Semi-active tuned mass damper building systems: Design

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SUMMARY

Passive and Semi-Active Tuned Mass Damper (PTMD and SATMD) building systems are proposed to mitigate structural response due to seismic loads. The structure’s upper portion self plays a role either as a tuned mass passive damper or a semi-active resetable device is adopted as a control feature for the PTMD, creating a SATMD system. Two-degree-of-freedom (2-DOF) analytical studies are employed to design the prototype structural system, specify its element characteristics and effectiveness for seismic responses, including defining the resetable device dynamics. The optimal parameters are derived for the large mass ratio by numerical analysis. For the SATMD building system the stiffness of the resetable device design is combined with rubber bearing stiffness. From parametric studies, effective practical control schemes can be derived for the SATMD system. To verify the principal efficacy of the conceptual system, the controlled system response is compared to the response spectrum of the earthquake suites used. The control ability of the SATMD scheme is compared to that of an uncontrolled (No TMD) and an ideal passive tuned mass damper (PTMD) building systems for multi-level seismic intensity.

KEY WORDS: tuned mass damper; semi-active control; resetable device; optimum parameters; seismic hazard; statistical assessment

1. INTRODUCTION

Tuned Mass Damper (TMD) systems are a practical strategy in the area of structural control for flexible structures such as tall buildings. Normally, TMD systems consist of added mass with properly functioned spring and damping elements that provides a frequency-dependent damping in the primary structure. The mechanism of suppressing structural vibrations by attaching a TMD to the structure is to transfer the vibration energy of the structure to the TMD and to dissipate the energy in the damper of the
However, overall performance is limited by the size of the additional mass (normally about 1% of building weight) and the sensitivity related to the narrow band control and the fluctuation in tuning the TMD frequency to the controlled frequency of a structure. The mistuning or off-optimum damping can significantly reduce the effectiveness of the TMD; therefore, the TMD system may be neither reliable nor robust. In addition, a TMD system may be more effective when the forcing function (from wind or earthquake excitation) has significant spectral content at the frequency of the TMD fundamental mode. Further away from this frequency a TMD may have much less effect. Therefore, it is difficult to draw general conclusions on the effectiveness of a TMD system, especially when the structure includes inelastic behavior for seismic excitation.

In an attempt to increase the performance of the TMD without incurring the problem of increasing structural weight, active tuned mass damper (ATMD) systems have been proposed. Chang and Soong [1] introduced an active force to act between the structure and the TMD system. Abdel-Rohman [2] proposed a process for designing an effective ATMD system to control a tall building subjected to stationary random wind forces by using the Pole-assignment method. The results suggested that the design of an optimal ATMD required at least a parametric study to select the ATMD parameters. Furthermore, common feedback control methods using displacement, velocity, complete and acceleration feedback for the ATMD have been studied by many researchers [3-6]. However, fully active systems may require significant power sources and entail implementation complexity, often beyond a level, which is available or manageable.

Meanwhile, semi-active (SA) control is emerging as an effective method of mitigating structural damage from large environmental loads, with two main benefits over active control and passive solutions. For the SA control, a large power or energy supply is not required to have a significant impact on the response. A broad feedback adaptive range of control can be provided. Semi-active systems are also strictly dissipative and do not add energy to the system, ensuring stability. Thus, SA control over time should be better able to respond to changes in the structural behavior, particularly due to non-linearity, damage or degradation.

Several researchers [7-9] have focused on the basic analytical techniques needed to characterize structural systems that use a resetable SA device for vibration suppression. Barroso et al. [10] and Hunt [11] presented an investigation of the ability of SA control methods utilizing resetable devices to mitigate structural response in the presence of hysteretic, geometric and yielding nonlinearities under various intensity level seismic hazard suites to define control efficiency and seismic hazard statistics. Yang et al. [12] suggested that a general resetting control law based on the Lyapunov theory for a resetting SA damper and compared this with a switching control method through extensive numerical simulations using different types of earthquake excitations.

Yang and Agrawal [13] presented the safety performances of various types of hybrid control systems, which consist of a base isolation system and resetting SA dampers for protecting nonlinear buildings against near-field earthquakes. Djajakesukma et al. [14] reported SA stiffness damper systems with various control laws, such as resetting control, switching control, LQR and modified LQR systems and the results were compared with no control and passive control cases. Similarly, Chase et al. [15, 16] proposed a series of SA control laws based on optimal control design, and presented results as cumulative hazard distribution based on responses to suites of ground motions. Abe [17] also presented the performance of SATMD with initial TMD displacement and variable damping subject to earthquake excitation. He found that the SATMD system
give higher reduction of structural response than conventional passive TMD.

To overcome the limitations of the TMD mass ratio, it has been suggested that using a portion of the building itself as a mass damper may be very effective. In particular, one idea is to use the building’s top storey as a tuned mass. The concept of an ‘expendable top storey’ introduced by Jagadish et al. [18], or the ‘energy absorbing storey’ presented by Miyama [19], is an effective alternative where the top storey acts as a vibration absorber for the other stories of the building. Another proposal is to convert a mega-structural system to a mega-sub-control system that exhibits structural efficiency by allowing a high rigidity of the system while keeping a minimum amount of structural materials [20]. Murakami et al. [21] described an example of the design of a multi-functional 14-storey building including apartments, office rooms, shops and parking lots where a seismic isolation system is installed on the middle-storey. Villaverde et al. [22] studied a 13-storey building to assess the viability and effectiveness of a ‘roof isolation’ system that aims at reducing the response of buildings to earthquakes, where the proposal to build a vibration absorber with a building’s roof has the potential to become an attractive way to reduce structural and nonstructural earthquake damage in low- and medium-rise buildings. Meanwhile, Pan and Cui [23], Pan et al. [24] and Charng [25] sought to evaluate the effect of using segmental structures where isolation devices are placed at various heights in the structure, as well as at the base, to reduce the displacements imposed on each of the devices. Thus, a variety of research has examined using segments of the structure itself as a tuned absorber.

This paper describes 2-DOF SATMD building system, in which resetable devices are incorporated for a structure divided into two segments. In this case, the interface represents or contains the isolation layer. For this study, the dynamic characteristics and seismic linear elastic responses are investigated and the response results are compared with those from the corresponding uncontrolled (No TMD) and ideal passive (PTMD) building systems. The control effects of the TMD (PTMD and SATMD) systems are represented in the combined graphical plots of the time history analysis (2-DOF) and response spectrum (SDOF) analysis.

To encompass a broad variety of earthquake ground motions, thirty earthquake events of three different probabilistic hazard intensity levels representing ground motions having low, medium and high probability of exceedance in 50 years for the Los Angeles area are used. Performance is thus evaluated statistically using lognormal distributions.

2. STRUCTURAL CONTROL CONCEPT

2.1. Semi-active resetable device

SA resetable devices are relatively reliable and simple devices, which can act autonomously. Described fundamentally as a non-linear pneumatic spring element, the equilibrium position or rest length can be reset to obtain maximum energy dissipation from the structural system [8]. Energy is stored in the device by compressing the working fluid, such as air, as the piston is displaced from its center position. When the piston reaches its maximum displaced position in a given cycle, the stored energy is also at a maximum and the device changes direction of motion. Thus, the reset criteria are
determined to be the point of zero velocity at displacement peaks. At this point, the stored energy is released by discharging the working fluid to the non-working side of the device, thus resetting the equilibrium position of the device. Figure 1 shows the conventional resetable device configuration [9], with a single valve connecting the two sides.

![Figure 1. Resetable device attached to a single-degree-of freedom system](image)

Unlike previous resetable devices, a recently developed design [26, 27] at the University of Canterbury eliminates the need to rapidly dissipate energy using a two-chambered design that utilizes each piston side independently, as shown in Figure 2. This new approach allows a wider variety of control laws to be imposed, as each valve can be operated independently allowing independent control of the pressure on each side of the piston. In this paper, resetable device, denoted as a ‘1-4 device’ providing damping in all four quadrants, is used for the SA control scheme as it provides dissipation over the entire SATMD motion. The detailed control law for the resetable device used in this research is well documented in several references [10, 26, 28].

![Figure 2. Newly designed resetable device with independent chamber](image)

2.2. Combined concept of TMD building system

The suggested TMD building system concept can be defined as an extension of the conventional TMD system, but using a large mass ratio. Due to the large mass ratio, the upper portion may experience large displacements. To avoid excessive lateral motion or stroke of the tuned mass, the upper portion is interconnected by the combined isolation system of rubber or elastomeric bearings and a viscous damper (for the PTMD) or a resetable device (for the SATMD). When the building frame is implemented with the proposed TMD (PTMD or SATMD) system, the upper portion is supported by elastomeric bearings that are attached on the top of the main frame’s columns. The system is shown schematically in Figure 3.
The overall mechanism of suppressing structural vibration induced by an earthquake is to transfer the vibration energy of the structure to the isolated upper storey(s). The transferred energy is dissipated at the isolation interface so that seismic force of the entire superstructure can be reduced. Thus, the overall effectiveness depends on the amount of energy transferred and the size of the tuned mass and the ability of the isolating elements (viscous damper or resetable device) to dissipate that energy via the relative motions at the interface. Additional trade-offs with respect to ease of tuning/design and ability to manage non-linearity and/or degradation may also be a factor.

3. MODELS

3.1. Motion characteristics and equations

Figure 4 presents three TMD-segmented structural systems that form the basis of the 2-DOF modeling strategy. Being characterized by its mass, tuning and damping ratios, the PTMD system consists of a TMD system, which is connected by a spring and a viscous damper, as shown in Figure 4(a). Figures 4(b) and 4(c) represent SATMD building systems including passive and resetable springs at the instants of rest and reset respectively. As the relative displacement between the main system and the SATMD increases, both springs (passive and resetable spring) stretch and work together against the relative motion of the SATMD. When the relative displacement reaches its maximum position, the velocity is zero and the resetable semi-active device resets, releasing the energy stored [29]. Thus, the equilibrium position or unstretched length of the resetable spring is time variant. In contrast, the viscous damper-based PTMD acts for all motion.
For the TMD (PTMD or SATMD) building systems, a 2-DOF system can be defined for design by a pair of coupled second-order ordinary differential equations. For the PTMD and SATMD building systems, the equations of motion of the systems subjected to the earthquake load can be defined respectively:

\[
\begin{bmatrix}
    m_1 & 0 \\
    0 & m_2
\end{bmatrix}
\begin{bmatrix}
    \ddot{x}_1 \\
    \ddot{x}_2
\end{bmatrix} + 
\begin{bmatrix}
    c_1 + c_2 & -c_2 \\
    -c_2 & c_2
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_1 \\
    \dot{x}_2
\end{bmatrix} + 
\begin{bmatrix}
    k_1 + k_2 & -k_2 \\
    -k_2 & k_2
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} = 
\begin{bmatrix}
    k_1 & 0 \\
    0 & k_2
\end{bmatrix}
\begin{bmatrix}
    x_g \\
    \dot{x}_g
\end{bmatrix}
\]  

(1)

\[
\begin{bmatrix}
    m_1 & 0 \\
    0 & m_2
\end{bmatrix}
\begin{bmatrix}
    \ddot{x}_1 \\
    \ddot{x}_2
\end{bmatrix} + 
\begin{bmatrix}
    c_1 & 0 \\
    0 & c_2
\end{bmatrix}
\begin{bmatrix}
    \dot{x}_1 \\
    \dot{x}_2
\end{bmatrix} + 
\begin{bmatrix}
    k_1 + k_{2(RB)} + k_{2(res)} & -k_{2(res)} \\
    -k_{2(res)} & k_{2(res)}
\end{bmatrix}
\begin{bmatrix}
    x_1 \\
    x_2
\end{bmatrix} = 
\begin{bmatrix}
    k_1 & 0 \\
    0 & k_{2(res)}
\end{bmatrix}
\begin{bmatrix}
    x_g \\
    \dot{x}_g
\end{bmatrix}
\]  

(2)

where \( m_1 \) = mass of main system; \( m_2 \) = mass of TMD; \( k_1 \) = stiffness of main system; \( k_{2(RB)} \) = stiffness of rubber bearings; \( k_{2(res)} \) = stiffness of resetable device; \( c_1 \) = damping coefficient of main system; \( c_2 \) = damping coefficient of TMD; \( x_1 \) = displacement of main system; \( x_2 \) = displacement of TMD; \( x_g \) = displacement of ground; and \( x_s \) = equilibrium position (unstretched length) of the resetable spring.

A resetable device non-linearly alters the stiffness as a function of its motion, creating a non-linear dynamic system with (implicit) feedback control, in contrast to the linear PTMD system model.

3.2. Parametric optimization

In this study, for large mass ratios, the equations from Sadek et al. are adopted to find the optimal parameters of frequency tuning and damping ratios. For high values of mass ratio, \( \mu \), it is likely that the TMD will not be an appendage added to the structure, but a portion of the structure itself, such as one or more of the upper stories. According to Sadek et al., the equation of the optimal frequency tuning ratio, \( f_{2opt} \), and the optimal
damping ratio, $\zeta_{2\text{opt}}$, of the TMD systems are defined:

$$f_{2\text{opt}} = \frac{1}{1 + \mu} \left( 1 - \zeta_1 \sqrt{\frac{\mu}{1 + \mu}} \right) \quad (3)$$

$$\zeta_{2\text{opt}} = \frac{\zeta_1}{1 + \mu} + \sqrt{\frac{\mu}{1 + \mu}} \quad (4)$$

For practical application, it is necessary to obtain the resulting optimal TMD stiffness, $k_{2\text{opt}}$, and optimal damping coefficient, $c_{2\text{opt}}$. These parameters can be derived using $f_{2\text{opt}}$ and $\zeta_{2\text{opt}}$.

$$k_{2\text{opt}} = m_2 \omega_1^2 f_{2\text{opt}}^2 = \frac{m_2 \omega_1^2}{(1 + \mu)} \left[ 1 - \frac{\zeta_1}{1 + \mu} \right]^{2} \quad (5)$$

$$c_{2\text{opt}} = 2m_2 \omega_1 f_{2\text{opt}} \zeta_{2\text{opt}} = \frac{2m_2 \omega_1}{1 + \mu} \left( 1 - \frac{\zeta_1}{1 + \mu} \right) \left( \frac{\zeta_1}{1 + \mu} + \sqrt{\frac{\mu}{1 + \mu}} \right) \quad (6)$$

where $\omega_1$ is the frequency of the main system.

Figure 5(a) shows the optimum TMD tuning and damping ratio versus mass ratios of 0 to 1 with different structural damping values ($\zeta_1$=0, 0.02, 0.05 and 0.1). The figure indicates that the higher the mass ratio, the lower the tuning ratio and the higher the TMD damping ratio. The higher the damping ratio ($\zeta_1$) of the main system, the lower the tuning ratio and the higher the TMD damping ratio. Figure 5(b) shows the optimum TMD stiffness and damping coefficient normalized to their equivalent values in the base structure. From this figure, as expected, it is observed that a TMD with both larger stiffness and larger damping is needed, the larger the mass ratio becomes. From these trends, it can be predicted that there is no more increase in the TMD stiffness when the mass ratio is over 1.0, which is an unrealistic value. The effects of the damping ratio of main system are lightly amplified with increase in mass ratio. A nearly linear increase in TMD damping coefficient is observed with increase in the mass ratio, and it is also observed that there is small effect of the damping ratio of the main structure ($\zeta_1$) on the TMD damping coefficient. Figure 6 shows the resulting optimal design process for the 2-DOF TMD system. The parametric results from the design process will be used as the basic references for the MDOF verification study on TMD building systems.
Figure 5. Optimum TMD parameters for different mass ratios and internal damping of main system

Figure 6. Design process for the TMD system
3.3. Modeling of TMD systems

To represent the effects of the TMD rubber bearing stiffness, the spring member that is incorporated to an inelastic dynamic analysis program, Ruaumoko [28], is used. In the transverse direction, the optimal TMD stiffness, \( k_{2_{\text{opt}}} \), is applied to the sum of the stiffness of the SA device and rubber bearings (SATMD) or to the whole stiffness of the rubber bearings (PTMD). Thus, the optimal stiffness of the semi-active system is assumed to be the same as for the passive TMD case. This may neglect or underuse certain qualities of the SA devices. Thus, the case of using some percentage of the optimal stiffness for the resetable device without rubber bearings could be compared to the previously studied cases mentioned above [27]. As the other component for the PTMD system, the added damping to the structure can be modeled using the damping or dashpot member in the program Ruaumoko to represent a local viscous energy dissipater. A linear elastic hysteresis has been used to represent the elastic properties of the TMD system.

To represent the idealized behavior modes of SA resetable device members used in Ruaumoko, Figure 7 shows the basic hysteresis loops; without saturation (a) and with saturation (b). For the saturable case, the force is proportional to the displacement until a saturation force is attained, \( F_{y^+} \) or \( F_{y^-} \) (the yield forces of the resetable device member or attached fuse) when the system employs an essentially perfectly plastic response. On any reversal of displacement the force is automatically reset to zero, the origin for spring forces moves to the existing displacement and the system will then behave as an elastic member until either saturation is achieved or the displacement again changes sign. In both cases, the drops to zero force representing device resetting.

This is an idealized element and several methods of further customizing these hysteresis loops have been presented [30-33], but are outside the scope of this work. Here, the primary focus is on the stiffness of the device in designing theses systems in comparison to passive approaches. Thus, further details on the dynamics of these devices is left in [30-33] to the reader.

![Figure 7. Hysteresis behavior of resetable device](image)

4. EARTHQUAKE SUITES AND STATISTICAL ASSESSMENT

Statistical assessment of structural response is an important step in performance-based seismic design. Most prior research into active or semi-actively controlled structures
employed either sinusoidal, random, single, or selected earthquake excitations to illustrate the benefits of control [8, 12, 34-37]. As the characteristics of seismic excitation are entirely random and vary significantly unlike other types of vibrational excitation, the use of a number of multiple time history records over a range of seismic levels is essential for effective controller evaluation. This approach has been used extensively to develop design guidelines and complete performance assessment of control [10, 11, 15, 16, 31, 38, 39].

The three ground motion acceleration suites used here were developed by Sommerville et al. [40] for the SAC Phase II project. Each suite has 10 pairs of recorded or generated ground motion accelerograms. These were selected to fit the magnitude and distance characteristics of the seismic hazard at the LA site. The first suite represents ground motions for which the structural demand has a 50% chance of being exceeded in 50 years (low suite). The second suite (medium suite) represents a 10% chance in 50 years and the final suite (high suite) a 2% chance in 50 years. To reduce the computational requirements, the first of each of the 10 pairs of records (odd half) were used in this paper. The earthquakes contained within the three suites are shown in Table 1.

To combine these results across the earthquakes in a suite, lognormal statistics are used [11, 41], since the statistical variation of many material properties and seismic response variables is well represented by this distribution provided one is not primarily concerned with the extreme tails of the distribution. More specifically, the central limit theorem states that a distribution of a random variable consisting of products and quotients of several random variables tends to be lognormal. Thus, results from within each earthquake suite are combined using the lognormal distribution geometric mean and variance.

5. 2-DOF MODEL IMPLEMENTATION

5.1. Method of analysis

To demonstrate the proposed control methodology, 2-DOF linear models including 5% internal structural damping with natural periods of 1.19, 1.52 and 1.88 seconds are investigated. Table 2 shows the dynamic properties of the main systems simulated. For these main systems, the mass ratio of 0.5 was used and this value is the mass ratio of the 1\textsuperscript{st} modal mass of the TMD to the total mass of the main system. To assess the control effects of the resetable device, the percentage ratio of the resetable device stiffness to the total stiffness are selected as 25%, 50%, 75%, 100% and 33% (without rubber bearing) of $k_{2,\text{opt}}$ the optimal value of the TMD stiffness. The TMD stiffness combination of resetable device and rubber bearings is shown in Table 3.

Performance with No TMD, optimum PTMD, and off-optimum PTMD are compared with the suggested SATMD cases. For the off-optimum PTMD, the TMD damping ratio ($\xi_2$) of 0.15 was used and this value is the realistic figure compared to the optimum one of 0.611, so that the reliability of the optimum parameters can be estimated. Also, this value represents a practical maximum amount of damping that can be obtained, and is thus reasonable for broad comparison to various SATMD cases. The maximum force of 27.7kN is selected for the SA resetable device, representing 13.8% [11] of the total system weight of 402kN multiplied by mass ratio ($\mu=0.5$). The TMD parameters used for each case obtained from Equations (9) to (12) are listed in Table 4.
To demonstrate the relative control effects of the TMD systems, the performance measures are evaluated statistically from the individual structural responses for the 10 seismic records within each suite (low, medium and high). All controlled displacement and acceleration values are presented and the reduction factors normalized to the uncontrolled (No TMD) result are evaluated. Reduction factors more clearly indicate effect and are more readily incorporated into performance-based design methods when using suites of probabilistically scaled events [31]. Thus, the response reduction factors for PTMD (off and on), SA33TMD* (without rubber bearing) and SATMDs for low, medium and high suites are presented.

To indicate the range of spread of results over a suite at a given natural period, the 16th, 50th and 84th percentiles are used. The values of median (50th percentile) and the width, which is the spread between the 16th and 84th percentiles, are taken for each period. Given the lognormally distributed results across a probabilistically scaled suite of events [31,41], lognormal statistics are appropriate and these percentiles represent 1 standard deviation of this distribution about the median value. As the results conform to a lognormal distribution, the lognormal standard deviation (commonly referred to as the dispersion factor, \( \beta \) is used herein to describe this spread resulting from randomness in response. The dispersion factor, \( \beta \), can be calculated as follows

\[
\beta = \ln\left(\frac{x_{84}}{x_{50}}\right) = \ln\left(\frac{x_{50}}{x_{16}}\right)
\]

in which, \( x_{84} \), \( x_{50} \) and \( x_{16} \) are the reduction factors for the 84th, 50th (median) and 16th percentiles, respectively. Finally, it can be found that which TMD case has the best trade-off between bandwidth (dispersion) reduction and overall response reduction for decision-making purposes.

5.2. Performance results

Figures 8 to 10 show the 50th percentile (median) of earthquake spectra for each suite and the maximum response results (displacement and acceleration) for the TMD systems examined. Each term of the TMD systems used in these figures is listed in Table 3. From the results, it is observed that the performance of the PTMD(on and off) and SATMD building systems is feasible. As expected, the No TMD values coincide with each spectrum line. The off-optimum PTMD system showed better response reductions than the optimum PTMD system in terms of displacement, while the optimum PTMD building system presented better reductions in acceleration response due to higher damping ratios under all suites of earthquake intensity. Even though the control efficiency is not so different, the SATMD systems around SA50TMD (SA25TMD to SA75TMD) showed marginally better displacement reductions than other SATMD cases in the lower plots of Figures 8-10. Note that these differences are less visible in the upper plots of Figures 8-10 where the diamonds effectively overlap for (all of the) SATMD systems due to the larger scales used, which makes the differences in the lower plots less visible. Overall, the balanced stiffness between the resetable device (50%) and the rubber bearings (50%) is a reasonable stiffness strategy for the generalized statistical aspects of TMD building systems to reduce the design parameters and make a more standared and simple comparison. Meanwhile, all SATMD cases reduced acceleration response of each main system, however, this reduction is less than that of the PTMD (both on and off) system, due to TMD damping provided.
Figure 8. Earthquake response spectra and maximum responses of main system by PTMD and SATMD – 50th percentile (median) and low suite

Figure 9. Earthquake response spectra and maximum responses of main system by PTMD and SATMD – 50th percentile (median) and medium suite
Figures 11 to 13 present the statistical final outcomes of the response (displacement and acceleration) reduction factors and the resulting dispersion factors from the uncontrolled main systems for each natural period and different earthquake suites used. Note that the upper, central and lower solid curves represent the 84th, 50th and 16th percentiles for each set of results, respectively. For all the TMD systems, the dispersion factor of the SA33TMD* systems shows remarkably small values when compared to any other system, indicating an improvement in performance and more predictable response of the system and the statistical properties are clear for the higher intensity of suites. Furthermore, it can be found that even if the TMD system is perfectly tuned for the structural system, the SA33TMD* has a better general performance. This is because the latter has a smaller bandwidth of response and is thus able to reduce the response of those earthquakes after systems struggle to cope with.

In reality, tuning the TMD system to perfection would be very difficult, if not impossible, due to uncertainty of structural design parameters and viable changes in the structure over time. Hence, the SATMD system without rubber bearings offers an alternative as it is easier to design with a certain resetable stiffness, a value that does not have to be exact for the system to have an improved performance. Thus, for the SATMD system, it is not necessary to either calculate the exact tuned stiffness required or demand that the devices produce the exact design stiffness. This fact saves time and effort in the design procedure and simplifies design, as any reasonable stiffness in the neighborhood of \( k_{opt} \) will produce an adequate design with satisfactory results for the SATMD system. Finally, the SA50TMD system may be regarded as the “preferred” TMD strategy to achieve the best performance trade-off.
In this parametric trade-off study, the efficacy of spreading stiffness between resetable devices and rubber bearings is illustrated. Spectral analysis of simplified 2-DOF model explores the efficacy of these modified structural control systems and the general validity of the optimal derived parameters is demonstrated. The end result of the spectral analysis is an optimally-based parametric design approach that fits into accepted design methods, rather than a non-linear, non-convex optimisation result that might be more optimal, but is not repeatable or applicable for the typical design engineer.

Figure 11. Earthquake response reduction factors and dispersion factors – Low suite
Figure 12. Earthquake response reduction factors and dispersion factors – Medium suite
Figure 13. Earthquake response reduction factors and dispersion factors – High suite

It should be noted that the results of Figures 8-13 reflect the response of the structure. However, they do not analyse the drift or relative magnitude of displacements between the TMD section of the structure and the base section. This analysis was not deemed necessary here as it was felt to be more application specific and is thus addressed in further works focused on specific application analyses. Such drifts were studied in the work of Mulligan et al [39], and found to be dependent on the control law used and the
specific ground motion or suite. A final conclusion of [39] was that this issue was manageable in design and could be managed by having the devices reset on differences in displacement, as done in this work and referenced to [33, 39]. Hence, the issue can be managed by design, but must be considered in this type of application, as with a PTMD as well.

6. CONCLUSIONS

PTMD and SATMD building concepts have been presented and implemented in a design simulation. The suggested system is the synthesis model of the TMD control and segmental building system using purposely separated seismic masses of a structure itself. A 2-DOF model explores the efficacy of these modified control system and the validity of the optimal parameters was demonstrated. To avoid erroneous conclusions being drawn due to a typical performance for a single earthquake, median response values were defined under three earthquake suites representing a multi-level seismic hazard analysis. For this parametric study, the reasonable efficacy of a stiffness combination between resetable device and rubber bearings was illustrated. Based upon the investigation described herein, the following conclusions can be drawn:

- SATMD system with reasonable combination of TMD parameters provides a better control strategy than PTMD systems, especially if the optimum stiffness of PTMD \(k_{2opt}\) is not ideal or perfect due to degradation or mis-modeling of the structure. Thus, more effective parameter combinations may be available beyond the scope of this initial parametric analysis. Overall, the SATMD systems indicate an improvement in performance and robustness. However, acceleration response reduction of PTMD systems (especially, optimum PTMD) is slightly greater than that of SATMD systems due to additional TMD damping provided at optimal damping values that may be unattainable.

- Semi-active solutions are not constrained to \(k_{2opt}\) and its control ability is improved when the value of less stiffness is used, providing robust and effective seismic energy management. Thus, the SATMD system is easier to design as the tuning of the system to the structure, by altering the stiffness value, is not as critical as for the PTMD system where some “out-of-tuning” may have a detrimental effect on the structural response.

- The entire use of semi-active control (without passive control) achieves a small control bandwidth (less dispersion) under various level of earthquake intensity. Again, narrower bandwidth results are expected by using less stiffness values for semi-active resetable device.

- There is good potential for SATMD building concept, especially in retrofit where lack of space constrains development to expand upward. It would be beneficial when additional stories are added to an existing structure, as these stories become part of the structure control system, thus alleviating the necessity for additional mass that is redundant for the majority of the time. For example, 12+2 or 12+4 story structural concept can be utilized to control 12-story structures. Such an analysis is the subject of a companion paper.

- Finally, note that this overall design approach using equivalent PTMD stiffnesses, or ratios thereof, is computationally simpler and proven. It also yields a parametric
optimisation problem that is convex compared to using non-linear optimization of each specific controlled structure over all possible variables, which is not guaranteed to return an optimal or useful result. It thus represents a more general and more easily employed (potential) design approach.

The numerical results from the 2-DOF design cases herein can be used as the basic reference for the design of multi-story applications mentioned above. Furthermore, the control concept presented here can be amenable to the base-isolation and hybrid (the TMD with base-isolation building system) control of the structures.

REFERENCES


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Table 2
Dynamic properties of main system

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<th>Item</th>
<th>Value</th>
<th>Unit</th>
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<td>Weight</td>
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<tr>
<td>1st Modal Mass</td>
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<tr>
<td>Natural period (Frequency)</td>
<td>1.19 (5.26) sec</td>
<td>rad/sec</td>
</tr>
<tr>
<td></td>
<td>1.52 (4.12)</td>
<td>sec (rad/sec)</td>
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<td>1.88 (3.34)</td>
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Table 3  
TMD stiffness combination of resetable device and rubber bearings

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<th>TMD</th>
<th>Resetable device (%)</th>
<th>Rubber bearing (%)</th>
<th>Total (%)</th>
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<td>75</td>
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<tr>
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<td>33</td>
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* without rubber bearing
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<tr>
<th>main system (sec)</th>
<th>TMD</th>
<th>$f_{2\text{opt}}$</th>
<th>$\xi_{2\text{opt}}$</th>
<th>$k_{2\text{opt}}$ (kN/m)</th>
<th>$c_{2\text{opt}}$ (kN-s/m)</th>
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<td>1.19</td>
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<td>0.647</td>
<td>0.150</td>
<td>158.7</td>
<td>14.0</td>
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<td>158.7</td>
<td>56.9</td>
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<td>-</td>
<td>158.7</td>
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<td>SA33TMD*</td>
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* without rubber bearing
138x35mm (120 x 120 DPI)
(a) TMD tuning and damping ratios

(b) Normalized TMD stiffness and damping coefficient
(a) Without saturation  (b) With saturation
138x181mm (120 x 120 DPI)