods [3-5].

This paper looks at the development of a simple modeling and system identification method for understanding the roll dynamics and the effect of disturbances on a small spin-stabilized sounding rocket, of about 1.3 m in length and 5kg in

weight. The simulation tool developed could then be used to develop and analyze various controllers to stabilize the rocket motion.

The maximum velocity of this type of rocket is approximately 600 km/h and it can reach up to 2 km in altitude. Sounding rockets are recoverable research rockets, designed to take measurements and perform scientific experiments during its flight. Figure 1 shows a model of the rocket, which describes the adopted nomenclature of the system. To simplify attitude control, this research concentrates on controlling the roll of the rocket. This approach effectively decouples the roll dynamics from the pitch and yaw avoiding complex controllers and advanced analysis (e.g. [6]).

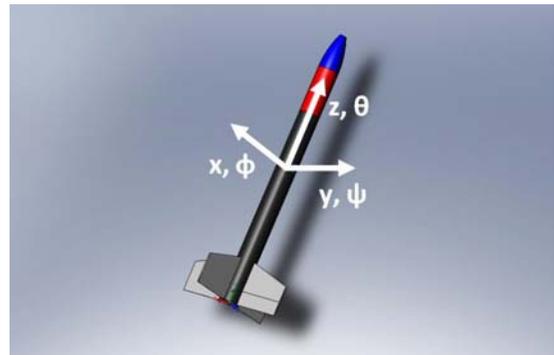


Figure 1: A CAD model of the rocket

To provide an intermediate step from simulation to a rocket launch, a vertical wind tunnel was built with a vacuum to suck air past a rocket airframe that was actuated by aluminum fins. Figure 2 shows the wind tunnel and a shot of the rocket airframe during operation with the fin extended.



Figure 2: (a) Vertical wind tunnel (b) Rocket with actuation

There were significant disturbances in the wind tunnel which made the task of analyzing the rocket’s roll dynamics challenging. Common methods of system identification include non-linear optimization [7-8] and Kalman filtering [9], which assume that noise and disturbance is Gaussian with zero mean. However, in the application for this paper, the disturbance is unknown and has a significantly high amplitude. Therefore, the goal was to create a method which assumed no prior knowledge on the distribution of the disturbance and could decouple the rocket frame roll response from the disturbance. Another goal was to ensure the methods were computationally fast and simple to implement.

## II. METHODOLOGY AND RESULTS

In order to provide spin-stabilization to the small sounding rocket, a static test was performed in a vertical wind tunnel. The rocket was attached to a fixed support which allowed the rocket to roll freely about its primary axis. Control of the rocket’s fins was provided by servos, which can be commanded to move to a set deflection (angle). The exact speed of the wind was unknown, but held constantly throughout the experiment. The roll of the rocket was captured by an optical encoder. Figure 3 shows the setup.

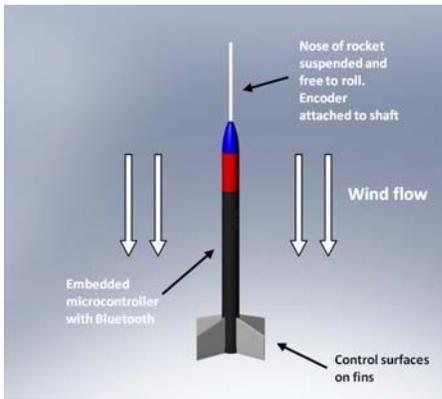


Figure 3: Experimental setup of the stationary rocket in

Data was captured and logged to file. Every 100 milliseconds, both the control input and the measured roll were recorded. The control input was logged as an angle in degrees of servo deflection from the fins neutral position, while the measured

roll was recorded as the angle in degrees since its initial start position. Throughout the test, the roll response was recorded for several different fin positions. Sufficient time between the transitions was provided to allow the system to reach steady state. Figures 4 (a) and (b) show the input control and output roll response.

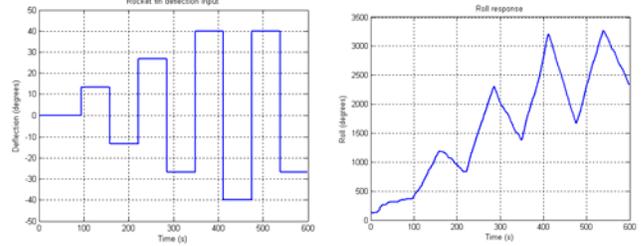


Figure 4: (a) Input control (b) Roll response

Once this raw data had been attained, it was then analyzed by the following key steps:

1. Fit a piecewise linear model to  $\theta$
2. Find a relationship between the steady state angular velocity and fin deflection
3. Create a model for velocity step response that ignores external disturbances
4. Apply integral method to identify time constant and torque constant
5. Fit disturbance model, to identify the disturbance present
6. Re-simulate model with external disturbance

### A. Fit piecewise model to $\theta$

The roll response of the rocket airframe is close to piecewise linear as shown in Figure 4(b). Thus, as an initial approximation, a piecewise linear model with eight steps was formulated for  $\theta$  :

$$\begin{aligned} \theta_{model} &= a_1 + b_1 t, \quad t_1 < t < t_2 \\ &\vdots \\ &= a_8 + b_8 t, \quad t_8 < t < t_{end} \end{aligned} \quad (1)$$

Equation (1) is solved by using a linearly constrained least squares approximation method, using a segment for each step input interval and having continuity between the segments. Figure 5 shows the closeness of fit of the linear piecewise model and the raw roll position data.

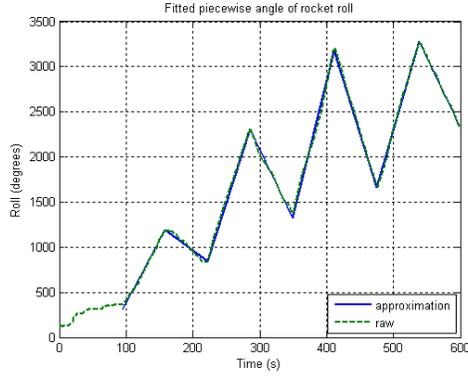


Figure 5: Piecewise linear model of the roll of the rocket

### B. Find approximation to steady state velocity

As shown in Figure 4 (a), eight step inputs of various magnitudes were carried out in the experiment. Provided that sufficient time was allowed after each step change for the system to reach steady state, this is enough data points to indicate a general trend between the deflection angles and roll rates. Figure 6 shows a strong linear relationship between steady state deflection (fss) and roll rate (vss), which is modeled by:

$$v_{ss,new} = \alpha f_{ss} + \beta \quad (2)$$

A line using least squares was fitted to this data, and the constants  $\alpha$  and  $\beta$  in Equation (2) were found to be 0.659 and 2.8856 respectively.

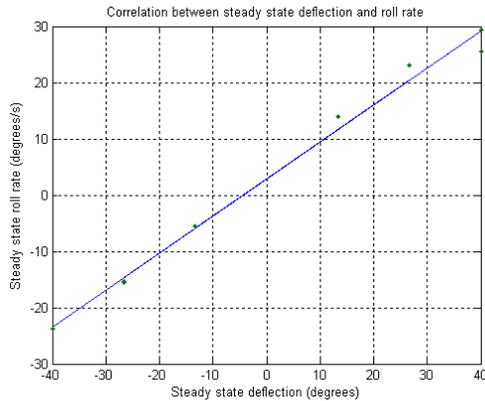


Figure 6: Correlation between steady state deflection and roll rate with a fitted line ( $R^2 = 0.991$ ).

### C. Create a model for velocity step response that ignores external disturbances

When subject to a step input, all real-world systems will have some transient response before reaching its steady state. Provided there is sufficient damping, often such systems can be modeled as a first order differential equation (DE).

Assuming that the rocket roll dynamics can be modeled by a first order differential equation, a bi-linear model as shown in Figure 7 can also model a first order DE. The general form of a bi-linear model is defined:

$$v_{model} = \begin{cases} \frac{v_{ss} - v_0}{t_0} t + v_0, & 0 < t < t_0 \\ v_{ss}, & t > t_0 \end{cases} \quad (3)$$

where it is assumed that  $v_{ss}$  and  $v_0$  are known and  $t_0$  is unknown.

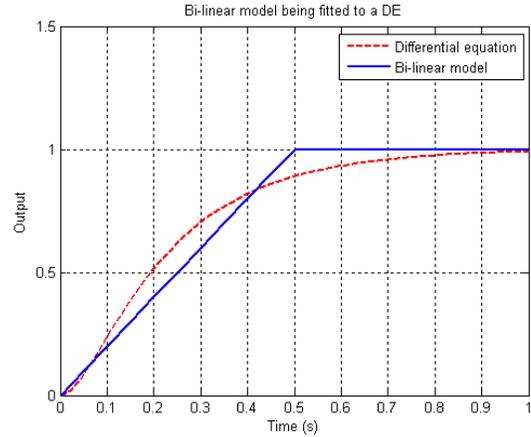


Figure 7: Bi-linear approximation to an over damped response modeled by an ordinary differential equation

In order to find the breakpoint ( $t_0$ ) for a particular velocity step, the breakpoint was incremented from 10ms to 4000ms in steps of 10ms. The values of  $v_{ss}$  and  $v_0$  at each velocity step are determined from the input deflection at the beginning and end of the step using Equation (2). The sum of squares was calculated between the model using a particular breakpoint and the velocity data. Figure 8 (a) shows the error for a range of breakpoints for modeling of the eighth velocity step input. Note that the subsequent values. The breakpoint with the least sum of squares error was chosen – in this case it was found to be 160.5ms.

Figure 8 (b) plots the approximated velocity (dotted blue line) using Equation (3) versus the true velocity (dashed green line). This result shows that the model of Equation (3) is sufficient to capture the overall steady state response even though there is a lot of external disturbance present in the raw velocity readings. This disturbance is not noise in the sensor, but is a real effect coming from turbulence in the wind tunnel.

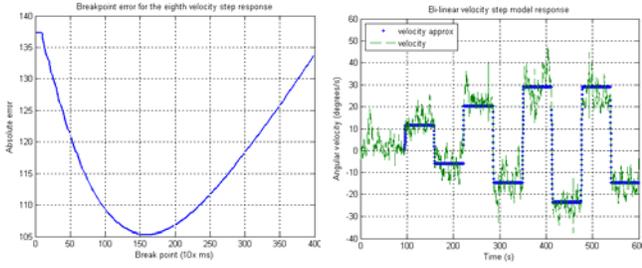


Figure 8: (a) Error between model and data while varying the breakpoint (b) bi-linear approximation velocity step response

#### D. Modeling and integral-based parameter identification

Initially, it is assumed that the velocity to be modeled only depends on inputs to the servo motors and is decoupled from the disturbance. To achieve this assumption, the velocity data used is obtained from the bi-linear approximation shown in Figure 8 (b), which effectively removes the disturbance. With no external disturbances, the rocket's roll dynamics are described by:

$$I\dot{v} = -cv + f_{ext} \quad (4)$$

where  $I$  is the rotational inertia,  $c$  is damping, and  $f_{ext}$  is the external torque resulting from a change in the fin angle  $\theta$ . Consider the following formulation of Equation (4):

$$I\dot{v} = -c(v - (\alpha\theta + \beta)) \quad (5)$$

At each steady state where  $\dot{v} = 0$ ,  $v_{ss}$  from Equation (5) satisfies Equation (2), and thus  $f_{ext} = c(\alpha\theta + \beta)$  in Equation (4). Equation (5) is rewritten in the form:

$$\dot{v} = -a(v - (\alpha\theta + \beta)) \quad (6)$$

where:

$$a = \frac{c}{I} \quad (7)$$

The model of Equation (6) model relates the angular acceleration to the rocket's current angular velocity ( $v$ ) and current fin deflection ( $\theta$ ). The constant  $a$  which is the inverse of the time constant, is therefore the unknown parameter, and the precise value of the rotational inertia  $I$  is not required to model  $v$ .

To identify the unknown parameter  $a$  in Equation (6), an integral-based method similar to [10] is formulated. The method is applied separately across each step input interval. Specifically, Equation (6) is integrated over each input step period, which yields:

$$\begin{aligned} v(t) &= -a \int_{t_1}^t (v - (\alpha\theta + \beta)) dt \quad t_1 < t < t_2 \\ &\vdots \\ &= -a \int_{t_8}^t (v - (\alpha\theta + \beta)) dt \quad t_8 < t < t_{end} \end{aligned} \quad (8)$$

where the integrals are numerically evaluated using the trapezium method. The resulting system of linear equations is solved for the unknown parameter  $a$  by linear least squares. Equation (6) can be further rewritten in the form:

$$\dot{v} = -av + b(\theta + \bar{\theta}) \quad (9)$$

where:

$$a = 1.34, \quad b = a\alpha = 0.88 \text{ s}^{-2}, \quad \bar{\theta} = a\beta = 4.38^\circ \quad (10)$$

The quantity  $b\bar{\theta}$  in Equation (9) represents the torque offset that occurs due to asymmetries across the rocket's primary axis. In other words, with no fin angle ( $\theta = 0$ ) the rocket will still spin. The parameter  $\bar{\theta}$  can be interpreted as the equivalent fin angle that would reproduce this spin if the rocket was perfectly symmetric. The parameter  $b$  is the torque constant that relates a movement in the fin angle to an applied torque on the rocket. The final result of this system identification process is plotted in Figure 9, which shows that the model captures the average response closely while ignoring the disturbances.

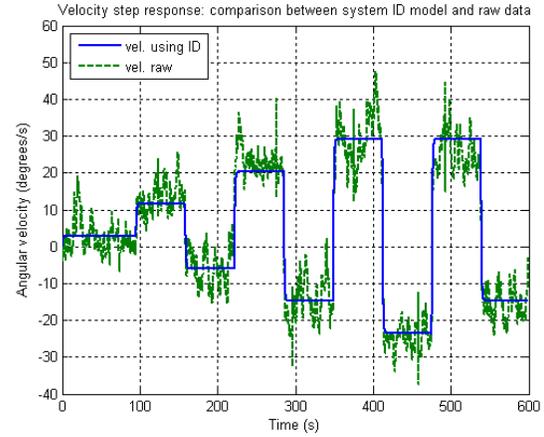


Figure 9: Response of the model of Equation (6) which does not account for disturbance.

#### E. Fit disturbance model

To capture the complete rocket roll response the external disturbances present in the experiment need to be taken into account. Equation (9) is thus reformulated:

$$\dot{v} = -av + b(\theta + \bar{\theta} + u_d) \quad (11)$$

where  $u_d$  is the unknown time-varying disturbance due to turbulence in the wind tunnel, and  $a$ ,  $b$  and  $\bar{\theta}$  are known from Equation (10). A piecewise linear model of  $u_d$  with a time interval of  $\Delta t$  is defined:

$$u_d(t) = u_{d,i-1} + \frac{u_{d,i} - u_{d,i-1}}{\Delta t} (t - t_{i-1}), \quad t_{i-1} \leq t \leq t_i \quad (12)$$

$$t_i = (i-1)\Delta t, \quad i = 1, \dots, n$$

where  $H(t)$  is the Heaviside function and  $n$  is the total number of points in the data set. Integrating Equation (11) from  $t_{i-1}$  to  $t_i$  yields:

$$\int_{t_{i-1}}^{t_i} \dot{v} dt = -a \int_{t_{i-1}}^{t_i} v dt + b \int_{t_{i-1}}^{t_i} \theta dt + b \int_{t_{i-1}}^{t_i} (\bar{\theta} + u_d) dt \quad (13)$$

Substituting  $u_d(t)$  from Equation (12) into Equation (13), and applying the trapezium rule yields:

$$v(t_i) - v(t_{i-1}) = -a \frac{(v(t_{i-1}) + v(t_i)) \Delta t}{2} + b \frac{(\theta(t_{i-1}) + \theta(t_i)) \Delta t}{2} + b \bar{\theta} \Delta t + b \frac{(u_{d,i-1} + u_{d,i}) \Delta t}{2} \quad (14)$$

Assuming that  $u_{d,i-1}$  is known from the previous time step, Equation (14) can be solved to determine  $u_{d,i}$  at the next time step:

$$u_{d,i} = \frac{1}{b} (2(v(t_i) - v(t_{i-1})) + a(v(t_{i-1}) + v(t_i)) \Delta t - (2\bar{\theta} + u_{d,i-1} + \theta(t_{i-1}) + \theta(t_i)) \Delta t) \quad (15)$$

For this experiment the time step is chosen to be  $\Delta t = 0.1s$ , and the result of the identified disturbance is shown in Figure 10. Note that to remove the effect of measurement error on the result, the velocity in Figure 9 and the resulting disturbance are smoothed several times with a 5-point moving average.

A further validation of the modeling approach is that the histogram of the disturbance is approximately a normal distribution centred about zero, as shown in Figure 10. Thus there is no bias in the model. Since the flow in the vertical wind tunnel is known to be turbulent, a random disturbance would be expected. Furthermore, the equivalent angles that represent the disturbance are around 10-20 degrees which is realistic on the rocket frame.

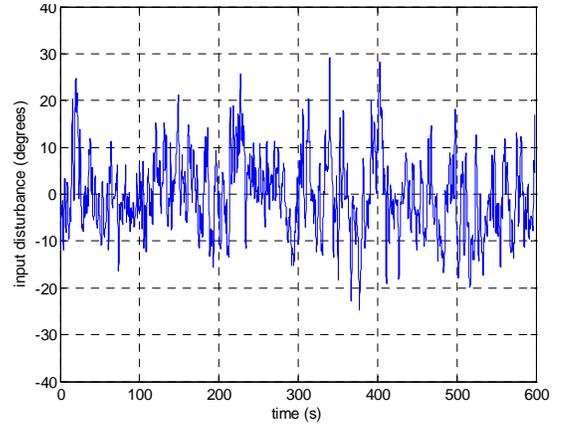


Figure 10: Identified external disturbance model

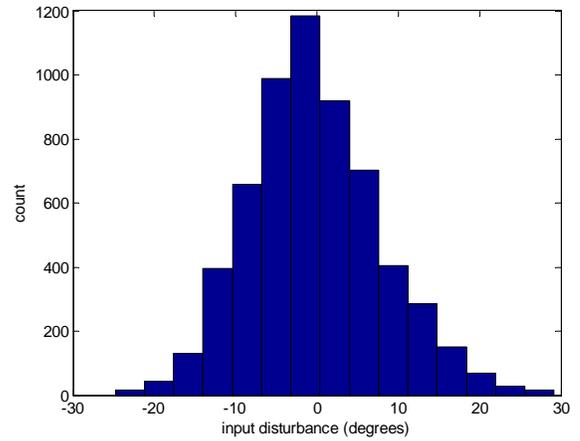


Figure 11: Distribution of the disturbance model

#### F. Re-simulate model with external disturbance

Using the identified disturbance from Equation (15) and Figure 9, the model of Equation (11) describing the full roll dynamics is numerically solved for the velocity and compared to the measured velocity in Figure 12. In addition, the angular position of the rocket is found by integrating the modeled velocity and is compared to the measured angular position in Figure 13. Both results show that the model closely captures the actual data obtained from the experiment. Specifically, the model predicted the angular velocity with a median absolute error of 0.51 degrees/s and a 90<sup>th</sup> percentile of 1.25 degrees/s. The model tracked the roll angle (as measured by an encoder) with a median absolute error of 0.25 degrees and a 90<sup>th</sup> percentile of 0.5 degrees. The error between the modeled and measured roll angle is plotted in Figure 14. Note that the spikes are likely due to the microprocessor missing counts in the encoder output.

### III. CONCLUSION

A minimal modeling approach and integral based parameter identification method were used to analyze roll dynamics of a sounding rocket airframe inside a vertical wind tunnel. By allowing a sufficient time to elapse after each step input, the disturbance was decoupled from the intrinsic dynamics of the rocket. This approach enabled an accurate calculation of the inertia and damping. The disturbance was then modeled by an effective fin angle and was directly identified. The overall modeled outputs closely matched the measured values for both the roll rate and angle. The identified disturbance gave realistic magnitudes and a close to normal distribution further validating the model and methods. Future work includes investigating how other key variables such as wind speed and inertia of the rocket affects its dynamics as well as extending the approach to pitch and yaw.

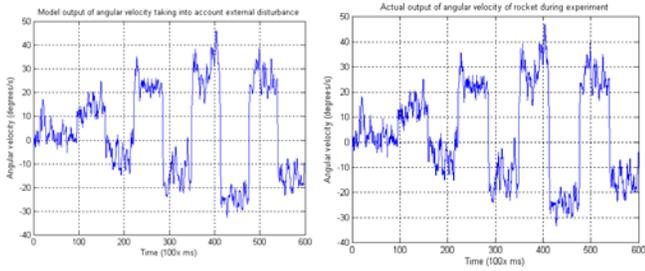


Figure 12: (a) Model output of angular velocity (b) Actual angular velocity from experiment obtained by differentiating the measured encoder output

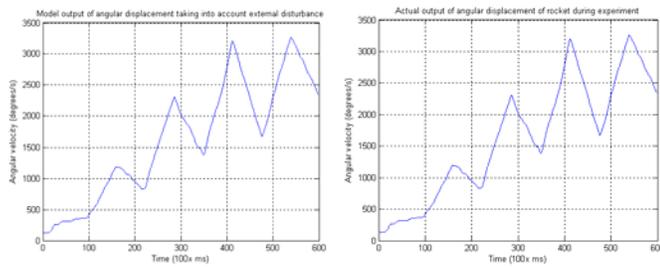


Figure 13: (a) Model output of angular displacement (b) Actual angular displacement as measured by an encoder

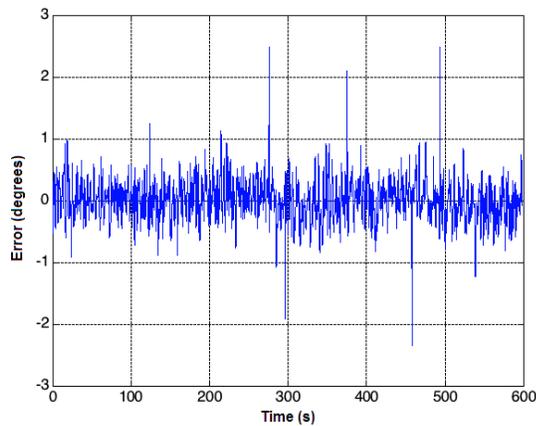


Figure 14: Error between the modeled and recorded roll angle of the rocket frame

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