Honours Report

Real-Time Quantitative Analysis and Surface Registration of Medical Scan Data

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Chapter 1

Introduction

Several medical scanning techniques provide data about a volume that is being scanned in a series of slices. The term *volume* refers to some object of interest being scanned, usually a closed three-dimensional object. For example, a volume could be a kidney or a bone fragment.

Edge analysis can be performed to obtain a *contour* of the volume's surface for each slice.

Operations such as quantitative analysis and surface registration are often performed only after the complete scanning of a volume. Quantitative analysis involves obtaining numerical information from the scanned data, such as calculating the volume and surface area of the scanned object. Surface registration refers to the process of aligning two or more sets of scan data obtained from the same or similar volumes. Differences between the aligned objects may then be analysed.

The objective of this project has been to investigate the possibility of providing operations such as those described above in real-time, as the scanning of a volume is performed. Also while a volume is being scanned, an estimate of the accuracy of such operations could be calculated. If the results of these operations are wanted to a certain level of accuracy, the scanning could continue until the accuracy reaches the required level.

Chapter 2

Problem Description

Each slice from the scan of a volume provides a single contour of the volume's surface. Each *contour* is a closed planar-curve represented by a list of three-dimensional points, or by a list of two-dimensional points and the orientation of the contour in three-dimensional space. These two contour representations are equivalent, with a mapping function between them.

The contours obtained from a scan may be at arbitrary orientations, and consecutive contours may not necessarily be parallel. The data from these contours, in an appropriate representation allows for the operations of quantitative analysis and surface registration to be performed. Also the accuracy of the results from such operations may be estimated.

Initially, in this report, contours in two-dimensions will be considered. Later, the ideas will be extended to contours in three-dimensions.

Chapter 3

Possible Approaches

3.1 Splines

Splines are commonly used in analytical geometry as approximate functions to describe curves and surfaces[3, 4]. A *spline* is a polynomial function of a certain degree. When splines are used to fit curves or surfaces to discrete points, the coefficients of the polynomial can be thought of as a description for the part of the curve or surface that the spline is modelling.

Often, a surface that is being modelled by splines has to be divided into many regions. Each region can then be modelled by a *spline patch.* With each spline patch there is a different set of describing coefficients. The model of a surface can then be thought of as a patchwork quilt, with the spline patches being woven together to form the complete surface.

The calculation of spline patches that model a surface has to be done in such a way that discontinuities do not occur in the surface, and so that the surface maintains a smooth appearance across the boundaries of the patches. These problems can be solved by using splines that take into account the first and higher order derivatives of the surface being modelled.

3.2 Fourier Transforms

Another more recent method method investigated, described by A. Seager and R. Grognard[?, ?, bb] uses Fourier transformations for describing curves and surfaces. Fourier transformations are applied to the function of a known curve or surface to generate a similar representation of the curve or surface being modelled.

A simple example would be where the curve to be modelled is an ellipse, and the known curve is the unit circle centred about a fixed point. A few simple transformations can be applied to the unit circle to map the known curve to the ellipse being modelled.

One advantage of this method is that it models a surface by a single function, and a series of Fourier transformations. Another advantage is the ease with which quantitative analysis can be performed on the surface being modelled. A quantitative analysis can be performed on a known surface. Then, the same Fourier transformations that generated the modelled surface can be applied to the quantitative result from the known surface, to transform this result into the equivalent quantitative result for the modelled surface.

For instance, if the known surface is a sphere then the integration of areas and volumes over that sphere can be easily calculated. The results from the sphere can have the same Fourier transformations that generate the modelled surface applied to them to calculate the equivalent results for the sample surface being modelled.

3.3 Renormalised Curvature Scale Space

The Renormalised Curvature Scale, RCSS, method described by Mackworth and Mokhtarian[1], provides a way of describing curves based on curvatures. Curves are described by RCSS images. An *RCSS image* for a curve, can be thought of as a graph with the curvature at every point plotted against the distance that the point occurs along the length of the curve.

The RCSS image is obtained by normalising the the scale space axis (the x-axis) of the graph, by dividing the distances along the scale space axis by the total length of the curve. Therefore, all the scale space values in a RCSS image range from zero to one.

The advantage of this method is that the description of a curve (the RCSS image) is independent of a curve's orientation, and is also independent of the scale of the curve. For example, two curves that are exactly the same (except that one curve is larger than the other) will have the same description in the RCSS method.

The curvature at a point on a curve is defined in terms of the second derivative of the curve, therefore curvature is independent of a curve's orientation in two- or three-dimensional space[7]. The RCSS image is independent of the scale of the curve being described, due to the length of every curve being normalised to the same unit one length.

The advantages of the RCSS method for describing curves, independent of scale and orientation, and the ease at which curve descriptions can be compared provide the most benefits of the three methods described for achieving the objectives of this project.

Chapter 4

Renormalised Curvature Scale Space

Figure 4.1 shows a simple example of curve and its related RCSS image. The values of curvature can be either positive of negative. In the example, point D is on a concave chapter of the curve, and in the graph has a negative curvature value. Whether a point on a curve has a positive or negative value is dependent on the choice of the function used to define curvature. The curvature function used the project is the same as the curvature function used in the paper by Mackworth and Mokhtarian[1]. The function uses the first and second discrete derivatives of a curve, to calculate the curvature at a point

$$curvature = \frac{\dot{x}.\ddot{y} - \ddot{x}.\dot{y}}{(\dot{x}^2 + \dot{y}^2)^{\frac{3}{2}}}$$

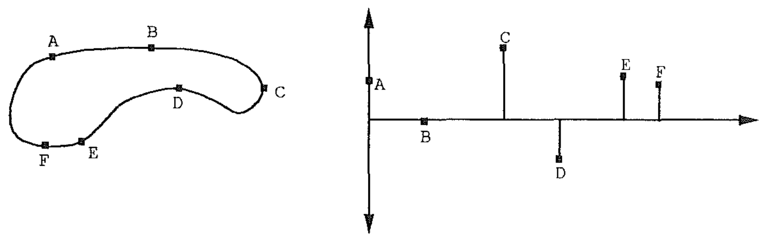

Figure 4.1: RCSS Image

Initially the list of data points that represent each contour were stored in a file. The data points for such contour files were either generated by a mathematical function (for example circular and elliptical contours), or by using a mouse to trace out a desired contour onto the screen. A program was written to convert a file of data points into the RCSS image of the contour. The RCSS image was stored in a file as a list of number pairs, one pair for each point in the contour. Each pair consisted of the distance at which the point occurred along the curve (normalised to between zero and one) and the curvature of the curve at that point.

Chapter 5

Comparing Contours

The RCSS method provides a framework in which curves can be compared, to determine how similar the contours are. Knowing whether two contours are similar is particularly useful if one of the contours being compared is known to have certain properties. For example, a known property of a contour could be that the contour is from the scan of a kidney. When two contours are found to be similar, the properties known about one contour can be inferred to also be properties of the other contour.

The process of comparing two curves involves using the two related RCSS images to calculate the difference in the curvatures between the contours. The *total curvature difference* between two contours is the sum of the difference in curvatures at each of the points around the contours. The comparison process can be thought of as moving around both contours simultaneously, calculating the difference between the curvatures along the length of the curves.

Table 1, shows the initial algorithm used in the project to compare contours. The root mean square, RMS, difference was calculated for the difference in curvatures between the contours, to provide a difference value that was independent of the number of comparisons in the calculation.

When comparing two contours, a total curvature difference that is zero or small indicates that the curves are the same or similar in shape. Several factors have to be considered when comparing curves, these are scale of the slices, orientation of the slices, incomplete contours, difference slice

```
total_difference = 0

for point1 = start point to end point of first contour image

    point2 = the point in the second contour image nearest to point1

    total_difference += ( curvature at point1 - curvature at point2 ) ^ 2

total_difference = total_difference /
                   number of points in the first contour

rms_difference = square root of total_difference
```

Table 5.1: Algorithm - Initial Comparison Method

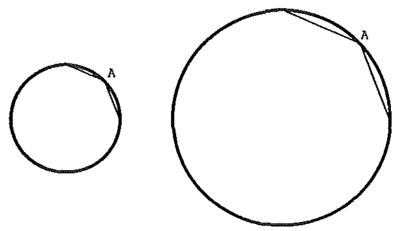

Figure 5.1: Scale Independence

resolutions, and the noise level of the data.

5.1 Scale

The RCSS method for representing curves is independent of scale, due to the normalisation of each curve's scale space, and the independence of the curvature function to scale. Curves of a similar shape but at varying scales will be considered similar by this method. For example, consider two circular contours either from volumes of different sizes, or from volumes scanned at different scales. The curvature at any point on a circle is constant no matter what the size of the circle is, as shown at point A in

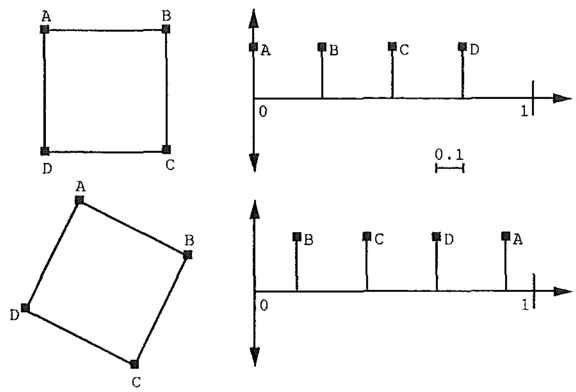

Figure 5.2: Rotated Contours

Figure 5.1. Therefore, the two circular contours will be considered to be similar by the RCSS comparison method.

5.2 Orientation

As the contours being compared are planar-curves, they treated as if they all lie in the same plane. Therefore, the only difference in orientation that can occur between contours is rotation within that plane. The RCSS image of a rotated contour is the same as the original contour image, except that a translation of the image has occurred along the scale space axis.

Comparison of two contours that may be at different orientations within the same plane, involves calculating the curvature difference of the two contours, while one of the images slides across the other in small steps. The data of the moving image is wrapped around, so that scale space values of the moving RCSS image still range from zero to one. The curvature difference of the two contours is calculated for every step as the images are moved relative to one another.

The *minimum total curvature difference*, is the smallest total curvature difference found in the comparison of two contours after a fixed number of orientations of the contours have been compared. Similarly as the initial comparison method, a minimum total curvature difference that is zero or

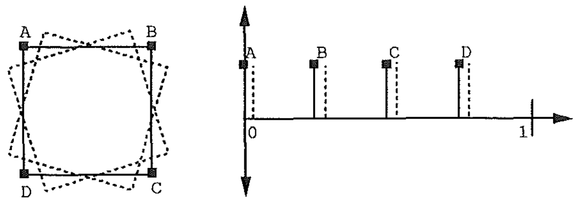

Figure 5.3: Translation Step Size

small indicates that the curves are the same or similar in shape.

Figure 5.2 shows two RCSS images for a square contour at different rotations. The translation distance between two images found to represent similar contours can be used to determine how much one contour is rotated relative to the other. In Figure 5.2, the translation distance is one tenth the length of the scale space axis. Therefore, the difference in rotation between the two square contours is 36 degrees.

The size of the translation step between consecutive comparisons of two RCSS images has to be small enough so the comparison of two contours that are similar does indicate their similarity by a small minimum total curvature difference. A small translation difference between RCSS images is equivalent to a small difference in rotation between the contours. The comparison of two similar contours will give a small total curvature difference when the contours are closely aligned. Therefore, the translation step has to be small enough to allow for similar contours to be compared at similar rotational orientations.

Figure 5.3 shows how, if a translation step is too large, two similar contours may not be considered similar by this comparison method. The solid square represents the contour of the image that remains still, while the dotted squares represent two consecutive contours of the moving image. The RCSS image represents the comparison of the solid contour and one of the dotted contours. Their orientations are too far apart to give a small total curvature difference.

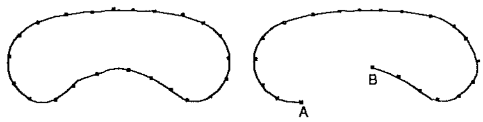

Figure 5.4: Incomplete Contours

5.3 Incomplete Contours

The RCSS method is not suitable for comparing two contours, if one or both of the contours are incomplete. An *incomplete contour* is one where a large length of the curve is not included in the curve's representation. Figure 5.4 shows two contours from a kidney shaped volume, with the contour on the right being incomplete. This type of problem could be due to a volume being partially obscured in a scan, or a fault in the scan or the edge-detection analysis.

The comparison of two similar contours, where one of the contours is incomplete, may result in the contours being considered different. The opposite effect may also happen, that is two different contours may be considered similar. For these reasons incomplete contours have not been used in this project, though for many other analysis methods the data from incomplete contours may be retained.

Incomplete contours are indicated by consecutive points along a curve's length occurring a large distance apart compared to the average distance between consecutive points on the curve. For example, in Figure 5.4 the consecutive points A and B would occur a large distance apart on the scale space axis of the RCSS image for the incomplete contour, compared to the average distance between consecutive points on the scale space axis.

5.4 Different Slice Resolutions

The term *Slice resolution* refers to the density of the data points that represent the contour obtained from a slice. A high slice resolution means that the average distance between consecutive points on the length of a curve is small compared to the total length of the curve. A contour where

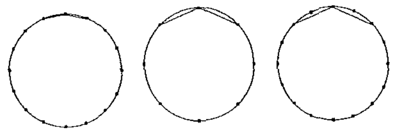

Figure 5.5: Different Slice Resolutions

the average distance between the points is relatively large compared to the the total length of the contour, has a low slice resolution.

Relatively large differences in slice resolutions can affect the accuracy of contour comparisons. Consider the left and middle circular contours shown in Figure 5.5. The contour on the left has a higher slice resolution than the middle contour. The curvature at the top point on the left circle would be calculated to be less than the curvature at the same point on the middle circle. Ignoring the different slice resolutions of these two contours could mean that the contours would be considered different curves when they are compared.

A method that deals with different slice resolutions involves retaining the original scan data, but still using the RCSS method for comparing contours. This method provides a more accurate calculation of the curvature difference at every point between two contours. When comparing two contours, the curvature at each point on a contour is calculated while the comparison is being done and not before. Calculating the curvature of a contour during the comparison process allows the curvature calculations of the higher slice resolution contour to be performed at the same lower slice resolution of the other contour. This means that the three points used to calculate the curvature on each contour are as similar as possible.

The right contour in Figure 5.5 shows an example of a curvature that is calculated at a lower resolution than the slice resolution of the contour.

Table 2 shows the modified comparison algorithm. The resolution of the first contour is assumed to be lower than the resolution of the second contour.

Table 3 shows the RMS curvature differences for the comparison of ellipses at varying slice resolutions for both the original and modified comparison algorithms. The major axis of the ellipse was twice as long as the minor axis. The results indicate that, at greatly different slice resolutions, the modified algorithm provides a more accurate comparison of of contours.

5.5 Noise

As no scanning method provides noise-free data, the effect that noise has on the RCSS method has to be considered. When two identical contours, except that noise exists in the data of one of the contours, are compared then the total curvature difference of the two contours would not be the zero value expected for identical contours, but some value greater than zero. Therefore, when comparing two contours, a total curvature difference that is greater than zero may still indicate that the curves are similar.

Choosing an appropriate cut-off point, below which a total curvature difference for the comparison of two contours is considered to indicate that the contours are similar, is important. If the cut-off value is too high, contours that are not similar may be considered similar by this RCSS method. Conversely, if the cut-off value is too small, similar contours may be considered different.

How closely the shape of one contour should match the shape of another contour before the RCSS comparison method considers that the two contours are similar, depends on the type and accuracy of the operations that are to be performed on the scan data of a volume. For example, if very accurate quantitative results are required, the quantitative analysis method may require that contours of very similar shapes to distinguishable by the RCSS comparison method. Therefore, a low cut-off value would be used to determine which total curvature difference values of the RCSS comparison method indicate that two contours are similar or not.

A method of blurring the RCSS image of a contour, can be used to reduce the effect that noise can have in the scan data when comparing contours. A blurring method used with the initial comparison algorithm (as describe in Table 1) was to apply a linear averaging function to the RCSS images. The curvature values of a group of consecutive points in an original image are averaged to calculate the curvature value of a single point in the blurred image. Blurring may be applied to an image several times.

The blurring method of averaging curvatures over a group of consecutive

```
total_difference = 0      count = 0

for point1 = start point to end point of first contour

    prev_point1 = the previous point to point1 in the first contour

    next_point1 = the next point to point1 in the first contour

    point2 = the nearest point to point1 in the second contour

    prev_point2 = the nearest point to prev_point1 in the second contour

    next_point2 = the nearest point to next_point1 in the second contour

    curve1 = the curvature at point1

    curve2 = the curvature at point2

    total_difference += ( curve1 - curve2 ) ^ 2

    increment count by 1

total_difference = total_difference / count

rms_difference = square root of the total_difference
```

Table 5.2: Algorithm - Modified Comparison Method

Slice Resolution	Old Comparison Method	Simulating Lower Resolutions
800	0.00000	0.00000
400	0.00194	0.00011
200	0.00576	0.00004
100	0.01340	0.00002
50	0.02831	0.00008
25	0.05566	0.00056

Table 5.3: Results - Simulating Lower Resolutions

points does not work well with the modified comparison algorithm (as described in Table 2). This algorithm calculates the curvatures at points around the contours during the comparison process, rather than in the initial comparison method, where all the curvature values of points on the contours have been pre-calculated. Therefore, in the original comparison method the averaging of curvatures could be done relatively quickly, compared to the numerically intensive operation of averaging curvatures in the modified comparison method.

A blurring method that does work well with the modified comparison method, is one that works in a similar way compared to how the modified comparison method deals with slices of different resolutions. The blurring method involves comparing only every *ith* point on the lower resolution contour to the higher resolution contour. Increasing the distance between comparison points on the lower resolution contour, increases the blurring of the RCSS images being compared.

The only change needed in the algorithm in Table 2, is to change the second, third and fourth lines to read

```
for point1 = start point to end point of the first contour in steps
             of the blur amount
```

Blur Amount	Points per Sample	Ellipse, noise 1%	Ellipse, noise 10%	Circle, no noise
1	100	0.00854	0.08528	0.12425
2	100	0.00194	0.01931	0.10710
3	100	0.00061	0.00611	0.10677
4	100	0.00048	0.00492	0.09629
5	100	0.00027	0.00275	0.09079
1	200	0.03507	0.34834	0.12941
2	200	0.00818	0.08168	0.10781
3	200	0.00372	0.03736	0.10729
4	200	0.00192	0.01914	0.10321
5	200	0.00112	0.01120	0.10229

Table 5.4: Results - Comparing Blurred Contours

```
prev_point1 = the ith previous point to point1 in the first sample

next_point1 = the ith next point to point1 in the first sample
```

The results in Table 5.4, show the effect that blurring of RCSS images has when comparing contours. The an ellipse was the shape that the contours in the table were compared against. All the ellipses in the comparisons had a major axis twice the length of its minor axis, and the circle's diameter was the same length as the major axis of the ellipse. All comparisons involved contours with the same number of points. The percentage noise values, refer to the amount of noise in the contours, relative to the slice resolution of the contours that had 100 points. So the percentage noise values for the contours with 200 points, is approximately 2% and 20% relative to their slice resolutions.

The results show that using a method that compares blurred images, does counteract the effect of noise in scan data. The method clearly distinguishes between the ellipse contours and the circle contours at blur amounts of 3 and higher for both noise levels, and for both slice resolutions.

The stability of the curvature difference between the circle and the ellipse, for increasing blur amounts indicates that the blurring of images in the comparison process does provide an valid method to compare contours that have been affected by noise.

The blur amount in the comparison process should not be too large compared to the number of points in the contours. If the blur amount is too large very simple contours are formed by the blurring process. These simplified contours will give an inaccurate comparison result. For instance, comparing the same 100 point ellipse and circle contours as in the Table 5.4, with a blur amount of 45, gives a curvature difference of just 0.00425. During experimentation, the the blurring method was found to be stable up to blur amounts of approximately 6% the number of points in the contour with the lowest slice resolution.

Chapter 6

Contour Libraries

Being able to recognise that one contour is similar to another is useful if some properties are already known about one of the contours. For instance, if one contour is known to come from the scan of a kidney, then any other contour that is found to be similar can be deduced to come also from a kidney. This leads to the idea of having a library of contours with known properties. The contours in the library could be recorded at a high slice resolution, perhaps with very accurate scanning equipment. The library could then be used with less accurate equipment, but because of the high slice resolution of the library contours, the accuracy of comparing contours should still be fairly high.

The contours in a library need not necessarily come from the scanning of an actual volume. For instance, generic kidney contours could be created by applying an averaging function to several contours from real instances of normal shaped kidneys. The generic contours could then be used to highlight anomalies in the kidney being scanned.

A library of contours could be used to determine what type of volume is being scanned, though this is an unlikely use as normally the operator would know what volume they are scanning in a patient. Rather than determining what type of volumes are being scanned, contour libraries would be more useful in helping with surface registration and quantitative analysis of scan data as discussed chapter 9. When contour libraries are used, the problems discussed in the previous chapter have to carefully considered, especially the problem of different slice resolutions as library contours would normally have a very high slice resolution.

Chapter 7

Extending to Three Dimensions

Quantitative analysis and surface registration are operations that are most often applied to three dimensional objects. The RCSS method has to be extended for dealing with objects in three dimensions while still retaining the properties of scale independence and ease of comparing objects at different orientations.

7.1 Extending the RCSS Image to Three Dimensions

The RCSS image could be extended by adding another scale space axis to the graph. Finding a function to represent curvature of a point on a surface is difficult, especially if that function is to be independent of the surface's orientation and scale. Also, mapping any three dimensional surface onto a plane is very difficult, even when the surface is a regular shape such as a sphere.

Consider the difficulties that cartographers have mapping the earth's surface compared to the more complex surfaces found in medical scanning. Cartographers have to distort the surface of the earth, so that they can represent surface in a two-dimensional map. Representing the more complex surfaces found in medical scanning, compared to the relatively simple spherical shape of the earth, is therefore more difficult task.

7.2 Three Dimensional Slicing

Three-dimensional information about a volume could be represented by a series of planar slices that intersect the volume at various angles. The three-dimensional orientation of a slice could be stored as part of the information of each contour. Representing a volume by several planar contours retains all the benefits of the RCSS method. A library of known contours could be used, as discussed in chapter 6, to represent the three-dimensional information about a single volume. Each library of a volume, would contain only the contours relating to that particular volume.

7.2.1 Regular Slicing

A volume could be sampled by a regular slicing pattern to build up a representation of the three-dimensional object. The term *slicing pattern* refers to the way in which a volume is sampled by a series of slices. For instance, a regular slicing pattern could be parallel slices of a volume at every 2mm. If a fixed regular slicing pattern was used some volume libraries may not accurately enough represent the volume, while other volume libraries may have redundant slices in their representation of the volume. Choosing a good slicing pattern and standard number of slices needed to represent any volume may reduce these problems.

7.2.2 Representative Slicing

Instead of slicing a volume in a regular sampling pattern, only the distinctive slices of a volume may be necessary to accurately represent the three-dimensional information of a volume. For many volumes, this could mean a very compact method for representing the volume in a library. Associated with each contour could be a list of orientations which indicate what regions of the volume the contour represents.

For example, consider the surface of a closed cylinder. Most of the contours that are obtained from slicing the cylinder are circles, ellipses and rectangles. The the list of orientations associated with the circle contour could indicate that the contour represents all the contours that would be obtained from slicing the cylinder at any point along the its length with a cross-section scan.

Representative slicing would require the identification of those slices that represent the three-dimension information of a volume. This would have to be done by the person creating the contour library for the volume.

Chapter 8

Generating Three-dimensional Data

To investigate the operations of performing quantitative analysis and surface registration on three dimensional scan data, a program was written to simulate a scanner. The program consists of five main modules, as shown in Figure 8.1.

The 3-D Surface Model, contains a description of the surface of the three-dimensional object that is to be scanned. The module consists of one function that takes a three-dimensional point as input, and returns a boolean result indicating if the point is on the surface of the object. Changing the type of volume being scanned is simply achieved by supplying the scanner with another function that describes a surface of a different volume.

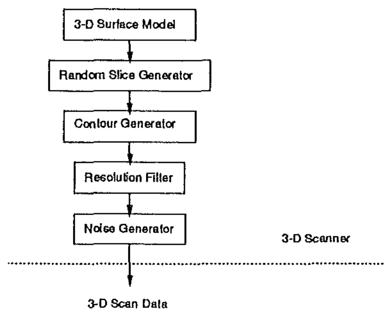

Figure 8.1: Generating Three-dimensional Data

The Slice Generator, chooses what slices of the volume are scanned. The slicing pattern of the volume can be a random pattern of a regular sampling pattern For example, sampling the volume in parallel slices at every millimetre.

The Contour Generator, calculates for all the points on a slice, at the orientation decided by the Slice Generator, whether the point occurs on the surface of the volume or not. The mesh size of the slice determines the slice resolution of the contour.

The Resolution Filter, can reduce the slice resolution of the contour by a regular or random removal of contour points. The random removal of contour points can be used to simulate the random distribution of contour points in a slice that can occur with some scanning methods.

The final module is the Noise Generator, which can introduce a specified noise level into the contour points of a slice, by adding random displacements to the points.

Chapter 9

Quantitative Analysis

Quantitative analysis involves obtaining numerical information from scan data, such as calculating the volume or surface area of a scanned object. Often quantitative analysis is performed only after the scanning of a volume is complete. One of the objectives of this project has been to perform quantitative analysis of volumes in real-time during the scanning process.

9.1 Quantitative Analysis Methods

Many of the quantitative analysis methods use integration to obtain numerical results[8]. In these methods the surface of the volume is modelled by one or more continuous functions. Integrals can then be calculated over the functions modelling the surface to obtain the results, such as the surface area and volume of the object.

The functions used to model the surface of a volume are approximations, and often several functions have to be used to describe the complete surface. The accuracy and resolution of the scan affects the accuracy of the surface model approximation, which in turn affects the accuracy of the result of a quantitative analysis. The higher the accuracy and the resolution of the scan, the more accurate the quantitative analysis should be.

Quantitative analysis cannot be done using this method during the scanning process if the scan resolution is not at a high enough level for a surface model approximation, or if scan data is not uniformly distributed about the surface of the volume being scanned. Consider the case where the first fifty scans of a volume only return information about the 'front' surface of the volume. Due to the uneven distribution of the scan data, the complete surface of the volume cannot be modelled. Therefore quantitative analysis such as to calculate the volume of the object cannot be accurately performed.

Another method for quantitative analysis uses numerical approximation to calculate results[8]. The numerical approximation method that is used depends on what quantitative analysis is to be performed. To calculate the surface area of a scanned volume, the scan data could be used to represent the surface of the volume as a mesh of triangles. The total area of all the triangles would be a numerical approximation for the surface area of the scanned volume.

In a similar way to the approximation of a surface area, the volume of a scanned object could be approximated by modelling the surface of the object as a mesh of triangles. A fixed point within the object could be used as the common apex point for all the triangle based pyramids whose bases lie on the surface of the object. The total volume of all the pyramids provides an approximate result for the volume of the scanned object.

Like the previous integration methods, numerical approximation methods can only provide quantitative analysis results for a scanned volume when the scan resolution is above a certain level, and the scan data is evenly distributed around the surface of the object. Therefore, like the integration methods, numerical approximation methods are not appropriate for performing quantitative analysis while a volume is being scanned.

9.2 Quantitative Analysis using the RCSS Method

In the RCSS method, each contour provides information about the volume being scanned. This information can be used to perform quantitative analysis, and with each contour more information can be used in the analysis.

A comparison with a library of contours can be used to determine where in the volume each scanned contour comes from. Knowing the location of a contour within a volume provides more information for the quantitative analysis. For example the shape of a 'standard' kidney could be stored in a contour library by the representative slicing method, as described in section 7.2.2. One of the representative contours in the library might be a length-wise cross-section. If a contour from a scan was found to be similar to the length-wise cross-section library contour, then the scanned contour could be used to estimate one numerical result - the length of the kidney being scanned.

For more complicated quantitative analysis, several contours would be used to estimate the numerical result, and as more contours are scanned the result would become more accurate. Also the relationship of orientation between scanned contours provides information for quantitative analysis.

For instance continuing the previous example, another scanned contour that was found to be perpendicular to the length-wise cross-section contour, would be known to be a width-wise cross-section contour. Also the position that the width-wise cross-section contour occurs along the length of the kidney could be estimated. If the volume of a kidney is known as a function of the kidney's length and the circumference of a width-wise cross-section, then the scanned kidney's volume could be estimated.

A function has to be known for every quantitative analysis that is to be performed on a volume. The function could involve several different contours of the volume, and use different information from each contour to estimate the result of a quantitative analysis. For example, the information from contours could be, the length of a contour, the circumference of a contour, the orientation of a contour within the volume being scanned, or the relative size of a scanned contour compared to the similar contour in the contour library.

The function to calculate quantitative analysis result of an object, could be found first by using post-scan analysis techniques to calculate the accurate results for several volumes that have a 'standard' shape. The results of this post-scan analysis could help to find a function that will allow the quantitative analysis to be performed in real-time during the scanning process.

An estimate, of the accuracy of the real-time quantitative analysis could be given, by defining a function that calculates the accuracy of a result by considering how many, and what type of contours have been used by the analysis so far.

Chapter 10

Surface Registration

Surface registration allows for operations such the differences between the aligned objects to be displayed or analysed. Like quantitative analysis, surface registration is usually a post-scan operation.

Surface registration between two or more sets of scan data is performed by finding the relative orientation of each set of data compared to the other sets.

10.1 Surface Registration Methods

One method of surface registration uses the distinctive surface features of a volume to align the sets of data. Distinctive features that are often used in the aligning process are high peaks or ridges, and low depressions and troughs. Choosing a relatively small number of distinctive features compared to the number of surface data points, means that an exhaustive matching of the data to be aligned can be performed.

For close matches, more accurate quantitative analysis may be performed. The volume of the difference between two sets of surface data may be calculated to provide a more accurate check as to whether the sets of data are properly aligned. A small difference volume between the two sets of data indicates that the surface registration is correct.

10.2 Surface Registration using the RCSS Method

Surface registration using the RCSS method for describing contours, uses a contour library to align sets of scanned data from the same or similar volumes. Therefore, the volume that the contour library represents must be the same or similar to the volumes that the sets of scan data are related to.

A scanned contour's orientation is found by comparing the contour with a contour library. Each scanned contour may at first be considered to come from from several possible orientations within the volume. As the number of scanned contours increases during the surface registration process, the more accurate the estimate of the surface registration will be.

Chapter 11

Conclusion

The RCSS method for describing and comparing curves was shown, in theory, to be appropriate for the use of performing of quantitative analysis and surface registration in real-time, during the scanning of a volume.

A algorithm was shown for the comparison of contours, that could deal with the contours having different slice resolutions. The algorithm also incorporated the idea of blurring RCSS images when comparing contours, to reduce the effect that noise in the data had on deciding if two contours were similar. Experiments indicated the the blur amount used in the comparison of two contours, should not be larger than approximately 6% of the number of points in the contour with the lowest slice resolution.

The methods and algorithms used for the comparison of contours within two-dimensions were extended to handle three-dimensional data. Using these extended methods and the idea of contour libraries, ways for performing quantitative analysis and surface registration were investigated.

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