Equivalent granular void ratio and state parameters for loose clean sand with small amount of fines

Md. Mizanur Rahman
University of New South Wales, ADFA campus, Canberra, Australia
Sik-Cheung Robert Lo
University of New South Wales, ADFA campus, Canberra, Australia

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ABSTRACT

Recent studies suggest that the undrained liquefaction behaviour of sand with fines is more complicated than clean sand, and their interpretation under Critical State Soil Mechanics (CSSM) is a challenge. It is also reported that void ratio is not a good state variable for sand with fines. The inactive contribution of fines to sand microstructure is responsible for the anomaly. To resolve this problem, it is proposed to use equivalent granular void ratio instead of void ratio. It is found that a unique Steady State Line (SSL) called equivalent granular SSL can be achieved for sand with fines by using of the equivalent granular void ratio. This equivalent granular SSL was used as the reference line to calculate state parameters. It is found that these state parameters can successfully predict stress path responses of sand with fines in undrained shearing.

1 INTRODUCTION

Void ratio is used as a state variable in Critical State Soil Mechanics (CSSM) in the modelling of soil behaviour. Along this line of thinking, Been and Jefferies (1985), in their study to predict Kogyuk sand behaviour, defined a state parameter, ψ, as the difference between the current void ratio and the void ratio on the Steady State Line at the same mean effective pressure. This state parameter is often used to predict liquefaction behaviour under CSSM. Later, a modified state parameter (Bobei and Lo 2005) has also been proposed for better prediction of liquefaction behaviour.

However, adopting void ratio as the state variable for sand with fines for liquefaction analysis becomes problematic. It is found that void ratio is no longer a suitable state variable of stress-strain response. A small amount of fines in sand decreases its void ratio but the stress-strain behaviour essentially remains the same. This leads to considerable scatter of the Steady State (SS) points in the e-log(p') space. Were, p' is the mean effective confining pressure.

The main reason for such behaviour is the inactive contribution of fines to the sand force structure. Sand force structure is the skeleton of sand particle that actively participate in transferring the normal force or sustain significant shear force. To represent this active contact-index among particles, it is proposed to use intergranular void ratio instead of void ratio within the CSSM framework to achieve a narrow trend of Steady State Line (SSL)/Critical State Line (CSL). Thevanayagam (2000) proposed the use of equivalent granular void ratio for relatively higher fines content. The concept of equivalent granular void ratio is an extension of the intergranular void for higher fines content in sand-fines mix. An additional parameter "*b*" is incorporated in the definition of equivalent granular void ratio so as to present the fraction of fines participating on force structure at higher fines content. However, the determination of the value of "*b*" is a controversial topic in the literature. Recently, Rahman and Lo (2007) presented a semi-empirical equation for "*b*" from the theoretical concept and experimental results on particle packing. It is found that SS points for sand with fines content come to a narrow band when "*b*" is calculated from the equation.

There is an obvious question: can we use the unique Equivalent Granular SSL in conjunction with the state parameter along the line of Been and Jefferies (1985) to predict the undrained shear strength behaviour within the CSSM framework? The authors have not been able to locate enough data in published literature to answer this question. The objective of this paper is to demonstrate that a unique Equivalent Granular SSL is obtained for Sydney sand with low-plasticity fines, and this unique equivalent granular SSL can be used to calculate state parameters for predicting undrained behaviour.

2 LITERATURE REVIEW

Ongoing development in soil mechanics has been found on or related to the concept of "state parameter". More recently, the "equivalent granular void ratio" has been proposed for interpreting the undrained behaviour of sand with fines. The detailed development of these concepts will be discussed below in order to combine these two concepts for better understanding of soil behaviour.

2.1 State Parameters

Schofield and Worth (1968) took critical states as a reference base for soil behaviour in effective pressure versus specific volume space. If the soil is looser than the critical state (above CSL), it will be contractive and if the soil state is denser than critical state (below CSL), it will be dilative. Later Polous (1981) proposed use of the steady state line instead of the critical state line. Recent literature (Li et al. 1999; Bobei and Lo 2005) showed that the CS line or SS line is not necessarily linear in the e-log(p'), and thus the possible curvature needs to be borne in mind, despite use of the word "line". Bobei and Lo (2005) reported that there is no theoretical distinction has been found between these two states, other than wording. Traditionally, critical states are measured from a drained test whereas steady states are measured from an undrained test, and therefore have different intrinsic errors. They highlighted that extremely strict experimental control is needed to reliably determine CS or SS data points in the e-log(p') space, and different intrinsic errors between drained and undrained tests may lead to an apparent difference between CSL and SSL. Therefore, CS and SS will be considered as equivalent in this paper.

Been and Jefferies (1985) chose SSL as a reference line to predict the undrained behaviour of soil. They realized soil behaviour was not only depend on its void ratio but also on stress level. Thus, combining the influence of void ratio and the stress level with the SSL, they proposed a "state parameter", ψ, defined by Eqn. (1) below:

$$\psi = e - e_s \tag{1}$$

where, ψ = state parameter, e = current void ratio of the sample, e_s = void ratio at SS for same effective confining pressure. A positive value of ψ indicates contractive tendency (limited liquefaction to liquefaction) of soil and negative value indicates dilation tendency (non-liquefaction) during undrained shearing.

Later, Wang et al. (2002) related mean effective pressure, p' to effective pressure at critical state, p'_{cr} by proposing a pressure index, I_p as:

$$I_p = \frac{p'}{p'_{cr}} \tag{2}$$

where, p' = current mean effective pressure and p'_{cr} = mean pressure at critical state at current void ratio, e. This parameter can reflect the effect of a curved SSL (Li et al. 1999).

If the SSL line is linear in the e-log(p') space, ψ and I_p are related. However, Bobei and Lo (2005) realized that for a curved SSL, the state parameter related to only void ratio or effective pressure is not enough to capture the real behaviour of the undrained shear test. Thus, they proposed a modified state parameter, ψ_m by combining the state parameter and pressure index defined as:

$$\psi_m = \psi \left| 1 - {}^{1}\!/_{I_p} \right|_{e_o} \tag{3}$$

2.2 Equivalent granular void ratio

Non-active contribution of fines is believed to have been first reported by Mitchell (1976). Later, Kenney (1977) also reported the inactive contribution of fines. However, Kuerbis et al. (1988) introduced the concept of intergranular void ratio by neglecting fines in the sand as they are inactive in the force structure. Georgiannou et al. (1990) also proposed essentially the same concept. Later, Thevanayagam (1998) simplified the idea and defined intergranular void ratio as:

$$e_g = \frac{e + f_c}{1 - f_c} \tag{4}$$

where, e_g = intergranular void ratio, f_c = fines content in decimal. Thevanayagam found that the SS points related to sand with non-plastic fines are located close to a single SSL when plotted in e_g-log(p') space.

With increasing fines content, it is realized that some fraction of fines may come in between the contact of sand grains and participate in the force structure of the solid skeleton. Therefore, a more general concept is to consider a fraction of fines actively contributing to the force structure. To take into account such a mechanism, Thevanayagam (2000) proposed the concept of equivalent granular void ratio, e_c defined as:

$$e_c = \frac{e + (1-b) f_c}{1 - (1-b) f_c} \tag{5}$$

where, b = fraction of fines which actively takes part in the sand force structure. Existing literature showed that for a given set of test data, a "b" value can be selected for Eqn. (5). But there is no literature available on prediction formula for "b".

A detailed synthesis of the relevant literatures and experimental results on binary particle packings, Rahman and Lo (2007) reported that "b" can be presented as a function of particles size ratio (r), percentage of fines (f_c) and the threshold fines content (f_{thre})by the following equation:

$$b = \left(1 - e^{-n(f_c)^n / k}\right) \cdot \left(\frac{r . f_c}{f_{thre}}\right)^r \tag{6}$$

where, r = (d_{50}/ D_{10}), D_{10} = sand particle size at 10% finer, d_{50} = fines size at 50% finer, n is an empirical constant determined to be 1.5, k = ($1-r^{0.25}$), f_{thre} = threshold fines content. Threshold fines content is the fines content where the behaviour of sand-fines mixture changes from fines in sand to sand in fines and it can be inferred from the reversal of sand-fines behaviour with increasing fines content. The Eqn. (5) is applicable for small fines content $f_c < f_{thre}$, so that the mixture remains as fines in sand.

Thus, the equivalent granular void ratio can be calculated using Eqn. (5) and (6), instead of being selected after a comprehensive analysis of extensive test data on Steady State or cyclic mobility responses for a range of fines content. This approach was tested on a wide range of sand with fines, with data from nine separate publications. Rahman and Lo (2007) reported that equivalent granular void ratio calculated from the proposed equations yielded a unique steady state line in the e_c-log(p') space or a unique correlation between cyclic resistance and e_c.

The concept of equivalent granular void ratio, recently, has been successfully applied to explain the behaviour of sand with fines. However, its integration into the CSSM framework based on state parameters is still limited. In the next section, equivalent granular void ratio will be used to calculate state parameters of sand with fines and these state parameters will be referred as equivalent granular state parameters. The unique relationship of SS points in the e_c-log(p') space is referred to as the equivalent granular Steady State Line.

3 TESTING PROGRAM

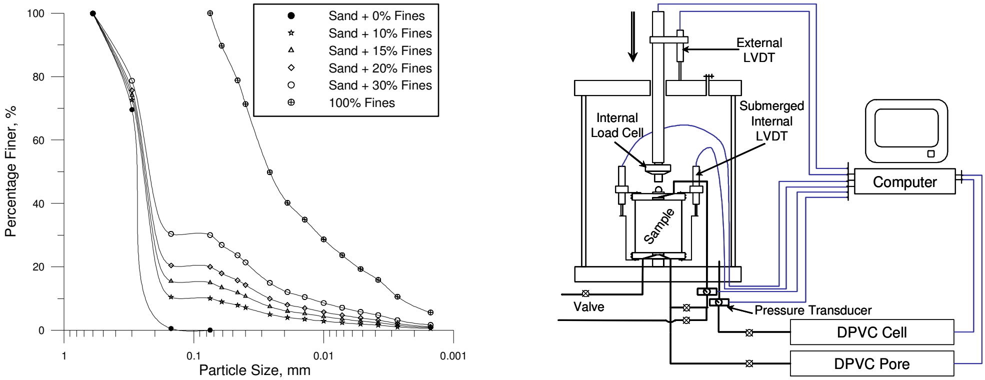
A series of experiments have been done on clean sand and sand with fines (0%, 15%, 20%, and 30%) to study the effect of fines on soil behaviour. The overall behaviour of clean sand and sand with fines is then explained under CSSM using the equivalent granular Steady State Line.

3.1 Material tested

Sydney sand, a medium quartz sand is a clean sand (SP) and it's index properties can be found in Lo et al. (1989). The fines used is a specially designed low plasticity fines (PI=11, LL=28) with a uniformity coefficient 12.56. It is composed of 2/3 of well graded silt from the Majura River and 1/3 commercial kaolin. The grain size distribution curves of sand, fines and sand-fines mixture are shown in Fig. 1a.

3.2 Experiment procedures

A strain controlled triaxial loading system with fully automated data logging facilities was used for this study. Axial load was measured with an internal load cell. The axial deformation was measured by a pair of internal LVDTs mounted directly across the top platen and an external LVDT. The former was used in the early stage of shearing whereas the latter was used at large deformation. Cell pressure was controlled by a large capacity Digital Pressure Volume Controller (DPVC). The pore pressure line was connected to a small capacity DPVC for controlling back pressure (and measuring the volume change) at the consolidation stage and for imposing an undrained condition and measuring the resultant pore pressure response. Two pressure transducers were also used to crosscheck the pressure measurement of the DPVC. The detailed schematic diagram of the testing program is shown in Fig. 1b.



(a) (b)

Figure 1: (a) Grain size distribution curve of Sydney sand with fines, (b) Schematic diagram of testing program

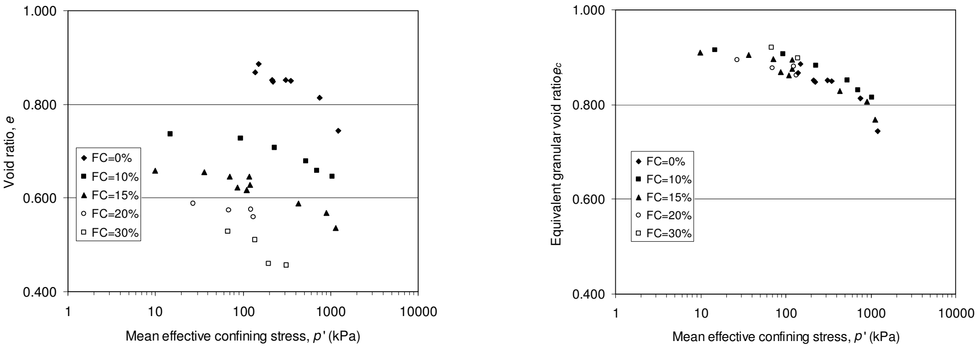
4 RESULTS AND DISCUSSIONS

Test results unambiguously showed that the SSL for sand with fines is a curved line in *e*-log(*p*′) space. This finding is consistent with recent literature (Wang et al. 2002; Bobei and Lo 2005). As shown in Fig. 2a, the SSL in *e*-log(*p*′) space moved downward with an increase in fines. However, if the SS points are plotted in e_c-log(p') space, an essentially unique relationship independent of fines content is obtained as shown in Figure 2b. Since the equivalent granular void ratio was calculated from Eqns. (5) and (6), this provides validation of Eqn. (5) in addition to those presented in Rahman and Lo (2007). Note that, the data for 10% fines was taken from Bobei and Lo (2005).

The unique equivalent granular SS curve can be modelled by the following equation proposed by Wang et al. (2002) for sand

$$e_c = 0.91 - 0.025\times\left(\frac{p'}{p_a}\right)^{0.7} \tag{7}$$

where, p_a is atmospheric pressure = 100kPa. The average deviation of equivalent granular void ratio from the best fit equation is only 0.013. Therefore, equivalent granular SSL for sand with fines will be used as the fundamental relationship under CSSM with the equivalent granular void ratio representing the soil grains that are active in the force structure.



(a) (b)

Figure 2: (a) Multiple SSLs for sand with fine,
(b) Equivalent granular SSL for sand with fines

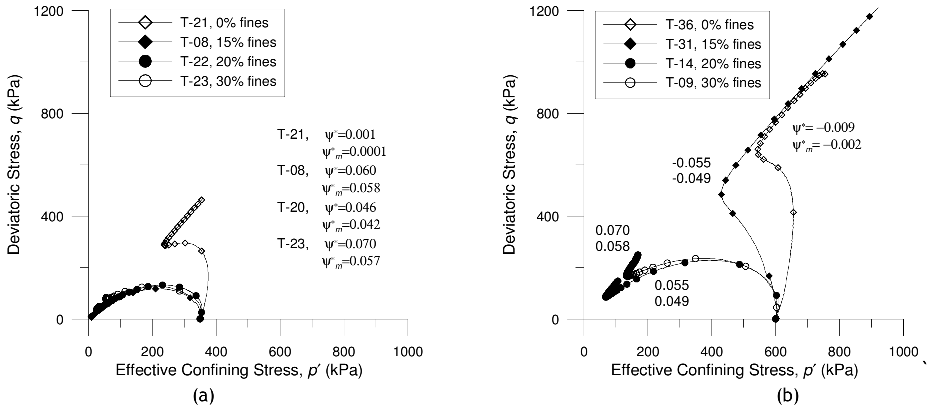
Considering equivalent granular SSL given by Eqn. (7) as the fundamental relationship, state parameters as defined in Been and Jefferies (1985) can be extended to an equivalent granular state parameter, ψ^* defined as:

$$\psi^* = e_c - e_{cs} \tag{8}$$

where, e_c = equivalent granular void ratio at current state and e_{cs} = equivalent granular void ratio at SS. On the same line of thinking, the modified state parameter as proposed by Bobei and Lo (2005) can be extended to a modified equivalent granular state parameter, ψ^*_m, define as:

$$\psi^*{}_m = \psi^* \left| 1 - \frac{1}{I_p} \right| e_c \tag{9}$$

To investigate the validity of the above concepts, the relationship between the equivalent granular state parameters and undrained response of sand with fines are studied. Preliminary results are presented below.



(a) (b)

Figure 3: (a) EG state parameters and q-p' response at p'_o= 350kPa
(b) EG state parameters and q-p' response at p'_o=600kPa

The effective stress path responses of eight tests are compared in Figure 3a & 3b. Despite both clean sand and sand with fines being included in the comparison, a clear relationship with ψ^* or ψ^*_m can be identified. In Figure 3a, T-08, T22 and T-23 had similar positive ψ^* or ψ^*_m values and manifested similar stress path responses. T-21 showed higher deviator response and that was captured by lower value of ψ^* or ψ^*_m. In Figure 3b, T-36 and T-31 had negative equivalent granular state parameters, ψ^* or ψ^*_m and both tests manifested non-flow behaviour with phase transformation points during undrained shearing. Thus, one can conclude that the concept of equivalent granular void ratio can be used for calculating equivalent granular state parameters for predicting the behaviour pattern of both sand and sand with fines behaviour under a single framework on CSSM.

5 CONCLUSIONS

The challenges in unifying the behaviour of sand with fines under the CSSM framework are examined and an experimental investigation has been done on the undrained behaviour of sand with a small amount of low plasticity fines ($f_c<f_{thre}$) so that sand-fines mix remains as fines in sand. The findings are:

- The proposed equation for predicting "*b*" can be used to calculate the equivalent granular void ratio of sand with fines. The SS points in the equivalent granular e_c-log(p') space follow a unique SSL. This unique SSL is referred to as the equivalent granular SSL.
- The equivalent granular SS curve can be used as the fundamental relationship so that equivalent granular state parameters can be defined.
- Equivalent granular state parameters so defined can be used to predict the behaviour pattern (flow, non-flow) of sand with fines under CSSM.

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