# Cosmic structure, averaging and dark energy

**David L. Wiltshire** (University of Canterbury, NZ)

DLW: **Class. Quan. Grav. 28 (2011) 164006**

**New J. Phys. 9 (2007) 377**

**Phys. Rev. Lett. 99 (2007) 251101**

**Phys. Rev. D78 (2008) 084032**

**Phys. Rev. D80 (2009) 123512**

B.M. Leith, S.C.C. Ng & DLW:

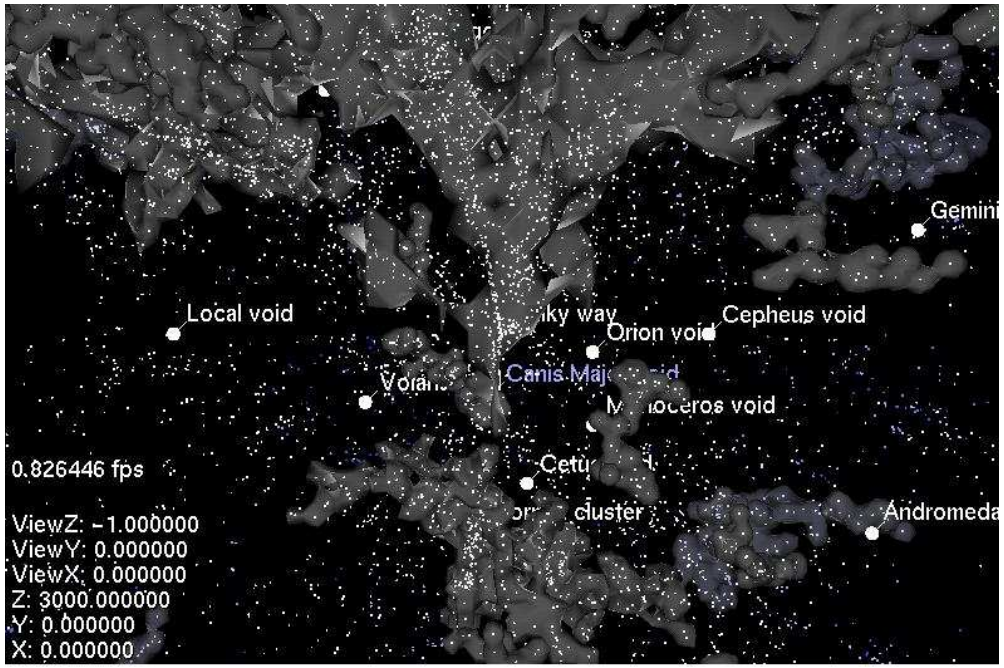
**ApJ 672 (2008) L91**

P.R. Smale & DLW, **MNRAS 413 (2011) 367**

P.R. Smale, **MNRAS 418 (2011) 2779**

DLW, P R Smale, T Mattsson and R Watkins

**arXiv:1201.5731, ApJ submitted**



# Plan of Lectures

1. What is dust? - Fitting, coarse-graining and averaging
2. Approaches to coarse-graining, averaging and backreaction
3. Timescape cosmology
4. Observational tests of the timescape cosmology
5. Variance of the Hubble flow

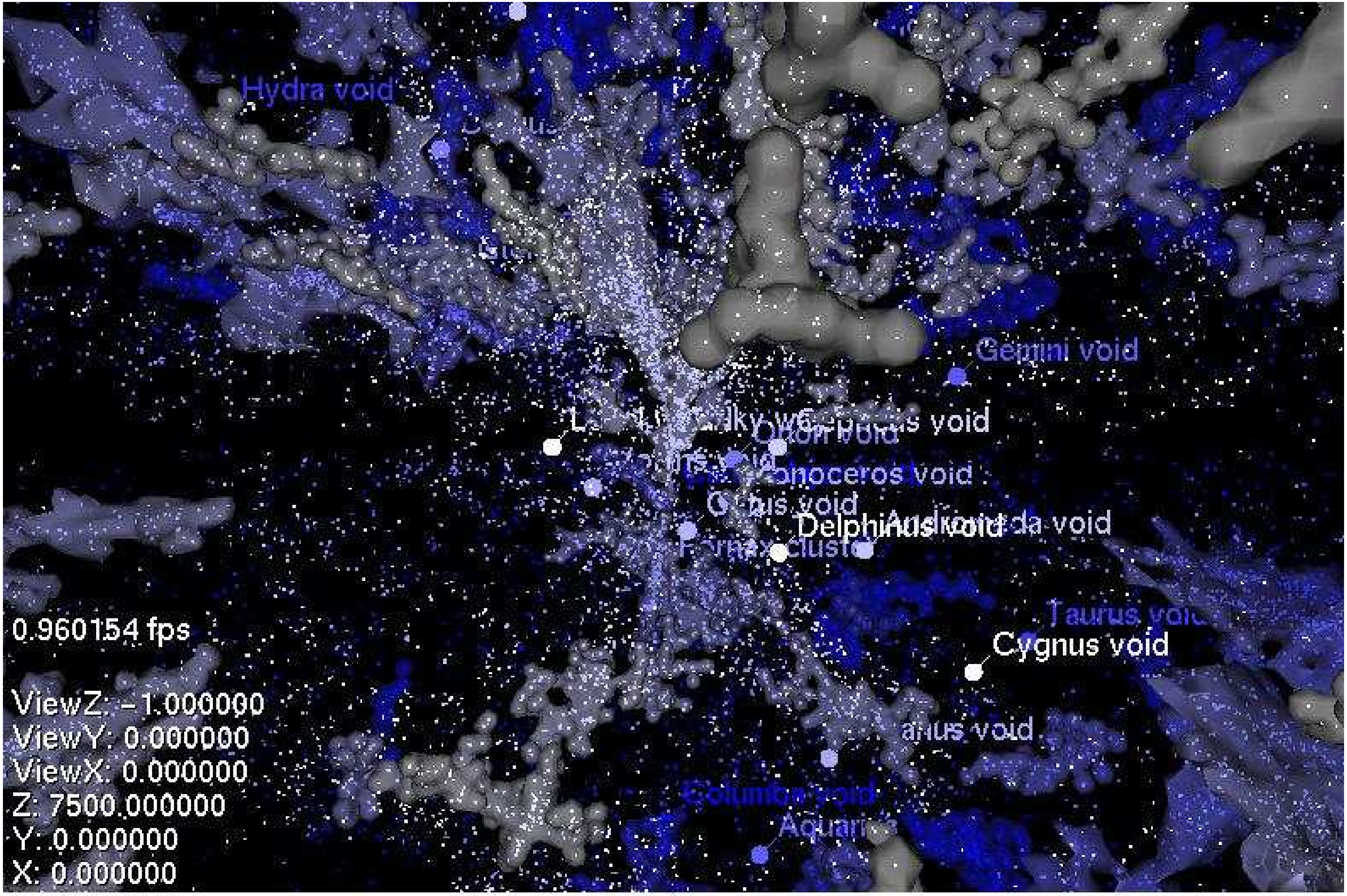
# Lecture 1

# What is dust?

# What is "dark energy"?

- Usual explanation: a homogeneous isotropic form of "stuff" which violates the strong energy condition. (Locally pressure $P = w\rho c^2$, $w < -\frac{1}{3}$.)
  Best-fit close to cosmological constant, $\Lambda$, $w = -1$.
- *Cosmic coincidence*: Why now? Why $\Omega_{\Lambda 0} \sim 2\Omega_{M0}$, so that a universe which has been decelerating for much of its history began accelerating only at $z \sim 0.7$?
- Onset of acceleration coincides also with the nonlinear growth of large structures
- Are we oversimplifying the geometry?
- Hypothesis: must understand nonlinear evolution with backreaction, AND gravitational energy gradients within the inhomogeneous geometry

# 6df: voids & bubble walls (A. Fairall, UCT)



# From smooth to lumpy

- Universe was very smooth at time of last scattering; fluctuations in the fluid were tiny ($\delta\rho/\rho \sim 10^{-5}$ in photons and baryons; $\sim 10^{-4}$,$10^{-3}$ in non–baryonic dark matter).
- FLRW approximation very good early on.
- Universe is very lumpy or inhomogeneous today.
- Recent surveys estimate that 40–50% of the volume of the universe is contained in voids of diameter $30h^{-1}$ Mpc. [Hubble constant $H_0 = 100h$ km/s/Mpc] (Hoyle & Vogeley, ApJ 566 (2002) 641; 607 (2004) 751)
- Add some larger voids, and many smaller minivoids, and the universe is *void–dominated* at present epoch.
- Clusters of galaxies are strung in filaments and bubbles around these voids.

# Coarse-graining, averaging, backreaction

- *Coarse-graining*: Replace the the microphysics of a given spacetime region by some collective degrees of freedom sufficient to describe physics on scales larger than the coarse-graining scale. BOTTOM UP.
- *Averaging*: Consider overall macroscopic dynamics and evolution, without direct consideration of the details of the course-graining procedure. TOP DOWN.
  - Often assumes the existence of a particular average, e.g., FLRW background, without showing that such an average exists.
  - *Backreaction*: Consider the effects of departures from the average, perturbative or nonperturbative, on the average evolution.

# Fitting problem (Ellis 1984)

- On what scale are Einstein's field equations (EFEs) valid?

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- Scale on which matter fields are coarse–grained to produce the energy–momentum tensor on r.h.s. not prescribed
- general relativity only well tested for isolated systems – e.g., solar system or binary pulsars – for which $T_{\mu\nu} = 0$
- Usual approach: just pretend
- Other approaches: cut and paste exact solutions, e.g., Einstein-Straus vacuole (1946) $\rightarrow$ Swiss cheese models; LTB vacuoles $\rightarrow$ meatball models

# Layers of coarse-graining in cosmology

1. Atomic, molecular, ionic or nuclear particles coarse-grained as fluid in early universe, voids, stars etc
2. Collapsed objects – stars, black holes coarse-grained as isolated objects;
3. Stellar systems coarse-grained as dust particles within galaxies;
4. Galaxies coarse-grained as dust particles within clusters;
5. Clusters of galaxies as bound systems within expanding walls and filaments;
6. Voids, walls and filaments combined as regions of different densities in a smoothed out expanding cosmological fluid.

# Coarse-graining: first steps

- $(1) \to (2)$ (fluid $\to$ universe, star etc): Realm of EFEs "matter tells space how to curve" – well-established for early universe; vacuum geometries; starting point for defining neutron stars etc
- $(2) \to (3)$ (isolated Schwarzschild, Kerr geometry $\to$ particle in fluid):
  - Replace Weyl curvature $\to$ Ricci curvature
  - No formal coarse-graining solution; but reasonable to assume possible in terms of ADM-like mass (see, e.g., Korzyński 2010)
  - Even for 2 particles, gravitational energy (binding energy etc) necessarily involved
  - Neglect inter-particle interactions $\to$ dust approximation

# Coarse-graining: further steps

- $(2) \to (3)$ (stars, gas $\to$ galaxies):
  - Newtonian gravity usually assumed
  - Cooperstock and Tieu (2006,2007): claimed non-Newtonian properties possible for rigidly rotating dust (van Stockum metrics)
  - Neill and DLW (in preparation): new van Stockum metrics for empirically observed density profiles do not solve galaxy rotation curve problem
- $(3) \to (4)$ (galaxies $\to$ galaxy clusters):
  - Newtonian gravity, viral theorem usually assumed
  - Virial theorem studied formally only in GR
  - Realistic solutions not known; given Lemaître–Tolman–Bondi (LTB) solutions inapplicable

# Coarse-graining: final steps

- $(4) \to (5)$ (bound galaxy clusters $\to$ expanding walls/filaments)
  - New ingredient: expanding space
- $(1) + (5) \to (6)$ (voids $+$ walls/filaments $\to$ universe)
  - New ingredient: “building blocks” themselves are expanding
- Effective hierarchy

$$\left.\begin{array}{l} g_{\mu\nu}^{\text{stellar}} \longrightarrow g_{\mu\nu}^{\text{galaxy}} \longrightarrow g_{\mu\nu}^{\text{cluster}} \longrightarrow g_{\mu\nu}^{\text{wall}} \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad \vdots \\ \qquad\qquad\qquad\qquad\qquad\qquad\quad g_{\mu\nu}^{\text{void}} \end{array}\right\} \longrightarrow g_{\mu\nu}^{\text{universe}}$$

- How does $\langle T^{\mu}{}_{\nu}(g_{\mu\nu}^{\text{x}})\rangle \to T^{\mu}{}_{\nu}(g_{\mu\nu}^{\text{y}})$ at each step?

# Dilemma of gravitational energy...

- In GR spacetime carries *energy* & *angular momentum*

$$G_{\mu\nu} = \frac{8\pi G}{c^4} T_{\mu\nu}$$

- On account of the strong equivalence principle, $T_{\mu\nu}$ contains localizable energy–momentum only
- Kinetic energy and energy associated with spatial curvature are in $G_{\mu\nu}$: variations are "quasilocal"!
- Newtonian version, $T - U = -V$, of Friedmann equation

$$\frac{\dot{a}^2}{a^2} + \frac{kc^2}{a^2} = \frac{8\pi G\rho}{3}$$

where $T = \frac{1}{2} m\dot{a}^2 x^2$, $U = -\frac{1}{2} kmc^2 x^2$, $V = -\frac{4}{3}\pi G\rho a^2 x^2 m$; $\mathbf{r} = a(t)\mathbf{x}$.

# Dilemma of gravitational energy...

- Each step of coarse-graining hierarchy involves coarse-graining gravitational degrees of freedom
- How do we define coarse-grained averages?

$$\langle G^{\mu}{}_{\nu}\rangle = \langle g^{\mu\lambda}R_{\lambda\nu}\rangle - \tfrac{1}{2}\delta^{\mu}{}_{\nu}\langle g^{\lambda\rho}R_{\lambda\rho}\rangle = \frac{8\pi G}{c^4}\langle T^{\mu}{}_{\nu}\rangle$$

- How do we relate coarse-grained “particle” mass etc to sub-system masses, angular momenta etc?
- How do we relate metric invariants (rulers, clocks) of subsystem to those of coarse-grained system?
- FLRW model success with $\Omega_{C0} = \Omega_{M0} - \Omega_{B0}$, $\Omega_{\Lambda 0}$, suggests simplifying physical principles to be found
- Steps 2 to 5 in hierarchy may shed light on dark matter; steps 5 to 6 on dark energy (only consider latter here)

# What is a cosmological particle (dust)?

- In FLRW one takes observers “comoving with the dust”
- Traditionally galaxies were regarded as dust. However,
  - Neither galaxies nor galaxy clusters are homogeneously distributed today
  - Dust particles should have (on average) invariant masses over the timescale of the problem
- Must coarse-grain over expanding fluid elements larger than the largest typical structures
- ASIDE: Taking galaxies as dust leads to flawed argument against backreaction (Peebles 0910.5142)

$$\Phi_{\rm Newton}(\text{galaxy}) \sim v_{\rm gal}^2/c^2 \sim 10^{-6}$$

$\Lambda$CDM self-consistent; but galaxies, clusters do not justify FLRW background

# Sandage-de Vaucouleurs paradox

- Matter homogeneity only observed at $\gtrsim 100/h$ Mpc scales
- If "the coins on the balloon" are galaxies, their peculiar velocities should show great statistical scatter on scale much smaller than $\sim 100/h$ Mpc
- However, a nearly linear Hubble law flow begins at scales above 1.5–2 Mpc from barycentre of local group.
- Moreover, the local flow is statistically "quiet"; despite a possible $65/h$ Mpc Hubble bubble feature.
- Peculiar velocities are isotropized in FLRW universes which expand forever (regardless of dark energy); but attempts to explain the paradox not a good fit to $\Lambda$CDM parameters (Axenides & Perivolaropoulos 2002).

# Largest typical structures

| Survey | Void diameter | Density contrast |
|---|---|---|
| PSCz | $(29.8 \pm 3.5)h^{-1}$Mpc | $\delta_\rho = -0.92 \pm 0.03$ |
| UZC | $(29.2 \pm 2.7)h^{-1}$Mpc | $\delta_\rho = -0.96 \pm 0.01$ |
| 2dF NGP | $(29.8 \pm 5.3)h^{-1}$Mpc | $\delta_\rho = -0.94 \pm 0.02$ |
| 2dF SGP | $(31.2 \pm 5.3)h^{-1}$Mpc | $\delta_\rho = -0.94 \pm 0.02$ |

Dominant void statistics in the Point Source Catalogue Survey (PSCz), the Updated Zwicky Catalogue (UZC), and the 2 degree Field Survey (2dF) North Galactic Pole (NGP) and South Galactic Pole (SGP), (Hoyle and Vogeley 2002,2004). More recent results of Pan et al. (2011) using SDSS Data Release 7 similar.

- Particle size should be a few times greater than largest typical structures (voids with $\delta_\rho \equiv (\rho - \bar{\rho})/\bar{\rho}$ near -1)
- Coarse grain dust “particles” – fluid elements – at Scale of Statistical Homogeneity (SSH) $\sim 100/h$ Mpc

# Scale of statistical homogeneity

- Coincides roughly with Baryon Acoustic Oscillation (BAO) scale
- Using Friedmann equation for pressureless dust

$$a_0^2 H_0^2 (\Omega_{M0} - 1) = a^2(t) H^2(t) [\Omega_M(t) - 1]$$

with $\delta_t \equiv \delta\rho/\rho = \Omega_M(t) - 1 \simeq A \times 10^{-4}$ at $z = 1090$, $t = 380$ kyr when $H \simeq 2/(3t)$, estimate on scales $>$ SSH

$$\delta_0 \equiv \left(\frac{\delta\rho}{\rho}\right)_0 \simeq \left(\frac{H}{H_0}\right)^2 \frac{\delta_t}{(1+z)^2} \simeq \frac{0.025\, A}{h^2}$$

$\delta_0 = 6$% if $A = 1$, $h = 0.65$

- Measurement 7% (Hogg et al, 2005), 8% (Sylos Labini et al, SDSS-DR7 2009)

# Scale of statistical homogeneity

- ASIDE: Sylos-Labini et al define SSH as $\delta\rho/\rho \to 0$, and disagree with Hogg et al who find SHS at $\gtrsim 70/h$ Mpc (but fractal dimension $D \simeq 2$ for galaxy distribution up to at least $20/h$ Mpc)
- Inflation and cosmic variance imply some large scale variation in average density, bounded on scales $>$ SSH
- BAO feature observed in linear regime of FLRW perturbation theory implying SSH $\lesssim$ BAO scale
- On scales below BAO scale density perturbations $\delta_t = A \times 10^{-4}$ amplified by acoustic waves, more so the smaller the scale, eventually becoming nonlinear
- Speculation: dominant void scale, diameter $30/h$ Mpc, is a rarefaction amplification set by 2nd acoustic peak?

# Lecture 2

# Approaches to coarse-graining, averaging and backreaction

# I. Coarse-graining at SSH

- In timescape model we will coarse-grain "dust" at SSH
- Scale at which fluid cell properties from cell to cell remain similar *on average* throughout evolution of universe
- Notion of "comoving with dust" will require clarification
- Variance of expansion etc relates more to internal degrees of freedom of fluid particle than differences between particles
- Coarse-graining over internal gravitational degrees of freedom means that we no longer deal with a single global geometry: *description of geometry is statistical*

# Coarse-graining: other approaches

- Lindquist-Wheeler (LW) (1959) lattice model: continuum FLRW model is only realised as an approximation
- Like Swiss cheese it assumes a simplified hierarchy

$$g_{\mu\nu}^{\text{sph symmetric}} \longrightarrow g_{\mu\nu}^{\text{universe}}$$

- Clifton & Ferreira studied light propagation in the spatially flat LW model, and initially concluded $1 + z \simeq (1 + z_{\text{FLRW}})^{7/10}$.
  However, after correcting a numerical error, the results were no different to the standard Friedman case (Clifton, Ferreira & O'Donnell, arXiv:1110.3191)
- Symmetry implies Friedmann evolution still; like Swiss cheese this limits the metric of the universe

# Korzyński's covariant coarse-graining

- Isometrically embed the boundary of a comoving dust-filled domain (with $S^2$ topology, positive scalar curvature) into a three-dimensional Euclidean space
- Construct a "fictitious" 3-dimensional fluid velocity which induces the same infinitesimal metric deformation on the embedded surface as "true" flow on domain boundary original spacetime
- Use velocity field to uniquely assign coarse-grained expressions for the volume expansion and shear. Using pushforward of ADM shift vector similarly obtain a coarse-grained vorticity.
- Coarse-grained quantities are quasilocal functionals which depend only on the geometry of the domain boundary.

Class. Quan. Grav. **27** (2010) 105015

# II. Averaging and backreaction

- In general $\langle G^\mu{}_\nu(g_{\alpha\beta})\rangle \neq G^\mu{}_\nu(\langle g_{\alpha\beta}\rangle)$
- $\langle G^\mu{}_\nu\rangle$ need not be Einstein tensor for an exact geometry

(1) $$\langle G^\mu{}_\nu\rangle = \langle g^{\mu\lambda}R_{\lambda\nu}\rangle - \tfrac{1}{2}\delta^\mu{}_\nu\langle g^{\lambda\rho}R_{\lambda\rho}\rangle = \frac{8\pi G}{c^4}\langle T^\mu{}_\nu\rangle$$

E.g., Zalaletdinov (1992,1993) works with the average inverse metric $\langle g^{\mu\nu}\rangle$ and the average Ricci tensor $\langle R_{\mu\nu}\rangle$, and writes

(2) $$\langle g^{\mu\lambda}\rangle\langle R_{\lambda\nu}\rangle - \tfrac{1}{2}\delta^\mu{}_\nu\langle g^{\lambda\rho}\rangle\langle R_{\lambda\rho}\rangle + C^\mu{}_\nu = \frac{8\pi G}{c^4}\langle T^\mu{}_\nu\rangle\,,$$

- Correlation functions $C^\mu{}_\nu$ defined by difference of the l.h.s. of (1) and (2): these are *backreaction* terms

# Averaging and backreaction

- Alternatively define $g_{\mu\nu} = \bar{g}_{\mu\nu} + \delta g_{\mu\nu}$, where $\bar{g}_{\mu\nu} \equiv \langle g_{\mu\nu} \rangle$, with inverse $\bar{g}^{\lambda\mu} \neq \langle g^{\lambda\mu} \rangle$
- Determine a connection $\bar{\Gamma}^{\lambda}{}_{\mu\nu}$, curvature tensor $\bar{R}^{\mu}{}_{\nu\lambda\rho}$ and Einstein tensor $\bar{G}^{\mu}{}_{\nu}$ based on the averaged metric, $\bar{g}_{\mu\nu}$, alone.
- Differences $\delta\Gamma^{\lambda}{}_{\mu\nu} \equiv \langle \Gamma^{\lambda}{}_{\mu\nu} \rangle - \bar{\Gamma}^{\lambda}{}_{\mu\nu}$, $\delta R^{\mu}{}_{\nu\lambda\rho} \equiv \langle R^{\mu}{}_{\nu\lambda\rho} \rangle - \bar{R}^{\mu}{}_{\nu\lambda\rho}$, $\delta R_{\mu\nu} \equiv \langle R_{\mu\nu} \rangle - \bar{R}_{\mu\nu}$ etc, then represent the *backreaction*
- Average EFEs (1) may be written

$$\bar{G}^{\mu}{}_{\nu} + \delta G^{\mu}{}_{\nu} = \frac{8\pi G}{c^4} \langle T^{\mu}{}_{\nu} \rangle$$

- Processes of averaging and constructing Einstein tensor do not commute

# Approaches to averaging

Three main types

1. Perturbative schemes about a given background geometry;
2. Spacetime averages;
3. Spatial averages on hypersurfaces based on a $1+3$ foliation.

- Perturbative schemes deal with *weak backreaction*
- Approaches 2 and 3 can be fully nonlinear giving *strong backreaction*
- No obvious way to average tensors on a manifold, so extra assumptions or structure needed

# Approach 1: Weak backreaction

- Much argument about backreaction from perturbation theory near a FLRW background (e.g., Kolb *et al*. 2006)
- Deal with gradient expansions of potentials and densities
- Debate is largely about mathematical consistency, and conclusions vary with assumptions made
- Debate shows there is a problem – perturbation theory does not converge – and if backreaction changes the background, then any single FLRW model may simply be the wrong background at present epoch
- Many reviews; e.g., Clarkson, Ellis, Larena and Umeh, Rep. Prog. Phys. 74 (2011) 112901 [arXiv:1109.2484]; Kolb, Class. Quan. Grav. 28 (2011) 164009

# Approach 2: Spacetime averages

- Any process of taking an average will in general break general covariance
- If average cosmological geometry on scales no longer satisfies EFEs, need to revisit the role of general covariance plays in defining spacetime structure on the largest scales from first principles
- How do coordinates of a “fine–grained manifold” relate to those of an average “coarse–grained manifold”?
- Zalaletdinov views general covariance as paramount; he introduces additional mathematical structure to perform averaging of tensors covariantly
- Aim: consistently average the Cartan equations from first principles, in analogy to averaging of microscopic Maxwell–Lorentz equations in electromagnetism

# Zalaletdinov's macroscopic gravity

- Define bilocal averaging operators, $\mathcal{A}^{\mu}{}_{\alpha}(x,x')$, with support at two points $x \in \mathcal{M}$ and $x' \in \mathcal{M}$
- Construct a bitensor extension, $\mathbf{T}^{\mu}{}_{\nu}(x,x')$, of tensor $T^{\mu}{}_{\nu}(x)$ according to

$$\mathbf{T}^{\mu}{}_{\nu}(x,x') = \mathcal{A}^{\mu}{}_{\alpha'}(x,x')T^{\alpha'}{}_{\beta'}(x')\mathcal{A}^{\beta'}{}_{\nu}(x',x)\,.$$

- Integrates bitensor extension over a 4-dimensional spacetime region, $\Sigma \subset \mathcal{M}$, with volume $\mathcal{V}_{\Sigma}$, to obtain regional average

$$T^{\mu}_{\nu}(x) = \frac{1}{\mathcal{V}_{\Sigma}}\int_{\Sigma} \mathrm{d}^4x'\,\sqrt{-g(x')}\,\mathbf{T}^{\mu}{}_{\nu}(x,x'),$$

- Bitensor transforms as a tensor at each point but is a scalar when integrated for regional average.

# Zalaletdinov’s macroscopic gravity

- Mathematical formalism which in the average bears a close resemblance to general relativity itself
- Macroscopic scale is assumed to be larger than the microscopic scale, there is no scale in the final theory
- Issues of coarse-graining of gravitational d.o.f. in cosmological relativity may make the problem subtly different
- Cosmological applications of Zalaletdinov’s formalism need additional assumptions
  - Assume a spatial averaging limit (Paranjape and Singh 2007)
  - Assume the average is FLRW: correlation tensors then take form of a spatial curvature (Coley, Pelavas & Zalaletdinov 2005)

# Approach 3: Spatial averages

- Average scalar quantities only; e.g., given a congruence of observers, with tangent vector $U^\mu$, and projection operator $h^\mu{}_\nu = \delta^\mu{}_\nu + U^\mu U_\nu$ then with $B_{\mu\nu} \equiv \nabla_\mu U_\nu$, acceleration $a^\mu = U^\nu \nabla_\nu U^\mu$, we have

$$\begin{aligned}
B^\perp_{\mu\nu} &\equiv h^\lambda{}_\mu h^\sigma{}_\nu B_{\lambda\sigma} = B_{\mu\nu} + a_\mu U_\nu \\
\theta_{\mu\nu} &= h^\lambda{}_\mu h^\sigma{}_\nu B^\perp_{(\lambda\sigma)} && \text{expansion} \\
\omega_{\mu\nu} &= h^\lambda{}_\mu h^\sigma{}_\nu B^\perp_{[\lambda\sigma]} && \text{vorticity} \\
\theta &= \theta^\mu{}_\mu = \nabla^\mu U_\mu && \text{expansion scalar} \\
\sigma_{\mu\nu} &= \theta_{\mu\nu} - \tfrac{1}{3} h_{\mu\nu}\theta && \text{shear}
\end{aligned}$$

- Buchert approach, assume $\omega_{\mu\nu} = 0$. Flow is hypersurface orthogonal, i.e., spacetime can foliated by spacelike hypersurfaces $\Sigma_t$, in standard ADM formalism.

# The $3+1$ decomposition

For a globally hyperbolic manifold, we can make a $3+1$-split

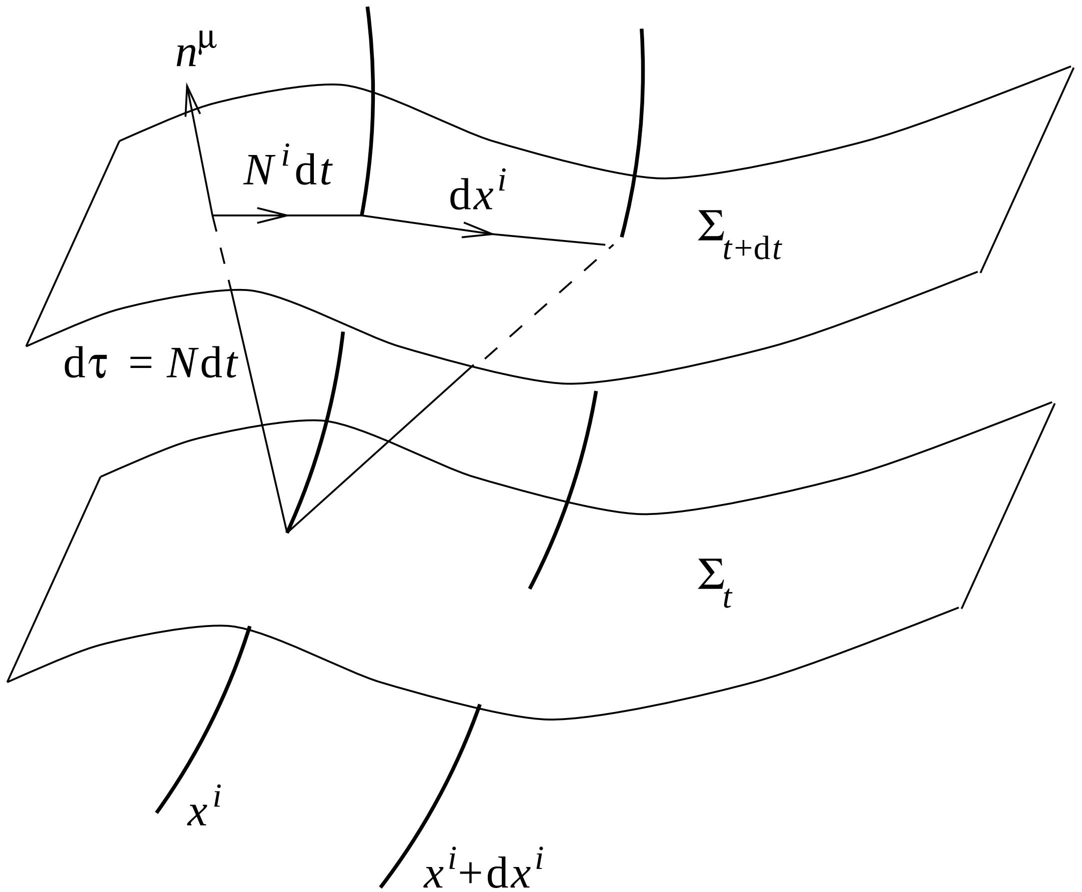
$$\mathrm{d}s^2 = -\boldsymbol{\omega}^0 \otimes \boldsymbol{\omega}^0 + g_{ij}(t,\mathbf{x})\,\boldsymbol{\omega}^i \otimes \boldsymbol{\omega}^j\,,$$

where

$$\begin{aligned}\boldsymbol{\omega}^0 &= \mathcal{N}(t,\mathbf{x})\,\mathrm{d}t\\ \boldsymbol{\omega}^i &= \mathrm{d}x^i + \mathcal{N}^i(t,\mathbf{x})\,\mathrm{d}t.\end{aligned}$$

- $\mathcal{N}(t,x^k)$ is the *lapse function*: measures difference between coordinate time, $t$, and proper time, $\tau$, on curves normal to hypersurfaces $\Sigma_t$, $n_\alpha = (-\mathcal{N},0,0,0)$
- $\mathcal{N}^i(t,x^k)$ is the *shift vector*: measures the difference between a spatial point, $p$, and the point reached by following the normal $\mathbf{n}$.

# The $3 + 1$ decomposition




When $P = 0$, i.e., for dust, in Buchert scheme may consistently choose

- $\mathcal{N}^i = 0$: comoving coordinates
- $\mathcal{N} = 1$: normalised, $n^\alpha n_\alpha = -1$ (i.e., $n^\mu \equiv U^\mu$)

# Buchert averaging

- Average scalar quantities only on domain in spatial hypersurface $\mathcal{D} \in \Sigma_t$; e.g.,

$$\langle \mathcal{R} \rangle \equiv \left( \int_{\mathcal{D}} \mathrm{d}^3 x \sqrt{{}^3\!g}\, \mathcal{R}(t, \mathbf{x}) \right) / \mathcal{V}(t)$$

where $\mathcal{V}(t) = \int_{\mathcal{D}} \mathrm{d}^3 x \sqrt{{}^3\!g}$, $\quad {}^3\!g \equiv \det({}^3 g_{ij}) = -\det({}^4 g_{\mu\nu})$.

- Now $\sqrt{{}^3\!g}\,\theta = \sqrt{-{}^4\!g}\,\nabla_\mu U^\mu = \partial_\mu(\sqrt{-{}^4\!g}\,U^\mu) = \partial_t(\sqrt{{}^3\!g})$, so

$$\langle \theta \rangle = (\partial_t \mathcal{V}) / \mathcal{V}$$

- Generally for any scalar $\Psi$, get commutation rule

$$\partial_t \langle \Psi \rangle - \langle \partial_t \Psi \rangle = \langle \Psi \theta \rangle - \langle \theta \rangle \langle \Psi \rangle = \langle \Psi\, \delta\theta \rangle = \langle \theta\, \delta\Psi \rangle = \langle \delta\Psi\, \delta\theta \rangle$$

where $\delta\Psi \equiv \Psi - \langle \Psi \rangle$, $\delta\theta \equiv \theta - \langle \theta \rangle$.

# Buchert-Ehlers-Carfora-Piotrkowska -Russ-Soffel-Kasai-Börner equations

For irrotational dust cosmologies, with energy density, $\rho(t,\mathbf{x})$, expansion scalar, $\theta(t,\mathbf{x})$, and shear scalar, $\sigma(t,\mathbf{x})$, where $\sigma^2 = \frac{1}{2}\sigma_{\mu\nu}\sigma^{\mu\nu}$, **defining** $3\dot{\bar{a}}/\bar{a} \equiv \langle\theta\rangle$, we find average cosmic evolution described by exact Buchert equations

$$
\begin{aligned}
&(3) \qquad & 3\frac{\dot{\bar{a}}^2}{\bar{a}^2} &= 8\pi G\langle\rho\rangle - \tfrac{1}{2}\langle\mathcal{R}\rangle - \tfrac{1}{2}\mathcal{Q} \\
&(4) \qquad & 3\frac{\ddot{\bar{a}}}{\bar{a}} &= -4\pi G\langle\rho\rangle + \mathcal{Q} \\
&(5) \qquad & \partial_t\langle\rho\rangle &+ 3\frac{\dot{\bar{a}}}{\bar{a}}\langle\rho\rangle = 0 \\
&(6) \qquad & \partial_t\left(\bar{a}^6\mathcal{Q}\right) &+ \bar{a}^4\partial_t\left(\bar{a}^2\langle\mathcal{R}\rangle\right) = 0 \\
& & \mathcal{Q} &\equiv \frac{2}{3}\left(\langle\theta^2\rangle - \langle\theta\rangle^2\right) - 2\langle\sigma^2\rangle
\end{aligned}
$$

# Backreaction in Buchert averaging

- *Kinematic backreaction* term can also be written

$$\mathcal{Q} = \tfrac{2}{3}\langle(\delta\theta)^2\rangle - 2\langle\sigma^2\rangle$$

  i.e., combines variance of expansion, and shear.
- Eq. (6) is required to ensure (3) is an integral of (4).
- Buchert equations look deceptively like Friedmann equations, but deal with *statistical* quantities
- The extent to which the back–reaction, $\mathcal{Q}$, can lead to apparent cosmic acceleration or not has been the subject of much debate (e.g., Ishibashi & Wald 2006):
  - How do statistical quantities relate to observables?
  - What about the time slicing?
  - How big is $\mathcal{Q}$ given reasonable initial conditions?

# III. Average spatial homogeneity

Many approaches simply assume FLRW as average:

- all perturbative calculations about the FLRW universe
- any “asymptotically FLRW” LTB models with core spherical inhomogeneity
- the Dyer-Roeder (1974) approach
- Swiss cheese and meatball models
- specific cosmological studies of spatial averaging (Russ et al 1997; Green & Wald 2011. . . )
- specific cosmological studies of Zalaletdinov’s macroscopic gravity (Coley et al 2005; Paranjape & Singh 2007,2008; van den Hoogen 2009)
- specific cosmological studies of general Constant Mean (extrinsic) Curvature (CMC) flows (Reiris 2009,2009)

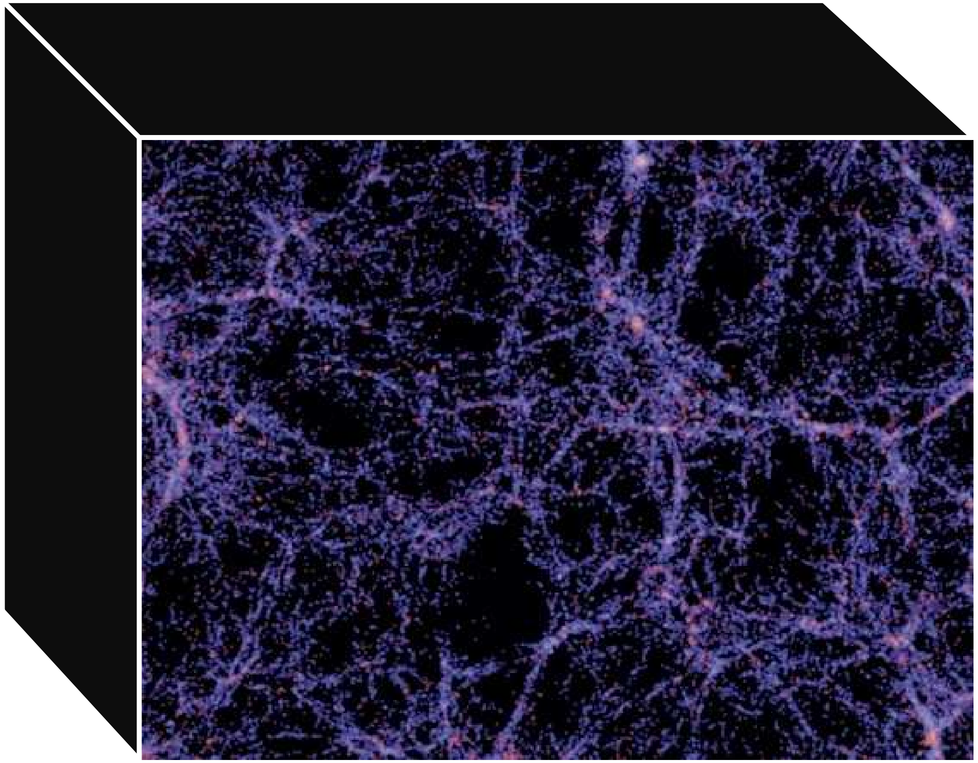
# Notions of average spatial homogeneity

- How do we define average spatial homogeneity?
- Assumption of FLRW average is restrictive, demanding 3 separate notions:
  1. Average spatial homogeneity is described by class of ideal comoving observers with synchronized clocks.
  2. Average spatial homogeneity is described by average surfaces of constant spatial curvature.
  3. The expansion rate at which the ideal comoving observers separate within the hypersurfaces of average spatial homogeneity is uniform.
- No need to demand all of these notions must hold together

# Perturbative average homogeneity

- Bardeen (1980) described gauge invariant FLRW perturbations in different foliations which take one or other property as more fundamental
  - *comoving hypersurfaces* (and related synchronous gauge) embody (1)
  - *minimal shear hypersurfaces* (and related Newtonian gauge) are one type of foliation related to (2)
  - *uniform Hubble flow hypersurfaces* embody (3)
- Bičak, Katz & Lynden-Bell (2007) consider Machian foliations (LIF coords uniquely determined by $\delta T^{\mu}{}_{\nu}$):
  - *uniform* $^{3}\mathcal{R}$ *hypersurfaces* – embody (2)
  - *minimal shear hypersurfaces*
  - *uniform Hubble flow hypersurfaces* plus *minimal shift distortion* gauge condition of Smarr and York (1978)

# Within a statistically average cell



- Need to consider relative position of observers over scales of tens of Mpc over which $\delta\rho/\rho \sim -1$.
- GR is a local theory: gradients in spatial curvature and gravitational energy can lead to calibration differences between our rulers & clocks and volume average ones

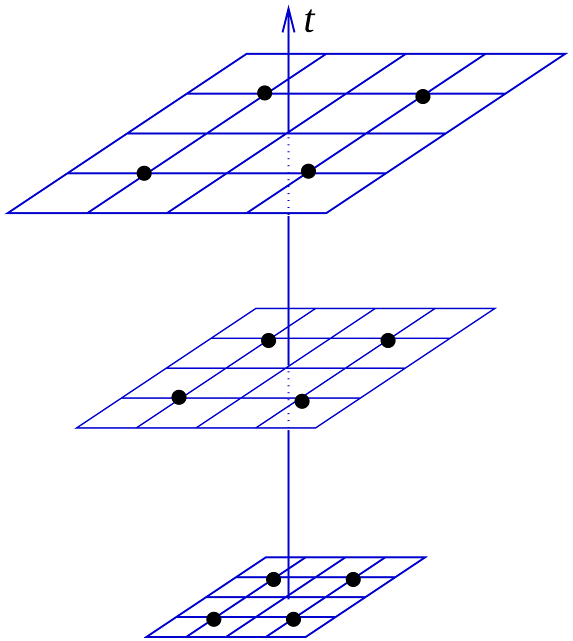
# The Copernican principle

- Retain Copernican Principle - we are at an average position *for observers in a galaxy*
- Observers in bound systems are not at a volume average position in freely expanding space
- By Copernican principle other average observers should see an isotropic CMB
- BUT *nothing in theory, principle nor observation demands that such observers measure the same mean CMB temperature nor the same angular scales in the CMB anisotropies*
- Average mass environment (galaxy) can differ significantly from volume–average environment (void)

# Back to first principles. . .

- Standard approach assumes single global FLRW frame plus Newtonian perturbations
- In absence of exact background symmetries, Newtonian approximation requires a weak field approximation about suitable static Minkowski frame
- What is the largest scale on which the Strong Equivalence Principle can be applied?
- Need to address Mach's principle: *"Local inertial frames are determined through the distributions of energy and momentum in the universe by some weighted average of the apparent motions"*
- How does coarse-graining affect relative calibration of clocks and rods, from local to global, to account for average effects of gravity?

# What expands? Can't tell!




- Homogeneous isotropic volume expansion is locally indistinguishable from equivalent motion in static Minkowski space; on local scales

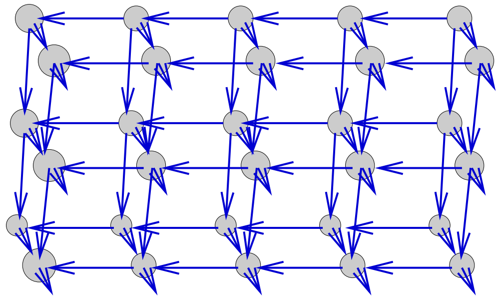
$$z \simeq \frac{v}{c} \simeq \frac{H_0 \ell_r}{c}, \qquad H_0 = \left.\frac{\dot{a}}{a}\right|_{t_0}$$

whether $z+1 = a_0/a$ or $z+1 = \sqrt{(c+v)/(c-v)}$.

# What expands? Can't tell!

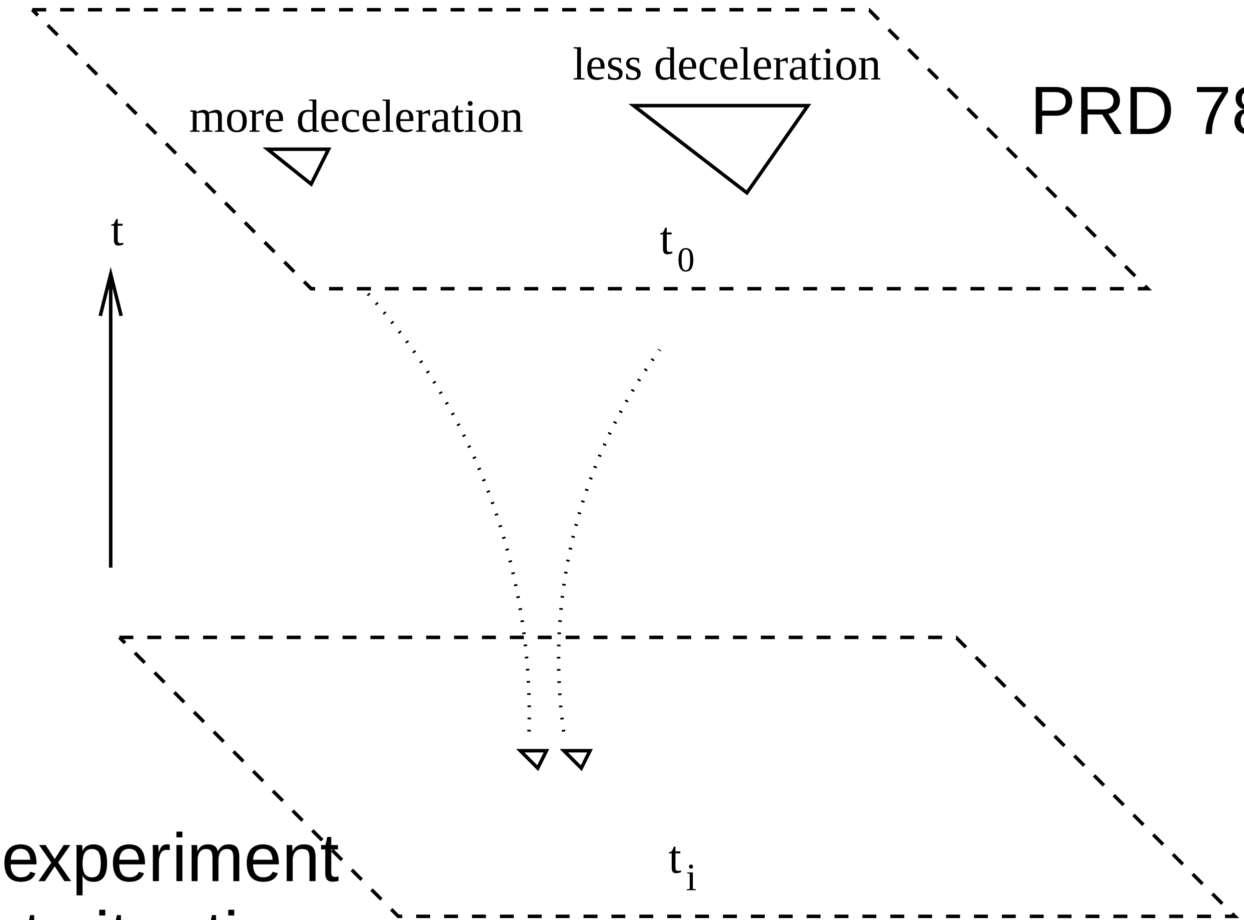
- For homogeneous isotropic volume expansion we cannot tell whether particles are at rest in an expanding space, or moving equivalently in a static Minkowski space.
- In the actual universe volume expansion decelerates because of the average regional density of matter
- Need to separate non-propagating d.o.f., in particular regional density, from propagating modes: shape d.o.f.
- Is there a Minkowski space analogue, like Galileo's ship, or Einstein's elevator, even accounting for the average density of matter? Yes. . .

# Semi-tethered lattice



- Extend to decelerating motion over long time intervals by Minkowski space analogue (semi-tethered lattice - indefinitely long tethers with one end fixed, one free end on spool, apply brakes syncronously at each site)
- Brakes convert kinetic energy of expansion to heat and so to other forms
- Brake impulse can be arbitrary pre-determined function of local proper time; but provided it is synchronous deceleration remains homogeneous and isotropic: *no net force on any lattice observer*.
- Deceleration preserves inertia, by symmetry

# Thought experiments

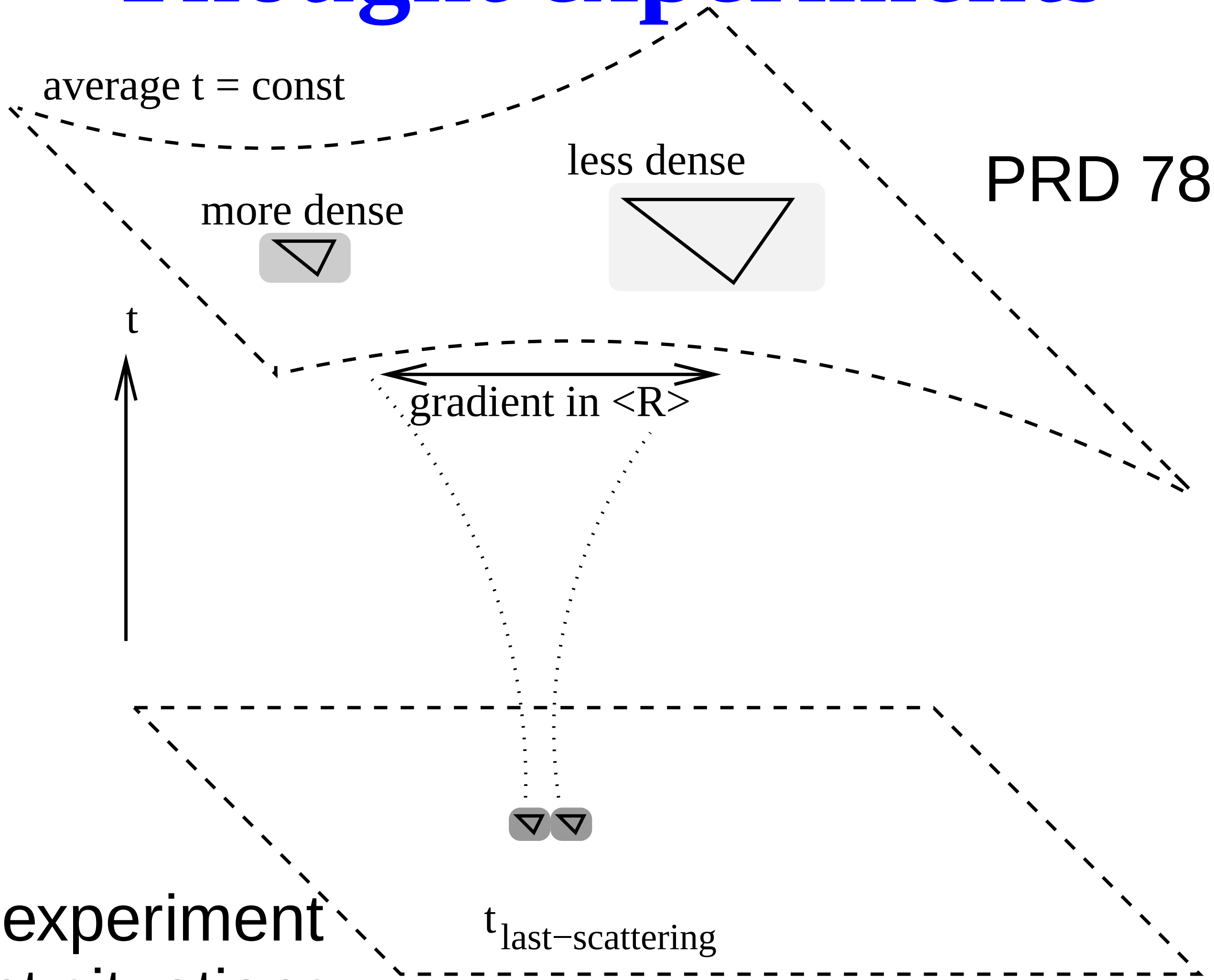



PRD 78, 084032 (2008)

Thought experiment equivalent situations:

- SR: observers in disjoint regional semi-tethered lattices volume decelerate at different rates
- Those who decelerate more age less

# Thought experiments




PRD 78, 084032 (2008)

Thought experiment equivalent situations:

- GR: regions of different density have different volume deceleration (for same initial conditions)
- Those in denser region age less

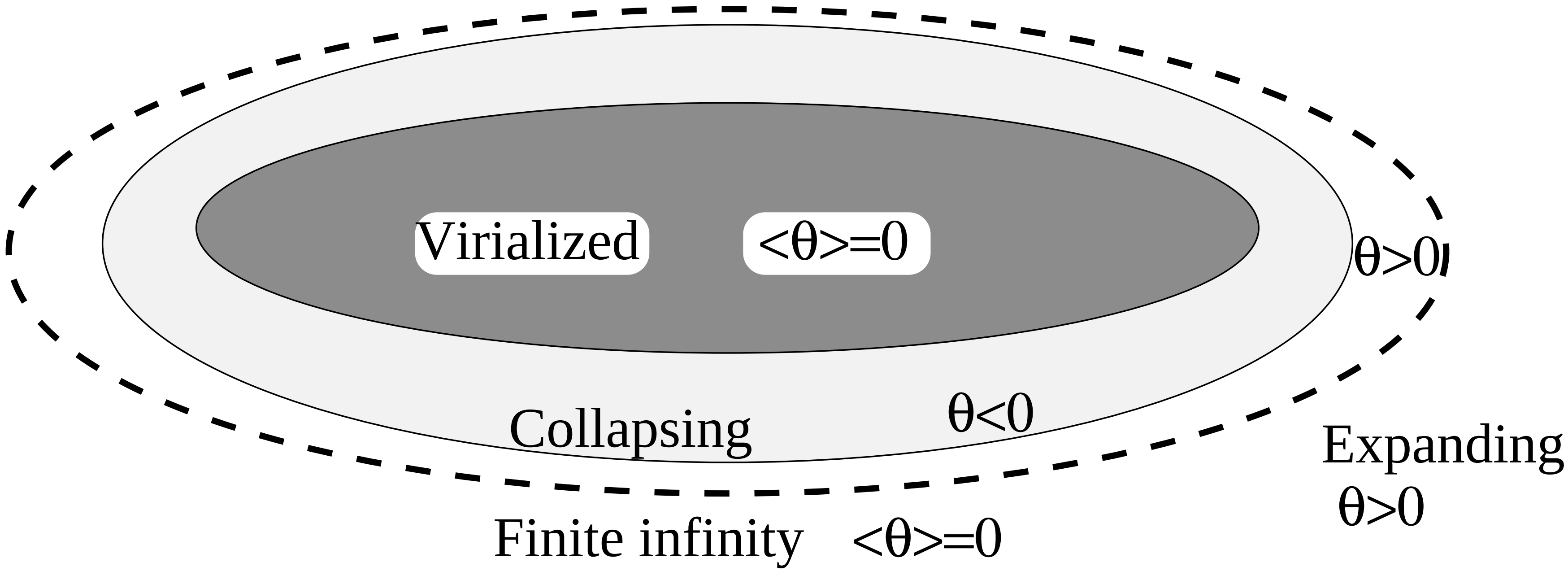
# Cosmological Equivalence Principle

- *At any event, always and everywhere, it is possible to choose a suitably defined spacetime neighbourhood, the cosmological inertial region, in which average motions (timelike and null) can be described by geodesics in a geometry which is Minkowski up to some time-dependent conformal transformation*,

$$\mathrm{d}s^2_{\mathrm{CIF}} = a^2(\eta)\left[-\mathrm{d}\eta^2 + \mathrm{d}r^2 + r^2\mathrm{d}\Omega^2\right],$$

- Defines Cosmological Inertial Frame (CIF)
- Accounts for regional average effect of density in terms of frames for which the state of rest in an expanding space is indistinguishable from decelerating expansion of particles moving in a static space

# Finite infinity




- Define *finite infinity*, "*fi*" as boundary to *connected* region within which *average expansion* vanishes $\langle\theta\rangle = 0$ and expansion is positive outside.
- Shape of *fi* boundary irrelevant (minimal surface generally): could typically contain a galaxy cluster.

# Cosmic "rest frame"

- Patch together CIFs for observers who see an isotropic CMB by taking surfaces of uniform volume expansion

$$\left\langle \frac{1}{\ell_r(\tau)} \frac{\mathrm{d}\ell_r(\tau)}{\mathrm{d}\tau} \right\rangle = \frac{1}{3}\langle\theta\rangle_1 = \frac{1}{3}\langle\theta\rangle_2 = \cdots = \bar{H}(\tau)$$

- Average over regions in which (i) spatial curvature is zero or negative; (ii) space is expanding at the boundaries, at least marginally.
- Solves the Sandage–de Vaucouleurs paradox implicitly.
- Voids appear to expand faster; but canonical rate $\tau_{\rm v}$ faster, locally measured expansion can still be uniform.
- Global average $H_{\rm av}$ on large scales with respect to *any one set of clocks* may differ from $\bar{H}$

# Better formalism?

- CEP should be associated with a statistical geometrical gauge principle
- Equivalent of general covariance for cosmological relativity, determined by initial state of universe
- Expect equivalent descriptions of internal d.o.f. of coarse-grained cell: (i) Buchert description; (ii) minimal shear description; (iii) uniform Hubble flow (CMC) description...
- Since more than one geometrical description is possible, patching goes beyond junction conditions for geometries with prescribed $T_{\mu\nu}$
- Principled “modification” of general relativity

# Lecture 3

# The timescape cosmology

# Two/three scale model

- Split spatial volume $\mathcal{V} = \mathcal{V}_i \bar{a}^3$ as disjoint union of negatively curved void fraction with scale factor $a_{\rm v}$ and spatially flat "wall" fraction with scale factor $a_{\rm w}$.

$$\begin{aligned} \bar{a}^3 &= f_{\rm wi} a_{\rm w}{}^3 + f_{\rm vi} a_{\rm v}{}^3 \equiv \bar{a}^3 (f_{\rm w} + f_{\rm v}) \\ f_{\rm w} &\equiv f_{\rm wi} a_{\rm w}{}^3 / \bar{a}^3, \qquad f_{\rm v} \equiv f_{\rm vi} a_{\rm v}{}^3 / \bar{a}^3 \end{aligned}$$

- $f_{\rm vi} = 1 - f_{\rm wi}$ is the fraction of present epoch horizon volume which was in uncompensated underdense perturbations at last scattering.

$$\bar{H}(t) = \frac{\dot{\bar{a}}}{\bar{a}} = f_{\rm w} H_{\rm w} + f_{\rm v} H_{\rm v}; \qquad H_{\rm w} \equiv \frac{1}{a_{\rm w}} \frac{{\rm d}a_{\rm w}}{{\rm d}t}, \quad H_{\rm v} \equiv \frac{1}{a_{\rm v}} \frac{{\rm d}a_{\rm v}}{{\rm d}t}$$

- Here $t$ is the Buchert time parameter, considered as a collective coordinate of dust cell coarse-grained at SSH.

# Phenomenological lapse functions

- According to Buchert average variance of $\theta$ will include internal variance of $H_{\rm w}$ relative to $H_{\rm v}$.
  Note $h_r \equiv H_{\rm w}/H_{\rm v} < 1$.
- Buchert time, $t$, is measured at the *volume average* position: locations where the local Ricci curvature scalar is the same as horizon volume average
- In timescape model, rates of wall and void centre observers who measure an isotropic CMB are fixed by the uniform quasilocal Hubble flow condition, i.e.,

$$\frac{1}{\bar a}\frac{{\rm d}\bar a}{{\rm d}t} = \frac{1}{a_{\rm w}}\frac{{\rm d}a_{\rm w}}{{\rm d}\tau_{\rm w}} = \frac{1}{a_{\rm v}}\frac{{\rm d}a_{\rm v}}{{\rm d}\tau_{\rm v}}; \qquad \text{or} \qquad \bar H(t) = \bar\gamma_{\rm w} H_{\rm w} = \bar\gamma_{\rm v} H_{\rm v}$$

where $\bar\gamma_{\rm v} = \frac{{\rm d}t}{{\rm d}\tau_{\rm v}}$, $\bar\gamma_{\rm w} = \frac{{\rm d}t}{{\rm d}\tau_{\rm w}} = 1 + (1-h_r)f_v/h_r$, are *phenomenological lapse functions* (NOT ADM lapse).

# Other ingredients

- $\langle\mathcal{R}\rangle = k_v/a_{\rm v}{}^3 = k_{\rm v} f_{\rm vi}{}^{2/3} f_{\rm v}{}^{1/3}/\bar{a}^3$ since $k_{\rm w} = 0$
- Assume that average shear in SSH cell vanishes; more precisely neglect $\mathcal{Q}$ *within* voids and walls *separately*

$$\langle\delta\theta^2\rangle_{\rm w} = \tfrac{3}{4}\langle\sigma^2\rangle_{\rm w} \qquad \langle\delta\theta^2\rangle_{\rm v} = \tfrac{3}{4}\langle\sigma^2\rangle_{\rm v}$$

  Justification: for spherical voids expect $\langle\sigma^2\rangle = \langle\omega^2\rangle = 0$; for walls expect $\langle\sigma^2\rangle$ and $\langle\omega^2\rangle$ largely self-canceling.
- Only remaining backreaction is variance of relative volume expansion of walls and voids

$$\mathcal{Q} = 6 f_{\rm v}(1 - f_{\rm v})\left(H_{\rm v} - H_{\rm w}\right)^2 = \frac{2\dot{f}_{\rm v}{}^2}{3 f_{\rm v}(1 - f_{\rm v})}$$

- Solutions known for:
  - dust (DLW 2007);
  - dust + $\Lambda$ (Viaggiu, 2012), taking $\bar{\gamma}_{\rm w} = \bar{\gamma}_{\rm v} = 1$;
  - dust + radiation (Duley, Nazer + DLW, in preparation)

# Bare cosmological parameters

- Buchert equations for volume averaged observer, with $f_{\rm v}(t)=f_{\rm vi}{a_{\rm v}}^3/\bar a^3$ (void volume fraction) and $k_{\rm v}<0$

$$\bar\Omega_M+\bar\Omega_R+\bar\Omega_k+\bar\Omega_{\mathcal Q}=1,$$
$$\bar a^{-6}\partial_t\left(\bar\Omega_{\mathcal Q}{\bar H}^2\bar a^6\right)+\bar a^{-2}\partial_t\left(\bar\Omega_k{\bar H}^2\bar a^2\right)=0\,.$$

where the *bare parameters* are

$$\bar\Omega_M=\frac{8\pi G\bar\rho_{M0}\bar a_0^3}{3{\bar H}^2\bar a^3},\qquad \bar\Omega_R=\frac{8\pi G\bar\rho_{R0}\bar a_0^4}{3{\bar H}^2\bar a^4},$$
$$\bar\Omega_k=\frac{-k_{\rm v}{f_{\rm vi}}^{2/3}{f_{\rm v}}^{1/3}}{\bar a^2{\bar H}^2},\qquad \bar\Omega_{\mathcal Q}=\frac{-{\dot f_{\rm v}}^{\,2}}{9f_{\rm v}(1-f_{\rm v}){\bar H}^2}\,.$$

# Dust model

- Specialize to dust only; in fact $\bar{\Omega}_R$ is negligible by the time $\bar{\Omega}_Q$ is significant, so this is good enough. Solution behaves like Einstein-de Sitter at early times, with $f_{\rm v}$ tiny and clock differences negligible.
- Buchert equations, in terms of bare (volume average) quantities are then

$$\frac{\dot{\bar{a}}^2}{\bar{a}^2} + \frac{\dot{f}_{\rm v}{}^2}{9f_{\rm v}(1-f_{\rm v})} - \frac{\alpha^2 f_{\rm v}{}^{1/3}}{\bar{a}^2} = \frac{8\pi G}{3}\bar{\rho}_0\frac{\bar{a}_0^3}{\bar{a}^3},$$

$$\ddot{f}_{\rm v} + \frac{\dot{f}_{\rm v}{}^2(2f_{\rm v}-1)}{2f_{\rm v}(1-f_{\rm v})} + 3\frac{\dot{\bar{a}}}{\bar{a}}\dot{f}_{\rm v} - \frac{3\alpha^2 f_{\rm v}{}^{1/3}(1-f_{\rm v})}{2\bar{a}^2} = 0,$$

where $\alpha^2 = -k_{\rm v} f_{\rm vi}{}^{2/3}$.

# General exact solution PRL 99, 251101

- Irrespective of physical interpretation the two scale Buchert equation can be solved exactly

$$a_{\rm w} = a_{\rm w0} t^{2/3}$$

$$a_{\rm w0} \equiv \bar{a}_0 \left[ \tfrac{9}{4} {f_{\rm wi}}^{-1} (1 - \epsilon_i) \, \bar{\Omega}_{M0} {\bar{H}_0}^2 \right]^{1/3}; \; C_\epsilon \equiv \frac{\epsilon_i \bar{\Omega}_{M0} {f_{\rm v0}}^{1/3}}{\bar{\Omega}_{k0}}$$

$$\sqrt{u(u + C_\epsilon)} - C_\epsilon \ln \left( \left| \frac{u}{C_\epsilon} \right|^{\frac{1}{2}} + \left| 1 + \frac{u}{C_\epsilon} \right|^{\frac{1}{2}} \right) = \frac{\alpha}{\bar{a}_0} (t + t_\epsilon)$$

where $u \equiv {f_{\rm v}}^{1/3} \bar{a} / \bar{a}_0 = {f_{\rm vi}}^{1/3} a_{\rm v} / \bar{a}_0$, $\alpha = \bar{a}_0 \bar{H}_0 \bar{\Omega}_{k0}^{1/2} / {f_{\rm v0}}^{1/6}$, and constraints relate many constants.

- 4 independent parameters: e.g., $\bar{H}_0$, $f_{\rm v0}$, $\epsilon_i$, $f_{\rm vi}$.

# Tracker solution limit

- Parameters $\epsilon_i$ and $f_{\rm vi}$ should be restricted by power spectrum at last scattering, e.g.,

$$\left(\frac{\delta\rho}{\rho}\right)_{\mathcal{H}\rm i} = f_{\rm vi}\left(\frac{\delta\rho}{\rho}\right)_{\rm vi} \sim -10^{-6}\ \text{to} - 10^{-5}$$

  E.g., $f_{\rm vi} \sim 10^{-3}$, $(\delta\rho/\rho)_{\rm vi} \sim -10^{-3}$; or $f_{\rm vi} \sim -10^{-2}$, $(\delta\rho/\rho)_{\rm vi} \sim -10^{-4}$; or $f_{\rm vi} \sim 3 \times 10^{-3}$, $(\delta\rho/\rho)_{\rm vi} \sim -3 \times 10^{-3}$.

- The general solution possesses a tracking limit, equivalent to the exact solution with $\epsilon_i = 0$, $t_\epsilon = 0$, with $a_{\rm v} = a_{\rm v0} t$ (i.e., voids expand like Milne solution in $t$)
- Tracker reached within 1% by redshift $z \sim 37$ for reasonable priors
- Effectively, there are only two free parameters

# Tracker solution

- PRL 99 (2007) 251101:

$$\bar{a} = \frac{\bar{a}_0 (3\bar{H}_0 t)^{2/3}}{2 + f_{\rm v0}} \left[ 3 f_{\rm v0} \bar{H}_0 t + (1 - f_{\rm v0})(2 + f_{\rm v0}) \right]^{1/3}$$

$$f_{\rm v} = \frac{3 f_{\rm v0} \bar{H}_0 t}{3 f_{\rm v0} \bar{H}_0 t + (1 - f_{\rm v0})(2 + f_{\rm v0})} \,,$$

- Other parameters (drop subscript w on $\bar{\gamma}_{\rm w}$):

$$\bar{\gamma} = 1 + \tfrac{1}{2} f_{\rm v} = \tfrac{3}{2} \bar{H} t$$

$$\bar{\Omega}_M = \frac{4(1 - f_{\rm v})}{(2 + f_{\rm v})^2} \,; \qquad \bar{\Omega}_k = \frac{9 f_{\rm v}}{(2 + f_{\rm v})^2} \,; \qquad \bar{\Omega}_{\mathcal{Q}} = \frac{-f_{\rm v}(1 - f_{\rm v})}{(2 + f_{\rm v})^2}$$

$$\tau_{\rm w} = \tfrac{2}{3} t + \frac{2(1 - f_{\rm v0})(2 + f_{\rm v0})}{27 f_{\rm v0} \bar{H}_0} \ln \left( 1 + \frac{9 f_{\rm v0} \bar{H}_0 t}{2(1 - f_{\rm v0})(2 + f_{\rm v0})} \right)$$

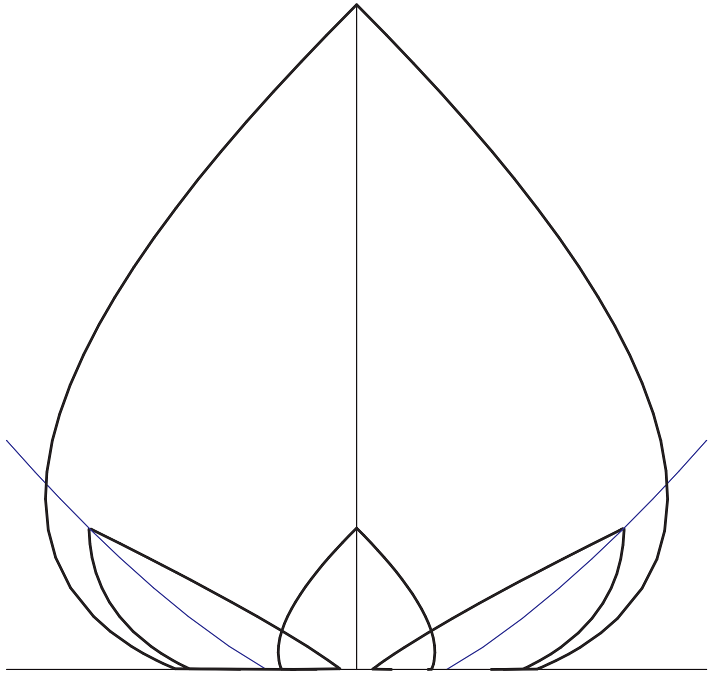
# Physical interpretation

- As pointed out by Ishibashi and Wald, need to relate average quantities to physical observables
- My interpretation assumes coarse–graining of dust at SSH; Buchert average domain $\mathcal{D}$ is horizon volume.
- Uniform quasi–local Hubble flow below this scale; volume average time, $t$, and average curvature, $\langle\mathcal{R}\rangle$, not local observable for all isotropic observers
- Observers within galaxies assumed to be within spatially flat finite infinity regions with geometry

$$\mathrm{d}s^2_{\mathit{fi}} = -\mathrm{d}\tau_{\mathrm{w}}^2 + a_{\mathrm{w}}{}^2(\tau_{\mathrm{w}})\left[\mathrm{d}\eta_{\mathrm{w}}^2 + \eta_{\mathrm{w}}^2\mathrm{d}\Omega^2\right]$$

- Determine dressed cosmological parameters, as if local clocks and rulers were extended to whole universe

# Past light cone average



- Interpret solution of Buchert equations by radial null cone average

$$\mathrm{d}s^2 = -\mathrm{d}t^2 + \bar{a}^2(t)\,\mathrm{d}\bar{\eta}^2 + A(\bar{\eta},t)\,\mathrm{d}\Omega^2,$$

where $\int_0^{\bar{\eta}_{\mathcal{H}}} \mathrm{d}\bar{\eta}\, A(\bar{\eta},t) = \bar{a}^2(t)\mathcal{V}_{\mathrm{i}}(\bar{\eta}_{\mathcal{H}})/(4\pi)$.

- LTB metric but NOT an LTB solution

# Physical interpretation

- Conformally match radial null geodesics of spherical Buchert geometry to those of finite infinity geometry with uniform local Hubble flow condition $\mathrm{d}t = \bar{a}\,\mathrm{d}\bar{\eta}$ and $\mathrm{d}\tau_{\mathrm{w}} = a_{\mathrm{w}}\mathrm{d}\eta_{\mathrm{w}}$. But $\mathrm{d}t = \bar{\gamma}\mathrm{d}\tau_{\mathrm{w}}$ and $a_{\mathrm{w}} = {f_{\mathrm{wi}}}^{-1/3}(1 - f_{\mathrm{v}})\,\bar{a}$. Hence *on radial null geodesics*

$$\mathrm{d}\eta_{\mathrm{w}} = \frac{{f_{\mathrm{wi}}}^{1/3}\mathrm{d}\bar{\eta}}{\bar{\gamma}\,(1 - f_{\mathrm{v}})^{1/3}}$$

  Define $\eta_{\mathrm{w}}$ by integral of above on radial null-geodesics.

- Extend spatially flat wall geometry to dressed geometry

$$\mathrm{d}s^2 = -\mathrm{d}\tau_{\mathrm{w}}^2 + a^2(\tau_{\mathrm{w}})\left[\mathrm{d}\bar{\eta}^2 + r_{\mathrm{w}}^2(\bar{\eta},\tau_{\mathrm{w}})\,\mathrm{d}\Omega^2\right]$$

  where $r_{\mathrm{w}} \equiv \bar{\gamma}\,(1 - f_{\mathrm{v}})^{1/3}\,{f_{\mathrm{wi}}}^{-1/3}\eta_{\mathrm{w}}(\bar{\eta},\tau_{\mathrm{w}})$, $a = \bar{a}/\bar{\gamma}$.

# Dressed cosmological parameters

- N.B. The extension is NOT an isometry

$$\text{N.B.}\qquad \begin{aligned} \mathrm{d}s^2_{\mathit{fi}} &= -\mathrm{d}\tau_{\mathrm{w}}^2 + {a_{\mathrm{w}}}^2(\tau_{\mathrm{w}})\left[\mathrm{d}\eta_{\mathrm{w}}^2 + \eta_{\mathrm{w}}^2 \mathrm{d}\Omega^2\right] \\ \to \mathrm{d}s^2 &= -\mathrm{d}\tau_{\mathrm{w}}^2 + a^2\left[\mathrm{d}\bar\eta^2 + r_{\mathrm{w}}^2(\bar\eta,\tau_{\mathrm{w}})\,\mathrm{d}\Omega^2\right] \end{aligned}$$

- Extended metric is an effective “spherical Buchert geometry” adapted to wall rulers and clocks.
- Since $\mathrm{d}\bar\eta = \mathrm{d}t/\bar a = \bar\gamma\,\mathrm{d}\tau_{\mathrm{w}}/\bar a = \mathrm{d}\tau_{\mathrm{w}}/a$, this leads to *dressed parameters* which do not sum to 1, e.g.,

$$\Omega_M = \bar\gamma^3 \bar\Omega_M \,.$$

- Dressed average Hubble parameter

$$H = \frac{1}{a}\frac{\mathrm{d}a}{\mathrm{d}\tau_{\mathrm{w}}} = \frac{1}{\bar a}\frac{\mathrm{d}\bar a}{\mathrm{d}\tau_{\mathrm{w}}} - \frac{1}{\bar\gamma}\frac{\mathrm{d}\bar\gamma}{\mathrm{d}\tau_{\mathrm{w}}}$$

# Dressed cosmological parameters

- $H$ is greater than wall Hubble rate; smaller than void Hubble rate measured by wall (or any one set of) clocks

$$\bar{H}(t) = \frac{1}{\bar{a}}\frac{\mathrm{d}\bar{a}}{\mathrm{d}t} = \frac{1}{a_{\mathrm{v}}}\frac{\mathrm{d}a_{\mathrm{v}}}{\mathrm{d}\tau_{\mathrm{v}}} = \frac{1}{a_{\mathrm{w}}}\frac{\mathrm{d}a_{\mathrm{w}}}{\mathrm{d}\tau_{\mathrm{w}}} < H < \frac{1}{a_{\mathrm{v}}}\frac{\mathrm{d}a_{\mathrm{v}}}{\mathrm{d}\tau_{\mathrm{w}}}$$

- For tracker solution $H = (4{f_{\mathrm{v}}}^2 + f_{\mathrm{v}} + 4)/6t$
- Dressed average deceleration parameter

$$q = \frac{-1}{H^2 a^2}\frac{\mathrm{d}^2 a}{\mathrm{d}\tau_{\mathrm{w}}^2}$$

Can have $q < 0$ even though $\bar{q} = \frac{-1}{\bar{H}^2\bar{a}^2}\frac{\mathrm{d}^2\bar{a}}{\mathrm{d}t^2} > 0$; difference of clocks important.

# Apparent cosmic acceleration

- Volume average observer sees no apparent cosmic acceleration

$$\bar{q} = \frac{2\,(1 - f_{\rm v})^2}{(2 + f_{\rm v})^2}\,.$$

As $t \to \infty$, $f_{\rm v} \to 1$ and $\bar{q} \to 0^+$.

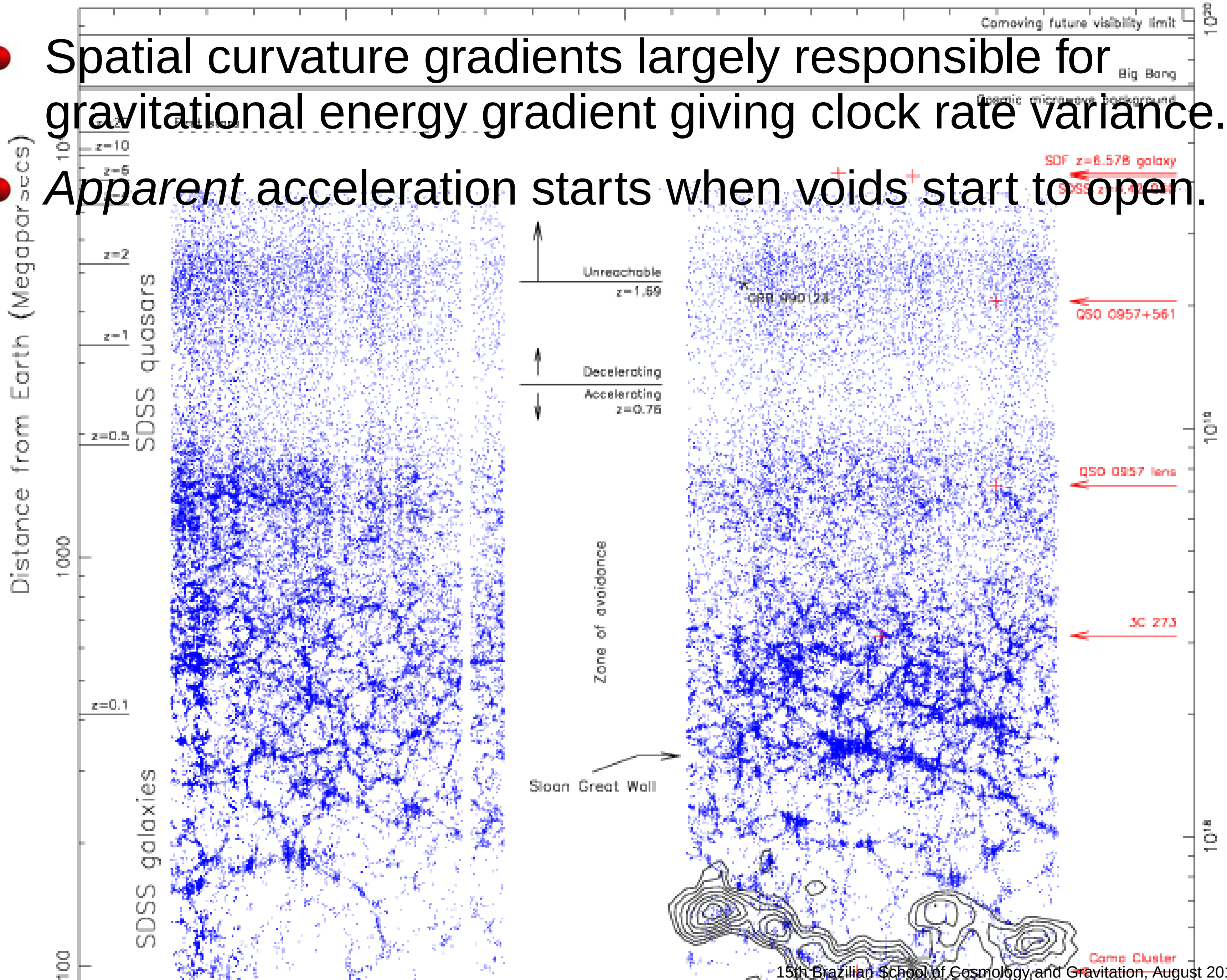
- A wall observer registers apparent cosmic acceleration

$$q = \frac{-\,(1 - f_{\rm v})\,(8 f_{\rm v}{}^3 + 39 f_{\rm v}{}^2 - 12 f_{\rm v} - 8)}{\left(4 + f_{\rm v} + 4 f_{\rm v}{}^2\right)^2}\,,$$

Effective deceleration parameter starts at $q \sim \frac{1}{2}$, for small $f_{\rm v}$; changes sign when $f_{\rm v} = 0.58670773\ldots$, and approaches $q \to 0^-$ at late times.

# Cosmic coincidence problem solved

- Spatial curvature gradients largely responsible for gravitational energy gradient giving clock rate variance.
- *Apparent* acceleration starts when voids start to open.



# Redshift, luminosity distance

- Cosmological redshift (last term tracker solution)

$$z+1=\frac{a}{a_0}=\frac{\bar a_0\bar\gamma}{\bar a\bar\gamma_0}=\frac{(2+f_{\rm v})f_{\rm v}{}^{1/3}}{3f_{\rm v0}{}^{1/3}\bar H_0 t}=\frac{2^{4/3}t^{1/3}(t+b)}{f_{\rm v0}{}^{1/3}\bar H_0 t(2t+3b)^{4/3}}\,,$$

  where $b=2(1-f_{\rm v0})(2+f_{\rm v0})/[9f_{\rm v0}\bar H_0]$

- Dressed luminosity distance relation $d_L=(1+z)D$ where the *effective comoving distance* to a redshift $z$ is $D=a_0 r_{\rm w}$, with

$$r_{\rm w}=\bar\gamma\,(1-f_{\rm v})^{1/3}\int_t^{t_0}\frac{{\rm d}t'}{\bar\gamma(t')(1-f_{\rm v}(t'))^{1/3}\bar a(t')}\,.$$

# Redshift, luminosity distance

- Perform integral for tracker solution

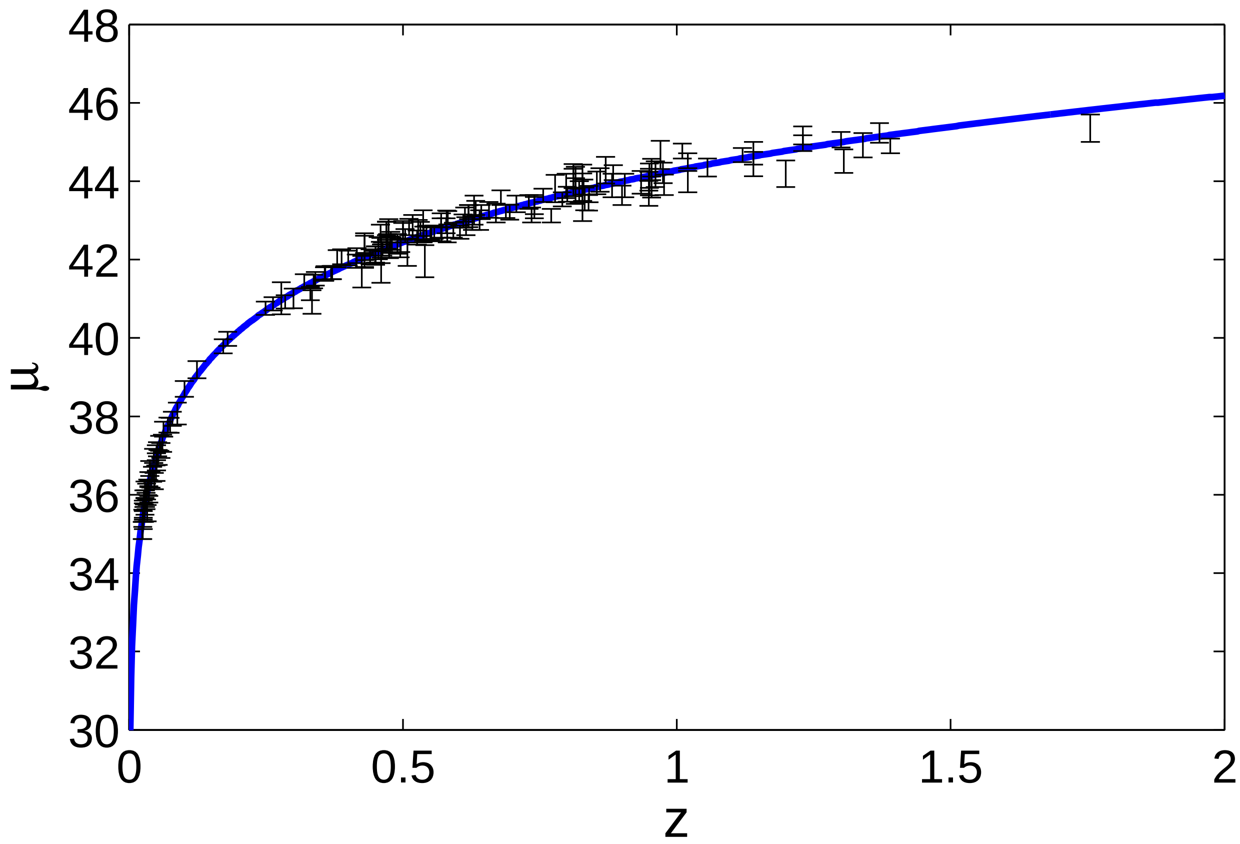
$$
\begin{aligned}
D_A = \frac{D}{1+z} = \frac{d_L}{(1+z)^2} &= (t)^{\frac{2}{3}} \int_t^{t_0} \frac{2\mathrm{d}t'}{(2+f_{\mathrm{v}}(t'))(t')^{2/3}} \\
&= t^{2/3}(\mathcal{F}(t_0) - \mathcal{F}(t))
\end{aligned}
$$

where

$$
\begin{aligned}
\mathcal{F}(t) &= 2t^{1/3} + \frac{b^{1/3}}{6} \ln\left( \frac{(t^{1/3}+b^{1/3})^2}{t^{2/3} - b^{1/3}t^{1/3} + b^{2/3}} \right) \\
&\quad + \frac{b^{1/3}}{\sqrt{3}} \tan^{-1}\left( \frac{2t^{1/3} - b^{1/3}}{\sqrt{3}\, b^{1/3}} \right).
\end{aligned}
$$

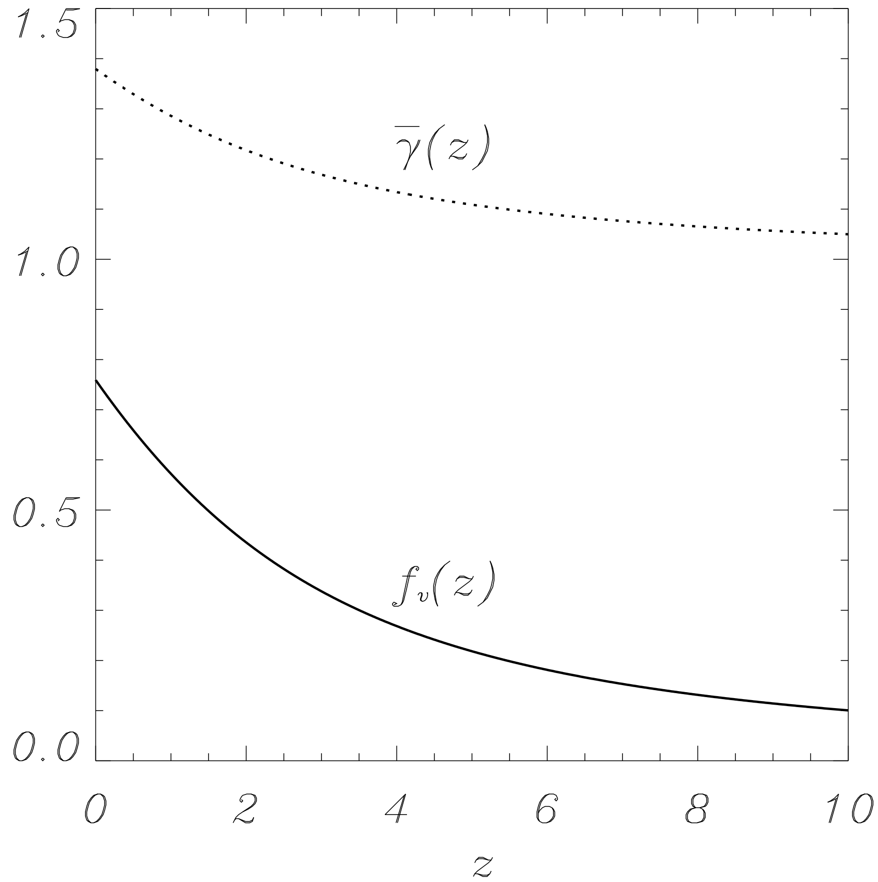
$t$ given implicitly in terms of $z$ by previous relation

# Sample Hubble diagram with SneIa



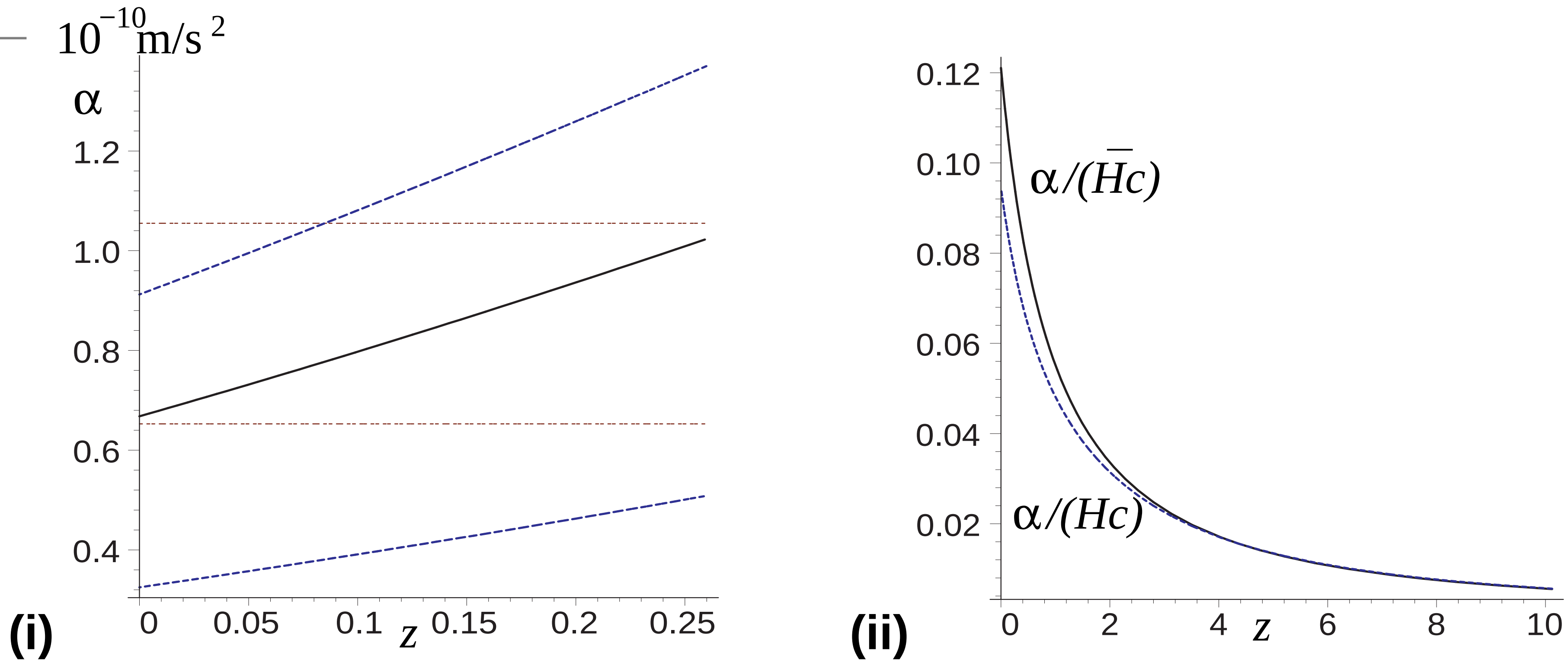

- Type Ia supernovae of Riess07 Gold data set fit with $\chi^2$ per degree of freedom $= 0.9$
- Statistically indistinguishable from $\Lambda$CDM.

# Void fraction, lapse function




The void fraction, $f_{\rm v}$, (solid line), and lapse of volume–average observers with respect to wall observers, $\bar{\gamma}$, (dotted line) as a function of redshift for the TS model with $H_0 = 62.0$ km/s/Mpc, $\bar{\gamma}_0 = 1.38$, $f_{\rm v0} = 0.759$.

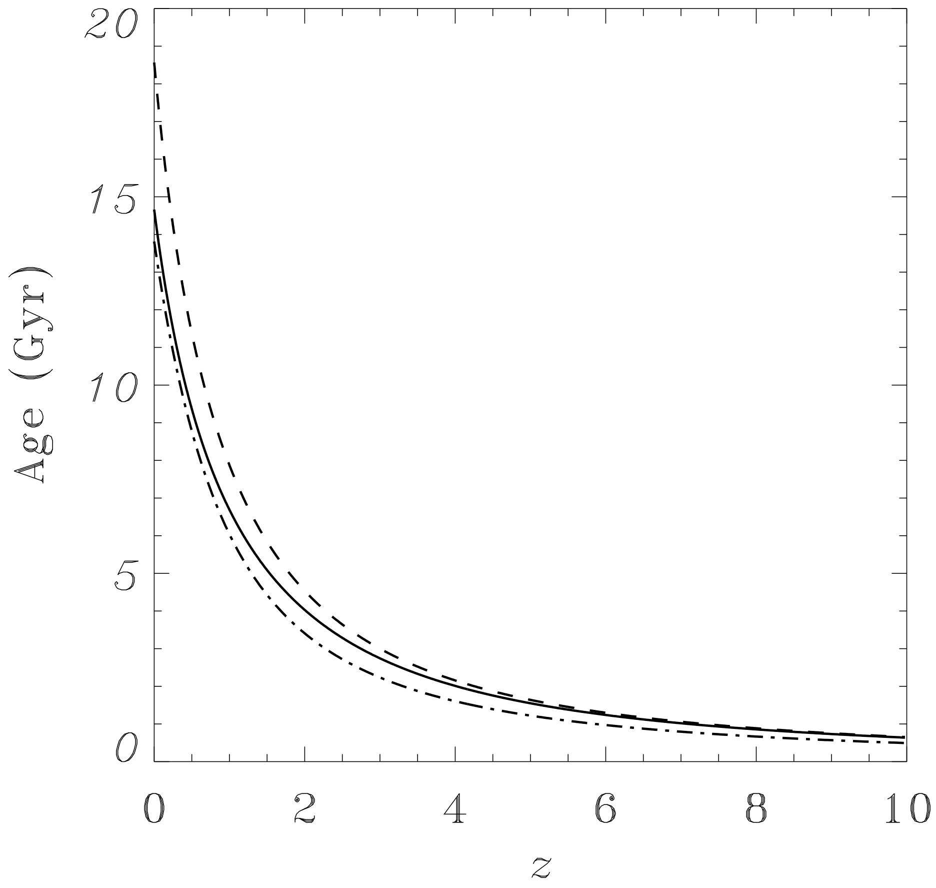
# CEP relative deceleration scale




By equivalence principle the instantaneous relative deceleration of backgrounds gives an instantaneous 4-acceleration of magnitude $\alpha = H_0 c\bar{\gamma}\dot{\bar{\gamma}}/(\sqrt{\bar{\gamma}^2 - 1})$ beyond which *weak field cosmological general relativity* will be changed from Newtonian expectations: **(i)** as absolute scale nearby; **(ii)** divided by Hubble parameter to large $z$.

- For $z \lesssim 0.25$, coincides with empirical MOND scale $\alpha_0 = 1.2^{+0.3}_{-0.2} \times 10^{-10}\,\text{ms}^{-2} h_{75}^2 = 8.1^{+2.5}_{-1.6} \times 10^{-11}\text{ms}^{-2}$ for $H_0 = 61.7$ km/s/Mpc.

# Age of universe

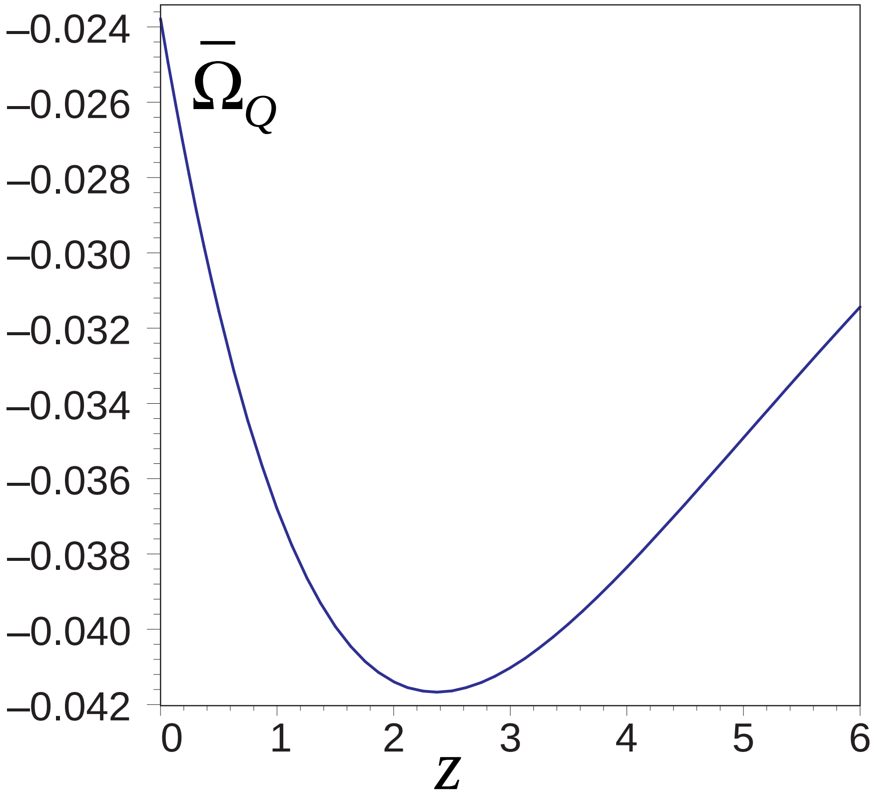



The expansion age $\tau_{\rm w}(z)$ for observers in galaxies (solid) and $t(z)$ for volume-average observers (dashed) for previous TS model. Comparison $\Lambda$CDM model: $H_0 = 71.0$ km/s/Mpc, $\Omega_{M0} = 0.268$, $\Omega_{\Lambda 0} = 0.732$ (dot–dashed). Note: $\tau_{\rm w0} = 14.6$ Gyr, $t_0 = 18.6$ Gyr.

# Alleviation of age problem

- Old structures seen at large redshifts are a challenge for $\Lambda$CDM.
- Problem alleviated here; expansion age is increased, by an increasingly larger relative fraction at larger redshifts, e.g., for best–fit values
  $\Lambda$CDM $\tau = 0.85$ Gyr at $z = 6.42$, $\tau = 0.365$ Gyr at $z = 11$
  TS $\tau = 1.14$ Gyr at $z = 6.42$, $\tau = 0.563$ Gyr at $z = 11$
- Present age of universe for best-fit is $\tau_0 \simeq 14.7$ Gyr for wall observer; $t_0 \simeq 18.6$ Gyr for volume–average observer.
- Would the under–emptiness of voids in Newtonian N-body simulations may be an issue in open universe with bare parameters $\bar{\Omega}_M = 0.125$, $t_0 \simeq 18.6$ Gyr?

# Magnitude of backreaction

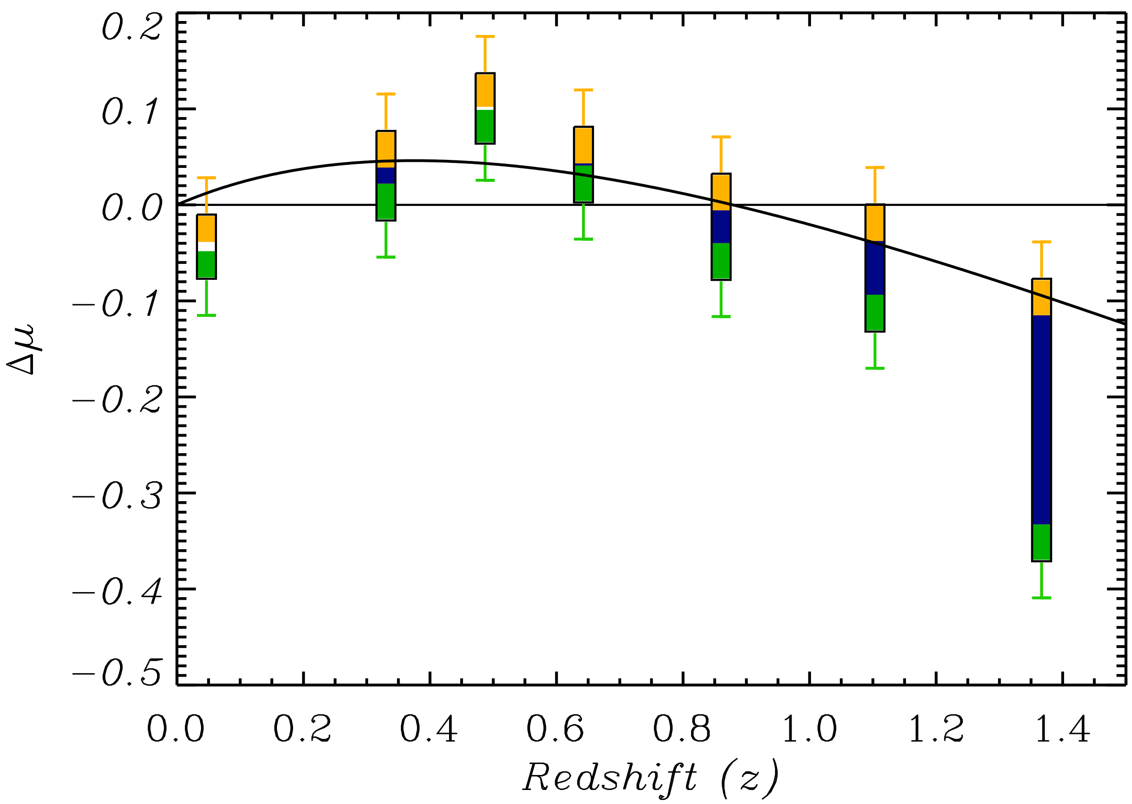



- Magnitude of $\bar{\Omega}_{\mathcal{Q}}$ determines departure from FLRW evolution: it is 4.2% at most
- There is a closest FLRW universe: open model with $\bar{\Omega}_{M0} = 0.125$, $t_0 = 18.6$ Gyr.

# Lecture 4

# Observational tests of the timescape cosmology

# Test 1: SneIa luminosity distances



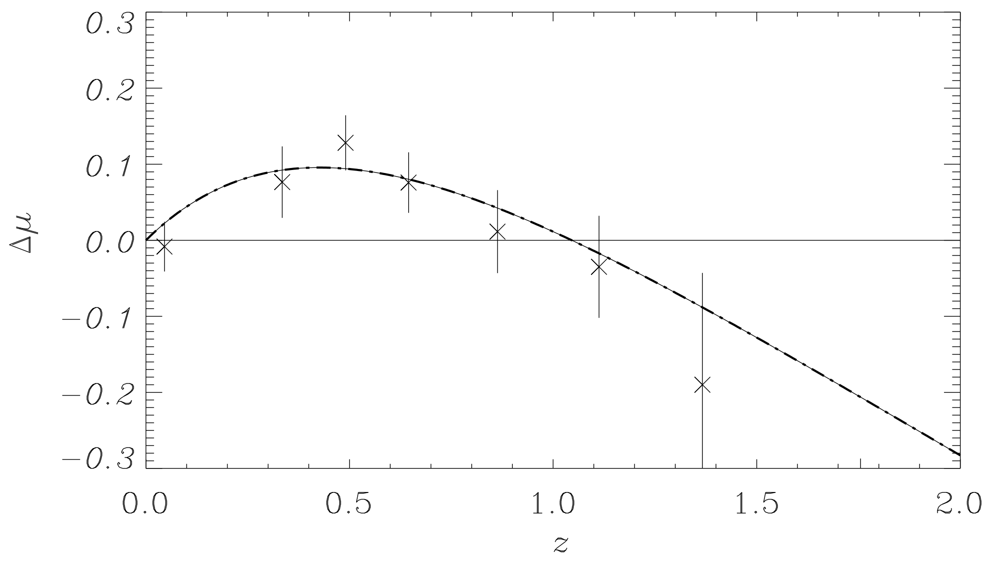

- Difference of model apparent magnitude and that of empty Milne universe of same $H_0 = 61.7$ km/s/Mpc, for Riess 2007 “gold data”. *Note*: residual depends on the expansion rate of the Milne universe subtracted ($2\sigma$ limits on $H_0$ indicated by whiskers)

# Comparison $\Lambda$CDM models

Best-fit spatially flat $\Lambda$CDM

$H_0 = 62.7$ km/s/Mpc,

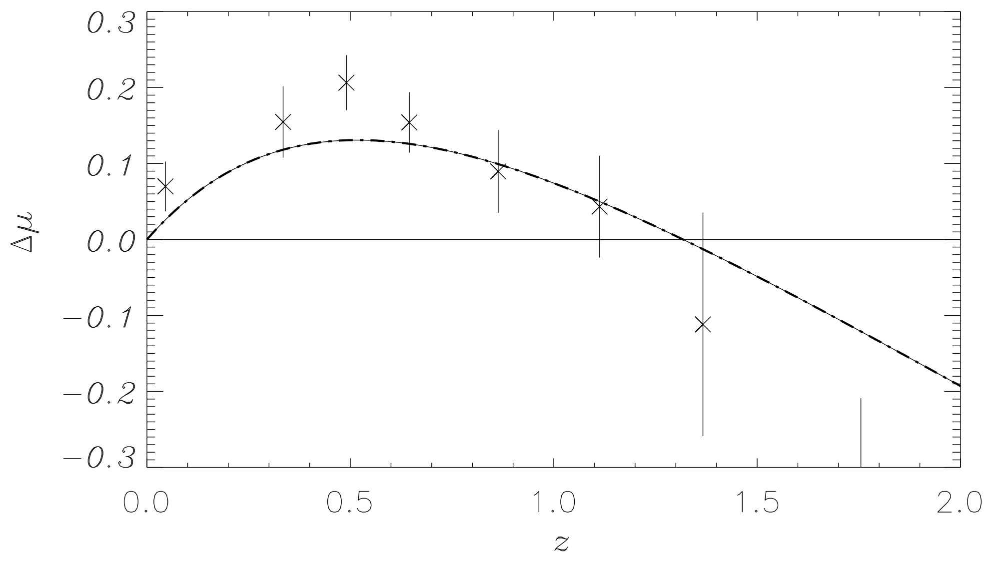
$\Omega_{M0} = 0.34$, $\Omega_{\Lambda 0} = 0.66$



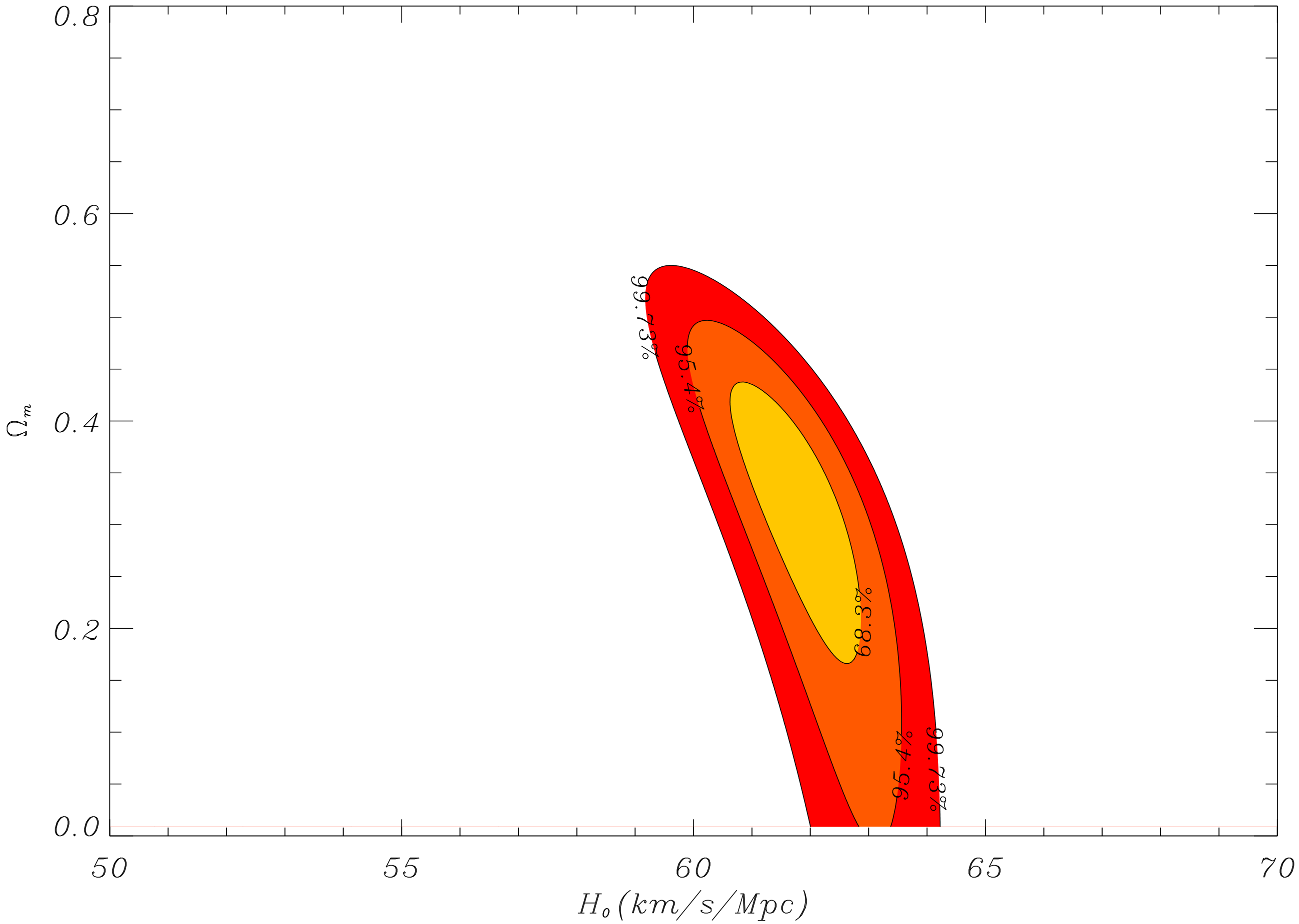

Riess astro-ph/0611572, p. 63

$H_0 = 65$ km/s/Mpc,

$\Omega_{M0} = 0.29$, $\Omega_{\Lambda 0} = 0.71$



# Test 1: SneIa luminosity distances




Two free parameters $H_0$ versus $\Omega_{M0}$ (dressed shown here), or alternatively “bare values”, constrained by Riess07 Gold data fit. (Normalization of $H_0$ not constrained by SneIa.)

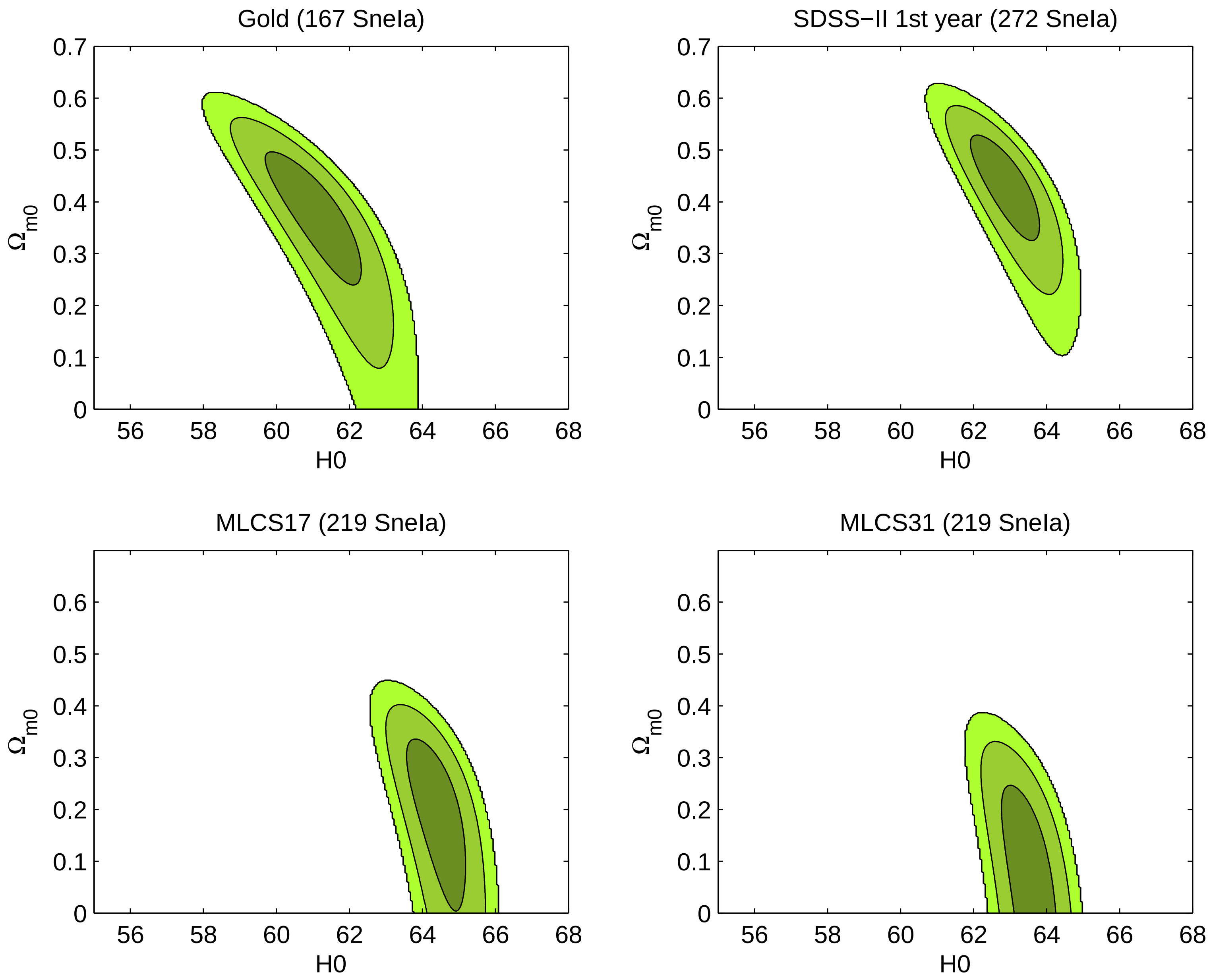
# Best fit parameters

- Hubble constant $H_0 + \Delta H_0 = 61.7^{+1.2}_{-1.1}$ km/s/Mpc
- present void volume fraction $f_{\rm v0} = 0.76^{+0.12}_{-0.09}$
- bare density parameter $\bar{\Omega}_{M0} = 0.125^{+0.060}_{-0.069}$
- dressed density parameter $\Omega_{M0} = 0.33^{+0.11}_{-0.16}$
- non–baryonic dark matter / baryonic matter mass ratio $(\bar{\Omega}_{M0} - \bar{\Omega}_{B0})/\bar{\Omega}_{B0} = 3.1^{+2.5}_{-2.4}$
- bare Hubble constant $\bar{H}_0 = 48.2^{+2.0}_{-2.4}$ km/s/Mpc
- mean lapse function $\bar{\gamma}_0 = 1.381^{+0.061}_{-0.046}$
- deceleration parameter $q_0 = -0.0428^{+0.0120}_{-0.0002}$
- wall age universe $\tau_0 = 14.7^{+0.7}_{-0.5}$ Gyr

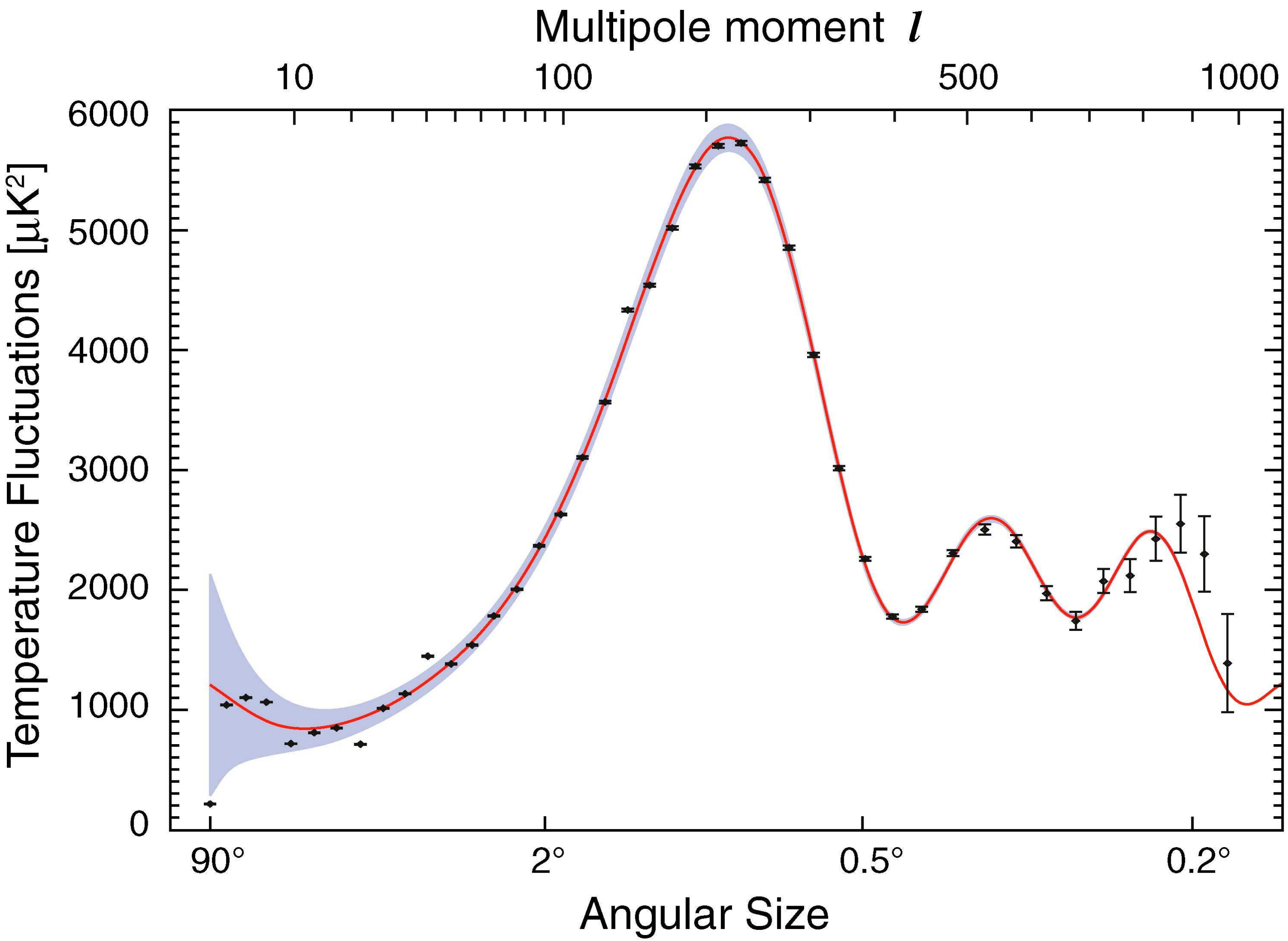
# Smale + DLW, MNRAS 413 (2011) 367

- SALT/SALTII fits (Constitution,SALT2,Union2) favour $\Lambda$CDM over TS: $\ln B_{\mathrm{TS:\Lambda CDM}} = -1.06, -1.55, -3.46$
- MLCS2k2 (fits MLCS17,MLCS31,SDSS-II) favour TS over $\Lambda$CDM: $\ln B_{\mathrm{TS:\Lambda CDM}} = 1.37, 1.55, 0.53$
- Different MLCS fitters give different best-fit parameters; e.g. with cut at statistical homogeneity scale, for MLCS31 (Hicken et al 2009) $\Omega_{M0} = 0.12^{+0.12}_{-0.11}$;
MLCS17 (Hicken et al 2009) $\Omega_{M0} = 0.19^{+0.14}_{-0.18}$;
SDSS-II (Kessler et al 2009) $\Omega_{M0} = 0.42^{+0.10}_{-0.10}$
- Supernovae systematics (reddening/extinction, intrinsic colour variations) must be understood to distinguish models
- Foregrounds, and inclusion of SneIa below SSH an important issue (more in next lecture)

# Supernovae systematics



# CMB anisotropies




Power in CMB temperature anisotropies versus angular size of fluctuation on sky

# CMB temperature calibration

- Volume average observers measure mean CMB temperature

$$\bar{T} = \bar{\gamma}^{-1} T$$

where $T$ is CMB temperature measured by wall observers; $T_0 = 2.725$ K, $\bar{T} = 1.975$ K.

- Number density of photons at the volume average is

$$\bar{n}_\gamma = \frac{2\,\zeta(3)}{\pi^2} \left( \frac{k_B \bar{T}}{\hbar c} \right)^3 = \frac{n_\gamma}{\bar{\gamma}^3}\,,$$

yielding $\bar{n}_{\gamma 0} = 4.105\,{\bar{\gamma}_0}^{-3} \times 10^8$ m$^{-3}$ at present.

- This is needed to calibrate light element abundances from primordial nucleosynthesis.

# Photon to baryon ratio

- Number density of baryons at the volume average is

$$\bar{n}_B = \frac{3\bar{H}_0^2\bar{\Omega}_{B0}}{8\pi G\, m_p} = \frac{11.2 f_B \bar{\Omega}_{M0} h^2}{(\bar{\gamma}_0 - \bar{\gamma}'_0)^2}\ \mathrm{m}^{-3},$$

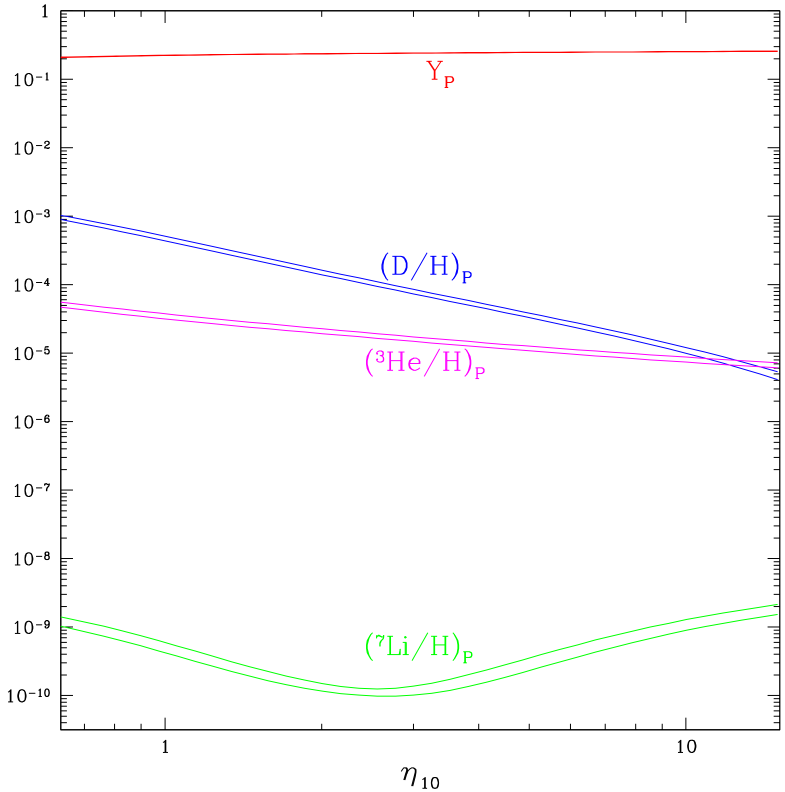
where $\bar{\gamma}'_0 \equiv \frac{1}{\bar{H}_0} \frac{\mathrm{d}\bar{\gamma}}{\mathrm{d}t}\Big|_{t_0}$, $m_p$ is the proton mass, $f_B \equiv \bar{\Omega}_{B0}/\bar{\Omega}_{M0}$.

- Hence the average photon to baryon ratio is

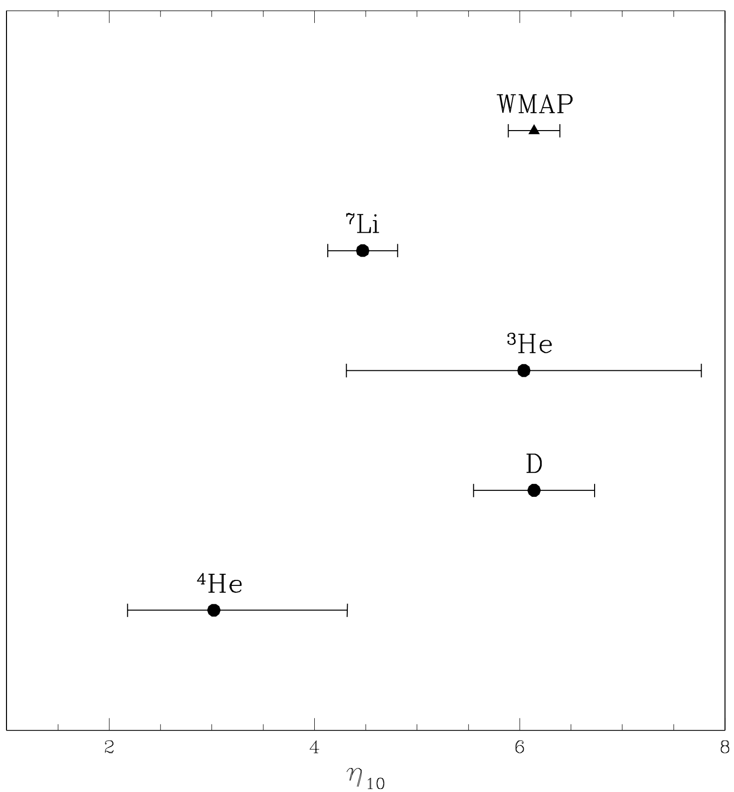
$$\eta_{B\gamma} = \frac{\bar{n}_B}{\bar{n}_\gamma} = \frac{2.736 \times 10^{-8} f_B \bar{\Omega}_{M0} \bar{\gamma}_0^3 h^2}{(\bar{\gamma}_0 - \bar{\gamma}'_0)^2},$$

as compared to $\eta_{\mathrm{FLRW}} = 2.736 \times 10^{-8} f_B \Omega_M h^2$.

# Li abundance anomaly




(i)

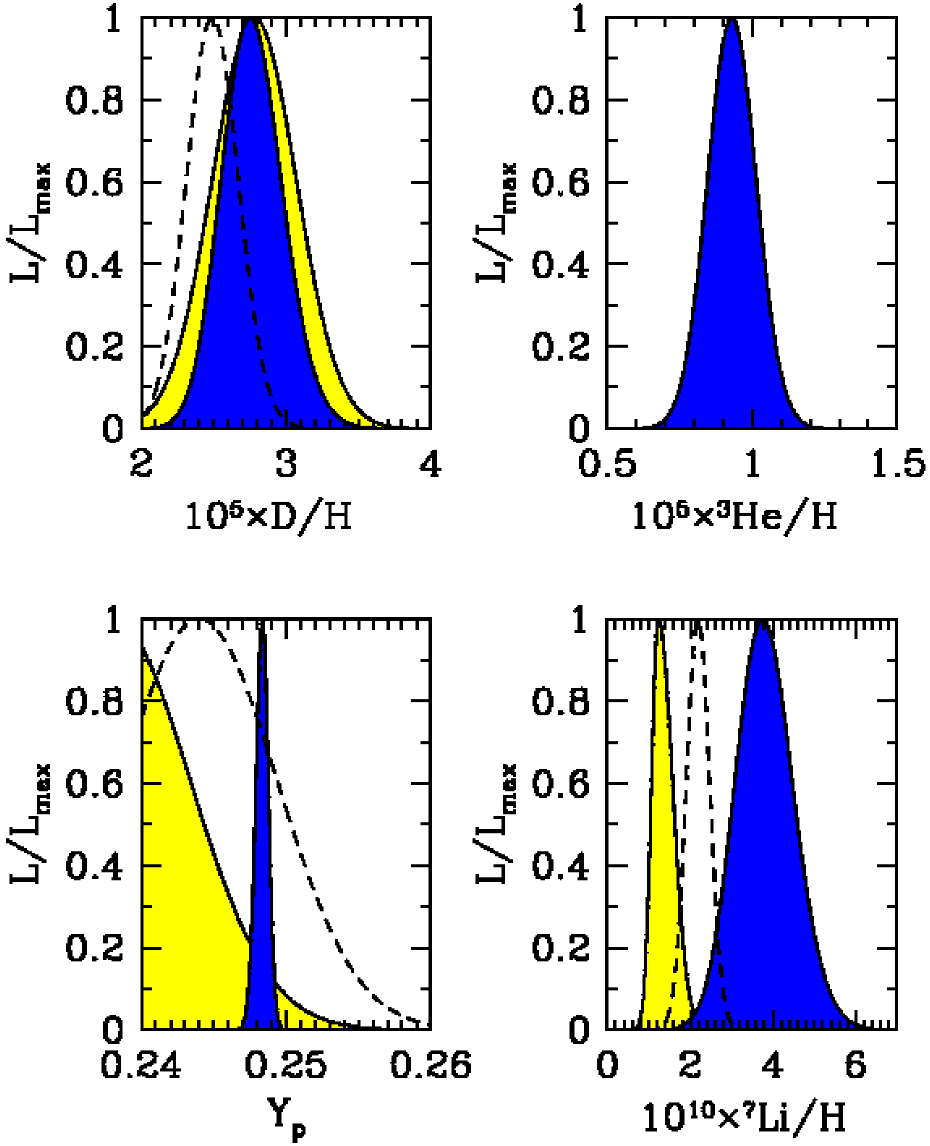



(ii)

- Expected abundances for different values of the parameter $\eta_{10} \equiv 10^{10}\eta_{B\gamma}$ (left), and measurements (right) (Steigman 2006), with $1\sigma$ uncertainties.

# Li abundance anomaly

Big-bang nucleosynthesis, light element abundances and WMAP with $\Lambda$CDM cosmology.



# Resolution of Li abundance anomaly?

- Prior to WMAP in 2003 favoured ratio was $\eta_{B\gamma} = 4.6–5.6 \times 10^{-10}$; after WMAP $\eta_{B\gamma} = 6.1^{+0.3}_{-0.2} \times 10^{-10}$
- Conventional dressed parameter $\Omega_{M0} = 0.33$ for wall observer means $\bar{\Omega}_{M0} = 0.125$ for the volume–average.
- Conventional theory predicts the *volume–average baryon fraction* – with old BBN favoured $\eta_{B\gamma}$:

  $\bar{\Omega}_{B0} \simeq 0.027–0.033$; but this translates to a conventional dressed baryon fraction parameter $\Omega_{B0} \simeq 0.072–0.088$
- The ratio of baryonic matter to non–baryonic dark matter is increased to 1:3.
- Need fit to 2nd acoustic peak to tighten $\Omega_{B0}$ further

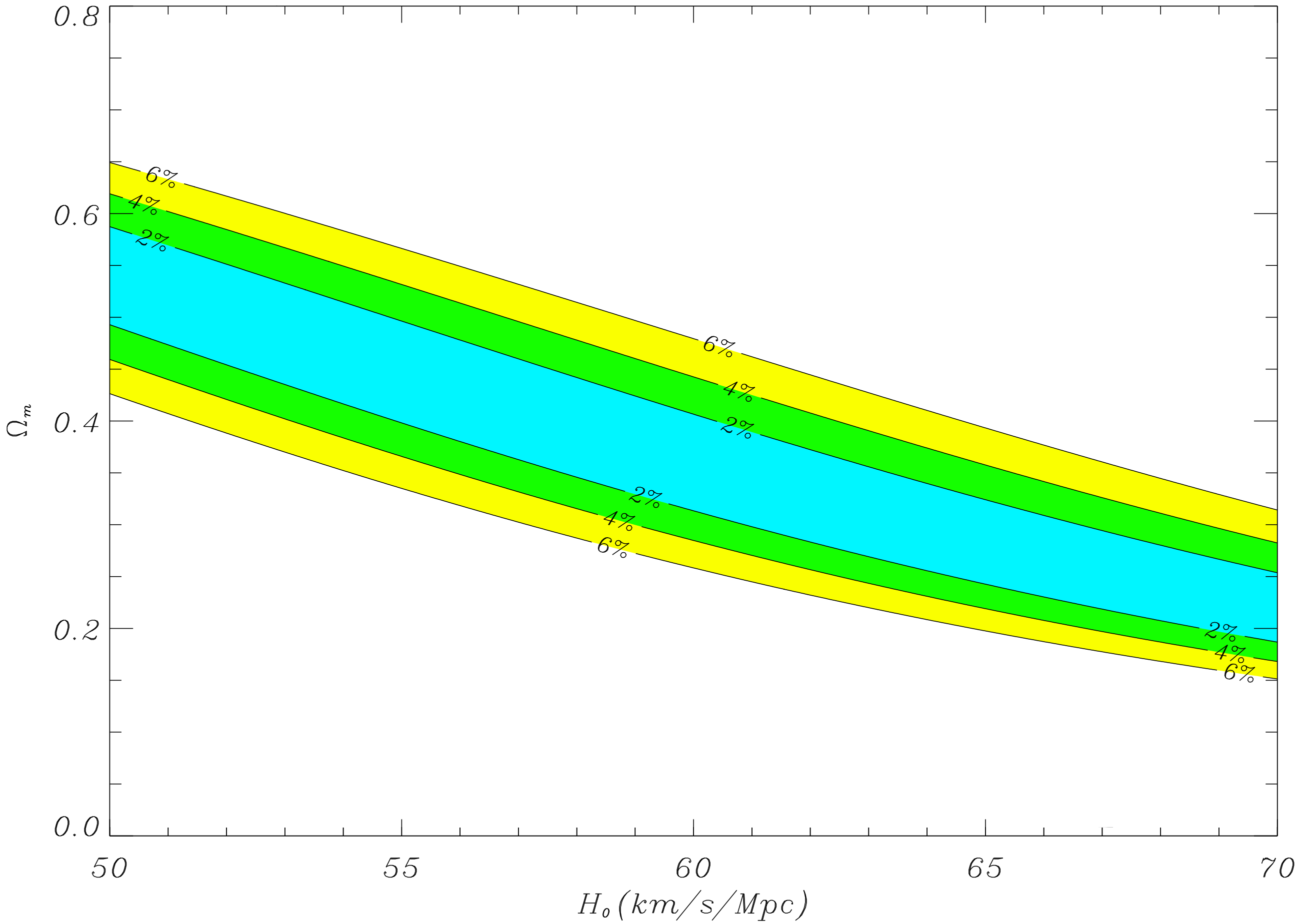
# CMB – calibration of sound horizon

- Physics at last–scattering same as perturbed Einstein–de Sitter model. What is changed is relative calibration between now and then.
- Estimated proper distance to comoving scale of the sound horizon at any epoch for volume–average observer [$\bar{x} = \bar{a}/\bar{a}_0$, so $\bar{x}_{\rm dec} = \bar{\gamma}_0^{-1}(1 + z_{\rm dec})^{-1}$]

$$\bar{D}_s = \frac{\bar{a}(t)}{\bar{a}_0} \frac{c}{\sqrt{3}\,\bar{H}_0} \int_0^{\bar{x}_{\rm dec}} \frac{{\rm d}\bar{x}}{\sqrt{(1 + 0.75\,\bar{\Omega}_{B0}\bar{x}/\bar{\Omega}_{\gamma 0})(\bar{\Omega}_{M0}\bar{x} + \bar{\Omega}_{R0})}},$$

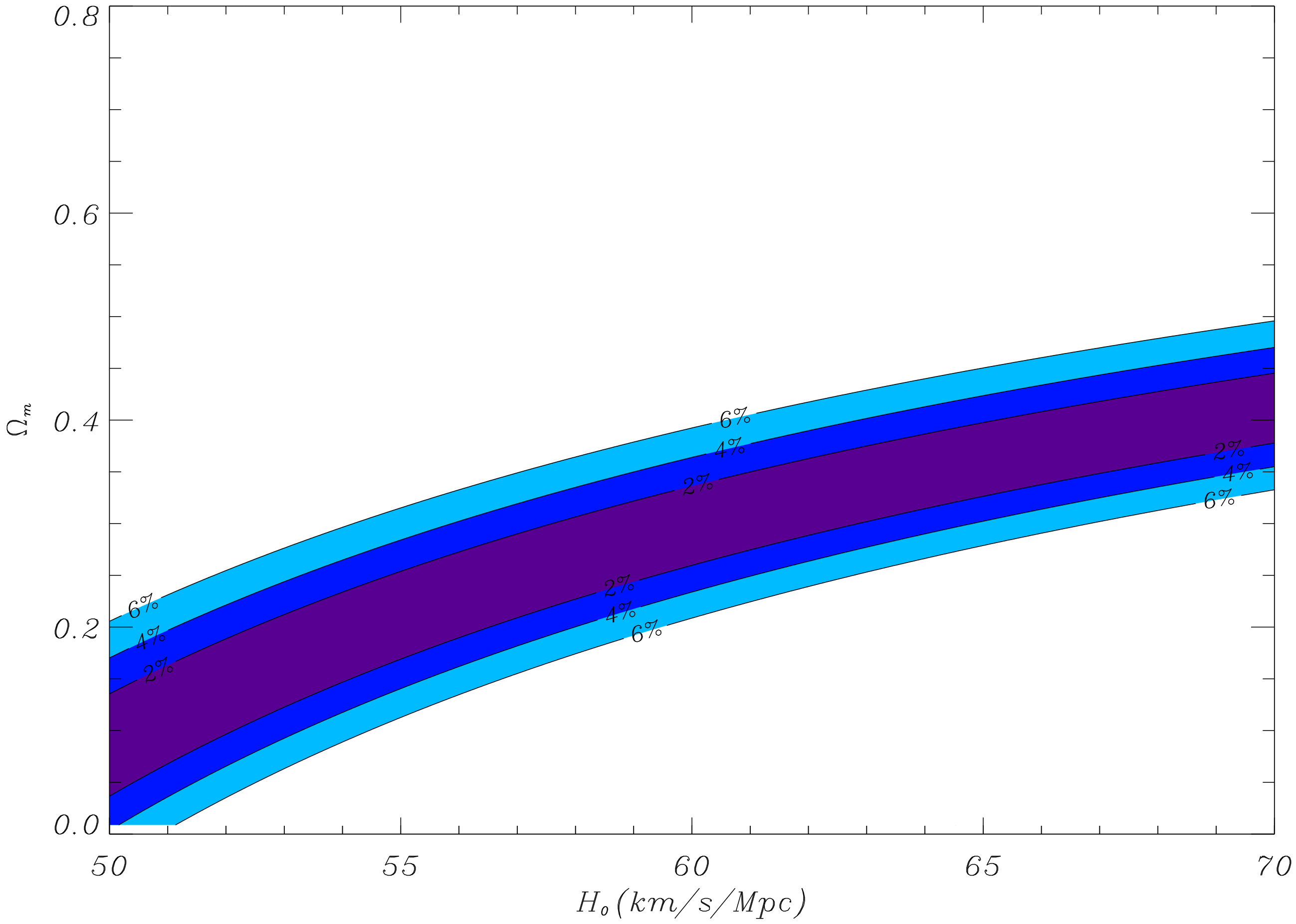
- For wall observer $D_s(\tau) = \bar{\gamma}^{-1} D_s$
- Volume–average observer measures lower mean CMB temperature ($\bar{T}_0 = T_0/\bar{\gamma}_0 \sim 1.98$ K, c.f. $T_0 \sim 2.73$ K in walls)

# Test 2: Angular diameter of sound horizon



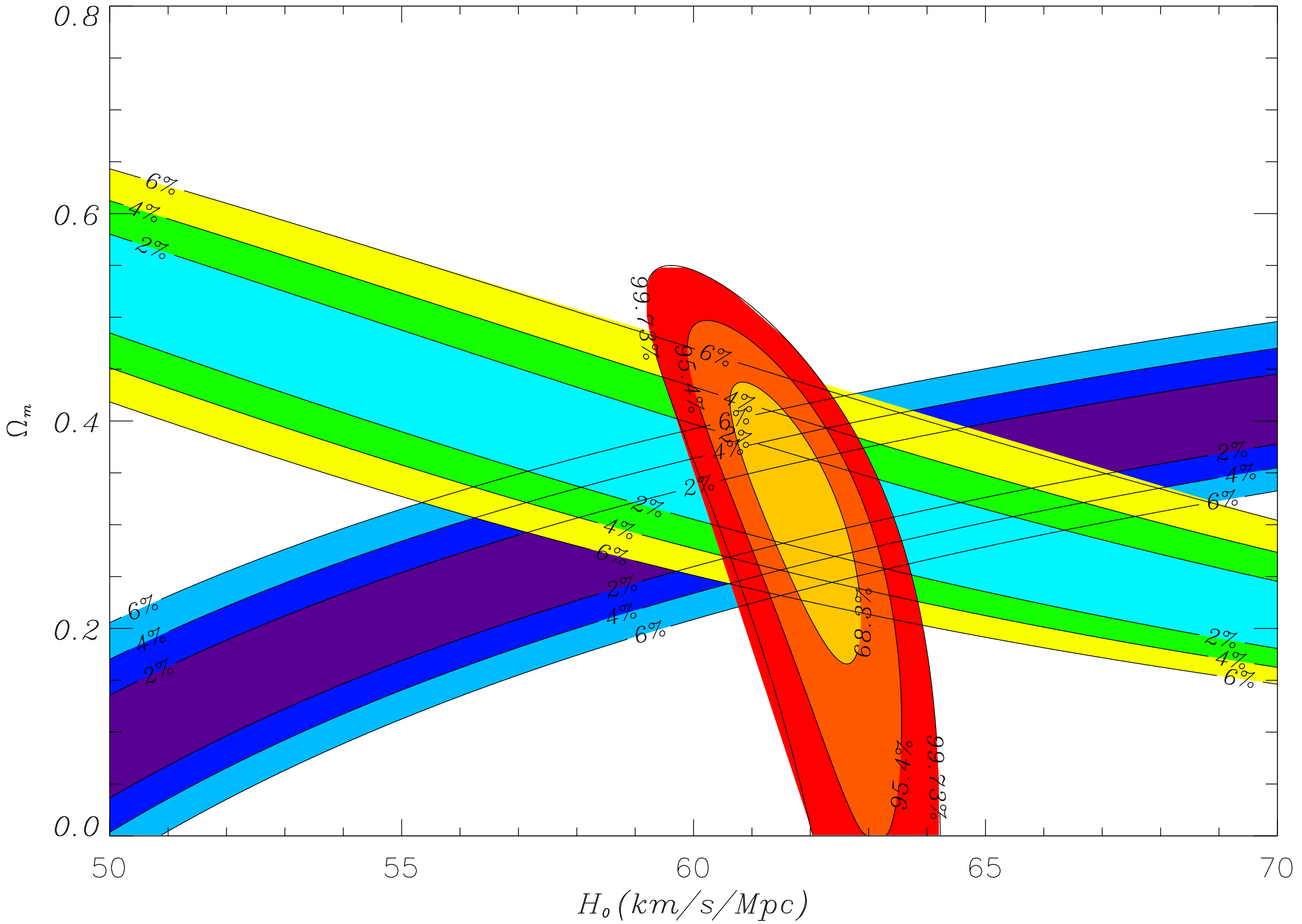

Parameters within the ($\Omega_{M0}$,$H_0$) plane which fit the angular scale of the sound horizon $\delta = 0.01$ rad deduced for WMAP, to within 2%, 4% and 6%, with $\eta_{B\gamma} = 4.6$–$5.6 \times 10^{-10}$.

# Test 3: Baryon acoustic oscillation scale



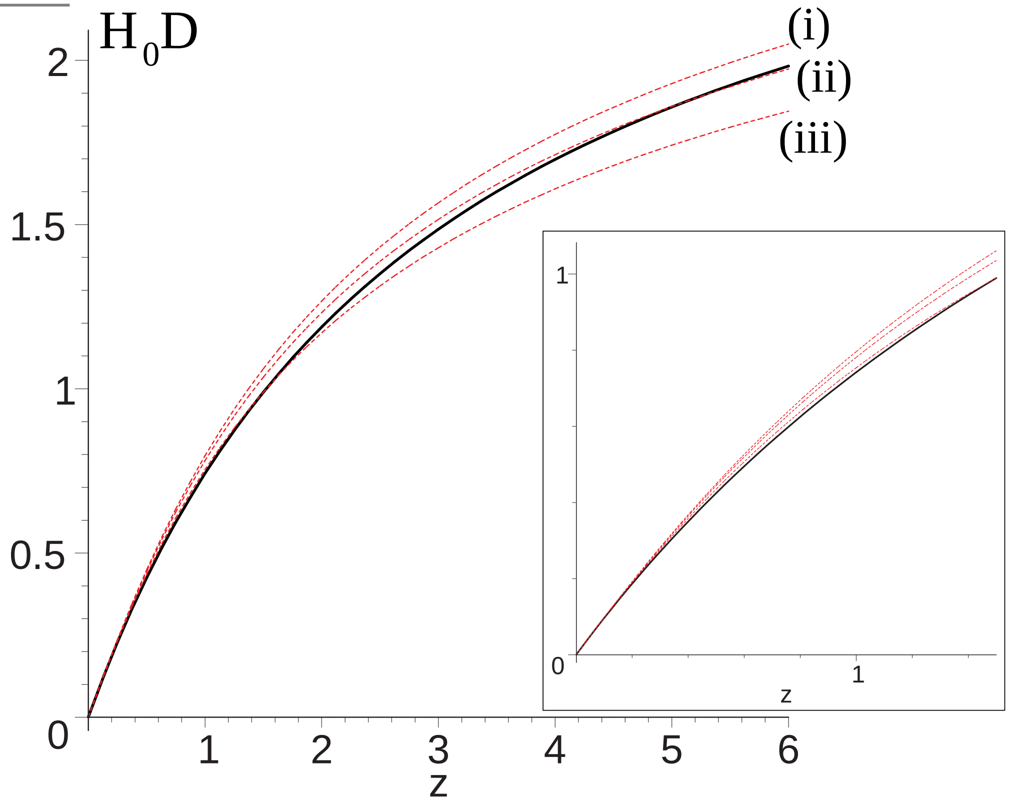

Parameters within the ($\Omega_m$,$H_0$) plane which fit the effective comoving baryon acoustic oscillation scale of $104h^{-1}$ Mpc, as seen in 2dF, SDSS etc. **Warning: crude estimate.**

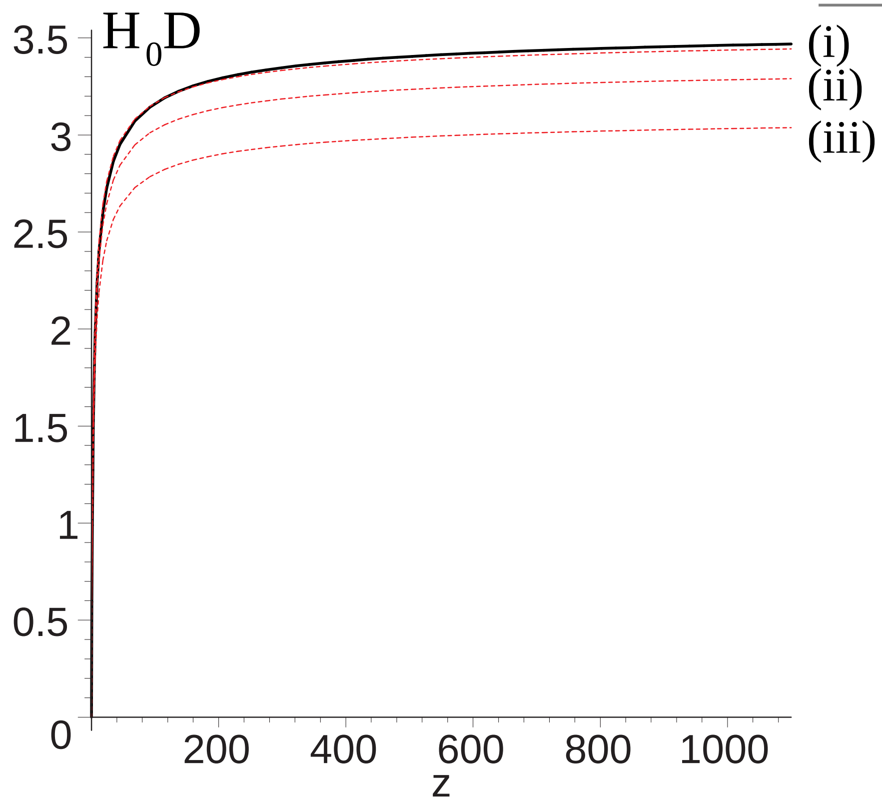
# Agreement of independent tests




Best–fit parameters: $H_0 = 61.7^{+1.2}_{-1.1}$ km/s/Mpc, $\Omega_m = 0.33^{+0.11}_{-0.16}$ ($1\sigma$ errors for SneIa only) [Leith, Ng & Wiltshire, ApJ 672 (2008) L91]

# Dressed "comoving distance" $D(z)$



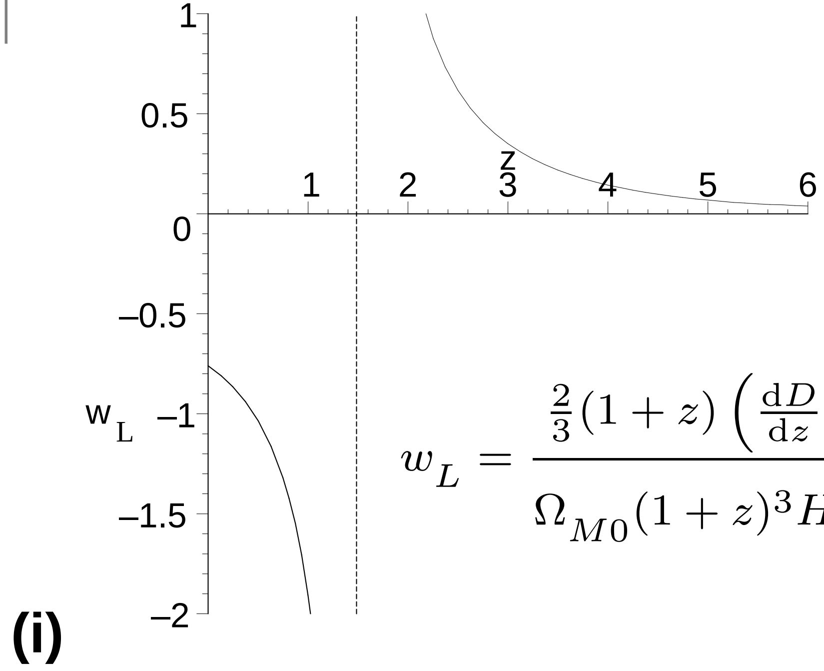



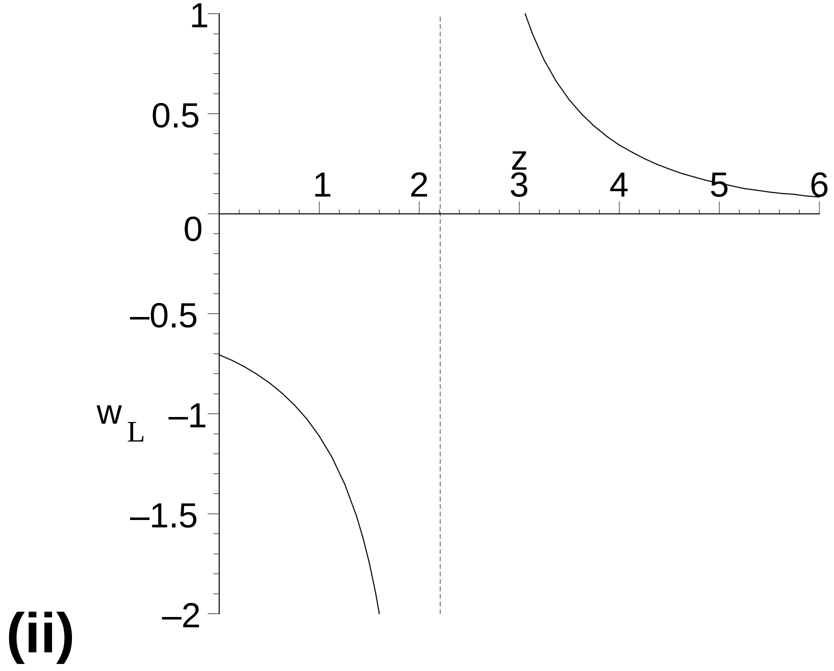

Best-fit TS model **(black line)** compared to 3 spatially flat $\Lambda$CDM models: **(i)** best–fit to WMAP5 only ($\Omega_\Lambda = 0.75$); **(ii)** joint WMAP5 + BAO + SneIa fit ($\Omega_\Lambda = 0.72$); **(iii)** best flat fit to (Riess07) SneIa only ($\Omega_\Lambda = 0.66$).

# Dressed "comoving distance" $D(z)$

- TS model closest to best–fit $\Lambda$CDM to *SneIa only* result ($\Omega_{M0} = 0.34$) at *low redshift*.
- TS model closest to best–fit *WMAP5 only* result, ($\Omega_{M0} = 0.249$) at *high redshift*
- Over the range $1 < z < 6.6$ tested by gamma ray bursters (GRBs) the TS model fits B. Schaefer's sample of 69 GRBs very slightly better than $\Lambda$CDM, not enough to be statistically significant - PR Smale, MNRAS 418 (2011) 2779
- Maximum difference in apparent magnitude between TS model and LCDM is only 0.18–0.34 mag at $z = 6$. Will need much data to get statistically significant test.

# Equivalent "equation of state"?



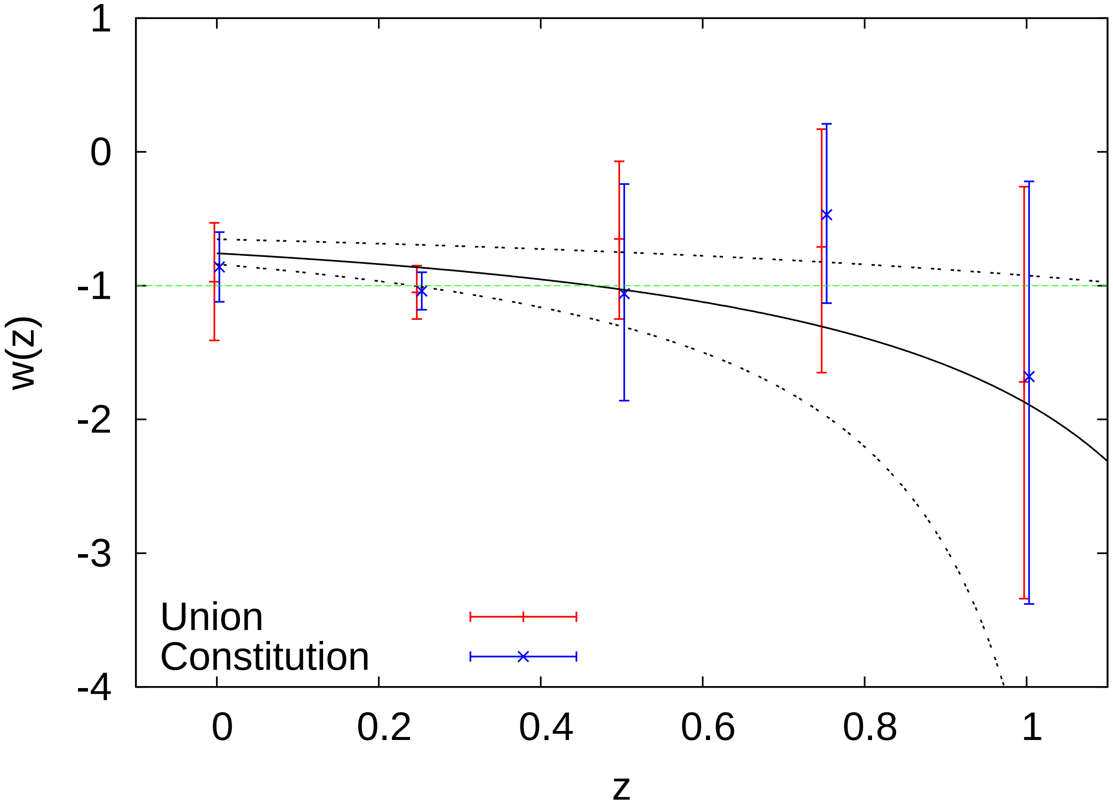




$$w_L = \frac{\frac{2}{3}(1+z)\left(\frac{\mathrm{d}D}{\mathrm{d}z}\right)^{-1}\frac{\mathrm{d}^2D}{\mathrm{d}z^2}+1}{\Omega_{M0}(1+z)^3H_0^2\left(\frac{\mathrm{d}D}{\mathrm{d}z}\right)^2-1}$$

A formal "dark energy equation of state" $w_L(z)$ for the best-fit TS model, $f_{v0} = 0.76$, calculated directly from $r_w(z)$: **(i)** $\Omega_{M0} = 0.33$; **(ii)** $\Omega_{M0} = 0.279$.

- Description by a "dark energy equation of state" makes no sense when there's no physics behind it; but average value $w_L \simeq -1$ for $z < 0.7$ makes empirical sense.

# Tests of "equation of state"




- Zhao and Zhang arXiv:0908.1568 find mild 95% evidence in favour of $w(z)$ crossing the phantom divide from $w > -1$ to $w < -1$ in the range $0.25 < w < 0.75$"
- Serra *et al*. arXiv:0908.3186 find "no evidence" of dynamical dark energy, but their analysis (above) also consistent with TS

# Sahni, Shafieloo and Starobinsky $Om(z)$

- Sahni, Shafieloo and Starobinsky propose a dark energy diagnostic

$$Om(z) = \frac{\frac{H^2(z)}{H_0^2} - 1}{(1+z)^3 - 1},$$

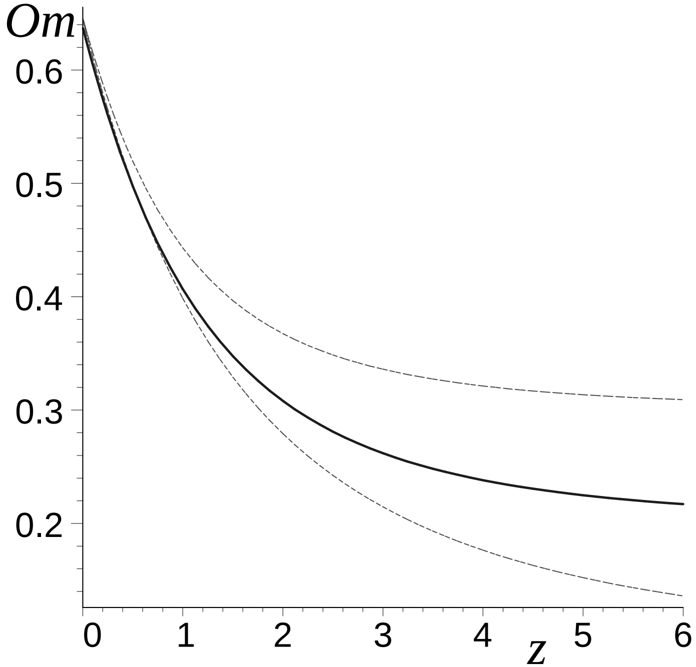
For FLRW models $OM(z) = \Omega_{M0}$ for $\Lambda$CDM $\forall z$ since

$$Om(z) = \Omega_{M0} + (1 - \Omega_{M0}) \frac{(1+z)^{3(1+w)} - 1}{(1+z)^3 - 1}.$$

but there are differences for other dark energy models.

- Note: $Om(z) \rightarrow \Omega_{M0}$ at large $z$ for all $w$.

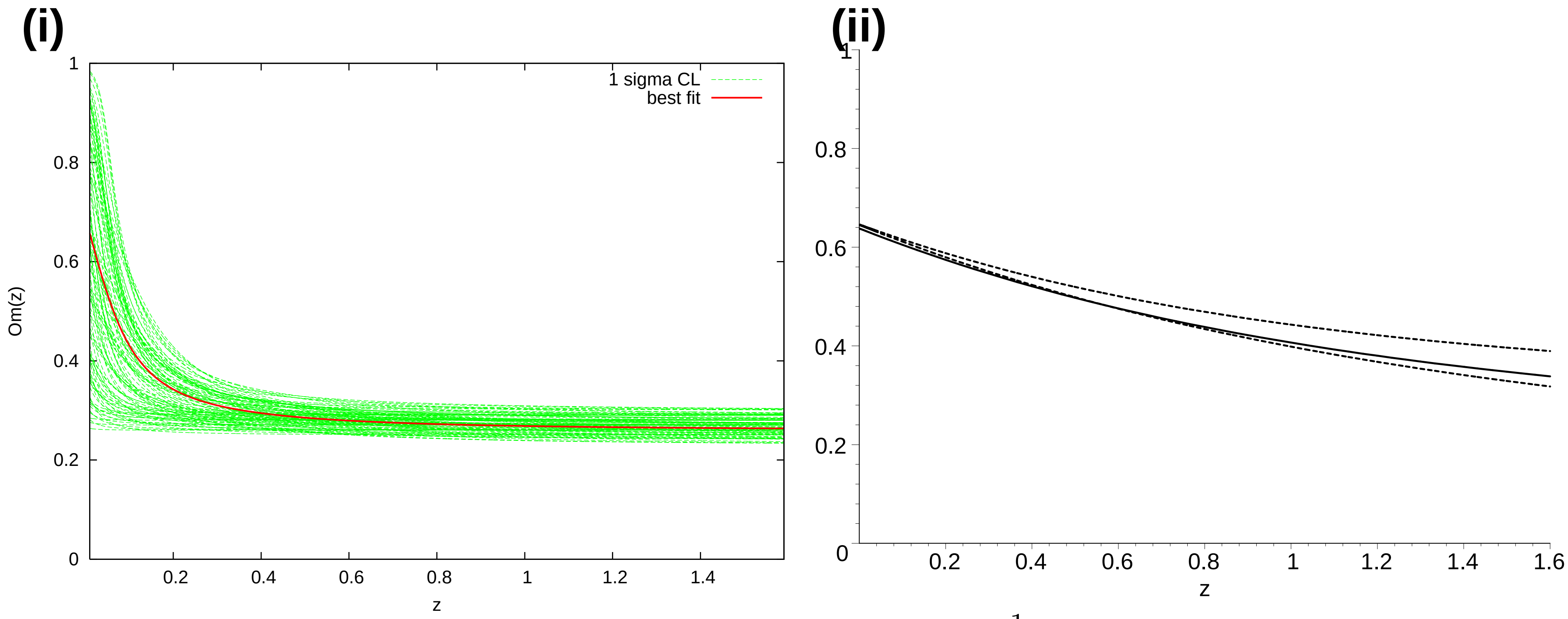
# $Om(z)$ test




$Om(z)$ for the tracker solution with best–fit value $f_{v0} = 0.762$ (solid line), and $1\sigma$ limits

- $Om(0) = \frac{2}{3}\, H'|_0 = \frac{2(8{f_{v0}}^3 - 3{f_{v0}}^2 + 4)(2 + f_{v0})}{(4{f_{v0}}^2 + f_{v0} + 4)^2}$ is larger than for DE models
- For large $z$, does not asymptote to $\Omega_{M0}$ but to $Om(\infty) = \frac{2(1 - f_{v0})(2 + f_{v0})^3}{(4{f_{v0}}^2 + f_{v0} + 4)^2}) < \Omega_{M0}$ if $f_{v0} > 0.25$.

# Sahni, Shafieloo and Starobinsky $Om(z)$

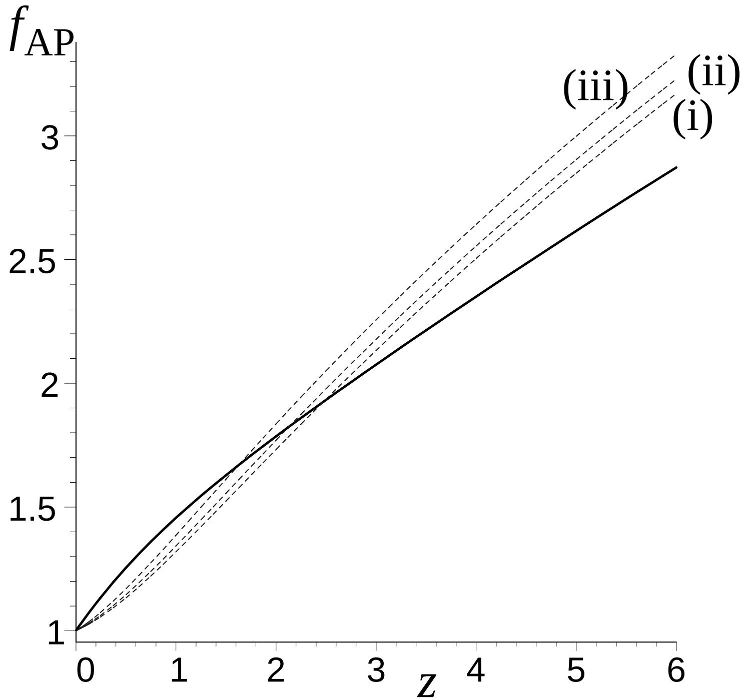



**(i)** Om(z) fit by Shafieloo *et al*. to SN+BAO+CMB with $w(z) = -\frac{1}{2}[1 + \tanh((z - z_t)\Delta)]$;

**(ii)** TS model prediction for Om(z) (NOT same w(z)) – best-fit and $1\sigma$ uncertainties

- Shafieloo *et al*., arxiv:0903.5141, fit $Om(z)$ with hint that "dark energy is decaying".
- Intercept $Om(0)$ agrees well with TS model expectation

# Alcock–Paczyński test

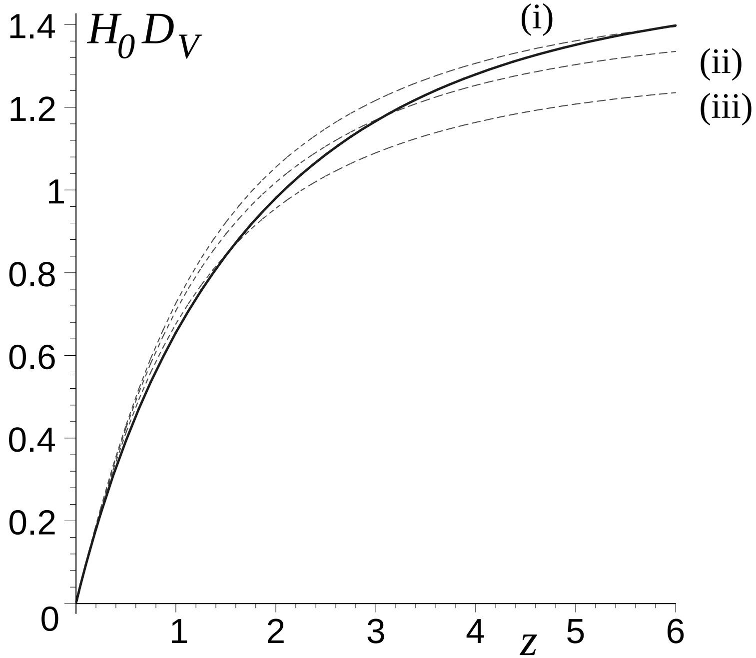



$f_{\mathrm{AP}} = \frac{1}{z}\left|\frac{\delta\theta}{\delta z}\right|$ for the tracker solution with $f_{\mathrm{v}0} = 0.762$ (solid line) is compared to three spatially flat $\Lambda$CDM models with the same values of $(\Omega_{M0}, \Omega_{\Lambda 0})$ as in earlier figures

- For a comoving standard ruler subtending and angle $\theta$,

$$f_{\mathrm{AP}} = \frac{1}{z}\left|\frac{\delta\theta}{\delta z}\right| = \frac{HD}{z} = \frac{3\left(2t^2 + 3bt + 2b^2\right)(1+z)D_A}{t\,(2t+3b)^2\, z}$$

# Baryon Acoustic Oscillation test function

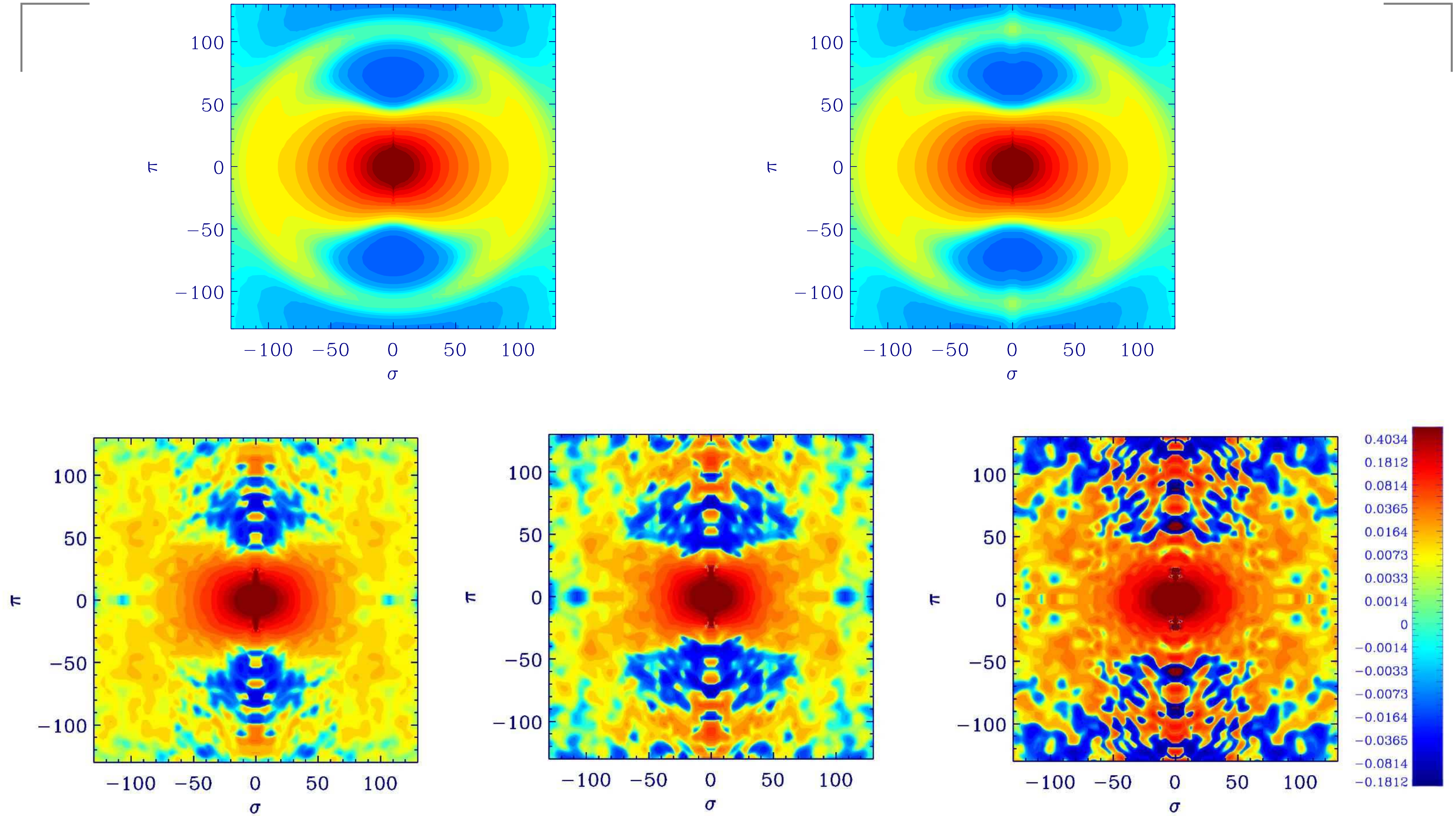



$H_0 D_V = H_0 D f_{\rm AP}^{-1/3}$ for the tracker solution with $f_{\rm v0} = 0.762$ (solid line) is compared to three spatially flat $\Lambda$CDM models with the same values of $(\Omega_{M0}, \Omega_{\Lambda 0})$ as in earlier figures

- BAO tests of galaxy clustering typically consider

$$D_V = \left[\frac{zD^2}{H(z)}\right]^{1/3} = D f_{\rm AP}^{-1/3}.$$

# Gaztañaga, Cabre and Hui 0807.3551



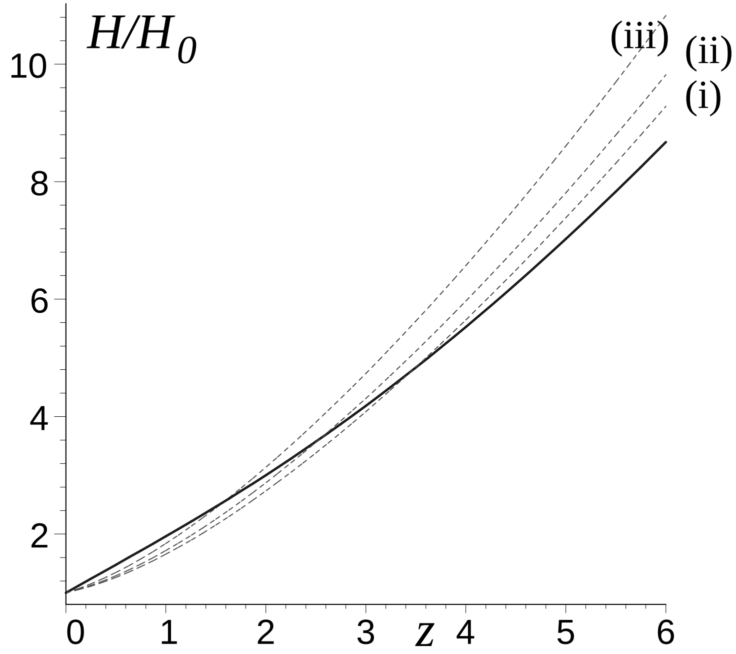

$z = 0.15\text{-}0.47$ $z = 0.15\text{-}0.30$ $z = 0.40\text{-}0.47$

# Gaztañaga, Cabre and Hui 0807.3551

| redshift range | $\Omega_{M0}h^2$ | $\Omega_{B0}h^2$ | $\Omega_{C0}/\Omega_{B0}$ |
|---|---|---|---|
| 0.15-0.30 | 0.132 | 0.028 | 3.7 |
| 0.15-0.47 | 0.12 | 0.026 | 3.6 |
| 0.40-0.47 | 0.124 | 0.04 | 2.1 |

- WMAP5 fit to $\Lambda$CDM: $\Omega_{B0} \simeq 0.045$, $\Omega_{C0}/\Omega_{B0} \simeq 6.1$
- GCH bestfit: $\Omega_{B0} = 0.079 \pm 0.025$, $\Omega_{C0}/\Omega_{B0} \simeq 3.6$.
- TS prediction $\Omega_{B0} = 0.080^{+0.021}_{-0.013}$, $\Omega_{C0}/\Omega_{B0} = 3.1^{+1.8}_{-1.3}$ with match to WMAP5 sound horizon within 4%.
- Blake et al (2012) now claim Alcock–Paczyński measurement in WiggleZ survey, fits $\Lambda$CDM well

# $H(z)/H_0$




$H(z)/H_0$ for $f_{v0} = 0.762$ (solid line) is compared to three spatially flat $\Lambda$CDM models: **(i)** $(\Omega_{M0}, \Omega_{\Lambda 0}) = (0.249, 0.751)$; **(ii)** $(\Omega_{M0}, \Omega_{\Lambda 0}) = (0.279, 0.721)$ **(iii)** $(\Omega_{M0}, \Omega_{\Lambda 0}) = (0.34, 0.66)$;.

- Function $H(z)/H_0$ displays quite different characteristics
- For $0 < z \lesssim 1.7$, $H(z)/H_0$ is larger for TS model, but value of $H_0$ assumed also affects $H(z)$ numerical value

# Redshift time drift (Sandage–Loeb test)

- $H(z)/H_0$ measurements are model dependent. However,

$$\frac{1}{H_0}\frac{\mathrm{d}z}{\mathrm{d}\tau} = 1 + z - \frac{H}{H_0}$$

- For $\Lambda$CDM

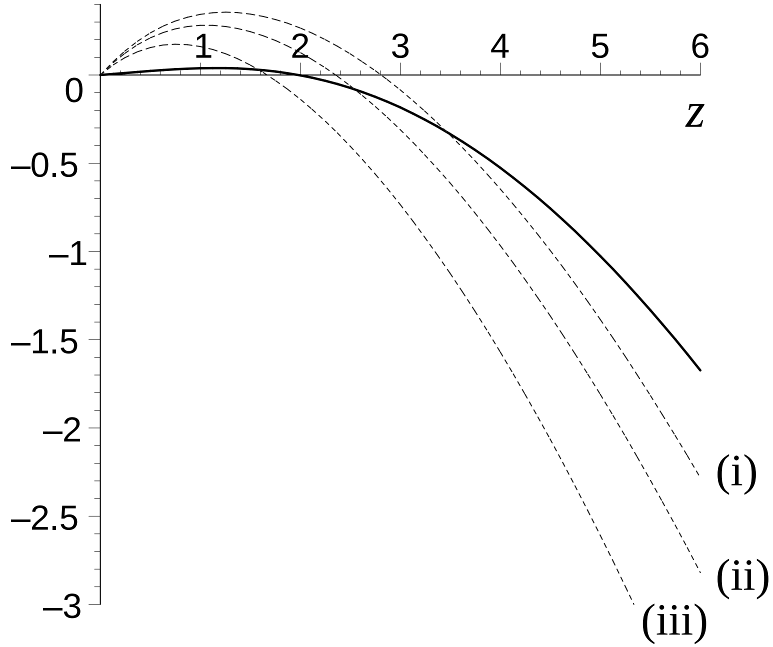
$$\frac{1}{H_0}\frac{\mathrm{d}z}{\mathrm{d}t} = (1+z) - \sqrt{\Omega_{M0}(1+z)^3 + \Omega_{\Lambda 0} + \Omega_{k0}(1+z)^2}.$$

- For TS model

$$\frac{1}{H_0}\frac{\mathrm{d}z}{\mathrm{d}\tau} = 1 + z - \frac{3\left(2t^2 + 3bt + 2b^2\right)}{H_0 t\left(2t + 3b\right)^2},$$

where $t$ is given implicitly in terms of $z$.

# Redshift time drift (Sandage–Loeb test)




$H_0^{-1}\frac{\mathrm{d}z}{\mathrm{d}\tau}$ for the TS model with $f_{\mathrm{v}0} = 0.762$ (solid line) is compared to three spatially flat $\Lambda$CDM models with the same values of $(\Omega_{M0}, \Omega_{\Lambda 0})$ as in previous figures.

- Measurement is extremely challenging. May be feasible over a 10–20 year period by precision measurements of the Lyman-$\alpha$ forest over redshift $2 < z < 5$ with next generation of Extremely Large Telescopes

# Clarkson, Bassett and Lu homogeneity test

- For FLRW equations, irrespective of dark energy model

$$\Omega_{k0} = \frac{\mathcal{B}}{[H_0 D(z)]^2} = \text{const}$$

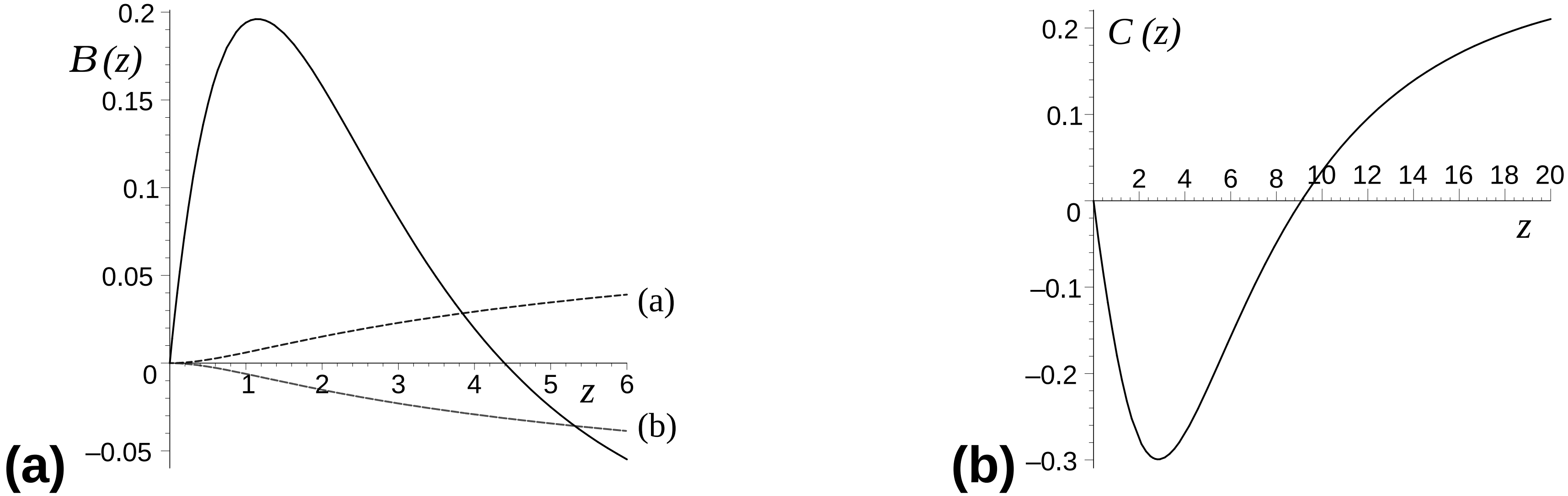
where $\mathcal{B}(z) \equiv [H(z)D'(z)]^2 - 1$. Thus

$$\mathcal{C}(z) \equiv 1 + H^2(DD'' - D'^2) + HH'DD' = 0$$

for any homogeneous isotropic cosmology, irrespective of DE.

- Clarkson, Bassett and Lu [PRL 101 (2008) 011301] call this a “test of the Copernican principle”. However, it is merely a test of (in)homogeneity.

# Clarkson, Bassett and Lu homogeneity test



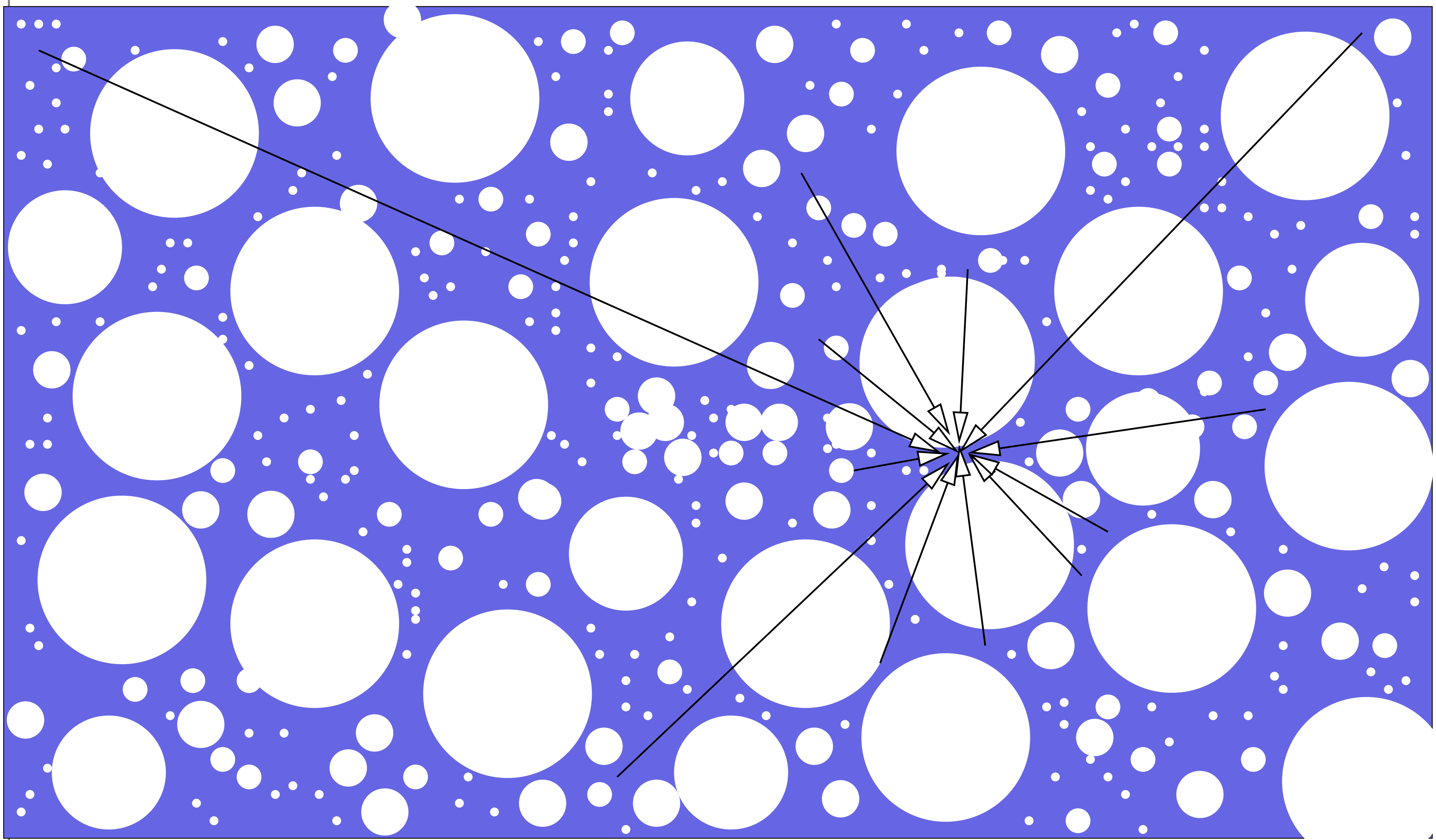

**(a)** $\mathcal{B} \equiv [H(z)D'(z)]^2 - 1$ for TS model with $f_{v0} = 0.762$ (solid line) and two $\Lambda$CDM models (dashed lines): **(i)** $\Omega_{M0} = 0.28$, $\Omega_{\Lambda 0} = 0.71$, $\Omega_{k0} = 0.01$; **(ii)** $\Omega_{M0} = 0.28$, $\Omega_{\Lambda 0} = 0.73$, $\Omega_{k0} = -0.01$; **(b)** $\mathcal{C}(z)$.

- Will give a powerful test of FLRW assumption in future, with quantitative different prediction for TS model.

# Lecture 5

# Variance of the Hubble flow

# Apparent Hubble flow variance



# Result: arXiv:1201.5371v2

- CMB dipole usually interpreted as result of a boost w.r.t. *cosmic rest frame*, composed of our motion w.r.t. barycentre of Local Group plus a motion of the Local Group of 635 $\mathrm{km\,s^{-1}}$ towards ? Great Attractor? Shapley Concentration ? ??
- But Shapley Supercluster, is at $\gtrsim 138\,h^{-1}$Mpc > Scale of Statistical Homogeneity
- We find Hubble flow is significantly more uniform in rest frame of LG rather than standard "rest frame of CMB"
- Suggests LG is not moving at 635 $\mathrm{km\,s^{-1}}$; but $\exists$ 0.5% foreground anisotropy in distance-redshift relation from foreground density gradient on $\lesssim 65\,h^{-1}$Mpc scales

# Peculiar velocity formalism

- Standard framework, FLRW + Newtonian perturbations, assumes peculiar velocity field

$$v_{\rm pec} = cz - H_0 r$$

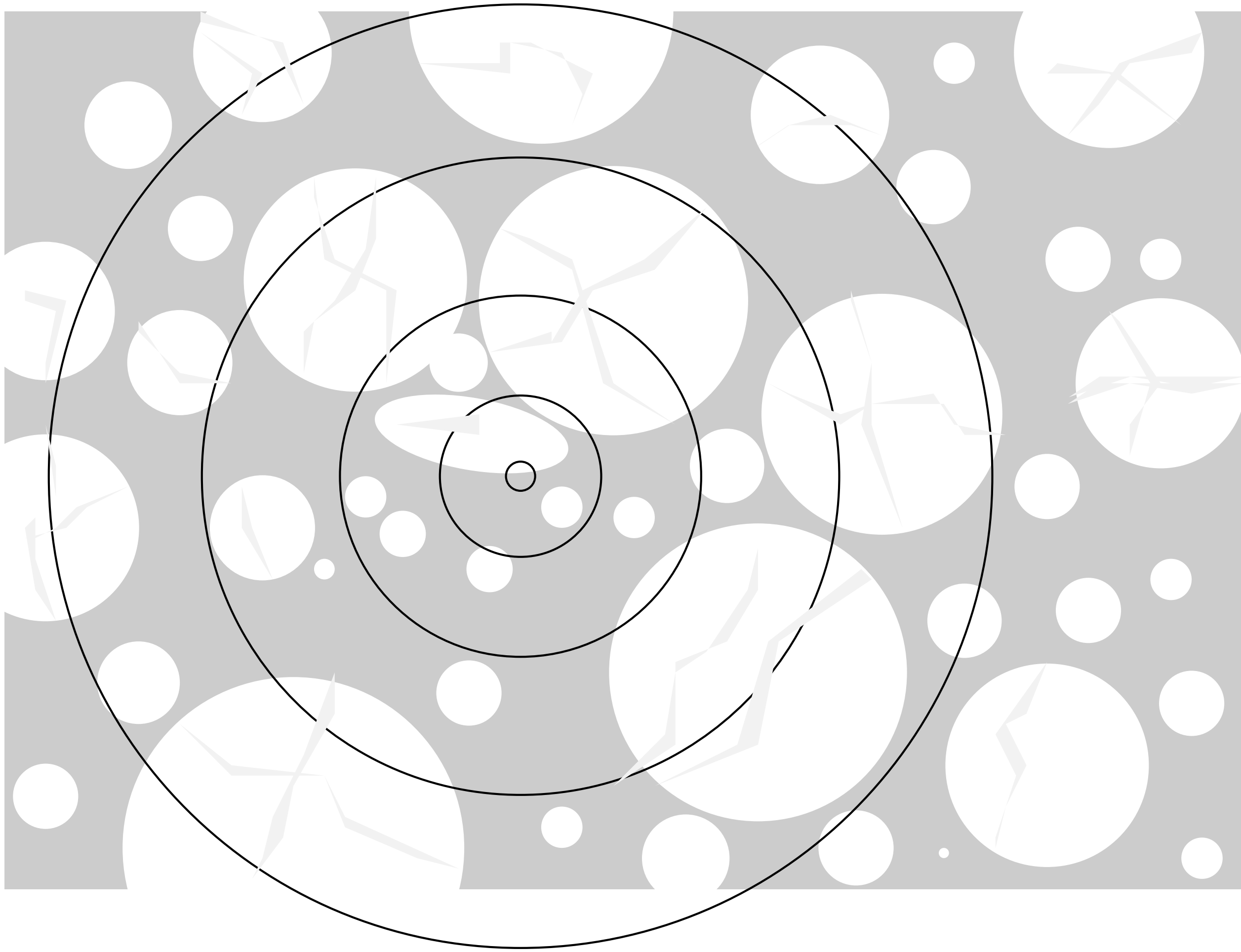
generated by

$$\mathbf{v}(\mathbf{r}) = \frac{H_0 \Omega_{M0}^{0.55}}{4\pi} \int \mathrm{d}^3\mathbf{r}'\, \delta_m(\mathbf{r}')\, \frac{(\mathbf{r}' - \mathbf{r})}{|\mathbf{r}' - \mathbf{r}|^3}$$

- After 3 decades of work, despite contradictory claims, the $\mathbf{v}(\mathbf{r})$ does not to converge to LG velocity w.r.t. CMB
- Agreement on direction, not amplitude or scale (Lavaux et al 2010; Bilicki et al 2011; . . . )
- Suggestions of bulk flows inconsistent with $\Lambda$CDM (Watkins, Feldman, Hudson 2009. . . )

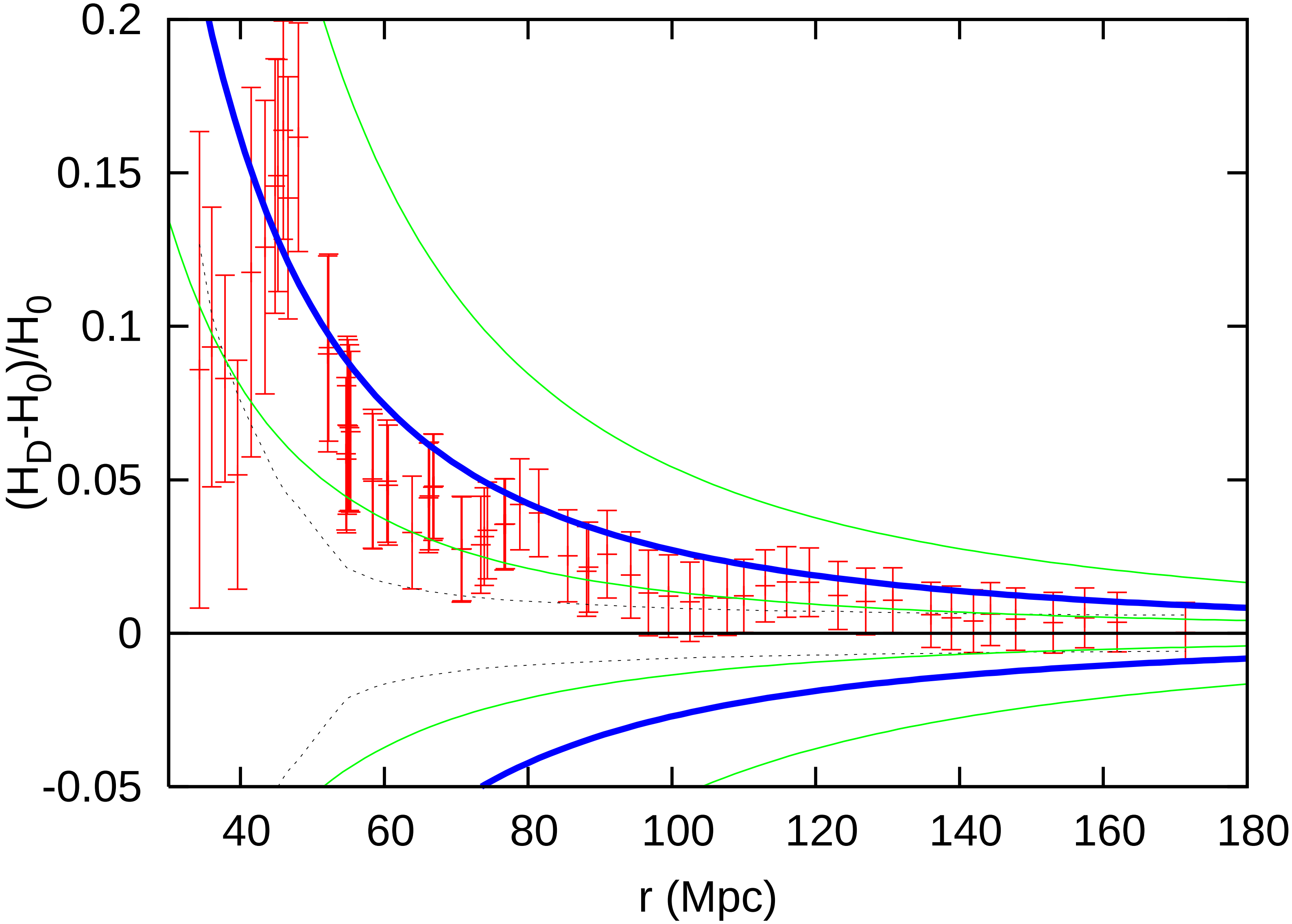
# Spherical averages

- Determine variation in Hubble flow by determining best-fit linear Hubble law in spherical shells



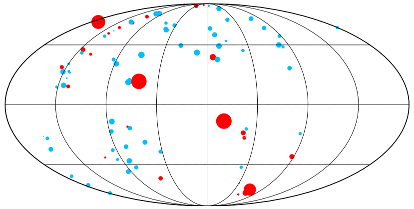
# N. Li & D. Schwarz, PRD 78, 083531

HST key data: 68 points, single shell (all points within r Mpc as r varied) – correlated result

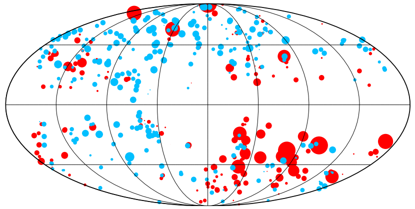


# Analysis of COMPOSITE sample

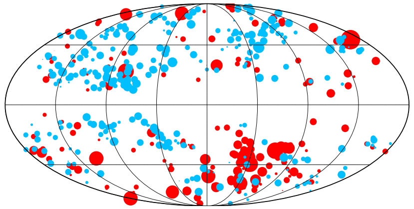
- Use COMPOSITE sample: Watkins, Feldman & Hudson 2009, 2010, with 4,534 galaxy redshifts and distances, includes most large surveys to 2009
- Distance methods: Tully Fisher, fundamental plane, surface brightness fluctuation; 103 supernovae distances.
- average in *independent spherical shells*
- Compute $H_s$ in $12.5\,h^{-1}$Mpc shells; combine 3 shells $> 112.5\,h^{-1}$Mpc
- Use data beyond $156.25\,h^{-1}$Mpc as check on $H_0$ normalisation – COMPOSITE sample is normalized to $100\,h$ km/s/Mpc



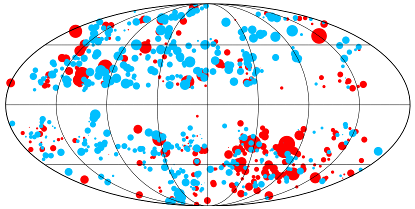
(a) 1: $0 - 12.5\ h^{-1}$ Mpc $N = 92$.



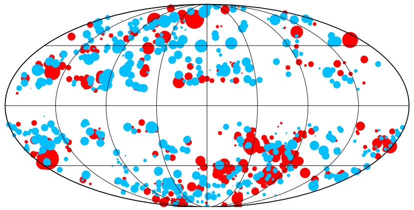
(b) 2: $12.5 - 25\ h^{-1}$ Mpc $N = 505$.



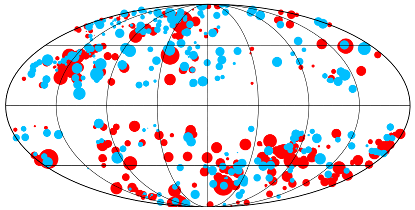
(c) 3: $25 - 37.5\ h^{-1}$ Mpc $N = 514$.



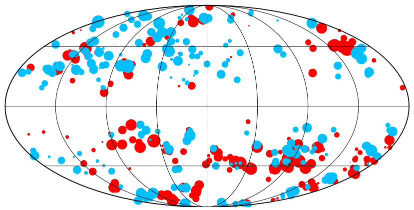
(d) 4: $37.5 - 50\ h^{-1}$ Mpc $N = 731$.



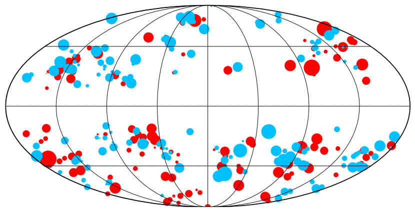
(e) 5: $50 - 62.5\ h^{-1}$ Mpc $N = 819$.



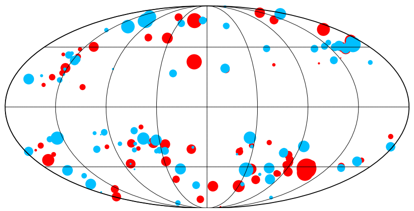
(f) 6: $62.5 - 75\ h^{-1}$ Mpc $N = 562$.



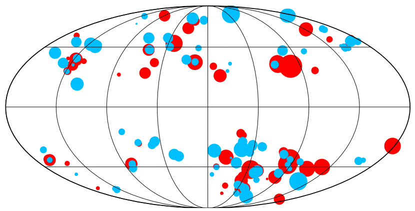
(g) 7: $75 - 87.5\ h^{-1}$ Mpc $N = 414$.



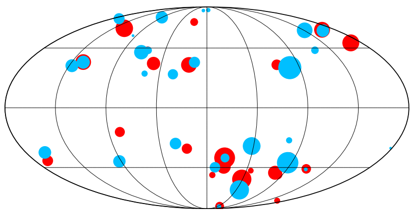
(h) 8: $87.5 - 100\ h^{-1}$ Mpc $N = 304$.



(i) 9: $100 - 112.5\ h^{-1}$ Mpc $N = 222$.

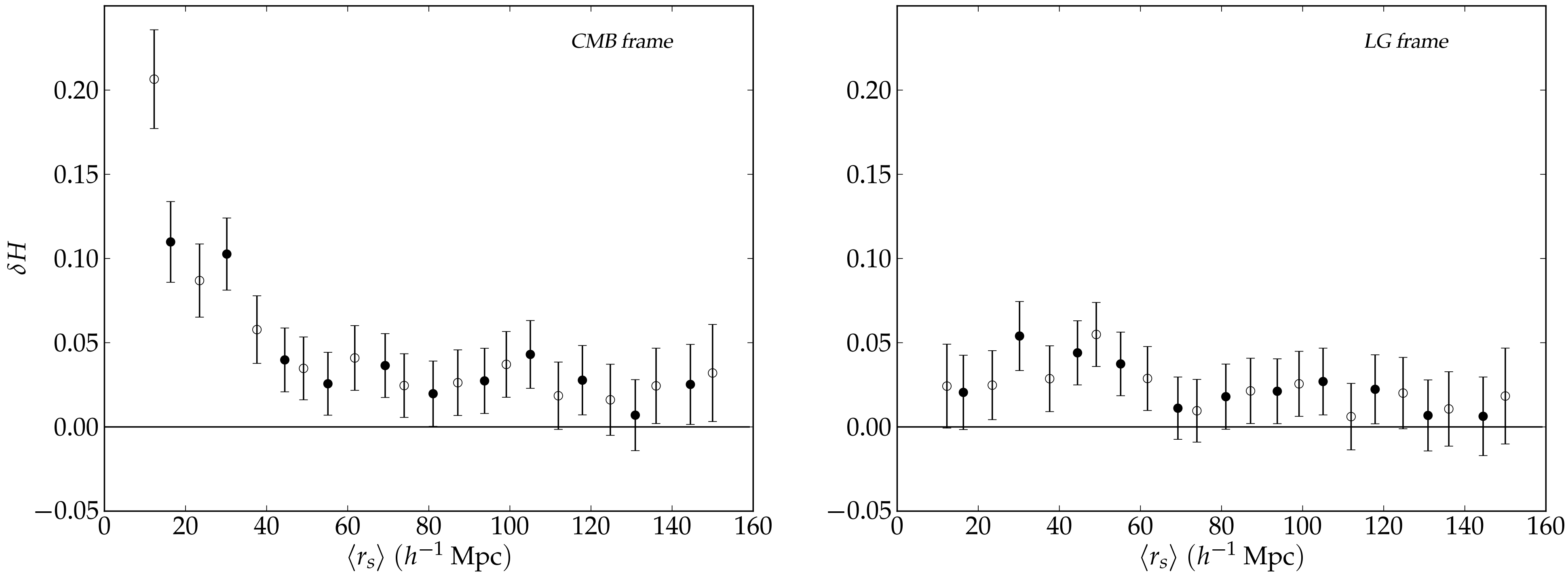


(j) 10: $112.5 - 156.25\ h^{-1}$ Mpc $N = 280$.



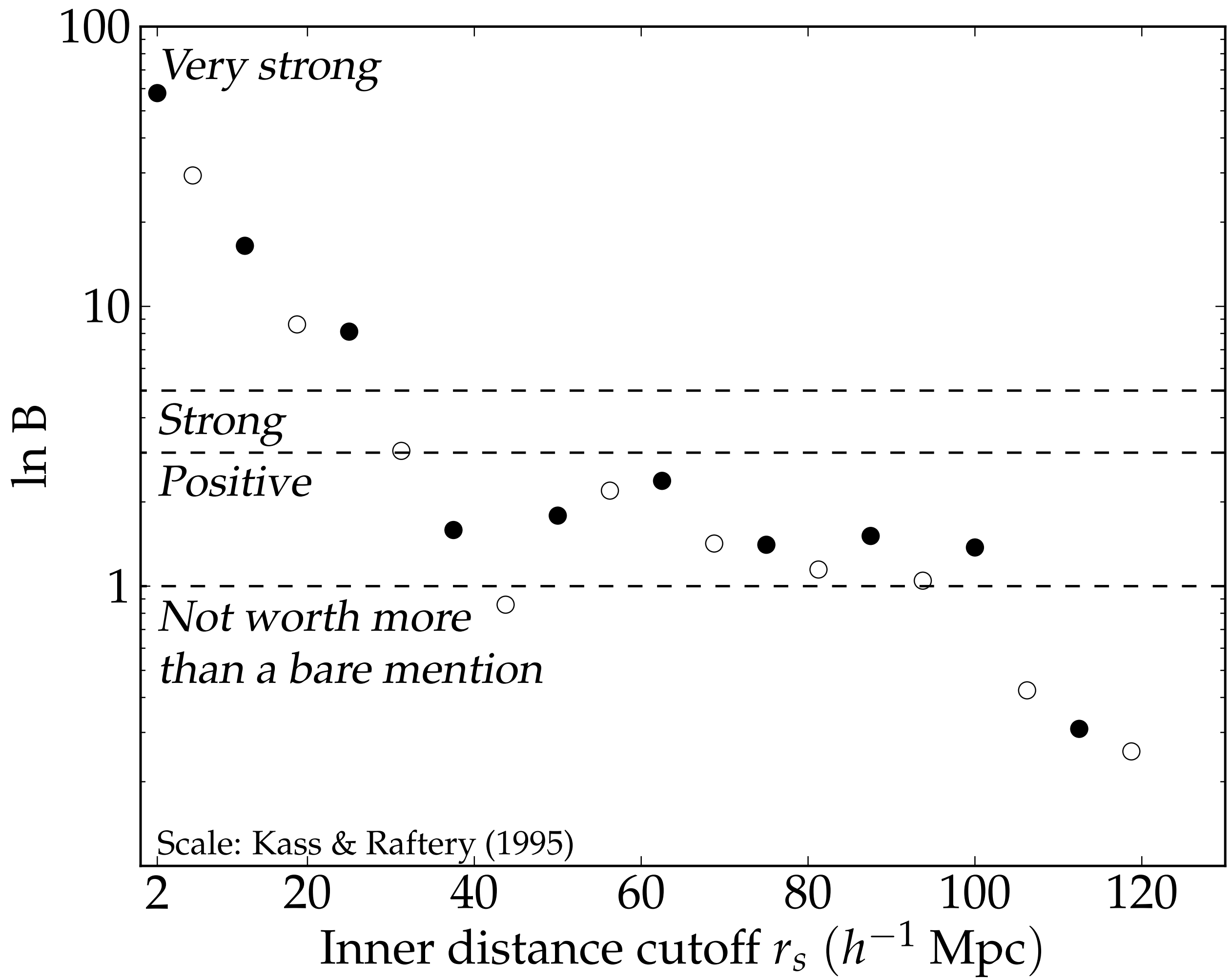
(k) 11: $156.25 - 417.4\ h^{-1}$ Mpc $N = 91$.

# Radial variance $\delta H_s = (H_s - H_0)/H_0$




- Two choices of shell boundaries (closed and open circles); for each choice data points uncorrelated
- Analyse linear Hubble relation in rest frame of CMB; Local Group (LG); Local Sheet (LS). LS result very close to LG result.

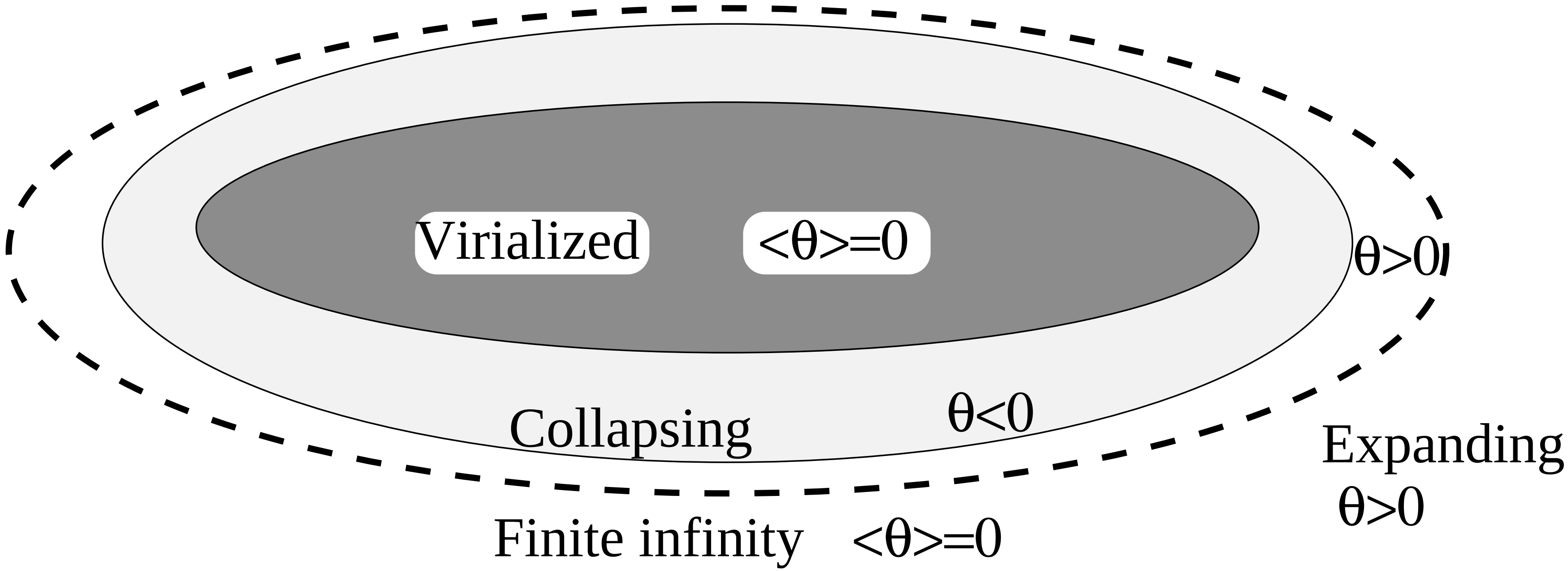
# Bayesian comparison of uniformity




- Hubble flow more uniform in LG frame than CMB frame with very strong evidence

# But why try the LG frame?

- From viewpoint of the timescape model and in particular the “Cosmological Equivalence Principle” in bound systems the *finite infinity* region (or *matter horizon*) is the standard of rest



# Boosts and spurious monopole variance

- $H_s$ determined by linear regression in each shell

$$H_s = \left( \sum_{i=1}^{N_s} \frac{(cz_i)^2}{\sigma_i^2} \right) \left( \sum_{i=1}^{N_s} \frac{cz_i r_i}{\sigma_i^2} \right)^{-1} ,$$

- Under boost $cz_i \to cz_i' = cz_i + v\cos\phi_i$ for uniformly distributed data, linear terms cancel on opposite sides of sky

$$\begin{aligned} H_s' - H_s &\sim \left( \sum_{i=1}^{N_s} \frac{(v\cos\phi_i)^2}{\sigma_i^2} \right) \left( \sum_{i=1}^{N_s} \frac{cz_i r_i}{\sigma_i^2} \right)^{-1} \\ &= \frac{\langle (v\cos\phi_i)^2 \rangle}{\langle cz_i r_i \rangle} \sim \frac{v^2}{3H_0 \langle r_i^2 \rangle} \end{aligned}$$

# Angular variance

Two approaches; fit

1. McClure and Dyer (2007) method – can look at higher multipoles

$$H_\alpha = \frac{\sum_{i=1}^{N} W_{i\,\alpha}\, cz_i\, r_i^{-1}}{\sum_{j=1}^{N} W_{j\,\alpha}}$$

   where with $\cos\theta_i = \vec{r}_{\rm grid}\cdot\vec{r_i}$, $\sigma_\theta = 25^\circ$ (typically)

$$W_{i\,\alpha} = \frac{1}{\sqrt{2\pi}\sigma_\theta}\exp\left(\frac{-\theta_i^2}{2\sigma_\theta^2}\right)$$

2. Simple dipole

$$\frac{cz}{r} = H_0 + b\cos\phi$$

# McClure-Dyer Gaussian window average

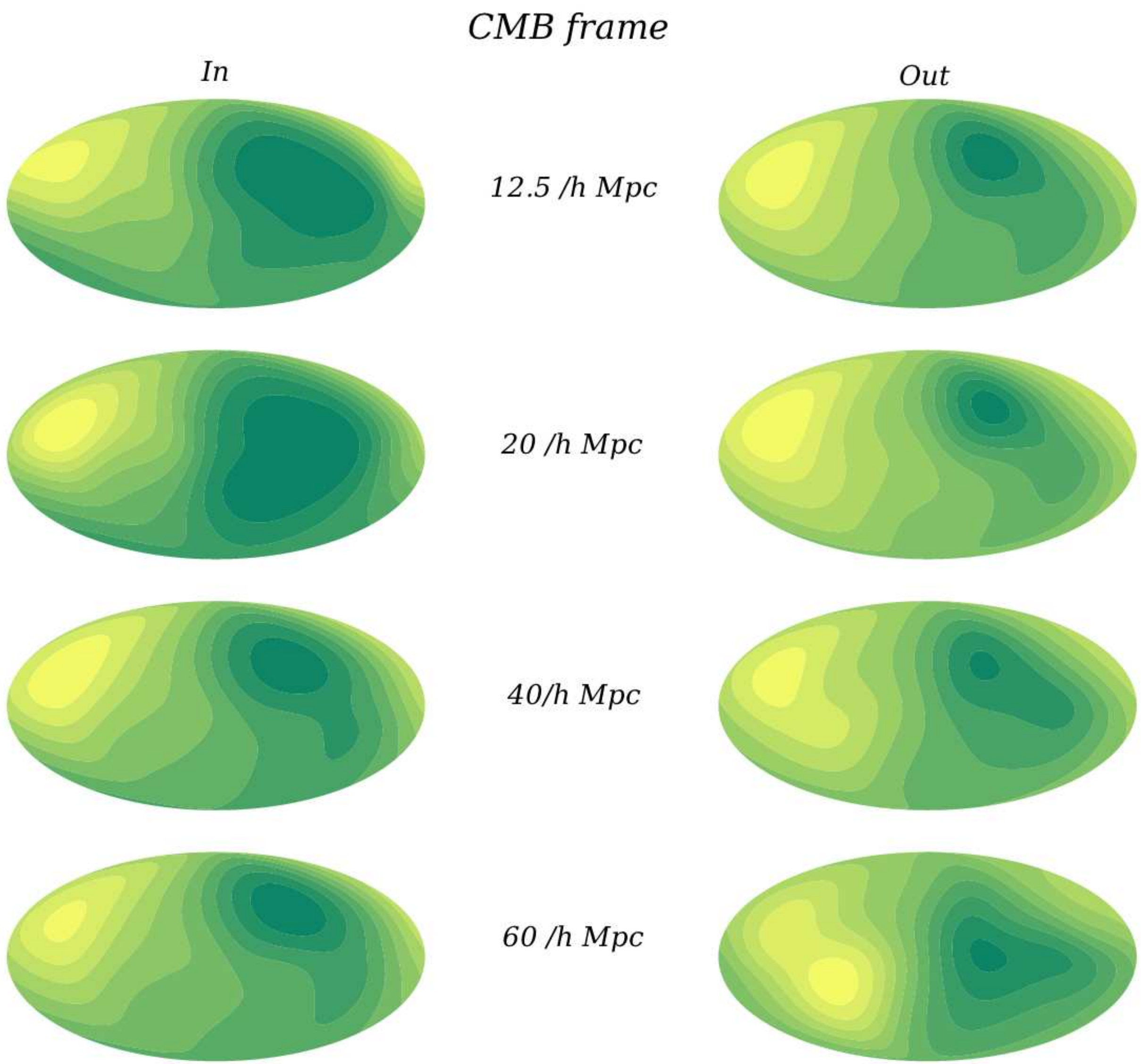
- Actually for COMPOSITE sample better to fit

$$H_\alpha^{-1} = \frac{\sum_{i=1}^{N} W_{i\,\alpha}\, r_i\, (cz_i)^{-1}}{\sum_{j=1}^{N} W_{j\,\alpha}},$$

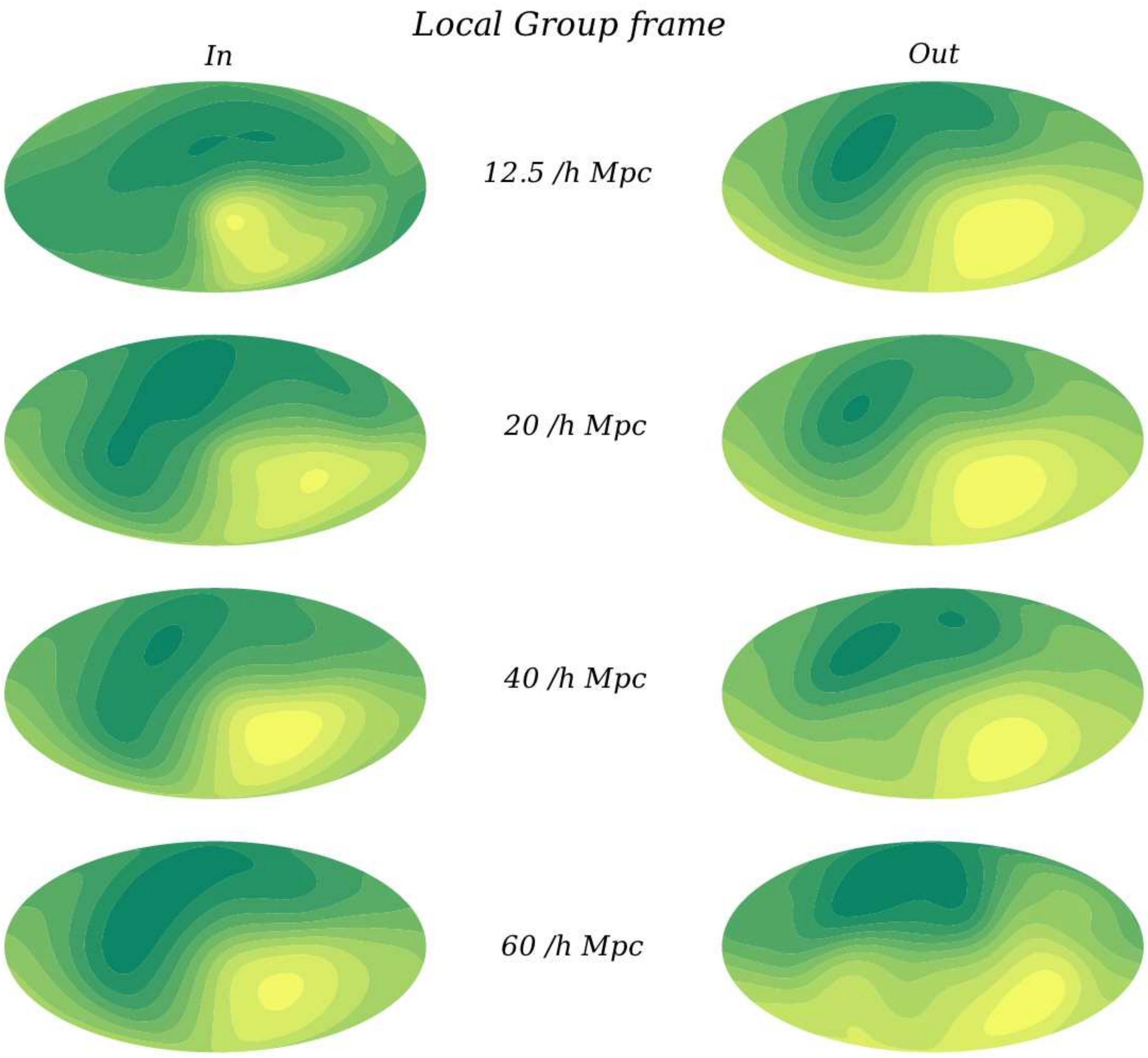
$$W_{i\,\alpha} = \frac{1}{\sigma^2_{H_i^{-1}}\sqrt{2\pi}\sigma_\theta} \exp\left(\frac{-\theta_i^2}{2\sigma_\theta^2}\right), \qquad \sigma_{H_i^{-1}} = \frac{\sigma_i}{cz_i}$$

- Canonical value of $\sigma_\theta = 25^\circ$ used, but varied $15^\circ < \sigma_\theta < 40^\circ$ with no significant change. However, $2\sigma_\theta$ must be greater than Zone of Avoidance diameter.

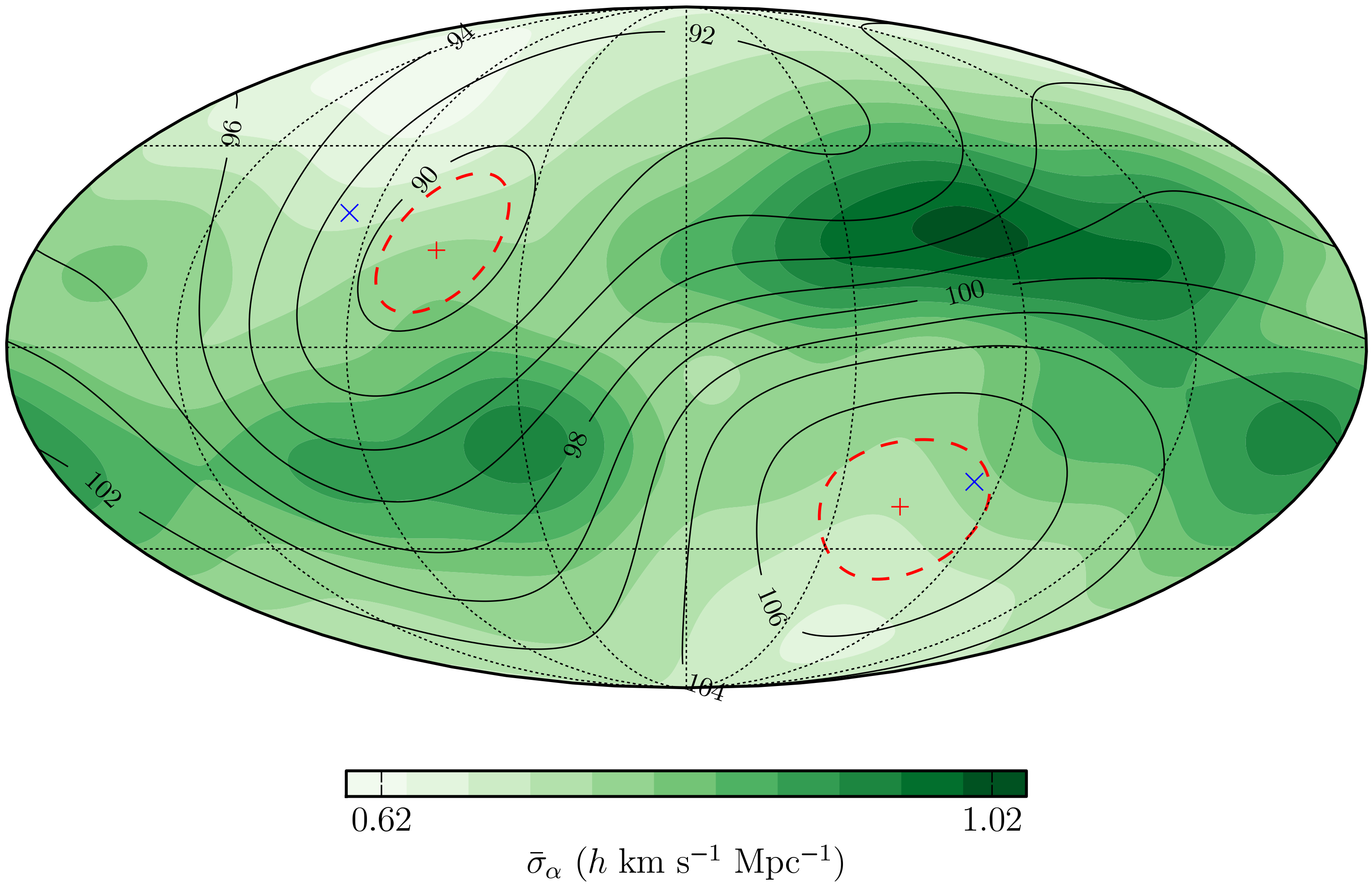
# Hubble variance: CMB frame



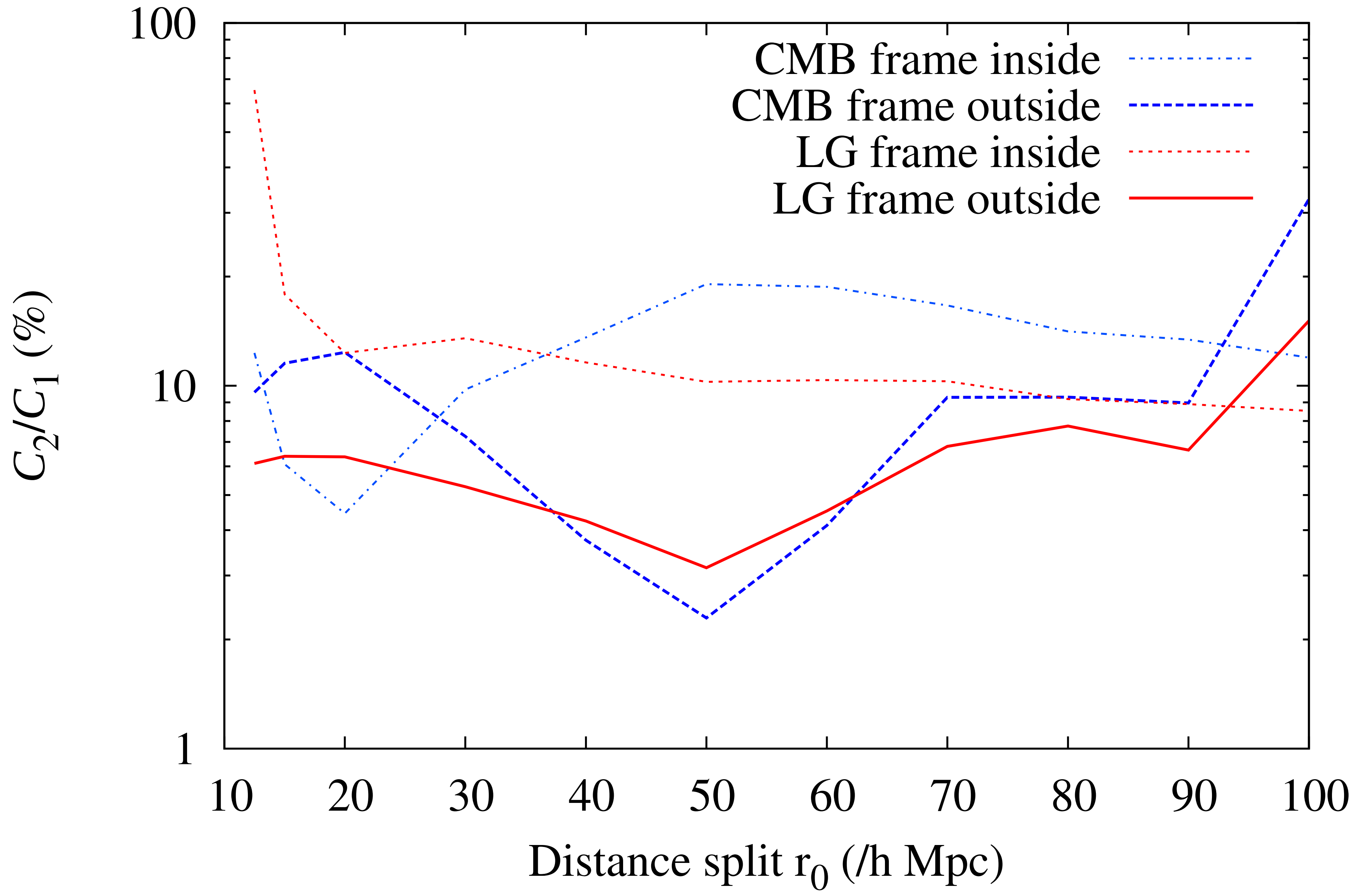
# Hubble variance: LG frame



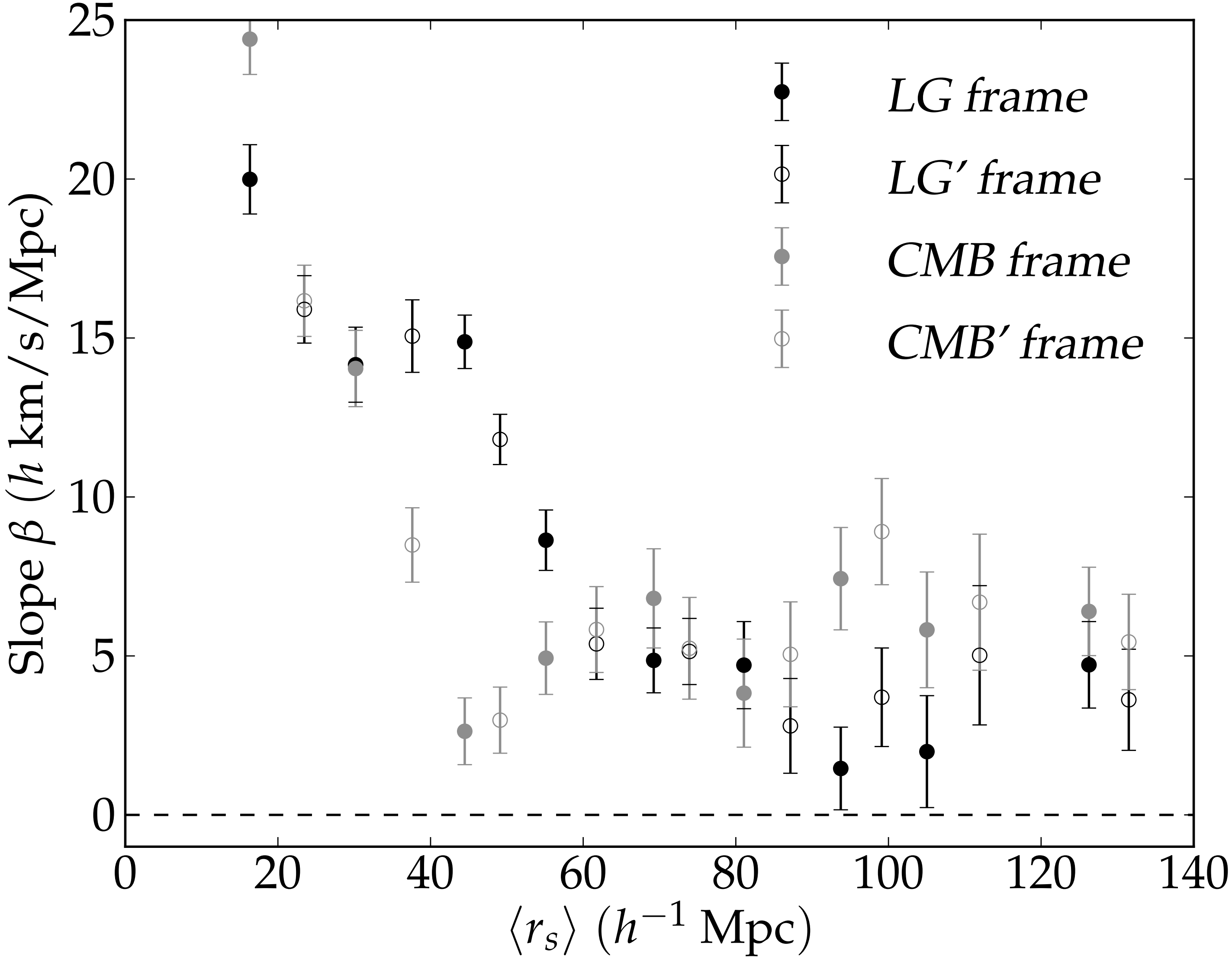
LG frame $r_0 \geq 15.0\ h^{-1}$ Mpc, $\Delta H$ : $18.4\ h$ km s$^{-1}$ Mpc$^{-1}$ ($N =$ 4359)



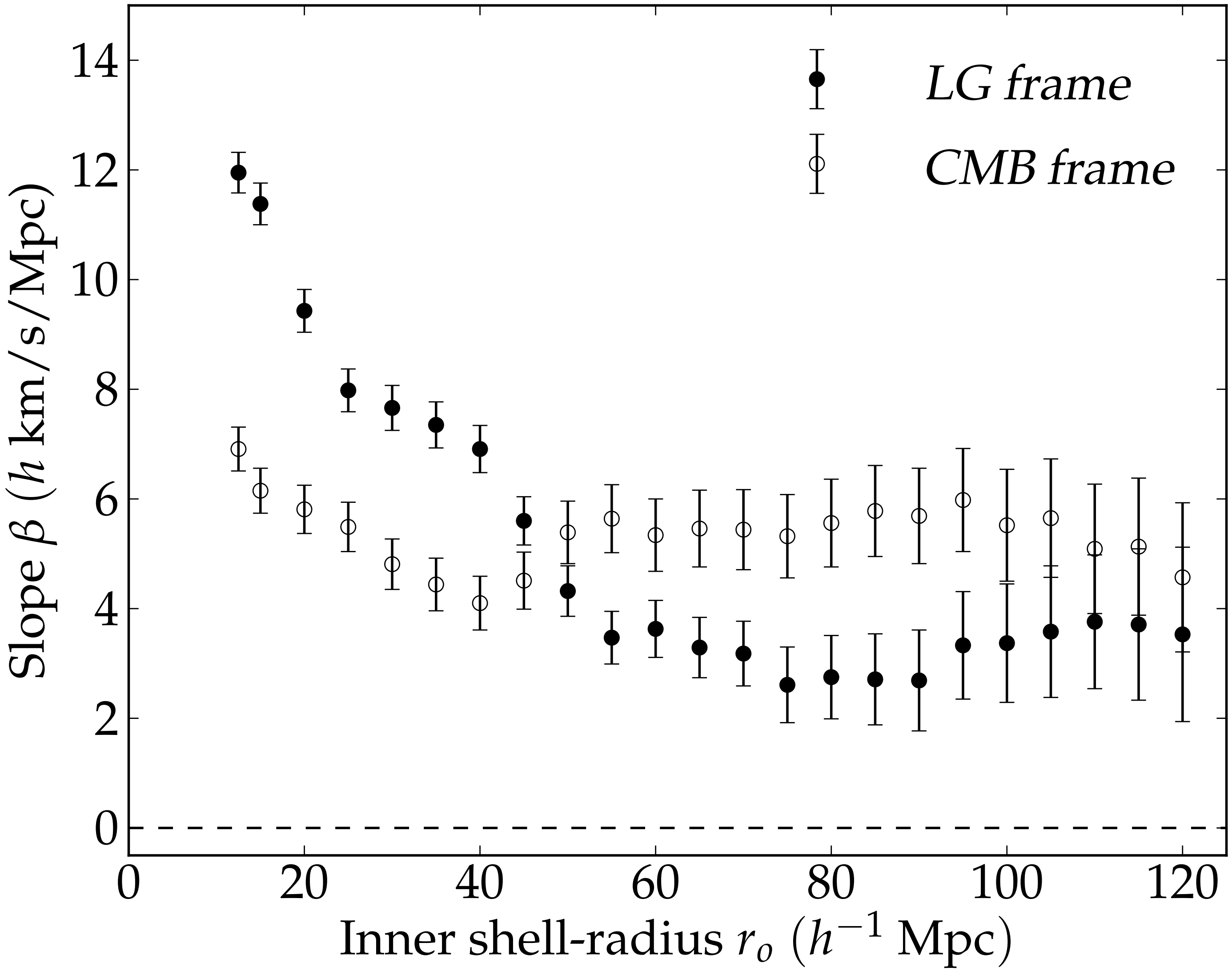
# Hubble variance quadrupole/dipole ratios



# Value of $\beta$ in $\frac{cz}{r} = H_0 + \beta \cos\phi$

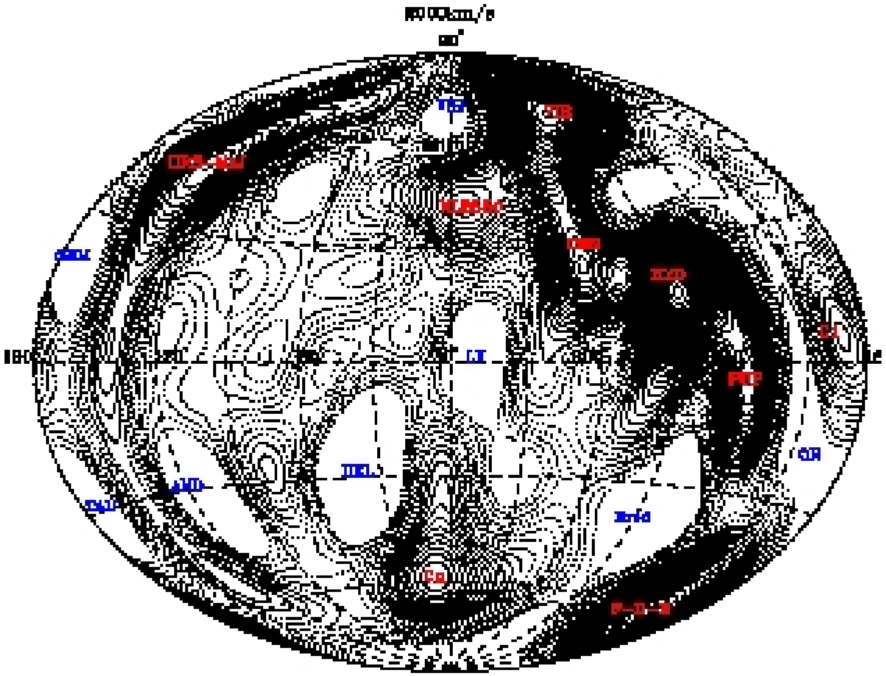


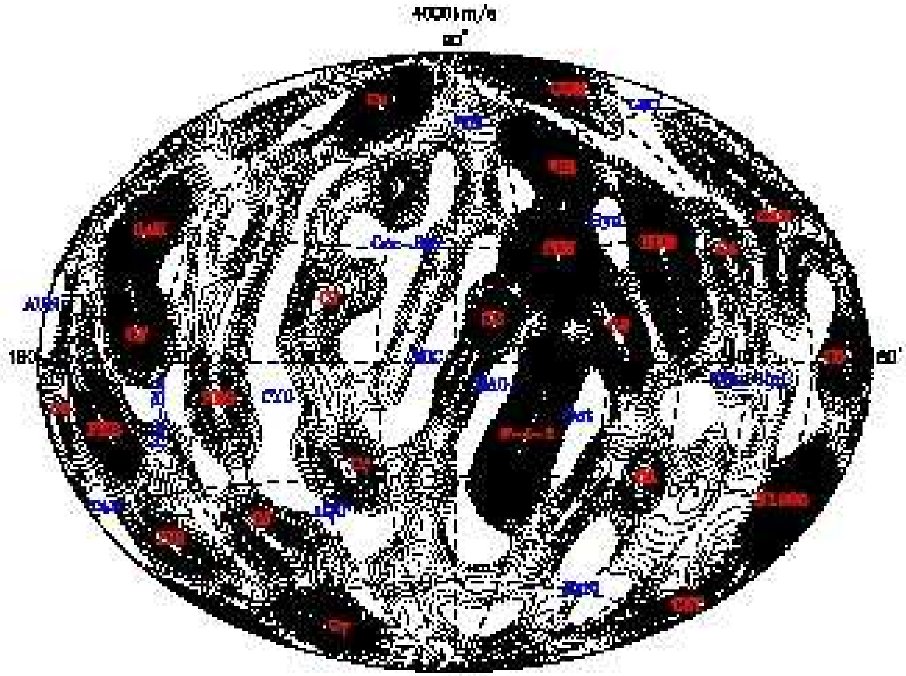
# Value of $\beta$ in $\frac{cz}{r} = H_0 + \beta \cos\phi$

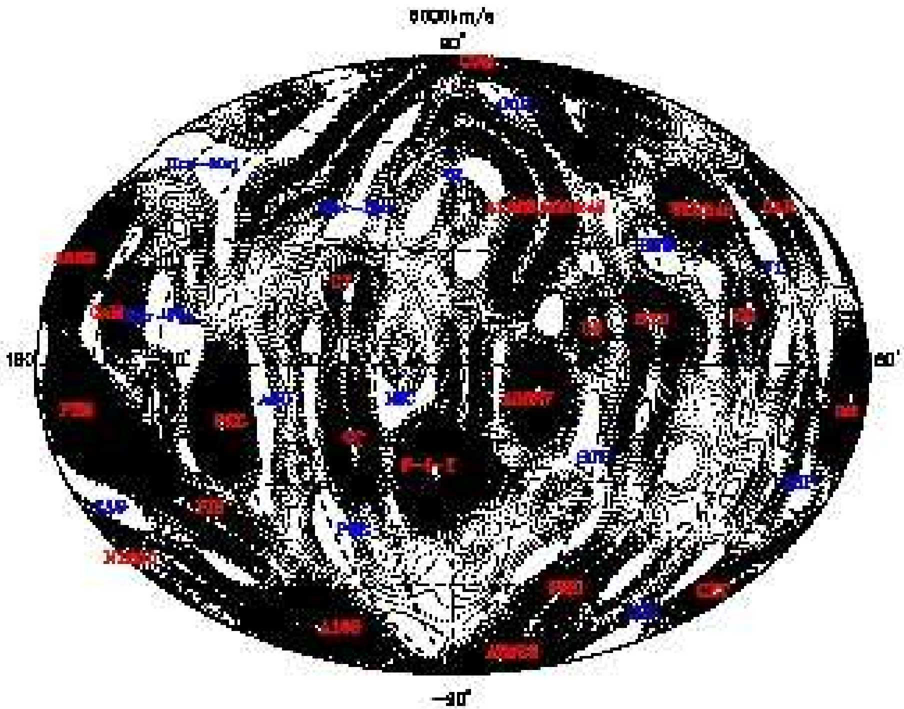


# Dipole direction

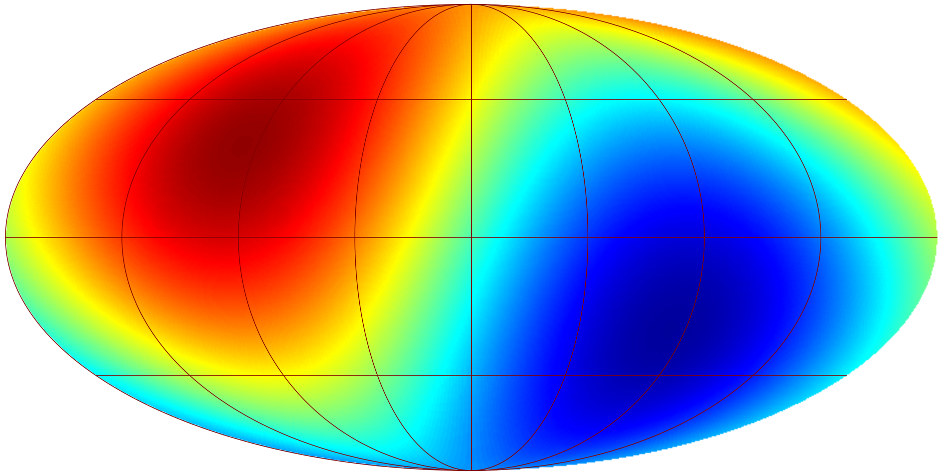
- CMB frame: direction $(\ell_d, b_d)$ remains within 1$\sigma$ of the bulk flow direction $(\ell, b) = (287^\circ \pm 9^\circ, 8^\circ \pm 6^\circ)$ found by Watkins et al (2009) for $20\,h^{-1} \le r_o \le 115\,h^{-1}$Mpc; for largest values remains consistent with bulk flow direction $(\ell, b) = (319^\circ \pm 18^\circ, 7^\circ \pm 14^\circ)$ of Turnbull et al (2012)
- CMB dipole drops to minimum at $40\,h^{-1}$Mpc but then increases and remains $4.0$–$7.0\sigma$ from zero.
- LG frame: For $20\,h^{-1} \lesssim r_o \lesssim 45\,h^{-1}$Mpc while the dipole is strong, direction is consistently in range $(\ell_d, b_d) = (83^\circ \pm 6^\circ, -39^\circ \pm 3^\circ)$ but angular position then wanders once magnitude reduced to residual levels. For $r_o \gtrsim 80\,h^{-1}$Mpc the typical position of residual dipole differs from that inner dipole by $80^\circ$ – $100^\circ$ in angle $\ell$.







# Correlation with residual CMB dipole

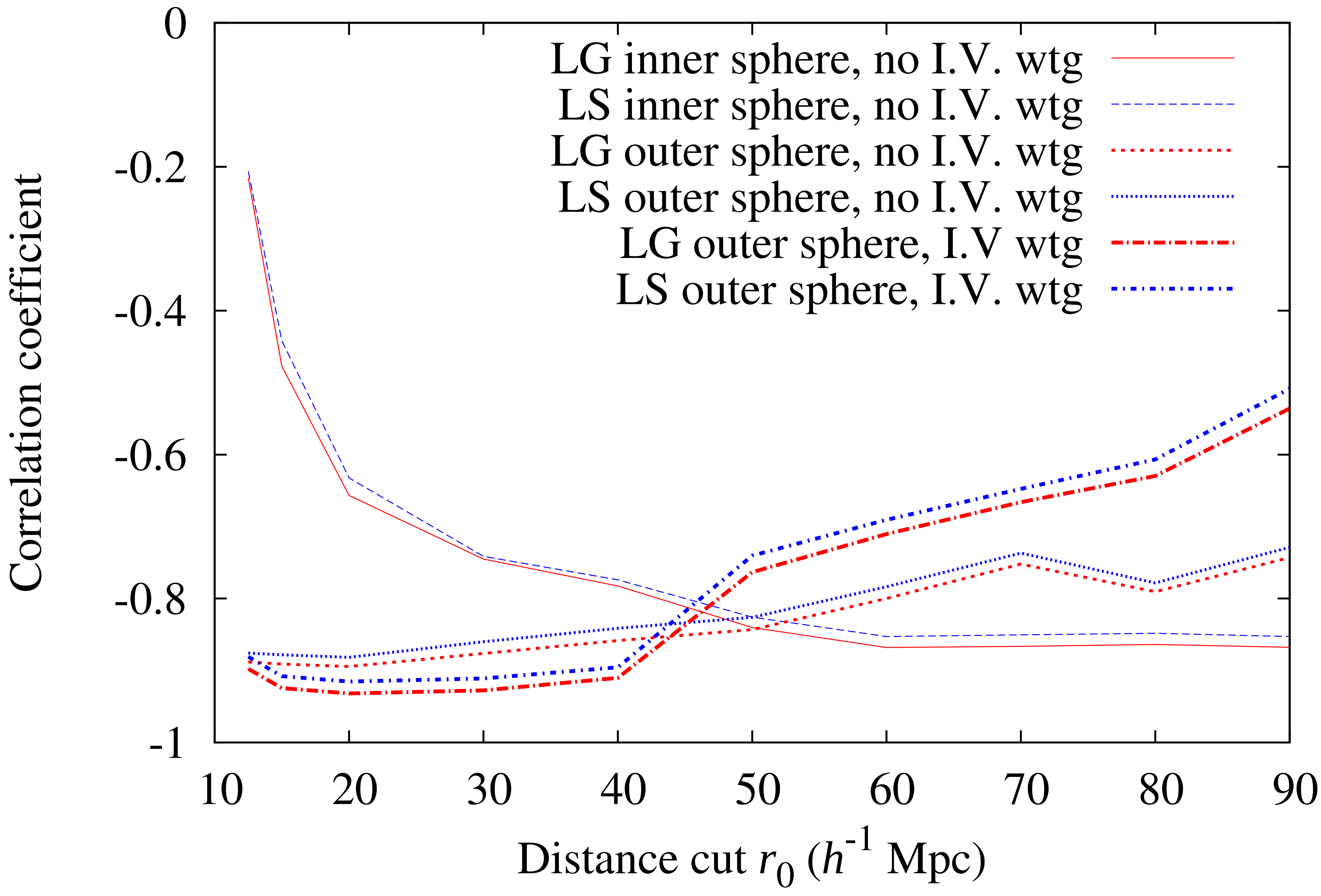



- Digitize skymaps with HEALPIX, compute

$$\rho_{HT} = \frac{\sqrt{N_p}\,\sum_\alpha \bar{\sigma}_\alpha^{-2}(H_\alpha - \bar{H})(T_\alpha - \bar{T})}{\sqrt{\left[\sum_\alpha \bar{\sigma}_\alpha^{-2}\right]\left[\sum_\alpha \bar{\sigma}_\alpha^{-2}(H_\alpha - \bar{H})^2\right]\left[\sum_\alpha (T_\alpha - \bar{T})^2\right]}}$$

- $\rho_{HT} = -0.92$, (almost unchanged for $15^\circ < \sigma_\theta < 40^\circ$)
- Alternatively, $t$-test on raw data: null hypothesis that maps uncorrelated is rejected at 24.4$\sigma$.

# Correlation with CMB dipole as $r_o$ varied



# Redshift-distance anisotropy

- As long as $T \propto 1/a$, where $a_0/a = 1 + z$ *for some appropriate average*, not necessarily FLRW, then small change, $\delta z$, in the redshift of the surface of photon decoupling – due to foreground structures – will induce a CMB temperature increment $T = T_0 + \delta T$, with

$$\frac{\delta T}{T_0} = \frac{-\delta z}{1 + z_{\rm dec}}$$

- With $z_{\rm dec} = 1089$, $\delta T = \pm(5.77 \pm 0.36)$ mK represents an increment $\delta z = \mp(2.31 \pm 0.15)$ to last scattering
- **Proposal**: rather than originating in a LG boost the $\pm 5.77$ mK dipole is due to a small anisotropy in the distance-redshift relation on scales $\lesssim 65\, h^{-1}$Mpc.

# Redshift-distance anisotropy

- For spatially flat $\Lambda$CDM

$$D = \frac{c}{H_0} \int_1^{1+z_{\rm dec}} \frac{\mathrm{d}x}{\sqrt{\Omega_{\Lambda 0} + \Omega_{M0} x^3 + \Omega_{R0} x^4}}$$

For standard values $\Omega_{R0} = 4.15 h^{-2} \times 10^{-5}$, $h = 0.72$

- $\Omega_{M0} = 0.25$, find $\delta D = \mp(0.33 \pm 0.02)\, h^{-1}$Mpc;
- $\Omega_{M0} = 0.30$, find $\delta D = \mp(0.32 \pm 0.02)\, h^{-1}$Mpc;
- timescape model similar.
- Assuming that the redshift-distance relation anisotropy is due to foreground structures within $65\, h^{-1}$Mpc then $\pm 0.35\, h^{-1}$Mpc represents a $\pm 0.5$% effect

# Why a strong CMB dipole?

- Ray tracing of CMB sky seen by off-centre observer in LTB void gives $|a_{10}| \gg |a_{20}| \gg |a_{30}|$ (Alnes and Amarzguioui 2006). E.g.,

$$\frac{a_{20}}{a_{10}} = \sqrt{\frac{4}{15}}\frac{(h_{\rm in} - h_{\rm out})d_{\rm off}}{2998\,\text{Mpc}}$$

  where $H_{\rm in\,0} = 100\,h_{\rm in}$ km/s/Mpc, $H_{\rm out\,0} = 100\,h_{\rm out}$ km/s/Mpc are Hubble constants inside/outside void, $d_{\rm off} =$ distance of the observer from centre in Mpc.

- Even for relatively large values $d_{\rm off} = 50\,h^{-1}$Mpc and $h_{\rm in} - h_{\rm out} = 0.2$, we have $a_{20}/a_{10} \lesssim$ 1%.

# Towards a new formalism

- For $z < z_{\rm hom}$ define $D_L = (1+z)D = (1+z)^2 D_A$, where

$$D(z) = c \int_0^z \frac{\mathrm{d}z_s}{H_s(z_s)} \,.$$

where by linear regression in shells, $z_s < z \le z_s + \sigma_z$

$$H_s = \left( \sum_{i=1}^{N_s} \frac{(cz_i)^2}{\sigma_i^2} \right) \left( \sum_{i=1}^{N_s} \frac{cz_i r_i}{\sigma_i^2} \right)^{-1} ,$$

- Smoothing scale $\sigma_z$ greater than largest typical bound structures, e.g., $\sigma_z = 0.0042$ for radial width $12.5\,h^{-1}$Mpc.
- This gives the monopole, or spherical Hubble bubble variance

# Towards a new formalism

- For each shell redshift $H(z_s, \theta, \phi) - H(z_s)$ will give angular corrections which should lead to an expansion of $D(z, \theta, \phi)$ in multipoles
- Convergence of Hubble flow variance to CMB dipole is then obtained if
  **(i)** dipole anisotropy in $D$ converges to fixed value for $z > z_{\rm conv}$;
  **(ii)** residual anisotropy in $D$ is of order order of $\pm 0.33\, h^{-1}$Mpc, with exact value depending on the cosmological model.
- Standard peculiar velocity formalism and Hubble variance formalism can then be directed compared and tested

# Type Ia supernova systematics?

- SneIa are standardizable candles only; two popular methods SALT/SALT–II, and MLCS2k2 yield different results when comparing cosmological results
- Degeneracy between intrinsic colour variations and reddening by dust
- Hubble bubble is seen if $R_V = 3.1$ (Milky way value); not if $R_V = 1.7$ (often includes data $0.015 \lesssim z \lesssim z_{\rm conv} \simeq 0.022$)
- Study independent of SneIa in 15 nearby galaxies gives $R_V = 2.77 \pm 0.41$ (Finkelman et al 2010, 2011)
- We find "Hubble bubble" *independently* of SneIa
- N.B. SneIa are standardized by minimizing $H_0$ residuals in CMB frame. *Union, Constitution compilations contain many SneIa in range* $0.015 \lesssim z \lesssim 0.022$ *where CMB boost compensates partly.*

# Large angle CMB anomalies?

Anomalies (varying significance) include

- power asymmetry of northern/southern hemispheres
- alignment of the quadrupole and octupole etc;
- low quadrupole power;
- parity asymmetry; . . .

Critical re-examination required; e.g.

- light propagation through Hubble variance dipole foregrounds may differ subtly from Lorentz boost dipole
- dipole subtraction is an integral part of the map-making; is galaxy correctly cleaned?
- Freeman et al (2006): 1–2% error in dipole subtraction may resolve the power asymmetry anomaly.

# Dark flow?

- Controversial claim of bulk flows by Kashlinsky et al (2009,2010) of $600 – 1,000\,\mathrm{km\,s^{-1}}$ over large scales, *coinciding with LG boost direction*, using the kinematic Sunyaev-Zel'dovich effect
- Claim to have subtracted all possible primordial dipoles, quadrupoles, octupoles etc, so measurement is made in "cluster rest frames"
- Not seen by Osborne et al (2011), Hand et al (2012), Lavaux et al (2012) who use different techniques
- However, we note Kashlinsky modelling of cluster temperature requires use of cluster redshift, *and an isotropic distance-redshift relation is assumed*

$$a_{1m} = a_{1m}^{KSZ} + a_{1m}^{TSZ} + a_{1m}^{CMB} + \frac{\sigma_{\rm noise}}{\sqrt{N_{\rm cl}}}$$

# Comments on ISW amplitude

- Integrated Sachs–Wolfe (Rees-Sciama) effect needs recomputation in timescape model
- Correlation of radio-galaxies, voids and superclusters etc with CMB positively detected and well established (Boughn and Crittenden 2004, . . . Granett, Neyrinck and Szapudi, 2008)
- Amplitude of effect consistently of order $2\sigma$ greater than LCDM prediction, or $3\sigma$ greater according to recent detailed calculation of Nathadur, Hotchkiss and Sarkar (2012)
- Does departure of local Hubble flow variance from $\Lambda$CDM expectations may give insights about features of inhomogeneities at high redshift?

# Conclusion/Outlook

- Variance of the Hubble flow over tens of megaparsecs cannot be reduced to a boost; i.e. *Eppur si espande!*, (Abramowicz et al 2007) space really is expanding
- Large CMB angle anomalies, and map-making procedures would need to be reconsidered ... are the cold spot etc foreground artifacts, or primordial
- “Dark flow” probably a systematic “error”
- Frame of minimum variance Hubble flow variance frame to be determined
- Impact of rest frame choice, e.g., on nearby measurements in setting distance scale etc, needs to be re-examined
- Opportunity to develop new formalism and approaches to observational cosmology

# Conclusion

- Apparent cosmic acceleration can be understood purely within general relativity; by (i) treating geometry of universe more realistically; (ii) understanding fundamental aspects of general relativity which have not been fully explored – *quasi–local gravitational energy*, of *gradients* in spatial curvature etc.
- “Timescape” model gives good fit to major independent tests of $\Lambda$CDM with new perspectives on many puzzles – e.g., primordial lithium abundance anomaly
- Many tests can be done to distinguish from $\Lambda$CDM.
- It is crucial that $\Lambda$CDM assumptions such as Friedmann equation are not used in data reduction.
- Many details – averaging scheme etc – may change, but fundamental questions remain