Material Strain Limits for Seismic Design of Concrete Structures

By

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Abstract

The design of structures to resist seismic actions to New Zealand Standards requires the level of detailing used in potential plastic regions to be based on the predicted material strain levels that they sustain in the ultimate limit state. For potential plastic hinges in reinforced concrete sections, curvatures are used rather than strains in reinforcement or concrete. This substitution is made as it is difficult to calculate realistic reinforcement and concrete strains in plastic hinges subjected to cyclic loading. Test results from 37 beams, 25 columns and 36 walls were examined. From each of these the maximum nominal curvature sustained when the lateral force reduced to 80 percent of the theoretical strength was found. From the results of the tests, design curvature limits for plastic hinges have been developed. These limits will be submitted to Standards with the proposal that they are incorporated in an amendment to the Structural Concrete Standard 3101:2006. They have a more rational basis than the existing curvature limits in the Standard and they are easier to use in design.

1. Background

The Standard, Earthquake Actions – New Zealand Standard, NZS 1170.5 [1], requires the level of detailing used in potential plastic regions to be based on the predicted material strain levels imposed on the region in the ultimate limit-state. For flexural members the Structural Concrete Standard, NZS 3101:2006 [2] specifies the material strain limits for each class of detailing in terms of nominal curvatures. Actual material strain levels in reinforcement and concrete cannot be easily determined and consequently curvature limits are calculated by simplified rules, which are given in the Standard [2] and described in a previous paper [3]. These values are in effect used as an index of the likely strain levels, which would be induced in plastic regions in the event of an ultimate limit state design level earthquake. This paper gives the background to a proposed revision of material strain limits for the Structural Concrete Standard [2]. These values, which as far as possible are based on test results, are generally less conservative than those currently (March 2006) in the Standard and they are simpler to apply in practice.

The approach followed in the Structural Concrete Standard, NZS 3101:2006 [2], is similar to that proposed by Baker in 1956, as described in reference [4]. Uniform plastic strains are assumed to develop in a beam, column or wall for an effective plastic hinge length. In actual flexural members the strains are far from uniform. However, by selecting an appropriate effective plastic hinge length, \( l_{\text{eff}} \), it was considered that the correct maximum curvature could be found and the total plastic rotation predicted. These assumptions are illustrated in Figure 1. It should be noted that the “effective plastic hinge length” over which the plastic curvature is assumed to
be uniform, is much shorter than the length of reinforcement that actually yields. Any detailing that is required to sustain plastic deformation should extend over the full length where yielding of reinforcement or spalling of concrete may be anticipated to prevent possible premature failure. This length, in the Structural Concrete Standard [2] is referred to as the “ductile detailing length”.

Figure 1: Actual and analytical curvatures in a beam sustaining plastic deformations

Two of the design objectives in the “Earthquake Actions Standard”, NZS 1170:2004 [1] are that a structure can sustain the design level earthquake (return period of 500 years for most buildings) for the ultimate limit state with a high margin of safety against collapse and the maximum credible earthquake (return period of 2,500 years) with a margin of safety. Typically the lateral displacements of a structure in the maximum credible earthquake are predicted to be approximately 1.8 times the corresponding values of ultimate limit state earthquake [1]. The material strain limits given in this paper are intended to satisfy both the ultimate limit state and maximum credible earthquake criteria. However, there is need for further research to refine and clarify these limits.
2. Method used to assess limiting material strains from test results

The actual strain levels in the longitudinal reinforcement and concrete in a plastic hinge cannot be accurately predicted. The length over which the longitudinal reinforcement yields, or the concrete spalls, varies with the characteristics of the reinforcement, the ratio of bending moment to shear at the critical section of the member, the proportion of shear reinforcement, the form of plastic hinge, namely unidirectional or reversing, the loading history and the structural details of the supporting member. The flexural rotation sustained in a plastic region depends on the longitudinal strains in the reinforcement and concrete and the pull out of longitudinal reinforcement in any supporting member (yield penetration). However, it should be noted that shear deformation associated with elongation and yielding of shear reinforcement has been found to account for an appreciable proportion of the total deflection in some test beams [5, 6], but this component of deformation does not involve any curvature in the plastic region. Some of the complexities noted above are described in more detail in section 4.

To get round the analytical problems described above, the proposed material strain limits are represented by curvatures, for ductile and limiting ductile members were derived from test results of structural members. For each type of member an effective plastic hinge length was assumed. Using this value an ultimate curvature was calculated from the test results for each structural member. This value was based on the assumption that all the inelastic deformation arose from curvature in the effective plastic hinge length. It was calculated from the displacement that was sustained when the strength of the member degraded to 80 percent of its theoretical value towards the end of the test. The process of assessing a limiting curvature from individual test results for a member is illustrated in Figure 2 and set out in the steps given below.

1. Each test unit was assessed to determine if the detailing in the potential plastic region satisfied the requirements for nominally ductile, limited ductile or ductile detailing, as set out in the Standard [2], or if it does not qualify for any of these.

2. From the pre-yield loading cycles the displacement sustained at approximately \( \frac{3}{4} \) of the theoretical strength was extrapolated linearly to the load level corresponding to the theoretical strength of the member. This is taken as the ductility one displacement, \( \delta_e \), as illustrated in Figure 2.

3. A scan of a large number of test results showed that in the majority of cases failure occurred in a load cycle which involved displacements ranging from a large negative displacement to a large positive displacement, or visa versa. In short this indicated it was the range of total displacement from the start to end of a half cycle that was responsible for failure, rather than the peak displacement measured from the initial position at the start of the test. In recognition of this the ultimate curvature was based on the average of the peak displacements sustained in the half cycle before failure occurred. With reference to Figure 2 this displacement is equal to \( \frac{1}{2} \left| (\hat{\delta}_{\text{max}} + \delta_{\text{max}}) \right| \), where \( \hat{\delta}_{\text{max}} \) and \( \delta_{\text{max}} \) are the maximum positive and negative displacements sustained in the half cycle before the load dropped below 80% of the theoretical strength.

4. In many tests several cycles of loading were applied between specific positive and negative displacements before failure occurred. Clearly in such cases the
Figure 2: Calculating curvatures from results of tests

member would have been capable of sustaining one or more larger displacement cycles before failure occurred if the previous load cycles had not been applied. From a scan of several beam tests it was concluded that this effect could be conservatively predicted by multiplying the critical displacement found in step 3 by $1.05^{(n-1)}$ but with $n$ not greater than 8, where $n$ was the number of times the positive and negative displacement peaks were sustained in the displacement cycles being considered before the applied force resisted at a peak displacement dropped below 80 percent of the theoretical
value. In a number of tests the member sustained several cycles of loading between the same displacement limits before it failed at the second peak displacement to a higher displacement range. Using the multiplier of $1.05^{(n-1)}$ on the previous set of displacement cycles appeared to give a reasonable estimate of the observed displacement sustained at failure. Further research using different forms of damage criteria should enable the limits to be assessed with greater confidence. This factor makes a crude allowance for the effect of damage caused in previous load cycles.

5. The process of finding a limiting curvature from a test result is illustrated in Figure 2, where in the load cycle (between $\delta_{\text{max}}^+$ and $\delta_{\text{max}}^-$) the load sustained at the peak positive and negative displacements exceeded 80 percent of the theoretical strength $(0.8H_i)$ four times ($n = 4$) before it dropped below this value. Consequently the critical displacement found in step 3 in this case would be multiplied by $1.05^{3}$.

The ultimate curvature due to plastic deformation obtained from a test, $\phi_p$, calculated as set out in steps 1 to 4 is given by:

$$\phi_p = \left(0.5(\delta_{\text{max}}^+ - \delta_{\text{max}}^-) - \delta_c\right)^{(n-1)}$$

$$= \left(\frac{M}{V} - \frac{l_{\text{eff}}}{2}\right) l_{\text{eff}}$$

but with the value of “$n$” not exceeding 8.

In equation 1 the terms $\delta_{\text{max}}^+$, $\delta_{\text{max}}^-$ are as defined in steps 1 to 4 above, $\delta_c$ is the ductility one displacement of the test unit and $l_{\text{eff}}$ is the effective plastic hinge length.

For beams and columns the effective plastic hinge length, $l_{\text{eff}}$, is taken as the smaller of:

- half depth of the member, $h_b$ for a beam and $h_c$ for a column, or
- the larger of $0.2 M/V$ or $h_b/4$ for a beam or $h_c/4$ for a column, where $M/V$ is the moment to shear ratio. For a member subjected to a single lateral point load $M/V$ is equal to the distance of this load from the critical section for flexure.

For walls the effective plastic hinge length is taken as the smaller of:

- half the length of the wall, $L_w$;
- $0.15 M/V$.

When a member is being designed the $M/V$ ratio is taken from the results of either an equivalent static or modal analysis for seismic actions together with an analysis for gravity loads. Changes in $M/V$ ratios due to higher mode effects (represented by dynamic magnification factors) should be neglected.

3. Proposed material strain limits in potential plastic regions

3.1 General

In the analysis of a structure the design loads are multiplied by structural performance factor, $S_p$, on the basis that the critical displacement for design is not the maximum value but one that is reached a few times. On this basis the peak displacement, and hence the maximum curvature, would be represented by $1/S_p$ times the value found from design calculations. However, no such allowance has been made in assessing
the critical curvatures as the implied increase in value is compensated by the conservative assumptions made in the assessment of the test results.

- In design the critical curvature at yield is based on the lower characteristic yield strength, while in analysing the tests the actual yield strength was used. On average this assumption results in the curvature at first yield used in the tests being 10\% higher than the corresponding value used in design.

- In the tests the critical displacement was taken as half the total displacement from the maximum negative displacement to the maximum positive displacement in the critical cycle or cycles. In actual structures the peak displacement occurs only once and it not preceded by a displacement of equal magnitude but opposite sign. Hence the assumption made in reducing the test results gives a conservative assessment of the critical displacement that can be sustained in an actual structure.

- Invariably in the tests the magnitude of the sets of displacement cycles applied to the test members increased as the test proceeded until failure occurred. Hence appreciable damage was sustained before the critical displacement cycle was applied. However, in time history analyses of structures it is generally found that the maximum displacement is not induced at the end of the record. Hence frequently the critical members in a structure have not sustained equivalent damage to the test unit when the maximum displacement is induced.

A major factor influencing the behaviour of plastic hinge zones is the type of deformation that they are required to sustain. As outlined in references [7, 8] plastic hinges may be subjected to reversing or unidirectional inelastic actions. The vast majority of tests on plastic regions, which may be classified as ductile or limited ductile, have been made on reversing actions. The few beams, which have been tested as unidirectional plastic hinges in ductile plastic regions, have indicated that these zones can sustain in excess of twice the rotation in a comparable reversing plastic hinge [7]. For nominally ductile plastic regions the situation is different and no suitable test results could be found in the readily available literature for the beams subjected to cyclic inelastic loading.

### 3.2 Nominally ductile beams

To fill the gap in our knowledge of the behaviour of nominally ductile plastic regions a research project has been started at the University of Canterbury. However, as it will be some time before these results are available the limits given below are proposed.

For members where the design strength is limited by flexure rather than shear the limiting curvatures in nominally ductile unidirectional plastic regions of beams are taken as the smaller of the values corresponding to;

- A compression strain in the concrete of 0.004, which is generally taken as a strain when spalling of the concrete may be expected;

- A tensile strain in the reinforcement of 0.016.

For reversing plastic regions it is proposed the corresponding limits are taken as approximately 60\% of the corresponding unidirectional limits, which, with a little rounding give a limiting concrete compressive strain of 0.0025 and a limiting tensile reinforcement strain of 0.010.
For nominally ductile plastic regions, where the design shear strength controls the design strength of the member, no inelastic deformation capacity should be assumed.

### 3.3 Ductile and limited ductile plastic regions

From an analysis of test results Priestley and Kowalsky [9] proposed that the curvature, $\phi_y$, in a plastic region in a member sustaining a displacement ductility of one could be taken as a multiple of a constant and the yield strain divided by the overall depth of the member. The constant was found to vary for different types of members and reinforcement arrangements, but it was generally close to 2.0, and for simplicity it has been assumed to be equal to 2.0 for all cases in this study. It was decided to give limiting curvature values in ductile and limited ductile regions as a multiple of $\phi_y$ as this avoids the need for detailed analysis of the section to find the curvature at first yield. In some columns the $\phi_y$ value defines the curvature limit at which appreciable inelastic deformation may occur due to crushing of the concrete. On this basis the curvature in a potential plastic region corresponding to the stage where significant inelastic deformation is initiated, $\phi_y$, is taken as:

$$\phi_y = \frac{2\varepsilon_y}{h}$$  \hspace{1cm} (2)

where $\varepsilon_y$ is the yield strain and $h$ is the member depth or wall length.

The maximum curvature for the ultimate limit state is taken as a product of $\phi_y$ and two factors, namely $K_d$, which allows for the type of member and level of detailing used in the plastic region, and $K_y$, which allows for the reinforcement grade. While $\phi_y$ increases with the yield stress of reinforcement, the ultimate curvature that can be sustained in many cases depends on the strain capacity of the concrete and the buckling resistance of the reinforcement. The grade of reinforcement does not have a major influence on these two properties. Analysis of the experimental results for the beams, columns and walls indicates that for yield stress levels above 425MPa there was no significant change in the ultimate curvature that could be sustained. The introduction of the $K_y$ factor allows for this observation and it effectively limits the ultimate plastic curvature that can be used with reinforcement that has a yield stress in excess of 425MPa to the value that would be sustained with a yield stress of 425MPa. With this adjustment the limiting ultimate limit-state curvature in a ductile or limited ductile plastic hinge is given by:

$$\phi_{\text{max}} = K_d K_y \phi_y$$  \hspace{1cm} (3)

Where $K_d$ is as defined above, $\phi_y$ is given by Equation 2 and $K_y$ is the factor allowing for reinforcement grade, which is given by:

$$K_y = \begin{cases} 1.0 & \text{for } f_y \leq 425\text{MPa} \\ \frac{425}{f_y} & \text{for } f_y \geq 425\text{MPa} \end{cases}$$  \hspace{1cm} (4)

### 3.4 Ductile and limited ductile beams

Limiting curvatures were calculated from 37 beam tests taken from the literature [5, 6, 10, 11 & 12]. Of these 19 were classified as containing ductile detailing and 18 tests were discarded as they contained details not representative of current practice. In particular many of these test units had shear reinforcement that was in excess of twice, and in some cases up to 7 times, the amount required by the Standard. Most of
these discarded beams sustained very high curvatures and they did not exhibit the shear pinching characteristic of the beams that were more representative of current practice. The results of the tests on beams with ductile plastic regions are summarised in Table 1 and shown in Figure 3, where the ultimate curvatures are plotted against the shear stress normalised in terms of the square root of the concrete strength. The shear stress level, within the range of tests that were examined, appeared to have little influence on the ultimate curvature. There were no test units that could be classified as limited ductile beams.

The beams tested in Auckland [5, 6] differed from the others in that yield penetration of the reinforcement into the supporting column was limited by welding additional bars onto the beam reinforcement in the anchorage zone in the supporting member. This reduced the pullout of the reinforcement and hence these results give a conservative value of curvature in the plastic regions.

![Figure 3: Ultimate curvatures sustained in beam tests](image)

**Table 1: Summary of ultimate curvatures from test results**

<table>
<thead>
<tr>
<th>Beams</th>
<th>Columns</th>
<th>Walls</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Ductile</td>
<td>Limited ductile</td>
</tr>
<tr>
<td>Average</td>
<td>27.3 $K_y$</td>
<td>18.5 $K_y$</td>
</tr>
<tr>
<td>Std. deviation</td>
<td>5.8 $K_y$</td>
<td>4.9 $K_y$</td>
</tr>
<tr>
<td>Lower Characteristic</td>
<td>17.7 $K_y$</td>
<td>10.4 $K_y$</td>
</tr>
<tr>
<td>Number of units</td>
<td>19</td>
<td>7</td>
</tr>
</tbody>
</table>

The correction factor for grade of reinforcement, $K_y$, is given Equation 4.

### 3.5 Columns

Limiting curvatures were calculated from 25 column tests from references [13, 14, 15 & 16]. Of these 9 were classified as having ductile plastic regions and 7 as limited ductile plastic regions. The remaining 9 test units did not satisfy the requirements for
either ductile or limited ductile detailing. Due to the low deformation capacity observed in tests and structural failures in earthquakes [17], which occurred due to the collapse of columns with minimal confinement reinforcement, the 1995 Structural Concrete Standard [18] required all columns to be detailed as either limited ductile or ductile. The same requirements have been carried through to the 2006 Standard [2]. The results of the analyses are summarised in Table 1 and Figure 4, where the ultimate curvatures are plotted against the axial load ratio. From the figure it appears that increasing axial load does not significantly reduce the ductility of the limited ductile plastic regions, which was one of the objectives on which the confinement criteria were based [18]. The same condition is assumed to apply to columns with ductile plastic regions. In this case the maximum axial load ratio \((N/A_g f'_c)\) of the test units considered was equal to 0.3.

![Figure 4: Ultimate curvatures sustained in column tests](image)

**3.6 Walls**

Two different sets of wall tests were examined, namely thin singly reinforced walls and ductile walls with two layers of reinforcement, one for each side face of the wall. Ultimate curvature values were determined from the results of 29 thin singly reinforced walls from references [19 to 23] together with results of tests carried out at the University of Auckland, which have not been published in the literature. Of these nine were rejected, five on the basis they were not representative of practice and 4 as they had aspect ratios less than 0.75 and failed by sliding shear. At present there is no codified method for assessing sliding shear failure in walls. The results obtained from these units have not been included in Figure 5 or in Table 1. The results from units with height to thickness ratios, which exceeded the permissible slenderness ratio by more than 35 percent, were excluded as were the results of two tests carried out in Auckland where the ultimate curvatures were more than 50 percent greater than those of similar companion units. The test walls described in references [20, 21 & 22] were constructed with vertically concave shape, so that the lack of vertical alignment corresponded with the maximum permissible out of true permitted for standard construction. The other feature to note is that three of the walls tested by McMenamin [22] failed, or partially failed, when some of the vertical reinforcement fractured. The results obtained from these walls gave limiting curvatures that were amongst the lowest observed in the series of tests. The reason for the apparent lack of ductility of this reinforcement is unknown. The reinforcement did not have a distinct yield point.
The nominal yield stress at an offset strain of 0.2 percent was 504MPa and the ultimate stress was 1.28 times the yield stress at a strain of 20 percent. The results of these three units have been included in Table 1 and in Figure 5, where the ultimate curvatures are plotted against the factor \((\frac{\rho f_y}{f_c'} + \frac{N}{A_g f_c'})\), which gives an assessment of the maximum compression force induced in the wall due to flexure and axial load.

The results of tests on 7 ductile rectangular walls reinforced with two layers of reinforcement, reported in references [24, 25 & 26] were analysed. Several of these walls formed wall elements in coupled walls. The axial load on these varied very significantly during the test. Due to the widely varying axial load levels and the limited number of tests the ultimate curvatures have not been shown in a figure. However, the average ultimate curvature, the standard deviation and the calculated lower characteristic curvature calculated from these tests are included in Table 1.

The addition of a boundary element at each end of a wall provides a zone, which can be effectively confined. This could be expected to increase the axial strain level and hence the curvature that can be sustained. On this basis the curvature limit for such walls could be expected to approach the corresponding limit for columns provided the boundary element resists the majority of the compression force in the wall and the boundary element satisfies the appropriate confinement requirements for columns.

### 3.7 Recommendations for material strain limits

Table 2 gives the recommended values for the coefficient, \(K_d\), used in Equation 3 for calculating curvature limits for reversing plastic hinge regions in beams, columns and walls. These are based on suitably rounded curvature limits corresponding to lower characteristic material strains summarised in Table 1. However, as there were no suitable tests for limited ductile plastic regions in beams the curvature limit has been placed approximately mid way between those for nominally ductile and ductile plastic regions. The effective plastic hinge lengths associated with these curvatures are given in section 2. The material strain limit for diagonally reinforced coupling beams is given in the Standard [2] as a shear deformation as these members deform with little or no flexural deformation. For these elements it is suggested that the current limit in
the Standard is not changed. This value was derived from an assessment of limited experimental work on coupling beams [25 & 27]. At present no material strain limits have been proposed for axial tension or compression.

**Table 2: Recommended $K_d$ values for determining curvature limits for reversing plastic regions**

<table>
<thead>
<tr>
<th></th>
<th>Nominally Ductile</th>
<th>Beams</th>
<th>Limited Ductile</th>
<th>Ductile</th>
<th>Columns</th>
<th>Nominally Ductile &amp; limited Ductile</th>
<th>Ductile</th>
<th>Walls</th>
<th>Nominally ductile</th>
<th>Limited ductile</th>
<th>Ductile**</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\varepsilon_c$</td>
<td>$\leq 0.0025$</td>
<td>10</td>
<td>17.5</td>
<td>10</td>
<td>$\varepsilon_c$</td>
<td>$\leq 0.0025$</td>
<td>5.0</td>
<td>$\varepsilon_s$</td>
<td>$\leq 0.010$</td>
<td>8.5*</td>
<td>12.5*</td>
</tr>
<tr>
<td>$\varepsilon_s$</td>
<td>$\leq 0.010$</td>
<td></td>
<td></td>
<td></td>
<td>$\varepsilon_s$</td>
<td></td>
<td></td>
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</tr>
</tbody>
</table>

* limited ductile doubly reinforced and singly reinforced walls
** two layers of reinforcement in each direction and confined as required by the Standard [2]
+ for walls with confined boundary elements this increased value may be used, see text below.

It is suggested that where each boundary element in a wall can resist 70 percent or more of the compression force in the ultimate limit state, and they are confined to meet the requirements for limited ductile columns in clause 10.3.10.6.2 in the Structural Concrete Standard [2], or for ductile columns in clause 10.4.7.5.1, the limiting section curvature could be increased to 85% of the corresponding value for columns. With the limited ductile walls the value of $N^*$ can be taken as the resultant axial load on the boundary element in the ultimate limit state, and with ductile walls $N^*_o$ may be taken as 1.2 times the resultant axial load acting on the boundary element calculated as for the ultimate limit state.

The recommended limiting curvatures for nominally ductile beams and walls are similar to the current values in the Structural Concrete Standard [2] (March 2006), though the maximum tensile strain limits have been changed. With the proposed values the curvature limit is approximately $2.5\phi_y$ to $4.0\phi_y$ for reinforcement Grades 500 and 300 respectively, where $\phi_y$ is given by Equation 2.

For unidirectional plastic regions the curvature limits may be doubled for ductile and limited ductile beams and columns and for nominally ductile plastic regions the strain limits may be increased to 0.004 and 0.016 for concrete and reinforcement respectively.
4. Strain and curvature distributions in a beam plastic region

Figure 6 illustrates the actions in a beam plastic region located close to a column. Part (a) of the figure shows the bending moments in the beam, while part (b) indicates a typical crack pattern. As shown in part (c) the longitudinal beam reinforcement yields over a length of “$g + e + f$” in the beam column joint zone and in the beam.
The distance “e” in the beam is a function of the increase in the maximum bending moment resisted at the critical section of the plastic region, $M_{\text{max}}$ above the moment which induces first yield of the longitudinal reinforcement, $M_{y1}$. The value of the length “e” is given by:

$$e = \left[ \frac{M_{\text{max}} - M_{y1}}{M_{\text{max}}} \right] \frac{M}{V}$$  \hspace{1cm} (5)$$

where $M/V$ is the ratio of moment to shear at the critical section. The increase in $M_{\text{max}}$ above the moment at first yield depends on the strain hardening characteristics of the reinforcement, the deformation history of the plastic region and the length over which the reinforcement yields ($g + e + f$). The additional extension of yielding in the beam, distance “f”, which is the tension lag, is associated with diagonal tension cracking in the beam, as illustrated on the free body shown on the right hand side of Figure 6 (b).

Moment equilibrium at section 2, which is at the head of the diagonal crack, leads to the equation-

$$M_2 = T_1 \, jd + \frac{V_s \, x_d}{2}$$  \hspace{1cm} (6)$$

where $M_2$ is the moment at section 2, $T_1$ is the flexural tension force at section 1 located at the end of the diagonal crack, $x_d$ is the distance measured along the beam between sections 1 and 2 and $V_s$ is the shear carried by the stirrups across the diagonal crack. If $V_s$ is negligible then the tension lag, $f$, equals $x_d$, the length of the diagonal crack. If all shear is resisted by stirrups then the tension lag is equal to $x_d/2$, which is equivalent to the tension force $T_1$ being induced by the moment sustained mid way between sections 1 and 2, that is half way along the diagonal crack. Generally at the end of the plastic region, provided the shear stress is sufficient to cause diagonal cracking, the value of $x_d$ is approximately equal to the effective depth.

The intersection of these cracks effectively destroys the shear that can be resisted by the concrete alone, that is $V_c$ is zero. In this situation the value of tension lag, $f$, for practical purposes is equal to half the effective depth, $d/2$. The formation of diagonal cracks in the beam column joint zone causes yielding to extend for some distance through the beam column joint zone. This distance, shown as “g” in Figure 6 (c), increases as bond resistance is lost due to yielding of the reinforcement and cyclic loading. The extent of yield penetration into or through a joint zone depends on many other factors. The most important of which are the depth of the column relative to the diameter of bar, the reinforcement yield stress, the axial load on the column and the stress in the column reinforcement.

Figure 6 (d) shows the curvature distribution along a plastic region in a beam based on the strains in the flexural tension reinforcement, for the case of a unidirectional plastic region. For comparison the assumed analytical curvature over the length $l_{\text{eff}}$ is shown. The yield penetration and anchorage pull out of the reinforcement in the joint zone results in cracks forming either at or close to the face of the column or/and at the face of the extreme reinforcement in the column. It may be noted that the yield extension of the reinforcement over the distance “$g + e + f$”, which is equal to the distance between C and A in Figure 6 (b), is associated with flexural compression of the concrete between the face of the column and point B. Hence the assumption of plane sections remaining plane is at best, even for unidirectional plastic regions, a very rough approximation. It should be noted that the analytical curvature limit was
found from displacements measured in tests on the basis of the assumption that the inelastic curvature accounted for the inelastic displacement. This is not strictly correct, as shear deformation does not induce any curvature and it has been found to account for 30 percent or more of the total displacement in beams subjected to extensive inelastic cyclic loading [5, 6 & 28]. The shear displacements in plastic regions in columns and walls are generally considerably smaller than the corresponding values in beams. For these two reasons, namely plane sections not remaining plane and shear deformation being ignored in the calculation of curvature limits, analytical curvatures in unidirectional plastic hinges cannot be used to predict realistic strain levels in either the reinforcement or concrete. With inelastic cyclic loading an additional complication arises due to elongation, as explained in the following paragraph.

With flexural cracking in beams, columns and walls, elongation occurs unless they are subjected to moderate to high axial load ratios. As illustrated in Figure 7 this elongation increases substantially when inelastic deformation is applied. In unidirectional plastic hinges elongation occurs as the tensile strains in the reinforcement are greater than the corresponding compression strains in the concrete. With reversing plastic hinges there are two causes of elongation.

1. When longitudinal reinforcement yields wide cracks develop. Micro cracks form round the bars and the slip of the bars through the concrete close to the cracks results in concrete chips and aggregate particles being pulled into the cracks. Additional material is dislodged from the crack surfaces due to shear displacements, which develop across cracks. This material tends to prevent the cracks from closing when the direction of moment and shear reverse. In effect the concrete dilates when subjected to inelastic cyclic loading.

2. Diagonal compression forces are sustained in the beam web due to the action of the shear reinforcement. The longitudinal component of these forces causes the flexural tension force at a section to be greater than the corresponding flexural compression force. As a result of this under inelastic cyclic loading rotation occurs predominately by tensile yielding of the flexural tension reinforcement rather than by compression yielding of the longitudinal reinforcement in the compression zone. As shown in Figure 7 elongation can more than double the extension of flexural tension reinforcement associated with curvature in the plastic hinge.

The behaviour of reversing and unidirectional plastic regions in beams and the causes of elongation are described elsewhere [28].

5. Discussion

1. A relatively simple failure criterion has been adopted in assessing the ultimate deformation capacity of members. Future research could look at using a more sophisticated approach based on damage indices and a larger range of test results. It should be noted that the displacement history imposed by an earthquake is likely to have a significant influence on the maximum deformation that can be sustained before failure. If the maximum deformation is imposed near the start of the ground motion, when the plastic regions have little damage, they are likely to be able to sustain greater deformation without failure than if the maximum deformation was imposed near the end of the ground motion. Analysis of test results on beams has shown that dissipated energy is not in itself a good guide to
deformation capacity. It was found that greater energy could be dissipated when a large number of relatively small displacement cycles were applied than when a relatively few large displacement cycles were applied [29].

Due to restraints on time, only test results which were readily available in the literature were assessed. Future research could examine a wider range of test data.

The limiting material strains listed in the paper are for the ultimate limit state where a high margin of safety is required. However, it is believed that plastic regions proportioned to meet these limits will be able to satisfy the greater deformation required for the maximum credible earthquake (return period of 2,500 years) with an adequate margin of safety for this event. There are two reasons for this.

Figure 7: Elongation of reinforcement in plastic hinge zones

2. Due to restraints on time, only test results which were readily available in the literature were assessed. Future research could examine a wider range of test data.

3. The limiting material strains listed in the paper are for the ultimate limit state where a high margin of safety is required. However, it is believed that plastic regions proportioned to meet these limits will be able to satisfy the greater deformation required for the maximum credible earthquake (return period of 2,500 years) with an adequate margin of safety for this event. There are two reasons for this.
The maximum deformation limit from each test was based on displacements cycles with ± displacements of nearly equal value. However, in an earthquake the peak displacement is only sustained in one direction.

A decrease in the flexural resistance of a plastic region in practice results in a redistribution of forces to other plastic regions in indeterminate structures. Hence the average strength and deformation capacities of plastic hinges in a region of the structure (such as the plastic hinges in a level in a moment resisting frame building) is the controlling factor rather than the ultimate design limits based on the lower characteristic values. The difference between the mean and lower characteristic values for deformation capacity is appreciable, as can be seen from Table 1. Allowance for this effect can make an appreciable difference to the total displacement that can be sustained before collapse occurs. The importance of indeterminacy on seismic performance is recognised in a number of major seismic design codes [30 & 31], but unfortunately it is not recognised in a numerical sense in the New Zealand Standard for Earthquake Actions [1].

6. Conclusions
1. The current (March 2006) Structural Concrete Standard [2] contains material strain limits for the design of seismic resistant potential plastic regions. This paper contains a new set of material strain limits, which will be submitted to Standards as a proposed amendment to the current values.

2. In developing the proposed material strain limits the test results from 37 beams, 25 columns and 36 walls were analysed. On the basis of these tests recommendations are made for material strain limits for limited ductile and ductile plastic regions.

3. Although the proposed curvature limits may appear at first glance to be more conservative than the values given in the March 2006 edition of the Standard [2], they are in fact less conservative for beams and columns due to the way they are calculated. In the proposal the limiting curvatures are given in terms of a multiple of a curvature corresponding to the initiation of significant inelastic deformation. This curvature limit is significantly larger than the first yield curvature on which the curvatures limits were based in the March 2006 edition of the Standard.

4. The use of a simple equation to estimate the curvature initiating significant inelastic behaviour for ductile and limited ductile plastic regions simplifies design calculations compared with the requirements given in the March 2006 edition of the Standard [2].

5. A relative simplistic method was used to assess the maximum material strain limit from each test. There is scope for further research looking at a wider range of test results and in assessing material strain limits by more sophisticated techniques based on damage indices.

6. No suitable test results were found for nominally ductile plastic regions subjected to inelastic cyclic loading. To fill this gap in our knowledge a research project has been started at the University of Canterbury. However, it will be some time before the results of this work are available. At this stage tentative
recommendations have been made to cover detailing of nominally ductile plastic regions.

7. The proposed material strain limits are intended to provide a high margin of safety against failure in the ultimate limit state earthquake and an adequate margin of safety against collapse for the maximum credible earthquake with a return period of 2,500 years.

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