PARAMETRIC STRUCTURE-SPECIFIC SEISMIC LOSS ESTIMATION

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Abstract: A loss estimation methodology is presented which provides several measures of seismic performance to decision-makers. The is methodology based on parametrically describing several key relationships between: (i) ground motion *intensity measure* (IM) and exceedance rate ν(IM); (ii) IM and *engineering demand parameter* (EDP); (iii) EDP and *damage states* (DS); and (iv) DS and economic loss (L). Both aleatoric and epistemic uncertainties are considered in each of the key relationships. A vector of EDPs is used and correlations between components are considered. The expected loss and standard deviation of loss as a function of the ground motion IM are obtained, allowing various percentile loss hazard curves to be computed. The distribution of Expected Annual Loss (EAL) is obtained for use in budgeting or assessing retrofit solutions. The methodology is applied to a case study of a typical bridge designed to New Zealand Standards.

1. INTRODUCTION

Performance Based Earthquake Engineering (PBEE) has emerged as a cornerstone of modern earthquake engineering as it attempts to capture the performance of structures over the full spectrum of structural behaviour, from initial elastic response through to global instability, when subjected to a range of ground motion excitations. Accurate quantification of seismic performance can be obtained by considering several key relationships, namely: (i) strong ground motion intensity vs. rate of exceedance; (ii) seismic intensity (demand) vs. structural response (capacity); (iii) structural response vs. structural damage; and (iv) structural damage vs. economic loss. Each of the aforementioned four relationships can be assessed separately and then integrated in various forms to provide useful information to key decision makers.

Loss estimation methodologies have emerged as way of considering seismic vulnerability of a structure in terms of a monetary value, allowing easy comparison with non-engineering decision makers.

Recently, the Pacific Earthquake Engineering Research (PEER) centre has developed the so-called 'triple integral formula'. This formula, based on the total probability theorem allows the mean annual frequency of a decision variable (i.e. economic loss, or closure) to be obtained. While the triple integral formula provides a benchmark for many loss estimation applications it may be advantageous to consider different sequences of application in order to provide useful intermediate information, and also consider correlation between various components.

In this paper, a fully parametric method of structure-specific loss estimation ,as opposed to regional loss estimation (HAZUS, 2003) is described, and then applied to a case study of a typical bridge structure designed to New Zealand Standards.

2. LOSS ESTIMATION METHODOLOGY

The loss estimation methodology is described first by considering each of the four key relationships separately and then how they are combined to provide useful information for decision makers.

2.1 Relation 1: Seismic intensity vs. rate of exceedance

In order to characterise the seismicity of a site a ground motion hazard plot is usually obtained via Probabilistic Seismic Hazard Analysis (PSHA). The hazard plot typically gives the rate of exceedance of a given ground motion parameter (herein referred to as *intensity measure*, IM) . Bradley et al., (2007) developed a parametric relationship for the ground motion hazard curve considering uncertainty given in Equation 1:

$$\ln(v) = \ln(v_{asy}) + \left[\alpha\left\{\ln\left(\frac{IM}{IM_{asy}}\right)\right\}^{-1}\right] + \varepsilon \qquad (1)$$

where IM = ground motion *intensity measure* (e.g. PGA, S_A, S_d); v = rate of exceedance of IM; ε = lognormal random variable; v_{asy}, α, and IM_{asy} are parameters that are determined based on a curve fitting technique.

There are several sources of uncertainty in the seismic hazard model. These can be grouped into uncertainty associated with obtaining the data points (β_{RH}), and the additional uncertainty introduced by fitting the curves parametrically (β_{UH}). These are discussed elsewhere (Bradley et al., 2007). The two uncertainties can be combined to give the total uncertainty associated with the seismic hazard curve (Kennedy et al., 1980):

$$\beta_H = \sqrt{\beta_{UH}^2 + \beta_{RH}^2} \qquad (2)$$

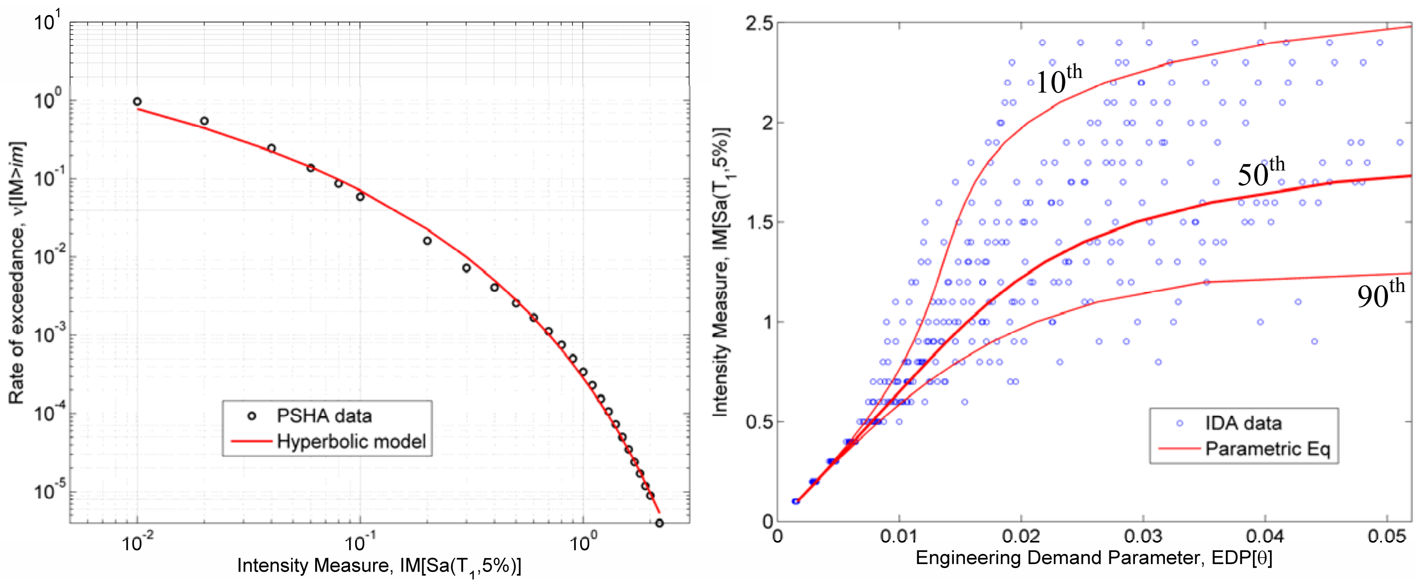
A comparison of the parametric fit given by Equation 1 and the raw data from the PSHA for a site in Wellington, New Zealand is given in Figure 1a.

2.2 Relation 2: Seismic intensity vs. structural response

Several methods (typically analytical) can be used to obtain the relationship between the IM and *engineering demand parameter* (EDP). A common method that has emerged is via Incremental Dynamic Analysis (IDA) (Vamvatsikos and Cornell, 2002). IDA involves carrying out non-linear dynamic time history analyses of an computational model of the structure subjected to ground motion records scaled to various levels of intensity, and monitoring a pre-determined EDP (e.g. interstorey drifts and/or floor accelerations). The resulting data from the IDA can then be used to define, probabilistically, a conditional IM-EDP relationship. The parametric form of this relationship used in this study is that derived by Jayaler (2002) which considers the separation of mutually exclusive and collectively exhaustive events of structural collapse and non-collaspe, and is given in Equation 3.

$$P(EDP > edp|IM) = P(EDP > edp|IM, NC)[1 - P(C|IM)] + P(C|IM) \qquad (3)$$

where $P(EDP>edp|IM,NC)$ = is the probability of exceeding an EDP of *edp* for a given IM and no collapse; $P(C|IM)$ = the probability of collapse given IM; $P(NC|IM)$ = the probability of no collapse given IM, note that $P(NC|IM) = 1-P(C|IM)$. A comparison of IDA data and the parametric relationship given by Equation 3 is given in Figure 1b.



(a) Ground motion hazard model (b) Structural (IM-EDP) model

Figure 1: Hazard model and IDA curves

2.3 Relation 3: Structural response vs. structural damage

This relationship (as relationship 4) is typically assumed to be discrete (but still probabilistic) because discrete repairs are required on damage (i.e. a partition is either patched or not patched) which cost discrete amounts of money. The relation is based on the concept of *damage states* (DS), which define the damage to a specific component in a structure (e.g. concrete spalling, bar buckling etc.). The DS are defined with both an expected value (mean) and uncertainty (variance) such that each DS boundary is represented as a fragility curve.

2.4 Relation 4: structural damage vs. economic loss

Economic losses associated with damage to a structure can be both direct and indirect. Direct losses due to damage for example are the cost to repair/replace damaged components of the structure, while indirect costs can be death/injury to occupants and downtime of the services/companies that cannot inhabit/use the structure while it is under repair. These losses are typically also measured discretely and associated with the DS boundaries based on the structural response. Losses associated with DS are presented as an expected value and standard deviation of the normalised cost of the component.

2.5 Integration of key relationships

A brief description of the loss estimation methodology is presented below. A thorough discussion on a similar framework is presented elsewhere (Aslani, 2005).

The expected loss in a structure conditioned on the ground motion intensity measure can be expressed as the combination of the mutually exclusive and collectively exhaustive cases of collapse and non-collapse:

$$E[L|IM] = E[L|IM, NC]P(NC|IM) + E[L|C]P(C|IM) \tag{4}$$

where $E[L|IM,NC]$ = the expected loss for a given ground motion intensity, IM, given that collapse has not occurred; $E[L|C]$ = the expected loss given collapse. Note that P(C|IM) =1- P(NC|IM), and can be obtained from the structural analysis (Figure 2a).

The expected loss in each component for a given EDP can be obtained as the summation of the expected loss for each DS multiplied by the probability of exceeding the given DS. The expected loss in each component for a given IM can be obtained by convoluting the expected loss in each component for a given EDP with the differential of the conditional IM-EDP relation. The above sentences can be expressed mathematically as:

$$E[L|IM,NC] = \sum_{j=1}^{N_c} a_j \left[\int_0^{\infty} \left\{ \sum_{i=1}^{N_{ds}} E\left[L_j|NC,DS_i\right] P\left(DS_i|NC,EDP_j\right) \right\} dP\left(EDP_j|NC,IM\right) \right] \tag{5}$$

where a_j = cost of component j; N_c = the number of components; N_{ds} = the number of damage states; $E[L_j|NC,DS_i]$ = the expected loss given DS_i; $P(DS_i|NC,EDP_j)$ = the probability of being in DS_i given the EDP for component j; and $dP(EDP_j|NC,IM)$ = the differential of the conditional EDP-IM relationship. The variance in the expected loss in each component conditional on the IM can be computed by:

$$\sigma^2_{[Lj|IM,NC]} = E\left[L_j^2|IM,NC\right] - \left\{E\left[L_j|IM,NC\right]\right\}^2 \tag{6}$$

where $E[L_j^2|IM,NC]$ is computed in a similar fashion to $E[L_j|IM,NC]$. The variance for the entire structure given no collapse can then be calculated as:

$$\sigma^2_{[L|IM,NC]} = \sum_{j=1:n} \sum_{j'=1:n} a_j a_{j'} \rho_{[Lj,Lj'|IM,NC]} \sigma_{[Lj|IM,NC]} \sigma_{[Lj'|IM,NC]} \tag{7}$$

where $\rho_{[LjLj'|IM,NC]}$ = the correlation coefficient for losses in components j and j'.

The annual frequency of exceeding a certain level of economic loss, l_T (or loss hazard) in a structure can be computed as follows:

$$\nu\left(L_T > l_T\right) = \int P\left(L_T > l_T|IM\right) \left|d\nu(IM)\right| \tag{8}$$

where $P\left(L_T > l_T|IM\right)$ = the probability of exceeding l_T for a given level of ground motion, IM, and can be calculated from:

$$P\left(L_T > l_T|IM\right) = P\left(L_T > l_T|IM,NC\right)P\left(NC|IM\right) + P\left(L_T > l_T|C\right)P\left(C|IM\right) \tag{9}$$

where $P\left(L_T > l_T|IM,NC\right)$ = the probability of exceeding l_T for a given level of ground motion, IM, and no collapse. $P\left(L_T > l_T|IM,NC\right)$ is assumed to be normally distributed based on the central limit theorem (assuming loss, given no collapse, is not dominated by a few components), while $P\left(L_T > l_T|C\right)$ is lognormally distributed (Aslani, 2005).

3. APPLICATION TO BRIDGE STRUCTURES

In the following section the loss estimation methodology is used to assess the seismic performance of a typical bridge designed to New Zealand Standards (Standards New Zealand, 1995). The prototype bridge is a typical 'long' multi-span highway bridge on firm soil with five 40m longitudinal spans, 10m transverse width, and 7m circular piers. The structure has an estimated replacement cost of \$9M with a coefficient of variation of 0.31 (assumed lognormal distribution). Further design details and experimental modelling of the pier can be found elsewhere (Mashiko, 2006; Solberg, 2007). The bridge was assumed to be located in the high seismicity region of Wellington, New Zealand. The fundamental period of the pier was 0.6 seconds.

Seismic hazard data for the site was obtained from Stirling et al. (2002). The IM selected was the spectral acceleration at the fundamental period of the structure, as it typically gives rise to low dispersion in the structural demand-response (IM-EDP) relationship (Shome and Cornell, 1999). From the hazard data, the parameters for Equation 1 were determined. A comparison of the data and the parametric fit are given in Figure 1. From the least squares regression β_{UH} was calculated to be 0.16. The uncertainty associated with the PSHA, β_{RH} is currently being investigated by the authors in conjunction with others, and was assumed to be equal to 0.2. Using Equation 2 therefore yields $\beta_H = 0.26$. A suite of ground motion records, previously used by Vamvatsikos and Cornell (2002) were adopted. These records, all of which were recorded on firm soil, have magnitude and direction ranges of 6.5-6.9 and 15.1-31.7 km, respectively.

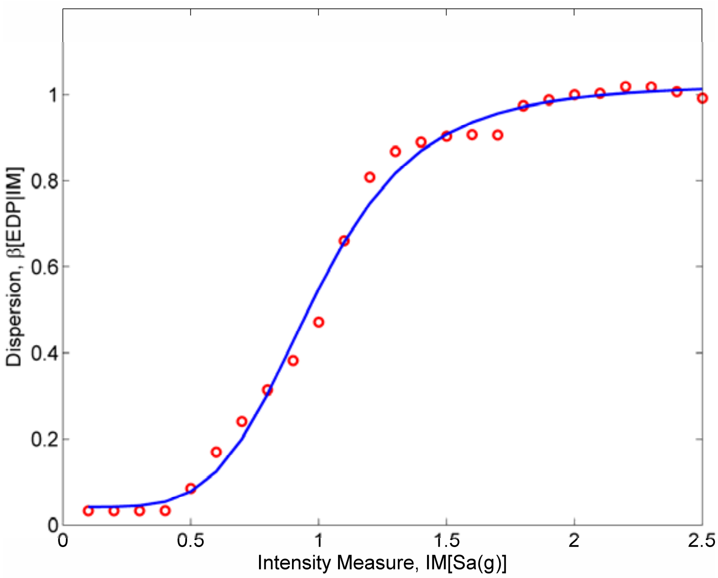
Using a finite element model of the bridge pier, IDA was used to provide the data to characterise the conditional IM-EDP relationship. The IDA was carried out using the spectral acceleration at the fundamental period of vibration as the intensity measure (IM), and the deck drift as the engineering demand parameter (EDP). The resulting IDA data from the structural analyses is presented in Figure 1b. The conditional IM-EDP relationship was then parameterised using Equation 2. $P(EDP>edp|IM,NC)$ was assumed to be lognormally distributed (Jalayer, 2002) with median EDP = aIM^b and lognormal standard deviation (dispersion) β_{US}, where a and b parameters were determined by regression on the IDA data. The 10^{th}, 50^{th} and 90^{th} percentile curves are shown in the figure. The bridge was deemed to collapse at a drift of 5.6% due to significant P-Δ effects from the superstructure. The epistemic uncertainty, β_{US}, due to the parametric fitting of the data was modeled using a hyperbolic tangent function of the form given in Equation 10.

$$\beta_{US} = \alpha_1 + \alpha_2 \tanh\{\alpha_3 IM\} \tag{10}$$

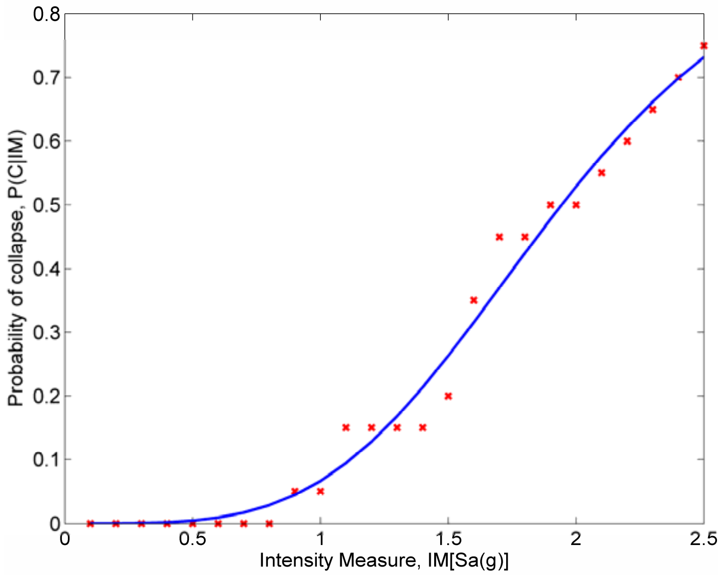
where α_1- α_3 are constants determined from the regression on the data.

The variation of the collapse probability with IM was assumed to follow a lognormal distribution (Aslani, 2005). A comparison of the parametric fits for dispersion and collapse probability and the raw data points is given in Figure 2.

Loss modelling of the bridge considered structural damage only, as non-structural damage was considered to be relatively insignificant. Downtime of the structure in some instances (when the bridge is on main arterial routes) may be significant. Downtime was not considered hereafter, although it is being investigated in a separate study by the authors. As is typical for structural damage, the EDP selected was the drift angle of the bridge deck relative to the abutment, as it correlates well with the curvature demand in the plastic hinge zone at the base of the pier. Damage for non-collapse cases was assumed to only occur in the pier, hence only during collapse was the deck assumed to be damaged.



(a) β_{US} variation with IM



(b) $P_{collapse}$ variation with IM

Figure 2: Parametric fitting of the standard deviation and Probability of collapse

Table 1 presents the mean EDP values and *loss ratios* (LR) at the onset of each of the *damage states* (DS) as well as the associated standard deviations. The mean values are based on actual repair data from the Northridge and Loma Prieta earthquakes as given by Mander and Basoz (1999), while the standard deviations were determined indirectly, with reference to the coefficient of variations used by Aslani (2005) for reinforced concrete columns.

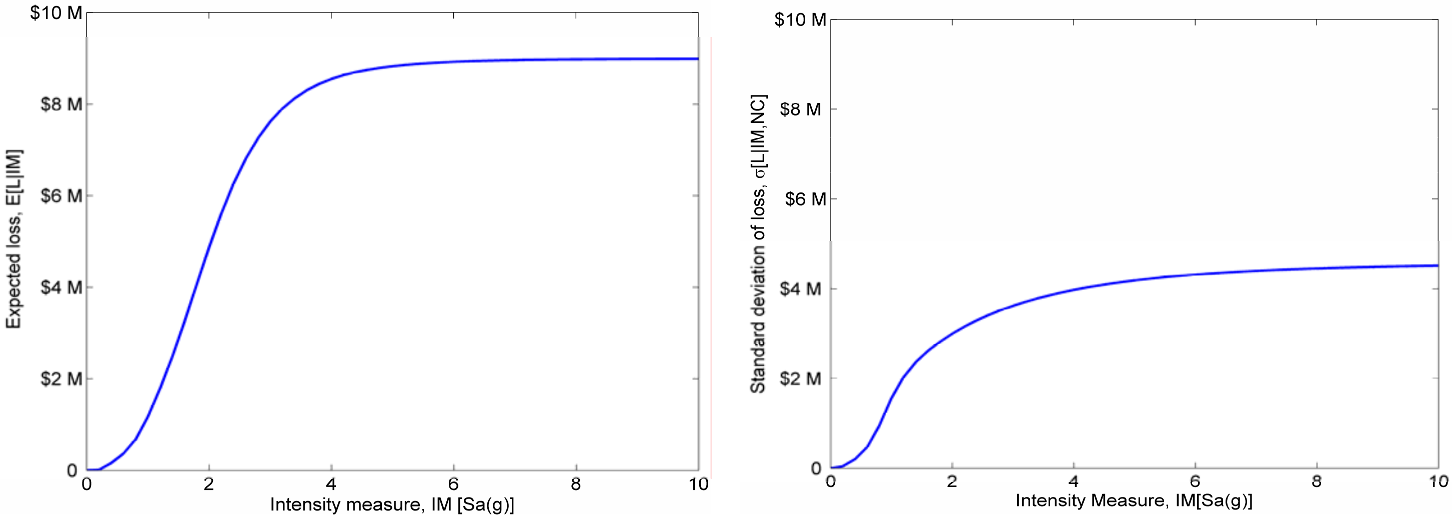
It was assumed that the direction of earthquake shaking was perpendicular to the longitudinal axis of the bridge, and it is also assumed that all piers are subjected to the same ground motion shaking in time (i.e. traveling wave effects are ignored). The correlation coefficients, $\rho_{[Li,Lj|IM,NC]}$ for the losses incurred at each of the piers can therefore be taken as unity. The correlation coefficients for when collapse occurs (i.e. requiring replacement of the structure), $\rho_{[CCIi,CCIj]}$, were also taken as one, with all of the bridge components being made primarily of concrete (Touran and Lerdwuthirong, 1997).

Table 1: Statistical parameters for the loss and damage functions in bridge piers

Damage State	$\overline{EDP}$	$\sigma_{\ln EDP}$	$E[L_j\|DS_i]$	$\sigma[L_j\|DS_i]$	$c.o.v[L_j\|DS_i]$
DS1:Yielding, cracking in hinge zone	0.0063	0.4	0.03	0.02	0.65
DS2: Spalling, bar buckling	0.016	0.45	0.08	0.05	0.65
DS3: Bar fracture/ significant strength degradation	0.046	0.6	0.25	0.18	0.7
DS4: Structural collapse	0.056	0.65	1.0	0.75	0.75

3.1 Loss estimation results

Equations 4 and 6 were computed to yield the expected loss and standard deviation in loss given no collapse as a function of the first mode spectral acceleration, which are plotted in Figure 3a and 3b, respectively. Note that the standard deviation given collapse is a constant ($4.5 M). As would be expected, it can be seen that the expected loss increases with IM and similarly for the standard deviation. Hence, the loss estimation methodology takes into account the variation in dispersion with IM.

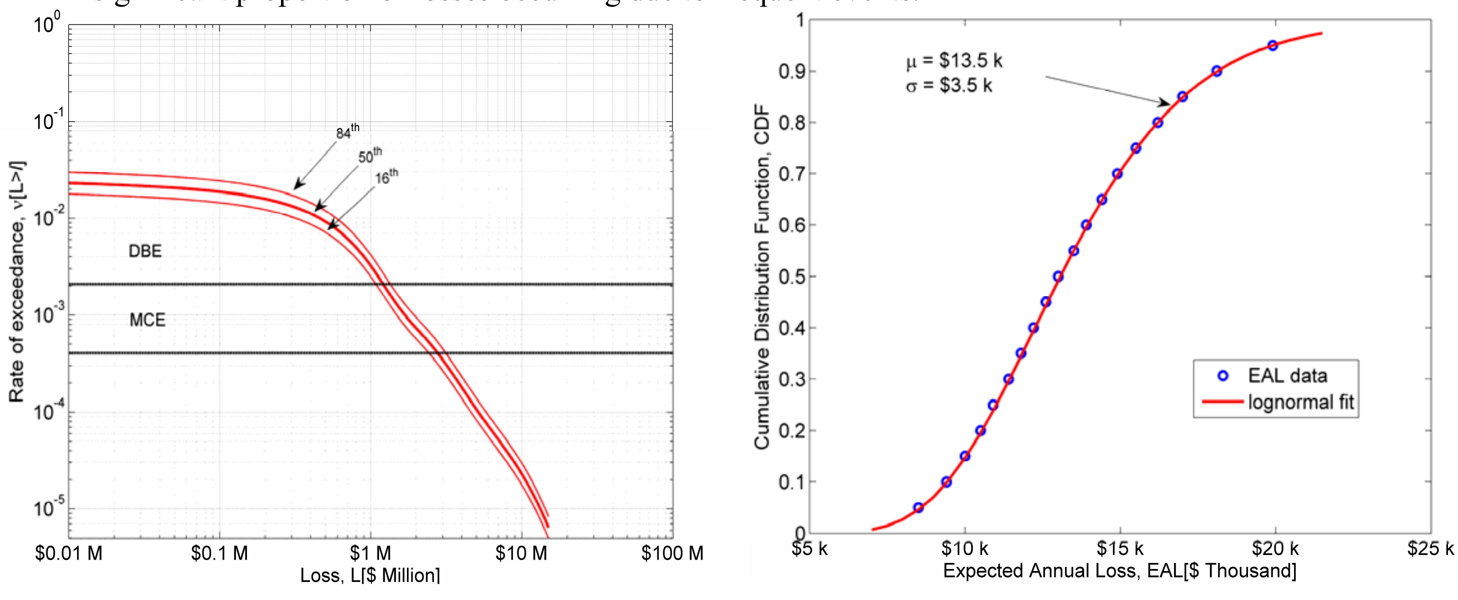


(a) Expected loss conditional on IM

(b) Standard deviation of loss given no collaspe

Figure 3 Expected losses and standard deviation conditional on IM

Equation 8 was then used to compute the loss hazard curves. As mentioned, uncertainty was also be incorporated into the seismic hazard model. By using the 16th, 50th, and 84th percentile seismic hazard curves (84th and 16th percentiles are ± one standard deviation), the corresponding loss-hazard curves shown in Figures 4a are obtained. The loss hazard curves show the typical concave from below shape, and at the DBE and MCE occurrence rates (475 and 2475 year return periods, respectively) the median economic losses were approximately \$1.5 M and \$3.0 M, respectively. The change in slope of the hazard curves at approximately \$0.5 M is due to a significant proportion of losses occurring due to frequent events.



(a) Median and ± 1σ loss hazard curves

(b) Distribution of Expected Annual Loss (EAL)

Figure 4: loss hazard curves and EAL with uncertainty

The Expected Annual Loss (EAL) is a convenient measure of economic loss, as it incorporates a range of seismic scenarios, return rate and expected damage into a single dollar value. Such a value can be used for incorporation into company budgets, or for use in evaluating the benefits of different retrofitting strategies. From the loss hazard curves given in Figure 4a the EAL can be obtained by first converting the vertical axis to probability of exceedance by the relationship: $P(L>l)= 1-\exp[-\nu(L>l)]$ (Bradley et al., 2007), and then integrating the area below the resulting curve. When this is conducted for a range of loss hazard curves of different percentiles, then the data labeled as 'EAL data' in Figure 4b results. This data can be accurately approximated by a lognormal distribution (which for the case study structure has a mean of $13.5 k and a standard deviation of $3.5 k). The EAL can therefore be given with a certain level of confidence, i.e. it allows one to make a statement such as "one can be 95% confident that the expected annual loss will not exceed $20 k". This distribution of EAL provides further information to decision makers, as opposed to just the mean EAL value. When considering EAL over several years a discount factor, λ (Wen et al., 2001) should be incorporated to account for the time value of money.

4. CONCLUSIONS

Based on the findings of this research the following conclusions can be drawn:

1. A fully parametric loss assessment method has been presented that provides various means of communicating seismic risk. The method incorporates uncertainty in all of the key relationships and considers correlations between components.
2. In particular, the method allows the Expected Annual Loss (EAL) to be given as a distribution, as opposed to a single value, providing EAL estimates with a certain level of confidence which is more insightful for decision makers.

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