What is General Relativity?

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Abstract. General relativity is a set of physical and geometric principles, which lead to a set of (Einstein) field equations that determine the gravitational field, and to the geodesic equations that describe light propagation and the motion of particles on the background. But open questions remain, including: What is the scale on which matter and geometry are dynamically coupled in the Einstein equations? Are the field equations valid on small and large scales? What is the largest scale on which matter can be coarse grained while following a geodesic of a solution to Einstein's equations? We address these questions. If the field equations are causal evolution equations, whose average on cosmological scales is not an exact solution of the Einstein equations, then some simplifying physical principle is required to explain the statistical homogeneity of the late epoch Universe. Such a principle may have its origin in the dynamical coupling between matter and geometry at the quantum level in the early Universe. This possibility is hinted at by diverse approaches to quantum gravity which find a dynamical reduction to two effective dimensions at high energies on one hand, and by cosmological observations which are beginning to strongly restrict the class of viable inflationary phenomenologies on the other. We suggest that the foundational principles of general relativity will play a central role in reformulating the theory of spacetime structure to meet the challenges of cosmology in the 21st century.

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1. Introduction

It is now one hundred years since Einstein formulated his General Theory of Relativity (or General Relativity or GR for short) in 1915 [1]. This past year there have been many articles written to celebrate this unique achievement. This paper is intended to briefly look at what theory Einstein proposed, what the theory has actually come to mean now to theoretical physicists, and what principles of GR have been abandoned in most attempts to tackle the unsolved challenges of cosmology and quantum gravity.

Public lectures on cosmology often motivate their descriptions of current theories as being built on the wonderful achievements of GR of the last century. However, such comments are misleading when they describe models that dispense with some of the most important innovations of GR, such as the dynamical coupling of matter and geometry. The choices we still typically make can be traced back to the precedents set by Einstein, who first applied GR to the Universe as a whole [2] within less than two years of formulating the theory. Back in 1917, at a time before the expansion of the Universe was known and when the very existence of separate galaxies was still a matter of debate, Einstein wrestled with many of the open foundational questions of his 1915 theory [2]. The choices he made in dealing with those open questions were informed by the observational knowledge and philosophical preconceptions of his day.

On the centenary of relativistic cosmology, it is important to ask to what extent current approaches to the theoretical frontiers of quantum gravity and cosmology still accurately reflect the essence of Einstein's 1915 theory. In light of the enormous developments of both quantum theory and observations of the Universe over the past 100 years, could we approach the open challenges in fresh ways while remaining true to the foundational principles of GR?

In particular, GR revolutionized our picture of spacetime, changing it from a fixed stage on which dynamics is played out to a relational structure in which the dynamics of geometry and matter are inextricably linked. As Wheeler famously stated: *"Matter tells space how to curve, and space tells matter how to move."* Yet standard cosmology as currently practised could be better summarized as: *"Friedmann tells space how to curve, and Newton tells matter how to move."* The standard Λ Cold Dark Matter (ΛCDM) cosmology assumes an average Friedmann-Lemaître-Robertson-Walker (FLRW) evolution in which space expands to maintain constant average spatial curvature, while the growth of structure in the regime when perturbations become non-linear is treated by Newtonian N–body numerical simulations‡.

Although the ΛCDM cosmology is extremely successful (up to various possible anomalies and tensions [4]), it requires sources of energy density—dark energy and non-baryonic CDM—that dominate the present epoch Universe, while never having

‡ Full GR computational cosmology is an immense technical challenge that has hitherto been simply too daunting. Nonetheless, in the past year the first investigations have begun [3], opening what we believe will become a major frontier for GR for decades to come.

been directly detected. Rather than abandoning the 100–year old innovations of GR, should we be thinking harder about its central principles in order to tackle these cosmic mysteries?

In many ways GR is not fully utilized in cosmology and modern physical theories that grapple with questions of the unification of gravity and gauge interactions, the very early Universe, dark matter and dark energy. It is perhaps true that certain approaches are more in the spirit of GR: e.g., loop quantum gravity (LQG) [5] and backreaction in cosmology [6]. However, it is possible that when facing fundamental puzzles at the interface of quantum gravity and cosmology in the very early Universe, the foundational principles of GR may resurface in any future redevelopments that are ultimately successful.

In this article we suggest that GR as Einstein originally envisaged is not a finished theory. As we point out in the final section, the latest cosmological observations and many independent lines of investigation in quantum gravity point to a need to rethink our ideas about the nature and origin of spacetime structure in ways which demand as much conceptual creativity and rigorous mathematical innovation as Einstein himself first applied 100 years ago.

2. What is General Relativity?

General relativity is based on a set of physical and geometrical principles:

- 1. Spacetime structure.
- 2. Equivalence principle.
- 3. Local causality.
- 4. Preferred local coordinate frames.

A very concise introduction to these topics for non-experts is given in the Appendix.§

In item 1 the structure includes the fact that the spacetime is a 4-dimensional differentiable manifold of Lorentzian signature and that the field equations are generally covariant (tensorial) The Strong Equivalence Principle leads to the postulate that spacetime is endowed with a metric and a pseudo-Riemannian structure. Other physical principles include notions of local energy–momentum conservation and an appropriate Newtonian limit. These together lead to:

I. The Einstein field equations.

Three aspects of this within GR are:

- Ia. The field equations are hyperbolic partial differential equations, and do not restrict the topology of spacetime a priori.

§ For a somewhat longer non-technical introduction to GR see, e.g., ref. [7].

- Ib. The geometry and matter are dynamical.
- Ic. Energy is not conserved in general. Instead the energy–momentum tensor is covariantly conserved, dynamically mixing the energy of matter and geometry.

In item 3 local causality includes the fact that GR subsumes special relativity (SR), in which local freely falling reference frames play an important role, and the fact that gravitational waves travel at the speed of light. It is then assumed that freely falling test particles follow timelike geodesics and light follows null geodesics of the spacetime geometry. This then leads to:

II. Particles and photons follow geodesics.

One very important aspect is: *what are the scales on which GR is valid?*. Clearly, GR and the field equations are certainly not valid on small scales when quantum effects must be taken into consideration. To obtain the classical theory of gravity, in principle one must then average the theory of Quantum Gravity (QG), and either obtain GR or an alternative. This leads to generalized classical field equations and necessarily a violation of Ia (loop quantum gravity) or a violation of Ib (string theory).

Perhaps of more interest here is whether GR is valid on large (cosmological) scales. In a similar way, we must average GR at the classical level to obtain a coarse grained theory on large (cosmological) scales. Averaging or coarse graining may well lead to a generalization of I (modified field equations) but also perhaps to II also. Most generalizations of GR are simple amendments of the field equations, but within the same mathematical framework and with similar physical principles. But this is not always the case with averaging; although we do expect corrected/modified field equations, in general the averaged geometry is not necessarily Riemannian (or even metric) and we must also average geodesics (which affect the interpretation of motions), which may lead to changes in the fundamental postulates of the theory.

3. Small scales: Quantum theory

Unlike in GR, time and space are not on an equal footing in quantum mechanics (time is treated classically whereas space is associated with a quantum description). Many modern theories of quantum gravity (QG) do not respect Einstein's revolutionary way of interpreting gravitational physics geometrically [8, 9]. Most approaches to QG are based on a covariant Lorentzian action, which entails an integral over the manifold plus an integral over the boundary. This is a global object and is only well defined when the topology is fixed. Unlike most field theories, in GR [8] the field variable is the spacetime metric, $g_{\mu\nu}$, defined on a four-dimensional (4D) manifold, $\mathcal{M}$. In this case, the natural volume element in the integrals itself depends on the field variables $g_{\mu\nu}$, and hence its variation must be taken into account when calculating functional derivatives. In the canonical approach a family of spacelike surfaces is introduced and used to construct a Hamiltonian.

To solve for the correct local evolution (the field equations or equations of motion) from the action, we need to know the topology, appropriate boundary conditions and (in an open manifold) the conditions at infinity [8]. Therefore, there needs to exist a preferred (global) timelike vector, and hence a global topology $\mathbb{R} \times \mathcal{M}^3$, for it to make sense. Regarding the boundary conditions, the action certainly makes more sense in a closed universe (i.e., a compact S^3). The surface integral is more complicated in open universes, in which boundary terms enter in a more fundamental way (and it is not known in general what these terms should be). Therefore, there are problems with boundary conditions at infinity for an open manifold: we need to know where infinity is (definition), and conditions at infinity (which might be timelike or null). In an open or closed universe we need to add surface terms on a case-by-case basis (e.g. different for each type of spacetime).

Therefore, in order to do canonical quantization, especially in the Hamiltonian formulation of QG, we need to know the topology, appropriate boundary conditions and (in an open manifold) the conditions at infinity [10]. In the *canonical* approach, the decomposition into three spatial dimensions and one time dimension seems to be contrary to the whole spirit of GR [8]. This is a valid concern, but successful background independent approaches to quantum GR [11] (such as LQG and causal dynamical triangulations) accept this price.

In GR, there is no background geometry. The space-time metric itself is the fundamental dynamical variable. In the *covariant* approach the emphasis is on field-theoretic techniques, and does not necessarily involve a $1 + 3$ decomposition of space-time but it is background dependent. In *covariant* approaches to QG, such as string theory [12], the spacetime metric is split into a kinematical background and dynamical fluctuations. The first step is to split the space-time metric $g_{\mu\nu}$ in two parts, $g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}$, where $\eta_{\mu\nu}$ is the background metric, often chosen to be flat, and it is only $h_{\mu\nu}$ that is quantized. That is, it is assumed that the underlying spacetime can be taken to be a continuum, endowed with a smooth background geometry, and the quantum gravitational field can be treated as any other quantum field evolving on this fixed background.

String theory is not restricted to perturbing about flat spacetime, however. The key advances from the 1990s onward have involved non-perturbative results which extend the scope of string theory to include additional extended objects such as D-branes, dualities between different higher-dimensional spacetime vacua in particular limits, and the anti-de Sitter space/conformal field theory (adS/CFT) correspondence. This led to a change from treating the spacetime metric as coupling constants in the quantum field theory of the 2-dimensional string world sheet to its reconstruction from a holographic, dual theory [9]. In either viewpoint, the spacetime metric has a geometric interpretation which is far more complicated than in classical GR.

In many theories of fundamental physics there are geometric classical corrections to GR. A Lorentzian spacetime with global topology $\mathbb{R} \times \mathcal{M}^3$ is completely classified by its set of scalar polynomial curvature invariants [13]. Thus, in the canonical approach to QG all gravitational degrees of freedom are curvature invariants. However, a Lorentzian degenerate Kundt spacetime‖ is not completely classified by its set of scalar polynomial curvature invariants [13]. It is perhaps within string theory that the full richness of Lorentzian geometry is realized, where the Kundt spacetimes may play a fundamental role [15].

Assuming the existence of two levels – microscopic and bulk¶ – of understanding classical physical phenomena, Lorentz formulated a microscopic theory of electromagnetism and showed Maxwell's theory to be its bulk version [16]. A space averaging is always necessary and unavoidable in all settings that deal with bulk matter fields. However, in electrodynamics the field operator is linear in the fields and it can be easily averaged, and models of continuous electromagnetic media which relate to the structure of averaged (bulk) fields can be constructed. So in the same sense that (linear) QED is averaged to obtain Maxwell equations, QG should be averaged to get GR. On larger scales, however, the results of coarse graining or averaging the geometry are expected to be far from trivial, since the Einstein field equations are highly non-linear. We shall return to this in the next Section.

4. Large scales: Cosmology

When one considers the whole Universe, then the open questions about spacetime topology and boundary conditions become inescapable. Furthermore, if the field equations are to be viewed as dynamical evolution equations that couple matter and geometry then the split of space and time in any metric decomposition also becomes a question with physical as well as technical mathematical implications.

In 1917 Einstein opted for a universe with a $\mathbb{R} \times S^3$ closed spatial topology to avoid the problem of unknown boundary conditions at spatial infinity [2]. Famously, Einstein had to introduce a finely tuned cosmological constant, Λ, in order to try to keep the universe static, when his equations were trying to tell him that a key feature of general relativity is that its solutions are dynamical.

In Einstein's day the idea that we could live in a universe where time could have a beginning was simply disregarded on philosophical grounds. In an eternally existing universe new information can always be entering our light cone from infinitely far away,

‖ A Lorentzian manifold admitting an indecomposable but non-irreducible holonomy representation, (i.e., with a one-dimensional invariant lightlike subspace) is a degenerate Kundt (degenerately reducible) spacetime [14], which contains the VSI and (non locally homogeneous) CSI subclasses (in which all of the scalar invariants are zero or constant, respectively) as special cases [13].

¶ The word "macroscopic" would be used here in particle physics. However, in the averaging problem in cosmology that we discuss in the next section there are so many different scales of averaging that such terminology becomes problematic. Thus we use the term bulk fields to refer to spatial averages of field theories involving purely non-gravitational physics.

posing a basic problem that completely changes its character if the universe had a beginning. In the latter case the universe is divided into observable and unobservable portions, and the question of spatial topology then becomes inextricably linked with initial conditions and quantum gravity (if one makes the reasonable assumption that quantum gravity is important in the very early universe).

The other foundational question about the dynamical coupling of matter and geometry—the fitting problem [17] and the scales of applicability of the field equations—did not stand out as a question in 1917. Given the prevailing view that nebulae were not separate galaxies at vast distances, it was reasonable to assume that the Universe existed as a material continuum consisting of stars with the density of the Milky Way in all directions. The idea that the energy–momentum tensor was described as dust coarse grained as stars, and statistically homogeneous, was observationally justified.

One hundred years on, our observations of the late epoch Universe reveal a significantly more complex picture, however. Stars and black holes form galaxies of a wide range of sizes, while groups and clusters of galaxies form the largest gravitationally bound structures. These structures themselves form knots, filaments and sheets that thread and surround very underdense voids, creating a vast cosmic web [18]. Some 40% of the volume of the present Universe is in voids of just one characteristic diameter [19], $\sim 30\,h^{-1}$Mpc, and density contrast $\delta_\rho = (\rho - \bar{\rho})/\bar{\rho} < -0.94$ which are close to being empty ($\delta_\rho = -1$). Once the distribution of all voids is accounted for, then by volume the present universe is void–dominated [20].

The *fitting problem* [17] – namely how does one coarse grain matter and geometry on a given scale to fit it into an effective geometry on larger scales – is perhaps the most important unsolved problem$^+$ in mathematical cosmology. The observed complex lumpy universe demands a hierarchy of steps in coarse graining [22], which can be depicted as:

$$\left.\begin{array}{r} g_{\mu\nu}^{\rm stellar} \to g_{\mu\nu}^{\rm galaxy} \to g_{\mu\nu}^{\rm cluster} \to g_{\mu\nu}^{\rm wall} \\ \vdots \\ g_{\mu\nu}^{\rm void} \end{array}\right\} \to g_{\mu\nu}^{\rm universe} \qquad (1)$$

if we assume a metric description. At one extreme we have discrete particles, ranging in size from isolated electrons and protons that fill the vastness of voids, where bound structures never formed, to stars and supermassive black holes that are the basic building blocks of more complex gravitationally bound systems. In the standard FLRW cosmology it is implicitly assumed that regardless of the gravitational physics in the coarse graining hierarchy (1), at the final step* the matter distribution can be approximated by an "effective averaged out" stress-energy tensor. Moreover, the

$^+$ The importance of this problem was first recognized by Einstein and Straus in their Swiss cheese model in 1945 [21]. At that time the existence of many of the structures in the coarse graining hierarchy (1) was still unknown, and they replaced it by the simpler scheme $g_{\mu\nu}^{\rm stellar} \to g_{\mu\nu}^{\rm universe}$, wherein a spatially homogeneous FLRW model is *assumed*, eliminating the possibility of backreaction on average expansion but allowing potential differences for light propagation as compared to the FLRW case.

* The smallest scale on which a notion of statistical homogeneity arises is 70–120 h^{-1}Mpc [23], based averaged stress-energy is assumed to be spatially homogeneous with dust equation of state, and to satisfy the field equations with a cosmological constant. Such an assumption is simply not justified from first principles, however.

The averaging of the Einstein field equations for local inhomogeneities on small scales can in general lead to very significant dynamical effects—*backreaction*—on the average evolution of the Universe [26]. Furthermore, averaging (and inhomogeneities in general) can affect the interpretation of cosmological data [27, 28, 29].

Almost all deductions about cosmology are based on null geodesics: light paths that traverse the greatest accessible distances. However, inhomogeneities bend null geodesics and can drastically alter observed distances when they are a sizeable fraction of the curvature radius. In the real Universe, voids occupy a much larger volume as compared to bound structures. Hence light preferentially travels much more through underdense regions and the effects of inhomogeneities on luminosity distances can be significant.

One further problem of interpreting observations is that it is necessary, in principle, to model properties of (not only a single photon) but of a 'narrow' beam of photons. Since the optical scalar equations [30] (which are non-linear and follow the geometric optics approximation) require integration along the beam of null geodesic congruences within a lumpy matter distribution, there may be important resulting averaged effects. Thus it is also of importance to study the effect of averaging on a beam of photons in the geometric optics limit. The small scale lensing effects of a lumpy matter distribution in an underdense universe principally involve Weyl curvature. However, since the FLRW geometry has zero Weyl curvature, if it is to provide an accurate effective description on cosmological scales, then the averaging of the optical equations must somehow replace the Weyl curvature due to isolated masses by a larger scale averaged Ricci curvature.

A theoretically conservative approach is to assume that GR is a (classical) *mesoscopic* theory♯ applicable on those small scales on which it has actually been tested, with a local metric field (the geometry) and matter fields. In this idealization, while strong field gravitational physics is required in the interaction of black holes, for the most part real particles are modelled as point particles moving along timelike geodesics in the absence of external forces, and photons move on null geodesics.

After coarse graining†† we obtain a smoothed out macroscopic geometry (with macroscopic metric) and macroscopic matter fields, valid on larger scales. In fact, (1)

on the two-point galaxy correlation function. However, variations of the number density of galaxies of the order 7–8% are still seen when sampling on the largest possible survey volumes [24, 25].

♯ Since GR incorporates the Strong Equivalence Principle, it already includes a relevant scale – local inertial frames – for the "microscopic physics" incorporating all the non-gravitational forces of the matter sector. Our use of the word "mesoscopic" here refers to scales possibly up to galactic scales, which is, of course, considerably larger than those envisaged with the use of this terminology in condensed matter physics.

†† The terms "coarse graining" and "averaging" are often used interchangeably, and should be understood as employing bottom-up versus top-down approaches to the same problem. Given the

indicates a succession of macroscopic scales. A photon follows a null geodesic in the local geometry. But what trajectories do photons follow in the averaged macro-geometry? The averaged vector is not necessarily null, geodesic (or affinely parametrized) in the macro-geometry [29]. This will affect cosmological observations. Similarly, the averaged matter does not necessarily move on timelike geodesics of the averaged metric. After all, in the final steps of the hierarchy (1) we are no longer dealing with particles, but "fluid" cells.

The coarse grained or averaged field equations need not take the same mathematical form as the original field equations. Indeed, in the case of Zalaletdinov's Macroscopic Gravity approach [16] the averaged spacetime is not necessarily even Riemannian. A rigorous mathematical definition of averaging in a cosmological model is necessary. Averaging often involves replacing variables by average values after integration over a domain. This is possible for scalars [26], but is problematic for non-scalars unless one introduces additional mathematical structures [16]. Numerous mathematically consistent approaches to averaging are possible, but any physically well-motivated approach should be both consistent with observations and with the principles of GR. Ideally, it should extend those principles in the most minimal fashion that seeks to understand open questions such as the nature of gravitational energy.

One profound consequence of the Strong Equivalence Principle is that gravitational energy cannot be localized at a point, but instead is non-local. The dynamical non-linear coupling of matter and geometry is therefore crucial to the definition of the "rest energy" assigned to regions at any level of the coarse graining hierarchy (1). It is also crucial to defining the relationship between the geodesic of any observer at the microscopic level of stars and the average effective geodesic of a coarse grained cosmic fluid cell.

In the standard approach to cosmology one implicitly assumes that irrespective of the dynamical coupling of matter and geometry, each step of coarse graining (1) will determine the velocity of one particle relative to the centre of mass of the coarse-grained particle at the next macroscopic scale by a pointlike boost, so that the whole succession of steps amount to a single boost with respect to the one global FLRW frame, explicable by an effective Newtonian gravitational potential.

Nothing in GR demands such an outcome from the physics of the unsolved fitting problem†. Furthermore, since the regional coarse graining of quasilocal gravitational energy is necessarily involved, we are talking about the very problem – the cosmological energy budget – for which unknown sources of dark matter and dark energy are added in the standard model of cosmology to make gravity stronger than our naïve expectation

historical importance of the FLRW models, the top-down approach has been much more widely studied.

† When Einstein's equations are applied to fluids, rather than to fundamental fields, one is assuming a notion of coarse graining of matter that is well-established in non-gravitational physics. In standard kinetic theory one coarse grains by *filtering*, i.e., neglecting stochastic fluctuations in phase space variables in favour of a mean field description. This is well understood for particles in the absence of spacetime curvature, but becomes an altogether different problem when coarse graining geometry.

on the level of bound structures on one hand, and weaker than our naïve expectation on the larger scale of unbound expanding structures on the other.

Setting aside the evidence of the CMB, the FLRW geometry is generally invoked "because it works". The fact that ΛCDM works so well, with just two essential parameters characterizing unknown physics, suggests that there are simplifying principles to be found in the unsolved gravitational physics of the fitting problem. While it is not our intention to debate the merits of particular proposals here, we point out that the timescape scenario [28, 31] attempts to explain how observed cosmic expansion is so close to uniform despite late epoch inhomogeneity, by applying a simplifying principle—the Cosmological Equivalence Principle [32]—that extends the Strong Equivalence Principle to cosmological averages. It results in a phenomenological description which is competitive with the ΛCDM model in independent tests that are currently possible [33], and it makes concrete predictions [34] for future tests that can distinguish it from ΛCDM, including the redshift time drift test [35] and the Clarkson–Bassett–Lu (CBL) test [36, 37].

The Euclid satellite will enable an extremely precise test of the validity of the FLRW geometry by the CBL test according to recent estimates [37], and also by the distance–sum–rule test [38]. Given the prospect of such tests in the next decade, and the fact that the fitting problem involves some of the deepest unsolved foundational questions in GR, we suggest that real progress can be made in the coming decades if researchers tackle the fundamental principles of the fitting problem while thinking hard about new data.

5. Large scales meet small scales: the very early Universe

The foundational questions faced by quantum theory and by cosmology become inextricably entangled in the very early Universe. The isotropy of the CMB on all scales larger than a degree indicates a prior state of thermal equilibrium between regions which cannot have been in causal contact given the expansion history of FLRW universes containing just non-relativistic matter and radiation. This is the *horizon problem.* The problem it poses for causal structure is one of the key reasons that the inflationary paradigm was introduced almost 40 years ago. Generically, with additional matter degrees of freedom, such as scalar fields, a very early period of de Sitter–like exponential expansion is produced, leading to vast changes in the structure of the past light cone. Inflation thereby not only resolves the horizon (and flatness) problems, but via quantum effects the energy of the new fundamental fields is converted to create all the particles of the Universe when inflation ends.

The phenomenology of inflation has been extraordinarily successful in accounting for a nearly scale invariant spectrum of density perturbations which in turn give rise to temperature fluctuations on the last scattering surface, with an anisotropy spectrum that well matches what is observed. Yet inflation remains a phenomenology, in search of a fundamental theory. Whether a given model inflates or not, and by how much, often depends on initial conditions, effectively pushing the foundational questions back to those of quantum gravity. Moreover, many models of inflation are now beginning to be ruled out by observations from the Planck satellite [39]: in particular, those that give rise to the production of copious primordial gravitational waves. The combined Planck/BICEP2/Keck observations [40] now yield an upper bound on the ratio of tensor to scalar power of $r < 0.07$ at 95% confidence, (with a pivot scale $0.05\,\mathrm{Mpc}^{-1}$). Typical models which survive are single field inflationary models with a long plateau, similar to the Starobinsky model [41], based on a $R + R^2$ extension of minimal Einstein gravity. Such models may naturally include "no scale" supergravity compactifications [42], or the Higgs boson itself as the inflaton provided it has a strong non-minimal coupling to gravity [43], which would pose challenges to standard field-theoretic approaches for the unification of gravity with gauge interactions [42].

Since the Higgs mechanism is associated with phase transitions in which standard model gauge bosons first acquire masses, from the first principles of GR it is natural to ask whether the simplistic picture of spacetime structure that underlies the conceptual framework of inflation is, in fact, the correct picture. In particular, does a single $(1+3)$–dimensional classical manifold (or a $1+3+6$ or $1+3+7$ higher-dimensional manifold) somehow nucleate at the Planck scale, giving us quantum field theoretic physics based on a fixed spacetime? (This is the picture that underlies most current very early Universe frameworks.) Or should the conventional notion of spacetime structure itself emerge at energy scales lower than the Planck scale?

After all, the Planck scale is derived by simple dimensional analysis extrapolated from the known fundamental constants. It therefore represents a limit in which we know our understanding of spacetime structure breaks down. However, between the electroweak scale and the Planck scale we simply extrapolate the idea that spacetime structure must be fixed, because that is the physics we are familiar with. All of this could change, in ways which change the effective couplings between gravity and the Higgs sector from the field theoretic view, if the problem has a more fundamental origin involving the dynamical coupling of matter and geometry at the quantum level.

It is remarkable that many independent approaches to quantum gravity seem to find an effective "spontaneous reduction to two dimensions" at high energies somewhat below the Planck scale [44]. Such approaches include causal dynamical triangulations [45], renormalization group methods [46] which invoke asymptotic safety, and loop quantum gravity [47], among others. Since one is dealing with an effective reduction of the spectral dimension, or the dimension of the space probed by typical geodesics, a "reduction to two dimensions" should be pictured in terms of an inherently quantum $(2+2)$–dimensional phase space rather than classical particles in a 2-dimensional configuration space.

As Carlip points out [44], the short-distance Wheeler-DeWitt equation may be dominated by "asymptotically silent spacetimes" in which light cones shrink to lines and nearby points become causally disconnected. This leads to Belinsky–Khalatnikov–

Lifschitz (BKL) asymptotics [48], in which the metric is locally Kasner with axes of anisotropy that vary chaotically. Each point effectively has a "preferred" spatial direction and geodesics effectively see only $1+1$ dimensions [44]. If one has asymptotic silence everywhere, then the small scale metric will have two length scales

$$ds^2 = \ell_{\|}^2 g_{\alpha\beta} dx^\alpha dx^\beta + \ell_\perp^2 h_{ij} dx^i dx^j \tag{2}$$

and the Einstein-Hilbert action becomes very nearly the action of a 2-dimensional conformal field theory for the transverse metric h_{ij} [44]. Such $2+2$ decompositions (2) have previously been studied in very high–energy gravitational scattering processes from a field theoretic viewpoint [49], and also give a useful framework for investigating quasilocal energy exchange in classical GR [50].

The spontaneous dimensional reduction to two dimensions [44] of course encompasses ideas which are still speculative, and not fully understood. But it illustrates a convergence between the geometric and field theoretic approaches to quantum gravity, and the phenomenology of the early Universe. In particular, we have an effective scale invariance at very high energies, and processes such as those based on the metric (2) which formally at least resemble those of string theory [49].

To make progress in the mathematical modelling of such ideas we need to change the conceptual picture of spacetime as a fixed manifold in which particles live as classical or quantum fields. The essence of GR is that spacetime is a relational structure between material particles. While our conventional picture works at low energies, at very high energies when the average relationship between particles is becoming lightlike, then the relational structure – i.e., spacetime structure – also appears to change in all the quantum gravity approaches that see an effective reduction to two dimensions.

Time is a property only measured by massive timelike particles undergoing the physical processes of particle physics that give rise to reaction rates, scattering amplitudes, and binding to form condensates. Massless particles do not carry clocks. The fact that successful inflationary phenomenologies are leading to models which are "scale free" in some sense, or which require unconventional non-minimal coupling between gravity and the Higgs sector, suggests that the high energy transition from a universe containing only massless particles to one in which particles acquire masses could also involve a phase transition in spacetime structure.

Such a scenario may appear radical when compared to current theories which generally invoke phase transitions in the very early Universe as occurring within field theories on a global FLRW spacetime. However, it is closer to the basic innovation of GR that matter and geometry are dynamically coupled. Moreover, it could lead to a deeper theoretical basis for explaining the phenomenology of inflation. The horizon problem is solved via a transition from a simpler asymptotically silent effectively 2–dimensional initial quantum state. The statistical spatial homogeneity and isotropy of the present epoch Universe could then be a consequence of a degree of scale invariance that survives from the very early Universe in the average geometry of the present Universe. A physical principle such as the Cosmological Equivalence Principle [32] may then be a constraining principle for relevant mathematical structures that relate the effective geometry of the early and late epoch Universes, without invoking a global FLRW geometry on the largest scales.

This paper is intended to initiate discussion about these questions, rather than advocating any particular approach. The questions are timely since current cosmological observations not only throw up fundamental mysteries such as dark energy and dark matter, but are now also strongly constraining models of inflation [39]. What survives are phenomenologies [42, 43] that have a strong resonance with findings from many different approaches to quantum gravity [44], and with the idea that gravity emerges as a breaking of conformal invariance [51]. There have already been studies of dynamical dimensional reduction in the early Universe, in terms of the effect of modified dispersion relations on cosmological perturbations [52].

It is, however, high time to rethink the mechanisms by which the spacetime of GR emerges from quantum gravity in the early Universe. String theory is already built on the conformal invariance of the 2-dimensional string worldsheet. However, it treats spacetime as a separate entity: a unique higher-dimensional continuum chosen somehow from a multiverse, a statistical collection of vacua – geometries of fixed finite dimension on which matter fields live, classified according to their (super)symmetries. Loop quantum gravity and other geometric approaches often focus purely on quantizing geometry, without reference to matter.

Here we offer the view that whatever ends up being retained in a theory of quantum gravity from either of these approaches, it is likely to more deeply embody the notion of GR as a theory of dynamical spacetime structure that *couples* matter and geometry at the quantum as well as the classical level. A logical extension of the principles of GR to a global theory of spacetime structure may well mean that while the Einstein field equations hold at small scales at late epochs, there is no need for these equations to hold on all scales at all epochs. Rather than having a single notion of geometry on all scales while modifying the Einstein-Hilbert action, we might retain the Einstein-Hilbert action on mesoscopic scales but change the notion of the geometrical structure of spacetime at early epochs, and on large scales. Rather than dealing with Modified Gravity we are then dealing with *Modified Geometry.*

GR is not a finished theory that was completely solved 100 years ago. The next 100 years of the development of relativistic cosmology are likely to be as exciting as the first 100 years. The problems we face also demand that we think equally rigorously about the foundations of GR as Einstein did 100 years before us.

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Appendix A. A brief overview of GR

GR is often described as both beautiful and elegant. To embody the universal nature of the gravitational interaction, Einstein re-envisioned gravity as a property of the relational structure between all matter particles (including massless ones). That is, gravity is a property of spacetime structure, rather than a force in a pre-existing spacetime. This distinguishes GR from the theories of the other fundamental interactions.

In GR it is assumed that all matter moves in an effective pseudo-Riemannian metric spacetime with a universal coupling, governed by the Einstein Equivalence Principle, consisting of two parts: (i) *The Weak Equivalence Principle:* Given the same initial positions and velocities, subject only to gravity particles will follow the same trajectories, or *geodesics*. In other words, particles all fall with the same acceleration regardless of composition and consequently gravity is universal. (ii) *The Strong Equivalence Principle:* The laws of physics take the same form in a freely-falling reference frame as in SR. Effectively, gravity can always be eliminated at a point. Instead gravity is related to tidal forces – the deviation of neighbouring geodesics from the SR expectation. Measurement of geodesic deviation, on a scale determined by a suitably large experiment, allows a direct determination of the Riemann curvature tensor [53].

On the road to the Strong Equivalence Principle, in 1907 Einstein formulated the more limited version that all motions in an external static homogeneous gravitational field are identical to those in the absence of a gravitational field if referred to a uniformly accelerated coordinate system. A common misconception that follows from this is that acceleration requires GR. It does not; accelerated frames can be treated by general coordinate systems within SR. As far as GR is concerned, it is the tidal forces that constitute the true gravitational field.

The Strong Equivalence Principle is satisfied in GR by assuming that all matter fields are minimally coupled to a single metric tensor, $g_{\mu\nu}$, with a torsion-free affine connection. Infinitesimal proper length and time intervals – those measured by physical rods and clocks – between two points (events) in 4-dimensional spacetime are given by the metric line element. On larger scales proper lengths and times are given by the integral curves of the geodesic equations, which follow from the line element by a variational principle. E.g., variation of the proper time, $\tau = \frac{1}{c}\int d\lambda\,\sqrt{-g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu}$, where $\dot{x}^\mu \equiv \frac{dx^\mu}{d\lambda}$ on curves with parameter λ, leads to the timelike geodesic equation $U^\nu\nabla_\nu U^\mu = 0$, where ∇ is the covariant derivative and $U^\mu \equiv \frac{dx^\mu}{d\tau}$ is the 4-velocity. Geometrically this is the equation of parallel transport for the 4-velocity.

The central idea in Einstein's GR is that gravity is described entirely in terms of the geometry determined from a metric, and that it is free of any "prior geometry" which is fixed immutably and independently of the distribution of the gravitating sources. Instead, the metric and the matter sources of energy–momentum in the Universe are dynamically coupled via the Einstein field equations,

$$G^{\mu\nu} = \frac{8\pi G}{c^4} T^{\mu\nu},$$

where $G^{\mu\nu}$ is the Einstein tensor which is related to the spacetime curvature (and is determined by the metric, $g_{\mu\nu}$, and its first and second partial derivatives), G is Newton's constant, and $T^{\mu\nu}$ is the energy-momentum tensor. The field equations can also be obtained in general from the Einstein-Hilbert action:

$$S_{\rm grav} = \frac{c^3}{16\pi G} \int d^4x \sqrt{-g}\, \mathcal{R},$$

where $\mathcal{R}$ is the Ricci scalar curvature, plus a minimally coupled action, $S_{\rm matter}$, for the matter fields on small scales. However, on mesoscopic scales the energy-momentum tensor is already assumed to be averaged as an effective fluid rather than being directly derived from an action.

There have been many successes of GR, from the predictions of the motion of planets, satellites (and GPS), the deflection of light and the Shapiro time delay in the solar system, to numerous strong field processes in astrophysics and cosmology, including in particular the prediction of black holes. Over the past century GR has been accurately tested at scales from 10^{-4} m in laboratories, up to 10^{14} m in the solar system and in strongly gravitating binary pulsar systems [54]. Perhaps, most excitingly, the first direct observation of gravitational waves was finally achieved by LIGO on the 100th anniversary of GR [55].

GR remains untested at both ends of the spectrum of distance scales, however. The largest scale on which GR is directly tested is still 12 orders of magnitude smaller than the present size of the observable Universe, $\sim 10^{26}$ m. Furthermore, on small scales gravitational effects are difficult to test directly due to the weakness of gravity relative to other fundamental interactions. Einstein never specified the scale of applicability of his equations, but given the universal nature of gravity we must assume that GR applies on all scales unless observational evidence for some modified theory is found. Quantum mechanics played no role in the original formulation of GR. However, since quantum mechanics also applies to every form of matter, we know from simple dimensional considerations that our classical understanding of spacetime structure must break down at the Planck scale. The development of a possible quantum theory of GR, and a discussion of the fundamental issues involved, has certainly garnered much attention. One point that is not often emphasized is that the Planck scale simply represents the limit of the "known unknowns". Quantization of the spacetime manifold itself may conceivably involve changes to the relations that define spacetime structure at scales somewhat larger than the Planck scale.

We may also question whether GR is valid on cosmological scales. Answering this question observationally is complicated since cosmological measurements of the evolution of large-scale structure depend strongly on the underlying cosmological model.

Most often this is taken to be the simple FLRW model based on the assumptions of spatial homogeneity and isotropy, with the matter content of the ΛCDM concordance model. However, this simple model leads to the very problematic issues of non-baryonic dark matter and dark energy (such as a cosmological constant, Λ) which have not been directly detected, and whose origin remains a mystery. There is much less discussion of the fundamental aspects of cosmology [7, 17, 22] than the fundamental aspects of field theory. In particular, if the Einstein field equations hold on large scales, they require an effective stress–energy tensor in which small scale inhomogeneities are averaged out. Any procedure of coarse-graining involves additional mathematical and physical assumptions, which in general give rise to *backreaction*: namely, average cosmic evolution that differs from FLRW evolution [6, 56].

The two regimes in which GR is not yet tested, the small and large scales, may be linked together in the very early Universe. For example, in a widely accepted framework quantum fluctuations just after the Plank regime are stretched by inflation to produce fluctuations on all spatial scales before the Universe entered a radiation-dominated regime. While baryons, electrons and photons remain tightly coupled, the density fluctuations are somewhat amplified on the small scales determined by the distance that sound waves can propagate. When the Universe cools enough that the first atoms form, at the last scattering epoch, the density perturbations are frozen in. The density perturbations grow via gravitational instability, giving rise to all the complex hierarchy of structures which are strongly inhomogeneous on scales smaller than the present comoving scale of the sound horizon, around $100\,h^{-1}$Mpc, where h is a dimensionless parameter related to the Hubble constant by $H_0 = 100\,h\,\mathrm{km\ sec^{-1}\,Mpc^{-1}}$.

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