Utility of Algebraic Connectivity Metric in Topology Design of Survivable Networks

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***Abstract*— In studies of survivable networks, it is important to be able to differentiate network topologies by means of a robust numerical measure that indicates the levels of immunity of these topologies to failures of their nodes and links. Ideally, such a measure should be sensitive to the existence of nodes or links which are more important than others, for example, if their failures cause the network's disintegration. In this paper, we suggest using an algebraic connectivity metric, adopted from spectral graph theory, namely the 2nd smallest eigenvalue of the Laplacian matrix of the network topology, instead of the average nodal degree that is usually used to characterize network connectivity in studies of the spare capacity allocation problem. Extensive simulation studies confirm that this metric is a more informative and more accurate parameter than the average nodal degree for characterizing network topologies in survivability studies.**

Keywords- Network survivability; spare capacity allocation; topology design; network connectivity; algebraic connectivity metric; 2nd smallest eigenvalue; Laplacian matrix; SBPP; ILP.

I. INTRODUCTION

Survivability is one of the most important design issues of future multi-service Next Generation Networks (NGNs). Survivable network design pre-plans the topology as well as the spare capacity allocation (SCA) of network links in case of failures. Performance of survivable routing protocols, robustness of the network under failures, traffic engineering, etc., depend crucially on the topology of the network. Network robustness can be characterized by the network topological connectivity, which expresses how well nodes are connected in a network. In general, as the network connectivity increases, there are more node- and link-disjoint paths between node pairs and both the predetermined working and protection path-pairs become shorter, it decreases both working and protection capacity. In addition, the more disjoint protection paths, the higher the spare capacity sharing that can be attained in shared backup path protection (SBPP) survivable routing scheme. These dependencies underlie the determination of an optimum topology in network survivability design.

The SCA design of survivable networks for given topologies has been subject to much research in recent years. Most previous studies [1-6] generally use the average nodal degree to reflect the effect of the network connectivity in determining the spare capacity allocation. The average nodal degree $\overline{d}$ is obtained by multiplying the number of links by two and dividing it by the number of nodes in a given network topology. Their simulation results have concentrated on showing how the working and spare capacity requirements of each network type vary with the network average nodal degree.

Despite of a wide adoption of the average nodal degree in such studies, we argue that this metric is only a coarse indicator of how sparse or dense a given topology is. It carries insufficient information on network topological structure. Furthermore, employing the average nodal degree for describing the network's connectivity may lead to misleading findings. We suggest using a more informative metric: algebraic connectivity, which is defined as the 2nd smallest eigenvalue of Laplacian matrix of a given topology, as it is more sensitive measure of connectivity in a broader spectrum of graphs [7-10]. The 2nd smallest eigenvalue of the Laplacian matrix of a network, known as the algebraic connectivity and defined in Section II, is one of the key invariants in a graph which is not only of theoretical interest but also has a wide range of applications. It has desirable properties, such as the larger the algebraic connectivity is, the greater the number of node- and link-disjoint paths to choose from.

Furthermore, we employ the notion of algebraic connectivity of a network to quantify the importance of a node or a link. Namely, the importance of a node or link is quantified by the algebraic connectivity of the remaining network after that particular node or link fails. The most important node or link from the network connectivity perspective is that which causes the most severe reduction in the remaining network's algebraic connectivity. Thus, such node or link needs more protection to ensure that the connectivity of the network always remains as large as possible.

The structure of this paper is outlined as follows. Section II introduces the definition on algebraic connectivity metric and presents the related theoretical results. In Section III, we introduce the ILP model of the shared backup path protection (SBPP) scheme, which is used to evaluate the impact of algebraic connectivity metric on capacity allocation. The extensive simulation studies and findings are presented in Section IV. Finally, conclusions are drawn in Section V.

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II. Algebraic Connectivity In Spectral Graph Theory

In this section, we introduce the definition of a Laplacian matrix, its eigenvalues and the relationship between the eigenvalues of Laplacian matrix and the algebraic connectivity of the associated network. We define that *G(V,E)* is the network with set of nodes *V* and set of links *E*. We recall the number of nodes as |*V* | = n and the number of links as |*E*| = m. Moreover, we define G_{-v_i} as a network resulted from removing node v_i and all of its adjacent links from the original network *G*. The Laplacian matrix associated with a network is defined as follow.

Definition 1 Laplacian matrix of a network

In a network *G(V,E)*, the Laplacian matrix associated with a network, **L**(*G*) =[*L(i,j)*], 1≤*i*≤n, 1≤*j*≤n, is an *n* x *n* matrix, defined as follows:

$$L(i,j)=\begin{cases} d_{v_i} & if\ v_i=v_j \\ -1 & if\ (v_{i,}v_j)\in E \\ 0 & otherwise \end{cases} \quad (1)$$

where $i,j\in\{1,\cdots,n\}$ are the indices of the nodes, and v_i is the *i*th node. Equivalently, the Laplacian matrix **L**(*G*) can be expressed as :

$$\mathbf{L}(G)=\mathrm{D}(G)-\mathrm{A}(G) \quad (2)$$

where D(*G*) is an *n* by *n* diagonal matrix associated with network *G,* with the (*i*,*i*)-th entry equal d_{v_i}, which represents the number of neighboring nodes. A(*G*) is a *n* x *n* adjacent matrix associated with network *G*. The eigenvalues of the Laplacian matrix **L**(*G*) are usually referred to as the network graph spectra and denoted as below:

$$0=\lambda_1\le\lambda_2\le\lambda_3\le\ldots\le\lambda_n \quad (3)$$

Since **L**(*G*) is real, symmetric and nonnegative semi-definite, thus all the eigenvalues of **L**(*G*) should be real and nonnegative. Hence the smallest eigenvalue of Laplacian matrix **L**(G) is zero. The 2nd smallest eigenvalue of the Laplacian matrix is referred to in [9] and [10] as the algebraic connectivity of the network graph *G*. It is also called the Fiedler value of a graph, which is related to the usual node and link connectivity. The reason for calling the 2nd smallest eigenvalue as the algebraic connectivity of network graph *G* comes from the following lemmas.

Lemma 1: If G_1 and G_2 are link disjoint network graphs with the same nodes, then

$$\lambda_2(G_1)+\lambda_2(G_2)\le\lambda_2(G_1\bigcup G_2) \quad (4)$$

Lemma 2: The algebraic connectivity $\lambda_2(G)$ is non-decreasing for graphs with the same set of nodes, i.e., $\lambda_2(G_1)\le\lambda_2(G)$, if $G_1(V,E_1)$, $G(V,E)$, and $E_1\subseteq E$.

We observe that G and G_1 have the same number of nodes. Since G_1 has fewer links compared to G and $E_1\subseteq E$, this implies that G_1 is more weakly connected than G. From Lemma 2, we have that the algebraic connectivity of G_1 smaller than that of G, i.e., $\lambda_2(G_1)\le\lambda_2(G)$. It is in this sense that the algebraic connectivity measures the degree of connectivity in a graph. In addition, the relation of algebraic connectivity for the graph obtained by removing a node and all of its adjacent links is given by the following lemma.

Lemma 3: Let G_{-v_i} be a graph obtained from removing node v_i from G and all of its adjacent links then

$$\lambda_2(G_{-v_i})\ge\lambda_2(G)-1 \quad (5)$$

According to the above lemma, we propose to quantify the importance of a node or a link based on the algebraic connectivity of the network's graph, because that the larger the algebraic connectivity of a graph is, the more connected the graph will be. Hence the degree of connectivity of the remaining graph can be quantified by the algebraic connectivity of the graph resulting from removing that particular node and all the links connected to that node from the original graph. Therefore, we can calculate the connectivity weight for each node or link. In this way, the node or link that causes more server reduction in the remaining algebraic network connectivity has higher importance and should need more protections. In addition, we can propose a principle that both working and spare capacity allocations benefit most from adding some critical nodes and links to maximize the algebraic connectivity of a current network.

III. ILP-based SBPP Model

In order to evaluate the impact of the algebraic connectivity versus the average nodal degree on capacity allocation in survivable network design, we use the shared backup path protection (SBPP) spare capacity allocation scheme. Our shared protection AMPL model is modified from [11].

Let *pF* denote the set of links whose failure disrupts working traffic for some *(o,d)* pair. For each $f\in pF$, pD_f denotes the set of *(o,d)* pairs affected by the failure of link *f*. Since the traffic for a demand pair, r_{od}, can be split among multiple paths, $q_{f(od)}$ denotes the total amount of traffic from *o* to *d* that must be restored when *f* fails.

We now define the decision variables. Let t_{pf} denote the protection traffic on path *p* when link *f* fails. Let t_e denote the total protection traffic on link *e*. Let w_e denote the working traffic and S_e denote the total traffic on link *e*. Let $J_{o,d}$ denotes the paths that can be used to satisfy demand *(o,d)* and Lp_e denotes the protection paths that use link *e*. Using these notations, the shared protection model uses three sets of constraints to determine the values for S_e. The demand for spare capacity is defined by the following constraints:

$$\sum_{p \in J_{od} \setminus Lp_f} t_{pf} \ = \ q_{f(od)} \qquad \forall f \in pF,\ (o,d) \in pD_f \qquad (6)$$

Note that a protection path containing link *f* cannot be used to protect against failures of link *f*. Conversion of protection path flows to link spare capacity requirements is accomplished by the following inequalities

$$\sum_{p \in Lp_e} t_{pf} \ \leq \ t_e \qquad \forall e \in E, f \in pF \qquad (7)$$

That is, the spare capacity must be sufficient to accommodate the failure that produces the largest traffic disruption. Provisioning of total traffic on a link is determined by the working traffic on the link plus the spare capacity on the link. The following $|E|$ = m equations provide this value

$$S_e \ = \ w_e + t_e, \qquad \forall e \in E \qquad (8)$$

The above AMPL model uses concepts of spare capacity sharing and path-based protection derived from SBPP algorithm. More details can be found in [11].

IV. SIMULATION RESULTS

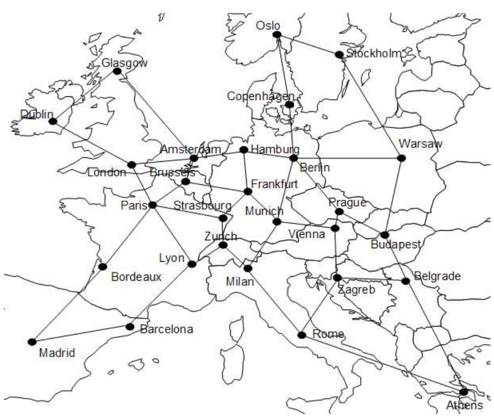

Fig.1. European Reference Network

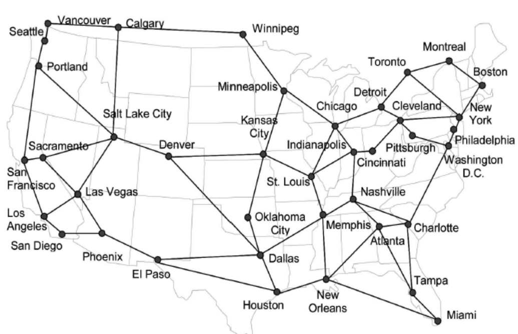

Fig.2. North-American Reference Network

Two network topologies are considered in the simulation studies. The first one is the pan-European network adopted in the project COST action 266 [12], see Figure 1. This network contains 28 nodes and 41 bidirectional links. The second network, shown in Figure 2, is based on the North-American Network [13] with 39 nodes and 61 bidirectional links. Without loss of generality, and for allowing easier comparison of results, we assume symmetrical traffic flows, i.e., one unit of bandwidth demand between any pair of nodes. The SBPP ILP model is solved using AMPL/CPLEX 11.1 [14], [15] on a PC with Intel(R) Celeron(R) 1.70GHz, 504MB of RAM.

Firstly, we investigate how the importance of each node affects the total capacity allocated. Simulations were conducted on two families of network topologies derived from the above two master networks by deleting one node at a time, together with all of its adjacent links. Afterwards, we calculate the algebraic connectivity and average nodal degree of the remaining graph. The SBPP algorithm is applied to evaluate each topology alternative, to find the optimal total capacity. Here, we ignore cases for which the SBPP cannot find feasible solutions since no node-disjoint paths exist for a traffic demand after the critical nodes have been deleted. For example, after deleting the Oslo node in Figure 1, its neighboring nodes have only one adjacent link left, so lack of solution, will be reported by the CPLEX solver. This is denoted as "impractical" in Total Capacity.

Deleted Node	Average Nodal Degree	Algebraic Connectivity	Total Capacity
Berlin	2.6667	0.0707	impractical
Paris	2.6667	0.0954	impractical
Hamburg	2.8148	0.1125	5694
Milan	2.8148	0.1128	5814
Zurich	2.8148	0.1197	impractical
Amsterdam	2.7407	0.1206	impractical
Frankfurt	2.7407	0.1297	5475
Munich	2.7407	0.1314	5616
Lyon	2.8148	0.1320	impractical
Rome	2.8148	0.1458	impractical
Warsaw	2.8148	0.1544	impractical
Copenhagen	2.8889	0.1632	impractical
Strasbourg	2.8148	0.1634	4685
Brussels	2.8148	0.1640	4724
Vienna	2.8148	0.1663	4853
Bordeaux	2.8889	0.1745	impractical
Prague	2.8148	0.1747	4572
Zagreb	2.8148	0.1761	4644
Barcelona	2.8889	0.1766	impractical
Athens	2.8889	0.1771	4671
London	2.8148	0.1773	impractical
Glasgow	2.8889	0.1774	impractical
Belgrade	2.8148	0.1787	impractical
Budapest	2.8148	0.1808	impractical
Dublin	2.8889	0.1822	impractical
Oslo	2.8889	0.1831	impractical
Stockholm	2.8889	0.1844	impractical
Madrid	2.8889	0.1891	impractical

Table 1. Total capacity, algebraic connectivity and average nodal degree after deleting specific nodes in COST266 reference network

Deleted Node	Average Nodal Degree	Algebraic Connectivity	Total Capacity
Chicago	3.0000	0.0730	12160
Minneapolis	3.0526	0.0775	impractical
El Paso	3.0526	0.0818	12403
Denver	3.0526	0.0846	11618
Salt Lake City	2.9474	0.0848	12396
Detroit	3.0526	0.0853	impractical
Charlotte	3.0000	0.0868	11254
Phoenix	3.0526	0.0877	impractical
Winnipeg	3.1053	0.0883	impractical
Calgary	3.0526	0.0888	impractical
New Orleans	3.0000	0.0910	impractical
Philadelphia	3.0526	0.0939	impractical
Dallas	2.9474	0.0956	impractical
Houston	3.0526	0.0959	10342
Kansas City	3.0000	0.0961	impractical
St. Louis	3.0000	0.0965	impractical
Memphis	3.0000	0.0978	10285
St Louis	3.0000	0.0979	10313
Cincinnati	3.1053	0.1034	10523
Cleveland	3.0000	0.1042	impractical
Nashville	3.0000	0.1056	10255
Toronto	3.0526	0.1058	impractical
Atlanta	3.0000	0.1068	9730
New York	3.0000	0.1069	impractical
Miami	3.1053	0.1090	9588
Tampa	3.0526	0.1094	impractical
Oklahoma	3.1053	0.1107	9638
Vancouver	3.1053	0.1120	impractical
Las Vegas	3.0000	0.1127	10068
Philadelphia	3.1053	0.1132	9724
San Diego	3.1053	0.1133	9594
Potland	3.0526	0.1142	impractical
Sacramento	3.0526	0.1145	9625
Pittsburgh	3.1053	0.1148	9659
Los Angeles	3.0526	0.1155	impractical
Seattle	3.1053	0.1163	impractical
San Francisco	3.0526	0.1165	9768
Montreal	3.1053	0.1169	impractical
Boston	3.1053	0.1173	impractical

Table 2 Total capacity, algebraic connectivity and average nodal degree after deleting specific nodes in North-American reference network

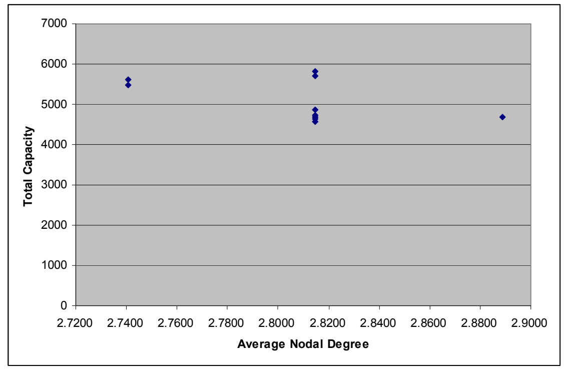

Fig.3. Total capacity vs average nodal degree after deleting specific nodes in COST266 reference network

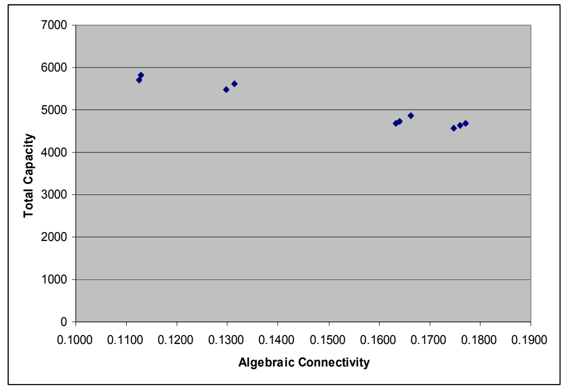

Fig.4. Total capacity vs algebraic connectivity after deleting specific nodes in COST266 reference network

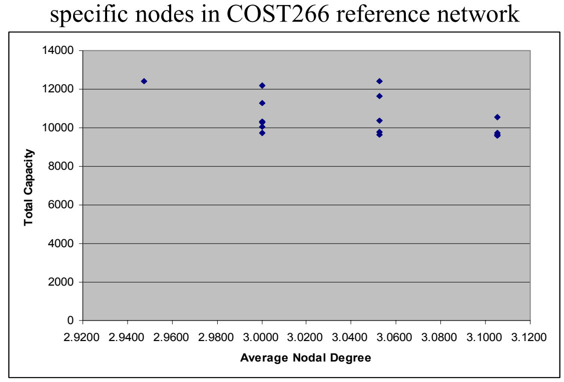

Fig.5. Total capacity vs average nodal degree after deleting specific nodes in North-American reference network

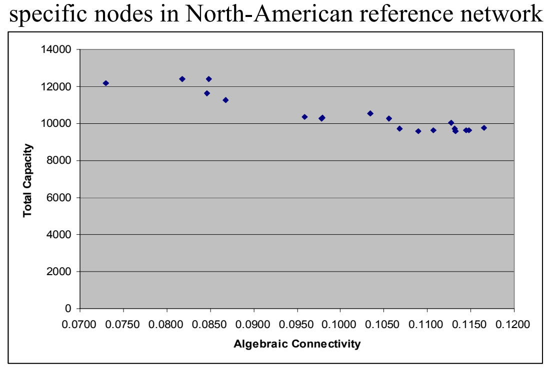

Fig.6. Total capacity vs algebraic connectivity after deleting specific nodes in North-American reference network

Tables 1 and 2 report the results on the optimal total capacity, average nodal degree and algebraic connectivity of the remaining graph after deleting specific nodes in the two reference networks. These results have been depicted in Figures 3-6, for showing how the total capacity varies with the average nodal degree and the algebraic connectivity, respectively. It is evident that total capacity is more strongly correlated with the algebraic connectivity than with the average nodal degree. There are two possible reasons for the SBPP algorithm's sensitivity to network connectivity. Firstly, as the network connectivity increases, both the predetermined working and protection path-pairs become shorter. This leads to a decrease of both working and protection capacity. Secondly, the potential for capacity sharing among protection paths is likely to increase as the network connectivity increases, and this leads to a further decrease of protection capacity

Looking at the results in detail, we can see almost linear dependence of the total capacity on the algebraic connectivity. By contrast, in the case of average nodal degree, its dependence on average nodal degree $\overline{d}$ is not monotonic. For example, in Figure 3, there are 7 different topologies with $\overline{d}$ =2.8148, while they have 7 different total capacities allocated ranging from 4572 to 5814 units. This shows that using of average nodal degree as a metric has a severe limitation as it is insensitive to the total capacity of a given topology. On the other hand, algebraic connectivity monotonically depends on total capacity of a given topological structure. We also can see that, if the traffic demands are uniform, the nodes in the core region e.g., Hamburg, Milan, Frankfurt and Munich, are more important than others, because they are more frequently being used by traffic flows. If any of these nodes is deleted, it will result in a severe reduction of algebraic connectivity. The similar phenomenon can be observed in the results obtained for North-American network: see Figures 5 and 6.

Further experiments have been carried out to analyze the properties of the algebraic connectivity metric and average nodal degree taking into account only slightly modified topological scenarios. We investigate how the importance of each link affects the total capacity allocated. Following the similar mechanism mentioned above, simulations were conducted on two families of network topologies derived from our two reference networks by deleting one link at a time. Here, we ignore cases for which the SBPP cannot find practical solutions since no node-disjoint paths existed for a given traffic demand after the critical links have been deleted, e.g., if the link between Oslo and Stockholm is deleted, see Figure 1. The simulation results are shown in Tables 3-4 and Figures 7-8.

Deleted Link		Algebraic Connectivity	Total Capacity
Hamburg	Berlin	0.1176	6110
Zurich	Milan	0.1198	6015
Copenhagen	Hamburg	0.1258	impractical
Bordeaux	Paris	0.1309	impractical
Amsterdam	Hamburg	0.1361	5402
Milan	Rome	0.1364	5778
Barcelona	Lyon	0.1401	impractical
Frankfurt	Munich	0.1402	5680
Lyon	Zurich	0.1449	impractical
Stockholm	Warsaw	0.1527	impractical
Munich	Vienna	0.1589	5545
Berlin	Warsaw	0.1589	4969
Brussels	Frankfurt	0.1591	4934
Glasgow	Amsterdam	0.1595	impractical
Oslo	Copenhagen	0.1611	impractical
Frankfurt	Strasbourg	0.1635	5030
Paris	Strasbourg	0.1639	4906
Berlin	Munich	0.1646	5029
Rome	Athens	0.1663	impractical
Paris	Brussels	0.1680	4922
Madrid	Bordeaux	0.1687	impractical
London	Amsterdam	0.1689	4911
Berlin	Prague	0.1707	4930
Madrid	Barcelona	0.1709	impractical
Prague	Budapest	0.1714	4909
Zagreb	Rome	0.1719	4965
Munich	Milan	0.1722	5091
Vienna	Zagreb	0.1722	4939
Hamburg	Frankfurt	0.1723	4855
Dublin	London	0.1724	impractical
Zagreb	Belgrade	0.1727	4828
Prague	Vienna	0.1729	5095
Strasbourg	Zurich	0.1733	4886
Dublin	Glasgow	0.1735	impractical
Belgrade	Athens	0.1741	impractical
Budapest	Belgrade	0.1747	5307
Amsterdam	Brussels	0.1748	5026
Paris	Lyon	0.1748	4918
London	Paris	0.1748	impractical
Warsaw	Budapest	0.1748	impractical
Oslo	Stockholm	0.1750	impractical

Table 3 Total capacity, algebraic connectivity and average nodal degree after deleting specific links in COST266 reference network

Deleted Link		Algebraic connectivity	Total capacity
Phoenix	EI Paso	0.0821	13106
Winnieg	Minneapolis	0.0826	impractical
Chicago	Detroit	0.0828	12159
Salt Lake City	Denver	0.0847	12162
Washington D.C.	Charlotte	0.0872	11901
Calgary	Winnipeg	0.0880	impractical
Minneapolis	Chicago	0.0897	11604
Houston	New Orleans	0.0970	11062
Indianapolis	Cincinnati	0.0976	impractical
Dallas	Memphis	0.0995	10598
Detroit	Toronto	0.0995	impractical
Kansas City	St. Louis	0.1006	10612
Vancouver	Calgary	0.1016	impractical
Denver	Kansas City	0.1019	10410
Cincinnati	Cleveland	0.1032	impractical
EI Paso	Houston	0.1035	10525
Philadelphia	Washington D.C.	0.1049	impractical
Memphis	Nashville	0.1055	10305
San Diego	Phoenix	0.1058	impractical
Denver	Dallas	0.1059	10312
New Orleans	Atlanta	0.1065	10305
St. Louis	Indianapolis	0.1066	10628
Cleveland	New York	0.1067	10582
EI Paso	Dallas	0.1068	10558
Portland	Salt Lake City	0.1070	10470
Toronto	Montreal	0.1070	impractical
New Orleans	Miami	0.1075	impractical
Pittsburgh	Washington D.C.	0.1076	impractical
Las Vegas	Phoenix	0.1080	10331
Nashville	Charlotte	0.1087	10624
Sacramento	Salt Lake City	0.1090	10354
Chicago	St. Louis	0.1090	10199
Tampa	Miami	0.1090	impractical
Atlanta	Charlotte	0.1091	10214
Vancouver	Seattle	0.1092	impractical
Detroit	Cleveland	0.1095	10206
Charlotte	Tampa	0.1097	10348
Los Angeles	Las Vegas	0.1100	10206
Las Vegas	Salt Lake City	0.1100	10281
Los Angeles	San Diego	0.1101	impractical
New York	Philadelphia	0.1101	impractical
San Francisco	Sacramento	0.1102	10359
Chicago	Indianapolis	0.1102	10342
Seattle	Potland	0.1102	impractical
Calgary	Salt Lake City	0.1103	10621
Dallas	Houston	0.1104	10093
St. Louis	Memphis	0.1104	10291
Kansas City	St. Louis	0.1104	impractical
Sacramento	Las Vegas	0.1105	10409
Cleveland	Pittsburgh	0.1105	impractical
Cleveland	New York	0.1105	impractical
San Francisco	Los Angeles	0.1106	10455
Minneapolis	Kansas City	0.1106	10320
Memphis	New Orleans	0.1106	10172
Indianapolis	Nashville	0.1106	10232
Atlanta	Tampa	0.1106	10358
Oklahoma City	Dallas	0.1106	impractical
Portland	San Francisco	0.1107	10348
Nashville	Atlanta	0.1107	10377
Toronto	New York	0.1107	10199
Montreal	Boston	0.1107	impractical

Table 4 Total capacity, algebraic connectivity and average nodal degree after deleting specific links in North-American reference network

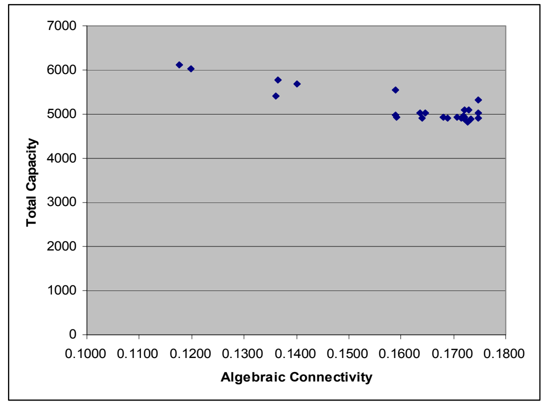

Fig. 7. Total capacity vs algebraic connectivity after deleting specific links in COST266 reference network

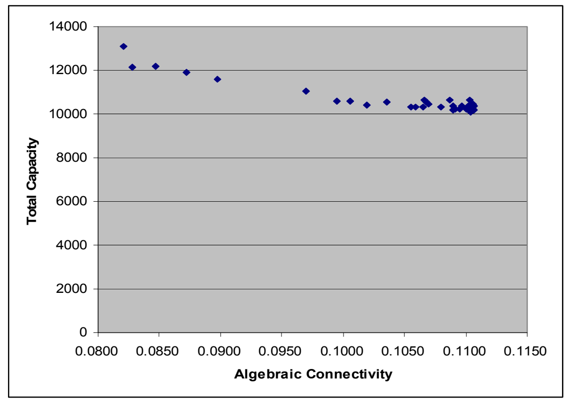

Fig. 8. Total capacity vs algebraic connectivity after deleting specific links in North-American reference network

From these results, one can see that the average nodal degree in two families of network topologies assumes constant value of $\overline{d}$ =2.8571 in COST 266 topologies with 40 links, and the value of $\overline{d}$ =3.0769 in North-American topologies with 60 links, respectively, while the total capacity solutions are significantly different. There are 25 solutions, with the total capacity ranging from 4828 to 6110 units in COST266 scenarios, and 40 solutions with the total capacity ranging from 10093 to 13106 units in North-American scenarios. This shows again that the average nodal degree has a severe limitation as it is insensitive to changes in total capacity caused by removals of single links. The algebraic connectivity remains sensitive to such changes. In addition, it can be seen that the links located in the network's core region are more important than those the network boundaries since they are more frequently used by the traffic flows. Thus deleting them cause severe decrease in network connectivity.

Additionally, we investigated the impact of algebraic connectivity metric and average nodal degree has on capacity allocation under links' repositions scenario. Seven sample networks derived from COST266 reference network by placing 4 links in different positions have been explored, see Figures 10-16 in the Appendix.

Topology	Algebraic Connectivity	Total Capacity
COST266-1	0.122	5794
COST266-2	0.126	5820
COST266-3	0.164	4951
COST266-4	0.175	4778
COST266-5	0.181	4925
COST266-6	0.187	4751
COST266-7	0.198	4571

Table 5 Total capacity vs algebraic connectivity for links repositions in COST266 reference network

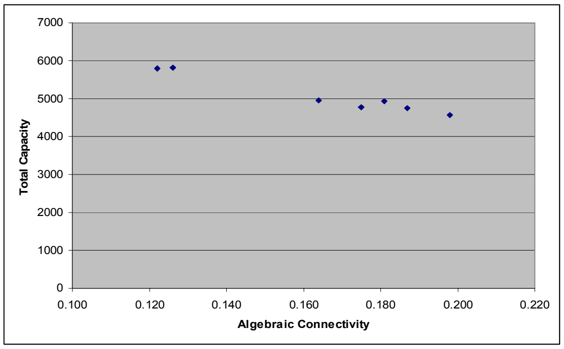

Fig.9. Total capacity vs algebraic connectivity for links repositions in COST266 reference network

As shown in Table 5 and Figure 9, while all the seven derived topologies have the same average nodal degree, i.e., $\bar{d}$ =2.9286, the resulted total capacity values are quite different from each of them. Note that total capacity decreases as algebraic connectivity increases. One can see that when four links are placed on the boundary of the network, see e.g., Figure 10 and 11, the total capacity is generally larger than deploying the links in the core region of the network, cf. Figures 15 and 16, because boundary links are less used in the SBPP solutions.

V. CONCLUSIONS AND FUTURE WORK

In this paper, we argue that the use of average nodal degree of a network for describing its connectivity is not sufficient on its own. We suggest using a more informative metric: the algebraic connectivity of the network, as it is a better numerical characteristic of a given topology and its dependence on the key network connectivity property. In general, a larger algebraic connectivity means better network connectivity i.e., more node- and link-disjoint paths exist and can be chosen from by pairs of communicating nodes, and so less network capacity need to be allocated.

More extensive studies on how the algebraic connectivity affects the amount of spare capacity to be allocated in more complex topologies are underway. A composite metric integrating the algebraic connectivity and network mean distance is considered to compare different types of topologies. Furthermore, capacitated versions of networks need to be studied, taking into account the fact that the network may have existing link capacities and/or link capacity limits to be respected with different traffic scenarios, which is also essential in the future work.

VI. ACKNOWLEDGEMENT

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APENDDIX

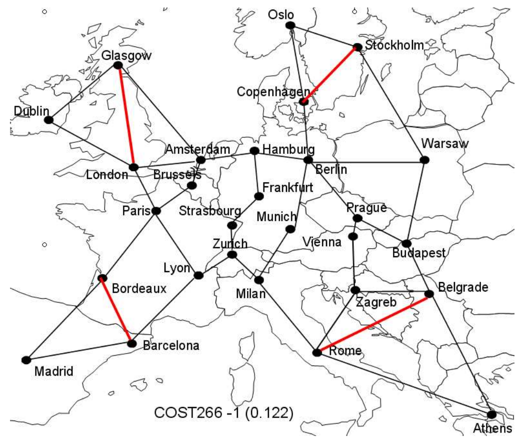

Fig.10. COST266-1

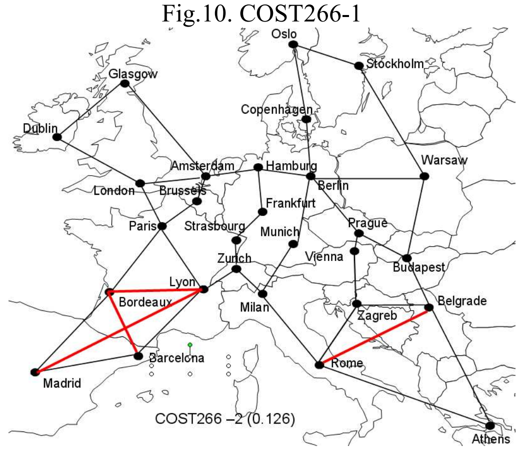

Fig.11. COST266-2

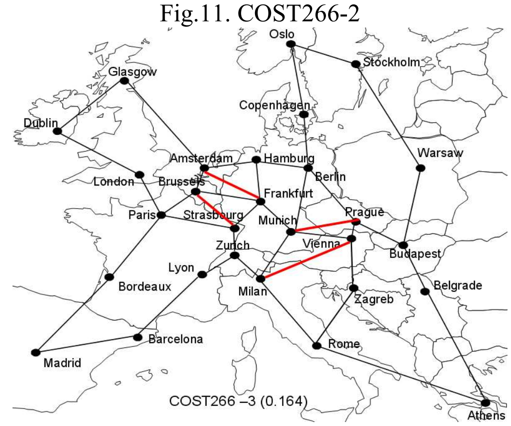

Fig.12. COST266-3

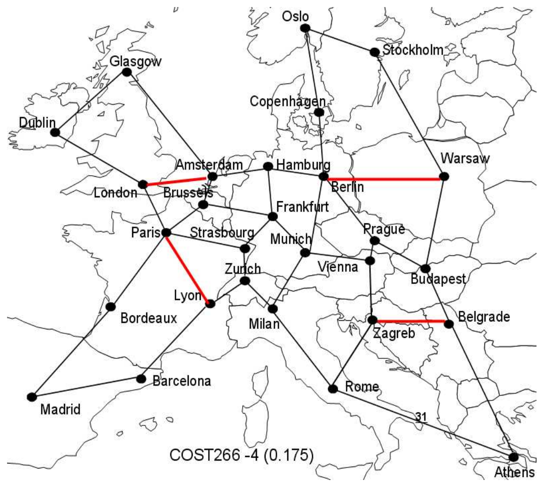

Fig.13. COST266-4

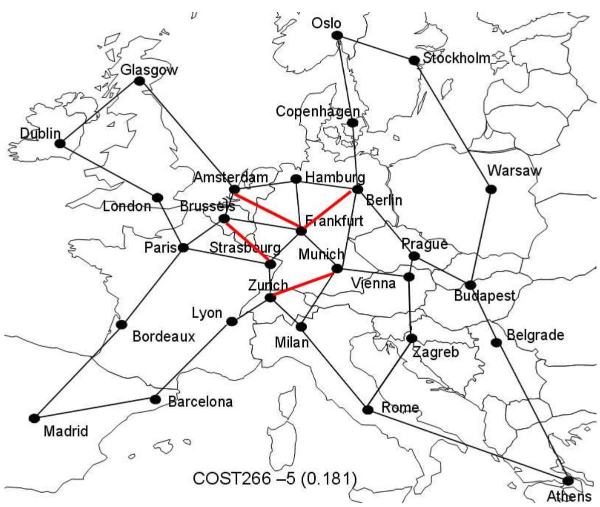

Fig.14. COST266-5

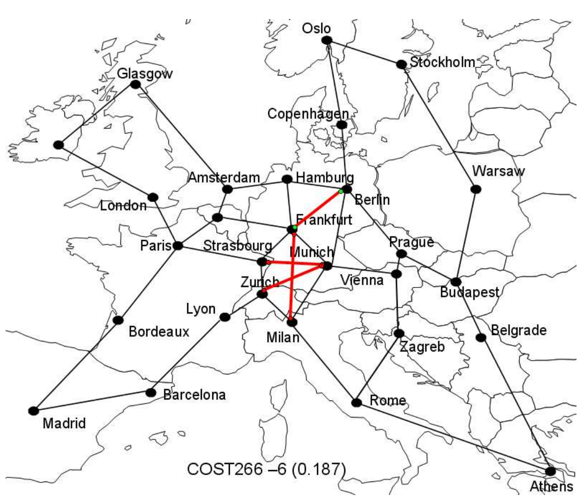

Fig.15. COST266-6

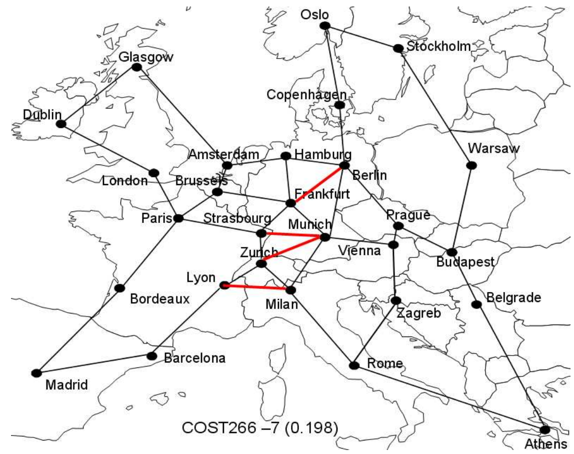

Fig.16. COST266-7

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