The Real Exchange Rate in Taylor Rules: A Re-Assessment

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Richard T Froyen          Alfred V Guender
Department of Economics    Department of Economics and Finance
University of North Carolina University of Canterbury
Chapel Hill, NC 27599-3305  Christchurch, New Zealand
USA

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Abstract:

Examining three flexible inflation targeting strategies, we find that a small concern for real exchange rate stability as a policy goal matters. First, it warrants the inclusion of the real exchange rate in Taylor rules and, second, it is sufficient to improve the performance of Taylor rules relative to optimal policy. Gains are substantial for domestic and REX inflation targets because a small weight on real exchange rate fluctuations makes optimal policy less aggressive. The gains under CPI inflation targeting are considerably lower.

E-mail: froyen@email.unc.edu (corresponding author); Alfred.Guender@canterbury.ac.nz

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Simple interest rate rules, often termed Taylor rules, have been used frequently to describe the implementation of monetary policy in closed economies. Before the onset of the Global Financial Crisis, central banks were typically viewed as responding to inflationary pressure and an overheating economy. The vagaries of financial markets since the crisis have, however, heightened the concern for financial stability. Greater awareness of the dangers of distorted asset prices and financial imbalances has produced more recent specifications of Taylor-type rules that include asset prices, interest rate spreads or credit aggregates.\(^1\)

A discussion of Taylor-type rules for small open economies in the current environment raises the question of whether monetary policy should respond to the (real or nominal) exchange rate. In the past, the case for including an exchange rate argument in simple interest rate rules for monetary policy has been weak. With the exception of economies characterized by financial fragility or dominated by the foreign sector, most of the literature supported the view of Taylor (2001, p.266) and Taylor and Williams (2011) that either there are small performance improvements from reacting to the exchange rate or that such reactions can make performance worse.\(^2\) Two sets of developments suggest the need to revisit this issue.

The first is the emergence of a new generation of open-economy macroeconomic models in which the real exchange rate plays a more fundamental role. The earlier generation of models, for example, the models in Gali and Monacelli (2005) and Clarida, Gali and Gertler (2001, 2002) had the implication that optimal monetary policy in the open economy was isomorphic to policy in the closed economy. This suggested that the Taylor rule for an open economy might not need to be extended beyond domestic inflation and the output gap.

The isomorphism is a distinctive feature of models where the Phillips curve has no direct real exchange rate channel. The real exchange rate affects domestic inflation only indirectly.

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1 In fact, an extensive debate about the response of monetary policy to developments in asset markets began at the turn of the millennium. Cecchetti et al. (2000, 2002) were in favor of rules that respond to financial asset prices while Bernanke and Gertler (1999, 2001) were opposed. For a recent overview of the debate see Kaefer (2014). More generally on the issue of reconciling financial stability with other macroeconomic goals, see Leeper and Nason (2015), Vredin (2015), Smets (2014), and Woodford (2012).

2 Empirical evidence cited by Taylor and Williams (2011) came from 1990s open economy econometric models. Their view receives support from studies within New Keynesian models. Examples are Batini et al. (2003 and Leitemo and Söderström (2005). Garcia et al. (2011) find that inclusion of the level of the real exchange rate in a Taylor-type rule increases the variability of inflation and the output gap. They do, however, find that smoothing the real exchange rate helps reduce financial volatility without adding to inflation or output variability.
through its effect on the output gap. A later generation of models extends the role of the real exchange rate. Imported inputs (Monacelli (2013)), incomplete exchange rate pass-through (Monacelli (2005)), and concern about foreign competitiveness as a factor in firm pricing (Froyen and Guender (2017)) all suggest direct exchange rate effects in the Phillips curve. In models with a real exchange rate channel in the Phillips curve the importance of the real exchange rate in the optimal target rule for monetary policy depends on the underlying inflation objective.

The recent literature also suggests that openness raises questions about the central bank’s inflation objective. Allsopp, Kara and Nelson (2006) favor CPI inflation targeting. Kirsanova, Leith, and Wren-Lewis (2006) and others support the earlier studies cited above in advocating a domestic rather than a CPI inflation objective. Froyen and Guender (2017), following Ball (1999), consider a real-exchange-rate-adjusted (REX) inflation target similar to core inflation measures employed by some central banks.

Recent papers such as Corsetti et al. (2011) and Monacelli (2013) probe the question of whether monetary policy in an open economy is fundamentally different from that of a closed economy; for a variety of reasons in the newer generation of open economy models the answer is yes. This suggests the need to re-examine open-economy Taylor rules.

The second set of developments concern changes in global financial markets following the financial crisis of 2007-2009. The policies of the Federal Reserve and other major central banks, consisting of large asset purchases and near zero policy interest rates, led to massive flows of capital to smaller open economies. Countries as diverse as Turkey, Brazil, South Africa, Malaysia, Mexico and Indonesia saw a substantial portion of their local currency government bonds purchased by foreign investors. In many of these countries the result was a sharp appreciation of their currency. These open economies are now left vulnerable to sudden stops and falling currency values with capital flowing back to the United States as the Federal Reserve pursues “lift-off”, returning U.S. interest rates to more normal levels. Raghuram Rajan, the former Governor of the Indian central bank, complained about the increased market volatility

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3 Corsetti et al. argue that monetary policy should stabilize the real exchange rate if import prices are sticky in the local currency. Monacelli advocates stabilizing the ratio of real marginal cost to the terms of trade friction.
caused by the sharp swings in capital flows. In 2016-17 concerns about possible devaluation of the Chinese yuan, Brexit, and then the prospect for a sharp rise in the U.S. dollar as a result of prospective Trump administration tax and trade policies led to further instability in global financial markets.

Because of the increased turbulence in world financial markets, central banks in small open economies may find it necessary to add exchange rate stability to the list of policy goals. Support for this comes from inside international supervisory agencies. Olivier Blanchard et al. (2010, p. 210) argue that “[c]entral banks in small open economies should explicitly recognize that exchange rate stability is part of their objective function.” One would think that expanding policy goals beyond the dual mandate that underlies the standard Taylor rule suggests expanding the rule itself to include a real exchange rate objective.

This paper evaluates Taylor-type rules in an open economy framework where the central bank views exchange rate stability as an added but secondary objective. We consider three potential central bank inflation objectives: domestic, CPI, and real-exchange-rate adjusted (REX) inflation. For each inflation measure we design a Taylor-type rule and compare its stabilizing properties to a benchmark where the central bank follows an optimal policy under Woodford’s (1999) timeless perspective defined for each of these inflation objectives.

Our most important findings can be briefly summarized here. Even a small weight on real exchange rate stability is sufficient to affect materially the performance of Taylor-type rules relative to the benchmark optimal policy. Gains are substantial particularly for domestic and REX inflation targets because even a small weight on real exchange rate fluctuations in the loss function inhibits the aggressive use of the policy instrument under optimal policy. As real exchange rate stability is a built-in feature of a CPI inflation objective, the gains under a CPI inflation target are considerably lower. A central bank that values real exchange rate stability to a degree and follows a Taylor-type rule should respond to the real exchange rate. Doing so reduces

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4 Concerns about the potential disruptive effect of a change in US monetary policy in the foreign exchange and international capital markets have been expressed by the World Bank (2015), the IMF (Lagarde (2015)), and individual BRICS countries. Not only emerging or developing countries are worried about exchange rate instability. Concerns about an unjustifiably high exchange rate and the consequences of a free fall of the exchange rate should the Federal Reserve tighten its policy stance were raised by the Reserve Bank of New Zealand in 2014. During the Global Financial Crisis in 2009 the Swiss National Bank took steps to stabilize the Swiss Franc vis-à-vis the Euro in a deflationary environment. It did so again in the wake of the Sovereign Debt Crisis in 2011.
relative losses further irrespective of the specification of the inflation objective. Relative losses decrease still further if the central bank can optimize over the coefficients in the Taylor rule. Indeed, relative losses hover around the 10 percent mark for all three inflation objectives. Only a complete disregard for exchange rate stability as an ultimate policy goal bears out the conventional view that there is no substantive role for the real exchange rate in Taylor-type rules.

The remainder of the paper is structured as follows. The next section introduces a New Keynesian model of a small open economy that serves as our frame of reference. This section also offers a brief analysis of flexible domestic, CPI, and REX inflation targeting under optimal policy from a timeless perspective. Section 3 compares the performance of various Taylor-type rules vis-à-vis optimal policy in two different scenarios, one where the central bank values exchange rate stability and the other where it does not. Section 4 discusses optimized Taylor rules and how they compare to optimal policy. A conclusion is offered in Section 5.

2. INFLATION OBJECTIVES IN A SMALL OPEN ECONOMY

The focus of this paper is a comparison of the performance of simple Taylor rules with optimal policy within a given flexible inflation targeting framework. The flexible inflation targeting frameworks considered include the output gap and inflation objectives actually in use in small open economies. These are expanded to include an exchange rate stability objective. This section lays out a stylized model of a small open economy which serves as a point of reference for our evaluation of Taylor rules in Section 3. This stylized model is based on uncovered interest rate parity holding in the asset market and perfect exchange rate pass-through.6

A. A Small Open Economy Model

The model consists of four equations which are briefly described below.7

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + b(q_t - q_{t-1}) - \beta b(E_t q_{t+1} - q_t) + u_t
\] (1)

\[
y_t = E_t y_{t+1} - a_1(R_t - E_t \pi_{t+1}^{CPI}) + a_2(q_t - E_t q_{t+1}) + a_3(y_t' - E_t y_{t+1}'') + v_t
\] (2)

6 Perfect exchange rate pass-through and uncovered interest rate parity can be thought of as a representative baseline case. An analysis of the sensitivity of the reported results to both conventions is left for future work.

7 The derivation of the Phillips curve is explained in a separate appendix. The rest of the model is a standard New Keynesian model based on optimizing behavior of households.
\[ R_t - E_t \pi_{t+1} = R^f_t - E_t \pi^f_{t+1} + E_t q_{t+1} - q_t + \varepsilon_t \]

\[ \pi^{CPI}_t = \pi_t + \gamma \Delta q_t \]

where

\( \pi_t \) = the rate of domestic inflation

\( E_t \pi^{CPI}_{t+1} \) = the expected rate of CPI inflation

\( q_t \) = the real exchange rate (an increase in \( q_t \) implies a depreciation)

\( y_t \) = the output gap

\( R_t \) = the nominal rate of interest (policy instrument)

\( R^f_t \) = the foreign nominal rate of interest

\( E_t \pi^f_{t+1} \) = the expected foreign rate of inflation

\( y^f_t \) = the foreign output gap

Lower case variables represent logarithms. All parameters are positive. The discount factor \( \beta \) is less than or equal to one.

Equation (1) is an open-economy Phillips curve that features a real exchange rate channel in addition to the standard output gap channel. Equation (2) is an open-economy IS relation with a real interest rate and a real exchange rate channel. A foreign output shock and an idiosyncratic domestic shock also affect the demand for domestic output. Equation (3) is the linearized uncovered interest rate parity (UIP) condition: apart from a stochastic risk premium \( (\varepsilon_t) \) agents are assumed to trade in a frictionless international bond market. More formally, the stochastic disturbances are modeled as follows: \(^8\)

\[ u_t \sim N(0, \sigma_u^2) \quad v_t \sim N(0, \sigma_v^2) \quad \varepsilon_t \sim N(0, \sigma_\varepsilon^2) \]

\[ R^f_t \sim N(0, \sigma_{R^f}^2) \quad \pi^f_t \sim N(0, \sigma_{\pi^f}^2) \quad y^f_t \sim N(0, \sigma_{y^f}^2) \]

All foreign variables are exogenous independent random variables. Finally, equation (4) describes the relationship between the CPI inflation rate, the domestic inflation rate, the real exchange rate, and consumption openness (\( \gamma \)) under perfect exchange rate pass-through.

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\(^8\) The property that all shocks are white noise follows Woodford (1999). Its purpose is to show that gradual adjustment of the output gap, the rate of inflation, etc. and the policy instrument under policy from a timeless perspective is not exclusively tied to the presence of autocorrelated disturbances in the model.
B. The Choice of a Flexible Inflation Target by an Optimizing Central Bank

In the following subsections we consider three flexible inflation targeting regimes. Each regime is associated with a particular definition of inflation: domestic inflation, exchange-rate adjusted inflation or CPI inflation.

A flexible inflation targeting regime is one where the central bank is concerned about variables other than just the rate of inflation. An optimizing central bank practicing flexible inflation targeting minimizes its objective function subject to the constraint imposed by the model economy. Woodford’s (1999) policy from a timeless perspective is the form of commitment the central bank adheres to.

In the academic literature a central bank engaging in flexible inflation targeting is typically modelled as having a dual mandate. The objective function consists of the squared deviations of the output gap and the particular rate of inflation the central bank targets. As mentioned previously, however, recent experience and ongoing developments cast doubt on whether this convention captures all objectives of a central bank in a small open economy. To articulate this point, we extend the central bank mandate to include stability of the real exchange rate.

1. Targeting Domestic Inflation

In the first strategy we consider, the rate of inflation is defined in terms of changes in the level of domestic prices. The explicit objective function that the central bank attempts to minimize is given by:

\[ E_t \sum_{i=0}^{\infty} \beta^i [y_{t+i}^2 + \mu \pi_{t+i}^2 + \delta q_{t+i}^2] \]  

(5)


With either specification, our approach is to assume that the central bank decides on (or is assigned) a set of objectives. The alternative is a utility-based endogenous loss function. The latter approach has much to recommend it in optimal policy analysis though each approach has pitfalls (cf. Blanchard (2016), Sims (2012), Walsh (2005) and with specific reference to Taylor rules Woodford (2001)). The approach here, where a Taylor rule is compared with optimal policy within a given flexible inflation targeting framework, seems better suited to the evaluation of simple rules, the underlying rationale for which lies with traditional central bank objectives. It is also the approach taken in almost all of the previous literature evaluating the role of the exchange rate in Taylor rules.
\( \beta \) is the discount factor and \( \mu \) represents the relative weight the policymaker attaches to the squared deviations of the rate of domestic inflation from target. In a similar vein, \( \delta \) is the relative weight accorded to the squared realizations of the real exchange rate.\(^{11}\)

To reduce the dimension of the central bank’s domestic inflation targeting strategy to one involving three choice variables, we need to take two additional steps. First, substitute for the rate of CPI inflation in equation (2). Second, substitute the UIP condition into the IS equation. The optimization problem can then be expressed as:

\[
\min_{\pi_t, y_t, q_t} E_t \sum_{i=0}^{\infty} \beta^i [y^2_{t+i} + \mu \pi^2_{t+i} + \delta q^2_{t+i}]
\]

s. t.

\[
\pi_t = \beta E_t \pi_{t+1} + \kappa y_t + b(q_t - q_{t-1}) - \beta b(E_t q_{t+1} - q_t) + u_t \quad (6)
\]

and

\[
y_t = E_t y_{t+1} - a_1(R^f_t - E_t \pi^f_{t+1} + \epsilon_t) + (a_1(y - 1) - a_2) (E_t q_{t+1} - q_t) + v_t
\]

Combining the first-order conditions yields the endogenous target rule. Under policy from a timeless perspective, the target rule is complex. The model is therefore solved numerically.

2. Targeting CPI Inflation

If the focus of policy rests on CPI inflation, then the policymaker minimizes

\[
E_t [\sum_{i=0}^{\infty} \beta^i [y^2_{t+i} + \mu \pi_{t+i}^2 + \delta q^2_{t+i}]]
\]

subject to the constraint which is represented by the model economy. After rewriting the structure of the economy in terms of the CPI inflation rate, we can restate the policy objective as:

\[
\begin{align*}
\min_{y_t, \pi_t^{\text{CPI}}, q_t} & \ E_t [\sum_{i=0}^{\infty} \beta^i [y^2_{t+i} + \mu \pi_{t+i}^{\text{CPI}}^2 + \delta q^2_{t+i}]] \\
\text{s. t.} & \end{align*}
\]

\(^{11}\) The target for the output gap, the rate of inflation, and the real exchange rate is zero, respectively.
\[ \pi^\text{CPI}_t = \beta E_t \pi^\text{CPI}_{t+1} + \kappa y_t + (1 + \beta)(\gamma + b)q_t - (\gamma + b)q_{t-1} - \beta(\gamma + b)E_t q_{t+1} + u_t \]

and

\[ y_t = E_t y_{t+1} - a_t(R^\ell_t - E_t \pi^\ell_{t+1} + \varepsilon_t) - (a_t(1 - \gamma) + a_2)(E_t q_{t+1} - q_t) + a_3(y^\ell_t - E_t y^\ell_{t+1}) + \nu_t \]

As in the case of domestic inflation the target rule for CPI inflation under policy from a timeless perspective proves complex and is therefore not reported. The variances of the endogenous variables are again determined by numerical solution.

3. Targeting “R(eal)-EX(change)-Rate-Adjusted” Inflation

This section introduces an alternative inflation target. This alternative target is domestic inflation stripped of the effects of changes in the real exchange rate. In small open economies central banks often target core or underlying inflation rather than headline inflation for the simple reason that headline inflation may be distorted due to severe temporary exchange rate movements. A REX inflation target transforms an open-economy Phillips curve into a closed-economy version.\(^{12}\) The exchange rate channel is shut down and the monetary policy transmission mechanism works solely through the output gap.

Both the current and expected change in the real exchange rate appear on the right-hand side of the Phillips curve (equation (1)), which can be rewritten as

\[ \pi_t - b(q_t - q_{t-1}) = \beta(E_t \pi_{t+1} - b(E_t q_{t+1} - q_t)) + \kappa y_t + u_t \]

Defining

\[ \pi^\text{REX}_t = \pi_t - b(q_t - q_{t-1}) \]

as the domestic rate of inflation purged of the real exchange rate effect allows us to rewrite the original open-economy Phillips curve as

\[ \pi^\text{REX}_t = \beta E_t \pi^\text{REX}_{t+1} + \kappa y_t + u_t \]

Written in this form, equation (10) looks like the original Phillips curve. The only difference between equation (1) and equation (10) pertains to the definition of the rate inflation.

\(^{12}\) Ball (1999) calls his version of real-exchange-rate-adjusted inflation in a backward-looking model “long-run inflation.” It is defined as domestic inflation purged of the effect of the lag (not the change) of the real exchange rate.
The remaining two equations of the model can be rewritten in terms of the real-exchange-rate-adjusted rate of inflation:

\[
\begin{align*}
\gamma_t &= E_t\gamma_{t+1} - a_1(R_t - E_t\pi^\text{REX}_{t+1}) + (a_1(b + \gamma) - a_2)(E_tq_{t+1} - q_t) + a_3(y_t - E_t\gamma_{t+1}) + \nu_t \\
R_t - E_t\pi^\text{REX}_{t+1} &= R^f_t - E_t\pi^f_{t+1} + (1 + b)(E_tq_{t+1} - q_t) + \varepsilon_t
\end{align*}
\]

(11) (12)

After substitution of equation (12) into equation (11) to eliminate the nominal interest rate, the optimization problem of the policymaker can be stated as:

\[
\begin{align*}
\min_{\pi^\text{REX}_t, \gamma_t, q_t} & E_t \sum_{i=0}^{\infty} \beta^i [y_{t+i}^2 + \mu \pi^\text{REX}_{t+i}^2 + \delta q_{t+i}^2] \\
\text{s. t.} & \pi^\text{REX}_t = \beta E_t \pi^\text{REX}_{t+1} + \kappa \gamma_t + u_t \\
& y_t = E_t\gamma_{t+1} - a_1(R^f_t - E_t\pi^f_{t+1} + \varepsilon_t) - (a_1(1 - \gamma) + a_2)(E_tq_{t+1} - q_t) + a_3(y_t - E_t\gamma_{t+1}) + \nu_t
\end{align*}
\]

(13)

Solving the optimization problem yields the target rule under REX inflation targeting:

\[
\frac{\delta \Delta q_t}{a_1(1 - \gamma) + a_2} + \Delta y_t + \mu \kappa \pi^\text{REX}_t = 0
\]

(14)

It is evident that the systematic relationship between the target variables depends on demand-side characteristics of the model economy: the denominator of the coefficient on the change in the real exchange rate depends on \(\gamma, a_1\) and \(a_2\). Together with \(\delta\) these parameters determine the relative importance of the real exchange rate in the target rule.\(^{13,14}\) Combining equation (14) with equations (10) – (12) yields the solutions for the endogenous variables and the policy instrument.

\(^{13}\) The target rule can also be written in level form as \(\frac{\delta q_t}{a_1(1 - \gamma) + a_2} + y_t + \mu \kappa \pi^\text{REX}_t = 0\). \(p^\text{REX}_t\) is the real-exchange-rate-adjusted price level.

\(^{14}\) Broadening the mandate of the central bank thus leads to a less parsimonious target rule. With \(\delta = 0\), only two parameters, \(\kappa\) and \(\mu\), appear in the target rule: \(\Delta y_t + \mu \kappa \pi^\text{REX}_t = 0\). Barring the definition of inflation, the target rule is the same as in a closed economy framework.
3. EVALUATION OF SIMPLE INTEREST RATE RULES

This section evaluates Taylor rules relative to optimal policy in flexible inflation targeting regimes for each of the three inflation objectives, as set out in the previous section. Simple Taylor rules will generate greater losses than the optimal policy for two general reasons: first the simple rule responds only to realized values of the target variables - not to the underlying shocks - and second, the coefficients, while chosen to be sensible, are arbitrary. We examine how these relative losses vary with the chosen inflation objective and with the role given to exchange rate stability as a policy objective and/or argument in the Taylor rule.

Initially the targeting regime includes only the goals of inflation and output stabilization. No weight is put on the variability of the real exchange rate ($\delta=0$). A response to the real exchange rate may still improve the performance of Taylor rules because in the model in Section 2 it has information content with respect to the other central bank objectives. Next, we allow for central bank concern with exchange rate volatility albeit with a small weight relative to the typical elements of the dual mandate ($\delta=0.1, \delta=0.2$).

To start, we evaluate the performance of Taylor’s original rule:

$$R_t = \tau_\pi \pi_t^i + \tau_y y_t$$  \hspace{1cm} (15)

$R$ is the interest rate; $\pi_t^i$ is the chosen inflation measure ($i = $ domestic, CPI or REX inflation); and $y$ is the output gap. The coefficients are set at the values suggested by Taylor: $\tau_\pi = 1.5$ and $\tau_y = 0.5$.

For each inflation objective we consider how the performance of this standard Taylor rule is affected by adding a response to the real exchange rate ($q$). The Taylor rule becomes

$$R_t = \tau_\pi \pi_t^i + \tau_y y_t + \tau_q q_t.$$  \hspace{1cm} (16)

The value of $\tau_q$ is set at either 0.25 or 0.50.

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16 These points have been stressed by Svensson (2003) and Woodford (2001). In Section 4 we examine Taylor rules with response coefficients that are chosen optimally.

17 The lower relative weight on the real exchange rate in the central bank’s loss function accords with Smets’ (2014) view whereby price (and output) stability dominate financial stability as final objectives. It also explains why we choose a low to moderate coefficient for the real exchange rate in the Taylor rule.
Next, for each inflation objective we evaluate a variant of the rule where we change the relative weight on the inflation and output objectives. Finally, we consider the implications of central bank concern for real exchange rate volatility ($\delta > 0$). In all cases the Taylor rule is evaluated relative to the optimal policy under commitment from the timeless perspective. Table 1 provides summary information about the parameter values and the distribution of the exogenous shocks used in the numerical calculations.

### A. Domestic Inflation

1. **The Standard Taylor Rule**

Table 2 shows results for a domestic inflation target ($\pi$). Panel A is for the standard Taylor rule. In the first column the rule has only the inflation and output gap variables. In the second and third columns the rule includes the real exchange rate ($q$) with weight of 0.25 and 0.5, respectively. The cells in the table show the variances of all three inflation objectives, the output gap, the real exchange rate ($q$) as well as the policy instrument ($R$). Also shown are the value of the loss function (Loss) and the loss relative to the optimal policy (Relative Loss (%)).

The loss under optimal policy is given in the first column of Panel C of Table 2—labelled TP for optimal policy from a timeless perspective. Panel C also shows the variances under the optimal policy of the same variables shown in Panel A.

The comparison of Panels A and C indicates that the loss under the Taylor rule exceeds that under the optimal policy by 75.9%. The Taylor rule results in much higher output variance (0.92 compared to 0.11 under optimal policy) with only a modest advantage in the variance of domestic inflation (0.74 compared to 0.83).

Columns 2 and 3 of Panel A show results for the cases where the real exchange rate is included in the Taylor rule with weights of 0.25 and 0.50, respectively. The performance deteriorates somewhat: loss relative to the optimal policy rises to 78.1% and then 82.2% as the weight on the exchange rate increases first to 0.25 and then to 0.5.

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18 The last four rows of the table can be ignored for the present.
2. The Modified Taylor Rule: a Higher Weight on the Output Gap

Previous studies [e.g. Ball (1999)] have found that a simple interest rate rule such as equation (15) or (16) can be improved by increasing the relative weight on output. That this might be the case in our model is suggested by the fact that the main reason why the Taylor rules performance falls short of the optimal policy outcome is the high variance of the output gap. Panel B explores this possibility by increasing $\tau_y$ from 0.5 to 1.0.

For equation (15), which excludes the real exchange rate, welfare loss relative to the optimal policy falls from 75.9% to 53.9%. The variance of output declines substantially with only a small rise in the variance of domestic inflation. In the case of equation (16) with the real exchange rate response included, increasing the weight on output is also welfare improving. It is still the case, however, that inclusion of the real exchange rate results in higher welfare loss.

3. The Real Exchange Rate as Policy Goal

To this point the loss function gives weights only to inflation and the output gap stabilization ($\delta=0$ in equation (5)). The results in Table 2 suggest that concern for exchange rate volatility is important for the evaluation of the desirability of Taylor rules relative to the optimal policy. A comparison of the first column of Panel A with the first column of Panel C reveals that the variance of the real exchange rate ($q$) under the optimal policy exceeds that under the Taylor rule by 83% for the specification where there is no response to the exchange rate (equation (15)). The optimal policy is much more aggressive in pursuing the inflation and output goals as can be seen by the much higher variance of the interest rate under this policy. The result is a better outcome for the output goal though not for domestic inflation, but also much more volatility of the real exchange rate.

The last two rows of Panels A and B of Table 2 show the welfare loss from following a Taylor rule relative to the optimal policy when the real exchange rate is given a weight as a policy goal. The weight on the real exchange rate ($\delta$) is set at 0.1 or 0.2 times the weight on inflation ($\mu$). The weights on inflation and output are then adjusted such that the weights in the loss function sum to 2.0 as before.\footnote{For all cases in this section the weights on inflation and the output gap are held equal. The method of calculating the scaled weights is explained in the notes accompanying Table 2.} The comparison here is now to the losses under the optimal
policy shown in the second and third columns of Panel C of the table. Even for these relatively small weights on exchange rate stability, the loss from following a Taylor rule relative to the optimal policy is diminished considerably. For the standard Taylor rule with no response to the real exchange rate, the relative loss with δ=0.2 is 42.8% compared to 75.9% percent. For the modified Taylor rule shown in column 1 of Panel B, the corresponding welfare loss diminishes from 53.9% to 28.4%.

Inclusion of real exchange rate volatility as a policy goal (δ > 0) also affects the desirability of including a response to the real exchange rate in the Taylor rule. For either weight attached to the output gap (0.5 or 1.0), it is now the case that adding the real exchange rate to the Taylor rule reduces the loss relative to the optimal policy. For the larger weight on the real exchange rate in the loss function (δ=0.2), a value of 0.50 for τ_q reduces this loss from 42.8% to 29.9% for an output weight of 0.5 and from 28.4% to 20.3% for a weight of 1.0.

Panel D of Table 2 summarizes the way in which the losses under a Taylor rule relative to the optimal policy change as we vary the weight on output (τ_y), the weight on the real exchange rate (τ_q) and the weight on the real exchange rate in the loss function (δ). The rule with (τ_y=1.0; τ_q=0.50; δ=0.2) has a loss of 20.3% relative to the optimal policy.

B. CPI Inflation

Table 3 presents results when CPI inflation (π^{CPI}) is the inflation objective.

1. The Standard Taylor Rule

Quantitatively, the losses from following a Taylor rule instead of the optimal policy are smaller for CPI than for domestic inflation targeting. From the first column of Panel A, it can be seen that for Taylor’s original rule the relative loss is 32.3% compared to 75.9% for domestic inflation. The primary reason for the difference is that optimal policy is less aggressive when CPI inflation is the target. The variance of the interest rate falls by more than 50% relative to that under domestic inflation targeting. The interest rate is less variable because the resulting volatility in the real exchange rate results in displacement of CPI inflation. Even if the variance of the exchange rate is given no weight in the central bank loss function, exchange rate volatility is costly. Thus, for a CPI inflation target the loss under optimal policy from a timeless
perspective is greater and closer to the Taylor rule outcome. Output is more variable under the Taylor rule but the difference is less marked than for the case of domestic inflation targeting.

A second pattern that differs for CPI relative to domestic inflation stabilization is that for a standard Taylor rule it is now the case that including a small response to the real exchange rate ($\tau_q=0.25$) improves the rule’s performance; the relative loss falls from 32.3% to 28.8%. Increasing $\tau_q$ to 0.5 raises this loss to 29.9%, still below the loss for the case of no response to the real exchange rate.

2. The Modified Taylor Rule: a Higher Weight on the Output Gap

Panel B of Table 3 shows results for the case where the weight on output stabilization ($\tau_y$) is increased from 0.5 to 1.0. As was the case with domestic inflation stabilization, this change reduces the relative loss from employing the Taylor-type rule. In the case where there is no real exchange rate response (Panel B, column 1), choosing the higher weight on output reduces the loss from 32.3% to 22.3%. From columns 2 and 3 of Panel B it can be seen that adding a response to the exchange rate results in a small improvement for $\tau_q=0.25$ but a minor deterioration of performance for $\tau_q=0.5$.

3. The Real Exchange Rate as a Policy Goal

If the central bank values real exchange rate stability ($\delta=0.1; \delta=0.2$), policy from a timeless perspective becomes less aggressive. Output is somewhat more variable and the real exchange rate is much less variable. The variance of CPI inflation is virtually unchanged. For comparison we begin with the standard Taylor rule ($\tau_\pi=1.5; \tau_y=0.5; \tau_q=0$). As can be seen from the first columns of Panels A and D, the relative loss from following the Taylor rule increases if the real exchange rate is given a weight in the loss function. The relative loss rises very slightly from 32.3% to 32.5% with a weight on the real exchange rate of 0.1 and to 37.3% for a weight of 0.2. The optimal policy adjusts with the change but the Taylor rule is unchanged. Adding a real exchange rate response to the Taylor rule in this case, however, improves its relative performance substantially as can be seen from the second and third columns of Panels A and D of Table 3. With a response coefficient ($\tau_q$) of 0.50 on the exchange rate, the relative loss for the Taylor rule falls from 37.3% to 19.1% for $\delta=0.2$.  

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Panel B and the right-hand portion of Panel D of Table 3 show results with a higher weight on output ($\tau_y=1.0$) in the Taylor rule. These results parallel those for the standard Taylor rule. Adding the real exchange rate to the loss function with a Taylor rule that does not include a real exchange rate response causes the loss relative to the optimal policy to increase (Panel B; column 1). When an exchange rate response is added to the Taylor rule, its relative performance improves substantially with losses relative to optimal policy falling as low as 14.3%.

C. REX Inflation

Table 4 presents results for the strategy of REX inflation ($\pi^{\text{REX}}$) targeting. As pointed out in Section 2, REX inflation targeting restores the isomorphism between optimal policy in the open economy and that in a closed economy. One result following from this property is that the optimal policy adjusts the interest rate to perfectly offset all demand-side shocks. Another implication is that the trade-off between output and inflation is least favorable under this inflation objective; reductions in inflation require the largest sacrifice in output. Optimal policy under this inflation objective thus results in a high degree of output stabilization. Moreover, there is an aggressive use of the interest rate instrument, resulting in a high variance of the real exchange rate if exchange rate stability is not a policy goal ($\delta=0$).

1. The Standard Taylor Rule

With Taylor’s original rule ($\tau_e=1.5$; $\tau_y=0.5$; $\tau_q=0$), the welfare loss from following a Taylor rule relative to the optimal policy is 99.5%, higher than with the other inflation targets (Panel A, Column 1). The loss comes mostly from higher output instability. The variance of the output gap under the standard Taylor rule exceeds that under the optimal policy by a factor of 20. Adding a real exchange rate response worsens the performance of the standard Taylor rule (Panel A, columns 2 and 3). Loss relative to optimal policy rises to 102.2% for $\tau_q=0.5$.

2. The Modified Taylor Rule: a Higher Weight on the Output Gap

Panel B of Table 3 reports results where a higher weight is attached to the output gap ($\tau_y=1.0$) in the Taylor rule. This change reduces the relative loss under the Taylor rule compared to the optimal policy from 99.5% to 72.8% when no response to the real exchange rate is
included in the rule. Including a response to the real exchange rate in this version of the Taylor rule again increases the relative loss.

3. The Real Exchange Rate as a Policy Goal

If real exchange rate stability is a policy goal (δ = 0.1 or 0.2), optimal policy under the timeless perspective becomes considerably less aggressive (Panel C, columns 2 and 3). Now at the margin the gains in output or REX inflation stability do not justify the costs in terms of increased real exchange rate volatility. With δ=0.2, the variance of the real exchange rate falls from 5.86 to 1.91. The variance of the policy instrument falls from 5.01 to 2.02. Optimal policy converges toward policy under the Taylor rule. The relative loss from following the standard Taylor rule falls substantially (Panel D). The fall is even more pronounced if a response to the real exchange rate is included in the Taylor rule. With a real exchange rate response coefficient of 0.5 and δ = 0.2, the relative loss for the Taylor rule is 37.3% compared to 99.5% for the standard Taylor rule when real exchange rate is not a policy goal (δ=0). As in previous cases putting a higher weight on the output gap (τ_y=1 instead of 0.5) in the Taylor rule improves its performance. With δ=0.2 in the loss function, relative loss is reduced to 26.4% if the response coefficient to the real exchange rate (τ_q) is 0.5.

D. Comparisons Across Inflation Targeting Strategies and Loss Functions

Table 5 presents comparisons of the relative losses that result from using a Taylor rule in place of the optimal policy from a timeless perspective across the three inflation targeting regimes. Table 6 compares the gains from adding a real exchange rate response to various specifications of the Taylor rule for different specifications of the loss function.

1. Relative Losses across Inflation Targeting Regimes

Employing the standard Taylor rule instead of optimal policy results in substantial losses for each of the three inflation targets if only output and inflation appear in the loss function. This can be seen from the first row of panel A in Table 5. This results from the fact that the Taylor rule responds only to realized values of the target variables, not to the underlying shocks, and that the weights in the Taylor rule are set arbitrarily. In the case of targeting domestic or CPI
inflation the optimal policy also takes advantage of the information content of the exchange rate.\footnote{Froyen and Guender (2017) employ an open economy Phillips curve (Eq. 1) to show that the real exchange rate enters the target rule for both a domestic and CPI inflation objective under optimal \textit{discretionary} policy.}

The relative losses are largest for REX inflation targeting (99.5\%) because in that case the optimal policy can, by responding to the demand-side shock directly, adjust the interest rate to offset its effect. The Taylor rule cannot. The relative loss under CPI inflation targeting is smallest (32.3\%). For this inflation objective the optimal policy is least aggressive because adjusting the interest rate with consequent effects on the real exchange rate displaces CPI inflation; thus the optimal policy is closer to the standard Taylor rule for this objective. Losses with domestic inflation as the inflation objective fall in between (75.9\%).

Relative losses in each case result mainly from higher output variability under the Taylor rule. Thus, in row 2 of Panel A the performance of the Taylor rule is improved considerably when the weight on the output gap ($\tau_y$) in the rule is increased from 0.5 to 1.0. The gains are approximately 30\% for each of the inflation objectives.

Panel B of Table 5 shows the relative losses from the Taylor rule when the variance of the real exchange rate ($q$) is also a policy goal. The case in the table is $\delta = 0.2$. The resulting scaled weights in the loss function ($\mu^{x}, \mu^{y}, \mu^{q}$) are (0.909, 0.909, 0.182); the weight on the real exchange rate is relatively small. For the results in lines 1 and 3 of Panel B, the Taylor rules are the same as those in Panel A. The interest rate settings are the same but loss is computed with the new weights. Moreover the optimal policy to which the Taylor rule is compared now is computed using these weights.

Relative losses from following the Taylor rules decline considerably with these alternative weights. The optimal policy becomes less aggressive when penalized for resulting real exchange rate volatility and comes closer to the behavior of the corresponding Taylor rule. The relative ranking across the three inflation objectives remains the same. The range of the relative losses, however, is far more compact. Losses relative to optimal policy decrease to 14.3\% for CPI inflation, 20.3\% for domestic inflation, and 26.4\% for REX inflation if policy follows a modified Taylor rule that smooths the real exchange rate.
2. Gains from Adding an Exchange Rate Response to the Taylor Rule

Rows 2 and 4 of Panel B in Table 5 show the losses for the Taylor rule relative to optimal policy when a real exchange rate response is added to the rule (τ_ω = 0.5) and the variance of the real exchange rate given a weight in the loss function (δ = 0.2). In Table 6 these losses are compared to the losses of the corresponding Taylor rules without the real exchange rate response (Panel B). Table 6 also shows the gain or loss from adding a real exchange rate response to the Taylor rules if the loss function gives a weight only to inflation and the output gap (Panel A).

In Table 6, if we confine ourselves to Panel A (δ=0), the results are consistent with Taylor’s (2001, p.266) description of previous studies: “they seem to be suggesting similar conclusions either that there are small performance improvements from reacting to the exchange rate or that such reactions can make performance worse.” Only for CPI inflation targeting with the standard rule and Taylor’s original weights is the gain (10.8%) not clearly “small.” In the cases of domestic and REX inflation targeting, performance of the Taylor rule deteriorates relative to the optimal policy when a real exchange rate response is added.

The situation is different when the real exchange rate variance is given a weight in the loss function. For each of the three inflation targets the gain is greatest for the higher real exchange rate response (τ_ω = 0.5). Across the three targeting strategies, the gains are highest for CPI inflation targeting: 48.8% for the rule with Taylor’s weights and 46.6% with the higher weight on the output gap (τ_y =1). Gains for the other two inflation targets are between 28% and 30%. If an even smaller weight is chosen for real exchange rate variance (δ = 0.1) such that the scaled weight (0.09524) comprises less than 5% of the sum total (2) of the weights in the loss function, adding an exchange rate response to the Taylor rule still appears desirable. For the rule with Taylor’s original weights, the gains are 33.5% for CPI inflation, 13.9% for REX inflation and 11.5% for domestic inflation.

What drives the results in Tables 5 and 6? With only inflation and the output gap as policy goals, optimal policy from a timeless perspective is very aggressive, resulting in high real exchange rate volatility. The Taylor rule is less aggressive. When the variance of the real exchange rate is also a policy goal, optimal policy takes volatility of the real exchange rate into
account and becomes less aggressive. Relative losses from using the Taylor rule are generally lower. Moreover in this case a Taylor rule that smooths the exchange rate is preferred.

4. RESULTS WITH OPTIMIZED TAYLOR-TYPE RULES

Taylor chose coefficients he believed were sensible but which were not tied to a specific model. Other papers have constructed Taylor-type rules with coefficients chosen to be optimal within a model. Examples within an open economy context are: Garcia, Restrepo and Roger (2011), Leitemo and Söderström (2005), and Batini, Harrison and Millard (2001).

Previous studies have found that, when unconstrained, the optimal response coefficients in Taylor rules have been too high to be economically sensible. These studies have addressed the problem by including the change in the interest rate as a cost in the policymaker’s loss function. There are valid reasons that justify interest rate stability as a policy objective.\(^{22}\) We follow this course. The resulting objective function for the central bank is given below Table 7. The real exchange rate only appears for \(\delta > 0\). The weight on the interest rate stability argument \((\varphi)\) is set at 0.1. The Taylor rules are unchanged from those in the previous section (equations (15) and (16)). The parameters are now chosen by joint optimization to minimize the new objective function. Because the objective function has changed the losses are not strictly comparable with those in Section 3.

A. Losses from Optimized Taylor Rules

Panel A of Table 7 shows the parameter values of the optimized Taylor rules without the real exchange rate for each of the three inflation targets. The loss score and relative loss, measured by losses above that of optimal policy from a timeless perspective are also shown.

The first row of the table provides results with \(\delta = 0\); the real exchange rate is given no weight in the loss function. The optimized Taylor rules show a stronger response to the output gap than Taylor’s original rule \((\tau_y=0.5)\) or the adjusted value \((\tau_y=1.0)\) used in the previous section for each of the three inflation targets. The response to inflation is smaller than Taylor’s

\(^{22}\) On the issues relevant to interest smoothing as a policy goal of an optimizing central bank see Rudebusch (2002) and Söderström et al. (2005).
value ($\tau_\pi=1.5$) for the domestic inflation or REX inflation target but higher for the CPI inflation target.

The pattern of losses relative to the optimal policy is the same as for the Taylor rules in the previous section. Losses for CPI inflation are smallest; those for REX inflation are largest with those for domestic inflation in between but closer to those for REX inflation. All losses relative to the optimal policy are smaller in Table 7A than those for the comparable rules in the previous section. But as noted the losses are not strictly comparable.

The second and third lines of panel A provide results where the central bank objective function includes stability of the real exchange rate. In the case of optimized Taylor rules, coefficients tend to decline as exchange rate volatility is given a larger weight. Optimal policy from a timeless perspective also becomes less aggressive with exchange rate volatility as an added policy goal. In the table the relative losses from using a Taylor rule are lower for the domestic and REX inflation target but higher for the CPI inflation target.

**B. Optimized Taylor Rules with a Real Exchange Rate Response**

Panel B of Table 7 shows results for optimized Taylor rules when the real exchange rate is added to the rule. The first row of the panel contains results for the case where the rule includes a response to the real exchange rate but the loss function does not give a weight to real exchange rate stability ($\delta = 0$). It is for this case that Taylor (2001) summarized the evidence as pointing to at best a small performance gain when a real exchange rate response is added to the rule. The table indicates no gain when domestic inflation is the target, a gain of 3.3% with the REX inflation target and 13.7% with a CPI inflation target.\(^{23}\) Only the last case might suggest need for a revision in Taylor’s summary of the evidence.

The situation is different when exchange rate stability is a policy goal even with a small weight ($\delta=0.1$, $\delta=0.2$). Results for these cases are shown in the second and third rows of Table 7B. The inclusion of exchange rate stability as a goal results in an increase in the optimal real exchange rate response in the Taylor rule. In a few cases the response to the real exchange rate exceeds the response to the inflation target. The gains to adding a real exchange rate response to

\(^{23}\) The gains are calculated in the same way as those reported in Table 6.
the rule, measured by the reduction in loss relative to the timeless perspective, are substantial for each inflation targeting strategy. With the higher weight on exchange rate stability ($\delta = 0.2$) the gains are 41.4% for domestic inflation targeting, 58.9% for CPI inflation targeting, and 50.2% for REX inflation targeting. Even with the smaller weight ($\delta=0.1$), the performance of Taylor rules improves markedly with the addition of a response to the real exchange rate.

5. CONCLUSION

In our final exercise assessing the losses from using Taylor rules in place of optimal policy from a timeless perspective (the bottom line of Table 7B), the relative losses are small, approximately 10% for each of the inflation targeting strategies. This is for a rule with optimized responses to the target variables including the real exchange rate and for the case where exchange rate stabilization is a secondary policy goal next to inflation and output gap stability. These losses are substantially lower than in the first case we considered in Section 3. There, the rule used Taylor’s original coefficients; there was no exchange rate response; and exchange rate stabilization was not a policy goal. The relative losses to employing the Taylor rule were 75.9% for domestic inflation targeting, 32.3% for CPI targeting, and 99.5% for REX inflation targeting.

Three factors are of importance in the improvement in the relative performance of the Taylor rule.

First, if real exchange rate stability is a minor goal, relative loss to a standard Taylor rule is reduced substantially for domestic and REX inflation targeting. The relative advantage of optimal policy for these strategies is that it is more aggressive, especially in stabilizing the output gap. But this advantage is achieved at the cost of increased real exchange rate volatility. If real exchange rate stability is a policy goal, this advantage is reduced. For CPI inflation targeting, exchange rate volatility affects the relevant inflation measure and is taken into account even without a real exchange rate goal in the loss function. Adding this goal does not reduce the relative loss from a standard Taylor rule under this strategy. The relative losses are therefore compressed across the three strategies.

Second, even if real exchange rate stability is merely a secondary policy goal, then adding a real exchange rate response to the Taylor-type rule substantially improves the rule’s performance relative to the optimal policy for each of the three inflation targeting strategies.
Third, within the model set out here, Taylor’s original coefficients are far from optimal. A simple adjustment of the output response (from $\tau_y = 0.5$ to $1.0$) improved the performance of the rule irrespective of the definition of the inflation target. If real exchange rate stability is a policy goal, then adding a response to the real exchange rate in the Taylor rule improves its performance for all three inflation objectives. Finally, we have seen that using values of the coefficients in the Taylor rule chosen optimally for each inflation targeting strategy and including a real exchange rate response further reduces and compresses the welfare loss relative to optimal policy from the timeless perspective.

If one believes that employing coefficients that are chosen to be optimal within a specific model violates the spirit of the Taylor rule, the relevant losses from employing Taylor rules are those in Section 3. There, when openness is taken into account with a minor weight given to stabilizing the real exchange rate and an exchange rate response included in the rule, the losses are: 20.3% for domestic inflation targeting; 14.3% for CPI inflation targeting; and 26.4% for REX inflation targeting.\(^{24}\)

Taylor and Williams (2011, p. 829-30) echo Taylor’s (2001) pessimistic conclusion on the merits of including an exchange rate response in simple interest rate rules. The literature on open economies that they had to survey in this regard, however, dated from the 1990s, focused on large economies and concentrated on the traditional elements of the dual mandate.\(^{25}\) Our results are based on a later generation of New Keynesian models in which the real exchange rate has a more fundamental role. This is important. Still, most crucial for the merits of including an exchange rate in the Taylor rules we consider is whether the central bank places a weight on stability of the real exchange rate. It is noteworthy that even with a small weight on real exchange rate stability, inclusion of the real exchange rate results in significant improvement in the rule’s performance relative to optimal policy under the timeless perspective. If the dual mandate is expanded to include real exchange rate stability even as a lesser policy goal, as seems

\(^{24}\) The losses are measured for $\tau_y = 1$ instead of $\tau_y = 0.5$ in Taylor’s original rule, an adjustment made in a number of previous studies.

\(^{25}\) See Bryant et al. (1993) and Henderson and McKibbin (1993). Several more recent studies, which we cite in the introduction, also support their pessimistic conclusion.
sensible given the current turbulence in world financial markets, pessimism concerning the usefulness of including a real exchange rate response in Taylor-type rules is unwarranted.

References:


Table 1: Calibration of Model

The following values for the parameters and variances of the stochastic disturbances are used in the numerical calculations of the variances of the endogenous variables of the model. Some of these were taken from Svensson (2000).

<table>
<thead>
<tr>
<th>Variable</th>
<th>Value</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\gamma$</td>
<td>0.3</td>
</tr>
<tr>
<td>$\sigma$</td>
<td>0.643</td>
</tr>
<tr>
<td>$\eta$</td>
<td>0.85</td>
</tr>
<tr>
<td>$\eta^f$</td>
<td>0.75</td>
</tr>
<tr>
<td>$\gamma^f$</td>
<td>0.075</td>
</tr>
<tr>
<td>$\xi^f$</td>
<td>0.9</td>
</tr>
<tr>
<td>$a_1 = (1 - \gamma)\sigma$</td>
<td>0.45</td>
</tr>
<tr>
<td>$a_2 = \gamma \left( (1 - \gamma)\eta + \eta^f \gamma^f \right)$</td>
<td>0.195</td>
</tr>
<tr>
<td>$a_3 = \gamma \xi^f$</td>
<td>0.27</td>
</tr>
<tr>
<td>$b$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>0.1</td>
</tr>
<tr>
<td>$\beta$</td>
<td>0.99</td>
</tr>
<tr>
<td>$\sigma_j^2$</td>
<td>$1$ for $j = v, u, y^f, R^f, \epsilon, \pi^f$</td>
</tr>
</tbody>
</table>

$\gamma$ = degree of consumption openness

$\sigma$ = intertemporal elasticity of substitution of consumption

$\eta$ = elasticity of substitution between domestic and foreign consumption good

$\eta^f$ = foreign elasticity of substitution between foreign and domestic consumption good

$\gamma^f$ = degree of consumption openness abroad

$\xi^f$ = share of foreign consumption in foreign income
Table 2: Domestic Inflation Target

<table>
<thead>
<tr>
<th>A.</th>
<th>Std. TR</th>
<th>Std.TR+.25q</th>
<th>Std.TR+.5q</th>
<th>B.</th>
<th>TR(τ_y = 1)</th>
<th>TR(τ_y = 1)+.25q</th>
<th>TR(τ_y = 1)+.5q</th>
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<tr>
<td>V(y)</td>
<td>0.9218</td>
<td>0.9103</td>
<td>0.9243</td>
<td>V(y)</td>
<td>0.6704</td>
<td>0.6940</td>
<td>0.7286</td>
</tr>
<tr>
<td>V((\pi^{REX}))</td>
<td>0.8939</td>
<td>0.9129</td>
<td>0.9266</td>
<td>V((\pi^{REX}))</td>
<td>0.9108</td>
<td>0.9244</td>
<td>0.9348</td>
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<tr>
<td>V((\pi))</td>
<td>0.7442</td>
<td>0.7763</td>
<td>0.8016</td>
<td>V((\pi))</td>
<td>0.7870</td>
<td>0.8083</td>
<td>0.8262</td>
</tr>
<tr>
<td>V((\pi^{CPI}))</td>
<td>0.9024</td>
<td>0.8082</td>
<td>0.7637</td>
<td>V((\pi^{CPI}))</td>
<td>1.0167</td>
<td>0.9190</td>
<td>0.8632</td>
</tr>
<tr>
<td>V(R)</td>
<td>1.2693</td>
<td>1.0411</td>
<td>0.9407</td>
<td>V(R)</td>
<td>1.2612</td>
<td>1.0573</td>
<td>0.9541</td>
</tr>
<tr>
<td>V(q)</td>
<td>2.8179</td>
<td>2.0138</td>
<td>1.5180</td>
<td>V(q)</td>
<td>2.7400</td>
<td>2.0673</td>
<td>1.6194</td>
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<tr>
<td>Loss</td>
<td>1.6660</td>
<td>1.6866</td>
<td>1.7259</td>
<td>Loss</td>
<td>1.4574</td>
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<tr>
<td>Relative Loss (%)</td>
<td>75.9</td>
<td>78.1</td>
<td>82.2</td>
<td>Relative Loss (%)</td>
<td>53.9</td>
<td>58.6</td>
<td>64.1</td>
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<tr>
<td>Loss(q,(\delta=.09524))</td>
<td>1.8851</td>
<td>1.7981</td>
<td>1.7883</td>
<td>Loss(q,(\delta=.09524))</td>
<td>1.6490</td>
<td>1.6277</td>
<td>1.6350</td>
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<td>Relative Loss (q,(\delta=.09524)) (%)</td>
<td>46.2</td>
<td>41.7</td>
<td>40.9</td>
<td>Relative Loss (q,(\delta=.09524)) (%)</td>
<td>30.0</td>
<td>28.3</td>
<td>28.9</td>
</tr>
<tr>
<td>Loss(q,(\delta=.1818))</td>
<td>2.0267</td>
<td>1.8992</td>
<td>1.8448</td>
<td>Loss(q,(\delta=.1818))</td>
<td>1.8229</td>
<td>1.7414</td>
<td>1.7077</td>
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<tr>
<td>Relative Loss (q,(\delta=.1818)) (%)</td>
<td>42.8</td>
<td>33.8</td>
<td>29.9</td>
<td>Relative Loss (q,(\delta=.1818)) (%)</td>
<td>28.4</td>
<td>22.7</td>
<td>20.3</td>
</tr>
</tbody>
</table>

Loss = \(V(\gamma_t) + \mu V(\pi_t)\) For all cases considered \(\mu = 1\). Loss (q,\(\delta\)) = \(\mu^q V(\gamma_t) + \mu^\pi V(\pi_t) + \delta V(q_t)\). For \(\delta = .1\mu^\pi\) and \(\mu^\gamma=\mu^\pi\) the loss function becomes:

Loss (q,\(\delta\)) = \(\mu^q V(\gamma_t) + \mu^\pi V(\pi_t) + .1\mu^\pi V(q_t)\). The sum of the weights must add up to 2.

Hence 2.1\(\mu^\pi\) = 2 or \(\mu^\pi = .9524\). From this it follows that \(\delta = .09524 = \mu^\gamma\). Follow the same procedure for the case of \(\delta = .2\mu^\pi\).

<table>
<thead>
<tr>
<th>C.</th>
<th>TP</th>
<th>TP(q,(\delta=.09524))</th>
<th>TP(q,(\delta=.1818))</th>
</tr>
</thead>
<tbody>
<tr>
<td>V(y)</td>
<td>0.1146</td>
<td>0.2104</td>
<td>0.3486</td>
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<tr>
<td>V((\pi^{REX}))</td>
<td>0.8392</td>
<td>0.8670</td>
<td>0.8858</td>
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<tr>
<td>V((\pi))</td>
<td>0.8327</td>
<td>0.8343</td>
<td>0.8465</td>
</tr>
<tr>
<td>V((\pi^{CPI}))</td>
<td>1.9618</td>
<td>1.3781</td>
<td>1.1378</td>
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<tr>
<td>V(R)</td>
<td>4.0318</td>
<td>2.5010</td>
<td>1.9121</td>
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<td>V(q)</td>
<td>5.1574</td>
<td>2.8755</td>
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<tr>
<td>Loss</td>
<td>0.9472</td>
<td>1.2688</td>
<td>1.4197</td>
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</tbody>
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Definitions: Std. TR: \(R_t = 1.5\pi_t + 0.5y_t\)

<table>
<thead>
<tr>
<th>D.</th>
<th>(\tau_\pi = 1.5, \tau_y = .5)</th>
<th>(\tau_\pi = 1.5, \tau_y = 1)</th>
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<tr>
<td>(\delta / \tau_q)</td>
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<td>.25</td>
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<tr>
<td>0</td>
<td>75.9</td>
<td>78.1</td>
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<tr>
<td>.1</td>
<td>46.2</td>
<td>41.7</td>
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<td>.2</td>
<td>42.8</td>
<td>33.8</td>
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</tbody>
</table>

Relative Loss = \(\frac{Loss^{TR} - Loss^{TP}}{Loss^{TP}} \times 100\)
Table 3: CPI Inflation Target

<table>
<thead>
<tr>
<th>A.</th>
<th>Std. TR</th>
<th>Std.TR+.25q</th>
<th>Std.TR+.5q</th>
<th>B.</th>
<th>TR((\tau_y = 1))</th>
<th>TR((\tau_y = 1))+.25q</th>
<th>TR((\tau_y = 1))+.5q</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V(y))</td>
<td>0.9945</td>
<td>0.9613</td>
<td>0.9686</td>
<td>(V(y))</td>
<td>0.7432</td>
<td>0.7573</td>
<td>0.7865</td>
</tr>
<tr>
<td>(V(\pi^{REX}))</td>
<td>0.8675</td>
<td>0.8978</td>
<td>0.9169</td>
<td>(V(\pi^{REX}))</td>
<td>0.8961</td>
<td>0.9150</td>
<td>0.9283</td>
</tr>
<tr>
<td>(V(\pi))</td>
<td>0.7213</td>
<td>0.7672</td>
<td>0.7988</td>
<td>(V(\pi))</td>
<td>0.7737</td>
<td>0.8023</td>
<td>0.8241</td>
</tr>
<tr>
<td>(V(\pi^{CPI}))</td>
<td>0.6759</td>
<td>0.6660</td>
<td>0.6714</td>
<td>(V(\pi^{CPI}))</td>
<td>0.8019</td>
<td>0.7734</td>
<td>0.7616</td>
</tr>
<tr>
<td>(V(R))</td>
<td>1.0510</td>
<td>0.9992</td>
<td>0.9874</td>
<td>(V(R))</td>
<td>1.0019</td>
<td>0.9666</td>
<td>0.9531</td>
</tr>
<tr>
<td>(V(q))</td>
<td>2.5792</td>
<td>1.7463</td>
<td>1.2894</td>
<td>(V(q))</td>
<td>2.3746</td>
<td>1.7555</td>
<td>1.3655</td>
</tr>
<tr>
<td>(Loss)</td>
<td>1.6704</td>
<td>1.6272</td>
<td>1.6399</td>
<td>(Loss)</td>
<td>1.5451</td>
<td>1.5307</td>
<td>1.5482</td>
</tr>
<tr>
<td>(Loss(q,\delta=.09524))</td>
<td>1.8365</td>
<td>1.7162</td>
<td>1.6847</td>
<td>(Loss(q,\delta=.09524))</td>
<td>1.6977</td>
<td>1.6250</td>
<td>1.6045</td>
</tr>
<tr>
<td>(Loss(q,\delta=.1818))</td>
<td>1.9875</td>
<td>1.7969</td>
<td>1.7253</td>
<td>(Loss(q,\delta=.1818))</td>
<td>1.8364</td>
<td>1.7107</td>
<td>1.6556</td>
</tr>
<tr>
<td>Relative Loss ((q, \delta=.09524)) (%)</td>
<td>32.3</td>
<td>28.8</td>
<td>29.9</td>
<td>Relative Loss ((q, \delta=.09524)) (%)</td>
<td>22.3</td>
<td>21.2</td>
<td>22.6</td>
</tr>
<tr>
<td>Relative Loss ((q, \delta=.1818)) (%)</td>
<td>37.3</td>
<td>24.1</td>
<td>19.1</td>
<td>Relative Loss ((q, \delta=.1818)) (%)</td>
<td>26.8</td>
<td>18.1</td>
<td>14.3</td>
</tr>
</tbody>
</table>

Loss = \(V(y_t) + \mu V(\pi^{CPI}_t)\) For all cases considered \(\mu = 1\). Loss (\(q, \delta\)) = \(V(y_t) + \mu V(\pi^{CPI}_t) + \delta V(q_t)\) See Table 2 for calculation of scaled weights.

<table>
<thead>
<tr>
<th>C.</th>
<th>TP</th>
<th>TP((q,\delta=.09524))</th>
<th>TP((q,\delta=.1818))</th>
</tr>
</thead>
<tbody>
<tr>
<td>(V(y))</td>
<td>0.6363</td>
<td>0.6752</td>
<td>0.7279</td>
</tr>
<tr>
<td>(V(\pi^{REX}))</td>
<td>0.8035</td>
<td>0.8335</td>
<td>0.8550</td>
</tr>
<tr>
<td>(V(\pi))</td>
<td>0.6690</td>
<td>0.7137</td>
<td>0.7472</td>
</tr>
<tr>
<td>(V(\pi^{CPI}))</td>
<td>0.6267</td>
<td>0.6198</td>
<td>0.6283</td>
</tr>
<tr>
<td>(V(R))</td>
<td>1.7628</td>
<td>1.6749</td>
<td>1.6262</td>
</tr>
<tr>
<td>(V(q))</td>
<td>2.3134</td>
<td>1.6017</td>
<td>1.1834</td>
</tr>
<tr>
<td>(Loss)</td>
<td>1.2629</td>
<td>1.3854</td>
<td>1.4480</td>
</tr>
</tbody>
</table>

Definitions:

Std. TR = \(R_t = 1.5\pi^{CPI}_t + 0.5 y_t\)

\[
\text{Relative Loss} = \frac{Loss^{TR} - Loss^{TP}}{Loss^{TP}} \times 100
\]

D.  \(\tau_\pi = 1.5, \tau_y = .5\)  \(\tau_\pi = 1.5, \tau_y = 1\)

<table>
<thead>
<tr>
<th>(\delta / \tau_q)</th>
<th>0</th>
<th>.25</th>
<th>.5</th>
<th>0</th>
<th>.25</th>
<th>.5</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>32.3</td>
<td>28.8</td>
<td>29.9</td>
<td>22.3</td>
<td>21.2</td>
<td>22.6</td>
</tr>
<tr>
<td>.1</td>
<td>32.5</td>
<td>23.8</td>
<td>21.6</td>
<td>22.5</td>
<td>17.3</td>
<td>15.8</td>
</tr>
<tr>
<td>.2</td>
<td>37.3</td>
<td>24.1</td>
<td>19.1</td>
<td>26.8</td>
<td>18.1</td>
<td>14.3</td>
</tr>
</tbody>
</table>
Table 4: REX Inflation Target

<table>
<thead>
<tr>
<th>A.</th>
<th>Std. TR</th>
<th>Std.TR+.25q</th>
<th>Std.TR+.5q</th>
<th>B.</th>
<th>TR(τ_π = 1)</th>
<th>TR(τ_π = 1)+.25q</th>
<th>TR(τ_π = 1)+.5q</th>
</tr>
</thead>
<tbody>
<tr>
<td>V(γ)</td>
<td>0.9193</td>
<td>0.9053</td>
<td>0.9167</td>
<td>V(γ)</td>
<td>0.6623</td>
<td>0.6826</td>
<td>0.7154</td>
</tr>
<tr>
<td>V(π_{REX})</td>
<td>0.9023</td>
<td>0.9181</td>
<td>0.9300</td>
<td>V(π_{REX})</td>
<td>0.9159</td>
<td>0.9278</td>
<td>0.9372</td>
</tr>
<tr>
<td>V(π)</td>
<td>0.7558</td>
<td>0.7821</td>
<td>0.8043</td>
<td>V(π)</td>
<td>0.7951</td>
<td>0.8127</td>
<td>0.8285</td>
</tr>
<tr>
<td>V(π_{CPI})</td>
<td>1.0243</td>
<td>0.8872</td>
<td>0.8162</td>
<td>V(π_{CPI})</td>
<td>1.1307</td>
<td>0.9969</td>
<td>0.9177</td>
</tr>
<tr>
<td>V(R)</td>
<td>1.5963</td>
<td>1.2046</td>
<td>1.0161</td>
<td>V(R)</td>
<td>1.5610</td>
<td>1.2219</td>
<td>1.0405</td>
</tr>
<tr>
<td>V(q)</td>
<td>2.9496</td>
<td>2.1377</td>
<td>1.6202</td>
<td>V(q)</td>
<td>2.9091</td>
<td>2.2065</td>
<td>1.7308</td>
</tr>
<tr>
<td>Loss</td>
<td>1.8216</td>
<td>1.8233</td>
<td>1.8467</td>
<td>Loss</td>
<td>1.5782</td>
<td>1.6104</td>
<td>1.6526</td>
</tr>
</tbody>
</table>

Relative Loss (%)

<p>| | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>V(γ)</td>
<td>99.5</td>
<td>99.7</td>
<td>102.2</td>
</tr>
<tr>
<td>V(π_{REX})</td>
<td>58.2</td>
<td>52.2</td>
<td>50.1</td>
</tr>
<tr>
<td>V(π)</td>
<td>2.1921</td>
<td>2.0461</td>
<td>1.9732</td>
</tr>
<tr>
<td>V(π_{CPI})</td>
<td>52.5</td>
<td>42.4</td>
<td>37.3</td>
</tr>
</tbody>
</table>

Loss = V(γ_t) + μV(π^*_{REX}) For all cases considered μ = 1.

Loss (q,δ)=V(γ_t)+μV(π^*_{REX})+δV(q_t) See Table 2 for calculation of scaled weights.

C. | TP | TP(q, δ = .09524) | TP(q, δ = .1818) |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>V(γ)</td>
<td>0.0456</td>
<td>0.1408</td>
<td>0.2962</td>
</tr>
<tr>
<td>V(π_{REX})</td>
<td>0.8675</td>
<td>0.8886</td>
<td>0.9035</td>
</tr>
<tr>
<td>V(π)</td>
<td>0.9473</td>
<td>0.9114</td>
<td>0.9058</td>
</tr>
<tr>
<td>V(π_{CPI})</td>
<td>2.5544</td>
<td>1.6954</td>
<td>1.3517</td>
</tr>
<tr>
<td>V(R)</td>
<td>5.0124</td>
<td>2.7977</td>
<td>2.0169</td>
</tr>
<tr>
<td>V(q)</td>
<td>5.8572</td>
<td>3.0888</td>
<td>1.9070</td>
</tr>
<tr>
<td>Loss</td>
<td>0.9131</td>
<td>1.2746</td>
<td>1.4371</td>
</tr>
</tbody>
</table>

D. | τ_π = 1.5, τ_y = .5 | τ_π = 1.5, τ_y = 1 |
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>δ /τ_q</td>
<td>0</td>
<td>.25</td>
</tr>
<tr>
<td>.1</td>
<td>99.5</td>
<td>99.7</td>
</tr>
<tr>
<td>.2</td>
<td>52.5</td>
<td>42.4</td>
</tr>
</tbody>
</table>

Definitions:

\[ \text{Std.TR} = 1.5\pi^*_{REX} + 0.5y_t \]

Relative Loss = \( \frac{\text{Loss}^{\text{TR}} - \text{Loss}^{\text{TP}}}{\text{Loss}^{\text{TP}}} \times 100 \)
**Table 5: Comparisons Across Inflation Targeting Strategies**

**A. Loss Function with Inflation and Output Gap**

<table>
<thead>
<tr>
<th>Relative Loss %</th>
<th>Domestic</th>
<th>CPI</th>
<th>REX</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Standard TR</td>
<td>75.9</td>
<td>32.3</td>
<td>99.5</td>
</tr>
<tr>
<td>2. Standard TR (τ_y=1)</td>
<td>53.9</td>
<td>22.3</td>
<td>72.8</td>
</tr>
</tbody>
</table>

**B. Loss Function Including Real Exchange Rate (δ=0.2)**

<table>
<thead>
<tr>
<th>Relative Loss %</th>
<th>Domestic</th>
<th>CPI</th>
<th>REX</th>
</tr>
</thead>
<tbody>
<tr>
<td>1. Standard TR</td>
<td>42.8</td>
<td>37.3</td>
<td>52.5</td>
</tr>
<tr>
<td>2. TR with τ_q=0.5</td>
<td>29.9</td>
<td>19.1</td>
<td>37.3</td>
</tr>
<tr>
<td>3. Standard TR (τ_y=1)</td>
<td>28.4</td>
<td>26.8</td>
<td>36.6</td>
</tr>
<tr>
<td>4. TR (τ_y=1) with τ_q=0.5</td>
<td>20.3</td>
<td>14.3</td>
<td>26.4</td>
</tr>
</tbody>
</table>

a. Entries in the table show the percentage by which the value of the loss function for policy by the Taylor rule exceeds that for the optimal policy from a timeless perspective.
Table 6: Gain (+) or Loss (-) from Adding a Real Exchange Rate Response to the Taylor Rule (Percent)\(^a\)

<table>
<thead>
<tr>
<th>Loss Funct. with only</th>
<th>Taylor’s Weights</th>
<th>Domestic</th>
<th>CPI</th>
<th>REX</th>
</tr>
</thead>
<tbody>
<tr>
<td>inflation &amp; output gap</td>
<td>(\tau_y=1)</td>
<td>-2.9</td>
<td>10.8</td>
<td>-0.2</td>
</tr>
<tr>
<td></td>
<td></td>
<td>-8.7</td>
<td>4.9</td>
<td>-5.2</td>
</tr>
</tbody>
</table>

B.

<table>
<thead>
<tr>
<th>Loss Funct. with inflation, output gap, and r. exch. rate</th>
<th>Taylor’s Weights</th>
<th>30.1</th>
<th>48.8</th>
<th>29.0</th>
</tr>
</thead>
<tbody>
<tr>
<td>(\tau_y=1)</td>
<td></td>
<td>28.5</td>
<td>46.6</td>
<td>27.9</td>
</tr>
</tbody>
</table>

\(^a\) Gain or loss is measured as the change in the loss relative to the optimal policy under the timeless perspective. The real exchange rate response (\(\tau_q\)) is set at 0.25 or 0.5 depending on which minimizes relative loss. For example, for CPI inflation with the loss function containing only \(x^{\text{CPI}}\) and \(y\), in Table 3 we have: Relative Loss (\(\tau_q=0\)) = 32.3\% and Relative Loss (\(\tau_q=0.25\)) = 28.8\%. Thus the gain reported in the table is \((32.3-28.8)/32.3 = 10.8\%\).
Table 7: Optimized Taylor Type Rules

A. Taylor Rule without the Real Exchange Rate

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\tau^*_\pi$</th>
<th>$\tau^*_y$</th>
<th>$\tau^*_q$</th>
<th>Loss</th>
<th>Rel. Loss (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.924</td>
<td>2.428</td>
<td>1.49</td>
<td>23.1</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>0.82</td>
<td>1.816</td>
<td>1.76</td>
<td>15.8</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>0.85</td>
<td>1.525</td>
<td>2.01</td>
<td>16.9</td>
<td></td>
</tr>
<tr>
<td>0</td>
<td>3.160</td>
<td>3.098</td>
<td>1.66</td>
<td>15.3</td>
<td></td>
</tr>
<tr>
<td>0.1</td>
<td>2.156</td>
<td>2.116</td>
<td>1.92</td>
<td>17.8</td>
<td></td>
</tr>
<tr>
<td>0.2</td>
<td>1.620</td>
<td>1.607</td>
<td>2.17</td>
<td>21.9</td>
<td></td>
</tr>
</tbody>
</table>

B. Taylor Rule with the Real Exchange Rate

<table>
<thead>
<tr>
<th>$\delta$</th>
<th>$\tau^*_\pi$</th>
<th>$\tau^*_y$</th>
<th>$\tau^*_q$</th>
<th>Loss</th>
<th>Rel. Loss (%)</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>0.928</td>
<td>2.442</td>
<td>0.006</td>
<td>1.49</td>
<td>23.1</td>
</tr>
<tr>
<td>0.1</td>
<td>0.818</td>
<td>2.413</td>
<td>0.496</td>
<td>1.72</td>
<td>13.2</td>
</tr>
<tr>
<td>0.2</td>
<td>0.819</td>
<td>2.606</td>
<td>1.074</td>
<td>1.89</td>
<td>9.9</td>
</tr>
<tr>
<td>0</td>
<td>3.898</td>
<td>3.970</td>
<td>0.619</td>
<td>1.63</td>
<td>13.2</td>
</tr>
<tr>
<td>0.1</td>
<td>3.340</td>
<td>3.534</td>
<td>1.350</td>
<td>1.81</td>
<td>11.0</td>
</tr>
<tr>
<td>0.2</td>
<td>3.017</td>
<td>3.290</td>
<td>1.974</td>
<td>1.94</td>
<td>9.0</td>
</tr>
</tbody>
</table>

1. Objective Function: $E_t \sum_{j=0}^{\infty} \beta^j (y_{t+j}^2 + \mu q_{t+j}^2 + \delta q_{t+j}^2 + \varphi \Delta R_{t+j}^2)$ where $\pi$ is domestic, CPI, or REX inflation. $\mu$ and $\varphi$ is fixed at 1 and 0.1, respectively. The calculation of losses is based on the unconditional variances of the variables that appear in the objective function. Percentage loss is measured relative to optimal policy from a timeless perspective.

2. Numbers in bold represent cases where it was not possible to optimize freely over the policy parameters in the Taylor rule. Doing so violated the rank condition. Satisfying this condition ensures the existence of a determinate rational expectations equilibrium. To meet this condition, we instead performed a search over values of $\tau_\pi$ to find the value which minimizes (along with the optimized coefficients on $\tau_y$ and $\tau_q$) the objective function. This procedure had to be followed in case of a REX inflation target irrespective of the inclusion of the real exchange rate in the Taylor rule and in the case of a domestic inflation target when the central bank cares about real exchange rate fluctuations but does not respond to the real exchange rate in the Taylor rule. A superscripted asterisk denotes a freely optimized coefficient.