DISCUSSION / DISCUSSION

Reply to the discussion by Wanatowski and Chu on "On equivalent granular void ratio and steady state behaviour of loose sand with fines"[1]

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The authors would like to thank the discussers for their interest in and contribution to our paper. The discussers have highlighted several important issues that we will address in detail below.

f_c normalized by f_{thre}

The authors recognized that it is the fines content, f_c, relative to the void space of the parent sand matrix that governs the participation of fines in the force structure. Therefore, in the prediction of the b value, the influence of fines is better represented, in principle, by f_c/f_{thre}. A generalized, and theoretically better, form of the prediction equation can be expressed as:

$$[R1a] \qquad b = \{1 - \exp[-\mu(f_c/f_{thre})^n/k]\}(rf_{thre}/f_c)^r$$

The equation can be rewritten as:

$$[R1b] \qquad b = \left\{1 - \exp\left[-\frac{\mu}{(f_{thre})^n}(f_c)^n/k\right]\right\}(rf_{thre}/f_c)^r$$

The parameter m in the original paper by Rahman et al. (2008) is in fact $\mu/(f_{thre})^n$ with $n = 2$. However, for several data sets, the values of f_{thre} were not reported or could not be reliably inferred from the source publications. An average value of 0.30 was adopted for these cases. There are also some ambiguities and uncertainties in the f_{thre} values reported in the published literature. These points were noted in the original paper. The expression $\mu/(f_{thre})^n$ is in the exponent and can influence the prediction significantly. To reduce the impact of the uncertainties in f_{thre} on the predicted b value, $\mu/(f_{thre})^n$ is treated as a single parameter "m" and its value was calibrated to be 2.5 (in conjunction with selecting $n = 2$).

As discussed in a subsequent section, further works have been completed to enable the prediction of f_{thre} using only the particle-size ratio, χ, as the input parameter. Based on this newly developed methodology, the generalized eq. [R1a] can be used, and μ can be explicitly determined by calibration with data sets. This gives $\mu = 0.30$ by selecting $n = 1$. Thus, the generalized prediction equation is given by:

$$[R2] \qquad b = \{1 - \exp[-\mu(f_c/f_{thre})/k]\}(rf_{thre}/f_c)^r$$

As demonstrated in a subsequent section in our discussion on f_{thre}, eq. [R2] gives slightly better predictions compared with those given in the original paper.

Prediction of f_{thre}

The discussers raised a pertinent question on "how can f_{thre} be determined if one does not have the data sets?" Further work on f_{thre} has been completed and the findings pertinent to this question are presented below.

As f_{thre} defines the transition from a fines-in-sand to a sand-in-fines soil fabric, it is defined by a "reversal" of the influence of f_c on the correlation examined. For steady-state (SS) data sets, the steady-state line (SSL) initially shifted downwards with an increase in f_c, but the trend reversed at a higher fines content. The reverse in the "movement" of the SSL occurs, by definition, at $f_c = f_{thre}$. To determine this value objectively, e_{100} was plotted against fines content as presented in Fig. R1 for the data set of Thevanayagam et al. (2002), where e_{100} is the void ratio on the SSL at $p' = 100$ kPa. The turning point gives $f_{thre} = 0.36$. To examine the sensitivity of f_{thre} on the selected p'_{ss} value for constructing the plot, the above exercise was repeated using e_{SS} at a p' of 50 and 200. The resultant plots presented in the same figure show that the fines content at the reversal point is only marginally affected by the choice of p'_{ss} for constructing the plot and thus, the proposed interpretation of the family of SSLs will reliably give f_{thre}.

For data sets on the effect of fines content on cyclic resistance, f_{thre}, is also defined by the fines content where the influence of fines content on cyclic resistance reverses in direction. A similar procedure can be used to determine the reversal point, hence f_{thre}.

Nine data sets, including that of the authors, contained sufficient information for deducing f_{thre} based on the above procedure. One can then examine the influence of χ on f_{thre}

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Fig. R1. Change in critical state void ratio, e_{100}, with increasing fines content (after Thevanayagam et al. (2002)).

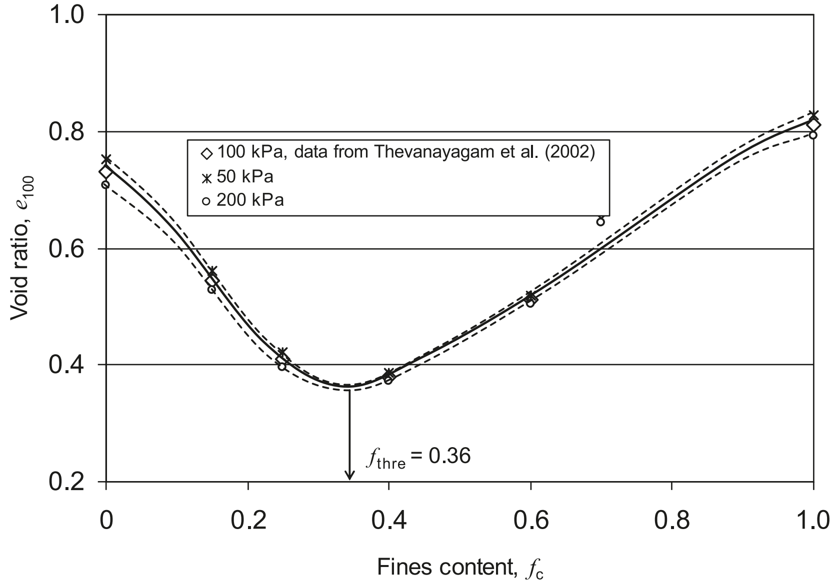
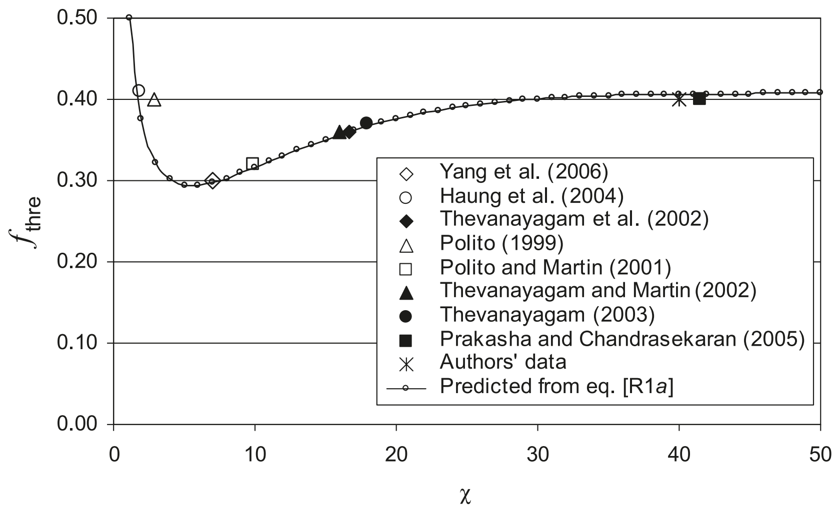


Fig. R2. Relation between threshold fines content, f_{thre}, and particle diameter ratio, χ.



as presented in Fig. R2. These data points follow a distinct pattern that can be characterized to three attributes:

- For a particle-size ratio (χ) in the range of 6.0 to 6.5, f_{thre} is at a minimum value. This is consistent with the binary packing consideration that at χ = 6.5, a minimum number of one "smaller particle'' can be fit in between the gap of larger particles.
- The increase of χ beyond the turning point (when f_{thre} is at a minimum) leads to a gradual increase of f_{thre} with χ to an asymptotic value. From a binary packing point of view, as the small particles get smaller in size, more particles can fit in the same void space between the large particles (thus giving a higher f_{thre}), but there is an asymptotic limit to this increase.
- Thirdly, when the diameter ratio is less than 5.0, f_{thre} increases rapidly with a reduction in χ. This is because the fines open up the gaps between the host sand and give more void space for the fines to get in.

These features can be modeled by the following equation with three constants:

$$[R3] \qquad f_{thre} = A\left(\frac{1}{1+e^{\alpha-\beta\chi}}+\frac{1}{\chi}\right)$$

The coefficient of A is the asymptotic value of 0.40 irrespective of the value of α and β. The other two parameters, α and β, are determined by curve-fitting and this gives α = 0.50 and β = 0.13. The resultant curve, in addition to fitting the data points, gives an approximately minimum value of f_{thre} at χ = 6.5

The two additional eqs. [R2] and [R3] proposed in this discussion were evaluated by re-analysing the data set of Huang et al. (2004). The original publication estimated a f_{thre} value of 0.30. Equation [R3] was first used to give f_{thre} of 0.41 and this value was then used in eq. [R2] for the prediction of b values to enable the conversion of e to e^*. Therefore, the data points could then be plotted in a e^*–log(p') space. As evident from Fig. R3, these data points can be

Fig. R3. Steady-state lines for Mai Liao sand with fines interpreted based on e^* using eqs. [R2] and [R3] (source data after Huang et al. (2004)).

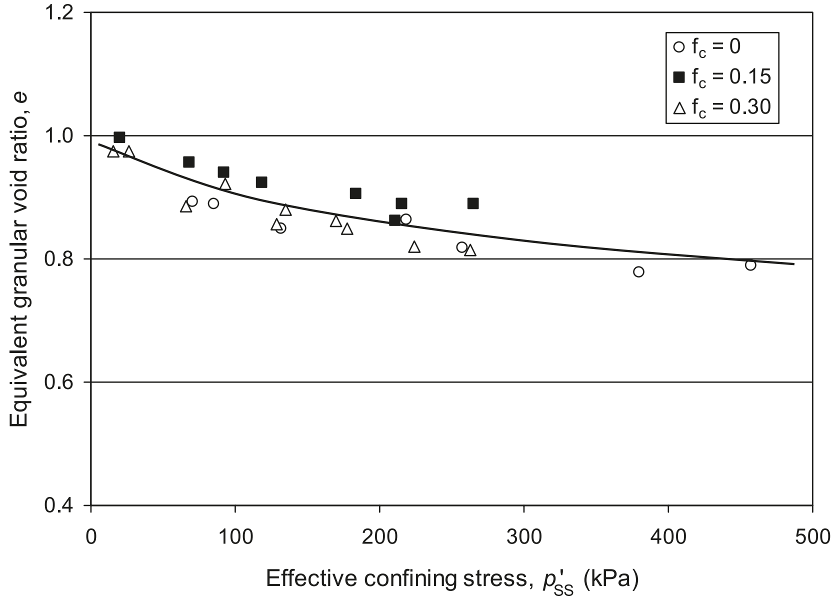


Table R1. Comparison of RMSD values.

Data sets	RMSD: Original paper (Rahman et al. 2008)	RMSD: With improved eqs. [R2] and [R3]
Yang et al. 2006	0.026	0.017
Huang et al. 2004	0.056	0.028
Ni et al. 2004	0.022	0.017
Zlatovic and Ishihara 1995	0.048	0.048
Thevanayagam et al. 2002	0.028	0.027
Vaid 1994	0.016	0.025
Polito 1999	0.067	0.044
Polito and Martin 2001	0.027	0.029
Thevanayagam and Martin 2002	0.054	0.045

represented by a single equivalent granular SSL. The root-mean-square deviation (RMSD) value, which measures the scatter of the data points about the trend line, is 0.028, and this is a very small value.

The reliability of the improved eqs. [R2] and [R3] was further examined by repeating the above synthesis for all other data sets. Thus, a new set of (e^*, p'_{ss}) data points, a new trend line, and a new RMSD value were available for all the data sets. These new RMSD values are compared with those reported in the original publication by Rahman et al. (2008) in Table R1. The improvement as indicated by the lower RMSD values is evident.

Function for describing the equivalent granular SSL

The discussers are correct in pointing out that the power function has "better mathematical attributes" than the quadratic eq. [6] (in the original paper) to represent the equivalent granular SSL. The quadratic function was chosen in the original paper because it yielded minimum scatter (as measured by the RMSD of SS data points from the trend line). This "criterion" was adopted to interpret tests performed with shearing commenced from states located slightly above (test T29) and slightly below (test T30) the equivalent granular SSL.

However, the difference between the RMSD values for the two function forms is slight. The authors recognize a power function, in general, is a better choice. Thus, the equivalent granular SSL can be expressed as:

$$[R4]\quad \cancel{\text{Equivalent granular SSL}} = \cancel{0.960} - 0.0266(p'_{ss}/p_{a})^{0.70}$$

The RMSD associated with the above function is 0.032, which is still very small.

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List of symbols

A ~~XXXX~~
b active fraction of fines in force structure
D_{10} sand particle diameter at 10% finer
d_{50} fines particle diameter at 50% finer
e void ratio
e_{100} void ratio on the SSL at $p' = 100$ kPa
e^* equivalent granular void ratio
e_{ss} void ratio at steady state
e^*_{ss} equivalent granular void ratio at steady state
f_c fines content in decimal
f_{thre} threshold fines content in decimal
k ~~XXXX~~
p_a atmospheric pressure, 100 kPa
p' mean effective stress, $(= (\sigma'_1 + 2\sigma'_3)/3)$
p'_{ss} mean effective stress at steady state
r particle size ratio, $(= (1/\chi) = d_{50}/D_{10})$
α ~~XXXX~~
β ~~XXXX~~
μ ~~XXXX~~
σ'_1, σ'_3 ~~XXXX~~
χ particle size ratio, $\chi = D_{10}/d_{50}$