Monte Carlo simulation of SSI effects using simple rheological soil model

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ABSTRACT: Most soil-structure interaction (SSI) studies use fairly simple rheological soil models that assume the soil-foundation system remains elastic. The goal of this study is to use these simplified models and systems to analyse almost exhaustively the impact of using these models on designing structures to account for SSI. To achieve this objective, an equivalent linear soil-structure model was examined in a comprehensive Monte Carlo simulation. This paper presents a study of 2.04 million simulations, over superstructures with periods ranging between 0.1 and 3.5 seconds, and over reasonable variations of soil, structure, and earthquake parameters. The overall finding shows in any case, there is over 70% of probability in which structural response is reduced due to SSI effects.

1 INTRODUCTION

Extensive research extending over more than 35 years has focused on explaining physical damage which was the result of soil-structure interaction (SSI) (Novak 1974, Gazetas 1983, Wolf 1985, Stewart 1999). Although the problem requires an appropriate non-linear soil-structure dynamic model, in most studies fairly simple rheological soil and structural models have been utilized (Aviles 2007, Ghanad 2008, Rodriguez 1999). To evaluate the impact of using these simplified models on structural design procedure accounting for SSI, a comprehensive Monte Carlo simulation followed by an exhaustive number of analyses was conducted. The emphasis is made on random selection of the most effective parameters in a soil-structure system, and using a large number of ground motions to cover various combinations of system parameters and ground motion record-to-record variability.

The soil-structure model adopted in this study is an idealization of a multi-story building located on a homogeneous half-space. In this model, the superstructure is a linear single-degree-of-freedom (SDOF) system with 5% equivalent viscous damping and the soil stratum is represented by equivalent linear springs, dashpots, and masses combining two principal concepts: (a) the cone model concept (Wolf 1994) and (b) the equivalent linear representation of nonlinear soil stress-strain behaviour. The generated soil-structure models are excited by a suite of earthquakes consisting of 30 ground motions recorded on stiff/soft soils. Through a statistical study with a large number of variables, the median and the related dispersion of the structural response were identified. These results provide valuable information revealing the SSI effects on the structural response using simple rheological soil models.

This study is the first step of a comprehensive investigation on the seismic SSI effects with the ultimate step being a fully non-linear soil-structure system analysis on more advanced yet practical models.

2 DESCRIPTION OF THE ADOPTED SOIL-STRUCTURE MODEL

Using a sub-structure methodology, the structure and the soil stratum can be modelled separately and then combined to constitute a soil-structure model as in Figure 1. The first stage of modelling is the representation of the multi-degree-of-freedom (MDOF) superstructure by an equivalent single-degree-of-freedom (SDOF) structure modelling the first mode of vibration (Figure 2). The superstructure is characterized by: (a) a structural mass participating in the fundamental mode of vibration $\mathbf{M_{str}}$, (b) a lateral stiffness $\mathbf{k_{str}}$, (c) a 5% equivalent viscous damping $\boldsymbol{\xi}$, and (d) an effective height **h**, considered from the foundation level to the centre of the inertial loads.

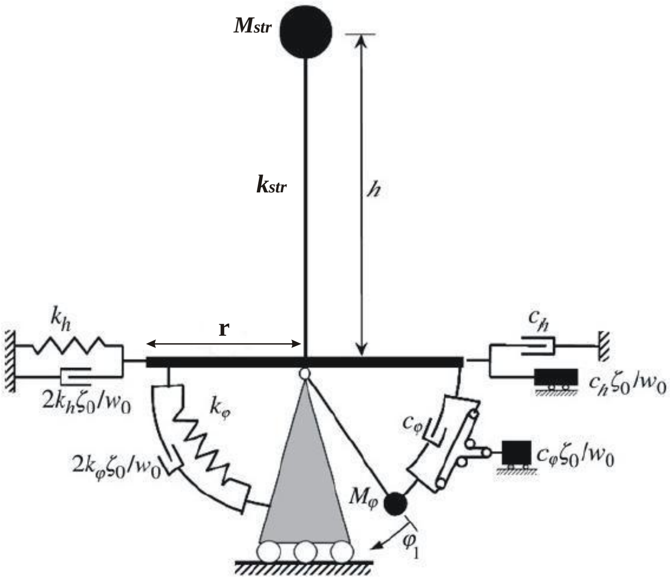

Figure 1. Coupled dynamic soil-structure model for horizontal and rocking ground motion (Wolf 1994)

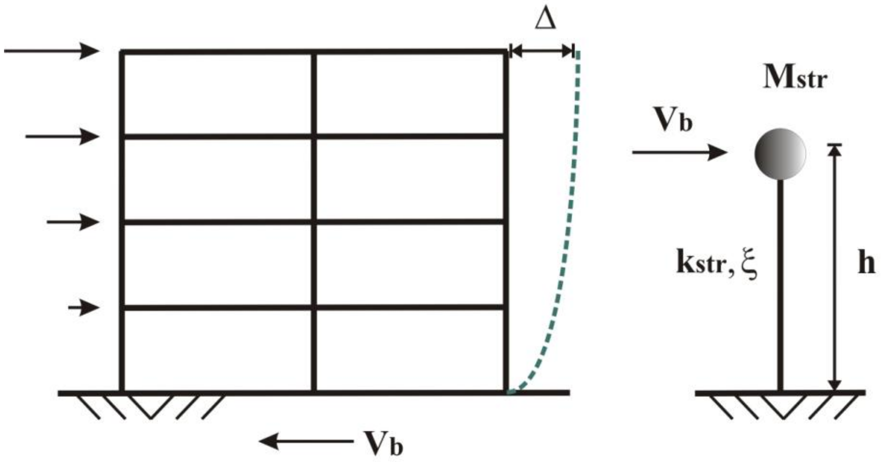

Figure 2. Equivalent SDOF representation of a MDOF structure

The soil-structure model is then completed by attaching the soil-foundation element at the base of the idealised SDOF superstructure. To define the coefficients of this element taking also into account soil nonlinearity, established parameters of the cone model (Wolf 1994) are modified based on the equivalent linear approach. The cone model is a macro element composed of springs, dampers, and masses with the frequency-independent coefficients and represents a surface foundation located on a homogeneous, linearly elastic, half-space. In this study, only horizontal and rocking motions of the foundation are investigated and since the foundation is located on the ground surface, the horizontal and rocking degrees of freedom are modelled independently. The soil material damping is also considered in this model, but to avoid further complications in time-domain analyses, the soil damping is assumed to be viscous instead of hysteretic. Expressions for the soil-foundation model properties such as spring stiffness, viscous damping coefficient and added masses are presented in Table 1.

Table 1. Properties of a soil-foundation element based on cone model concept (Wolf 1994)

Motion		Spring Stiffness	Viscous damper	Added mass
Horizontal		$k_h = \frac{8Gr}{2-\nu}$	$c_h = \rho V_s A$	-
Rocking	$\nu \le 1/3$	$k_\varphi = \frac{8Gr^3}{3(1-\nu)}$	$c_\varphi = \rho V_p I_r$	-
	$1/3 \le \nu \le 1/2$		$c_\varphi = \rho(2V_s)I_r$	$\Delta M_\varphi = 1.2(\nu - 1/3)\rho I_r r$
	Added monkey tail mass			
	$\nu \le 1/3$	$M_\varphi = \frac{9\pi}{32}\rho I_r r(1-\nu)\left(\frac{V_p}{V_s}\right)^2$		
	$1/3 \le \nu \le 1/2$	$M_\varphi = \frac{9\pi}{8}\rho I_r r(1-\nu)$		

The parameters utilised in this table are defined as:

- **r**: Equivalent radius of the foundation
- **A**: Area of the foundation, $A=\pi r^2$
- $\mathbf{I_r}$: Mass moment of inertia for rocking motion, $I_r=\pi r^4/4$
- **ρ**, **υ**, $\mathbf{V_s}$, and $\mathbf{V_p}$ are soil mass density, Poisson's ratio, dilatational wave velocity and shear wave velocity, respectively

As a main approximation in the construction of the cone model, the soil is idealized as a linearly elastic medium. To incorporate soil nonlinearity into this model, the equivalent linear approach may be utilized. The equivalent linear approach is a well-known method for site-specific response analysis and the evaluation of earthquake effects on soil deposits (e.g. implemented in the computer program SHAKE). This approach is illustrated with reference to Figure 3, which considers degraded secant stiffness (an equivalent linear stiffness) and equivalent damping (indicating the hysteretic damping) as a representation of nonlinear stress-strain relationship of the soil at each specified shear strain level. To define degraded secant stiffness and its equivalent damping, modulus reduction curves and related damping ratio curves presented in the literature (e.g. Vucetic and Dobry 1991) are used. As shown in Figure 3b, knowing shear strain level **γ** and initial shear modulus $\mathbf{G_{max}}$, the value of $\mathbf{G_{sec}}$ is simply determined. Similarly, equivalent damping ratio **ξ** is calculated by implementing the shear strain level into the damping ratio curves (Figure 3c). The assumed shear strain level can be estimated on the basis of the anticipated maximum ground acceleration (ATC 40).

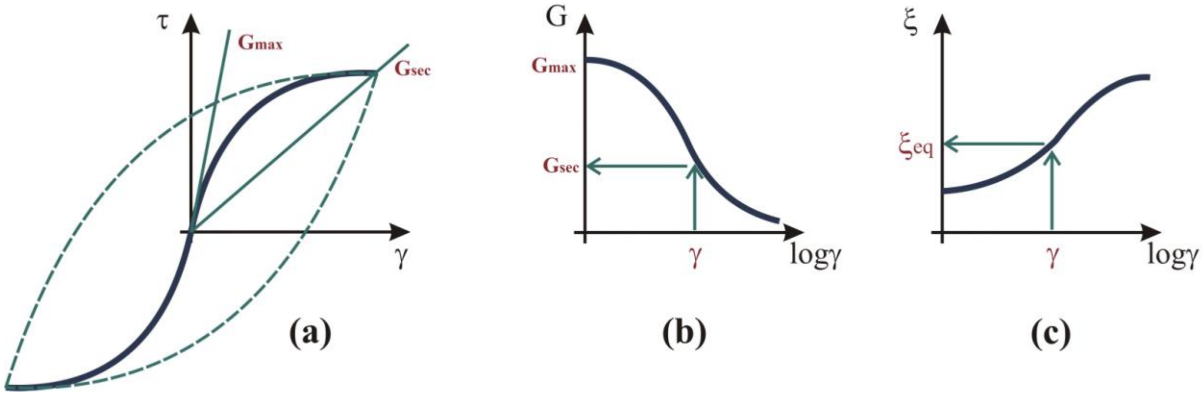

Figure 3. Equivalent linear idealization of soil stress-strain hysteresis loop

3 METHOD OF ANALYSIS

3.1 *Monte Carlo simulation to cover main uncertainties*

In seismic analysis, there are two recognized principal sources of uncertainty which need to be addressed: (a) model parameters and (b) input ground motion, typically categorised as epistemic and aleatory uncertainties, respectively. In this research, a systematic approach was developed for taking into account most of the uncertainties which may be related to the effect of foundation flexibility on the structural response. First, 68000 soil-structure models with randomly generated parameters were developed using the following three main steps:

- Degraded soil shear wave velocity $\mathbf{V_s}$, soil mass density $\boldsymbol{\rho}$, Poisson's ratio $\boldsymbol{\upsilon}$, and soil hysteretic damping $\boldsymbol{\zeta_0}$ at the effective period of the soil-structure system $\mathbf{T_{ssi}}$ were selected as the soil parameters need to be varied. For each of these parameters, 2000 random values were generated to reasonably cover their relevant range of variation. The description of how these values were selected is presented in Sec. 3.2.
- 34 groups of soil-structure models, which categorized based on the predominant periods of the superstructure $\mathbf{T_{str}}$ (T_{str}=0.1, 0.15 … 1, 1.1… 2, 2.25… 2.75, 3, 3.5 sec), were defined. This period set covers a wide range of superstructures with very stiff to very soft lateral stiffness. For each of these groups, depending on the randomly defined soil parameters, 2000 relevant structural parameters including structural mass $\mathbf{M_{str}}$, stiffness $\mathbf{K_{str}}$, height **h**, and foundation radius **r** were defined. The procedure followed for defining structural parameters is explained in Sec. 3.3.
- Finally, using the defined soil parameters and foundation radiuses, the coefficients needed for the soil-foundation element were constructed and the 68000 soil-structure models were completed.

All the developed models were then analysed using a suite of 30 earthquakes. Therefore, in total 2.04 millions of scenarios are considered to study the effects of SSI on the structural response.

3.2 *Random selection of soil parameters*

To define random values for each considered soil parameter ($\mathbf{V_s}$, $\boldsymbol{\rho}$, $\boldsymbol{\upsilon}$, $\boldsymbol{\zeta_0}$), a range of variation was assumed first. Subsequently, 2000 randomly generated values following a uniform distribution were picked up from that range. In this study, the values introduced in ATC40 1996, Fang 1999, Smoltczyk 2003 for soil type **C** with V_s=180-360 m/sec and soil type **D** with Vs<180 m/sec (based on NEHRP categorization) were applied to define the range of variation for different soil parameters.

Based on the equivalent linear approach explained in Sec. 2 (Figure 3b) and using the fact that $G=\rho V_s^2$ for linear soil, shear wave velocity corresponding to degraded shear modulus is calculated through multiplication of initial shear wave velocity $\mathbf{V_{s,initial}}$ and shear wave velocity degradation ratio ($\mathbf{V_{s,degraded}/V_{s,initial}}$). A range of 80-360 m/sec was chosen for initial shear wave velocity; and since all the selected ground motions have magnitude within the range of 6.2-7.6 and recorded at a distance less than 40 km distant from the fault, soil shear strain was presumed to vary within the range of 0.01-1 % and causes shear wave velocity degradation ratio to be between 0.15-0.7. Soil mass density and Poisson's ratio were also considered to be within the range of 1.6-1.9 ton/m^3 and 0.3-0.45 respectively. After defining soil mass density and degraded shear wave velocity, degraded shear modulus was calculated using the equation $G=\rho V_s^2$. The applied distribution of degraded shear wave velocity, soil density, and degraded soil shear modulus (definition of soil models) within the presumed range of variation are presented in Figure 4.

To define soil damping $\boldsymbol{\zeta_0}$, a rational approach was utilized. The expected amount of damping within the soil is directly relevant to the level of soil degradation, i.e. the more degradation that happens, the more damping is expected. Therefore, the damping is assumed to linearly varying between 10% and 25% for the minimum and maximum shear wave velocity degradation ratio, respectively.

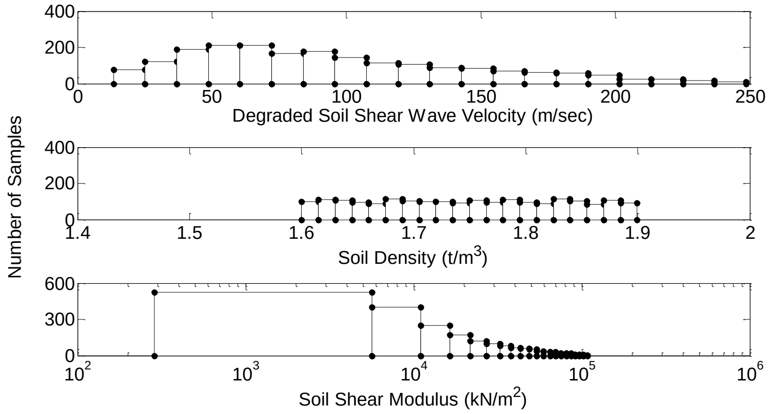

Figure 4. Distribution of degraded soil shear wave velocity, soil mass density, and degraded soil shear modulus

3.3 *Random selection of structural parameters*

Structural parameters for each soil-structure model were selected based on the assumed predominant period of the superstructure and soil parameters. The first calculated parameter was the height of the superstructure **h**. For each group of soil-structure models, 2000 uniformly distributed samples were picked up from the range of variation defined based on a typical period-height relationship adopted in NZS 1170.5 (Eq. 1). It was also assumed that for most common buildings (approximately 1 to 10 storeys), **h** is limited between 3 and 30 m.

$$10T_{str}^{1.33} \le h \le 27T_{str}^{1.33} \quad (1)$$

After defining the height of the superstructure, the building aspect ratio **h/r** was used to calculate the foundation radius **r**. **h/r** for ordinary (residential/commercial) structures was assumed to vary from 1 to 4 and **r** was limited to 2-12 m representing structures having 1 to 3 bays with the length of 4-8 (m) each. Considering the adopted variation range (Max{2,h/4} to Min{12,h}), 2000 uniformly distributed samples were generated for foundation radius. For each model, the foundation radius along with the selected soil parameters was used to calculate the coefficients of soil-foundation element. To define the best relevant structural mass $\mathbf{M_{str}}$ to the assumed **h**, **r**, and **ρ**, structure-to-soil mass ratio index $\overline{m}$ was used. This index is defined by Equation 2 and varies between 0.4 and 0.6 for ordinary structures (Ghanad 2008). From that range, 2000 uniformly distributed samples were selected and using the previously defined **h**, **r**, and **ρ**, structural mass was defined.

$$\overline{m} = \frac{M_{str}}{\rho r^2 h} \quad (2)$$

By knowing $\mathbf{T_{str}}$ and $\mathbf{M_{str}}$, the lateral stiffness of the superstructure $\mathbf{k_{tr}}$ was calculated:

$$k_{str} = \frac{4\pi^2}{T_{str}^{\ 2}} M_{str} \quad (3)$$

Supposing that the superstructure has an inherent 5% structural damping, $\mathbf{c_{str}}$ was calculated:

$$c_{str} = 2(0.05)\sqrt{M_{str} k_{str}} \quad (4)$$

Finally, the period of the soil-structure system was simply calculated as (Veletsos & Meek 1974):

$$T_{ssi} = T_{str}\sqrt{1 + \frac{k_{str}}{k_h} + \frac{k_{str} h^2}{k_\varphi}} \quad (5)$$

The distribution of the structural mass, stiffness, and damping for T_{str}=1.0 (sec) is presented in Figure 5.

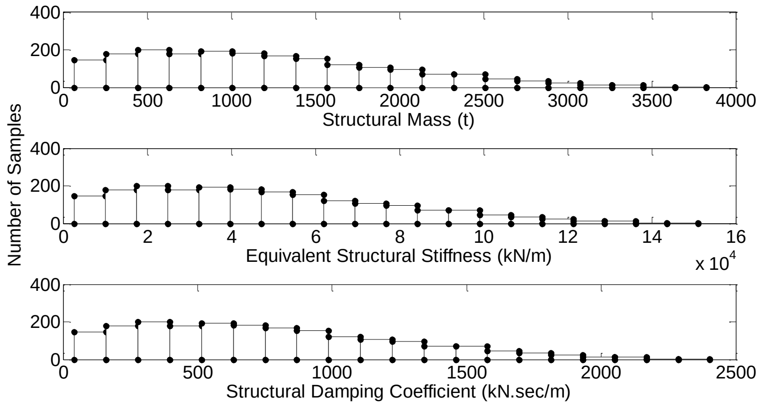

Figure 5. Distribution of structural mass, stiffness, and damping coefficients for T_{str}=1.0 sec

3.4 *Input ground motions*

Similar to most of soil-structure interaction analyses, kinematic interaction was neglected in this study. Therefore, a suite of free-field ground motions recorded on soil type **C** and **D** (based on NEHRP categorization) were directly used as input excitation to the system. These ground motions, with PGA ranging between 0.116 and 1.298 g, have a Magnitude of 6.2-7.5 and are recorded at a source distance of less than 25 km. The normalized (to PGA=1g) elastic acceleration response spectra (5% damping) of the records are shown in Figure 6.

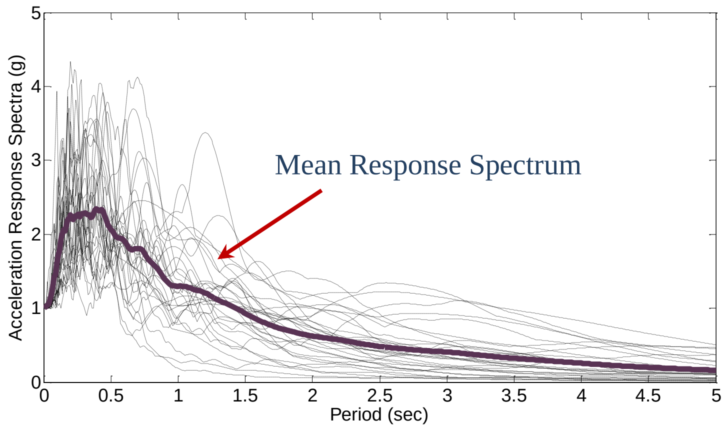

Figure 6. Normalized elastic acceleration response spectra for the suite of ground motions (5% damping)

4 RESULTS AND DISCUSSION

In this Monte Carlo simulation, the wide range of selected soil-structure models (68000) and ground motions (30), i.e. 2.04 million (30X68000) analyses in total, allows for a comprehensive statistical study of the SSI effects on the structural response. To characterize the central tendency of the interested response, the median value is selected as the statistical measure. In addition, for more comprehensive statistical evaluation, the level of dispersion exiting around that expected median value is quantified. Theses values are presented in a box and whisker plot in which the box has lines at 25th percentile (1st quartile), median, and 75th percentile (3rd quartile) values. Whiskers extend from each end of the box to the adjacent values in the range of 5th percentile and 95th percentile, and outliers are the data with values beyond the ends of the whiskers.

The following parameters are selected as an indicator of SSI effects on the structural response:

- **U_{str}**: Structural relative displacement (since the behaviour of superstructure was assumed to be linear, only U_{str} represents the dynamic behaviour of the superstructure)
- **U_t**: Structural total displacement

The examined parameters are expressed as normalized maximum values resulting from the time-history analyses. Normalization is used to represent the relative response of soil-structure system to equivalent fixed-base system for the same ground motion.

4.1 *Structural relative displacement*

Figure 7 displays the median and dispersion of the normalised maximum structural relative displacement for different groups of soil-structure models (categorized in terms of predominant period of superstructure). At each specific period, the resulting responses from 60000 different scenarios are presented (30 ground motions and 2000 soil-structure models). In this figure, values higher than 1 on the vertical axis indicate SSI consideration can cause an increase in stress and deformation within the superstructure.

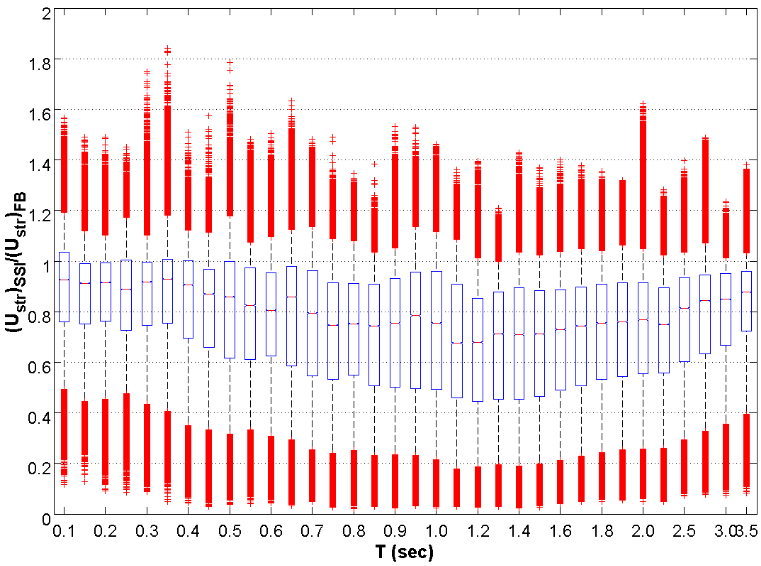

Figure 7. Median response and dispersion representation of structural relative displacement

As observed, the median response of all the considered scenarios is less than 1, i.e. consideration of flexible foundation in seismic analysis, in general, causes a decrease in structural relative response. However, for 10-20% of the examined cases increased responses can be recognized (Figure 9). The large tail distribution of the outliers is also justified noting that normally distributed variations of inputs and model parameters in many cases can lead to lognormal responses (Limpert et al 2001).

4.2 *Structural total displacement*

Structural total displacement is a factor which needs to be controlled to prevent pounding between structures. Total displacement includes horizontal foundation displacement, lateral displacement due to foundation rocking, and structural relative displacement. Figure 8 shows the median and dispersion of the normalised maximum structural total displacement. As clearly illustrated, considering SSI causes the structural total displacement to increase approximately up to 10 times; however, the median value is still around 1. Comparing the structural relative displacement in Figure 7 with the structural total displacement in Figure 8 reveal that SSI consideration in extreme cases can only show an amplification factor in the order of 1.5-2.0 within the superstructure, but in the order of 10 for the whole system. This increase is obviously due to foundation horizontal movement and rocking, and does not cause the same effects on the stresses and deformations within the superstructure.

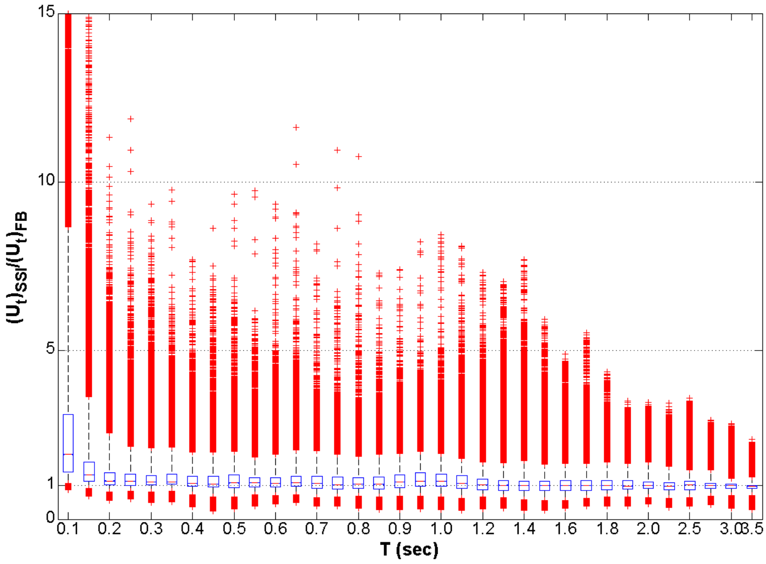

Figure 8. Median response and dispersion representation of structural total displacement

4.3 *Quantification of increased structural response due to SSI effects*

To quantify the SSI amplification effect on the response of the superstructure, two main aspects should be investigated simultaneously: (a) the probability of the cases in which SSI causes the superstructure to show a higher response and (b) the corresponding ratio of the increase. In Figure 9, the probability of the cases with normalized maximum relative displacement greater than 1 is presented. As shown, the percentage of increase is around 20% for stiff structures and almost 10% for the other types of structures.

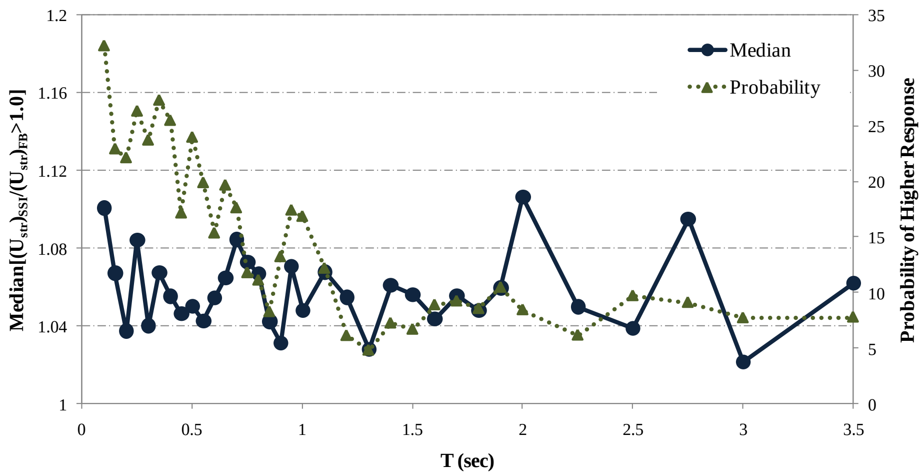

Figure 9. Probability of structural relative displacement increase due to SSI consideration

In addition, the median of the resulting values which are above 1 is shown in Figure 9 to quantify the ratio of increase. The median values are in the order of 5-10% depending on the predominant period of the superstructure. Considering the probability of increasing SSI effects in conjunction with the amount of increase in structural relative response, it can well be explained that SSI may increase the stress and deformation of the structure but the risk of this increase is limited. Therefore, depending on the importance of the project and the acceptable risk of the damage, the SSI effect could be either considered or ignored. Either way, consideration of SSI effects will clearly increase the accuracy in the prediction of structural response and seismic performance.

5 CONCLUSIONS

A comprehensive Monte Carlo simulation covering a wide range of parameters for a soil-structure model was conducted. Through this numerical simulation, the SSI effects on dynamic structural response were investigated. In this analysis, the structure was simplified as an equivalent SDOF system and the nonlinear stress-strain relationship of the soil was modelled by an equivalent linear approach. Based on the observed results the following conclusions are made:

1. In a median sense, consideration of a flexible foundation for the structural analysis reduces the relative deformations and stresses in the superstructure. However, it should be considered that for 10-20% of the considered cases, the deformations within the superstructure increased due to SSI effects.
2. The total displacement of structures with foundation flexibility is similar to the displacement of the corresponding fixed-base systems. But, in extreme cases considering SSI causes the structural total displacement to increase approximately up to 10 times.
3. The percentage of increase in structural relative response is around 20% for stiff structures and almost 10% for the other types of structures, on average. The median value for the increase ratios is in the order of 5-10% depending on the predominant period of the superstructure.

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