A PROBABILISTIC FRAMEWORK FOR PERFORMANCE-BASED SEISMIC ASSESSMENT OF STRUCTURES CONSIDERING RESIDUAL DEFORMATIONS

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SUMMARY

Recent advances in performance based design and assessment procedures have highlighted the importance of considering residual deformations in addition to maximum deformations as a complementary damage indicator. A combined 3-dimensional performance matrix, where maximum and residual deformations are combined to identify performance levels coupled with various seismic intensity levels is presented within a probabilistic formulation of a performance based assessment procedures. Combined fragility curves providing the probability of exceedence of performance levels defined by pairs of maximum-residual deformations are derived using bivariate probability distributions, due to the statistical dependence of the two parameters. Numerical examples on equivalent SDOF systems with extensive non-linear time history analyses under a properly selected suite of earthquakes are performed to derive the fragility curves for various performance levels. The effects of hysteretic systems and strength ratios on fragility curves are examined. The significance of accounting for residual deformations in addition to maximum deformation indices when evaluating the actual performance level is confirmed by using joined fragility curves. In conclusion, for a given strength ratio and performance level, joined fragility spectra are generated for a range of effective secant periods of SDOF systems providing a measure of confidence in achieving the targeted performance.

1. INTRODUCTION

Performance Based Earthquake Engineering (PBEE) approaches typically assesses the performance of a structure using one or multiple structural response indices, usually based on maximum responses. Recent developments in performance-based design and assessment concepts [Pampanin et al., 2002 Christopoulos and Pampanin, 2004], have highlighted the limitations and inconsistencies related to these traditional approaches. Reports from past earthquake reconnaissance observations, from shake table tests, as well as from analytical studies, indicate that most structures designed according to current codes will sustain residual deformations in the event of a design level earthquake, even if they perform exactly as expected.

Assessing the residual deformations in the structure in the event of a major earthquake is very important with regard to the difficulty and cost associated with the straightening of structures [Priestley, 1993]. A number of researchers [MacRae and Kawashima, 1997; Borzi et al., 2001; Christopoulos et al., 2003] have identified the post yielding stiffness as the main parameter influencing the residual deformations of non-linear Single Degree of Freedom (SDOF) oscillators. A first attempt to introduce limits on residual deformation/drift in design guidelines or code provisions is found in the 1996 Japanese seismic design specifications for highway bridges, which, as reported by Kawashima [1997], imposes an additional design check for important bridges in terms of residual displacements which are required to be smaller than 1% of the bridge height. In recent draft guidelines for performance evaluation of earthquake resistant reinforced concrete buildings under preparation by the

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Architectural Institute of Japan (AIJ), limits on residual crack widths are tentatively indicated and associated to ranges of maximum drift/ductility and damage level.

A residual deformation damage index (RDDI), which measures the degree of permanent deformations and drifts of SDOF or MODF structures, has been proposed in Pampanin et al., [2002, 2003] and Christopoulos et al. [2003] as an additional damage indicator to fully quantify the performance level of buildings under seismic loading. In these studies, as part of a more refined framework for performance-based design and assessment procedures, the concept of joined performance levels, combining maximum and residual deformation, coupled with seismic intensity in the form of 3-dimensional performance matrix has been suggested.

More recently, a direct displacement-based design approach which includes an explicit consideration of the expected residual deformations has also been proposed by Christopoulos and Pampanin [2004]. Building on the aforementioned framework, extensive numerical analyses have been carried out by Garcia and Miranda [2006] to propose an empirical relationship to evaluate the ratios of residual displacement demand to the peak elastic displacement demand for SDOF systems with known strength ratios. It has been observed that residual displacement ratios exhibit larger levels of record-to-record variability when compared to peak inelastic displacements. With the recent developments of probabilistic approaches for performance based earthquake engineering, considering the uncertainties on the seismic hazard and on the structural capacities, preliminary suggestions to describe the design objectives in the form of fragility curves representing the probabilities of exceedence of different damage states for various seismic intensity levels have been given by the authors [Uma et al, 2006].

In this contribution, as part of the on-going development for a refined framework for performance based seismic design and assessment procedures, a conceptual description of probabilistic formulation of the performance based matrix is briefly given. Non-linear dynamic analyses are carried out on a SDOF system under a properly chosen suite of earthquake records to derive fragility curves where combined maximum-residual performance objectives associated with a targeted probability of exceedence are established. Also, the effects of hysteretic systems and strength ratios on fragility curves are studied and suggestions for design/assessment are proposed. Joined fragility spectra are also derived and suggested to assess the confidence in achieving the desired performance level.

2. PERFORMANCE DESIGN OBJECTIVE MATRIX BASED ON MAXIMUM AND RESIDUAL DEFORMATIONS

The objective of Performance Based Seismic Engineering (PBSE) is to design, construct and maintain facilities with better damage control. A comprehensive document has been prepared by the SEAOC Vision 2000 Committee [1995] that includes interim recommendations. The performance design objectives couple expected or desired performance levels with levels of seismic hazard as illustrated by the Performance Design Objective Matrix shown in Fig.1.

![Figure 1 Seismic Performance Design Objective Matrix [SEAOC Vision 2000, 1995]](image)

Recognising the importance of accounting for residual deformations in assessment of the actual performance of structures, for a given seismic intensity, the aforementioned joined performance levels [Pampanin et al., 2002]
can capture the joined occurrence of maximum and residual responses. As a result, 2-D performance domain (Fig. 2a, X-Y plane) can be used consisting of Performance Levels, PL(i,j), defined by the combination of Maximum Deformation or Drift, MD, (index i) and Residual Deformation or Drift, RD, (index j). By accounting for the effect of seismic intensity, a 3-dimensional performance matrix (Fig. 2b) can be visualised as a set of pre-defined joined performance domains (“masks”) for different seismic intensity level, IM (Z-axis). It should be noted that for a given value of maximum response parameter the performance levels would thus be poorer when combined with higher level of residual responses leading to increased damage and repair costs.

Analogous to the Performance Design Objective Matrix developed by the SEAOC Vision 2000 document, alternative Performance Objectives associated with different structural systems can be defined within the 3-D performance matrix by connecting a set of performance levels/domains PL(i,j) belonging to different intensity levels (Fig. 2b).

![Figure 2 Framework for Performance Based Design and Assessment Approach [Pampanin et al., 2002]](image)

**3. PROBABILISTIC FORMULATION OF PERFORMANCE BASED MATRIX CONCEPT**

In principle, either a deterministic or probabilistic approach could be used within a performance based design or assessment procedure, with preference to the latter approach when a particular level of confidence of achieving performance objective is of interest. More recently, a probabilistic framework for performance-based design and assessment evaluation has been proposed by the Pacific Earthquake Engineering Research Center (PEER) [Cornell, 2000]. The PEER performance-based design framework utilizes the total probability theory to de-aggregate the problem into several interim probabilistic models (namely seismic hazard, demand, capacity and loss models), to account for the randomness and uncertainty in a more rigorous way. The basic and necessary assumption is that all interim models are statistically independent. The mean annual frequency of a decision variable (DV) can be expressed within the framework of performance based design as

\[
\nu(DV) = \int \int \int G(DV|DM) \cdot dG(DM|EDP) \cdot dG(EDP|IM) \cdot d\lambda(IM)
\]

It should be noted that all the interim models are handling only one parameter conditioned on one other parameter. In this study, the development of demand and corresponding capacity models with reference to the performance based matrix concept is discussed. The demand models reported in literature have typically consisted of prediction of the probability of exceeding a given value of one engineering demand parameter (EDP) for a given level of Intensity Measure (IM). When implementing the concept of a joined performance-based matrix, the performance levels are defined using a pair of EDPs, i.e. residual and maximum deformations. Hence, a new Probabilistic Seismic Demand Model (PSDM) relating the effects of the selected IM to two EDPs has to be developed.

The probabilistic assessment of seismic structural performance of a given structure for a given seismic environment is performed using suitable probabilistic seismic demand model (PSDM)s which represent the relationship between EDPs and ground motion IMs [Jankovic and Stojadinovic, 2004]. Considering the parameters of the PSDM as continuous Random Variables (RV), the uncertainty involved in the prediction of the values of EDPs can be accounted for by associating suitable probability distributions to the RVs. Let us consider three RVs, namely X, Y and Z corresponding to residual drift (RD), maximum drift (MD) and seismic intensity measure (IM), respectively. Let their individual (marginal) Probability Density Function (PDF) be \(f_X(x), \)
The function \( f_{X,Y}(x,y) \) is geometrically represented in 3 dimensions by a surface above the (x-y) plane with RD and MD along the x and y axes and whose range is the set of probability values corresponding to the ordered pairs of \((x,y)\) in its domain as shown in Fig 3. At any seismic intensity level, the probability of joint occurrence of the two RVs \((X,Y)\) with values corresponding to a performance level, say PL(2,2) could be obtained from the joint PDF and is represented by volume beneath this surface as illustrated in Fig.3.

\[
\exp \left\{ -0.5 \left[ \frac{(\log x - \lambda_X)^2}{\varsigma_X^2} + \frac{2\rho(\log x - \lambda_X)(\log y - \lambda_Y)}{\varsigma_X \varsigma_Y} + \frac{(\log y - \lambda_Y)^2}{\varsigma_Y^2} \right] \right\}
\]

where \( \lambda_X, \lambda_Y, \varsigma_X \) and \( \varsigma_Y \) are the location and scale parameters of the marginal PDF of \( X \) (residual drift) and \( Y \) (maximum drift), respectively. The parameter \( \rho \) forms a linear correlation coefficient between the two variables.

4. DEFINITION OF PERFORMANCE OBJECTIVES ACCORDING TO THE 3-D PERFORMANCE MATRIX CONCEPT: A PROBABILISTIC APPROACH

4.1 Probabilistic procedure adopted on 3-dimensional performance-based matrix

A PSDM is appropriately selected and seismic response analyses are carried out for the chosen structural system for a suite of earthquake records varying the levels of IM. The EDPs the residual and maximum drifts at every level of IM are analysed for their statistical parameters to describe the joint PDF. These data pairs correspond to a single 2-D performance domain.

The probability of achieving a PL(i,j) specific to certain domain of RD and MD is obtained by performing double integration over the joint PDF as in Eq. 3, with respective values of the variables as upper and lower limits of integration. For example, the probability associated with PL(2,2) is given by

\[
\int_{MD}^{MD} \int_{RD}^{RD} f_{X,Y}(x,y) \ dx\ dy
\]

The probability of exceeding a generic PL(i,j), for example PL(2,2), is given by the volume under shaded portion of the surface area as shown in Figure 4, which may also be expressed as 1 minus the probability of reaching or being within PL(2,2). At this stage, it may be of interest to know the contribution to the probability distribution by alternative pairs of RD and MD. As shown in Fig. 4, the zone A may be interpreted as contribution mostly governed by MD, zone B as that mostly governed by RD and zone C as that governed by both parameters.
Joint PDF enables the computation of total probability of exceedence for PL(i,j) on a performance domain associated with a given level of IM. A cumulative distribution of probability of exceedence at increasing intensity levels gives fragility curves. Performance objectives connecting various performance levels with increasing seismic hazard levels can be established using these fragility curves.

### 5. NUMERICAL EXAMPLES ON SDOF SYSTEMS

The performance evaluation procedure according to a probabilistic approach is herein illustrated with numerical examples on equivalent Single Degree of Freedom (SDOF) systems. The exercise essentially includes selecting ground motion records, performing seismic response analyses and developing fragility curves for performance levels representing damage limit states. A spectrum of fragility curves representing a particular performance level can be established by choosing SDOF oscillators with a range of periods. A parametric study has been conducted with respect to hysteretic models and strength ratios defined as the ratio of the base shear at yielding to the building weight.

#### 5.1 Properties of Single Degree of Freedom Systems

An equivalent SDOF system representing a 4 storey reinforced concrete building designed according to a displacement based design approach with a target maximum inter-storey drift of 2.5% and a peak spectrum acceleration of 0.5g [Priestely, 1998] has been considered. The dynamic properties of the equivalent SDOF systems are: (i) initial elastic period, $T_i = 1.11\text{s}$; (ii) Strength ratio $S_y$ between the base shear at yielding, $F_y = 1040 \text{kN}$ and the building weight, $W = 4000 \text{kN}$ is 0.26; (iii) Effective heights, $H_{eff} = 9 \text{m}$; (iv) Effective weights (first mode), $W_{eff} = 3333 \text{KN}$ [Pampanin et al., 2002]. The SDOF systems are modelled in Ruaumoko [Carr, A.J., 2005] based on a lumped plasticity approach.

#### 5.2 Selection of Ground Motions and Representative Intensity Measure

A total of thirty earthquake ground motions were utilised in this study. They were extracted from two sources: the database used by Pampanin et al. [2002] and the Pacific Earthquake Research database [PEER, 2000]. The records represent magnitude ranging from 6.5 to 7.2, closest distance to fault rupture varying from 15 km to 30 km and soil category C and D (according to NEHRP provisions [1997]). The response spectra with 5% damping for each 30 earthquake records scaled to 0.1g is shown in Figure 5a. A significant degree of record to record variation can be observed with respect to the mean spectral curve. The degree of variation is plotted as lognormal coefficient of variation as shown in Figure 5b. The average coefficient of variation is 0.46 for periods shorter than 2 s. It can be noted that the mean spectrum is in good agreement with the NZS 1170.5 (2004) code design spectrum for PGA of 0.1g with soil category C except for very short periods, less than 0.5 s.
In this numerical study, the spectral acceleration ($S_a$) corresponding to the initial period of the building ($T_1$) is chosen as the intensity measure (IM) to satisfy the statistical independence of the hazard model with respect to the magnitude (M) and the distance (R) in predicting the engineering demand parameters. This has been verified by conducting a multivariate linear regression analysis [Mackie and Stojadinovic, 2003]. It was observed that the regression coefficients corresponding to the magnitude and distance variables were not as significant as the one corresponding to the IM, thus ensuring the sufficiency of the model to relate independently $S_a(T_1)$ with EDPs.

Similar studies on Equivalent SDOF system representing an eight story building with spectral velocity $\nu_s$ has been reported by the authors elsewhere [Uma et al., 2006].

![Figure 5 Response spectra with 5% damping for the earthquake records normalised to 0.1g](image)

**Figure 5 Response spectra with 5% damping for the earthquake records normalised to 0.1g**

5.3 Definition of Limit States for Engineering Demand Parameters

As mentioned, structural and non-structural damage limit states or performance levels have been typically related to maximum transient responses. However, recent publications have emphasized the need to check the permanent (residual) deformations in structures and have suggested tentative residual drift limits based on percentage of the maximum expected drift of the structure [NEHRP, 1997, Kawashima, 1997, FEMA 356, 2004]. In this study, referring to the previous research work [Pampanin et al., 2002] and the draft guidelines of AIJ [2004], tentative values for the limit states based on residual drift, RD, are taken as 0.2%, 0.4% and 0.6% and 1.0% while, more traditional values for the limit states based on maximum drift, MD, are considered as 0.5%, 1.0%, 2.0% and 4.0%. The limit states can be typically referred to as “Serviceability”, “Repairable Damage”, “Irreparable Damage” and “Collapse prevention”.

5.4 Seismic Response Analysis

A series of inelastic time history analyses using the selected suite of earthquake records was performed on SDOF systems based on a lumped plasticity approach for two hysteretic models namely elasto-plastic (EP) and Takeda (TK) as shown in Fig. 6. The performance of the building in terms of maximum and residual drift ratio (with respect to the effective height) is studied at various seismic intensity levels by scaling up the IM ($S_a(T_1)$) from 0.2g to 2g.

![Figure 6 Hysteretic Models used in the Analyses](image)

**Figure 6 Hysteretic Models used in the Analyses**

The analyses are performed for 30 records for a chosen level of intensity measure and repeated for 10 levels of intensity measure. The distribution of RD and MD at spectral acceleration of 0.8g for EP and TK systems are shown in Fig 7a,b and the joint PDF for EP system is shown in Fig 7c. The lower values of residual with less scatter is shown by TK systems compared to EP systems.
5.5 Development of fragility curves

The residual and maximum drift ratios of SDOF systems were evaluated for all 30 records at each level of intensity measure, IM, and the corresponding statistical parameters were derived. The distribution of RD and MD at each intensity level is described as a bivariate lognormal joint PDF using their respective lognormal mean, lognormal standard deviation and their correlation coefficient. The total probability of reaching or exceeding a desired performance level for a given intensity can be computed using the corresponding damage limit states as integration limits, as described in section 5.1. A smooth fragility can be fitted to the computed probability of exceedence values assuming a lognormal distribution.

5.5.1 Significance of including residual deformations as a complementary damage indicator

As mentioned, residual deformation has been proposed by Pampanin et al., [2002] as a complementary damage indicator parameter in addition to maximum deformation indices for assessing the actual performance level of a structure. In other words, the PL defined by a given maximum drift ratio limit with lower residuals might represent a lower damage state when compared to the PLs corresponding to same maximum drift limit but larger residual drift limits. The significance has been illustrated in Figure 8, where fragility curves obtained for PLs corresponding to maximum drift limits (for i = 3 and 4) combined with residual drift limits (for j = 1, 2, 3) are presented. It can be seen that for a chosen intensity level, i.e. $S_a = 0.6g$, the probability of exceedence ‘(d)’ of PL(4,1) is higher than that corresponding to ‘(b)’ of PL(3,2) and ‘(c)’ of PL(3,3). It should in fact be noted that for a given intensity level, the fragility curve with higher probability of exceedence indicates a lower level of damage whereas the curve with lower probability of exceedence represents a higher level of damage. Thus, it is evident that, although being subjected to similar maximum drift demands, the systems should be assigned substantially different levels of performance, depending on the value of the residual response indices.
5.5.2. Effect of hysteretic systems on joined fragility curve

The nonlinear systems were assumed to exhibit two hysteretic behaviours: the EP and TK models. Both these systems were assigned a zero post yield stiffness ratio. The hysteresis coefficients describing the unloading and reloading stiffness behaviour of a TK model (typically known as $\alpha$ and $\beta$) were taken as 0.3 and 0.2 respectively. Figure 9 shows the probability of exceedence of PL(4,2) for the two systems. It can be noted that EP systems show higher probability of exceedence than TK systems regardless of the intensity level. Fig 9 also illustrates the contributions to the total probability of exceedence, referred to A, B and C as per Fig. 4, showing some difference in the behaviour of the two systems. The contributions of residual (B zone) and combined max. and res. (C zone) are in fact predominant in both EP and TK system, whereas some contribution from the maximum drift (A zone) can be noted in the TK system for higher level of IM. From this, the influence of the response parameters to the probability of exceedence of a PL can be recognised.

![Figure 9 Contribution “zones” to the total probability of exceedence of PL(4,2) for EP and TK systems](image)

5.5.3. Effect of strength ratios on fragility curve

The effects of a variation of the system strength ratio, defined as yield strength normalised by the effective weight of the SDOF system, on the total probability of exceedence of performance levels has been investigated. Four strength ratios were considered for the SDOF systems keeping the dynamic and structural properties (i.e mass, damping ratio and fundamental period) the same. Figure 10 (a and b) show the fragility curves for PL (4,2) for the two hysteretic systems considered. In general, higher probabilities of exceedence are observed for lower strength ratios up to a critical intensity level, after which, the probability of exceedence increases as the strength ratio increases. This critical point is observed at a much lower intensity level in the case of TK systems than in the case of EP systems.

![Figure 10 Effect of strength ratio on fragility for PL (4,2)](image)

A meaningful interpretation can be made from these fragility curves with regard to the required strength ratios for the two hysteretic systems to achieve the same targeted probability of exceedence at a given design intensity level. For example, at a chosen level of intensity, say 0.8g, and a targeted probability of exceedence of 40% for the PL(4,2), the EP system should be designed for a strength ratio of 0.5 whereas the TK system requires only a strength ratio of 0.26. Figure 10 c shows the percentage of variation of the difference in probability of
exceedence of two systems with respect to EP system for varying intensity levels. This quantity decreases as the intensity level increases for a certain strength ratio. It can be observed that for all the strength ratios considered, at intensity levels, e.g. 0.4g-0.8g, the difference in probability of exceedence between EP and TK systems is approximately 50%. At an intensity level of 1.2g, it is approximately 25%. At higher intensity levels, the difference is less than 15%. Hence, if PL(4,2) is the targeted PL, the advantage of designing a TK system with a lower strength ratio than an equivalent EP system is more significant at lower intensity levels.

5.5.4. Fragility spectrum for performance levels based on combination of maximum and residual

Christopoulos et al., [2004] suggested a modified direct displacement based design method which includes an explicit consideration of residual deformations in the early stages of the design procedure. Inelastic (ductility constant) design spectra based on residual/maximum drift ratios as a function of effective secant period were also derived. In a probabilistic formulation, a joined ‘fragility spectrum’ generated for a range of effective periods for different performance levels, defined based on a combination of maximum and residual, would enable the designer to target a performance with certain confidence corresponding to the effective period of the system. Figure 11 shows the fragility spectrum for PL(3,3) for a constant ductility level of 4.

5.5.5 Definition of performance objectives within probabilistic approach

In a design or assessment phase, the performance objectives can be obtained by connecting the performance levels with targeted probabilities of achieving them. Fig. 11 shows the visualization of typical performance objectives when adopting fragility curves. A major increase in the confidence of the design could be for example achieved by targeting a defined level of probability of exceedence of different performance levels for increase levels of intensity. Within a complete probabilistic formulation (including the seismic hazard model), a “uniform risk” design approach could be suggested to be followed, consisting of targeting the same probability of exceeding different PLs belonging to a predefined performance objective, as shown in a solid line, UR, in Fig. 12. Alternatively, a variable level of acceptable probability of exceedence (i.e. possibly referred to as “multiple risk” approach in a general formulation) associated to different intensity and performance levels can be adopted in the design phase (as indicated with a dashed line, MR). For example, the MR curve connects PL(2,2), PL(3,3) and PL(4,4) with respective probabilities of achieving them of 40%, 30% and 30%.

It would be thus possible to define and target performance objectives with the associated probability of occurrence by connecting various performance levels with increasing seismic intensity within the 3-D performance matrix framework.

Figure 11 Jointed MD-RD fragility spectrum (PL(4,2))  Figure 12 Definition of performance objectives using fragility curves

6. CONCLUSIONS

A probabilistic formulation of a performance matrix combining maximum and residual deformation to define performance levels and performance objectives at increasing level of seismic intensity has been presented. The joint occurrence of residual and maximum deformations within a chosen performance domain is described by a bivariate lognormal probability density functions. Fragility curves representing the probabilities of achieving or exceeding different maximum-residual performance levels are derived. Numerical examples on SDOF systems confirmed that the contributions of the maximum or residual response parameters to the total probability of exceedence of a performance level, PL(i,j) are largely influenced by the hysteresis models. Given the intensity level, the EP systems display a higher probability of exceedence of a PL governed by RD whereas the TK
systems show lower probability of exceedence governed by MD. The amount contributed by each response parameters would be an important factor to suggest a mitigation strategy in a design phase as well as a suitable retrofitting intervention to achieve higher performance. The effect of strength ratios on the two hysteretic models with different targeted performance levels is currently being further examined to fully assess the advantages of designing a TK system with a lower strength ratio than the corresponding EP systems at different intensity levels.

In conclusion, preliminary suggestions for a “uniform risk design” (or controlled “multiple risk”) approach, whereby targeted probability of exceeding predefined performance levels, at increasing intensity levels, are considered in the design phase, have been given. The concept of a fragility spectrum for various performance levels is tentatively introduced as useful tool within a probabilistic approach of performance-based seismic design/assessment of structures considering residual deformations.

7. REFERENCES


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