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Macroprudential tools and monetary policy interactions in New Zealand

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Abstract

This thesis constructs a DSGE model of the New Zealand economy based on Iacoviello (2005). Its key innovation is the introduction of an investor property sector that allows consideration of the loan-to-value (LVR) restrictions that the Reserve Bank of New Zealand (RBNZ) recently implemented. These tools were employed in order to mitigate the risks associated with highly leveraged investors with multiple exposures to the housing market.

With the introduction of an investor sector, both patient and impatient households are given the choice between owning and renting property. This decision is affected by house prices relative to rental prices, the expected capital gains for home ownership, and any LVR restrictions that the impatient households may face. By contrast, property investors are assumed to have the choice between investing in capital or investing in housing. This model therefore shows the impact of the trade off between investing in housing and investing in capital on other variables in the economy.

The model shows that LVR restrictions work to stabilise output in the event of a monetary policy or housing preference shock. In addition, the presence of LVR restrictions affects the distribution of housing to different agents in the economy. Although risk is not explicitly modelled in this framework, this suggests LVR restrictions may increase financial stability by encouraging greater activity in the housing market from less risky agents (such as patient households in this framework).

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Chapter 1

Introduction

This thesis introduces a framework for examining the impact of loan-to-value (LVR) restrictions on the transmission of monetary policy and the business cycle. It is timely as many central banks, including the Reserve Bank of New Zealand, have started adding macroprudential tools to their financial stability toolkits. These tools are aimed at mitigating the build-up of systemic risk during the business cycle and ensuring systemic resilience in the event of a crisis. Due to the impact these tools have on both the availability and price of credit, they have clear implications for both the business cycle and the transmission of monetary policy through interest rates.

With the growing popularity of macroprudential tools at central banks, there has been a recent growth in literature that considers the impact of these tools on monetary policy transmission and the business cycle. Given the popularity of general equilibrium frameworks for modelling the business cycle, many papers have incorporated these tools into dynamic stochastic general equilibrium (DSGE) models.

The model presented in this thesis contributes to the literature by constructing a DSGE model based on Iacoviello (2005) that is able to consider how the application of LVR restrictions on different agents affects the business cycle. As the name implies, LVR restrictions limit the amount a borrower can borrow to a certain percentage of their underlying collateral, often housing. High LVR lending is considered riskier as the lender has less of a buffer on the loan in the event of a downturn in the price of the underlying asset. If a high LVR borrower is unable or unwilling to continue servicing the loan, it is more likely that the lender will suffer a loss, as the value of the borrower's equity in the collateral plus whatever can be earned from its sale may not meet the value of the loan.

This thesis builds on Iacoviello (2005)'s model by introducing a rental market for housing and investors to that model. In this model, investors have the choice between investing in capital and investing in housing, with the choice determined by which has the highest expected capital gain and rental yield. As investors do not gain utility from housing itself, they treat houses as financial assets. From a policy perspective, the inclusion of investors allows the model to consider the effect of investor-specific LVR restrictions on the business cycle, as well as evaluating the impact of standard LVR restrictions that do not distinguish borrower type.

By contrast, households gain utility from housing and are subject to exogenous housing preference shocks. These shocks approximate *bubble*-like behaviour to the extent that they cause an idiosyncratic deviation of the utility gained from housing with no corresponding increase in the asset value of housing.

As noted above, this analysis is particularly pertinent to New Zealand where the Reserve Bank of New Zealand (RBNZ) recently implemented a macroprudential toolkit to enable it to combat the financial stability risks associated with high domestic house price inflation. In 2013, the RBNZ implemented an LVR restriction that limited the amount of new lending banks could undertake above an 80% LVR. At the time, it was noted that this restriction would be unlikely to bind investors who could withdraw equity from current residential holdings to invest in additional properties.

Despite meeting the residual LVR thresholds, such borrowers are nevertheless risky, as their exposure to single markets may make them more likely to withdraw quickly in the event of a sudden downturn in property prices. In addition, if mortgage repayments are reliant on rental returns, a downturn in rental prices could also affect borrowers' ability to service their mortgages. Given these concerns, the RBNZ implemented a further wave of LVR restrictions targeted specifically at housing investors in 2015.

This thesis is organised as follows. Section II reviews the literature on housing in DSGE models, before discussing the findings from papers that explicitly incorporate macroprudential tools. Section III introduces a simple Bayesian VAR (BVAR) to highlight the empirical relationship between housing and the business cycle in New Zealand and to note key relationships that the DSGE model should capture.

Section IV introduces the model, highlighting its key features, with further exposition and derivations available in the appendix. Section V discusses the model's calibration, comparing its steady state results to key relationships in the New Zealand data and to the impulse responses derived in the BVAR.

Section VI of the thesis analyses the impulse responses for key model variables to different shocks in the model. Some of these shocks are standard, such as the monetary policy shock, while others are more idiosyncratic to the literature, such as the housing preference shock. This analysis allows us to explore the transmission of shocks originating in different sectors through the model. The housing preference shock is of particular interest as it affects the utility patient and impatient households gain from owning housing. This allows us to observe how an idiosyncratic increase in demand for housing, which is unrelated to model fundamentals, affects other real variables.

Most importantly, this section explores the impact of different levels of LVR restrictions on the transmission of shocks to agents within the economy in order to explore the business cycle properties of LVR restrictions. The baseline case is assumed to be a 95% LVR restriction imposed on both impatient households and investors. The next case is a restriction of 80% on impatient households while leaving the restriction for investors unchanged from the baseline. This is equivalent to the LVR restriction imposed by the Reserve Bank of New Zealand in 2013. The final scenario retains the 80% LVR restriction on impatient households and in addition imposes a 70% LVR restriction on investors. This last scenario is a model equivalent to the LVR restrictions imposed by the Reserve Bank of New Zealand in 2015. The final section of the paper concludes.

The scenario analysis in Section VI reveals that LVR restrictions reduce output fluctuations in response to monetary policy shocks. This is especially the case with investor-specific LVR restrictions, where the investor quickly divests property in response to the increased interest rate and then only gradually rebuilds their housing stock following the shock. As a result of these dynamics, increased interest rates suppress investor activity far longer in the presence of an investor-specific LVR restriction than without it.

With a positive housing preference shock, the constraint on the capacity of investors to borrow in response to the shock results in smoother consumption and output paths. Crucially, as consumption remains higher with the investor-specific LVR restriction, capital investment is higher and smoother than in other scenarios. Although risk analysis is beyond the scope of the model, the redistribution of housing toward patient and impatient households in response to the shock implies an improvement in financial stability. This assertion is supported by Kelly and O'Malley (2014)'s analysis that suggests that investors are at higher risk of default in the event of a downturn.

In response to a positive technology shock, an investor-specific LVR is similarly effective at smoothing investor consumption and output by moderating investor activity in the housing market. Although there is a sustained decline in capital investment following the technology shock, there is a slightly stronger increase in housing investment in the medium term in the presence of LVR restrictions. The paths for house prices, inflation and the interest rate in response to this shock remain broadly unchanged by the use of LVR instruments. Again, this implies that LVR restrictions allow the interest rate to moderate inflation while minimising fluctuations in output.

The presence of LVR restrictions do not materially alter the transmission of a positive cost-push shock to real variables. However, as with other shocks, the presence of LVR restrictions has important implications for the distribution of housing among agents. In particular, investor housing ownership is significantly reduced in the presence of an investor-specific LVR restriction following a cost-push shock relative to scenarios without an LVR restriction.

Comparing the steady state of the model under different LVR assumptions also provides insight how permanent application of an LVR restriction would affect key housing market variables in the long term. From steady state analysis, it is possible to derive the following stylised facts about the long term impacts of LVR restrictions on the economy:

(a) Regardless of the agent that an LVR restriction constrains, it leads to a decline in residential investment to GDP, leading to higher consumption in the economy.

(b) Investor specific LVR restrictions lead to decreased rental supply and increased rents.

(c) Impatient household LVR restrictions lead to a severe contraction in the percentage of housing with a mortgage.

(d) Implementation of any LVR restriction causes investors' borrowing to fall, regardless of the agent the restriction constrains.

These stylised facts and their policy implications are discussed further in the concluding remarks.

Chapter 2

Literature review

The dynamic stochastic general equilibrium (DSGE) framework has become a standard means of modelling in central banks around the world. The key advantage of this framework is that it allows one to observe how a shock in one sector flows through to, and affects, the rest of the economy.

Starting from the simple three equation model described in Gertler et al. (1999), DSGE models have grown in size and complexity in order to capture the impact of a variety of shocks on different sectors in the economy. Since the GFC, there has been increased focus on incorporating a housing market and the risks associated with credit booms in a general equilibrium framework. Recent empirical studies have noted the interconnection between house prices and GDP, with busts in the housing market being associated with large and persistent falls in economic activity (Jorda et al. (2015)).

With the GFC came a call for improved financial regulation to reduce the risk of, and mitigate the losses arising from, another financial crisis. These measures have included changes to the instruments that can be considered bank capital for regulatory purposes and enhanced liquidity requirements (Basel Committee on Banking Supervision (2010)).

Coupled with these measures, many countries have considered supplementary policies aimed at minimising systemic risk in the financial system in general. Where the traditional focus of prudential regulation has been the risks faced by individual institutions, the GFC forced regulators to recognise that there may be broader risks that need to be recognised and addressed contemporaneously. *Macro*-prudential regulation recognises that the interconnectedness of the financial system means that even if a bank is well-capitalised for the risks it faces, it may be assuming a sub-optimal level of risk from a system-wide point of view (Borio (2003)).

Macroprudential regulation is aimed at improving financial stability either by mitigating the impact of external shocks on the system (Allen and Wood (2006)) or by increasing the resilience of the system to shocks that are endogenously generated in the system (Schinasi (2004)). Although some have argued that avoiding 'bubbles' could be a mandate for macroprudential policy (Landau (2009)), it is generally accepted that the aim should be the stable provision of financial services rather than avoiding bubbles or credit imbalances. As the dot-com bubble illustrated, such imbalances do not automatically result in a reduction of credit supply that precipitates a real contraction. As such, it is not appropriate to conclude that a bubble in one sector is by nature a financial stability risk to be addressed (Bank of England (2009)).

In this context, the greater emphasis on regulations that explicitly address housing market risks is unsurprising given the GFC was driven by the collapse of house prices in the United States and the default of many mortgage backed securities (MBS). Many regulators have introduced policies to reduce risky loans to the housing sector and/or minimise the systemic losses that could result from a collapse in house prices. These tools are applied similarly across all institutions, rather than traditional prudential policies that may be applied differentially depending on the size or systemic importance of the institution.

2.1 Systemic risk and macroprudential policy

As an example of systemic risk, consider a period of elevated house price inflation due to low interest rates. In such circumstances, lower rates would reduce the cost of borrowing while lowering the discount factor applied to the income stream from rental, which would increase the asset value of housing. As a result of the increased underlying return to housing, house prices and rental prices would rise. Note that the expected return from housing would be the rental yield and any expected capital appreciation of the house.

At the same time, low interest rates would render the yield from holding money low, which would increase incentives for those that own housing to withdraw equity from their current housing loans in order to invest in more housing. Such circumstances could lead to greater borrowing and increased demand for housing, which would in turn increase house prices and the expected returns from capital gain.

However, if circumstances were to reverse and interest rates were to rise, those that borrowed heavily may not be able to continue servicing their loans, especially if their income is a small fraction of their mortgage. As borrowers start to default on their loans, more houses would become available on the market as banks attempt to sell them in order to recover the value of the loans. The added supply of house in the market would reduce house prices. Contemporaneously, the yield on housing as an investment would fall as the discount rate applied to rents would be higher, which would put further downward pressure on house prices.

If borrowers expect future interest rate increases and lower house prices, even those that could afford to continue servicing their loans may choose to sell while house prices are still relatively high. This would especially be the case for investors as their only return from owning the house is rental and the potential capital gain. Empirical evidence suggests that investors are more likely to strategically default on their loans than owner-occupiers (McCann (2014)).

Increased supply of housing would further depress house prices, reducing the value of all houses as underlying assets. This would increase the risk associated with these loans. As house prices fall, the value of houses as security would decrease for all residential loans in banks' books.

A falling property value relative to value of the loan will increase key measures of risk associated with these loans (for example, loan to value ratios), which will in turn require banks to hold more capital against these loans. This could have two related consequences. First, to the extent that bank capital is 'expensive' (see Admati et al. (2010) for discussion) banks will be disincentivised to continue holding these loans, and may attempt to refocus their lending on a different sector. Second, the increased risk associated with such loans may be outside the risk appetite of a bank's board, which may then require the bank to reduce their exposure to these types of loans.

In either case, if a bank cannot sell the loans to another mortgage provider (likely the case if all banks are facing the same market) then they will have to foreclose on them in order to recover what they can from the loans. This creates a cycle that could threaten the stability of the system if banks are highly exposed to the collapsing sector.

Macro-prudential policies are aimed at guarding against the build-up of this excessive risk in the system. Such policies may include counter-cyclical capital requirements and LVR restrictions. Counter-cyclical capital requirements require banks to hold more capital at the peak of the economic cycle that can then be used to absorb losses in a downturn. As described in the introduction, LVR restrictions limit the volume of lending that a bank may undertake above a given LVR.

In New Zealand, in response to the increasing house price inflation, the Reserve Bank has implemented a LVR restriction on banks' lending that requires that no more than 10% of new residential mortgage lending be above 80% LVR (Reserve Bank of New Zealand (2013)). Recognising the additional risk associated with investor lending, the RBNZ added an additional restriction on new residential mortgage lending that requires investors (as opposed to owner-occupiers) to have a minimum 30% deposit on residential properties (Reserve Bank of New Zealand (2015)).

2.2 Interaction between policies

This paper seeks to explore the likely impact of different agent-specific LVR restrictions in a DSGE framework. Although there are other frameworks that may be more ideal for considering the impact of these policies in isolation (i.e. frameworks that allow borrower default through time) the benefit of using a DSGE model is that it allows one to observe the interactions with monetary policy and the business cycle.

In some cases, monetary and macro-prudential policies move in the same direction and are complementary to each other's purposes. For example, during a booming economy with a high output gap, increased inflationary pressure and high house prices, the central bank would likely respond by tightening monetary policy and tightening macroprudential policy if there is risk of excessive credit growth.

However, it may also be the case that monetary policy and macroprudential policy may operate in a way where they are in conflict with each other. This could be the case where there is a slowing economy, with potentially negative output gap and low inflationary pressure, which requires stimulus through an easing in monetary policy. At the same time, limitations in supply of, or increased demand for, housing may lead to higher house prices, which could require more restrictive macroprudential policy if there were excessive risks associated with lending to that sector.

In such case, the monetary policy easing required to address the decline in demand and inflation in the rest of the economy may also worsen issues associated with house price inflation. Lower interest rates lower the cost of servicing mortgages as well as increasing the present-value of future rental payments so would be expected to increase house prices. In such circumstances, the interest rate would be in conflict with the macroprudential obligations of the central bank. These circumstances described are similar (at a high level) to the issues currently facing the Reserve Bank of New Zealand.

Papers including Angelini et al. (2012) have explored welfare implications of cooperation or autonomous decision making by independent macroprudential and monetary policy authorities. But where macroprudential and monetary policy making powers are vested in the same institution it is possible to consider the application of both policies jointly. The purpose of this paper is to develop a framework to consider the impacts of exogenously set LVR restrictions on monetary policy and the business cycle.

Other papers have considered alternative models for macroprudential and monetary policy decision making. These studies have sought to investigate whether these policies should be conducted within the same institution, and if so, by the same decision makers (see Svensson (2016), De Paoli and Paustian (2013), and Collard et al. (2012)). For the purposes of this thesis, I take the joint decision making model of the Reserve Bank as given, but I note that alternative models could be an area of further research, especially if alternative policy rules were to be considered.

2.3 DSGE in modelling systemic risk

DSGE models have a number of advantages for use in modelling the business cycle implications of the application of macroprudential tools, but they also have limitations that should be acknowledged. One of the key advantages of DSGE models is that the short term and long term impact of different policy measures can be quantified both at equilibrium and in response to different exogenous shocks. This allows them to take into account spillovers through time (dynamic effects) and in different markets (general equilibrium effects). Crucially, as DSGE models include different markets within the economy, they allow one to see the transmission of policy changes across variables and markets. This is very useful as it enables us to vary monetary policy and macroprudential rules within the model in order to assess the impact of different combinations of rules across sectors.

Another benefit of applying a DSGE approach is that it allows for micro-founded identification of the channels through which different effects take place. These structural foundations provide internal consistency to the model, although at the cost of less flexibility than in purely data-based approaches. In contrast, unrestricted multivariate data models, such as VARs, may fit data very well but fail to provide any economic intuition as to why certain relationships are observed to exist between variables. Given the very short time series of data available on the application of LVR restrictions in New Zealand, such models would offer little insight into business cycle dynamics in this case.

The limitations of DSGE models in matching data should not be overstated. Although unrestricted multivariate models may better fit the data in sample, Smets and Wouters (2003) show that an estimated DSGE model is able to compete with standard unrestricted time series models in out-of-sample forecasting.

However, the general equilibrium properties of DSGE models do place limitations on their use for modelling financial frictions. In particular, a model has to be calibrated or estimated based on parameters at a 'normal' point in the business cycle so as to allow a unique equilibrium. This may limit the usefulness of the model for describing impacts if there is a change in underlying parameters, as may be the case following a major financial crisis. Additional complications arise when estimating a model. For example, data is detrended when fitting the model to the data, which eliminates outliers and results in a stable series with no structural breaks.

These models are also typically solved using linearisation techniques that mean there can be no discontinuity between normal times and crisis periods - a result that is at odds with the observation that we would expect for nonlinearities to be prevalent during crisis periods. Therefore, some argue that systemic risks are outliers that are not able to be considered in this framework (Freixas et al. (2015)).

Although the DSGE framework may not be amenable to the discussion of systemic risk, it should be emphasised that the purpose of this paper is to highlight the impact of LVR restrictions on the business cycle during normal times. Therefore, even though the efficacy of the instruments themselves in addressing systemic risk may be beyond the scope of analysis in a DSGE model, DSGE is an appropriate framework for evaluating their impact on the business cycle. Furthermore, even with the limitations of DSGE, it is possible to draw conclusions about the impact different policies have on the distribution of property among different agents and therefore the instruments' risk mitigation properties.

2.3.1 Other areas of research exploring housing and the business cycle

It should be noted that DSGE is not the only framework used to consider the impacts financial frictions on housing and risk. There is a developing literature that uses incomplete markets models to explore effects of changing house prices on other real and financial variables. An extensive review of this literature is beyond the scope of this thesis, but there are some contributions that should briefly be noted.

Favilukis et al. (2010) incorporate aggregate business cycle risk and heterogeneity in bequests into an incomplete markets framework with collateral constraints. Within this framework they find that a relaxation of collateral constraints leads to a large boom in house prices due to declining housing risk premia. In this framework, they find that low interest rates cannot explain high house prices.

Similarly, Iacoviello and Pavan (2013) use an incomplete markets framework to consider the impact housing and debt have on the volatility of the macroeconomy in a model that includes aggregate risk. However, as that model has no risk-free asset, it does not consider the impact of risk premia on the economy. Likewise, Campbell and Hercowitz (2005) consider the impact of different collateral constraints in a general equilibrium model with collateralised debt and heterogeneity in time preferences, but again there is no role for risk premia in explaining economic fluctuations in the model.

The finding of Favilukis et al. (2010) that collateral constraints have a large impact on house prices is doubted by Kiyotaki et al. (2011). Using a life-cycle model of a productive economy, they find that expected productivity and world interest rates have large impacts on house prices, but that changing financing constraints only has a limited effect on house prices. Likewise, Sommer et al. (2013) similarly find that low interest rates and higher incomes can account for much of the observed house price-to-rent ratio, while relaxing collateral constraints only plays a minor role.

There are few papers that incorporate a rental market for housing into a DSGE model. Ortega et al. (2011) incorporates a rental market using a CES aggregator similar to that used in this paper but with the aim of evaluating the impact of different housing subsidies on price in the rental market. In my research, I have not found a paper that includes a rental market in order to evaluate the impact of macroprudential tools on rental prices.

2.4 Financial frictions in DSGE models

Since the GFC there has been an increased interest in models that explicitly incorporate financial frictions into general equilibrium models. This has developed the literature in three related ways.

First, there has been a proliferation of models that include credit effects on the real economy, typically through the introduction of external finance premia. This literature developed from Bernanke and Gertler (1989) and Bernanke et al. (1999) who incorporated a financial accelerator mechanism into a standard real business cycle model. This mechanism makes the cost of credit depend on the underlying assets of the borrower. Aoki et al. (2004) applied this literature to the housing market by using housing as the underlying asset rather than focussing strictly on entrepreneurs' net worth. Note that the only limitation on borrowing in these external finance models is the cost of credit or external finance premium that is applied given the agents' underlying assets. That is, provided the borrower can finance the loan, there is no limitation on the quantity of borrowing that may take place within the model.

The other popular method of including financing constraints in these models has been through use of collateral constraints. Such models have been particularly useful for answering questions about the impact that a contraction in the consumer and financial sectors may have on the real economy. Building on Kiyotaki and Moore (1997), these models explicitly limit the amount of borrowing to a certain percentage of the underlying collateral of the loan. Thus, a downturn in the price of underlying assets limits borrowing in the current period, which delays the return to equilibrium in future periods. The addition of collateral to these models allows one to directly evaluate the impact of LVR restrictions.

Third, there has been development in models that incorporate credit spreads through the explicit modelling of a banking sector. These models allow the inclusion of credit spreads through modelling the wedge between bank lending rates and the risk-free rate. Some models go further to consider the role of bank capital in business cycle fluctuations and the propagation of systemic risk. Each of these strands of literature will be discussed in detail below.

These strands of literature have grown in popularity at central banks that have sought to better understand the supply and demand influences of house prices, housing investment, and the wealth effect of rising house prices on consumption. Central banks and international organisations that have developed models to address these matters include the Federal Reserve in the United States (Liu et al. (2013)), the European Central Bank (Lombardo and McAdam (2012)), Bank of England (Tayler and Zilberman (2016)), Bank of Canada (Alpanda et al. (2014)), the International Monetary Fund (Prakash et al. (2012) and Benes et al. (2014)), Riksbank (Walentin (2014)) and Central Bank of Ireland (Clancy and Merola (2014)).

2.4.1 The External Finance Premium Channel

The external finance premium is the the difference between the price that debtors must pay to creditors and the opportunity cost of using internal funds. Empirical evidence suggests that the cost of external funding is almost always higher than using internal resources (De Graeve (2008)). One cause of this premium is the information asymmetry faced by lenders who have less information than borrowers about the status or prospects of success of an investment project.

The cost of remedying this information asymmetry can be described with project monitoring costs, as assumed by Diamond (1965). Akerlof (1970) showed that the presence of information asymmetries could in the extreme prevent a market from existing at all.

Responding to this work, Townsend (1979) proposed an optimal contract to address information asymmetries and the associated monitoring costs. In this contract, the borrower reports the outcome of the project to the lender, who then only requires an audit of the project result if the reported profit is lower than the monitoring threshold. The monitoring threshold is a decreasing function of the borrower's capital, as a wealthier borrower could pay the agreed return on investment regardless of losses on the project. Conversely, the monitoring threshold is increasing in the risk-free rate, which represents the opportunity cost of investing in the project.

This contract design was incorporated into Bernanke and Gertler (1989). In that paper, entrepreneurs borrow from lenders in order to produce capital through risky investment projects in an overlapping generations model. The outcome of a project is only freely visible to the entrepreneurs, while lenders have to pay monitoring costs to observe the outcome. The combination of uncertainty and monitoring costs in this model generates a positive external finance premium on loanable funds, which limits the ability of entrepreneurs to borrow. Crucially, this paper showed that the agency costs of borrowing are inversely related to borrower's net worth. That is, in good times, improved borrower balance sheets increase investment demand as the cost of credit falls. The opposite effects amplify the resulting contraction in bad times. With its counter-cyclical external finance premium, this model laid the foundations of the financial accelerator literature that would follow.

Following the overlapping generations model of Bernanke and Gertler (1989), Carlstrom and Fuerst (1997) embedded the frictions introduced by Townsend (1979) into an infinite-horizon setting in a standard real business cycle (RBC) model.

This literature was extended again by Bernanke et al. (1999) (BGG) who incorporated an external finance premium into a standard New-Keynesian DSGE model. BGG showed that there is an increasing relationship between the premium on external funds and the capital to wealth ratio. The paper abstracts from firm reputation by assuming that the producers of capital face random termination through time. Given the model's relative simplicity, the authors noted there were many possible extensions that could be made to the model, including the incorporation of nominal debt, a banking sector and open economy features.

One of the first papers to explore these additional dimensions of research was Christiano et al. (2004). That paper expanded the BGG model to include labour market features and a banking system. Using their model, they successfully replicate many features of the data from the Great Depression. Rather than focussing on capital, Aoki et al. (2004) modify the framework to incorporate housing that produces housing services.

Motto et al. (2010) incorporated nominal debt into the model to evaluate the impact of nominal debt contracts on the financial accelerator mechanism. They find that the financial accelerator mechanism is only of minor importance in explaining business cycle fluctuations where there were nominal debt contracts.

Despite the theoretical attractiveness of the financial accelerator mechanism, some studies have doubted its empirical relevance. Meier and Muller (2005) use a minimum distance strategy (similar to Rotemberg and Woodford (1998) and Christiano et al. (2005)) to match the impulse responses of a medium scale DSGE with a financial accelerator mechanism to those estimated from data in response to a monetary policy shock. Using the estimated parameters, they find that their model is capable of reproducing the shape and magnitude of the empirical impulse responses.

Although the estimated parameterisation of their model implies an important role for nominal price and wage rigidities as well as capital adjustment costs, the estimate for the parameter governing the financial accelerator mechanism is not statistically significant. This suggests that the financial accelerator does not improve the empirical performance of DSGE models in response to monetary policy shocks and, as such, may be of limited relevance.

Conversely, Christensen and Dib (2008a) find that the inclusion of a financial accelerator mechanism improves the empirical fit of output and investment volatilities in a model with a modified Taylor rule. In particular, they note the importance of the financial accelerator for spreading of investment shocks to the wider economy.

From these studies, there is evidence that inclusion of a financial accelerator can improve empirical fit in DSGE models in some cases. However, its significance in the model will depend on the monetary policy rule used and the source of the shock under consideration, among other characteristics.

Models containing an external finance premium also have other limitations that should be noted. First, they are not capable of generating the asymmetric dynamics observed in the data where the premium rises much faster in response to deteriorating credit quality, as model implied premia are normally assumed to rise and fall linearly. In addition, they typically do not take account of expectations of future asset prices in determining the current level of the premium, as the premium is derived only from the current net worth of the entrepreneurs.

Furthermore, models with a financial accelerator are not apt to capture increased bankruptcy rates during economic downturns due to the assumption of a constant probability of bankruptcy. For these reasons, models that incorporate an external finance premium are best suited to evaluating business cycle dynamics during normal times rather than for hypothesising the likely consequences of severe financial distress.

2.4.2 The Collateral Constraint Channel

The inability of the external finance premium to capture asymmetric credit dynamics is remedied in the collateral constraint literature. Unlike in models containing an external finance premium where the supply of funds is only limited by the price of credit, agents facing a collateral constraint are directly limited in the amount they can borrow. Typically, models specify an exogenous or endogenous constraint on the amount an agent can borrow to a certain percentage of their underlying assets that are offered as collateral for the loan. As such, this methodology limits the availability of funding to borrowers in the model.

In collateral based models, in the event of the borrower's bankruptcy, the underlying assets (or collateral) used to secure the loan are transferred to the creditor. It follows that creditors can only recover the nominal value of the assets used to secure the loan in the event of default. As a result, creditors in these models are very sensitive to changes in the underlying value of collateral. It is this feature that makes these models better able to capture the observed asymmetry in the propagation of negative financial shocks in the economy.

Following Brazdik et al. (2012), consider a firm that allocates its resources optimally between financial capital (deposits at the bank) and productive capital (machinery). Assume this firm has no access to external funding. In response to a temporary income shock, the firm will either choose to consume the additional revenue or increase its financial capital (as it is assumed to have already optimised its production factors).

Suppose now that the firm faces a negative income shock. If the shock is small enough, the firm will absorb the shock by reducing its expenditure or the amount of financial capital it holds, or both. However, if the shock is too large to be absorbed by the firm's liquid resources, it will have to reduce its productive capital as well, which will affect its output in future periods.

In contrast, where the firm has the option of borrowing outside financial capital, it will prefer to use external funding in order to absorb the shock and will gradually return to the optimal debt allocation over time. However, with a collateral constraint, the firm's ability to obtain external financing will be limited by its underlying assets.

It is this limitation that allows for the asymmetric impact of shocks: even with the option of obtaining external finance, a sufficiently large negative shock could exhaust a firm's financing options. This forces the firm to reduce its productive capital, which results in a real economic contraction. In this way, limiting external financing through a collateral constraint allows for the asymmetric propagation of shocks.

The collateral constraint literature developed from Hart and Moore (1994). That paper introduced a model of external funding where there is a risk that the project will be unsuccessful in which case the debtor will not be able to repay the loan. This leads the lender to impose an upper limit on the amount of credit made available to the borrower, with the limit depending positively on the assets the borrower can use to secure the loan.

Kiyotaki and Moore (1997) expanded the literature by introducing a model that included both patient and impatient firms, with the latter facing credit constraints on the amount they could borrow to finance expenditure. These credit constraints limit the amount the impatient firm could borrow to the value of their durable assets, which are also used as factors in production. It follows that changing underlying collateral values amplifies and propagates shocks through the economy as falling collateral values limit firms' ability to borrow and invest in both current and future periods.

Similarly to the external finance premium literature, the finding that collateral constraints have important amplification effects on the business cycle has been doubted. Cordoba and Ripoll (2004) show that the financial accelerator effect observed in Kiyotaki and Moore (1997) was driven by two key assumptions: the risk neutrality of investors and constant returns to scale in production.

With more realistic assumptions of risk-averse lenders and decreasing returns to scale in production, Cordoba and Ripoll (2004) find that credit constraints only have a small amplification impact on responses in the model. Through simulations they conclude that most parameterisations will produce some amplification compared to perfect-markets models, but that a large amplification from the financial accelerator effect can only be achieved with few combinations of parameters.

Similarly, Kocherlakota (2000) finds that the amplification effect of collateral constraints is sensitive to the quantitative specification of the underlying economy, especially factor shares.

Like Christensen et al. (2011) for the external finance premium literature, Iacoviello (2005) was sceptical of the use of real contracts in the collateral constraint literature. To explore the impact of nominal contracts, he included nominal debt in a DSGE with housing as collateral. In this model, there is a financial accelerator response to positive demand shocks where increased asset prices buoy borrowing and consumption. This mirrors the observed empirical relationship between house prices and consumption.

However, the model also shows that there is a decelerator mechanism in response to an adverse supply shocks, as increased inflationary pressure decreases the real value of obligations, improving borrowers' net worth and minimising the resulting economic contraction. Finally, he notes that including asset prices in the central bank's policy rule has a negligible impact on the variation of output and inflation in the model.

Although Iacoviello (2005) was capable of matching empirical responses on the demand side of the housing market, the model assumed a fixed supply of housing that limited the supply side analysis that could be performed. Iacoviello and Neri (2010) developed this model by incorporating a heterogeneous supply side that allows business investment, consumption and housing to be produced from capital, labour and land, the latter of which is only used for housing production. As expected, this creates a trade-off between business investment and housing in the model that allows for endogenous dynamics between house prices, business investment, and consumption.

Iacoviello and Neri (2010) find that housing demand shocks and housing technology shocks together explain roughly half of the cyclical variation in housing investment and house prices. Monetary policy shocks contribute between 15-20 percent of the volatility. Based on this result, it is arguable that macroprudential tools may be effective to the extent that they reduce cyclical demand for housing, but nevertheless monetary policy remains a strong cyclical driver of house prices. Crucially, they find that specifying a collateral constraint on household borrowing amplifies the impact of housing demand and interest rate shocks on consumption.

Given the importance of the housing market to the persistent depression following the GFC , Liu et al. (2010) were puzzled by the earlier finding of Kocherlakota (2000) and Cordoba and Ripoll (2004) that credit constraints only have a muted impact on the transmission of shocks. Using a model in which entrepreneurs can use both land and capital as collateral, as well as being used as inputs in production, they show that credit constraints can substantially amplify and propagate macroeconomic fluctuations. However, this model does not address supply side dynamics as housing supply is fixed in the model.

2.5 Financial intermediation

Another key area for development of financial frictions in DSGE models has been through the inclusion of a banking sector in these models. This allows the modeller to examine the role of banks in the financial market, especially to understand: the wedge between interbank lending rates and the risk-free rate; the role of bank capital in business cycle fluctuations; and the propagation of systemic risk.

Goodfriend and McCallum (2007) pioneered a model that incorporated a banking sector with multiple types of interest rates and a financial accelerator mechanism as in Bernanke et al. (1999). This model shows that use of a single interest rate in a DSGE model may cause the central bank to miss appropriate policy settings by up to 4% per annum.

Goodfriend and McCallum (2007) also note the two conflicting impacts of the inclusion of a banking sector on the financial accelerator mechanism. First, the spread between the risk-free rate and the loan rate increases procyclically, so the external finance premium grows in booms and drops in recessions. This attenuates the impact of a monetary policy shock, as it reduces demand for loans in a boom, which limits consumption growth.

At the same time, the increased loan servicing cost raises the opportunity cost of investment. This causes the price of capital to increase, in turn raising the collateral value of capital, implicitly decreasing the monitoring costs and amplifying the impact of the monetary policy shock. Goodfriend and McCallum (2007) conclude that for reasonable parameterisations, the attenuating impact will dominate the amplification effect of the financial accelerator for a monetary policy shock.

In a series of papers, Curdia and Woodford developed a model with a stylised banking sector and heterogenous agents that can change type. This allowed them to consider whether the incorporation of credit spreads changes the impact of different monetary policy rules. In their first paper, Curdia and Woodford (2009) incorporate a credit spread that can vary both endogenously and exogenously in the model. They find that the existence of positive spread in the model does not significantly alter the effects of different monetary policy settings, but that variations in this spread over time have implications for the relationship between real activity and inflation. Nevertheless, they conclude that inclusion of a credit spread does not significantly change optimal monetary policy in this model.

In the second paper, Curdia and Woodford (2010) extend the analysis to consider whether including a measure of financial conditions in the standard Taylor rule improves the central bank's response to various disturbances. Although they find that an adjustment for variations in the credit spread can improve the performance of a Taylor rule, the optimal size of adjustment depends on the source of the shock. As such, it is difficult to prespecify an adjustment that would invariably improve on the performance of a standard Taylor rule.

Christiano et al. (2011) incorporate a banking sector into an estimated standard New Keynesian open economy model of the type proposed by Adolfson et al. (2007). In this model, they find that financial shocks are important drivers of fluctuations in investment and GDP. Additionally, they note that the model with financial frictions has better forecasting performance for CPI and the interest rate than other versions of the model that do not include financial frictions.

Although it has rich labour market dynamics, the banking sector in Christiano et al. (2011)'s model is still relatively simple. In order to explore the impact of competition and endogenous capital on the transmission, Gerali et al. (2010) extend Iacoviello (2005)'s model to include a monopolistically competitive banking sector. They find that sticky loan and deposit rates attenuate the impact of a monetary policy shock, while incorporating financial intermediaries amplifies the propagation of supply shocks in the model. They conclude that shocks originating in the banking sector explain the largest share of the contraction in economic activity that occurred during the GFC in 2008, with a far smaller share being ascribed to macroeconomic shocks.

Dib (2010) synthesised much of this earlier work by incorporating two different types of banks and endogenous default in a DSGE model. In his paper, one of the banks is given a portfolio choice between investing in government bonds or in the interbank market. As it is costly for banks to raise capital to satisfy the regulatory requirements, he finds that the inclusion of a banking sector dampens the impact of financial shocks on the real economy. This in turn reduces macroeconomic volatility. Consistent with the results of previous studies, he finds that inclusion of a financial sector amplifies the impact of supply side shocks but attenuates the propagation of demand side shocks to real variables.

Note that where the primary objective of the external finance premium and collateral constraint literature was to explain observed fluctuations in output, the financial intermediation literature has developed primarily to help explain financial crises. But, as noted above, inclusion of a financial sector has important implications for the transmission of shocks to the real economy, especially when focussing on crisis periods. Given the focus of this thesis is the business cycle implications of LVR restrictions during normal times, I have decided to not include a banking sector for parsimony. Nevertheless, the inclusion of a banking sector into the model could be an area of future research.

Chapter 3

BVAR Evidence of House Prices and the Business Cycle

Figure 3.1 presents the impulse responses (standardised with one standard deviation confidence bands) from a structural vector autoregression model estimated using Bayesian techniques (BVAR) containing key business cycle variables in New Zealand. The model is estimated using a loose Litterman prior. It includes change in the output gap, change in headline inflation, change in detrended real house prices, and the overnight cash rate from the first quarter of 1993 to the last quarter of 2015. This period was selected as it represents the time of low and stable inflation following changes to the Reserve Bank of New Zealand Act in 1989.

Following the approach by Sims (1980) and others, the shocks are orthogonalised using a Choleski decomposition, with the ordering given by $[R, \Delta\pi, \Delta q, \Delta Y]$. This ordering can be motivated both by Granger causality tests and consistency with the model. In particular, note that these variables are ordered from most exogenous to the least. The interest rate is directly influenced by the central bank so is exogenously controlled by a central authority, while output is a result of all agents acting in the economy. Note that responses are robust to alternative orderings of the variables

Confidence intervals have been generated using Monte Carlo techniques to resample the data 10,000 times.

From this analysis we can make several observations about the business cycle that we should capture in a model.

(a) First, a model should deliver a negative response of real house prices and output to tight monetary policy (Figure 3.1). Note that although we would also expect prices to fall in response to a positive shock to the interest rate, this is not captured in the VAR, likely due to the lagged response of inflation to monetary policy.

(b) Second, the model should capture a negative response in real house prices and output to a positive inflation disturbance (Figure 3.1, second column).

(c) Third, there should be a significant positive response of output to a house price shock (Figure 3.1, third column). Note that we would also expect house prices to rise in response to a demand shock. Theory suggests that when output rises, households become wealthier and spend more on housing as a result, but this impact is not clear in the data. This is likely due to the endogeneity in the data.

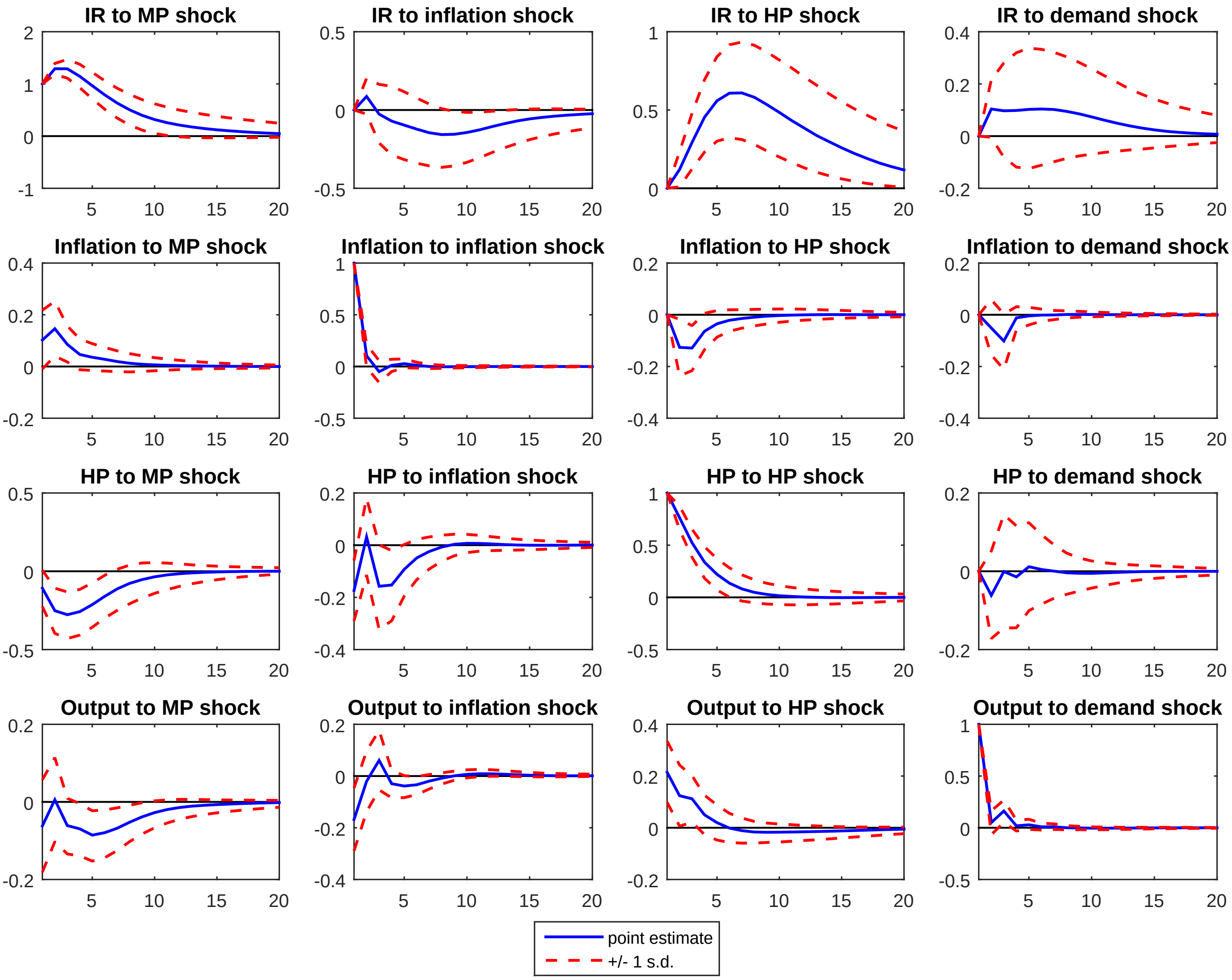
When output rises, interest rates would also be expected to rise, which would increase the cost of borrowing and may depress housing demand and prices. These offsetting impacts on house prices mean that the channel from increased demand to house prices is less well defined in the data than we would expect. Nevertheless, we expect the model to capture a two-way positive interaction between house prices and output, as in Iacoviello (2005).

Note the persistence of the interest rate response to a house price shock (top row, third column). There is a strong wealth effect in New Zealand data, with high house prices encouraging greater consumption in the economy. This effect is likely driving the interest rate response in this VAR, as output is likewise high over this period.

Given the strength and persistence of the interest rate response relative to the output and inflation response, it is arguable that a macroprudential tool that acted to moderate house prices would diminish the on-going pressure on the interest rate. However, separate macroprudential intervention would only be necessary where there was a misalignment between the strength of the economy and the strength of the housing market. This matter will be discussed in further detail below.

The rest of this paper develops a model based on Iacoviello (2005) that is consistent with these business cycle facts, but that can be used for policy analysis of the implications of LVR restrictions on the business cycle and for the transmission of monetary policy.

Figure 3.1: BVAR evidence from New Zealand



Chapter 4

Model Description

The model developed in this paper is a standard New Keynesian DSGE model with a sophisticated housing sector and collateral constraints. It is intended as a tool to assist in the discussion of the interaction between monetary and macroprudential policies in New Zealand.

There are two types of households: patient (denoted with a superscript 'P') and impatient (denoted with a superscript 'I'), each with unit mass. The difference between patient and impatient households is the discount factor that is applied to their utility. Patient households have a higher discount factor than impatient households ($\beta_P > \beta_I$) so impatient households value consumption relatively more today than in the future. As a result, patient households act as savers in the model while impatient households are borrowers.

Figure 4.1 depicts the model framework. The components in blue are consistent with Iacoviello (2005)'s original framework while those in pink are the new components that have been added to the model. Red arrows indicate the transfer of stocks in the model while blue arrows indicate real prices for these components.

This model differs from Iacoviello (2005) as entrepreneur's functions have been transferred into two different agents in the model: intermediate goods producers and investors. There is a continuum of monopolistically competitive intermediate goods producers that produce differentiated goods. Their inputs in production are labour from patient and impatient households and capital, which is hired from investors each period at the rate of r_t^k.

Unlike in Iacoviello (2005), housing is not an input in production of the final good in the economy, so housing has no productive use in the economy. Rather, investors have the option of making housing available to patient and impatient households as rental accommodation at the rate q_t^r.

Intermediate goods firms are subject to Calvo-pricing such that each firm is only able to update its price in a given period with a probability $(1-\theta)$, which generates nominal price rigidities in the economy. There is also a perfectly competitive final goods producer that aggregates the goods from intermediate goods producers and sells them to patient households, impatient households and investors at a price P_t.

Investors are distinct from patient and impatient households as housing is not in their utility function. Rather, investors obtain utility solely from consumption, which they seek to maximise by investing in housing and capital. As noted above, the return on housing is the rent q_t^r, which is the rate that patient and impatient households pay to rent the accommodation. Similarly, the return on capital is r_t^k, which is the rental price that intermediate goods producers pay to use capital in production. Investors also benefit from any increase in the price of houses or capital from one period to the next (i.e. capital gains). Investors have a discount rate (γ) that sits between that of patient and impatient households in the economy, with the result that they are net borrowers in the model $(\beta_P > \gamma > \beta_I)$.

Capital is produced by a capital goods producer that takes the undepreciated portion of capital from investors and combines it with the final good to produce the new capital stock in the economy. This capital stock is sold back to investors for a price of q_t^k. Note that this process does not directly require labour inputs, which simplifies the analysis.

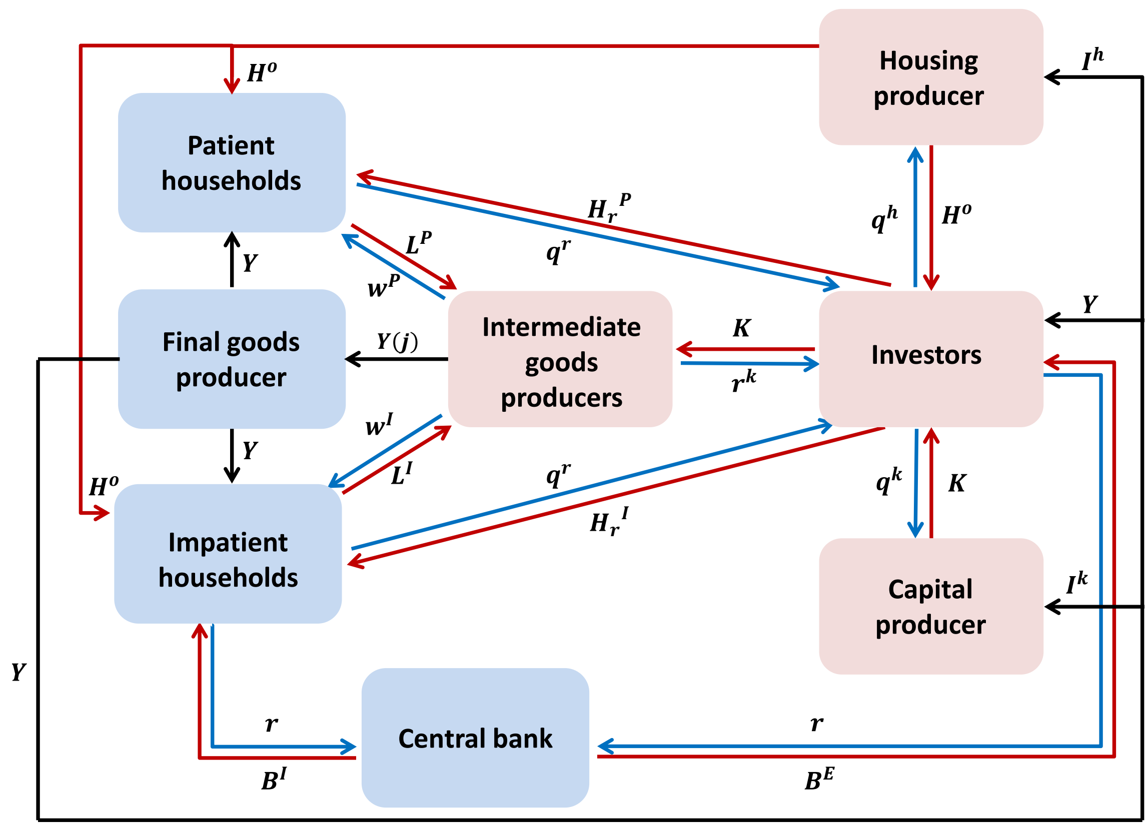
Housing is likewise produced by a separate housing producer that combines the existing undepreciated housing stock with the final good to produce the current period's housing stock. This housing stock is sold back to patient households, impatient households and investors at a price of q_t^h. Again, the production of housing does not directly require labour as an input.

Both impatient households and investors are subject to a borrowing constraint that limits the amount that they can borrow in the current period to a fraction, m^I and m^E respectively, of the expected value of their housing the following period. These loan-to-value ratios are exogenously set by the central bank in this model. This specification allows us to run policy experiments that test the impact of different levels of LVR restriction on the transmission of shocks within the model.

The central bank in the model follows a Taylor rule with smoothing that aims to minimise the variation in output and inflation. Following the rule as specified in Iacoviello (2005), the central bank only responds to lagged inflation and lagged output rather than to these variables contemporaneously. This is consistent with central bank practice where data is released with a lag so the only available data for the central bank to use (other than estimates based on component results) is lagged results.

A full description of the technical components of this model follows in the next section.

Figure 4.1: Model framework



4.1 Description of agents in the model

4.1.1 Patient Households

Patient households gain utility from consumption c_t^P, housing h_t^P, and lose utility from supplying labour to firms, L_t^P. Therefore, their utility maximisation problem can be represented as:

$$\max_{c_t^P, B_t^P, h_t^P, L_t^P} E_0 \sum_{t=0}^{\infty} (\beta^P)^t \left(\log c_t^P + j_t^P \log h_t^P - \frac{(L_t^P)^{\eta^P}}{\eta^P} \right) \tag{4.1}$$

where β^P is the rate at which patient households discount future utility.

Patient households have the option of consuming housing either through ownership $h_{o,t}^P$ or by renting housing $h_{r,t}^P$ from period to period. Ownership comes at the nominal cost of Q_t^h while rental is available at the nominal cost of Q_t^r. The utility provided from each of these forms of housing is given by the following composite utility function:

$$h_t^P = \left[\omega_P^{\frac{1}{v_P}} (h_{o,t}^P)^{\frac{v_P-1}{v_P}} + (1-\omega_P)^{\frac{1}{v_P}} (h_{r,t}^P)^{\frac{v_P-1}{v_P}}\right]^{\frac{v_P}{v_P-1}} \tag{4.2}$$

where v_P measures the elasticity of substitution between rental and home ownership and ω_P measures the share of each type of housing that they consume. Their consumption of housing is also subject to an exogenous preference shock j_t^P and depreciation of δ_h per period.

Patient households earn income through wages (w_t^P), lending money to impatient households and entrepreneurs (b_t^P) and through their ownership of the intermediate goods producers. The intermediate goods producers rebate their profits to the patient household in full as F_t.

Patient households face a budget constraint that limits their amount of spending on consumption, housing and deposits (b_t^P) to their income in the current period. This can be expressed in real terms as follows:

$$c_t^P + q_t^h(h_{o,t}^P - (1-\delta_h)h_{o,t-1}^P) + q_t^r h_{r,t}^P + \frac{R_{t-1}}{\pi_t} b_{t-1}^P = b_t^P + w_t^P L_t^P + F_t \tag{4.3}$$

Patient households also face an adjustment cost associated with changing the amount of housing they use each period. To reflect the differences in costs associated with home ownership (for example, solicitor's fees and building inspections) we assume the cost is only incurred for changing the amount of home ownership. The adjustment cost function $\xi_t^{h,P}$ is specified as follows:

$$\xi_t^{h,P} = \frac{\phi_o^P q_t^h}{2}\left(\frac{h_{o,t}^P - h_{o,t-1}^P}{h_{o,t-1}^P}\right)^2 h_{o,t-1}^P \tag{4.4}$$

where ϕ_o^P is the parameter that governs the adjustment costs.

Given the budget constraint and adjustment costs specified above, we can write the maximisation problem for patient households as a Lagrangian as follows:

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \Bigg[(\beta^P)^t \left(\log c_t^P + j_t^P \log h_t^P - \frac{(L_t^P)^{\eta^P}}{\eta^P} \right)$$
$$- \lambda_t^P \left(c_t^P + q_t^h(h_{o,t}^P - (1-\delta_h)h_{o,t-1}^P) + q_t^r h_{r,t}^P + \frac{R_{t-1}}{\pi_t} b_{t-1}^P + \xi_t^h - b_t^P - w_t^P L_t^P \right) \Bigg] \tag{4.5}$$

Differentiating with respect to the patient households' choice variables (consumption, house ownership, house rental, borrowing and labour supply) we obtain the following first order conditions:

For consumption:

$$\frac{\partial \mathcal{L}}{\partial c_t^P} = 0 \Leftrightarrow \lambda_t^P = \frac{1}{c_t^P} \tag{4.6}$$

For house ownership:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial h_{o,t}^P} = 0 \Leftrightarrow \frac{1}{c_t^P}\left(q_t^h + \phi_o^P q_t^h \left(\frac{h_{o,t}^P - h_{o,t-1}^P}{h_{o,t}^P}\right)\right) = \frac{j_t^P}{h_t^P}\left(\frac{\omega_P h_t^P}{h_{o,t}^P}\right)^{\frac{1}{v_P}} \\ + E_t\left[\frac{\beta^P}{c_{t+1}^P}(1-\delta_h)\left(q_{t+1}^h + \frac{\phi_o^P}{2} q_{t+1}^h \left(\frac{(h_{o,t+1}^P)^2 - (h_{o,t}^P)^2}{(h_{o,t}^P)^2}\right)\right)\right]\end{aligned} \tag{4.7}$$

For house rental:

$$\frac{\partial \mathcal{L}}{\partial h_{r,t}^P} = 0 \Leftrightarrow \frac{1}{c_t^P}\left(q_t^r + \phi_r^P q_t^r \left(\frac{h_{r,t}^P - h_{r,t-1}^P}{h_{r,t}^P}\right)\right) = \frac{j_t^P}{h_t^P}\left(\frac{(1-\omega_P) h_t^P}{h_{r,t}^P}\right)^{\frac{1}{v_P}} \tag{4.8}$$

For labour:

$$\frac{\partial \mathcal{L}}{\partial L_t^P} = 0 \Leftrightarrow \frac{w_t^P}{c_t^P} = (L_t^P)^{\eta^P - 1} \equiv w_t^P = (L_t^P)^{\eta^P - 1} c_t^P \tag{4.9}$$

For borrowing:

$$\frac{\partial \mathcal{L}}{\partial b_t^P} = 0 \Leftrightarrow \lambda_t^P = E_t \lambda_{t+1}^P \beta^P \frac{R_t}{\pi_{t+1}} \Leftrightarrow \frac{1}{c_t^P} = \beta^P E_t\left(\frac{R_t}{c_{t+1}^P \pi_{t+1}}\right) \tag{4.10}$$

4.1.2 Impatient Households

Similar to patient households, impatient households choose consumption (c_t^I), borrowing (b_t^I), housing (h_t^I), and labour supply (L_t^I)in order to maximise their utility:

$$\max_{c_t^I b_t^I, h_t^I, L_t^I} E_0 \sum_{t=0}^{\infty} (\beta^I)^t \left(\log c_t^I + j_t^I \log h_t^I - \frac{(L_t^I)^{\eta^I}}{\eta^I} \right) \tag{4.11}$$

We assume that the the impatient households have a lower discount factor on future utility than patient households (giving rise to the description 'impatient') so $\beta^I < \beta^P$.

Impatient households have the option of consuming housing either through ownership or by renting housing from period to period. Ownership comes at the nominal cost of Q_t^h while rental is available at the nominal cost of Q_t^r. The utility provided from each of these forms of housing is given by the following composite utility function:

$$h_t^I = \left[\omega_I^{\frac{1}{v_I}}(h_{o,t}^I)^{\frac{v_I-1}{v_I}} + (1-\omega_I)^{\frac{1}{v_I}}(h_{r,t}^I)^{\frac{v_I-1}{v_I}}\right]^{\frac{v_I}{v_I-1}} \tag{4.12}$$

where v_I measures the elasticity of substitution between rental and home ownership and ω_I measures the share of each type of housing that they consume. Their consumption of housing is also subject to an exogenous preference shock j_t^I and depreciation of δ_h per period.

Borrowing (b_t^I) is available from the patient household at the central bank specified nominal rate of R_t. Impatient households earn income each period through labour supply (at wage w_t^I) and borrowing, which they use to pay for consumption, housing and to repay previous period's borrowing. Dividing the nominal prices by the price level P_t allows their real budget constraint to be summarised as follows:

$$c_t^I + q_t^h(h_{o,t}^I - (1-\delta_h)h_{o,t-1}^I) + q_t^r h_{r,t}^I + \frac{R_{t-1}}{\pi_t}b_{t-1}^I = b_t^I + w_t^I L_t^I \tag{4.13}$$

Impatient households are also subject to a borrowing constraint that limits the amount they can borrow to a certain proportion (m^I) of the expected value of the housing $h_{o,t}^I$ that they own:

$$R_t b_t^I \leq E_t m^I q_{t+1}^h (1-\delta_h) h_{o,t}^I \pi_{t+1} \tag{4.14}$$

This proportion (m^I) of the expected value of the housing is the loan-to-value (LVR) restriction in the model. It is exogenously fixed so that we may conduct policy experiments by varying the level of this restriction.

It follows that in addition to the utility that house ownership provides, house ownership also allows the impatient household to borrow more as the collateral constraint is relaxed. As with patient households, impatient households face an adjustment cost associated with changing the amount of house ownership that they hold each period.

$$\xi_t^{h,I} = \frac{\phi_o^I q_t^h}{2}\left(\frac{h_{o,t}^I - h_{o,t-1}^I}{h_{o,t-1}^I}\right)^2 h_{o,t-1}^I \tag{4.15}$$

By writing the budget constraint in terms of consumption, then substituting this definition into the objective function, the maximisation problem for impatient households can be written as a Lagrangian as follows:

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \Bigg[(\beta^I)^t \Bigg(\log \Bigg(b_t^I + w_t^I L_t^I - q_t^h (h_{o,t}^I - (1-\delta_h) h_{o,t-1}^I) - q_t^r h_{r,t}^I - \frac{R_{t-1}}{\pi_t} b_{t-1}^I - \xi_t^{h,I} \Bigg) \\ + j_t^I \log \left(\left[\omega_I^{\frac{1}{v_I}} (h_{o,t}^I)^{\frac{v_I-1}{v_I}} + (1-\omega_I)^{\frac{1}{v_I}} (h_{r,t}^I)^{\frac{v_I-1}{v_I}} \right]^{\frac{v_I}{v_I-1}} \right) - \frac{(L_t^I)^{\eta^I}}{\eta^I} \Bigg) - \lambda_t^I \Big(R_t b_t^I - E_t(m^I q_{t+1} h_t^I \pi_{t+1}) \Big) \Bigg] \tag{4.16}$$

The first order conditions with respect to the impatient household's choice variables (borrowing, house ownership, house rental, and labour) can therefore be solved as:

For borrowing:

$$\frac{\partial \mathcal{L}}{\partial b_t^I} = 0 \Leftrightarrow \frac{1}{c_t^I} - \beta^I \frac{1}{c_{t+1}^I} \frac{R_t}{\pi_{t+1}} - \lambda_t^I R_t = 0 \Leftrightarrow \frac{1}{c_t^I} = \beta^I E_t \left(\frac{R_t}{c_{t+1}^I \pi_{t+1}} \right) + \lambda_t^I R_t \tag{4.17}$$

For housing ownership:

$$\frac{\partial \mathcal{L}}{\partial h_{o,t}^I} = 0 \Leftrightarrow \frac{1}{c_t^I} \left(q_t^h + \phi_o^I q_t^h \left(\frac{h_{o,t}^I - h_{o,t-1}^I}{h_{o,t}^I} \right) \right) = \frac{j_t^I}{h_t^I} \left(\frac{\omega_I h_t^I}{h_{o,t}^I} \right)^{\frac{1}{v_I}} \\ + E_t \left[\frac{\beta^I}{c_{t+1}^I} (1-\delta_h) \left(q_{t+1}^h + \frac{\phi_o^I}{2} q_{t+1}^h \left(\frac{(h_{o,t+1}^I)^2 - (h_{o,t}^I)^2}{(h_{o,t}^I)^2} \right) \right) \right] + \lambda_t^I m^I E_t (q_{t+1}^h \pi_{t+1})(1-\delta_h) \tag{4.18}$$

This means that when the impatient household buys housing $h_{o,t}^I$, the cost is q_t^h plus the adjustment cost associated with changing its level of housing use from the previous period. The benefits of acquiring housing in the current period include any capital gain the impatient household attains on the property, possibly avoiding higher adjustment costs in the future, and the increased collateral value of the underlying housing (that relaxes the borrowing constraint for the impatient household).

For housing rental:

$$\frac{\partial \mathcal{L}}{\partial h_{r,t}^I} = 0 \Leftrightarrow \frac{1}{c_t^I} q_t^r = \frac{j_t^I}{h_t^I} \left(\frac{(1-\omega_I) h_t^I}{h_{r,t}^I} \right)^{\frac{1}{v_I}} \tag{4.19}$$

For labour:

$$\frac{\partial \mathcal{L}}{\partial L_t^I} = 0 \Leftrightarrow \frac{w_t^I}{c_t^I} = (L_t^I)^{\eta^I - 1} \equiv w_t^I = (L_t^I)^{\eta^I - 1} c_t^I \tag{4.20}$$

4.1.3 Investors

Investors aim to maximise their consumption by investing in houses, making houses available for rental to patient and impatient households, and by renting capital to the intermediate goods producer. Note that investors are different from impatient and patient households as they only have consumption in their utility function. They therefore treat houses as a financial investment for which the return is the rent received in one period and the expected capital gain. This specification likely captures the motives of housing investors in the New Zealand market as distinct from owner-occupiers per the LVR policy.

There is an argument that there is not a clear real world analogue to investors in the model as this specification implies that investors do not work. However, with a growing proportion of retirees and overseas investors in New Zealand, both groups of which are active in the housing market, it is fair to assume there are some 'investors' active in the economy that do not supply labour.

Houses are transformed into rental accommodation by the investor using the following transformation process:

$$h_t^r = A_r h_{r,t}^E \tag{4.21}$$

where A_r is a parameter that acts as a proxy for the efficiency of the rental market as in Ortega et al. (2011). This parameter covers things such as risk of vacancy, damage, enforceability of contracts, etc. Rental accommodation is made available to households at the nominal price of Q_t^r (or the real price of $q_t^r = \frac{Q_t^r}{P_t}$). It follows that in order to close the rental market it must be the case that the housing made available for rental by the investor is equal to the total amount of housing rented to the patient and impatient households: $h_t^E = \frac{(h_{r,t}^P + h_{r,t}^I)}{A_r}$.

Investors also own all of the capital K_t in the economy, which they rent to intermediate goods producers at the rate of r_t^k. Similar to housing, capital depreciates each period at the rate of δ_k. As with impatient households, investors may borrow from patient households at the nominal rate R_t that is set by the central bank.

Investors choose their level of borrowing (b_t^I), capital, and housing in order to maximise their consumption:

$$\max_{b_t^E, K_t, h_t^E} E_0 \sum_{t=0}^{\infty} \gamma^t \ln c_t^E \tag{4.22}$$

where γ is the discount rate that applies to investors' utility. Consistent with Iacoviello (2005), this parameter is calibrated to sit between the respective discount rates of patient and impatient households.

This maximisation problem is subject to a budget constraint. Investors' costs include consumption, housing, and inflation-adjusted repayment of borrowing from the previous period ($\frac{R_{t-1}}{\pi_t} b_{t-1}^E$). The budget constraint states that these costs must be equal to the entrepreneur's revenue from housing and capital rental in that period (stated in terms of the price of the consumer/investment good) plus any borrowing. That is,

$$b_t^E + q_t^r A_r h_t^E + r_t^k K_{t-1} = c_t^E + q_t^h \Big[(h_t^E - (1-\delta_h) h_{t-1}^E) \Big] + q_t^k \Big[(K_t - (1-\delta_k) K_{t-1}) \Big] + \frac{R_{t-1}}{\pi_t} b_{t-1}^E + \xi_t^{h,e} \tag{4.23}$$

where δ_k and δ_h are the depreciation rates of capital and housing respectively. As with patient and impatient households, investors also face adjustment costs for changing the level of housing that they own, which are denoted $\xi_t^{h,E}$. Adjustment costs for housing are given by:

$$\xi_t^{h,E} = \frac{\phi_h^E q_t^h}{2} \left(\frac{h_t^E - h_{t-1}^E}{h_{t-1}^E} \right)^2 h_{t-1}^E \tag{4.24}$$

where ϕ^E is the parameter that governs the level of the adjustment costs.

Finally, as with impatient households, entrepreneurs are subject to a borrowing constraint that restricts their level of borrowing to a certain proportion of the expected value of their housing the following period. This collateral constraint is specified as follows:

$$R_t b_t^E \leq m^E q_{t+1}^h (1-\delta_h) h_t^E \pi_{t+1} \tag{4.25}$$

Borrowing is pre-multiplied by the interest rate as the total amount due to the lender in the following period will be the amount borrowed multiplied by the interest rate. This borrowing constraint, m^E, is the investor LVR restriction in the model that is exogenously specified so that we may conduct policy scenarios within the model.

Note that allowing investors to choose between capital and housing investment has important implications for the transmission of shocks in the model. In particular, where there is a shock that causes house prices and rental yield to rise, housing investment will increase and capital investment will fall.

Empirical findings suggest that housing and capital investment have a positive correlation (see Davis and Heathcote (2005) and Leung (2004)), but the negative correlation generated in this model is a key assumption for capturing the trade off between housing investment and capital investment. With the demographic of asset-rich retirees growing in New Zealand, it is interesting to model how their choices between interesting in housing as a financial asset and investing in capital would affect output and inflation.

There has long been a school of thought that housing is a non-productive resource and that the cost of excess investment in housing is lack of growth in other sectors (Burns and Grebler (1976)). This could cause inflation, exert pressure on the balance of payments as resources were diverted away from export sectors, and tie up resources for a long period of time given housing's very high capital to output ratio (Harris and Gillies (1963)).

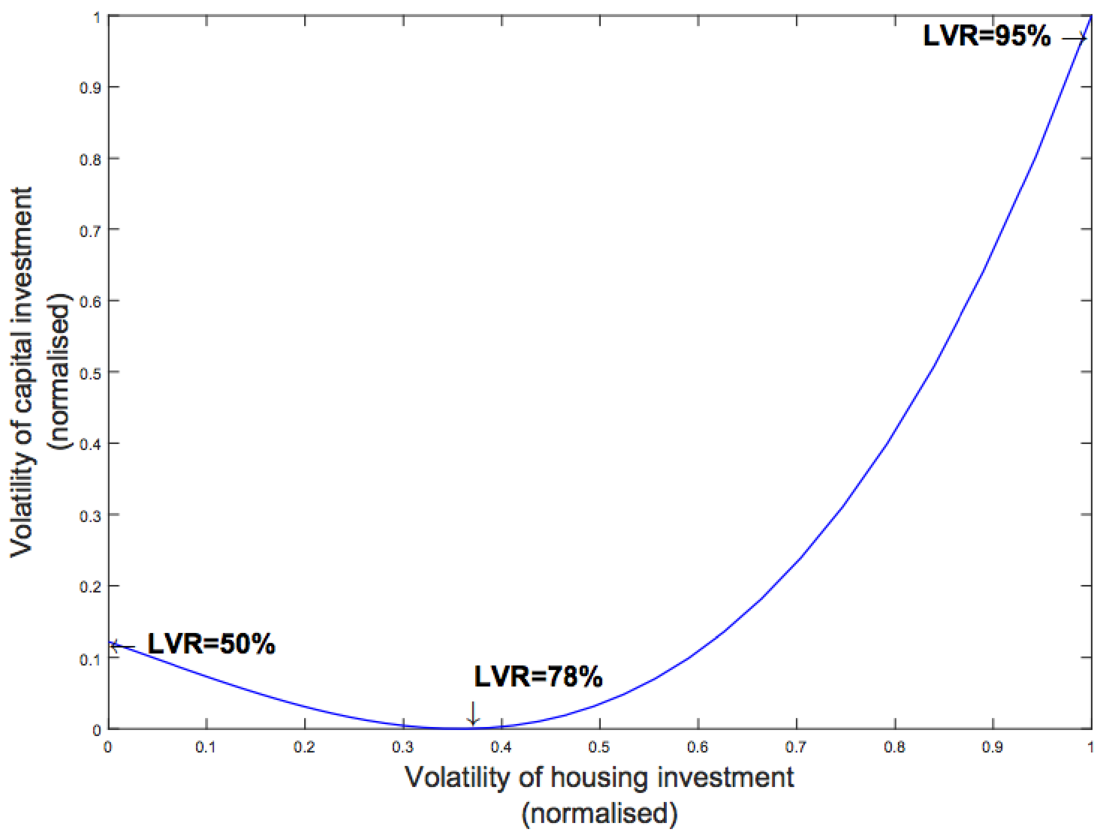
Business cycle models have historically treated business capital and housing investment as perfect subsitutes (Kydland and Prescott (1982) and Christiano (1988)) such that the allocation between the two is indeterminate in the model. However, when the two types of capital are disentangled, Greenwood and Hercowitz (1991) observe a negative comovement between housing and business investment in response to a productivity shock.

In addition, the differential tax treatment of housing favours the accumulation of housing capital at the expense of business capital (see Greenwood and Hercowitz (1991)). In New Zealand, Henry (1982) argued that the differential tax treatment of housing has diverted resources away from productive sectors.

Figure 4.2 shows how this model captures the trade off between housing investment and capital investment at different levels of investor-specific LVR restrictions. In particular, it shows while the volatility of housing investment is decreasing with reduced LVR restrictions, the volatility of capital investment is minimised at an LVR of 78%. This result is intuitive. As the investor LVR restriction decreases, investors must retain more of their housing stock in order to retain a given level of borrowing. This decreases the volatility of demand for housing and volatility of housing investment accordingly.

Simultaneously, as the investor divests (or acquires) less housing in response to shocks due to the borrowing constraint, they likewise acquire (or divest) less of the consumption good in response to the same shock. This means that a falling LVR constraint also reduces the volatility of demand for the output good, and accordingly capital investment. This impact is minimised at an investor LVR of 78%. Beyond that point, the LVR restriction binds the investor's behaviour regarding housing and borrowing so tightly that much of the impact of the shock needs to be absorbed by the investor's consumption. This increases the volatility of the investor's demand for the output good. This seems to imply that a LVR greater than 78% is undesirable as it increases both housing and capital investment volatility all things equal.

Figure 4.2: Comparison of volatility of capital and housing investment at different investor-specific LVRs



By solving the budget constraint for consumption, we can summarise the utility maximisation problem for investors in the following Lagrangian:

$$\mathcal{L} = E_t \sum_{t=0}^{\infty} \Bigg[(\gamma)^t \Bigg(\log \Bigg(b_t^E + q_t^r A_r h_t^E + r_t^k K_{t-1} - q_t^h \Big[(h_t^E - (1-\delta_h) h_{t-1}^E) \Big] - q_t^k \Big[(K_t - (1-\delta_k) K_{t-1}) \Big] - \frac{R_{t-1}}{\pi_t} b_{t-1}^E - \frac{\phi^E q_t^h}{2\delta_h} \left(\frac{h_t^E - h_{t-1}^E}{h_{t-1}^E} \right)^2 h_{t-1}^E \Bigg) - \lambda_t^E \Bigg(R_t b_t^E - E_t (m^E (1-\delta_k) q_{t+1} h_t^E \pi_{t+1}) \Bigg) \Bigg] \quad (4.26)$$

The first order conditions with respect to the entrepreneurs' choice variables of borrowing, housing and capital are:

For borrowing:

$$\frac{\partial \mathcal{L}}{\partial b_t^E} = 0 \Leftrightarrow \frac{1}{c_t^E} - \gamma \frac{1}{c_{t+1}^E} \frac{R_t}{\pi_{t+1}} - \lambda_t^E R_t = 0 \Leftrightarrow \frac{1}{c_t^E} = \gamma E_t \left(\frac{R_t}{c_{t+1}^E \pi_{t+1}} \right) + \lambda_t^E R_t \tag{4.27}$$

For housing:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial h_t^E} = 0 \Leftrightarrow \frac{q_t^h}{c_t^E} \left(1 + \frac{\phi^E}{\delta_h} \left(\frac{h_t^E - h_{t-1}^E}{h_{t-1}^E} \right) \right) = \frac{q_t^r A_r}{c_t^E} \\ + E_t \left[\frac{\gamma q_{t+1}^h}{c_{t+1}^E} \left((1-\delta_h) + \frac{\phi^E q_{t+1}^h}{2} \left(\frac{(h_{t+1}^E)^2 - (h_t^E)^2}{(h_t^E)^2} \right) \right) \right] + \lambda_t^E m^E E_t [q_{t+1}^h \pi_{t+1}](1-\delta_k) \end{aligned} \tag{4.28}$$

This means that when the investor buys housing h_t^E, the cost is q_t^h plus the adjustment cost associated with changing its level of housing use from the previous period. The benefits of acquiring housing in the current period include the rental value of housing, any capital gain attained on the property, avoidance of higher adjustment costs in the future, and the increased collateral value of the underlying housing that relaxes the borrowing constraint for the investor.

For capital:

$$\frac{\partial \mathcal{L}}{\partial K_t} = 0 \Leftrightarrow \frac{q_t^k}{c_t^E} = E_t \left[\frac{\gamma}{c_{t+1}^E} \left(r_{t+1}^k + q_{t+1}^k (1-\delta_k) \right) \right] \tag{4.29}$$

4.1.4 Final Goods Producers

The final goods sector aggregates the output of the intermediate goods producers, $Y_t(j)$ into a single final good, Y_t, using the following CES aggregator:

$$Y_t = \left(\int_0^1 Y_t(j)^{\frac{\epsilon_p - 1}{\epsilon_p}} dj \right)^{\frac{\epsilon_p}{\epsilon_p - 1}} \tag{4.30}$$

where $\epsilon_p > 1$ is the elasticity of substitution between the different types of goods.

The final goods producer seeks to maximise its profits from the sale of Y_t by minimising the expenditure on inputs $P_t(j)$ from the intermediate goods producers given the production constraint. Therefore, we can write the Lagrangian for the final goods producer as follows:

$$\mathcal{L} = \int_0^1 P_t(j)Y_t(j)dj + P_t\Big(Y_t - \Big(\int_0^1 Y_t(j)^{\frac{\epsilon_p-1}{\epsilon_p}} dj\Big)^{\frac{\epsilon_p}{\epsilon_p-1}}\Big) \tag{4.31}$$

The optimal choice of inputs $Y_t(j)$ is given by the first order condition:

$$\frac{\partial \mathcal{L}}{\partial Y_t(j)} = 0 \Leftrightarrow P_t(j) - P_t\Big(\frac{\epsilon_p}{\epsilon_p - 1}\Big(\int_0^1 Y_t(j)^{\frac{\epsilon_p-1}{\epsilon_p}} dj\Big)^{\frac{1}{\epsilon_p-1}}\Big)\frac{\epsilon_p - 1}{\epsilon_p} Y_t(j)^{-\frac{1}{\epsilon_p}}\Big) = 0 \tag{4.32}$$

That is,

$$P_t(j) = P_t\Big(\frac{Y_t}{Y_t(j)}\Big)^{\frac{1}{\epsilon_p}} \equiv Y_t(j) = \Big(\frac{P_t(j)}{P_t}\Big)^{\epsilon_p} Y_t \tag{4.33}$$

Combining this definition of $Y_t(j)$ with the CES aggregator we obtain:

$$Y_t = \Big(\int_0^1 \Big(\Big(\frac{P_t(j)}{P_t}\Big)^{\epsilon_p} Y_t\Big)^{\frac{\epsilon_p-1}{\epsilon_p}} dj\Big)^{\frac{\epsilon_p}{\epsilon_p-1}} = Y_t\Big(\int_0^1 \Big(\frac{P_t(j)}{P_t}\Big)^{1-\epsilon_p} dj\Big)^{\frac{\epsilon_p}{\epsilon_p-1}} \tag{4.34}$$

Y_t therefore drops from both sides of the expression, so it can be solved for P_t as:

$$P_t = \Big(\int_0^1 P_t(j)^{1-\epsilon_p} dj\Big)^{\frac{1}{1-\epsilon_p}} \tag{4.35}$$

As there is constant returns to scale, P_t represents the minimum cost of producing one unit of the final good regardless of the total quantity produced. This allows P_t to be interpreted as the aggregate price index.

4.1.5 Intermediate Goods Producers

There is a continuum of monopolistically competitive firms that produce the intermediate goods. These firms are owned by the patient households and all profits are rebated lump sum as F_t.

These intermediate goods producers use labour from patient households, labour from impatient households, and capital to produce a differentiated final output, $Y_t(j)$ that they sell to the final goods producer at price $P_t(j)$.

The production function for each intermediate goods producer is therefore given by:

$$Y_t(j) = A_t K_t(j)^{\mu}(L_t^P(j))^{\alpha(1-\mu)}(L_t^I(j))^{(1-\alpha)(1-\mu)} \tag{4.36}$$

where A_t is an AR(1) technology shock.

Each firm faces the following demand function from the final goods producer for their good $Y_t(j)$:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t \tag{4.37}$$

where ϵ_p is the degree of substitution between the different types of goods in the market.

Therefore each intermediate good producer seeks to minimise the cost of their inputs to production per the following minimisation problem:

$$\min_{K_t(j), L_t^P(j), L_t^I(j)} = w_t^P L_t^P(j) + w_t^I L_t^I(j) + r_t^k K_t(j) \tag{4.38}$$

which is subject to the following production condition:

$$A_t K_t(j)^{\mu}(L_t^P(j))^{\alpha(1-\mu)}(L_t^I(j))^{(1-\alpha)(1-\mu)} \geq \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t \tag{4.39}$$

Substituting in the demand function from above, we can therefore set up the intermediate goods producers maximisation problem as a Lagrangian optimisation:

$$\begin{aligned}\mathcal{L} = E_t \sum_{t=0}^{\infty} & -w_t^P L_t^P(j) - w_t^I L_t^I(j) - r_t^k K_t(j) \\ & + RMC_t\left(A_t K_t(j)^{\mu}(L_t^P(j))^{\alpha(1-\mu)}(L_t^I(j))^{(1-\alpha)(1-\mu)} - \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t\right)\end{aligned} \tag{4.40}$$

where the Lagrangian multiplier RMC_t is the real marginal cost of producing an additional good.

Differentiating with respect to the choice variables of capital, labour from patient households and labour from impatient households we have:

For capital:

$$\frac{\partial \mathcal{L}}{\partial K_t(j)} = 0 \Leftrightarrow -r_t^k + RMC_t A_t \mu K_t(j)^{\mu-1}(L_t^P(j))^{\alpha(1-\mu)}(L_t^I(j))^{(1-\alpha)(1-\mu)} = 0 \tag{4.41}$$

$$\Leftrightarrow r_t^k = \frac{RMC_t \mu Y_t(j)}{K_t(j)} \tag{4.42}$$

For labour supplied by patient households:

$$\frac{\partial \mathcal{L}}{\partial L_t^P(j)} = 0 \Leftrightarrow -w_t^P + RMC_t\bigg(A_t K_t(j)^{\mu}\alpha(1-\mu)(L_t^P(j))^{\alpha(1-\mu)-1}(L_t^I(j))^{(1-\alpha)(1-\mu)}\bigg) = 0 \tag{4.43}$$

$$\Leftrightarrow w_t^P = \frac{RMC_t \alpha(1-\mu) Y_t(j)}{L_t^P(j)} \tag{4.44}$$

For labour supplied by impatient households is:

$$\frac{\partial \mathcal{L}}{\partial L_t^I(j)} = 0 \Leftrightarrow -w_t^I + RMC_t\bigg(A_t K_t(j)^{\mu}(L_t^P(j))^{\alpha(1-\mu)}(1-\alpha)(1-\mu)(L_t^I(j))^{(1-\alpha)(1-\mu)-1}\bigg) = 0 \tag{4.45}$$

$$\Leftrightarrow w_t^I = \frac{RMC_t(1-\alpha)(1-\mu)Y_t(j)}{L_t^I(j)} \tag{4.46}$$

Note that as intermediate firms face the same factor prices, they will hire capital and labour in the same ratios, which will in turn be equal to the aggregate ratios. Therefore we can drop the j that differentiates the different firms in production. The Lagrangian multiplier in this case is the real marginal cost, which is equal to:

$$RMC_t = \frac{(r_t^k)^{\mu}(w_t^I)^{(1-\alpha)(1-\mu)}(w_t^P)^{\alpha(1-\mu)}}{[\mu]^{\mu}[(1-\alpha)(1-\mu)]^{(1-\alpha)(1-\mu)}[\alpha(1-\mu)]^{\alpha(1-\mu)}} \tag{4.47}$$

The real profit flow to each firm j can be written as the amount of output it produces multiplied by the price it charges, minus the marginal cost per unit multiplied by the amount of output. Therefore, intermediate goods producers seek to maximise:

$$\max_{P_t(j)} \frac{P_t(j)}{P_t} Y_t(j) - RMC_t Y_t(j) \tag{4.48}$$

We assume that intermediate goods producers are subject to Calvo (1983) pricing such that only a fraction $(1-\theta)$ of intermediate goods firms can adjust their prices each period. It follows that there is a probability θ that a firm will be stuck with its previous price for one period. As such, the average price level in the economy will be an aggregate of the all prices in the economy:

$$P_t^{1-\epsilon_p} = \theta P_{t-1}^{1-\epsilon_p} + (1-\theta)(P_t^*)^{1-\epsilon_p} \tag{4.49}$$

where P_{t-1} is the price from the previous period and P_t^* is the average price charged by firms that reset their prices in that period.

The demand function of a firm that has the opportunity to reset its price in a period t is therefore:

$$Y_{t+s}^*(j) = \left(\frac{P_t^*(j)}{P_{t+s}}\right)^{-\epsilon_p} Y_{t+s} \tag{4.50}$$

for any period $s \geq 0$ for which the firm will retain the same price. With this demand function, the profit maximisation condition for a firm can be written as:

$$\max_{P_t(j)} E_t \sum_{s=0}^{\infty} (\beta^P \theta)^s \Delta_{t,s} \left(\frac{P_t^*(j)}{P_{t+s}} Y_{t+s}^*(j) - RMC_t Y_{t+s}^*(j) \right) \tag{4.51}$$

where $\Delta_{t,s}$ is the patient households stochastic discount factor given by: $\Delta_{t,s} = \frac{u'(c_t^P)}{u'(c_{t+s}^P)}$

Taking the first order condition with respect to the intermediate goods producers price we have:

$$E_t \sum_{s=0}^{\infty} (\beta^P \theta)^s \Delta_{t,s} \frac{Y_{t+s}^*(j)}{P_{t+s}} \left(P_t^*(j) - \frac{\epsilon_p}{\epsilon_p - 1} RMC_{t+s} P_{t+s} \right) = 0 \tag{4.52}$$

Therefore all updating firms in a given period will update to the same price so that $P_t^*(j) = P_t^*$.

Define the steady state mark-up as $X = \frac{\epsilon_p}{\epsilon_p - 1}$ so that the above can be written as:

$$E_t \sum_{s=0}^{\infty} (\beta^P \theta)^s \Delta_{t,s} \frac{Y_{t+s}^*(j)}{P_{t+s}} P_t^* = X E_t \sum_{s=0}^{\infty} (\beta^P \theta)^s \Delta_{t,s} RMC_t Y_{t+s}^*(j) \tag{4.53}$$

Substituting in the definition of Y_{t+s}^* from above, this can be written as:

$$P_t^* = X \frac{E_t \sum_{s=0}^{\infty} (\beta^P \theta)^s \Delta_{t,s} RMC_{t+s} P_{t+s}^{\epsilon} Y_{t+s}}{E_t \sum_{s=0}^{\infty} (\beta^P \theta)^s \Delta_{t,s} P_{t+s}^{\epsilon-1} Y_{t+s}} \tag{4.54}$$

If $\theta = 0$ then the equation for the optimal price reduces to $P_t^* = \frac{\epsilon_p}{\epsilon_p - 1} RMC_t P_t$. That is, if firms are able to update their prices every period then the the optimal real price would be a fixed mark up $\frac{\epsilon_p}{\epsilon_p - 1}$ over nominal marginal cost.

The above first order condition can be log-linearised as (for details, see technical appendix):

$$\pi_t = \beta^p E_t \hat{\pi_{t+1}} + \kappa R\hat{M}C_t \tag{4.55}$$

where $\kappa = \frac{(1-\theta)(1-\beta^P \theta)}{\theta}$.

4.1.6 Capital producers

Similar to Alpanda et al. (2014) and Jacob et al. (2014), capital producers combine the stock of capital from the previous period with capital investment goods to produce capital. Capital producers purchase the capital investment good (I_t^k) from final goods producer at the price of P_t after final goods production has taken place. They combine this with the undepreciated capital (K_{t-1}) from investors that they purchase at the relative price of q_t^k to produce the capital stock for the next period. This allows production to be described by the following law of motion:

$$K_t = (1 - \delta_k) K_{t-1} + Z_t^k I_t^k - \frac{\kappa_k}{2} \left(\frac{I_t^k}{I_{t-1}^k} - 1 \right)^2 Z_t^k I_t^k \tag{4.56}$$

where κ_k is the parameter governing the adjustment costs that apply to changing the level of capital investment from period to period. Z_t^k is an investment-specific technological change shock which is assumed to be exogenous and follow an AR(1) process.

Following production, the capital producer sells its capital stock back to investors at the price of q_t^k. Therefore, the capital producers' objective function can be written as follows:

$$\max_{I_t^k} E_t \sum_{s=0}^{\infty} (\beta^P)^s \frac{\lambda_{t+s}^P}{\lambda_t^P} \left[q_{t+s}^k Z_{t+s}^k I_{t+s}^k \left(1 - \frac{\kappa_k}{2} \left(\frac{I_{t+s}^k}{I_{t+s-1}^k} - 1 \right)^2 \right) - I_{t+s}^k \right] \tag{4.57}$$

Differentiating with respect to the choice variable of capital investment we have:

$$\frac{\partial}{\partial I_t^k} = 0 \Leftrightarrow q_t^k Z_t^k - q_t^k Z_t^k \frac{\kappa_k}{2}\left(\frac{I_t^k}{I_{t-1}^k} - 1\right)^2 - q_t^k Z_t^k \kappa_k \left(\frac{I_t^k}{I_{t-1}^k} - 1\right)\frac{I_t^k}{I_{t-1}^k} - 1$$
$$+ E_t\left[\beta^P \frac{\lambda_{t+1}^P}{\lambda_t^P} q_{t+1}^k Z_{t+1}^k \kappa_k \left(\frac{I_{t+1}^k}{I_t^k} - 1\right)\left(\frac{I_{t+1}^k}{I_t^k}\right)^2\right] = 0 \quad (4.58)$$

Log-linearising this expression we obtain:

$$\hat{I_t^k} = \frac{1}{\kappa_k(1+\beta^P)}(\hat{q_t^k} + \hat{Z_t^k}) + \frac{1}{(1+\beta^P)}\hat{I_{t-1}^k} + \frac{\beta^P}{(1+\beta^P)} E_t \hat{I_{t+1}^k} \quad (4.59)$$

4.1.7 Housing Producers

Housing producers function in a similar way to the capital goods producers. At the end of the period they purchase the existing housing stock off patient households, impatient households and investors at the market price q_t^h. They combine this housing stock with the housing investment good (I_t^h) to produce the final stock of housing for that period. As with the capital investment good, the housing investment good is purchased from the final goods producer at the price P_t.

Therefore, the evolution of the housing stock can be described by the following law of motion:

$$h_t = (1-\delta_h)h_{t-1} + Z_t^h I_t^h - \frac{\kappa_h}{2}\left(\frac{I_t^h}{I_{t-1}^h} - 1\right)^2 Z_t^h I_t^h \quad (4.60)$$

where κ_h is the parameter that governs the adjustment costs associated with changing the investment in housing. Z_t^h is an exogenous housing-investment-specific shock that is assumed to follow an AR(1) process.

Following production, the housing producer sells the newly produced and existing housing stock back to the patient households, impatient households and investor at the real price of q_t^h. Therefore, analogously to the capital producer, the profit maximisation problem for the housing producer can be written as:

$$\max_{I_t^h} E_t \sum_{t=0}^{\infty} (\beta^E)^t \left[q_t^h \left(Z_t^h I_t^h - \frac{\kappa_h}{2}\left(\frac{I_t^h}{I_{t-1}^h} - 1\right)^2 Z_t^h I_t^h\right) - I_t^h\right] \quad (4.61)$$

The first order condition with respect to the investment in housing is:

$$\frac{\partial}{\partial I_t^h} = 0 \Leftrightarrow q_t^h Z_t^h - q_t^h Z_t^h \frac{\kappa_h}{2}\left(\frac{I_t^h}{I_{t-1}^h} - 1\right)^2 - q_t^h Z_t^h \kappa_h \left(\frac{I_t^h}{I_{t-1}^h} - 1\right)\frac{I_t^h}{I_{t-1}^h} - 1$$
$$+ E_t\left[\beta^E q_{t+1}^h Z_{t+1}^h \kappa_h \left(\frac{I_{t+1}^h}{I_t^h} - 1\right)\left(\frac{I_{t+1}^h}{I_t^h}\right)^2\right] = 0 \quad (4.62)$$

Log-linearising this first order condition we obtain the following equation for housing investment demand:

$$\hat{I_t^h} = \frac{1}{\kappa_h(1+\beta^P)}(\hat{q_t^h} + \hat{Z_t^h}) + \frac{1}{(1+\beta^P)}\hat{I_{t-1}^h} + \frac{\beta^P}{(1+\beta^P)}E_t\hat{I_{t+1}^h} \quad (4.63)$$

4.1.8 The Central Bank

The central bank sets monetary policy in the model according to a Taylor rule so that the nominal interest rate responds to lagged inflation, lagged output and its own lag. It is also subject to a monetary policy shock ϵ_r. The log-linearised monetary policy rule is therefore:

$$\hat{R}_t = (1-r_r)(1+r_\pi)\hat{\pi_{t-1}} + r_y(1-r_r)\hat{Y_{t-1}} + r_r\hat{R_{t-1}} + \epsilon_r \quad (4.64)$$

where r_r measures the degree of interest rate smoothing, r_π is the weight on lagged inflation, and r_y is the weight on lagged output.

4.1.9 Market clearing conditions and equilibrium

In order to close the model, it must be the case that consumption and investment equal output in a given period. Therefore, aggregate output must equal the sum of all agents' consumption, housing investment and capital investment in a given period:

$$Y_t = c_t^P + c_t^I + c_t^E + I_t^h + I_t^k \quad (4.65)$$

The housing market clears if two conditions are satified. First, it needs to be the case that the housing stock in the current period equals the housing stock from the previous period plus the housing produced through investment:

$$I_t^h = h_{o,t}^P + h_{o,t}^I + h_t^E - (1-\delta_h)h_{o,t-1}^P - (1-\delta_h)h_{o,t-1}^I - (1-\delta_h)h_{t-1}^E \quad (4.66)$$

Second, for the rental market to clear, it must also be the case that the total rental made available by investors equals the rental occupied by patient and impatient households in that period:

$$A_r h_t^E = h_{r,t}^P + h_{r,t}^I \tag{4.67}$$

All saving in the model is provided by patient households who lend money to the impatient households and investors at the risk free rate. Therefore, in order for the financial sector to clear, savings from patient households must equal borrowing from impatient households and investors.

$$0 = b_t^P + b_t^I + b_t^E \tag{4.68}$$

Finally, two of the agents' flow of funds constraints must be satisfied as, per Walrus' law, satisfying two is sufficient to satisfy the third.

Therefore, the investors' flow of funds constraint must be satisfied:

$$b_t^E + q_t^r A_r h_t^E + r_t^k K_{t-1} = c_t^E + q_t^h \Big[(h_t^E - (1-\delta_h) h_{t-1}^E)\Big] + q_t^k \Big[(K_t - (1-\delta_k) K_{t-1})\Big] + \frac{R_{t-1}}{\pi_t} b_{t-1}^E + \xi_t^{h,e} \tag{4.69}$$

Likewise, the impatient households' flow of funds constraint must be satisfied:

$$c_t^I + q_t^h (h_{o,t}^I - (1-\delta_h) h_{o,t-1}^I) + q_t^r h_{r,t}^I + \frac{R_{t-1}}{\pi_t} b_{t-1}^I = b_t^I + w_t^I L_t^I \tag{4.70}$$

Intermediate goods producers profits are rebated in lump sum to patient households so have no impact on dynamics in the model. These are given by:

$$F_t = \left(\frac{X_t - 1}{X_t}\right) Y_t \tag{4.71}$$

Chapter 5

Calibration

I have calibrated a subset of the parameters in the model in order to match a number of key ratios in the New Zealand economy, mostly based on data available in the 2013 New Zealand census.

The parameter that governs the efficiency of the rental market, A_r, has been selected so that the rent-to-house price ratio, which is given by $\frac{\bar{q_r}}{\bar{q^h}} = \frac{\left(1-(1-\delta_h)-(\beta^p-\gamma)m^E(1-\delta_h)\right)}{A_r}$, matches that observed in the New Zealand economy. The data used to calculate this ratio is annual average rental payments divided by the average value of house prices. The weights of housing ownership for patient and impatient households, ω_P and ω_I respectively, are likewise selected to match New Zealand data.

Similarly, the elasticities of substitution between home and rental accommodation for patient and impatient households, $\upsilon_P = 1.45$ and $\upsilon_I = 1.40$, have been calibrated so consumption of ownership and housing rental match proportions observed in the economy. The share of housing with a mortgage in the model also closely matches that observed in New Zealand data (see Table 5.1).

The calibration selected also yields ratios of residential investment to GDP, business investment to GDP, and consumption to GDP that closely match those observed in the domestic economy. For the purposes of calculating these ratios, I have included government expenditure with consumption due to its aggregate demand impact on the economy. Excluding government expenditure only slightly modifies the distribution between these different components in the New Zealand economy.

A summary of these key steady state ratios in the model compared to New Zealand data is provided in Table 5.1.

Table 5.1: Comparison of key steady state values to New Zealand data

	Data	Model	Data Sources
Rent over house price, $\frac{\bar{q^r}}{\bar{q^h}}$	0.009333	0.009333	New Zealand Census 2013
Housing rental share, $\frac{\bar{h_r^P}+\bar{h_r^I}}{\bar{h^P}+\bar{h^I}}$	41.38%	46.6%	New Zealand Census 2013
Share of housing with mortgage, $\frac{\bar{h^I}}{\bar{h}}$	0.5407	0.5531	New Zealand Census 2013
Residential investment to GDP	4.48 %	6.20 %	New Zealand GDP 2016
Capital investment to GDP	17.58 %	18.75%	New Zealand GDP 2016
Consumption to GDP	76.17%	76.77%	New Zealand GDP 2016

Other parameters are set according to the literature. A complete list of parameter values is given in Table 5.2.

The discount factor for patient households is set to a level that corresponds to an annual interest rate of 4% in steady state. This is close to the Reserve Bank of New Zealand's assumption for the neutral rate, which is currently set at 4.25% (Richardson and Williams (2015)). Consistent with Iacoviello (2005) and the related housing DSGE literature, the discount factor for impatient households is set at a lower level, with the discount factor for housing investors sitting between the two.

The labour supply aversion, η, and the capital depreciation δ_k is set following Iacoviello (2005). Housing depreciation is then set to match capital depreciation so there is no bias toward one or the other due to relative depreciation weights. The variable capital share, μ, is set equal to 0.3, which implies a 70% income share of labour. The wage share of patient households is set equal to 64%.

Firms are assumed to be able to change prices every four quarters on average, which implies a probability of 0.75. The steady state mark up for firms is set at 1.05, consistent with Iacoviello (2005).

The parameters of the Taylor Rule are set to 0.82 for the lagged interest rate, 1.90 for response to inflation and 0.32 for response to output, consistent with Jacob et al. (2014)'s estimated model of the New Zealand economy. These parameters are higher than those found from non-linear regressions of the New Zealand economy (Kendall and Ng (2013)), which is likely due to the period of price stability in the sample used in that study.

A long period of price stability would have anchored expectations so that the real interest rate would be less affected by contemporaneous changes to the inflation rate. As a result of this, the nominal interest rate response to inflation required to generate a given response in the real interest rate may be lower than in the pre-inflation targeting period.

However, to impose these parameters upon a DSGE model may not accurately reflect how the central bank responds to agents in a model context, so I have chosen to use those estimated within Jacob et al. (2014)'s model instead. It is of note that the calibration selected is similar to that proposed by Taylor (1993).

The LVR parameters for impatient households, m^I, and investors, m^E, are modified per the scenarios described in the policy analysis section below. Three different calibrations are used

(a) No LVR restriction scenario: $m^I = m^E = 95\%$

(b) Impatient household LVR restriction scenario: $m^I = 80\%$ and $m^E = 95\%$

(c) Investor specific LVR restriction and impatient household LVR restriction: $m^I = 80\%$ and $m^E = 70\%$

These scenarios have been selected as they correspond to the levels of the LVR restrictions that the Reserve Bank of New Zealand implemented in New Zealand in 2013 and 2015. More generally, these scenarios allow consideration of how varying the LVR restrictions imposed on different agents affects agents' trade offs and business cycle dynamics. A complete analysis of different parameterisations of these LVR restrictions (and their interactions) is available in the appendix.

Table 5.2: Calibration of parameter values

Parameter	Symbol	Value
Preferences: Discount factors		
Patient households discount factor	β^P	0.99
Impatient households discount factor	β^I	0.95
Investors discount factor	γ	0.98
Other preference parameters		
Labour supply aversion	η	1.01
Weight on housing services	j	0.1
Ownership/rental preferences		
Weight on housing ownership for patient households	ω_P	0.71
Weight on housing ownership for impatient households	ω_I	0.71
Elasticity of substitution between home ownership and rental for patient households	υ_P	1.45
Elasticity of substitution between home ownership and rental for impatient households	υ_I	1.4
Technology: Factors productivity	μ	0.3
Variable capital share	α	0.64
Share of patient labour		
Depreciation	δ_h	0.03
Housing depreciation	δ_k	0.03
Capital depreciation		
Adjustment costs		
Housing adjustment cost	ϕ_h	1
Capital adjustment cost	ψ_k	1
Housing producers adjustment cost	κ_h	1
Capital producers adjustment cost	κ_k	1
Efficiency of conversion to rental property	A_r	0.56
Sticky prices		
Probability of fixed price	θ	0.75
Steady-state gross markup	X	1.05
Monetary policy		
Interest rate smoothing	r_r	0.73
Weight on lagged inflation	r_π	0.27
Weight on lagged output	r_Y	0.13
Shocks		
Monetary policy shock	ϵ_r	$N(0, 0.0841)$
Housing preference shock	ϵ_j	$N(0, 619.51)$
Technology shock	ϵ_A	$N(0, 5.018)$
Cost-push shock	ϵ_u	$N(0, 0.0289)$

5.1 Comparison to Iacoviello (2005)

For robustness, it is useful to compare the impulse responses from Iacoviello (2005) to see that the modifications made to the model have not fundamentally altered the business cycle properties of that model. This comparison is provided in the appendix.

5.2 Comparison to the VAR analysis

The calibration provided in the above section can be compared to the VAR that we estimated based on New Zealand data. A comparison of the impulse responses from the model to the SVAR with one standard deviation confidence bands is shown in Figure 5.1.

From this chart, it is clear that the model replicates many of the responses observed in the data:

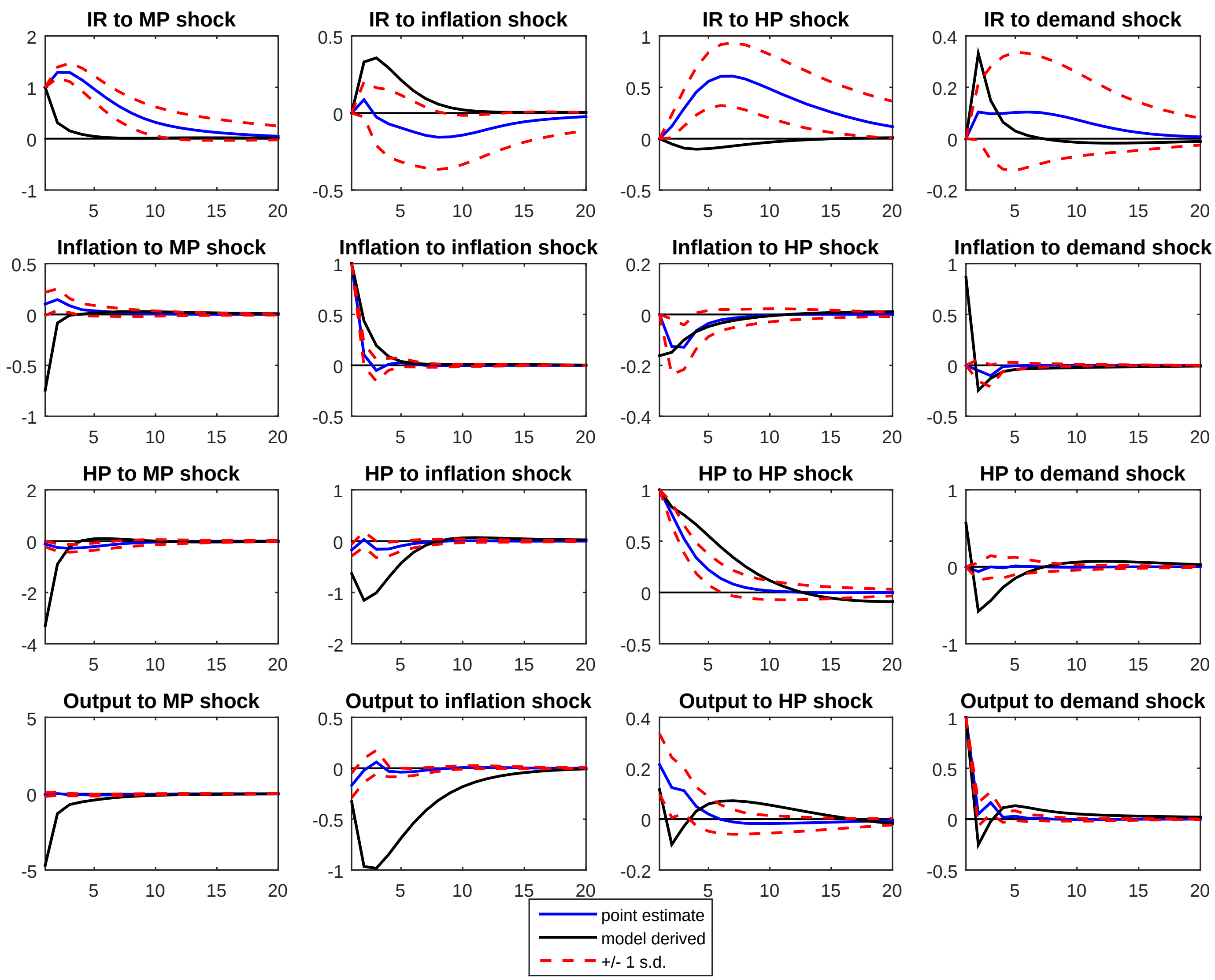
(a) Figure 5.1 shows that in response to a monetary policy shock, output and house prices fall, though to a greater magnitude than what is predicted by the data. Inflation in the model also falls in response to a monetary policy shock. This is inconsistent with the data but likely reflects that the VAR specification does not capture the length of time monetary policy takes to affect inflation.

(b) A cost-push shock has a positive impact on interest rates and a negative impact on house prices and output in the model (Figure 5.1, second column). The negative effect on house prices and output captures the dynamics observed in the data well, but again due to lags in monetary policy, the response of the interest rate in the data is not well defined. This may also be due to the observed rate of inflation being affected by exchange rate movements. As New Zealand dollar is a carry-trade currency, an increase in the policy rate may cause the New Zealand dollar to appreciate.

(c) The housing preference shock in the model matches the data very well (Figure 5.1, third column). Demand rises in response to the increased house prices, and inflation falls as the interest rate rises. Note that the interest rate response in the model is slightly negative rather than the positive response we would expect. This is likely due to the instantaneous disinflationary pressure coming through the model as agents substitute into housing from consumption. This causes a temporary decline in output, which recovers as the wealth effect increases consumption and output in the medium term.

(d) Finally, the fourth column of Figure 5.1 shows that a shock to output in the model captures the dynamics of the data. An increase in output has a wealth effect that increases demand for housing. Likewise, increased demand raises inflationary

Figure 5.1: Comparison of model impulse responses with VAR evidence from New Zealand



pressure, which causes the interest rate to rise. Higher interest rates raise borrowing costs and decrease demand for housing so house prices fall in the medium term.

The magnitude of the impulse responses from the model reasonably matches the responses in the data for most variables. However, the magnitude of the response of output and house prices to a monetary policy shock is greater than would be expected given the data. Nevertheless, as the model is intended for qualitative analysis of the impact of different LVR policies rather than to provide a precise quantitative impact of a particular LVR specification, deviations in magnitude of impulse responses are not of overriding concern.

Chapter 6

Policy analysis

This section considers the impulse responses of the model that arise with three different calibrations of LVR policy. The baseline scenario is an economy without any LVR restrictions. In this case we assume that $m^E = m^I = 95\%$ such that investors and impatient households can borrow up to 95% the value of their housing stock. Although it is not unheard of for banks to lend at $LVR \geq 95\%$, we assume a moderate level of prudence in this model so that agents cannot borrow above 95% of the expected value of their underlying collateral in the following period.

The second scenario that is considered is aligned with the first wave of macroprudential intervention that the Reserve Bank implemented in 2013. This limited banks lending above an LVR of 80% to 10% of new mortgage lending. This LVR restriction is implemented in the model as a constraint on any lending to impatient households above 80% LVR or $m^I = 0.8$.

It is arguable that this constraint should be implemented on housing investors as well in this scenario, but I have chosen not to do so for two reasons. First, as noted above, the constraint is applied as a complete restriction on borrowing above 80% in the model while in New Zealand it was applied with a 10% buffer, so to include investors may overstate its efficacy. Second, from a policy perspective, part of the reason why the second LVR restriction was introduced was due to a perception that the initial restriction was insufficient to target investors due to their ability to draw capital from alternative sources. Therefore, it may be inconsistent with observed behaviour to impose this in the model.

The final scenario incorporates an LVR restriction on both impatient households and investors in the model. The constraint on impatient households remains at $m^I = 0.8$ in the model, while the constraint on investors is imposed at 70% LVR or $m^E = 0.7$. This mirrors the level of the policy imposed by the second set of LVR restrictions that were implemented in 2015.

6.1 Monetary policy shock

The impulse responses to a monetary policy shock are given in Figure 6.1. The typical channels are consistent with what would be expected. Consumption falls across all agents as the heightened interest rate raises the cost of consuming in the current period. This causes a reduction in demand for output that leads to a fall in capital investment. As demand for the output good falls, the final goods producer decreases its price, leading to a decline in inflation in the economy.

Likewise, the increased interest rate makes borrowing more expensive with the consequence that both investors and impatient households decrease their borrowing. Reduced borrowing precipitates a decline in housing ownership for both investors and impatient households. This reduces demand for housing in the economy, so house prices and housing investment fall. As a result of the decreased supply of rental accommodation from investors, rental prices simulaneously rise. Facing increased rental prices and decreased house prices, patient households increase their housing ownership in response to the shock.

As the interest rate falls following the shock, investors and impatient households start increasing their borrowing and housing ownership. At the same time, all agents start consuming more as the opportunity cost of consumption falls. Likewise, capital and housing investment increase as demand for output and housing increases. Output rises as demand returns to pre-shock levels, along with inflation.

6.1.1 Policy scenarios

The responses indicate that the imposition of investment LVR restriction causes a marked decrease in the volatility of output while delivering broadly similar interest rate and inflation profiles. This is due to the change in investor behaviour with the presence of the LVR restriction. The impact of each set of LVR restrictions will be described in detail below.

Note that LVR restrictions are another tool with which the Reserve Bank can achieve its stabilisation objectives. The greater flexibility provided by this additional lever is a clear benefit to having macroprudential tools within the Reserve Bank's powers.

Impatient household LVR restriction

First consider the impact of an impatient household LVR restriction. Although impatient housholds reduce their borrowing to a similar extent in response to the increased interest rates as in the case of no LVR restrictions, impatient households do not divest as much of their housing ownership. This is due to the need to hold more housing to meet the LVR restriction in this scenario. This extra housing comes from patient households whose ownership of housing does not increase as much in response to the shock.

As a result of reduced patient household ownership, patient households demand more rental accommodation. Investors meet this demand by divesting less of their property in response to the shock. With the increased availability of rental accommodation, rental prices are slightly lower in the near term in this scenario than in the baseline scenario with no LVR restrictions.

As impatient households hold more housing with the restriction, they use less rental housing, which enables them to have higher consumption than without the restriction. With consumption for patient households and investors unchanged, demand is slightly higher and capital investment falls by slightly less in the near term than without the restriction.

However, this is counterbalanced by housing investment, which is slightly reduced due to the reduced demand for housing from all agents. The net impact of these offsetting effects is that output is only marginally higher than in the scenario with no LVR restriction.

Although an impatient household LVR restriction alters the distribution of housing between different agents in the economy, it only has a marginal impact on other model variables. Note that the resulting allocation of housing between agents is undesirable from a financial stability perspective, as patient households' ownership falls in favour of impatient household and investor housing ownership. Based on empirical studies, impatient households and investors are riskier than patient households as they are net borrowers. However as risk is not modelled in this framework, a full analysis of the risk or welfare implications of a particular housing allocation is beyond the scope of this model.

Impatient household and investor LVR restriction

Consider an investor-specific LVR restriction applied together with the impatient household LVR discussed above. With the higher interest rate, investors significantly reduce their borrowing, and their investment in housing as a result. As they are constrained in the amount they can borrow to a certain percentage of their housing stock, even as the interest rate falls and borrowing becomes less expensive, they cannot increase their borrowing rapidly. Rather, they continue to be constrained by the LVR restriction, which limits their borrowing to a certain percentage of their now significantly depleted housing stock. Therefore, investor borrowing and housing remains depressed for significantly longer than in other policy scenarios.

The inability of investors to quickly begin borrowing again due to the LVR constraint sustains the impact of the contractionary monetary policy shock in the model. With reduced borrowing, investor housing ownership, and accordingly house prices, remain significantly depressed. The reduced supply of rental accommodation from investors results in higher rental prices.

Facing lower house prices and higher housing rental rates, patient and impatient households increase their housing ownership well into the medium term. Due to the offsetting housing demand from patient and impatient households, housing investment follows a similar path to that which occurs in the presense of an impatient household LVR restriction alone.

With the higher rental prices earned from the housing they still own, investors' consumption falls by less in the near term than in other policy scenarios. As the path for patient and impatient household consumption remains unchanged, this buoys demand and likewise output falls by less in the near term. The extra demand also encourages slightly higher capital investment in the near term, though it follows a broadly similar path to other scenarios in the medium term.

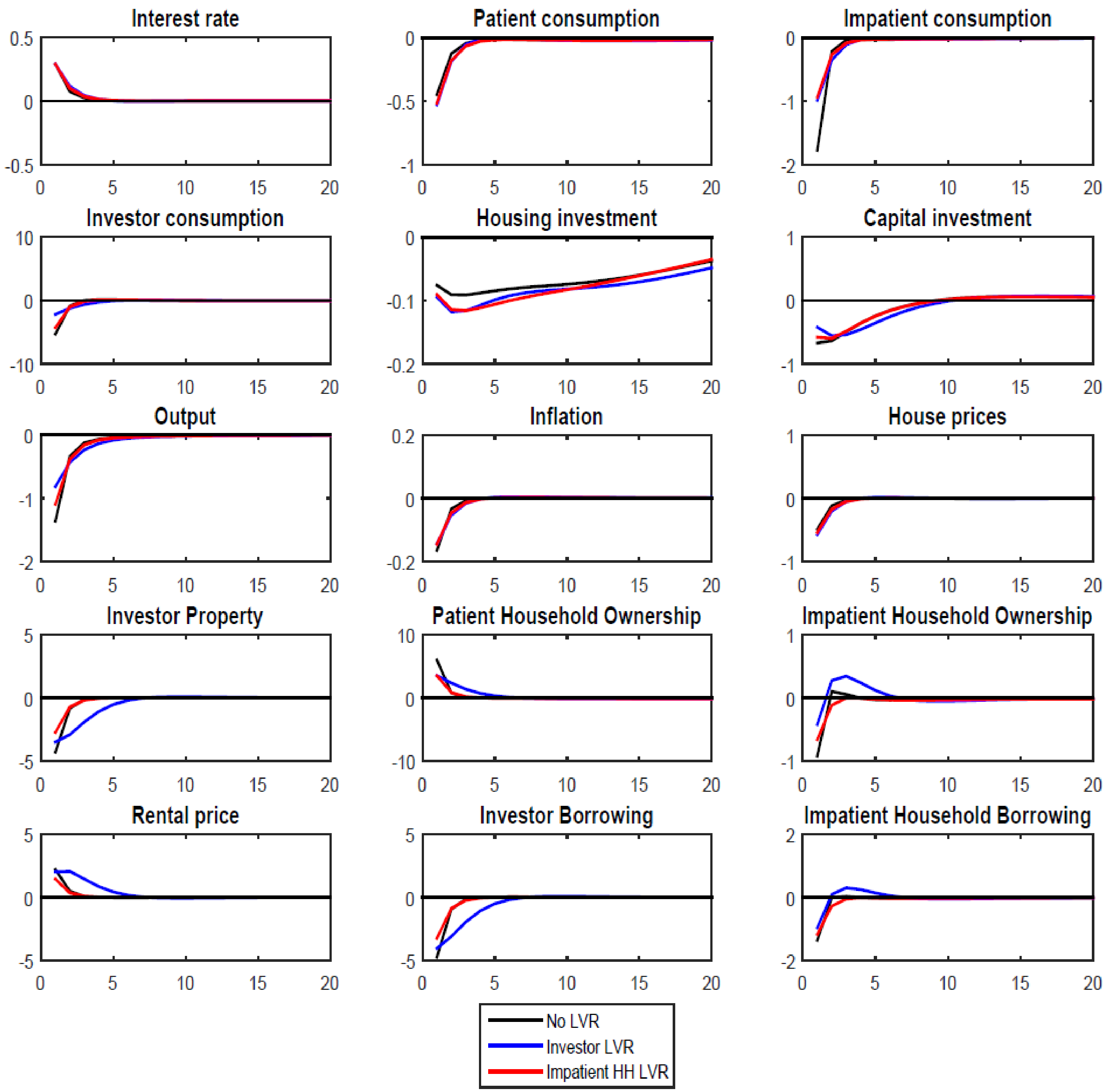
6.1.2 Conclusions

LVR restrictions do not change the nature of behaviour in response to a monetary policy shock, but they do alter the distribution of housing between agents. More importantly, they encourage higher near term consumption by the constrained agents, so output falls less in response to the shock. In stabilising the volatility of output, monetary policy shocks have less of a contractionary impact on the economy in the presence of LVR restrictions with the same inflation impact.

Investor specific LVR restrictions are powerful in propagating the impact of the initial shock. Given the divestment of housing that takes place in response to the shock, investors take many more periods to recover their housing stock, and ability to borrow, which changes the distribution of housing ownership toward households for many periods.

From a financial stability perspective, this is a very attractive feature of this restriction as it means that borrowing to investors is constrained and their housing ownership dampened for periods well beyond the length of the initial shock. In particular, households own more housing than investors relative to steady state values, for many periods beyond the intial shock. This is achieved with minimal disruption to the transmission of the shock to real variables - in fact, such a restriction minimises the contractionary impact of the initial shock, with no change to its efficacy in curbing inflation.

Figure 6.1: Responses to monetary policy shock



6.2 Housing preference shock

Figure 6.2 shows the impulse responses of key model variables to a housing preference shock. A positive housing preference shock increases patient and impatient households' utility gained from housing consumption, and in turn their demand for housing. Due to increased demand for housing, house prices rise considerably in response to the shock. This affects investors' profit maximising decision as the expected capital gain from owning housing from one period to the next is now substantially higher. As a result, investors increase their home ownership significantly. This additional investor housing is rented to patient households whose house ownership decreases considerably as a result of the increased house prices. However, with the money saved through renting, patient households are able to increase their consumption.

On the other hand, impatient households, who benefit from the increased value of housing in relaxing their collateral constraint, increase their house ownership significantly and reduce their consumption accordingly. Both impatient households and investors borrow heavily to finance their additional acquisition of houses. This demand for houses also encourages greater housing investment, which rises significantly in response to the shock.

Investors experience an initial surge in consumption due to the increased value of their collateral enabling them to borrow more, but their consumption likewise falls as resources are instead allocated to further housing investment. Capital investment falls due to the declining demand for the output good as both investors and impatient households reduce their consumption. As a result of the lack of consumption and declining capital investment, output falls after an initial surge due to the wealth effect. This causes interest rates and inflation to fall in the model.

6.2.1 Policy scenarios

Impatient Household LVR restriction

Consider the impact of an impatient-household-only LVR restriction on the transmission of a housing preference shock in the model. The restriction constrains impatient households' borrowing, and as a result they are less able to purchase houses in the periods following the shock. As a result, their consumption falls by less in than it did when there was no LVR restriction. The reduction in demand for housing ownership by impatient households is offset by patient households, who reduce their housing ownership by less in response to the shock.

Due to patient households owning more houses, demand for rental property reduces with the imposition of this restriction. Investors reduce their ownership of housing slightly accordingly, with this reduced supply offsetting the reduced demand from patient households. As a result, the rental price does not change significantly. With reduced housing ownership, the investor borrows less and their consumption falls as a result.

Reduced demand for housing results in lower housing investment. Conversely, capital investment contracts less in the presence of an LVR restriction due to the higher consumption from impatient households and investors in the medium term. However, this impact is partially offset by reduced consumption by patient households. As output and inflation are slightly higher in the medium term, the interest rate path is smoother and higher than without an LVR restriction.

As the impact of the shock dissipates, housing demand returns to steady state levels, which allows house prices and rental rates to fall. Investor and impatient household housing ownership and borrowing likewise return to steady state levels as house prices fall.

Reduced demand for housing results in lower housing investment and higher capital investment as the utility trade-off for households between consumption and housing starts to return to steady state levels. With increased output demand, inflation rises along with the interest rate as the economy returns to equilibrium.

Impatient Household and Investor LVR restrictions

Now consider the added impact of an investor specific LVR restriction. This constrains investors' ability to borrow to acquire more housing in response to the shock, so investor borrowing and housing are significantly lower in the periods following the shock in the presence of an investor-specific LVR restriction.

The resulting slack in housing demand is absorbed by patient households whose housing ownership falls by less in the near term in this scenario. Impatient household borrowing and housing ownership is broadly unchanged from the imposition of the impatient household restriction. The reduction in investor housing ownership limits the supply of rental accommodation causing the rental price to rise.

Despite the higher rental price, investors' reduced return from providing rental housing forces them to decrease their consumption. With patient and impatient households' consumption unchanged from the impatient LVR scenario, the reduction in investor consumption causes a commensurate fall in output in the near term. As a result, capital investment falls more in the near term than with no LVR restriction or an impatient household LVR restriction.

The central bank responds to the reduced inflationary pressure and decreased output with a swifter and greater decrease in the interest rate. This fall in the interest rate reduces the cost of borrowing. Investors respond to this decline in interest rates by borrowing more and acquiring a higher amount of housing. Increased returns from renting housing allows their consumption to rise rapidly, which leads to a swift recovery in output and capital investment. Therefore, the impulse responses in the medium term match those from the impatient household LVR, but with a swifter recovery in capital investment.

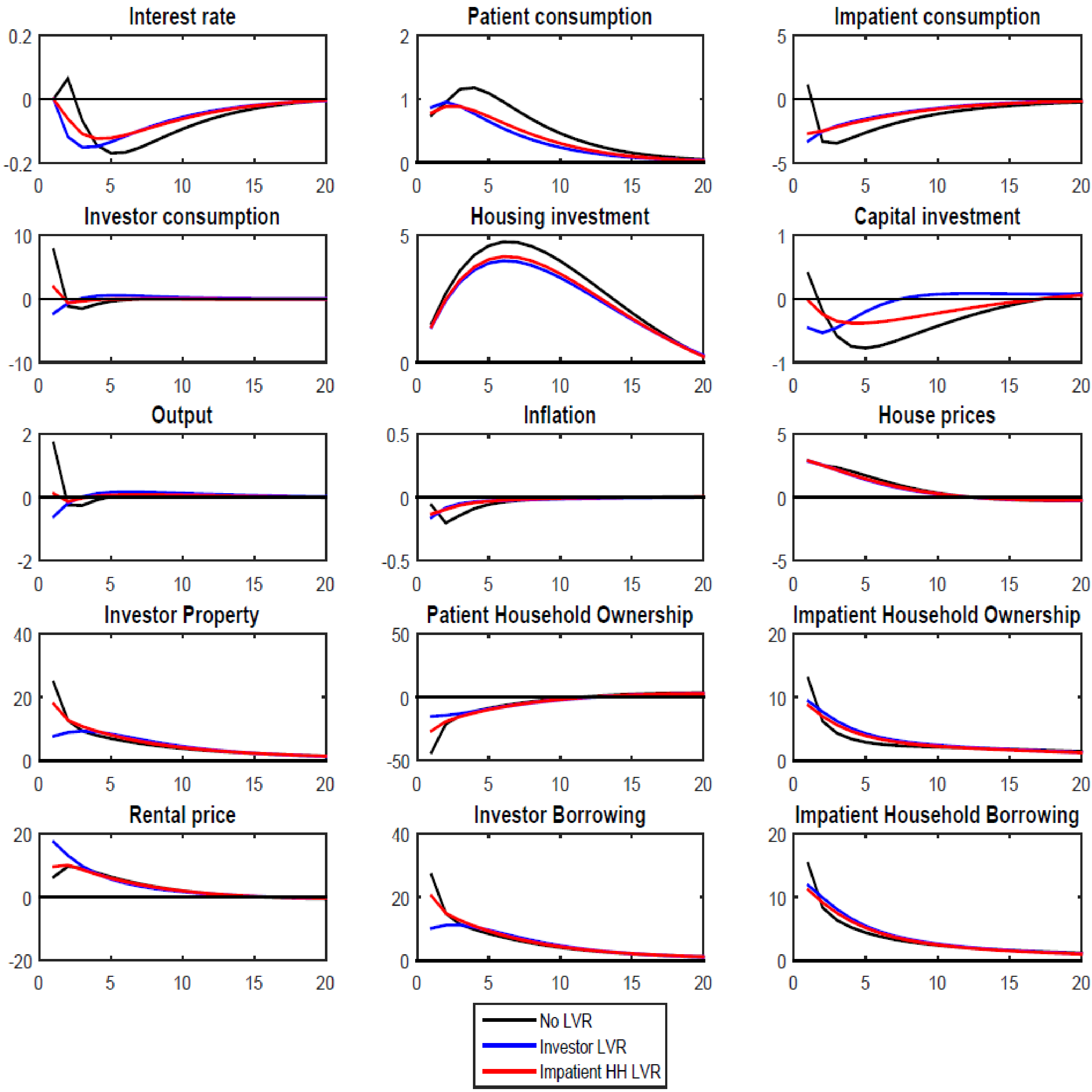
6.2.2 Conclusions

Although the profiles for house prices and output are broadly similar in the medium term in all the scenarios, the imposition of an investor specific LVR has a marked impact on investor housing. Unable to borrow as much against the value of their property, investors are less active in the property market in this scenario, which allows patient households' home ownership to remain higher than in other scenarios.

As a result, the profiles for consumption and output are both smoother despite a small contraction in the near term. Capital investment is higher (except in the near term) and smoother than in the absence of an investor specific LVR restriction.

The reallocation of housing from investors to patient households in the presence of an LVR restriction is particularly relevant from a financial stability perspective. Given the concerns regarding investors' greater likelihood of default (a feature not captured in the model framework), this suggests that LVR restrictions can provide extra stability to the extent that they moderate the growth of investors' housing portfolios in favour of patient households in response to a housing preference shock. Although full analysis of this issue is beyond the scope of the paper, these redistributional effects would indicate that LVR restrictions are effective in altering the composition and associated risk of the lending undertaken in response to exogenous housing demand.

Figure 6.2: Responses to housing preference shock



6.3 Technology shock

Figure 6.3 shows the impulse responses of key model variables to a technology shock. A positive technology shock allows more output to be produced for a given level of inputs. Therefore, in response to a technology shock, demand for capital falls as the intermediate goods producer can now produce the same amount of output with a reduced level of capital. Decreased demand for capital causes investors' income to fall and they reduce their consumption and investment in housing accordingly.

Investors reduced consumption causes demand for the output good to fall. This precipitates a fall in prices and the final and intermediate goods producers to reduce their production accordingly. At the same time, decreased demand for capital causes capital investment to fall.

With the depressed price of the output good, patient and impatient households increase their consumption. Likewise, the reduction in rental property as a result of investors selling their stock of housing encourages patient and impatient households to increase their housing ownership. As a result of the increased demand for housing, house prices rise along with housing investment.

Reduced output demand and decreased inflationary pressure cause the central bank to reduce interest rates, making borrowing less costly for impatient households and investors. This encourages impatient households to borrow more to finance more consumption and the acquisition of further housing ownership.

With increased demand for the final good in the economy, both prices and output increase in the medium term. This increases demand for capital, which provides more income to investors as the owners of capital. In turn, this enables investors' consumption to return to steady state levels.

At the same time, patient and impatient households reduce their consumption as prices rise. Likewise, as investors' increased income allows them to increase their housing ownership to steady state levels, households substitute into rental property again. Decreased demand for housing allows house prices and housing investment to fall as the economy returns to steady state.

6.3.1 Policy scenarios

The primary difference in response to the technology shock in the presence of an LVR restriction is how the falling interest rate is transmitted through the economy. As demand for output falls following the technology shock, prices fall and the central bank lowers interest rates in order to stimulate the economy. This makes it more attractive for impatient households and investors to borrow more given it is now less costly. However, where there is an LVR restriction, impatient households and investors are limited in the amount they can borrow to a certain percentage of their housing ownership. The impacts of each type of restriction will be discussed in detail below.

Impatient household LVR restriction

In the case of an LVR restriction that applies only to impatient households, impatient households can borrow less in response to the falling interest rate. As a result, impatient household consumption and housing ownership rises by less in response to the shock than with no LVR restriction.

Therefore, output and the interest rate fall by slightly more than in other scenarios. With reduced demand for housing from impatient households, housing investment is lower in this scenario than with other LVR restrictions. However, other variables are not materially affected by the presence of this restriction.

Impatient household and investor LVR restrictions

As noted above, investors' income falls as demand for capital falls as a result of the increased productivity of current capital. With the LVR restriction limiting the amount that investors can borrow, investors reduce their housing ownership even more in response to the technology shock. This reduction in housing ownership constrains their borrowing in future periods, so it takes longer for investors' level of housing to return to steady state than in the other scenarios.

With less invested in housing, investors are able to consume more in this scenario, which increases demand in the economy. As a result of increased demand, output falls by considerably less in response to the shock than in other scenarios.

With less housing rental being supplied by investors, both patient and impatient households experience larger and more persistent increases in housing ownership. This more than offsets the reduction in demand from investors, so housing investment is stronger in this scenario than in others.

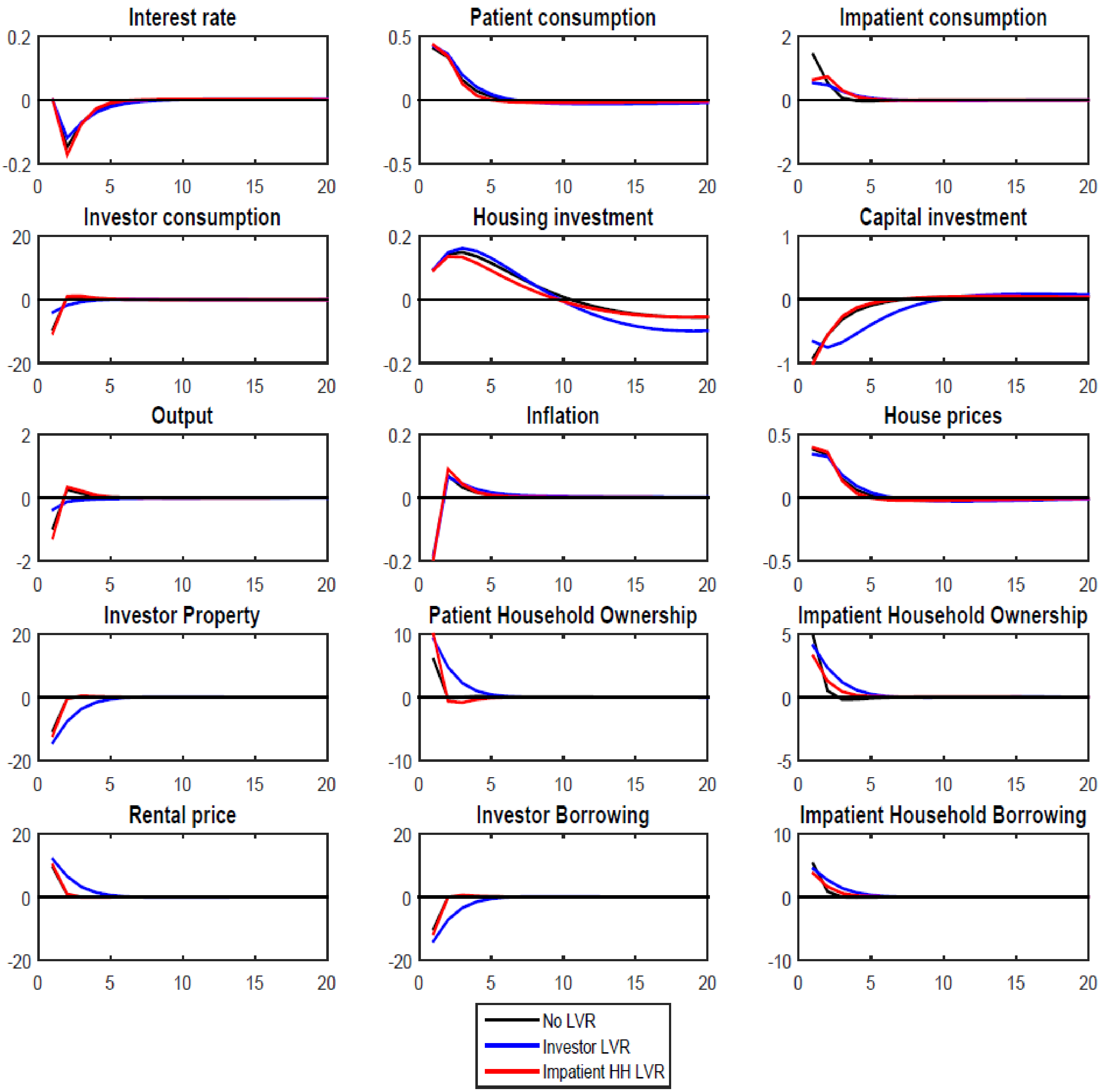
For impatient households who are constrained in their borrowing by an LVR restriction, additional housing ownership comes at the cost of consumption. With decreased consumption by impatient households, there is decreased output demand in the medium term and capital investment remains lower than in the other scenarios. The consumption path for patient households is largely unchanged by the LVR restriction being applied.

6.3.2 Conclusions

To conclude, investor-specific LVR restrictions cause a pronounced and sustained decline in investor property ownership in response to a technology shock. This has the result of smoothing the path for output by reducing the loss of investor consumption in response to the shock.

Impatient households substitute out of consumption into housing ownership in the presence of an investor specific LVR restriction combined with an impatient household LVR restriction. This results in a sustained reduction in capital investment in response to the shock, but a slight increase in housing investment in the medium term. The path for house prices, inflation and the interest rate is largely unchanged by the presence of LVR restrictions.

Figure 6.3: Responses to technology shock



6.4 Cost-push shock

Figure 6.4 shows the impulse responses of key model variables to a cost-push shock. A positive cost-push shock increases the cost of the output good in the economy, causing all agents to decrease their consumption in the economy. This reduces demand for the output good so that final and intermediate goods producers decrease production, which reduces demand for the inputs of labour and capital. As demand for capital falls, so does capital investent.

In response to the added inflationary pressure, the central bank increases its interest rate. The higher interest rate makes borrowing more expensive so impatient households and investors decrease their borrowing and further reduce their consumption. At the same time, patient households earn less from their ownership of the final good firm and from lending to impatient households and investors, despite the increased cost of both. As a result, patient households' income falls and they decrease their demand for housing in response to the cost-push shock.

With the marked reduction in demand from patient households, house prices fall, encouraging investors and impatient households to increase their housing ownership. Nevertheless, the net softening in demand for housing reduces housing investment.

As inflation returns to steady state, the interest rate likewise falls to steady state level. Falling prices encourage all agents to increase their consumption and production of the output good increases as a result. The resulting demand for capital causes capital investment to return to steady state levels.

With the falling interest rate, investors and impatient houseolds are able to increase their borrowing. Coupled with increased consumption, this increases the profits earned by patient households, who increase demand for housing ownership. This causes house prices to rise and impatient households and investors to decrease their housing ownership. As a result of the offsetting impacts on housing demand, housing investment takes longer to return to steady state levels than other variables.

6.4.1 Policy scenarios

The profiles for most variables do not vary materially with different LVR specifications in response to a cost-push shock. However, different LVR settings do change the way the shock affects housing ownership for all agents in the economy. The presence of an LVR restriction also affects the transmission of the shock to housing investment and consumption, for impatient households in particular. These impacts are detailed in full in Figure 6.4.

Impatient household LVR restrictions

First consider an LVR restriction that applies to impatient households only. In response to this restriction, impatient households consume less as housing ownership becomes more valuable due to its collateral value. They instead increase their housing ownership in the medium term, which in turn allows them to increase their borrowing as the interest rate falls.

The demand effect of this increased housing ownership by impatient households is offset by patient households, whose housing ownership is reduced in the medium term relative to in a scenario with no LVR restrictions. Investors' housing ownership is only marginally impacted in the near term by an impatient household LVR restriction. All other variables are largely unaffected by the imposition of an LVR restriction with a cost-push shock.

Impatient household and investor LVR restrictions

The impacts discussed above are magnified in the case of LVR restrictions that apply to both impatient households and investors. The most marked change in the presence of an investor specific LVR restriction is to investors' housing ownership. Due to the constraint, the investor cannot borrow enough in the near term to accumulate housing in response to the decreased house prices as a result of the shock.

As a result, patient households do not divest their housing ownership suddenly as they did in other scenarios in response to the cost-push shock, but instead reduce their ownership over the medium term. Impatient households accumulate even more housing in the near term than in other scenarios, and simultaneously reduce their consumption accordingly. Housing investment is broadly unchanged, though its profile decreases slightly with the imposition of each type of LVR restriction.

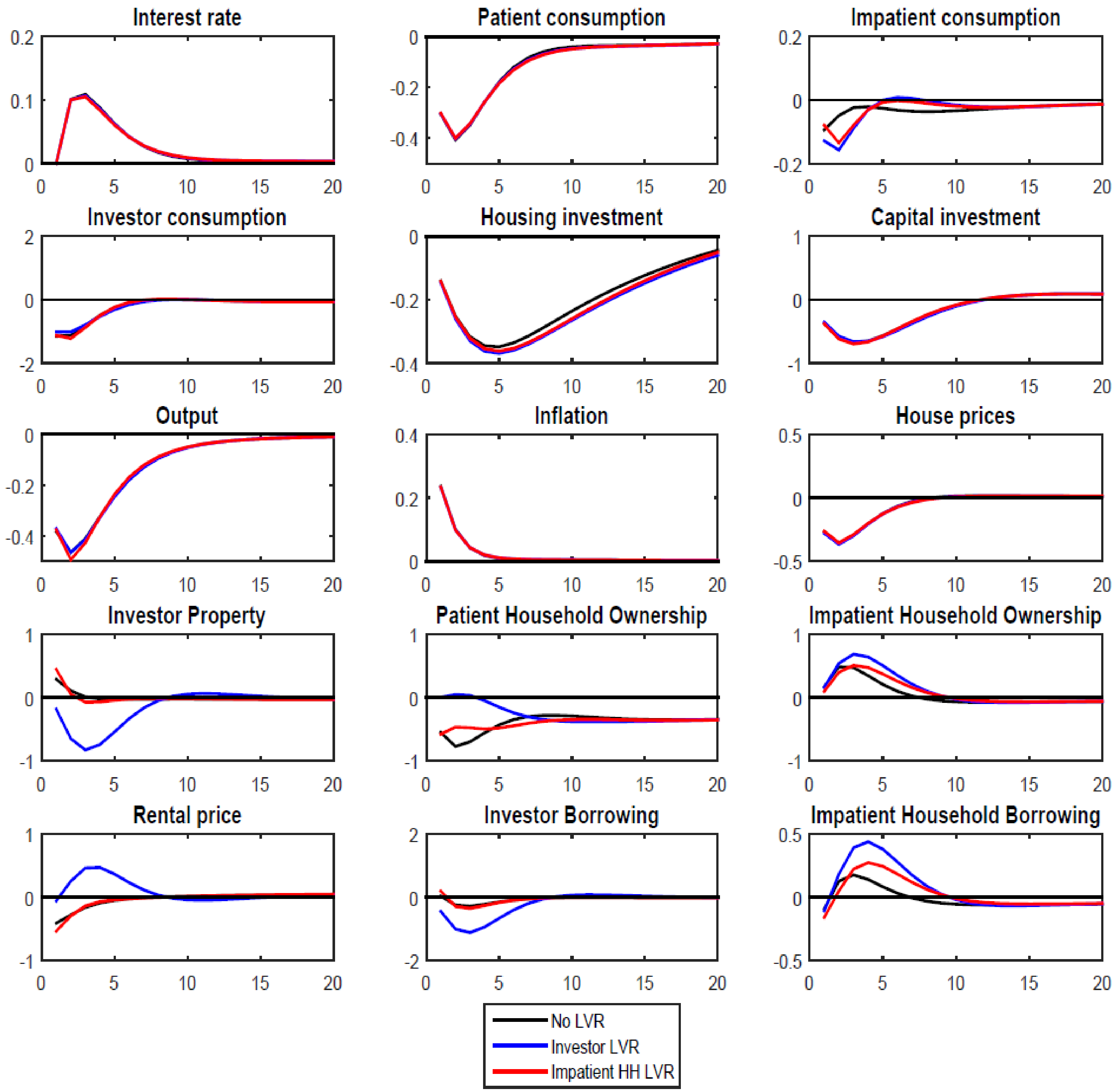
Other variables are largely unaffected.

6.4.2 Conclusions

The presence of LVR restrictions does not fundamentally alter the propagation of a cost-push shock to most real variables, but they do have important distributional impacts. In particular, investor specific LVR restrictions significantly reduce investor activity in the housing market in response to a cost-push shock. As a result, housing remains held by patient households in the periods immediately following the shock.

To the extent that housing investors are riskier than patient households in the real world, this distribution of housing in response to the inflation shock could be argued to be favourable from a financial stability perspective (though rigorously proving this assertion is well beyond the scope of this paper).

Figure 6.4: Responses to a cost-push shock



6.5 Steady state impacts of different policy measures

Assuming an LVR restriction becomes permanent, its implementation would modify the steady state levels of some variables in the economy. LVR restrictions have direct impacts on housing-related steady state conditions, but other variables (such as capital investment/GDP) do not vary with different specifications of the LVR restriction.

Therefore, the impacts on the housing market result in a redistribution from ownership to rental and from mortgagees to unencumbered households. These redistribution effects are particularly interesting as they give insight into how the structure of the housing market changes in response to these restrictions.

A summary of the steady state impacts of different LVR measures on housing related variables are available in Table 6.1.

Table 6.1: Steady state impacts of alternative LVR measures

	Impatient HH LVR	**Investor LVR**	**Both**
Residential investment/GDP	−0.92%	−0.28%	−1.19%
Rental price	0%	12.2%	12.2%
Rental share	2.38%	−4.04%	−1.75%
Share of housing with mortgage	−6.06%	0.17%	−5.92%

6.5.1 Impatient household LVR restrictions

Consider first the impact of an impatient household LVR restriction. The LVR limits the ability of impatient households to borrow, so as a result their demand for housing falls. With this reduction in housing demand, residential investment/GDP contracts −0.92% to 3.56% of GDP. With the reduced demand for housing ownership from impatient households, the rental share of the housing market increases 2.38% to 48.98% housing. With increased demand for rental offset with reduced supply of housing, the steady state rental price does not change with this scenario. Finally, the LVR restriction makes borrowing less attractive, so the share of housing with a residential mortgage falls −6.06% to 49.25%.

6.5.2 Investor LVR restrictions

Next consider the impact of an investor specific LVR restriction in the model. In the presence of an investor specific LVR restriction, investors are constrained in the amount they can borrow so their ownership of housing decreases relative to patient and impatient households. As investors' borrowing contracts, impatient households borrow slightly more, which they use to fund additional acquisition of housing.

With reduced housing ownership by investors, the supply of rental propoerty decreases, so rental prices rise 12.19% and the rental share of the market falls −4.04% to 42.56%. As a result of the reduction in availability of rental houses, demand for housing ownership from patient and impatient households increases, so residential investment/GDP only falls by 0.28% to 4.2%. At the same time, the share of housing with a mortgage increases marginally by 0.17% to 55.48% of the housing market.

6.5.3 Both impatient household and investor specific LVR restrictions

Finally consider the case of both an investor specific LVR restriction and an impatient household LVR restriction imposed at the same time. In that case, the investors are constrained in the amount they can borrow so their housing ownership decreases relative to that of patient households. However, at the same time, impatient households are also constrained in the amount they can borrow so their demand for housing ownership similarly falls. As a result, impatient households and investors' borrowing declines considerably. This results in a contraction in consumption for patient households whose profits have fallen as the owners of the goods firm.

Decline in demand for new housing precipitates a decline in residential investment/GDP, which falls 1.19% to 3.29%. Offsetting their reduced demand for housing ownership, impatient households demand more rental property and housing investors do not divest as much of their housing stock as a result. Higher investor ownership results in a relatively higher supply of rental property, so the rental share of the market only declines by 1.75% to 44.85%. With higher demand for rental accommodation from impatient households, the rental price rises by 12.19% relative to the steady state of no LVR restrictions.

6.5.4 Summary

From this analysis, it is clear that permanent application of LVR restrictions has impacts on the steady state distribution of the housing market. This has important implications for how money is spent and invested in the economy. Based on what we know of the risk profiles of different classes of borrowers in the real world, these changes also have implications for risk in the economy. We can conclude the following stylised facts from the above steady state analysis:

(a) Imposition of LVR restrictions will lead to a decline in residential investment regardless of the agent they constrain. However, the magnitude of the decline is far greater when the restriction constrains the impatient household compared to when it constrains investors.

(b) As impatient household LVR restrictions do not directly constrain the supply of rental accommodation, they have no impact on the rental price. However, investor LVRs lead to a decline in the supply of rental accommodation that increases rents.

(c) Impatient households demand greater rental accommodation when constrained by an impatient household LVR restriction. This heightened demand leads to an increase in the rental share of the market, whereas an investor LVR restriction constrains supply such that the rental share of the housing market also declines.

(d) Investor LVR restrictions have a negligible impact on the share of housing with a mortgage, whereas impatient household restrictions lead to a severe constraction in the share of housing with a mortgage.

(e) Investors' borrowing invariably falls in response to any LVR restriction, regardless of the agent it constrains. Impatient households' borrowing is significantly reduced where there is an impatient household LVR restriction, but increases slightly where there is only an investor specific LVR restriction.

(f) Patient households' consumption invariably falls slightly in response to the imposition of an LVR restriction, while investors and impatient households increase their consumption regardless of the agent constrained by the restriction. The net impact of this is that there is a substitution from residential investment to consumption in response to an LVR restriction in this framework.

Although financial stability and housing supply issues are beyond the scope of what can be examined within this model, the framework is sufficiently broad to provide insight into how the permanent application of LVR restrictions would impact the optimising decisions of the agents within the model. If we assume there is an exogenous driver that is increasing demand for housing then implementation of a permanent LVR restriction has desirable properties: it diverts investment away from housing and increases consumption while reducing risky agents' borrowing.

However, if the exogenous driver increasing demand for housing is a supply constraint, then this framework implies that the response of limiting residential investment is the opposite of what is necessary to redress the imbalance. Nevertheless, the reduction in risky borrowing is desirable from a financial stability perspective, but the long-term structural change required to redress the supply imbalance would not come endogenously. That is, residential investment would need to occur outside the model framework.

Chapter 7

Conclusions

This paper introduces a framework for examining the impact of LVR restrictions on the transmission of monetary policy and the business cycle. It is timely as many central banks around the world have started adding macroprudential tools to their toolkit, but how these tools interact with the transmission of monetary policy is yet not broadly understood.

The scenario analysis in Section VI showed that LVR restrictions reduce output fluctuations in response to monetary policy shocks. This is especially the case with investor specific LVR restrictions, where the investor quickly divests property in response to the increased interest rate and then only gradually rebuilds their housing stock following the shock. As a result of these dynamics, increased interest rates suppress investor activity far longer in the presence of an investor specific LVR restriction than without it.

In terms of transmission of monetary policy, these responses indicate that LVR restrictions are advantageous to a flexible inflation targeting central bank as they allow the central bank to respond to inflation more given the reduced variation in output. This is consistent with the responses to a technology shock where an investor specific LVR is similarly effective at smoothing investor consumption and, accordingly, output by moderating investor activity in the housing market.

With a housing preference shock, the constraint on investors' capacities to borrow in response to the shock results in smoother consumption and output paths. Crucially, as consumption remains higher with the investor specific LVR restriction, capital investment is higher and smoother than in other scenarios. This reflects the earlier finding that below an LVR of 78%, there is a trade off between volatility of capital investment and volatitility of housing investment.

Although risk analysis is beyond the scope of the model, the redistribution of housing toward patient and impatient households in response to a housing preference shock implies an increase in financial stability, given analysis suggests that investors are at higher risk of default in the event of a downturn (Kelly and O'Malley (2014)).

This redistribution of housing ownership toward different agents also occurs in response to a cost-push shock in the presence of an LVR restriction. In particular, investor housing ownership is significantly reduced in the presence of an investor specific LVR restriction following a cost-push shock relative to the scenario of a cost-push shock without an LVR restriction.

Comparing the steady state of the model under different LVR assumptions also provides insight how permanent application of an LVR restriction would affect key housing market variables in the longer term. From steady state analysis, it is possible to derive the following stylised facts about the long term impacts of LVR restrictions on the economy:

(a) Regardless of the agent that an LVR restriction constrains, it leads to a decline in residential investment to GDP, leading to higher consumption in the economy.

(b) Investor specific LVR restrictions lead to decreased rental supply and increased rents.

(c) Impatient household LVR restrictions lead to a severe contraction in the percentage of housing with a mortgage.

(d) Implementation of any LVR restriction causes investors borrowing to fall, regardless of the agent the restriction constrains.

Overall, this model suggests that LVR restrictions can be effective in moderating the business cycle by curbing swings in demand for housing in response to a variety of shocks. This allows a flexible inflation targeting central bank to more directly set the interest rate to target inflation.

There are many areas of future development for this research. First, one could consider extending this paper to include risk premia and a banking sector, to explore how risk-related spreads coupled with LVR restrictions affect the transmission of monetary policy. Another possible area would be to extend the model to make it an open economy model to allow consideration of how the availability of credit from abroad changes the business cycle impacts of LVR restrictions.

Appendix A

The steady state

$$
\begin{aligned}
\pi &= 1 \\
R &= \frac{1}{\beta^P} \\
\lambda^E &= \frac{\beta^P - \gamma}{\bar{c^E}} \\
\lambda^I &= \frac{\beta^P - \beta^I}{\bar{c^I}} \\
F &= (1 - \frac{1}{X})Y \\
\bar{X} &= \frac{\epsilon}{\epsilon - 1} \\
\bar{r^k} &= \frac{1 - \gamma(1 - \delta_k)}{\gamma} \\
\frac{\bar{K}}{\bar{Y}} &= \frac{\mu}{\bar{r^k}\bar{X}} \\
\bar{w^P}\bar{L^P} &= \big(\alpha(1-\mu) + X - 1\big)\frac{\bar{Y}}{\bar{X}} = s^P\bar{Y} \\
\bar{w^I}\bar{L^I} &= (1-\alpha)(1-\mu)\frac{\bar{Y}}{\bar{X}} = s^I\bar{Y} \\
\bar{q_r} &= \frac{q^h\Big(1 - (1-\delta_h) - (\beta^p - \gamma)m^E(1-\delta_h)\Big)}{A_r} \\
\frac{\bar{h^P}}{\bar{h_o^P}} &= \left[\omega_p^{1/v_p} + (1-\omega_p)^{1/v_p}\Big(\frac{1-\omega_p}{\omega_p}\Big)^{\frac{v_p-1}{v_p}}\Big(\frac{\bar{q^h}(1-\beta_p(1-\delta_h))}{\bar{q_r}}\Big)^{v_p-1}\right]^{\frac{v_p}{v_p-1}} = \Delta_o^p \\
\frac{\bar{h^P}}{\bar{h_r^P}} &= \left[\omega_p^{1/v_p}\Big(\frac{\omega_p}{1-\omega_p}\Big)^{\frac{v_p-1}{v_p}}\Big(\frac{\bar{q_r}}{\bar{q^h}(1-\beta_p(1-\delta_h))}\Big)^{v_p-1} + (1-\omega_p)^{1/v_p}\right]^{\frac{v_p}{v_p-1}} = \Delta_r^P \\
\bar{q^h} &= \frac{j^P}{\bar{h^P}}\frac{1}{1-\beta^P(1-\delta_h)}(\omega_p\Delta_o^P)^{\frac{1}{v_p}}\bar{c^P} = \xi_3\frac{\bar{c^P}}{\bar{h^P}}
\end{aligned}
$$

$$\frac{\bar{h^I}}{\bar{h_o^I}} = \left[\omega_I^{1/v_I} + (1-\omega_I)^{1/v_I}\left(\frac{1-\omega_I}{\omega_I}\right)^{\frac{v_I-1}{v_I}}\left(\frac{\bar{q^h}(1-\beta_I(1-\delta_h)(1-m^I)-\beta_p m^I(1-\delta_h))}{\bar{q_r}}\right)^{v_I-1}\right]^{\frac{v_I}{v_I-1}} = \Delta_o^I$$

$$\frac{\bar{h^I}}{\bar{h_r^I}} = \left[\omega_I^{1/v_I}\left(\frac{\omega_I}{1-\omega_I}\right)^{\frac{v_I-1}{v_I}}\left(\frac{\bar{q_r}}{\bar{q^h}(1-\beta_I(1-\delta_h)(1-m^I)-\beta_p m^I(1-\delta_h))}\right)^{v_I-1} + (1-\omega_I)^{1/v_I}\right]^{\frac{v_I}{v_I-1}} = \Delta_r^I$$

$$\bar{q^h} = \frac{j^I}{\bar{h^I}}\frac{1}{1-\beta_I(1-m^I(1-\delta_h))-\beta_p m^I(1-\delta_h)}(\omega_I\Delta_o^I)^{\frac{1}{v_I}}\bar{c^I} = \xi_4\frac{\bar{c^I}}{\bar{h^I}}$$

$$\bar{b^I} = \beta^P m^I \bar{q^h}\bar{h_o^I}(1-\delta_h) = \frac{\beta^P m^I(1-\delta_h)\xi_4\bar{c^I}}{\Delta_o^I}$$

$$\bar{c^I} = \bar{w^I}\bar{L^I} + \bar{b^I} - \frac{\bar{R}\bar{b^I}}{\bar{\pi}} - q^h h_o^I - q^r h_r^I \Leftrightarrow \bar{c^I} = s^I\bar{Y} - (1-\beta^p)\frac{m^I(1-\delta_h)\xi_4\bar{c^I}}{\Delta_o^I} - q^h\xi_4\frac{\bar{c^I}}{\Delta_o^I} - q^r\xi_4\frac{\bar{c^I}}{\Delta_r^I}$$

$$\bar{c^I} = \frac{s^I}{1+\frac{((1-\beta^p)m^I(1-\delta_h)+\delta_h)\xi_4}{\Delta_o^I}+\frac{q^r\xi_4}{\Delta_r^I}}\bar{Y} = \xi_5\bar{Y}$$

$$\bar{c^P} = \bar{w^P}\bar{L^P} + F + \bar{b^P} - \frac{\bar{R}\bar{b^P}}{\bar{\pi}} - q^r h_r^P - \delta_h q^h h_o^P$$

$$c^P = \left(\frac{1}{1-\left(1-\beta^P\right)m^E(1-\delta_h)\frac{\xi_3}{\Delta_r^P A_r}+\frac{q^r\xi_3}{\Delta_r^P}+\frac{\delta_h\xi_3}{\Delta_o^P}}\right)$$
$$\left(s^P + \left(1-\beta^P\right)\left(\frac{m^I(1-\delta_h)\xi_4\xi_5}{\Delta_o^I}\right) + \left(1-\beta^P\right)\left(m^E(1-\delta_h)\frac{\xi_4\xi_5}{\Delta_r^I A_r}\right)\right)Y$$

$$c^P = \xi_6\bar{Y}$$

$$\bar{h^E} = \frac{\xi_4\bar{c^I}}{\Delta_r^I A_r} + \frac{\xi_3\bar{c^P}}{\Delta_r^P A_r} = \left(\frac{\xi_4\xi_5}{\Delta_r^I A_r} + \frac{\xi_3\xi_6}{\Delta_r^P A_r}\right)\bar{Y} = \xi_7\bar{Y}$$

$$\bar{b^E} = \beta^P m^E\bar{q^h}\bar{h^E}(1-\delta_h)$$

$$\bar{c^E} = \left(\bar{q^r}A_r - \delta_h - m^E q^h(1-\delta_h)(1-\beta^P)\right)\bar{h^E} + \left(\bar{r^k} - \delta_k\right)\bar{K}$$

Appendix B

The log-linearised model

B.1 Aggregate demand equations

$$
\begin{aligned}
\hat{Y_t} &= \frac{\bar{c^P}}{\bar{Y}}\hat{c^P_t} + \frac{\bar{c^I}}{\bar{Y}}\hat{c^I_t} + \frac{\bar{c^E}}{\bar{Y}}\hat{c^E_t} + \frac{\bar{I^k}}{\bar{Y}}\hat{I^k_t} + \frac{\bar{I^h}}{\bar{Y}}\hat{I^h_t} \\
\hat{c^P_t} &= E_t\hat{c^P_{t+1}} - \hat{R}_t + E_t\hat{\pi_{t+1}} \\
\beta^P\hat{c^I_t} &= \beta^I E_t\hat{c^I_{t+1}} - (\beta^P - \beta^I)\hat{\lambda^I_t} - \beta^P\hat{R}_t + \beta^I E_t\hat{\pi_{t+1}} \\
\beta^P\hat{c^E_t} &= \gamma E_t\hat{c^E_{t+1}} - (\beta^P - \gamma)\hat{\lambda^E_t} - \beta^P\hat{R}_t + \gamma E_t\hat{\pi_{t+1}} \\
\hat{q^k_t} &= \hat{c^E_t} + \gamma\frac{\bar{r^k}}{\bar{q^k}}\hat{r^k_t} + \gamma(1-\delta_k)E_t\hat{q^k_{t+1}} - \gamma\left(\frac{\bar{r^k}}{\bar{q^k}} + (1-\delta_k)\right)E_t\hat{c^E_{t+1}}
\end{aligned}
$$

B.2 Borrowing constraints

$$
\begin{aligned}
\hat{b^E_t} &= E_t\hat{q^h_{t+1}} + \frac{\bar{h^P}}{\bar{h^E}}\frac{1}{\Delta_p r A_r}\hat{h^P_{r,t}} + \frac{\bar{h^I}}{\bar{h^E}}\frac{1}{\Delta_i r A_r}\hat{h^I_{r,t}} + \hat{\pi_{t+1}} - \hat{R}_t \\
\hat{b^I_t} &= E_t\hat{q^h_{t+1}} + \hat{h^I_{o,t}} + E_t\hat{\pi_{t+1}} - \hat{R}_t
\end{aligned}
$$

B.3 Aggregate supply equations

$$
\begin{aligned}
\hat{Y}_t &= \hat{A}_t + \mu \widehat{K_{t-1}} + \alpha(1-\mu)\widehat{L_t^P} + (1-\alpha)(1-\mu)\widehat{L_t^I} \\
\hat{Y}_t &= \eta \widehat{L_t^I} + \widehat{c_t^I} - \hat{X}_t \\
\hat{Y}_t &= \eta \widehat{L_t^P} + \widehat{c_t^P} - \hat{X}_t \\
\hat{\pi}_t &= \beta_p E_t \widehat{\pi_{t+1}} + \kappa \hat{X}_t + \hat{u}_t \\
\left(1 + \frac{1}{\bar{X}}\right)\hat{X}_t &= \frac{\hat{Y}_t}{\bar{X}} + \bar{r^k}\bar{K}(\widehat{r_t^k} + \widehat{K_{t-1}}) \\
\widehat{I_t^k} &= \frac{1}{(1+\gamma)}\widehat{q_t^k} + \frac{1}{(1+\gamma)}\widehat{I_{t-1}^k} + \frac{\gamma}{(1+\gamma)}E_t \widehat{I_{t+1}^k}
\end{aligned}
$$

B.4 Housing market equations

$$
\begin{aligned}
\widehat{q_t^h} &= \gamma^E E_t \widehat{q_{t+1}^h} + (1 - \beta^P m_e(1-\delta_h) - \bar{q^r} A_r)\widehat{c_t^E} + \bar{q^r} A_r \widehat{q_t^r} - \gamma(1-\delta_h)(1-m_e)E_t \widehat{c_{t+1}^E} \\
&\quad - \beta^P m_e(1-\delta_h)(\hat{R}_t - E_t \widehat{\pi_{t+1}}) \\
\widehat{q_t^h} &= \gamma^h E_t \widehat{q_{t+1}^h} + (1-\gamma^h)\Big(\hat{j}_t + \Big(\frac{1}{v_I} - 1\Big)\widehat{h_t^I} - \frac{1}{v_I}\widehat{h_{o,t}^I}\Big) - m_i \beta^P (1-\delta_h)(\hat{R}_t - E_t \widehat{\pi_{t+1}}) \\
&\quad + (1 - \beta^P m_i(1-\delta_h))\widehat{c_t^I} - \beta^I (1-\delta_h)(1-m_i)E_t \widehat{c_{t+1}^I} \\
\widehat{q_t^r} &= \widehat{c_t^I} + \hat{j}_t + (\frac{1}{v_I} - 1)\widehat{h_t^I} - \frac{1}{v_I}\widehat{h_{r,t}^I} \\
\widehat{h_t^I} &= \frac{1}{\Delta_i o}\widehat{h_{o,t}^I} + \frac{1}{\Delta_i r}\widehat{h_{r,t}^I} \\
\widehat{q_t^h} &= \beta^P E_t \widehat{q_{t+1}^h} + (1-\beta^P)\Big(\hat{j}_t + \Big(\frac{1}{v_p} - 1\Big)\widehat{h_t^P} - \frac{1}{v_I}\widehat{h_{o,t}^P}\Big) + \widehat{c_t^P} - \beta^P(1-\delta_h)E_t \widehat{c_{t+1}^P} \\
\widehat{q_t^r} &= \widehat{c_t^P} + \hat{j}_t + (\frac{1}{v_p} - 1)\widehat{h_t^P} - \frac{1}{v_p}\widehat{h_{r,t}^P} \\
\widehat{h_t^P} &= \frac{1}{\Delta_p o}\widehat{h_{o,t}^P} + \frac{1}{\Delta_p r}\widehat{h_{r,t}^P} \\
\bar{h}\hat{h}_t &= \frac{\bar{h^P}}{\Delta_p r A_r}\widehat{h_{r,t}^P} + \frac{\bar{h^I}}{\Delta_i r A_r}\widehat{h_{r,t}^I} + \frac{\bar{h^P}}{\Delta_p o}\widehat{h_{o,t}^P} + \frac{\bar{h^I}}{\Delta_i o}\widehat{h_{o,t}^I}
\end{aligned}
$$

B.5 Flow of funds/ evolution of state variables

$$
\begin{aligned}
\hat{K_t} &= \delta_k \hat{I_t^k} + (1-\delta_k)\hat{K_{t-1}} \\
\hat{h_t} &= \delta_h \hat{I_t^h} + (1-\delta_h)\hat{h_{t-1}} \\
\frac{\bar{b^E}}{\bar{Y}}\hat{b_t^E} &= \frac{\bar{c^E}}{Y}\hat{c_t^E} - \frac{\bar{q^r}A_r\bar{h^e}}{\bar{Y}}(\hat{q_t^r} + \hat{h_t^E}) - \frac{\bar{r^k}\bar{K}}{Y}(\hat{r_t^k} + \hat{K_{t-1}}) + \frac{\bar{h^E}}{Y}(\delta_h\hat{q_t^h} + \hat{h_t^E} - (1-\delta_h)\hat{h_{t-1}^E}) \\
&\quad + \frac{\bar{K}}{\bar{Y}}(\delta_k\hat{q_t^k} + \hat{K_t} - (1-\delta_k)\hat{K_{t-1}}) + \frac{\bar{b^E}\bar{R}}{\bar{Y}}(\hat{R_{t-1}} - \hat{\pi}_t + \hat{b_{t-1}^E}) \\
\frac{\bar{b^I}}{\bar{Y}}\hat{b_t^I} &= \frac{\bar{c^I}}{Y}\hat{c_t^I} - \frac{\bar{q^r}A_r\bar{h^I}}{\bar{Y}\Delta_i r}(\hat{q_t^r} + \hat{h_{r,t}^I}) + \frac{\bar{h^I}}{\bar{Y}\Delta_i o}(\delta_h\hat{q_t^h} + \hat{h_{o,t}^I} - (1-\delta_h)\hat{h_{o,t-1}^I}) \\
&\quad + + \frac{\bar{b^I}\bar{R}}{\bar{Y}}(\hat{R_{t-1}} - \hat{\pi}_t + \hat{b_{t-1}^I}) - s_i(\hat{Y}_t - \hat{X}_t)
\end{aligned}
$$

B.6 Monetary policy rule and shock processes

$$
\begin{aligned}
\hat{R_t} &= (1-r_r)(1+r_\pi)\hat{\pi_{t-1}} + r_y(1-r_r)\hat{Y_{t-1}} + r_r\hat{R_{t-1}} + \epsilon_r \\
\hat{j_t} &= \rho_j\hat{j_{t-1}} + \epsilon_j \\
\hat{u_t} &= \rho_u\hat{u_{t-1}} + \epsilon_u \\
\hat{A_t} &= \rho_A\hat{A_{t-1}} + \epsilon_A
\end{aligned}
$$

Appendix C

Technical derivations

This section provides the derivations of the steady state and log-linearisations for the equations in the paper.

C.1 Patient Households

Patient households gain utility from consumption c_t^P, housing h_t^P, and lose utility from supplying labour to firms, L_t^P. Therefore, their utility maximisation problem can be represented as:

$$\max_{c_t^P, B_t^P, h_t^P, L_t^P} E_0 \sum_{t=0}^{\infty} (\beta^P)^t \left(\log c_t^P + j_t^P \log h_t^P - \frac{(L_t^P)^{\eta^P}}{\eta^P} \right)$$

where β^P is the rate at which patient households discount future utility.

Patient households have the option of consuming housing either through ownership $h_{o,t}^P$ or by renting housing $h_{r,t}^P$ from period to period. Ownership comes at the nominal cost of Q_t^h while rental is available at the nominal cost of Q_t^r. The utility provided from each of these forms of housing is given by the following composite utility function:

$$h_t^P = \left[\omega_P^{\frac{1}{v_P}} (h_{o,t}^P)^{\frac{v_P - 1}{v_P}} + (1 - \omega_P)^{\frac{1}{v_P}} (h_{r,t}^P)^{\frac{v_P - 1}{v_P}} \right]^{\frac{v_P}{v_P - 1}}$$

where v_P measures the elasticity of substitution between rental and home ownership and ω_P measures the share of each type of housing that they consume. Their consumption of housing is also subject to a preference shock j_t^P and depreciation of δ_h per period.

Patient households also face a budget constraint that limits the amount of spending of consumption, housing and deposits to wages earned in the previous period, and deposits returned this period. This can be expressed in real terms as follows:

$$c_t^P + q_t^h(h_{o,t}^P - (1-\delta_h)h_{o,t-1}^P) + q_t^r h_{r,t}^P + \frac{R_{t-1}}{\pi_t} b_{t-1}^P = b_t^P + w_t^P L_t^P + F_t$$

Patient households also face an adjustment cost associated with changing the amount of housing that it uses each period. To reflect the differences in costs associated with home ownership (for example, solicitor's fees and building inspections) we assume the cost is only incurred for changing the amount of home ownership. The adjustment cost function $\xi_t^{h,P}$ is specified as follows:

$$\xi_t^{h,P} = \frac{\phi_o^P q_t^h}{2}\left(\frac{h_{o,t}^P - h_{o,t-1}^P}{h_{o,t-1}^P}\right)^2 h_{o,t-1}^P$$

where ϕ_o^P is the parameter that governs the adjustment costs.

Given this budget constraint and adjustment cost, we can write the maximisation problem as a Lagrangian as follows:

$$\begin{aligned}\mathcal{L} = E_t \sum_{t=0}^{\infty} \Bigg[&(\beta^P)^t \left(\log c_t^P + j_t^P \log h_t^P - \frac{(L_t^P)^{\eta^P}}{\eta^P} \right) \\ &- \lambda_t^P \left(c_t^P + q_t^h(h_{o,t}^P - (1-\delta_h)h_{o,t-1}^P) + q_t^r h_{r,t}^P + \frac{R_{t-1}}{\pi_t} b_{t-1}^P + \xi_t^h - b_t^P - w_t^P L_t^P \right) \Bigg]\end{aligned}$$

Differentiating with respect to the patient households' choice variables (consumption, house ownership, house rental, borrowing and labour supply) we obtain the following first order conditions:

For consumption:

$$\frac{\partial \mathcal{L}}{\partial c_t^P} = 0 \Leftrightarrow \lambda_t^P = \frac{1}{c_t^P}$$

For house ownership:

$$\frac{\partial \mathcal{L}}{\partial h^P_{o,t}} = 0 \Leftrightarrow \frac{1}{c^P_t}\left(q^h_t + \phi^P_o q^h_t \left(\frac{h^P_{o,t} - h^P_{o,t-1}}{h^P_{o,t}}\right)\right) = \frac{j^P_t}{h^P_t}\left(\frac{\omega_P h^P_t}{h^P_{o,t}}\right)^{\frac{1}{v_P}}$$
$$+ E_t\left[\frac{\beta^P}{c^P_{t+1}}(1-\delta_h)\left(q^h_{t+1} + \frac{\phi^P_o}{2} q^h_{t+1}\left(\frac{(h^P_{o,t+1})^2 - (h^P_{o,t})^2}{(h^P_{o,t})^2}\right)\right)\right]$$

For house rental:

$$\frac{\partial \mathcal{L}}{\partial h^P_{r,t}} = 0 \Leftrightarrow \frac{1}{c^P_t}\left(q^r_t + \phi^P_r q^r_t \left(\frac{h^P_{r,t} - h^P_{r,t-1}}{h^P_{r,t}}\right)\right) = \frac{j^P_t}{h^P_t}\left(\frac{(1-\omega_P) h^P_t}{h^P_{r,t}}\right)^{\frac{1}{v_P}}$$

For labour:

$$\frac{\partial \mathcal{L}}{\partial L^P_t} = 0 \Leftrightarrow \frac{w^P_t}{c^P_t} = (L^P_t)^{\eta^P - 1} \equiv w^P_t = (L^P_t)^{\eta^P - 1} c^P_t$$

For borrowing:

$$\frac{\partial \mathcal{L}}{\partial b^P_t} = 0 \Leftrightarrow \lambda^P_t = E_t \lambda^P_{t+1} \beta^P \frac{R_t}{\pi_{t+1}} \Leftrightarrow \frac{1}{c^P_t} = \beta^P E_t\left(\frac{R_t}{c^P_{t+1}\pi_{t+1}}\right)$$

C.1.1 The steady state

From the first order conditions, we can calculate the following steady state relationships. Assuming a steady state of $\pi = 1$ we have from the patient household's first order condition with respect to borrowing:

$$\frac{c^P}{c^P} = \frac{\beta^P R}{\pi} \Leftrightarrow R = \frac{1}{\beta^P}$$

From the patient household's first order condition with respect to housing ownership we have:

$$\frac{q^h}{c^P}(1 - \beta^P(1-\delta_h)) = \frac{j^P}{h^P}\left(\frac{\omega_P h^P}{h^P_o}\right)^{\frac{1}{v_P}}$$

From the patient household's first order condition with respect to housing rental we have:

$$\frac{q^r}{c^P} = \frac{j^P}{h^P}\left(\frac{(1-\omega_P)h^P}{h_r^P}\right)^{\frac{1}{v_P}}$$

Equating these two conditions with respect to consumption we have:

$$\frac{j^P}{h^P}\left(\frac{\omega_P h^P}{h_o^P}\right)^{\frac{1}{v_p}}\frac{1}{q^h(1-\beta^P(1-\delta_h))} = \frac{j^P}{h^P q^r}\left(\frac{1-\omega_p)h^P}{h_r^P}\right)^{\frac{1}{v_P}} \tag{C.1}$$

That is:

$$\frac{h_o^P}{h_r^P} = \left(\frac{q^r}{q^h(1-\beta^P(1-\delta_h))}\right)^{v_P}\frac{\omega_P}{1-\omega_P}$$

From the definition of h_t^P we have:

$$h_t^P = \left[\omega_P^{\frac{1}{v_P}}(h_{o,t}^P)^{\frac{v_P-1}{v_P}} + (1-\omega_P)^{\frac{1}{v_P}}(h_{r,t}^P)^{\frac{v_P-1}{v_P}}\right]^{\frac{v_P}{v_P-1}}$$

Raising both sides to $\frac{v_P-1}{v_P}$ and dividing through by $(h_r^P)^{\frac{v_P-1}{v_P}}$ we have:

$$\begin{aligned}\frac{h^P}{h_r^P} &= \left[\omega_P^{\frac{1}{v_P}}\left(\frac{h_o^P}{h_r^P}\right)^{\frac{v_P-1}{v_P}} + (1-\omega_P)^{\frac{1}{v_P}}\right]^{\frac{v_P}{v_P-1}}\\ &= \left[\omega_P^{\frac{1}{v_P}}\left(\left(\frac{q^r}{q^h(1-\beta^P(1-\delta_h))}\right)^{v_P}\frac{\omega_P}{1-\omega_P}\right)^{\frac{v_P-1}{v_P}} + (1-\omega_P)^{\frac{1}{v_P}}\right]^{\frac{v_P}{v_P-1}}\\ &\equiv \Delta_r^P\end{aligned}$$

We can similarly solve for the ratio of total housing consumption to housing ownership as:

$$\begin{aligned}\frac{\bar{h^P}}{\bar{h_o^P}} &= \left[\omega_p^{1/v_p} + (1-\omega_p)^{1/v_p}\left(\frac{1-\omega_p}{\omega_p}\right)^{\frac{v_p-1}{v_p}}\left(\frac{\bar{q^h}(1-\beta_p(1-\delta_h))}{\bar{q_r}}\right)^{v_p-1}\right]^{\frac{v_p}{v_p-1}}\\ &= \Delta_o^p\end{aligned}$$

Using this condition we can solve for steady state house prices as a function of steady state consumption and housing consumption as follows:

$$\bar{q^h} = \frac{j^P}{\bar{h^P}} \frac{1}{1-\beta^P(1-\delta_h)} (\omega_p \Delta_o^P)^{\frac{1}{v_p}} \bar{c^P} = \xi_3 \frac{\bar{c^P}}{\bar{h^P}}$$

From the budget constraint for patient households we have:

$$c^P + \frac{Rb^P}{\pi} = b^P + w^P L^P + F - q^r h_r^P - q^h \delta_h h_o^P$$

with the steady state for profits from from the final good firm given as $F = \frac{X-1}{X} \Leftrightarrow F = Y\left(1 - \frac{1}{X}\right)$.

Using the first order condition with respect to the intermediate good producers' use of labour from patient households, $w^P L^P = \frac{\alpha(1-\mu)Y}{X}$ the budget constraint can be written as:

$$c^P = b^P\left(1 - \frac{1}{\beta^P}\right) + s^P Y - q^r h_r^P - q^h \delta_h h_o^P$$

where $s^P = \frac{\alpha(1-\mu)+X-1}{X}$.

From the resource constraint for borrowing ($b^P + b^I + b^E = 0$), this can be rewritten as:

$$c^P = s^P Y - (b^E + b^I)\left(1 - \frac{1}{\beta^P}\right) - \frac{q^r \xi_3 c^P}{\Delta_r^P} - \frac{\delta_h \xi_3 c^P}{\Delta_o^P}$$
$$c^P = s^P Y - \left(1 - \frac{1}{\beta^P}\right)\left(\frac{\beta^P m^I (1-\delta_h)\xi_4 c^I}{\Delta_o^I}\right) - \left(1 - \frac{1}{\beta^P}\right)\left(\beta^P m^E (1-\delta_h) q^h h^E\right) - \frac{q^r \xi_3 c^P}{\Delta_r^P} - \frac{\delta_h \xi_3 c^P}{\Delta_o^P}$$

Using the steady state of the entrepreneur's housing rental this becomes:

$$
\begin{aligned}
c^P &= s^P Y + \Big(1-\beta^P\Big)\Big(\frac{m^I(1-\delta_h)\xi_4 c^I}{\Delta^I_o}\Big) + \Big(1-\beta^P\Big)\Big(m^E(1-\delta_h)q^h\frac{h^P_r+h^I_r}{A_r}\Big) - \frac{q^r\xi_3 c^P}{\Delta^P_r} - \frac{\delta_h\xi_3 c^P}{\Delta^P_o} \\
c^P &= s^P Y + \Big(1-\beta^P\Big)\Big(\frac{m^I(1-\delta_h)\xi_4 c^I}{\Delta^I_o}\Big) + \Big(1-\beta^P\Big)\Big(m^E(1-\delta_h)\frac{\xi_3 c^P}{\Delta^P_r A_r}\Big) \\
&\quad + \Big(1-\beta^P\Big)\Big(m^E(1-\delta_h)\frac{\xi_4 c^I}{\Delta^I_r A_r}\Big) - \frac{q^r\xi_3 c^P}{\Delta^P_r} - \frac{\delta_h\xi_3 c^P}{\Delta^P_o} \\
c^P&\Bigg(1-\big(1-\beta^P\big)m^E(1-\delta_h)\frac{\xi_3}{\Delta^P_r A_r} + \frac{q^r\xi_3}{\Delta^P_r} + \frac{\delta_h\xi_3}{\Delta^P_o}\Bigg) \\
&= s^P Y + \Big(1-\beta^P\Big)\Big(\frac{m^I(1-\delta_h)\xi_4 c^I}{\Delta^I_o}\Big) + \Big(1-\beta^P\Big)\Big(m^E(1-\delta_h)\frac{\xi_4 c^I}{\Delta^I_r A_r}\Big)
\end{aligned}
$$

Substituting in the steady state value of c^I written in terms of steady state output this is:

$$
\begin{aligned}
c^P&\Bigg(1-\big(1-\beta^P\big)m^E(1-\delta_h)\frac{\xi_3}{\Delta^P_r A_r} + \frac{q^r\xi_3}{\Delta^P_r} + \frac{\delta_h\xi_3}{\Delta^P_o}\Bigg) \\
&= s^P Y + \big(1-\beta^P\big)\Big(\frac{m^I(1-\delta_h)\xi_4\xi_5 Y}{\Delta^I_o}\Big) + \big(1-\beta^P\big)\Big(m^E(1-\delta_h)\frac{\xi_4\xi_5 Y}{\Delta^I_r A_r}\Big) \\
c^P &= \Bigg(\frac{1}{1-\big(1-\beta^P\big)m^E(1-\delta_h)\frac{\xi_3}{\Delta^P_r A_r} + \frac{q^r\xi_3}{\Delta^P_r} + \frac{\delta_h\xi_3}{\Delta^P_o}}\Bigg) \\
&\quad \Bigg(s^P + \big(1-\beta^P\big)\Big(\frac{m^I(1-\delta_h)\xi_4\xi_5}{\Delta^I_o}\Big) + \big(1-\beta^P\big)\Big(m^E(1-\delta_h)\frac{\xi_4\xi_5}{\Delta^I_r A_r}\Big)\Bigg)Y \\
c^P &= \xi_6 Y
\end{aligned}
$$

C.2 Impatient Households

Similar to patient households, impatient households choose consumption (c^I_t), borrowing (b^I_t), housing (h^I_t), and labour supply (L^I_t)in order to maximise their utility:

$$
\max_{c^I_t b^I_t, h^I_t, L^I_t} E_0 \sum_{t=0}^{\infty} (\beta^I)^t \Bigg(\log c^I_t + j^I_t \log h^I_t - \frac{(L^I_t)^{\eta^I}}{\eta^I}\Bigg)
$$

We assume that the the impatient households have a lower discount factor on future utility than patient households (giving rise to the description 'impatient') so $\beta^I < \beta^P$.

Impatient households have the option of consuming housing either through ownership or by renting housing from period to period. Ownership comes at the nominal cost of Q_t^h while rental is available at the nominal cost of Q_t^r. The utility provided from each of these forms of housing is given by the following composite utility function:

$$h_t^I = \left[\omega_I^{\frac{1}{v_I}}\left(h_{o,t}^I\right)^{\frac{v_I-1}{v_I}} + (1-\omega_I)^{\frac{1}{v_I}}\left(h_{r,t}^I\right)^{\frac{v_I-1}{v_I}}\right]^{\frac{v_I}{v_I-1}}$$

Impatient households earn income in one period through labour supply and borrowing that they use to pay for consumption, housing and to repay previous period's borrowing. Dividing the nominal prices by the price level P_t allows their budget constraint can therefore be summarised as follows:

$$c_t^I + q_t^h(h_{o,t}^I - (1-\delta_h)h_{o,t-1}^I) + q_t^r h_{r,t}^I + \frac{R_{t-1}}{\pi_t}b_{t-1}^I = b_t^I + w_t^I L_t^I$$

where δ_h is the depreciation of houses from one period to the next. Impatient households are also subject to a borrowing constraint that limits the amount they can borrow to a certain proportion (m^I) of the expected value of the housing $h_{o,t}^I$ that they own:

$$R_t b_t^I \leq E_t m^I q_{t+1}^h(1-\delta_h)h_{o,t}^I\pi_{t+1}$$

This means that in addition to the utility that house ownership provides, house ownership also allows the impatient household to borrow more as the collateral constraint is relaxed. As with patient households, impatient households face an adjustment cost associated with changing the amount of house ownership that it uses each period.

$$\xi_t^{h,I} = \frac{\phi_o^I q_t^h}{2}\left(\frac{h_{o,t}^I - h_{o,t-1}^I}{h_{o,t-1}^I}\right)^2 h_{o,t-1}^I$$

By writing the budget constraint in terms of consumption, then substituting this definition into the objective function, this maximisation problem can be written as a Lagrangian as follows:

$$\mathcal{L} = E_t\sum_{t=0}^{\infty}\Bigg[(\beta^I)^t\Bigg(\log\left(b_t^I + w_t^I L_t^I - q_t^h(h_{o,t}^I - (1-\delta_h)h_{o,t-1}^I) - q_t^r h_{r,t}^I - \frac{R_{t-1}}{\pi_t}b_{t-1}^I - \xi_t^{h,I}\right)$$
$$+ j_t^I\log\left(\left[\omega_I^{\frac{1}{v_I}}\left(h_{o,t}^I\right)^{\frac{v_I-1}{v_I}} + (1-\omega_I)^{\frac{1}{v_I}}\left(h_{r,t}^I\right)^{\frac{v_I-1}{v_I}}\right]^{\frac{v_I}{v_I-1}}\right) - \frac{(L_t^I)^{\eta^I}}{\eta^I}\Bigg) - \lambda_t^I\left(R_t b_t^I - E_t(m^I q_{t+1} h_t^I \pi_{t+1}\right)\Bigg]$$

The first order conditions with respect to the impatient household's choice variables can therefore be solved as:

For borrowing:

$$\frac{\partial \mathcal{L}}{\partial b_t^I} = 0 \Leftrightarrow \frac{1}{c_t^I} - \beta^I \frac{1}{c_{t+1}^I} \frac{R_t}{\pi_{t+1}} - \lambda_t^I R_t = 0 \Leftrightarrow \frac{1}{c_t^I} = \beta^I E_t \left(\frac{R_t}{c_{t+1}^I \pi_{t+1}} \right) + \lambda_t^I R_t$$

For housing ownership:

$$\frac{\partial \mathcal{L}}{\partial h_{o,t}^I} = 0 \Leftrightarrow \frac{1}{c_t^I} \left(q_t^h + \phi_o^I q_t^h \left(\frac{h_{o,t}^I - h_{o,t-1}^I}{h_{o,t}^I} \right) \right) = \frac{j_t^I}{h_t^I} \left(\frac{\omega_I h_t^I}{h_{o,t}^I} \right)^{\frac{1}{\upsilon_I}}$$
$$+ E_t \left[\frac{\beta^I}{c_{t+1}^I} (1 - \delta_h) \left(q_{t+1}^h + \frac{\phi_o^I}{2} q_{t+1}^h \left(\frac{(h_{o,t+1}^I)^2 - (h_{o,t}^I)^2}{(h_{o,t}^I)^2} \right) \right) \right] + \lambda_t^I m^I E_t \pi_{t+1} q_{t+1}^h (1 - \delta_h)$$

This means that when the impatient household buys housing $h_{o,t}^I$, the cost is q_t^h plus the adjustment cost associated with changing its level of housing use from the previous period. The benefits of acquiring housing in the current period include any capital gain the impatient household attains on the property, possibly avoiding higher adjustment costs in the future, and the increased collateral value of the underlying housing (that relaxes the borrowing constraint for the impatient household).

For housing rental:

$$\frac{\partial \mathcal{L}}{\partial h_{r,t}^I} = 0 \Leftrightarrow \frac{1}{c_t^I} q_t^r = \frac{j_t^I}{h_t^I} \left(\frac{(1 - \omega_I) h_t^I}{h_{r,t}^I} \right)^{\frac{1}{\upsilon_I}}$$

For labour:

$$\frac{\partial \mathcal{L}}{\partial L_t^I} = 0 \Leftrightarrow \frac{w_t^I}{c_t^I} = (L_t^I)^{\eta^I - 1} \equiv w_t^I = (L_t^I)^{\eta^I - 1} c_t^I$$

C.2.1 The steady state

From the impatient household's first order condition with respect to borrowing:

$$\frac{1}{c^I} = \frac{\beta^I R}{c^I} + \lambda^I R \Leftrightarrow \lambda^I = \frac{\beta^P - \beta^I}{c^I}$$

From the impatient household's first order condition with respect to housing ownership we have:

$$\frac{q^h}{c^I} = \frac{j^I}{h^I}\left(\frac{\omega_I h^I}{h_o^I}\right)^{\frac{1}{v_I}} + \frac{q^h(1-\delta_h)\beta^I}{c^I} + \lambda^I m^I q^h(1-\delta_h)\pi \rightarrow$$
$$\frac{q^h}{c^I} = \frac{j^I}{h^I(1-(1-m^I)(1-\delta_h)\beta^I - \beta^P m^I(1-\delta_h))}\left(\frac{\omega_I h^I}{h_o^I}\right)^{\frac{1}{v_I}}$$

and from the impatient household's first order condition with respect to housing rental we have:

$$\frac{q^r}{c^I} = \frac{j^I}{h^I}\left(\frac{h^I(1-\omega_I)}{h_r^I}\right)^{\frac{1}{v_I}}$$

Note that the above two conditions with respect to impatient household's housing ownership and housing rental can be solved for c^I and equated to each other as follows:

$$\frac{j^I}{q^h h^I(1-(1-m^I)(1-\delta_h)\beta^I - \beta^P m^I(1-\delta_h))}\left(\frac{\omega_I h^I}{h_o^I}\right)^{\frac{1}{v_I}} = \frac{j^I}{q^r h^I}\left(\frac{h^I(1-\omega_I)}{h_r^I}\right)^{\frac{1}{v_I}}$$

Simplifying we obtain:

$$\frac{h_o^I}{h_r^I} = \left[\frac{q^r}{q^h h^I(1-(1-m^I)(1-\delta_h)\beta^I - \beta^P m^I(1-\delta_h))}\right]^{v_I}\frac{\omega_I}{1-\omega_I}$$

From the definition of h_t^I we can calculate the ratio of total housing to housing ownership for impatient households:

$$h_t^I = \left[\omega_I^{\frac{1}{v_I}}(h_{o,t}^I)^{\frac{v_I-1}{v_I}} + (1-\omega_I)^{\frac{1}{v_I}}(h_{r,t}^I)^{\frac{v_I-1}{v_I}}\right]^{\frac{v_I}{v_I-1}}$$
$$\left(\frac{h^I}{h_o^I}\right)^{\frac{v_I-1}{v_I}} = \omega_I^{\frac{1}{v_I}} + (1-\omega_I)^{\frac{1}{v_I}}\left(\frac{h_r^I}{h_o^I}\right)^{\frac{v_I-1}{v_I}}$$
$$\frac{h^I}{h_o^I} = \left[\omega_I^{\frac{1}{v_I}} + (1-\omega_I)^{\frac{1}{v_I}}\left([\frac{q^h h^I(1-(1-m^I)(1-\delta_h)\beta^I - \beta^P m^I(1-\delta_h))}{q^r}]^{v_I}\frac{1-\omega_I}{\omega_I}\right)^{\frac{v_I-1}{v_I}}\right]^{\frac{v_I}{v_I-1}}$$
$$\frac{\bar{h^I}}{\bar{h_o^I}} = \left[\omega_I^{1/v_I} + (1-\omega_I)^{1/v_I}\left(\frac{1-\omega_I}{\omega_I}\right)^{\frac{v_I-1}{v_I}}\left(\frac{\bar{q^h}(1-\beta_I(1-\delta_h)(1-m^I) - \beta_p m^I(1-\delta_h))}{\bar{q_r}}\right)^{v_I-1}\right]^{\frac{v_I}{v_I-1}}$$
$$= \Delta_o^I$$

Similarly we can solve for the ratio of total housing to rental for impatient households as:

$$
\begin{aligned}
h^I_t &= \left[\omega_I^{\frac{1}{v_I}}(h^I_{o,t})^{\frac{v_I-1}{v_I}} + (1-\omega_I)^{\frac{1}{v_I}}(h^I_{r,t})^{\frac{v_I-1}{v_I}}\right]^{\frac{v_I}{v_I-1}} \\
\left(\frac{h^I}{h^I_r}\right)^{\frac{v_I-1}{v_I}} &= \omega_I^{\frac{1}{v_I}}\left(\frac{h^I_o}{h^I_r}\right)^{\frac{v_I-1}{v_I}} + (1-\omega_I)^{\frac{1}{v_I}} \\
\frac{h^I}{h^I_r} &= \left[\omega_I^{\frac{1}{v_I}}\left([\frac{q^r}{q^h h^I(1-(1-m^I)(1-\delta_h)\beta^I - \beta^P m^I(1-\delta_h))}]^{v_I}\frac{\omega_I}{\omega_I-1}\right)^{\frac{v_I-1}{v_I}} + (1-\omega_I)^{\frac{1}{v_I}}\right]^{\frac{v_I}{v_I-1}} \\
\frac{\bar{h^I}}{\bar{h^I_r}} &= \left[\omega_I^{1/v_I}\left(\frac{\omega_I}{1-\omega_I}\right)^{\frac{v_I-1}{v_I}}\left(\frac{\bar{q_r}}{\bar{q^h}(1-\beta_I(1-\delta_h)(1-m^I) - \beta_p m^I(1-\delta_h))}\right)^{v_I-1} + (1-\omega_I)^{1/v_I}\right]^{\frac{v_I}{v_I-1}} \\
&= \Delta^I_r
\end{aligned}
$$

Rearranging the impatient household's first order condition with respect to house ownership we can derive the following equation:

$$
\bar{q^h} = \frac{j^I}{\bar{h^I}}\frac{1}{1-\beta_I(1-m^I(1-\delta_h)) - \beta_p m^I(1-\delta_h)}(\omega_I\Delta^I_o)^{\frac{1}{v_I}}\bar{c^I} = \xi_4\frac{\bar{c^I}}{\bar{h^I}}
$$

We can substitute this into the steady state of the impatient household's borrowing constraint to give:

$$
\begin{aligned}
\bar{b^I} &= \beta^P m^I \bar{q^h}\bar{h^I_o}(1-\delta_h) \\
&= \frac{\beta^P m^I(1-\delta_h)\xi_4\bar{c^I}}{\Delta^I_o}
\end{aligned}
$$

From the intermediate good firm's first order condition with respect to labour supplied by impatient households we have:

$$
\bar{w^I}\bar{L^I} = (1-\alpha)(1-\mu)\frac{\bar{Y}}{\bar{X}} = s^I\bar{Y}
$$

Substituting both this and the steady state borrowing constraint for impatient households into their budget constraint we have:

$$
\begin{aligned}
\bar{c^I} &= \bar{w^I}\bar{L^I} + \bar{b^I} - \frac{\bar{R}\bar{b^I}}{\bar{\pi}} - q^h h_o^I - q^r h_r^I \\
\bar{c^I} &= s^I\bar{Y} - (1-\beta^p)\frac{m^I(1-\delta_h)\xi_4\bar{c^I}}{\Delta_o^I} - q^h\xi_4\frac{\bar{c^I}}{\Delta_o^I} - q^r\xi_4\frac{\bar{c^I}}{\Delta_r^I} \\
\bar{c^I} &= \frac{s^I}{1 + \frac{((1-\beta^p)m^I(1-\delta_h)+\delta_h)\xi_4}{\Delta_o^I} + \frac{q^r\xi_4}{\Delta_r^I}}\bar{Y} = \xi_5\bar{Y}
\end{aligned}
$$

C.3 Investors

Investors aim to maximise their consumption by investing in houses, making houses available for rental for patient and impatient households and by renting capital to the intermediate goods producer. Note that investors are different from impatient and patient households in only having consumption in their utility function. They therefore treat houses as a financial investment for which the return is the rent received in one period and the expected capital gain. This specification likely captures the motives of housing investors in the New Zealand market as distinct from owner-occupiers per the LVR policy.

Houses are transformed into rental accommodation by the investor using the following transformation process:

$$h_t^r = A_r h_{r,t}^E$$

where A_r is a parameter that acts as a proxy for the efficiency of the rental market. This parameter covers things such as risk of vacancy, damage, enforceability of contracts, etc. Rental accommodation is made available to households at the nominal price of Q_t^r (or the real price of $q_t^r = \frac{Q_t^r}{P_t}$). It follows that in order to close the rental market it must be the case that the housing made available for rental by the investor is equal to the total amount of housing rented to the patient and impatient households: $h_t^E = \frac{(h_{r,t}^P + h_{r,t}^I)}{/} A_r$.

Investors choose their level of borrowing, capital, and housing in order to maximise their consumption:

$$\max_{B_t^E, K_t, h_t^E} E_0 \sum_{t=0}^{\infty} \gamma^t \ln c_t^E$$

This maximisation problem is subject to a budget constraint. Investors' costs include consumption, housing, and inflation-adjusted repayment of borrowing from the previous period ($\frac{R_{t-1}}{\pi_t}b^E_{t-1}$). The budget constraint states that these costs must be equal to the entrepreneur's revenue from housing and capital rental in that period (stated in terms of the price of the consumer/investment good) plus any borrowing.

$$b^E_t+q^r_t A_r h^E_t+r^k_t K_{t-1} = c^E_t+q^h_t\Big[(h^E_t-(1-\delta_h)h^E_{t-1})\Big]+q^k_t\Big[(K_t-(1-\delta_k)K_{t-1})\Big]+\frac{R_{t-1}}{\pi_t}b^E_{t-1}+\xi^{h,e}_t$$

where δ_k and δ_h are the depreciation rates of capital and housing respectively. As with patient and impatient households, investors also face adjustment costs for changing the level of housing that they own, which are denoted $\xi^{h,E}_t$. Adjustment costs for housing are given by:

$$\xi^{h,E}_t = \frac{\phi^E_h q^h_t}{2}\left(\frac{h^E_t - h^E_{t-1}}{h^E_{t-1}}\right)^2 h^E_{t-1}$$

where ϕ^E is the parameter that governs the level of the adjustment costs.

Finally, as with impatient households, entrepreneurs are subject to a borrowing constraint that restricts their level of borrowing to a certain proportion of the expected value of their housing the following period. This collateral constraint is specified as follows:

$$R_t b^E_t \leq m^E q^h_{t+1}(1-\delta_h)h^E_t \pi_{t+1}$$

By solving the budget constraint for consumption, we can summarise the utility maximisation problem for investors in the following Lagrangian:

$$\mathcal{L} = E_t\sum_{t=0}^{\infty}\Bigg[(\gamma)^t\Bigg(\log\Bigg(b^E_t+q^r_t A_r h^E_t+r^k_t K_{t-1}+F-q^h_t\Big[(h^E_t-(1-\delta_h)h^E_{t-1})\Big]-q^k_t\Big[(K_t-(1-\delta_k)K_{t-1})\Big]$$
$$-\frac{R_{t-1}}{\pi_t}b^E_{t-1}-\frac{\phi^E q^h_t}{2\delta_h}\left(\frac{h^E_t-h^E_{t-1}}{h^E_{t-1}}\right)^2 h^E_{t-1}\Bigg)-\lambda^E_t\Bigg(R_t b^E_t - E_t(m^E(1-\delta_k)q_{t+1}h^E_t\pi_{t+1}\Bigg)\Bigg]$$

The first order conditions with respect to the entrepreneurs' choice variables are:

For borrowing:

$$\frac{\partial\mathcal{L}}{\partial b^E_t} = 0 \Leftrightarrow \frac{1}{c^E_t} - \gamma\frac{1}{c^E_{t+1}}\frac{R_t}{\pi_{t+1}} - \lambda^E_t R_t = 0 \Leftrightarrow \frac{1}{c^E_t} = \gamma E_t\left(\frac{R_t}{c^E_{t+1}\pi_{t+1}}\right) + \lambda^E_t R_t$$

For housing:

$$\frac{\partial \mathcal{L}}{\partial h_t^E} = 0 \Leftrightarrow \frac{q_t^h}{c_t^E}\left(1 + \frac{\phi^E}{\delta_h}\left(\frac{h_t^E - h_{t-1}^E}{h_{t-1}^E}\right)\right) = \frac{q_t^r A_r}{c_t^E}$$
$$+ E_t\left[\frac{\gamma q_{t+1}^h}{c_{t+1}^E}\left((1-\delta_h) + \frac{\phi^E q_{t+1}^h}{2}\left(\frac{(h_{t+1}^E)^2 - (h_t^E)^2}{(h_t^E)^2}\right)\right)\right] + \lambda_t^E m^E q_{t+1}^h \pi_{t+1}(1-\delta_h)$$

This means that when the investor buys housing h_t^E, the cost is q_t^h plus the adjustment cost associated with changing its level of housing use from the previous period. The benefits of acquiring housing in the current period include the rental value of housing, any capital gain attained on the property, avoidance of higher adjustment costs in the future, and the increased collateral value of the underlying housing that relaxes the borrowing constraint for the investor.

For capital:

$$\frac{\partial \mathcal{L}}{\partial K_t} = 0 \Leftrightarrow \frac{q_t^k}{c_t^E} = E_t\left[\frac{\gamma}{c_{t+1}^E}\left(r_{t+1}^k + q_{t+1}^k(1-\delta_k)\right)\right]$$

C.3.1 The steady state

From the investor's first order condition with respect to borrowing:

$$\frac{1}{c^E} = \frac{\gamma R}{\pi c^E} + \lambda^E R \Leftrightarrow \lambda^E = \frac{1}{c^E}(\beta^P - \gamma)$$

From the investor's first order condition with respect to housing we have:

$$\frac{q^h}{c^E} = \frac{q^r A_r}{c^E} + \frac{\gamma q^h(1-\delta_h)}{c^E} + \lambda^E m^E q^h \pi(1-\delta_h)$$
$$\frac{q^h}{c^E} = \frac{q^r A_r}{c^E} + \frac{\gamma q^h(1-\delta_h)}{c^E} + \frac{(\beta^P - \gamma)}{c^E} m^E q^h(1-\delta_h)$$
$$q^r = \frac{q^h\left(1 - \gamma(1-\delta_h) + (\beta^P - \gamma)m^E(1-\delta_h)\right)}{A_r}$$

Likewise, the steady state for investor's rental with respect to patient and impatient house rental is given by:

$$h_r^E = \frac{h_r^P + h_r^I}{A_r}$$

From the investor's first order condition with respect to capital:

$$
\begin{aligned}
\frac{q^k}{c^E} &= \frac{\gamma}{c^E}\Big(r^k + q^k(1-\delta_k)\Big) \\
r^k &= q^k \frac{\big(1-\gamma(1-\delta_k)\big)}{\gamma}
\end{aligned}
$$

The investor's borrowing constraint has the following steady state:

$$
b^E = \beta^P m^E q^h h^E (1-\delta_h)
$$

This can be combined with the investor's flow of funds constraint to obtain:

$$
\begin{aligned}
c^E &= b^E\Big(1-\frac{1}{\beta^P}\Big) + q^r A_r h^E + r^k K - q^h\delta_h - q^k\delta^k \\
c^E &= q^r A_r h^E + r^k K - q^h\delta_h - q^k\delta^k - (1-\beta^P)m^E q^h h^E(1-\delta_h) \\
c^E &= \Big(q^r A_r - \delta_h - m^E q^h(1-\delta_h)(1-\beta^P)\Big)h^E + \Big(r^k - \delta_k\Big)K
\end{aligned}
$$

Finally the steady state value of housing for investors can be found by using the steady state conditions for housing rental for patient and impatient households:

$$
\begin{aligned}
h^E &= \frac{h_r^P + h_r^I}{A_r} \\
h^E &= \frac{\xi_3 c^P}{\Delta_r^P A_r} + \frac{\xi_4 c^I}{\Delta_r^I A_r} \\
h^E &= \frac{\xi_3\xi_6}{\Delta_r^P A_r} + \frac{\xi_4\xi_5}{\Delta_r^I A_r}
\end{aligned}
$$

C.4 Intermediate Goods Producers

There is a continuum of monopolistically competitive firms that are owned by households that produce the intermediate goods. These intermediate goods producers use labour from patient households, labour from impatient households and capital to produce a differentiated final output, $Y_t(j)$ that they sell to the final goods producer at price $P_t(j)$. The production function for each intermediate goods producer is therefore given by:

$$Y_t(j) = A_t K_t(j)^{\mu} (L_t^P(j))^{\alpha(1-\mu)} (L_t^I(j))^{(1-\alpha)(1-\mu)}$$

where A_t is an AR(1) technology shock.

Each firm faces the following demand function from the final goods producer for their good $Y_t(j)$:

$$Y_t(j) = \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t$$

where $epsilon_p$ is the degree of substitution between the different types of goods in the market.

Therefore each intermediate good producer seeks to minimise the cost of their inputs to production per the following minimisation problem:

$$\min_{K_t(j), L_t^P(j), L_t^I(j)} = w_t^P L_t^P(j) + w_t^I L_t^I(j) + r_t^k K_t(j)$$

which is subject to the following production condition:

$$A_t K_t(j)^{\mu} (L_t^P(j))^{\alpha(1-\mu)} (L_t^I(j))^{(1-\alpha)(1-\mu)} \geq \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t$$

We can therefore set up the Lagrangian multiplier as:

$$\begin{aligned} \mathcal{L} = E_t \sum_{t=0}^{\infty} & -w_t^P L_t^P(j) - w_t^I L_t^I(j) - r_t^k K_t(j) \\ & + RMC_t \Bigg(A_t K_t(j)^{\mu} (L_t^P(j))^{\alpha(1-\mu)} (L_t^I(j))^{(1-\alpha)(1-\mu)} - \left(\frac{P_t(j)}{P_t}\right)^{-\epsilon_p} Y_t \Bigg) \end{aligned}$$

Differentiating with respect to the choice variables of capital, labour from patient households and labour from impatient households we have:

For capital:

$$\begin{aligned} \frac{\partial \mathcal{L}}{\partial K_t(j)} = 0 &\Leftrightarrow -r_t^k + RMC_t A_t \mu K_t(j)^{\mu-1} (L_t^P(j))^{\alpha(1-\mu)} (L_t^I(j))^{(1-\alpha)(1-\mu)} = 0 \\ &\Leftrightarrow r_t^k = \frac{RMC_t \mu Y_t(j)}{K_t(j)} \end{aligned}$$

This has the following steady state (using the steady state relationship derived below that the real marginal cost is simply the inverse of the mark up):

$$\frac{K}{Y} = \frac{\mu}{r^k X}$$

For labour supplied by patient households the first order condition is:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial L_t^P(j)} = 0 \Leftrightarrow -w_t^P + RMC_t\bigg(A_t K_t(j)^{\mu} \alpha(1-\mu)(L_t^P(j))^{\alpha(1-\mu)-1}(L_t^I(j))^{(1-\alpha)(1-\mu)}\bigg) = 0 \\ \Leftrightarrow w_t^P = \frac{RMC_t \alpha(1-\mu)Y_t(j)}{L_t^P(j)}\end{aligned}$$

which has the steady state that:

$$w^P = \frac{\alpha(1-\mu)Y}{L^P X}$$

For labour supplied by impatient households is:

$$\begin{aligned}\frac{\partial \mathcal{L}}{\partial L_t^I(j)} = 0 \Leftrightarrow -w_t^I + RMC_t\bigg(A_t K_t(j)^{\mu} (L_t^P(j))^{\alpha(1-\mu)}(1-\alpha)(1-\mu)(L_t^I(j))^{(1-\alpha)(1-\mu)-1}\bigg) = 0 \\ \Leftrightarrow w_t^I = \frac{RMC_t (1-\alpha)(1-\mu)Y_t(j)}{L_t^I(j)}\end{aligned}$$

which has the steady state that:

$$w^I = \frac{(1-\alpha)(1-\mu)Y}{L^I X}$$

Note that as intermediate firms face the same factor prices, they will hire capital and labour in the same ratios, which will in turn be equal to the aggregate ratios. Therefore we can drop the j that differentiates the different firms in production. The Lagrangian multiplier in this case is the real marginal cost, which is equal to:

$$RMC_t = \frac{(r_t^k)^{\mu}(w_t^I)^{(1-\alpha)(1-\mu)}(w_t^P)^{\alpha(1-\mu)}}{[\mu]^{\mu}[(1-\alpha)(1-\mu)]^{(1-\alpha)(1-\mu)}[\alpha(1-\mu)]^{\alpha(1-\mu)}}$$

The real profit flow to each firm j can be written as the amount of output it produces multiplied by the price it charges, minus the marginal cost per unit multiplied by the amount of output. Therefore, intermediate goods producers seek to maximise:

$$\max_{P_t(j)} \frac{P_t(j)}{P_t} Y_t(j) - RMC_t Y_t(j)$$

We assume that intermediate goods producers are subject to Calvo (1983) pricing where only a fraction $(1-\theta)$ of intermediate goods firms can adjust their prices each period, then there is a probability θ that a firm will be stuck with its previous price for one period. Therefore, the average price level in the economy will be an aggregate of the all prices in the economy:

$$P_t^{1-\epsilon_p} = \theta P_{t-1}^{1-\epsilon_p} + (1-\theta)(P_t^*)^{1-\epsilon_p}$$

where P_{t-1} is the price from the previous period and P_t^* is the average price charged by firms that have their opportunity to reset in that period. The demand function faced by a given firm that has the opportunity to reset its price in a period t is therefore:

$$Y_{t+s}^*(j) = \left(\frac{P_t^*(j)}{P_{t+s}}\right)^{\epsilon_p} Y_{t+s}$$

for any period $s \geq 0$ for which the firm will retain the same price. With this demand function, the profit maximisation condition for a firm can be written as:

$$\max_{P_t(j)} E_t \sum_{s=0}^{\infty} (\beta^P \theta)^s \Delta_{t,s} \left(\frac{P_t^*(j)}{P_{t+s}} Y_{t+s}^*(j) - RMC_t Y_{t+s}^*(j)\right)$$

where $\Delta_{t,s}$ is the patient households stochastic discount factor given by: $\Delta_{t,s} = \frac{u'(c_t^P)}{u'(c_{t+s}^P)}$

Taking the first order condition with respect to the intermediate goods producers price we have:

$$E_t \sum_{s=0}^{\infty} (\beta^P \theta)^s \Delta_{t,s} \left(\frac{Y_{t+s}^*(j)}{P_{t+s}} + \frac{P_t^*(j)}{P_{t+s}} \frac{\partial Y_{t+s}^*(j)}{\partial P_t^*(j)} - RMC_t \frac{\partial Y_{t+s}^*(j)}{\partial P_t^*(j)}\right) = 0$$

That is,

$$
\begin{aligned}
&E_t \sum_{s=0}^{\infty} (\beta^P \theta)^s \Delta_{t,s} \frac{Y_{t+s}^*(j)}{P_{t+s}} \Bigg(1 + \frac{P_t^*(j)}{Y_{t+s}^*(j)} \frac{\partial Y_{t+s}^*(j)}{\partial P_t^*(j)} \\
&- \frac{RMC_{t+s} P_{t+s}}{P_t^*(j)} \frac{P_t^*(j)}{Y_{t+s}^*(j)} \frac{\partial Y_{t+s}^*(j)}{\partial P_t^*(j)} \Bigg) = 0 \\
&E_t \sum_{s=0}^{\infty} (\beta^P \theta)^s \Delta_{t,s} \frac{Y_{t+s}^*(j)}{P_{t+s}} \left(1 - \epsilon_p + \epsilon_p RMC_{t+s} \frac{P_{t+s}}{P_t^*(j)} \right) = 0 \\
&E_t \sum_{s=0}^{\infty} (\beta^P \theta)^s \Delta_{t,s} \frac{Y_{t+s}^*(j)}{P_{t+s}} \left(P_t^*(j) - \frac{\epsilon_p}{\epsilon_p - 1} RMC_{t+s} P_{t+s} \right) = 0
\end{aligned}
$$

This shows that all updating firms in a given period will update to the same price so that $P_t^*(j) = P_t^*$.

Define the steady state mark-up as $X = \frac{\epsilon_p}{\epsilon_p - 1}$ so that the above can be written as:

$$
E_t \sum_{s=0}^{\infty} (\beta^P \theta)^s \Delta_{t,s} Y_{t+s}^* \left(\frac{P_t^*}{P_{t+s}} - XRMC_{t+s} \right) = 0
$$

That is,

$$
E_t \sum_{s=0}^{\infty} (\beta^P \theta)^s \Delta_{t,s} Y_{t+s}^* \frac{P_t^*}{P_{t+s}} = XE_t \sum_{s=0}^{\infty} (\beta^P \theta)^s \Delta_{t,s} Y_{t+s}^* RMC_{t+s} \Big)
$$

Let $MC_{t+s} = RMC_{t+s} P_{t+s}$ so that solving for P_t^* the above expression can be written as:

$$
P_t^* = X \frac{\sum_{k=0}^{\infty} (\beta^P \theta)^s E_t \Big[\Delta_{t,s} Y_{t+s}^* MC_{t+s} P_{t+s}^{-1} \Big]}{\sum_{k=0}^{\infty} (\beta^P \theta)^s E_t \Big[\Delta_{t,s} Y_{t+s}^* P_{t+s}^{-1} \Big]}
$$

Substituting in the definition of Y_{t+s}^{ast} this can be written as:

$$
P_t^* = X \frac{\sum_{s=0}^{\infty} (\beta^P \theta)^s E_t \Delta_{t,s} MC_{t+s} P_{t+s}^{\epsilon - 1} Y_{t+s}}{\sum_{s=0}^{\infty} (\beta^P \theta)^s E_t \Delta_{t,s} P_{t+s}^{\epsilon - 1} Y_{t+s}}
$$

If $\theta = 0$ then the equation for the optimal price reduces to $P_t^* = \frac{\epsilon_p}{\epsilon_p - 1} MC_t$. That is, if firms are able to update their prices every period then the the optimal price would be a fixed mark up $\frac{\epsilon_p}{\epsilon_p - 1}$ over nominal marginal cost. This gives the steady state $P = XMC$.

Rearranging the above expression and dividing by the price level yields:

$$\frac{P_t^*}{P_t}\sum_{s=0}^{\infty}(\beta^P\theta)^s E_t\Big[\Delta_{t,s}Y_{t+s}P_{t+s}^{\epsilon-1}\Big] = \frac{1}{P_t}X\sum_{s=0}^{\infty}(\beta^P\theta)^s E_t\Big[\Delta_{t,s}Y_{t+s}P_{t+s}^{\epsilon-1}MC_{t+s}\Big]$$

Log-linearising the above expression we obtain:

$$\sum_{s=0}^{\infty}(\beta^P\theta)^s E_t\Big[\Delta Y P^{\epsilon-1}\Big](\hat{P}_t^* - \hat{P}_t + \hat{\Delta_{t,s}} + \hat{Y_{t+s}} + (\epsilon-1)\hat{P_{t+s}}) =$$
$$\sum_{s=0}^{\infty}(\beta^P\theta)^s E_t\Big[\Delta Y P^{\epsilon-1}\Big](-\hat{P}_t + \hat{\Delta_{t,s}} + M\hat{C}_{t+s} + \hat{Y_{t+s}} + (\epsilon-1)\hat{P_{t+s}})$$

Now using the log-linearised definition of marginal cost $M\hat{C}_{t+s} = \hat{P_{t+s}} + RM\hat{C}_{t+s}$ we have:

$$\sum_{s=0}^{\infty}(\beta^P\theta)^s(\hat{P}_t^*) = \sum_{s=0}^{\infty}(\beta^P\theta)^s E_t(RM\hat{C}_{t+s} + \hat{P_{t+s}})$$

As $\sum_{s=0}^{\infty}(\beta^P\theta)^s$ is an infinite series, this can be approximated by $(1-\beta^P\theta)$ so that:

$$\hat{P}_t^* = (1-\beta^P\theta)\sum_{s=0}^{\infty}(\beta^P\theta)^s E_t(RM\hat{C}_{t+s} + \hat{P_{t+s}})$$

That is,

$$\hat{P}_t^* = (1-\beta^P\theta)(RM\hat{C}_t + \hat{P}_t) + \beta^P\theta E_t\hat{P_{t+1}^*}$$

Multiplying both sides by $(1-\theta)$:

$$(1-\theta)\hat{P}_t^* = (1-\theta)[(1-\beta^P\theta)(RM\hat{C}_t + \hat{P}_t) + \beta^P\theta E_t\hat{P_{t+1}^*}$$

Substituting in the log-linearised definition of the aggregate price level $((1-\theta)\hat{P}_t^* = \hat{P}_t - \theta\hat{P_{t-1}})$ this is:

$$\begin{aligned}
\hat{P}_t - \theta \hat{P_{t-1}} &= (1-\theta)(1-\beta^P\theta)(R\hat{M}C_t + \hat{P}_t) + \beta^P\theta(E_t\hat{P_{t+1}} - \theta\hat{P}_t) \\
\hat{P}_t - \hat{P_{t-1}} &= -(1-\theta)\hat{P_{t-1}} + (1-\theta)(1-\beta^P\theta)(R\hat{M}C_t + \hat{P}_t) + \beta^P\theta(E_t\hat{P_{t+1}} - \theta\hat{P}_t) \\
\hat{P}_t - \hat{P_{t-1}} &= (1-\theta)(\hat{P}_t - \hat{P_{t-1}}) - \beta^P\theta(1-\theta)\hat{P}_t + (1-\theta)(1-\beta^P\theta)R\hat{M}C_t + \beta^P\theta(E_t\hat{P_{t+1}} - \hat{P}_t) \\
&\quad + \theta\beta^P(1-\theta)\hat{P}_t
\end{aligned}$$

From the log-lineasrised definition of inflation $\hat{\pi}_t = \hat{P}_t - \hat{P_{t-1}}$ this is:

$$\hat{\pi}_t = (1-\theta)\hat{\pi}_t + \theta\beta^P E_t\hat{\pi_{t+1}} + (1-\theta)(1-\beta^P\theta)R\hat{M}C_t\hat{\pi}_t \quad = \beta^P E_t\hat{\pi_{t+1}} + \frac{(1-\theta)(1-\beta^P\theta)}{\theta}R\hat{M}C_t$$

Substituting $\kappa = \frac{(1-\theta)(1-\beta^P\theta)}{\theta}$, this is simply an expectations augmented Phillips curve.

$$\pi_t = \beta^p\hat{\pi_{t+1}} + \kappa R\hat{M}C_t$$

C.5 Capital producers

Capital producers purchase the final good from firms at the price of P_t after final goods production has taken place. They combine this with the undepreciated capital from investors that they purchase at the relative price of q_t^k to produce the capital stock for the next period. This allows production to be described by the following law of motion:

$$K_t = (1-\delta_k)K_{t-1} + Z_t^k I_t^k - \frac{\kappa_k}{2}\left(\frac{I_t^k}{I_{t-1}^k} - 1\right)^2 Z_t^k I_t^k$$

where κ_k is the parameter governing the adjustment costs that apply to changing the level of investment from period to period. Z_t^k is an investment-specific technological change shock which is assumed to be exogenous and follow an AR(1) process.

Following production, the capital producer sells its capital stock back to investors at the price of q_t^k. Therefore, the capital producers' objective function can be written as follows:

$$E_t\sum_{t=0}^{\infty}(\beta^E)^t\left[q_t^k K_t - q_t^k(1-\delta_k)K_{t-1} - P_t I_t^k\right]$$

This is subject to the law of motion for capital production that can be rewritten in terms of investment as follows:

$$K_t - (1-\delta_k)K_{t-1} = Z_t^k I_t^k \left(1 - \frac{\kappa_k}{2}\left(\frac{I_t^k}{I_{t-1}^k} - 1\right)^2\right)$$

Substituting this condition into the above, the profit maximisation problem can be written as follows:

$$\max_{I_t^k} E_t \sum_{s=0}^{\infty} (\beta^P)^s \frac{\lambda_{t+s}^P}{\lambda_t^P}\left[q_{t+s}^k Z_{t+s}^k I_{t+s}^k \left(1 - \frac{\kappa_k}{2}\left(\frac{I_{t+s}^k}{I_{t+s-1}^k} - 1\right)^2\right) - I_{t+s}^k\right]$$

Differentiating with respect to the choice variable of investment we have:

$$\frac{\partial}{\partial I_t^k} = 0 \Leftrightarrow q_t^k Z_t^k - q_t^k Z_t^k \frac{\kappa_k}{2}\left(\frac{I_t^k}{I_{t-1}^k} - 1\right)^2 - q_t^k Z_t^k \kappa_k \left(\frac{I_t^k}{I_{t-1}^k} - 1\right)\frac{I_t^k}{I_{t-1}^k} - 1$$
$$+ \beta^P \frac{\lambda_{t+1}^P}{\lambda_t^P} q_{t+1}^k Z_{t+1}^k \kappa_k \left(\frac{I_{t+1}^k}{I_t^k} - 1\right)\left(\frac{I_{t+1}^k}{I_t^k}\right)^2 = 0$$

Log-linearising this expression we obtain:

$$0 = \bar{q^k}\bar{Z^k}(\hat{q_t^k} + \hat{Z_t^k}) - 2\bar{q^k}\bar{Z^k}\kappa_k\left(\frac{I^k}{I^k}\right)^2 \hat{I_t^k} + 2\bar{q^k}\bar{Z^k}\kappa_k\left(\frac{I^k}{I^k}\right)^3 \hat{I_{t-1}^k} + \bar{q^k}\bar{Z^k}\kappa_k\left(\frac{I^k}{I^k}\right)\hat{I_t^k} - \bar{q^k}\bar{Z^k}\kappa_k\left(\frac{I^k}{I^k}\right)^2 \hat{I_{t-1}^k}$$
$$+3\beta^E\bar{q^k}\bar{Z^k}\kappa_k\left(\frac{I^k}{I^k}\right)^3 \hat{I_{t+1}^k} - 3\beta^E\bar{q^k}\bar{Z^k}\kappa_k\left(\frac{I^k}{I^k}\right)^4 \hat{I_t^k} - 2\beta^E\bar{q^k}\bar{Z^k}\kappa_k\left(\frac{I^k}{I^k}\right)^2 \hat{I_{t+1}^k} + 2\beta^E\bar{q^k}\bar{Z^k}\kappa_k\left(\frac{I^k}{I^k}\right)^3 \hat{I_t^k}$$

This can be simplified to:

$$\hat{I_t^k} = \frac{1}{\kappa_k(1+\beta^P)}(\hat{q_t^k} + \hat{Z_t^k}) + \frac{1}{(1+\beta^P)}\hat{I_{t-1}^k} + \frac{\beta^P}{(1+\beta^P)}\hat{I_{t+1}^k}$$

C.6 Housing Producers

Housing producers function in a similar way to the capital goods producers. At the end of the period they purchase the existing housing stock off the patient households, impatient households and investors at the market price q_t^h. They combine this housing stock with investment the investment good to produce the final stock of housing for that period. Therefore, the evolution of the housing stock can be described by the following law of motion:

$$h_t = (1-\delta_h)h_{t-1} + Z_t^h I_t^h - \frac{\kappa_h}{2}\left(\frac{I_t^h}{I_{t-1}^h} - 1\right)^2 Z_t^h I_t^h$$

where κ_h is the parameter that governs the adjustment costs associated with changing the investment in housing. Z_t^h is an exogenous housing-investment-specific shock that is assumed to follow an AR(1) process.

Following production, the housing producer sells the new housing stock back to the patient households, impatient households and investor at the price of q_t^h. Therefore, analogously to the capital producer, the profit maximisation problem for the housing producer can be written as:

$$\max_{I_t^h} E_t \sum_{t=0}^{\infty} (\beta^E)^t \left[q_t^h \left(Z_t^h I_t^h - \frac{\kappa_h}{2}\left(\frac{I_t^h}{I_{t-1}^h} - 1\right)^2 Z_t^h I_t^h \right) - I_t^h \right]$$

The first order condition with respect to the investment in housing is:

$$\begin{aligned}\frac{\partial}{\partial I_t^h} = 0 \Leftrightarrow q_t^h Z_t^h - q_t^h Z_t^h \frac{\kappa_h}{2}\left(\frac{I_t^h}{I_{t-1}^h} - 1\right)^2 - q_t^h Z_t^h \kappa_h \left(\frac{I_t^h}{I_{t-1}^h} - 1\right)\frac{I_t^h}{I_{t-1}^h} - 1 \\ + \beta^E q_{t+1}^h Z_{t+1}^h \kappa_h \left(\frac{I_{t+1}^h}{I_t^h} - 1\right)\left(\frac{I_{t+1}^h}{I_t^h}\right)^2 = 0\end{aligned}$$

Log-linearising this first order condition we obtain the following equation for housing investment demand:

$$\hat{I_t^h} = \frac{1}{\kappa_h(1+\beta^P)}(\hat{q_t^h} + \hat{Z_t^h}) + \frac{1}{(1+\beta^P)}\hat{I_{t-1}^h} + \frac{\beta^P}{(1+\beta^P)}\hat{I_{t+1}^h}$$

Appendix D

Comparison to Iacoviello (2005)

Figure D.1 shows a comparison of the impulse responses of model proposed in this paper to Iacoviello (2005)'s paper in response to a monetary policy shock. Where there is no analogue to Iacoviello (2005)'s paper, only the responses from the new model are shown. This analysis is useful to show the ways in which the model is consistent with Iacoviello (2005) and to check that the areas where it deviates from Iacoviello (2005)'s basline model can be explained by changes to the model structure or calibration.

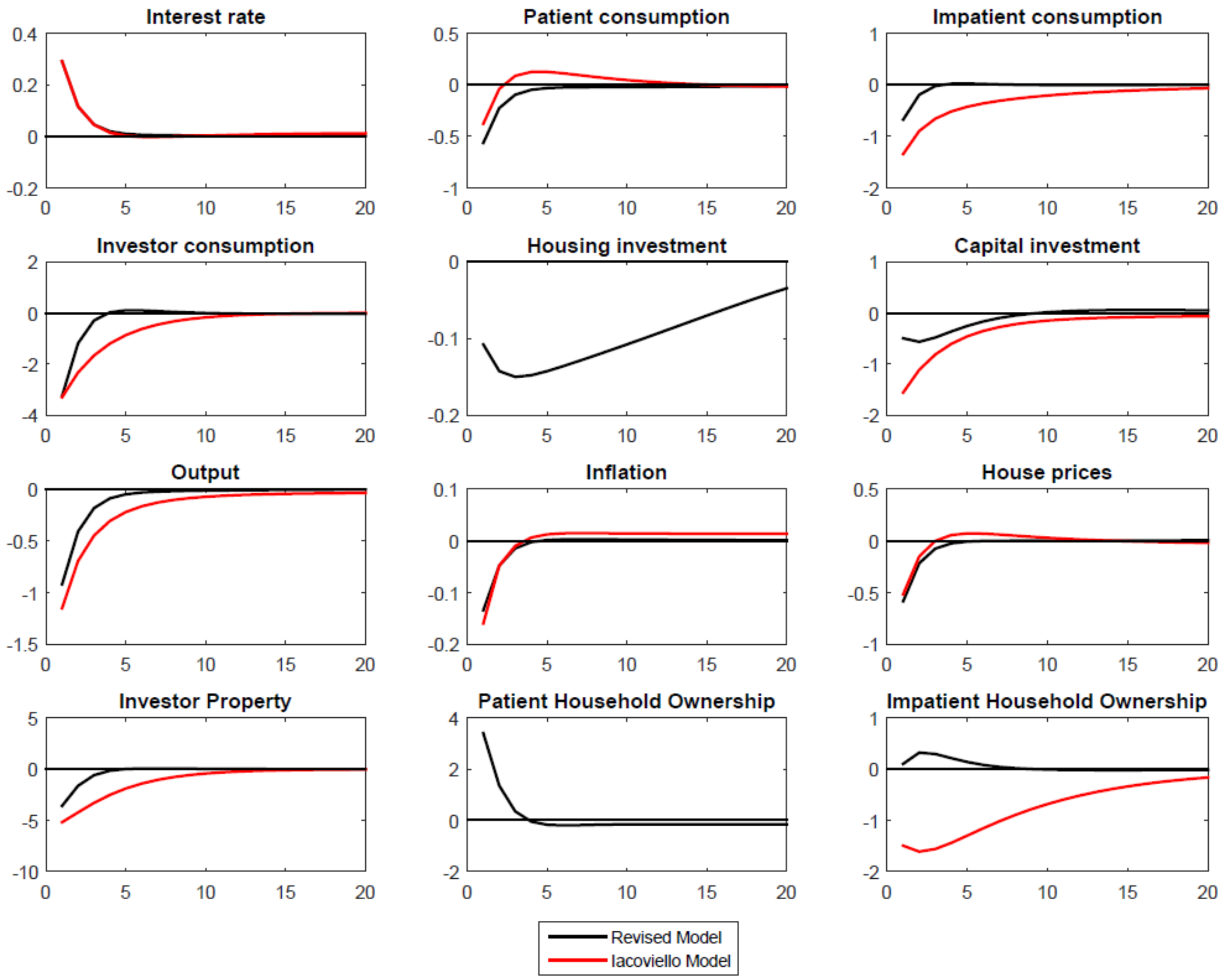
Note that the calibration for the LVR restrictions used in this scenario match those in the Iacoviello (2005)'s paper for more precise comparability. That is, the impatient household LVR restriction is assumed to be very tight at 55% while the investor-specific LVR restriction is very loose at 89%. This means the analysis is most directly comparable to the impatient household LVR scenario outlined in the paper.

The responses are similar in both direction and magnitude to the Iacoviello paper. The responses of inflation and house prices are very similar to the Iacoviello paper, as one would expect given there has been little modification to the components of the model that directly calculate these variables. On the other hand, the responses of investment, housing ownership, consumption and output differ from Iacoviello (2005)'s model.

In particular, the proposed model results in a different distribution of property and consumption between different agents in the economy. As the cost of borrowing increases, investors reduce their demand for housing, which increases the demand from patient households and impatient households, which face a very tight borrowing constraint from the LVR restriction. Unlike Iacoviello (2005)'s model, the housing supply is not fixed, so although housing investment falls in the near term, impatient households and investors are able to retain a higher level of housing than in the baseline model.

Patient households reduce their consumption by more in the near term relative to the Iacoviello (2005) model. Conversely, investors consume more as they sell their housing stock, which buoys demand and encourages more capital investment. Given

Figure D.1: Comparison of responses with Iacoviello (2005) to a monetary policy shock



the increased housing purchased by impatient households, they are able to borrow more in the presence of the LVR restriction, so likewise increase their consumption relative to the baseline model. As there is more demand for consumption in the revised model, output falls by less in the near term than in Iacoviello (2005).

From this analysis, it is clear that the key mechanisms driving the changed behaviour of agents in the revised model is the housing production sector that allows the economy to respond to increased housing demand and the investor's choice between capital and housing investment. Therefore, the model is consistent with Iacoviello (2005)'s with appropriate deviations given the changes to the structure of the model.

Appendix E

Alternative calibrations for the LVR restriction

This section considers the impact of alternative calibrations of the LVR restriction on the transmission of a monetary policy shock through the model. It first shows the impact of different levels of impatient household LVR restrictions varying in increments of 5% from 50% to 95% assuming no investor LVR restriction is present (i.e. a 95% investor LVR is assumed). In a similar vein, the second section considers the impact of different levels of the investor specific LVR restriction assuming that there is no impatient household restriction. Finally, the third section will consider how the two restrictions interact with each other.

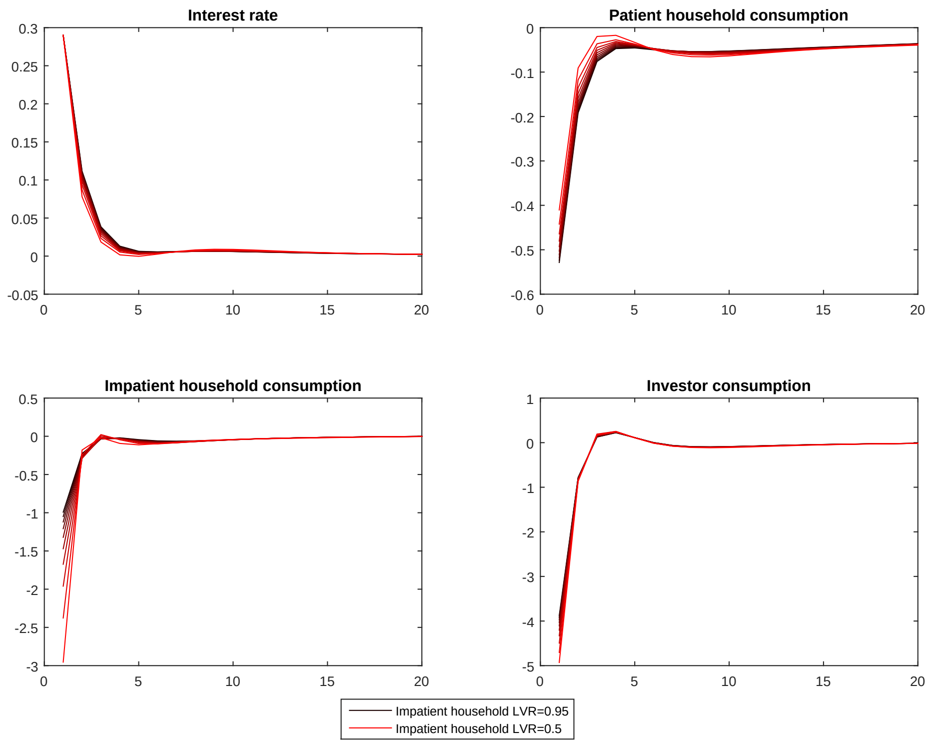
E.1 Impatient household LVR restriction

This section considers the impact of varying the level of the impatient household LVR restriction assuming that there is no investor specific LVR restriction imposed in the model, which is equivalent to assuming that the investor housing restriction is fixed at 95%.

E.1.1 Interest rate and consumption responses

From Figure E.1, it is clear that the primary channel through which the impatient household LVR restriction impacts the model is through impatient household consumption, with consumption varying from -1% in response to the monetary policy response where the impatient household LVR restriction is as its loosest to -3% when the LVR restriction is at its tightest. Investor consumption similarly falls more in the presence of a tight impatient household LVR restriction than it does in the

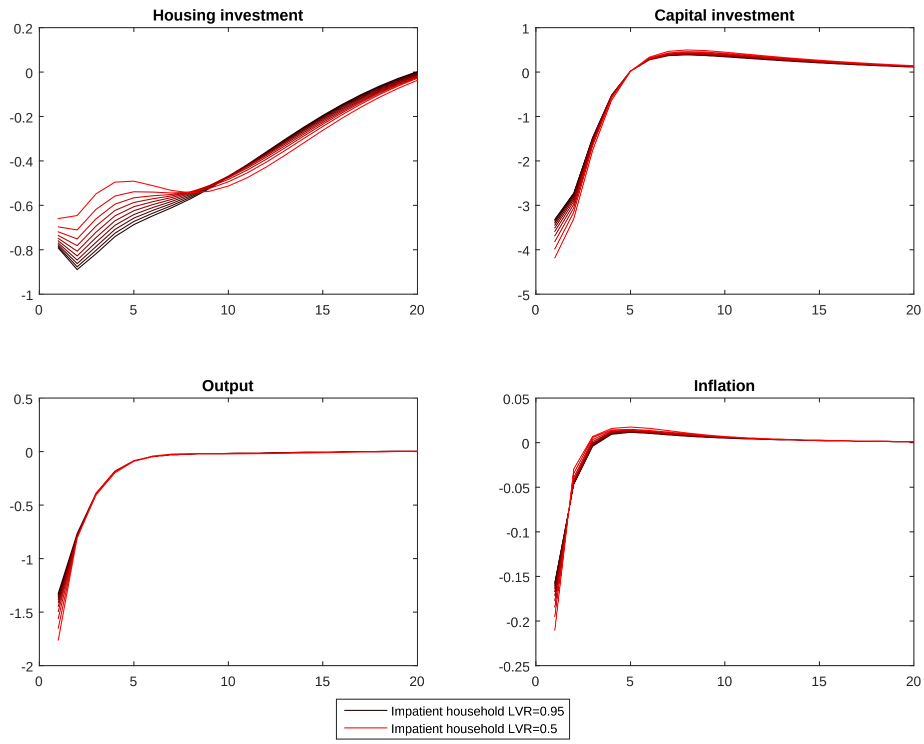
Figure E.1: Responses to a monetary policy shock with different impatient household LVR restrictions



E.1.2 Investment, output and inflation responses

Figure E.2 shows that the fall in impatient household and investor consumption in the presence of a tighter impatient household LVR restriction leads to a greater fall in capital investment and output. However, as impatient households increase their housing with a tighter LVR restriction, housing investment falls by slightly less in the presence of a tighter impatient household LVR. Given these offsetting impacts, the path for inflation is broadly unchanged regardless of the LVR restriction.

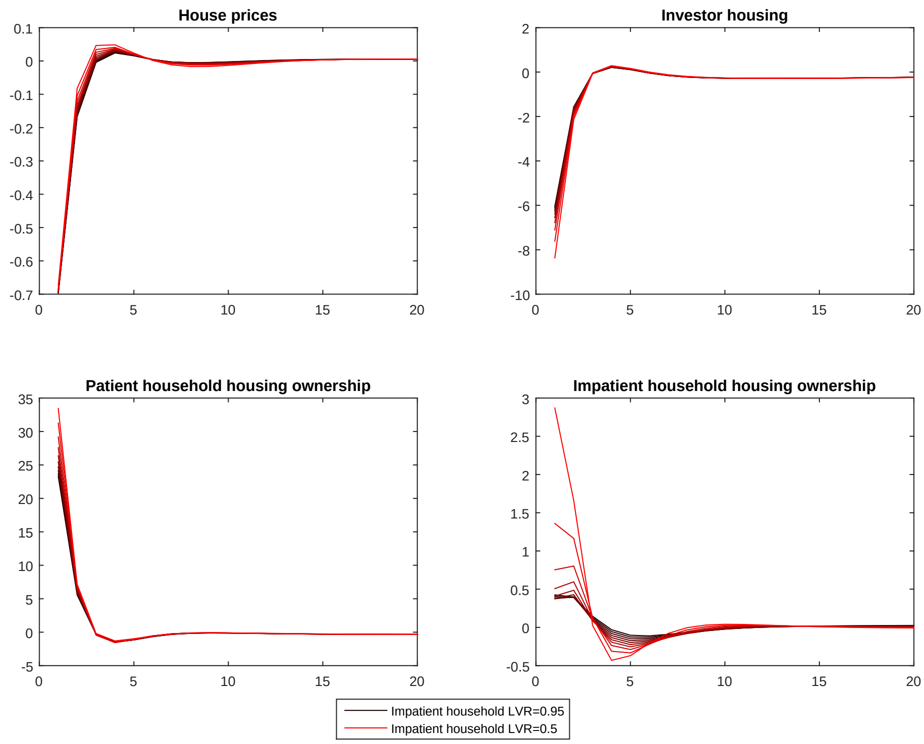
Figure E.2: Responses to a monetary policy shock with different impatient household LVR restrictions



E.1.3 Housing market responses

As noted above, Figure E.3 shows that impatient household housing ownership increases rapidly as the LVR restriction tightens. This allows them to undertake greater borrowing in the current period. Weak demand for capital due to reduced demand for output causes investors to have significantly less income to invest in housing, which leads them to divest their housing stock. The reduced demand from investors causes house prices to fall, leading patient households to likewise increase their housing ownership considerably. With patient and impatient households demanding less rental accommodation, investors further reduce their interests in housing. Consequently, patient and impatient household ownership is higher, and investor ownership significantly reduced, in the presence of an LVR restriction on impatient households.

Figure E.3: Responses to a monetary policy shock with different impatient household LVR restrictions



E.2 Investor LVR restriction

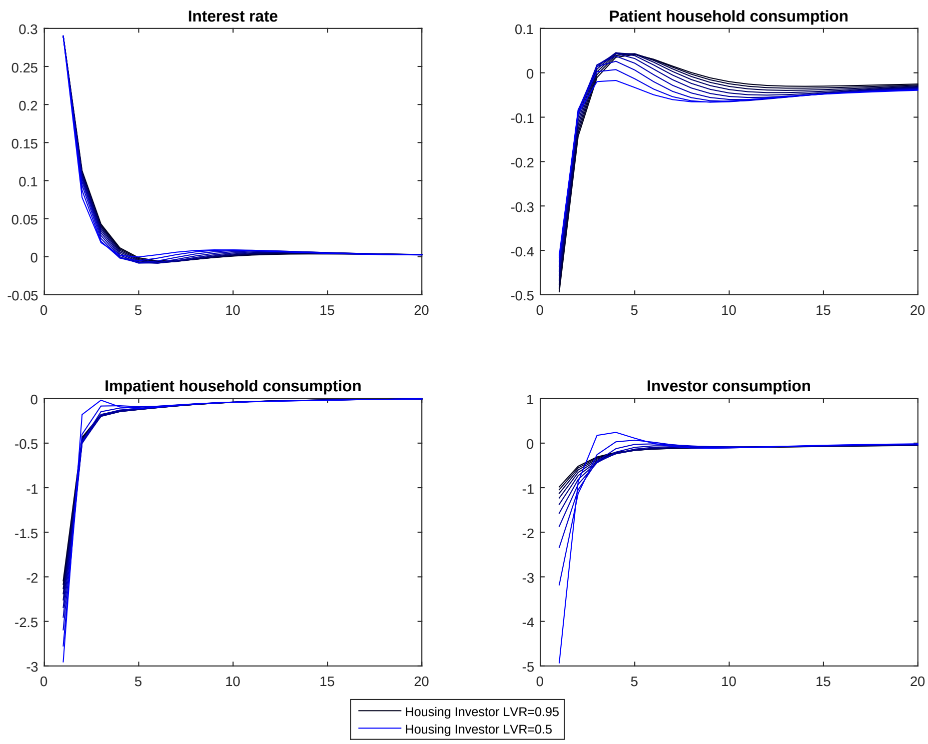
This section considers the impact of varying the level of the investor LVR restriction assuming that there are no impatient household LVR restrictions imposed in the model, which is equivalent to assuming that the impatient household housing restriction is fixed at 95%.

E.2.1 Interest rate and consumption responses

From Figure E.4, it is unsurprising that the level of the investor LVR restriction has little impact on the transmission of the monetary policy shock to the interest rate in the model. The pass-through of the monetary policy shock to consumption is strongly impacted by the level of the LVR restriction, especially for patient households and investors. As the LVR restriction tightens, investors have to reduce consumption by a greater degree in order to rebalance toward investment in housing, which is discussed below. This also impacts the trade off between housing and consumption for patient households, who consume less in the medium term when there is an investor specific

LVR restriction in place.

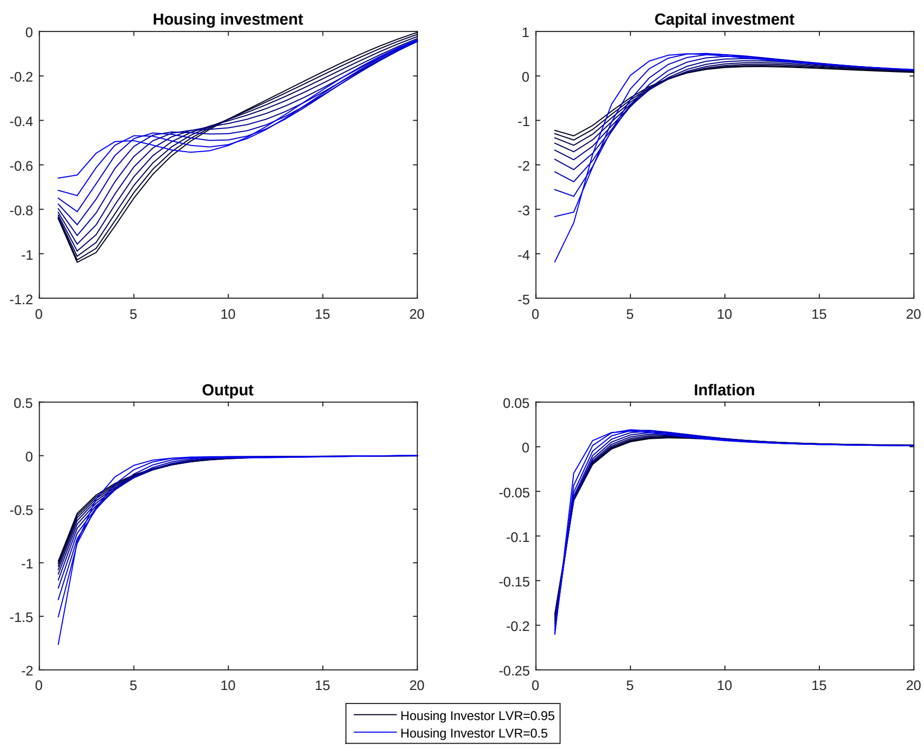
Figure E.4: Responses to a monetary policy shock with different investor LVR restrictions



E.2.2 Investment, output and inflation responses

With agents consuming less with tighter LVR restrictions, Figure E.5 shows that output falls more where there is a tighter LVR restriction. With reduced demand, capital investment also falls by more where there is a tighter LVR restriction. This impact is partially offset by housing investment, which falls by less in the near term in response to the monetary policy shock in the presence of a tighter investor-specific LVR restriction, though it takes longer to return to steady state than in the case of no LVR restriction. The inflation profile is largely unchanged, although inflation does recover slightly faster for tighter levels of the LVR restriction.

Figure E.5: Responses to a monetary policy shock with different investor LVR restrictions

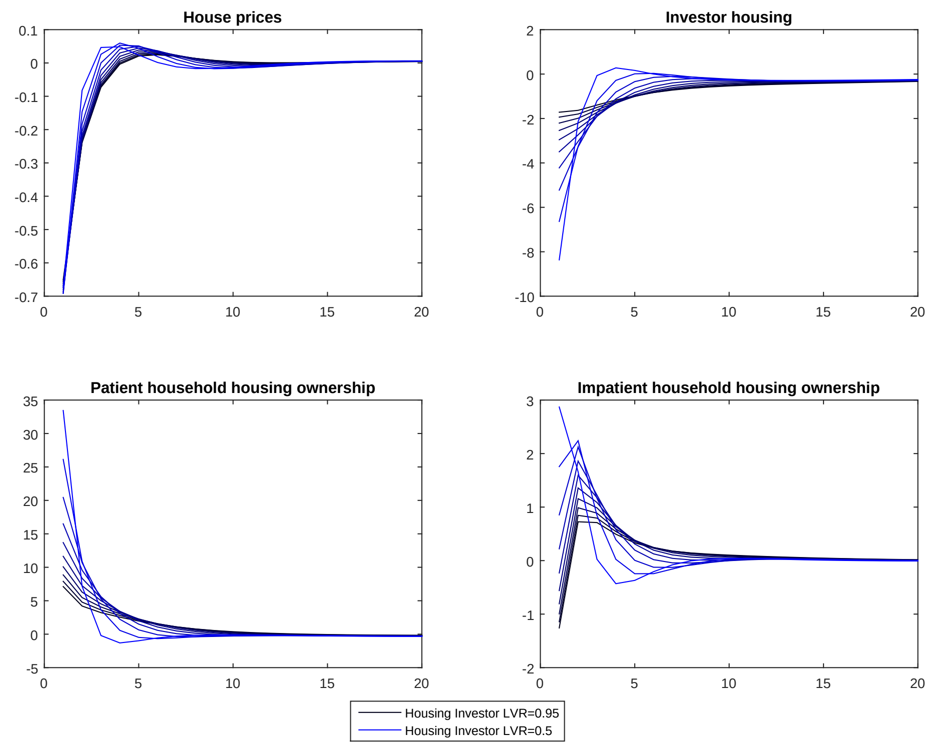


E.2.3 Housing market responses

Figure E.6 shows that patient households, the net savers in the economy, benefit from the increased interest rates as a response of the monetary policy shock so increase their housing ownership. With reduced demand for capital from the intermediate goods producer, the investor's income falls, so it divests some of its housing ownership. Reduced demand from investors causes house prices to fall. With reduced house prices and increased rental costs as a result of the reduced supply of investor housing available for rental, it becomes more favourable for the impatient household to purchase housing, depite the higher interest rates. The initial drop in impatient household owenrship followed by the quick recovery in response to the shock reflects these conflicting dynamics at work.

It is interesting to note the dynamics in the housing market with different levels of LVR restriction. Tighter investor LVR restrictions mean that investor housing is reduced more in the near term as investors have less capacity to borrow, but recovers more quickly as owning houses becomes more valuable due to their ability to be used as collateral. Similarly, patient household housing rises even more in the near term where there is a tight investor LVR restriction, but likewise falls more swiftly to steady state levels as investor housing ownership recovers to steady state levels. Impatient household housing ownership has similar dynamics to patient households, with a swifter rise and then fall in housing ownership in the presence of a tighter LVR restriction.

Figure E.6: Responses to a monetary policy shock with different investor LVR restrictions



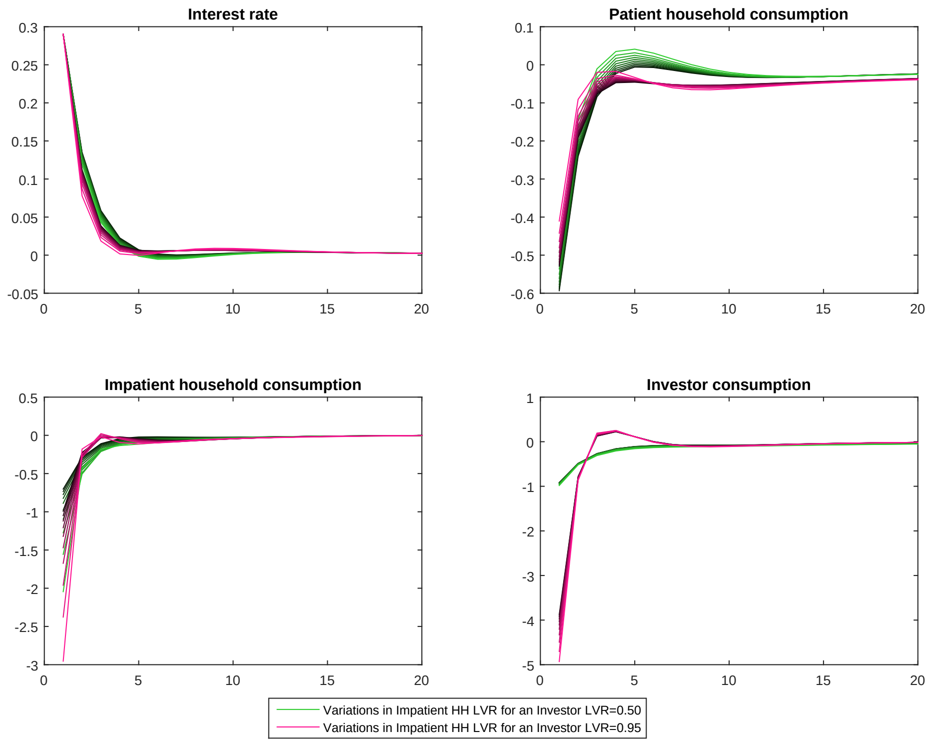
E.3 Both LVR restrictions simultaneously

The next section focusses on the impact of varying both types of LVR restrictions simulataneously to observe how interactions between the two restrictions affects the transmission of the monetary policy shock. For ease of analysis, we will hold one LVR restriction at either its minimum and its maximum level while allowing the other to varying along a spectrum of all plausible LVR restrictions (i.e. 50% to 95%) in increments of 5%. First we will consider the impact of different levels of the impatient household LVR restriction with the investor specific LVR restriction set at 50% and 90%. Next, we will consider the impact of different levels of the investor household LVR restriction with the impatient household restriction set at 50% and 90%.

E.3.1 Responses to variations in the impatient household LVR at fixed values of the investor LVR

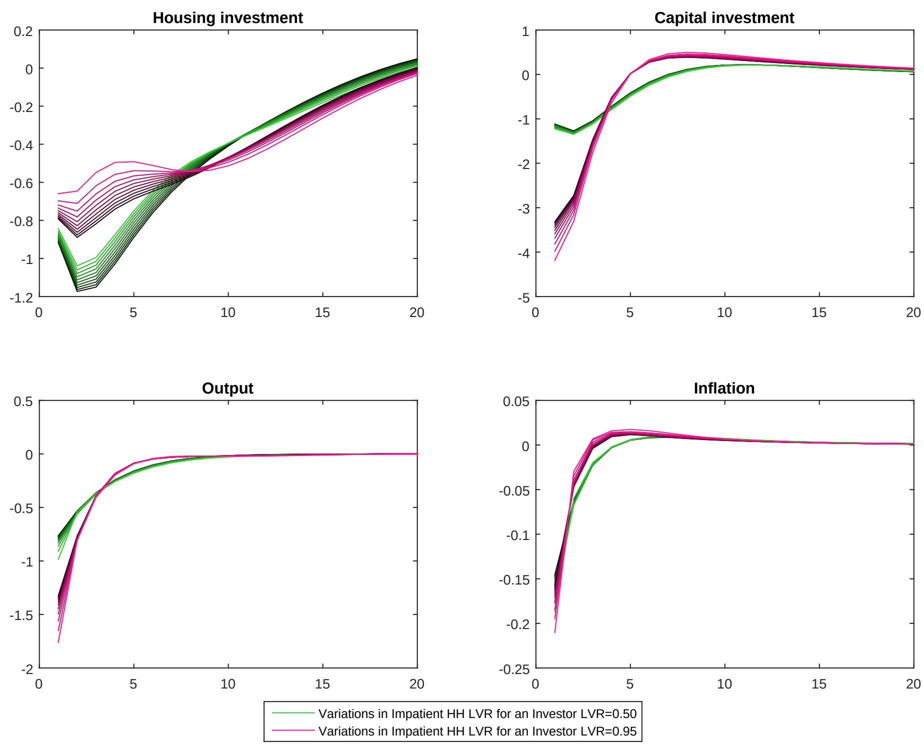
Interest rate and consumption responses

Figure E.7: Responses to a monetary policy shock with different impatient LVR restrictions holding investor restrictions fixed at extremes



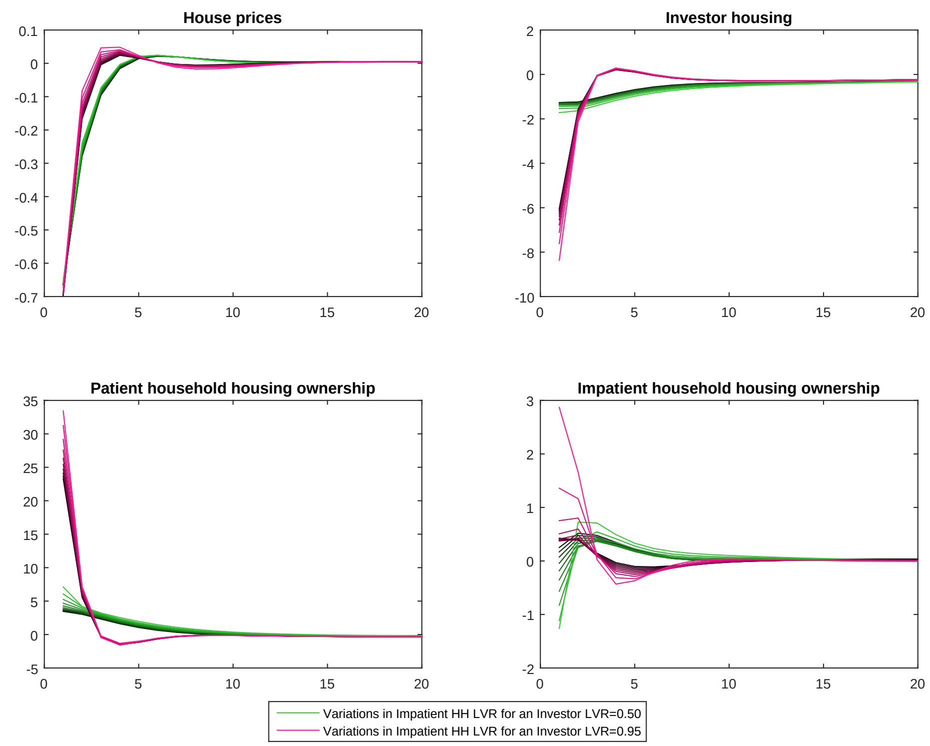
Investment, output and inflation responses

Figure E.8: Responses to a monetary policy shock with different impatient LVR restrictions holding investor restrictions fixed at extremes



Housing market responses

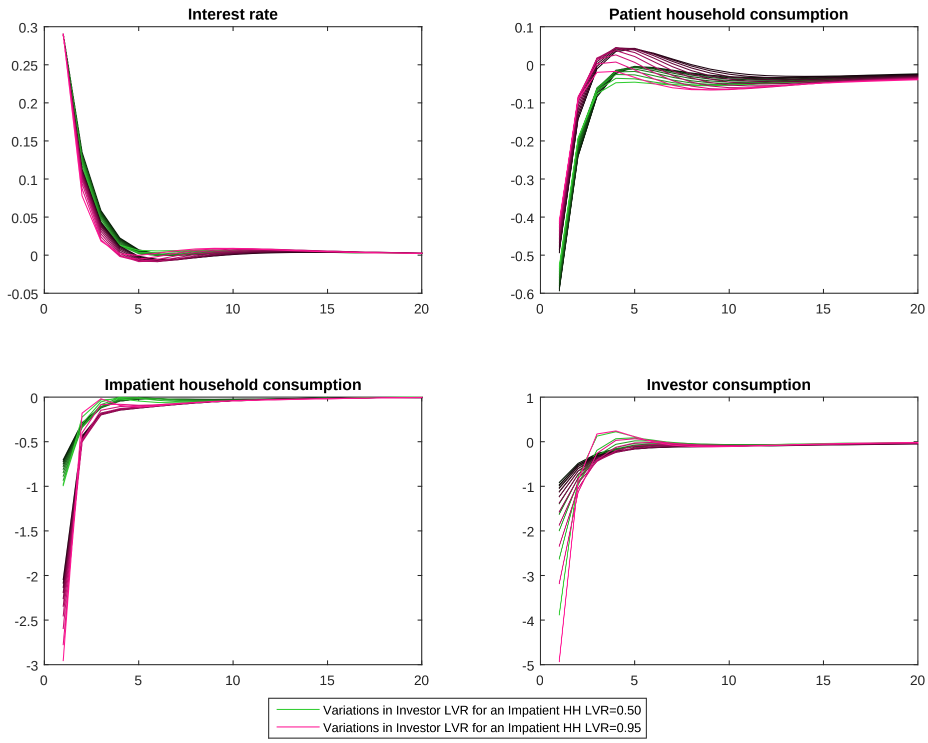
Figure E.9: Responses to a monetary policy shock with different impatient LVR restrictions holding investor restrictions fixed at extremes



E.3.2 Responses to variations in the investor LVR at fixed values of the impatient household LVR

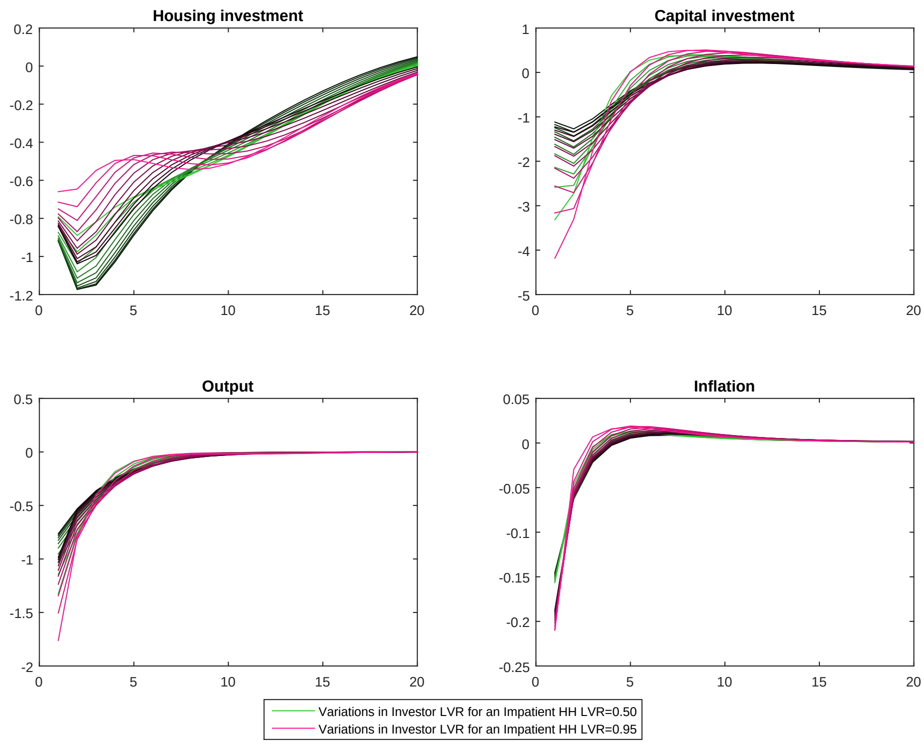
Interest rate and consumption responses

Figure E.10: Responses to a monetary policy shock with different investor LVR restrictions holding impatient household restrictions fixed at extremes



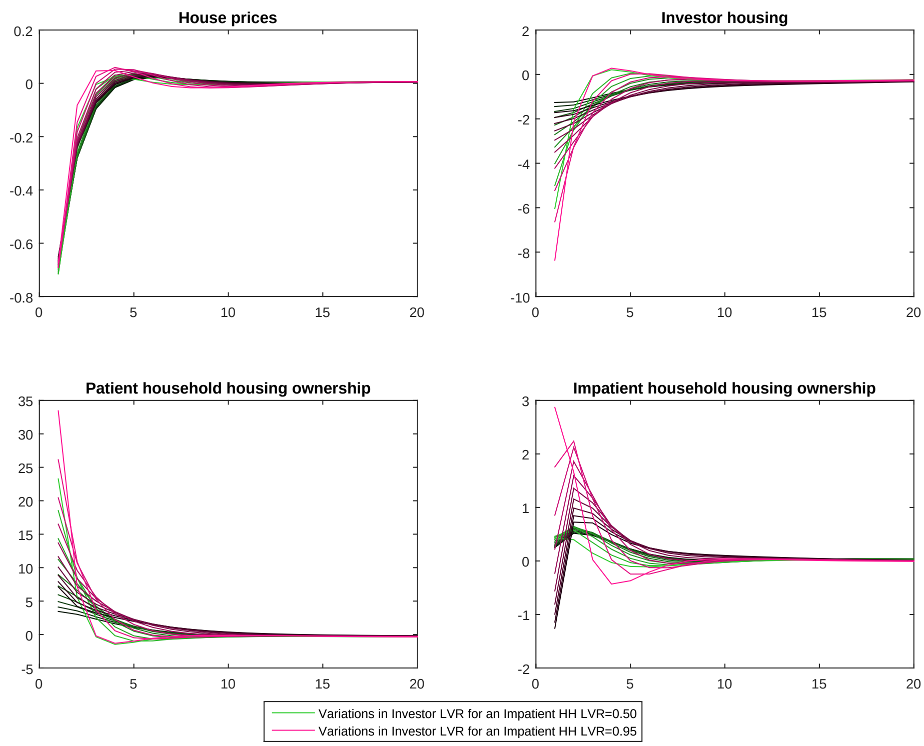
Investment, output and inflation responses

Figure E.11: Responses to a monetary policy shock with different investor LVR restrictions holding impatient household restrictions fixed at extremes



Housing market responses

Figure E.12: Responses to a monetary policy shock with different investor LVR restrictions holding impatient household restrictions fixed at extremes



Appendix F

Summary of key papers in the literature

F.1 The Financial Accelerator Channel

F.1.1 Bernanke and Gertler (1989)

One of the early contributions to this literature was the seminal model of Bernanke and Gertler (1989) that introduced borrowers and lenders into an overlapping generations model. In this model, entrepreneurs produce capital using a combination of their assets and borrowed funds, and only they are able to observe the success of their individual projects. The lenders who they borrow from have to pay monitoring costs to verify whether the entrepreneurs' investment projects have been successful. In the overlapping generations framework, each agent only lives a limited time so the model abstracts from issues of reputation and the consequences of differing rates of capital investment over time.

Households in this model decide how many goods to consume and how much to save. Household savings are lent to the financial agent (lender) at the risk free rate, and are then used by entrepreneurs in investment projects. These investment projects produce capital goods. Uncertainty over the outcome of these investment projects, coupled with monitoring costs, forces entrepreneurs to pay a positive premium to lenders in order to secure funding. This restricts the availability of funding to entrepreneurs to produce capital. This premium is essential to the financial accelerator mechanism, with its procyclical properties creating the strong supply shocks to the rest of the economy.

If households choose to spend rather than save they consume an output good. This good is produced from a combination of capital and labour. If not consumed in one period, this output good is transformed into a capital good according to the investment technology of the entrepreneur.

A positive productivity shock in this model increases the expected growth in the price of capital. The higher price of capital would increase profit expectations for entrepreneurs, who increase their demand for input goods to produce capital and offer an increased returns from investment projects. Higher profits for entrepreneurs increase their wealth and decrease consumers' monitoring costs for entrepreneurs' investments (as entrepreneurs are lower risk due to higher wealth). Lower monitoring costs lower the price for entrepreneurs to borrow from households in turn stimulating further growth of investment and capital production. These secondary round impacts of a productivity shock leading to higher investment have been called the 'financial accelerator' mechanism in subsequent literature.

F.1.2 Carlstrom and Fuerst (1997)

Carlstrom and Fuerst (1997) consider a real business cycle (RBC) model with costly verification of the outcome of the entrepreneurs' investment. They use this model to analyse the optimal lending contract between entrepreneurs and financial intermediaries. This paper finds that net worth in the presence of agency costs causes a delay in the response of variables to different kinds of shocks. For example, in response to a positive technology shock, net worth increases slightly as the increased productivity boots entrepreneurs' wage and rental income. However, as capital is initially fixed, net worth is unable respond instantly. Instead, increased demand for capital in subsequent periods increases the price of capital, driving up the return to internal funds.

This has both direct and indirect effects on entrepreneurs' net worth as the increased return also causes entrepreneurs to reduce their consumption. This gradual increase in investment leads investment to have a hump-shaped impulse response to a technology shock. In turn, this generates hump-shaped responses to consumption (negative after the initial increase), hours worked, and output. Hump-shaped impulse responses are empirically appealing as they better match what is observed in the data (Cogley and Nason (1995)). However, being an RBC model with fully flexible prices, this model did not test the impact of agency costs on the efficacy of monetary policy or other nominal variables.

F.1.3 Bernanke, Gertler and Gilchrist 1999

Bernanke et al. (1999)(hereafter BGG) answered this limitation by incorporating this financial accelerator mechanism into an infinitely lived New Keynesian DSGE model with price rigidities. In this framework, households are net savers that transfer resources to net-borrowing entrepreneurs. Unlike households, entrepreneurs have a probability of surviving until the next period. This captures the dynamics of firms entering and leaving the market and also prevents entrepreneurs from accumulating enough wealth to become self-funding.

Entrepreneurs use their net worth and funds borrowed from households to purchase physical capital that they use to produce intermediate goods. Entrepreneurs accumulate wealth from profits (including capital gains) on capital investments and income from labour that they supply to the market. In this mechanism, the cost of external funding to entrepreneurs depends on the fundamental price of capital and the accumulated net worth of the entrepreneurs. This means that the external finance premium is reduced as an entrepreneur's net worth (and ability to fund capital purchases out of the its own income) increases, and the commensurate agency costs decrease.

Entrepreneurs produce wholesale goods in a competitive market that they sell to retailers who are monopolistically competitive. These retailers repackage these goods at no cost and re-sell them to households. Retailers' ability to differentiate goods and sell them at a mark-up over marginal cost in a monopolistically competitive framework introduces nominal stickiness in prices. These retailers are owned by households so that all profits are rebated lump-sum to households.

The key finding of this paper is that credit market frictions amplify the response of real variables to shocks. Compared to the baseline model with no financial frictions, an unanticipated 25 basis point decline in the nominal interest rate increases output by 50% more than the baseline and almost doubles the effect on investment. Furthermore, these impacts are much more persistent than in the baseline. This difference can be explained by the second round effects the external finance premium generates in the model. Common to the baseline and financial accelerator model, the decline in the interest rate stimulates the demand for capital, increasing the price of capital and investment. In the financial accelerator model, the increased price of capital raises net worth, reducing the external finance premium, which stimulates even further investment.

This has a multiplier (or accelerator) effect in the model as increased investment further buoys asset prices, increasing net worth, thereby reducing the external finance premium and encouraging further investment. Entrepreneurial net worth gradually returns to trend as firms leave the market, with the external finance premium likewise gradually increasing to trend. Nevertheless, it is the persistence in net worth and the external finance premium that causes these additional dynamics in the model.

This paper also shows that a reduction in interest rates leads to greater output persistence than in the baseline model without relying on modifying the elasticity of labour supply (namely, by making it much higher). This is due to the countercyclical movement of the external finance premium flattening the marginal cost curve. The paper therefore argues that introducing credit-market market effects can help explain the observed strength and persistence of an economy's response to monetary policy - a result that replicates VAR evidence reasonably well (citation).

BGG also answered a limitation of the Carlstrom and Fuerst (1997) model by explicitly incorporating the role of asset prices as collateral into a framework with agency costs. In Carlstrom and Fuerst (1997), capital (which is subject to agency costs) is produced from the output good, but the output good itself is produced (combining capital and labour) by firms that do not face agency problems in obtaining external finance. Therefore, changes in net worth affect the economy primarily through changes in the supply of capital (i.e. low net worth leads to reduced production of capital that period). However, in BGG, agency costs are faced by entrepreneurs who produce the intermediate good and also own the capital stock. Therefore, changes in the price of capital directly affect the entrepreneurs' net worth and ability to borrow, which more explicitly incorporates the price asset price effects of Kiyotaki and Moore (1997). It also means that shocks are amplified in BGG relative to Carlstrom and Fuerst (1997).

F.1.4 Aoki, Proudman and Vlieghe (2004)

Aoki et al. (2004) expand on the BGG model to allow for investment in housing. In that framework, households have the split roles of being both homeowners and consumers. Homeowners purchase houses using a combination of their net worth and borrowing from financial intermediaries that they rent to consumers. When borrowing from financial intermediaries, they face an external finance premium caused by information asymmetries, similar to entrpreneurs in BGG. Households in this model also consume goods and housing services, which are financed through the wage earned by supplying labour.

Crucially, the homeowner side of the household is also linked to the consumer side via a transfer that homeowners pay consumers. This captures the trade off that households face when house prices rise: they can either increase the transfer, in which case consumption today increases; or they can keep transfer payments constant, allowing net worth to increase, therefore reducing the future external finance premium. How the household chooses to allocate funds between periods would depend on the elasticity of intertemporal substitution, the sensitivity of the external finance premium to household net worth, and future income uncertainty. Aoki et al. (2004) assume that there is a target level of net worth to debt (i.e. leverage). Transfers depend on the deviation from this target and are increasing in leverage (i.e. as net worth increases relative to a household's level of debt).

Households are further divided into patient and impatient households in this model. Patient households are assumed to have accumulated enough wealth that their consumption can be approximated by the permanent income hypothesis. In contrast, impatient households are subject to borrowing constraints so limit their consumption to their current income each period (that is, their income from labour and from transfers). Impatient households can only borrow when the value of their house increases, giving them increased access to borrowing opportunities.

Similar to the dynamics in the BGG model, an expansionary monetary policy shock causes a rise in housing demand, increasing house prices and homeowners' net worth. This decreases the external finance premium, which increases housing demand and the transfer paid to consumers. This higher transfer supports higher consumption (though the expansion in consumption is muted relative to the increase in housing investment compared to the model without the financial accelerator). Therefore, the existence of credit market frictions in this economy amplifies the shocks in the economy.

In addition to embedding housing in a DSGE model with a financial accelerator, this model also explores the effect of changing access to home equity (i.e. through reduced transaction costs on home equity withdrawal). This is modelled through changing the adjustment parameter on the transfer stream between homeowners and consumers (that is, reducing transaction costs increases the elasticity of the transfer with respect to housing equity). With an increased elasticity of transfer, the response of house prices and housing investment to an expansionary monetary policy shock is dampened, while the impact on consumption is amplified. This shows that when transaction costs are lower, households use the increased housing equity to finance larger amounts of current consumption than in the baseline case. This reduces the impact of their improved balance sheet position on housing investment and house prices.

F.1.5 Christensen and Dib(2008) and Christiano et al (2010)

Christensen and Dib (2008b) develop a model that introduces nominal interest rate contracts into the financial accelerator framework. They also specify the monetary policy rule as an adjusted Taylor rule, where the central bank responds to inflation, output and changes in money growth. With the addition of this type of rule, they conclude that the role of the financial accelerator in amplifying the propagation of business cycles diminishes with the ability of monetary policy to stabilise output. This is consistent with the findings of BGG who further note that the inclusion of the financial accelerator means that smaller countercyclical movements in interest rates are required to dampen fluctuations in output.

Christensen et al. (2011) consider a model with nominal debt contracts alone. Comparing the model with and without the financial accelerator, they note only minor variation in the responses due to the inclusion of the financial accelerator. In particular, they find that the impact of including an external financing premium on variables' responses to demand shocks is negligible and that the fluctuations in investment and output are only minor compared to the model without the financial accelerator. On this basis, they conclude that in the presence of nominal debt contracts, the financial accelerator mechanism can only explain a small part of observed business cycle fluctuations. This is an important finding as the majority of contracts in developed countries as specified in nominal terms. In particular, this paper raises questions about the real world applicability of the magnitude impact of the accelerator mechanism on the propagation of shocks found in earlier work in the financial accelerator literature.

F.1.6 Limitations of the financial accelerator literature

There are two key limitations of the financial accelerator literature outlined above. First, the nature of the external finance premium in these models means that there is no asymmetry in impact of shocks, whereas in the real world we observe credit channels causing much greater downturn than upturn amplification. Second, because the external finance premium faced by borrowers depends entirely on their current net worth in these models, it does not directly take into account expectations of future economic conditions. In particular, expectations of future downturn conditions should further increase the external finance premium and amplify the downturn as borrowers struggle to obtain financing. Moreover, the assumptions of constant length of investment projects and rates of bankruptcy further limit these models' ability to evaluate conditions in a severe economic downturn.

F.2 The Collateral Constraint Channel

DSGE models with a collateral constraint models limit the amount an agent can borrow to a certain percentage of their underlying assets that they can offer as collateral for the loan. Therefore, this methodology limits the availability of funding for borrowers in the model. This can be contrasted with the financial accelerator literature that does not limit the supply of funds to borrowers but uses price to control demand (i.e. the cost of external finance is increasing in the quantity required).

F.2.1 Kiyotaki and Moore (1997)

One of the seminal models in the collateral constraint literature was introduced by Kiyotaki and Moore (1997). In this model, there are both patient and impatient firms, with the latter facing credit constraints on the amount they can borrow to finance expenditure. These credit constraints arise as lenders cannot force borrowers to replay their debts unless the debts are secured by underlying collateral (i.e. durable assets). Therefore, the durable assets in the model play the dual role of serving as factors in production as well as collateral for loans. The dynamic between the credit constraint and asset prices provides a powerful intertemporal mechanism for the propagation of shocks in this model.

Consider a temporary productivity shock that reduces the net worth of all firms in this economy. Assume that some firms have borrowed heavily against their underlying assets and therefore are credit constrained, while other firms in the economy are not.

The credit constrained firms are unable to borrow more in response to the shock, so instead must cut back on their investment expenditure, including their investment in durable assets (that provides collateral). As a result, in the following period they have fewer durable assets (a lower net worth) and earn less revenue as a result of the reduced investment the previous period, causing them to reduce investment again. This impact persists, reducing credit constrained firms' demand for durable goods in subsequent periods.

In order for markets to clear, unconstrained firms must increased their demand for durable goods. In order for this to happen, the user cost of the durable goods must fall (where the user cost in one period is equal to the price of the land that period minus the discounted value of the land in the following period). It follows that the anticipated decline in future user costs causes the value of the durable goods in the current period to fall (as price is equal to the discounted sum of future user costs).

The reduction in the price of durable goods in the current period also has an immediate impact on the value of the constrained firms' collateral. Due to their high net worth, this decline in land value causes credit constrained firms' net worth to decline sharply, causing further tightening of investment expenditure. This causes an intertemporal multiplier effect: the shock causes credit constrained firms' to reduce their demand for durable goods in subsequent periods, reducing the user cost of durable goods for unconstrained firms in those periods, causing a further reduction in the price of durable goods (and the value of collateral) in the initial period, reducing the net worth of constrained firms even further. These persistence and amplification effects reinforce each other showing that even small or short-term shocks to productivity can have prolonged impacts on production, consumption and the price of capital.

F.2.2 Iacoviello (2005)

Like Christensen et al. (2011) for the financial accelerator literature, Iacoviello (2005) was sceptical of the use of real contracts in the collateral constraint literature. To explore the impact of nominal contracts, he includes nominal debt in a DSGE with housing as collateral. In his model, there are patient households that are net lenders, and impatient households and entrepreneurs that are net borrowers. Entrepreneurs produce a homogeneous good (using labour and collateralised real estate) that they sell to retailers. Retailers repackage the good to differentiate it, and re-sell it in a monopolistically competitive market, thereby providing the nominal rigidity in the model. There is also a central bank that follows an interest rate rule.

The transmission of the model is intuitive. Consider a positive demand shock that causes consumer and asset prices to increase. The rising asset prices increase the value of collateral, allowing impatient households and entrepreneurs to spend and invest more. Likewise, the increased consumer prices reduces the real value of outstanding debt in the model, improving borrowers' net worth. This allows borrowers to spend even more, which further buoys demand (as impatient households and entrepreneurs have a higher marginal propensity to consume by construction). Higher demand further increases consumer and asset prices, providing the amplification in the model.

However, for shocks that induce a negative correlation between output and inflation, consumer price inflation works to dampen the impact of these shocks. For example, an adverse supply shock increases inflation, reducing the real value of obligations, thereby improving borrowers' net worth and dampening the adverse impact of the shock. In this way, the model features both a financial accelerator for demand shocks and a financial "decelerator" for demand shocks.

Iacoviello notes that this transmission mechanism allows the model to capture two key features of the data. First, the observed positive relationship between house prices and spending is captured by positive demand shocks amplifying spending through higher collateral values. Second, the use of nominal debt contracts generates hump-shaped impulse responses that better reflect the dynamics of spending in response to an inflation shock.

The paper also explores the impact of explicitly including asset prices in the central bank's policy rule. Interestingly, Iacoviello finds that this has a negligible impact on the variation of output and inflation. However, he does find that the introduction of nominal debt contracts improves the output-variance trade off for the central bank. This is because during a downturn resources are transferred from lenders to borrowers, so that the trade offs in the model are not amplified in a downturn scenario.

F.2.3 Iacoviello and Neri (2010)

Developing the model of Iacoviello (2005), Iacoviello and Neri (2010) adds two features to the housing DSGE literature. First, it incorporates a heterogeneous supply side that allows either business investment and consumption or housing to be produced from capital, labour and land (only used for housing production). This creates a trade-off between business investment and housing in the model that allows for endogenous dynamics between house prices, business investment, and consumption.

On the demand side, the model follows Iacoviello (2005) in allowing housing to be used as collateral for loans. This relaxes the household's budget constraint as house prices rise so that they may increase their expenditure. Likewise, rising house prices increase the profitability of producing new houses, encouraging firms to increase the resources used in housing investment.

This model finds that housing demand shocks and housing technology shocks together explain roughly half of the cyclical variation in housing investment and house prices. Monetary policy shocks contribute between 15-20 percent of the volatility. Based on this result, it is arguable that macroprudential tools may be effective to the extent that they reduce cyclical demand for housing, but nevertheless monetary policy remains a strong driver of cyclical house prices.

Iacoviello and Neri also comments on the impact of spillovers from the housing market (defined as the nominal, real, and financial frictions that alter housing investment) have on the rest of the economy. First, they find that the wage and price rigidities they include in the model more than double the impact of changing housing preferences on GDP (by making housing investment more sensitive to housing demand). Crucially, they find that the collateral constraint on household borrowing amplifies the impact of housing demand and interest rate shocks on consumption. They conclude that dynamics in the housing market can explain 15 percent of consumption growth in the latter half of their sample period.

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