

Trip Assignment under Energy and Environmental Constraints

Kenneth D. Kuhn

Department of Civil and Natural Resources Engineering
University of Canterbury
Christchurch, New Zealand
kenneth.kuhn@canterbury.ac.nz

Abstract

Limits on available energy and allowable environmental impacts may soon restrict transportation systems. Relatively little work has been done to investigate how this will impact the techniques used in the field of transportation engineering. This research considers one such technique, all-or-nothing trip assignment. A process that today involves solving thousands of shortest path problems may involve solving thousands of constrained shortest path problems in the future. This research examines the impact such a shift will have on computational burdens.

The results of computational studies involving energy-constrained trip assignment are presented here. It is found that solving constrained shortest path problems can take several orders of magnitude more time than solving traditional shortest path problems. This is worrying given that such problems will have to be solved large numbers of times in order to use one of the simplest techniques of transportation engineering. A specialized algorithm (Carlyle and Wood 2003) typically outperforms a generic solver, but occasionally takes an excessively long amount of time to select a path. The results indicate that it may become increasingly important for transportation engineers to be well versed in optimization.

Keywords: energy-constrained transportation, constrained shortest path problem, trip assignment, transportation planning.

1 Introduction

Research suggests that constraints on energy use and environmental impacts may soon restrict transportation. This relates to both individual vehicles, which may have reduced range, and whole systems, which may face new local, regional, or global restrictions. Transportation engineering as a discipline is, by and large, unprepared for a future where energy and environmental constraints take on increased prominence (Dantas et al. 2005).

This research investigates how energy and environmental constraints could be incorporated in a relatively simple fashion into one technique in transportation engineering, all-or-nothing trip assignment. Essentially, a process that involves solving thousands of shortest path problems would be replaced by a process that involves solving thousands of constrained shortest path problems. The implications of such a change, in terms of computational burden, are analyzed here.

The next section describes why energy and environmental concerns may seriously constrain transportation in the future. The following section defines trip assignment in the presence of energy and environmental constraints. Algorithms for the constrained shortest path problem are described next, including an efficient algorithm presented at a previous ORSNZ conference (Carlyle and Wood 2003). Computational studies are introduced and conclusions drawn in subsequent sections of this paper.

2 Transportation in a constrained future

The overwhelming majority of vehicles in use today are powered by burning petroleum-based fuels. A number of researchers are predicting serious declines in the availability of petroleum and/or a transition to alternate fuels in the next ten to twenty years (Farrell and Brandt 2006). At the same time, significant environmental concerns are being raised. For example, one study found that child asthma prevalence rates were associated with local rates of carbon monoxide and nitrogen oxides production from automobile traffic (Guo et al. 1999). Other studies have indicated that the burning of fossil fuels is causing dangerous and irreversible climate change (see www.ipcc.ch).

A significant and growing number of vehicles are being powered by fuels extracted from biological materials. The production and consumption of biofuels releases pollutants at a lower rate than that of petroleum, and carbon is stored in materials as they are grown to make biofuels. However, the widespread use of biofuels globally would require dramatic land use changes and new distributional systems that may not be feasible. Many biofuels are produced on land that could otherwise be used for food production, setting up a *food vs. fuel* dynamic that could constrain production of both. Finally, evidence indicates that the net result of a large-scale switch from petroleum to biofuels, when incorporating land use changes, may be a dramatic *increase* in pollutant emissions (Searchinger et al. 2008).

It now looks likely that many transportation systems of the not-too-distant future will be largely electric. When power is produced in wind, solar, hydroelectric, tidal, or geothermal power plants, zero harmful pollutants are emitted. Unfortunately, current global electricity production is insufficient to meet current demands and power transportation. Electricity produced by renewable resources is particularly limited. Many sustainable modes of electric energy production depend on factors beyond our control. One effort to design a bus route in Christchurch, New Zealand using sustainable resources concluded that “no amount of investment in wind and solar energy capacity can provide the same service as the fossil fuel system” (Dantas et al. 2005). Pumped-storage hydroelectricity could provide zero-emissions energy when wind and sun are not strong enough. However, the land area and funds required to establish a wind, solar and pumped-storage hydroelectricity system capable of

providing the continuous power required by just one bus route (operating in a manner consistent with current practice) are impractical (Ibid.). On a smaller scale, personal electric vehicles will likely be powered by batteries and have a range substantially lower than current vehicles.

Summarizing the points listed above, research suggests that transportation systems and individual vehicles may be significantly more constrained in terms of energy use and environmental impacts in the future. This would clearly have substantial implications for the practice of transportation engineering. Yet little has been written on this point. In the words of one of the articles cited above, “a survey of transportation engineering texts illustrates that current modelling and planning techniques do not include any method to consider constraints in natural resources, emissions, or, most importantly, energy” (Dantas et al. 2005)

3 Trip assignment

Transportation engineering often requires forecasting vehicle loads on different sections of a transportation network. One example would be estimating the traffic impacts of the opening of a new shopping centre. Another example would be planning the future maintenance requirements of an airport taxiway. The most commonly used method for forecasting vehicle loads is known as the *four-step method*, the roots of which can be traced back to the seminal Chicago Area Transportation Study (CATS 1959). First, *trip generation* estimates the numbers of trips that will depart from different origins as well as the numbers of trips that will arrive at different destinations. Next, *trip distribution* links origins and destinations, forecasting the numbers of trips between location pairs. Where relevant, *mode choice* breaks down trip counts by mode of transportation. Finally, *trip assignment* forecasts the numbers of vehicles that will travel on individual sections of a given transportation network, based on the preceding analyses. Trip assignment is one of the most researched areas of transportation engineering, and one of the most important to practitioners.

The simplest form of trip assignment follows what is known as the *all-or-nothing approach*. This technique assigns fixed travel times to different sections of the transportation network. All trips between given origins and destinations are then assigned to the sections of the network that allow the trips to be completed in the shortest possible time. Travel times can be replaced with generalized cost estimates without substantively altering the process. The all-or-nothing approach does not consider congestion, the idea that travel times (or costs) increase as the number of vehicles on the relevant section of the network increase. However, the all-or-nothing approach remains useful when congestion is relatively unimportant or when estimating the costs of congestion.

The research presented here considers trip assignment using the all-or-nothing approach in scenarios involving energy and environmental constraints. The assumption is made that vehicles using individual sections of a transport network consume fixed amounts of various *resources*. Resource consumption is additive across the sections of a transport network. There are budgets of the various resources available, and a vehicle may not be assigned to a route that would require it to consume

more of any resource than the available budget. This is a relatively simple way to imagine integrating energy and environmental constraints into trip assignment. Essentially, we will be solving large numbers of constrained shortest path problems. Mathematical details are presented below.

Let a transportation network consisting of a set V of vertices and another set E of edges be given. Each element in E will consist of a start and an end vertex. Let o and d be the origin and destination, respectively, of the trip currently being assigned. The parameter c_{ij} is the generalized cost associated with the edge (i, j) . Let R be the set of resources constraining us. The parameters b^r and q_{ij}^r are the budget (available amount) of resource r and the quantity of resource r used when travelling on the edge (i, j) respectively. The decision variable α_{ij} is binary and is to be set to 1 if we travel on edge (i, j) and 0 otherwise. The nominal formulation of the problem of interest, for one trip, follows.

Model Nominal formulation

$$\min \sum_{(i,j) \in E} c_{ij} \alpha_{ij} \quad (1)$$

s.t.

$$\sum_{i:(i,j) \in E} \alpha_{ij} - \sum_{k:(j,k) \in E} \alpha_{jk} = \begin{cases} -1 & j = o \\ 0 & \forall j \in V \setminus \{o, d\} \\ 1 & j = d \end{cases} \quad (2)$$

$$\sum_{(i,j) \in E} q_{ij}^r \alpha_{ij} \leq b^r \quad \forall r \in R \quad (3)$$

$$\alpha_{ij} \in \{0, 1\} \quad \forall (i, j) \in E \quad (4)$$

The objective function, (1), minimizes the total generalized cost. Constraint set (2) ensures that the edges chosen represent a logical path, departing only from the origin and arriving only at the destination. Constraint set (3) captures energy and environmental constraints. Finally, constraint set (4) ensures decision variables take on values consistent with the desired interpretation.

4 Solution techniques

The formulation presented above is well known in operational research and has been labelled the *constrained shortest path problem* (Irnich and Desaulniers 2004). This problem can be computationally challenging, and is NP hard (Garey and Johnson 1979). The constrained shortest path problem is a special case of the *shortest path problem with resource constraints*, which is often solved via labelling algorithms (Irnich and Desaulniers 2004). Indeed, a specialized labelling algorithm has been proposed for the problem studied here (Dumitrescu and Boland 2003). Efficient techniques based on Lagrangian relaxation have also been proposed (Handler and Zang 1980; Carlyle and Wood 2003). A number of researchers have proposed special techniques applicable when there is only one resource constraint (Handler and Zang 1980; Santos, Coutinho-Rodrigues, and Current 2007). This work considers two approaches for solving the identified problem. One approach encodes the nom-

inal formulation shown above and asks the generic *glpk* solver to find the optimal solution. An alternate approach uses a Lagrangian relaxation and enumeration algorithm presented at an earlier ORSNZ conference (Carlyle and Wood 2003) to find a solution. The two approaches were used to see the ranges of solution times we might expect depending upon our choice of solution technique.

The Carlyle and Wood approach begins by finding optimal, or near-optimal values for Lagrange multipliers associated with resource constraints. A formulation for the Lagrangian relaxation is as follows.

Model Lagrangian relaxation

$$\max_{\lambda \geq 0} \min_{\alpha} \sum_{(i,j) \in E} (c_{ij} + \sum_{r \in R} \lambda_r q_{ij}^r) \alpha_{ij} - \sum_{r \in R} \lambda_r b^r \quad (5)$$

s.t.

$$\sum_{i:(i,j) \in E} \alpha_{ij} - \sum_{k:(j,k) \in E} \alpha_{jk} = \begin{cases} -1 & j = o \\ 0 & \forall j \in V \setminus \{o, d\} \\ 1 & j = d \end{cases} \quad (6)$$

$$\alpha_{ij} \in \{0, 1\} \quad \forall (i, j) \in E \quad (7)$$

Note that the inner problem of finding values for α is a traditional shortest path problem where the cost of taking edge (i, j) is $c_{ij} + \sum_{r \in R} \lambda_r q_{ij}^r$. The outer problem of finding values for λ can be solved using relatively simple techniques making use of solutions of the inner problem. Such techniques include subgradient optimization, or bisection search when there is only a single resource constraint. Note that it is possible to begin such techniques by setting all λ terms equal to zero and solving the resulting shortest path problem for α . In other words, the first step for solving the constrained shortest path problem is to solve the (unconstrained) shortest path problem. This is helpful in comparing the two problems.

Once optimal, or near-optimal, values for the Lagrange multipliers have been found, the Carlyle and Wood approach enumerates paths that are feasible in terms of resource budgets, as well as being near-optimal in terms of the nominal and Lagrangian objective functions. The algorithm can be set up to finish when a solution is found within a set distance of a lower bound previously identified. A complete description of the algorithm including pseudo-code has been presented at this conference previously (Carlyle and Wood 2003). The referenced text claims that the proposed approach was able to solve constrained shortest path problems “an order of magnitude faster” than arguably the most promising alternate approach (Ibid.).

5 Computational studies

The results of computational studies are presented here. In these computational studies we considered only a single resource constraint on energy use. Considering a single constraint keeps analysis simple, although the techniques used in this study could easily be applied in situations involving multiple constraints. 1,000 vehicles were assigned paths from an origin to a destination. In each case, 1,000 vertices were

set up in a 4 by 250 grid. Edges linked vertices adjacent to one another vertically or horizontally. Edges went up and down vertically, but only to the right horizontally. In total there were 2,496 edges. The vertex at the top left was chosen to be the origin and the vertex at the bottom right was chosen to be the destination. The generalized costs and energy use associated with individual edges were independent identically distributed random variables uniformly distributed between 0 and 1. It is unlikely that costs and energy use would be independent in real life, but the assumption of independence simplifies problem parameterization. The cost and energy use data were randomized each time a new vehicle was to be assigned a path. Other researchers have used vertex grids similar to the one used here to test constrained shortest path algorithms (Carlyle and Wood 2003; Dumitrescu and Boland 2003). The grid used here is somewhat unique in that one dimension is significantly longer than the other. It is believed that this reflects many transportation path planning problems, where there are a handful of parallel routes to take and the decision boils down to when to switch between routes.

Vehicles were only allowed to use 155 units of energy to get from the origin to the destination. Vehicles had to transverse at least 252 edges to get from the origin to the destination, and a shortest cost path could include significantly more edges. The constraint on energy use was binding around half the time. Paths proposed by a simple shortest path algorithm (ignoring the energy constraint), the Carlyle-Wood algorithm, and the glpk solver were saved. The Carlyle-Wood algorithm was set up to quit and return the best feasible path found if the optimality gap was reduced to 0.1 units of cost or if 600 seconds had elapsed. There were cases where 600 seconds elapsed and a solution not proven optimal or near-optimal was returned (more on this later). Figure 1 shows histograms of energy use and generalized costs on paths found by the various algorithms.

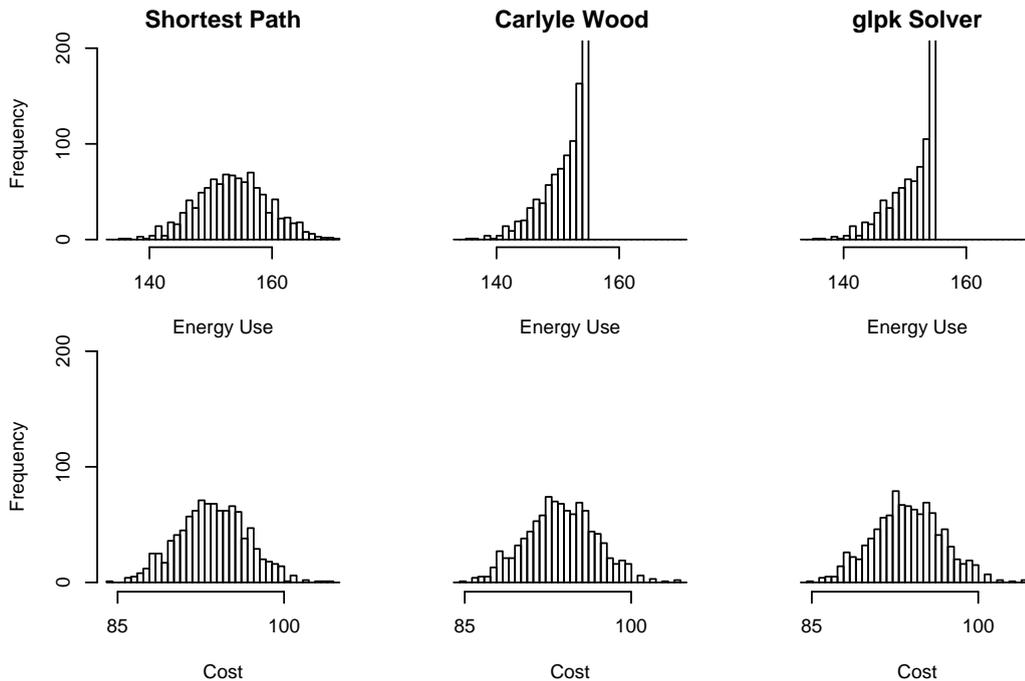


Figure 1: Energy use and generalized costs associated with chosen paths.

The Carlyle-Wood algorithm and glpk solver produced close to identical results. Each was able to limit energy use to below 155 units for each vehicle without substantially altering the costs of travel. The results reflect the assumption that energy use and cost are independent. Given a good deal of independence, large enough choice sets and good decision making, it is possible to constrain resource use without dramatically increasing costs. This is encouraging, although it seems likely that resource use and costs would be highly correlated in reality. Further research could investigate the link between resource use and generalized travel cost estimates in reality, or look at the sensitivity of simulation results to resource-cost correlation.

Special attention was paid to the solution times of the various algorithms. On the 1,000 test problems, the shortest path algorithm took between 0.01 and 0.02 seconds while the glpk solver took between 27 and 35 seconds. These results highlight how much more complicated the constrained shortest path problem is than the shortest path problem. The Carlyle-Wood algorithm was able to finish after the shortest path problem had been solved in cases where the budget for energy use was not fully used. This meant more than half the time, the Carlyle-Wood algorithm took only around 0.01 seconds to find a vehicle path. Over ninety percent of the time, the Carlyle-Wood algorithm took less than 0.25 seconds to find a solution, significantly outperforming the glpk solver. However, on 28 out of the 1,000 trials, the Carlyle-Wood algorithm had not found a solution after ten minutes and had to be stopped. Some summary statistics regarding the distribution of computation times are presented in Table 1.

	minimum	median	75th perc.	90th	95th	99th
Shortest Path	0.0112	0.0113	0.0113	0.0113	0.0113	0.0193
Carlyle Wood	0.0112	0.0113	0.242	0.243	11.1	600*
glpk Solver	27.2	27.6	27.9	28.3	28.7	35.0

* - Algorithms were forcibly stopped after 600 seconds.

Table 1: Distributions of computation times, in seconds, by algorithm.

It is not clear why the Carlyle-Wood algorithm performed so well most of the time, but so poorly occasionally. The results mirror those of another study investigating the biobjective shortest path problem (Raith and Ehrgott 2009). Raith and Ehrgott conclude that a specialized enumerative algorithm “is a very successful approach to solve some problem instances, but the run-time on others is very long” (Ibid.). The data presented in Figure 1 and an informal analysis show that the Carlyle-Wood algorithm was always able to identify a high quality feasible solution, but not to verify that the solution was optimal or near-optimal.

One of the difficulties associated with the constrained shortest path problem is that it is difficult to obtain good lower bounds on the optimal objective function value. Actually, one of the strengths of the Lagrangian approach is that the relaxed problem provides a reasonable lower bound. Further research investigating ways to dynamically determine stronger lower bounds and incorporate their use into an enumerative algorithm may be worthwhile. It’s worth mentioning that the author of this paper is a casual programmer and there may have been inefficiencies, unknown to the author, in the code used here to test the various algorithms.

Extra long computation times are likely associated with problems where there are large optimality gaps. In such situations, there would also likely be a large separation between the energy used on the shortest cost path and the available energy budget. Figure 2 displays the computation times of the Carlyle-Wood algorithm as a function of the energy used on the shortest cost path. There is some, but not a dramatic amount of, evidence for the hypothesis that computation times increase with energy use on the shortest cost path.

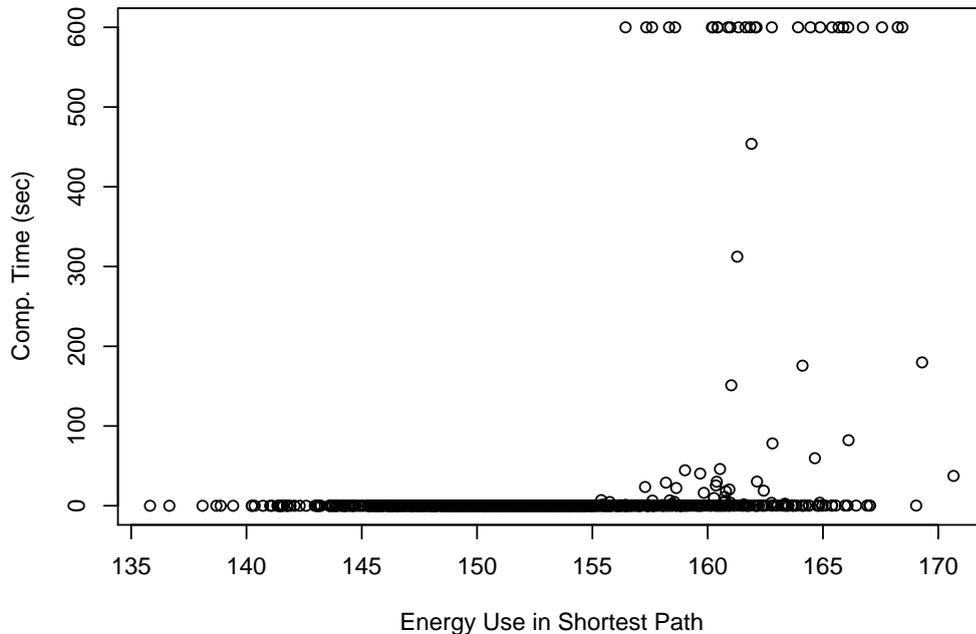


Figure 2: Exploring the performance of the Carlyle-Wood algorithm.

6 Conclusion

This article discussed the possibility of there being significant new energy use or environmental impact constraints on transportation in the next ten to twenty years. Consideration was given to how such constraints would impact techniques used in the field of transportation engineering. One of the simplest of these techniques, all-or-nothing trip assignment, was used as an example. It was hypothesized that what today involves solving thousands of shortest path problems may, in the future, involve solving thousands of constrained shortest path problems. It was found that such a change would increase computation times by more than three orders of magnitude if a generic solver was used to solve constrained shortest path problems. Using a specialized algorithm resulted in computation times that were typically only 20 to 30 times those associated with the nominal shortest path problem, in cases involving binding resource constraints. However, the specialized algorithm chosen occasionally took an exceptionally long amount of time to select an optimal path. Further research is needed to identify what caused these outlying results. Overall, the results indicate that it may become increasingly important for transportation engineers to be well versed in optimization.

Acknowledgments

The author would like to thank Andrea Raith for providing technical guidance regarding shortest path problems. Andrea, Stephan Hassold, and Matthias Ehrgott convinced the author to attend this conference. Susan Krumdieck, Shannon Page, André Dantas, and Judith Wang interested the author in transportation planning for an energy-constrained future. The author wishes to thank those mentioned for several enjoyable discussions.

References

- Carlyle, W.M., and R.K. Wood. 2003. "Lagrangian relaxation and enumeration for solving constrained shortest-path problems." *Proceedings of the 38th Annual ORSNZ Conference*.
- CATS. 1959. *Chicago Area Transportation Study*. Volume 1. Chicago.
- Dantas, A., S. Krumdieck, A. Hamn, M. Saunders, and S. Minges. 2005. "Performance-objective design for energy constrained transportation system." *Journal of the Eastern Asia Society for Transportation Studies* 6:3276–3292.
- Dumitrescu, I., and N. Boland. 2003. "Improved preprocessing, labeling and scaling algorithm for the weight-constrained shortest path problem." *Networks* 42:135–153.
- Farrell, A.E., and A.R. Brandt. 2006. "Risks of the oil transition." *Environmental Research Letters* 1:1–8.
- Garey, M.R., and D.S. Johnson. 1979. *Computers and intractability: A guide to the theory of NP-completeness*. San Francisco: Freeman.
- Guo, Y.L., Y.C. Lin, F.C. Sung, S.L. Huang, Y.C. Ko, J.S. Lai, H.J. Su, C.K. Shaw, R.S. Lin, and D.W. Dockery. 1999. "Climate, traffic-related air pollutants, and asthma prevalence in middle school children in Taiwan." *Environmental Health Perspectives* 107:1001–1006.
- Handler, G.Y., and I. Zang. 1980. "A dual algorithm for the constrained shortest path problem." *Networks* 10:293–310.
- Irnich, S., and G. Desaulniers. 2004. "Shortest Path Problems with Resource Constraints." *Les Cahiers du GERAD*.
- Raith, A., and M. Ehrgott. 2009. "A comparison of solution strategies for biobjective shortest path problems." *Computers & Operations Research* 36:1299–1331.
- Santos, L., J. Coutinho-Rodrigues, and J.R. Current. 2007. "An improved solution algorithm for the constrained shortest path problem." *Transportation Research Part B* 41:756–771.
- Searchinger, T., R. Heimlich, R.A. Houghton, F. Dong, A. Elobeid, J. Fabiosa, S. Tokgoz, D. Hayes, and T.H. Yu. 2008. "Use of U.S. croplands for biofuels increases greenhouse gases through emissions from land-use changes." *Science* 319:1238–1240.