LOWER IONOSPHERIC IRREGULARITIES

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by

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1. **INTRODUCTION.**

This thesis is a study of the structure of that part of the ionosphere lying between 60 and 120 km. In the usual terminology the ionized parts of the atmosphere in this altitude range are called the ionospheric D and E regions, the boundary between them occurring at a height of 90 km. Above this height the E region extends upwards to 140 km, the base of the F region. Since the ionization below 50 km is not enough to effect the propagation of radio waves this height effectively marks the bottom of the D region. There is also another system of nomenclature based on the neutral gas temperatures of the atmosphere. The mesosphere lies in the altitude range 50 to 85 km, which is a region of decreasing temperature with height. Because they refer to the same height range the terms mesosphere and D region are often used synonymously in the following work. Above 90 km the temperature increases, rapidly at first and then more slowly, in the thermosphere.

The daytime electron densities \( N(h) \) in the E and F regions are sufficiently large to strongly reflect even weakly radiated signals of frequencies the order of 1 MHz or higher. Ionosondes which measure the height of reflection of pulsed radio waves as a function of frequency,
have been used for a number of years to produce electron density profiles as functions of height and time. The widespread use of ionosondes and other techniques utilizing both single and multifrequency soundings from the ground and satellites has led to a large measure of understanding of the behaviour of normal E and F layers, as well as irregularities which exist at these heights.

Observationally little is known about the physical processes occurring in the D region, particularly between 60 and 70 km. What information has been obtained comes from insitu rocket probes and relatively sophisticated ground based radio soundings. For example the study by radar systems of wind induced distortions in meteor trails (Greenhow and Neufeld (1961), Elford (1959)) together with propagation of sound measurements using rockets (Nordberg et al. (1965)) has enabled the circulation and temperatures in the mesosphere to be outlined in some detail.

Further information about the D region has come from a study of the behaviour of radio waves reflected from these heights. Because of low N(h) values, ranging from N = 10 e/cc at 60 km to a few thousand at 90 km, only low frequency waves (< 300 kHz) are totally reflected from the D region. However if pulsed radio waves (~ 2 MHz)
are transmitted at sufficiently high powers weak partial reflections are returned from the D region as well as the totally reflected E region signal. The use of these echoes forms the basis of one of the better techniques for measuring D region electron densities, the Differential Absorption Experiments or D.A.E. (Gardner and Pawsey (1963), Fejer and Vice (1959), Belrose and Burke (1964)).

Partial reflections occur when the refractive index of the medium changes rapidly with height. The $N(h)$ values in the lower ionosphere change by four orders of magnitude between 60 and 100 km and if the electron density changed smoothly with height such gradients could cause partial reflections. However it has been shown (Gardner and Pawsey (1953)) that to explain the strength of the observed echoes the reflections must come from sharp discontinuities in refractive index, changes occurring in a fraction of a radio wavelength. It is usually assumed that partial reflections are caused by sharply bounded irregularities in electron density, although collision frequency changes may also be important.

Although the D.A.E. method has been used for a number of years very little is known about the irregularities which produce partial reflections except that they are known to exist in all seasons (Gregory (1961)),
usually at heights above 60 km. What information there is about the dimensions of the inhomogeneities has been inferred mainly from studies of amplitude fading values (Gardner and Pawsey (1953)). Before any mechanisms suggested for the formation of irregularities can be assessed more information is required about the physical properties of the irregularities such as spatial scales and whether these are a function of season or not. Further observations of ionospheric structure would give some insight into the dynamics of the mesosphere since the irregularities are probably produced by some kind of mechanical redistribution of ionization.

One reason for the lack of knowledge about the reflections can be attributed to the relatively poor spatial resolution inherent in the normal methods of pulsed soundings. The usual technique is to transmit a packet of radio waves and observe the delay time \( \Delta t \), between transmission and reception, thus giving the group or virtual height \( h' \) of the reflections

\[
h' = \frac{c\Delta t}{2} = c \int_0^{z_r} \frac{dr}{u} = \int_0^{z_r} \mu' \, dz = \frac{v'}{2}
\]

where
- \( c \) = free space velocity of light
- \( u \) = group velocity of pulse over path \( dz \)
- \( \mu' \) = group refractive index
- \( z_r \) = height of reflection
\[ p' = \text{group path} \]

Provided \( zr \) is not too close to total reflection \( h' \) is a good measure of the true height of reflection. Because of fading and other complications it is difficult to measure \( At \) very accurately, and measured changes \( Ah' \) of order \( \pm 1 \) km are considered good under normal circumstances.

An alternative measure of the distance between an ionospheric reflector and a radio frequency sender can be given in terms of double the number of free space radio wavelengths (\( \lambda \)) occurring between the two. This measure defines the phase or optical path \( P \) defined in a similar fashion to \( P' \) as

\[
P = 2c \int_0^{zr} \frac{dz}{v} = 2 \int_0^{zr} \mu dz \quad (1.1)
\]

\( v \) = phase velocity of the wave over interval \( dz \)
\( \mu \) = phase refractive index of medium over \( dz \).

Unlike \( P' \) the phase path cannot be measured absolutely but changes in \( P \) can. For example the phase of continuous wave, c.w., transmissions returned from the ionosphere can be compared with the phase of the wave received directly from the sender. In this way
changes in phase height (ΔP), caused by change in μ or zr, can be measured to fractions of a wavelength. Findlay (1951a) has described how changes in phase path of reflections using pulsed radio waves may be measured in a similar fashion by comparing the phase of the returned signal with the phase of a reference oscillation on the ground. This technique combines both the advantages of the pulse and c.w. systems since the observations of the group height of an echo gives the height of reflection and the measurements of changes in P indicate how the reflector is behaving with time. Findlay's technique is both simple and accurate since changes in phase path can be estimated to within a fraction of a radio wavelength. For frequencies the order of 2.5 MHz (λ ~ 120 m) this means changes of the order of a few tens of meters at least can be detected. The method also has certain other advantages over straightforward group path measurements (McNicol and Thomas (1960)), for example a better ability to resolve overlapping echoes by observing their different phase structure.

Because of its sensitivity Findlay's method for observing small phase changes is potentially a very useful one for studying the irregularities which produce partial reflections. For this reason it was used in this study of the lower ionosphere. Most of the basic
requirements for a study of D region partial reflections (high power pulse transmitter and low noise field site) were already available at the Universities' field site at Birdling's Flat (43°50'S, 172°40'E). Experiments already in operation were the D.A.E. method of obtaining D region electron densities (Gregory 1964), Manson (1965) and a spaced receiver experiment for observing lower ionospheric drifts (Fraser (1965)). Results from these experiments were found to be very useful in the present project the aims of which were+-

(i) to investigate the detail structure of the lower ionosphere

(ii) to assess some of the theories put forward to explain the formation of the D region irregularities in the light of any seasonal changes observed.
2. MEASUREMENTS OF CHANGES IN PHASE PATH.

Findlay (1951a) described a relatively simple method whereby changes in phase path of pulsed radio waves can be measured easily and accurately. His basic technique has been used by a number of other workers, although with some modification to experimental detail (e.g. McNicol and Thomas (1960)). The present experiment used another variation of the original method.

2.1 OUTLINE OF EXPERIMENTAL METHOD.

Basically the experiment consists of transmitting a pulse of radio waves (frequency $f_s$) and comparing the phase of the reflected signal with a reference oscillation (P.R.O.) (frequency $f_r$) which bears some fixed phase relationship with the transmitted wave. The result of mixing the two signals will be a series of beats or fringes of frequency $\delta f = (f_s - f_r)$, the number of fringes being equal to the product of the pulse width and $\delta f$. If the sequence is observed on an oscilloscope triggered at the time of pulse transmission the beats will appear after a time delay corresponding to the virtual height of the echo. (See Diag. 1. for an example).
Idealized A' scope display, echo at a group height
\[ h' = c\Delta t/2 \]

Representation of phase oscillations for single echo

D region phase-height video

FIGURE 1
If the phase between the reflector and observer is \( P \) the equation of the "beats" can be represented by

\[
E(t) = A(t) \cos \left[ 2\pi \left( \delta f t - \frac{P fs}{c} \right) \right]
\]  
(2.1)

- see Appendix 1 for a derivation. The position of a fringe peak is given by

\[
\delta f t_m = \frac{P fs}{c}
\]  
(2.2)

Provided \( \delta f > 0 \) then as \( P \) increases (or decreases) the beats will move up (or down) the time base. The phase path change \( \Delta P \), is easily obtained by following one fringe. When it has moved by a distance equal to \( N \) times the distance between successive beats \( P \) has changed by

\[
\Delta P = N \frac{c}{fs} = N\lambda
\]  
(2.3)

2.2 EXPERIMENTAL ARRANGEMENT.

As the transmitted pulse is derived from a continuously running oscillator of frequency 2.404 MHz it can be modified to supply the phase reference signal as well. This is achieved by mixing the waves with
another continuous oscillation of frequency 100 KHz and selecting and amplifying the lower side band of frequency 2.304 MHz. This is used as the phase reference signal fr.

The received signal and the phase reference oscillation are then converted to their respective intermediate frequencies of 3.166 MHz and 3.266 MHz by mixing them with the same local oscillator frequency 5.570 MHz (in order to preserve phase continuity). It is these intermediate frequencies which were finally mixed to produce the beats of frequency \( \delta f = 100 \text{ kHz} \). With a pulse width of 25 \( \mu \text{sec} \) two to three fringes were obtained for each echo, with correspondingly more fringes as the retarded echoes broaden due to group retardation.

2.3 THE PHASE RELATIONSHIP BETWEEN THE TRANSMITTED RADIO FREQUENCY AND THE PHASE REFERENCE OSCILLATION.

Normally the transmitter was triggered every 1/50th or 1/25th sec., the pulse repetition rate being derived from the mains. Since this mains frequency at the field station exhibited considerable variations in comparison with the stability of the transmitter and phase reference oscillations some care was taken to ensure the transmitted radio frequency pulse bore some
fixed phase relationship to the reference signal.

Since it is the 100 kHz difference in frequencies which is finally observed it was to the 100 kHz waveform used in generating the P.R.O. that the transmitter trigger was synchronized. This was achieved by using a simple diode logic 'AND' gate the inputs to which were the 50 or 25 c/s timing pulses and the series of "spikes" produced by differentiating the squared off 100 kHz reference signal. The first spike coincident with, or present just after, the beginning of the timing pulse triggered a transistorised pulse generator producing a long output pulse. The leading edge of the pulse was thus locked to one particular part of the reference waveform and so used to define the timing of all subsequent operations in the cycle. Both the transmitter and oscilloscope trigger pulses were derived from this master pulse.

The timing was not perfect because of the finite rise times of the pulses used, meaning that the output pulses jittered slightly with respect to the reference waveform. This "jitter" was found to be the order of only ±0.04 μsec or ±$\frac{1}{250}$ th of a cycle of the 100 kHz oscillation.
2.4 PRESENTATION OF PHASE INFORMATION.

Since each echo from the ionosphere had its own set of fringes (and since reflections could be expected from between 70 and 120 km) some simple method had to be devised to display and record all the information simultaneously. This was achieved by intensity modulating an oscilloscope tube with the phase video signal, each fringe peak producing a bright spot on the tube. With a 35 m.m. film moving continuously at right angles across the time base these spots traced lines on the film, each echo tracing its own set of lines or phase pattern.

The strength of the observed reflections covered a wide dynamic range (~80 dB from 65 km up to E region) and the echo amplitude could be expected to fade considerably. To ensure constant intensity traces on the film the output phase video was clipped giving constant amplitude pulses. It was these pulses which were actually used to brightness modulate the cathode ray tube. Noise was also limited by this method but because it was randomly phased with respect to the reference oscillation it only produced a background on the film. One of the advantages of a phase coherent system is an improvement in signal to noise.
ratio (McNicol and Thomas (1960)).

Ten km height marker pips were also supplied by a separate generator and mixed with the 100 kHz video and simultaneously displayed. A film speed of 75 cm/hour and a time base sweep corresponding to the altitude range 60 to 140 km gave adequate spatial and time resolution of fringes.

A block diagram of the experimental system is shown in Fig. 2 and details of the actual electronics are discussed in Appendix 2.

2.5 CHART RECORDING OF PHASE PATH CHANGES.

As well as using film another technique was devised whereby the ΔP for a given echo could be tracked electronically and displayed directly on a chart recorder.

A height gate of width equal to one fringe spacing (10 μsec) was centered on the echo so that only one fringe at a time was gated. An output pulse was generated of width equal to the delay between the leading edge of the gate and the gated fringe. The variations in output pulse width were thus proportional
BLOCK DIAGRAM.

FIGURE 2
to the changes of phase path of the echo in wavelength units. The variable pulse width was converted to a proportional voltage which drove the pen recorder. Everytime the pulse width was zero or 10 µsec a change in phase path of one radio wavelength had occurred and a discontinuity appeared in the trace. The pen recorder trace is thus roughly sawtooth in form and the direction of phase change (i.e. whether increasing or decreasing) can be found by the sense of the pen's deflection.

Recording on chart had the disadvantage that as the group height changed the gate had to be moved correspondingly thus causing a break in the phase record. However the chart technique was used as a convenient monitor of long lived and slowly changing reflections such as from E region. It had the advantage that phase path changes could be obtained almost immediately by algebraically counting the fringe changes with time. See Appendix 2 for further details.

2.6 TRANSMITTING AND RECEIVING ARRANGEMENTS.

Transmission is at a frequency of 2404 MHz (wavelength 125 m) with a pulse width of 25 µsec and a peak power of 150 kw at repetition rates of 25 or 50 sec⁻¹. The transmitting array consists of 8 half
wave dipoles in a broadside array, with a calculated gain of 13.8 dB over an isotropic radiator.

The receiving array is composed of 4 folded dipoles arranged in the form of a square. Discrimination between the ordinary and extraordinary magnetoionic components is thus possible. However because of general aerial availability and the need to run the phase experiment for long uninterrupted periods only two parallel sides of the array connected in phase were used, meaning that both components were present in the received signal. The possible effects of this mixture on the phase record is discussed in Sec. 4.2 and 5.5

2.7 SUPPRESSION OF 2.4 MHz CARRIER.

In Sec. 2.3 it was implied that the P.R.O. was obtained by mixing two signals and extracting their difference, all other harmonics and the carrier signal being suppressed. In general this was true but some leakage of the carrier signal (2.4 MHz) did occur occasionally, resulting in weak beating with the 2.3 MHz P.R.O. Since all signals were clipped these beats appeared as fringes parallel to the time axis on the film. They were a slight nuisance for the weaker
echoes but did have the advantage of giving an indication of the stability of the phase system.
3. ACCURACY OF PHASE PATH MEASUREMENTS.

3.1 EFFECTS OF VARIATIONS IN SENDER AND REFERENCE FREQUENCIES.

Findlay (1951) in his original paper on phase path measurements considered the effects of \( f_s \) and \( f_r \) changing with time producing spurious changes in \( P \). Using equation (2.2) changes of the time of occurrence of a fringe with respect to changes in either \( f_s \) or \( f_r \) can be estimated by evaluating \( \frac{\partial t}{\partial f_s} \) max and \( \frac{\partial t}{\partial f_r} \) max.

\[
\frac{\partial t}{\partial f_s} \text{ max} = \frac{1}{c} \left( P + f_s \frac{dP}{df_s} \right) - t \text{ max}
\]

(3.1)

and \( \frac{\partial t}{\partial f_r} \) max = \( \frac{t \text{ max}}{\partial f} \)

(3.2)

But \( P' = P + f_s \frac{dP}{df_s} \)

(3.3)

Hence \( \frac{\partial t \text{ max}}{\partial f_s} = \frac{(P' - t \text{ max})}{\partial f} \)

(3.4)

Now \( \frac{(P' - t \text{ max})}{\partial f} \) is the time between the leading edge of the echo and the peak and is therefore small (~ the order of the pulse width at the most). Since the transmitter oscillator is crystal controlled \( \Delta f_s \) is
only a few parts in $10^7$ so that $\Delta t_{\text{max}}$ produced by fluctuations in $f_s$ were very small.

From (3.2) changes in $t_{\text{max}}$ due to the phase reference oscillator changing frequency are proportional to the reflection height. The largest range considered in the experiment corresponds to a delay time of approximately 1000 $\mu$sec. The phase oscillator was also crystal controlled and fluctuations in $f_r$ were found to be the order of $\pm 1$ in $10^5$ over long periods so that $\Delta t_{\text{max}}$ would be the order of $10^{-3}$ of one cycle of the reference signal.

From the above it is obvious that instabilities in the sender and local oscillator frequencies produced negligible changes in any phase path measurements. In Sec. 2.3 it was pointed out that the 'jitter' of the transmitted pulse with respect to the P.R.O. was $\sim 0.04$ $\mu$sec. which again only produces very small fluctuations in the phase comparison.

3.2 PHASE COHERENT ECHOES.

The previous section has shown that potentially the experimental system was capable of detecting fluctuations in $P$ the order of $\pm 0.01$ wavelength at
least, but the final determiner of accuracy (other than the actual reading of $\Delta P$ from film or chart) is the ionosphere itself. Because only changes in $P$ can be measured, and not $P$ absolutely, if the reflection point changes discontinuously there is no way of knowing by how much the phase path has changed.

The important terms phase coherence and phase incoherence are now introduced. An echo is said to be phase coherent when at any instant in time the phase over the whole echo is constant and during the period of reception of the echo the phase changes smoothly with time. A phase incoherent echo is one in which the phase changes randomly over the duration of the echo. Obviously only phase coherent echos can have their phase path changes with time measured. Examples of phase coherent and incoherent echoes are seen in Fig. 5a.

3.3 MEASUREMENT OF $\Delta P$ FROM FILM OR CHART.

Some simple method had to be used to extract the phase information from the film, but reasonable accuracy was required otherwise most of the reasons for measuring $\Delta P$ are lost. Photographic enlargement and printing of long pieces of film was tedious and time consuming. The final arrangement used was to project
the film onto a horizontal surface using an ordinary 35 m.m. strip projector, displaying 2.5 mins of record at a time. The horizontal surface was ruled in lines parallel to the time axis of film and the phase height of any given phase coherent echo could be found by algebraically counting the number of fringes which crossed a given datum line passing through the echo trace. Readings taken every ½ minute gave good time resolution of ΔP starting from some arbitrary time zero. If the echo was longer than 2½ minutes the film was wound on so that the next section of record was displayed. To ensure no lateral displacement of the film had occurred the height marker lines were used as guides.

Changes in P by this method could be estimated to about ±0.1 of a fringe spacing - that is a change of approximately ±10-20 meters. The chart recording could be read with better accuracy, but again ±0.1 fringe was adequate.

All coherent records of any length do show phase "jumps", that is the phase over the whole echo changes discontinuously by some apparent fraction of a fringe spacing. To deal with these discontinuities it was decided that if ΔP/Δt was the same on either side of the point in question the break could be ignored,
the total value of $\Delta P$ immediately before the break being assigned to the trace just after.
4. \textbf{IONOSPHERIC STRUCTURE AND PHASE PATH CHANGES.}

In the previous chapter most of the experimental limitations of measuring changes in phase path were analysed and shown to be small. However no consideration has yet been given to the effects on the phase records of receiving with a linearly polarized array. In the first section of this chapter the causes of phase incoherence are discussed with particular reference to the possibility of interference between the magneto-ionic components. This Section is followed by an analysis of how changes in ionospheric structure, such as in electron content, will influence phase path measurements. Finally the phase patterns produced by certain types of ionospheric irregularities are briefly discussed.

4.1 \textbf{CAUSES OF PHASE INCOHERENCE.}

A phase incoherent echo has been described as one in which the phase changes discontinuously during the duration of the echo. To see how incoherence is caused consider the case of a number of discrete reflections all situated at approximately the same range. If the amplitude of the radio wave components reflected from each scatterer are comparable and their phases
are randomly distributed strong amplitude "beating" or fading will occur. Every time the resultant amplitude passes through zero the phase of the overall signal is likely to change abruptly. Since the components will be received with differing time delays the phase of the resultant signal at the commencement of the echo is unlikely to be the same as the phase at the end meaning substantial phase jumps will eventuate.

A phase coherent reflection from a given range will occur when only one reflection is dominant. Most of the recording in the present experiment was made with linearly polarized aerials so there was some probability of phase incoherence resulting from interference between the two magneto-ionic components. Because their phase refractive indices are different their phase paths will differ as they penetrate into the ionosphere. Furthermore because of the presence of the earth's magnetic field the directions of propagation of phase and energy are different. (i.e. the medium is anisotropic). Budden (1961) and others have shown that the ray or energy path of a vertically incident wave lies in a plane defined by the magnetic meridion, the rays for the two magneto-ionic components diverging in opposite directions. In the southern hemisphere the 'o' ray diverges towards the south and the 'E' ray towards the north.
Interference is most likely when the amplitude of the two components are comparable and their phases substantially different. The relative amplitudes of the ordinary and extraordinary components as a function of height have been studied at Birdling's Flat in connection with Differential Absorption Experiment (Manson (1965)). In general near noon the extraordinary signal is the stronger of the two at the lowest heights. However because the extraordinary ray suffers greater absorption than the ordinary ray with increasing height the latter eventually dominates. Table 1 shows a typical equinoctial profile of the ratio of the amplitude of the extraordinary and ordinary components. One of the most noticeable features of the lower echoes is the high correlation in fading between the magneto-ionic components (Belrose and Burke (1964), Manson (1965)). To see by how much the phase paths and the reflection points might differ average electron density and collision frequency profiles were used to calculate the theoretical phase path differences and horizontal deviations of the two components at various heights.

As a number of records were taken in the equinoxes the electron densities profile for September, October and November 1966 were averaged to give the electron densities in the 70-80 km region. During this
period total reflection at noon occurred at a virtual
height of about 105 km. The true height was estimated
from ionograms produced by the local ionosonde at Godley
Head (24 km to the north) using the Kelso 10 point method
(Wright and Norton 1959) and was found to be about 100 km.
An exponential profile was fitted from the electron
concentration at 80 km to the electron concentration
required for total reflection (72,000 e/cc) at 100 km,
the composite profile being shown in Fig. 3. The
collision frequency profile used was found by averaging
the summer and winter profiles presented by Kane (1962)
and fitting an exponential function to this, giving

\[ v = 3.2 \times 10^{11} \exp(-z/6.2) \]  

(4.1)

The refractive indices were derived from the full
Appleton Hartree expression

\[ n^2 = (\mu - i\chi)^2 \]

\[ = 1 - \frac{x}{1 - i_z - \frac{\gamma_T^2}{2(1 - x - i_z)}} + \frac{\gamma_T^4}{4(1 - x - i_z)^2} + \gamma_L^2 \]  

(4.2)

\[ n = \text{complex refractive index} \]
\[ \mu = \text{phase refractive index} \]
AVERAGE ELECTRON DENSITY PROFILE
NOON
SPRING 1966

$N = 4175 \exp(0.24z)$

FIG. 3
$\chi = \text{imaginary part of refractive index}$

$\chi = \frac{N e^2}{\varepsilon_0 m^2}$

$Y_L = \frac{eB_0 \cos \theta}{m \omega}$

$Y_T = \frac{eB_c \sin \theta}{m \omega}$

$Z = \frac{v}{\omega}$

where $N$ is the electron density

$\varepsilon_0 = \text{permittivity of free space}$

$e, m = \text{charge and mass of the electron}$

$\omega = 2\pi f$ where $f = 2.404 \text{ MHz}$

$\nu = \text{collision frequency}$

$B_0 = \text{magnetic field}$

$\theta = \text{angle between direction of magnetic field and direction of phase propagation.}$

At Birdling's Flat for vertical incidence $\theta = 22^\circ$.

The difference in phase paths at some height $h(\text{km})$ was taken to be
The base of the ionosphere was taken to be 60 km because the small electron densities below that height would produce negligible differences in phase path.

The lateral deviations were found using

\[ D = \int_{60\ km}^{h} \tan \alpha \ dh \]  

(4.4)

where \( \alpha \) is the angle between the directions of phase and energy propagation - (Appendix 3).

The result of the calculations are summarized in Table 2. This shows that substantial phase differences (greater than a radian) will occur above 75 km. As Table 1 shows the amplitudes of the components also become comparable between 75 and 80 km. It is concluded that because of the high correlation in fading for the lowest echoes noted above (probably due to the small differences in phase) most interference will occur between 75 and 80 km in this model. Above this height the recorded phase path changes will be those of the ordinary components. These statements are justified
in Sec. 5 when phase records taken using the components individually are considered.

### TABLE 1

<table>
<thead>
<tr>
<th>HT. (km)</th>
<th>72</th>
<th>75</th>
<th>77</th>
<th>80</th>
<th>90</th>
</tr>
</thead>
<tbody>
<tr>
<td>$A_\infty/A_o$</td>
<td>1.8</td>
<td>1.2</td>
<td>1.0</td>
<td>0.52</td>
<td>0.1</td>
</tr>
</tbody>
</table>

### TABLE 2

PHASE PATH DIFFERENCES AND HORIZONTAL SEPARATION OF MAGNETO-IONIC COMPONENTS FOR MODEL IONOSPHERE

<table>
<thead>
<tr>
<th>HT. (km)</th>
<th>64</th>
<th>68</th>
<th>72</th>
<th>76</th>
<th>80</th>
<th>84</th>
<th>88</th>
</tr>
</thead>
<tbody>
<tr>
<td>PHASE PATH DIFFERENCE (WAVELENGTHS)</td>
<td>0.0013</td>
<td>0.006</td>
<td>0.063</td>
<td>0.252</td>
<td>0.64</td>
<td>1.8</td>
<td>5.4</td>
</tr>
<tr>
<td>HORIZONTAL SEPARATION (METERS)</td>
<td>0.5</td>
<td>1</td>
<td>6</td>
<td>13</td>
<td>25</td>
<td>60</td>
<td>154</td>
</tr>
</tbody>
</table>
4.2 THE EFFECT OF CHANGES IN REFRACTIVE INDEX ON PHASE PATH MEASUREMENTS.

As equation 1.1 shows the phase height of a reflection is proportional to the integrated phase refractive index up to the point of reflection. Changes in $P$ are therefore caused by either the point of reflection moving or by variations in the phase refractive index $\mu$ of the intervening medium. To see what order of magnitude changes the latter might produce in the D region the quasi-longitudinal approximation of (4.2) for $\mu$ was used (Davies (1966))

$$\mu_0^2 = 1 - \frac{X}{1 \pm Y_L}$$

(4.5)

$\mu_{o,e}$ = phase refractive index for ordinary and extraordinary components, which are associated with the plus and minus signs respectively. (4.5) holds when the angle between the magnetic field and the direction of propagation is small. At Birdling's Flat at vertical incidence the angle is $22^\circ$. Since it ignores collisional effects (4.5) is not valid for D region heights where collision frequencies are high but will suffice for order of magnitude changes.
For a frequency of $2.4\text{MHz}$ and local gyro-frequency of $1.65\text{MHz}$

$$X = xN \text{ where } x = 1.4 \times 10^{-5} \text{ cm}^{-3}$$

$$y_L = 0.6$$

Differentiating (4.5) with respect to $N$ gives

$$2 \mu_0 e \delta \mu_o e = -x_\pm \delta N \quad (4.6)$$

where $$x_\pm = \frac{x}{1 \pm y_L}$$

Therefore the fractional change in the refractive index for a change $\delta N$ in electron content $N$ at height $h$ is given by

$$\frac{\delta \mu_o e}{\mu_o e} = -\frac{x_+}{\xi^2 (1 - x_+ N)} \delta N \quad (4.7)$$

now $$x_+ = 9.10^{-6} \text{ cm}^3$$

$$x_\pm = 3.5 \times 10^{-5} \text{ cm}^3$$

For values of $N$ less than 3000 $e/\text{cc}$

$$x_\pm N \ll 1$$
It is noted from (4.8) that $\delta \mu / \mu$ is negative for positive $\delta N$ implying a decrease in phase path for increases in electron density. For $N(h)$ equal to 1000 e/cc and a fractional change of the order of 100% the percentage changes in $\mu_{0,e}$ are only one part in 100. Changes of this order in $\mu$ are unlikely to produce significant changes in $P$, unless integrated over a large height range. Since the electron densities are considerably smaller than $10^3$ e/cc in the lower D region and $\delta N/N = 1$ are very unlikely, changes in electron content below the point of reflection will not produce significant changes in $P$. This may not be true under solar flare conditions when $N(h)$ can change considerably. (Belrose and Cetiner (1962)).

The above conclusions do not hold near total reflection where $\mu$ becomes very small. Fooks (1962) using a linear profile for $N(h)$ and irregularities with gaussian cross-section has shown how fluctuations of a few percent in $N$ will produce phase deviations the order of a wavelength in the totally reflected signal.
TABLE 3

DEVIATIONS IN PHASE PATH CAUSED BY 100% N(h) IRREGULARITIES AT HEIGHT z₀

<table>
<thead>
<tr>
<th>z₀ (km)</th>
<th>ΔP (WAVELENGTHS)</th>
</tr>
</thead>
<tbody>
<tr>
<td>75.0</td>
<td>-0.1</td>
</tr>
<tr>
<td>80.0</td>
<td>-0.2</td>
</tr>
<tr>
<td>85.0</td>
<td>-0.4</td>
</tr>
</tbody>
</table>

TABLE 4

PHASE DEVIATIONS (WAVELENGTHS) IN TOTALLY REFLECTED SIGNAL

<table>
<thead>
<tr>
<th>a (km)</th>
<th>z₀ (km)</th>
<th>% AMPLITUDE</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>1</td>
</tr>
<tr>
<td>0.5</td>
<td>96</td>
<td>-</td>
</tr>
<tr>
<td></td>
<td>97</td>
<td>-0.04</td>
</tr>
<tr>
<td></td>
<td>98</td>
<td>-0.06</td>
</tr>
<tr>
<td>1.0</td>
<td>96</td>
<td>-0.06</td>
</tr>
<tr>
<td></td>
<td>97</td>
<td>-0.08</td>
</tr>
<tr>
<td>2.0</td>
<td>96</td>
<td>0.14</td>
</tr>
<tr>
<td></td>
<td>97</td>
<td>0.22</td>
</tr>
</tbody>
</table>
Since Findlay's (1951) technique is a sensitive method for measuring $\Delta P$ a calculation similar to Fooks was carried out for D region irregularities. $\mu$ was derived using the full Appleton Hartree equation and the irregularities were assumed to be of the form

$$\frac{\delta N}{N} \sim A \exp \left( \frac{(z - z_0)^2}{\sigma^2} \right) \quad (4.9)$$

$z_0$ = height of the irregularity
$\sigma$ = half width of layer
$A$ = peak amplitude = $\Delta N/N (z_0)$

The electron density profile in Fig. 2 was used and $\Delta P$ is given by the difference between the phase path calculated to a height of 90 km with and without the irregularity at some height $z_0$. Table 3 shows the changes in wavelengths of the ordinary ray for $\delta N/N$ equal to one and $\sigma = 2.5$ km.

As expected the changes are barely detectable even for a large change in $N$. For comparison Table 4 shows the calculated phase deviations for various irregularities close to the point of total reflection (assumed to be 100 km). In this situation even small changes in $N$ produce measurable changes in $P$. The closer a given irregularity is to the point of reflection
the greater the effect, although changes in the height of reflection also have to be considered.

4.3 **Phase Patterns Produced by Idealized Irregularities.**

It was shown in the previous section that changes in the refractive index are unlikely to produce significant changes in the phase paths of D region reflections so that only changes in the reflection point need be considered. This section discusses what phase path variations can be expected when the reflecting surface changes in some idealized fashion. For example, if the reflection is from a distorted surface of constant electron density it is of interest to know how the phase path varies as the distortions move overhead.

One simple model which can be easily treated is that of a sinusoidal oscillation of amplitude $a$ and horizontal wavelength $\lambda$ in a layer at height $h$. Bramley (1953), Austin (1967). If the reflections are assumed to be specular it is easy to compute the changes in range from graphical methods (Appendix 4). The necessary condition for only one specular reflecting point is that the factor $A = (\lambda^2/4\pi^2 ah)$ be greater than one (see Appendix 4). For $A < 1$ three or more specular reflecting points will occur for some positions of the
undulation and phase incoherence will be caused by the resulting interference between the components. Fig. 4 shows the phase plots for different values of \( A \). The main feature is that for \( A \) large the phase path changes are similar to the shape of the sinusoid but become more distorted as \( A \rightarrow 1 \). However for all cases \( A > 1 \) the peak to peak amplitude of the \( \Delta P \) trace is twice the peak to peak amplitude of the oscillation.

A similar case has been analysed by Munro (1953) who used a simple concave mirror type of perturbation to explain the group path changes produced by moving disturbances in the F region. Again multiple reflecting points can occur for certain cases of amplitude and horizontal scale depending on the height of reflection.

The winds in the mesosphere are known to be the order of some meters/sec and it can be expected that small irregularities will show signs of horizontal movement. It is therefore of interest to know what kind of phase pattern irregularities of this type will produce. Findlay (1953) has investigated the idealized case of a specular reflection from a "point reflector" moving horizontally overhead with constant speed \( v \). If the shortest phase path from the reflecting point to the observer is \( P_0 \), then the change in phase as a function
Reflecting Shape

Theoretical range changes produced by sinusoidal reflecting surface.

Fig. 4,

$A = 3.2$

$A = 1.6$

$A = 0.6$
of time is given by

$$\Delta P = \frac{2v^2 t^2}{P_0}$$  \hspace{1cm} (4.10)

where \( t \) is the time measured from when the irregularity is at its shortest range. Essentially the range changes parabolically with time, firstly decreasing with the rate of change of \( P \) becoming zero at \( t = 0 \) and then increasing. Findlay (1953) and Thomas and Burke (1956) have used this relationship (4.10) to measure the speed of moving clouds of ionization in E region.

### 4.4 SUMMARY

In this chapter the cause of phase incoherence was discussed. This occurs when multiple reflections from roughly the same range have comparable amplitudes. The components from each individual signal add in random fashion to produce a resultant signal whose amplitude and phase vary substantially over the total echo giving rise to phase discontinuities. Incoherence caused by interference between the magneto-ionic components was considered and shown to be of the most likely consequence at heights between 75 and 80 km where the component amplitude are of roughly equal amplitude.
In the second section deviations in the phase paths of D region reflections caused by changes in the refractive index of the underlying medium were shown to be negligible, although near total reflection even small percentage changes in electron content can produce measurable fluctuations in P.

Finally the phase changes produced by specular reflection from an undulation in a reflecting surface at constant height were examined. The phase plots are shown to be reasonable representations of the oscillation provided the tilts of the reflecting surface do not become large. If they do multiple specular reflecting points occur and interference will be produced. This is of interest to the present experiment since it shows that phase incoherence is not necessarily caused by interference between rays from separate irregularities but may be produced by multiple reflections from a single irregularity.
Phase measurements were taken from mid September 1966 to mid July 1967, observations being made in all months except January. The majority of records were taken over periods near noon but some evening and nighttime observations were also made. In general all records were made under the same conditions except that the transmitter power was down by a factor of 3 in November 1966, and June and July 1967.

This chapter is devoted to a discussion of the phase observation of echoes received from below 100 km. Since the total recording time extended over many hours every feature of the individual phase records cannot be discussed. Instead sections of records representative of each season are presented, except where special phenomena were observed.

5.1 EXAMPLES OF RECORDS.

The ionization below a height of approximately 90 km seems to be under solar control. After dawn reflections gradually appear at lower heights, the lower
boundary moving downwards as the solar zenith angle decreases. Usually near midday echoes are observed down to 70 km, but occasionally down to 60 km, particularly in winter.

5.1(a) **DAYTIME RECORD.**

Fig. 5(a) shows part of a daytime record taken in November 1966. Because the transmitter power was down on this particular day very few reflections were observed below 80 km and in this sense it is not a typical record. However it does have the advantage of clearly illustrating the terms phase coherence and phase incoherence. The discrete echo at 80 km is very phase coherent and its phase path changes as a function of time are easily measured. The totally reflected E region echo is at 105 km virtual height and is reasonably coherent although there are a number of phase jumps.

Between the 80 km echo and E region the phase structure is very random or incoherent. However a little phase continuity is exhibited, that is the phase persists for a few seconds at least before changing discontinuously. McNicol and Thomas (1960) have pointed out that since the line of sight motion of the individual scatterer is unlikely to be more than some

**FIG. 5a**

November 2, 1966. Nighttime scattering stratum +90 km.

**FIG. 5b**
PHASE PATH CHANGES MOVING IRREGULARITY
80 km, 4 NOV 1966

\[ \Delta P \] (wavelengths)

\[ t \] (mins)

\[ V = 30 \text{ m.s}^{-1} \]

FIGURE 6
meters/sec an individual contribution to the total signal will not change substantially in less than the time the reflector takes to move a tenth of a radio wavelength (a few seconds). The phase over the whole signal will not change by very much in the same time.

The phase pattern of the 80 km echo is a good example of that produced by a horizontally moving irregularity as discussed in Sec. 4.3. Fig. 6 shows the phase path changes for the whole echo together with a plot of $\Delta P$ against $t^2$. From the slope of the graph and using the relationship $4\cdot 10^{-10}$ the speed is found to be 30 m/sec.

5.1(b) NIGHTTIME RECORD.

At night there is usually strong broadcast interference because of low absorption but during the nights and evenings for which observations are available reflections were obtained from heights above about 90 km. From the observations made there appear to be two distinct types of structure present at night. Often reflections seem to come from weak, partially reflecting irregularities situated above a base at about 90 km. Fig 5(b) shows a typical phase record, the echoes being mainly incoherent. The lower boundary in this case extends to just below the 90 km marker but reflections
were occasionally observed down to 85 km. The upper boundary of the scatters is less well defined but reflections appear to terminate between 100 and 110 km.

However these echoes are often marked by very strong reflections from clouds of high electron density, presumably sporadic E layers (Smith and Matsushita (1962)). These phase records are quite coherent and they show signs of horizontal movement, usually appearing at heights between 100 and 120 km. These layers are usually transitory in nature, but the stratum at 90 km seems more permanent.

5.2 MOVING IRREGULARITIES IN THE D REGION.

Before going on to discuss the detailed structure of the ionosphere it is of interest to know over what angles reflections were effectively received. The beamwidth to the half power points of the transmitting aerial array was $50^\circ \times 30^\circ$ so that at a height of 80 km the aerial aperture would have dimensions the order of 75 by 43 km. An estimate of the actual semi-cone angle over which the transmitter and receiver effectively detect energy can be made using the speed and duration of moving reflections of the type shown in Fig. 5(a). Findlay (1953) showed that a linear
relationship existed between the speed and inverse of the duration of moving irregularities observed in the E region. The slope of the graph giving the distance over which the reflections effectively return energy. Fig. 7 shows a similar plot for moving echoes observed in the present experiment at heights between 70 and 80 km and although there is some scatter a reasonable straight line of slope 13.5 km can be drawn through the points. Taking the mean height as 75 km this gives approximately 10° as the semi-cone angle.

As far as is known this is the first time that horizontal moving irregularities have been observed directly in the D region. The maximum measured change in phase path has been only 2-3 km, implying total changes in slant range the order of 1 km, which probably explains why they have not been detected before, by even sensitive group height measurements (Gregory (1961)).

The seasonal variations in speed and scale size of moving reflections measured in the present experiment are discussed in 5.4.
Fig. 7

SPEED vs. \((\text{SEMIDURATION})^{-1}\) of Moving Irregularities (70-80 km. Region)

Slope = 13.5 km.
5.3 PHASE COHERENCE OF REFLECTIONS FROM HEIGHTS BELOW 100 KM.

In Sec. 4.1 it was pointed out that a phase coherent echo results when only one reflector at a given height is dominant. However it was shown above that reflections from scatterers in an area of radius 14 km at 75 km, and proportionally at other heights, can be detected. This suggests that coherent reflections come from either irregularities of some horizontal extent, so that they fill an appreciable part of the aperture, or they come from isolated "blobs" of electrons. Whatever the situation, observations of how the "average" coherence of the echoes at a given height changes with season should relate to changes in ionospheric structure. For example the coherence decreases from one month to the next this would imply an increase in small scale structure leading to increased scattering.

To see how the phase coherence varied with season at different heights below 100 km the records were quantitatively assessed in the following fashion. Each film record was divided into 5 km height groups, the lowest commencing at 70 km, and into consecutive 2 minute intervals. The individual echoes in each group and interval were classified as being:-
(a) **Phase Coherent**: if the phase over the echo at any instant was constant and showed good continuity with time.

(b) **Partially Coherent**: if some phase pattern were obvious but showed a number of discontinuities.

(c) **Incoherent**: no apparent phase pattern.

### TABLE 5

**PERCENTAGE TIME ECHOES PRESENT BELOW 80 KM FOR HOURS NEAR NOON.**

<table>
<thead>
<tr>
<th>MONTH</th>
<th>70-75 KM</th>
<th>75-80 KM</th>
<th>REMARKS</th>
</tr>
</thead>
<tbody>
<tr>
<td>SEPT 1966</td>
<td>85</td>
<td>100</td>
<td></td>
</tr>
<tr>
<td>OCT</td>
<td>65</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>NOV</td>
<td>10</td>
<td>50</td>
<td>Tx power down</td>
</tr>
<tr>
<td>DEC</td>
<td>75</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>FEB 1967</td>
<td>75</td>
<td>95</td>
<td></td>
</tr>
<tr>
<td>MARCH</td>
<td>32</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>APRIL</td>
<td>-</td>
<td>90</td>
<td></td>
</tr>
<tr>
<td>MAY</td>
<td>60</td>
<td>75</td>
<td></td>
</tr>
<tr>
<td>JUNE</td>
<td>85</td>
<td>100</td>
<td>Tx power down</td>
</tr>
<tr>
<td>JULY</td>
<td>90</td>
<td>100</td>
<td>Tx power down</td>
</tr>
</tbody>
</table>
For any particular height range estimations of coherence were only made when echoes were present. As Table 5 shows this was of consequence only for reflections between 70 and 75 km. To lessen any dependence on solar zenith angle only records taken for at least an hour near midday were evaluated.

For easier presentation of results a "coherence coefficient" was devised. The coherence types (a), (b) and (c) were given numerical weights of 2, 1, and 0 and if the numbers of each type for a given range and record were found to be $n_1$, $n_2$ and $n_3$ respectively the coefficient $c$ was given by

$$c = \frac{2n_1 + n_2}{2(n_1 + n_2 + n_3)}$$

The closer $c$ is to unity the more coherent the echoes in the total time interval $2(n_1 + n_2 + n_3)$ minutes. The monthly plots of coherence coefficient against height are shown in Fig. 8 and the values are replotted in Fig. 9 for each height range as a function of time. The vertical bars show the standard deviation of the daily values of $c$ for each month.

The main features of the coherence plots are:
Monthly plots of coherence coefficient as a function of height

FIGURE 8
Figure 9

Coherence plots as a function of time

<table>
<thead>
<tr>
<th>Season</th>
<th>70-75 km</th>
<th>75-80 km</th>
<th>80-85 km</th>
<th>85-90 km</th>
<th>90-95 km</th>
<th>95-100 km</th>
</tr>
</thead>
<tbody>
<tr>
<td>S</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>O</td>
<td></td>
<td></td>
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<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>N</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>D</td>
<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>J</td>
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<tr>
<td>F</td>
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<td>M</td>
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<tr>
<td>A</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>J</td>
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<td></td>
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<td></td>
<td></td>
</tr>
<tr>
<td>J</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Coherence plots as a function of time

FIGURE 9
(i) There is in general fairly steady decrease of coherence with height, echoes from above 85 km being mainly incoherent at all times.

(ii) Seasonal variations in coherence are strongest for reflections from altitudes below 85 km. Echoes, when they are present, being most coherent in September to November (Southern Hemisphere Spring) and in March (Autumn). In local summer the echoes tend to be more incoherent but not to the same extent as they are in winter, when coherence falls to zero in the 70-80 km range in June.

Although the coherence coefficient is a somewhat crude device it does summarize the phase information contained in the records. If as, discussed above phase coherence can be taken as a reasonable index of ionospheric structure, it would seem that above 85 km the structure is random at all times of the year, and below this height at the solstices.

5.4 PHASE PATTERNS OF LOWER IONOSPHERE REFLECTIONS.

5.4(a) SPRING AND AUTUMN.

An example of a phase record made in spring
(Fig. 5(a)) has already been discussed in 5.1, the main features being the very coherent phase pattern caused by a moving reflector. Echoes of this type were only observed in spring, autumn and to a lesser extent in summer. The measured speeds ranged from 20 m/sec. (Sept., Oct. and March) to 100 m/sec. (late February), the most probable velocity being about 60 m/sec.

The phase patterns observed in spring (September to November) and early autumn (March) often suggested the reflections came from drifting patches of ionization. Figs. 10 (a) and (b) show typical records made in September and October with phase coherent echoes from near 75 km, the tilted fringes suggesting irregularities drifting overhead with quite low velocities. Reflections are coherent for periods up to 15 minutes or more and even with the lowest observed speed of 1 km/min (17 m/sec) this suggest horizontal scales the order of 15 km at least. These dimensions are possibly an underestimate since as the irregularities pass overhead the reflections often become incoherent for short intervals. It is difficult to decide whether the incoherence is caused by multiple reflections from the same irregularity or from separate patches. However the inferred scales are consistent
Phase-path records showing discrete coherent reflections occurring between 70 & 80 km, the downward tilted fringe patterns indicating decreasing phase paths.
with the belief that phase coherent reflections come from irregularities having some horizontal extent. As well as the moving irregularities stable reflecting regions also seem to exist at the equinoxes. Fig. 11(a) shows a record made on the 26th October 1966, the discrete, phase coherent echo between 70 and 80 km shows a distinct oscillation over a period of roughly 10 minutes. Fig. 11(b) shows the plot of its phase path changes while it remained coherent, the fluctuations being quite slow. This phase record was terminated by a period of incoherence and then followed by a further period of slow phase changes at the same height. The same type of echo structure was also evident on the 25th and 27th October, suggesting the reflections came from a stable stratification having an undulating lower boundary.

For the particular phase record shown in Fig 11(a) a comparison can be made with measurements of the fading period of the echoes made using the spaced receiver experiment at Birdling's Flat (Fraser (1965a)). This technique measures the drifts of partially reflecting irregularities at various discrete heights in the D region by the well known Mitra (1949) method. For a single receiver if the amplitude as a function of time is \( A(t) \) then the autocorrelation coefficient as a function of lag \( \tau \) is defined as,
October 26, 1966. Phase coherent echo at 75km with undulating phase pattern. Incoherent sporadic-E reflection at 100 km.

FIG. 11a.

Phase path changes of 75km echo. 26 October 1966

FIG. 11 b.
\[ r(\tau) = \frac{\langle A(t) A(t + \tau) \rangle}{\langle A(t)^2 \rangle} \]

\( \langle \rangle \) denotes time average.

By definition \( r(0) = 1 \) and the fading period is given by the lag \( \tau_0 \) such that \( r(\tau_0) = 0.5 \).

**TABLE 6**

<table>
<thead>
<tr>
<th>HEIGHT (KM)</th>
<th>70</th>
<th>75</th>
<th>80</th>
<th>85</th>
<th>90</th>
<th>95</th>
</tr>
</thead>
<tbody>
<tr>
<td>Fading period (sec)</td>
<td>5</td>
<td>4</td>
<td>3.2</td>
<td>1.3</td>
<td>0.8</td>
<td>1</td>
</tr>
<tr>
<td>Coherence coefficient</td>
<td>0.9</td>
<td>0.8</td>
<td>0.5</td>
<td>0.1</td>
<td>0.08</td>
<td>0.1</td>
</tr>
</tbody>
</table>

Table 6 shows the fading rates as measured by the drifts experiment and the coherence coefficients obtained from the phase records taken over half an hour, the phase structure not changing significantly in that time. The echo amplitudes were sampled at 5 km height intervals in sequence for periods of 5 minutes each.

The table shows the high correlation between
fading period and coherence at each height. This is to be expected since the more scattering irregularities present the more phase incoherent the resultant echoes and the higher the fading rates. Although this is the only comparison available at present, it does give some confidence in the use of the coherence coefficient as a measure of ionospheric structure.

A discrete phase stable layer at 75 km was also a feature of one of the records made in early May. The phase path changing slowly over a few minutes. However the strength of the reflections was appreciably less than the corresponding echoes observed in October. This weakness in signal strength seems a characteristic of the echoes returned from below 80 km in late autumn. No echoes were observed below 75 km on the 3 observing days in April for instance. The echoes were also more incoherent than those observed in March.

5.4 (b) SUMMER.

Although a few coherent morning irregularities were observed the echoes from all heights in summer were mainly incoherent. Fig.12(a) shows part of a record made in early February, which is also representative of the December record.
February 9, 1967. Typical summer conditions. Total reflection at 110 km, partially reflecting stratum at 92 km, continuous incoherent echoes down to 70 km. Note coherent 'sea return' below 70 km.

FIG 12a

JUNE 16, 1967. Phase and h(t) records showing strong reflecting region at 82 km. Note coherent region E echo replaced by incoherent Es reflection

FIG. 12 b.
The regular downward tilted fringe pattern at the bottom of the record is caused by reflections from sea waves. Because of the closeness of the field site to the sea, "sea return" is often obtained out to ranges of 75 km. Sea echoes are always distinguishable from the ionospheric reflections by the phase pattern shown in this case.

Because no phase records were obtained in January care must be taken in assuming the "random" structure shown in Fig. 12(a) is typical of the summer months but the similarity between the early February records and December records suggests it is. At the end of February more coherent echoes were observed at the lowest heights and there was a gradual change into the coherent structure typical of March.

The general weakening in the strength of reflections already noted for autumn was also observed in summer, although not to the same extent.

5.4(c) WINTER.

Despite a decrease in transmitter power the winter months of June and July were characterized by a sharp increase in echo strength and incoherent phase
records, particularly between 70 and 80 km. A distinct stratified region also seemed to exist between 80 and 85 km. The type of reflections observed are best typified by Fig. 12(b) which shows a phase record taken in conjunction with a swept gain (h'(t)) recording. The h'(t) record was made by intensity modulating the oscilloscope with the detected output of the receiver and automatically decreasing the receiver gain by 10 steps of 6 dBs each and the last one of 12 dBs.

By noting the difference in gain reductions required to remove the E region echo and a given lower echo the h'(t) records were used to estimate the effective voltage reflection coefficients of the lower reflection. The E region signal was also corrected for the absorption it suffered, this being found by comparing the amplitude of the E layer echo with that of its first multiple reflection (Beynon and Brown (1957)). Absorption suffered by the lower echoes was neglected.

Swept gain records can also be used to estimate the "thickness" of the reflecting regions by measuring how much their lower boundary moves up as the receiver sensitivity is decreased. This"apparent rise h" is due partly to the finite rise time of the transmitter pulse (2 km) and to the scattering charact-
eristics of the layer (Gregory (1964)). If the region effectively scatters power back over a height interval \( \Delta h \) the total upward movement will be

\[
\Delta h' = \Delta h + 2 \text{ kms}
\]

The outstanding features of Fig. 12(b) are the region E echo at 110 km and the strong reflection with its base at 82 km. On this occasion the measured reflection coefficient was \( 10^{-3} \) and the layer "thickness" approximately 2 km. The 80 km echo is also very apparent in the accompanying phase record, its fringe showing phase continuity for periods up to 1 minute. In comparison the echoes above and below are very incoherent and diffuse.

The 80 km region was observed near noon on 13 out of 17 observing days in June and July. Its lower boundary was normally near 85 km and on any particular day the phase records showed that it remained at the same height (with in \( \pm 2 \) km) over the hours that observations were taken. In general the measured reflection coefficients were the order of \( 10^{-3} \) and thickness ranged from 1 to 5 km (2 km on the average). On the days that a distinct 80 km region was not observed at midday (Fig. 13) it tended to appear near ground sunset as the lower
July 7, 1967. Total reflection occurs at 120km and continuous reflections down to 70km are present.

FIG. 13
echoes disappeared. This suggests it had a continued existence but was marked by stronger scattering from lower heights.

As far as could be ascertained no long enduring regions with recognizable lower boundary were observed in the 80-90 km altitude range in other seasons, although partial reflecting stratifications (presumed to be sporadic E) were often observed between 90 and 100 km.

As noted above the strength of the reflections received from below 80 km was variable and the echoes generally incoherent, although they were slightly more coherent in July than in June and on occasions stratifications similar to the 80 km region seemed to be present. The average reflection coefficient for the lower echoes was about $10^{-4}$.

5.5 **EFFECTS OF INTERFERENCE BETWEEN THE MAGNETO-IONIC COMPONENTS.**

Evidence presented in the above sections shows that the reflections from the 70-80 km region are more phase incoherent in summer and winter than the equinoxes. In Sec. 4.1 it was shown that when
recording was made using a linear polarized receiving array some incoherence could be caused by interference between the magneto-ionic components. To see what effect this interference had, recording using only the ordinary or the extraordinary rays were made from time to time.

Fig. 13 shows a winter record made with the 'o' and 'x' components respectively and for comparison a recording made shortly before using a linear polarization. It is noticeable that the echoes on the x record start at 70 km, with a little phase persistence at this height, but the 'o' record echoes only commence at 75 km, that is, below 75 km the extraordinary component is the stronger. (The actual measured ratios were $A_x/A_o = 2$ at 72 km and 0.8 at 77 km). The 'x' component is normally heavily absorbed above 90 km and the phase patterns observed above this height in the 'x' record are caused by leakage of the 'o' component into the system (the rejection was about 30 dB).

The records made with the linear array are similar to those of the 'x' component below 75 km and is similar to the 'o' record above this height and on all other days that comparisons were made the same situation prevailed, the lowest phase records were those of
the 'K' ray. As this example shows the increased incoherence between 70 and 80 km in winter is not caused by increased interference between the magnetoionic components and similar records made in summer indicate the same thing. The increased incoherence is therefore caused by a change in ionospheric structure.

5.6 COMPARISON WITH OTHER RESULTS.

At the time of writing only one other study of phase path measurements from the lower ionosphere is known. Smith et al (1965) briefly mention a phase experiment conducted in 1962 at Armidale (31°S), Australia. They report very phase coherent reflections in winter from a stable discrete stratum situated at 72 km; phase path changes the order of one radio wavelength in 30 minutes being recorded. Otherwise there was a remarkable absence of reflection below 90 km except for occasional reflections from another stable region at 67 km. These findings are very opposite to what the present work has shown to be the case in winter at Birdling's Flat. However these phase stable echoes seem to be similar to the discrete reflections observed in October and May. In summer Smith et al report a more random phase structure which agrees with the present work. They do not mention
Observing any extended moving irregularities as reported here.

Very extensive measurements of the heights of occurrence and strengths of partial reflections have been reported by Gregory (1956, 1961, 1964) who used a swept gain technique at a frequency of 1.75 mHz. In winter and spring Gregory observed reflections down to 55 km but during late summer and autumn there was a decrease in the number of reflections from the lowest heights, which seems to agree with the present findings. From mid-May to August Gregory reports a marked increase in the strength and thickness of reflecting regions between 70 and 80 km. The phase incoherence of the lower echoes confirms Gregory's statement that "scattering in depth" is a feature of the winter mesosphere.

This strengthening of reflections in winter is also observed in temperate latitudes in the Northern Hemisphere. (Dieminger (1955), Belrose (1963)). The latter states that amplitudes of scattered echoes near 80 km show a strong day to day variability, partly due to increased gradients in the N(h) profiles but mainly due to an increase in irregularities.

Although the increased strength of winter
reflections seems mainly due to more irregularities, the ionization in the D region can increase sufficiently to produce marked absorption of radio waves (Thomas (1961), Gregory (1965)). As far as is known none of the measurements in the present experiment were made on days of "high absorption" (Thorpe (private communication)). However even on "quiet days" the winter N(h) densities are anomalous. Manson (1965) comparing average quiet summer and winter profiles measured at Birdling's Flat reports somewhat high densities in the 80-90 km region in winter but higher densities in summer below 80 km. The N(h) values in winter are however much larger than the production rates predict.

From the above discussion there seems general agreement that there is an increase in small scale ionospheric structure in winter leading to increased scatter. There is less information on the scale sizes of the irregularities at other times of the year. Gardner and Pawsey (1953) from observations of the fading characteristics of the partial reflections from 70 km suggest that these echoes are of some horizontal extent (not less than 1 km) and from only one or two irregularities at a time. It is of interest that Gardner and Pawsey made most of their measurements
in autumn and spring and it is in these periods that the greatest phase coherence at 70 km was observed in the present experiment. Fraser (private communication) also reports slow fading echoes at the equinoxes as measured using the spaced receiver experiment. In general this suggests that large scale irregularities are a feature of the lower D region in the spring and autumn.

5.7 SUMMARY.

From the phase height measurements it appears that there is a distinct seasonal variation in echoes structure at the lower height of observation (70-80km). In spring and to a lesser extent in autumn the lowest reflections are often discrete and phase stable. From their phase path changes these reflections seem to come from drifting patches of ionization having some horizontal extent. On the other hand at the solstices the echoes are more incoherent, particularly in winter. This has been interpreted as meaning an increase in scattering irregularities.

At heights above 85 km the phase records are generally incoherent at all times, except that between
95 and 100 km occasional phase coherent sporadic E layers are observed. This phase incoherence seems to persist into the night when a weakly scattering stratum is observed between 90 and 110 km (as well as $E_s$ layers up to 120 km). Layer type regions also seem to exist in the lower D region. The phase and swept gain records suggest the presence of one such a region near 85 km in winter which scatters power back over heights of 2-3 km. As well as this stratification very phase stable reflections also occur in the equinoxes from more sharply bounded irregularities near 70 km.

The general phase stability of the lower reflections at the equinoxes supports Gardner and Pawsey's (1953) conclusions that these reflections come from irregularities which are anisotropic (horizontal extent much larger than vertical) and that the use of Fresnel reflection coefficients, which assumes reflections from regions having dimensions comparable with the 1st Fresnel zone, $\sim 3$ km) is valid approximation. On the other hand the very incoherent reflections observed in winter suggest that reflection coefficients based on Booker scatter theory (Booker (1959)) are of more use (Belrose and Burke (1964)). In the latter theory the dominant
scatterers have spatial periods the order of half a radio wavelength (~60 km).

Overall the phase records suggest that at heights near 70 km the structure of the ionosphere varies with season, a more random structure existing at the solistics and at the equinoxes. Above 85 km the structure is mainly random at all seasons.
6. **LOWER IONOSPHERIC IRREGULARITIES.**

This chapter attempts to analyze some of the phase observations of D region irregularities in terms of the theories proposed for their formation.

Because of the weak strength of the partial reflections it is inferred they come from irregularities whose electron densities are only a few percent different from the background concentrations. Such inhomogeneities can be generated by the turbulent processes mechanically redistributing the ionization. (Gardner and Pawsey 1953, Gregory 1961). However it is also possible for density perturbations to be produced by more organised motions, such as propagating atmospheric waves. Hines (1960) has proposed that one such wave type, internal atmospheric gravity waves (Section 6.2) can cause many of the observed irregularities in the upper atmosphere, including some originally ascribed to turbulence.

One notable feature of the phase measurements is the seasonal fluctuations in echo phase coherence of reflection from 70 \( \mu \text{m} \), this aspect is considered when the possible seasonal fluctuations in gravity wave energy and turbulence generation are...
ZONAL WIND COMPONENTS (M. SEC^-1) AT 45°S
From Murgatroyd 1965
FIG.14.
examined in general - Section 6.4. Since the meteorology of the middle atmosphere is of some importance this is outlined in Section 6.1.

Some of the more specific phase observations such as the nighttime observations, are discussed in Sections 6.4 to 6.6.

6.1 METEOROLOGY OF THE ATMOSPHERE BETWEEN 30 and 100 KM.

6.1(i) WINDS.

Below 75 km the winds are generally west-east orzonal in direction, the main features being summarized in Fig. 14, which is taken from a meridional cross section presented by Murgatroyd (1965). In both winter and summer the winds peak near 65 km, but are westerly (from the west) in winter and the reverse in summer. At the equinoxes the winds are generally very light over most of the lower atmosphere.

Above 75 km the meridional winds grow rapidly and can become larger than the zonal winds (Groves (1964), Nordberg et al (1965)). Much of this growth is due to stronger solar tidal components but regular motions equatorwards in winter and polewards in summer
Temperature profiles from rocket-grenade experiments at Wallops Island, 38°N. Individual profiles after Nordberg and Smith (1963). Seasonal averages from Nordberg et al (1965)

Fig. 15.
have been inferred by Kochanski (1963).

6.1(ii) **TEMPERATURES.**

Reliable mid-latitude temperature measurements came from rocket grenade soundings. To illustrate the main difference between the winter and summer temperatures, Fig. 15 shows both individual and average profiles presented by Nordberg and Smith (1963), Nordberg et al. (1965). These measurements were made at Wallops Island (38°N) and should indicate likely variations at corresponding latitudes in the southern hemisphere.

The average profiles show that the mesosphere is warmer in winter than in summer, the maximum difference (about 35°K occurring at 85 km (the mesopause). Conversely, the temperatures at the stratopause vary in the opposite manner with season, so that the average temperature gradients are -3°K/km in summer and -1°K/km in winter (Webb (1966)).

However comparison of individual soundings show that the most striking disparity between seasons is in the stability of the profile. The summer profiles vary smoothly with height (up to 80 km) and show
consistency from day to day. In winter the profiles often exhibit multiple peaks or inversions which seem to be unstable (e.g. compare the profile made on March 1 and March 2 1962).

6.2 INTERNAL GRAVITY WAVES.

In this section the properties of gravity waves are briefly reviewed, (a more detailed analysis and derivation of required equations is given in Appendix 5). Internal atmospheric gravity waves (i.a.g.w) are a class of hydrodynamic waves, (similar to sound waves) whose spatial scales vary from a few hundred meters up to a few hundred km and whose periods range from some minutes up to hours. As they propagate through the atmosphere they perturb the air particles producing fluctuations in density and pressure and cause horizontal and vertical wind motions - (Fig.16).

The dispersion relation which links the frequency and horizontal and vertical wavenumbers of a wave propagating in a motionless atmosphere can be approximated by:

\[ k_z^2 = k_x^2 \left( \omega_B^2 - \omega^2 \right)/\omega^2 \]  

(6.1)
Instaneous velocity vectors depicted by arrows, density variations are depicted by a background of parallel lines lying in surfaces of constant phase. Deformations in level of constant electron density shown by solid line.

REPRESENTATION OF A GRAVITY WAVE (AFTER HINES, 1960)

FIGURE 16
Internal gravity waves exist when $\omega^2 < \omega_B^2$ i.e. $k_z^2 > 0$

where $\omega_B^2 = g(\Gamma + dT/dz)/T $ \hspace{1cm} (6.2)

$\omega_B$ = Vaisälä Brunt frequency
$\Gamma$ = adiabatic lapse rate (+ 100 K/km)
$T$ = temperature

$\omega_B^2$ can be considered a measure of the stability of the atmosphere (Eckart 1960) and is shown as a function of height in Fig. 17 using temperature data taken from the U.S. Standard Atmosphere (1962) (with suitable smoothing).

The main features of gravity waves can be summarized as:

(i) If $k_z^2 < 0$ over a substantial part of the atmosphere the wave is reflected at the height where $k_z^2 = 0$.

(ii) To ensure that the wave kinetic energy (proportional to the atmospheric density) remains constant, the wave parameters vary as $\int dz/H$ in the vertical direction. This means a wave propagation upwards is
HEIGHT DISTRIBUTION OF VAISALA FREQUENCY

FIGURE 17
amplified until eventually the wave is no longer of perturbation magnitude and it breaks down. However viscous damping forces, such as molecular viscosity and thermal conductivity, also act to limit wave growth. Essentially the smaller scale modes are affected most and there is a scale at any given height, below which waves cannot exist (Pitteway and Hines 1963, Golitsyn 1965, Midgley and Liemohn 1966).

(iii) In a moving atmosphere the wave frequency relative \((\omega)\) to an observer on the ground becomes
\[ \nu = (\omega - k_x U_0) \]
in the coordinate system moving with the mean wind, which has speed \(U_0\) in the direction of horizontal phase progression. \(\nu\) replaces \(\omega\) in (6.1). If the phase velocity of the wave \((\omega / k_x)\) matches \(U_0\) the effective wave frequency \(\nu \rightarrow 0\) and there is a singularity in (6.1) and \(\lambda_z (2\pi/k_z)\) becomes small, the wave travelling horizontally. Analysis shows that at this singular or critical level the wave is removed from the overall wave spectrum (Hines and Reddy, (1967), Booker and Bretherton (1967)). Also waves propagating against the mean flow have their \(\nu\) raised and it is possible for it to become greater than \(\omega_B\) producing reflection. Hence background wind shears and temperature gradients can severely modify and initial spectrum of waves, those waves having critical levels
being removed from the spectrum. Since wave modes with azimuths of wave progression perpendicular to the mean flow are least affected the ambient winds tend to produce directional filtering.

As they propagate through the ionosphere gravity waves will produce electron density fluctuations which can give rise to partial reflections and it is of interest to the present experiment to know which waves are the most important for radio soundings. The dominant gravity waves at 90 km seem to have horizontal wavelengths the order of a few hundred km. (Hines 1960, 1963a), but Hines (1960, 1963a, 1963b) considers that medium frequency radio techniques are selective and respond most to the presence of small scale modes. Qualitatively this is because they produce the largest horizontal gradient of electron density. As far as considerations of multiple scattering, and hence phase incoherence are concerned the smaller scale modes should be the most important. As was discussed in 4.3 wave like perturbations in levels of constant electron density produce multiple reflecting surfaces when

\[ a > \lambda_x^2 \left(4\pi^2 h \right)^{-1} \]
The smaller the wavelength $\lambda_x$ the smaller the value of amplitude required. For the present study the spectrum of waves which will influence phase coherence most will probably have spatial scales ranging from approximately 0.5 km up to about 25 km. The lower limit is set by the effects of viscous damping (Pitteway and Hines 1963) and the upper limit by the estimated diameter of the cone of acceptance at this height. Since the periods would be 10 minutes or greater these waves will have quite small horizontal phase velocities ($V_x \approx 30\text{m/sec}$). The seasonal variations in the phase coherence may therefore be partly explained by seasonal changes in the spectrum of gravity waves at D region heights. The more small scale waves present the greater the phase incoherence expected from multiple scatter.

6.3 TURBULENCE.

The existence of turbulence in the mesosphere and lower thermosphere is inferred from the anomalous growth of meteor and vapour trails, whose rate of expansion is larger than can be supported by molecular diffusion alone. Turbulent expansion ceases somewhere between 100 and 110 km (Blamont and Jaeger 1960, Bedinger and Knaflieh 1966).
Since turbulence is dissipative it requires an external energy source. This is usually taken to be the mechanical breakdown of shear or the modification of vertical motions due to buoyancy forces (Nawrocki and Papa (1963)). These factors are combined in the Richardson expression

\[ R_i = \frac{\omega_B^2}{(\partial u_x / \partial z)^2} \]

where \( \partial u_x / \partial z \) is the mean vertical shear of the horizontal wind.

To maintain turbulence values of \( R_i \leq 1 \) seem reasonable but to actually generate turbulence critical values the order of \( R_i \approx 0.04 - 0.08 \) may be required (Townsend 1958, Layzer 1962, Nawrocki and Papa 1963). The lowest value of the Vaisala frequency \( \omega_B \) occurs near 70 km (Fig. 17 which is probably indicative of average conditions in summer) with \( \omega_B \approx 0.016 \) sec\(^{-1}\). To generate turbulence shear values as large as 0.08 sec\(^{-1}\) may be required (if Layzer's criterion is correct). The largest observed shears seem to be about 0.05 sec\(^{-1}\) (Kochanski, 1964) although there is no differentiation between summer and winter conditions. The mean shears are about 0.01 sec\(^{-1}\), although Roper (1966a) considers that even these may
be too large for average conditions. If values of \( R_i = 1 \) are required to produce turbulence then the observed shears are just adequate near 70 km. Values of \( R_i \) as small as one are only observed again near 105 km (Justus 1967). Since observed shear values seem inadequate to account for the observed turbulence small scale variations in temperature have to be considered (see Section 6.4).

It is noted that because gravity waves provide shears they can be expected to produce turbulence and it has been suggested (Hines 1963a) that small scale modes may be particularly important in this respect. As consideration of 6.1 shows waves of small period (10 - 20 minutes, \( \omega \approx \omega_B \)) will have comparable horizontal and vertical wavelengths and hence wind systems which can be titled at angles up to \( 45^\circ \). Because of this Hines suggests that the strong vertical winds may act to overcome the stabilizing influence of gravity and, in conjunction with the horizontal shears generate turbulence more easily than by shears alone.

6.4 SEASONAL VARIATIONS OF SCATTERING IN IONOSPHERIC IRREGULARITIES.

In Section 6.2 the aspects of gravity wave
theory pertinent to the present discussion were outlined. In particular it was suggested that some of the seasonal fluctuations in phase coherence may be explained by corresponding variations in the spectrum of small scale modes \( \lambda_x \approx 10 \text{ to } 20 \text{ km} \) at mesospheric heights and possibly the amount of turbulence if Hines' hypothesis is correct. Recent evidence shows that these waves may be produced both in the ionosphere itself and in the lower atmosphere. In the following sections each of the sources and the probable seasonal variations of wave activity are discussed in turn.

Internal waves have been shown to exist in the troposphere (Gossard (1963), Gossard and Munk (1953)) and this is a potentially important source for waves in the ionosphere because of the large decrease in density over the height range. The amplitudes of waves propagating upwards can be expected to increase by a factor of 350 between the ground and 75 km (providing there is no dissipation or reflection). The flux of energy out of the troposphere has been shown to be more than sufficient to account for the observed waves in the ionosphere (Gossard (1962)) and that the outward flux peaks at periods between 10 and 20 minutes. Furthermore the observed phase velocities are small, \( V_x \approx 20 \text{ m/sec} \), (Gossard (1962), Gossard and Munk (1954))
and although they only made a limited number of observations this suggests that an appreciable amount of the energy lies in short period low phase velocity waves. This may not apply to any waves generated by the troposphere jet streams.

Recently Hines and Reddy (1967) have calculated the energy transmission coefficients for gravity waves propagating upwards to the lower ionosphere through typical middle atmospheric wind and temperature systems (their model WT-2). Their results for small scale, low phase velocity modes may be summarized as:

(i) There is increased transmission of waves to 80 km at the solistices

(ii) The spectrum is anisotropic in azimuth (provided the incident flux is isotropic). Those waves propagating in a westerly direction having critical levels are removed, leaving a spectrum with a westerly bias and similarly an opposite bias in summer.

At the equinoxes practically no energy in slow phase velocity waves reaches the 80 km region because of
strong reflections from the stratospheric and mesospheric temperature gradients, the reflection coefficient being proportional to the ratio of the change of sound speed to the effective phase velocity \((v/k_x)\). As discussed in 6.2 waves travelling against the wind have their effective frequency raised, implying increased transmission when the winds are strongest.

The temperature model used by Hines and Reddy (1967) is more representative of summer conditions. In winter when the average lapse rates are lower a marked increase in transmission of small scale modes can be expected.

As well as being generated in the troposphere waves may also be generated in the ionosphere. Lindzen (1967) in very thorough analysis of the diurnal tide suggests that its temperature fields become unstable near 90 km at mid-latitudes, generating both turbulence and gravity waves. Good experimental evidence supports this theory, meteor trail measurements of diurnal tidal winds at Adelaide (35°S) show an amplitude growth of only 1.2 between 83 and 97 km whereas from density decrease amplifications of the order of 3.2 are expected (Roper (1966b)). Over the same height range the semi-diurnal tide grows unim-
peded. Furthermore seasonal fluctuations in turbulence power correlate with the variation in the strength of the diurnal tide (Roper (1966c)), and there should be corresponding changes in the gravity wave flux from this source.

Since the diurnal tide is present at all seasons it should be a constant source of gravity waves in the ionosphere. However the flux of wave energy may be seasonally dependent. Elford (1959) reports the amplitude of the diurnal tide is strongest in late summer. Wave generation in the mesopause itself has also been proposed (Hines (1960), MacDonald (1963)), and this appears likely in winter. Webb (1965) reports an increase in small scale wind structure above 55 km in winter as measured by falling sphere observations. He interprets this as indicating a source of waves or turbulence in the winter mesosphere. The wave generating mechanism is probably associated with the unstable winter temperature profiles, possibly with the inversion since the presence of these seems a pre-requisite for internal wave generation in the troposphere (Gossard (1962)).

The seasonal variations in turbulence must also be considered. Above 85 km the temperature
instabilities caused by the diurnal tide seems to account for the turbulence observed in the lower thermosphere. However as discussed in Section 6.3 the observed shears in the D region may be too small to generate turbulence. This will not be true in winter. As Fig. 15 shows on some winter days over certain height ranges the lapse rates approach the adiabatic. Under these conditions convective turbulence seems inevitable (i.e. $\omega^2_\text{B} \to 0$) which probably accounts for the day to day variability in scatter in winter (Gregory (1961)).

The above has been a general discussion of the sources of gravity waves and turbulence. However it suggested that on a qualitative basis the general phase incoherence of reflections from 70 km in summer may be accounted for by an increased flux of small scale waves propagating upwards from the lower atmosphere. The downward propagation of wave energy from the lower thermosphere may also be strongest in the summer months.

The very strong incoherent reflections in winter seem more than accounted for by the increased generation of turbulence and waves in the mesosphere. Furthermore there will be an increased propagation of
waves upwards from the troposphere, particularly since wave generationmaybe strongest in the winter (Hines (1963a)).

The general low coherence of partial reflections from above 85 km seems caused by scatter from small scale turbulent irregularities generated by tidal breakdown near 90 km.

One aspect not discussed so far is the directional filtering imposed on small scale modes and the consequence for spaced receiver experiments. These are used to measure the drifts of ionospheric irregularities or, more correctly, the radio wave diffraction patterns they produce on the ground. As they propagate through the ionization gravity waves will produce apparent movements in the diffraction patterns, the velocities derived will be horizontal phase velocities of the waves and not that of the true wind (Hines (1960, 1963a, 1963b, 1964)). If the waves are normally randomly orientated the mean wind direction can be extracted by averaging a number of individual drift observations, the gravity waves merely adding to the variance of the mean. However if there is a seasonal bias in azimuth the averages will show variations in response to the wind structure at underlying levels and
not at the height of observation. Spaced receiver observations of D region drifts have been reported by Fraser (1965a, 1965b, 1966). Since his measurements were made above the peak of the stratospheric winds directional filtering should be strong and the effects might appear in the records. Instead Fraser reports the directions of the mean monthly drifts to be in good agreement with meteor and rocket observations of mesospheric winds. However as discussed above gravity waves are generated in the ionosphere itself and since these waves are most likely to be produced by wind shear they will travel with the wind (Martyn (1950)). In which case the directional filtering imposed on wave propagation upwards from the lower atmosphere will be downgraded.

It is emphasized that this discussion has been mainly general and does not seek to explain everything. One particular anomaly is why the reflections are weaker in autumn than in other seasons? Perhaps this may in some way be associated with the height at which gravity waves become large enough to produce either turbulence and or density perturbations.
6.5 ANISOTROPIC IRREGULARITIES.

The equinoctial phase records show that the reflections from the 70 km region often come from irregularities which are appreciably anisotropic (their horizontal dimensions are much larger than their vertical). To explain similar observations of anisotropy (Gardner and Pawsey 1953, Fejer and Vice 1959) Hines (1960) suggested the scattering was caused by electron density perturbations produced by gravity waves with spatial periods of half the radio wavelength \((\lambda \approx 70 \text{ m})\). However it is most doubtful that such waves could exist at D region heights. Examination of later calculations of viscous damping effects (Pitteway and Hines (1963)) show that at 70 km only waves with wavelengths 200 m or greater would not be severely damped.

There seems little doubt that these phase coherent reflections are from large scale irregularities, dimensions of 15 km being inferred (Sec. 5.4). If these measurements are confirmed these moving irregularities seem much larger than have been observed before in the D region. Furthermore the measured speeds are lowest in early spring and autumn, at which times the mesospheric winds are lowest, which suggests irregular-
ities are drifting with the mean wind. If these irregularities are generated by turbulence their dimensions seem appreciably larger than the upper limit of 3 km for the turbulence spectrum which Hines (1963a) suggests. However his calculations were for the 90 km region and may not apply to the 70 km region.

Some idea of the electron density changes involved can be found using theoretical Fresnel reflection coefficients which are proportional to change in electron concentration across a boundary (Gardner and Pawsey (1953)). Comparison of the measured reflection coefficient of these irregularities (≈10^{-4}) with calculated Fresnel coefficients (Manson (1965)) using electron densities appropriate to the 70 km region, it would seem that approximately 10% changes in electron concentration are required, neglecting possible collision frequency changes (Manson (1966), Piggott and Thrane (1966)). If these density fluctuations are associated with gravity waves then the wave motions are becoming appreciably non-linear (Hines (1960)).

Obviously these irregularities need to be studied further before any firm conclusions can be drawn. Because of their coherence the reflections from
these irregularities should be particularly suitable for direction finding techniques and it is hoped to carry out such an experiment in conjunction with the phase observations.

6.6 NIGHT OBSERVATIONS.

The phase observations made at night indicate the presence of a weak scattering layer existing between 90 and 100 and 110 km. This stratum also seems to be a feature of low frequency ionograms, appearing as patchy layers with very little group retardation (Belrose (1963)).

Electron density measurements of the night E region (Smith L.G. (1966), Bowhill (1966), Smith et al (1965), show that between 80 and 90 km there is a rapid increase in electrons from low values near 85 km to values the order of $10^3 \text{ cm}^{-3}$ at 90 km. It would appear that the lower boundary of the stratum coincides with a natural boundary in ionization. However it seems more difficult to similarly explain the upper height limit of the stratum. The rocket profiles are somewhat contaminated by sporadic layers but generally the measured densities remain near 1000 cm$^{-3}$ up to about 120 km. Above this height they decay
to reach values the order of a few hundred per cm$^3$. (Smith L.G. (1966). Hence there seem adequate densities above 110 km to support scattering, provided sufficient irregularities are present. It is suggested that the cut-off is related to the 'turbopause' which is such a feature of sodium trail observations of thermospheric winds. (Blamont and Jaeger (1960), Bedinger and Knaflich (1966)). A single vapour trail shows an irregular growth up to some height between 100 and 110 km and a laminar expansion above. The beginning of the turbopause has been placed at 106 ± 4 km (Justus (1966) which is in good agreement with the observed cessation of scattering. One possible discrepancy is that individual trail observations show an abrupt turbopause whereas the boundary of the radio soundings is more diffuse, however this is not a serious objection since oblique echoes will tend to obscure any sharp height cut-off in scattering. It is felt that the night-time observations of a thin stratum seem well explained, the lower boundary corresponding to the height at which density changes become detectable and the upper one coincides with a natural decrease in scattering irregularities.
6.7 D REGION STRATIFICATION.

The previous sections have been concerned mainly with the smaller scale irregularities in the lower mesosphere but stratifications also seem to be present on occasions and some of the possible explanations for these are briefly considered.

Phase coherent reflections from discrete layers at 75 km in October and May have been reported in (Sec. 5.4). These observations seem similar to the echoes reported by Smith et al (1965) which occurred from the same region in winter. They suggest the reflections come from a thin dust layer trapped below an inversion and because of the similarity of the echo structures their explanation can perhaps be applied to the present observations. If so, it would seem that on these occasions the inversions were stable since the October layer was observed on three consecutive days.

It is unlikely that dust layers explain the 80 km stratification which is a feature of the phase height and $h'(t)$ records made in winter. The observed thickness (approximately 2 km) perhaps indicates scattering from a strong electron density
gradient. The individual N(h) profiles produced by the D.A.E. method at Birdling's Flat are not detailed enough to confirm the existence of such a gradient. However as already mentioned the average electron concentrations are larger in winter than in summer above 80 km, but are less below this height and also above 90 km (Manson 1965). This does suggest a large density gradient does exist in the 80 km region in winter only. To ascertian the exact nature of the stratification a rocket experiment is probably required.

In the context of higher electron concentrations near 85 km a recent suggestion by Sechrist (1967) must be mentioned. To explain the anomalously large N(h) densities in winter he suggests they may be caused by enhanced production of nitric oxide NO, the principle ionizable constituent in the D region (Nicolet and Aiken (1960)). Specifically it appears that the concentration of NO is very dependent on temperature.

Near the mesopause (85 km) the main chemical reactions for the formation and loss of NO are thought to be (following Nicolet (1965))
production
\[ N + O_2 \rightarrow NO + O \quad b_7 = 2 \cdot 10^{-13} \cdot T^\frac{1}{2} \cdot e^{-3000/T} \cdot cm^3 \cdot sec^{-1} \]

\[ N + O \rightarrow NO \quad b_1 = 10^{-7} \cdot cm^3 \cdot sec^{-1} \]

loss
\[ N + NO \rightarrow N_2 + O \quad b_6 = 1.5 \cdot 10^{-12} \cdot T^\frac{1}{2} \cdot cm^3 \cdot sec^{-1} \]

\[ N_2, N = \text{molecular and atomic nitrogen} \]
\[ O_2, O = \text{molecular and atomic oxygen}. \]

Under equilibrium conditions the concentration of NO is given by

\[ n(NO) = (b_7 \cdot n(O_2) + b_1 \cdot n(O)) \cdot b_6^{-1} \]

(6.3)

The important rate coefficient is \( b_7 \) which is very sensitive to temperature changes.

The mesosphere is warmer in winter than in summer, the largest difference occurring near 85 km. (215°K in winter and 180°K in summer). With these values the ratio of \( b_7 \) (winter) to \( b_7 \) (summer) is 15 which seems more than sufficient to account for the smaller production rates in winter.

Sechrist (1967) has calculated the D region
equilibrium values of NO, and hence electrons, using 6.3 for specific models of oxygen concentration and the average temperature profiles of Nordberg et al (1968). Above 80 km his calculated electron concentrations (his Figs. 11 and 12) seem in agreement with those measured at Birdling's Flat (approximately $10^3$ cm$^{-3}$) and as expected the densities are larger in winter than in summer. (The amount depending on the $n(O)$ profile assumed). However below 80 km Sechrist's predicted densities seem far too large for "quiet" day conditions, approximately $10^3$ cm$^{-3}$ at 70 km compared with observed values of about 200 cm$^{-3}$ (Belrose (1963), Belrose et al (1966), Manson (1965)).

Possibly the discrepancies can be accounted for by other reactions being important in the mesosphere. Nicolet (1965) suggests that (6.3) will not hold below 80 km because the concentrations of atomic nitrogen become too small. Unfortunately the values of $n(N)$ are unknown in the D region (Nicolet (1965), Young (1967)).

Although the situation is not clear it may be possible that the enhanced densities and perhaps the stratifications in winter are associated with this observed temperature dependence of the formation of
NO. It may also account for the irregular day to day variations in electron density in the D region (Belrose (1963)), however other minor constituents, such as atomic oxygen and negative ions may also be important to D region morphology (Belrose et al (1966)).
7. E REGION PHASE PATH CHANGES.

Findlay (1951) first used the present technique as early as the late 1940's in his study of the diurnal and seasonal behaviour of the E layer height changes. Since then there have been comparatively few phase path studies of the E region, among them Landmark (1957) and Fooks (1962) who were primarily concerned with the E region irregularities.

In this chapter results of a study of the phase path changes of the totally reflected E region signal as well as partial and total reflections from irregularities at E region heights are presented and compared with previous work on these subjects.

7.1 E LAYER PHASE CHANGES.

During the present study the totally reflected signal was coherent for less than 50% of the time, which is probably why phase observations of E layer are rare. Reflections from sporadic E clouds producing multipath effects are the usual causes of E region incoherence. Few E layer records are of suitable length for statistical analysis, since on only three days were coherent records obtained of durations three
hours or longer. Figs. 18 and 19 show the phase height changes of region E for two of these days. They show the large regular diurnal changes in layer height, superimposed on which are irregular fluctuations. Two dips due to flares are also evident. (Section 7.4).

7.1.1 DIURNAL PHASE CHANGES.

The diurnal changes in E layer height have been shown to be approximately those associated with a classical Chapman layer (Robinson (1959)). However there are discrepancies, closer inspection of the Fig. 19 shows that the overall phase height changes more rapidly before local noon than after. This behaviour was noticed by Findlay (1951a) who ascribed a large part of the anomaly to lunar tidal effects but later studies (Matsushita (1962)) suggest that lunar tidal effects are negligible in the E region (except for Es layers).

Instead the height assymetry and the related depression in E region peak ionization seem to be due to vertical drifts in ionization (Robinson (1959), Appleton and Lyon (1961) and Brown et al (1963)). These drifts are induced by the interaction between
E REGION PHASE PATH CHANGES  5. OCT. 1966.

FIG. 18.

FIG. 19.

ΔP

(Wavelengths)
the horizontal component $H$ of the earth's magnetic field and the quiet day $S_q$ currents which flow at these heights. Maximum drift occurs when the current is flowing east-west, at which time $H$ will be a minimum. Brown et al found a good correlation between the time of maximum height distortion of the E region peak and the time of minimum $H_0$ (usually 1130 L.T.).

Inspection of magnetograms taken at Amberly (40 km to the north) shows that on the 14/10 minimum $H$ occurred at 11.30 L.T. but the 5/10 was a magnetically disturbed day and no conclusion could be made. The other long record finished before local noon and again no comparison was possible.

7/1/(ii) IRREGULAR FLUCTUATIONS IN THE E LAYER.

As well as the regular diurnal changes all the E region phase height curves show what appear to be quasi-sinusoidal oscillations fo amplitude one radio wavelength or more. Short period phase height fluctuations have been studied by Landmark (1957) and Fooks (1962). The latter concluded that if the phase deviations were caused by variations in electron content it would be produced by those irregularities closest to the point of reflection (see Sec. 4.2).
Landmark investigating the amplitude "bursts" which often accompany phase path changes showed that they were caused by an effective concavity in the reflecting surface with a duration of 5 to 10 minutes. Using a relationship between amplitude and $\frac{d^2p}{dt^2}$ devised by Whitehead (1956) Landmark found the speeds of these irregularities to be of the order of 50 to 100 m/sec. and deduced spatial scales lying between 10 and 40 km (most probable size 25 km).

Regular "focusing" fading seems to be a feature of the normal ionospheric layers (Rawer 1962) and from the work of Fooks, Landmark and also Jones (1958) it would appear that the normal phase height fluctuations are caused by irregularities close to, or at, the point of total reflection.

As discussed in Chapter 6 Hines (1960, 1963a) has argued that these short term fluctuations in layers of constant electron density are manifestation of internal gravity waves. If these oscillations are caused by gravity waves then they should have periods greater than the relevant Vaisälä-Brunt period at 100 km (the true height of reflection) which is 4.3 mins. (Fig. 17). Inspection of the phase plots does suggest that the shortest periods present are of this order.
As a check the power spectra of these phase height records were computed.

As the phase records were already in digital form normal digital computation techniques were followed (Blackman and Tukey 1958, Jenkins 1961). Details can be found in Appendix 6. However it should be noted that estimations of spectra are based on the assumption that the series have zero averages and an absence of trends, which the records as they are plotted in Fig. 18-19, certainly have not. Firstly the dips due to the flares are not typical fluctuations and must be removed. For the 14/10 this was achieved by drawing a smooth curve through the beginning and ends of the dip (shown dotted in Fig. 19). However on 5/10 the flare effect occurred later in the day when the height of E region was changing rapidly, so this record was only analysed up to the time of the flare. Also to provide some compensation for the non-stationarity introduced by the large diurnal changes a sine curve of long period was least squares fitted to, and then subtracted from, the data. The period used was the order of 10 hours.

The autocorrelation functions and power spectra of the 3 records are shown plotted in Figs. 20
AUTOCORRELATION FUNCTIONS

FIG 20
Smoothed power spectra of phase height fluctuations

FIGURE 21
and 21. The spectra are not plotted completely, the power estimates at the higher frequencies being even lower before finally falling to a constant "noise" level. The cut off period shown is where the power changes are not significant (at the 96% confidence level).

As visual inspection of the records suggested all significant power lies in phase height fluctuations having periods greater than some minutes, comparable with the Vaisälä period. Because $\tau_B$ increases with height, to reach a steady value of about 15 minutes at the thermopause (200 km), the higher the point of reflection and the greater should be the cut-off period in the small scale height undulation. F region height variations were not measured in the present experiment but Davies (1962) reports small height oscillations with quasi-periods the order of 15 minutes. Furthermore direction finding studies of the tilts of the reflecting surface in the daytime F layer (Bramley 1953) show the tilts are on the average uncorrelated after a period of 5 minutes. This again suggests quasi-periods of approximately 20 minutes. At night the periods are about twice as great. Bramley also reports the undulations have spatial periods the order of 200 km.
However the best demonstration of how the minimum period changes with height has been presented by Georges (1967). Spectrum analyzing, using an analogue technique, doppler measurements of radio waves reflected from 100 km by day and 250 km at night. Georges showed the minimum quasi-periods were 5 and 15 minutes respectively. He also suggests that his findings are consistent with gravity wave theory as presented by Hines (1960).

For full agreement with theory a knowledge of the spatial scales of the irregularities is required. An estimate of the horizontal wavelengths expected in the E region comes from an inspection of Pitteway and Hines (1963 Fig. 2). Combining the effects kinematic viscosity of and thermal conductivity to give an effective viscosity \( \eta' = 2.26 \eta \) and taking a value of \( \eta' = 100 \text{ m}^2/\text{sec} \) to be appropriate at 100 km the allowed horizontal scales for waves of 5 minute period range from 10 to 50 km and between 5 and 150 km for 10 minute period modes. For comparison the horizontal dimensions of the wavelike perturbations observed by Landmark vary between 10 and 40 km.

In the lower F region Hines (1964) suggests
that for waves of period 20 minutes, the minimum wavelengths should be the order of 100 km, hence the spatial periods of 200 km deduced by Bramley (1953) are also within the limits imposed by viscous dissipation.

From the above evidence there appear good reasons for assuming that the small height fluctuations are produced by internal gravity waves. As discussed in Chapter 6 these waves are generated in the troposphere and the ionosphere by tidal unstabilities but on occasions other sources may be operative. Comparison of Figs. 18 and 19 show that on the 5 October the amplitudes of the short period fluctuations are greater than those occurring on the 14 October. This is also true for the other E region record analysed (31 October) which suggests that on the 5 and the 31 October there was greater wave activity than on the 14th. Some confirmation of this comes from an inspection of ionograms made at Godley Head (24 km to the north). During the hours of phase observations numerous travelling ionospheric disturbances (T.I.D's) were a feature of the ionograms, for the 5th and 31st whereas on the 14th very few T.I.D's were noted. These disturbances appear as cusps or "kinks" which travel down the ionogram trace to low
virtual heights and frequencies usually disappearing near E region (see Heisler 1958). Hines (1960), has put forward a satisfactory explanation of these T.I.D’s as being manifestations of partially ducted gravity waves (Friedman 1966), the downward motion being associated with the downward phase progression of the wave, energy propagating upwards, (Appendix 5).

The ionograms of the 5th and 31st also showed conditions typical of ionospheric storms, enhanced stratification of the F1 peak and depressed F2. It has been suggested (King 1966, 1967) that the higher recombination rates (and hence lower electron densities) may be caused by waves or T.I.D’s becoming non-linear in the F region. Under magnetically disturbed conditions internal waves are produced in the auroral zone near 100 km (Piddington 1964). If this additional source is operative on the 5th and the 31st then the waves may travel to mid-latitudes by being partially ducted under the strong thermospheric temperature gradients (Friedman 1966, Hines and Reddy 1967).

7.2 **GRAVITY WAVES AND SPACED RECEIVER OBSERVATIONS.**

It has been suggested in the preceding
section that both the temporal and spatial periods of
the irregularities in the E and F layers are compatible
with gravity wave theory. If they are then, because
gravity waves are dispersive, some sort of dispersion
of the waveforms should be noticeable in measurements
made at spaced observing points. The equation
linking the horizontal group velocity $U_x$ and the
horizontal phase velocity $V_x$ is (Pitteway and Hines
1965)

$$U_x = V_x \frac{(\omega_B^2 - \omega^2)}{(\omega_B^2 - V_x^2 \omega^2/c^2)}$$

At low frequencies ($\omega^2 \ll \omega_B^2$) the group and phase
velocities are equal, but as $\omega \to \omega_B$ the group velocity
becomes small. For example at 100 km with
$\omega_B = 2.3$ rad/sec and for a wave of period 5 minutes
with phase velocity 67 m/sec ($\lambda_x \approx 20$ km) the group
velocity is 23 m/sec.

One way in which dispersion may be measured
is by the spaced receiver method for measuring iono-
ospheric drifts. If the fading pattern on the ground
is composed of a number of Fourier components, each
of which is derived from a corresponding component
or irregularity in the ionosphere, then if the
irregularities move with different speeds the Fourier component in the ground pattern will also move with different speeds. The speeds of the various components can be found by Fourier transforming the cross correlation functions of the fading patterns and comparing the phases of the cross spectra.

Jones and Maude (1965) carried out such an analysis and suggested their results indicated the presence of surface gravity waves (hydrodynamic waves propagating horizontally at an interface) with \( \lambda_x \approx 100 \) meters. However, McGee (1966) analyzing long fading records could find no evidence for dispersive waves in either the E or F region. He pointed out that because of the smallness of the postulated wavelengths, comparable to the aerial spacing, the results of Jones and Maude could be explained by ambiguities of \( 2\pi \) in the phase records from each aerial.

Although McGee's analysis suggests that dispersive surface gravity waves are not present in the ionosphere this does not entirely rule out internal modes. As McGee points out if the ionospheric diffracting screen is deeper than one radian the phase deviations imposed on the radio waves will introduce
short period fading components into the amplitude spectrum (Ratcliffe 1956). These higher frequency components will move with the velocity of the large scale irregularity, as the Fourier analysis suggests they do. Examination of the E region phase records shows that the peak to peak amplitudes of the phase fluctuations are indeed the order of 1 radio wavelength or more (Fooks 1962). The phase deviations would then be \( \pi/2 \) radians or greater. This suggests that the dispersion of short period waves would be difficult to detect by normal spaced receiver measurements, particularly since the fading records required would have to be of long duration.

It is felt that to measure adequately the dispersion of the gravity waves at E region heights an array should be used which has spacings comparable with the spatial scale of the waves (\( \lambda \approx 10 \) km). Probably phase height measurements would give the best indication of when the ionosphere is stable.

7.3 SPOPADIC E.

In this section the structure characteristics of the observed sporadic E clouds are briefly discussed. As mentioned in 7.1 blanketing from moving E\(_5\) layers
was the main reason why few long phase continuous records were obtained from the E layer.

The presence of a sporadic layer on a record was usually denoted by strong oblique echoes having tilted fringe patterns followed by an incoherent or partially coherent "mess" as the cloud moved overhead. A typical layer is shown in Fig. 11a at 100 km. The regular obliques suggest reflections from a fairly well defined cloud edge but the phase incoherence implies multiple scattering from density variation in the layer. This seems to agree with the structure of typical $E_s$ as deduced from ionosonde records; Rawer (1962) suggests clouds having graduations in electron content comparable in size with the Fresnel zone (approximately 5 km at 100 km).

Less than 25% of the sporadic layers observed produced coherent records, but when they were present their lower boundaries showed the same quasi-periodic height fluctuations as observed in the normal E region phase records. Occasionally these layers appear to be semi-transparent with the regular E region signal appearing through the $E_s$ cloud (Fig. 22a). This type of echo structure appears to be rare, (Briggs 1951) and in the present study this example is the only
observation where E region has been visible for any length of time. The cloud was overhead for 50 minutes and from the measured speed of 1.5 km/min (deduced from changes in slant range) horizontal dimensions of 75 km can be inferred. E region appeared half way through the record and its phase height increased almost monotonically until the sporadic layer moved away, which suggests the gradual decrease in its electron content. (Although the regular E region phase height variations are unknown).

As well as the blanketing sporadic clouds partially reflecting ledges with distinct lower boundaries are a feature of the lower E region, usually appearing near 95 km (Fig. 12a). These layers have been observed in all seasons for durations of up to 3 hours and are probably identical with the '95 km regions' reported by Gregory (1956).

Most of the sporadic E reflections were observed for less than one hour and the traces suggest reflections from isolated 'clouds' (which may themselves have a floculent structure) rather than extended layers. The exceptions are the '95 km' partially reflecting regions, (30% of these were observed to exist for periods of 2 hours or greater) and a structure
March 9, 1967. Coherent reflection from a transparent $E_s$ layer at 95 km, $E$ region is visible at 110 km and a partially coherent echo at 75 km.

FIG. 22a.

October 5, 1966. Record showing phase path changes caused by a solar flare (f)

FIG. 22b.
which was observed above 100 km continuously for 3 hours on one of the night records.

A number of theories have been proposed for the formation of temperate latitude sporadic E. One way in which the stronger layers originate through the convergence of ionization through the interaction of vertical shears in the horizontal wind with the geomagnetic field (Whitehead 1966, Axford and Gunnold 1966, MacLeod 1966) or by discontinuities in the vertical drifts of positive ions (Layzer 1967). Probably the blanketing clouds observed in the present experiment can be produced by the above process.

Although the partially reflecting layers may be associated with small electron density peaks formed by wind shear effects their comparatively long duration suggests a different process may be operative. As far as can be ascertained the 95 km regions remained at the same height (to within ± 1 km) for periods up to 3 hours and did not show the vertical motion reported for \( E_s \) layers formed by wind shear mechanisms at higher altitudes (MacLeod 1966, Wright et al. 1967). Possibly the 95 km regions may be identified with the scattering sheets reported by Bowhill (1966) which had \( N(h) \) variations occurring in
scale comparable with the vertical half-wavelength of the radio wave.

In the context of vertical motions it must be noted that the night-time $E_s$ layer mentioned above did appear to progress downwards from $107 \pm 1$ km to $100 \pm 1$ km over the period of observation (24 April 1967 0100-0400 NZST), giving a vertical movement comparable to that reported by MacLeod (1966). Whether this single observation is a confirmation of wind shear theory is doubtful. Phase records made on succeeding nights show only horizontally moving $E_s$ clouds of short duration; which may indicate a different neutral wind structure on these nights.

7.4 **SOLAR FLARE EFFECTS.**

Both the E region phase height curves (Figs. 18 and 19) show abrupt decreases in phase path of the order of 8 wavelengths (1 km). These sudden changes are typical of a number of other ionospheric phenomena, collectively known as Sudden Ionospheric Disturbances or S.I.D. (Collins and Hertzberg 1965). Representative of these anomalies in radio propagation are sudden decreases in the received amplitude of H.F. waves (S.W.F.) and sudden changes in the phase of long waves.
transmitted over long distances. (S.P.A.). All these S.I.D. are caused by transient increases in electron content produced by abrupt enhancement of the ultraviolet and x-ray radiations from the sun. In this section the possible electron density changes which caused the phase height deviations are briefly discussed.

On the 14 October an optical flare of importance -N was observed between 0027 and 0107 U.T. with a maximum brightness occurring at 0036 U.T. A S.P.A. was also reported on the transmission of V.L.F. signals (18.6 kHz), Seattle to Hawaii lasting between 0034 and 0106 U.T. On the 5 October there was no flare patrol at the time of the phase changes but again a S.P.A. was observed on the Seattle-Hawaii link (0205 - 0308 U.T.). All information was taken from the monthly bulletins of Solar and Geophysical Data (Space Disturbances Laboratory 1966). New Zealand local time is twelve hours ahead of universal time.

The phase height changes could have been produced by either increases in electron content below the point of reflection or by a decrease in the height of reflection and from a single frequency observation it is difficult to differentiate between the two mechanisms. It was hoped to get information about the
extra ionization changes by observations of how the partially reflecting irregularities behaved. Unfortunately on the 14 October at the time of the flare transmission was at low power so only E region was being observed, but partial reflection data was obtained on the 5 October. Fig. 22o shows the record at the beginning of the flare. Only echoes down to 90 km seem affected, no fluctuations in phase height being observed below this height. It is noticeable that the E region phase height changes are irregular, showing very short period fluctuations superimposed on the overall decrease. These are probably associated with variations in the strength of the ionizing radiations. By contrast the decrease on the 14 October was very smooth.

Since S.P.A. were reported some extra ionization was produced in the D region, V.L.F. waves being reflected from heights near 75 km (Collins and Hertzberg 1965). However as discussed in Section 4.3 even large percentage increases low in the D region produce only small decreases in total phase path. It is felt that to produce the observed decreases (1 km) at 2.4 MHz the relevant ionization was produced near the point of total reflection, and possibly by a single decrease in reflection height of \( \frac{1}{2} \) km. Having the
electron profile in Fig. 3 as a guide this would require only a 10% increase in \( N(h) \) in E region (\( \Delta N(h) \sim 6000 \text{ cm}^{-3} \) at 100km).

Some confirmation of this comes from a comparison of the times taken to decay from the time of maximum amplitude of the S.P.A and the phase deviations (e.g. 0040-0106 U.T. for the S.P.A on the 14 October compared with a decay time of 5 minutes for that at 2.4 mHz). These are explicable by the different relaxation times of the various parts of the ionosphere, \( \tau = (2 \alpha N)^{-1} \) where \( \alpha \) is the effective recombination coefficient (Appleton 1953). Taking \( \alpha \approx 10^{-6} \text{ cm}^3 \text{ sec}^{-1} \) at 75 km (Belrose 1965) and \( N \approx 300 \text{ cm}^{-3} \) gives \( \tau \approx 30 \text{ minutes} \), comparable with the decay time of the S.P.A. In the E region the relaxation times are the order of a minute (Baker and Davis 1966) which, although smaller than the observed 5 minutes, is the right order of magnitude.

Although not conclusive the E region phase height changes seem to be caused by ion production at the point of reflection rather than in the D region. Extensive multifrequency experiments do show that for 'average' small flares the bulk of the extra ionization is deposited in the E and lower F regions (Morriss 1960,
Kanellekos 1963, Winkleman and Dyce 1964). The ionizing radiations responsible are probably soft x-rays with wavelengths $10^{-8}$ - $100\,\text{Å}$, since these show greatest increases in flares (Collins and Hertzberg 1965).
8. CONCLUSION.

This investigation has used phase path techniques of pulsed transmissions at vertical incidence to study lower ionospheric irregularities. From an analysis of partial echo structure it has been shown that:

(i) The reflections received from above approximately 85 km are mainly incoherent at all times of the year and at all heights in summer and winter.

(ii) The echoes from the 70 km region at the equinoxes often exhibit a high degree of phase coherence, good phase continuity with time and are slowly fading.

Point (i) indicates that generally the reflections come from randomly distributed scattering centers or from two or more points on a reflecting surface. However the second feature shows the echoes can come from discrete reflecting surfaces, the associated phase path changes suggesting that these irregularities are horizontally moving and have some horizontal extent. The scale implied by these
characteristics shows that the Booker scattering model is inapplicable in some circumstances.

The phase records do show that partially reflecting strata also exist occasionally, particularly above 90 km. Although their reflections are generally phase incoherent these layers do have clearly distinguishable lower boundaries.

To explain the seasonal variation in D region scatter, possible changes in the generating mechanisms of the irregularities were qualitatively examined, with particular regard to fluctuations in the turbulence and gravity wave spectrums.

Small scale gravity modes \((\lambda_x \approx 10 - 20 \text{ km})\) having low phase velocities are most likely to cause scatter directly. As discussed in Section 6, the dynamic coupling of the mesosphere to tropospheric and thermospheric sources of these modes appears to be least at the equinoxes.

Because the temperature gradients near 70 km are generally negative, the atmosphere there is unstable and should be sensitive to energy input in the form of shears from atmospheric waves. Since phase coherence
is greatest when the middle atmospheric circulation is reversing (mid September - mid November, Late February-March) this suggests the instabilities are least at these times. This could be explained if the bulk energy of the tropospheric gravity waves lies in low phase velocity modes, irrespective of size. Diurnal tidal input is operative at all times but appears to be greatest in summer. Tidal breakdown at altitudes above 85 km, generating small scale phenomena, seems to be the cause of the general low coherence observed at these heights.

There does seem qualitative agreement between energy input and reported incoherence with the atmospheric temperature profile being the controlling feature. The origin of the D region layers is less certain but these also may be connected indirectly with temperature changes.
Let the transmitter emit a pulse of radio waves of frequency $f_s$ which is reflected from an irregularity at a phase height $P$. Then the received signal can be written as

$$A_r = A(t) \cos f_s (t - P/c)$$

where $c$ is the velocity of light.

Let this be transformed to an intermediate frequency of $(f_L - f_s)$ by mixing with a local oscillation of frequency $f_L$. The main term of the resultant signal will be (Tucker (1953))

$$T = k A(t) \cos (f_L t) \cos f_s (t - P/c)$$

expanding which gives

$$T= \frac{1}{2} k A(t) \left[ \cos((f_L - f_s)t + f_s P/c) + \cos ((f_L + f_s)t - f_s P/c) \right]$$

Filtering out the wave of frequency $(f_L + f_s)$ gives

$$T = k_1 \cos ((f_L - f_s)t + f_s P/c)$$

(1)
If the phase reference signal has a frequency 
\( \delta f = (f_s - f_r) \) then it can be represented by

\[
A_p = A_p \cos (\delta f \cdot t) \tag{2}
\]

which transforming to an intermediate frequency
\( (f_L - \delta f) \) similar to above gives

\[
A_{pi} = k_2 \cos (f_L - \delta f) t \tag{3}
\]

Finally mixing signals (1) and (3) and extracting the difference frequency gives

\[
A_0 = k (t) \cos (f_r t - f_s P/c)
\]
APPENDIX 2.

EXPERIMENTAL EQUIPMENT FOR PHASE PATH MEASUREMENTS.

The problem of measuring the phase path changes of pulsed reflections from the ionosphere can be broken into 4 parts. Firstly a reference waveform is required against which the phase of the received pulse can be compared. In order to show whether the oscillation is increasing or decreasing the p.r.o. must be of different frequency to that of the transmitted signal. Secondly the p.r.o. must bear some fixed phase relationship to that of the transmitted signal. Thirdly the phase of the received echo has to be compared with the p.r.o. and finally the phase changes have to be displayed.

The ways in which each of these requirements were met are now described in detail. For an overall block diagram of the layout see Fig. 2.

A.2(i) GENERATION OF PHASE REFERENCE OSCILLATION.

A free running crystal controlled oscillator of frequency 2.404 MHz provides the basis for both the transmitted and the phase reference oscillations.
For transmission the signal is amplified and finally pulsed, the output pulse width being 25 µsec.

To get a suitable number of fringes to a pulse a frequency of 100 kHz was chosen as the difference frequency \( f_r - f_s \). Hence the p.r.o. had to have a frequency of 2.3 MHz. To obtain this oscillation a small part of the 2.404 MHz signal was brought from the transmitter caravan via a 75 µ coaxial cable, mixed with a continuous 100 kHz oscillation and the difference frequency extracted.

Since the 2.4 MHz signal contained a large amount of harmonic content and was only 50 m.v.p.p. it was first filtered and amplified. This was achieved in one operation using a single stage tuned collector transistor amplifier (Fig. 23a) having a gain of approximately 200. To avoid possible interference with other experiments this amplifier was heavily shielded.

The 100 kHz modulating signal was generated by a free running Colpitts oscillator (Fig. 23b); with crystal control this had a frequency stability of ± 1 part in \( 10^5 \) Hz over a period of one hour.
2.4 MHz TUNED AMPLIFIER

FIGURE 23a

100 kHz OSCILLATOR

FIGURE 23b
To generate the p.r.o. the switching signal (2.4 mhz) and the modulating signal (100 khz) were mixed in a diode ring modulator (Fig. 24a). Under idealized conditions (perfect diodes) the modulating function is given by (Tucker 1953)

\[ \Phi(t) = \sum_{n=1}^{\infty} (-1)^n (2n-1)^{-1} \cos (2n - 1)p t \]

\[ p = 2.4 \text{ MHz}. \]

It is noted that \( \Phi(t) \) contains no constant terms so that when the modulating signal is absent there is no output and the ring modulator is said to be a double-balanced modulator. Under real conditions the forward and back resistances for each diode will be slightly different so as compensation variable resistors (1 k\( \Omega \)) were placed in opposite sides of the diode bridge.

The switching signal was fed into the bridge via the split capacitors of the parallel side of a band pass impedance transformer, which stepped the bridge impedance (1 k\( \Omega \)) down to the input impedance (470 \( \Omega \)) of the following stage (phase reference amplifier). The modulating waveform (1 vpp) was injected into the bridge via a single transistor.
Bridge balance was achieved by removing the modulating signal and adjusting the variable resistors and tuning capacitors until no switching signal was observed at the output.

With the modulator working the output voltage consists of the upper and lower sidebands (2.504 and 2.304 Mhz respectively) plus other harmonics. As was shown in Appendix 1 it is the lower side band which is required. Furthermore since the receiver signal (2.404 Mhz) is converted to an intermediate frequency the p.r.o. must also be transferred using the same local oscillator (5.570 Mhz) as the receiver amplifier. The selection, frequency conversion and amplification of the lower sideband was effected using a conventional 3 stage tuned pentode amplifier with a triode mixer fed by a grounded grid input. The output was via a cathode follower.

With proper adjustment of the rejection of the 1st upper side band was well over 40dB. The output p.r.o. was then at a frequency of 3.266Mhz.
DIODE RING MODULATOR

FIGURE 24a

VIDEO AMPLIFIER

FIGURE 24b
A.2(ii) PHASE LOCKED TRIGGER GENERATOR.

The transmitter was pulsed every 25 sec$^{-1}$ so to ensure good phase continuity it was required that each trigger pulse be in some way locked to the 100 kHz waveform (the oscillation finally observed). Fig. 25 shows the circuit which provided the requisite triggers, the inputs being the 100 kHz c.w. oscillation taken from the 100 kHz oscillator (Fig. 23b) and timing pulses derived from the mains frequency.

Immediately after input the 50 v peak amplitude, 20 µsec wide timing pulses were converted to positive going 12 v steps starting at -12 v by a diode pulse converter and clamp circuit. Hence at point B (Fig. 25) the voltage level was always -12 v unless the timing pulse was present, when it became 0v for the duration of the pulse.

The 100 kHz signal was turned into a 12 v pp square wave by a Schmitt trigger circuit and the square wave differentiated by the R.C. circuit $C_1 R_1$ (Fig. 25) producing positive and negative going spikes. $C_1 R_1$ and diode D (Fig. 25) constitute a
FROM 100kHz OSCILLATOR TO TRANSMITTER

100kHz OSCILLATOR

2.7k

18k

10k

22k

33pF

75Ω

1k

10k

100pF

C

10k

10k

22

18k

47k

10k

47pF

10k

0.02

+12V

All Transistors ASZ21

PHASE LOCKED TRIGGER GENERATOR

FIGURE 25
simple diode AND gate. Only when the timing pulse is present (level zero) did the diode pass a positive spike. The latter circuit produced a negative going pulse of width 40 µsec. The leading edge of this master pulse was thus locked to the 100 khz waveform and occurred only 25 sec\(^{-1}\).

A. 2(iii) **PHASE COMPARISON CIRCUITS.**

The receiver output was a series of echo pulses of approximate width 25 µsec and intermediate frequency 3.166 Mhz. To produce the 100 khz beats the receiver signal was mixed with the continuous P.R.O. at a frequency of 3.266 Mhz in a single stage transistor detector. Fig 26a shows the mixing circuit with the inputs passing through two grounded emitter stages to prevent any loading of preceding valve equipment.

The P.R.O. was used as the switching signal for the single stage transistor detector, its amplitude being large enough (> 2v pp) to make the transistor act as a linear detector with a modulating function (Tucker 1953).

\[ \Phi(t) = h_{ot} h_1 \left[ \cos pt - \frac{1}{3} \cos 3 pt + \frac{1}{5} \cos 5 pt \ldots \right] \]
RECEIVED I.F. (3.166 MHz)

PHASE REFERENCE IN (3.266 MHz)

All Transistors AF118's

DETECTOR AND FILTER CIRCUIT

FIGURE 26a

INPUT FROM VIDEO AMPLIFIER

All Transistors ASY27's

SQUARING CIRCUIT

FIGURE 26b
\[ p = 3.266 \text{ Mhz}. \]

The modulating input \( A(t) \cos qt \) (\( q = 3.166 \text{ Mhz} \)) was via the collector of the transistor producing an output \( \Phi(t) A(t) \cos qt \).

To remove the unwanted high frequency components and any stray low frequency ones the output was passed through a series of high and low pass R.C. filters, the overall response designed to have minimum effect at 100 khz. Simple L.C. filters could not be used because if they were of high 'Q' they introduced 'ringing' and possible unwanted phase shifts.

The filter was followed by a two stage R.C. coupled video amplifier (Fig. 24b). Both stages had capacitive feed back from the collector to the base to limit the high frequency response, further reducing any 3 mhz carrier signals. At 100 khz the combined filter - amplifier had a gain of 25 and the overall rejection at 3 mhz with reference to the 100 kz signal was 52 dB.

The output at this stage consisted of sinusoidal 100 khz 'beats' which were amplitude dependent. To remove this dependence the output was 'clipped' to
GATING AND COMPARATOR CIRCUITS

**FIGURE 27a**

CONVERSION AND AMPLIFIER CIRCUITS

**FIGURE 27b**
produce equal amplitude square waves (Fig. 26b).
The squaring circuit consisted of an input emitter follower, as an isolating stage, followed by a preamplifier and Schmitt trigger. The overall circuit converted any signal of amplitude greater than 40mv pp into a square wave of amplitude 12v pp. This was found to be more than adequate for all work.

A. 2(iv) RECORDING OF PHASE CHANGES.

(a) PHOTOGRAPHIC.

The output of the squaring circuit was used to intensity modulate an oscilloscope. Every time the output was -12v (corresponding to a fringe peak) the bright spot appeared on the time base. As a 35 m.m. film moved perpendicularly past the time base each spot traced a line on the film. A film speed of 75 cm/hr was found adequate.

(b) ELECTRONIC.

To sample the phase path changes of a given echo independently of the photographic method an electronic sampling technique was devised.
Essentially the fringes of the required echo were gated by a narrow gate of width equal to one fringe spacing (10 µsec), ensuring that only one fringe at a time was covered. The phase changes were found by comparing the position of the gated fringe relative to the leading edge of the gate.

Fig. 27a shows the gate generating circuit consists of two monostable multivibrators in tandem and pulse inverting circuit (NOR gate). The first multivibrator was triggered by the transmitter trigger pulse producing a variable width pulse. The trailing edge of this pulse turned on the second monostable giving a 10µsec wide negative going pulse. This was inverted by the NOR gate (which acted as a buffer stage) to give the required positive going, from -12v, pulse gate. The time delay of this gate, relative to the transmitter trigger, was controlled by the variable resistor $R_1$ (1 K $+ 100 K$) enabling any required echo to be selected.

The phase comparison was effected by a bistable multivibrator or flip flop. One input was connected directly to the gate generating circuit. Every time the 2nd multivibrator by triggered the flip flop also changed state, or was 'SET'. The
other input to the flip flop was via a diode AND gate, inputs to which were the 10 µsec gate and 100 khz fringes. With the gating pulse acting as a voltage level control the first positive going fringe present after the leading edge of the gate RE-SET the flip flop.

The net output of the bistable was a narrow pulse whose width depended on the changes of the echoes' fringe relative to gate and hence the phase path changes of the reflection. The use of high speed transistors (ASZ 21S) in the bistable circuit ensured that the rise time of the output pulse was less than 0.5 µsec. To display the phase changes the variable width pulse was converted to one whose amplitude varied in proportion to the input width. The conversion and amplifier circuit are shown in Fig. 27b. The amplitude transformation was made by using a single R.C. integrating circuit of time constant 10 µsec. If the input pulse width was \( \Delta t \) (\( 0 \leq \Delta t \leq 10 \) µsec) then the output voltage \( V_o \) was proportional to \((1 - \exp(-\Delta t/10))\) and was almost linearly dependent on the pulse width. The output was passed to a 'hold' or Box-car circuit which holds the voltage \( V_o \) until the circuit is discharged by the next transmitter trigger. The amplitude
changes were then recorded on an Esterline Angus chart recorder driven by a differential amplifier.
APPENDIX 3.

At vertical incidence the angle between the wave normal and the ray direction is $\alpha$, which is given by (Davies 1966 Sec. 2.5.2)

$$\tan \alpha = \frac{1}{\mu} \frac{d\mu}{d\theta}$$

$\theta$ = angle between the wave normal and the magnetic field (22° at Birdling's Flat). Using equation ignoring collisions ($z = 0$) gives

$$\tan \alpha = \pm \frac{(\mu^2 - 1) Y_T Y_L}{\sqrt{Y_T^4 + 4 (1-X)^2 Y_L^2}}$$

where the + and - signs refer to the ordinary and extraordinary rays respectively.
APPENDIX 4.

REFLECTIONS FROM A SINUSOIDAL LAYER.

The effects of specular reflections from a sinusoidal layer have been discussed by Bramley (1953) and Austin (1967) who were concerned with direction finding and amplitude variations respectively. This appendix considers the changes in range as the reflecting ripple moves overhead.

Let the layer be at height average height \( h \) and the undulation have the form

\[ \phi = a \sin \left( k \frac{x}{h} - \omega t \right) \]

where \( a = \) amplitude

\[ \lambda = \frac{2\pi}{k} = \text{horizontal wavelength} \]

Provided \( h \) is very much greater than the amplitude (which in this case means a \( \approx 10 \) km) the range to the specular point is given by

\[ r^2 = x^2 + h^2 \]

(A4.1)

The angle between the ray and the vertical is given by
\[ \tan \theta = \frac{x}{h} = -\frac{d\varphi}{dx} \]

so that \[ \cos y = -A(y + \omega t) \hspace{1cm} (A4.2) \]

where \[ y = \frac{k}{x} - \omega_L \]
\[ A = \frac{\lambda^2}{4\pi^2} ah \]

\( x \) can be found by a graphical solution of the trans-cidental equation \( A4.2 \) and so \( r \) can be found from \( A4.1 \). The phase path is assumed to be \( P = 2r \).

It is noted that two specular reflections will occur when the line \( t = -A(y + \omega t) \) just touches the curve \( t = \cos y \) (i.e. \( A = 1 \)). Since the greatest value of the slope of \( \cos y \) is one if \( A \) is less than one there will be at least 3 specular points for some values of \( (\omega t) \).

The range changes for different values of the parameter are shown plotted in Fig.4.
APPENDIX 5.

SOME ASPECTS OF GRAVITY WAVE THEORY.

In previous chapters it has been suggested that a class of atmospheric motions known as internal gravity waves are an important feature of the atmosphere. It has been argued that some of the observations made in the present experiment are well explained by the properties of these waves, although many of the discussions were of a qualitative nature. This appendix attempts to present the theory of gravity wave propagation in a more rigorous fashion with special emphasis on those features which the author feels are more important for the ionosphere. For other works on internal gravity waves, the reader is referred to Hines (1960) and Midgley and Liemohn (1966). The wider aspects of hydrodynamic wave motion in general have been discussed in a fairly abstract sense by Eckart (1960) but a good understanding of these waves may be obtained from an excellent paper by Tolstoy (1964).

A. 6.1 NOTATION.

The important symbols used in this work are:-
Cartesian co-ordinates, \( z \) vertical upwards

\[ \rho_1, p_1 \]

Perturbed atmospheric density and pressure.

\[ p_1 = p_z (1 + \rho) \text{ where } p_z \text{ is the equilibrium pressure and } \rho \text{ is the fractional perturbation} \]

\[ (\delta P/p) \text{ produced by a superimposed wave motion,} \]

similarly for \( \rho_1 = p_z (1 + \rho) \)

\[ g = (0, 0, -g) \]

Velocity of sound, \( c^2 = \gamma p_z / \rho_z \)

Scale height \( c^2 / \gamma g \)

Ratio of specific heat

\( U_0 \)

Steady background wind in \( x \) direction

\[ \frac{D}{Dt} \]

Operation \( \frac{\partial}{\partial t} + U_0 \frac{\partial}{\partial x} \)

\( \omega, T \)

Angular wave frequency and period

\( k_x \)

\( x \) component of wave number, assumed real

\( \nu \)

Doppler shifted wave number = \( \omega - k_x U_0 \)

\( u_x, u_z \)

Horizontal and vertical particle perturbation velocities

\( X \)

Velocity divergence \( \frac{\partial u_x}{\partial x} + \frac{\partial u_z}{\partial z} \)

\( \omega_B \)

Vaisālā Brunt frequency

\( U_x, U_z \)

Horizontal and vertical group velocities,

\[ \frac{\partial \nu}{\partial k_x}, \frac{\partial \nu}{\partial k_z} \]

\( V_x, V_z \)

Horizontal and vertical phase velocities,

\[ \nu / k_x, \nu / k_z \]
A.5.2 BASIC EQUATIONS.

The theory of hydrodynamic wave motions usually assumes that the waves are of perturbation magnitude only, second order effects being negligible. For present purposes the basic assumption is made that the motions are taking place in a fluid which is moving with a constant velocity and the only external force is that due to gravity acting vertically downwards. The neglect of the Coriolis force restricts the discussion to motions with periods a few hours or less. Since the only preferred direction is imposed by gravity the wave motions can be discussed in a two dimensional co-ordinate system, the axis $x$ being horizontal and that of $z$ vertical positive upwards. The wave motions are then described by the following equations

\[ \frac{Dv}{Dt} + \frac{1}{\rho_z} \nabla P_1 - \frac{\rho_1}{\rho_z} g = 0 \]  \hspace{1cm} (1)

\[ x + \frac{1}{P_z} \left( \frac{D\rho_1}{Dt} + u \frac{\partial \rho_z}{\partial z} \right) = 0 \]  \hspace{1cm} (2)

\[ \left( \frac{D\rho_1}{Dt} + u \frac{\partial \rho_z}{\partial z} \right) = \frac{1}{c^2} \left( \frac{D\rho_1}{Dt} + u \frac{\partial \rho_z}{\partial z} \right) \]  \hspace{1cm} (3)
These are the linearized Eulerian equations of motion, continuity and energy (Lamb 1945). The basic assumption also made that the atmosphere is in hydrostatic equilibrium

\[
\frac{\partial p}{\partial z} = -\rho_z g \tag{4}
\]

From equation (4) and using the relationship

\[
c^2 = \gamma \frac{p}{\rho} \frac{z}{z}
\]

\[
p = e^{-z/H}
\]

and \( \rho = \frac{1}{H} e^{-z/H} \)

Hence \( p = -\frac{\rho}{H} \)

and \( \rho = -\rho \frac{(1 + Hz)}{H} \)

Where a prime indicates differentiation with respect to \( z \). Rearranging equations (1) to (3) such that \( p_1, o_1 \) are replaced by terms in \( p, p_z \) and \( \rho, \rho_z \) and assuming that the waves propagate harmonically in the \( x \) direction and with respect to time such that

\[
u_x, u_z, p, \rho \alpha e^{i(\omega t - k_x x)}
\]
the equations (1) to (3) reduce to

\[ \nu U_x = k_x g H \rho \] (5)

\[ i \nu U_z = g (p - H \rho') - g \rho \] (6)

\[ \frac{i \nu \rho - U_z}{1} = \frac{\gamma (i \nu \rho - U_z (1 + H)}{H} \] (7)

\[ x = - (i \nu \rho - U_z (1 + H)} \] (8)

Eliminating \( U_x, p, \rho \) from equations (5) and (7) gives the equations

\[ U_z^2 = \frac{(\nu^2 - k_x^2 c^2)}{\nu^2} x + \frac{k_x^2 g}{\nu^2} U_z \] (9)

\[ X' = \left( \frac{1}{H} - \frac{g k_x^2}{c^2} \right) x - \left( \nu^2 - \frac{g^2 k_x^2}{\nu^2} \right) \frac{U_z}{c^2} \] (10)

Eliminating \( U_z \) from (9) and (10) gives

\[ X'' + \left( \frac{H - 1}{H} \right) X' + \left( k_x^2 \left( \omega_B^2 - \nu^2 \right) + \frac{\omega_B^2}{c^2} \right) x = 0 \] (11)

The term \( \omega_B \) is the Väisälä Brunt frequency and is the frequency at which a small parcel of air will oscillate if it is displaced from its equilibrium position.
(Tolstoy 1963). In the notation used above

\[ \omega_B = g \left[ \frac{\gamma H' + \gamma - 1}{\gamma H} \right] \]

which is directly equivalent to (6.2) provided only changes in the velocity of sound \( (c^2 \propto T) \) are considered.

A5.3 PROPAGATION OF ATMOSPHERIC WAVES.

Using equation (11) it is now possible to discuss the propagation of waves in the vertical direction. So far the effects of temperature variations have been included but if these are neglected, the atmosphere assumed isothermal, the coefficients in the equations are constant which simplify the discussion considerably. As a first approximation this assumption is made.

(i) ISOThERMAAL ATMOSPHERE.

Equation (11) is of the form

\[ X'' + a(z) X' + b(z) X + 0 \]

(12)

Using the transformation
\[ X = \eta e^{z/2H} \]

gives \[ \eta'' + k_z^2 \eta = 0 \]

where \[ k_z^2 = k^2 \left( \frac{\omega_B^2 - \nu^2}{\nu^2} \right) + \frac{\nu^2 - \omega^2}{c^2} \]

and \[ \omega_a = \frac{c}{2H} = \text{acoustic resonant frequency} \]

(Tolstoy 1963) and is the highest frequency at which the atmosphere will oscillate as a whole. The first thing that should be noticed is that \( X \) is proportional to \( \exp(z/2H) \) so that a wave propagating upwards will experience a growth in amplitude by this amount. The physical reason is that the kinetic energy of the waves is \( \frac{1}{2} p_z U^2 \), where \( U \) is the perturbation velocity, and so the amplification ensures that the kinetic energy remains constant as the atmospheric density decreases. There is a corresponding amplitude decrease for waves propagating downwards. (13) is the usual form of the wave equation and has an oscillating solution when \( k_z^2 > 0 \), so that \( k_z \) can be regarded as the vertical wave number. The wave will propagate in the vertical direction with wavelength \( \lambda_z = (2\pi/k_z) \). When \( k_z^2 < 0 \) the wave will not propagate vertically but merely decay by a factor \( \exp|2\pi/k_z| \).
If \( k_z^2 > 0 \) the waves are usually called internal (or cellular) and if \( k_z^2 < 0 \) these are referred to as evanescent (or external or non-cellular). Examination of (14) show that the waves will be internal provided \( v^2 > \omega_a^2 \) or \( v^2 < \omega_B^2 \). These are classed as acoustic and gravity modes respectively. The division between the classes can best be seen by referring to what is usually termed the diagnostic which shows \( k^2 \) curves plotted in \( (\omega, k_x) \) space - Fig. 28. \( \omega_a \) is the low frequency cut-off for the internal acoustic modes and \( \omega_B \) is the corresponding high frequency cut-off for internal gravity modes. It is not proposed to discuss acoustic modes further but to concentrate on gravity waves.

For most purposes \( \frac{v^2 - \omega^2}{c^2} \) is much less than

\[
k_x^2 \frac{(\omega_B^2 - v^2)}{\omega^2} \text{ which is equivalent to saying that}
\]

\[v_x^2 \ll c^2.\] In the atmosphere for heights up to 100 km this restricts the discussion to waves with \( V_x \leq 100 \text{m/sec}.\)

\[
k_z^2 = \frac{\omega_B^2 - v^2}{v^2} k_x^2 \tag{15}
\]

Gravity waves are dispersive, that is the
INTERNAL WAVES

DISPERSION OF PLANE WAVES IN A COMRESSIBLE MEDIUM

FIGURE 28
direction of energy flow is different from the direction of phase propagation. It is quite straightforward to show that the flux of energy is in the direction of the group or packet velocity or \( \frac{\partial v}{\partial k} \) (Eckart 1960). Using the dispersion relation (14) the horizontal and vertical components of the group velocity are given by

\[
U_x = \left( \frac{\partial v}{\partial k} \right)_{k_z} = V_x \left( \frac{\omega - v^2 c^2}{\omega^2 - v^2 V_x^2} \right) \tag{16}
\]

\[
U_z = \left( \frac{\partial v}{\partial k} \right)_{k_x} = V_z \left( \frac{\omega - v^2 c^2}{\omega^2 - v^2 V_x^2} \right) \tag{17}
\]

(16) shows that the phase and energy propagate in the same horizontal directions but (17) indicates that the vertical direction of energy propagation is opposite to the vertical phase progression. This is one of the peculiarities of gravity waves. In the low frequency limit \( \nu^2 \ll \omega_B^2 \) the group and phase velocities are equal in magnitude but as \( \nu \to \omega_B \) the horizontal group velocity becomes very small.

Some feeling for the physical behaviour of the wave parameters may be obtained from an examination
of equations (5) to (8). These may be re-written by replacing the height derivatives of \( p \) and \( u_z \) by \( \frac{1}{2H} - i k_z \) as

\[
\left( \frac{v^2}{\gamma^2 \omega_B^2} - \frac{1}{2} + ikz \right) \rho = \left( \frac{v^2}{\omega_B^2} - 1 \right) \rho \quad (18)
\]

\[
iki \left( \frac{x^2}{c^2} - 1 \right) U_x = \left( \frac{1}{\gamma H} - \frac{1}{2H} + ikz \right) U_z \quad (19)
\]

Two limits are examined

(i) \( v^2 \ll \omega_B^2 \) and \( v_x^2 \ll c^2 \)

Below 100 km these approximations apply to waves with periods greater than about 20 minutes and phase velocities less than 100 m/sec. Under these circumstances

\[
k_z \approx k_x \frac{\omega_B}{v} \quad (20)
\]

\[
p \left( \frac{1}{2} + ikz \right) \sim \rho \quad (21)
\]

\[
U_x \sim \frac{\omega_B}{v} U_z \quad (22)
\]

In the lower atmosphere \( H \approx 6 \) km and \( \lambda_z \approx 30 \) km so that
hence the fractional pressure changes are always less than the density fluctuations. The lower the horizontal phase velocity the weaker the changes in pressure. This suggests that gravity waves will cause partial reflections through their density perturbations rather than their pressure (or collision frequency) changes.

From (22) \( U_x \gg U_z \) and so the perturbation winds are almost horizontal, parallel to the energy flow.

\[(ii) \quad \nu^2 \rightarrow \omega_B^2; \quad \nu_x^2 \ll c^2.\]

\[k_z^2 \rightarrow 0 \quad (k_x \text{ finite})\]

\[\frac{p}{\rho} \rightarrow 0\]

\[\left| \frac{U_x}{U_z} \right| \rightarrow \frac{2 - \gamma}{2\gamma \frac{H}{k_x}} = \frac{0.03\lambda}{H} \quad (23)\]

For short wave length modes \( \lambda_x \ll 2\pi H \) equation 23 shows that the air particles oscillate mainly in the vertical plane, and the wave motion is essentially transverse.
(ii) VARIABLE ATMOSPHERE.

When the temperature varies with height the discussion of the propagation of gravity waves becomes more complicated. Examination of (11) shows that terms in $H'$ are now present in the equation for $X$ and furthermore equations similar to (11) for parameters other than $X$ are different again for example the equation for $p$ is found to be

$$p'' + \left(\frac{H' - 1}{H}\right)p' + \left[k_x^2 \frac{N^2 - v^2}{v^2} + v^2 \frac{c^2}{\gamma H^2} \frac{H'}{H^2}\right]p$$

so this differs from (11) in a term $-H'/\gamma H^2$ in the last coefficient. Similarly other equations in $u_x$ and $a_z$ can be derived, all slightly different.

Since the equations are all of the form of (12) solutions of the W.K.B. type can be found using the transformation

$$X = \eta e^{-\int d_z/2 a(z)}$$

Giving an equation of the form

$$\eta'' + q^2 \eta = 0$$
\[ q^2 = k_x^2 \left( \frac{\omega_b^2 - \psi^2}{\psi^2} \right) + \frac{\psi^2 - \omega^2}{c^2} + \text{terms in } H' \]  

(24)

Each parameter will have its own \( q^2 \) all differing by terms in \( H' \) (ignoring terms in \( H'' \) and \( H'^2 \)). For a further discussion on W.K.B approximations and gravity waves the reader is referred to Pitteway and Hines (1965).

In the atmosphere the wave will propagate vertically in regions where \( q^2 > 0 \) but will be reflected at heights where \( q^2 = 0 \) (Martyn 1950, Hines 1960).

A5.4 GRAVITY WAVES IN A REALISTIC ATMOSPHERE.

In the real atmosphere temperature and the background winds will all vary with height and viscous damping must be considered. The inclusion of the latter introduce great complexity into the equations and will be ignored here; for a treatment see Pitteway and Hines (1963), Golitsyn (1965), Midgley and Liemohn (1966). In this section a brief description of some work carried out in 1965 on the effects of temperature and winds on the propagation of gravity waves is presented and compared with the more detailed analysis of Hines and Reddy (1967).

The earlier work was based on the W.K.B.
approximation (24), All terms in $H'$ being ignored save that in $\omega_B^2$ and also wind shear terms. This equation for $q^2$ is identical to that discussed by Hines (1966) except that $v^2$ replaces $\omega^2$. The work was an attempt mainly to see what effects the stratospheric winds had on the spectrum of waves reaching the ionosphere. The values of $q^2$ were found as a function of height for given models of temperature (hence $\omega_B^2$) and the winds in summer and winter. The temperature profile given in the U.S. Standard Atmosphere (1962), after suitable smoothing was used as a basis of $\omega_B^2$ - (Fig. 17). The wind profiles for latitude 45°S were taken from Murgatroyd (1965) - Fig. 14.

As an example of the work consider the propagation of a gravity wave of period 20 minutes and horizontal phase velocity 50m/sec. Figs. 29,30 and 31 show plots of $q^2$ versus height for the winter cases of (a) no wind present, (b) wave propagating in the direction of the wind and (c) wave propagating against the wind. When no wind is present the $q^2$ plot is similar to the profile of $\omega_B^2$, which is to be expected since the approximation $q^2 = \omega_B^2 / v_x^2$ holds.

For (b) the most noticeable feature in Fig. 30
Height distribution of vertical wavenumber squared assuming a phase velocity of 50 m/sec and a period of 20 mins.

**FIGURE 29**
Height distribution of vertical wavenumber squared assuming a phase velocity of 50 m/sec and a period of 20 mins. Wave propagating with the wind (winter profile).

FIGURE 30
Height distribution of vertical wavenumber squared assuming a phase velocity of 50 m/sec and a period of 20 mins. Wave propagating against the wind (winter profile).

FIGURE 31
are the two very large increases in $q^2$ at heights of 47 and 85 km respectively. These are associated with the singularities which exist in (24) when $v = 0$. This means that the horizontal phase velocity of the wave just matches the wind velocity, the heights at which this occurs being known as the singular (Hines and Reddy 1967) or critical levels (Booker and Bretherton 1967). As the wave propagates towards a critical level its vertical group and phase velocities tend to zero and the wave propagates horizontally. Initially it was thought that the wave was reflected at these heights (Martyn 1950) but it now seems that it is absorbed into the mean flow instead (Booker and Bretherton 1967).

Finally Fig. 31 indicates the situation for propagating against the mean wind. Here the effective frequency ($= \omega + k_x u_o$) is increased and $q^2$ has its smallest value at the height of maximum wind (65 km). At somewhat higher frequencies $q^2 < 0$ over a range of heights and an upward propagating wave would be reflected.

Similar plots were constructed for other values of $\omega$ and $k_x$ and for the case of summer winds. It was generally concluded that temperature effects
were not great (for most modes of interest) but
directional filtering was possible - that is the mean
winds would tend to remove gravity waves propagating
against or with the mean flow and so leave an initially
isotropic spectrum with a meridional bias.

It is interesting to compare the above
mentioned work with that of Hines and Reddy (1967).
They calculated the transmission coefficients for
gravity waves propagating to the ionosphere from the
lower atmosphere using similar temperature and wind
profiles. As expected they show the effect of direct­
ional filtering, especially for modes of low phase
velocity. However the main feature that Hines and
Reddy show is the strong thermal reflections that waves
with low phase speed suffer. From the approximation
\[ q^2 = \omega_B^2 \nu_x^{-2} \]
it might be expected that the lower the
phase speed the larger \( q^2 \) and the less chance a wave
has of being reflected (neglecting wind effects).

Probably the reason why the earlier calcu­
lations do not show these strong reflections may be
attributed to the limitations of the W.K.B. approx­
imation. This only holds when \( q^2 \) does not change rap­
idly in a small height distance, that is \((1/q)(d_q/d_z)\ll 1\). Near \( q = 0 \) the wave is totally reflected but for large
\[ \frac{d q}{d z} \] strong partial reflections occur in an analogous fashion to radio waves. How \( \frac{d q}{d z} \) varies may be seen using \( q^2 = \frac{\omega_B^2}{V_x^2} \) and the further approximation
\[ \omega_B^2 \approx g(\gamma - 1)/\gamma H, \] where \( H = \gamma gc^2 \), giving

\[ \frac{d q}{d z} \approx -\frac{\omega_B}{c^2 V_x} \frac{d c}{d z} - \frac{q}{\gamma} \frac{dy}{d z} \]  \( (25) \)

Although a number of terms have been neglected they are all fairly small in comparison with the terms retained. The main feature of \( (25) \) is that \( \frac{d q}{d z} \) is largest when \( V_x \) is small so that it is the ratio \( \frac{d c}{V_x} \) which is important rather than \( \frac{d c}{c^2} \). Hines and Reddy point out that the term \( \frac{d c}{V_x} \) is also most important in the calculations of the reflection coefficients.

Both winds and temperature variations are therefore important when considering the coupling of the lower atmosphere to the ionosphere through gravity waves. Further work is still required to elucidate fully their respective roles in wave propagation. One possible drawback to the work of Hines and Reddy (1967) is their use of a multilayer atmosphere, with temperature and wind held constant in each segment. This may lead to constructive and destructive
interference caused by the vertical wavelengths being comparable to artificial layer thicknesses. It is hoped to investigate this effect further, perhaps by replacing the layers with ones in which wind and temperature vary linearly.
APPENDIX 6.

POWER SPECTRUM ANALYSIS

The power spectra of the fluctuations in the E region plots were found using normal digital techniques. Following Blackman and Tukey (1958) the d.c. component was removed by taking the mean of the raw data \( z_t \) and subtracting to give the new series \( x_t \):

\[
x_t = z_t - \frac{1}{N} \sum_{i=1}^{n} z_i
\]

\( n = \text{number of observations} \)

The autocovariance function was then found giving

\[
\gamma_p = \frac{1}{(n-p)} \sum_{t=0}^{n-1} x_t x_{t+p}
\]

\( p = \text{lag} \)

The line powers were then calculated by the finite Fourier transform

\[
V_p = \left[ C_0 + 2 \sum_{q=1}^{m-1} C_q \cos \frac{q \pi}{m} \cos \frac{p \pi}{m} + C_m \cos p \pi \right]
\]

where \( m \) is the maximum lag.
Finally the powers were smoothed using the Hamming function

\[ U_o = 0.46 \ V_1 + 0.46 \ V_o \]
\[ U_p = 0.23 \ (V_{p-1} + V_{p+1}) + 0.46 \ V_p \]
\[ U_m = 0.46 \ V_{m-1} + 0.46 \ V_m \]

There seems to be some disagreement as to how big \( m \) should be, Blackman and Tukey suggesting \( m \leq 10\% \) of \( N \). However Jenkins (1961) suggests that \( m \) is best chosen by plotting the auto correlation coefficient (the normalized autocovariance function) and using the lag where the correlation falls to zero (provided \( m \) is less than 30\% of \( N \) and the number of degrees of freedom is not too large). From the plot of the autocorrelation functions (Fig. 21) the best compromise seemed to be a lag of \( m = 60 \) since all 3 functions are very close to zero at this lag.

The number of data points was 536, 436 and 360 on the 14/10, 5/10 and 31/10 respectively. The number of degrees of freedom is found from

\[ k = 2\left(\frac{N}{m} - \frac{1}{2}\right) \]
giving 17, 14, and 12 degrees of freedom with resultant stabilities at the 96% confidence level of $\pm 3\, \text{dB}$, $\pm 3.5\, \text{dB}$ and $\pm 3.7\, \text{dB}$ respectively.
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