Nonlinear Phenomena in AFM Arrays

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Summary. Cantilever arrays offer a promising alternative to standard single cantilever atomic force microscopy (AFM) imaging techniques. The successful implementation and operation of such an AFM array, however, is determined by the collective, nonlinear dynamics of the array. There exists two key parameters that influence the AFM array response, namely the mechanical coupling strength and nonlinear interaction forces. We present a mathematical model of a coupled array of cantilevers subjected to nonlinear force interactions and demonstrate how changes in modal response can be used to enhance the sensitivity of AFM technology.

Extended Abstract

In comparison to current AFM technology based on single beam dynamics, an array has a larger number of degrees of freedom and offers a greater level of flexibility with regards to parameter selection and tuning. Tuning the coupling parameter in particular can generate different system eigenmodes, ranging from completely unsynchronised cantilevers oscillating at individual frequencies for a small coupling parameter to perfectly synchronised array modes governed by phase shifts between beams for a large coupling parameter [1]. Introduction of a nonlinear external force to each cantilever tip further alters the observed system response, and can be shown to result in bifurcation events with the creation/destruction of resonant modes. Changes in array eigenmodes present new opportunities with regards to AFM and related applications, allowing for the measurement of multiple feedback parameters from the system simultaneously as well as to enhance sensitivity to changes in sample topography and material properties.

In this paper, we use a recently developed mathematical model of a coupled cantilever array subjected to nonlinear tip forces to demonstrate changes in eigenmode response and to show how the array response is influenced by system parameters. The model is developed from a continuum mechanics approach, utilising Hamilton's method to solve the system energy balance and Galerkin's method to spatially discretise the mode shapes. Each cantilever in the array is modelled using Euler-Bernoulli beam theory with a shared base structure providing mechanical coupling and an external forcing term. A nonlinear force term is applied at the tip of each cantilever, which represents the force interactions between the cantilever tip and a sample surface in the context of AFM. The full derivation of the model can be found in [2], and the final form of the equation of motion for an arbitrary sized array is as follows.

$$\ddot{\mathbf{\Phi}} + \mathbf{W_c}\dot{\mathbf{\Phi}} + \mathbf{W_k}\mathbf{\Phi} - \mathbf{W_F}\mathbf{F} - \mathbf{W_{NL}}\mathbf{NL} = \mathbf{0}, \tag{1}$$

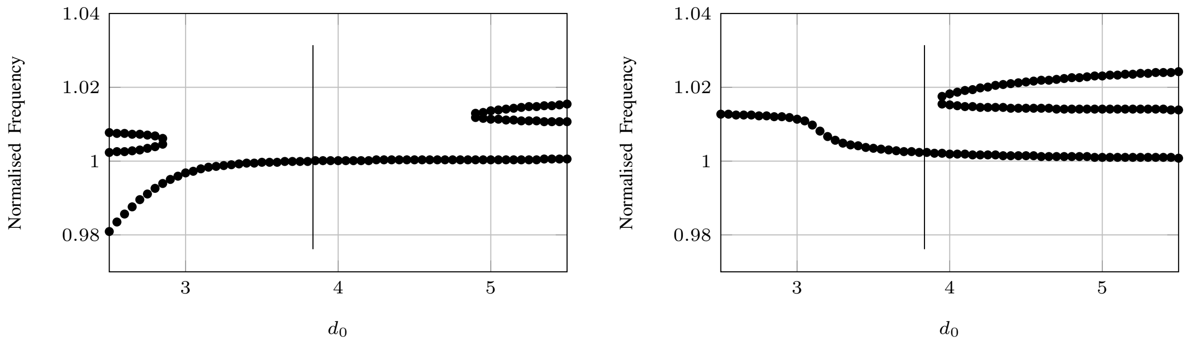
where $\mathbf{\Phi}$ is a vector of the time dependent mode functions of the array, $\mathbf{W_c}$ and $\mathbf{W_k}$ are diagonal matrices containing the damping and stiffness coefficients respectively for each mode and $\mathbf{W_F}\mathbf{F}$ and $\mathbf{W_{NL}}\mathbf{NL}$ contain the non-zero terms that represent the actuation and tip force coupling between modes. The analysis conducted using the mathematical model is done primarily for the case of a two beam array, for which the equation of motion can be written as follows with two array modes, Φ_1 and Φ_2.

$$\begin{aligned} &\begin{bmatrix}\ddot{\Phi}_1\\ \ddot{\Phi}_2\end{bmatrix} + \begin{bmatrix}W_{c11} & 0\\ 0 & W_{c22}\end{bmatrix}\begin{bmatrix}\dot{\Phi}_1\\ \dot{\Phi}_2\end{bmatrix} + \begin{bmatrix}W_{k11} & 0\\ 0 & W_{k22}\end{bmatrix}\begin{bmatrix}\Phi_1\\ \Phi_2\end{bmatrix} \\ &- \begin{bmatrix}W_{F11} & W_{F12}\\ W_{F21} & W_{F22}\end{bmatrix}\begin{bmatrix}AC_1\cos(\Omega\tau)\\ AC_2\cos(\Omega\tau)\end{bmatrix} - \begin{bmatrix}\dfrac{\tau_m}{(\hat{d}_{01} - \overline{W}_{11}\Phi_1 - \overline{W}_{12}\Phi_2)^2}\\ \dfrac{\tau_m}{(\hat{d}_{02} - \overline{W}_{21}\Phi_1 - \overline{W}_{22}\Phi_2)^2}\end{bmatrix}, \end{aligned} \tag{2}$$

where W_{c11} and W_{c22} are damping coefficients, W_{k11} and W_{k22} are stiffness coefficients, W_{F11} to W_{F22} are excitation coefficients, AC_1 and AC_2 are the excitation input amplitudes to beams 1 and 2, respectively, Ω is the forcing frequency, τ is nondimensionalised time, τ_m is the nonlinear force coefficient, $\hat{d}_{01}$ and $\hat{d}_{02}$ are the distances between the cantilever tips and sample surface and $\overline{W}_{11}$ to $\overline{W}_{22}$ are the spatial mode shapes of the array at the cantilever tips. Solving (2) numerically produces the time response of the array modes, which can be recombined with the spatial mode shapes using (3) to find the response of each cantilever, where m is the cantilever number and n is the mode number.

$$\hat{w}_m(\hat{x}, \tau) = \sum_{n=1}^{N} W_{mn}(\hat{x})\Phi_n(\tau) \tag{3}$$

The mathematical model is used to show how variations in coupling strength and tip-sample separation alter the modal response of the array system. Specifically, we show how the resonant modes of the system (where a 90° phase shift exists between the response and input signal) change. In certain parameter space bifurcation events are observed to occur, where array modes converge and annihilate. In addition, steep changes in the frequency gradient occur for certain ratios of cantilever frequency. Both of these phenomena can be seen in Figure 1, which depicts resonant frequencies as a function of tip-sample displacement (d_0). To simulate the depicted responses, the Computational Continuation Core (COCO) software framework [3] was used in conjunction with MATLAB® to perform continuation simulations using the mathematical model. Simulations were performed with a two beam array and applying external excitation and nonlinear tip forces to one beam only, leaving the other beam passive.



(a) Observed resonant frequencies (90° phase shift) in a weakly coupled two beam array. The vertical line represents the location of the cross-over point.

(b) Observed resonant frequencies (90° phase shift) in a strongly coupled two beam array. The vertical line represents the location of the cross-over point.

Figure 1: Resonant frequency as a function of tip-sample separation.

The observed bifurcation events in Figure 1 are strongly linked to the amount of coupling between cantilevers as expected. Of particular interest is the location of the bifurcation points, which occur in the vicinity of the cross-over point of the cantilevers. The cross-over point is the point at which the uncoupled resonant frequencies of the two cantilevers approach and cross each other due to the influence of nonlinear tip forces [2]. It is near this region that discontinuities can occur in cantilever response during standard AFM operation, which is undesirable and a smooth relationship between response feedback and tip-sample separation is necessary for reliable imaging. It is advantageous to be able to predict when these discontinuities will occur for a given parameter space, so as they may be avoided.

An additional feature observable in Figure 1b near the cross-over point is a sharp increase in the gradient of the frequency curve near a d_0 value of 3. Such high feedback gradients could be utilised to increase the achievable measurement sensitivity over standard single beam methods. The mathematical model can be shown to predict the gradient and optimise the system parameters to achieve maximum sensitivity whilst ensuring continued stability.

References

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