National Conference on Research and Development in Computer Science and its Applications
School of Computer Sciences, Universiti Sains Malaysia, Penang, Malaysia
27 - 29 November 1997

A CONTOUR INTEGRATION METHOD FOR THE COMPUTATION OF ZERNIKE MOMENTS OF A BINARY IMAGE

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ABSTRACT

Zernike moments are widely used in several pattern recognition applications, as invariant descriptors of the image shape. Zernike moments have proved to be superior than other moment functions in terms of their feature representation capabilities. The major drawback with Zernike moments is the computational complexity. This paper presents a fast algorithm for the computation of Zernike moments of a binary image. The Zernike moment integrals are evaluated along the object boundary points using a discrete version of the Green's theorem. The real-valued Zernike radial polynomials are computed with the help of a recursive procedure. The performance of the algorithm based on contour integration is faster and more accurate than the moments evaluated using the whole image. Experimental results are presented for the image of a ellipse shape. The exact (analytically derived) values of the Zernike moments are compared with the numerically evaluated moments of the discretized image, using both the conventional direct sum method and the contour integration method.

Keywords: Feature descriptors, Zernike moments, Image ellipse, Contour integration.

1 INTRODUCTION

Moment functions of an image intensity distribution are used to characterize various features of the image shape. Commonly used moment functions are geometric moments, complex moments, Legendre moments and Zernike moments. Moments have also been widely used in pattern recognition and object identification applications, as feature descriptors of an image that are invariant under translation, rotation and scale variations[1-3].

This paper presents an algorithm for the fast computation of Zernike moments of a binary image. Zernike moments are defined based on the orthogonal functions of Zernike radial polynomials[4-5]. The orthogonality property helps in obtaining shape features which represent independent geometrical characteristics, leading to minimum amount of information redundancy in a set of Zernike moments. The high degree of information content also facilitates the reconstruction of the image from a finite set of moments, using the Fourier expansion theorem on orthogonal functions. Zernike moments have also shown to be more robust in the presence of image quantization and noise; and have performed better than geometric moments in several pattern recognition applications[6-7].

The evaluation of Zernike moments is however computationally more complex compared to geometric and Legendre moments. A fast method to compute the Zernike moments of gray-level images can be found in reference [8]. The evaluation of moment integrals for a binary image can be speeded up by using only the boundary points. Contour integration methods for fast computation of geometric moments of binary images using only the boundary information have already appeared in literature [9-11]. This paper details a similar approach using contour integration, to compute the Zernike moments of a binary image. A recursive scheme to compute the radial Zernike polynomials is also given. The exact moment values of an ellipse shape with arbitrary rotation and position parameters are derived, and compared with the moments computed using the ellipse boundary on a discrete pixel grid, for validating the algorithm.

2 ZERNIKE MOMENTS

The Zernike moments have complex kernel functions based on Zernike polynomials, and are often defined with respect to a polar coordinate representation of the image intensity function $f(r, \theta)$ as

$$Z_{nl} = \frac{(n+1)}{\pi}\int_0^{2\pi}\int_0^1 V_{nl}^*(r,\theta)\, f(r,\theta)\, r\, dr\, d\theta\ , \quad r \le 1 \qquad \textbf{(1)}$$

where the functions $V_{nl}(r, \theta)$ denote Zernike polynomials of order n and repetition l, and * denotes complex conjugate. In the above equation n is a non-negative integer, and l is an integer such that $n-|l|$ is even, and $|l| \le n$. The Zernike polynomials are defined as

$$V_{nl}(r, \theta) = R_{nl}(r)\ e^{jl\theta}\ , \qquad \textbf{(2)}$$

where $j = (-1)^{½}$, and $R_{nl}()$ are the real-valued Zernike radial polynomials given by

$$R_{nl}(r)= \sum_{s=0}^{(n-|l|)/2}(-1)^s \frac{(n-s)!\ r^{n-2s}}{s!\left(\frac{n-2s+|l|}{2}\right)!\left(\frac{n-2s-|l|}{2}\right)!}\ . \qquad \textbf{(3)}$$

By introducing a transformation of the index $k=n-2s$, the above expression can be rewritten as

$$R_{nl}(r) = \sum_{k=|l|}^{n} B_{nlk}\, r^k\ , \qquad (n-k \text{ is even}) \qquad \textbf{(4)}$$

where

$$B_{nlk} = \frac{(-1)^{(n-k)/2}\left(\frac{n+k}{2}\right)!}{\left(\frac{n-k}{2}\right)!\left(\frac{k+|l|}{2}\right)!\left(\frac{k-|l|}{2}\right)!}\ . \qquad \textbf{(5)}$$

Equation (4) is generally preferred to equation (3) for the evaluation of the Zernike polynomials.

3 CONTOUR INTEGRATION

A binary image with pixel positions in polar coordinate form can be represented by the set

$$\{(r,\theta) :\ f(r, \theta) = 1 \} \qquad \textbf{(6)}$$

and assuming that the image region is of convex shape, without holes, and containing the origin of the coordinate system; the boundary of the image region can be represented by the set of the radial distance functions r_θ at angles θ as

$$\{ r_\theta \ : \ 0 \le \theta \le 2\pi \}. \tag{7}$$

Substituting equations (3-5) in (1) , and using the conditions (6-7) for the boudary of a binary image, we get

$$Z_{nl} = \frac{(n+1)}{\pi} \int_0^{2\pi} \left\{ \sum_{k=|l|}^{n} B_{nlk} \int_0^{r_\theta} r^{k+1} dr \right\} e^{-jl\theta} d\theta \ . \tag{8}$$

If the image boundary is sampled at discrete pixel positions, and if (r_i, θ_i) $i = 1,2,...N$, denote the coordinates of the i^{th} boundary point, and N is the total number of boundary points, then equation (8) can be expressed in the discrete form

$$Z_{nl} = \frac{(n+1)}{\pi} \sum_{k=|l|}^{n} \frac{B_{nlk}}{k+2} \left\{ \sum_{i=1}^{N} r_i^{k+2} (\cos l\theta_i - j \sin l\theta_i) \Delta\theta_i \right\} \tag{9}$$

where $\Delta\theta_i = \theta_{i+1} - \theta_i$. The above equation is nothing but the discrete version of Green's theorem on contour integration, applied to the Zernike moment integral in (1); and provides a computational advantage over the direct sum approximation of(1), by using only the boundary points.

The coefficients B_{nlk} can be computed using the following recursive relations:

$$B_{nnn} = 1 \ ; \tag{10}$$

$$B_{n(l-2)k} = B_{nlk} \ \frac{k+l}{k-l+2} \ ; \tag{11}$$

$$B_{nl(k-2)} = (-1) \ B_{nlk} \ \frac{(k+l)(k-l)}{(n+k)(n-k+2)} \ . \tag{12}$$

The Pseudo-code for the recursive computation of the coefficients is given in Figure 1. Note that the evaluation of Z_{nl} requires the coefficients B_{nln} , $B_{nl(n-2)}$, …. B_{nll}.

1. Set $p = n$, $q = n$.
   Initialize $B_{npq} = 1$.

2. Use (11) to compute $B_{n(p-2)q}$.
   $p = p - 2$
   If $p < l$, Go to Step 2 (Repeat).

3. The value of B_{nlq} is now available.

4. Use (12) to compute $B_{nl(q-2)}$.
   $q = q - 2$
   If $q < l$, Go to Step 4 (Repeat).

5. The values of B_{nln} , $B_{nl(n-2)}$, …. B_{nll} are now available.

FIGURE 1.
PSEUDO-CODE FOR EVALUATING THE COEFFICIENTS OF ZERNIKE POLYNOMIALS.

The above set of equations together with the contour integral form in (9) can significantly reduce the computation time required for Zernike moment evaluation on a binary image.

4 EXPERIMENTAL RESULTS

A general ellipse shape with the following parameters was used to compute the Zernike moments by the methods of direct summation and contour integration.

Pixel coordinates of the center of the ellipse: $x_0 = 7$, $y_0 = 5$.

Semi-major axis of the ellipse : 10 pixels

Semi-minor axis of the ellipse : 8 pixels

Angle made by the semi-major axis with the x-axis: 45 degrees.

The discretized image coordinates of the ellipse was further normalised to the range [-1,1] by dividing each pixel coordinate by 60. The comparison between the exact (theoretical) values of the Zernike moments of an ellipse shape, the Zernike moments computed from the whole image of the ellipse (1), and the Zernike moments computed using only the image boundary points (9); is given in Table 1. The average percentage error ε of the numerically computed values with respect to the theoretical value for each of the methods, is also given. A comparison of the CPU time required for computing the Zernike moments on a VAX-Station 3100 Workstation is also shown in Table 1.

	Exact values		**Using whole image**		**Using contour points**	
	Real	Imaginary	Real	Imaginary	Real	Imaginary
Z_{00}	0.0222	0.0000	0.0237	0.0000	0.0221	0.0000
Z_{11}	0.0052	–0.0037	0.0055	–0.0040	0.0052	–0.0038
Z_{22}	0.0004	–0.0015	0.0005	–0.0016	0.0005	–0.0016
Z_{20}	–0.0624	0.0000	–0.0664	0.0000	–0.0620	0.0000
Z_{33}	–0.0001	–0.0003	–0.0001	–0.0004	–0.0001	–0.0004
Z_{31}	–0.0193	0.0138	–0.0206	0.0147	–0.0195	0.0142
ε (%)	–		8.1	8.5	2.6	5.9
CPU	–		29 msec.		8 msec.	

TABLE 1.

PERFORMANCE COMPARISON OF CONVENTIONAL AND CONTOUR INTEGRATION METHODS

5 CONCLUSIONS

Zernike moments are used as orthogonal feature descriptors in several pattern recognition applications. A method based on contour integration to compute the Zernike moments of a binary image was presented. This method also used a recursive algorithm to compute the real-valued Zernike radial polynomials. The contour integration scheme showed better accuracy compared to the discrete summation method to compute the moments using the whole image. The contour integration method is also faster than other algorithms as it uses only the boundary point coordinates of the image.

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