Reverse Design Transformer Modelling Technique with Particular Application to Partial Core Transformers

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A thesis presented for the degree of Doctor of Philosophy in Electrical and Electronic Engineering at the University of Canterbury, Christchurch, New Zealand.

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Dedicated to my dear parents and brothers,

With wisdom, there no delusion;
with benevolence, there is no worry;
with courage, there is no fear.

To gain knowledge quietly;
to learn without losing interest;
to instruct other relentlessly.

Meet the virtuous and think how to be their match;
meet those not virtuous and examine oneself.

Learning without thought is labour in vain;
thought without learning is desolation.

To acknowledge what is known as known;
and what is not known as not known;
that is knowledge.

Walking in a company of three,
there is always something I can learn;
choose to follow what is good in them,
and correct what is not good.

-Confucius (551 B.C.)
This thesis first describes the conventional transformer design method used for design iron-core transformers. Limitations associated with this design method is highlighted. In this thesis, an alternative transformer design method is presented. It is called the reverse design method. This new design technique is compared against the conventional design method, and validated with experimental results.

The reverse design method is applied to partial core transformers. Modifications made to accommodate full-core equivalent circuit components to partial core transformers are discussed. Particular attention is given to the derivation of core loss resistance, core magnetising reactance and winding leakage reactances. The new reverse design partial core model is applied at 50Hz normal operating temperature applications. The model is verified with experimental results.

Next the reverse design model is applied to transformers when immersed in liquid nitrogen. The accuracy of the model derived previously for normal operating temperatures is investigated. Necessary modifications are made to the model. The corrected model is again justified with experimental results.

Finally, the model is used for the harmonic frequency analysis of partial core transformers. Capacitive components are included as part of the analysis. Frequency responses of transformers with relatively low turn ratio are analysed, followed by high voltage partial core transformers with large turn ratio. Comparisons are made between the model calculated and test results.
LIST OF PUBLICATIONS

The following papers have either been published, accepted or submitted during the course of the research described in this thesis:


In addition, a patent application has also been filed in conjunction with this research:

ACKNOWLEDGEMENTS

Writing these last paragraphs are probably the happiest moment of my postgraduate life, and yet one of the most difficult ones. I am now leaving the work behind. The past four years have always been packed with learning, questions, stress and emotions. With this I can now look forward to the next challenge of my life.

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GLOSSARY

General notation

In the following, $Q$ and $K$ are generic variables. Frequent use is made of other variables for local explanations, but these are explained in the immediate context.

\[
\begin{align*}
V_Q, v_Q & : \text{Voltage of } Q \\
I_Q, i_Q & : \text{Current of } Q \\
R_Q, p_f Q, S_Q & : \text{Real power loss, power factor, volt-ampere of } Q \\
R_Q, L_Q, C_Q & : \text{Resistance, inductance, capacitance of } Q \\
X_Q, Y_Q, Z_Q & : \text{Reactance, admittance, impedance of } Q \\
G_Q, B_Q & : \text{Conductance, susceptance of } Q \\
\Re(Q) & : \text{Real part of } Q \\
\Im(Q) & : \text{Imaginary part of } Q \\
Q', Q'' & : Q \text{ referred to the primary, secondary} \\
\hat{Q}, Q^* & : \text{Complex value, complex conjugate of } Q \\
|Q| & : \text{Magnitude of } Q \\
\mu_Q & : \text{Relative permeability of } Q \\
\epsilon_Q & : \text{Relative permittivity of } Q \\
\gamma_Q & : \text{Material density of } Q \\
\rho_Q & : \text{Resistivity of } Q \\
\Delta \rho_Q & : \text{Thermal resistivity coefficient of } Q \\
\rho_Q, K^\circ \text{C} & : \text{Resistivity of } Q \text{ at } K^\circ \text{C} \\
Wgt_Q & : \text{Weight of } Q \\
V_Q & : \text{Volume of } Q \\
\text{Cost}_{Q}, \text{Cost}_{\text{wgt}} Q & : \text{Cost, cost per weight of } Q \\
T_Q & : \text{Operating temperature } (^\circ \text{C}) \text{ of } Q \\
W_Q & : \text{Energy stored in } Q \\
\kappa_Q & : \text{Reluctance of } Q \\
\bar{l}_Q & : \text{Mean circumferential length of } Q \\
w_i Q, w_o Q, w_A Q & : \text{Inner, outer, average width of } Q
\end{align*}
\]
Transformer in general

\( \mu_0 \) Permeability of free space = \( 4\pi \times 10^{-7}\text{Hm}^{-1} \)

\( \varepsilon_0 \) Permittivity of free space = \( 8.854 \times 10^{-12}\text{Fm}^{-1} \)

\( w \) Angular frequency

\( f \) Frequency

\( t \) Time

\( a \) Turn ratio

\( \theta \) Phase angle between the voltage and current

\( \eta, \text{EFF} \) Transformer efficiency

\( V_{\text{REG}} \) Transformer voltage regulation

\( V_{\text{TF}} \) Voltage per turn factor

\( W_W \) Winding window width

\( W_H \) Winding window height

\( W_{WF} \) Window width factor

\( \beta_a \) Transformer aspect ratio

\( R \) Average radius of the transformer

\( \tau_{12} \) Winding thickness factor

\( \phi_{\text{air}} \) Magnetic flux in the air

\( A_{\text{air}} \) Effective cross-sectional area of the magnetic flux in the air

\( B_{\text{air}} \) Magnetic flux density in the air

\( x_a \) Vertical distance where flux travels in the air

\( a_a \) Horizontal distance where flux travels in the air

\( l_{\text{air}} \) Effective air path length

\( l_T \) Overall flux path length

\( A_T \) Overall cross-sectional area of the transformer

\( \Gamma \) Rogowski factor

\( \Gamma(\beta_a) \) Leakage function

\( \zeta \) A function determining the rate of saturation of \( \Gamma(\beta_a) \)

Transformer core

\( \phi \) Magnetic flux

\( \phi_m \) Peak magnetic flux value

\( B \) Magnetic flux density

\( B_{\text{crms}} \) rms value of magnetic flux density

\( B_{\text{core}} \) Peak value of magnetic flux density

\( H_{\text{core}} \) Magnetic field strength
GLOSSARY

$S_F_c$  Core stacking factor
$A'_{core}$  Cross-sectional area of the core
$A_{core}$  Cross-sectional area of the core including stacking factor
$l_{core}$  Transformer core length
$w_{core}$  Transformer core width
$b_{core}$  Transformer core breadth
$n$  Number of laminations
$\delta_{ec}$  Skin depth of the eddy current in the core
$l_{ec}$  Effective eddy current path
$p_{ec}$  Loss density of the eddy current in the lamination
$c_{lam}$  Lamination thickness
$E_{lam}$  Electric field in the lamination
$J_{lam}$  Current density in the lamination
$A_{ecn}$  Effective cross-sectional area of the lamination
$A_{eff\ n}$  Effective area of one lamination
$A_{eff}$  Total lamination area
$k_h$  Hysteresis constant
$x$  Steinmetz factor
$IC1$  Core insulation thickness
$\varnothing$  Partial core saturation factor
$k_\varnothing$  Partial core saturation constant
$\alpha_\varnothing$  Partial core saturation power constant
$r_{core}$  Effective radius of the core
$\hat{A}$  An area constant
$\hat{t}$  A length constant
$k_{hpc}$  Partial core hysteresis loss constant
$\alpha_{hpc}$  Partial core factor
$\Upsilon(\beta_a)$  Magnetising function
$\zeta$  A function determining the rate of saturation of $\Upsilon(\beta_a)$

Transformer primary winding

$e_1$  Induced voltage in the primary winding
$E_1$  rms value of $e_1$
$N_1$  Number of primary turns
$VT_1$  Primary voltage per turn ratio
$J_1$  Primary winding current density
$SFP_1$  Primary winding space factor
\( A_1 \)  Primary winding wire cross-sectional area
\( t_1 \)  Primary winding wire thickness
\( A_{W1} \) Primary winding window area
\( A_{cu1} \) Primary copper cross-sectional area
\( L_{y1} \) Number of primary winding layers
\( w_1 \) Primary winding conductor thickness
\( w_{i1} \) Primary winding wire insulation thickness
\( l_1 \) Effective length of primary winding conductor
\( d_1 \) Primary winding thickness factor

**Transformer secondary winding**

\( e_2 \) Induced voltage in the secondary winding
\( E_2 \) rms value of \( e_2 \)
\( N_2 \) Number of secondary turns
\( VT_2 \) Secondary voltage per turn ratio
\( J_2 \) Secondary winding current density
\( SF_2 \) Secondary winding space factor
\( A_2 \) Secondary winding wire cross-sectional area
\( t_2 \) Secondary winding wire thickness
\( A_{W2} \) Secondary winding window area
\( A_{cu2} \) Secondary copper cross-sectional area
\( L_{y2} \) Number of secondary winding layers
\( w_2 \) Secondary winding conductor thickness
\( w_{i2} \) Secondary winding wire insulation thickness
\( l_2 \) Effective length of secondary winding conductor
\( d_2 \) Secondary winding thickness factor

**Transformer interwinding space**

\( \Delta d \) Interwinding thickness factor
\( I12 \) Primary-secondary interwinding insulation thickness
Chapter 1

INTRODUCTION

1.1 GENERAL OVERVIEW

Alternating current (AC) supply systems have long become an integral part of the power system network. This is due fundamentally to the ease with which AC can be stepped up or down in voltage. This greatly facilitates the transmission of high powers over considerable lengths of line. The piece of equipment which makes this possible and in fact contributes more than any other to the success of AC systems is the transformer.

A transformer is a device by which power supplied at one voltage is converted to another voltage. It was first developed more than a century ago by Nikola Tesla [Lowdon, 1989]. Transformers can only operate on an alternating, or, at least a fluctuating supply. Running a transformer from a steady d.c supply would yield no output voltage.

Transformers are made in sizes ranging from tiny coupling types used in hearing aids and portable electrical equipment, to the enormous ones seen in power networks as well as high voltage testing laboratories. In heavy electrical engineering many transformers are used to step up voltages to transmission levels and to step down to distribution voltages for consumers. In addition, the system uses large numbers of instrument transformers which enable high voltages and heavy currents to be measured by meters at distant points, without taking the main cables there and without exposing attendants to the risk of shock from high voltages. The protection circuits of power systems also use many instrument type transformers for relay operation. Furthermore, they are also used for electrical isolation, to block DC signals, to eliminate electromagnetic noise, and to provide safety in electrical appliances [Nasar, 1984]. In laboratories, transformers are frequently required for feeding rectifiers for small d.c supplies, for regulating and adjusting voltages, and for special applications, such as the production of peaky or flattened waveforms.
1.2 THESIS OBJECTIVES

Power system and high voltage testing transformers are usually fully iron-cored. Their construction is generally big and complex in order to minimise energy losses, to produce a significant flux flow in the iron core by a current in the primary winding, and to see that as much of that flux as possible links as many of the turns as possible of the secondary windings on the core [Sarma, 1985]. They are very bulky in nature, not to mention the tremendous weight they can have. This is because of the large amount of laminated steel core material needed. Also, the amount of copper conductors used is immense. The conductor size is also massive to allow for the enormous current capacity as high as 20,000A [Heathcote, 1998]. For AC voltages as high as 500kV, adequate clearance between the core and the winding, between windings and between the winding and the transformer tank is critical. Insulation, in the form of transformer oil, is also necessary to prevent breakdown of transformers.

There are limitations on the conventional approach to transformer design. In the conventional transformer design method, the terminal voltages, VA rating and frequency are entered as design specifications which determine the materials to be used and their dimensions. It estimates a number of factors for the core and winding arrangement, using values that are generally only known to experienced design engineers. Core and winding material characteristics are known from standard values or physical measurements. The resultant design may not match what is actually available in materials and hence the predicted performance can be in error.

In this thesis, an alternative transformer design method called the reverse design method is presented. This reverse design method is developed for fundamental frequency transformers and tested against the conventional design method. While it is possible to design a full-scale large transformer using this method, building it would be a tremendous task. Substantial amounts of time and resources would have to be spent for the assembly of the transformer. Therefore, the feasibility of the reverse design method is studied using small transformers, which cost much less and are faster to build. Hence, all the transformers designed and tested in this thesis have ratings less than 10kVA.

There is also an intention of developing a transformer which is lighter in comparison with the full-core transformer and hopefully more economical to manufacture. The partial core transformer, which has usually been utilised for applications at a higher frequency range (10 – 100kHz) [Connelly, 1965], is revisited. It is designed for power system frequencies. The feasibility of the partial core transformer operating at harmonic frequencies (50 – 5kHz) is also explored. Compared to full-core transformers,
partial core transformers have a relatively higher magnetising current, and they can be used as shunt reactors for the purpose of reactive power compensation, especially in very high voltage transmission systems [Yamaguchi et al., 1995b].

The reverse design method is applied to the partial core transformers and models are tested under various conditions, namely:

- at 50Hz
- at normal operating temperatures
- at liquid nitrogen temperature
- at harmonic frequencies between 50Hz – 5kHz

High temperature superconducting (HTS) transformers have been studied extensively in recent years because of their potential superiority over conventional oil-filled transformers. They are more efficient, much lighter and environmentally safer compared to their conventional counterparts. In this thesis, an attempt is made to explore the feasibility of a partial core transformer operating at extremely low temperatures. This will lay the platform for the intended development of an HTS partial core transformer within the department.

1.3 THESIS OUTLINE

Chapter 2 gives a brief introduction to the conventional full-core transformer. The theory of a full-core transformer is covered. The conventional transformer design method, including detailed formulations, is also presented.

Chapter 3 details the work done on the new transformer design method, the reverse design method. The computer program routine for this reverse design approach is presented. Detailed derivations for each module are provided. It is then applied to two sample high voltage transformers. The performance of the reverse design method is compared and validated against the conventional transformer design method. Both methods are compared to data measured on the transformers as they were built.

Chapter 4 describes the application of the reverse design method to partial core transformers. It briefly outlines the difference between partial core and conventional full-core transformers. It covers the modifications made to the equivalent circuit components of full-core transformers in order to model partial core transformers. These include the magnetising reactance, the winding leakage reactances, and
the core loss resistance. New formulations are derived for these components. The new reverse design partial core model is validated with experimental results.

Chapter 5 details further application of the reverse design method to partial core transformers at a different operating temperature. The accuracy of the partial core model is tested in an extremely low-temperature environment. Three sample transformers were built and then tested under liquid nitrogen ($N_2$) conditions. Further modifications to the model are discussed. The modified model is again validated with experimental results.

Chapter 6 describes a harmonic frequency analysis of the partial core model. The accuracy of the resistance and inductive reactance components is compared to test results for frequencies up to 5kHz. The chapter details necessary modifications made to the equivalent circuit components of the partial core model. Once comparisons are made between the model and the measured values, the capacitive reactance components are incorporated into the model. The modified partial core model is verified against experimental results by designing a partial core transformer with a very high turn ratio.

Chapter 7 discusses the directions for future research and development.

Chapter 8 contains the main conclusions to this thesis.
Chapter 2

CONVENTIONAL TRANSFORMER DESIGN

2.1 OVERVIEW

In this chapter, the theory of a full-core transformer is presented. The theory of an ideal transformer is detailed, followed by the inclusion of non-ideal components to the transformer. Phasor representations of the transformer under no-load and loaded conditions are outlined. Finally, the conventional design method, including detailed formulations, is presented.

2.2 BACKGROUND

Transformers take many different forms according to the work they have to do, but the basic features are always recognisable.

![Diagram of a conventional full-core transformer](image)

**Figure 2.1** A conventional full-core transformer

With respect to Figure 2.1, there is a primary or input winding connected to the power supply, a core of magnetic material around which the primary is wound, and a secondary winding from which the output is taken, also wound on the core. There is no electrical connection between the input and output sides — power is transferred from one to the other solely by electro-magnetic induction. The two electrical circuits
on the input and output sides are linked by the magnetic field in the core [Hindmarsh, 1977] [Steinmetz, 1895].

A transformer has been regarded as an arrangement of two windings which are coupled together magnetically through an iron core. The primary winding is connected to a voltage source, while the secondary winding delivers power to the load at a different voltage. The core is formed of a stack of steel laminations. The steel has a high magnetic permeability and provides a high-permeance path for the flux, which is mutual to the primary and secondary. The thin laminations are electrically insulated from each other. This limits the flow of eddy current due to voltages induced in the core itself [McPherson and Laramore, 1990].

Heat will be produced when the transformer is in operation. Therefore, the heat generated in the transformer must be removed in order to avoid overheating and consequently breakdown of insulation. For small transformers with ratings typically up to a few kVA’s, natural air cooling is satisfactory. The heat is removed by radiation and convection either immediately from the core and coil surfaces or from the protective enclosure surrounding them. However, for larger transformers, they are immersed in oil-filled tanks. Oil is used for cooling, and also for insulation since it has a higher breakdown strength than air. The heat is passed to the oil which circulates round the tank by natural convection, thus carrying the heat to the walls of the tank whence it is dissipated. In some very large transformers, radiators and forced circulation of oil in the tank and air outside it are employed.

The tank provides mechanical protection to the transformer. It must be able to withstand the stresses imposed by jacking and lifting the transformer. It should be no larger than is necessary to accommodate the core, windings and internal connections with appropriate clearance [Say, 1983]. Figure 2.2(a) shows a picture of a 3-phase transformer under assembly, with the tank being lowered to enclose the core and windings. The ends of the windings are brought to a terminal block, from which leads are brought to the outside of the tank through insulating bushings mounted in holes in the sides of the tank or in the lid. The high voltage bushings are clearly seen in Figure 2.2(b).

At first glance, it would appear to be quite possible to calculate the performance of a transformer under any specified conditions by means of the coupled-circuit theory. However, since there is variation in the permeability of the iron core material over the cycle of the applied voltage, the magnetic flux distribution within the core becomes non-uniform. In addition, there are various power losses within the core and windings, which are also dependent on the frequency of the applied voltage. As a result, the calculations would be very laborious. Consequently, it is better to start by assuming that a transformer is ideal in which the core is assumed to have a very high and constant
permeability, and free from power losses. Later, external circuit elements can be added to represent departures of the actual transformer from the ideal [Slemon and Straughen, 1980].

Figure 2.2 Pictures of 3-phase transformers (taken from [Heathcote, 1998])

2.2.1 The Ideal Transformer

In an ideal transformer, it is assumed that

1. the resistances of the windings are negligible,
2. the capacitances of the windings can be neglected,
3. all the magnetic flux links all the turns of both primary and secondary windings,
4. the permeability of the core material is extremely high and constant,
5. the hysteresis loss in the core is negligible,
6. there are no eddy current losses in the core, fittings, or windings.

These assumptions are equivalent to the statement that the transformer has no winding resistances, no magnetising current, no core losses, and no magnetic leakage.
Figure 2.3 shows the schematic of a two-winding transformer and the equivalent circuit of an ideal transformer.

![Schematic and Equivalent Circuit of a Transformer](image)

(a) The schematic

(b) Equivalent circuit

**Figure 2.3** An ideal full-core transformer

The basic components are the core, primary winding and the secondary winding with \( N_1 \) and \( N_2 \) turns respectively. According to Faraday’s law of electromagnetic induction [Bell, 1988], *emf’s* \( e_1 \) and \( e_2 \) are induced in \( N_1 \) and \( N_2 \) owing to a finite rate of change of flux \( \phi \) such that

\[
e_1 = -N_1 \frac{d\phi}{dt} \tag{2.1}
\]

and

\[
e_2 = -N_2 \frac{d\phi}{dt} \tag{2.2}
\]

With the transformer being ideal, \( e_1 = v_1 \) and \( e_2 = v_2 \). The direction of \( e_1 \) is such that it produces a current which opposes the flux change, according to Lenz’s law. Using the dot convention, from Equations 2.1 and 2.2,

\[
\frac{e_1}{e_2} = \frac{N_1}{N_2} \tag{2.3}
\]
If \( E_1 \) and \( E_2 \) are the RMS values of \( e_1 \) and \( e_2 \) respectively, then

\[
\frac{E_1}{E_2} = \frac{N_1}{N_2} = a
\]  
(2.4)

where \( a \) is the nominal turns ratio.

Since \( e_1 = v_1 \) and \( e_2 = v_2 \), the flux and voltage are related by

\[
\phi = \frac{1}{N_1} \int v_1 \, dt = \frac{1}{N_2} \int v_2 \, dt
\]  
(2.5)

In general terms, if the flux varies sinusoidally such that

\[
\phi = \phi_m \sin \omega t
\]  
(2.6)

then the corresponding induced voltage \( e \) linking an \( N \)-turn winding is given by Faraday's law as

\[
e = \omega N \phi_m \cos \omega t
\]  
(2.7)

From, Equation 2.7, the RMS value of the induced voltage is thus

\[
E = \frac{\omega N \phi_m}{\sqrt{2}} = 4.44fN \phi_m
\]  
(2.8)

where \( \omega = 2\pi f \)

\( f \) is the frequency (Hz).

Equation 2.8 is known as the \textit{emf} or Transformer equation [Nasar, 1984].

Assumption (4) is equivalent to saying that the \textit{mmf} required to produce the working flux is negligibly small. This \textit{mmf} is the resultant of the \textit{mmf} due to the primary current and that due to the secondary current so that

\[
N_1 i_1 = N_2 i_2
\]  
(2.9)

Therefore,

\[
\frac{i_1}{i_2} = \frac{N_2}{N_1} = \frac{1}{a}
\]  
(2.10)

Multiplying Equations 2.3 and 2.10 together,

\[
e_1 i_1 = e_2 i_2
\]  
(2.11)

The primary and secondary apparent powers at any instant are thus equal. The primary absorbs power from the sources, whereas the secondary delivers power. In the ideal
transformer, no power is lost internally so that the two quantities are equal.

From Equations 2.3 and 2.10, it can be shown that if an impedance \( Z_2 = \frac{E_2}{I_2} \) is connected to the secondary, the impedance \( Z'_2 \) seen at the primary is given by

\[
\frac{Z'_2}{Z_2} = \left( \frac{N_1}{N_2} \right)^2 = a^2
\]

(2.12)

### 2.2.2 Departures From the Ideal

If the operation of an actual transformer is examined, it will be evident that it departs from the ideal [Paul et al., 1986]. The most obvious defect is the existence of a current in the primary, even when the secondary is open circuited. This current comprises two components: one, which is generally known as the magnetising current, results from a core of finite permeability. Therefore, it requires an appreciable magnetising force to produce the operating flux. This can be represented as a shunt reactance path (designated by \( X_m \)) on the primary side of the transformer. In addition, in practice the iron core has hysteresis and eddy current losses, so that some real power is absorbed at no-load. This can be represented by the addition of a shunt resistance (designated by \( R_{core} \)) at the primary side, through which a core loss current flows.

Both primary and secondary windings have finite resistances which mean that there are real power losses in these windings. These can be represented by a series resistance in each winding (\( R_1 \) and \( R_2 \), respectively). Also, when primary and secondary currents flow, not all the flux set up by the currents passes through the iron core. Instead, there is some leakage flux which passes through the air surrounding each winding. Since the magnetic path through the iron core has a very small reluctance compared to that of the air path around each winding, the leakage flux is usually quite small. However, the leakage flux links with the turns in each winding, and sets up emf’s that oppose the flow of current through each winding. Thus, the leakage flux produces the same effect as an unwanted inductance in series with each winding. The unwanted inductance is also termed the leakage inductance, which is represented by a reactance \( X_1 \) and \( X_2 \) in the primary and secondary windings respectively.

One other factor that contributes to the losses and other undesirable phenomena is the stray capacitance. Stray capacitances inevitably exist between turns, between one winding and another, between windings and the core, as well as between windings and the tank. These capacitances form shunt paths that modify the performance of the transformer. They can be represented by the addition of a shunt capacitance at each winding. Generally, these stray capacitances need to be considered only at
relatively high frequencies, for then the capacitive reactance is relatively low compared to the resistance and inductive reactance. Relatively high levels of stray currents flow through these capacitances and shunt out the other components, and therefore affect the resonance characteristics of the transformer.

Taking all these factors into account, the practical transformer equivalent circuit accordingly takes the form in Figure 2.4.

![Figure 2.4 Practical transformer equivalent circuit](image)

In Figure 2.4, the resistance \( R_{\text{core}} \) is of such a value that the power dissipated in it is equal to the combined hysteresis and eddy current losses [Slemon, 1966]. The values of \( X_m \) and \( R_{\text{core}} \) vary with the operating frequency, owing to the variation of the iron characteristics with different magnetisation levels.

The introduction of the shunt paths through \( X_m \) and \( R_{\text{core}} \) illustrates the conception of the primary current being composed of several components; the magnetising component, the iron-loss component, and the load component. The first two components are present, practically unchanged in magnitude, regardless of whether the transformer is supplying a load or not. Together, they make up the no-load current. On the other hand, the load component is proportional to the secondary current. The effect of the winding resistances \( R_1 \) and \( R_2 \) and leakage reactances \( X_1 \) and \( X_2 \) is to cause a fall in output voltage with increasing secondary current.

### 2.2.3 Transformer Equivalent Circuits

The equivalent circuit of a non-ideal transformer, neglecting the capacitances \( C_1, C_2 \) and \( C_{12} \), is shown in Figure 2.5. This circuit is relevant for low frequency modelling of transformers.

In Figure 2.5, it can be seen that the circuit components denoting imperfections of the transformer are coupled by the ideal transformer of Figure 2.3(b). This ideal
Figure 2.5  Equivalent circuit of a non-ideal transformer

transformer may be removed and the entire equivalent circuit may be referred either to the primary or secondary of the transformer, depending on the problem that is being solved [McPherson and Laramore, 1990]. By using Equations 2.4, 2.10 and 2.12, the equivalent circuit of Figure 2.5 can be referred to the primary, as shown in Figure 2.6. By referring the entire circuit to the primary, the ideal transformer is eliminated, and therefore the transformer can be represented solely by an RL circuit. Such a representation involves a simpler circuit analysis than that of the circuit of Figure 2.5. Moreover, referring the secondary impedance ($Z_2$) to the primary side ($Z'_2$) implies that the real and reactive powers in $Z'_2$ through which the secondary current $I'_2$ flows, remain the same when the primary current $I'_1$ flows through $Z'_2$. This ensures that the performance of a transformer as calculated from the circuit of Figure 2.6 remains the same as the results obtained from the circuit of Figure 2.5. The operation of the secondary components and the load can be viewed from the primary side.

Using the same reasonings as above, the primary circuit components can be referred to the secondary circuit components. This is shown in Figure 2.7. By referring the primary and core impedances $Z_1$ and $Z_{core}$ to the secondary side ($Z''_1$ and $Z''_{core}$ respectively), the real and reactive powers in $Z_1$ and $Z_{core}$ through which currents $I_1$ and $I_o$ flow remain the same when $aI_1$ and $aI_o$ flow through $Z''_1$ and $Z''_{core}$ respectively. The performance of a transformer as calculated from the circuit of Figure 2.7 thus remains the same as that obtained from the circuit of Figure 2.5. In this case, the operation of the primary components can be viewed from the secondary side.
Figure 2.7 Transformer equivalent circuit referred to the secondary side

The equivalent circuit of a transformer is particularly useful in determining its performance and characteristics. For instance, its voltage regulation, efficiency and frequency response\textsuperscript{1} can be determined from the equivalent circuit.

2.2.4 Transformer on No-Load

A concise statement of the mode of operation of a transformer can very conveniently be made in the form of a vector diagram of currents and voltages [Franklin et al., 1983]. The vector diagram shows not only the relative magnitudes of the various quantities but also the phase angles between them. It thus provides a very easy way of adding together components of current flowing in the same circuit. The vector diagrams of the transformer can be examined under different operating conditions.

A transformer is said to be on no-load when the secondary (output) terminals are open-circuited. In this condition there is no current flowing in the secondary winding. If the flux $\phi$ is taken as the starting point from which to build up a vector diagram of an unloaded transformer, the magnetising current $I_m$ will be in phase with $\phi$. The vector diagram of a transformer on no-load is shown in Figure 2.8.

From Figure 2.8, the loss component $I_c$ of the no-load current will be $90^\circ$ ahead of the flux, so that the no-load primary current $I_o$ will be the resultant of $I_m$ and $I_c$.\textsuperscript{2} The induced rms voltages of Equations 2.1 and 2.2 lag $90^\circ$ behind the flux. The primary voltage $-E_1$ will accordingly be $180^\circ$ ahead of the back emf, i.e. $90^\circ$ ahead of the flux. The small component $I_oR_1$ represents the voltage drop in the primary winding due to the flow of $I_o$ through the primary resistance. It will be in phase with $I_o$. In addition, the component $I_oX_1$ represents the voltage drop in the primary leakage reactance. This will be $90^\circ$ out of phase with $I_o$. The resultant of $-E_1$, $I_oR_1$, and $I_oX_1$ will give $v_1$, the

\textsuperscript{1}The frequency response analysis can be performed using the equivalent circuit model of Figure 2.4.

\textsuperscript{2}This is only strictly true if $I_m$ is a sinusoidal quantity; if it is not, it cannot be represented by a vector.
applied voltage from the supply. The angle \( \theta \) between \( v_1 \) and \( I_o \) is the no-load phase angle. \( \cos \theta \) is thus the no-load power factor. In the secondary, the induced voltage \( E_2 \) lags 90° behind the flux. There is no secondary current. The mathematical derivation of these vector components under no-load conditions is detailed in Chapter 3.

### 2.2.5 Transformer on Load

When a load (resistive, inductive, capacitive, or a combination of these) is connected to the secondary (output) terminals, a secondary current \( I_2 \) flows. The load component of the primary current \( I_1 \) will be in phase opposition to the secondary current \( I_2 \). The total primary current \( I_1 \) is the resultant of \( I_1' \) and \( I_o \). The vector diagrams of a transformer for various loads are shown in Figure 2.9.

For the diagram in Figure 2.9(a), even though the transformer is only supplying a purely resistive load, the presence of the leakage reactance in the secondary winding causes the secondary load current to lag the output voltage by an angle \( \theta_2 \). This angle will be relatively small compared to that for an inductive load (see Figure 2.9(b)). \( \cos \theta_2 \) is thus the lagging power factor of the load. Conversely, when a capacitive load
Legend:

- $u_1$ – Primary terminal voltage (supply voltage)
- $E_1$ – Primary induced emf
- $u_2$ – Secondary terminal voltage (output voltage)
- $E_2$ – Secondary induced emf
- $I_1 R_1$ – Primary resistance voltage drop
- $I_1 X_1$ – Primary reactance voltage drop
- $I_2 R_2$ – Secondary resistance voltage drop
- $I_2 X_2$ – Secondary reactance voltage drop
- $\phi_m$ – Maximum value of magnetic flux
- $I_o$ – Primary no-load current
- $I_c$ – Primary core loss current
- $I_m$ – Primary magnetising current
- $I_2$ – Secondary load current
- $I'_1$ – Load component of total primary current
- $I_1$ – Total primary current (including $I_1$ and $I_c$)
- $\cos \theta_1$ – Primary total load power factor
- $\cos \theta_2$ – Secondary load power factor

Figure 2.9 Transformer on load
is connected to the secondary terminals, the secondary current, which flows in the output, leads the output voltage. This results in the vectors of Figure 2.9(c). The mathematical derivation of these vector components is also detailed in Chapter 3.

2.2.6 Transformer Voltage Regulation and Efficiency

It is apparent from the transformer equivalent circuit in Figure 2.6 that the secondary current $I_2$ produces voltage drops $I_2R_2$ and $I_2X_2$ across the secondary winding resistive and reactive components respectively. In addition, the primary current $I_1$ causes primary winding voltage drops $I_1R_1$ and $I_1X_1$. As a result, the effective primary voltage $E_1$ is less than the input $v_1$ and the output voltage $v_2$ is less than the secondary induced voltage $E_2$.

When there is no load connected to the output terminals of the transformer, no secondary current $I_2$ flows. Hence, there is no voltage drop across $R_2$ and $X_2$. With no secondary current, the primary current drops to the no-load current $I_0$, and the voltage drops across $R_1$ and $X_1$ becomes very small. Thus, in the no-load situation, $E_1$ is almost equal to $v_1$ and $v_2$ equals $E_2$.

The transformer output voltage is greatest on no-load. Under loaded conditions the voltage drops across the resistive and reactive components of the equivalent circuit increase and cause $v_2$ to drop below its no-load level. The percentage change in output voltage from no-load to full-load is termed the voltage regulation of the transformer.

\[
\text{voltage regulation (\%)} = \frac{v_2(\text{no-load}) - v_2(\text{full-load})}{v_2(\text{full-load})} \times 100\% \tag{2.13}
\]

where $v_2(\text{no-load}) = \text{no-load output voltage}$
$v_2(\text{full-load}) = \text{full-load output voltage}$

Ideally for power transformers, there should be no change in $v_2$ from no-load to full-load (i.e. voltage regulation = 0\%). Thus for the best possible performance, the transformer should have the lowest possible regulation.

The efficiency of a transformer, like that of any other device, is defined as the ratio of output real power to input real power:

\[
\text{efficiency, } \eta = \frac{P_2}{P_1} \tag{2.14}
\]

\[3\text{This is only true for a resistive and inductive loads, and a certain level of capacitive loading. Depending on the power factor of the load, the output full-load voltage may actually be larger than the no-load voltage. This can be seen in Figure 2.9(c).}\]
where $P_1$ = input power  \\
$P_2$ = output power

The output power is always less than the input power because of losses ($P_{losses}$). Thus, in terms of these losses, the efficiency can be expressed as

$$\eta = \frac{P_1 - P_{losses}}{P_1}$$

$$= 1 - \frac{P_{losses}}{P_1}$$

$$= 1 - \frac{P_{losses}}{P_2 + P_{losses}}$$

(2.15) \hspace{1cm} (2.16)

Alternatively,

$$\eta = \frac{P_2}{P_2 + P_{losses}}$$

$$= \frac{1}{1 + \frac{P_{losses}}{P_2}}$$

(2.17)

The losses, which are referred to in the expressions above, are those occurring under the particular operating conditions under consideration. The losses are made up of two types; those which are independent of the load on the transformer, and those which increase with load. The losses which are independent of the load, are usually called the core losses. These are the hysteresis and eddy current losses in the transformer core. On the other hand, the copper losses, which are essentially the $I^2R$ losses in both the primary and secondary windings, are load dependent. Thus, the efficiency will vary according to the output load of the transformer.

The condition for maximum efficiency can be derived by letting $P_{core}$ and $P_{wind}$ represent the core and copper losses of the windings respectively. $P_{core}$ is constant, while $P_{wind} = I_2^2 R''_e$, where $R''_e$ is the equivalent resistance of the whole of the windings referred to the secondary:

$$R''_e = R''_1 + R_2$$

(2.18)

It is desired to find the relationship between $P_{core}$ and $P_{wind}$ at which the curve of efficiency against load current reaches its highest value. From Equation 2.17, the
efficiency will be a maximum when the ratio $P_{\text{losses}}/P_2$ is a minimum. Thus,

$$\frac{d}{dI_2} \left( \frac{P_{\text{losses}}}{P_2} \right) = 0$$

$$\frac{d}{dI_2} \left( \frac{P_{\text{core}} + \frac{I_2^2 R''}{v_2 I_2}}{v_2 I_2} \right) = 0$$

$$\frac{d}{dI_2} \left( \frac{P_{\text{core}}}{v_2 I_2^{-1}} + \frac{R''}{v_2 I_2} \right) = 0$$

Hence,

$$-\frac{P_{\text{core}}}{v_2 I_2^{-2}} + \frac{R''}{v_2} = 0$$

$$P_{\text{core}} = I_2^2 R'' = P_{\text{wind}} \quad (2.19)$$

Thus, a transformer reaches its highest efficiency at the particular load at which the copper loss is equal to the core loss.

2.3 CONVENTIONAL DESIGN METHOD

From a manufacturer’s perspective it is convenient to design and produce a set range of transformer sizes. The manufacturers offer transformers of preferred sizes and ratings, particularly so for smaller units. This reflects their manufacturing capabilities in forming tools and instruments. It is not economic to offer customers any size, shape and rating they require. Usually, the voltage, power ratings and frequency are specified. These specifications determine the materials to be used and their dimensions. This conventional approach to transformer design has been utilised and presented in detail in standard textbooks [Lowdon, 1989] [McLyman, 1988] [Feinberg, 1979] [Slemon and Straughen, 1980]. It has been used as a design tool for teaching undergraduate power system courses at universities. [Rubai, 1994] and [Jewell, 1990] have developed single phase transformer design programs for undergraduate students. Programs which incorporate single and three phase transformer design have also been developed and used in the classroom [Shahzad and Shwehd, 1997] as well as in the power electronics industry [Grady et al., 1992]. Another program has been written for single/three phase transformers, with the incorporation of harmonic distortion analysis [Makram et al., 1988]. [Poloujadoff and Findlay, 1986] have developed a transformer design program to achieve minimum cost of production. A unified method of designing not only transformers, but electrical machines in general, has also been developed [Andersen, 1967]. The conventional design method has been used extensively in designing transformers for
switched mode power supplies [Hurley et al., 1998] [Judd and Kressler, 1977] [Petkov, 1995] [A S. Gilmour, 1991] [O’Connor, 1997]. Finite Element Analysis has also been applied, concurrent with the above approach, to aid the overall design process [Asensi et al., 1994] [Miri et al., 1993] [Basak et al., 1994] [Allcock et al., 1995] [Saravolac, 1998].

In the conventional design method, consideration is given to the layout of the core and windings of a two winding transformer, as depicted in Figure 2.10.

![Cross-sectional view of a full-core transformer](image)

**Figure 2.10** Cross-sectional view of a full-core transformer

The laminated core occupies the central space. The windings are wrapped around the core, with the primary winding inside the secondary winding. Insulation is used between the core and windings, between windings, around winding wire and in between each layer of winding if required. However, no explicit account of the insulation, which determines the capacitance, is usually made in the performance calculations for fundamental frequency transformers. Never-the-less, insulation thickness does affect spatial displacements of windings and hence equivalent circuit parameters.

The yokes and limbs of the core are additional to this. They depend on whether the transformer is a “core” or “shell” type [Slemen and Straughen, 1980] and have dimensions determined by the boundaries of the windings and cross-section of the core. Usually, for smaller transformers, the core laminations come in discrete sizes. For shell type cores they may be fabricated to eliminate waste from stamping from rolled strip. Such “scrapless” EI cores [Flanagan, 1986] have specific ratios for their window dimensions and magnetic path sizes. For larger transformers it may be economic to select set sizes which minimise lamination material wastage through stepped circular cores.
In the conventional approach to designing transformers, the terminal voltages, \( V_1, V_2 \), volt-ampere rating, \( S \), and frequency, \( f \), are specified. Material characteristics then lead to calculation of core and winding dimensions.

Based on the designer's experience, core steel with known relative permeability, \( \mu_r \), and knee point flux density, \( B \), is chosen. A stacking factor, \( SF_c \), is assigned to account for the lamination's metal and insulation composition. A window width factor, \( WWF \), (the ratio of winding space height to width) is also selected, again on experience.

For the primary and secondary windings, acceptable current densities, \( J_1, J_2 \), volts per turn, \( VT_1, VT_2 \), and space factors, \( SF_1, SF_2 \) (amount of copper to total winding cross-sectional area) are chosen. The current density estimates are made based on generally accepted thermal capacities of transformer winding material. Typically this is 1 - 2 A/mm\(^2\) for non-water or forced cooled copper or aluminium.

The volts per turn reflect a designer's experience and may differ from one manufacturer to another. In practice the values vary from under unity to more than 50, with inside this range being most typical. An empirical formula cited in the literature [Slemon and Straughen, 1980] is

\[
VT = \frac{\sqrt{S}}{VT_F}
\]

(2.20)

where \( VT_F = \text{voltage per turn factor} \)

All this achieves is to move the problem of estimating the volts per turn to the factor. No calculation is presented for the latter, so once more this is selected on experience.

The space factors depend on voltage ratings and the insulation systems used. It is difficult to find any general rules for specifying this. Again experience determines the values.

Having specified the ratings and made estimates of the other factors listed above, the design procedure for the transformer then follows a more calculated path. The current ratings are

\[
I_1 = \frac{S}{V_1}
\]

(2.21)

and

\[
I_2 = \frac{S}{V_2}
\]

(2.22)
Hence the areas of the winding wires are

\[ A_1 = \frac{I_1}{J_1} \]  \hspace{1cm} (2.23)

and

\[ A_2 = \frac{I_2}{J_2} \]  \hspace{1cm} (2.24)

Thicknesses, \( t_1 \) and \( t_2 \), can be calculated for circular cross-section wires from (in general)

\[ t = \sqrt{\frac{4A}{\pi}} \]  \hspace{1cm} (2.25)

From the chosen volts per turn, the numbers of turns per winding are

\[ N_1 = \frac{V_1}{V T_1} \]  \hspace{1cm} (2.26)

and

\[ N_2 = \frac{V_2}{V T_2} \]  \hspace{1cm} (2.27)

from which the winding ratio is

\[ a = \frac{N_1}{N_2} \]  \hspace{1cm} (2.28)

This is only the same as the voltage ratio if the volts per turn are the same for both windings.

From the 'Transformer Equation' of Equation 2.8,

\[ V_1 = \frac{2\pi f N_1 \phi}{\sqrt{2}} \]  \hspace{1cm} (2.29)

where

\[ \phi = BA'_{\text{core}} \]  \hspace{1cm} (2.30)

\( A'_{\text{core}} \) is the area of the iron in the core and can be calculated from

\[ A'_{\text{core}} = \frac{\sqrt{2V_1}}{2\pi f N_1 B} \]  \hspace{1cm} (2.31)
The actual core dimensions include the stacking factor:

\[ A_{\text{core}} = \frac{A'_{\text{core}}}{SF_c} \]  
(2.32)

The primary winding window area is defined as

\[ A_{W1} = \frac{A_{\text{cu1}}}{SF_1} \]  
(2.33)

where \( A_{\text{cu1}} = \) primary copper cross-sectional area
\[ = N_1 A_1 \]

The primary full-load current is

\[ I_1 = A_1 J_1 = \frac{A_{W1} SF_1}{N_1} J_1 \]  
(2.34)

The transformer volt-ampere rating is

\[ S = V_1 I_1 = \frac{2\pi f B A'_{\text{core}}}{\sqrt{2}} \times A_{W1} SF_1 J_1 \]  
(2.35)

Therefore,

\[ A_{W1} = \frac{\sqrt{2} S}{2\pi f B A'_{\text{core}} J_1} \times \frac{1}{SF_1} \]  
(2.36)

Also, the secondary winding window area is defined as

\[ A_{W2} = \frac{A_{\text{cu2}}}{SF_2} \]  
(2.37)

where \( A_{\text{cu2}} = \) secondary copper cross-sectional area
\[ = N_2 A_2 \]

The secondary full-load current is

\[ I_2 = A_2 J_2 = \frac{A_{W2} SF_2}{N_2} J_2 \]  
(2.38)

Alternatively, the transformer volt-ampere rating is

\[ S = V_2 I_2 = \frac{2\pi f B A'_{\text{core}}}{\sqrt{2}} \times A_{W2} SF_2 J_2 \]  
(2.39)

Therefore,

\[ A_{W2} = \frac{\sqrt{2} S}{2\pi f B A'_{\text{core}} J_2} \times \frac{1}{SF_2} \]  
(2.40)
The winding window width is

\[ WW = \sqrt{\frac{A_{W1} + A_{W2}}{WWF}} \]  \hspace{1cm} (2.41)

and height

\[ WH = WWF \times WW \]  \hspace{1cm} (2.42)

from which the number of winding layers can be calculated.

Having obtained the dimensions of the transformer windings and core, and the material characteristics being known, the components of a transformer equivalent circuit can be calculated. The equivalent circuit shown in Figure 2.11 is often used for supply frequencies [Paul et al., 1986].

![Transformer equivalent circuit](image)

**Figure 2.11** Transformer equivalent circuit, referred to the primary winding

The performance of the transformer under open circuit, short circuit and loaded conditions can then be estimated by putting an impedance \( Z_L = R_L + jX_L \) across the output and varying its value.

### 2.4 CONCLUSIONS

The modelling and operational theory of a full-core transformer has been presented. Firstly, the concept of an ideal transformer was introduced. Loss components were added to represent the departures of the actual transformer from the ideal. Equivalent circuits, which are often used in determining a transformer's performance, have been presented. Phasor diagrams representing the operation of a transformer, under both no-load and loaded conditions, have also been given.

The conventional approach to transformer design has been widely used in research institutions as well as in the industry. It has been presented in this chapter. The performance of this conventional design approach will be presented in Chapter 3.
Chapter 3

REVERSE TRANSFORMER DESIGN

3.1 OVERVIEW

In this chapter a new approach to designing full-core transformers is described. It is designated the reverse transformer design method. The physical characteristics, such as material resistivities, permeabilities etc., and dimensions of the windings and core are the specifications. By manipulating the amount and type of the material actually to be used in the construction of the transformer, its performance can be determined. Such an approach is essentially the opposite of the conventional transformer design method, which was described in Chapter 2.

The computer program routine for this reverse design approach is detailed. The program can be divided into a number of modules in order to calculate the transformer equivalent circuit parameters and other information. Each module is further subdivided into subroutines. Certain components of the equivalent circuit parameters have alternative calculation methods. Consequently, the alternatives are looked at. Comparisons are then made between the alternatives to resolve which method will be used throughout the thesis.

The reverse design method is applied to two sample high voltage transformers. The measured performance of the as-built transformers verifies the usefulness of using this reverse design approach. Moreover, they also highlight the limitations in using the conventional method, which are overcome using the reverse design approach.

3.2 REVERSE DESIGN: A NEW CONCEPT

From Chapter 2, in the conventional transformer design method, the terminal voltages, VA rating and frequency are entered as design specifications. These specifications
determine the materials to be used and their dimensions. However, by designing to rated specifications, consideration is not explicitly given to what materials and sizes are actually available. It is not always possible to procure the materials with the exact characteristics and dimensions that conventional design approaches determine. Core and winding material suppliers offer catalogues of preferred sizes, particularly so for smaller rating devices. This reflects the supplier’s manufacturing capabilities in extrusion, rolling and forming tools and equipment. It is not economic to offer customers any size and shape they require. It is possible that an engineer, having designed a transformer, may then find the material sizes do not exist. The engineer may then be forced to use available materials and consequently the performance of the actual transformer that is built is likely to be significantly different from that of the design calculations. Due to the inexactness of estimating the actual winding dimensions, the leakage reactance in particular may be miscalculated. Moreover, conventional transformer design miscalculates the spatial arrangement of the windings which significantly affects the leakage reactance. The leakage reactance is a dominant parameter in power system studies involving transformers.

The conventional design method is usually followed by transformer manufacturers. Even though they may well have more accurate modelling techniques, these are likely to be custom designed by their engineers, rather than what is freely available in the public literature.

Given this perspective, a new transformer design method has been looked at. It is called the reverse design approach [Bodger and Liew, 2000]. Given the material characteristics and dimensions, the performance of the transformer is determined and it is seen if this is what is required. Physical characteristics and dimensions of the windings and core are the specifications. By manipulating the amount and type of material actually to be used in the construction of the transformer, its performance can be determined. Such an approach lends itself to designing transformers using what is available from suppliers. This is essentially the opposite of the conventional transformer design method. It allows for customised design, as there is considerable flexibility in meeting the performance required for a particular application. This can allow matched rated, cost effective solutions to power supply requirements, as against the traditional practice of overrating the transformer component of a power supply because of batch construction and fixed sized transformer ratings.

A comparison between the conventional and reverse design methods can be summarised using the flowcharts in Figures 3.1 and 3.2.

A profile of the transformer central limb, showing known material characteristics and dimensions, is depicted in Figure 3.3.
3.2 REVERSE DESIGN: A NEW CONCEPT

START
CONVENTIONAL
DESIGN PROCESS

PRINCIPAL INPUT SPECIFICATIONS
Voltages, rating and operating frequency - $V_1, V_2, S, f$

AUXILIARY INPUT SPECIFICATIONS
(i) Core - $B_{core}, \mu_{core}, SF_1, WWF$
(ii) Primary winding - $V_{T1}, J_1, SF_1$
(iii) Secondary winding - $V_{T2}, J_2, SF_2$

CALCULATIONS
(i) Core - $A_{core}, n, WW, WH$
(ii) Primary winding - $I_p, A_p, w_p, N_p, A_{pp1}$
(iii) Secondary winding - $I_p, A_p, w_p, N_p, A_{pp2}, a$

EQUIVALENT CIRCUIT PARAMETERS
(i) Core - $R_{core}, R_p, R_{core}, X_m$
(ii) Primary winding - $R_p, X_I$
(iii) Secondary winding - $R_p, X_2$

ELECTRICAL PERFORMANCE CALCULATIONS
(i) Primary - $P_p, Bf_1$
(ii) Secondary - $P_p, Bf_2$
(iii) Overall - $VREG, EFF$

Is this the required design?
No
Change Auxiliary Input Specifications
Yes

END

Figure 3.1 Conventional design process flowchart
Figure 3.2 Reverse design process flowchart
Figure 3.3  Centre limb of a transformer showing component dimensions and material properties
With respect to Figure 3.3(b), the winding resistivities as well as core material resistivity and permeability become specifications. The core cross-sectional dimensions (diameter for a circular core and side lengths for a rectangular core) are selected from catalogues of available materials. A core length is chosen. Laminations that are available can be specified in thickness and a stacking factor estimated from the ratio of iron to total volume. Other factors such as material density and cost per unit weight assist in material ordering when totals are known.

For each winding, the wire size can be selected, again from catalogues. They also specify insulation thickness. The designer can then specify how many layers of each winding are wound. Insulation space between layers allows for high voltage applications. Again, material density and cost per unit weight assist in material ordering when totals are known.

The only rating requirements are the primary voltage and frequency. The secondary voltage and transformer VA rating are a consequence of the construction of the transformer. Also, instead of 'guessing' the values of space factors $S_{F1}$, $S_{F2}$, and the window width factor $WWF$, as required in the conventional approach in Chapter 2, these are accounted for by knowledge of the actual dimensions of materials used. In addition, winding current densities and volts per turn become a consequence of the design, rather than a design specification.

Having obtained the dimensions of the transformer windings and core, and the material characteristics being known, the components of the transformer equivalent circuit of Figure 2.11 can be solved to calculate the performance of the device.

Open circuit, short circuit and loaded circuit performances can then be obtained. From this information the secondary voltage and VA rating of the transformer are derived. Further, performance measures of voltage regulation and power transfer efficiency for any load condition can be readily calculated. Finally, the current flows and densities in the windings can be calculated and compared to the desired levels to check that the wires selected can withstand the rated current densities. These are a result of the design rather than being initial design specifications.

With this reverse approach to transformer design, if the calculated performance is not what is required, material dimensions can be altered to change the performance. With experience, a few iterations are all that is required before an acceptable solution can be obtained. While an automatic performance optimisation routine may seem relevant, this may not necessarily be appropriate as the criteria depends on the function for which the transformer is being designed.
An as-built transformer based on the design will give a different measured performance because of magnetic material non-linearity, minor construction differences, material characteristic variations, and because the equivalent circuit does not exactly model real processes. In the development of new devices, information gained on the performance of the first as-built device can be used to improve the design of the next device built.

3.3 REVERSE DESIGN METHOD

Consideration is given to the transformer as shown in Figure 3.4.

![Diagram of a full-core transformer]

**Figure 3.4** A full-core transformer

The reverse design model for the transformer contains the following modules:

1. **Main** module
2. **Data** module
3. **Calculation** module
4. **Inductance** module
5. **Resistance** module
6. **Output** module

Figure 3.5 presents the flow diagram of the reverse transformer design program.
Figure 3.5 Reverse transformer design program
3.4 MAIN MODULE

The Main module is essentially the hub of the model. It calls upon all the subroutines from within the other modules, as shown in Figure 3.5. The descriptions of each subroutine are detailed in the subsequent sections. In addition, Main creates 3 files to be used for the model. They are given in Table 3.1.

<table>
<thead>
<tr>
<th>File names</th>
<th>Descriptions</th>
</tr>
</thead>
<tbody>
<tr>
<td>DATA.DAT</td>
<td>Lists all the physical parameters that have been entered.</td>
</tr>
<tr>
<td>OUTPUT.DAT</td>
<td>Lists all the input parameters as well as output parameters.</td>
</tr>
<tr>
<td>OUTPUT.D.DAT</td>
<td>Lists the output parameters, used in fabrication.</td>
</tr>
</tbody>
</table>

Table 3.1 Files created by the Main module

3.5 DATA MODULE

This module contains a subroutine DATA.IN. It reads the physical parameter data from the file DATA.DAT, and then writes the data to the screen upon execution. Moreover, it also writes the data to OUTPUT.DAT and OUTPUT.D.DAT. Table 3.2 lists all the data that are read by the subroutine.

<table>
<thead>
<tr>
<th>(a) Supply Characteristics</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>V</td>
</tr>
<tr>
<td>Frequency</td>
<td>Hz</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b) Core</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Length of the core</td>
<td>m</td>
</tr>
<tr>
<td>Width of the core</td>
<td>m</td>
</tr>
<tr>
<td>Breadth of the core</td>
<td>m</td>
</tr>
<tr>
<td>Stacking factor</td>
<td></td>
</tr>
<tr>
<td>Lamination thickness</td>
<td>m</td>
</tr>
<tr>
<td>Thickness of the core insulation</td>
<td>m</td>
</tr>
<tr>
<td>Relative permeability of the core</td>
<td></td>
</tr>
<tr>
<td>Resistivity of the core material at 20°C</td>
<td>Ωm</td>
</tr>
<tr>
<td>Coefficient of thermal resistivity of the core</td>
<td>°C⁻¹</td>
</tr>
<tr>
<td>Operating temperature of the core</td>
<td>°C</td>
</tr>
<tr>
<td>Core material density</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Core material cost</td>
<td>$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(c) Primary Winding</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total number of layers of the primary winding</td>
<td></td>
</tr>
<tr>
<td>Thickness of the primary interlayer insulation</td>
<td>m</td>
</tr>
</tbody>
</table>

continued on next page
continued from previous page

<table>
<thead>
<tr>
<th>Metric</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness of the primary winding wire insulation</td>
<td>m</td>
</tr>
<tr>
<td>Thickness of the primary winding wire</td>
<td>m</td>
</tr>
<tr>
<td>Relative permeability of the primary winding material</td>
<td></td>
</tr>
<tr>
<td>Resistivity of the primary winding wire at 20°C</td>
<td>Ωm</td>
</tr>
<tr>
<td>Coefficient of thermal resistivity of the primary winding</td>
<td>°C⁻¹</td>
</tr>
<tr>
<td>Operating temperature of the primary winding</td>
<td>°C</td>
</tr>
<tr>
<td>Primary winding material density</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Primary winding material cost</td>
<td>$</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Metric</th>
<th>Unit</th>
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<tbody>
<tr>
<td>(d) Primary–Secondary Interwinding Space</td>
<td>unit</td>
</tr>
<tr>
<td>Thickness of the primary-secondary interwinding insulation</td>
<td>m</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Metric</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(e) Secondary Winding</td>
<td>unit</td>
</tr>
<tr>
<td>Total number of layers of the secondary winding</td>
<td></td>
</tr>
<tr>
<td>Thickness of the secondary interlayer insulation</td>
<td>m</td>
</tr>
<tr>
<td>Thickness of the secondary winding wire insulation</td>
<td>m</td>
</tr>
<tr>
<td>Thickness of the secondary winding wire</td>
<td>m</td>
</tr>
<tr>
<td>Relative permeability of the secondary winding material</td>
<td></td>
</tr>
<tr>
<td>Resistivity of the secondary winding wire at 20°C</td>
<td>Ωm</td>
</tr>
<tr>
<td>Coefficient of thermal resistivity of the secondary winding</td>
<td>°C⁻¹</td>
</tr>
<tr>
<td>Operating temperature of the secondary winding</td>
<td>°C</td>
</tr>
<tr>
<td>Secondary winding material density</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Secondary winding material cost</td>
<td>$</td>
</tr>
</tbody>
</table>

Table 3.2: Data read in by the program

3.6 CALCULATION MODULE

This module contains eleven subroutines, which are detailed in the following sections. Figure 3.6 presents the flow diagram of this module.

3.6.1 A_FREQ Subroutine

A_FREQ computes the angular frequency, ω:

\[
\omega = 2\pi f
\]  

(3.1)

where \( f \) = the fundamental frequency of the system

\footnote{For a cylindrical core, the diameter of the core is used.}
3.6 CALCULATION MODULE

Figure 3.6 Implementation of the Calculation module
3.6.2 PRIM\_TURNS Subroutine

PRIM\_TURNS calculates the number of turns for the primary winding:

\[
N_1 = L_{y1} \frac{l_{core}}{t_1}
\]

where

- \(L_{y1}\) = number of primary winding layers
- \(l_{core}\) = length of the core
- \(t_1\) = primary winding wire thickness, including wire insulation
  = \(w_1 + 2w_{i1}\)
- \(w_1\) = primary winding conductor thickness
- \(w_{i1}\) = primary winding wire insulation thickness

This is done by first calculating the number of turns per layer for each winding, and then multiplying by the total number of layers for each winding.

3.6.3 SECNDRY\_TURNS Subroutine

In a similar manner, SECNDRY\_TURNS calculates the number of turns for the secondary winding:

\[
N_2 = L_{y2} \frac{l_{core}}{t_2}
\]

where

- \(L_{y2}\) = number of secondary winding layers
- \(t_2\) = secondary winding wire thickness, including wire insulation
  = \(w_2 + 2w_{i2}\)
- \(w_2\) = secondary winding conductor thickness
- \(w_{i2}\) = secondary winding wire insulation thickness

3.6.4 TURN\_RATIO Subroutine

Having obtained the number of turns for both the primary and secondary windings, TURN\_RATIO calculates the turns ratio for the transformer:

\[
a = \frac{N_1}{N_2}
\]
3.6.5 FLUX_DEN Subroutine

FLUX_DEN computes the rms and maximum values of the magnetic flux density ($B_{crms}$ and $B_{core}$, respectively) in the transformer core. Equations 3.5 and 3.6 give the values of the rms and the maximum magnetic flux density in the transformer core [McPherson and Laramore, 1990]:

$$B_{crms} = \frac{V_1}{\omega N_1 A_{core}}$$  \hspace{1cm} (3.5)

$$B_{core} = \frac{\sqrt{2}V_1}{\omega N_1 A_{core}}$$  \hspace{1cm} (3.6)

where $V_1$ = input voltage

$A_{core}$ = cross sectional area of the core

Once computed, the magnitude of $B_{core}$ can be checked against an optimal value for the core material.

3.6.6 CORE_WGT_COST Subroutine

CORE_WGT_COST calculates the weight and the cost of the transformer core. The weight is calculated using the following:

$$W_{gtcore} = \gamma_{core} V_{core}$$  \hspace{1cm} (3.7)

where $\gamma_{core}$ = core material density

$V_{core}$ = volume of the core material used

The core volume is calculated using

$$V_{core} = A_{core} \times l_{core}$$  \hspace{1cm} (3.8)

The cost is calculated as

$$Cost_{core} = W_{gtcore} \times C_{wgt_{core}}$$  \hspace{1cm} (3.9)

where $C_{wgt_{core}}$ = core material cost per weight
3.6.7 CORE_RESTVTY_SDEPTH Subroutine

CORE_RESTVTY_SDEPTH computes the effective resistivity and the skin depth of the eddy current ($\delta_{ec}$) in the transformer core. The variation of resistivity with temperature of the core material should be accounted for, since the transformer will be heated up under operation. The operating resistivity at temperature $T_{core} \degree C$ is calculated as

$$\rho_{core} = \frac{(1 + \Delta \rho_c T_{core}) \rho_{c,20\degree C}}{(1 + 20\Delta \rho_c)}$$  \hspace{1cm} (3.10)

where $\Delta \rho_c = \text{thermal resistivity coefficient of the core material}$

$\rho_{c,20\degree C} = \text{core material resistivity at 20\degree C}$

The skin depth of the eddy current is computed using [Snelling, 1988]:

$$\delta_{ec} = \sqrt{\frac{2\rho_{core}}{\mu_0 \mu_{rc} \omega}}$$  \hspace{1cm} (3.11)

where $\rho_{core} = \text{resistivity of the core material}$

$\mu_0 = \text{permeability of free space} = 4\pi \times 10^{-7}\text{Hm}^{-1}$

$\mu_{rc} = \text{relative permeability of the core material}$

At a power frequency of 50Hz and under an ambient temperature of 20\degree C, the skin depth of the laminated steel core is often significantly greater than half the lamination thickness. Therefore, the magnetic flux can be considered to be uniformly distributed throughout the lamination and indeed the entire core cross section. However, as will be detailed in Chapters 5 and 6, changes in the transformer operating temperature (Chapter 5) as well as frequency (Chapter 6) reduce the skin depth to less than half the lamination thickness. As a result, the distribution of flux within the laminated core is no longer uniform. The flux will have a tendency to concentrate towards the outside surfaces of the laminations. This affects the core loss components.

3.6.8 PRIM_WGT_COST Subroutine

PRIM_WGT_COST calculates the weight and the cost of the primary winding. The weight is calculated using

$$W_{gt_1} = \gamma_1 V_1$$  \hspace{1cm} (3.12)

where $\gamma_1 = \text{primary winding material density}$

$V_1 = \text{volume of the primary winding material used}$
The volume of the primary winding is calculated using

\[ V_1 = A_1 \times l_1 \]  

(3.13)

where \( A_1 \) = Cross-sectional area of the primary winding conductor

\( l_1 \) = effective length of the primary winding conductor

The cost is calculated as

\[ Cost_1 = Wgt_1 \times C_{wgt_1} \]  

(3.14)

where \( C_{wgt_1} \) = primary winding material cost per weight

3.6.9 SECNDRY.WGT.COST Subroutine

SECNDRY.WGT.COST calculates the weight and the cost of the secondary winding. The weight is calculated using

\[ Wgt_2 = \gamma_2 V_2 \]  

(3.15)

where \( \gamma_2 \) = secondary winding material density

\( V_2 \) = volume of the secondary winding material used

The volume of the secondary winding is calculated using

\[ V_2 = A_2 \times l_2 \]  

(3.16)

where \( A_2 \) = Cross-sectional area of the secondary winding conductor

\( l_2 \) = effective length of the secondary winding conductor

The cost is calculated as

\[ Cost_2 = Wgt_2 \times C_{wgt_2} \]  

(3.17)

where \( C_{wgt_2} \) = secondary winding material cost per weight

3.6.10 ASPECT_RATIO Subroutine

This module determines the aspect ratio of the transformer. Aspect ratio is used to determine the leakage reactance of the transformer. Consideration is given to the transformer profile shown in Figure 3.7.
When calculating the leakage reactance of an air-cored superconducting transformer, [Yamaguchi et al., 1995b] defines the aspect ratio as

\[ \beta_a = \frac{l_{core}}{2R} \]  \hspace{1cm} (3.18)

where \( R \) = average radius of the transformer.

It takes the radius of the air core into account. Hence, the aspect ratio in Equation 3.18 is dependent on the core size. It does not account for the true width of the windings. As detailed in Section 3.10, the spatial arrangement of the windings affects the leakage reactance calculation.

On the other hand, a reference in [Jiang, 1987] defines the aspect ratio as

\[ \beta_a = \frac{l_{core}}{\tau_{12}} \]  \hspace{1cm} (3.19)

where \( \tau_{12} \) = winding thickness factor

\[ = d_1 + \Delta d + d_2 \]

The aspect ratio of Equation 3.19 is solely dependent on the spatial arrangement of both windings. Hence, for the purpose of this thesis Equation 3.19 is used to compute the aspect ratio.

3.6.11 XFORDER_WGT_COST Subroutine

The total weight of the metallic parts of the transformer is computed in this subroutine. It is the sum of the weights of the core, primary winding and secondary winding. The
insulation is extra but generally a minor component of the overall weight.

\[ Wg_{\text{total}} = Wg_{\text{core}} + Wg_{t1} + Wg_{t2} \]  

(3.20)

In a similar manner, the total cost of the metallic parts of the transformer can be obtained by summing the costs of all the individual parts:

\[ Cost_{\text{total}} = Cost_{\text{core}} + Cost_{t1} + Cost_{t2} \]  

(3.21)

In this case, the cost of insulation may be a significant extra.

### 3.7 INDUCTANCE MODULE

The purpose of this module is to calculate the core magnetising current reactance and the leakage reactances of the transformer. It consists of four subroutines. The flow diagram of this module is presented in Figure 3.8.

![Flow diagram of the Inductance module](image)

**Figure 3.8** Implementation of the Inductance module
3.7.1 MAG_X Subroutine

This subroutine calculates the core magnetising current reactance of the transformer. From Faraday's law, the induced voltage in the primary winding is given by

\[ e_1 = N_1 \frac{\delta \phi}{\delta t} \]  \hspace{1cm} (3.22)

Also,

\[ \phi = B_{\text{core}} A_{\text{core}} \]  \hspace{1cm} (3.23)

\[ B_{\text{core}} = \mu_0 \mu_{\text{rc}} H_{\text{core}} \]  \hspace{1cm} (3.24)

\[ H_{\text{core}} l_{\text{core}} = N_1 i_1 \]  \hspace{1cm} (3.25)

where

- \( B_{\text{core}} \) = the maximum flux density in the transformer
- \( H_{\text{core}} \) = the magnetic field strength
- \( i_1 \) = the primary current

Substituting Equations 3.23, 3.24 and 3.25 into Equation 3.22,

\[ e_1 = \frac{N_1^2 A_{\text{core}} \mu_0 \mu_{\text{rc}}}{l_{\text{core}}} \frac{\delta i_1}{\delta t} \]  \hspace{1cm} (3.26)

Alternatively, \( e_1 \) can be defined in terms of the magnetising inductance:

\[ e_1 = L_m \frac{\delta i_1}{\delta t} \]  \hspace{1cm} (3.27)

Thus, from Equations 3.26 and 3.27, the magnetising inductance is

\[ L_m = \frac{N_1^2 \mu_0 \mu_{\text{rc}} A_{\text{core}}}{l_{\text{core}}} \]

\[ = \frac{N_1^2}{R_{\text{core}}} \]  \hspace{1cm} (3.28)

where

\[ R_{\text{core}} = \frac{l_{\text{core}}}{\mu_0 \mu_{\text{rc}} A_{\text{core}}} \]

\( R_{\text{core}} \) is the reluctance of the core. Finally, the core magnetising current reactance is given by

\[ X_m = \omega L_m \]  \hspace{1cm} (3.29)
3.7.2 LEAK_X1 Subroutine

This subroutine calculates the leakage reactance of the transformer based on the derivation by [Slemon, 1966, chap. 3]. Consideration is given to the axial view of the transformer as shown in Figure 3.9.

![Diagram](image)

(a) Leakage field  
(b) The mmf diagram

**Figure 3.9** Calculation of leakage reactance based on [Slemon, 1966]

An elementary section of the primary winding, of thickness dx, is situated at a distance x from the inner surface of the primary winding. The magnetic field intensity along the flux path, which includes this section, depends on the number of ampere-turns linked by the path:

$$\int H_{core} ds = N_1 i_1 \frac{x}{d_1}$$

where $d_1$ = the primary winding thickness factor  
$s$ = distance along the flux path

(3.30)

Since the reluctance of the path within the magnetic core is negligible compared to that of the path in the winding, $H_{core}$ may be taken as the field strength in the winding section dx with the path length $l_{core}$. It is assumed to be constant along the section. Thus,

$$H_{core} = \frac{N_1 i_1 x}{l_{core} d_1}$$

(3.31)
As shown in Figure 3.9(b), the magnetomotive force (mmf) will vary linearly from zero when \( x = 0 \) to \( N_1 i_1 \) when \( x = d_1 \). It will be constant across the interwinding space since the ampere-turns embraced by the line integral in this region are constant. Since the secondary winding space is traversed, using the dot convention the magnitude of the mmf will fall linearly to zero as \( N_1 i_1 = -N_2 i_2 \).

The energy stored in a winding is generally given by

\[
W = \frac{1}{2} Li^2
\]  

(3.32)

With

\[
L = \frac{N \phi}{i} \\
\phi = \int B \text{core} dA \text{core} \\
B \text{core} = \mu_0 H \text{core} \\
N i = H \text{core} \text{core}
\]

then,

\[
W = \frac{\mu_0 \phi \text{core}}{2} \int H^2 \text{core} dA \text{core}
\]  

(3.33)

The volume of the elementary layer in this analysis is \( \bar{l}_w \text{core} dx \). Therefore the energy stored in the field is

\[
W = \frac{\mu_0 \bar{l}_{w \text{core}}}{2} \int_0^d H^2 \text{core} dx
\]  

(3.34)

The mean circumferential length between the two windings, \( \bar{l}_w \), is given by

\[
\bar{l}_w = 4 \left( \frac{w_{A1} + w_{A2}}{2} \right)
\]  

(3.35)

where

- \( w_{A1} = \) average width of primary winding
- \( w_{A2} = \) average width of secondary winding

The energy in the primary winding space is given by

\[
W_p = \frac{\mu_0 \bar{l}_w \text{core}}{2} \int_0^{d_1} \left( \frac{N_1 i_1 x}{l \text{core} d_1} \right)^2 dx
\]

\[
= \frac{\mu_0 \bar{l}_w d_1}{2l \text{core} \frac{3}{3}} (N_1 i_1)^2
\]  

(3.36)
The energy in the interwinding space is given by

\[
W_{ps} = \frac{\mu_0 l_w l_{core}}{2} \int_{0}^{\Delta d} \left( \frac{N_1 i_1}{l_{core}} \right)^2 \, dx
= \frac{\mu_0 l_w}{2l_{core}} \Delta d (N_1 i_1)^2
\]  \hspace{1cm} (3.37)

And the energy in the secondary winding space is given by

\[
W_s = \frac{\mu_0 l_w l_{core}}{2} \int_{0}^{d_2} \left( \frac{N_2 i_2 x}{l_{core} d_2} \right)^2 \, dx
= \frac{\mu_0 l_w d_2}{2l_{core} 3} (N_2 i_2)^2
\]  \hspace{1cm} (3.38)

Hence, by definition, the energy in the total winding space is

\[
\frac{1}{2} L_{12} i_1^2 = W_p + W_{ps} + W_s
\]  \hspace{1cm} (3.39)

where \( L_{12} \) = the transformer leakage inductance

Since \( |N_1 i_1| = |N_2 i_2| \), Equation 3.39 reduces to

\[
\frac{1}{2} L_{12} i_1^2 = \frac{\mu_0 l_w}{2l_{core}} \left( \frac{d_1 + d_2}{3} + \Delta d \right) (N_1 i_1)^2
\]  \hspace{1cm} (3.40)

The total transformer leakage inductance is

\[
L_{12} = \frac{\mu_0 N_1^2 i_w}{l_{core}} \left( \frac{d_1 + d_2}{3} + \Delta d \right)
\]  \hspace{1cm} (3.41)

And the leakage reactance is thus

\[
X_{12} = \omega L_{12}
\]  \hspace{1cm} (3.42)

The leakage reactance found by Equation 3.42 is the total leakage reactance for both the primary as well as the secondary windings. In order to obtain the individual leakage reactance of each winding, the total leakage reactance can be divided equally between the primary and secondary windings.

\[
X_1 = X_2 = \frac{X_{12}}{2}
\]  \hspace{1cm} (3.43)
where $X_1 =$ primary winding leakage reactance
$X_2 =$ secondary winding leakage reactance

### 3.7.3 LEAK-X2 Subroutine

This subroutine also calculates the leakage reactance of the transformer. It is based on the derivation by [Connelly, 1965, chap. 8]. Consider the axial view of the transformer in Figure 3.10.

**Figure 3.10** Calculation of leakage reactance based on [Connelly, 1965]

From Equation 3.33, the energy stored in a winding is given by

$$ W = \frac{\mu_0 l_{\text{core}}}{2} \int H_{\text{core}}^2 dA_{\text{core}} \tag{3.44} $$

Each individual winding space has its own mean circumferential length. Hence, the energy in the primary winding space is given by

$$ W_p = \frac{\mu_0 l_{\text{core}}}{2} \int_0^{d_1} \left( \frac{N_1 i_1 x}{l_{\text{core}} d_1} \right)^2 dx $$

$$ = \frac{\mu_0 l_{\text{core}}}{2 l_{\text{core}}} \frac{d_1}{3} (N_1 i_1)^2 \tag{3.45} $$
where $\bar{l}_p = 4 \left( \frac{w_{i1} + w_{o1}}{2} \right) = \text{mean circumferential length of the primary winding}$

$w_{i1} = \text{inner width of the primary winding}$

$w_{o1} = \text{outer width of the primary winding}$

The energy in the interwinding space is given by

$$W_{ps} = \frac{\mu_o \bar{l}_{ps} l_{core}}{2} \int_0^{\Delta d} \left( \frac{N_1 i_1}{l_{core}} \right)^2 dx$$

$$= \frac{\mu_o \bar{l}_{ps}}{2l_{core}} \Delta d (N_1 i_1)^2$$

where $\bar{l}_{ps} = 4 \left( \frac{w_{o1} + w_{i2}}{2} \right) = \text{mean circumferential length of the interwinding space}$

$w_{i2} = \text{inner width of the secondary winding}$

And the energy in the secondary winding space is given by

$$W_s = \frac{\mu_o \bar{l}_s l_{core}}{2} \int_0^{d_2} \left( \frac{N_2 i_2}{\bar{l}_{core} d_2} \right)^2 dx$$

$$= \frac{\mu_o \bar{l}_s d_2}{2l_{core}} \frac{d_2}{3} (N_2 i_2)^2$$

where $\bar{l}_s = 4 \left( \frac{w_{i2} + w_{o2}}{2} \right) = \text{mean circumferential length of the secondary winding}$

$w_{o2} = \text{outer width of the secondary winding}$

By definition, the energy in the total winding space is

$$\frac{1}{2} L_{12} i_1^2 = W_p + W_{ps} + W_s$$

Since $N_1 i_1 = N_2 i_2$, Equation 3.40 reduces to

$$\frac{1}{2} L_{12} i_1^2 = \frac{\mu_o}{2l_{core}} \left( \frac{l_p d_1}{3} + \bar{l}_s d_2 + \bar{l}_{ps} \Delta d \right) (N_1 i_1)^2$$

The total transformer leakage inductance and reactance are

$$L_{12} = \frac{\mu_o N_1^2}{l_{core}} \left( \frac{l_p d_1}{3} + \bar{l}_s d_2 + \bar{l}_{ps} \Delta d \right)$$

$$X_{12} = \omega L_{12}$$
The primary and secondary leakage reactances are thus

\[ X_1 = X_2 = \frac{X_{12}}{2} \]  

(3.52)

### 3.7.4 LEAK_X3 Subroutine

This subroutine also calculates the leakage reactance of the transformer, which is based on the derivation by [Snelling, 1988, chap. 11]. Consider the axial view of the transformer in Figure 3.11.

![Figure 3.11](image)

**Figure 3.11** calculation of leakage reactance based on [Snelling, 1988]

The derivation by [Snelling, 1988, chap. 11] is similar to that of [Slemen, 1966, chap. 3]. Thus, reproducing Equations 3.31, 3.34, 3.39, 3.40, 3.41, 3.42 and 3.43,

\[ H_{\text{core}} = \frac{N_1 i_1 x}{l_{\text{core}} d_1} \]  

(3.53)

\[ W = \frac{\mu_0 l_{\text{core}}}{2} \int_0^d H_{\text{core}}^2 dx \]  

(3.54)

\[ \frac{1}{2} L_{12} i_1^2 = W_p + W_{ps} + W_s \]  

(3.55)
\[
\frac{1}{2} L_{12} i_1^2 = \frac{\mu_0 i_w}{2 l_{core}} \left( \frac{d_1 + d_2}{3} + \Delta d \right) (N_1 i_1)^2
\]  \hspace{1cm} (3.56)

\[
L_{12} = \frac{\mu_0 N_1^2 i_w}{l_{core}} \left( \frac{d_1 + d_2}{3} + \Delta d \right)
\]  \hspace{1cm} (3.57)

\[
X_{12} = \omega L_{12}
\]  \hspace{1cm} (3.58)

\[
X_1 = X_2 = \frac{X_{12}}{2}
\]  \hspace{1cm} (3.59)

The only exception is that of the mean circumferential length between the two windings, \( \bar{l}_w \). For Snelling, the mean length is taken as a constant for the winding as a whole, i.e. it is based on \( \tau_{12} \). Thus,

\[
\bar{l}_w = 4 \left( \frac{w_{11} + w_{22}}{2} \right)
\]  \hspace{1cm} (3.60)

3.8 RESISTANCE MODULE

This module calculates the core loss resistance and the winding resistances. The core loss resistance includes both the eddy current loss and the hysteresis loss components. The flow diagram of this module is presented in Figure 3.12.

3.8.1 EDDY.R1 Subroutine

EDDY.R1 calculates the eddy current resistance. It is based on the derivation made by [Slemon, 1966, chap. 2]. Consider the diagram in Figure 3.13.

Figure 3.13(b) shows an enlarged cross section of the lamination used in the core of Figure 3.13(a). From Section 3.6.7, at a power frequency of 50Hz and under an ambient temperature of 20° C, the skin depth of the laminated steel core is greater than half the lamination thickness, \( \frac{c_{lam}}{2} \). Thus, the magnetic flux density \( B_{core} \) is considered to be uniformly distributed throughout the entire core cross section.

Consider a closed path within the lamination, as shown in Figure 3.13(b). The sides of this path are at a distance \( x \) from the centre line of the lamination. This path encloses
Figure 3.12  Implementation of the Resistance module

Figure 3.13  Determination of eddy current loss in a lamination
a magnetic flux

\[ \phi = 2xyB_{\text{core}} \]  

(3.61)

Since \( y \gg x \), the change of this magnetic flux can be assumed to produce an electric field of constant magnitude down one side of the path and up the other. By Faraday's law,

\[ E_{\text{lam}} = \frac{d\phi}{dt} \]  

(3.62)

where \( E_{\text{lam}} \) = electric field produced in the lamination

Combining Equations 3.61 and 3.62,

\[ E_{\text{lam}} = x \frac{dB_{\text{core}}}{dt} \]  

(3.63)

Thus, the current density \( J_{\text{lam}} \) [A/m²] at a distance \( x \) from the centre plane of the lamination is

\[ J_{\text{lam}} = \frac{E_{\text{lam}}}{\rho_{\text{core}}} = \frac{x}{\rho_{\text{core}}} \frac{dB_{\text{core}}}{dt} \]  

(3.64)

The total power loss in a lamination of thickness \( c_{\text{lam}} \), height \( y \), and length \( l_{\text{core}} \) is found by integrating the loss density

\[ p_{\text{ec}} = \rho_{\text{core}} J_{\text{lam}}^2 \]  

(3.65)

over the volume of the lamination. Thus,

\[
\begin{align*}
    p_{\text{ec}} &= \int \rho_{\text{core}} J_{\text{lam}}^2 dV_{\text{lam}} \\
    &= \int_{-c_{\text{lam}}/2}^{c_{\text{lam}}/2} \rho_{\text{core}} \left( \frac{x}{\rho_{\text{core}}} \frac{dB_{\text{core}}}{dt} \right) y l_{\text{core}} dx \\
    &= \frac{c_{\text{lam}}^3 y l_{\text{core}}}{12 \rho_{\text{core}}} \left( \frac{dB_{\text{core}}}{dt} \right)^2
\end{align*}
\]  

(3.66)

Averaged over the volume \( c_{\text{lam}} y l_{\text{core}} \) of the lamination, the instantaneous eddy current
The power loss per unit volume is

\[ p_{ec} = \frac{c_{lam}^2}{12 \rho_{core}} \left( \frac{dB_{core}}{dt} \right)^2 \]  

(3.67)

For a core of cross-sectional area \( A_{core} \), mean length of flux path \( l_{core} \), lamination thickness \( c_{lam} \), and primary turns \( N_1 \), the total eddy current power loss is, from Equation 3.67,

\[ P_{ec} = \frac{c_{lam}^2}{12 \rho_{core}} \left( \frac{dB_{core}}{dt} \right)^2 A_{core} l_{core} \]  

(3.68)

The induced voltage in the primary winding is

\[ e_1 = N_1 \frac{d\phi}{dt} \]

\[ = N_1 A_{core} \frac{dB_{core}}{dt} \]  

(3.69)

Substituting Equation 3.69 into 3.68, the power loss can be expressed in terms of the induced voltage as

\[ P_{ec} = \frac{c_{lam}^2}{12 \rho_{core}} \frac{l_{core}}{N_1^2 A_{core}} e_1^2 \]  

(3.70)

And since

\[ P_{ec} = \frac{e_1^2}{R_{ec}} \]  

(3.71)

therefore the eddy current effect can be included in the model as a resistance \( R_{ec} \):

\[ R_{ec} = \frac{N_1^2 A_{core} l_{core}}{12 \rho_{core} c_{lam}^2} \]  

(3.72)

This resistance applies for any value of frequency for which the assumption of uniform flux density in the core is valid.

### 3.8.2 EDDY\_R2 Subroutine

EDDY\_R2 also calculates the eddy current resistance. Consider a transformer core, with \( n \) laminations, as depicted in Figure 3.14.

From Figure 3.14(a), the eddy current flows in each of the lamination, the direction of
the flow being opposite to that of the input current. Figure 3.14(b) shows an enlarged section of one of the laminations. At power frequencies, the skin depth $\delta_{ec}$ is greater than half the lamination thickness so that the magnetic flux is uniformly distributed throughout the entire core cross section:

$$\delta_{ec} > \frac{w_{\text{core}}}{2n}$$ (3.73)

Therefore, the skin effect is negligible, and $\delta_{ec}$ can be set equal to half the lamination thickness:

$$\delta_{ec} = \frac{w_{\text{core}}}{2n}$$ (3.74)

The effective eddy current path in the lamination is also shown in Figure 3.14(b). The effective path length is calculated as

$$l_{ec} = 2 \left( b_{\text{core}} - \frac{w_{\text{core}}}{2n} \right) + 2 \left( \frac{w_{\text{core}}}{2n} \right)$$

$$= 2b_{\text{core}}$$ (3.75)

The effective eddy current resistance for one lamination is

$$R_{ecn} = \frac{\rho_{\text{core}} l_{ec}}{A_{ecn}}$$

where $A_{ecn}$ = effective cross-sectional area of the lamination

$$= l_{\text{core}} \times \frac{w_{\text{core}}}{2n}$$

This is no longer true when $\delta_{ec} < \frac{w_{\text{core}}}{2n}$. Please refer to Section 5.3.3 for further detail.
The power dissipated across the lamination is

\[
P_{\text{ecn}} = \left( \frac{V_{\text{core}}}{n} \right)^2 \frac{1}{R_{\text{ecn}}} \tag{3.77}
\]

where \( V_{\text{core}} = \) induced voltage across the core
\[
= \frac{V_1}{N_1}
\]

Therefore,

\[
P_{\text{ecn}} = \frac{1}{n^2} \frac{V_1^2}{N_1^2} \frac{l_{\text{core}} w_{\text{core}}}{4n \rho_{\text{core}} b_{\text{core}}} \tag{3.78}
\]

The total power for \( n \) laminations is

\[
P_{\text{ec}} = nP_{\text{ecn}}
\]

\[
= \frac{V_1^2 l_{\text{core}} w_{\text{core}}}{n^2 N_1^2 4n \rho_{\text{core}} b_{\text{core}}} \tag{3.79}
\]

The equivalent eddy current resistance of the core, referred to the primary, is thus

\[
R_{\text{ec}} = \frac{V_1^2}{P_{\text{ec}}}
\]

\[
= \frac{\rho_{\text{core}} N_1^2}{l_{\text{core}}} \left( \frac{4n^2 b_{\text{core}}}{w_{\text{core}}} \right) \tag{3.80}
\]

### 3.8.3 PRIMRY.R Subroutine

This subroutine calculates the primary winding resistance:

\[
R_1 = \frac{\rho_1 l_1}{A_1} \tag{3.81}
\]

where \( \rho_1 = \) resistivity of the primary winding conductor, the value of which depends on the operating temperature

\( l_1 = \) effective length of the primary winding conductor

\( A_1 = \) cross-sectional area of the primary winding conductor
The primary winding operating resistivity at temperature $T_1^\circ C$ is calculated as

$$
\rho_1 = \frac{(1 + \Delta \rho_1 T_1) \rho_{1,20^\circ C}}{(1 + 20\Delta \rho_1)}
$$

(3.82)

where $\Delta \rho_1$ = thermal resistivity coefficient of the primary winding material
$\rho_{1,20^\circ C}$ = primary winding material resistivity at $20^\circ C$

The effective length of the primary winding conductor is estimated by calculating the approximate length of the conductor on each layer of the winding, and then adding them all up.

### 3.8.4 SECNDRY R Subroutine

This subroutine calculates the secondary winding resistance:

$$
R_2 = \frac{\rho_2 l_2}{A_2}
$$

(3.83)

where $\rho_2$ = resistivity of the secondary winding conductor, whose value depends on the operating temperature
$l_2$ = effective length of the secondary winding conductor
$A_2$ = cross-sectional area of the secondary winding conductor

The secondary winding operating resistivity at temperature $T_2^\circ C$ is calculated as

$$
\rho_2 = \frac{(1 + \Delta \rho_2 T_2) \rho_{2,20^\circ C}}{(1 + 20\Delta \rho_2)}
$$

(3.84)

where $\Delta \rho_2$ = thermal resistivity coefficient of the secondary winding material
$\rho_{1,20^\circ C}$ = secondary winding material resistivity at $20^\circ C$

The effective length of the secondary winding conductor is also estimated by calculating the approximate length of the conductor on each layer of the winding, and then adding them all up.

### 3.8.5 HYS.TOTAL.CORE.R Subroutine

In this subroutine, the hysteresis loss can be empirically derived using [Nasar and Boldea, 1990]

$$
P_h = k_h f B_{core}^z
$$

(3.85)
where \( k_h = \) a constant (material dependant)
\[ x = \text{Steinmetz factor}^3 \]

The hysteresis loss can be expressed in term of the induced voltage \( e_1 \) as

\[ P_h = \frac{e_1^2}{R_h} \quad (3.86) \]

where \( R_h = \) hysteresis loss equivalent resistance

As shown in Figure 3.15, both \( R_h \) and \( R_{ec} \) can be included in the model as the core loss resistance \( R_{core} \) [Slemen and Straughen, 1980] [Sato and Sakaki, 1990].

\begin{figure}[h]
\centering
\includegraphics[width=0.5\textwidth]{core_equivalent_circuit.png}
\caption{Core equivalent circuit}
\end{figure}

\( R_{core} \) is expressed as

\[ R_{core} = \frac{R_h R_{ec}}{R_h + R_{ec}} \quad (3.87) \]

\section{3.9 OUTPUT MODULE}

This final module compiles and displays the calculated components of the equivalent circuit parameters. In addition, the open circuit, short circuit and the loaded circuit performances are calculated. The flow diagram of this module is presented in Figure 3.16.

\subsection{3.9.1 EQUI.CCT.PARAM Subroutine}

The equivalent circuit parameters, based on the calculations from Sections 3.7 and 3.8, are compiled and presented to the user in this subroutine. The equivalent circuit used for later analyses is shown in Figure 3.17.

\footnote{\text{The Steinmetz factor has a value between 1.8 to 2.5.}}
Figure 3.16 Implementation of the Output module

Figure 3.17 The transformer equivalent circuit used for analyses
3.9.2 OPEN.CCT Subroutine

An open circuit analysis from the equivalent circuit parameters is performed in this subroutine. The equivalent circuit used for the open circuit analysis is shown in Figure 3.18.

![Equivalent Circuit](image)

**Figure 3.18** The open circuit transformer equivalent circuit

The primary winding impedance is

\[ Z_1 = R_1 + jX_1 \]  
(3.88)

The core admittance is

\[ Y_{\text{core}} = \frac{1}{R_{\text{core}}} - j \frac{1}{X_m} \]  
(3.89)

where \( R_{\text{core}} = \frac{R_h R_{\text{nc}}}{R_h + R_{\text{nc}}} \)

from which the core impedance is calculated:

\[ Z_{\text{core}} = \frac{1}{Y_{\text{core}}} \]  
(3.90)

The total open circuit impedance looking from the primary side is

\[ Z_{\text{oc}} = Z_1 + Z_{\text{core}} \]  
(3.91)

The open circuit admittance is thus

\[ Y_{\text{oc}} = \frac{1}{Z_{\text{oc}}} \]  
(3.92)

So the open circuit conductance and susceptance are

\[ G_{\text{oc}} = \Re(Y_{\text{oc}}) \]  
(3.93)
and

\[ B_{oc} = \Im(Y_{oc}) \]  

(3.94)

where \( \Re(\ ) \) denotes the real part
\( \Im(\ ) \) denotes the imaginary part

The equivalent open circuit components are thus

\[ R_{oc} = \frac{1}{G_{oc}} \]  

(3.95)

and

\[ X_{oc} = -\frac{1}{B_{oc}} \]  

(3.96)

The complex open circuit primary current is calculated as

\[ \tilde{I}_{oc1} = V_1 Y_{oc} \]  

(3.97)

The magnitude of open circuit primary current is

\[ I_{oc1} = |\tilde{I}_{oc1}| \]  

(3.98)

The complex open circuit volt-ampere is

\[ \tilde{S}_{oc} = V_1 \tilde{I}_{oc1}^* \]  

(3.99)

where \( \tilde{I}_{oc1}^* \) is the complex conjugate of \( \tilde{I}_{oc1} \)

The open circuit real power loss is

\[ P_{oc} = \frac{V_1^2}{R_{oc}} \]  

(3.100)

The open circuit power factor is

\[ pf_{oc} = \frac{P_{oc}}{|\tilde{S}_{oc}|} \]  

(3.101)

The induced emf across the core is

\[ \tilde{e}_1 = V_1 - \tilde{I}_{oc1} Z_1 \]  

(3.102)

The open circuit performance parameters, as derived from the circuit of Figure 3.18, are tabulated in Table 3.3.
### Table 3.3 Open circuit performance data

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Description</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>Primary voltage (V)</td>
</tr>
<tr>
<td>$I_1$</td>
<td>Primary current (A)</td>
</tr>
<tr>
<td>$V_2$</td>
<td>Secondary voltage (V)</td>
</tr>
<tr>
<td>$S_{oc}$</td>
<td>Volt-ampere (kVA)</td>
</tr>
<tr>
<td>$P_{oc}$</td>
<td>Real power (kW)</td>
</tr>
<tr>
<td>$\psi_{oc}$</td>
<td>Power factor</td>
</tr>
<tr>
<td>$R_{oc}$</td>
<td>Open circuit equivalent shunt resistance (Ω)</td>
</tr>
<tr>
<td>$X_{oc}$</td>
<td>Open circuit equivalent shunt reactance (Ω)</td>
</tr>
</tbody>
</table>

#### 3.9.3 SHORT_CCT Subroutine

A short circuit analysis is performed in this subroutine. The equivalent circuit used for the short circuit analysis is shown in Figure 3.19.

![Equivalent Circuit](image)

**Figure 3.19** The short circuit transformer equivalent circuit

For a short circuit condition, the load impedance is zero:

$$Z_L = 0$$  \hspace{1cm} (3.103)

The secondary winding impedance is

$$Z'_2 = R'_2 + jX'_2$$  \hspace{1cm} (3.104)

from which the corresponding admittance is

$$Y'_2 = \frac{1}{Z'_2}$$  \hspace{1cm} (3.105)

It can be seen that $Z_2$ is in parallel with $Z_{core}$. Thus the equivalent admittance is calculated as

$$Y_{core,2} = Y_{core} + Y'_2$$  \hspace{1cm} (3.106)
The corresponding impedance is
\[ Z_{core,2} = \frac{1}{Y_{core,2}} \]  
(3.107)

The total short circuit impedance looking from the primary side is
\[ Z_{sc} = Z_1 + Z_{core,2} \]  
(3.108)

So the equivalent short circuit components are
\[ R_{sc} = \Re(Z_{sc}) \]  
(3.109)

and
\[ X_{sc} = \Im(Z_{sc}) \]  
(3.110)

The complex short circuit primary current is calculated as
\[ \tilde{I}_{sc1} = \frac{V_1}{Z_{sc}} \]  
(3.111)

The magnitude of short circuit primary current is calculated using
\[ I_{sc1} = |\tilde{I}_{sc1}| \]  
(3.112)

The complex short circuit volt-ampere is
\[ \tilde{S}_{sc} = V_1 \tilde{I}_{sc1} \]  
(3.113)

where \( \tilde{I}_{sc1}^* \) is the complex conjugate of \( \tilde{I}_{sc1} \)

The short circuit real power loss is
\[ P_{sc} = I_{sc1}^2 R_{sc} \]  
(3.114)

The short circuit power factor is
\[ pf_{sc} = \frac{P_{sc}}{|\tilde{S}_{sc}|} \]  
(3.115)

The short circuit performance parameters, as derived from the circuit of Figure 3.19, are tabulated in Table 3.4.
### 3.9.4 LOADED_CCT Subroutine

A loaded circuit analysis is performed in this subroutine. A load \( Z_L = R_L + jX_L \) is placed across the secondary terminals. The load, referred to the primary side, \( Z'_L \), is calculated as:

\[
Z'_L = a^2 Z_L
\]  
(3.116)

The equivalent circuit used for the loaded circuit analysis is shown in Figure 3.20.

![Figure 3.20 The loaded circuit transformer equivalent circuit](image)

The secondary winding impedance \( Z'_2 \) is in series with \( Z'_L \).

\[
Z'_{2,L} = Z'_2 + Z'_L
\]  
(3.117)

from which the corresponding admittance is

\[
Y'_{2,L} = \frac{1}{Z'_{2,L}}
\]  
(3.118)

It can be seen that \( Z'_{2,L} \) is in parallel with \( Z_{core} \). Thus the equivalent admittance is calculated as

\[
Y_{core,2,L} = Y_{core} + Y'_{2,L}
\]  
(3.119)
The corresponding impedance is

\[ Z_{\text{core},2,L} = \frac{1}{Y_{\text{core},2,L}} \]  

(3.120)

The loaded circuit impedance looking from the primary side is

\[ Z_{\text{loaded}} = Z_1 + Z_{\text{core},2,L} \]  

(3.121)

The complex loaded circuit primary current is calculated as

\[ \tilde{I}_1 = \frac{V_1}{Z_{\text{loaded}}} \]  

(3.122)

The magnitude of loaded circuit primary current is calculated using

\[ I_1 = |\tilde{I}_1| \]  

(3.123)

The complex volt-ampere is

\[ \tilde{S}_1 = V_1 \tilde{I}_1^* \]  

(3.124)

where \( \tilde{I}_1^* \) is the complex conjugate of \( \tilde{I}_1 \)

The total real power loss is

\[ P_1 = I_1^2 \Re(Z_{\text{loaded}}) \]  

(3.125)

The power factor is

\[ pf_1 = \frac{P_1}{|\tilde{S}_1|} \]  

(3.126)

The induced *emf* across the core is

\[ \tilde{e}_1 = V_1 - \tilde{I}_1 Z_1 \]  

(3.127)

The referred complex secondary current is

\[ \tilde{I}_2 = \frac{\tilde{e}_1}{Z_{2,L}} \]  

(3.128)

The magnitude of the referred secondary current is therefore

\[ I_2' = |\tilde{I}_2'| \]  

(3.129)
The referred complex load voltage is

\[ \tilde{V}_L' = \tilde{I}_2'Z_L' \]  

from which the magnitude is calculated as

\[ V_L' = |\tilde{V}_L'| \]  

The corresponding secondary current and load voltage are

\[ I_2 = aI'_2 \]  

and

\[ V_L = \frac{V'_L}{a} \]  

The voltage regulation is calculated using

\[ V_{REG}(\%) = \frac{V_1 - V_L'}{V_1} \times 100 \]  

The real power dissipated across the load is

\[ P_L' = I_2'^2R_L' \]  

The transformer efficiency is therefore

\[ EFF(\%) = \frac{P_L'}{P_1} \times 100 \]  

The loaded circuit performance parameters are tabulated in Table 3.5.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V_1)</td>
<td>Primary voltage (V)</td>
</tr>
<tr>
<td>(I_1)</td>
<td>Primary current (A)</td>
</tr>
<tr>
<td>(S_1)</td>
<td>Primary volt-ampere (kVA)</td>
</tr>
<tr>
<td>(P_1)</td>
<td>Primary real power (kW)</td>
</tr>
<tr>
<td>(pf_1)</td>
<td>Primary power factor</td>
</tr>
<tr>
<td>(V_L)</td>
<td>Load voltage (V)</td>
</tr>
<tr>
<td>(I_2)</td>
<td>Secondary current (A)</td>
</tr>
<tr>
<td>(V_{REG})</td>
<td>Voltage regulation (%)</td>
</tr>
<tr>
<td>(EFF)</td>
<td>Transformer efficiency (%)</td>
</tr>
</tbody>
</table>

*Table 3.5* Loaded circuit performance data
3.10 COMPARISON BETWEEN THE VARIOUS CALCULATION METHODS

Several parameters in Sections 3.7 and 3.8 have alternative calculation methods. They are the primary and the secondary windings leakage reactances \((X_1, X_2)\) as well as the eddy current loss resistance \((R_{ec})\). Each calculation method is compared to determine their merits.

3.10.1 Leakage Reactances

From Equations 3.41, 3.50 and 3.57, the total transformer leakage reactance can be generalised as follows:

\[
X_{12} = \frac{\omega \mu_0 N_1^2}{I_{core}} \cdot g' \tag{3.137}
\]

where

\[
g' = \begin{cases} 
2 \left( w_{A1} + w_{A2} \right) \left( \frac{d_1 + d_2}{3} + \Delta d \right) & \text{for LEAK.X1} \\
\left( \frac{\bar{I}_p d_1 + \bar{I}_s d_2}{3} \right) + \bar{I}_{ps} \Delta d & \text{for LEAK.X2} \\
2 \left( w_{i1} + w_{i2} \right) \left( \frac{d_1 + d_2}{3} + \Delta d \right) & \text{for LEAK.X3}
\end{cases}
\]

\[
\bar{I}_p = 2 \left( w_{i1} + w_{o1} \right) \\
\bar{I}_s = 2 \left( w_{i2} + w_{o2} \right) \\
\bar{I}_{ps} = 2 \left( w_{o1} + w_{i2} \right)
\]

For the subroutines LEAK.X1 and LEAK.X3, the function \(g'\) is very similar. The difference is the calculation of the circumferential length between the two windings. Once the mean circumferential length is obtained, it is then multiplied by the winding factor. For the subroutine LEAK.X2, circumferential lengths of the primary and secondary windings as well as the interwinding space are computed separately. They are then multiplied by their respective thickness factors.

To assess the differences, consider three transformers with square cores as shown in Figure 3.21. The detailed winding specifications are shown in Table 3.6.

The function \(g'\) of each subroutine is calculated for each transformer, and is presented in Table 3.7.

From Table 3.7, it can be seen that for LEAK.X3 the value of \(g'\) does not change with the different winding thicknesses. Relative to the other models, LEAK.X1 appears to underestimate \(g'\) when dealing with a transformer with a narrow primary winding. It
Figure 3.21 Three transformers with different winding thicknesses

<table>
<thead>
<tr>
<th>$w_{core}$ (mm)</th>
<th>Transformer A</th>
<th>Transformer B</th>
<th>Transformer C</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{c1}$ (mm)</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>$w_1$ (mm)</td>
<td>125</td>
<td>75</td>
<td>175</td>
</tr>
<tr>
<td>$w_2$ (mm)</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>$I_{12}$ (mm)</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>$d_1$ (mm)</td>
<td>35</td>
<td>10</td>
<td>60</td>
</tr>
<tr>
<td>$d_2$ (mm)</td>
<td>35</td>
<td>60</td>
<td>10</td>
</tr>
<tr>
<td>$\Delta d$ (mm)</td>
<td>2.5</td>
<td>2.5</td>
<td>2.5</td>
</tr>
<tr>
<td>$w_{11}$ (mm)</td>
<td>55</td>
<td>55</td>
<td>55</td>
</tr>
<tr>
<td>$w_{d1}$ (mm)</td>
<td>125</td>
<td>75</td>
<td>175</td>
</tr>
<tr>
<td>$w_{12}$ (mm)</td>
<td>130</td>
<td>80</td>
<td>180</td>
</tr>
<tr>
<td>$w_{d2}$ (mm)</td>
<td>200</td>
<td>200</td>
<td>200</td>
</tr>
<tr>
<td>$w_{A1}$ (mm)</td>
<td>90</td>
<td>65</td>
<td>115</td>
</tr>
<tr>
<td>$w_{A2}$ (mm)</td>
<td>165</td>
<td>140</td>
<td>190</td>
</tr>
</tbody>
</table>

Table 3.6 Winding details of the transformers

<table>
<thead>
<tr>
<th>Subroutines</th>
<th>Transformer A</th>
<th>Transformer B</th>
<th>Transformer C</th>
</tr>
</thead>
<tbody>
<tr>
<td>LEAK_X1</td>
<td>13175</td>
<td>10592</td>
<td>15758</td>
</tr>
<tr>
<td>LEAK_X2</td>
<td>13175</td>
<td>12842</td>
<td>13508</td>
</tr>
<tr>
<td>LEAK_X3</td>
<td>13175</td>
<td>13175</td>
<td>13175</td>
</tr>
</tbody>
</table>

Table 3.7 Calculation of $g'$ for each transformer
tends to relatively overestimate \( g' \) when handling a transformer with a narrow secondary winding. This highlights the differences between the two methods in estimating the leakage reactance under extreme winding conditions.

For LEAK.X2, which takes into account the area function of the windings and the interwinding space separately, the changes in \( g' \) are more moderate. This is because all three transformers have exactly the same overall dimensions. LEAK.X2 is thus chosen for calculating the total leakage reactance throughout the thesis. It should be mentioned that generally, for a designed transformer, the winding window areas are usually evenly divided between the primary and the secondary windings. Therefore, all three methods for determining the total leakage reactance are often valid (refer to the result of Transformer A in Table 3.7), except when transformers with extreme winding conditions are considered. It is not the intention of the thesis to comment on and make comparisons between different methods of transformer leakage reactance calculation.

### 3.10.2 Eddy Current Loss Resistance

The value of eddy current loss resistance \( R_{ec} \) is usually relatively large at power frequencies and at room temperatures, compared to the other equivalent circuit parameters. Therefore, it has no significant effect on the overall transformer performance at this stage. However, when the operating temperature is changed, as well as when the supply frequency is altered, the effect of \( R_{ec} \) becomes more prominent. Subroutine EDDY.R2 has been developed to account for these changes. These effects on the transformer performance are further dealt with in Chapters 5 and 6. In this chapter and Chapter 4, subroutine EDDY.R1 (Slemon's derivation) is used for calculating \( R_{ec} \) at power frequencies.

### 3.11 COMPARISON BETWEEN CONVENTIONAL AND REVERSE DESIGN METHODS

Since formulae for determining the equivalent circuit parameters have been determined, comparisons can be made between the conventional and reverse design methods in terms of predicting the performance of transformers.
3.11.1 Design Data for Sample Transformers

To illustrate the two approaches to transformer design, two single phase, 50Hz, high voltage transformers have been designed, built and tested [Bodger et al., 2000]. Their nominal ratings are listed in Table 3.8. Using the reverse design approach, only the frequency and nominal primary voltage are entered as rated data. The secondary voltage and VA rating are a consequence of the design.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>FC1</th>
<th>FC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary voltage (V)</td>
<td>240</td>
<td>14</td>
</tr>
<tr>
<td>Secondary voltage (kV)</td>
<td>6.24</td>
<td>4.56</td>
</tr>
<tr>
<td>VA rating (VA)</td>
<td>200</td>
<td>617</td>
</tr>
</tbody>
</table>

Table 3.8 Nominal ratings of high voltage transformers

Transformer FC1 was designed for the power supply of an electric, water purification device [Johnstone and Bodger, 1997]. Transformer FC2 was a model, designed to evaluate the harmonic performance of capacitive voltage transformers. Both transformers were built as shell types with rectangular cores. Both transformers were for special applications and not procurable directly from a manufacturer. Furthermore, in both cases, size, weight and cost were premium requirements and influenced the design.

Using the conventional design approach, in addition to the rating data above, estimates of the core maximum flux density, stacking factors, current densities, volts per turn factors, and the winding width factor were specified, as listed in Table 3.9.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>FC1</th>
<th>FC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Peak Flux density (T)</td>
<td>1.5</td>
<td>1.65</td>
</tr>
<tr>
<td>Window width factor</td>
<td>3</td>
<td>5</td>
</tr>
<tr>
<td>Primary winding:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current density (A/mm²)</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Voltage per turn factor</td>
<td>24</td>
<td>49</td>
</tr>
<tr>
<td>Space factor</td>
<td>0.35</td>
<td>0.5</td>
</tr>
<tr>
<td>Secondary winding:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Current density (A/mm²)</td>
<td>2</td>
<td>3</td>
</tr>
<tr>
<td>Voltage per turn factor</td>
<td>24</td>
<td>49</td>
</tr>
<tr>
<td>Space factor</td>
<td>0.35</td>
<td>0.5</td>
</tr>
</tbody>
</table>

Table 3.9 Data for conventional transformer design

The window width factor for Transformer FC1 was based on available EI core steel, while that for Transformer FC2 was determined by the first natural resonance characteristics required [Jiang and Bodger, 1991]. The voltage per turn and space factors were values prescribed by an experienced small transformer manufacturer.
Standard physical values of material permeabilities, resistivities and thermal resistivity coefficients were also entered as data, for the two different steel cores used, and the copper windings, as shown in Table 3.10. However, the relative permeability values for steels do depend on the metallurgical processes that are used in their manufacture and may not be readily available in practice. They may have to be measured. A core stacking factor of 0.95 was estimated for both transformers.

<table>
<thead>
<tr>
<th>Relative permeability:</th>
<th>Core</th>
<th>Primary winding</th>
<th>Secondary winding</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transformer FC1</td>
<td>3000</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Transformer FC2</td>
<td>9000</td>
<td>1</td>
<td>1</td>
</tr>
<tr>
<td>Resistivity at 20°C (Ωm)</td>
<td>$1.8 \times 10^{-7}$</td>
<td>$1.76 \times 10^{-8}$</td>
<td>$1.76 \times 10^{-8}$</td>
</tr>
<tr>
<td>Coefficient of thermal resistivity ($\times 10^{-8}$°C⁻¹)</td>
<td>0.006</td>
<td>0.0039</td>
<td>0.0039</td>
</tr>
<tr>
<td>Operating temperature (°C)</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Density (kg/m³)</td>
<td>7870</td>
<td>8960</td>
<td>8960</td>
</tr>
</tbody>
</table>

**Table 3.10** Material constants

The dimensions of the various components entered as data for the reverse design method are shown in Table 3.11. Here the final values used to build the transformer are listed.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>FC1</th>
<th>FC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>Core:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length (mm)</td>
<td>66</td>
<td>114</td>
</tr>
<tr>
<td>Width (mm)</td>
<td>51</td>
<td>152</td>
</tr>
<tr>
<td>Breadth (mm)</td>
<td>44</td>
<td>44</td>
</tr>
<tr>
<td>Core/LV winding insulation thickness (mm)</td>
<td>2</td>
<td>3.25</td>
</tr>
<tr>
<td>LV winding:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>5</td>
<td>1</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>0.8</td>
<td>3.55</td>
</tr>
<tr>
<td>Interlayer insulation thickness (mm)</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>LV/HV winding insulation thickness (mm)</td>
<td>0.7</td>
<td>6.5</td>
</tr>
<tr>
<td>HV winding:</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>0.125</td>
<td>0.212</td>
</tr>
<tr>
<td>Interlayer insulation thickness (mm)</td>
<td>0</td>
<td>0.09</td>
</tr>
</tbody>
</table>

**Table 3.11** Data for reverse transformer design

With the conventional design approach there is a lack of precision in including the interlayer, interwinding and intercore/winding insulation. They are not input parameters. This can have the effect of squashing up the winding space which affects the calculation of leakage reactances. They are underestimated in value, which affects the calculated performance of the transformer.
3.11.2 Calculated and Measured Performances

The equivalent circuit parameters, referred to the primary, calculated for the transformers using both conventional and reverse design methods, are presented in Table 3.12. The calculation methods of the equivalent circuit parameters are essentially the same for both the conventional and reverse design approaches, which have been described in Sections 3.7 and 3.8.

While there is generally close agreement in the calculation of resistances and magnetising reactance for Transformer FC1, the leakage reactance calculation is quite different. However, all the equivalent circuit parameters for Transformer FC2, calculated by the conventional and reverse design methods, show significant differences.

<table>
<thead>
<tr>
<th>Equivalent Circuit Parameters</th>
<th>Transformer FC1</th>
<th>Transformer FC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_e (\Omega)$</td>
<td>3076</td>
<td>3419</td>
</tr>
<tr>
<td>$X_m (\Omega)$</td>
<td>1755</td>
<td>1982</td>
</tr>
<tr>
<td>$R_{\text{wind}} (\Omega)$</td>
<td>11.5</td>
<td>8.6</td>
</tr>
<tr>
<td>$X_{12} (\Omega)$</td>
<td>0.2</td>
<td>1.8</td>
</tr>
</tbody>
</table>

Table 3.12 Calculated and measured equivalent circuit parameters for sample transformers

The measured values of these equivalent circuit parameters, as determined by open circuit and short circuit tests are also shown in Table 3.12. These show that the reverse design method, with its particular accounting of actual dimensions, most accurately models the equivalent circuit parameters. However, the notable exception is that of the calculations of leakage reactance of both transformers. This is due to the spatial arrangement mismatches between the calculated and the measured results.

A resistance was placed across the secondary of Transformer FC1 to the give rated load conditions at unity power factor. On the other hand, since Transformer FC2 was designed for capacitive loads, an open circuit condition was used to compare calculated and measured values. All the results are given in Table 3.13.

The values listed in Table 3.13 show that despite the variation in equivalent circuit parameter estimation, both the conventional and the reverse design methods give performance results which are useful in predicting the actual performance of as-built transformers. The difference in regulation in Transformer FC1 reflects the difference in the calculated and measured values of the leakage reactance. The zero efficiency entered for Transformer FC2 is a matter of definition for this transformer feeding a purely capacitive load where no real power is transferred to the load. A greater than zero efficiency will be recorded when the transformer feeds resistive loads.
<table>
<thead>
<tr>
<th>Performance Parameters</th>
<th>Transformer FC1</th>
<th>Transformer FC2</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$ (V)</td>
<td>240</td>
<td>240</td>
</tr>
<tr>
<td>$I_1$ (A)</td>
<td>0.87</td>
<td>0.84</td>
</tr>
<tr>
<td>$V_2$ (kV)</td>
<td>6.1</td>
<td>6.02</td>
</tr>
<tr>
<td>$I_2$ (mA)</td>
<td>30</td>
<td>30</td>
</tr>
<tr>
<td>$P_1$ (W)</td>
<td>205</td>
<td>200</td>
</tr>
<tr>
<td>Efficiency (%)</td>
<td>89</td>
<td>89</td>
</tr>
<tr>
<td>Regulation (%)</td>
<td>2.2</td>
<td>2.9</td>
</tr>
</tbody>
</table>

Table 3.13 Calculated and measured rated load performance

In the case of Transformer FC2, it can be seen that the conventional method doesn’t predict the actual performance well. The primary current and hence the total input power are significantly different to the actual values. For Transformer FC2, the conventional method specifies a flux density of 1.65T. On the other hand, the reverse design method calculates a flux density of only 0.35T, which approximates that of the actual transformer under operating conditions. In this case the conventional method is not accurate, and highlights a limitation in estimating correct flux densities and hence performance. Usually, the load on a transformer in operation varies so the design is most about size and ultimate ratings. Either of the approaches can be taken depending on the limitations present.

### 3.12 CONCLUSIONS

Conventional transformer design starts from a consideration of required frequency, voltage and VA ratings. It estimates a number of factors for the core and winding arrangement, using values that are generally only known to experienced design engineers. They reflect the “art” of transformer manufacture. Core and winding material characteristics are known from standard values or physical measurements. The resultant design may not match what is actually available in materials and hence the predicted performance can be in error.

An alternative approach to designing full-core transformers has been described. It is called the reverse transformer design method. The dimensions of core and winding materials are entered based on what is available. The overall size, ratings and performance of the transformer can then be predicted.

The program routine for this reverse design approach has been detailed. The program was divided into modules in order to calculate the transformer equivalent circuit parameters and other information.
Sample high voltage transformers have been designed, built and tested. The results highlight the problems associated with the conventional design and show the usefulness of the reverse design approach. Such a design philosophy allows for the exploration in the design of transformers with alternative construction options, where flexibility in shape and size is required.
Chapter 4

PARTIAL CORE TRANSFORMERS

4.1 OVERVIEW

In this chapter the reverse design method, as presented in Chapter 3, is applied to partial core transformers. The difference between partial core and conventional full-core transformers is outlined. Modifications are made to the equivalent circuit components of full-core transformers to model partial core transformers.

Equivalent circuit components which require special consideration include the magnetising reactance \( (X_m) \), the winding leakage reactances \( (X_1 \text{ and } X_2) \), and the core loss resistance \( (R_{core}) \). New formulations are derived for these components. The new reverse design partial core model is validated with experimental results.

4.2 PARTIAL CORE VERSUS FULL-CORE TRANSFORMERS

Figure 4.1(a) shows a cross-sectional view of a partial core transformer.

The laminated core occupies the central space. The windings are wrapped around the core, with the primary winding inside the secondary winding. This was a convenient arrangement for the intended use of the step-down transformers as detailed in this chapter. The yokes and limbs, which usually form the rest of the core in full-core transformers (see Figure 4.1(b)), are not present. Therefore, the performance of these transformers is expected to be different. Partial core transformers are being studied because of their potential use with superconducting windings where the size of the core can be dramatically reduced albeit by an increase in winding turns. The combination gives a better magnetisation than a coreless transformer and maintains the leakage flux at an acceptably low level. The combination also means that the overall weight of the partial core units is significantly reduced, and they are easier to manufacture.
Figure 4.1 Transformer's cross-sectional views
Conventional transformer equivalent circuit components, which are commonly derived, do not readily represent partial core transformers [Enright and Arrillaga, 1998]. As a result, modifications have to be made to these equivalent circuit components to model partial core transformers.

With respect to the transformer equivalent circuit presented in Figure 2.11, the winding resistances $R_1$ and $R'_2$, which are derived in Sections 3.8.3 and 3.8.4, are maintained as they are not influenced by the partial core configuration. However, modifications to the calculation of $X_m$, $X_{12}$ (a combination of $X_1$ and $X_2$) and $R_{core}$ are required as the core is no longer a closed loop, which affects the magnetic field characteristics of the transformer.

To illustrate the differences between partial core transformers and full-core transformers, 3 partial core transformers were designed using the reverse design model as applied to full-core transformers described in Chapter 3. Table 4.1 shows a summary of the transformers’ specifications. The specifications and construction details of these partial core transformers are shown in Appendices A and B.

<table>
<thead>
<tr>
<th>Transformer Ratings:</th>
<th>$PC_1$</th>
<th>$PC_2$</th>
<th>$PC_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Primary voltage (V)</td>
<td>120</td>
<td>230</td>
<td>20</td>
</tr>
<tr>
<td>Secondary voltage ($V$)</td>
<td>30</td>
<td>24</td>
<td>20</td>
</tr>
<tr>
<td>VA rating (VA)</td>
<td>5400</td>
<td>4800</td>
<td>30</td>
</tr>
<tr>
<td>Operating frequency (Hz)</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td>Core:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length (mm)</td>
<td>400</td>
<td>150</td>
<td>133</td>
</tr>
<tr>
<td>Width (mm)</td>
<td>31</td>
<td>39</td>
<td>38</td>
</tr>
<tr>
<td>Breadth (mm)</td>
<td>38</td>
<td>38</td>
<td>44</td>
</tr>
<tr>
<td>Primary winding:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>3</td>
<td>11</td>
<td>4</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>2.5</td>
<td>1.9</td>
<td>0.8</td>
</tr>
<tr>
<td>Secondary winding:</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>2</td>
<td>5</td>
<td>4</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>4.5</td>
<td>4.5</td>
<td>0.8</td>
</tr>
</tbody>
</table>

Table 4.1 Partial core transformer design data

Transformers $PC_1$ and $PC_2$ were designed for high load current applications ($\approx 100A$), each operated at a different supply voltage. Transformer $PC_3$ was a model for a 1 : 1 isolating transformer.

The transformers were built and tested. The open circuit and short circuit performances, calculated for the transformers using the reverse design method, are presented.

---

1The secondary voltage and the VA rating are not part of the reverse design input specifications. They are only shown to indicate the operation of the transformers.
in Table 4.2. Open circuit and short circuit tests were carried out on the transformers. For Transformers PC1 and PC2 the open circuit tests were carried out at rated primary input voltages. The short circuit tests were then carried out at current levels lower than the rated currents due to the limitation of test instruments. The variac used had a maximum output current of 20A. For Transformer PC3, both the open and short circuit tests were carried out at rated primary input voltage. The test details are shown in Table 4.2. The complete open circuit and short circuit test results of these transformers are shown in Appendix C.1.

<table>
<thead>
<tr>
<th>Open circuit and short circuit test results</th>
<th>Transformers</th>
<th>PC1</th>
<th>Meas.</th>
<th>PC2</th>
<th>Meas.</th>
<th>PC3</th>
<th>Meas.</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{1oc}$ (V)</td>
<td></td>
<td>120</td>
<td>120</td>
<td>230</td>
<td>230</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$I_{1oc}$ (A)</td>
<td></td>
<td>0.28</td>
<td>8.2</td>
<td>0.06</td>
<td>8.0</td>
<td>0.0065</td>
<td>1.0</td>
</tr>
<tr>
<td>$P_{1oc}$ (W)</td>
<td></td>
<td>15</td>
<td>80</td>
<td>6</td>
<td>183</td>
<td>0.06</td>
<td>6</td>
</tr>
<tr>
<td>$p_{1oc}$</td>
<td></td>
<td>0.46</td>
<td>0.08</td>
<td>0.46</td>
<td>0.1</td>
<td>0.47</td>
<td>0.3</td>
</tr>
<tr>
<td>$R_{core}$ ($\Omega$)</td>
<td></td>
<td>955.1</td>
<td>180.9</td>
<td>8359</td>
<td>289.0</td>
<td>6445</td>
<td>67.3</td>
</tr>
<tr>
<td>$X_m$ ($\Omega$)</td>
<td></td>
<td>490.3</td>
<td>14.7</td>
<td>4304</td>
<td>29.1</td>
<td>3474</td>
<td>20.8</td>
</tr>
<tr>
<td>$V_{1sc}$ (V)</td>
<td></td>
<td>20</td>
<td>20</td>
<td>143</td>
<td>143</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>$I_{1sc}$ (A)</td>
<td></td>
<td>29.4</td>
<td>25.2</td>
<td>11.6</td>
<td>19.0</td>
<td>2.5</td>
<td>2.3</td>
</tr>
<tr>
<td>$P_{1sc}$ (W)</td>
<td></td>
<td>528</td>
<td>470</td>
<td>439</td>
<td>1010</td>
<td>50</td>
<td>46</td>
</tr>
<tr>
<td>$p_{1sc}$</td>
<td></td>
<td>0.90</td>
<td>0.95</td>
<td>0.27</td>
<td>0.37</td>
<td>1.0</td>
<td>1.0</td>
</tr>
<tr>
<td>$R_{wind}$ ($\Omega$)</td>
<td></td>
<td>0.61</td>
<td>0.74</td>
<td>3.28</td>
<td>2.77</td>
<td>7.96</td>
<td>8.5</td>
</tr>
<tr>
<td>$X_{12}$ ($\Omega$)</td>
<td></td>
<td>0.30</td>
<td>0.25</td>
<td>11.92</td>
<td>6.96</td>
<td>0.57</td>
<td>1.6</td>
</tr>
</tbody>
</table>

Table 4.2 Experimental results for sample partial core transformers

From Table 4.2, it can be seen that the core loss resistance ($R_{core}$), magnetising reactance ($X_m$) and the total winding leakage reactance ($X_{12}$) are notably different between the calculated and measured results. This highlights the limitations the full-core model has in predicting the performance of partial core transformers. Therefore, modifications have to be made to these equivalent circuit components to model partial core transformers [Liew and Bodger, 2001].

### 4.3 MAGNETISING REACTANCE COMPONENT

In Chapter 3, when calculating the magnetising current reactance of a full-core transformer, Equations 3.28 and 3.29 were used. They are combined to give

$$
X_m = \frac{\omega N^2 \mu_{0} \mu_{rc} A_{core}}{I_{core}}
$$

which is valid for a full-core transformer [Connelly, 1965] [Slemon, 1966]. The relative permeability of the core material was used to calculate this magnetising current...
4.3 MAGNETISING REACTANCE COMPONENT

reactance. However, for a partial core transformer, the formulation no longer holds.

Consider the axial view of a cylindrical partial core transformer as shown in Figure 4.2.

For the partial core transformer, the magnetic flux generated by the energised primary winding flows through the core and returns via the air, back to the core. Hence the reluctance of the magnetic flux path is significantly greater that if constrained in a full steel core. This implies that the overall relative permeability of the transformer should be much lower than that of the core material ($\mu_{rc} = 2000$). Therefore, a new overall relative permeability, which takes into account the magnetic flux path in the air, has to be determined.

4.3.1 Derivation of a New Air Reactance Expression

With respect to Figure 4.2, if the flux lines are uniform within the core, then the reluctance of the core is [Paul et al., 1986]

$$R_{core} = \frac{l_{core}}{\mu_0 \mu_{rc} A_{core}} \quad (4.2)$$

where $l_{core} = \text{length of core}$

$\mu_0 = \text{permeability of free space}$

$\mu_{rc} = \text{relative permeability of the core}$

$A_{core} = \text{cross sectional area of the core} = \pi r_{core}^2$
To calculate the reluctance of the air path, it is assumed that the flux density in the air 
\( B_{\text{air}} = \frac{\phi_{\text{air}}}{A_{\text{air}}} \) is only significant near both ends of the core. It is considered negligible elsewhere. Thus, only the air reluctances at both ends of the core are considered.

It is initially assumed that at both ends of the core, the flux lines flow into the air and expand as an exponential function which reaches an asymptotic value of \( l_{\text{air}} \). This is an approximation to reality which allows a simple or naive calculation of the reluctance of the air path of the magnetic circuit. While field analysis using 2D and 3D packages may yield more accurate results, the added complexity in achieving this is over-ridden by the simplicity of the exponential function used, and ultimately the accuracy of the overall performance results. The reluctance is used in the calculation of the magnetising reactance which has less effect on results than the leakage reactance. The exponential function has the form:

\[
x_a = l_{\text{air}} \left( 1 - \exp \left( -\frac{a_a}{l_{\text{air}}} \right) \right)
\]  
(4.3)

where \( x_a \) = vertical distance where the flux travels in the air  
\( l_{\text{air}} \) = effective air path length  
\( a_a \) = horizontal distance from the edge of the core where the flux travels in the air

This is an approximation to which further factors can be added. Rearranging Equation 4.3 gives

\[
a_a = l_{\text{air}} \ln \left( \frac{l_{\text{air}}}{l_{\text{air}} - x_a} \right)
\]  
(4.4)

Hence the flux cross-sectional area expands as

\[
A_{\text{air}} = \pi a^2_{\text{air}}
\]  
(4.5)

where \( a_{\text{air}} \) = \( r_{\text{core}} + a_a \)  
\[ = r_{\text{core}} + l_{\text{air}} \ln \left( \frac{l_{\text{air}}}{l_{\text{air}} - x_a} \right) \]

Therefore, Equation 4.5 becomes

\[
A_{\text{air}} = \pi \left( r_{\text{core}} + l_{\text{air}} \ln \left( \frac{l_{\text{air}}}{l_{\text{air}} - x_a} \right) \right)^2
\]  
(4.6)
Equation 4.6 can be expanded to become

\[
A_{\text{air}} = \pi r_{\text{core}}^2 + 2\pi r_{\text{core}} l_{\text{air}} \ln \left( \frac{l_{\text{air}}}{l_{\text{air}} - x_a} \right) + \pi l_{\text{air}}^2 \left[ \ln \left( \frac{l_{\text{air}}}{l_{\text{air}} - x_a} \right) \right]^2
\]

\[
= A_{\text{core}} - 2\pi r_{\text{core}} l_{\text{air}} \ln \left( \frac{l_{\text{air}} - x_a}{l_{\text{air}}} \right) + \pi l_{\text{air}}^2 \left[ \ln \left( \frac{l_{\text{air}} - x_a}{l_{\text{air}}} \right) \right]^2
\]  

(4.7)

The reluctance of the air can be calculated by integrating over the entire air path length:

\[
R_{\text{air}} = \int_{0}^{l_{\text{air}}} \frac{dx_a}{\mu_0 \mu_r \mu_{\text{air}} A_{\text{air}}}
\]

(4.8)

where \( \mu_r = \) relative permeability of air (= 1)

Substituting Equation 4.7 into Equation 4.8,

\[
R_{\text{air}} = \int_{0}^{l_{\text{air}}} \frac{dx_a}{\mu_0 \left( A_{\text{core}} - 2\pi r_{\text{core}} l_{\text{air}} \ln \left( \frac{l_{\text{air}} - x_a}{l_{\text{air}}} \right) + \pi l_{\text{air}}^2 \left[ \ln \left( \frac{l_{\text{air}} - x_a}{l_{\text{air}}} \right) \right]^2 \right)}
\]

(4.9)

The equivalent magnetic circuit of the transformer is given in Figure 4.3.

\[\text{Figure 4.3 Magnetic circuit of the transformer}\]

From Figure 4.3, the overall reluctance of the transformer is

\[
R_T = R_{\text{core}} + 2R_{\text{air}}
\]

(4.10)
The overall relative permeability of the transformer can thus be calculated from

$$R_T = \frac{l_T}{\mu_0 \mu_{rT} A_T}$$  \hspace{1cm} (4.11)

where $l_T = \text{overall flux path length}$
\[ = l_{\text{core}} + 2l_{\text{air}} \]
\[ \approx l_{\text{core}} \text{ (since } l_{\text{core}} \gg 2l_{\text{air}}) \]

$A_T = \text{overall cross-sectional area of the transformer}$
\[ \approx A_{\text{core}} \]

Rearranging Equation 4.11 gives

$$\mu_{rT} = \frac{l_{\text{core}}}{\mu_0 R_T A_{\text{core}}}$$  \hspace{1cm} (4.12)

### 4.3.2 Finding a Solution for the Air Reluctance

There is no definite solution for the integral in Equation 4.9, so numerical integration has to be performed for a given transformer to calculate the air reluctance.

With consideration to Transformer $PC1$, it has the relevant data listed in Table 4.3.

<table>
<thead>
<tr>
<th>Transformer $PC1$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Effective cross sectional area, $A_{\text{core}}$ (mm$^2$)</td>
<td>1113</td>
</tr>
<tr>
<td>Radius, $r_{\text{core}}$ (mm)</td>
<td>18.99</td>
</tr>
<tr>
<td>Length of core, $l_{\text{core}}$ (mm)</td>
<td>400</td>
</tr>
</tbody>
</table>

Table 4.3 Transformer $PC1$ data

The values in Table 4.3 can be substituted into Equation 4.9 to calculate the air reluctance. Table 4.4 shows the results of the calculation of air reluctance, total reluctance and the overall relative permeability of the transformer. This shows that virtually all the reluctance is associated with the air path.

<table>
<thead>
<tr>
<th>Transformer $PC1$</th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Core reluctance, $R_{\text{core}}$ (H$^{-1}$)</td>
<td>$0.014 \times 10^7$</td>
</tr>
<tr>
<td>Air reluctance, $R_{\text{air}}$ (H$^{-1}$)</td>
<td>$1.334 \times 10^7$</td>
</tr>
<tr>
<td>Overall reluctance, $R_T$ (H$^{-1}$)</td>
<td>$2.682 \times 10^7$</td>
</tr>
<tr>
<td>Overall $\mu_r$, $\mu_{rT}$</td>
<td>10.48</td>
</tr>
</tbody>
</table>

Table 4.4 Transformer $PC1$ results

From Table 4.2, the magnetising current reactance of Transformer $PC1$ obtained from the open circuit test is 14.7 $\Omega$. Therefore, by using Equation 4.1, the experimental $\mu_{rT}$

\[ \text{Transformer } PC1 \text{ has a rectangular core. Thus, the equivalent radius of its core is calculated by equating the core cross sectional area using } \pi r_{\text{core}}^2 = A_{\text{core}}. \]
can be calculated. Rearranging Equation 4.1,

\[ \mu_{rT1} = \frac{X_{m1} l_{core1}}{\omega N_{11}^2 \mu_o A_{core1}} \]  

(4.13)

\[ = 60.27 \]  

(4.14)

where the subscript 1 at the end of each term denotes Transformer PC1

Comparing the above value against that from Table 4.4, there is a significant difference between the test and the calculated values. Examining Equation 4.3, it can be seen that the rate of saturation of the expression is defined by the length of air path \( l_{air} \). This in turn affects the level at which the air reluctance reaches an asymptote. As a first stage in providing a better comparison between the calculated and measured values of \( \mu_{rT1} \), Equation 4.3 is redefined as

\[ x_a = l_{air} \left( 1 - \exp \left( - \frac{a_a}{\theta l_{air}} \right) \right) \]  

(4.15)

where \( \theta \) = partial core saturation factor

\( \theta \) is assumed to be a constant value. The expression for the reluctance of the air path thus becomes

\[ R_{air} = \int_0^{l_{air} \to \infty} \frac{dx_a}{\mu_o \left( A_{core} - 2\pi r_{core} \theta l_{air} \ln \left( \frac{l_{air} - x_a}{l_{air}} \right) + \pi \theta^2 l_{air}^2 \left[ \ln \left( \frac{l_{air} - x_a}{l_{air}} \right) \right]^2 \right)} \]  

(4.16)

The effect \( \theta \) has on the calculation of air reluctance, and therefore the overall relative permeability \( \mu_{rT} \), is tabulated in Table 4.5.

<table>
<thead>
<tr>
<th>( \theta )</th>
<th>( R_{air} ) (H(^{-1}))</th>
<th>( R_T ) (H(^{-1}))</th>
<th>( \mu_{rT} )</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>( 1.334 \times 10^6 )</td>
<td>( 2.682 \times 10^7 )</td>
<td>10.48</td>
</tr>
<tr>
<td>2</td>
<td>( 6.669 \times 10^6 )</td>
<td>( 1.348 \times 10^7 )</td>
<td>20.84</td>
</tr>
<tr>
<td>3</td>
<td>( 4.446 \times 10^6 )</td>
<td>( 9.032 \times 10^6 )</td>
<td>31.10</td>
</tr>
<tr>
<td>4</td>
<td>( 3.334 \times 10^6 )</td>
<td>( 6.809 \times 10^6 )</td>
<td>41.25</td>
</tr>
<tr>
<td>5</td>
<td>( 2.667 \times 10^6 )</td>
<td>( 5.475 \times 10^6 )</td>
<td>51.30</td>
</tr>
<tr>
<td>* 6</td>
<td>( 2.223 \times 10^6 )</td>
<td>( 4.586 \times 10^6 )</td>
<td>61.25</td>
</tr>
<tr>
<td>7</td>
<td>( 1.905 \times 10^6 )</td>
<td>( 3.951 \times 10^6 )</td>
<td>71.10</td>
</tr>
<tr>
<td>8</td>
<td>( 1.667 \times 10^6 )</td>
<td>( 3.475 \times 10^6 )</td>
<td>80.84</td>
</tr>
</tbody>
</table>

| Table 4.5 The effect of \( \theta \) on the overall \( \mu_{rT} \) |

In order to obtain the desired air reluctance and hence the accurate \( \mu_{rT} \), by referring to
the row marked with "\(\ast\)" in Table 4.5, the partial core saturation factor is approximately
\[
\vartheta \approx 6 \quad (4.17)
\]

Thus,
\[
x_a = l_{\text{air}} \left( 1 - \exp \left( -\frac{a_a}{6l_{\text{air}}} \right) \right) \quad (4.18)
\]

Equation 4.18 is used in Equation 4.9 to compute the air reluctance of the two partial core transformers, \(PC2\) and \(PC3\), to verify the use of the partial core saturation factor.

### 4.3.3 Verification of the Air Reluctance Formula

Having developed the air reluctance formula in Equation 4.9, and verified the result for Transformer \(PC1\), it is necessary to verify the use of the partial core saturation factor \((\vartheta = 6)\), for Transformers \(PC2\) and \(PC3\). Table 4.6 shows the tabulated result of the air reluctance needed to obtain the required \(\mu_{rT}\).

<table>
<thead>
<tr>
<th>(\mu_{rT}) from test</th>
<th>Transformer (PC2)</th>
<th>Transformer (PC3)</th>
</tr>
</thead>
<tbody>
<tr>
<td>(R_T) (H(^{-1}))</td>
<td>(6.005 \times 10^6)</td>
<td>(5.546 \times 10^6)</td>
</tr>
<tr>
<td>(R_{\text{air}}) (H(^{-1}))</td>
<td>(2.981 \times 10^6)</td>
<td>(2.757 \times 10^6)</td>
</tr>
<tr>
<td>(\vartheta)</td>
<td>4.0</td>
<td>3.8</td>
</tr>
</tbody>
</table>

**Table 4.6** Finding the appropriate \(\vartheta\) values for Transformers \(PC2\) and \(PC3\)

In addition, the appropriate values of \(\vartheta\) for each transformer have also been approximated, and shown in the last row of Table 4.6. Clearly, the value of \(\vartheta\) for each transformer differs. \(\vartheta\), which was initially assumed to be a fixed constant, is a function of the physical dimensions of the transformer. These factors must be taken into account.

\(\vartheta\) is thus redefined as
\[
\vartheta = k_\vartheta \left( \frac{l_{\text{core}}}{r_{\text{core}}} \right)^{\alpha_\vartheta} \quad (4.19)
\]

where
- \(l_{\text{core}}\) = length of the core
- \(r_{\text{core}}\) = effective radius of the core
- \(k_\vartheta\) = partial core saturation constant
- \(\alpha_\vartheta\) = partial core saturation power constant

\(\vartheta\) is therefore a dimensionless quantity. It determines the rate at which \(x_a\) reaches its asymptotic value.
The values of $\alpha_\theta$ and $k_\theta$ are found by solving, pair by pair, Equations 4.20–4.22.

\[ \vartheta_1 = k_\theta \left( \frac{l_{\text{core}1}}{r_{\text{core}1}} \right)^{\alpha_\theta} \]  
(4.20)

\[ \vartheta_2 = k_\theta \left( \frac{l_{\text{core}2}}{r_{\text{core}2}} \right)^{\alpha_\theta} \]  
(4.21)

\[ \vartheta_3 = k_\theta \left( \frac{l_{\text{core}3}}{r_{\text{core}3}} \right)^{\alpha_\theta} \]  
(4.22)

where the subscripts 1, 2 and 3 denote Transformers PC1, PC2 and PC3 respectively.

The solutions are

\[ \alpha_\theta = \frac{\log (\vartheta_n/\vartheta_{n+1})}{\log \left( \frac{l_{\text{core}(n)}}{l_{\text{core}(n+1)}}/\frac{r_{\text{core}(n)}}{r_{\text{core}(n+1)}} \right)} \]  
(4.23)

and

\[ k_\theta = \vartheta_n \left( \frac{r_{\text{core}(n)}}{l_{\text{core}(n)}} \right)^{\alpha_\theta} \]  
(4.24)

where $n = 1, 2, 3$ denotes the transformer number.

Table 4.7 shows the calculated values of $\alpha_\theta$ and $k_\theta$ for the three transformers.

<table>
<thead>
<tr>
<th>Solving equations between</th>
<th>Transformers PC1 and PC2</th>
<th>Transformers PC2 and PC3</th>
<th>Transformers PC1 and PC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_\theta$</td>
<td>0.31</td>
<td>0.29</td>
<td>0.32</td>
</tr>
<tr>
<td>$k_\theta$</td>
<td>2.27</td>
<td>2.19</td>
<td>2.18</td>
</tr>
</tbody>
</table>

Table 4.7 Estimating $\alpha_\theta$ and $k_\theta$

The values obtained are very consistent despite the variations in dimensions. The average values of $\alpha_\theta$ and $k_\theta$ are

\[ \alpha_\theta = 0.31 \]  
(4.25)

and

\[ k_\theta = 2.21 \]  
(4.26)

The $\vartheta$ value for each transformer is recalculated using Equation 4.19 with values from Equations 4.25 and 4.26. From this, $R_{\text{air}}$, $R_T$ and $\mu_{rT}$ are then calculated. The results
are shown in Table 4.8. The measured $\mu_{rT}$ from the test is also inserted in the table to compare against the calculated value for each transformer.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>$PC1$</th>
<th>$PC2$</th>
<th>$PC3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\vartheta$ value</td>
<td>5.65</td>
<td>4.03</td>
<td>3.80</td>
</tr>
<tr>
<td>$R_{air}$ (H$^{-1}$)</td>
<td>$2.336 \times 10^6$</td>
<td>$2.931 \times 10^6$</td>
<td>$2.889 \times 10^6$</td>
</tr>
<tr>
<td>$R_{T}$ (H$^{-1}$)</td>
<td>$4.855 \times 10^6$</td>
<td>$5.956 \times 10^6$</td>
<td>$5.863 \times 10^6$</td>
</tr>
<tr>
<td>$\mu_{rT}$</td>
<td>57.86</td>
<td>14.14</td>
<td>10.97</td>
</tr>
<tr>
<td>$\mu_{rT}$ from test</td>
<td>60.27</td>
<td>13.14</td>
<td>11.60</td>
</tr>
</tbody>
</table>

Table 4.8 Calculated and measured results of $\mu_{rT}$

The $\mu_{rT}$'s obtained analytically are very close to those obtained from tests. Thus, the use of $\vartheta$ (Eqn 4.19), $k_\varphi$ (Eqn 4.25) and $\alpha_\varphi$ (Eqn 4.26) in Equation 4.15 to find the air reluctance (Equation 4.16) for a given transformer, thus leading to the computation of $\mu_{rT}$, is justified. The value of $\mu_{rT}$ found can be used to predict the magnetising reactance for a particular partial core transformer.

### 4.3.4 Generalisation of the Air Reluctance Formula

An air reluctance expression for partial core transformers has been developed. A numerical integration technique is required to solve Equation 4.16. An attempt is now made to try to derive a simpler expression, which can approximate Equation 4.16, hence saving computation time performing the numerical integration.

#### 4.3.4.1 $R_{air}$ against $A_{core}$

The air reluctance $R_{air}$ is plotted against the transformer core cross-sectional area $A_{core}$, in the generalised form shown in Figure 4.4.

The air reluctance $R_{air}$ is an inversely proportional function of $A_{core}$:

$$R_{air} = \frac{a}{A_{core}^b} \quad (4.27)$$

where $a$ and $b$ are constants

Alternatively, Equation 4.27 can be modified to

$$R_{air} = \frac{a}{(A_{core}/\hat{A})^b} \quad (4.28)$$
where $\hat{A}$ is an area constant

The introduction of $\hat{A}$ makes the denominator of Equation 4.28 a dimensionless quantity. Thus, $a$ has a unit (H$^{-1}$), and $b$ is dimensionless. The constants $a$ and $b$ can be obtained by solving Equations 4.29 and 4.30 simultaneously:

$$R_{\text{air}(i)} = \frac{a}{\left(\frac{A_{\text{core}(i)}}{\hat{A}}\right)^b} \quad (4.29)$$

$$R_{\text{air}(j)} = \frac{a}{\left(\frac{A_{\text{core}(j)}}{\hat{A}}\right)^b} \quad (4.30)$$

The subscripts $i$ and $j$ denote different $R_{\text{air}}$, calculated for different core cross-sectional areas $A_{\text{core}}$ while the other parameters in the equation are kept constant. The solutions to Equations 4.29 and 4.30 are

$$a = R_{\text{air}(i)} \left(\frac{A_{\text{core}(i)}}{\hat{A}}\right)^b \quad (4.31)$$

and

$$b = \frac{\log(R_{\text{air}(i)}/R_{\text{air}(j)})}{\log(A_{\text{core}(j)}/A_{\text{core}(i)})} \quad (4.32)$$
Using the data for Transformer PC1 from Table 4.3, $\hat{A}$ is set as

$$\hat{A} = 1.113 \times 10^{-3} \text{ m}^2$$  \hspace{1cm} (4.33)

The constants are

$$a = 2.33578 \times 10^6 \text{ H}^{-1}$$  \hspace{1cm} (4.34)

and

$$b = 0.345$$  \hspace{1cm} (4.35)

4.3.4.2 $R_{\text{air}}$ against $l_{\text{core}}$

Similarly, the air reluctance $R_{\text{air}}$ is plotted in general form against the transformer core length $l_{\text{core}}$.

![Graph showing $R_{\text{air}}$ against $l_{\text{core}}$.](image)

**Figure 4.5** $R_{\text{air}}$ against $l_{\text{core}}$

It can be seen from Figure 4.5 that the air reluctance $R_{\text{air}}$ is also inversely proportional to $l_{\text{core}}$:

$$R_{\text{air}} = \frac{c}{l_{\text{core}}^d}$$  \hspace{1cm} (4.36)
where \( c \) and \( d \) are constants

Equation 4.36 can be modified to be

\[
R_{\text{air}} = \frac{a}{(l_{\text{core}}/\hat{l})^b} \tag{4.37}
\]

where \( \hat{l} \) is a length constant

This also makes the denominator of Equation 4.37 a dimensionless quantity. Thus, \( c \) has a unit \( (H^{-1}) \), and \( d \) is dimensionless. The constants \( c \) and \( d \) can again be obtained by solving the simultaneous equations 4.38–4.39:

\[
R_{\text{air}(i)} = \frac{c}{(l_{\text{core}(i)}/\hat{l})^b} \tag{4.38}
\]

\[
R_{\text{air}(j)} = \frac{c}{(l_{\text{core}(j)}/\hat{l})^b} \tag{4.39}
\]

This time the subscripts \( i \) and \( j \) denote different \( R_{\text{air}} \), calculated for different transformer core lengths \( l_{\text{core}} \) while the other parameters in the equation are kept fixed. The solutions to Equations 4.38 and 4.39 are

\[
c = R_{\text{air}(i)} \left( \frac{l_{\text{core}(i)}}{\hat{l}} \right)^b \tag{4.40}
\]

and

\[
d = \frac{\log(R_{\text{air}(j)}/R_{\text{air}(i)})}{\log(l_{\text{core}(j)}/l_{\text{core}(i)})} \tag{4.41}
\]

Using the same data from Table 4.3 for Transformer \( PC1 \), \( \hat{l} \) is set as

\[
\hat{l} = 0.4 \text{ m} \tag{4.42}
\]

The constants are

\[
c = 2.33578 \times 10^6 \text{ H}^{-1} \tag{4.43}
\]

and

\[
d = 0.31 \tag{4.44}
\]
4.3.4.3 Combining $A_{core}$ and $l_{core}$ into $R_{air}$

Having developed the air reluctance $R_{air}$ in terms of the transformer core cross-sectional area $A_{core}$ and core length $l_{core}$ separately, it is necessary to express $R_{air}$ with a combination of both $A_{core}$ and $l_{core}$. From the previous two sections, it can be seen that the constants $a$ (Equation 4.34) and $c$ (Equation 4.43) are identical. Therefore, the denominators of Equations 4.28 and 4.37 are independent of each other. They can be combined as

$$R_{air} (A_{core}, l_{core}) = 2.33578 \times 10^6 \left( \frac{\hat{A}}{A_{core}} \right)^{0.345} \left( \frac{\hat{l}}{l_{core}} \right)^{0.31}$$  \hspace{1cm} (4.45)

$$= 1.69356 \times 10^5 \left( \frac{1}{A_{core}} \right)^{0.345} \left( \frac{1}{l_{core}} \right)^{0.31}$$  \hspace{1cm} (4.46)

The validity of Equation 4.46 is checked for Transformers PC1, PC2 and PC3. Table 4.9 shows the comparison of the calculated $R_{air}$ between the numerical integration method (Equation 4.16) and Equation 4.46.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>$R_{air}$ from Eqn 4.16 (H⁻¹)</th>
<th>PC1</th>
<th>PC2</th>
<th>PC</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{air}$ from Eqn 4.16 (H⁻¹)</td>
<td>$2.336 \times 10^6$</td>
<td>$2.931 \times 10^6$</td>
<td>$2.889 \times 10^6$</td>
<td></td>
</tr>
<tr>
<td>$R_{air}$ from Eqn 4.46 (H⁻¹)</td>
<td>$2.336 \times 10^6$</td>
<td>$2.931 \times 10^6$</td>
<td>$2.889 \times 10^6$</td>
<td></td>
</tr>
</tbody>
</table>

Table 4.9 Comparing the calculated and measured results of $R_{air}$.

From Table 4.9, it can be seen that the air reluctances calculated by the two methods are identical. Therefore, the validity of Equation 4.46 is confirmed. It is effectively a much simpler calculation technique compared to the numerical integration method, and hence is used in the reverse design model from herein after.

4.4 LEAKAGE REACTANCE COMPONENT

Equations 3.50 and 3.51 in Chapter 3 were initially selected and used to calculate the total leakage inductance and the leakage reactance of the partial core transformer. However, as shown in Table 4.2, the accuracy of these equations was not as good. Hence, a factor needs to be included to improve the equation for partial core transformers. For full-core transformers, a Rogowski factor [Jiang and Bodger, 1991] was introduced to improve the calculation of the total leakage reactance. The conventional equivalent circuit and the concept of leakage reactance for a full-core transformer stem from the difficulty in analysing the behaviour of very tightly-coupled coils in terms of self and mutual inductances. The leakage-based approach is well-suited to conventional trans-
formers where the closed magnetic circuit leads to coupling coefficients very close to 1. This conventional approach is maintained, but modified, for the partial core transformer because the closely packed multilayered windings act as guides which channel the flux up the middle and around the outside such that in open circuit tests, almost perfect coupling between windings still exists. A similar approach can be used to model induction heaters [Davies, 1990], and has been the basis of computer modelling of a successful commercial line of fluid heaters [Bodger et al., 1996] which uses combined induction and transformer heating, and which yielded the conceptual ideas behind the partial core transformer.

However, no further specific information was given on what the exact Rogowski factor should be, given a transformer with a certain aspect ratio $\beta_a$ defined in Equation 3.19. Consequently, it is necessary to derive a new expression that governs the Rogowski factor.

### 4.4.1 The Rogowski Factor

The Rogowski factor $\Gamma$ [Jiang, 1987] is introduced to improve the calculation of the total leakage reactance:

\[
\Gamma \approx 1 \text{ when } \beta_a \gg 1 \tag{4.47}
\]

\[
\Gamma < 1 \text{ when } \beta_a \approx 1 \tag{4.48}
\]

Thus, the expression for leakage reactance originally derived in Equations 3.50 and 3.51 now becomes

\[
X_{12} = \Gamma X_{12}' \tag{4.49}
\]

where

\[
X_{12}' = \text{ the total leakage reactance } X_{12} \text{ originally derived in Equation 3.51}
\]

The total leakage reactance developed by [Connelly, 1965] in Equations 3.50 and 3.51 is modified to become

\[
X_{12} = \frac{\omega \mu_0 N_1^2 \Gamma}{I_{\text{core}}} \left( \frac{I_{p1}d_1 + I_{s2}d_2}{3} + I_{ps} \Delta d \right) \tag{4.50}
\]

From Equations 4.47 and 4.48 it is obvious that no specific information has been given on what the exact Rogowski factor should be for a transformer with an aspect ratio $\beta_a$. Equation 4.47 does not state what value $\beta_a$ should be in order for $\Gamma$ to be approximately
equal to 1, i.e. what is the minimum limit of $\beta_a$ for Equation 4.47 to still hold. Also, Equation 4.48 does not state what the value of $\Gamma$ should be when $\beta_a \approx 1$. Thus the Rogowski factor is ill-defined. Therefore, it is necessary to express the Rogowski factor in terms of the dimensions of the transformer which are known.

### 4.4.2 Deriving a New Leakage Function

Table 4.10 shows the total leakage reactance derived from the test results of Transformers PC1, PC2, and PC3.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>PC1</th>
<th>PC2</th>
<th>PC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$X_{12}$</td>
<td>0.25</td>
<td>6.96</td>
<td>1.6</td>
</tr>
<tr>
<td>$\Gamma$ from test</td>
<td>0.876</td>
<td>0.541</td>
<td>0.867</td>
</tr>
<tr>
<td>$\beta_a$</td>
<td>20.0</td>
<td>2.26</td>
<td>16.9</td>
</tr>
</tbody>
</table>

Table 4.10 Total leakage reactance, with the corresponding $\Gamma$ and $\beta_a$ for each transformer

Rearranging Equation 4.50, the Rogowski factor can be expressed as

$$\Gamma = \frac{X_{12}l_{core}}{\omega \mu_0 N_1^2} \left( \frac{l_p d_1 + l_s d_2}{3} + l_{ps} \Delta d \right)^{-1}$$  \hspace{1cm} (4.51)

Therefore, an "experimental" $\Gamma$ can be obtained for each transformer. These are also shown in Table 4.10 along with their respective aspect ratios. $\Gamma$ is plotted against $\beta_a$ in Figure 4.6, shown as '□'. Moreover, three additional partial core units have been built and tested, but not reported on in detail here. Their respective experimental $\Gamma$'s are also plotted against $\beta_a$ in Figure 4.6 (shown as '○').

From Figure 4.6 it can be deduced that the Rogowski factor is a non-linear function of the aspect ratio.

From the definition of the Rogowski factor in Equations 4.47 and 4.48, it is evident that the maximum value of $\Gamma$ is 1. Thus,

$$\Gamma \leq 1$$  \hspace{1cm} (4.52)

This makes the assumption that for large aspect ratios, i.e. long thin cores, there will be very little leakage. However, the smaller the aspect ratio, the greater the leakage.

It is assumed that the new leakage function $\Gamma$ has an exponential distribution function against $\beta_a$, which saturates at $\Gamma = 1$, signifying a perfect coupling between windings. This is depicted in Figure 4.7.
Figure 4.6 Rogowski factor $\Gamma$ against aspect ratio $\beta_a$

Figure 4.7 Exponential function of $\Gamma$
The leakage function has the form:

$$\Gamma(\beta_a) = 1 - \exp\left(-\frac{\beta_a}{\zeta}\right)$$

(4.53)

where $\zeta$ determines the rate of saturation of the function.

Thus, for any aspect ratio, it is possible to determine the appropriate values of $\Gamma$ and $\zeta$ for each transformer. Table 4.11 details the aspect ratio of the transformers, and the corresponding $\Gamma$ and $\zeta$ values.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>$PC_1$</th>
<th>$PC_2$</th>
<th>$PC_3$</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_a$</td>
<td>20.0</td>
<td>2.26</td>
<td>16.9</td>
</tr>
<tr>
<td>$\Gamma$</td>
<td>0.876</td>
<td>0.541</td>
<td>0.867</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>9.59</td>
<td>2.76</td>
<td>8.37</td>
</tr>
</tbody>
</table>

Table 4.11 Finding the appropriate $\zeta$ value

From Table 4.11, it can be seen that once again $\zeta$ is not a constant quantity. Therefore, it is necessary to find an expression for $\zeta$, which fits the above values, based on the physical quantities of the corresponding transformer. The $\zeta$ value is plotted against the aspect ratio $\beta_a$, as shown in Figure 4.8.

![Figure 4.8 Finding the relationship between $\zeta$ and $\beta_a$](image)

From Figure 4.8, $\zeta$ shows a linear relationship with $\beta_a$. This can be expressed as

$$\zeta = 0.4\beta_a + 1.59$$

(4.54)
Thus, the leakage function of Equation 4.53 can be finalised and expressed as

\[ \Gamma(\beta_a) = 1 - \exp\left(-\frac{\beta_a}{0.4\beta_a + 1.59}\right) \]  \hspace{1cm} (4.55)

The expression was programmed into the model, and short circuit tests were simulated for the transformers. Table 4.12 gives a comparison of \( X_{12} \) of the test and the calculated results.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>PC1</th>
<th>PC2</th>
<th>PC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( X_{12} ) from test</td>
<td>0.25</td>
<td>6.96</td>
<td>1.6</td>
</tr>
<tr>
<td>( X_{12} ) from model</td>
<td>0.25</td>
<td>6.77</td>
<td>1.3</td>
</tr>
</tbody>
</table>

**Table 4.12** Comparing \( X_{12} \) between test and model

From the table, it can be seen that the difference between the test and calculated values is small. Thus, the derivation of the new Rogowski expression, termed the leakage function, is justified for partial core transformers.

### 4.5 CORE LOSS COMPONENT

When calculating the total core loss resistance of a partial core transformer, both the eddy current and hysteresis loss components are taken into account. However, as can be seen in Table 4.2, the core loss resistances \( R_{\text{core}} \) calculated by the model are very different compared to the results obtained from the open circuit tests.

Hence, the existing Steinmetz’s hysteresis loss model [Connelly, 1965] [McPherson and Laramore, 1990], which estimates the hysteresis loss component for any full-core transformer, is not accurate when applied to partial core transformers. As a result, modifications have to be made to provide accurate values of this component in the partial core model.

In addition, the measured open circuit core resistance \( R_{\text{core}} \) is actually a consequence of a combination of the primary winding components \((R_1 \text{ and } X_1)\) as well as the core components \((R'_{\text{core}} \text{ and } X'_{m})\). This is depicted in Figure 4.9. In full-core transformers, \( R_{\text{core}} \text{ and } X_{m} \) are usually much greater than \( R_1 \text{ and } X_1 \). Hence in an open circuit test, \( R_1 \text{ and } X_1 \) are usually ignored and \( R_{\text{core}} \text{ and } X_{m} \) are taken to be \( R'_{\text{core}} \text{ and } X'_{m} \) respectively. However, for a partial core transformer, \( X_{m} \) is significantly lower than its full-core equivalent and hence cannot be decoupled from \( R_1 \text{ and } X_1 \) in the circuit calculations.

Therefore, \( R_{\text{core}} \) obtained directly from the open circuit tests cannot be used to de-
termine the actual core loss component. Instead, $R'_{\text{core}}$ is used. The hysteresis and eddy current loss components can be represented independently, each by an equivalent resistance [Slemon, 1966]. This is shown in Figure 4.9. From Figure 4.9, the hysteresis resistance $R_h$ and the eddy current resistance $R_{ec}$ are in parallel, thus forming the actual total core resistance $R'_{\text{core}}$:

$$R'_{\text{core}} = \frac{R_h}{R_{ec}} = \frac{R_h R_{ec}}{R_h + R_{ec}}$$  \hfill (4.56)

From [Say, 1983], the hysteresis loss component accounts for 70–80% of the total transformer core loss. This means that the eddy current loss component is actually a secondary loss component.

Steinmetz formulated the hysteresis loss formula for the full iron core transformer [McPherson and Laramore, 1990] as

$$P_h = \nu_{\text{core}} \gamma_{\text{core}} k_h f B_{\text{core}}^x$$  \hfill (4.57)

where
- $\nu_{\text{core}}$ = volume of the core
- $\gamma_{\text{core}}$ = density of the core material
- $f$ = input supply frequency
- $B_{\text{core}}$ = core maximum flux density
- $k_h$ = a constant dependent on the core material
- $x$ = Steinmetz’s factor, whose value ranges between 1.8 to 2.5 [Paul et al., 1986]

A new evaluation of this hysteresis loss component is necessary to minimise the mismatches between the model calculations and tests. For the partial core transformer, a similar expression can be derived. This ignores any effects that the flux remote from
the core-ends may have in inducing losses in conducting materials in close proximity to
the transformer. This is a common problem in induction heating, where judicial design
and shielding minimise any significance of this effect. A similar approach has been used
for the fluid heating apparatus of [Bodger et al., 1996]. In the end, the numerical value
of the core loss component is dominated by the magnetising reactance so any error due
to proximity effects is not significant.

4.5.1 Determining the Constants $k_h$ and $x$

The core of Transformers PC1, PC2 and PC3 are all made from the same laminated
iron material. Thus, the constants $k_h$ and $x$ should be the same for these transformers.
Data sheets and specifications of the core material used were not available. Consequently, the values of $k_h$ and $x$ were determined experimentally. From Figure 4.9,

$$P_h = \frac{e_1^2}{R_h}$$  \hspace{1cm} (4.58)

where $e_1 = \text{emf}$ across the core components

$$R_h = \text{hysteresis loss resistance}$$

From Equation 4.56,

$$R_h = \frac{R_{ec} R'_{core}}{R_{ec} - R'_{core}}$$  \hspace{1cm} (4.59)

The eddy current resistance, $R_{ec}$, can be determined from the model using the Subroutine EDDY.R1. Table 4.13 shows $R_{ec}$, $R'_{core}$, and the respective $R_h$ calculated using Equation 4.59, for Transformers PC1, PC2 and PC3.

<table>
<thead>
<tr>
<th></th>
<th>Transformer PC1</th>
<th>Transformer PC2</th>
<th>Transformer PC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{ec}$ ($\Omega$)</td>
<td>1788</td>
<td>15686</td>
<td>12657</td>
</tr>
<tr>
<td>$R'_{core}$ ($\Omega$)</td>
<td>249.0</td>
<td>390.8</td>
<td>158.2</td>
</tr>
<tr>
<td>$R_h$ ($\Omega$)</td>
<td>259.4</td>
<td>393.6</td>
<td>158.8</td>
</tr>
<tr>
<td>$e_1$ (V)</td>
<td>117.4</td>
<td>198.6</td>
<td>15.6</td>
</tr>
<tr>
<td>$P_h$ (W)</td>
<td>55.8</td>
<td>100.2</td>
<td>2.54</td>
</tr>
</tbody>
</table>

Table 4.13 Finding $R_h$ and $P_h$

Table 4.13 also shows $e_1$, which is obtained by subtracting the voltage across the
primary winding components $(R_1 + jX_1)$ from the primary input voltage $V_1$. Thus,
using the values of $R_h$'s and $e_1$'s of Table 4.13, the hysteresis loss for each transformer
can be calculated using Equation 4.58. These are shown in the last line of Table 4.13.

$P_h$ is found for each transformer for various input voltage levels.\(^3\) The values of $k_h$ and

\(^3\)Refer to Appendix C.2 for detailed calculations.
are calculated by solving Equations 4.60 and 4.61 simultaneously:

\[ P_{h(i)} = V_{\text{core}} \gamma_{\text{core}} k_h f B_{\text{core}(i)}^x \]  
\[ P_{h(j)} = V_{\text{core}} \gamma_{\text{core}} k_h f B_{\text{core}(j)}^x \]  

where the subscripts \( i \) and \( j \) denote different input voltage levels in Tables C.14, C.15 or C.16 in Appendix C.2, respectively.

Solving Equations 4.60 and 4.61 for \( k_h \) and \( x \) yield

\[ x = \frac{\log \left( \frac{P_{h(i)}}{P_{h(j)}} \right)}{\log \left( \frac{B_{\text{core}(i)}}{B_{\text{core}(j)}} \right)} \]  

and

\[ k_h = \frac{P_{h(i)}}{V_{\text{core}} \gamma_{\text{core}} f B_{\text{core}(i)}^x} \]  

Values of \( k_h \)'s and \( x \)'s are found for various values of \( P_h \) for each transformer.\(^4\) A summary of the results is presented in Table 4.14.

<table>
<thead>
<tr>
<th></th>
<th>Transformer PC1</th>
<th>Transformer PC2</th>
<th>Transformer PC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x )</td>
<td>1.84</td>
<td>1.83</td>
<td>1.86</td>
</tr>
<tr>
<td>( k_h )</td>
<td>0.31</td>
<td>1.58</td>
<td>3.32</td>
</tr>
</tbody>
</table>

Table 4.14 The constants of the hysteresis loss formula

It can be seen that the Steinmetz factors \( x \)'s for the three transformers are in very good agreement with each other, and are within the range of 1.8 and 2.5 as specified by [Paul et al., 1986]. Thus, for the core material used in the transformer, the average value is

\[ x = 1.84 \]  

However, the values of \( k_h \) for the three transformers are significantly different from one another. This indicates that the Steinmetz’s hysteresis loss model does not apply to the partial core transformers directly. As a result, a new expression for \( k_h \) is required for partial core transformers.

\(^4\)The detailed results can be seen in Appendix C.2.
4.5.2 Steinmetz’s Model for Partial Core Transformers

From Equation 4.57, it can be seen that despite the constants \( k_h \) and \( x \), \( P_h \) is dependent on \( V_{\text{core}}, \gamma_{\text{core}}, f \) and \( B_{\text{core}} \). Also, \( B_{\text{core}} \) is dependent on the transformer input characteristics, i.e., the input voltage and the operating supply frequency. These values can be set to be equal for all three transformers by adjusting the supply feeding each of them. Thus the only factor that is different between the transformers is the core volume. To account for this dependence, \( k_h \) can be defined as

\[
k_h = (V_{\text{core}} k_{h_{\text{pc}}})^{\alpha_{h_{\text{pc}}}}
\]

where \( \alpha_{h_{\text{pc}}} \) = partial core factor
\( k_{h_{\text{pc}}} \) = partial core hysteresis loss constant

(4.65)

The values of \( \alpha_{h_{\text{pc}}} \) and \( k_{h_{\text{pc}}} \) can be found by solving, pair by pair, Equations 4.66 to 4.68:

\[
k_{h1} = (V_{\text{core1}} k_{h_{\text{pc}}})^{\alpha_{h_{\text{pc}}}}
\]

(4.66)

\[
k_{h2} = (V_{\text{core2}} k_{h_{\text{pc}}})^{\alpha_{h_{\text{pc}}}}
\]

(4.67)

\[
k_{h3} = (V_{\text{core3}} k_{h_{\text{pc}}})^{\alpha_{h_{\text{pc}}}}
\]

(4.68)

where the subscripts 1, 2 and 3 denote Transformers PC1, PC2 and PC3 respectively.

The solutions are

\[
\alpha_{h_{\text{pc}}} = \frac{\log \left( k_{h(n)}/k_{h(n+1)} \right)}{\log \left( V_{\text{core}(n)}/V_{\text{core}(n+1)} \right)}
\]

(4.69)

and

\[
k_{h_{\text{pc}}} = \frac{k_{h(n)}}{V_{\text{core}(n)}}^{1/\alpha_{h_{\text{pc}}}}
\]

(4.70)

where \( n = 1, 2, 3 \) denotes the transformer number.

The results are shown in Table 4.15. The value of \( x \) in Equation 4.64 is also shown.

It can be seen that the partial core factors \( \alpha_{h_{\text{pc}}} \)'s and the hysteresis loss constant \( k_{h_{\text{pc}}} \) are very consistent. Thus, from the open circuit test results of the three transformers, the inclusion of the factors \( \alpha_{h_{\text{pc}}} \) and \( k_{h_{\text{pc}}} \) are validated. The average values used in the
Table 4.15 Computing $\alpha_{hpc}$ and $k_{hpc}$

<table>
<thead>
<tr>
<th>Transformer</th>
<th>PC1</th>
<th>PC2</th>
<th>PC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$x$</td>
<td>1.84</td>
<td>1.83</td>
<td>1.86</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Solving between Transformers</th>
<th>PC1 and PC2</th>
<th>PC2 and PC3</th>
<th>PC1 and PC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\alpha_h$</td>
<td>-2.46</td>
<td>-2.62</td>
<td>-2.47</td>
</tr>
<tr>
<td>$k_{hpc}$</td>
<td>$3.5 \times 10^3$</td>
<td>$3.6 \times 10^3$</td>
<td>$3.5 \times 10^3$</td>
</tr>
</tbody>
</table>

Component modelling are

$$\alpha_{hpc} = -2.5 \quad (4.71)$$

and

$$k_{hpc} = 3.5 \times 10^3 \quad (4.72)$$

4.5.3 Justifying the Constants $\alpha_{hpc}$ and $k_{hpc}$

Combining Equations 4.57, 4.64, 4.65, 4.71 and 4.72, the hysteresis loss for a partial core transformer becomes

$$P_{hpc} = V_{core}^{1+\alpha_{hpc}} \gamma_{core} k_{hpc}^{\alpha_{hpc}} f B_{core}^{2} \quad (4.73)$$

$$= 1.38 \times 10^{-9} V_{core}^{-1.5} \gamma_{core} f B_{core}^{1.84} \quad (4.74)$$

Equation 4.74 is only valid for the core material used in the transformers tested, while Equation 4.73 is the general hysteresis loss formula for the partial core transformer. Equation 4.73 is a much improved model over the Steinmetz’s model for full-core transformer (Equation 4.57), when applied to partial core transformers. The hysteresis loss of each transformer was calculated using Equation 4.74. Table 4.16 details the calculated hysteresis loss of each transformer.

Table 4.16 also shows the hysteresis resistance $R_h$ calculated using Equation 4.58, actual core resistance $R'_{core}$ from Equation 4.56, the calculated $R_{core}$, and $R_{core}$ from the open circuit tests. It can be seen that $R_{core}$’s obtained from both the model and the tests agree reasonably well with one another for each transformer. Therefore, the use of Equation 4.73, which calculates the hysteresis loss for partial core transformers, is justified with the above results.
### Table 4.16 Comparing $R_{core}$ obtained from the new hysteresis model with test results

<table>
<thead>
<tr>
<th></th>
<th>Transformer PC1</th>
<th>Transformer PC2</th>
<th>Transformer PC3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$P_{hpc}$ (W)</td>
<td>58.7</td>
<td>102.6</td>
<td>2.1</td>
</tr>
<tr>
<td>$e_1$ (V)</td>
<td>118.6</td>
<td>198.9</td>
<td>18.9</td>
</tr>
<tr>
<td>$R_h$ (Ω)</td>
<td>224.6</td>
<td>225</td>
<td>167.4</td>
</tr>
<tr>
<td>$R_{ex}$ (Ω)</td>
<td>1788</td>
<td>15686</td>
<td>12657</td>
</tr>
<tr>
<td>$R'_{core}$ (Ω)</td>
<td>199.5</td>
<td>222.2</td>
<td>165.5</td>
</tr>
<tr>
<td>$R_{core}$ from model (Ω)</td>
<td><strong>155.1</strong></td>
<td><strong>212.5</strong></td>
<td><strong>62.1</strong></td>
</tr>
<tr>
<td>$R_{core}$ from tests (Ω)</td>
<td><strong>180.9</strong></td>
<td><strong>289.0</strong></td>
<td><strong>67.3</strong></td>
</tr>
</tbody>
</table>

### 4.6 VERIFICATION OF THE PARAMETERS

Having developed the equivalent circuit parameters and determined the components within the parameters, the effectiveness of the reverse approach to designing partial core transformers could be examined. Another transformer, Transformer PC4, was designed and built. It was also designed for high load current applications. Its physical and electrical specifications are listed in Table 4.17.

<table>
<thead>
<tr>
<th></th>
<th>Transformer PC4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ratings:</strong></td>
<td></td>
</tr>
<tr>
<td>Primary voltage (V)</td>
<td>230</td>
</tr>
<tr>
<td>Secondary voltage (V)</td>
<td>25</td>
</tr>
<tr>
<td>VA rating (VA)</td>
<td>5000</td>
</tr>
<tr>
<td>Operating frequency (Hz)</td>
<td>50</td>
</tr>
<tr>
<td><strong>Core:</strong></td>
<td></td>
</tr>
<tr>
<td>Length (mm)</td>
<td>195</td>
</tr>
<tr>
<td>Width (mm)</td>
<td>39</td>
</tr>
<tr>
<td>Breadth (mm)</td>
<td>43</td>
</tr>
<tr>
<td><strong>Primary winding:</strong></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>9.5</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>1.9</td>
</tr>
<tr>
<td><strong>Secondary winding:</strong></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>3.5</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>4.0</td>
</tr>
</tbody>
</table>

### Table 4.17 Transformer design data

The equivalent circuit parameters, referred to the primary, and calculated using the new derivations, are presented in Table 4.18. The measured values, as determined by standard open circuit and short circuit tests, are also shown.

The core resistances obtained from both the model and the test agree well with each other. Therefore, the use of Equation 4.73, which calculates the hysteresis loss for partial core transformers, is confirmed. The use of earlier derivations of $R_1$ and $R_2$ from previous work [Bodger and Liew, 2000], which combine to give $R_{wind}$, are also
Table 4.18 Calculated and measured equivalent circuit parameters for the sample transformer validated.

Comparing the values for $X_m$, it can be seen that the calculated value is very close to that obtained from the test. Thus, the use of $\theta$ (Eqn 4.19), $k_o$ (Eqn 4.25) and $\alpha_o$ (Eqn 4.26) in Equation 4.3, to find the air reluctance, thus leading to the computation of $\mu_rT$ and $X_m$, is justified.

Also, from Table 4.18, it can be seen that there is virtually no difference between the test and calculated values of the total leakage reactance $X_{12}$. Thus, the validity of the derivation of the new leakage function is corroborated for partial core transformers.

Transformer $PC4$ was operated at rated conditions to compare calculated and measured values. The results are given in Table 4.19.

The closeness of the calculated and measured results for the transformer in Table 4.19 further confirms the validity of using the formulations derived in the thesis.

4.7 CONCLUSIONS

Current methods in determining equivalent circuit components for full-core transformers have notable limitations when applied to partial core transformers. Consequently, improvements in some of these components have been derived and presented. The modified components include the core loss resistance, magnetising reactance and the
winding leakage reactances.

For partial core transformers, in order to calculate the core magnetising reactance \( X_m \), a new overall relative permeability \( \mu_{rT} \), which takes into account the magnetic flux path in the air, has been developed. An expression for the reluctance of the air path in Equation 4.16 has been derived, which incorporates a new partial core saturation factor \( \vartheta \). Numerical integration was performed to calculate the air reluctance. A simplification to the air reluctance expression was developed, in the form of Equation 4.46, to provide computational simplicity and speed.

A new leakage function \( \Gamma(\beta_a) \) has been introduced to calculate the total leakage reactance \( X_{12} \). It has been derived to improve the Rogowski factor, which has previously been ill-defined. The leakage function has been expressed in terms of the transformer’s aspect ratio (Equation 4.55).

The existing Steinmetz’s hysteresis loss model for full-core transformers was also found to have limitations when applied to partial core transformers. Consequently, the full-core hysteresis loss constant \( k_h \) has been replaced by new partial core constants, \( \alpha_{hpc} \) and \( k_{hpc} \). The new partial core hysteresis loss formula becomes

\[
P_{hpc} = V_{\text{core}}^{1+\alpha_{hpc}} \gamma_{\text{core}} k_{hpc}^{\alpha_{hpc}} f B_{\text{core}}^2
\]

Three sample transformers have been designed, built and tested to determine the equivalent circuit parameters derived. Components within the parameters have also been determined. A fourth transformer was designed, built and tested to verify the component models. Significant agreement has been achieved between the values of the transformer equivalent circuit components as determined through calculation and test. In addition, calculated and measured operational performances of the transformer show good agreement. This opens the way for innovative new designs of transformers with partial cores, such as harmonic transformers and superconducting units.
Chapter 5

LOW TEMPERATURE APPLICATIONS

5.1 OVERVIEW

In Chapter 4, a partial core transformer model using the reverse design method has been developed. The model is valid at operating temperatures in the vicinity of room temperature (20°C) and above. In this chapter the accuracy of this new partial core model is tested for transformers in an extremely low-temperature environment. Three sample transformers were built and then tested under liquid nitrogen (N₂) conditions. The measured results are compared to model calculated values. The mismatches are used to modify the reverse design model in order to more accurately determine the calculated equivalent circuit parameters. Based on the newly modified parameters, a fourth transformer was then designed and built. The new partial core model, which accommodates low temperature applications, is validated with experimental results.

5.2 LOW TEMPERATURE APPLICATIONS OF TRANSFORMERS

Studies of superconducting transformers at extremely low temperatures started as early as the 60's. [McFee, 1961] first looked at the feasibility of superconducting power transformers. Detailed design of power transformers with the use of hypothetical superconductive windings was then presented [Wilkinson, 1963]. Further studies of power transformers with superconducting windings have been made by [Wilkinson, 1966], [Borcherds, 1966] and [Harrowell, 1970]. The work has since been intensified, with the designs of high temperature superconducting (HTS) transformers by [Riemersma et al., 1981], [Yamamoto et al., 1986], [Singh et al., 1988], [Fevrier et al., 1988], [Kito et al., 1991] and [Hörnfeldt et al., 1993]. The last few years have seen a tremendous progress in the design techniques of HTS transformers, with the availability of high-quality low-loss HTS tapes [Funaki et al., 1997] [Sykulski et al., 1999] [Schwenterly
et al., 1999] [Godeke et al., 2000]. While all the above references contemplated iron-core HTS transformers, a conceptual design of an air-core HTS transformer has been looked at [Yamaguchi et al., 1995b]. A comparison has been made between air-core and iron-core HTS transformers as well as conventional power transformers [Yamaguchi et al., 1995a]. Furthermore, the analysis of a 3-phase air-core HTS power transformer has also been presented [Yamaguchi et al., 1999].

The use of HTS in transformers can reduce transformer losses, thus provide substantial energy savings and reduced lifetime ownership costs for power utilities. HTS transformers are very appealing because they require a relatively small amount of conductor compared to conventional transformers. Therefore, they are much reduced in weight, and are of a more compact size. Moreover, they are more environmental friendly compared to conventional transformers due to the elimination of transformer oil. Hence, environmental and fire hazards can be reduced.

While the use of iron-core HTS transformers has notable advantages over their conventional counterparts, air-core HTS transformers have been proposed which have significantly reduced weight over the iron-core HTS transformers [Yamaguchi et al., 1995a]. Therefore, air-core transformers are much cheaper to produce. However, there is a trade-off — air-core transformers have a relatively high magnetising current, resulting in large magnetising loss. From this perspective, partial core HTS transformers provide a further alternative. Partial core HTS transformers have reduced weight over the full iron-core transformers, and reduced magnetising current over the air-core transformers.

Before the feasibility of partial core HTS transformers can be studied, it is essential to examine the validity of the reverse-design partial core model developed in Chapter 4 when operated at extremely low temperatures.

Conventional full-core transformers have been known to perform differently under different operating temperatures [Fallou et al., 1974]. The resistivity of metals, in particular copper and aluminium, is temperature dependent. The copper losses in liquid nitrogen filled full-core transformers have been found to be approximately 30% of that of traditional oil filled full-core transformers [O’Neill et al., 2000]. But while the overall losses have been decreased, the efficiency of the liquid nitrogen generator is such that the combined transformer/generator efficiency is worse than that for oil insulated transformers. Moreover, the electrical and mechanical breakdown strengths of selected insulation are of the same order, if not better, under liquid nitrogen as compared to oil impregnated insulation [O’Neill et al., 2000]. No further specific information in the literature has been found regarding the effect of the temperature change to the other parameters.
In this chapter, partial core transformers are operated in liquid nitrogen to further confirm the accuracy and to illustrate the usefulness and flexibility of the reverse design method.

5.3 INITIAL INVESTIGATIONS

In order to further investigate the precision of the partial core model developed in Chapter 4, three partial core transformers were immersed in liquid nitrogen and their performance measured. Their nominal ratings for normal operating temperatures, physical dimensions and material characteristics are listed in Table 5.1. The specifications and construction details of these partial core transformers are shown in Appendices A and B.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>PC1</th>
<th>PC5</th>
<th>PC6</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ratings:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary voltage (V)</td>
<td>110</td>
<td>230</td>
<td>230</td>
</tr>
<tr>
<td>Secondary voltage (V)</td>
<td>28</td>
<td>23</td>
<td>230</td>
</tr>
<tr>
<td>VA rating (kVA)</td>
<td>4.4</td>
<td>5.3</td>
<td>2.0</td>
</tr>
<tr>
<td>Operating frequency (Hz)</td>
<td>50</td>
<td>50</td>
<td>50</td>
</tr>
<tr>
<td><strong>Core:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length (mm)</td>
<td>400</td>
<td>140</td>
<td>258</td>
</tr>
<tr>
<td>Width (mm)</td>
<td>31</td>
<td>44</td>
<td>42</td>
</tr>
<tr>
<td>Breadth (mm)</td>
<td>38</td>
<td>44</td>
<td>43</td>
</tr>
<tr>
<td>Lamination thickness (mm)</td>
<td>0.5</td>
<td>0.5</td>
<td>0.5</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>2000</td>
<td>2000</td>
<td>2000</td>
</tr>
<tr>
<td><strong>Primary winding:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>3</td>
<td>8</td>
<td>6</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>2.5</td>
<td>2.25</td>
<td>2.5</td>
</tr>
<tr>
<td><strong>Secondary winding:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>2</td>
<td>2</td>
<td>7</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>4.5</td>
<td>4.5</td>
<td>2.5</td>
</tr>
</tbody>
</table>

Table 5.1 Transformer design data

Transformers PC1 and PC5 were arc welders designed for high secondary load current (≈ 100A) applications, each operated at a different supply voltage. Transformer PC6 was a prototype 1 : 1 isolating transformer.

The relative permeabilities of the steel are estimates of open circuit rated values. For design purposes they are considered constant.

Open circuit tests were carried out on the transformers at rated primary input voltages, while short circuit tests were carried out at rated input currents. The test details are shown in Table 5.2. The complete open circuit and short circuit test results of these transformers are shown in Appendix C.3.
The open circuit and short circuit tests results, calculated for the transformers using the reverse design method, are also presented in Table 5.2 for comparison. It can be seen that the reactive components $X_m$ and $X_{12}$ do not alter significantly. Therefore, changes in the operating temperature of the transformers have a negligible effect on the reactive components. This is an expected result since the magnetic flux is independent of the variation of the ambient temperature. However, the core loss component $R_{core}$ is notably different between the calculated and measured values. Therefore, a better calculation is required to account for low temperature operation. Moreover, while it can been seen from Table 5.2 that the calculated and measured values of the combined winding resistance $R_{wind}$ appear to be in close agreement, this is misleading. This is because the resistivity of copper wire at low temperatures is different than that calculated using conventional formulae. Also, the actual wire temperature will be higher than the immersion liquid temperature due to the heat being generated and dissipated. An alternative calculation is proposed.

### Table 5.2 Experimental results for the transformers

<table>
<thead>
<tr>
<th>Test results</th>
<th>Transformer #1</th>
<th>Transformer #2</th>
<th>Transformer #3</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{1oc}(V)$</td>
<td>110</td>
<td>230</td>
<td>230</td>
</tr>
<tr>
<td>$I_{1oc}(A)$</td>
<td>8.1</td>
<td>19.8</td>
<td>10.7</td>
</tr>
<tr>
<td>$P_{1oc}(W)$</td>
<td>60</td>
<td>235</td>
<td>76</td>
</tr>
<tr>
<td>$p_{1oc}$</td>
<td>0.07</td>
<td>0.05</td>
<td>0.03</td>
</tr>
<tr>
<td>$R_c(\Omega)$</td>
<td>202.4</td>
<td>225.3</td>
<td>695.0</td>
</tr>
<tr>
<td>$X_m(\Omega)$</td>
<td>13.6</td>
<td>11.6</td>
<td>21.4</td>
</tr>
<tr>
<td>$V_{1sc}(V)$</td>
<td>11</td>
<td>40</td>
<td>12.2</td>
</tr>
<tr>
<td>$I_{1sc}(A)$</td>
<td>40</td>
<td>24.1</td>
<td>8.2</td>
</tr>
<tr>
<td>$P_{1sc}(W)$</td>
<td>217</td>
<td>343</td>
<td>15</td>
</tr>
<tr>
<td>$p_{1sc}$</td>
<td>0.49</td>
<td>0.36</td>
<td>0.15</td>
</tr>
<tr>
<td>$R_{wind}(\Omega)$</td>
<td>0.14</td>
<td>0.59</td>
<td>0.22</td>
</tr>
<tr>
<td>$X_{teak}(\Omega)$</td>
<td>0.24</td>
<td>1.55</td>
<td>1.47</td>
</tr>
</tbody>
</table>

5.3.1 Effects of Temperature Changes on the Windings

$R_{wind}$ is a combination of the primary and secondary winding resistances ($R_1$ and $R_2'$), referred to the primary side. This is depicted in the equivalent circuit model in Figure 2.11. $R_1$ and $R_2'$ can be calculated using

$$R = \frac{ho l}{A} \quad (5.1)$$
where $\rho$ = resistivity of the conductor

$l$ = length of the conductor

$A$ = cross-sectional area of the conductor

The resistivity of a given conductor varies with its temperature. It is proportional to the temperature. Consequently, by immersing the transformers into liquid nitrogen, which has a temperature of 77°C or $-196^\circ$C, the windings' resistivities drop significantly. In Chapter 3, the operating resistivity at temperature $T^\circ$C is given as

$$\rho = \frac{(1 + \Delta \rho T) \rho_{20^\circ C}}{(1 + 20\Delta \rho)}$$  \hspace{1cm} (5.2)

where $\Delta \rho$ = thermal resistivity coefficient

$\rho_{20^\circ C}$ = material resistivity at $20^\circ$C

For a copper conductor, the resistivity at $20^\circ$C is

$\rho_{20^\circ C} = 1.76 \times 10^{-8}\Omega\text{m}$

and the thermal resistivity coefficient is

$\Delta \rho = 0.0039 \times 10^{-8}^\circ\text{C}^{-1}$

Using Equation 5.2, the corresponding resistivity at $-196^\circ$C is calculated to be

$\rho_{-196^\circ C} = 3.85 \times 10^{-9}\Omega\text{m}.$

A plot of the copper resistivity against temperature is found in [Dyos and Farrell, 1992]. It is verified by a number of references quoted within. A portion of this plot, between $-203^\circ$C ($70^\circ$K) and $127^\circ$C ($400^\circ$K) is reproduced in Figure 5.1. The values calculated using Equation 5.2 are superimposed on Figure 5.1. It can be seen that the resistivity values between the two lines are very close at room temperature ($20^\circ$C) and above. Therefore, the use of Equation 5.2 for temperatures above $20^\circ$C is justified. However, there is a significant difference between the two lines at extremely low temperatures. The relative error is as much as 125%. As a result, Equation 5.2 needs to be modified to account for the difference at extremely low temperatures.

From Figure 5.1, it is observed that the line from [Dyos and Farrell, 1992] is almost straight between $-203^\circ$C and $127^\circ$C. A linear regression fit of the line is found to give

$$\rho = 6.99 \times 10^{-11}T + 1.57 \times 10^{-8}$$  \hspace{1cm} (5.3)

where $T$ = the operating temperature ($^\circ$C)
The relative errors between Equation 5.3 and the data line [Dyos and Farrell, 1992] of Figure 5.1 are very small. This is shown in Table 5.3. Thus, this equation is used to replace Equation 5.2 in the model. In Sections 3.8.3 and 3.8.4, Equations 3.82 and 3.84 thus become

\[ \rho_1 = 6.99 \times 10^{-11}T_1 + 1.57 \times 10^{-8} \]  

(5.4)

and

\[ \rho_2 = 6.99 \times 10^{-11}T_2 + 1.57 \times 10^{-8} \]  

(5.5)

<table>
<thead>
<tr>
<th>Temperature</th>
<th>Data from [Dyos and Farrell, 1992] ((\times 10^{-9}\Omega m))</th>
<th>Data from Eqn. 5.3 ((\times 10^{-9}\Omega m))</th>
<th>Error (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>70 (-203)</td>
<td>1.52</td>
<td>1.51</td>
<td>0.64</td>
</tr>
<tr>
<td>77 (-196)</td>
<td>2.00</td>
<td>2.00</td>
<td>0.02</td>
</tr>
<tr>
<td>100 (-173)</td>
<td>3.41</td>
<td>3.61</td>
<td>5.79</td>
</tr>
<tr>
<td>110 (-163)</td>
<td>4.26</td>
<td>4.31</td>
<td>1.09</td>
</tr>
<tr>
<td>120 (-153)</td>
<td>5.18</td>
<td>5.01</td>
<td>3.37</td>
</tr>
<tr>
<td>200 (-73)</td>
<td>10.5</td>
<td>10.6</td>
<td>1.31</td>
</tr>
<tr>
<td>300 27</td>
<td>17.6</td>
<td>17.6</td>
<td>0.07</td>
</tr>
<tr>
<td>400 127</td>
<td>24.5</td>
<td>24.6</td>
<td>0.19</td>
</tr>
</tbody>
</table>

Table 5.3 Comparing resistivity between the data from [Dyos and Farrell, 1992] and Equation 5.3
5.3.2 Effects of Temperature Changes on the Core Components

5.3.2.1 Core Resistivity

In a similar manner, the resistivity of the laminated steel core is dependent on the temperature. In Section 3.6.7, the operating resistivity of the core at temperature $T_{\text{core}}$°C is given as

$$\rho_{\text{core}} = \frac{(1 + \Delta\rho_c T_{\text{core}}) \rho_{\text{c.}20^\circ C}}{(1 + 20\Delta\rho_c)} \tag{5.6}$$

where $\Delta\rho_c$ = thermal resistivity coefficient of the core

$\rho_{\text{c.}20^\circ C}$ = core resistivity at 20°C

The core resistivity at 20°C is

$$\rho_{\text{c.}20^\circ C} = 1.8 \times 10^{-7}\Omega \text{m}$$

and

$$\Delta\rho_c = 0.006 \times 10^{-8}\text{°C}^{-1}$$

Using Equation 5.6, at $T_{\text{core}} = -196^\circ C$, the corresponding resistivity of the core becomes an unrealistic negative value:

$$\rho_{\text{c.}(-196^\circ C)} = -2.83 \times 10^{-8}\Omega \text{m}$$

Therefore, Equation 5.6 cannot be used when estimating the core resistivity at low temperatures. For the core material in the sample transformers, the exact composition of the steel material is unknown. Hence, the exact resistivity of the core material at −196°C could not be determined. Instead, the core resistivity at −196°C was estimated by trial and error. Table 5.4 shows $R_{\text{core}}$'s calculated at different resistivities for Transformer $P1$.

<table>
<thead>
<tr>
<th>$\rho_{\text{c.}(-196^\circ C)}$ ($\times 10^{-8}\Omega \text{m}$)</th>
<th>Calculated $R_{\text{core}}$ (Ω)</th>
</tr>
</thead>
<tbody>
<tr>
<td>3.0</td>
<td>129.1</td>
</tr>
<tr>
<td>3.5</td>
<td>136.8</td>
</tr>
<tr>
<td>4.0</td>
<td>143.3</td>
</tr>
<tr>
<td>4.5</td>
<td>148.7</td>
</tr>
<tr>
<td>5.0</td>
<td>153.4</td>
</tr>
</tbody>
</table>

**Table 5.4** $R_{\text{core}}$'s at different resistivity values for Transformer $P1$

It can be seen that when $\rho_{\text{c.}(-196^\circ C)} = 4.5 \times 10^{-8}\Omega \text{m}$, the calculated $R_{\text{core}}$ has improved from a 17% error to 14%. The resistivity value was then used to compute $R_{\text{core}}$'s of
the other two transformers, as shown in Table 5.5.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>PC1</th>
<th>PC5</th>
<th>PC6</th>
</tr>
</thead>
<tbody>
<tr>
<td>( \rho_{c,-196^\circ C} \times 10^{-8}\Omega m )</td>
<td>4.5</td>
<td>4.5</td>
<td>4.5</td>
</tr>
<tr>
<td>Calculated ( R_{core} ) (( \Omega ))</td>
<td>148.7</td>
<td>197.3</td>
<td>562.2</td>
</tr>
<tr>
<td>Measured ( R_{core} ) (( \Omega ))</td>
<td>173.1</td>
<td>188.9</td>
<td>440.8</td>
</tr>
<tr>
<td>Percentage Error (%)</td>
<td>14</td>
<td>4</td>
<td>27</td>
</tr>
<tr>
<td>Percentage Error before modification (%)</td>
<td>17</td>
<td>19</td>
<td>58</td>
</tr>
</tbody>
</table>

Table 5.5 \( R_{core}'s \) for Transformers PC1, PC5 and PC6

The percentage errors of \( R_{core}'s \) for Transformers PC5 and PC6 have also shown significant improvement. Therefore, the resistivity value has been set as

\[
\rho_{c,-196^\circ C} = 4.5 \times 10^{-8}\Omega m
\]  

(5.7)

It should be cautioned that the core resistivity value in Equation 5.7 is only valid for the laminated steel core used in the transformers at \(-196^\circ C\). For other laminated core materials at different temperatures, the appropriate core resistivity data sheet should be referred to.

### 5.3.2.2 Skin Effect in the Core

The skin depth of eddy currents and flux in the steel core are also affected by the change in the resistivity of the core with temperature. The skin depth is defined in Equation 3.11 as

\[
\delta_{ec} = \sqrt{\frac{2\rho_{core}}{\mu_0\mu_r\omega}}
\]  

(5.8)

At a power frequency of 50Hz and under an ambient temperature of 20°C, the skin depth of the laminated steel core is calculated to be 0.68mm. It is significantly greater than half the lamination thickness, \( \frac{c_{lam}}{2} = 0.25mm \). Therefore, the magnetic flux can be considered to be uniformly distributed throughout the lamination and indeed the entire core cross section. This will also be true for normal operating temperatures which are higher than ambient. This is illustrated in Figure 5.2(a).

However, at a temperature of \(-196^\circ C\), the calculated skin depth significantly drops to 0.23mm. The distribution of flux within the laminated core is no longer uniform. This can be seen in Figure 5.2(b). The flux will have a tendency to concentrate towards the outside surfaces of the laminations. Hence, immersing the transformers in liquid nitrogen causes the skin depth in the laminations of the transformer cores to decrease below half the lamination thickness. This affects the core loss components, as detailed
in the subsequent two sections.

5.3.3 Eddy Current Loss

From Figure 3.15, the total core loss consists of eddy current and hysteresis losses. They can each be represented by an equivalent shunt resistance. In the reverse design model, Equation 3.72 is used to calculate the eddy current resistance $R_{ec}$:

$$R_{ec} = \frac{12\rho_{core} N_1^2 A_{core}}{c_{lam}^2 l_{core}}$$  \hspace{1cm} (5.9)

Equation 5.9 assumes that the skin depth is significantly greater than half the lamination thickness. However, when the skin depth drops to below half the lamination thickness, the effective cross-sectional area of the core is reduced, the flux distribution and hence the eddy current flow can no longer be considered uniform. Therefore Equation 5.9 must be modified when the skin depth is less than $\frac{c_{lam}}{2}$.

To provide a more realistic calculation of the eddy current losses, a revisit to the eddy current resistance of Equation 3.80 is required. This was initially derived for the case of $\delta_{ec} \geq \frac{c_{lam}}{2}$. Consideration is again given to a transformer core, with $n$ laminations, as depicted in Figure 3.14. In Figure 3.14(a), eddy current flows in each lamination, the direction of the flow being opposite to that of the current in the excitation winding due to Lenz's law. Figure 3.14(b) shows an enlarged section of one of the laminations. When
the skin depth is less than \( \frac{\varepsilon_{\text{lam}}}{2} \), the effective eddy current path length is calculated as

\[
l_{\text{ec}} = 2 \left( b_{\text{core}} - \delta_{\text{ec}} \right) + 2 \left( \frac{w_{\text{core}}}{n} - \delta_{\text{ec}} \right)
\]

\[
= 2 \left( b_{\text{core}} + \frac{w_{\text{core}}}{n} \right) - 4\delta_{\text{ec}}
\]

(5.10)

where \( w_{\text{core}}, b_{\text{core}} \) = width and breadth of the core, respectively.

The effective eddy current resistance for one lamination now becomes

\[
R_{\text{ecn}} = \rho_{\text{core}} \frac{l_{\text{ec}}}{A_{\text{ecn}}}
\]

(5.11)

where \( A_{\text{ecn}} \) = effective cross-sectional area of the eddy current flow path

\[
= l_{\text{core}} \times \delta_{\text{ec}}
\]

The power dissipated in the lamination is thus

\[
P_{\text{ecn}} = \left( \frac{V_{\text{core}}}{n} \right)^2 \frac{1}{R_{\text{ecn}}}
\]

(5.12)

where \( V_{\text{core}} \) = induced voltage in the core

\[
= \frac{V_1}{N_1}
\]

Therefore,

\[
P_{\text{ecn}} = \frac{V_1^2}{n^2 N_1^2 \rho_{\text{core}}} \left( \frac{l_{\text{core}} \delta_{\text{ec}}}{2 \left( b_{\text{core}} + \frac{w_{\text{core}}}{n} \right) - 4\delta_{\text{ec}}} \right)
\]

(5.13)

The total power for \( n \) laminations is

\[
P_{\text{ec}} = nP_{\text{ecn}}
\]

\[
= \frac{V_1^2}{n N_1^2 \rho_{\text{core}}} \left( \frac{l_{\text{core}} \delta_{\text{ec}}}{2 \left( b_{\text{core}} + \frac{w_{\text{core}}}{n} \right) - 4\delta_{\text{ec}}} \right)
\]

(5.14)
The equivalent eddy current resistance of the core, referred to the primary, is thus

\[ R_{ec} = \frac{V_1^2}{P_{ec}} = \frac{2N_1^2 \rho_{core}}{l_{core}} \left( \frac{n b_{core} + w_{core} - 2n \delta_{ec}}{\delta_{ec}} \right) \]  

(5.15)

Hence, Equation 5.15 replaces both 3.72 and 3.80 in the partial core model when calculating \( R_{ec} \).

### 5.3.4 Hysteresis Loss

The hysteresis loss resistance of a partial core transformer can be calculated using

\[ R_h = \frac{e_1^2}{P_{hpc}} \]  

(5.16)

where \( e_1 = \) induced primary winding voltage

\( P_{hpc} = \) partial core hysteresis power loss defined by Equation 4.73

The flux density \( B_{core} \) is calculated as

\[ B_{core} = \frac{\sqrt{2V_1}}{\omega N_1 A_{core}} \]  

(5.17)

where \( V_1 = \) input voltage

Since the effective cross-sectional area of the core is affected by the skin depth, the flux density is also affected. The effective cross-sectional area is less than the actual core cross-sectional area. This can be seen in Figure 5.2(b).

The effective area for one lamination is

\[ A_{effn} = b_{core} \frac{w_{core}}{n} - (b_{core} - 2\delta_{ec}) \left( \frac{w_{core}}{n} - 2\delta_{ec} \right) \]  

(5.18)

Thus the total effective area is

\[ A_{eff} = n \times A_{effn} = b_{core} w_{core} - (b_{core} - 2\delta_{ec}) (w_{core} - 2n\delta_{ec}) \]  

(5.19)

The area \( A_{core} \) in Equation 5.17 in which the flux flows is replaced by Equation 5.19.
5.4 TESTING THE MODEL

A comparison was again made between the modified calculated and experimental results. This is presented in Table 5.6.

<table>
<thead>
<tr>
<th>o/c and s/c test results</th>
<th>Transformer PC1</th>
<th>Transformer PC5</th>
<th>Transformer PC6</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{1oc}$ (V)</td>
<td>110</td>
<td>110</td>
<td>0</td>
</tr>
<tr>
<td>$I_{1oc}$ (A)</td>
<td>8.2</td>
<td>8.4</td>
<td>14</td>
</tr>
<tr>
<td>$P_{1oc}$ (W)</td>
<td>81</td>
<td>70</td>
<td>16</td>
</tr>
<tr>
<td>$p_f_{1oc}$</td>
<td>0.09</td>
<td>0.08</td>
<td>13</td>
</tr>
<tr>
<td>$R_c$ ($\Omega$)</td>
<td>148.7</td>
<td>173.1</td>
<td>14</td>
</tr>
<tr>
<td>$X_m$ ($\Omega$)</td>
<td>13.6</td>
<td>13.1</td>
<td>17</td>
</tr>
<tr>
<td>$V_{1sc}$ (V)</td>
<td>11</td>
<td>11</td>
<td>0</td>
</tr>
<tr>
<td>$I_{1sc}$ (A)</td>
<td>44</td>
<td>40</td>
<td>10</td>
</tr>
<tr>
<td>$P_{1sc}$ (W)</td>
<td>138</td>
<td>200</td>
<td>31</td>
</tr>
<tr>
<td>$p_f_{1sc}$</td>
<td>0.28</td>
<td>0.44</td>
<td>36</td>
</tr>
<tr>
<td>$R_{wind}$ ($\Omega$)</td>
<td>0.07</td>
<td>0.13</td>
<td>46</td>
</tr>
<tr>
<td>$X_{leak}$ (V)</td>
<td>0.24</td>
<td>0.24</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 5.6 Experimental results for the transformers after resistance calculation modifications

The new calculated $R_{core}$ components determined from the open circuit tests are much closer to the experimental values than those of Table 5.2. Therefore, the new formulations are justified. However, the calculated values of the winding resistance $R_{wind}$'s are significantly different to the experimental values, and indeed worse than the calculated values of Table 5.2. When performing the short circuit tests, the power dissipated as heat could be observed with the transformers constantly “boiling” in the liquid nitrogen, generating gas bubbles. Therefore, even though the ambient temperature surrounding the transformers was approximately $-196^\circ$C, the transformer windings actually operated at a relatively higher temperature. Due to the limitations of the test instruments, measuring the actual transformers’ temperatures was not possible.

<table>
<thead>
<tr>
<th>Temperature ($^\circ$C)</th>
<th>Calculated $R_{wind}$ ($\Omega$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>-150</td>
<td>0.184</td>
</tr>
<tr>
<td>-155</td>
<td>0.171</td>
</tr>
<tr>
<td>-160</td>
<td>0.159</td>
</tr>
<tr>
<td>-165</td>
<td>0.147</td>
</tr>
<tr>
<td>-170</td>
<td>0.135</td>
</tr>
<tr>
<td>-175</td>
<td>0.123</td>
</tr>
<tr>
<td>-180</td>
<td>0.110</td>
</tr>
</tbody>
</table>

Table 5.7 $R_{wind}$'s at different temperatures for Transformer PC1

Using the model, the actual operating temperature of Transformer PC1 was predicted
in order to match the experimental results. Table 5.7 shows $R_{\text{wind}}$'s calculated at different temperatures for Transformer \textit{PC1}.

It can be seen that when the temperature is $-170^\circ\text{C}$, the calculated $R_{\text{wind}}$ is very close to the measured $R_{\text{wind}}$ which has a value of 0.13\,$\Omega$. The same temperature was then applied to the other two transformers to calculate their performance. Another comparison was made between the calculated and the experimental results. This is shown in Table 5.8.

<table>
<thead>
<tr>
<th>$s/c$ test results</th>
<th>Transformer \textit{PC1}</th>
<th>Transformer \textit{PC5}</th>
<th>Transformer \textit{PC6}</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{10c}(V)$</td>
<td>110 110 0</td>
<td>230 230 0</td>
<td>230 230 0</td>
</tr>
<tr>
<td>$I_{10c}(A)$</td>
<td>8.2 8.4 2</td>
<td>19.8 21.9 9</td>
<td>10.7 10.25 4</td>
</tr>
<tr>
<td>$P_{10c}(W)$</td>
<td>83 70 18</td>
<td>284 280 1</td>
<td>97 120 19</td>
</tr>
<tr>
<td>$p_{f_{10c}}$</td>
<td>0.09 0.08 13</td>
<td>0.06 0.06 0</td>
<td>0.04 0.05 20</td>
</tr>
<tr>
<td>$R_{c}(\Omega)$</td>
<td>145.0 173.1 16</td>
<td>186.6 188.9 1</td>
<td>546.3 440.8 23</td>
</tr>
<tr>
<td>$X_{m}(\Omega)$</td>
<td>13.6 13.1 4</td>
<td>11.6 10.5 10</td>
<td>21.5 22.5 4</td>
</tr>
<tr>
<td>$V_{1sc}(V)$</td>
<td>11 11 0</td>
<td>40 40 0</td>
<td>12.2 12.2 0</td>
</tr>
<tr>
<td>$I_{1sc}(A)$</td>
<td>40 40 0</td>
<td>24.1 23 5</td>
<td>8.2 8.7 6</td>
</tr>
<tr>
<td>$P_{1sc}(W)$</td>
<td>216 200 8</td>
<td>342 280 20</td>
<td>15 18 17</td>
</tr>
<tr>
<td>$p_{f_{1sc}}$</td>
<td>0.49 0.44 11</td>
<td>0.35 0.30 35</td>
<td>0.15 0.18 17</td>
</tr>
<tr>
<td>$R_{\text{wind}}(\Omega)$</td>
<td>0.14 0.13 7</td>
<td>0.59 0.53 11</td>
<td>0.22 0.23 4</td>
</tr>
<tr>
<td>$X_{\text{leak}}(\Omega)$</td>
<td>0.24 0.24 0</td>
<td>1.55 1.66 7</td>
<td>1.47 1.38 6</td>
</tr>
</tbody>
</table>

Table 5.8 Experimental results for the transformers: final modifications

The calculated results can be seen to agree reasonably well with the experimental results for all three transformers. They are very similar to the values listed in Table 5.2 calculated using the original model but at an operating temperature of $-170^\circ\text{C}$. Therefore, it is justified to set the operating temperature of the transformer windings to $-170^\circ\text{C}$, an increase of 26$^\circ\text{C}$.

5.5 VERIFICATION OF THE MODEL

Having modified the resistance parameters in the reverse design model as described, and determined the actual operating temperature of the windings in three transformers, the effectiveness of the reverse design approach under different operating temperatures can be examined. Another transformer, Transformer \textit{PC4}, was designed and built. It was also designed for high load current applications. Its physical and electrical specifications for normal operating temperatures are listed in Table 5.9.

The equivalent circuit parameters, referred to the primary, and calculated for different
Table 5.9 Transformer #4 design data

<table>
<thead>
<tr>
<th></th>
<th>Transformer PC4</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ratings:</strong></td>
<td></td>
</tr>
<tr>
<td>Primary voltage (V)</td>
<td>230</td>
</tr>
<tr>
<td>Secondary voltage (V)</td>
<td>25</td>
</tr>
<tr>
<td>VA rating (VA)</td>
<td>5000</td>
</tr>
<tr>
<td>Operating frequency (Hz)</td>
<td>50</td>
</tr>
<tr>
<td><strong>Core:</strong></td>
<td></td>
</tr>
<tr>
<td>Length (mm)</td>
<td>195</td>
</tr>
<tr>
<td>Width (mm)</td>
<td>39</td>
</tr>
<tr>
<td>Breadth (mm)</td>
<td>43</td>
</tr>
<tr>
<td>Lamination thickness (mm)</td>
<td>0.5</td>
</tr>
<tr>
<td>Relative permeability</td>
<td>2000</td>
</tr>
<tr>
<td><strong>Primary winding:</strong></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>9.5</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>1.9</td>
</tr>
<tr>
<td><strong>Secondary winding:</strong></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>3.5</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>4.0</td>
</tr>
</tbody>
</table>

operating temperatures using the new derivations, are presented in Table 5.10. The measured values, as determined by the corresponding standard open circuit and short circuit tests, are also shown.

<table>
<thead>
<tr>
<th>o/c and s/c test results</th>
<th>Normal Temperature (Calc.</th>
<th>Meas.</th>
<th>Err (%)</th>
<th>Liquid Nitrogen (Calc.</th>
<th>Meas.</th>
<th>Err (%))</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_{1oc}(V)$</td>
<td>230</td>
<td>230</td>
<td>0</td>
<td>230</td>
<td>230</td>
<td>0</td>
</tr>
<tr>
<td>$I_{1oc}(A)$</td>
<td>5.5</td>
<td>5.4</td>
<td>2</td>
<td>5.9</td>
<td>5.7</td>
<td>4</td>
</tr>
<tr>
<td>$P_{1oc}(W)$</td>
<td>101</td>
<td>96</td>
<td>5</td>
<td>84</td>
<td>80</td>
<td>5</td>
</tr>
<tr>
<td>$p_{f1oc}$</td>
<td>0.08</td>
<td>0.08</td>
<td>0</td>
<td>0.06</td>
<td>0.06</td>
<td>0</td>
</tr>
<tr>
<td>$R_c(\Omega)$</td>
<td>522</td>
<td>551</td>
<td>5</td>
<td>625</td>
<td>661</td>
<td>5</td>
</tr>
<tr>
<td>$X_m(\Omega)$</td>
<td>42.0</td>
<td>42.6</td>
<td>1</td>
<td>39.0</td>
<td>40.3</td>
<td>3</td>
</tr>
<tr>
<td>$V_{1sc}(V)$</td>
<td>100</td>
<td>100</td>
<td>0</td>
<td>100</td>
<td>100</td>
<td>0</td>
</tr>
<tr>
<td>$I_{1sc}(A)$</td>
<td>17.2</td>
<td>17.2</td>
<td>0</td>
<td>22.7</td>
<td>21.4</td>
<td>6</td>
</tr>
<tr>
<td>$P_{1sc}(W)$</td>
<td>1093</td>
<td>1030</td>
<td>6</td>
<td>429</td>
<td>390</td>
<td>10</td>
</tr>
<tr>
<td>$p_{f1sc}$</td>
<td>0.64</td>
<td>0.60</td>
<td>7</td>
<td>0.19</td>
<td>0.18</td>
<td>6</td>
</tr>
<tr>
<td>$R_{wind}(\Omega)$</td>
<td>3.69</td>
<td>3.49</td>
<td>6</td>
<td>0.83</td>
<td>0.85</td>
<td>2</td>
</tr>
<tr>
<td>$X_{leak}(\Omega)$</td>
<td>4.49</td>
<td>4.65</td>
<td>3</td>
<td>4.32</td>
<td>4.59</td>
<td>6</td>
</tr>
</tbody>
</table>

Table 5.10 Calculated and measured equivalent circuit parameters for Transformer PC4 under different operating temperatures

It can be seen that the results obtained from both the model and the tests agree well with each other. Transformer PC4 was then operated at rated conditions to compare calculated and measured values. The results are given in Table 5.11.

The closeness of the calculated and measured results for the transformer in Table 5.11,
Table 5.11  Calculated and measured rated load performance at different operating temperatures

both at normal and liquid nitrogen temperatures, further emphasises the formulations derived, and the effectiveness and flexibility of the reverse design method.

5.6 CONCLUSIONS

The reverse design method has been applied to partial core transformers when immersed in liquid nitrogen. Three sample transformers were designed, built and tested. From the results obtained, modifications were made to the resistance parameters of the reverse design model to account for the changes in the resistance of the materials at very low temperatures. A fourth transformer was then designed, built and tested to verify the component models. Significant agreement has been achieved between the values of the transformer equivalent circuit components as determined through calculation and test. In addition, calculated and measured operational performances of the transformer show good agreement. This paves the way for the feasibility study of the design of high temperature partial core superconducting transformers, particularly for distribution units of various sizes and ratings.
Chapter 6

HIGH FREQUENCY HARMONIC CHARACTERISTICS

6.1 OVERVIEW

In this chapter harmonic frequency analysis of the partial core transformers is performed. Alterations are made to the calculation of the core magnetising reactance as well as the primary and secondary winding resistances. The modified parameters are validated with experimental results.

Capacitive components were briefly introduced in Section 2.2.2, as shown in Figure 2.4, and were neglected for power frequency analysis. In this chapter these are added into the reverse design partial core model as the effect of these stray capacitances can no longer be neglected at high frequencies. Also, for transformers with high turns ratio, the secondary winding capacitive component in particular, has a significant impact on the calculation of the transformer's natural frequencies. Modifications are also made to certain subroutines in the model to accommodate for these capacitive components.

The harmonic frequency responses of partial core transformers with relatively low turns ratio are analysed with the model. Capacitive loads of various magnitudes are then connected to the secondary terminals in order to calculate loaded resonant frequencies. The calculated values are validated with test results.

Next, harmonic frequency responses of two high voltage partial core transformers with large turns ratio are analysed. Calculations of the transformers' natural frequency are validated with test results. Finally, a capacitive load is placed across each of the secondary terminals. The loaded resonant frequencies calculated by the model are also compared against measured values.
6.2 VALIDATION OF RESISTANCE AND INDUCTIVE REACTANCE COMPONENTS

To verify the resistance and inductive reactance components derived in Chapters 4 and 5 when applied to harmonic frequencies, consideration is given to three partial core transformers; PC2, PC4 and PC7. Their design and rating specifications are shown in Table 6.1. The specifications and construction details of these partial core transformers are listed in Appendices A and B.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>PC2</th>
<th>PC4</th>
<th>PC7</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ratings:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Primary voltage (V)</td>
<td>230</td>
<td>230</td>
<td>230</td>
</tr>
<tr>
<td>Secondary voltage (V)</td>
<td>24</td>
<td>25</td>
<td>60</td>
</tr>
<tr>
<td>VA rating (VA)</td>
<td>4800</td>
<td>5000</td>
<td>5300</td>
</tr>
<tr>
<td><strong>Core:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Length (mm)</td>
<td>150</td>
<td>195</td>
<td>200</td>
</tr>
<tr>
<td>Width (mm)</td>
<td>39</td>
<td>39</td>
<td>44</td>
</tr>
<tr>
<td>Breadth (mm)</td>
<td>38</td>
<td>43</td>
<td>44</td>
</tr>
<tr>
<td><strong>Primary winding:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>11</td>
<td>9.5</td>
<td>8.5</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>1.9</td>
<td>1.9</td>
<td>2.3</td>
</tr>
<tr>
<td><strong>Secondary winding:</strong></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>5</td>
<td>3.5</td>
<td>6</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>4.5</td>
<td>4.0</td>
<td>4.5</td>
</tr>
</tbody>
</table>

Table 6.1 Partial core transformers design data for harmonic frequency analysis

6.2.1 Primary and Secondary Winding Resistances

From Equation 5.8, at a power frequency of 50Hz, the skin depth of the copper conductor is calculated to be 9.44mm.\(^1\) It is significantly greater than half the conductor thickness of the three transformers as tabulated in Table 6.2. Therefore, the AC current flowing in the conductor is uniformly distributed within the diameter of the conductor. This is illustrated in Figure 6.1(a).

<table>
<thead>
<tr>
<th>Transformer</th>
<th>PC2</th>
<th>PC4</th>
<th>PC7</th>
</tr>
</thead>
<tbody>
<tr>
<td>(w_1) (mm)</td>
<td>1.9</td>
<td>1.9</td>
<td>2.3</td>
</tr>
<tr>
<td>(\frac{w_1}{2}) (mm)</td>
<td>0.95</td>
<td>0.95</td>
<td>1.15</td>
</tr>
<tr>
<td>(w_2) (mm)</td>
<td>4.5</td>
<td>4.0</td>
<td>4.5</td>
</tr>
<tr>
<td>(\frac{w_2}{2}) (mm)</td>
<td>2.25</td>
<td>2.0</td>
<td>2.25</td>
</tr>
</tbody>
</table>

Table 6.2 Conductor thickness of the three transformers

\(^1\)For copper, \(\rho_{\text{copper}} = 1.76 \times 10^{-8}\Omega m\) and \(\mu_{r, \text{copper}} = 1\)
However, as the frequency increases, the calculated skin depth drops. The conductors’ skin depths at various frequencies for the three transformers are shown in Table 6.3.

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>( \delta_{sc} ) (mm)</th>
</tr>
</thead>
<tbody>
<tr>
<td>50</td>
<td>9.44</td>
</tr>
<tr>
<td>100</td>
<td>6.68</td>
</tr>
<tr>
<td>200</td>
<td>4.72</td>
</tr>
<tr>
<td>500</td>
<td>2.99</td>
</tr>
<tr>
<td>1000</td>
<td>2.11</td>
</tr>
<tr>
<td>2000</td>
<td>1.49</td>
</tr>
<tr>
<td>5000</td>
<td>0.94</td>
</tr>
</tbody>
</table>

Table 6.3 Comparison of skin depths at different frequencies

The skin depth drops below half the conductor thickness at different frequencies for the three transformers, for both the primary and secondary conductors. As a result, the current distribution within the conductors is no longer uniform. The current concentrates towards the outside surfaces of the conductor. The effective current carrying cross-sectional area can be seen in Figure 6.1(b). In reality the current distribution in a semi-infinite conductor decays from the surface value exponentially. The skin depth in Figure 6.1(b) is an approximation of this. The effective cross-sectional area is calculated as

\[
A_{eff} = \pi \left( \frac{w}{2} \right)^2 - \pi \left( \frac{w - 2\delta_{sc}}{2} \right)^2 \\
= \pi \delta_{sc} (w - \delta_{sc})
\]  

(6.1)

The primary and secondary winding resistances \( R_1 \) and \( R'_2 \) referred to the primary side can be calculated using the general form of Equation 5.1. When taking into account
the skin depth, the cross-sectional area in Equation 5.1 is replaced by Equation 6.1. The resistance thus calculated can be combined together as \( R_{\text{wind}} = R_1 + R'_2 \) to allow comparison to measured values using a short circuit test.

6.2.2 Magnetising Reactance

Equation 4.1 was used in Chapter 4 to calculate the magnetising reactance. \( X_m \) is directly proportional to the operating frequency. The graphs of measured magnetising reactance against frequency for the three transformers are plotted in Figure 6.2.

It can be observed that the relative percentage errors between the calculated and measured values are linear within the frequency range of interest. This is true for all three transformers tested. Therefore, the magnetising reactance derived earlier in Chapter 4 needs improvement.

The magnetising reactance of a partial core transformer is

\[
X_m = \frac{\omega N_1^2 \mu_0 \mu_{rT} A_{\text{core}}}{l_{\text{core}}}
\]

(6.2)

where \( \mu_{rT} = \) the overall relative permeability of the transformer defined by Equation 4.12

\( A_{\text{core}} = \) the effective core cross-sectional area defined by Equation 5.19

Approximations to the distribution of the flux within partial core transformers with different winding thickness factors are depicted in Figure 6.3. The two transformers have the same core length \( l_{\text{core}} \). The flux path is uniform within each core. The air flux path is dependent on the winding thickness factor \( \tau \). For a partial core transformer with a smaller thickness factor, \( \tau_1 \), the flux flows through a relatively shorter air path (Figure 6.3(a)) compared to the one with a larger thickness factor \( \tau_2 \) (Figure 6.3(b)). Therefore, the aspect ratio of the transformer affects the air flux path and hence the overall relative permeability and the magnetising reactance of the partial core transformer.

The overall relative permeability is redefined as

\[
\mu_{rT} = \Upsilon(\beta_a)\mu_{rT}'
\]

(6.3)

where \( \mu_{rT}' \) is defined earlier in Equation 4.12.

The expression \( \Upsilon(\beta_a) \) introduced is called the magnetising function. It is expressed in
(a) $PC2$

(b) $PC4$

(c) $PC7$

Figure 6.2 $X_m$ against frequency
Figure 6.3  Flux distribution within the partial core transformer with different winding thickness factors
terms of the transformer aspect ratio. The aspect ratio is defined in Equation 3.19 as

\[
\beta_a = \frac{l_{\text{core}}}{\tau_{12}} = \frac{l_{\text{core}}}{d_1 + \Delta d + d_2} \tag{6.4}
\]

where \(d_1\) = primary winding thickness factor
\(\Delta d\) = interwinding thickness factor
\(d_2\) = secondary winding thickness factor

For a fixed core length, the flux will take a relatively shorter return air path back to the core if the aspect ratio \(\beta_a\) is large. Therefore \(\mu_{rT}\) should be near its maximum value and hence, \(\Upsilon(\beta_a)\) approaches unity. Conversely, the flux will take a very long return path if the aspect ratio is very small. Hypothetically, if \(\beta_a\) tends to zero, the air flux path will tend to infinity. The air reluctance will also tend to infinity, and as a result the overall relative permeability becomes zero. Thus, \(\Upsilon(\beta_a)\) is equal to zero:

\[
0 \leq \Upsilon(\beta_a) \leq 1, \text{ for } 0 \leq \beta_a \leq \infty \tag{6.5}
\]

Substituting Equation 6.3 into 6.2, and rearranging Equation 6.2, the magnetising function can be expressed as

\[
\Upsilon(\beta_a) = \frac{X_m l_{\text{core}}}{\omega N_l^2 \mu_0 \mu'_{rT} A_{\text{core}}} \tag{6.6}
\]

Table 6.4 shows measured values of the magnetising reactance for transformers PC2, PC4 and PC7 at the operating frequency of 2kHz, and their corresponding aspect ratio. From these, an “experimental” \(\Upsilon(\beta_a)\) was obtained for each transformer. The values are also shown in Table 6.4.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>PC2</th>
<th>PC4</th>
<th>PC7</th>
</tr>
</thead>
<tbody>
<tr>
<td>(X_m) ((\Omega))</td>
<td>1120</td>
<td>1540</td>
<td>105</td>
</tr>
<tr>
<td>(\beta_a)</td>
<td>2.26</td>
<td>4.81</td>
<td>3.03</td>
</tr>
<tr>
<td>(\Upsilon(\beta_a))</td>
<td>0.81</td>
<td>0.86</td>
<td>0.85</td>
</tr>
</tbody>
</table>

Table 6.4 Finding \(\Upsilon(\beta_a)\)'s for the three transformers

\(\Upsilon(\beta_a)\) is plotted against \(\beta_a\) in Figure 6.4, shown as '□'. Three additional partial core units have also been built and tested, but not reported on in detail in this chapter. Their respective experimental \(\Upsilon(\beta_a)\)'s are also plotted in Figure 6.4 (shown as '○').

From Figure 6.4 it can be concluded that the magnetising function is a non-linear function of the aspect ratio. To find an expression for \(\Upsilon(\beta_a)\), it is assumed that it has an exponential distribution function with respect to the aspect ratio, which has an asymptotic value of \(\Upsilon = 1\). This is depicted in Figure 6.5.
Figure 6.4 Magnetising function against aspect ratio

Figure 6.5 Exponential function of $Y(\beta a)$
The magnetising function thus has the form:

$$\Upsilon(\beta_a) = 1 - \exp \left( -\frac{\beta_a}{\zeta} \right)$$  \hspace{1cm} (6.7)

where $\zeta$ determines the rate of saturation of the magnetising function.

Thus, for a calculated aspect ratio it is possible to determine the appropriate value of $\Upsilon(\beta_a)$ and $\zeta$ for each transformer. Table 6.5 details the aspect ratio of the transformers, and the corresponding $\Upsilon(\beta_a)$ and $\zeta$ values.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>PC2</th>
<th>PC4</th>
<th>PC7</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\beta_a$</td>
<td>2.26</td>
<td>4.81</td>
<td>3.03</td>
</tr>
<tr>
<td>$\Upsilon(\beta_a)$</td>
<td>0.81</td>
<td>0.86</td>
<td>0.85</td>
</tr>
<tr>
<td>$\zeta$</td>
<td>1.38</td>
<td>2.44</td>
<td>1.60</td>
</tr>
</tbody>
</table>

**Table 6.5** Finding the values of $\zeta$

Plotting $\zeta$ against the aspect ratio $\beta_a$, as shown in Figure 6.6, $\zeta$ shows a linear relationship with $\beta_a$.

![Figure 6.6 Finding the relationship between $\zeta$ and $\beta_a$](image)

Using Figure 6.6, $\zeta$ can be expressed as

$$\zeta = 0.32\beta_a + 0.8$$  \hspace{1cm} (6.8)

Thus, the magnetising function of Equation 6.7 becomes

$$\Upsilon(\beta_a) = 1 - \exp \left( -\frac{\beta_a}{0.32\beta_a + 0.8} \right)$$  \hspace{1cm} (6.9)
6.2.3 Verification of Results

Both the modifications detailed in Sections 6.2.1 and 6.2.2 were programmed into the model. Open circuit and short circuit tests were simulated for the transformers at selected frequencies of 50Hz, 100Hz, 200Hz, 500Hz, 1kHz, 2kHz and 5kHz. An oscilloscope was used to measure the primary voltage and current waveforms, and the corresponding phase shift between the two waveforms. Other performance quantities such as the power factors \((p\beta's)\), real powers \((P's)\) and the volt-ampere's \((S's)\) were calculated directly from these three quantities. Figures 6.7 – 6.10 show graphs of \(X_m\), \(X_{12}\), \(R_{\text{wind}}\) and \(R_{\text{core}}\) against frequency for the three transformers.

It can be seen that both the magnetising reactance (Figure 6.7) and leakage reactance (Figure 6.8) components have acceptable matches between model calculation and test values. Therefore, incorporation of the magnetising function in the calculation of the overall relative permeability is justified. Also, the original leakage function derived in Chapter 4 is also justified at higher frequencies. However, from Figures 6.9 and 6.10, the measured winding resistance and core resistance components are very different to the calculated values at high frequencies.

As frequency increases, magnitudes of both reactive components \(X_m\) and \(X_{12}\) increase. This is because inductive reactance is directly proportional to the frequency. Consequently with increasing frequency the phase angle between the current and voltage tends towards 90° lagging as the inductive reactances dominate the resistances. At phase angles near 90°, a small error in the angle can result in erroneous calculations in the power factor, and hence the real power loss. Table 6.6 shows phase angles and their corresponding power factors. A change of 5% in the phase angle between 85° and 89.5° results in a change of 90% in the power factor.

<table>
<thead>
<tr>
<th>Phase angle (°)</th>
<th>Power factor</th>
</tr>
</thead>
<tbody>
<tr>
<td>85</td>
<td>0.087</td>
</tr>
<tr>
<td>86</td>
<td>0.070</td>
</tr>
<tr>
<td>87</td>
<td>0.052</td>
</tr>
<tr>
<td>88</td>
<td>0.035</td>
</tr>
<tr>
<td>89</td>
<td>0.017</td>
</tr>
<tr>
<td>89.5</td>
<td>0.009</td>
</tr>
<tr>
<td>90</td>
<td>0</td>
</tr>
</tbody>
</table>

Table 6.6 Comparison of the relative error between phase angle and power factor

A \(±10\%) error was added to the measured phase angles. The power factors, real power losses and hence the resistance components were recalculated. The graphs showing these errors are also depicted in Figures 6.9 and 6.10. From Figure 6.9 it can be seen that the calculated winding resistance values all lie within the 10% error band.
Figure 6.7 $X_m$ against frequency
Figure 6.8 $X_{12}$ against frequency
Figure 6.9 $R_{\text{wind}}$ against frequency
Figure 6.10 $R_{core}$ against frequency
However, in Figure 6.10 this is only true for \( R_{\text{core}} \) up to about 1kHz. Above this frequency the calculated values lie below the \(-10\%\) error line. The calculated curves of \( R_{\text{core}} \) tend to flatten as the frequency increases.

Examining Equation 4.74, the hysteresis loss for a partial core transformer is proportional to the operating frequency and the core flux density as

\[
P_{\text{hpc}} \propto f B_{\text{core}}^{1.84}
\]

(6.10)

From Equations 5.17 and 5.19, \( B_{\text{core}} \) is inversely proportional to the frequency and the effective area of the core \( A_{\text{eff}} \). Thus,

\[
P_{\text{hpc}} \propto f \left( \frac{1}{f A_{\text{eff}}} \right)^{1.84}
\]

\[
\propto \frac{1}{f^{0.84}} \frac{1}{A_{\text{eff}}^{1.84}}
\]

(6.11)

Since the hysteresis loss resistance \( R_{h} \) is inversely proportional to \( P_{\text{hpc}} \) (Equation 5.16), \( R_{h} \) is thus directly proportional to \( f \) and \( A_{\text{eff}} \):

\[
R_{h} \propto f^{0.84} A_{\text{eff}}^{1.84}
\]

(6.12)

From Equation 5.19, the core effective area \( A_{\text{eff}} \) is a function of the skin depth \( \delta_{ec} \), which decreases when the frequency increases (Equation 5.8):

\[
A_{\text{eff}} \propto \frac{1}{f^{0.5}}
\]

(6.13)

Therefore,

\[
R_{h} \propto f^{0.84} \left( \frac{1}{f^{0.5}} \right)^{1.84}
\]

\[
\propto \frac{1}{f^{0.08}}
\]

(6.14)

For the eddy current resistance of the core in Equation 5.15, \( R_{ec} \) is also a function of \( \delta_{ec} \). Since \( \left| nb_{\text{core}} + w_{\text{core}} \right| \gg \left| 2n\delta_{ec} \right| \), Equation 5.15 is approximately

\[
R_{ec} \approx \frac{2N_{i}^{2} \rho_{\text{core}}}{l_{\text{core}}} \left( \frac{nb_{\text{core}} + w_{\text{core}}}{\delta_{ec}} \right)
\]

\[
\propto \frac{1}{\delta_{ec}}
\]

\[
\propto \frac{1}{f^{0.5}}
\]

(6.15)
The total core resistance $R_{core}$ is $R_h$ in parallel with $R_{ec}$ (Equation 3.87). Consequently, it is a combination of all the above factors which cause the calculated $R_{core}$ curves to become flat as the frequency increases.

While further analysis could be done on the model to improve its accuracy, and high-precision equipment could be used to accurately measure the core resistance component at high frequencies, it has not been carried out. This is because the magnitude of $R_{core}$ at high frequencies is significantly larger than $X_m$, $X_{12}$ and $R_{wind}$ in the equivalent circuit. Thus, it does not cause any critical changes to the analysis of other components. Hence the accuracy of $R_{core}$ is not further pursued in this thesis. The calculated resistance components are justified and accepted to within 10% of the measured values.

6.3 INCORPORATION OF CAPACITIVE REACTANCE COMPONENTS

A new module, Capacitance, has been incorporated into the reverse design partial core model to account for the capacitive components of the transformer. The module was merged into the Main module described in Section 3.4. It was inserted after the Resistance module and before the Output module. The enlarged portion of the new Main module which shows the inclusion of the Capacitance module is depicted in Figure 6.11.

![Figure 6.11 Inclusion of Capacitance module](image)

The Capacitance module calculates all the inherent capacitances that exist in a partial core transformer. Using these inherent capacitances, the primary winding capacitance, interwinding capacitance and the referred secondary winding capacitance are then calculated. The flow diagram of this module is presented in Figure 6.12.
Figure 6.12 Implementation of Capacitance module
6.3.1 INDIVIDUAL-CAP Subroutine

This subroutine calculates all the inherent capacitances in the partial core transformer. Consideration is given to the axial view of the partial core transformer shown in Figure 6.13.

![Figure 6.13 Axial view of the transformer showing individual capacitances](image)

Thus far, the transformer tank, although mentioned briefly in Chapter 2 for full-core transformers, has not been incorporated anywhere in the analysis of partial core transformers in this thesis. This is because all the transformers tested so far do not have enclosed tanks. However, for capacitive reactance analysis, it is imperative that there always exists a capacitance between the outmost layer of the secondary winding and an earthed plane, whether the plane's orientation is horizontal (usually just the ground plane) or vertical (in this case the transformer tank). In Figure 6.13 a tank is incorporated to act merely as an earthed plane at some uniform distance away from the transformer's outmost winding. However, inclusion of a metal tank with a partial core transformer will pose problems of induction heating in the tank due to its close proximity. Non metallic tanks may need to be considered to address this.

From Figure 6.13, five capacitive components can be identified:

1. $C_{c1}$ — capacitance between the core and the first layer of the primary winding
2. $C_{w1}$ — self-capacitance of the primary winding
3. $C_{12}$ — interwinding capacitance
4. $C_{w2}$ — self-capacitance of the secondary winding
5. $C_{2T}$ — capacitance between the last layer of the secondary winding and the tank
6.3.1.1 Calculation of $C_{cl}$

The calculation of any capacitance can be done using the parallel plate theory [Snelling, 1988]. Consider two adjacent surfaces shown in Figure 6.14.

![Diagram showing two adjacent conductive surfaces with linear potential distribution.](image)

**Figure 6.14** Two adjacent conductive surfaces, each having a linear potential distribution

The voltage of surface A is assumed to be varying linearly from $V_{AO}$ at the lower end to $V_{AP}$ at the top end. Similarly, the voltage of surface B varies linearly from $V_{BO}$ to $V_{BP}$. The voltage difference between the two surfaces should vary linearly along the length $l$ from $V_O = V_{BO} - V_{AO}$ at the bottom to $V_P = V_{BP} - V_{AP}$ at the top. The voltage difference at the element $dx$ is

$$dV = V_O + (V_P - V_O) \frac{x}{l} \quad (6.16)$$

The associated electrical energy within the element $dx$ is

$$dW = \frac{1}{2} dC dV^2$$

$$= \frac{1}{2} C_I \frac{dx}{l} \left( V_O + (V_P - V_O) \frac{x}{l} \right)^2 \quad (6.17)$$

$C_I$ is the capacitance between the two surfaces within the elemental length $dx$.

---

2 This is strictly true only if the electric field can be considered linear between the plates. This is a reasonable approximation if the electrode surface separation dimension is small compared to the electrode dimensions.
The total energy is calculated by integrating over the whole length:

\[
\int dW = \int_0^l \frac{1}{2} C_l \frac{dx}{l} \left( V_O + (V_P - V_O) \frac{x}{l} \right)^2
\]

\[
W = \frac{1}{2} C_l \int_0^l \left\{ V_O^2 + \frac{2V_O(V_P - V_O)}{l} x + (V_P - V_O)^2 \frac{x^2}{l^2} \right\} dx
\]

\[
= \frac{1}{6} C_l \left( V_O^2 + V_O V_P + V_P^2 \right)
\]

(6.18)

If one of the surfaces represents a layer of a winding having a total voltage of \( V \) across it, then the capacitance \( C \) appearing across the terminals of the winding due to the distributed capacitance of that layer can be calculated by equating energies:

\[
\frac{1}{2} CV^2 = \frac{C_l}{6} \left( V_O^2 + V_O V_P + V_P^2 \right)
\]

(6.19)

Therefore,

\[
C = \frac{C_l}{3V^2} \left( V_O^2 + V_O V_P + V_P^2 \right)
\]

(6.20)

A primary winding with \( L_{y_1} \) layers, is shown in Figure 6.15.

![Figure 6.15 Calculating the primary winding self-capacitance](image)

If \( C_m \) is the parallel-plate capacitance between the first layer and the core, and \( C_l \) is the capacitance between layers, then the primary winding equivalent capacitance is

\[
C_p = \frac{C_m}{3V^2} \left( \frac{V_1}{L_{y_1}} \right)^2 + \frac{C_l}{3V^2} \left( \frac{2V_1}{L_{y_1}} \right)^2 + \cdots + \frac{C_l}{3V^2} \left( \frac{2V_1}{L_{y_1}} \right)^2
\]

\[
= \frac{C_m}{3L_{y_1}^2} + \frac{4C_l}{3L_{y_1}^2} (L_{y_1} - 1)
\]

(6.21)

Therefore, the capacitance between the core and the first layer of the primary winding
is

\[ C_{cl} = \frac{C_m}{3L_y^2} \]  \hspace{1cm} (6.22)

To find \( C_m \), consider the enlarged portion of the transformer showing the core, the first two layers of the primary winding, and the space between the core and the winding.

![Diagram of transformer showing core and primary winding](image)

**Figure 6.16** Space between the core and the primary winding

From Figure 6.16, \( C_m \) is a series of capacitances caused by different insulation layers:

\[ C_m = \frac{1}{\frac{1}{C_{IC1}} + \frac{1}{C_{WI1}}} \]  \hspace{1cm} (6.23)

where \( C_{IC1} \) = capacitance of the insulation between the core and the primary winding \( IC1 \)

\( C_{WI1} \) = capacitance of the primary winding wire insulation \( WI1 \)

Each of these insulation capacitances are calculated using the parallel-plate capacitance formula:

\[ C = \frac{\varepsilon_r \varepsilon_0 A}{d} \]  \hspace{1cm} (6.24)

where \( \varepsilon_r \) = the corresponding relative permittivity of the insulating material

\( \varepsilon_0 \) = permittivity of free space = \( 8.854 \times 10^{-12} \) Fm\(^{-1}\)

\( A \) = area of the insulation surface

\( d \) = effective thickness of the insulation

### 6.3.1.2 Calculation of \( C_{W1} \)

The self-capacitance of the primary winding is calculated using the second term of Equation 6.21:

\[ C_{W1} = \frac{4C_l}{3L_y^2} (L_y - 1) \]  \hspace{1cm} (6.25)
From Figure 6.16, $C_t$ is calculated as

$$C_t = \frac{1}{\frac{1}{C_{t1}} + 2 \left( \frac{1}{C_{W11}} \right)}$$

(6.26)

**6.3.1.3 Calculation of $C_{12}$**

The calculation of the interwinding capacitance depends on whether the number of layers in the primary winding is odd or even.

![Diagram](a) $L_{y1}$ odd 
(b) $L_{y1}$ even

**Figure 6.17 Determining the interwinding capacitance $C_{12}$**

Using Equation 6.20 and the schematics of Figure 6.17, $C_{12}$ can be determined:

$$C_{12} = \frac{1}{3(V_1 - V_2)^2 C_w} \left[ (V_1 - \frac{L_{y1} - 1}{L_{y1}}) V_1 + \left( \frac{L_{y1} - 1}{L_{y2}} \right) V_1 + \frac{V_2}{L_{y2}} - V_1 \right]$$

for $L_{y1}$ odd

$$= \frac{1}{3(V_1 - V_2)^2 C_w} \left[ \left( \frac{L_{y1} - 1}{L_{y1}} \right) V_1 - \frac{V_2}{L_{y2}} \right] V_1 + \left( \frac{V_2}{L_{y2}} - \frac{L_{y1} - 1}{L_{y1}} V_1 \right)$$

for $L_{y1}$ even

(6.27)

Examining the enlarged portion of the interwinding space between the primary and secondary windings shown in Figure 6.18, the parallel-plate capacitance $C_w$ between
the two windings is

\[
C_w = \frac{1}{\frac{1}{C_{W11}} + \frac{1}{C_{I12}} + \frac{1}{C_{W12}}}
\]  
(6.28)

where \( C_{I12} \) = capacitance of the primary-secondary interwinding insulation \( I12 \)

\( C_{W12} \) = capacitance of the secondary winding wire insulation \( w_{I2} \)

**Secondary**

**Primary**

![Diagram showing the space between the primary and secondary windings](image)

*Figure 6.18* Space between the primary and secondary windings

### 6.3.1.4 Calculation of \( C_{w2} \)

In a similar way to \( C_{w1} \), the secondary winding self-capacitance is calculated as

\[
C_{w2} = \frac{4C_t}{3L_{y2}^2 (L_{y2} - 1)}
\]  
(6.29)

\( C_t \) for the secondary winding, referring to Figure 6.18, is

\[
C_t = \frac{1}{\frac{1}{C_{I2}} + \frac{1}{C_{W12}}} \frac{1}{2}
\]  
(6.30)

### 6.3.1.5 Calculation of \( C_{2T} \)

To calculate the capacitance between the last layer of the secondary winding and the tank, consider the following diagram of a secondary winding with \( L_{y2} \) layers.
Using Equation 6.20 and the schematic of Figure 6.19, $C_{2T}$ is calculated as [Macfayden, 1953]

$$C_{2T} = \frac{C_t}{3V_2^2} \left( V_0^2 + V_O V_P + V_P^2 \right)$$

$$= \frac{C_t}{3V_2^2} \left( V_2^2 + \left( \frac{L y_2 - 1}{L y_2} \right) V_2^2 + \left( \frac{L y_2 - 1}{L y_2} \right) V_2^2 \right)$$

$$= \frac{C_t (3 L y_2 (L y_2 - 1) + 1)}{3 L y_2^2}$$ \hfill (6.31)

Once again, an enlarged portion of the transformer showing the last two layers of the secondary winding, the tank, and the space between the last layer and the tank is shown in Figure 6.20.

The parallel-plate capacitance of $C_t$ is thus

$$C_t = \frac{1}{1 + \frac{1}{C_{W2}} + \frac{1}{C_{I2T}}}$$ \hfill (6.32)

$C_{I2T}$ = capacitance of the insulation between secondary winding and the tank $I2T$
6.3.2 CAP1 Subroutine

This subroutine calculates the equivalent primary winding capacitance. Consideration is given to the transformer equivalent circuit of Figure 6.21.

![Transformer equivalent circuit with capacitive components](image)

**Figure 6.21** Transformer equivalent circuit with capacitive components

Current in the interwinding capacitance is obtained from

\[ i_{12} = \omega C_{12} (V_1 - V_2) \]
\[ = \omega C_{12} \left( V_1 - \frac{V_2'}{a} \right) \]
\[ = \frac{\omega C_{12}}{a} (V_1 - V_2') + \omega C_{12} \left( \frac{a - 1}{a} \right) V_1 \quad (6.33) \]

where \( V_2' = \) secondary voltage referred to the primary

Alternatively, the interwinding capacitance current entering the secondary side may be expressed as

\[ i_{12} = \omega C_{12} (V_1 - V_2') + \omega C_{12} \left( \frac{a - 1}{a} \right) V_2' \quad (6.34) \]

When referred to the primary side, Equation 6.34 becomes

\[ \frac{i_{12}}{a} = \frac{\omega C_{12}}{a} (V_1 - V_2') + \omega C_{12} \left( \frac{a - 1}{a^2} \right) V_2' \quad (6.35) \]

From Figure 6.21, referring the secondary capacitance to the primary and using Equations 6.33 and 6.35 to refer the interwinding capacitance to the primary side yield the equivalent circuit shown in Figure 6.22.

From Figure 6.22, the total primary winding capacitance is

\[ C_1' = C_1 + C_{12} \left( \frac{a - 1}{a} \right) \quad (6.36) \]
The primary winding capacitive reactance is thus

\[ X_{C1} = \frac{1}{\omega C_1'} \]

(6.37)

### 6.3.3 CAP2 Subroutine

From Figure 6.22, the total secondary winding capacitance is thus

\[ C_2' = \frac{C_2}{a^2} + C_{12} \left( \frac{a - 1}{a^2} \right) \]

(6.38)

from which its capacitive reactance is

\[ X_{C2} = \frac{1}{\omega C_2'} \]

(6.39)

### 6.3.4 CAP12 Subroutine

Finally, from Figure 6.22, the referred interwinding capacitance is

\[ C_{12}' = \frac{C_{12}}{a} \]

(6.40)

The interwinding capacitive reactance is thus

\[ X_{C12} = \frac{1}{\omega C_{12}'} \]

(6.41)

### 6.4 MODIFICATIONS TO OTHER SUBROUTINES

Incorporation of the capacitive components will affect the calculations in some subroutines, notably the EQUILCCT_PARAM, OPEN_CCT, SHORT_CCT and LOADED_CCT.
subroutines of Sections 3.9.1 – 3.9.4. Hence they need to be modified to account for the capacitive effects.

For EQUIL.CCT.PARAM, in addition to the equivalent circuit parameters from Sections 3.7 and 3.8, the capacitive reactance components calculated in Section 6.3 are also included and presented in this subroutine. The equivalent circuit used for the open, short and loaded circuit analyses is shown in Figure 6.23.

![Circuit Diagram](image)

*Figure 6.23* The new transformer equivalent circuit used for analyses

For OPEN.CCT, the detailed open circuit analysis is presented in Appendix D.1. Secondly, for SHORT.CCT the detailed short circuit analysis is presented in Appendix D.2. Finally, for LOADED.CCT the detailed loaded circuit analysis is presented in Appendix D.3.

### 6.5 TRANSFORMER HARMONIC FREQUENCY ANALYSIS

With the incorporation of the capacitive components into the reverse design partial core model, as detailed in Section 6.3, it has become possible to carry out harmonic frequency analysis of the partial core transformers.

#### 6.5.1 Open Circuit Resonance Tests

Transformers PC2, PC4 and PC7 were used to compare the harmonic frequency response results between model calculations and experiments. The equivalent circuit parameters of the transformers calculated by the model are summarised in Table 6.7.

The circuit of Figure 6.23 was used in MicroSim Schematics circuit simulation package, \(^3\)

---

\(^3\)The resistance values presented here are calculated for a 50Hz supply. When the supply frequency is changed, so are the skin depths in the core and the winding. This affects the resistive components, as detailed in Sections 5.3.3, 5.3.4 and 6.2.1. This will affect the damping of the circuit but not the resonant frequencies.


<table>
<thead>
<tr>
<th>Circuit parameters</th>
<th>PC2</th>
<th>PC4</th>
<th>PC7</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance (Ω)³</td>
<td>223.08</td>
<td>680.47</td>
<td>613.68</td>
</tr>
<tr>
<td>$R_{core}$</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$R_1$</td>
<td>1.03</td>
<td>1.18</td>
<td>0.70</td>
</tr>
<tr>
<td>$R_2'$</td>
<td>2.26</td>
<td>2.84</td>
<td>1.32</td>
</tr>
<tr>
<td>Inductance (mH)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$L_m$</td>
<td>76.66</td>
<td>116.91</td>
<td>69.47</td>
</tr>
<tr>
<td>$L_1$</td>
<td>11.37</td>
<td>7.08</td>
<td>7.95</td>
</tr>
<tr>
<td>$L_2'$</td>
<td>11.37</td>
<td>7.08</td>
<td>7.95</td>
</tr>
<tr>
<td>Capacitance (nF)</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>$C_1'$</td>
<td>5.81</td>
<td>5.79</td>
<td>5.86</td>
</tr>
<tr>
<td>$C_2'$</td>
<td>0.97</td>
<td>1.04</td>
<td>1.41</td>
</tr>
<tr>
<td>$C_{12}$</td>
<td>1.02</td>
<td>1.15</td>
<td>1.81</td>
</tr>
</tbody>
</table>

Table 6.7 Equivalent circuit parameters of the three transformers

Together with the values in Table 6.7, to determine the frequency response of each transformer. These are shown in Figure 6.25. It can be seen that the transformers' natural frequencies are well in excess of the frequency range of interest (50Hz - 5kHz) for power systems. Frequency responses between 50Hz and 10kHz are flat.

The frequency response of each of the transformers was measured. The test circuit is shown in Figure 6.24.

![Frequency response test circuit](image)

The frequency was measured up to 10kHz, within the bandwidth of the audio amplifier. The measured data of the transformers was also plotted in Figure 6.25, shown as '□' with dotted lines. The measured responses of the three transformers are also flat between 50Hz and 10kHz. Therefore acceptable matches were achieved within these frequencies. The slight magnitude discrepancies between measured and simulated frequency response curves are due to sensitivities of the entered circuit parameters in the simulation program. In power systems analysis, there is generally no interest for frequencies higher than 5kHz, hence there is no intention in this thesis to pursue transformers' natural frequencies beyond 5kHz further. However, it is still important to verify the validity of the harmonic model developed. This is detailed in the next
Figure 6.25  Simulated normalised gain frequency response of the three transformers
6.5.2 Loaded Circuit Resonance Tests

In order to confirm the validity of the model developed for frequencies up to 5kHz, capacitive loads were connected across the secondary terminals to force the resonant frequencies of the transformers down to within power systems harmonic frequency ranges. This setup is shown in Figure 6.24. The tests were conducted at three different capacitive loads for the transformers, to force resonant frequencies at 1kHz, 2kHz and 5kHz respectively. Table 6.8 shows the capacitive load $C_L$ values used for the transformers to give the three frequencies.

<table>
<thead>
<tr>
<th>Frequency (kHz)</th>
<th>$PC2 \ C_L$ ($\mu$F)</th>
<th>$PC4 \ C_L$ ($\mu$F)</th>
<th>$PC7 \ C_L$ ($\mu$F)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>36</td>
<td>48</td>
<td>19</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>13</td>
<td>5</td>
</tr>
<tr>
<td>5</td>
<td>1.9</td>
<td>2.3</td>
<td>0.9</td>
</tr>
</tbody>
</table>

Table 6.8 $C_L$ values to give specified frequencies

Using the $C_L$ values determined from the tests, each value was then put into the model to determine the corresponding calculated resonant frequency. Table 6.9 shows the resonant frequencies calculated.

<table>
<thead>
<tr>
<th>Measured frequency (kHz)</th>
<th>Calculated frequency (kHz)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>$PC2$</td>
</tr>
<tr>
<td>1</td>
<td>1.05</td>
</tr>
<tr>
<td>2</td>
<td>2.02</td>
</tr>
<tr>
<td>5</td>
<td>5.00</td>
</tr>
</tbody>
</table>

Table 6.9 Calculated frequencies

From Table 6.9 it is evident that frequencies calculated by the model are very accurate.

6.6 TRANSFORMER WITH VERY HIGH TURNS RATIO

Having verified the harmonic model by putting a capacitive load on the secondary winding does not necessarily imply that the inherent capacitive components of the transformer developed earlier in Section 6.3 are correct. This is because the magnitude of the transformers’ inherent capacitances are much smaller than the capacitive loads $C_L$’s. The accuracy of the results obtained in Section 6.5.2 merely further confirmed the validity of the inductive reactance components, especially the leakage reactance. This is because the leakage reactance usually determines the transformer's first natural
resonant frequency [Jiang, 1987]. It is therefore imperative to check the accuracy of the capacitive components developed against experimental results.

Examining Equation 6.38, it can be seen that if a transformer with a very high secondary to primary turns ratio is employed, the transformer’s turns ratio, α, becomes a very small value. As a result, the secondary winding capacitance, when referred to the primary, becomes significantly large. This affects and therefore lowers the transformer’s first natural frequency. In this section, high turns ratio transformers are looked at.

### 6.6.1 High Voltage Partial Core Transformer

A high voltage partial core transformer had been built prior to the development of the partial core model described in this thesis. It has the specifications shown in Table 6.10. The specifications and construction details of the transformer are shown in Appendix E.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>HVPC1</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ratings:</strong></td>
<td></td>
</tr>
<tr>
<td>Primary voltage (V)</td>
<td>14</td>
</tr>
<tr>
<td>Secondary voltage (kV)</td>
<td>4.56</td>
</tr>
<tr>
<td>VA rating (VA)</td>
<td>620</td>
</tr>
<tr>
<td>Number of primary turns</td>
<td>45</td>
</tr>
<tr>
<td>Number of secondary turns</td>
<td>14,605</td>
</tr>
<tr>
<td><strong>Core:</strong></td>
<td></td>
</tr>
<tr>
<td>Length (mm)</td>
<td>110</td>
</tr>
<tr>
<td>Diameter (mm)</td>
<td>76</td>
</tr>
<tr>
<td>Core/LV winding insulation thickness (mm)</td>
<td>6</td>
</tr>
<tr>
<td><strong>LV winding:</strong></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>2</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>3.5</td>
</tr>
<tr>
<td>Interlayer insulation thickness (mm)</td>
<td>0.0</td>
</tr>
<tr>
<td>LV/HV winding insulation thickness (mm)</td>
<td>0.5</td>
</tr>
<tr>
<td><strong>HV winding:</strong></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>20</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>0.11</td>
</tr>
<tr>
<td>Interlayer insulation thickness (mm)</td>
<td>0.15</td>
</tr>
</tbody>
</table>

**Table 6.10** Specifications and design data for Transformer HVPC1

It was built as an attempt to compare with the performance of Transformer FC2, the full-core transformer described in Chapter 3. The equivalent circuit parameters, as determined by the model, are shown in Table 6.11.

From Table 6.11, the referred secondary winding capacitance $C'_2$ has a value of 47.2μF, which is a lot larger in magnitude than $C'_1$ and $C'_{12}$, and also the $C'_2$'s of transform-
ers PC2, PC4 and PC7. The open circuit frequency response of the transformer is simulated and plotted in Figure 6.26.

![Figure 6.26 Open circuit frequency response of HVPC1](image)

From Figure 6.26, the natural frequency is found to be 4.4kHz, significantly lower than those of transformers PC2, PC4 and PC7. The frequency response was also measured, and plotted in Figure 6.26. The measured resonant frequency was 4.5kHz, which matched that of the calculated one. The fact that the measured amplitude of the resonant peak is much lower than that calculated from the model is an expected outcome. This is because in all tuned filters using magnetic core inductances, the losses increase considerably near the resonant frequency. The quality factor of the circuit,

<table>
<thead>
<tr>
<th>Circuit parameters</th>
<th>HVPC1</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance (Ω)</td>
<td></td>
</tr>
<tr>
<td>$R_{core}$</td>
<td>44.4</td>
</tr>
<tr>
<td>$R_1$</td>
<td>0.024</td>
</tr>
<tr>
<td>$R_2$</td>
<td>0.088</td>
</tr>
<tr>
<td>Inductance (mH)</td>
<td></td>
</tr>
<tr>
<td>$L_m$</td>
<td>0.474</td>
</tr>
<tr>
<td>$L_1$</td>
<td>0.014</td>
</tr>
<tr>
<td>$L_2$</td>
<td>0.014</td>
</tr>
<tr>
<td>Capacitance (μF)</td>
<td></td>
</tr>
<tr>
<td>$C'_1$</td>
<td>$1.675 \times 10^{-3}$</td>
</tr>
<tr>
<td>$C'_2$</td>
<td>47.2</td>
</tr>
<tr>
<td>$C'_{12}$</td>
<td>$0.272 \times 10^{-3}$</td>
</tr>
</tbody>
</table>

**Table 6.11** Equivalent circuit parameters of HVPC1
hence the magnitude of the peak, is decreased [Olivier et al., 1980].

A capacitive load of 1000pF was then connected to the secondary winding. 1000pF was chosen to represent the loading effect of a capacitive voltage transformer. When referred to the primary, the load becomes

\[ C'_L = \frac{1}{a^2} C_L \]
\[ = \left( \frac{14605}{45} \right)^2 \times 1000 \text{ pF} \]
\[ = 105.3 \ \mu\text{F} \]

The frequency response of the transformer with the capacitive load was simulated. It is shown in Figure 6.27.

![Figure 6.27](image)

Again, it can be seen that the magnitude of the measured resonant peak is much lower than that of the model. The resonant frequency was estimated to be 2.4kHz. The loaded frequency response was also measured and plotted in Figure 6.27. The measured resonant frequency was 1.9kHz. The simulated and measured frequencies did not quite match, although they were close. There could be a number of reasons for the differences in the measured and the calculated loaded resonant peaks for the two transformers. This could be due to the fact that the data put into the model did not match that of the actual values. This is because exact construction details of the transformer were not available. While the size of the conductors used, the number of primary winding layers, and the total number of winding turns were known, details of the insulation thickness, especially on the HV winding, were unknown. The values
entered into the model were estimates using crude measurement tools. This would have affected the spatial arrangement of the transformers, and hence the performance of the transformer.

In addition, it could initially be presumed that the transformer internal capacitances of $C'_1$, $C'_2$ and $C'_{12}$ may have caused such an error. However, as seen in Figure 6.26, under the open circuit condition, the measured and calculated values of the resonant frequency agree well. Therefore the internal capacitance calculations were accurate. Since the error occurred under loaded conditions, it could be anticipated that the load capacitance caused the error. The load capacitance could have varied with frequency under the HV environment. This could also be true for the parasitic capacitance associated with the load.

### 6.6.2 Design of Another HV Partial Core Transformer

While Section 6.6.1 almost confirmed the validity of the capacitive components developed, a final validation is to design and test another HV partial core transformer. The designed transformer has specifications and physical data shown in Table 6.12. The specifications and construction details of the transformer are shown in Appendix F.

<table>
<thead>
<tr>
<th>Transformer</th>
<th>HVPC2</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ratings:</strong></td>
<td></td>
</tr>
<tr>
<td>Primary voltage (V)</td>
<td>12</td>
</tr>
<tr>
<td>Secondary voltage (kV)</td>
<td>6</td>
</tr>
<tr>
<td>VA rating (VA)</td>
<td>1000</td>
</tr>
<tr>
<td>Number of primary turns</td>
<td>50</td>
</tr>
<tr>
<td>Number of secondary turns</td>
<td>24853</td>
</tr>
<tr>
<td><strong>Core:</strong></td>
<td></td>
</tr>
<tr>
<td>Length (mm)</td>
<td>132</td>
</tr>
<tr>
<td>Diameter (mm)</td>
<td>64</td>
</tr>
<tr>
<td>Core/LV winding insulation thickness (mm)</td>
<td>6</td>
</tr>
<tr>
<td><strong>LV winding:</strong></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>2</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>4.25</td>
</tr>
<tr>
<td>Interlayer insulation thickness (mm)</td>
<td>0</td>
</tr>
<tr>
<td>LV/HV winding insulation thickness (mm)</td>
<td>0.8</td>
</tr>
<tr>
<td><strong>HV winding:</strong></td>
<td></td>
</tr>
<tr>
<td>Number of layers</td>
<td>29</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
<td>0.11</td>
</tr>
<tr>
<td>Interlayer insulation thickness (mm)</td>
<td>0.1</td>
</tr>
</tbody>
</table>

**Table 6.12** Specifications and design data for HVPC2

Transformer HVPC2 was designed for the power supply of an electric water purification
device running at 12Vac. The equivalent circuit parameters for \( HVPC2 \), as determined by the partial core model, are given in Table 6.13.

<table>
<thead>
<tr>
<th>Circuit parameters</th>
<th>( HVPC2 )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Resistance (( \Omega ))</td>
<td>( R_{core} )</td>
</tr>
<tr>
<td></td>
<td>( R_1 )</td>
</tr>
<tr>
<td></td>
<td>( R_2 )</td>
</tr>
<tr>
<td>Inductance (mH)</td>
<td>( L_m )</td>
</tr>
<tr>
<td></td>
<td>( L_1 )</td>
</tr>
<tr>
<td></td>
<td>( L_2 )</td>
</tr>
<tr>
<td>Capacitance (( \mu F ))</td>
<td>( C'_1 )</td>
</tr>
<tr>
<td></td>
<td>( C'_2 )</td>
</tr>
<tr>
<td></td>
<td>( C'_{12} )</td>
</tr>
</tbody>
</table>

Table 6.13  Equivalent circuit parameters of \( HVPC2 \)

The open circuit frequency response of this transformer was simulated and is depicted in Figure 6.28. The simulated resonant frequency is 2.1kHz. The frequency response of \( HVPC2 \) was measured, and is also plotted in Figure 6.28. The simulated and measured results again match, confirming the validity of the capacitive components developed.

![Figure 6.28 Open circuit frequency response of HVPC2](image)

Finally, to make a relative comparison with \( HVPC1 \), a capacitive load of 1000pF was
connected on the secondary winding. The load, when referred to the primary, becomes

\[
C'_L = \frac{1}{a^2} C_L \\
= \left( \frac{24853}{50} \right)^2 \times 1000 \, \text{pF} \\
= 247.1 \, \mu\text{F}
\]

The frequency response of \textit{HVPC2} under the loaded condition was simulated. The results are shown in Figure 6.29. The simulated frequency is approximately 1.3kHz.

![Figure 6.29](image)

The loaded frequency response was measured, and the resonant frequency was determined to be 1.1kHz. The result is also plotted in Figure 6.29 for comparison. The simulated and measured frequencies are close. The validity of the harmonic reverse design partial core model is therefore confirmed. Again, the magnitude of each of the measured resonant peaks is much lower than that of the model due to reduced quality factors of the circuits.

6.7 CONCLUSIONS

In order to incorporate the harmonic frequency analysis of a partial core transformer, further modifications were made to the reverse design model. The skin effect at high frequencies was included into the calculation of the effective cross-sectional areas of both winding conductors. This reduced the effective conductor cross-sectional areas and therefore increased the winding resistances. In addition, a magnetising function
has been added to the calculation of the overall relative permeability. This function takes into account the effect of the transformer aspect ratio on the flux distribution. Acceptable matches were achieved between model calculation and test values.

Capacitive components have been added into the model. Necessary modifications have also been made to other subroutines within the model to account for these capacitive components. The harmonic frequency response of three partial core transformers with relatively low turns ratio have been analysed. Test results showed that the transformers' natural resonant frequencies were much higher than the highest frequency of interest, 5kHz. Capacitive loads were then connected to the secondary terminals in order to calculate loaded resonant frequencies. Acceptable matches between the calculated and the test values were achieved. Two high voltage partial core transformers with large turns ratios were designed, built and analysed. Calculations of the transformers' resonant frequencies, under both open circuit and loaded conditions, were verified by experimental results. This has strengthened the use of the reverse design partial core model developed.
Chapter 7

FUTURE WORK

7.1 IMPROVEMENT ON THE ACCURACY OF THE $R_{\text{CORE}}$ COMPONENT

In Section 6.2.3 the core loss component $R_{\text{core}}$ is not accurate, especially at high frequencies. The relative errors between the calculated and measured values are higher than 10%. Therefore, it would be useful to repeat the high frequency response tests using high-precision equipment. Then, further comparison could be made between the test and the model to determine the validity of the model. Modifications could be made to the model should there be any discrepancies between the model and the test.

7.2 INCLUSION OF DIFFERENT CORE/WINDING CONSTRUCTIONS

At present, two core configurations for the reverse design partial core model have been developed:

1. Rectangular core
2. Cylindrical core

The conductors of the windings are cylindrical in both models. These kind of conductors are usually used for small rating transformers. For large transformers in power systems, the conductors are usually rectangular in shape. Moreover, a number of these rectangular conductors are often bundled together in parallel to increase the current carrying capacity in the windings. Therefore, alternative winding wire configurations are needed if the reverse design model is to be used in the future for designing large transformers. Among the possible winding configurations are:
1. Cylindrical wires
2. Rectangular wires — single wire
3. Rectangular wires — bundled wires
4. A combination of the above in both the primary and secondary windings

Also, for large transformers, the windings are usually sectionalised into various configurations, some of which are shown in Figure 7.1.

The reverse model developed is based on the configuration of Figure 7.1(a). In addition, most large transformers’ windings are also interleaved because of ease of assembly and also to reduce the capacitance to ground [Bean et al., 1959]. Hence, alternative winding configuration options could be incorporated into the model to give more versatility to partial core transformer designs using the reverse design method.

In some transformers, an earthed shield is present between the primary and secondary windings. The purpose of the shield is to protect the primary winding and any attached equipment from any short circuits emanating from the high voltage secondary winding. By introducing the shield, as shown in Figure 7.2, the interwinding capacitance is suppressed. Instead, two additional capacitive components appear:

1. $C_{1S}$ — capacitance between the primary winding and the shield
2. $C_{S2}$ — capacitance between the shield and secondary winding

The inclusion of a shield therefore affects the calculations of the transformer primary and secondary winding capacitances. For the primary winding, the two capacitances to ground are in series, as depicted in Figure 7.3 [Snelling, 1988].

Therefore,

$$C_{wind} = C_a + \frac{C_b C_c}{C_b + C_c}$$

$$C_1 = C_{w1} + \frac{C_{c1} C_{1S}}{C_{c1} + C_{1S}} \tag{7.1}$$

Similarly, the secondary winding capacitance is

$$C_2 = C_{w2} + \frac{C_{S2} C_{2T}}{C_{S2} + C_{2T}} \tag{7.2}$$

Hence the inclusion of the interwinding shield could be programmed into the model to provide further alternatives in the partial core transformer design.
Figure 7.1 Different winding configurations
Figure 7.2 A transformer with an interwinding earthed shield

Figure 7.3 Representation of a winding capacitance
7.3 ASSOCIATION WITH A CIRCUIT SIMULATION PACKAGE

At present, in order to perform the frequency response analysis, equivalent circuit parameters of the transformer have had to be entered into a circuit analysis package such as MicroSim Schematics. It would be convenient to directly associate output files of the model with input files of MicroSim Schematics. Alternatively, a module could be added to the model, calculating the frequency response of a designed transformer.

7.4 COMPARISON WITH THE UMEC MODEL

A unified magnetic equivalent circuit (UMEC) model for power transformers under transient behaviour and harmonics has been developed [Enright, 1996]. It was compared against Steinmetz equivalent circuits [Steinmetz, 1895]. The Steinmetz shunt and series branches will not accurately represent the mutual leakage flux circuit of either an air or partial core transformer. This can be overcome by using the UMEC model [Enright and Arrillaga, 1998]. Therefore, it would be useful to compare the magnetising and leakage reactive components derived from the UMEC model and the reverse design model for partial core transformers.

7.5 PARTIAL CORE HTS TRANSFORMERS

The feasibility of designing a partial core HTS transformer could be studied using this reverse design method. Formulae relevant to HTS tapes, such as the transport current and hysteresis losses [Yamaguchi et al., 1992] [Sykulski et al., 1997] [Goddard et al., 1999] [Honjo et al., 1999], could be included into winding component calculations.

7.6 HIGH VOLTAGE HARMONIC TESTING TRANSFORMERS

The department currently has a high voltage testing transformer used for high voltage generation, and calibrations of instruments such as capacitive voltage transformers (CVT's). It has the specifications [Jiang and Bodger, 1991] shown in Table 7.1.

With a no-load bandwidth of only 500Hz, the transformer is not suitable for the harmonic testing and calibration of power system components. Using the reverse design partial core model, the feasibility of designing a HV harmonic testing transformer could be carried out. Using similar ratings as for the transformer of Table 7.1, a designed
oil-filled harmonic testing transformer has the following specifications:

<table>
<thead>
<tr>
<th>HV harmonic testing transformer</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Ratings:</strong></td>
</tr>
<tr>
<td>Primary voltage (V)</td>
</tr>
<tr>
<td>Secondary voltage (kV)</td>
</tr>
<tr>
<td>VA rating (kVA)</td>
</tr>
<tr>
<td><strong>Core:</strong></td>
</tr>
<tr>
<td>Length (mm)</td>
</tr>
<tr>
<td>Diameter (mm)</td>
</tr>
<tr>
<td>Core/LV winding insulation thickness (mm)</td>
</tr>
<tr>
<td>Lamination thickness (mm)</td>
</tr>
<tr>
<td><strong>LV winding:</strong></td>
</tr>
<tr>
<td>Number of layers</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
</tr>
<tr>
<td>Inter-turn insulation thickness (mm)</td>
</tr>
<tr>
<td>LV/HV winding insulation thickness (mm)</td>
</tr>
<tr>
<td><strong>HV winding:</strong></td>
</tr>
<tr>
<td>Number of layers</td>
</tr>
<tr>
<td>Wire diameter (mm)</td>
</tr>
<tr>
<td>Interlayer insulation thickness (mm)</td>
</tr>
<tr>
<td><strong>Tank:</strong></td>
</tr>
<tr>
<td>Diameter (mm)</td>
</tr>
</tbody>
</table>

The frequency response of the designed transformer is shown in Figure 7.4. The natural frequency of the transformer is 2.5kHz, which is higher than the 500Hz of the existing testing transformer. When a high capacitive load of 2,000pF is placed across the secondary winding, representing a CVT, the frequency response of Figure 7.5 is obtained.

The resonant frequency is approximately 850Hz. Therefore, the designed transformer has a significantly wider bandwidth than the existing transformer. Moreover, the overall dimension, hence the cost and weight of the designed transformer, is much less than the existing transformer.
Figure 7.4 Normalised gain frequency response of the harmonic transformer

Figure 7.5 Normalised gain loaded frequency response of the harmonic transformer
The above design is conceptual only. None-the-less, the design gives an indication of the extent of the transformer; its size, the frequency response, and ultimately, the versatility of using the reverse design partial core model.
Chapter 8

CONCLUSIONS

This thesis started by presenting the modelling and operational theory of a full-core transformer. Equivalent circuits, which are often used in determining a transformer’s performance, have been presented. These include the core loss components $R_{cc}$, $R_h$ and $X_m$; the primary winding components $R_1$ and $X_1$, and the secondary winding components $R_2$ and $X_2$. The conventional approach to transformer design has been presented in detail. Test results indicated that although the conventional design approach is useful in predicting the actual performance of as-built transformers, there were differences in the equivalent circuit parameter estimation. This is because the conventional transformer design starts from a consideration of required frequency, voltage and VA ratings. It estimates a number of factors for the core and winding arrangement, using values that are generally only known to experienced design engineers. Core and winding material characteristics are known from standard values or physical measurements. The resultant design therefore may not match what is actually available in materials, causing variation between calculated and measured values.

Given this perspective, an alternative approach to designing full-core transformers has been described in this thesis. It is called the reverse transformer design method. The dimensions of core and winding materials are based on what is available. The overall size, ratings and performance of the transformer can then be predicted. Such an approach lends itself to designing transformers using what is available from suppliers. It allows for customised design, as there is considerable flexibility in meeting the performance required for a particular application.

The detailed program routines for this reverse design approach have been presented. The performance of this reverse design approach was verified with experimental data, and compared with the conventional design approach. Results from the designed transformers showed the usefulness of the reverse design approach.

This philosophy allows for exploration in the design of transformers with alternative
construction options, where flexibility in shape and size is required. Hence, the design of partial core transformers for power systems applications has been looked into. Alterations were made to accommodate full-core equivalent circuit components of partial core transformers. These included the hysteresis loss resistance $R_h$, magnetising reactance $X_m$ and the winding leakage reactances $X_1$ and $X_2$. The reverse design partial core transformer model was validated with experimental results. Significant agreement has been achieved between the values of the transformer equivalent circuit components as determined through calculation and test.

The reverse design method has also been applied to partial core transformers immersed in liquid nitrogen. Modifications were made to the equivalent circuit parameters to account for the changes in the resistance of the materials at very low temperatures. The components were the core loss resistances ($R_{cc}$ and $R_h$) and the winding resistances ($R_1$ and $R_2$). The modified model was verified with experimental results, and acceptable agreement has been achieved between the values of the transformer equivalent circuit components.

The reverse design partial core model was extended to account for harmonic frequencies. Further modifications were made to the winding resistances to include the skin effects at high frequencies, and to the core magnetising reactance to take into account the effect the transformer aspect ratio has on the flux distribution. Capacitive components were also added into the reverse design partial core model. The harmonic frequency response of partial core transformers with relatively low turns ratios were analysed. Test results showed that the transformers’ natural resonant frequencies were higher than the highest frequency of interest. Capacitive loads were then connected to the secondary terminals in order to calculate loaded resonant frequencies. Acceptable matches between the calculated and test values were achieved. Two high voltage partial core transformers with large turns ratios were designed, built and tested. Calculations of the transformers’ resonant frequencies under open circuit and loaded conditions were verified with experimental results, thus reinforcing the reverse design partial core model developed in the thesis.

Finally, a number of options for future work to improve the versatility and validity of the modelling techniques and design of transformers was presented.
REFERENCES


REFERENCES


Slemon, G. R. (1966), Magnetoelcetric Devices: Transducers, Transformers, and Machines, John Wiley and Sons, Inc., USA.


Appendix A

PARTIAL CORE TRANSFORMERS PHYSICAL DETAILS

Figure A.1 A partial core transformer: isometric view

Note: For a cylindrical core, the respective widths and breadths are replaced by the corresponding diameters.
Figure A.2  Axial view of a partial core transformer
Appendix B

PARTIAL CORE TRANSFORMERS DESIGN PARAMETERS

The specifications and construction details of all the partial core transformers utilised in this thesis are presented here.

B.1 TRANSFORMER INPUT DATA

Table B.1 shows all the data read by the reverse design partial core model.

<table>
<thead>
<tr>
<th>(a) Supply Characteristics</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>V</td>
</tr>
<tr>
<td>$f$</td>
<td>Hz</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(b) Core</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{core}$</td>
<td>m</td>
</tr>
<tr>
<td>$w_{core}$</td>
<td>m</td>
</tr>
<tr>
<td>$b_{core}$</td>
<td>m</td>
</tr>
<tr>
<td>$SF_c$</td>
<td></td>
</tr>
<tr>
<td>$c_{lam}$</td>
<td>m</td>
</tr>
<tr>
<td>$I_{cl}$</td>
<td>m</td>
</tr>
<tr>
<td>$\mu_{rc}$</td>
<td></td>
</tr>
<tr>
<td>$\rho_{core,20^\circ C}$</td>
<td>$\Omega m$</td>
</tr>
<tr>
<td>$T_{core}$</td>
<td>$^\circ C$</td>
</tr>
<tr>
<td>$\gamma_{core}$</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$Cost_{core}$</td>
<td>$$/kg</td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>(c) Primary Winding</th>
<th>unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{prim}$</td>
<td>m</td>
</tr>
<tr>
<td>$L_{y1}$</td>
<td></td>
</tr>
</tbody>
</table>

continued on next page
### Table B.1: Data read in by the program

<table>
<thead>
<tr>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Thickness of the primary winding wire</td>
<td>m</td>
</tr>
<tr>
<td>Thickness of the primary winding wire insulation</td>
<td>m</td>
</tr>
<tr>
<td>Thickness of the primary winding insulation</td>
<td>m</td>
</tr>
<tr>
<td>Relative permeability of the primary winding material</td>
<td></td>
</tr>
<tr>
<td>Resistivity of the primary winding wire at 20°C</td>
<td>Ωm</td>
</tr>
<tr>
<td>Coefficient of thermal resistivity of the primary winding</td>
<td>°C⁻¹</td>
</tr>
<tr>
<td>Operating temperature of the primary winding</td>
<td>°C</td>
</tr>
<tr>
<td>Primary winding material density</td>
<td>kg/m³</td>
</tr>
<tr>
<td>Primary winding material cost</td>
<td>$/kg</td>
</tr>
<tr>
<td>Thickness of the primary-secondary winding insulation</td>
<td>m</td>
</tr>
<tr>
<td>Axial length of the secondary winding</td>
<td>m</td>
</tr>
<tr>
<td>Total number of layers of the secondary winding</td>
<td></td>
</tr>
<tr>
<td>Thickness of the secondary winding wire</td>
<td>m</td>
</tr>
<tr>
<td>Thickness of the secondary winding wire insulation</td>
<td>m</td>
</tr>
<tr>
<td>Thickness of the secondary winding insulation</td>
<td>m</td>
</tr>
<tr>
<td>Relative permeability of the secondary winding material</td>
<td></td>
</tr>
<tr>
<td>Resistivity of the secondary winding wire at 20°C</td>
<td>Ωm</td>
</tr>
<tr>
<td>Coefficient of thermal resistivity of the secondary winding</td>
<td>°C⁻¹</td>
</tr>
<tr>
<td>Operating temperature of the secondary winding</td>
<td>°C</td>
</tr>
<tr>
<td>Secondary winding material density</td>
<td>kg/m³</td>
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<tr>
<td>Secondary winding material cost</td>
<td>$/kg</td>
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</table>

<table>
<thead>
<tr>
<th>Description</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>Transformer PC1</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th>Description</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Voltage</td>
<td>120 V</td>
</tr>
<tr>
<td>Frequency</td>
<td>50 Hz</td>
</tr>
<tr>
<td>Axial length of the core</td>
<td>400 mm</td>
</tr>
<tr>
<td>Thickness of the core</td>
<td>31 mm</td>
</tr>
<tr>
<td>Thickness of the core</td>
<td>38 mm</td>
</tr>
<tr>
<td>Secondary winding factor</td>
<td>0.96</td>
</tr>
<tr>
<td>Thickness of the lamina</td>
<td>0.5 mm</td>
</tr>
<tr>
<td>Internal insulation factor</td>
<td>2 mm</td>
</tr>
</tbody>
</table>

### B.1.1 Transformer PC1

(a) Supply Characteristics

- Voltage $V_1 = 120$ V
- Frequency $f = 50$ Hz

(b) Core

- Axial length of the core $l_{core} = 400$ mm
- Thickness of the core $w_{core} = 31$ mm
- Thickness of the core $b_{core} = 38$ mm
- Secondary winding factor $SF_c = 0.96$
- Thickness of the lamina $c_{lam} = 0.5$ mm
- Internal insulation factor $L_{c1} = 2$ mm
### B.1 TRASFORMER INPUT DATA

<table>
<thead>
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<th>Value</th>
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</thead>
<tbody>
<tr>
<td>( \mu_{rc} )</td>
<td>2000</td>
</tr>
<tr>
<td>( \rho_{core,20^\circ C} )</td>
<td>( 1.8 \times 10^{-7} ) ( \Omega m )</td>
</tr>
<tr>
<td>( \Delta \rho_{core} )</td>
<td>( 0.006 \times 10^{-8} ) ( ^\circ C^{-1} )</td>
</tr>
<tr>
<td>( T_{core} )</td>
<td>20 ( ^\circ C )</td>
</tr>
<tr>
<td>( \gamma_{core} )</td>
<td>7870 ( \text{kg/m}^3 )</td>
</tr>
<tr>
<td>( Cost_{core} )</td>
<td>6.7 $/kg</td>
</tr>
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</table>

(c) Primary Winding

<table>
<thead>
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<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_{prim} )</td>
<td>400 mm</td>
</tr>
<tr>
<td>( L_{y1} )</td>
<td>3</td>
</tr>
<tr>
<td>( w_1 )</td>
<td>2.5   mm</td>
</tr>
<tr>
<td>( w_{i1} )</td>
<td>0.03  mm</td>
</tr>
<tr>
<td>( I_1 )</td>
<td>0     mm</td>
</tr>
<tr>
<td>( \mu_{r1} )</td>
<td>1.0</td>
</tr>
<tr>
<td>( \rho_{1,20^\circ C} )</td>
<td>( 1.76 \times 10^{-8} ) ( \Omega m )</td>
</tr>
<tr>
<td>( \Delta \rho_1 )</td>
<td>( 0.0039 \times 10^{-8} ) ( ^\circ C^{-1} )</td>
</tr>
<tr>
<td>( T_1^\dagger )</td>
<td>20 ( ^\circ C )</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>8960  ( \text{kg/m}^3 )</td>
</tr>
<tr>
<td>( Cost_1 )</td>
<td>12 $/kg</td>
</tr>
</tbody>
</table>

(d) Primary–Secondary Interwinding Space

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_{12} )</td>
<td>0.18 mm</td>
</tr>
</tbody>
</table>

(e) Secondary Winding

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_{sec} )</td>
<td>400 mm</td>
</tr>
<tr>
<td>( L_{y2} )</td>
<td>2</td>
</tr>
<tr>
<td>( w_2 )</td>
<td>4.5   mm</td>
</tr>
<tr>
<td>( w_{i2} )</td>
<td>0.3   mm</td>
</tr>
<tr>
<td>( I_2 )</td>
<td>0     mm</td>
</tr>
<tr>
<td>( \mu_{r2} )</td>
<td>1.0</td>
</tr>
<tr>
<td>( \rho_{2,20^\circ C} )</td>
<td>( 1.76 \times 10^{-8} ) ( \Omega m )</td>
</tr>
<tr>
<td>( \Delta \rho_2 )</td>
<td>( 0.0039 \times 10^{-8} ) ( ^\circ C^{-1} )</td>
</tr>
<tr>
<td>( T_2^\dagger )</td>
<td>20 ( ^\circ C )</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>8960  ( \text{kg/m}^3 )</td>
</tr>
<tr>
<td>( Cost_2 )</td>
<td>12 $/kg</td>
</tr>
</tbody>
</table>

### B.1.2 Transformer PC2

(a) Supply Characteristics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( V_1 )</td>
<td>230 V</td>
</tr>
<tr>
<td>( f )</td>
<td>50 Hz</td>
</tr>
</tbody>
</table>

\*When immersed in liquid nitrogen, the operating temperature of the core is \(-196^\circ C\)

\dagger When immersed in liquid nitrogen, the operating temperatures of the windings are \(-170^\circ C\)
(b) Core
\begin{align*}
l_{\text{core}} & : 150 \text{ mm} \\
w_{\text{core}} & : 39 \text{ mm} \\
b_{\text{core}} & : 38 \text{ mm} \\
S_{F_c} & : 0.96 \\
c_{\text{lam}} & : 0.5 \text{ mm} \\
L_{c1} & : 2 \text{ mm} \\
\mu_{rc} & : 2000 \\
p_{\text{core, } 20^\circ C} & : 1.8 \times 10^{-7} \Omega \text{m} \\
\Delta p_{\text{core}} & : 0.006 \times 10^{-8} \degree C^{-1} \\
T_{\text{core}} & : 20 \degree C \\
\gamma_{\text{core}} & : 7870 \text{ kg/m}^3 \\
Cost_{\text{core}} & : 6.7 \$/kg
\end{align*}

(c) Primary Winding
\begin{align*}
l_{\text{prim}} & : 140 \text{ mm} \\
L_{y1} & : 11 \\
w_1 & : 1.9 \text{ mm} \\
w_{i1} & : 0.08 \text{ mm} \\
I_1 & : 0 \text{ mm} \\
\mu_{r1} & : 1.0 \\
p_{1,20^\circ C} & : 1.76 \times 10^{-8} \Omega \text{m} \\
\Delta p_1 & : 0.0039 \times 10^{-8} \degree C^{-1} \\
T_1 & : 20 \degree C \\
\gamma_1 & : 8960 \text{ kg/m}^3 \\
Cost_1 & : 12 \$/kg
\end{align*}

(d) Primary–Secondary Interwinding Space
\begin{align*}
I_{12} & : 0.18 \text{ mm}
\end{align*}

(e) Secondary Winding
\begin{align*}
l_{\text{sec}} & : 115 \text{ mm} \\
L_{y2} & : 5 \\
w_2 & : 4.5 \text{ mm} \\
w_{i2} & : 0.05 \text{ mm} \\
I_2 & : 0 \text{ mm} \\
\mu_{r2} & : 1.0 \\
p_{2,20^\circ C} & : 1.76 \times 10^{-8} \Omega \text{m} \\
\Delta p_2 & : 0.0039 \times 10^{-8} \degree C^{-1} \\
T_2 & : 20 \degree C \\
\gamma_2 & : 8960 \text{ kg/m}^3 \\
Cost_2 & : 12 \$/kg
B.1.3 Transformer PC3

(a) Supply Characteristics
\[ V_1 = 20 \text{ V} \]
\[ f = 50 \text{ Hz} \]

(b) Core
\[ l_{\text{core}} = 133 \text{ mm} \]
\[ w_{\text{core}} = 38 \text{ mm} \]
\[ b_{\text{core}} = 44 \text{ mm} \]
\[ SF_c = 0.984 \]
\[ c_{\text{lam}} = 0.5 \text{ mm} \]
\[ I_{c1} = 2 \text{ mm} \]
\[ \mu_{rc} = 2000 \]
\[ \rho_{\text{core,}20^\circ C} = 1.8 \times 10^{-7} \Omega \text{m} \]
\[ \Delta \rho_{\text{core}} = 0.006 \times 10^{-8} \text{°C}^{-1} \]
\[ T_{\text{core}} = 20 \text{ °C} \]
\[ \gamma_{\text{core}} = 7870 \text{ kg/m}^3 \]
\[ Cost_{\text{core}} = 6.7 \$/kg \]

(c) Primary Winding
\[ l_{\text{prim}} = 133 \text{ mm} \]
\[ L_{y1} = 4 \]
\[ w_1 = 0.8 \text{ mm} \]
\[ w_{11} = 0.045 \text{ mm} \]
\[ I_1 = 0 \text{ mm} \]
\[ \mu_{r1} = 1.0 \]
\[ \rho_{1,20^\circ C} = 1.76 \times 10^{-8} \Omega \text{m} \]
\[ \Delta \rho_1 = 0.0039 \times 10^{-8} \text{°C}^{-1} \]
\[ T_1 = 20 \text{ °C} \]
\[ \gamma_1 = 8960 \text{ kg/m}^3 \]
\[ Cost_1 = 12 \$/kg \]

(d) Primary–Secondary Interwinding Space
\[ I_{12} = 0.2 \text{ mm} \]

(e) Secondary Winding
\[ l_{\text{sec}} = 133 \text{ mm} \]
\[ L_{y2} = 4 \]
\[ w_2 = 0.8 \text{ mm} \]
\[ w_{22} = 0.045 \text{ mm} \]
\[ I_2 = 0 \text{ mm} \]
\[ \mu_{r2} = 1.0 \]
\[ \rho_{2,20^\circ C} = 1.76 \times 10^{-8} \Omega \text{m} \]
\[\Delta \rho_2 \quad \frac{0.0039 \times 10^{-8}}{\circ C^{-1}}\]
\[T_2 \quad 20 \quad \circ C\]
\[\gamma_2 \quad 8960 \quad \text{kg/m}^3\]
\[\text{Cost}_2 \quad 12 \quad \$/$kg\]

### B.1.4 Transformer PC4

(a) Supply Characteristics
\[V_1 \quad 230 \quad V\]
\[f \quad 50 \quad Hz\]

(b) Core
\[l_{core} \quad 195 \quad \text{mm}\]
\[w_{core} \quad 39 \quad \text{mm}\]
\[b_{core} \quad 43 \quad \text{mm}\]
\[S_{F_c} \quad 0.96\]
\[c_{lam} \quad 0.5 \quad \text{mm}\]
\[I_{c1} \quad 2 \quad \text{mm}\]
\[\mu_{rc} \quad 2000\]
\[\rho_{core,20^\circ C} \quad 1.8 \times 10^{-7} \quad \Omega m\]
\[\Delta \rho_{core} \quad 0.006 \times 10^{-8} \quad \circ C^{-1}\]
\[T_{core} \quad 20 \quad \circ C\]
\[\gamma_{core} \quad 7870 \quad \text{kg/m}^3\]
\[\text{Cost}_{core} \quad 6.7 \quad \$/$kg\]

(c) Primary Winding
\[l_{prim} \quad 195 \quad \text{mm}\]
\[L_{y1} \quad 9.5\]
\[w_1 \quad 1.9 \quad \text{mm}\]
\[w_{i1} \quad 0.15 \quad \text{mm}\]
\[I_1 \quad 0 \quad \text{mm}\]
\[\mu_{r1} \quad 1.0\]
\[\rho_{1,20^\circ C} \quad 1.76 \times 10^{-8} \quad \Omega m\]
\[\Delta \rho_1 \quad 0.0039 \times 10^{-8} \quad \circ C^{-1}\]
\[T_1 \quad 20 \quad \circ C\]
\[\gamma_1 \quad 8960 \quad \text{kg/m}^3\]
\[\text{Cost}_1 \quad 12 \quad \$/$kg\]

(d) Primary–Secondary Interwinding Space
\[I_{12} \quad 0.18 \quad \text{mm}\]

(e) Secondary Winding
\[l_{sec} \quad 195 \quad \text{mm}\]
\[L_{y2} \quad 3.5\]
B.1 Transformer Input Data

\[
\begin{array}{ll}
  w_2 & 4 \quad \text{mm} \\
  w_{i2} & 0.15 \quad \text{mm} \\
  I_2 & 0 \quad \text{mm} \\
  \mu_{r2} & 1.0 \\
  \rho_{2,20^\circ C} & 1.76 \times 10^{-8} \quad \Omega \text{m} \\
  \Delta \rho_2 & 0.0039 \times 10^{-8} \quad ^\circ \text{C}^{-1} \\
  T_2 & 20 \quad ^\circ \text{C} \\
  \gamma_2 & 8960 \quad \text{kg/m}^3 \\
  Cost_2 & 12 \quad $/\text{kg}
\end{array}
\]

B.1.5 Transformer PC5

(a) Supply Characteristics

\[
\begin{array}{ll}
  V_1 & 210 \quad \text{V} \\
  f & 50 \quad \text{Hz}
\end{array}
\]

(b) Core

\[
\begin{array}{ll}
  l_{\text{core}} & 140 \quad \text{mm} \\
  w_{\text{core}} & 44 \quad \text{mm} \\
  b_{\text{core}} & 44 \quad \text{mm} \\
  S_{P_c} & 0.96 \\
  c_{\text{lam}} & 0.5 \quad \text{mm} \\
  I_{c1} & 3 \quad \text{mm} \\
  \mu_{r_c} & 2000 \\
  \rho_{\text{core},20^\circ C} & 1.8 \times 10^{-7} \quad \Omega \text{m} \\
  \Delta \rho_{\text{core}} & 0.006 \times 10^{-8} \quad ^\circ \text{C}^{-1} \\
  T_{\text{core}} & 20 \quad ^\circ \text{C} \\
  \gamma_{\text{core}} & 7870 \quad \text{kg/m}^3 \\
  Cost_{\text{core}} & 6.7 \quad $/\text{kg}
\end{array}
\]

(c) Primary Winding

\[
\begin{array}{ll}
  l_{\text{prim}} & 140 \quad \text{mm} \\
  L_{y1} & 8 \\
  w_1 & 2.26 \quad \text{mm} \\
  w_{i1} & 0.12 \quad \text{mm} \\
  I_1 & 0 \quad \text{mm} \\
  \mu_{r1} & 1.0 \\
  \rho_{1,20^\circ C} & 1.76 \times 10^{-8} \quad \Omega \text{m} \\
  \Delta \rho_1 & 0.0039 \times 10^{-8} \quad ^\circ \text{C}^{-1} \\
  T_1 & 20 \quad ^\circ \text{C} \\
  \gamma_1 & 8960 \quad \text{kg/m}^3 \\
  Cost_{1} & 12 \quad $/\text{kg}
\end{array}
\]
(d) Primary–Secondary Interwinding Space

\[ I_{12} = 0.2 \text{ mm} \]

(e) Secondary Winding

\[ l_{sec} = 125 \text{ mm} \]
\[ L_{y2} = 2 \]
\[ w_2 = 4.5 \text{ mm} \]
\[ w_{i2} = 0.07 \text{ mm} \]
\[ I_2 = 0 \text{ mm} \]
\[ \mu_{r2} = 1.0 \]
\[ \rho_{220^\circ C} = 1.76 \times 10^{-8} \text{ } \Omega \text{m} \]
\[ \Delta \rho_2 = 0.0039 \times 10^{-8} \text{ } ^\circ \text{C}^{-1} \]
\[ T_2 = 20 \text{ } ^\circ \text{C} \]
\[ \gamma_2 = 8960 \text{ kg/m}^3 \]
\[ \text{Cost}_2 = 12 \text{ } \$/kg \]

B.1.6 Transformer PC6

(a) Supply Characteristics

\[ V_1 = 230 \text{ V} \]
\[ f = 50 \text{ Hz} \]

(b) Core

\[ l_{core} = 258 \text{ mm} \]
\[ w_{core} = 42 \text{ mm} \]
\[ b_{core} = 43 \text{ mm} \]
\[ S_{F_c} = 0.96 \]
\[ c_{lam} = 0.5 \text{ mm} \]
\[ I_{c1} = 3 \text{ mm} \]
\[ \mu_{rc} = 2000 \]
\[ \rho_{core,20^\circ C} = 1.8 \times 10^{-7} \text{ } \Omega \text{m} \]
\[ \Delta \rho_{core} = 0.006 \times 10^{-8} \text{ } ^\circ \text{C}^{-1} \]
\[ T_{core} = 20 \text{ } ^\circ \text{C} \]
\[ \gamma_{core} = 7870 \text{ kg/m}^3 \]
\[ \text{Cost}_{core} = 6.7 \text{ } \$/kg \]

(c) Primary Winding

\[ l_{prim} = 258 \text{ mm} \]
\[ L_{y1} = 6 \]
\[ w_1 = 2.5 \text{ mm} \]
\[ w_{i1} = 0.07 \text{ mm} \]
\[ I_1 = 0 \text{ mm} \]
\[ \mu_{r1} = 1.0 \]
B.1 TRANSFORMER INPUT DATA

\[ \rho_{1,20^\circ C} = 1.76 \times 10^{-8} \quad \Omega m \]
\[ \Delta \rho_1 = 0.0039 \times 10^{-8} \quad ^\circ C^{-1} \]
\[ T_1 = 20 \quad ^\circ C \]
\[ \gamma_1 = 8960 \quad \text{kg/m}^3 \]
\[ Cost_1 = 12 \quad \$ / \text{kg} \]

(d) Primary–Secondary Interwinding Space
\[ I_{12} = 0.2 \quad \text{mm} \]

(e) Secondary Winding
\[ l_{sec} = 258 \quad \text{mm} \]
\[ Lg_2 = 7 \]
\[ w_2 = 2.5 \quad \text{mm} \]
\[ w_{12} = 0.07 \quad \text{mm} \]
\[ I_2 = 0 \quad \text{mm} \]
\[ \mu_r = 1.0 \]
\[ \rho_{2,20^\circ C} = 1.76 \times 10^{-8} \quad \Omega m \]
\[ \Delta \rho_2 = 0.0039 \times 10^{-8} \quad ^\circ C^{-1} \]
\[ T_2 = 20 \quad ^\circ C \]
\[ \gamma_2 = 8960 \quad \text{kg/m}^3 \]
\[ Cost_2 = 12 \quad \$ / \text{kg} \]

B.1.7 Transformer PC7

(a) Supply Characteristics
\[ V_1 = 230 \quad \text{V} \]
\[ f = 50 \quad \text{Hz} \]

(b) Core
\[ l_{core} = 200 \quad \text{mm} \]
\[ w_{core} = 44 \quad \text{mm} \]
\[ b_{core} = 44 \quad \text{mm} \]
\[ SF_c = 0.96 \]
\[ a_{lam} = 0.5 \quad \text{mm} \]
\[ I_{c1} = 3 \quad \text{mm} \]
\[ \mu_{rc} = 2000 \]
\[ \rho_{core,20^\circ C} = 1.8 \times 10^{-7} \quad \Omega m \]
\[ \Delta \rho_{core} = 0.006 \times 10^{-8} \quad ^\circ C^{-1} \]
\[ T_{core} = 20 \quad ^\circ C \]
\[ \gamma_{core} = 7870 \quad \text{kg/m}^3 \]
\[ Cost_{core} = 6.7 \quad \$ / \text{kg} \]

(c) Primary Winding
\[ l_{prim} = 195 \quad \text{mm} \]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$L_{y_1}$</td>
<td>9 mm</td>
</tr>
<tr>
<td>$w_1$</td>
<td>2.25 mm</td>
</tr>
<tr>
<td>$w_{i1}$</td>
<td>0.12 mm</td>
</tr>
<tr>
<td>$I_1$</td>
<td>0 mm</td>
</tr>
<tr>
<td>$\mu_{r1}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$\rho_{1,20^\circ C}$</td>
<td>$1.76 \times 10^{-8}$ Ωm</td>
</tr>
<tr>
<td>$\Delta \rho_1$</td>
<td>$0.0039 \times 10^{-8}$ °C$^{-1}$</td>
</tr>
<tr>
<td>$T_1$</td>
<td>20 °C</td>
</tr>
<tr>
<td>$\gamma_1$</td>
<td>8960 kg/m$^3$</td>
</tr>
<tr>
<td>$Cost_1$</td>
<td>12 $$/kg</td>
</tr>
</tbody>
</table>

(d) Primary–Secondary Interwinding Space

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$I_{12}$</td>
<td>0.2 mm</td>
</tr>
</tbody>
</table>

(e) Secondary Winding

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{sec}$</td>
<td>150 mm</td>
</tr>
<tr>
<td>$L_{y_2}$</td>
<td>6 mm</td>
</tr>
<tr>
<td>$w_2$</td>
<td>4.5 mm</td>
</tr>
<tr>
<td>$w_{i2}$</td>
<td>0.2 mm</td>
</tr>
<tr>
<td>$I_2$</td>
<td>0 mm</td>
</tr>
<tr>
<td>$\mu_{r2}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$\rho_{2,20^\circ C}$</td>
<td>$1.76 \times 10^{-8}$ Ωm</td>
</tr>
<tr>
<td>$\Delta \rho_2$</td>
<td>$0.0039 \times 10^{-8}$ °C$^{-1}$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>20 °C</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>8960 kg/m$^3$</td>
</tr>
<tr>
<td>$Cost_2$</td>
<td>12 $$/kg</td>
</tr>
</tbody>
</table>
Appendix C

PARTIAL CORE TRANSFORMERS EXPERIMENTAL DATA

The complete open circuit and short circuit performances of all the partial core transformers utilised in this thesis are presented here. All the data in Sections C.1 and C.3 have been used to determine the resistive and reactive components.

C.1 50HZ SUPPLY AT ROOM TEMPERATURE

This section presents the detailed results of the open and short circuit tests. In addition, the loaded test result performed on Transformer PC4 is also presented. Figure C.1 shows the open and short circuit equivalent circuits, together with the measured quantities.

![Open and Short Circuit Diagrams](image)

(a) Open circuit  (b) Short circuit

**Figure C.1** Transformer equivalent circuits

The open circuit components $R_{oc}$ and $X_{oc}$ are calculated using

\[ R_{oc} = \frac{V^2_h}{P_c} \quad (C.1) \]
\[ X_{oc} = \frac{V_1}{I_m} \]  \hspace{1cm} (C.2)

By plotting \(V_1^2\) against \(P_c\) and \(V_1\) against \(I_m\) using values obtained from the tests, \(R_{oc}\) and \(X_{oc}\) can each be acquired from the gradient of the corresponding line.

In a similar manner, the short circuit components \(R_{sc}\) and \(X_{sc}\) are calculated using

\[ R_{sc} = \frac{P_1}{I_1^2} \]  \hspace{1cm} (C.3)

\[ X_{sc} = \sqrt{\left(\frac{V_1}{I_1}\right)^2 - R_{sc}^2} \]  \hspace{1cm} (C.4)

\(R_{sc}\) is obtained from the gradient of the plot of \(P_1\) against \(I_1^2\), while the magnitude of \(X_{sc}\) is obtained by subtracting the magnitudes of \(R_{sc}\) from \(\frac{V_1}{I_1}\).
C.1.1 Transformer PC1

<table>
<thead>
<tr>
<th>$V_1$(V)</th>
<th>$I_1$(A)</th>
<th>$P_c$(W)</th>
<th>$p_f$</th>
<th>$V_2$(V)</th>
<th>$I_c$(A)</th>
<th>$I_m$(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>0.13</td>
<td>0</td>
<td>0.08</td>
<td>0.6</td>
<td>0.00</td>
<td>0.13</td>
</tr>
<tr>
<td>20.4</td>
<td>1.37</td>
<td>1</td>
<td>0.12</td>
<td>6.8</td>
<td>0.15</td>
<td>1.36</td>
</tr>
<tr>
<td>40.4</td>
<td>2.7</td>
<td>11</td>
<td>0.1</td>
<td>13.3</td>
<td>0.27</td>
<td>2.69</td>
</tr>
<tr>
<td>61.2</td>
<td>4.1</td>
<td>23</td>
<td>0.09</td>
<td>20.1</td>
<td>0.38</td>
<td>4.08</td>
</tr>
<tr>
<td>81.8</td>
<td>5.51</td>
<td>40</td>
<td>0.09</td>
<td>26.9</td>
<td>0.49</td>
<td>5.49</td>
</tr>
<tr>
<td>100.4</td>
<td>6.8</td>
<td>60</td>
<td>0.09</td>
<td>33</td>
<td>0.60</td>
<td>6.77</td>
</tr>
<tr>
<td>120.3</td>
<td>8.21</td>
<td>80</td>
<td>0.08</td>
<td>39.5</td>
<td>0.67</td>
<td>8.18</td>
</tr>
<tr>
<td>139.8</td>
<td>9.68</td>
<td>110</td>
<td>0.08</td>
<td>45.8</td>
<td>0.79</td>
<td>9.65</td>
</tr>
<tr>
<td>159</td>
<td>11.61</td>
<td>140</td>
<td>0.08</td>
<td>52.2</td>
<td>0.88</td>
<td>11.58</td>
</tr>
<tr>
<td>179</td>
<td>14.5</td>
<td>200</td>
<td>0.07</td>
<td>58.7</td>
<td>1.12</td>
<td>14.46</td>
</tr>
<tr>
<td>199</td>
<td>18.7</td>
<td>280</td>
<td>0.07</td>
<td>65.3</td>
<td>1.41</td>
<td>18.65</td>
</tr>
<tr>
<td>219</td>
<td>25.3</td>
<td>410</td>
<td>0.07</td>
<td>71.7</td>
<td>1.87</td>
<td>25.23</td>
</tr>
</tbody>
</table>

Table C.1 Transformer PC1 open circuit results

$R_{oc} = 180.8\Omega$ and $X_{oc} = 14.5\Omega$

<table>
<thead>
<tr>
<th>$V_1$(V)</th>
<th>$I_1$(A)</th>
<th>$P_c$(W)</th>
<th>$p_f$</th>
<th>$I_2$(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.7</td>
<td>0.61</td>
<td>0</td>
<td>0.84</td>
<td>1.78</td>
</tr>
<tr>
<td>3.3</td>
<td>4.25</td>
<td>13</td>
<td>0.95</td>
<td>12.66</td>
</tr>
<tr>
<td>6.1</td>
<td>8.02</td>
<td>46</td>
<td>0.95</td>
<td>23.9</td>
</tr>
<tr>
<td>9.6</td>
<td>12.74</td>
<td>116</td>
<td>0.95</td>
<td>38.1</td>
</tr>
<tr>
<td>12.2</td>
<td>16.1</td>
<td>184</td>
<td>0.95</td>
<td>47.6</td>
</tr>
<tr>
<td>16.5</td>
<td>21.6</td>
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<td>63.9</td>
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<td>0.95</td>
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Table C.2 Transformer PC1 short circuit results

$R_{sc} = 0.74\Omega$ and $X_{sc} = 0.25\Omega$
C.1.2 Transformer $PC2$

<table>
<thead>
<tr>
<th>$V_1$(V)</th>
<th>$I_1$(A)</th>
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<th>$pf$</th>
<th>$V_2$(V)</th>
<th>$I_c$(A)</th>
<th>$I_m$(A)</th>
</tr>
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<tr>
<td>30.4</td>
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<td>4.8</td>
<td>0.13</td>
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<td>0.17</td>
<td>2.04</td>
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<td>0.1</td>
<td>14.2</td>
<td>0.33</td>
<td>3.04</td>
</tr>
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<td>119.6</td>
<td>4.09</td>
<td>55</td>
<td>0.1</td>
<td>18.9</td>
<td>0.46</td>
<td>4.06</td>
</tr>
<tr>
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<td>5.12</td>
<td>80</td>
<td>0.1</td>
<td>23.4</td>
<td>0.54</td>
<td>5.09</td>
</tr>
<tr>
<td>180</td>
<td>6.18</td>
<td>110</td>
<td>0.1</td>
<td>28.3</td>
<td>0.61</td>
<td>6.15</td>
</tr>
<tr>
<td>200</td>
<td>6.88</td>
<td>140</td>
<td>0.09</td>
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<td>0.70</td>
<td>6.84</td>
</tr>
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<td>0.09</td>
<td>33</td>
<td>0.71</td>
<td>7.20</td>
</tr>
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<td>34.6</td>
<td>0.77</td>
<td>7.56</td>
</tr>
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<td>230</td>
<td>7.96</td>
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<td>36.2</td>
<td>0.80</td>
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Table C.3 Transformer $PC2$ open circuit results

$R_{oc} = 289\Omega$ and $X_{oc} = 29.1\Omega$

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<th>$I_2$(A)</th>
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<tbody>
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<td>6.48</td>
<td>114</td>
<td>0.36</td>
<td>30.8</td>
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<td>63.2</td>
<td>8.43</td>
<td>190</td>
<td>0.36</td>
<td>40.1</td>
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<td>50.1</td>
</tr>
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<td>0.36</td>
<td>60.9</td>
</tr>
<tr>
<td>110.9</td>
<td>15</td>
<td>600</td>
<td>0.36</td>
<td>69.6</td>
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<tr>
<td>128.4</td>
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<td>810</td>
<td>0.37</td>
<td>80.5</td>
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<tr>
<td>143.1</td>
<td>19.1</td>
<td>1010</td>
<td>0.37</td>
<td>89.9</td>
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<tr>
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Table C.4 Transformer $PC2$ short circuit results

$R_{sc} = 2.77\Omega$ and $X_{sc} = 6.96\Omega$
C.1.3 Transformer PC3

<table>
<thead>
<tr>
<th>$V_1$(V)</th>
<th>$I_1$(A)</th>
<th>$P_c$(W)</th>
<th>$p_f$</th>
<th>$V_2$(V)</th>
<th>$I_c$(A)</th>
<th>$I_m$(A)</th>
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<td>0.48</td>
</tr>
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<td>14.9</td>
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<td>0.34</td>
<td>14.2</td>
<td>0.20</td>
<td>0.72</td>
</tr>
<tr>
<td>20.1</td>
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<td>6</td>
<td>0.32</td>
<td>19.1</td>
<td>0.30</td>
<td>0.96</td>
</tr>
<tr>
<td>25</td>
<td>1.27</td>
<td>9</td>
<td>0.31</td>
<td>23.8</td>
<td>0.36</td>
<td>1.22</td>
</tr>
<tr>
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<td>1.54</td>
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<td>28.6</td>
<td>0.43</td>
<td>1.48</td>
</tr>
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<td>1.79</td>
<td>18</td>
<td>0.29</td>
<td>33.2</td>
<td>0.52</td>
<td>1.71</td>
</tr>
<tr>
<td>39.9</td>
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<td>1.97</td>
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<td>43</td>
<td>0.64</td>
<td>2.24</td>
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<td>50.1</td>
<td>2.58</td>
<td>36</td>
<td>0.29</td>
<td>47.5</td>
<td>0.72</td>
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</tr>
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<td>0.80</td>
<td>2.73</td>
</tr>
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<td>60.1</td>
<td>3.11</td>
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<td>57</td>
<td>0.88</td>
<td>2.98</td>
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<td>61.7</td>
<td>0.97</td>
<td>3.23</td>
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<td>73</td>
<td>0.29</td>
<td>66.5</td>
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Table C.5 Transformer PC3 open circuit results

$R_{oc} = 67.3\Omega$ and $X_{oc} = 20.1\Omega$

<table>
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<tr>
<th>$V_1$(V)</th>
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<th>$P_c$(W)</th>
<th>$p_f$</th>
<th>$I_2$(A)</th>
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<tr>
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<td>0.51</td>
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<tr>
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<td>11</td>
<td>1</td>
<td>1.09</td>
</tr>
<tr>
<td>14.9</td>
<td>1.73</td>
<td>25</td>
<td>1</td>
<td>1.61</td>
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<tr>
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<td>2.68</td>
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Table C.6 Transformer PC3 short circuit results

$R_{sc} = 8.86\Omega$ and $X_{sc} = 1.3\Omega$
C.1.4 Transformer PC4

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<th>$V_1$(V)</th>
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<th>$P_c$(W)</th>
<th>$p_f$</th>
<th>$V_2$(V)</th>
<th>$I_c$(A)</th>
<th>$I_m$(A)</th>
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<td>0.09</td>
<td>10.9</td>
<td>0.12</td>
<td>1.35</td>
</tr>
<tr>
<td>90</td>
<td>2.07</td>
<td>16</td>
<td>0.09</td>
<td>16.4</td>
<td>0.18</td>
<td>2.06</td>
</tr>
<tr>
<td>120</td>
<td>2.78</td>
<td>27</td>
<td>0.08</td>
<td>21.8</td>
<td>0.23</td>
<td>2.77</td>
</tr>
<tr>
<td>150</td>
<td>3.48</td>
<td>40</td>
<td>0.08</td>
<td>27.2</td>
<td>0.27</td>
<td>3.47</td>
</tr>
<tr>
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<td>0.08</td>
<td>32.6</td>
<td>0.33</td>
<td>4.19</td>
</tr>
<tr>
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<td>80</td>
<td>0.08</td>
<td>38.2</td>
<td>0.38</td>
<td>4.91</td>
</tr>
<tr>
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<td>5.16</td>
<td>90</td>
<td>0.08</td>
<td>40</td>
<td>0.41</td>
<td>5.14</td>
</tr>
<tr>
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<td>95</td>
<td>0.08</td>
<td>4108</td>
<td>0.41</td>
<td>5.39</td>
</tr>
<tr>
<td>238</td>
<td>5.59</td>
<td>105</td>
<td>0.08</td>
<td>43.3</td>
<td>0.44</td>
<td>5.57</td>
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</table>

Table C.7 Transformer PC4 open circuit results

$R_{sc} = 551\Omega$ and $X_{sc} = 43\Omega$

<table>
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<th>$V_1$(V)</th>
<th>$I_1$(A)</th>
<th>$P_c$(W)</th>
<th>$p_f$</th>
<th>$I_2$(A)</th>
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<tr>
<td>6.6</td>
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<td>5.55</td>
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<td>85</td>
<td>0.57</td>
<td>25.6</td>
</tr>
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<td>57.9</td>
<td>9.99</td>
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<td>0.57</td>
<td>50</td>
</tr>
<tr>
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<td>750</td>
<td>0.58</td>
<td>74.5</td>
</tr>
<tr>
<td>105.7</td>
<td>18.1</td>
<td>1110</td>
<td>0.59</td>
<td>89.8</td>
</tr>
<tr>
<td>118.4</td>
<td>20</td>
<td>1410</td>
<td>0.6</td>
<td>98.9</td>
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</table>

Table C.8 Transformer PC4 short circuit results

$R_{sc} = 3.5\Omega$ and $X_{sc} = 4.72\Omega$

<table>
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<th>$V_1$(V)</th>
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<th>$P_1$(W)</th>
<th>$p_f_1$</th>
<th>$V_2$(V)</th>
<th>$I_2$(A)</th>
<th>$P_2$(W)</th>
<th>$p_f_2$</th>
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<td>109</td>
<td>2500</td>
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Table C.9 Transformer PC4 loaded circuit results
C.1.5 Transformer *PC5*

<table>
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<th>$I_1$ (A)</th>
<th>$P_c$ (W)</th>
<th>$pf$</th>
<th>$V_2$ (V)</th>
<th>$I_c$ (A)</th>
<th>$I_n$ (A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30.3</td>
<td>2.59</td>
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<td>0.1</td>
<td>3.2</td>
<td>0.23</td>
<td>2.58</td>
</tr>
<tr>
<td>59.6</td>
<td>5.17</td>
<td>27</td>
<td>0.09</td>
<td>6.1</td>
<td>0.45</td>
<td>5.15</td>
</tr>
<tr>
<td>80.3</td>
<td>7</td>
<td>50</td>
<td>0.09</td>
<td>8.2</td>
<td>0.62</td>
<td>6.97</td>
</tr>
<tr>
<td>100.4</td>
<td>8.79</td>
<td>70</td>
<td>0.09</td>
<td>10.2</td>
<td>0.70</td>
<td>8.76</td>
</tr>
<tr>
<td>120.5</td>
<td>10.58</td>
<td>110</td>
<td>0.08</td>
<td>12.2</td>
<td>0.91</td>
<td>10.54</td>
</tr>
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</table>

*Table C.10* Transformer *PC5* open circuit results

$R_{oc} = 134\Omega$ and $X_{oc} = 11.3\Omega$

<table>
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<th>$V_1$ (V)</th>
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<th>$P_c$ (W)</th>
<th>$pf$</th>
<th>$I_2$ (A)</th>
</tr>
</thead>
<tbody>
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<td>16.2</td>
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<tr>
<td>12.4</td>
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<td>0.73</td>
<td>37.1</td>
</tr>
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<td>17.9</td>
<td>6.35</td>
<td>82</td>
<td>0.73</td>
<td>54</td>
</tr>
<tr>
<td>25.2</td>
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<td>75.3</td>
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</table>

*Table C.11* Transformer *PC5* short circuit results

$R_{sc} = 2.07\Omega$ and $X_{sc} = 1.96\Omega$
C.1.6 Transformer \textit{PC6}

<table>
<thead>
<tr>
<th>$V_1$(V)</th>
<th>$I_1$(A)</th>
<th>$P_c$(W)</th>
<th>$p_f$</th>
<th>$V_2$(V)</th>
<th>$I_c$(A)</th>
<th>$I_m$(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>30</td>
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<td>3</td>
<td>0.09</td>
<td>32.9</td>
<td>0.10</td>
<td>1.36</td>
</tr>
<tr>
<td>60</td>
<td>2.71</td>
<td>12</td>
<td>0.09</td>
<td>66.4</td>
<td>0.20</td>
<td>2.70</td>
</tr>
<tr>
<td>90</td>
<td>4.06</td>
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<td>99.7</td>
<td>0.29</td>
<td>4.05</td>
</tr>
<tr>
<td>120</td>
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<td>133</td>
<td>0.33</td>
<td>5.41</td>
</tr>
<tr>
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<td>6.76</td>
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<td>0.07</td>
<td>200</td>
<td>0.56</td>
<td>8.14</td>
</tr>
<tr>
<td>210</td>
<td>9.53</td>
<td>130</td>
<td>0.07</td>
<td>232</td>
<td>0.62</td>
<td>9.51</td>
</tr>
<tr>
<td>230</td>
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<td>254</td>
<td>0.70</td>
<td>10.45</td>
</tr>
<tr>
<td>240</td>
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<td>0.07</td>
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</table>

Table C.12 Transformer \textit{PC6} open circuit results

$R_{oc} = 325.9\Omega$ and $X_{oc} = 21.9\Omega$

<table>
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<tr>
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<th>$p_f$</th>
<th>$I_2$(A)</th>
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<tbody>
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<td>3.5</td>
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<td>1.6</td>
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<tr>
<td>7.1</td>
<td>4</td>
<td>18</td>
<td>0.65</td>
<td>3.25</td>
</tr>
<tr>
<td>10.6</td>
<td>6</td>
<td>40</td>
<td>0.64</td>
<td>4.88</td>
</tr>
<tr>
<td>14.3</td>
<td>8</td>
<td>73</td>
<td>0.64</td>
<td>6.54</td>
</tr>
<tr>
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<td>112</td>
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<td>8.14</td>
</tr>
<tr>
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<td>12</td>
<td>160</td>
<td>0.64</td>
<td>9.52</td>
</tr>
<tr>
<td>25.3</td>
<td>14</td>
<td>230</td>
<td>0.64</td>
<td>11.38</td>
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Table C.13 Transformer \textit{PC6} short circuit results

$R_{sc} = 1.14\Omega$ and $X_{sc} = 1.37\Omega$

C.2 DATA ANALYSIS

This section presents the analyses performed to calculate the hysteresis loss parameters $x$ and $k_h$. 


<table>
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<tr>
<th>$V_1$(V)</th>
<th>$I_1$(A)</th>
<th>Total Loss(W)</th>
<th>$P^2R_1$ Loss(W)</th>
<th>Core Loss(W)</th>
<th>$e_1$(V)</th>
<th>$R_{core}$(Ω)</th>
<th>$R_h$(Ω)</th>
<th>$P_h$(W)</th>
<th>$B_{core}$(T)</th>
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Table C.14 Transformer PC1

Average values

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**Table C.15** Transformer PC2
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<th>Core Loss (W)</th>
<th>$e_1$ (V)</th>
<th>$R_{core}$(Ω)</th>
<th>$R_h$(Ω)</th>
<th>$P_h$(W)</th>
<th>$B_{core}$(T)</th>
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Table C.16  Transformer $PC3$
C.3 50HZ SUPPLY AT LIQUID NITROGEN TEMPERATURE

This section presents the detailed results of the open and short circuit tests performed at liquid nitrogen temperatures. In addition, the loaded test results performed on Transformer PC4, under both the room and liquid nitrogen temperatures, are also presented.

C.3.1 Transformer PC1

<table>
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<th>$I_1$ (A)</th>
<th>$P_c$ (W)</th>
<th>$p_f$</th>
<th>$V_2$ (V)</th>
<th>$I_c$ (A)</th>
<th>$I_m$ (A)</th>
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<td>0.71</td>
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Table C.17 Transformer PC1 open circuit results

$R_{oc} = 174.0\Omega \text{ and } X_{oc} = 13.1\Omega$

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<th>$P_c$ (W)</th>
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<td>10</td>
<td>12</td>
<td>0.45</td>
<td>29</td>
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Table C.18 Transformer PC1 short circuit results

$R_{sc} = 0.13\Omega \text{ and } X_{sc} = 0.24\Omega$
C.3.2 Transformer PC4

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<th>$I_1$ (A)</th>
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<td>0.10</td>
<td>1.48</td>
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<tr>
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<td>0.18</td>
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**Table C.19** Transformer PC4 open circuit results

$R_{oc} = 661\Omega$ and $X_{oc} = 40.3\Omega$

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<th>$P_c$ (W)</th>
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<th>$I_2$ (A)</th>
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<td>66</td>
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<td>43.1</td>
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<td>150</td>
<td>0.19</td>
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**Table C.20** Transformer PC4 short circuit results

$R_{sc} = 0.85\Omega$ and $X_{sc} = 4.59\Omega$

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<th>$P_1$ (W)</th>
<th>$pf_1$</th>
<th>$V_2$ (V)</th>
<th>$I_2$ (A)</th>
<th>$P_2$ (W)</th>
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**Table C.21** Transformer PC4 loaded circuit results
C.3.3 Transformer PC5

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<th>$p_f$</th>
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<th>$I_2$(A)</th>
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<td>0.07</td>
<td>6.2</td>
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<td>5.68</td>
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<td>150</td>
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<td>0.06</td>
<td>15.2</td>
<td>0.80</td>
<td>14.18</td>
</tr>
<tr>
<td>180</td>
<td>17.1</td>
<td>180</td>
<td>0.05</td>
<td>18.2</td>
<td>1.00</td>
<td>17.07</td>
</tr>
<tr>
<td>210</td>
<td>20.1</td>
<td>240</td>
<td>0.06</td>
<td>21.2</td>
<td>1.14</td>
<td>20.07</td>
</tr>
<tr>
<td>230</td>
<td>21.9</td>
<td>280</td>
<td>0.06</td>
<td>23.3</td>
<td>1.22</td>
<td>21.87</td>
</tr>
</tbody>
</table>

Table C.22 Transformer PC5 open circuit results

$R_{oc} = 188.9\Omega$ and $X_{oc} = 10.5\Omega$

<table>
<thead>
<tr>
<th>$V_1$(V)</th>
<th>$I_1$(A)</th>
<th>$P_e$(W)</th>
<th>$p_f$</th>
<th>$I_2$(A)</th>
</tr>
</thead>
<tbody>
<tr>
<td>9.9</td>
<td>5.52</td>
<td>21</td>
<td>0.4</td>
<td>44.3</td>
</tr>
<tr>
<td>20.2</td>
<td>11.56</td>
<td>79</td>
<td>0.34</td>
<td>93.1</td>
</tr>
<tr>
<td>29.9</td>
<td>17.3</td>
<td>170</td>
<td>0.32</td>
<td>139.5</td>
</tr>
<tr>
<td>40</td>
<td>23</td>
<td>285</td>
<td>0.3</td>
<td>186</td>
</tr>
</tbody>
</table>

Table C.23 Transformer PC5 short circuit results

$R_{sc} = 0.53\Omega$ and $X_{sc} = 1.66\Omega$
C.3.4 Transformer PC6

\[
\begin{array}{|c|c|c|c|c|c|c|}
\hline
V_1 (V) & I_1 (A) & P_c (W) & pf & V_2 (V) & I_c (A) & I_m (A) \\
\hline
30.5 & 1.36 & 3 & 0.08 & 33.8 & 0.10 & 1.36 \\
60.3 & 2.63 & 9 & 0.06 & 66.8 & 0.15 & 2.63 \\
89.9 & 3.91 & 19 & 0.06 & 99.6 & 0.21 & 3.90 \\
120.6 & 5.26 & 30 & 0.05 & 133.6 & 0.25 & 5.25 \\
150 & 6.58 & 50 & 0.05 & 166 & 0.33 & 6.57 \\
180 & 7.93 & 75 & 0.05 & 199 & 0.42 & 7.92 \\
210 & 9.33 & 100 & 0.05 & 233 & 0.48 & 9.32 \\
230 & 10.25 & 120 & 0.05 & 254 & 0.52 & 10.24 \\
\hline
\end{array}
\]

*Table C.24* Transformer PC6 open circuit results

\[R_{oc} = 440\Omega \text{ and } X_{oc} = 22.5\Omega\]

\[
\begin{array}{|c|c|c|c|c|c|}
\hline
V_1 (V) & I_1 (A) & P_c (W) & pf & I_2 (A) \\
\hline
4.2 & 2.98 & 2 & 0.19 & 2.38 \\
8.5 & 6.04 & 9 & 0.18 & 4.87 \\
12.2 & 8.7 & 18 & 0.18 & 7 \\
16.8 & 12.02 & 35 & 0.17 & 9.73 \\
20.9 & 15.1 & 54 & 0.17 & 12.14 \\
25.3 & 18 & 80 & 0.17 & 14.8 \\
28.1 & 20 & 100 & 0.17 & 16.3 \\
30.5 & 21.7 & 110 & 0.17 & 17.8 \\
\hline
\end{array}
\]

*Table C.25* Transformer PC6 short circuit results

\[R_{sc} = 0.23\Omega \text{ and } X_{sc} = 1.38\Omega\]
Appendix D

INCORPORATING CAPACITIVE COMPONENTS TO SUBROUTINES

Circuit analyses with the incorporation of capacitive components to subroutines OPEN_CCT, SHORT_CCT and LOADED_CCT are presented.

D.1 OPEN_CCT SUBROUTINE

An open circuit analysis from the equivalent circuit parameters of Figure 6.23 is performed in this subroutine. The new equivalent circuit used for the open circuit analysis is shown in Figure D.1.

![Diagram of open circuit transformer equivalent circuit](image)

**Figure D.1** The open circuit transformer equivalent circuit

Let the voltages at Nodes 1, 2 and 3 be \( \tilde{V}_{Node1} \), \( \tilde{V}_{Node2} \) and \( \tilde{V}_{Node3} \) respectively. At Node 2, using Kirchhoff’s current law (KCL):

\[
\frac{\tilde{V}_{Node2} - \tilde{V}_{Node1}}{Z_1} + \frac{\tilde{V}_{Node2}}{Z_{core}} + \frac{\tilde{V}_{Node2} - \tilde{V}_{Node3}}{Z_2} = 0
\]

\[
\tilde{V}_{Node2} \left( \frac{1}{Z_1} + \frac{1}{Z_{core}} + \frac{1}{Z_2} \right) = \tilde{V}_{Node1} \frac{1}{Z_1} + \tilde{V}_{Node3} \frac{1}{Z_2}
\]
\[ \tilde{V}_{\text{Node}2} = \left( \frac{\tilde{V}_{\text{Node}1}}{Z_1} + \frac{\tilde{V}_{\text{Node}3}}{Z_2} \right) \frac{1}{Y_{\text{Node}2}} \]  
(D.1)

where \( Y_{\text{Node}2} = \frac{1}{Z_1} + \frac{1}{Z_{\text{core}}} + \frac{1}{Z_2} \)

And at Node 3:

\[ \frac{\tilde{V}_{\text{Node}3} - \tilde{V}_{\text{Node}2}}{Z_2} + \frac{\tilde{V}_{\text{Node}3} - \tilde{V}_{\text{Node}1}}{Z_{12}} + \frac{\tilde{V}_{\text{Node}3} - \tilde{V}_{\text{Node}1}}{Z_{C2}} = 0 \]

\[ \tilde{V}_{\text{Node}3} \left( \frac{1}{Z_2} + \frac{1}{Z_{12}} + \frac{1}{Z_{C2}} \right) = \frac{\tilde{V}_{\text{Node}2}}{Z_2} + \frac{\tilde{V}_{\text{Node}1}}{Z_{12}} \]

\[ \tilde{V}_{\text{Node}3} = \left( \frac{\tilde{V}_{\text{Node}2}}{Z_2} + \frac{\tilde{V}_{\text{Node}1}}{Z_{12}} \right) \frac{1}{Y_{\text{Node}3}} \]  
(D.2)

where \( Y_{\text{Node}3} = \frac{1}{Z_2} + \frac{1}{Z_{12}} + \frac{1}{Z_{C2}} \)

\( Z_{C2} = -jX_{C2} \)

Substituting Equation D.1 into D.2,

\[ \tilde{V}_{\text{Node}3} = \tilde{V}_{\text{Node}1} \left( \frac{1}{Z_1 Z_2 Y_{\text{Node}2}} + \frac{1}{Z_1 Z_2 Y_{\text{Node}3}} - \frac{1}{Z_1 Z_2 Y_{\text{Node}1}} \right) \]  
(D.3)

Since

\[ \tilde{V}_{\text{Node}1} = V_1 + j0 \]  
(D.4)

the complex open circuit primary current is the sum of all the currents at Node 1:

\[ I_{oc1} = \sum \text{currents at Node 1} \]

\[ = \frac{\tilde{V}_{\text{Node}1}}{Z_{C1}} + \frac{\tilde{V}_{\text{Node}1} - \tilde{V}_{\text{Node}2}}{Z_1} + \frac{\tilde{V}_{\text{Node}1} - \tilde{V}_{\text{Node}3}}{Z_{12}} \]  
(D.5)

where \( Z_{C1} = -jX_{C1} \)

The transformer’s open circuit impedance is

\[ Z_{oc} = \frac{\tilde{V}_{\text{Node}1}}{I_{oc1}} \]  
(D.6)

The equivalent open circuit components, \( R_{oc} \) and \( X_{oc} \), can be calculated using Equations 3.92 – 3.96. Similarly, the performance parameters under open circuit conditions, i.e. \( I_1, S_{oc}, P_{oc} \) and \( pf_{oc} \) can all be obtained using Equations 3.98 – 3.101.
D.2 SHORT_CCT SUBROUTINE

This subroutine re-calculate the short circuit analysis based on the equivalent shown in Figure D.2.

![Equivalent circuit diagram]

**Figure D.2** The short circuit transformer equivalent circuit

At Node 2, using KCL:

\[
\frac{\tilde{V}_{\text{Node}_2}}{Z_1} - \frac{\tilde{V}_{\text{Node}_1}}{Z_{\text{core}}} + \frac{\tilde{V}_{\text{Node}_2}}{Z_2} = 0
\]

\[
\tilde{V}_{\text{Node}_2} \left(\frac{1}{Z_1} + \frac{1}{Z_{\text{core}}} + \frac{1}{Z_2}\right) = \frac{\tilde{V}_{\text{Node}_1}}{Z_1}
\]

\[
\tilde{V}_{\text{Node}_2} = \left(\frac{\tilde{V}_{\text{Node}_1}}{Z_1}\right) \frac{1}{Y_{\text{Node}_2}}
\]

where \( Y_{\text{Node}_2} \) is as defined earlier in Section D.1

The complex short circuit primary current is the sum of all currents at Node 1:

\[
\tilde{I}_{\text{scl}} = \sum \text{currents at Node 1}
\]

\[
= \frac{\tilde{V}_{\text{Node}_1}}{Z_{C1}} + \frac{\tilde{V}_{\text{Node}_1} - \tilde{V}_{\text{Node}_2}}{Z_1} + \frac{\tilde{V}_{\text{Node}_1}}{Z_{12}}
\]

\[(D.8)\]

The equivalent short circuit impedance is

\[
Z_{sc} = \frac{\tilde{V}_{\text{Node}_1}}{\tilde{I}_{\text{scl}}}
\]

\[(D.9)\]

The equivalent short circuit components, \( R_{sc} \) and \( X_{sc} \), can be calculated using Equations 3.109 and 3.110. The corresponding performance parameters, namely \( I_1, S_{sc}, P_{sc} \) and \( pf_{sc} \), can be calculated using Equations 3.112 – 3.115.
D.3 LOADED_CCT SUBROUTINE

Finally, the loaded circuit analysis is re-calculated in this subroutine. The corresponding equivalent circuit used is shown in Figure D.3.

![Figure D.3 The loaded circuit transformer equivalent circuit](image)

The load impedance, referred to the primary side, $Z'_L$, is defined in Equation 3.116. Therefore the load admittance referred to the primary is

$$Y'_L = \frac{1}{Z'_L}$$  \hspace{1cm} (D.10)

At Node 3, using KCL:

$$\frac{\tilde{V}_{Node3} - \tilde{V}_{Node2}}{Z_2} + \frac{\tilde{V}_{Node3} - \tilde{V}_{Node1}}{Z_{12}} + \frac{\tilde{V}_{Node3}}{Z_{C2,L}} = 0$$

$$\tilde{V}_{Node3} \left( \frac{1}{Z_2} + \frac{1}{Z_{12}} + \frac{1}{Z_{C2,L}} \right) = \frac{\tilde{V}_{Node2}}{Z_2} + \frac{\tilde{V}_{Node1}}{Z_{12}}$$

$$\tilde{V}_{Node3} = \left( \frac{\tilde{V}_{Node2}}{Z_2} + \frac{\tilde{V}_{Node1}}{Z_{12}} \right) \frac{1}{Y'_{Node3,L}}$$  \hspace{1cm} (D.11)

where $Y'_{Node3,L} = \frac{1}{Z_2} + \frac{1}{Z_{12}} + \frac{1}{Z_{C2,L}}$

The output admittance is a combination of $\frac{1}{Z_{C2}}$ and $Y'_L$. Therefore,

$$Y_{C2,L} = \frac{1}{X_{C2}} + Y'_L$$  \hspace{1cm} (D.12)

from which

$$Z_{C2,L} = \frac{1}{Y_{CAP,L}}$$  \hspace{1cm} (D.13)
Solving Equations D.1 and D.11 simultaneously, the solutions are

\[
\tilde{V}_{\text{Node}3} = \tilde{V}_{\text{Node}1} \left( \frac{1}{Z_1 Z_2 Y_{\text{Node}2}} + \frac{1}{Z_{12}} \right) \left( \frac{1}{Y_{\text{Node}3,L}} - \frac{1}{Z_2^2 Y_{\text{Node}2}} \right)
\]  \hspace{1cm} (D.14)

and

\[
\tilde{V}_{\text{Node}2} = \left( \frac{\tilde{V}_{\text{Node}1}}{Z_1} + \frac{\tilde{V}_{\text{Node}3}}{Z_2} \right) \frac{1}{Y_{\text{Node}2}}
\]  \hspace{1cm} (D.15)

The complex loaded primary current is the sum of all currents at Node 1.

\[
\tilde{I}_1 = \sum \text{ current at Node } 1
\]
\[
= \frac{\tilde{V}_{\text{Node}1}}{Z_{C1}} + \frac{\tilde{V}_{\text{Node}1} - \tilde{V}_{\text{Node}2}}{Z_1} + \frac{\tilde{V}_{\text{Node}1} - \tilde{V}_{\text{Node}3}}{Z_{12}}
\]  \hspace{1cm} (D.16)

The loaded performance parameters, for instance \(I_1, S_1, P_1, p f_1, I_2', V_L', I_2, V_L, VREG, P_L'\), and \(EFF\) can be calculated using Equations 3.123 – 3.136.
Appendix E

HVPC1 HIGH VOLTAGE TRANSFORMER DATA

The specifications and construction details of all the high voltage partial core transformer utilised in this thesis are presented here. The frequency response test results under no-load and loaded conditions are also presented.

E.1 REVERSE DESIGN PARAMETERS

(a) Supply Characteristics

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$V_1$</td>
<td>14 V</td>
</tr>
<tr>
<td>$f$</td>
<td>50 Hz</td>
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</table>

(b) Core

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{core}$</td>
<td>110 mm</td>
</tr>
<tr>
<td>$D_{core}$</td>
<td>76 mm</td>
</tr>
<tr>
<td>$SF_c$</td>
<td>0.94</td>
</tr>
<tr>
<td>$a_{lam}$</td>
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</tr>
<tr>
<td>$I_{cl}$</td>
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</tr>
<tr>
<td>$\epsilon_{IC1}$</td>
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</tr>
<tr>
<td>$\mu_{rc}$</td>
<td>2000</td>
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<tr>
<td>$\rho_{core,20^\circ C}$</td>
<td>$1.8 \times 10^{-7}$ $\Omega m$</td>
</tr>
<tr>
<td>$\Delta \rho_{core}$</td>
<td>$0.006 \times 10^{-8}$ $^{\circ C}^{-1}$</td>
</tr>
<tr>
<td>$T_{core}$</td>
<td>20 $^{\circ C}$</td>
</tr>
<tr>
<td>$\gamma_{core}$</td>
<td>7870 kg/m$^3$</td>
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<tr>
<td>$Cost_{core}$</td>
<td>6.7 $$/kg$$</td>
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</table>

(c) Primary Winding

<table>
<thead>
<tr>
<th>Parameter</th>
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<tbody>
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<td>$l_{prim}$</td>
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<tr>
<td>$L_{y1}$</td>
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<tr>
<td>$w_1$</td>
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<tr>
<td>$w_{i1}$</td>
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<tr>
<td>$\epsilon_{WI1}$</td>
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</tr>
<tr>
<td>Symbol</td>
<td>Value</td>
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<tr>
<td>--------</td>
<td>-------</td>
</tr>
<tr>
<td>$I_1$</td>
<td>0</td>
</tr>
<tr>
<td>$\varepsilon_{11}$</td>
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<tr>
<td>$\mu_{r1}$</td>
<td>1.0</td>
</tr>
<tr>
<td>$\rho_{1,20^\circ C}$</td>
<td>$1.76 \times 10^{-8}$</td>
</tr>
<tr>
<td>$\Delta \rho_1$</td>
<td>$0.0039 \times 10^{-8}$</td>
</tr>
<tr>
<td>$T_1$</td>
<td>20</td>
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<tr>
<td>$\gamma_1$</td>
<td>8960</td>
</tr>
<tr>
<td>$Cost_1$</td>
<td>12</td>
</tr>
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</table>

**(d) Primary–Secondary Interwinding Space**

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
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</thead>
<tbody>
<tr>
<td>$I_{12}$</td>
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</tr>
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<td>$\varepsilon_{112}$</td>
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</table>

***(e) Secondary Winding***

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$l_{sec}$</td>
<td>102</td>
<td>mm</td>
</tr>
<tr>
<td>$L_{y2}$</td>
<td>20</td>
<td></td>
</tr>
<tr>
<td>$w_2$</td>
<td>0.11</td>
<td>mm</td>
</tr>
<tr>
<td>$w_{i2}$</td>
<td>0.015</td>
<td>mm</td>
</tr>
<tr>
<td>$\varepsilon_{W12}$</td>
<td>4.3</td>
<td></td>
</tr>
<tr>
<td>$I_2$</td>
<td>0.15</td>
<td>mm</td>
</tr>
<tr>
<td>$\varepsilon_{f2}$</td>
<td>5.0</td>
<td></td>
</tr>
<tr>
<td>$\mu_{r2}$</td>
<td>1.0</td>
<td></td>
</tr>
<tr>
<td>$\rho_{2,20^\circ C}$</td>
<td>$1.76 \times 10^{-8}$</td>
<td>Ωm</td>
</tr>
<tr>
<td>$\Delta \rho_2$</td>
<td>$0.0039 \times 10^{-8}$</td>
<td>°C$^{-1}$</td>
</tr>
<tr>
<td>$T_2$</td>
<td>20</td>
<td>°C</td>
</tr>
<tr>
<td>$\gamma_2$</td>
<td>8960</td>
<td>kg/m$^3$</td>
</tr>
<tr>
<td>$Cost_2$</td>
<td>12</td>
<td>$$/kg</td>
</tr>
</tbody>
</table>
E.2 EXPERIMENTAL DATA

E.2.1 O/C Resonance Test

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>$V_{1pk-pk}$ (V)</th>
<th>$V_{2pk-pk}$ (kV)</th>
<th>Normalised Gain Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>36</td>
<td>9.4</td>
<td>0.80</td>
</tr>
<tr>
<td>200</td>
<td>35.2</td>
<td>9.2</td>
<td>0.80</td>
</tr>
<tr>
<td>500</td>
<td>32</td>
<td>8.6</td>
<td>0.83</td>
</tr>
<tr>
<td>1000</td>
<td>31.2</td>
<td>8.6</td>
<td>0.85</td>
</tr>
<tr>
<td>1500</td>
<td>31.2</td>
<td>9.2</td>
<td>0.91</td>
</tr>
<tr>
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<td>30.8</td>
<td>9.8</td>
<td>0.98</td>
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<td>10.0</td>
<td>1.03</td>
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<td>10.0</td>
<td>1.05</td>
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<td>1.13</td>
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</tr>
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<tr>
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<tr>
<td>4800</td>
<td>18.8</td>
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<td>2.32</td>
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</tbody>
</table>

Table E.1 HVPC1 open circuit resonance results

E.2.2 Loaded Resonance Test

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>$V_{1pk-pk}$ (V)</th>
<th>$V_{2pk-pk}$ (kV)</th>
<th>Normalised Gain Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>28.4</td>
<td>7.4</td>
<td>0.80</td>
</tr>
<tr>
<td>200</td>
<td>30.8</td>
<td>8.2</td>
<td>0.82</td>
</tr>
<tr>
<td>500</td>
<td>31.2</td>
<td>8.8</td>
<td>0.87</td>
</tr>
<tr>
<td>1000</td>
<td>31.6</td>
<td>11.4</td>
<td>1.11</td>
</tr>
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<td>24</td>
<td>13.2</td>
<td>1.69</td>
</tr>
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<td>1700</td>
<td>23.2</td>
<td>16.2</td>
<td>2.15</td>
</tr>
<tr>
<td>1800</td>
<td>18</td>
<td>13.2</td>
<td>2.26</td>
</tr>
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<td>1900</td>
<td>20</td>
<td>15.6</td>
<td>2.40</td>
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<tr>
<td>2000</td>
<td>22.8</td>
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<td>32.4</td>
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<tr>
<td>4000</td>
<td>36</td>
<td>3.4</td>
<td>0.29</td>
</tr>
<tr>
<td>5000</td>
<td>39.2</td>
<td>2.8</td>
<td>0.22</td>
</tr>
</tbody>
</table>

Table E.2 HVPC1 loaded resonance results
Appendix F

HVPC2 HIGH VOLTAGE TRANSFORMER DATA

The specifications and construction details of all the designed high voltage partial core transformer utilised in this thesis are presented here. The frequency response test results under no-load and loaded conditions are also presented.

F.1 REVERSE DESIGN PARAMETERS

(a) Supply Characteristics
\[ V_1 = 12 \text{ V} \]
\[ f = 50 \text{ Hz} \]

(b) Core
\[ l_{\text{core}} = 132 \text{ mm} \]
\[ D_{\text{core}} = 64 \text{ mm} \]
\[ SF_c = 0.94 \]
\[ a_{\text{lam}} = 0.3 \text{ mm} \]
\[ L_{c1} = 6 \text{ mm} \]
\[ \varepsilon_{IC1} = 4.3 \]
\[ \mu_{\text{rc}} = 2000 \]
\[ \rho_{\text{core,20}^\circ C} = 1.8 \times 10^{-7} \text{ } \Omega \text{m} \]
\[ \Delta \rho_{\text{core}} = 0.006 \times 10^{-8} \text{ } ^\circ \text{C}^{-1} \]
\[ T_{\text{core}} = 20 \text{ } ^\circ \text{C} \]
\[ \gamma_{\text{core}} = 7870 \text{ kg/m}^3 \]
\[ C_{\text{ost,core}} = 6.7 \text{ } \$ / \text{kg} \]

(c) Primary Winding
\[ l_{\text{prim}} = 120 \text{ mm} \]
\[ L_{y1} = 2 \]
\[ w_1 = 4.25 \text{ mm} \]
\[ w_{i1} = 0.2 \text{ mm} \]
\[ \varepsilon_{W11} = 4.3 \]
<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_1 )</td>
<td>0 mm</td>
</tr>
<tr>
<td>( \epsilon_{I1} )</td>
<td>0</td>
</tr>
<tr>
<td>( \mu_{r1} )</td>
<td>1.0</td>
</tr>
<tr>
<td>( \rho_{1,20^\circ C} )</td>
<td>( 1.76 \times 10^{-8} ) Ωm</td>
</tr>
<tr>
<td>( \Delta\rho_1 )</td>
<td>( 0.0039 \times 10^{-8} ) °C(^{-1})</td>
</tr>
<tr>
<td>( T_1 )</td>
<td>20 °C</td>
</tr>
<tr>
<td>( \gamma_1 )</td>
<td>8960 kg/m(^3)</td>
</tr>
<tr>
<td>( Cost_1 )</td>
<td>12 $/kg</td>
</tr>
</tbody>
</table>

(d) Primary–Secondary Interwinding Space

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( I_{12} )</td>
<td>0.8 mm</td>
</tr>
<tr>
<td>( \epsilon_{I12} )</td>
<td>4.3</td>
</tr>
</tbody>
</table>

(e) Secondary Winding

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>( l_{sec} )</td>
<td>110 mm</td>
</tr>
<tr>
<td>( L_{y2} )</td>
<td>29</td>
</tr>
<tr>
<td>( w_2 )</td>
<td>0.11 mm</td>
</tr>
<tr>
<td>( w_{i2} )</td>
<td>0.009 mm</td>
</tr>
<tr>
<td>( \epsilon_{W12} )</td>
<td>4.3</td>
</tr>
<tr>
<td>( I_2 )</td>
<td>0.1 mm</td>
</tr>
<tr>
<td>( \epsilon_{I2} )</td>
<td>7.6</td>
</tr>
<tr>
<td>( \mu_{r2} )</td>
<td>1.0</td>
</tr>
<tr>
<td>( \rho_{2,20^\circ C} )</td>
<td>( 1.76 \times 10^{-8} ) Ωm</td>
</tr>
<tr>
<td>( \Delta\rho_2 )</td>
<td>( 0.0039 \times 10^{-8} ) °C(^{-1})</td>
</tr>
<tr>
<td>( T_2 )</td>
<td>20 °C</td>
</tr>
<tr>
<td>( \gamma_2 )</td>
<td>8960 kg/m(^3)</td>
</tr>
<tr>
<td>( Cost_2 )</td>
<td>12 $/kg</td>
</tr>
</tbody>
</table>
F.2 EXPERIMENTAL DATA

F.2.1 O/C Resonance Test

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>$V_{1 pk-pk}$ (V)</th>
<th>$V_{2 pk-pk}$ (kV)</th>
<th>Normalised Gain Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>21.2</td>
<td>10.4</td>
<td>0.99</td>
</tr>
<tr>
<td>200</td>
<td>28.8</td>
<td>14.4</td>
<td>1.01</td>
</tr>
<tr>
<td>500</td>
<td>27.6</td>
<td>14.8</td>
<td>1.08</td>
</tr>
<tr>
<td>1000</td>
<td>24.0</td>
<td>13.0</td>
<td>1.09</td>
</tr>
<tr>
<td>1900</td>
<td>21.6</td>
<td>12.0</td>
<td>1.12</td>
</tr>
<tr>
<td>2000</td>
<td>16.0</td>
<td>11.8</td>
<td>1.48</td>
</tr>
<tr>
<td>2100</td>
<td>14.4</td>
<td>10.4</td>
<td>1.45</td>
</tr>
<tr>
<td>2500</td>
<td>19.6</td>
<td>9.4</td>
<td>0.96</td>
</tr>
<tr>
<td>3000</td>
<td>24.4</td>
<td>7.2</td>
<td>0.59</td>
</tr>
</tbody>
</table>

Table F.1 HVPC2 open circuit resonance results

F.2.2 Loaded Resonance Test

<table>
<thead>
<tr>
<th>Frequency (Hz)</th>
<th>$V_{1 pk-pk}$ (V)</th>
<th>$V_{2 pk-pk}$ (kV)</th>
<th>Normalised Gain Ratio</th>
</tr>
</thead>
<tbody>
<tr>
<td>100</td>
<td>13.4</td>
<td>6.48</td>
<td>0.97</td>
</tr>
<tr>
<td>200</td>
<td>23.8</td>
<td>11.4</td>
<td>0.96</td>
</tr>
<tr>
<td>500</td>
<td>21.6</td>
<td>10.6</td>
<td>0.99</td>
</tr>
<tr>
<td>1000</td>
<td>10.6</td>
<td>9.6</td>
<td>1.82</td>
</tr>
<tr>
<td>1100</td>
<td>8.8</td>
<td>8.8</td>
<td>2.01</td>
</tr>
<tr>
<td>1200</td>
<td>12.2</td>
<td>9.6</td>
<td>1.58</td>
</tr>
<tr>
<td>1300</td>
<td>10.2</td>
<td>6.8</td>
<td>1.34</td>
</tr>
<tr>
<td>1400</td>
<td>11.4</td>
<td>6.2</td>
<td>1.09</td>
</tr>
<tr>
<td>1500</td>
<td>13.4</td>
<td>5.8</td>
<td>0.87</td>
</tr>
<tr>
<td>2000</td>
<td>12.4</td>
<td>2.8</td>
<td>0.45</td>
</tr>
<tr>
<td>3000</td>
<td>15.0</td>
<td>2.0</td>
<td>0.27</td>
</tr>
</tbody>
</table>

Table F.2 HVPC2 loaded resonance results