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Effective Stress Nonlinear Model Parameters and Simulation of Stress-Strain for Expansive Soil

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Abstract

An experimental investigation of the stress-strain and volume change behavior of normally consolidated expansive (Barind) soil characteristics are studied using a computer controlled triaxial cell. This paper presents a procedure for estimating the effective nonlinear stress-strain parameters for Barind soils from the results of triaxial, consolidation and direct shear tests. In this study compacted soil is tested to different stress level. Since field tests are very costly, it is essential to develop suitable model parameters to predict the behavior of Barind soils based on the triaxial, consolidation and direct shear tests. From the results of the drained triaxial compression tests, the Young's modulus and modulus exponent, defined for their dependences on shear stress level and strain rate were quantified. A procedure for estimating the bulk modulus number, the bulk modulus exponent and the unload-reload modulus number is also presented. Finally, a comparison is made between the predictions and experimental results using model constants and the predictions are found to be satisfactory.

Keywords: expansive soil, triaxial test, effective stress, non-linear parameters

1 Introduction

Mathematical models that describe the behaviour of materials are called constitutive laws or models. Development of a viable constitutive law for successful implementation in numerical solution techniques consist of five main steps: (a) mathematical formulation, (b) identification of significant parameters, (c) determination of parameters and verification, (d) successful prediction of observed data from which the parameters were determined, and (e) satisfactory comparisons between predicted and measured response. Once a mathematical constitutive law consistent with the physical laws is derived, it is necessary to identify and choose all significant parameters that are needed to define it. Then, it is essential to perform appropriate tests to evaluate the model parameters. This is an extremely important step and comprehensive consideration should be given to this step. The increasing demand seen today for a better capability in design and analysis in geotechnical engineering practice are reflected in the corresponding requirements for more accurate predictions of the performance of soils under load. The hyperbolic stress-strain relationships (Duncan and Chang, 1970) were developed for use in non-linear incremental analyses of soil deformations. The nonlinear parameters can be determined from conventional triaxial tests and an extensive database of total and effective stress nonlinear (hyperbolic) parameters has been developed by Duncan et al. (1980). The model has been successfully applied to open excavations, braced excavations, embankment, dams and a variety of soil-structure interaction problems (Duncan et al. 1990; Mana and Clough 1981; Seed and Duncan 1986). In this study, the stress-strain and volume change behavior of normally consolidated expansive (Barind) soil characteristics are studied using a computer controlled triaxial machine. This paper presents a procedure for estimating the effective nonlinear stress-strain parameters for expansive soils from the results of triaxial, consolidation and direct shear tests.

2 Assessments of Nonlinear Model Parameters

Kondner (1963) suggested that the stress-strain curves for a number of soils could be approximated reasonably accurate by hyperbolas as shown in Figure 1(a). This hyperbola can be represented by the following equation:

$$(\sigma_1' - \sigma_3') = \sigma_d' = \frac{\varepsilon_a}{\dfrac{1}{E_i} + \dfrac{\varepsilon_a}{(\sigma_1' - \sigma_3')_{ult}}} \tag{1}$$

where $(\sigma'_1-\sigma'_3)$ is the incremental increase of deviatoric stress, ε_a is the axial strain, E_i is the initial tangent modulus and $(\sigma'_1-\sigma'_3)_{ult}$ is the asymptotic value of stress difference. If the hyperbolic equation is transformed as shown in Figure 1(b), it represents a linear relationship between $\varepsilon_a/(\sigma'_1-\sigma'_3)$ and ε_a.

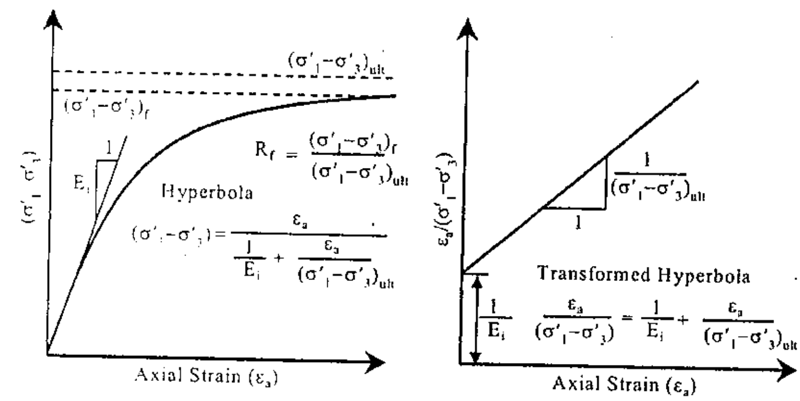

Figure 1. Schematic representation of non-linear model (a) hyperbolic equation and curve (b) transformed hyperbola

The stress-dependency is taken into account by using empirical equations to represent the variations of E_i and $(\sigma'_1-\sigma'_3)_{ult}$ with confining pressure. The variation of E_i with σ'_3 is represented by an equation of the following form, which was suggested by Janbu (1963):

$$E_i = KP_a\left(\frac{\sigma_3'}{P_a}\right)^n \tag{2}$$

where K is the modulus number, n is the modulus exponent number and P_a is the atmospheric pressure. The values of K and n are the same for any system of units, and the units E_i are the same as the units of P_a. The different constants in above equation are obtained by conducting triaxial tests on Barind soil at varying consolidation pressures. The values of K and n are determined by plotting the experimental data of E_i/P_a and σ'_3/P_a on a log-log scale. The modulus number equals (E_i/P_a) at a value of (σ'_3/P_a) equal to one and n is the slope of the resulting line.

The variation of $(\sigma'_1-\sigma'_3)_{ult}$ with σ'_3 is accounted by relating $(\sigma'_1-\sigma'_3)_{ult}$ to the stress difference at failure, $(\sigma'_1-\sigma'_3)_f$, and then using the Mohr-Coulomb strength equation to relate to σ'_3. The values of $(\sigma'_1-\sigma'_3)_{ult}$ and $(\sigma'_1-\sigma'_3)_f$ can be expressed as:

$$(\sigma'_1-\sigma'_3)_f = R_f(\sigma'_1-\sigma'_3)_{ult} \tag{3}$$

where R_f is the failure stress ratio as shown in Figure 1. The value of R_f is always less than or equal to 1.0 and varies from 0.5 to 1.0 for most soils.

The variation of $(\sigma'_1-\sigma'_3)_f$ with σ'_3 is represented by the familiar Mohr-Coulomb strength relationship, which can be expressed as:

$$(\sigma'_1-\sigma'_3)_f = \frac{2c'\cos\phi' + 2\sigma'_3\sin\phi'}{1-\sin\phi'} \tag{4}$$

where c′ is the cohesion intercept, and ϕ' is the friction angle of the soil. The instantaneous slope of the stress-strain curve is the tangent modulus, E_t, is obtained by differentiating Eq 1 with respect of axial strain ε_a and substituting the expressions of Eqs 2, 3, and 4 into the resulting expression for E_t, the following equation can be derived:

$$E_t = \left[1 - \frac{R_f(1-\sin\phi')(\sigma'_1-\sigma'_3)}{2c'\cos\phi' + 2\sigma'_3\sin\phi'}\right]^2 KP_a\left(\frac{\sigma'_3}{P_a}\right)^n \tag{5}$$

This expression can be used to calculate the appropriate value of tangent modulus for any stress conditions if the values of the parameters K, n, c′, ϕ', and R_f are known. The nonlinear stress-strain model accounts for the nonlinear volume change behavior of soils by assuming that the bulk modulus is independent of stress level, $(\sigma'_1-\sigma'_3)$, and that it varied with confining cell pressure (Duncan et al. 1980). The variation of B with confining pressure can be expressed in the following form:

$$B = \frac{(\sigma'_1+\sigma'_2+\sigma'_3)}{3\varepsilon_v} \tag{6}$$

where σ'_1, σ'_2, σ'_3 are the principal stresses, and ε_v is the volumetric strain corresponding to the stress condition. Also, the bulk modulus may be found by an empirical equation if the same soil is tested at various confining pressures, as follows:

$$B = K_bP_a\left(\frac{\sigma'_3}{P_a}\right)^m \tag{7}$$

where B is the bulk modulus, K_b is the bulk modulus number, m is the bulk modulus exponent, and P_a is the atmospheric pressure. The variation of *B* is linear when the logarithm of (B/P_a) and (σ'_3/P_a) are plotted against each other. The bulk modulus number equals (B/P_a) at a value of (σ'_3/P_a) equal to one and m is the slope of the resulting line.

In the hyperbolic relationships, the same value of unloading-reloading tangent modulus, E_{ur}, is used for both unloading and reloading. The value of E_{ur} is related to the confining pressure and can be expressed as:

$$E_{ur} = K_{ur} P_a \left(\frac{\sigma'_3}{P_a} \right)^n \tag{8}$$

where K_{ur} is the unloading-reloading modulus number. The value of K_{ur} is always greater than the value of K. Duncan et al. (1980) suggested that the value of K_{ur} may be 1.2 times of K value for stiff soil and for soft soils, K_{ur} may be three times as large. The value of the exponent n is always very similar for primary loading and unloading, and in the hyperbolic relationships it is assumed to be the same.

3 Effective Stress Nonlinear Stress-Strain Parameters

Clough and Duncan (1969) developed a procedure for estimating the effective stress hyperbolic stress-strain parameters for clays using the results of consolidation and direct shear tests. The direct shear tests are used to determine the effective stress cohesion c', and friction angle, ϕ'. Clough and Duncan (1969) derived the following expression for determining the initial tangent modulus, E_i, in terms of E_t for any load increment in a consolidation test

$$E_i = \frac{E_t}{\left[1 - \frac{R_f(1-K_o)}{K_o\left\{\tan^2(45+\phi'/2)-1\right\}}\right]^2} \tag{9}$$

where K_o is the coefficient of earth pressure at rest. The major assumption incorporated into Eq 9 is that c' equals zero, which is a typical value for normally consolidated soils. Values of (E_i/P_a) and (σ_3/P_a) for different load increments are plotted on logarithmic scales to obtain a straight line from which modulus number K, and modulus exponent, n can be determined. The tangent modulus can be derived from a consolidation curve using the following equation developed by Chang (1969)

$$E_t = \frac{1+e_o}{a_v}\left[1 - \frac{2K_o^2}{(1+K_o)}\right] \tag{10}$$

where e_o is the void ratio at the beginning of a load increment, and a_v is the coefficient of compressibility. The bulk modulus can be approximated from a consolidation test using the following equation

$$B = \frac{\Delta\sigma'_1(1+2K_o)}{3\Delta\varepsilon} \tag{11}$$

where $\Delta\sigma'_1$ is the change in effective stress and $\Delta\varepsilon$ is the change in vertical strain.

4 Nonlinear Parameters from Consolidated Drained Triaxial Tests

Seven CD triaxial tests were performed on the expansive (Barind) soil from the Godagari Upazilla and is located in the Rajshahi Division approximately 40 Km southwest of Rajshahi District. The Barind soil is high plasticity clay with liquid and plastic limits equal to approximately 70 and 37, respectively. The Barind soil is classified as a CH according to the Unified Soil Classification system. The specimens used in the CD triaxial, consolidation, and direct shear tests were obtained from the block sample. An axial strain rate of 0.003% per minute was used in the CD triaxial tests to ensure drained conditions during loading. The strain rate was determined using the procedure presented by Gibson and Henkel (1954) and a coefficient of consolidation equal to 0.51 cm^2/min. The CD Triaxial tests were continued until the axial strain reached approximately 20%, which took about four to five days. After saturation, the specimens were then consolidated using confining pressures, which ensured the specimens would exhibit a normally consolidated behavior. The resulting stress-strain and volume change curves from the seven CD triaxail tests are show in Figure 2. The effective nonlinear and Mohr-Coulomb strength parameters obtained from these tests using the procedure outlined by Duncan et al. (1980) are shown in Table 1. The modulus number and bulk modulus number were obtained from Figures 3 and 4, respectively.

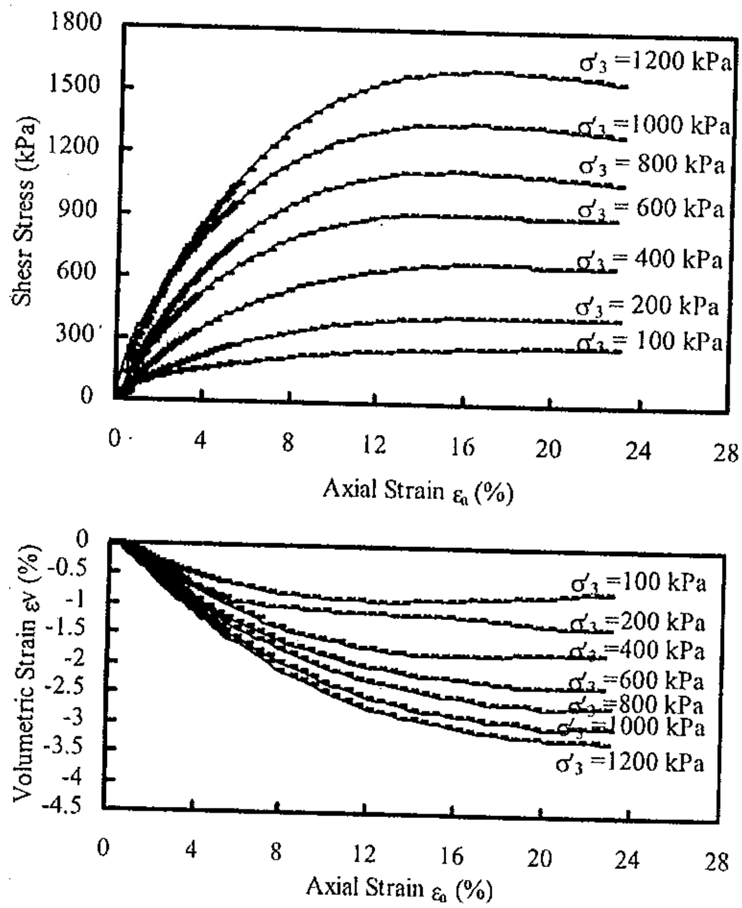

Figure 2. Stress-strain and volumetric strain representation of CD triaxial tests for Barind soil.

Table 1. Effective stress non-linear and Mohr-coulomb parameters from CD triaxial tests.

Description of parameters	Symbol	Value
Stress range	σ'_3	100-1200 kPa
Modulus number	K	182
Modulus exponent	n	0.84
Failure ratio	R_f	0.81
Bulk modulus number	K_b	94
Bulk modulus exponent	m	0.48
Cohesion	c'	27 kPa
Angle of internal friction	ϕ'	29

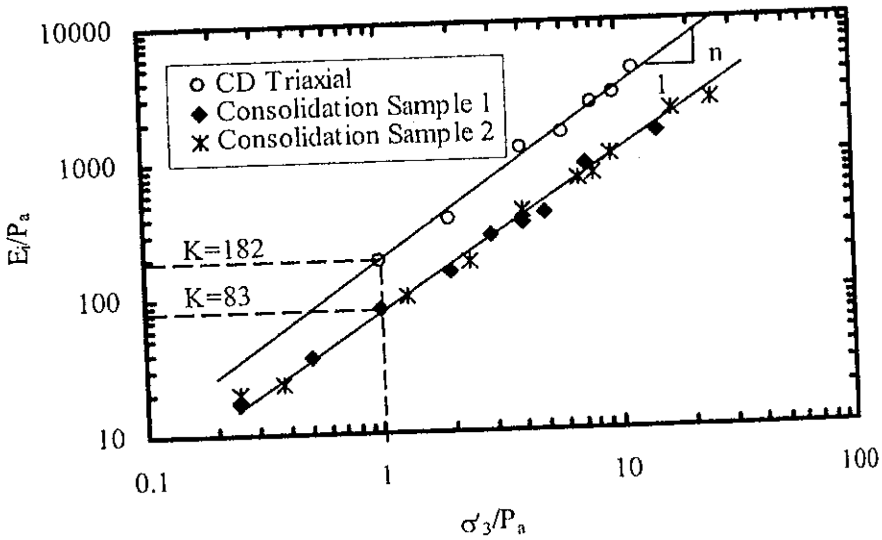

Figure 3. Non-linear model parameters from CD triaxial and consolidation tests.

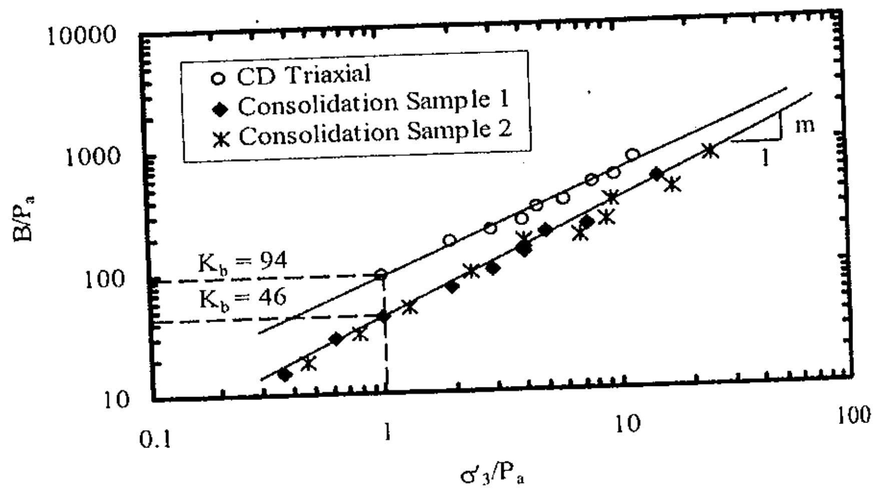

Figure 4. Bulk modulus parameters from CD triaxial and consolidation tests.

5 Nonlinear Parameters from Consolidation and Direct Shear Tests

Using Equations 9 and 10 and the results of two consolidation tests (Figure 5) and six direct shear tests on undisturbed specimens, the nonlinear stress-strain and Mohr-Coulomb strength parameters shown in Table 2 were obtained. Figures 3 and 4 were also used to determine the nonlinear stiffness and volume change parameters shown in Table 2. It can be seen from Tables 1 and 2 that the modulus number obtained from the consolidation and direct shears test results significantly underestimated the triaxial modulus number. The difference in the modulus numbers is probably due to the different boundary conditions and stress paths inherent in these tests. However, the value of the modulus exponent was approximately unity for both cases. It can be shown using critical state soil mechanics that the modulus exponent should be unity for soils undergoing primary loading.

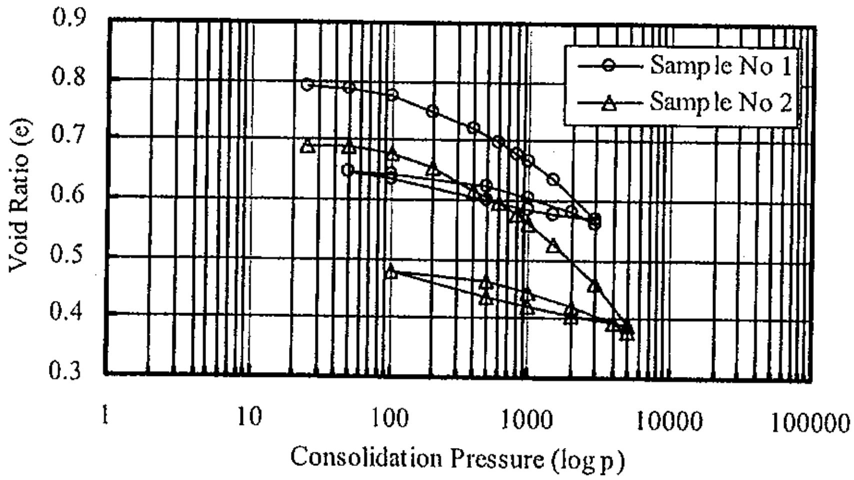

Figure 5. Void ratio versus consolidation pressure for oedometer tests.

Table 2. Effective stress non-linear and Mohr-coulomb parameters from consolidation and direct shear tests.

Description of parameters	Symbol	Value
Stress range	σ'_3	50-3000 kPa
Modulus number	K	83
Modulus exponent	n	0.83
Failure ratio	R_f	0.92
Bulk modulus number	K_b	46
Bulk modulus exponent	m	0.56
Cohesion	c'	22 kPa
Angle of internal friction	ϕ'	27

6 Model Parameters and Simulation of Stress-Strain for Expansive Soil

A numerical study interpret that the influence of the friction angle on initial tangent modulus E_i using Eq.9 is very low. The friction angle for the Barind (expansive) soil was measured to be 27 and 29 in direct shear and CD triaxial tests, respectively. The friction angle was also found to have no effect on the modulus exponent. It was concluded that using the friction angle obtained from direct shear tests would not significantly affect the estimates of modulus number and modulus exponent. The parametric study also interpret that variations in K_o has a significant effect on K and insignificant effect on modulus exponent, n. This is due to the importance of K_o in Eqs 9 and 10, which are used to calculate E_i and E_t, respectively. The values of bulk modulus were calculated using Eq 11 and the consolidation test results. It can be seen from Figure 4 that estimate of K_b and m from the consolidation test data were also in poor agreement with the CD triaxial values. For consolidation test, the value of σ'_3 used in Figure 4 is the average value of σ'_3 over a particular load increment. The major differences between consolidation and CD triaxial tests are: (i) the value of lateral stress increases with each load increment in the consolidation test and remains approximately constant in the triaxial test; and (ii) the volumetric strain in the consolidation test is equal to the axial strain, whereas the volumetric strain in the triaxial test is the sum of the three dimensional strains. These fundamental differences in the boundary conditions and stress paths in these two tests are believed to contribute significantly to the differences observed in K_b and m in Figure 4. For tests at confining pressure 200 kPa, the samples were unloaded at stress levels (ratio of maximum deviator stress to unloading stress) of 0.90 for drained test. The E_{ur} were obtained from the best-fit straight line at the stress-strain plot (Figure 6). The values of K_{ur} were then calculated using Eq 8. The unload-reload modulus number, bulk modulus number and bulk modulus exponent of barind soil are K_{ur} = 256, K_b = 94 and m = 0.5 respectively.

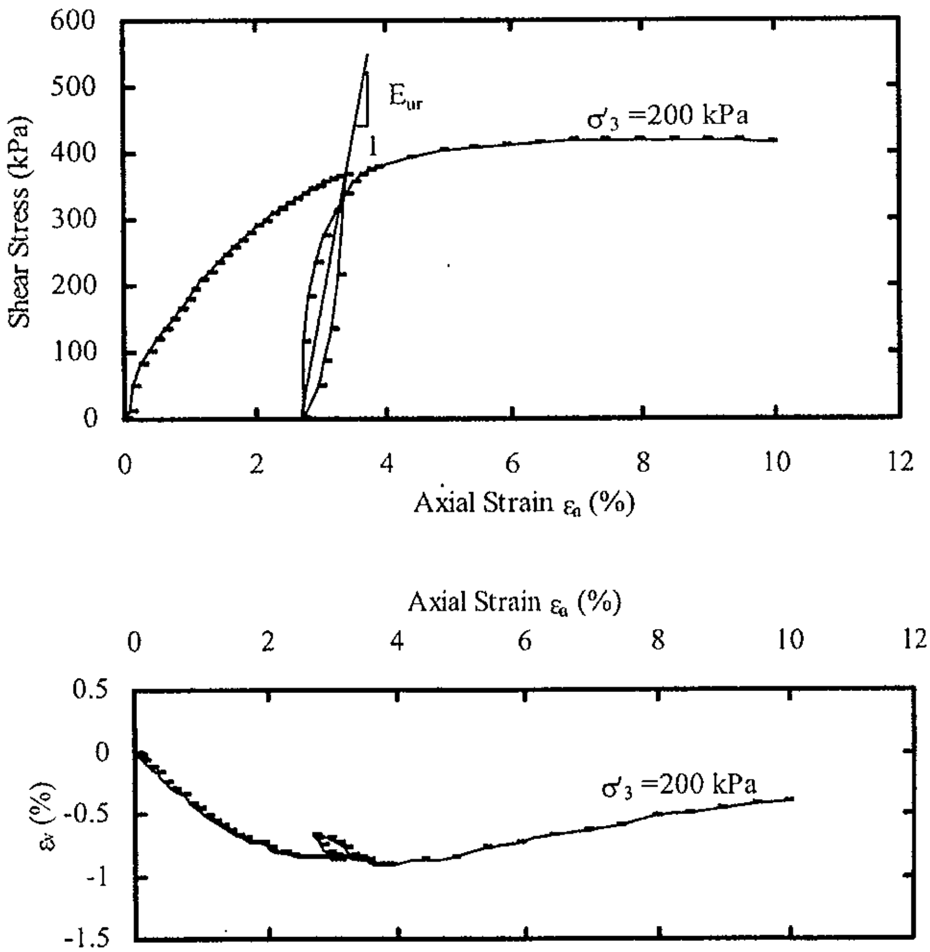

Figure 6. Stress-strain and volumetric representation of unload-reload triaxial tests.

Analysis was made to verify the model parameters by comparing numerical predictions with the experimental triaxial test results. The measured and predicted stress-strain curve for drained triaxial tests is shown in Figure 7. In order to check the sensitivity of the model parameters, the modulus number and modulus exponent were varied up to ± 5 %. Within this range, the results exhibit that there is no significance difference in the predicted response. On the other hand, if this tolerance is more than ± 5 %, then the predicted behavior deviated from the measured values. Thus, it is important that these parameters be obtained as accurate as possible since deviation of 5% or more can lead to deviation in the predicted behavior.

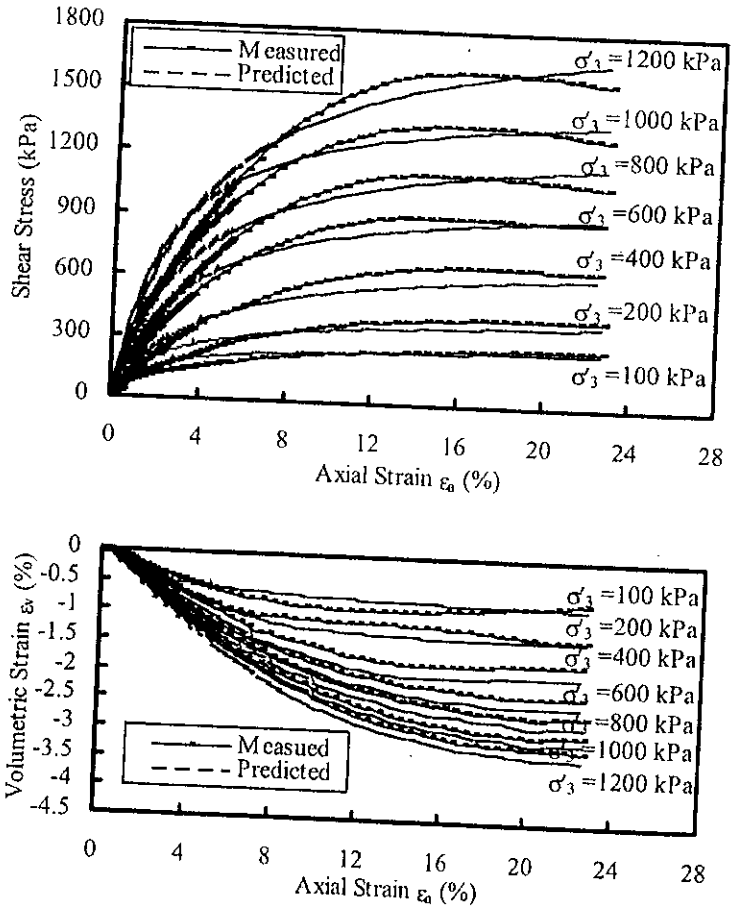

Figure 7. Measured and predicted behavior of stress-strain and volumetric representation of CD triaxial tests.

7 Conclusions

Based on the results of different tests conducted on expansive soil and numerical calculation, the following conclusions may be drawn:

1. To investigate the stress-strain and volumetric characteristics of expansive (Barind) soil using consolidated drained triaxial, consolidation and direct shear tests.
2. This study presents a comparison of the effective stress nonlinear stress-strain parameters determined from CD triaxial tests, consolidation and direct shear tests.

The comparison revealed that the results of consolidation and direct shear tests underestimate the modulus number determined from triaxial test results.

3. The modulus exponent equal to unity for both the CD triaxial tests and consolidation tests.
4. The consolidation and direct shear test results also provided poor estimates of the bulk modulus number and exponent determined from the CD triaxial test results. The differences in the modulus and bulk modulus numbers are probably due to the different boundary conditions and stress paths inherent in the consolidation and CD triaxial tests.

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