Probabilistic pseudo-static analysis of pile foundations in liquefiable soils

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ABSTRACT: In this paper the pseudo-static method for assessing the seismic performance of pile foundations in liquefying soils is implemented in a probabilistic framework. The framework allows for rigorous consideration and propagation of the large uncertainties which exist in the pseudo-static method. The key features of the framework are outlined, as well as further research requirements in the inputs to the analysis. The results of applying the pseudo-static method to a case study bridge structure are compared to those obtained by using effective stress analysis within a similar framework.

1 INTRODUCTION

The seismic response of pile foundations embedded in liquefiable soils involves complex non-linear soil and pile behaviour as well as soil-structure-foundation interaction. While the effective stress method with advanced constitutive models is capable of capturing many of these complex features, it requires detailed knowledge to setup, perform, and interpret the analysis, as well as quality field data to calibrate constitutive models. An alternative to seismic effective stress analysis is the simplified pseudo-static method of analysis. The pseudo-static method involves applying static displacements and forces to a beam-spring model of the pile and soil to represent, in an average sense, the response of the pile foundation during ground motion shaking. As the pseudo-static analysis uses conventional geotechnical and structural parameters, it is conceptually simple and therefore widely applicable for conventional design. However, simplifications in the pseudo-static method cause uncertainty in the response of the pile, in addition to that already inherent in a complex model of the soil-structure-foundation system. In this paper the pseudo-static method is implemented in a probabilistic framework to allow for rigorous consideration and propagation of these uncertainties. The results of applying the simplified method to a case study are compared to those obtained by using effective stress analysis within a similar framework.

2 THE PSEUDO-STATIC MODEL

In order to be a successful practical approach, the pseudo-static analysis should be relatively simple, based on conventional geotechnical parameters and engineering concepts, and applicable without requiring significant computational resources. To be applicable for performance-based design the pseudo-static analysis of piles in liquefying soils must: (i) capture the essential features of pile behaviour in liquefying soils, and (ii) permit estimation of the inelastic response and damage to piles. In what follows, a recently developed method for pseudo-static analysis of piles in liquefying soils (Cubrinovski, et al., 2004, Cubrinovski, et al., 2008b) is discussed.

2.1 *Computational model and input parameters*

A typical beam-spring model representing the soil-pile system in the simplified pseudo-static analysis is shown in Figure 1, which was developed in the OpenSees platform (McKenna, et al., 2004). The model can easily incorporate a stratified soil profile with different thickness of the liquefied layer and a crust of non-liquefiable soil at the ground surface. In the model, the soil is represented using bilinear springs in which effects of nonlinear behaviour and liquefaction are accounted for through the degradation of stiffness and strength of the soil. The pile is modelled with a series of beam elements which have a general nonlinear moment-curvature relationship. Parameters of the model are illustrated in Figure 2 where a typical three-layer configuration is shown with a liquefied layer sandwiched between a surface layer and base layer of non-liquefiable soils. All model parameters are based on conventional geotechnical data (e.g. SPT blow count) and concepts (e.g. subgrade reaction coefficient, Rankine passive pressure). Two equivalent static loads can be applied to the pile in this model: a lateral force at the pile-head representing the inertial load due to vibration of the superstructure and pile cap, and a lateral ground displacement applied at the free end of the soil springs representing the kinematic load imposed by the cyclic ground displacements. As discussed by Cubrinovski and Ishihara (2007) the key uncertainties in the pseudo-static analysis are the strength and stiffness of the liquefied layer, and the magnitude and combination of the kinematic ground displacements and superstructure inertia force.

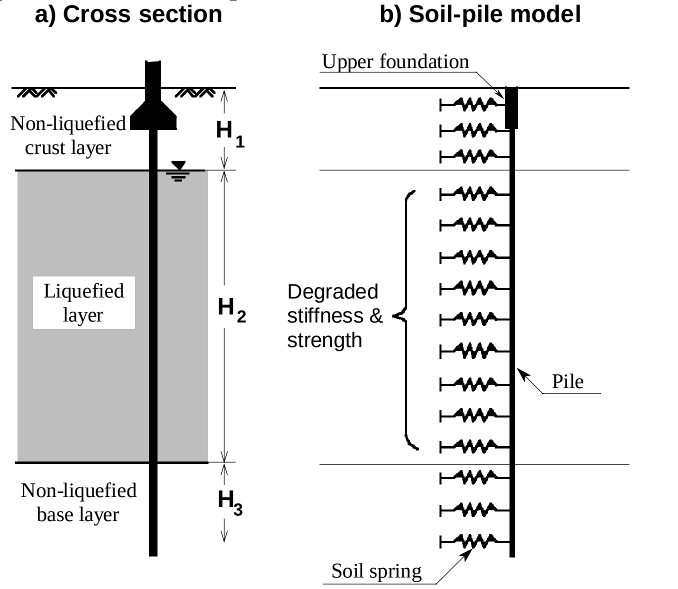

Figure 1: Schematic illustration of the pseudo-static method of analysis

3 CONSIDERATION OF UNCERTAINTIES

The seismic response of pile foundations in liquefiable soils is clearly a complex phenomenon, and therefore inherently burdened by many uncertainties such as ground motion variability, and modelling uncertainty (i.e. parameters in numerical models). The simplifications made when adopting the pseudo-static analysis introduce additional uncertainties into the problem. Such is the magnitude of these uncertainties, that it becomes necessary to consider a range of parameter values in the analysis to capture the range of possible seismic response for a given level of ground motion. Owing to the nature of the simplified approach, it is both conceptually and computational simple to account for such uncertainties. Therefore, the analyst should not be faced with the decision of whether or not to consider uncertainties, but is instead faced with the just as significant problem of: (i) what are the key uncertainties; and (ii) how to consider such uncertainties and propagate them to determine the uncertainty in the seismic response of the soil-pile system. The above two questions can generally be treated in either a deterministic or probabilistic fashion. While this paper is devoted to the latter, the relative merits of the two approaches are briefly discussed below.

The authors believe that deterministic and probabilistic approaches are complementary in that they each provide different information that offer insight into the phenomenon under consideration. Within the particular application of pile foundations in liquefying soils, the methodology in Cubrinovski and Bradley (2008a) is an example of a deterministic approach. In Cubrinovski and Bradley (2008a) each parameter is defined by its mean (best-estimate) value and an upper and lower bound value, typically based on experience. The pseudo-static analysis is first performed using all parameters at their mean value to obtain what Cubrinovski and Bradley (2008a) call the 'reference-model'. Each of the parameters is then individually varied to the upper and lower bound values, while other parameters are kept at their mean value. The resulting information obtained is the variation in the system response (typically in terms of peak pile curvature or peak pile head displacement) when varying each parameter while others are kept constant (at their mean values). The deterministic consideration uncertainties for this problem is therefore highly transparent with users easily able to determine which are the key parameters affecting the solution, and therefore where attention can be directed to reduce (if possible) such uncertainties. The deterministic framework on the other hand becomes inept when one starts to ask questions like, how likely is that parameter X will take that value?; and what will happen if multiple parameters take values different than their mean? The answer to the second problem is not-trivial due to the typically non-linear nature of such problems. Probability theory provides a consistent and rigorous approach to handle the two problems noted above, although with this rigour some of the transparency afforded in the deterministic approach is lost. The following section outlines the proposed probabilistic approach for consideration of the seismic response of pile foundations in liquefiable soils via pseudo-static analysis.

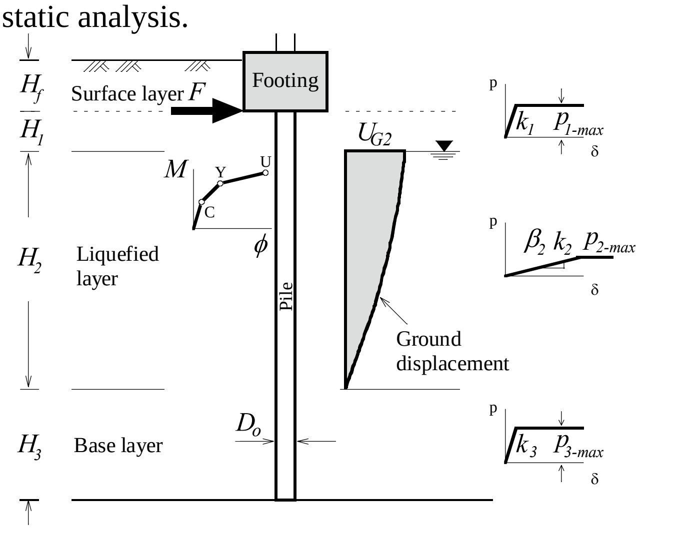

Figure 2: Input parameters required in pseudo-static model

4 PROBABILISTIC PSEUDO-STATIC ANALYSIS OF PILES

The final goal of seismic assessment of structures is a relationship between consequences associated with a seismic event and there likelihood of occurrence (commonly referred to as a loss hazard curve). For the seismic assessment of piles based on the pseudo-static approach, since the pile(s) are generally considered in isolation from the seismic performance of the superstructure, then the pseudo-static approach is not compatible with consideration of the seismic performance of the entire soil-foundation-structure system. Therefore, when using the pseudo-static method the primary consideration is the response of the pile(s) alone, and hence the relationship between pile demand and its likelihood of occurrence should be the key output of the pseudo-static analysis (of course, the deformational response can be related to consequences which will require post-earthquake repair etc).

Using the conditional independence assumption, the rate of exceedance of a specific level of pile demand (generally referred to as an engineering demand parameter, EDP) is given by:

$$\lambda_{EDP}(edp) = \int_{allI\ IM} G_{EDP|IM}(edp|im)\left|\frac{d\lambda_{IM}(im)}{dIM}\right| dIM \qquad (1)$$

where *EDP* and *IM* are the engineering demand parameter and ground motion intensity measure, respectively; $\lambda_Z(z)$ is the (annual) rate of $Z > z$; and $G_{EDP|IM}(edp|im)$ is the probability of $EDP > edp$ given $IM = im$. Thus, $\lambda_{IM}(im)$ gives the rate of exceeding a specific level of ground motion IM, which is the typical output of a probabilistic seismic hazard analysis (PSHA); while determination of the distribution of pile demand for a given IM, $G_{EDP|IM}(edp|im)$ can be computed from:

$$G_{EDP|IM}(edp|im) = \int G_{EDP|\boldsymbol{\Theta}}(edp|\boldsymbol{\theta}) f_{\boldsymbol{\Theta}|IM}(\boldsymbol{\theta}|im) d\boldsymbol{\Theta} \qquad (2)$$

where $\boldsymbol{\Theta}$ is a vector representing all of the uncertainties in the problem (discussed further below); $f_{\boldsymbol{\Theta}|IM}(\boldsymbol{\theta}|im)$ is the (joint) probability density function of $\boldsymbol{\Theta} = \boldsymbol{\theta}$ given $IM = im$; and $G_{EDP|\boldsymbol{\Theta}}(edp|\boldsymbol{\theta})$ is the probability of $EDP > edp$ given $\boldsymbol{\Theta} = \boldsymbol{\theta}$. Because the dimensionality of the integral in Equation (2) could be many-fold, Monte Carlo simulation will be employed here. That is, for each simulation $\boldsymbol{\Theta} = \boldsymbol{\theta}_i$ is obtained from $f_{\boldsymbol{\Theta}|IM}(\boldsymbol{\theta}|im)$, and then the pseudo-static analysis is executed to obtain the result EDP=edp_i. For N simulations, the N edp_i values can then be used to form an empirical distribution for $G_{EDP|IM}(edp|im)$. Since each pseudo-static analysis will take in the order of several seconds to solve then it is not impractical to perform hundreds of simulations. Furthermore, since the primary aim of this probabilistic approach is to determine the uncertainty in the pile response in proximity to the mean response (i.e. particular attention to the tails of the distribution is not necessary), then it suffices to use the so-called 'crude' Monte Carlo procedure without any variance-reduction techniques.

5 QUANTIFICATION OF UNCERTAINTIES

For the pseudo-static model employed in this paper there are several key uncertainties, as noted by Cubrinovski and Ishihara (2007) which can be classified into: (i) ground motion induced loads and displacements; (ii) soil constitutive relations (strength and stiffness); and (iii) in-situ soil measurements used to develop the model. The dynamic ground motion causes dynamic kinematic ground displacements, and dynamic inertial loading which have uncertain magnitudes, as well as an uncertain combination when applied as equivalent static loads. The soils are characterised as elastic-perfectly plastic materials, and have significant uncertainties in their equivalent stiffness and strength of the soil to account for non-linearity and pore pressure development. Finally, based on field measurements there is uncertainty about the actual in-situ values of the soil properties (e.g. SPT blow count). No other uncertainties are considered in the model, although additional uncertainties could easily be incorporated if further research suggests they are significant.

5.1 *Ground motion induced loads and displacements*

The prediction of free-field ground displacements is in itself a difficult task, as the displacements are a function of the intensity, frequency content and duration of the ground motion, as well as constitutive relationship of the soil strata. While there are several relationships for predicting lateral spreading displacements (e.g. Baska, 2002, Youd, et al., 2002) currently limited means are available to predict the magnitude of cyclic ground displacements (e.g. Tokimatsu, et al., 1998), in particular, no regression equations exist which can give the required distribution of cyclic ground displacements for use in the probabilistic approach presented herein.

Determination of the magnitude of the inertial load applied to the head of the pile(s) from the vibration of the superstructure and pile cap are also difficult to estimate as the ground motion is modified as it passes through the soil strata. In particular, liquefaction of soil layers may have a significant effect on reducing the high frequency content of the ground motion at the surface, which usually corresponds to the peak accelerations. Furthermore, as the maximum values of the inertial load and kinematic ground displacements are unlikely to occur simultaneously then only some fraction of these loads should be applied as static loads in the pseudo-static analysis.

Recently, Boulanger *et al*, (2007) proposed that the inertial load be obtained from:

$$I_{cc\_liq} = C_{cc} C_{liq} I_{\max\_nonliq} \tag{3}$$

where $I_{\max\_nonliq}$ is the maximum inertial load in the presence of no liquefaction; C_{liq} is a factor of account for liquefaction; and C_{cc} is a factor to account for the portion which is occurs at the same time as the peak kinematic ground displacements. Boulanger *et al*, (2007) also provided some preliminary values for C_{liq} and C_{cc} useful for design. Within the probabilistic framework, if it is assumed that all of $I_{\max\_nonliq}$, C_{liq} and C_{cc} are lognormal random variables then the distribution of I_{cc_liq} is:

$$I_{cc\_liq} \sim LN\left(\mu_{\ln C_{cc}} \mu_{\ln C_{liq}} \mu_{\ln I_{\max\_nonliq}}, \sigma^2_{\ln I_{\max\_nonliq}} + \sigma^2_{\ln C_{cc}} + \sigma^2_{\ln C_{liq}}\right) \tag{4}$$

where $LN(\mu, \sigma^2)$ means "is lognormal with mean, μ, and variance, σ^2" (Ang, et al., 1975). Alternatively to Equation (4), one can simply simulate C_{cc}, C_{liq} and $I_{\max\_nonliq}$ then use Equation (3) directly.
There is also a need to understand the correlation between the uncertain values of the peak kinematic ground displacements, and inertia force applied in the analysis. That is, if the kinematic ground displacements are larger than expected is it more likely that the inertia force will be larger than expected. Details on such correlations are addressed later in the manuscript.

5.2 *Soil constitutive relations*

The constitutive relationships for modelling liquefiable soils are typically based within the theory of plasticity, to account for complex non-linearity and dilatancy. It should be no surprise that representing such complex behaviour with an equivalent bilinear model will cause significant uncertainties in the stiffness and strength values of the simple model.
In the adopted model (Figure 2), effects of liquefaction on stiffness of liquefied soil are taken into account through the degradation parameter β. Figure 3a illustrates a comparison from observations of full-size experiments and back- calculations from well-documented case histories (Cubrinovski, et al., 2006) (best-estimate and ± 10% values), those used by Bowen (2007) and the empirical expression used herein given by:

$$\beta \sim LN\left(\mu_{\ln \beta} = \ln[0.06\exp(-8\gamma)], \sigma^2_{\ln \beta} = 0.8^2\right) \tag{5}$$

The large lognormal standard deviation of 0.8 represents the large uncertainty in the prediction of β, and was calibrated from the ± 10% values shown in Figure 3a.
Similar uncertainty exists regarding the ultimate pressure from the liquefied soil on the pile or the value of p_{max} in the model (see Figure 2). The ultimate lateral pressure p_{max} is often approximated using the undrained residual strength of liquefied soil (S_u), e.g. $p_{max} = \alpha S_u$, where α accounts for the finite width of the pile foundation (Cubrinovski, Kokusho and Ishihara, 2006). Ledezma and Bray (2008) developed a regression equation (Equation (6)) for the residual strength of soil as a function of SPT blow-count.

$$\frac{S_u}{\sigma'_v} \sim LN\left(\mu_{\ln S_u/\sigma'_v} = \frac{N_{1,60-cs}}{8} - 3.5, \sigma^2_{\ln S_u/\sigma'_v} = 0.4^2\right) \tag{6}$$

As the expressions for β, Su and alpha are empirical then it is not possible to estimate any correlations between them. Based on engineering intuition and judgement it can be assumed that they are perfectly correlated (a conservative assumption).

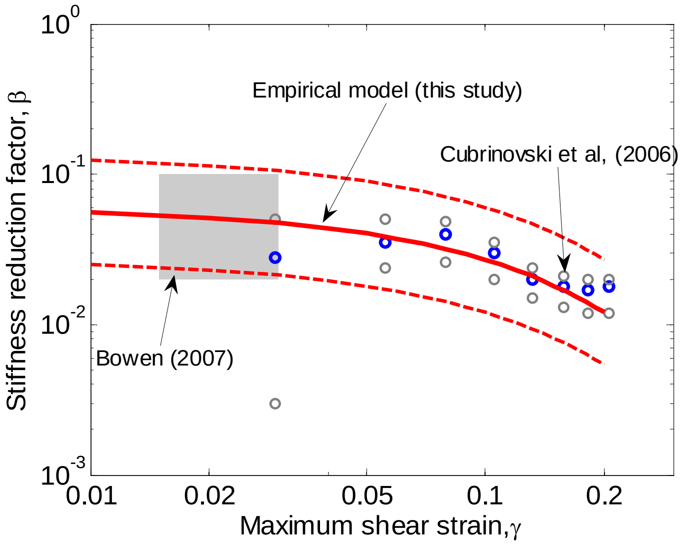

Figure 3: Regression relationships for: (a) soil stiffness; and (b) soil residual strength.

5.3 *In-situ soil measurements*

Input parameters used to characterise soil behaviour (stiffness and strength) in the pseudo-static analysis are typically based on field measures such as the Standard Penetrometer Test (SPT). There is inevitably some uncertainty in the results of SPT tests for a single site due to spatial variation in soil properties and measurement uncertainty. This uncertainty differs from those mentioned above in primarily two ways. Firstly, unlike the other uncertainties, the SPT uncertainty related to measurement limitations can be easily reduced by carrying out additional testing at different locations. Secondly, unlike the other uncertainties which are assumed to be random (and ergodic in time), the (epistemic) uncertainty in the SPT values is likely to be non-ergodic in time (Der Kiureghian, 2005). Because of the different nature of this uncertainty one could use a two-part Monte Carlo algorithm to keep aleatory (random) and epistemic uncertainties separate. Considering the many other simplifications made so far, such rigorous uncertainty analysis is not considered here.

6 CASE STUDY: BRIDGE FOUNDATION

The Fitzgerald Avenue Bridge over the Avon River in Christchurch, New Zealand, are used herein as a case study. Since the bridge has been identified as an important lifeline for post-disaster emergency services, a structural retrofit has been considered in order to avoid failure or loss of function of the bridge in the event of a strong earthquake. In conjunction with the bridge widening, the retrofit involves strengthening of the foundation with new large diameter piles. A cross section of the bridge through the central pier is shown in Figure 4. For simplicity and compatibility with the effective stress analyses of the same structure, no uncertainties were considered in the SPT blowcount of the soil.

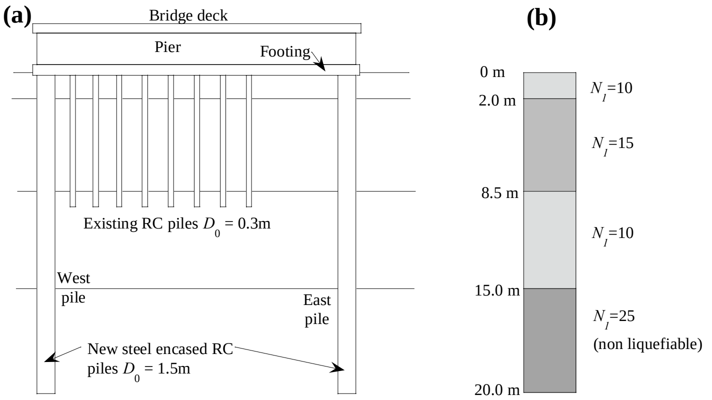

Figure 4: The Fitzgerald St bridge case study: (a) cross-section; (b) soil properties with depth

6.1 *Back-analysis of static loads using effect stress analysis*

The seismic performance of the Fitzgerald bridge has been assessed by Cubrinovski and Bradley (2008a), using effective stress analysis in conjunction with the Pacific Earthquake Engineering Research (PEER) Centre framework. The effective stress analysis utilised an elastic-plastic constitutive model developed specifically for modelling sand behaviour and liquefaction problems (Cubrinovski, et al., 1998a, Cubrinovski, et al., 1998b). A suite of 40 ground motions were used for peak ground accelerations from 0.1-1.0g resulting in a total of 400 effective stress analyses. As general models do not exist for some features required in the pseudo-static analysis (e.g. prediction of ground displacements) the results of the effective stress analyses are used here to back-calculate the necessary input values.

Figure 5a and Figure 5b illustrate the variation in the peak ground displacement and peak ground acceleration observed in the free-field (i.e. at a horizontal distance from the superstructure which has negligible interaction effects). Each of the points in these figures represents the result of a single analysis while the trend lines represent the mean and ± one standard deviation values obtained from regression. Figure 5c illustrates the distribution of the ground displacements with depth at the time of the peak ground displacement. Clearly, the shear strains are largest in the weakest layers, with the there contribution increasing as the PGA level of the input motion increases (due to the occurrence of significant liquefaction). As stated earlier, in the pseudo-static analysis only some fraction of the peak inertia load should be applied since it is unlikely that it will occur simultaneously with the peak ground displacements. Figure 5d illustrates the fraction of the peak free-field acceleration (Figure 5b) which occurs at the time of the peak ground displacements (Figure 5a). It is clear that there is significant scatter in this relationship (even for a single soil-pile-structure system), with the ± one standard deviation values typically ranging from 0.1-0.8. Detailed inspection of the responses also suggests that individual ground motions show no consistent tendency to produce high or low values in Figure 5d, which further illustrates the significance of the nonlinear response.

6.2 *Distribution of pile response for a given level of ground motion*

Figure 6 illustrates the resulting statistics of the pile and ground displacement and pile moment obtained using 30 Monte Carlo simulations, with the back-calculated input load at PGA = 0.6g. For the purposes of discussion one may consider the mean values of the analysis (solid lines in Figure 6) to represent the result that might occur if only a single 'reference' analysis with best-guess values for all the parameters was obtained (although strictly speaking the 'reference' analysis may be significantly different than the mean value). Clearly, the values for one standard deviation either side of the mean illustrate that there is significant uncertainty in the response of the pile for PGA = 0.6g, with pile head displacements of 0.08-0.41 m and the moment at the pile cap going from just above yielding to beyond failure. It is therefore evident, considering the magnitude of the uncertainty in the pile response, that making judgements based on a single 'reference' analysis could lead to potentially erroneous decisions.

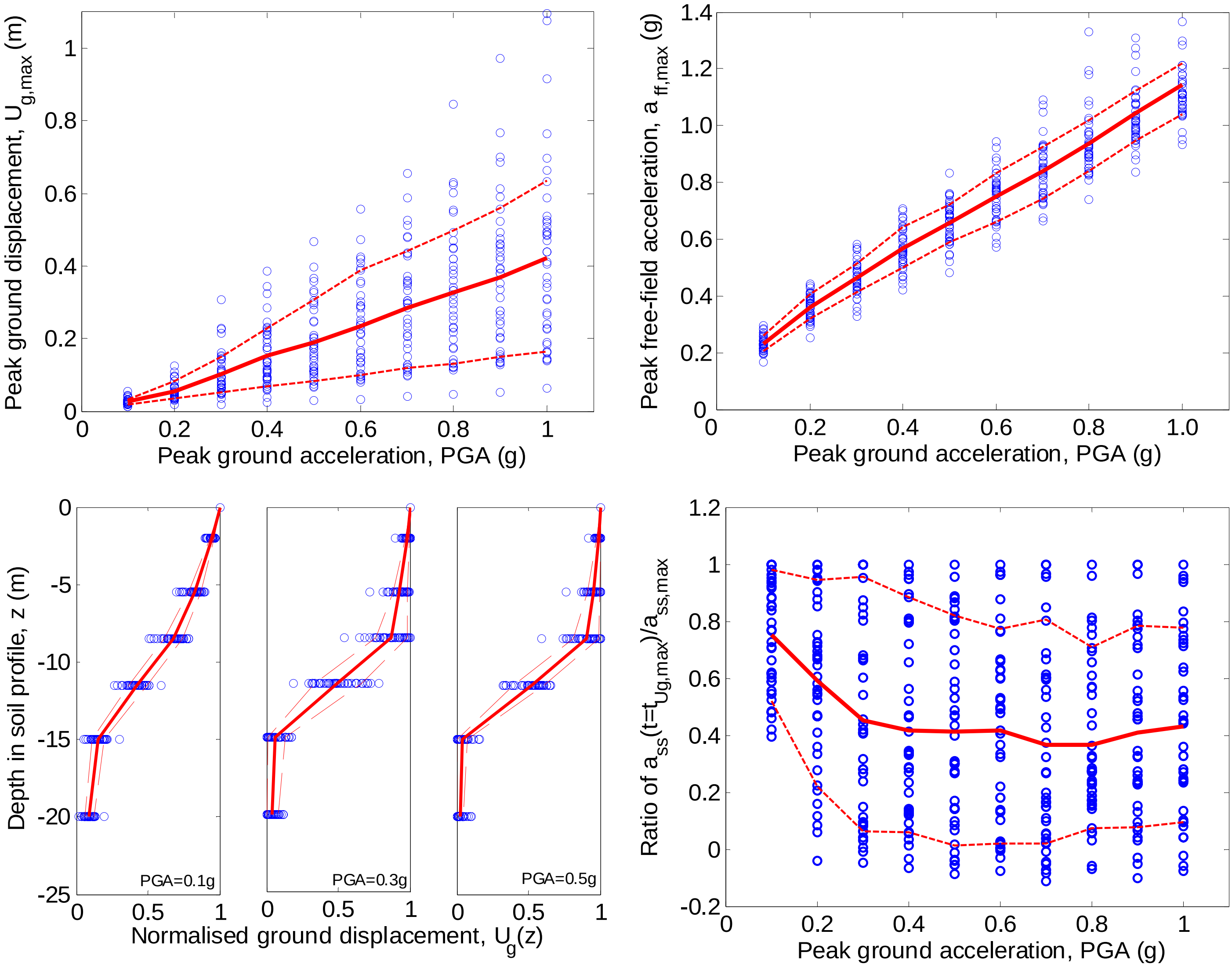

Figure 5: Back-analysed responses from effective stress analysis: (a) free-field ground displacements; (b) peak free-field accelerations; (cnormalised free-field displacements with depth; (c) peak inertia load; (d) fraction of peak inertia load at time of peak ground displacement; (e) correlations between peak ground displacement and peak inertial load

6.3 *Prediction at various PGA levels and the demand hazard curve*

Figure 7 provides a comparison of the statistics of the pile head displacement obtained using the effective stress and pseudo-static methods of analysis. It can be seen that the mean values are similar between the two methods with a minor conservative bias of the pseudo-static approach compared to the more accurate effective stress analysis. Furthermore, the large over-prediction of the 84th percentile value by the pseudo-static approach indicates that there is a larger uncertainty in the pile head displacement for a given PGA level. This larger uncertainty in the pseudo-static method is consistent with the many simplifications made in this simplified method.

By combining the relationship between pile head displacement and PGA in Figure 7 with the PGA hazard curve for the site (Stirling, et al., 2002), the pile head displacement hazard curve can be computed (via Equation (1)), as illustrated in Figure 8.

As would be expected, the hazard is over-predicted using the pseudo-static method compared to that obtained via effective stress analysis. This is because both the mean and uncertainty in the pile response are over-predicted using the pseudo-static analysis. As previously mentioned, the demand hazard curve, whether it be for peak pile head displacement or some other measure such as pile curvature, represents the final aim of the pseudo-static analysis. This demand hazard curve can be used to determine the pile response for a specified annual rate of exceedance (which will depend on the importance of the structure).

7 DISCUSSION: INPUT REQUIREMENTS

There is clearly additional research needed to allow the pseudo-static analysis to be applied consistently as a design method for pile foundations. It is important to note that these needs are not specific to the probabilistic approach but are requirements whether the pseudo-static analysis is conducted in a probabilistic or deterministic framework.

As is evident from Figure 3, Figure 5 and Figure 6, the uncertainty in the liquefied soil stiffness and strength, and the magnitude and combination of kinematic and inertia loads are the key uncertainties. While there are several relationships for predicting residual strength upon which the relationship used here is based there is currently little data to calibrate the reduction in stiffness, β (which had a large dispersion of 0.8). Clearly further experimental data to calibrate the stiffness reduction factor will lead to some reduction in this uncertainty.

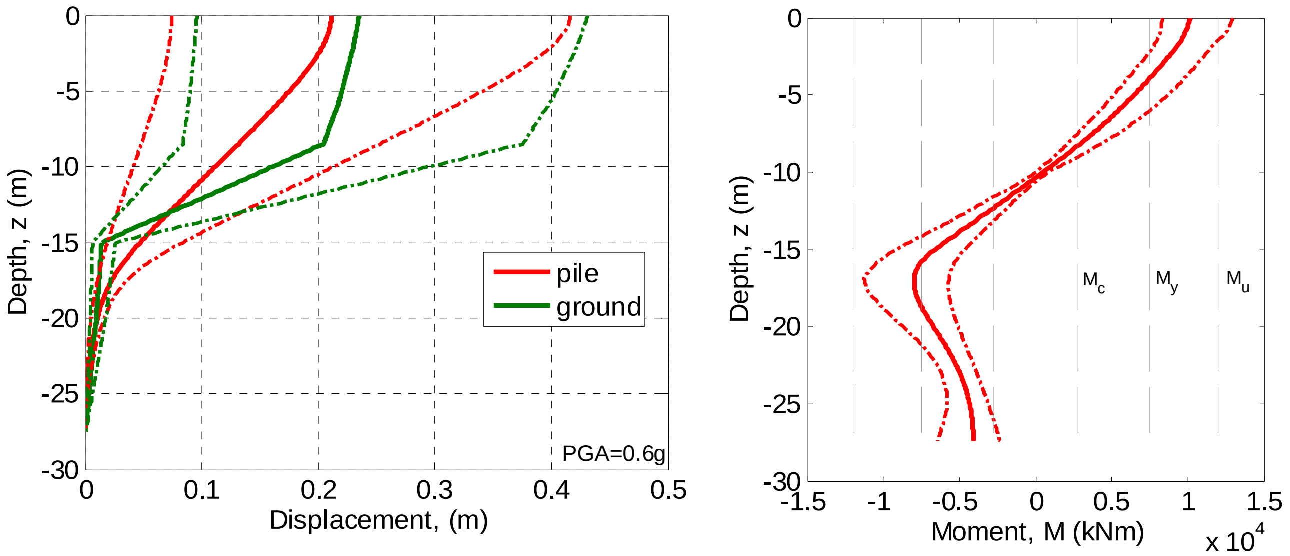

Figure 6: Mean and ± one standard deviation results of the probabilistic analysis for PGA = 0.6g: (a) distribution of pile and ground displacements with depth; and (b) distribution of pile moment with depth.

The uncertainty in the prediction of the kinematic and inertia loads will be a function of the ground motion IM used (PGA used here). That is, while PGA predicts the peak acceleration at the free surface well, its prediction of the peak ground displacement is very poor. Bradley *et al*, (2008) and Kramer and Mitchell (2006) have illustrated that velocity-based intensity measures provide a significantly better correlation with ground displacements and liquefaction prediction, which may be a better choice as the measure of ground motion intensity when endeavouring to develop prediction equations for cyclic ground displacements.

The use of a probabilistic framework is also beneficial in that the 'quality' of any prediction equations are explicitly incorporated in the analysis. Such relationships can be easily updated as further data (be it analytical or field) becomes available, which will result in the reduction of uncertainties (and a reduction in the demand hazard curve).

8 CONCLUSIONS

This paper has proposed a probabilistic framework for pseudo-static analysis of pile foundations in liquefiable soils. The framework allows for rigorous consideration and propagation of the large uncertainties which are inherent in the pseudo-static approach. Probabilistic relationships for several of the key uncertainties were proposed, while others were back-calculated for a specific bridge structure used as a case study.

Comparisons with effective stress analyses conducted within a similar framework illustrated that the pseudo-static analysis produces reasonable results.

Several aspects require further research before the pseudo-static analysis, in either a deterministic or probabilistic framework, can be applied consistently in a design environment.

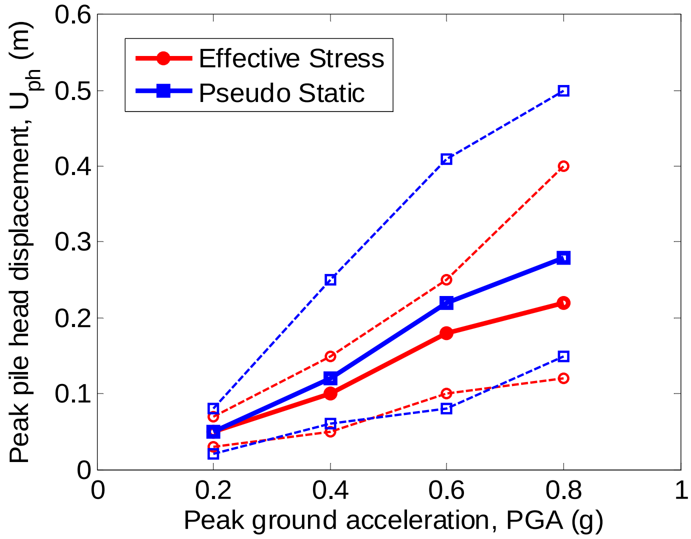

Figure 7: Comparison of pile head displacements obtained via effective stress and pseudo static analyses. Solid lines are means and dashed lines are ± one standard deviation.

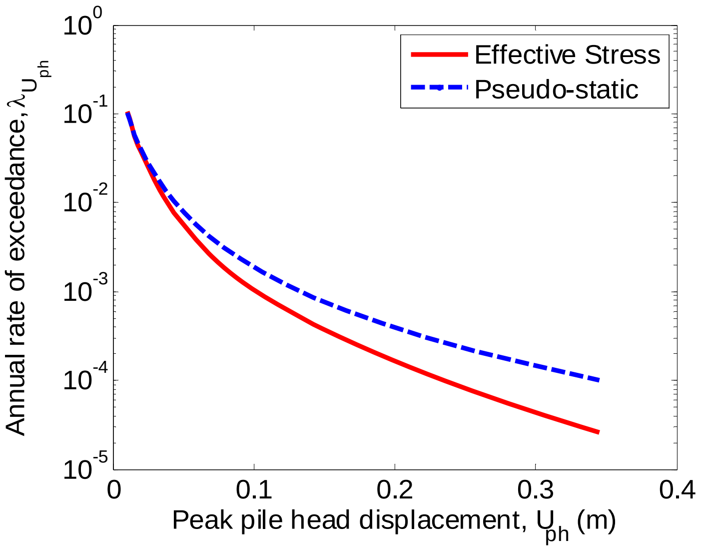

Figure 8: Comparison of demand hazard curve obtained using effective stress and pseudo-static analysis results.

9 ACKNOWLEDGEMENTS

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