How much do Employee Stock Options really cost?*

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Abstract

From 2007, New Zealand firms must report the cost of granting employee stock options (ESOs). Market-based option pricing models assume that options are continuously tradable and thus that option holders are indifferent to the specific risk of the firm. ESOs, by contrast, cannot be traded and so their cost depends on the risk aversion and under-diversification characteristics of the recipient. Using hypothetical ESOs, we show that ESO cost is extremely sensitive to employee characteristics, thereby casting doubt on the usefulness of any market-based model. Incorporating early exercise in the latter does nothing to resolve this problem, because the optimal exercise policy is itself dependent on holder characteristics which are typically unobservable. Vesting restrictions help reduce the magnitude of error.
How much do Employee Stock Options really cost?

1. Introduction
In recent years, employee stock options (ESOs) have been the subject of much criticism. To many, the seemingly attractive link between pay and performance that ESOs offer has been soured by their association with exorbitant compensation packages, repricing in favour of ESO recipients, and general corporate malfeasance. In essence, they have come to be seen as a symptom of the owner-manager agency problem rather than as a solution to it.

One commonly cited reason for the failure of ESOs to live up to their potential is the absence of an accounting standard requiring firms to treat ESOs as a compensation expense. As far as reported profits are concerned, ESOs have been a free lunch, thereby encouraging the granting of too many of them on too generous terms. In response to these concerns, accounting authorities have recently taken steps to ensure that the costs of ESOs will in future be recognised in financial statements. The principal authority on this issue is International Financial Reporting Standard 2 (IFRS 2), issued in March 2004 by the International Accounting Standards Board. In November 2004, the Institute of Chartered Accountants of New Zealand adopted the same requirements in NZ IFRS 2. In brief, this specifies that all NZ firms must begin recognising ESOs at their grant date 'fair value' (essentially the value the ESOs would trade at in the marketplace) by no later than 2007, and provides principles-based guidance on how fair value should be determined.

On the latter issue, the value of market-traded options has traditionally been calculated using the famous model of Black and Scholes (1973) and Merton (1973) or some variant thereof, but IFRS 2 notes that ESOs typically have a number of more complex features than the type of option envisaged by those authors (henceforth BSM) and that an appropriate calculation of ESO cost should reflect these differences. For example, IFRS 2 suggests that employees' inability to trade ESOs combined with their ability to exercise prior to the expiration date can drive a wedge between the BSM
value and actual ESO cost. However, because IFRS 2 contains no explicit examples of fair value calculation, the extent to which these differences are material for valuation purposes is unclear.¹

In this paper, we attempt to shed some light on this issue and assess its implications for accounting recognition of ESO costs. Using a pricing model that explicitly incorporates non-tradability and early exercise, we calculate the 'true' cost of two hypothetical ESOs and compare these with the values generated by the sorts of methods discussed in IFRS 2. Because ESOs cannot be traded, holders have an incentive to exercise these earlier than they would otherwise-equivalent traded options, and the strength of this incentive increases with employee risk aversion and under-diversification. Since the market value of any option depends on the exercise policy applied to it, ESO cost is thus a function of employee characteristics, in contrast to market-based models like BSM. Moreover, ESO cost is quite sensitive to these characteristics, so any estimate from a market-based model approaches the true value of any particular ESO only by good luck.

In recent years, a number of studies have developed methods for valuing ESOs. Our analysis builds on the model of Ingersoll (2003), while others include Carpenter (1998), Hall and Murphy (2002), Huddart (1994), Kulatilaka and Marcus (1994), and Maller et al (2002). However, with the indirect exception of Carpenter, none of these explicitly calculate the sensitivity of ESO cost to employee characteristics and the implications of this for accurate financial reporting.

In the next section, we discuss in more detail some of the issues that arise in valuing ESOs and describe the method we use for calculating the true cost of ESOs. Section 3 contains our results, and section 4 offers some concluding remarks.

¹ IFRS 2 does, however, contain several examples of how a given ESO value should be recognised in financial statements.
2. What determines the cost of ESOs?

ESOs cannot be traded (since allowing employees to trade their options would undo the reasons for granting them in the first place) and usually offer a choice of exercise dates, a combination that has potentially significant implications for ESOs. When all assets are tradable, investors diversify their portfolios and thus are essentially risk-neutral with respect to firm-specific risk. Hence, the value of an option equals the expected option payoff discounted at a rate that includes a premium for systematic (non-diversifiable) risk only. However, ESO holders typically have a large over-investment in the firm that grants the ESOs: many have (and in some cases are required to have) substantial stock holdings and, more importantly, all have a large amount of human capital tied up in the firm, at least some of which is unique to that firm. Consequently, ESO recipients are under-diversified and thus exposed to the unsystematic risk of the issuing firm. As a result, the value (to the recipient) of ESOs equals the expected option payoff discounted at a rate that includes a premium for both systematic and firm-specific risk. In short, not being able to trade ESOs lowers the value placed on them by their holders relative to that of traded options.

However, this phenomenon is not directly relevant to the value that investors and accountants are interested in - the cost incurred by the firm in granting ESOs. To understand the difference, suppose an employee's remuneration contract includes the use of a $50,000 car. If the employee does not drive, or lives only a short distance from the workplace, then the value he places on the car is likely to be considerably less than $50,000, and may approach zero. But the cost to the firm is still $50,000. And exactly the same principle applies to ESOs. The opportunity cost to the firm is the value of the ESO in the marketplace (i.e., the expected ESO payoff discounted at a rate including a premium for systematic risk only) since the funds potentially used to pay out the option could otherwise have been invested elsewhere in the market; the particular circumstances of the ESO holder are irrelevant.

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2 Models of option pricing are typically cast in a certainty-equivalent, rather than risk-adjusted discount rate, framework (although see Arnold and Crack, 2004), but the latter's intuition is better suited to our purpose here.
Nevertheless, the preferences and diversification of the ESO holder do have an indirect effect on the cost of ESOs to the firm. Returning to the company car analogy, the total cost to the firm depends on the employee's usage policy, insofar as a car that has been only lightly used will generally have a greater resale value than one with many kilometres on the clock. With ESOs, exposure to the firm's unsystematic risk leads holders to pursue an exercise policy that differs from the one they would have chosen if the options were able to be traded. This typically results in the ESO being exercised earlier than an otherwise-equivalent traded option, as exercise represents the only way for under-diversified holders to liquidate their position. Early exercise changes the expected option payoff, and so the cost to the firm equals this revised expected payoff discounted at the systematic-risk-adjusted rate used by the market. Thus, the cost to firms of granting ESOs differs from the value of otherwise-equivalent options traded in the market not because the characteristics of ESOs make them less valuable to recipients, but rather because these characteristics affect the optimal exercise policy of recipients, thereby changing the option payoff distribution.

Of course, it may be that employee characteristics have only a minor impact on exercise policy and ESO fair value, and thus can safely be ignored for reporting purposes, but this cannot simply be assumed. Instead, we need some mechanism for assessing the sensitivity of fair value to employee characteristics, which in turn requires that employee exercise policy be determined simultaneously with the option price. The model of Ingersoll (2003), as applied and modified by Clyne (2004), is particularly suitable for this purpose: it determines the optimal exercise policy for an ESO and then calculates the marketplace cost of the ESO to the firm given this policy.

Intuitively, this model proceeds in two steps. First, a risk-averse employee with excess holdings in the firm (from ESO grants, stock holdings and human capital) in which he is employed chooses the ESO exercise policy that maximises the expected present value of the option payoff. The implicit discount rate used in this optimisation,

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3 For a full description of the model, which uses continuous time mathematics, see Ingersoll (2003). The VBA code that we use to solve this model is available on request.
and hence the optimal exercise policy, depends on the employee's risk aversion and under-diversification. Second, the cost to the firm is calculated by discounting, at the appropriate market rate, the expected ESO payoff generated by the employee's exercise policy. Somewhat loosely, we can think of the first step as identifying the optimal employee-specific exercise date, with the second step then calculating the market value of exercising the option at this date. The latter is the theoretically correct value of a non-traded American option; we henceforth refer to this as the actual cost model.4

A simple example may help illustrate this approach. Consider an ESO that (i) is written on a stock currently trading at $10, (ii) is exercisable in either of the next two years, (iii) has an exercise price of $10.30. In addition, as shown in Figure 1, the price $S_t$ of the stock (which does not pay dividends) either rises by 20% or falls by 15% with equal probability in each of the next two years.

[Insert Figure 1 about here]

The value of this ESO is determined by, and simultaneously with, the optimal exercise policy. The general nature of the latter is obvious: keep the ESO alive if its value is greater than the exercise payoff, otherwise exercise. At date 2, the value from retaining the ESO is zero (since it expires at that date), so a necessary and sufficient condition for exercise is simply that the exercise payoff be positive. As a result, exercise occurs at date 2 if and only if $S_2 = $14.40, since this is the only state in which the stock price exceeds the exercise price. At date 1, however, retaining the ESO is a viable alternative to exercising it, so the latter is justified if and only if the payoff exceeds the value of retaining the option until date 2. Clearly, this is possible if and only if $S_1 = $12; whether or not it is optimal depends on the date 1 value of the ESO. In turn, the current (date 0) ESO value depends on whether or not exercise will occur at date 1.

4 We focus solely on the issues associated with early exercise and do not consider possible dynamic effects resulting from time-varying parameters.
To obtain the date 0 ESO value, we proceed in three discrete steps. First, we calculate the date 1 ESO value, given the exercise policy that will prevail at date 2. Second, we use this value to determine the optimal exercise policy at date 1. Third, we use the date 1 payoffs implied by this policy to calculate the date 0 ESO value. The date 1 value is obviously crucial to this process since it determines the date 1 exercise policy and hence the current value. Because the ESO is not tradable, a risk-averse employee is unable to hedge its firm-specific risk, and hence, in maximising his personal expected utility, requires additional compensation for bearing that risk. Accordingly, the date 1 value of the ESO to the employee will be less than its market value, and thus the employee is more likely to exercise at date 1 than purely market considerations would suggest.

To make this point concrete and demonstrate its implications, we assume that the implicit discount rate embedded in the market value of the ESO is 10%, while that used by a hypothetical, risk-averse, and under-diversified employee is 25%.\(^5\) Then the market value of the ESO at date 1 is

\[
V_{m1} = \frac{\{\text{probability of } S_2 = 14.40\} \times \{\text{exercise payoff if } S_2 = 14.40\}}{1.1} \\
= \frac{\{0.5\} \times \{14.40 - 10.30\}}{1.1} \\
= 1.86
\]

Since $1.86 > $1.70 (the payoff from exercising the ESO at date 1), the ESO is worth more alive than dead at date 1 and so the optimal policy from the market's perspective is to delay exercise until date 2. As a result, the date 0 market value of the ESO is simply the present value of an asset that is worth $1.86 in one year's time with probability 0.5 and zero otherwise. That is

\[^5\text{Note that Figure 1 implies that the discount rate on the stock itself is 2.5\%, so the ESO is significantly riskier than the stock. This is a standard feature of call options such as ESOs - see Boyle and Irwin (2004). In the next section, we allow all these discount rates to be determined endogenously as part of the equilibrium process, rather than arbitrarily set as in this example.}\]
\[
V_{m0} = \frac{\{\text{probability of } S_1 = 12\} \times \{\text{ESO value if } S_1 = 12\}}{1.1} \\
= \frac{0.5 \times \{1.86\}}{1.1} \\
= 0.85
\]

The calculation from the under-diversified employee's perspective is similar, but all payoffs are discounted at 25% rather than 10%. His subjective date 1 valuation of the ESO is

\[
V_{e1} = \frac{\{\text{probability of } S_2 = 14.40\} \times \{\text{exercise payoff if } S_2 = 14.40\}}{1.25} \\
= \frac{0.5 \times \{14.40 - 10.30\}}{1.25} \\
= 1.64
\]

which is less than the date 1 exercise payoff (1.70), so the optimal policy from the employee's perspective is to exercise at date 1. Therefore, the employee's date 0 value of the ESO is equal to his subjective present value of an asset that is worth $1.70 in one year's time with probability 0.5 and zero otherwise. That is

\[
V_{e0} = \frac{\{\text{probability of } S_1 = 12\} \times \{\text{ESO value if } S_1 = 12\}}{1.25} \\
= \frac{0.5 \times \{1.70\}}{1.25} \\
= 0.68
\]

which is 20% less than the market value of 0.85. However, neither \(V_{m0}\) nor \(V_{e0}\) represents the cost of the ESO to the firm; the former assumes an exercise policy, and hence an expected payout, different from that actually followed by the employee, while the latter assumes an opportunity cost of funds different from that faced by the firm. Instead, the actual cost to the firm is the present market value of the expected liability created by the employee's exercise policy. That is
\[
C_0 = \frac{\{\text{probability of } S_1 = 12\} \times \{\text{ESO market value if } S_1 = 12\}}{1.1} \\
= \frac{0.5 \times \{1.7\}}{1.1} \\
= 0.77
\]

which is less than the market value (0.85), but greater than the employee value (0.68). Under-diversification and risk aversion lower the value of the ESO to the employee, but are irrelevant for market discount rates and hence for the option's market value. But by inducing early exercise of the ESO, they also lower the cost of the ESO to the firm, to a level between between the market value and the employee value. Figure 2 summarises this outcome.

[Insert Figure 2 about here]

Our focus in this paper is on calculation of the actual cost \(C_0\). In contrast to the simple example above, the procedure we follow endogenously determines the market and employee discount rates as functions of market conditions and employee characteristics respectively, so the values we obtain are consistent with market equilibrium (which may not be the case in the example). Of course, the usual difficulties in observing employee characteristics of risk aversion and under-diversification mean that the actual cost model may ultimately have little practical impact. However, by calculating the actual costs of hypothetical ESOs, it can tell us something about the likely accuracy of practical methods that ignore employee-specific parameters. This is the goal of the remainder of this paper.

3. Calculating the cost of ESOs

To illustrate the impact of non-tradability and early exercise on ESO value, we use two hypothetical ESOs, the details of which appear in Table 1. ESO I is an in-the-money option granted by a firm with moderate volatility and dividend yield; ESO II is
an out-of-the-money option granted by a firm with high volatility that pays no dividends. If these two options were traded, exercise prior to expiration would potentially be optimal for ESO I (because of its positive dividend yield), but not for ESO II.

[Insert Table 1 about here]

A. Non-tradability and early exercise

Ignoring any complications created by vesting (i.e., we initially assume the ESOs have vested), we begin by comparing the actual cost of each ESO with its corresponding BSM value. We calculate and report the actual cost for all values of $\alpha$ (employee under-diversification) between 0.00 and 1.00, and for three values of $\gamma$ (employee risk aversion): $\gamma = 1, 5, 10$. A value of $\alpha = 0.5$, for example, means that the employee holds 50% more of his wealth in the firm that employs him than he would if he were unconstrained; given the importance of human capital in the wealth of most employees, such a figure is not especially high. Turning to the risk aversion parameter, the return means and volatilities reported in Lally and Marsden (2004) imply that an unconstrained NZ investor with $\gamma$ equal to one should invest approximately 130% of his wealth in the stockmarket; the corresponding values for $\gamma$ equal to five and ten are 30% and 15% respectively. Cochrane (2001) notes that an average value of $\gamma$ is usually taken to be between three and five.

Figure 3 plots the value of ESO I. For low employee risk aversion ($\gamma = 1$), the cost to the firm is very close to the BSM value of $0.26$. Indeed, unless the employee has virtually all of his wealth tied up in the firm, the actual ESO cost is greater than the BSM value. This reflects the optimality of early exercise in the presence of a positive dividend yield. For more risk-averse employees, however, the desired exercise date is sufficiently early to be significantly sub-optimal from the market's

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6 The BSM model is sufficiently well known to not require explicit description here; discussions are standard fare in all introductory finance textbooks. For a detailed explanation, see Crack (2004).
perspective and the actual cost falls below the BSM value. The deviation can be significant: a risk-intolerant employee ($\gamma = 10$) with an additional 50% of his wealth tied up in his firm creates an ESO cost that is little more than half the BSM value. Intuitively, such an employee chooses to eliminate the diversification risk associated with the ESO by exercising it particularly early. Although this maximises the expected payoff discounted at the employee's (high) subjective discount rate, it results in a lower (relative to holding until expiration) discounted expected payoff from the market's perspective.

[Insert Figure 3 about here]

The situation with ESO II is somewhat different. With much higher stock price volatility, even risk-tolerant employees desire exercise that is sub-optimally early from the market's perspective (especially as the stock pays no dividends), and so the cost to the firm is always, and often substantially, less than the BSM value. Even a moderately risk-averse ($\gamma = 5$) and moderately over-exposed ($\alpha = 0.3$) employee costs the firm only 40% of the BSM value of $0.59$. For such an ESO, the BSM model provides inaccurate estimates of the cost to the firm.

[Insert Figure 4 about here]

The differences between the various curves in Figures 3 and 4 reflect differences in exercise policy. For example, the market value-maximising expected time to exercise for ESO I is 4.4 years (resulting in a current value of $0.28$), but an employee with $\gamma = 5$ and $\alpha = 0.25$ expects to exercise before three years are up (lowering the ESO cost to $0.25$); increasing $\alpha$ to 0.75 means that exercise is expected to occur in approximately fifteen months (with a resultant ESO cost of $0.17$). Although the positive dividend yield on the ESO I stock means that some degree of early exercise is likely to be optimal from the market's perspective, the combined
effects of risk aversion and under-diversification lead holders to exercise earlier still, thereby lowering the market value of the ESO.

Figures 3 and 4 confirm the commonly held view that the BSM value may differ significantly from the actual cost incurred by firms in issuing ESOs. This raises the question of whether the BSM model can be adjusted in some simple way to approximate the effects of under-diversification and early exercise. In this regard, the usual recommendation - contained in IFRS 2 (para B17) - is to use the expected exercise date in place of the expiration date in the BSM model.

To evaluate this suggestion, we recalculate the BSM value using the expected date of exercise implied by the actual cost model. In panel (a) of Figure 5, we plot the resulting estimates for ESO I and compare these with its actual cost; panel (b) depicts the corresponding comparison for ESO II. To avoid cluttering up the figures, we include only the curves for moderate employee risk aversion ($\gamma = 5$).

The accuracy of the exercise date adjustment to the BSM model differs significantly across the two ESOs. For ESO I, the adjusted BSM model approximates the actual cost very closely, but the divergence is substantial for ESO II. The difference reflects the much greater volatility of the stock covered by ESO II; the value of an option is a non-linear function of the time to expiration, so the accuracy of a linear approximation decreases with volatility.

[Insert Figure 5 about here]

While the comparisons depicted in Figure 5 might suggest that the standard adjustment to the BSM model for non-tradability and early exercise is accurate only so long as the underlying stock is of no more than moderate volatility, even this overstates the case. The adjusted BSM values in Figure 5 are calculated using employee-specific expected times to exercise, which are themselves dependent on employee $\gamma$ and $\alpha$.

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7 The expected exercise date is calculated with respect to the risk-neutral distribution.
values. As these parameters are generally unobservable (indeed, if they were available, then there would be no reason to assess the BSM adjustment as the actual cost model could be used directly), the BSM adjustment used in Figure 5 is infeasible in practice.

The standard, and probably the only practical, way of estimating the expected time to exercise is to extract some measure, typically the average, from data on previous exercise decisions. Thus, for example, if past experience revealed that the average exercise date was one month after vesting, then the expected time to exercise in the BSM adjusted model would be 19 months for ESO I and seven months for ESO II. However, such a procedure ignores employee-specific variation in the expected time to exercise and hence in ESO cost. In terms of Figures 3-5, any BSM estimate based on a generic expected exercise date is a horizontal line, and so will approach the actual cost of any particular ESO only by good fortune.

To illustrate this point explicitly, we re-calculate the BSM value for both ESOs using an expected exercise date equal to one month after vesting, and compare this with the actual cost. As can be seen in Figure 6, the adjustment arguably makes matters worse. Whereas previously (Figures 3 and 4) the BSM value was close to the actual cost for employees with low risk aversion and generally an upper bound for less risk tolerant holders, now it is a significant under-estimate for employees with low risk aversion and subject to pricing errors of both sign for others. In other words, the adjustment has increased the absolute magnitude of error in some cases, while in others it has introduced ambiguity, where none previously existed, about the sign of error.

Note that this outcome is not an artifact of the particular expected exercise date we have assumed. *Any* exercise date used in *any* model that generates an option value which is independent of employee characteristics will have similar problems. Given that actual ESO costs are so sensitive to employee characteristics, it is obvious that any method or model that ignores these characteristics will produce cost estimates that are subject to substantial error.

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8 Of course, it also ignores variation over time in stock price volatility and the terms on which ESOs are granted, but we do not address these issues here.
Option pricing models that are more flexible than BSM, such as the Binomial model, cannot overcome this problem. If the exercise policy embedded in the Binomial model is not related to employee characteristics, then the resulting value is no more useful than the BSM value precisely because it ignores inter-employee variation. And if employee characteristics are incorporated in the Binomial model calculation, then the problem of observing these characteristics in order to apply the model in practice arises. In the absence of reliable information about employee characteristics, firms have the opportunity to manipulate ESO valuations in self-interested ways, thereby negating the intention that these valuations should provide more information to investors.

B. Vesting

Vesting restrictions have two effects on ESO cost. First, they increase the probability that some options will never become eligible for exercise because, for example, workers cease employment with the firm before the vesting period ends. Second, they reduce the period of time in which exercise can occur. IFRS 2 requires that the first effect be recognised by adjusting the quantity, rather than the price, of ESOs. Here we focus on the second effect as this relates most closely to the exercise policy issue analysed above.

To do so, we recalculate the actual costs of our hypothetical ESOs incorporating the vesting restrictions listed in Table 1 (i.e., that ESO I cannot be exercised for 18 months and ESO II for six months). In Figure 7, we illustrate the effects of this restriction for the latter option. The solid curve depicts the actual cost function for a vested ESO II held by an employee with $\gamma = 5$; the dashed curve shows

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9 In this situation, the Binomial model is essentially the actual cost model with longer time steps.
10 The vertical difference between the two curves is even more pronounced for the longer vesting period of plan I.
the same function when vesting occurs in six months time. The latter lies on or above the former, so imposing vesting restrictions raises the cost of ESO II.

[Insert Figure 7 about here]

This result seems surprising, insofar as restricting the dates on which ESOs can be exercised unambiguously reduces their value to employees, but the reason is straightforward: employees who would otherwise have exercised their options sub-optimally early from the market's perspective (i.e., prior to the vesting date) are now forced to wait until a later date that offers an expected payoff with a higher market value. Note, however, that this need not always be the case: if market value maximisation required exercise before the vesting date (as might be the case for a firm with a high dividend yield, for example), then vesting would enforce sub-optimal late exercise and thus lower the ESO cost.

Regardless of whether it raises or lowers ESO cost, vesting always reduces the sensitivity of ESO cost to employee characteristics. As Figure 7 indicates, the slope of the curve depicting the relationship between ESO cost and holder under-diversification is lower with a six-month vesting period. Similarly, the vertical gap between curves of different risk aversion (not drawn) is also smaller with vesting. In effect, vesting places restrictions on employees' abilities to exercise early and thus on their freedom to choose an exercise policy that optimally reflects their individual circumstances.

The importance of this point is that vesting helps reduce the inaccuracy of market-based option pricing models that ignore employee characteristics. As the vesting period increases, the importance of these characteristics for ESO cost becomes smaller and the BSM figure approaches the actual cost.
4. Concluding Remarks

Using models of market-traded options to determine the cost of ESOs is a task fraught with difficulty. Because ESOs cannot be traded, employees choose to exercise earlier than would otherwise be the case, the extent of which depends on risk aversion and under-diversification stemming from over-exposure to the issuing firm. Although such early exercise is optimal from the employee's perspective, it is frequently premature from the perspective of the market, and hence ESOs are generally less costly than otherwise-equivalent traded options.

Because the cost of ESOs is sensitive to characteristics of the employee to whom they are granted, standard market-based models that ignore these characteristics are likely to produce value estimates that differ substantially from the true cost. Commonly-cited adjustments, such as using the expected exercise date in place of the expiration date in the BSM model, or determining the optimal exercise date endogenously within the Binomial model, leave this fundamental problem unresolved. The difficulty with applying market-based models to ESOs is not early exercise per se, but rather the factors underlying such early exercise.

The principal difficulty facing ESO valuation is straightforward. On the one hand, standard market-based models of option pricing ignore features integral to the cost of ESOs. On the other hand, quantification of these features is necessarily subjective, thereby creating the potential for manipulation.

Fortunately, to end on an optimistic note, there are practical methods for mitigating this problem. First, it should be possible to 'back out' information about employee characteristics from data on actual exercise decisions. If such analysis revealed that these characteristics were clustered in a fairly tight range, then the issues highlighted in this paper are of less importance. Second, as shown in section 3B, lengthy vesting periods reduce the sensitivity of ESO cost to holder characteristics. Third, the actual cost model assumes that ESO holders adopt an exercise policy that is optimal given their own particular circumstances. However, Carpenter (1998) shows that incorporating an exogenous stopping process in the standard market-based model
for pricing American options explains actual exercise decisions about as well as a more
general employee-specific model. This suggests that a simple extension to market-
based models may provide accurate ESO valuations without the need to incorporate
subjective estimates of employee characteristics. Future research should investigate
these issues in more detail.
References
Today (at date 0), the stock price is $10. In subsequent years, it either rises by 20% or falls by 15%, both with probability 0.5.
Today (at date 0), the stock price is $10. In subsequent years, it either rises by 20% or falls by 15%, both with probability 0.5. The ESO can be exercised at either date 1 or date 2 in return for a payment of $10.30. $V_{mt}$ is the date $t$ market value of the ESO; $V_{et}$ is the corresponding subjective employee value; $C_0$ is the date 0 cost to the firm.

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<th>Date 2</th>
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Table 1

**ESO Details**
Parameter values for two hypothetical ESOs. In addition, the riskless interest rate is 5%, market volatility is 20%, and the beta of both underlying stocks is 0.9.

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<th>ESO II</th>
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Figure 3

ESO I Values
This figure plots the cost of ESO I for various combinations of employee risk aversion ($\gamma$) and under-diversification ($\alpha$), and compares these with the Black-Scholes-Merton (BSM) value. Parameter values for ESO I are in Table 1.
Figure 4

ESO II Values
This figure plots the cost of ESO II for various combinations of employee risk aversion ($\gamma$) and under-diversification ($\alpha$), and compares these with the Black-Scholes-Merton (BSM) value. Parameter values for ESO II are in Table 1.
Figure 5

Actual Cost versus BSM Value when Time to Expiration equals True Expected Exercise Date

For an employee with $\gamma = 5$, this figure plots the ESO value from the BSM model using the expected exercise date implied by the actual cost model, and compares it with the value obtained from the actual cost model.

Panel (a): ESO I

Panel (b): ESO II
Figure 6

Actual Cost versus BSM Value when Time to Expiration equals Vesting Date plus One Month

This figure plots the ESO cost to the firm for various combinations of employee risk aversion ($\gamma$) and under-diversification ($\alpha$), and compares these with the Black-Scholes-Merton (BSM) value when the time to expiration is set equal to one month after the vesting date. Parameter values for the two ESOs are in Table 1.

Panel (a): ESO I

Panel (b): ESO II
Figure 7

The Effect of Vesting on Actual ESO Cost
For an employee with $\gamma = 5$, this figure plots the actual cost of ESO II under conditions of vesting (six months) and no vesting.